A Study of Metacognitive Skill as Influenced by Expressive Writing in College Introductory Algebra Classes.

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A study of metacognitive skill as influenced by expressive writing in college introductory algebra classes

Allen, Naomi Barbara R., Ph.D.
The Louisiana State University and Agricultural and Mechanical Col., 1991
A STUDY OF METACOGNITIVE SKILL
AS INFLUENCED BY EXPRESSIVE WRITING
IN COLLEGE INTRODUCTORY ALGEBRA CLASSES

A Dissertation

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Louisiana State University and
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in

The Department of Curriculum and Instruction

by

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iii
TABLE OF CONTENTS

ACKNOWLEDGEMENTS ......................................ii
LIST OF TABLES .......................................vi
LIST OF FIGURES .......................................vii
ABSTRACT .............................................viii

CHAPTER

1. INTRODUCTION TO THE PROBLEM. ..........................1
   Theoretical Issues ...................................2
   Purpose of the Study ..................................11
   Questions to be Investigated ..........................13
   Organization of the Study. ...........................14

2. REVIEW OF THE LITERATURE ...............................15
   Developmental Education................................16
   Role of Metacognition in Learning ........................21
   Metacognition and Mathematics ........................36
   Writing to Learn ....................................45
   Writing in Mathematics ................................52
   Using Writing to Promote Metacognition.................60

3. METHODOLOGY .........................................63
   Method ...............................................63
   Data Selection .......................................66
   Treatments ..........................................68
LIST OF TABLES

Table                                                                 Page

1.  Point Assignment for Codings. . . . . . . . 75
2.  Student Perception of Correct Answers . . 81
3.  Definition of Ranges for
Achievement and Awareness . . . . . . . . . . 83
4.  Cell Means for Awareness. . . . . . . . . . 87
5.  Tukey's Pairwise Comparisons. . . . . . . . 88
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Data Collected from Each Group</td>
<td>72</td>
</tr>
<tr>
<td>2.</td>
<td>Frequencies of Codings per Student on Quizzes</td>
<td>82</td>
</tr>
<tr>
<td>3.</td>
<td>Comparison of Achievement Scores with Awareness Scores for Each Quiz</td>
<td>84</td>
</tr>
<tr>
<td>4.</td>
<td>Graph of Interaction of Cell Means</td>
<td>87</td>
</tr>
</tbody>
</table>
ABSTRACT

The quality of mathematics education has become a major concern to mathematics educators. As a result, increased attention is being given to identifying the abilities that underlie competent performance. An outcome of this effort is an increasing belief that the development of metacognitive skills is an essential component of proficient mathematics performance. Writing, because it promotes reflective thinking, is believed to be the vehicle for this development.

Writing in the mathematics classroom has previously received anecdotal support for its benefits to the learner and to the instructor, and limited quantitative benefits in problem-solving ability and attitude toward mathematics. This study examined the effect of expressive writing on self-awareness and would suggest quantitative support that writing is beneficial in promoting student ability to assess the correctness of work. If metacognitive skills are a necessary condition for successful mathematics performance, the use of writing may provide the process for attaining these essential skills. Further research in the benefits of writing is warranted by this study.
CHAPTER 1
INTRODUCTION

The present quality of mathematics education will not support the realization of personal expectations for career goals and quality of life for many students. This belief is reflected in publications titled, A Nation at Risk: The Imperative for Educational Reform (National Commission on Excellence in Education, 1983), The Mathematics Report Card: Are We Measuring Up? (Dossey, Mullis, Lindquist, & Chambers, 1988) and Everybody Counts: A Report on the Future of Mathematics Education (National Research Council, 1989). These publications express a need for a commitment to life-long learning which will enable workers to adapt to changing job descriptions mandated by our rapidly changing technology.

Life-long learning asserts the need of greater student responsibility for and participation in the learning process. This idea is not new as indicated by the following quote from Jerome Bruner (1972): "To instruct someone in these disciplines is not a matter of getting him to commit the results to mind; rather, it is to teach him to participate in the
process that makes possible the establishment of knowledge" (p. 72).

This calls for a transference of power from the instructor to the pupil. Rather than in the traditional direct instruction, interest is towards a more dynamic learning environment characterized by an interactive process (Brown, 1987). This interest is confirmed in the Curriculum and Evaluation Standards for Mathematics Education, a publication by the National Council of Teachers of Mathematics (1989) which reflects a consensus of university mathematicians and mathematics educators (classroom teachers, supervisors, educational researchers and teacher educators). As a result of these concerns, how students learn is receiving increasing attention with an emphasis on identifying the abilities that underlie competent performance.

Theoretical Issues

Cognitive scientists have drawn distinctions between novice and expert learners and have begun to investigate the differences in the way these individuals process information. The novice is
believed to process new information as individual, unrelated statements rather than embedding this information into appropriate and relevant contexts. Generally, these students are not confident in controlling their performance and tend to be relatively passive in their learning (Campione, Brown, & Connell, 1989). An expert, on the other hand, displays the ability to reflect upon and assess a situation, and to regulate the strategies employed (Brown & DeLoache, 1978; Long, 1986; Schoenfeld, 1987). However, these activities become more covert and less observable as the learner develops expertise. An attempt to identify and classify this ability has led to the adoption of the term "metacognition" which serves as an umbrella for all activities concerning either knowledge of cognition or control of cognition.

**Metacognition**

Flavell (1976), a pioneer in the field of metacognition, views metacognition as the monitoring of cognitive enterprises which subsumes four classes of phenomena: (a) metacognitive knowledge, (b) metacognitive experiences, (c) goals or tasks, and
(d) actions or strategies. An example of (a) would be a person's belief about his or her ability to do mathematics, and an example of (b) would be a sudden feeling that something that had just been said was not understood. A goal refers to the objective of a cognitive enterprise such as knowing a chapter of a text for a test; the strategy would be the behavior implemented to achieve that goal (outlining or rereading the chapter).

Long (1986) views metacognition as a conscious aspect of thinking and learning, referring to "purposeful actions that monitor progress in a task, and regulate the procedures used to perform it" (p. 8). Developing awareness increases monitoring which, in turn, increases the ability to make productive decisions at the right time and place. These abilities are believed to be essential components of proficient mathematical performance (Campione et al., 1989; Garofalo & Lester, 1985; Long, 1986; Narode, 1985; and Schoenfeld, 1987).

Flavell (1979) sees investigations pointing to an important role for metacognition in reading comprehension, language acquisition, memory, problem
solving, social cognition, attention, various types of self-control and instruction, and oral persuasion, comprehension and communication of information. Current literature affirms the view of metacognitive skills as an essential component of learning (Brown, 1987; Campione, 1987; Henderson, 1986; Long, 1986; Pramling, 1988; Schoenfeld, 1985b; Slife, Weiss, & Bell, 1985).

Available research studies on teaching strategies which address reflection about one's thinking and self-regulation span various content areas as well as diverse ability groups. Examples include studies of a metacognitive model for solving the problem of skill generalization in learning-disabled students (Borkowski, Estrada, Milstead, & Hale, 1989), the effects of self-management strategies on journal writing (Hull, 1981), and the effects of programming with Logo on metacognitive skills (Clements, 1986).

In the area of mathematics, the focus on metacognition is predominately in the area of problem-solving. Included are studies which have investigated self-regulation as a problem-solving strategy (Schoenfeld, 1987; Narode, 1985). Schoenfeld (1987)
describes the main point of self-regulation in the following: "It's not only what you know, but how you use it (if at all) that matters" (p. 192).

Schoenfeld actively pursues the development of metacognitive skills through using videos of problem-solving sessions to make students aware of metacognitive issues and of their own behavior, or he models metacognitive behavior through what he terms "problem resolution," which demonstrates the planning, exploration, and evaluation that occurs during true problem solving. He also employs "whole class" discussions, in which the instructor records and moderates, to focus on control decisions. Group work is also incorporated with the instructor serving only as a consultant to direct a student's thinking toward questions that ask what (is being done), why (it is being done), and how (it will help) when searching for a solution.

Using a variation of group work, Narode (1985) applies paired problem solving in which one student solves the problem while the second student listens carefully or asks questions until both clearly follow the solution. Believing that metacognition is
"facilitated through oral and written communication of think as-it-happens (p. 14)," he requires students to write about their thoughts, to read them, and to reflect upon them. While Narode feels this method to be useful, he also believes more research into the benefits of writing is needed and opens the door for that possibility.

Writing to learn

Writing in the classroom has traditionally been used to demonstrate learning rather than to promote it. During the last decade, the focus on the use of writing has shifted to its use for promoting cognitive and affective benefits for the student. Rose (1989a) provides a review of the literature that cites two broad categories: transactional and expressive. Britton, Burgess, Martin, McLeod, and Rosen (1975) describe transactional writing as public writing, intended for an audience. They contrast transactional writing with expressive writing, personal writing for the purpose of exploring and recording thoughts. While not supported by hard research, the benefits of using such writing in the classroom have been
documented by many researchers (Applebee, 1984; Rose, 1989a).

Geeslin (1977) views this writing as a "diagnostic tool" for the instructor and a "learning tool" for the student. The use of writing as a strategy for active learning encourages students to explore, conjecture, and reason logically as concepts and procedures are developed. The emphasis is shifted to the process by which students learn rather than the product of their learning.

Writing to learn is based on the belief that there are strong connections between writing and learning (Applebee, 1984; Britton et al., 1975; Emig, 1977; Rose, 1989a). Emig's theoretical work on writing, thought to parallel powerful learning strategies, has been the underpinning of much of the current literature on writing to learn. She identifies four learning strategies: multi-representational and integral, connective, personally engaged, and those which generate self-provided feedback. All can be specifically achieved by writing. Her provision for self-provided feedback is particularly important to this inquiry.
Instructional Implications

The use of expressive writing allows an approximation of the learner's thoughts to be recorded, making them available for both visible reflection by the student and for inspection by the instructor (Emig, 1977). Consequently, writing has the potential for making learners aware of their cognition as realized by Narode (1985), Goodkin (1982) and Rose (1989a). Narode states the importance of communication, either by discussion or writing, in developing self-awareness:

The communication of ideas is a sure attempt at presenting a problem or concept to another but more importantly it is the representation of an idea to the thinker him/herself. No sooner than we begin to describe our ideas do we evaluate them, alter or elaborate them and effectively monitor ourselves with a new awareness; the self-consciousness which asks questions like: How can I say what I mean? Will this particular method of solution help me? Is this problem clear? and many other such questions which cause us to reflect and to analyze what we think. (p.5)
Hence, the relationship between writing to learn and metacognition appears to be naturally convergent.

**Background to the Research Questions**

In this researcher's use of writing in a college-level remedial mathematics classroom (Allen, 1989), students wrote that they thought they had understood a concept until they began to write about it. Students also expressed an inability to evaluate their understanding of a concept before they encountered it on a quiz or a test. When asked if their efforts were matched by results, their responses disclosed that students can unknowingly work an entire assignment incorrectly. Students indicated they viewed writing as an effective means of self-evaluation.

Insights into the role of the instructor as the perceived source of knowledge were also revealed through the writings. In summation, these informal findings from student writings support this researcher's belief that the use of expressive writing has the potential to promote the metacognitive skills displayed by an expert learner.
Purpose of the Study

Blais (1988) states that education must provide the process which will transform a novice into an expert. As recent studies have suggested, experts seem to possess self-awareness and employ certain managerial skills which are believed to promote improved performance. Researchers believe these skills can be taught (Long, 1986; Narode, 1985; Schoenfeld, 1987), allowing the novice to make the desired transition. Campione et al. (1989) suggest alternatives to traditional teaching which encourage active student participation. An example is reciprocal teaching, a form of cooperative learning, which features guided practice in applying four concrete strategies: questioning, clarifying, summarizing, and predicting. These authors believe these strategies provide students with concrete methods for monitoring their understanding.

This study focused on the aspect of metacognition described as the "students' conscious and statable knowledge about cognition, about themselves as learners..." (Campione et al., 1989, p. 94). The aim of this study will be twofold: (a) to examine the
self-awareness possessed by remedial introductory algebra students and (b) the effect of in-class expressive writing on their ability to further develop this awareness.

Self-awareness was students to indicate their assessment of both the accuracy of the method chosen to answer a mathematical question and the accuracy of the implementation of the chosen method. Comparing the accuracy of the indications to the accuracy of the work will yield information on the student's awareness.

The process of writing, because it promotes reflection, is sought as a vehicle for enhancing self-awareness. In-class writings, consisting of asking students to respond daily for five minutes to a given prompt, focus on questions which promote reflection and self-awareness, and which motivate monitoring of study habits. Three-part paper, which provides two carbonless copies of the student's writing, was used to allow the student, the instructor, and the researcher to each have a copy of these writings for later examination.
The findings include implications for identifying: (a) the metacognitive skill of self-awareness these students possess, (b) the effect of reflective questioning on this metacognitive skill, and (c) the possible role of the writing process in enhancing self-awareness or achievement.

Questions to be Investigated

The following research questions will be addressed:

1. When asked to do so, can students in a college-level elementary algebra class consistently assess the accuracy of the method chosen and of its implementation when determining an answer?

2. Does the ability to accurately assess the correctness of the method and its implementation increase over time and with experience?

3. Does the intervention of daily in-class expressive writing or lecture containing reflective questioning affect the ability of students to assess the accuracy of the method chosen and its implementation when determining an answer?

4. Does the intervention of daily in-class expressive writing or lecture containing reflective questioning affect student achievement?
Organization of the Study

Following this introduction, the second chapter will present a review of the literature. In the area of metacognition, the review will address the meanings given to this phenomenon by theorists, the current research related to mathematics education, and the status of current instructional practices relative to metacognition. To address the concept of writing to learn, the review will discuss the purported role of the writing process in learning, current uses of writing in the mathematics classroom, and current research on writing-to-learn in the mathematics classroom.

Chapter 3 will describe the research procedures, and Chapter 4 will present the findings of the study. Chapter 5 will summarize the study through drawing conclusions and suggesting implications for further investigation.
Researchers today stress the importance of metacognitive skills in learning and successful mathematics performance and urge mathematics educators to help their students develop a metacognitive posture when performing mathematics (Campione et al., 1989; Flavell, 1979; Garofalo & Lester, 1985; Long, 1986; Schoenfeld, 1987,1989b; Weinert, 1987). Studies reveal that the novice does not display the necessary metacognitive skills for successful learning but further research supports the belief these skills can be taught (Brown, 1987; Campione et al., 1989). Developmental mathematics students, who are clearly novices, provide a population appropriate for this study. Writing, because it promotes reflection (Emig, 1977), can provide an instructional activity for promoting the necessary metacognitive skills (Klein & Vukovich, 1991). This chapter presents evidence which supports this conclusion by summarizing a review of current literature and research in the following areas: (a) the developmental education, (b) metacognition and learning, (c) metacognition and mathematics, (d) writing to learn, (e) writing in
mathematics, and (f) writing and metacognition in mathematics.

Developmental Education

Nation-wide large numbers of students are entering college (public and private) without the skills or knowledge needed to perform college-level work (Abraham, 1988), resulting in a widespread presence of developmental education programs in post-secondary institutions (National Center for Educational Statistics, 1991). In the Southern Regional Education Board (SREB) states, about one-third of the first-time freshmen required at least one remedial course. These numbers are indicative of national averages (32%) and only slightly lower than other regional averages (Abraham, 1991).

Brief Historical Notes

The advent of programs to meet the needs of underprepared students is not new. Abraham (1991) cites the presence of remedial courses at Yale University as early as 1828. In 1849, the University of Wisconsin (Abraham, 1991) established a Department
of Preparatory Studies followed by Iowa State College in 1862 (Mickler & Chapel, 1989). By the 1900s, 84% of the colleges and universities in the United States had similar preparatory schools (Abraham, 1991) including Harvard, Yale, Princeton, and Columbia Universities (Maxwell, 1979). In the 1970s, this number increased to 90% (Boylan, 1986). Although the goal of the SREB for the year 2000 is to reduce the number of underprepared students to one in five, the present need for developmental programs is not declining (Abraham, 1991).

Developmental Students

Generally, developmental students can be classified as those who (a) made adverse academic decisions, (b) have been out of academia for an extended time, (c) have learning or physical deficits which may or may not have been identified in high school, (d) have acquired pre-college education in foreign countries, and (e) lack clear-cut or well-formed academic goals (Hardin, 1988). Because these students also have diverse backgrounds, many are culturally deprived and may have learned helplessness
(Spears, Atkinson & Longman, 1986). As a result of having difficulty locating and using available support, developmental students often set high goals but have only a limited understanding of how to go about achieving them.

For these students, varied instructional styles and methods are required to meet their needs. In the 1970s, programs began to include courses designed by content specialists and taught by faculty who were specially trained for remedial education (Mickler & Chapel, 1989). Rounds and Anderson (1985) cited instructional innovations such as individualized, self-paced, mastery, and programmed instruction. Spears et al. (1986) suggest instruction that is traditional or taught in large groups will not be effective for the developmental student. They also suggest these students do not work well independently often relying on external motivation such as instructor expectations. They cite the importance of having students feel the instructor cares about their success in a course.
Developmental Mathematics

Developmental programs typically offer additional preparation in mathematics, writing, and reading as well as academic skills. Studies (Abraham, 1991) of the percentage of SREB students in the three academic disciplines revealed the need for mathematics (38.5%) to be substantially higher than writing (27.5%) and reading (26.7%). An examination of the population represented in this study revealed, in the fall of 1990, 56% of the entering freshman were required to enroll in developmental mathematics. These needs are reflected in A Nation at Risk (National Commission on Excellence in Education, 1983), which reported one fourth of all mathematics courses offered in 1980 at four-year institutions were remedial.

Developmental mathematics students often see the course as the "greatest academic hurdle" that must be crossed (Darken, 1990, ix). Because these students have not been successful in mathematics, they often develop fears which are manifested in learned helplessness (Spears et al., 1986) and anxiety (Nolte, 1991). Overcoming these barriers represents the first instructional step.
In the developmental mathematics classroom, the need for emotional support and a well-organized program constitutes critical components for successful mathematics remediation (Darken, 1990). To meet these needs, McDonald (1988) stresses the importance of employing caring faculty and maintaining small class sizes. Darken (1990) suggests that developmental mathematics students respond best when passivity is avoided by actively involving the students in the instructional process. She sees small groups and instructor-to-student conferences as generating excitement and interest and providing a vehicle for individualization of instruction. She further suggests using a variety of methods for providing students opportunities to learn at their own pace and style. Feedback should be given individually, both in verbal and written forms (Spears et al., 1986) and on a daily basis (Long, 1986).

Powell (1986) uses three pedagogical devices—journals, creative writing and research problems, and explorations and problem-generating activities—with his underprepared mathematics students. He views one of the purposes of mathematics education as assisting
"learners in becoming aware of the powers they possess and how they can construct meaning and procedural know-how" (p. 183).

Garofalo (1986) suggests that developmental mathematics students are particularly deficient in aspects of mathematical performance which include not only procedural and declarative content knowledge but also knowledge that is linguistic and factual, heuristic and strategic, and metacognitive. In this article, Garofalo focuses on some instructional practices that would develop metacognitive skills, and these will be discussed later in this chapter in the section on teaching for metacognition.

Role of Metacognition in Learning

At the turn of this century, educational psychologists such as Dewey used the terms "active monitoring," "critical evaluation," and "seeking after meanings and relationships" to depict reflective reading activities now subsumed under the rubric "metacognition" (Brown, 1987). In the 1970s, terms included "reflective intelligence," which referred to consciously observing one's own thoughts, and
"reflexive abstraction," a mechanism for extracting, reorganizing, and consolidating knowledge. Obviously, the concept underlying the adoption of the term metacognition is not new and is recognized in many areas.

Garofalo & Lester (1985) see metacognition as originally stemming from an article criticizing the lack of research on memory which particularly noted no one was considering the fact that people have knowledge and beliefs about their memory processes. Flavell began to study children's "metamemory" and went on to become a pioneer in the field of metacognition.

Defining Metacognition

Flavell (1976) generally defines metacognition as "one's knowledge concerning one's own cognitive processes and....the active monitoring and consequent regulation and orchestration of these processes" (p. 232).

Although Flavell's definition is often used, Brown (1987) focuses on metacognition as being "stable and stateable." A review of current literature
immediately reveals that the definition of metacognition is often neither. Campione et al. (1989) aptly describes the current state of affairs in the following:

One of the most salient features about metacognition is that the term means different things to different people, with the result that there is considerable confusion in the literature about what is and what is not metacognitive. This confusion leads to apparently contradictory viewpoints, ranging from claims that the concept is too ill-defined or fuzzy to be the object of scientific inquiry to assertions that things metacognitive are the driving force of learning, and therefore the major aspects of learning we should be studying. (p. 93)

Brown (1987) believes the confusion evolves from both the historical development of the term and the difficulty in distinguishing between cognition and metacognition. An historical consideration reveals roots originating in four separate strands of inquiry which Brown discusses at length under the headings:

(a) verbal reports as data restated as a reflective
access of one's own thinking, (b) executive control or performing intelligent evaluation of one's own operating, planning, and monitoring, (c) self-regulation which can be described as decision-making actions influenced interpersonally, and (d) other-regulation which results from actions fostered by external influences or activities, that is, intrapersonal. She points out that initial theorizing about metacognition was goal oriented. She asserts the need to now develop workable theories and procedures for the separate areas that emerge from a concentration on metacognition.

Garofalo and Lester (1985) attempt to clarify the confusion by generalizing cognition as endeavors involved in doing whereas metacognition is activities involved in planning and monitoring what is being done. In Pramling's (1988) research, she merely describes metacognition as a child's awareness of his learning and proceeds with her study, while authors such as Brown (1987), Campione et al. (1989), Garofalo and Lester (1985) devote pages to the term's development and current use.
Metacognition is sometimes stated very informally as "thinking about one's thinking." Schoenfeld (1987) uses this explanation but he further classifies metacognition into three separate but related areas of intellectual behavior. The first area identifies how accurately one's own thinking can be described (self-awareness), the second area centers on monitoring one's thinking (control or self-regulation), and the third examines the experiences (beliefs and intuitions) that shape the way one thinks. Schoenfeld's research focuses on the second area, self-regulation, which he sees as the ability to manage time and effort when working on complex tasks. This management task encompasses understanding, planning, monitoring, and allocating of resources. Simply stated, Schoenfeld states the main point is not only what you know but how you use it, if at all.

Collectively, metacognition is generally viewed by many as falling under the two non-distinct headings of knowledge of cognition and control of cognition. Examples given for knowledge of cognition include beliefs about oneself as a learner (whether factual or not); knowledge about the scope, requirements, and
difficulties of tasks; and knowledge of general and specific cognitive strategies and their potential usefulness for certain tasks.

Control or regulation of cognition is concerned with a variety of decisions and strategic activities which have been influenced by the knowledge of one's cognition as described above. These activities include, but are not limited to, predicting, checking, planning, selecting, revising, etc.

Flavell (1976) perceives metacognition as the monitoring of cognitive enterprises which subsumes four classes of phenomena: (a) metacognitive knowledge, (b) metacognitive experiences, (c) goals or tasks, and (d) actions or strategies. An example of (a) would be beliefs about one's personal ability to do mathematics while an example of (b) would be perhaps the feeling of not understanding something that had been said. A goal would refer to the objective of a cognitive enterprise such as knowing a chapter of a text for a test while the strategy would be the behavior implemented to achieve that goal such as outlining or rereading the chapter.
Learning and Metacognition

Metacognitive knowledge and metacognitive experiences are seen to interact where knowledge can influence the control of metacognitive experiences and, likewise, these experiences can shape the acquisition of metacognitive enterprises. This dynamic interplay is illustrated by the following example from Flavell (1979).

Let us begin at the point where some self-imposed or externally imposed task and goal are established. Your existing metacognitive knowledge concerning this class of goals leads to the conscious metacognitive experience that this goal will be difficult to achieve. That metacognitive experience, combined with additional metacognitive knowledge, causes you to select and use the cognitive strategy of asking questions of knowledgeable other people. Their answers to your questions trigger additional metacognitive experiences about how the endeavor is faring. These experiences, again informed and guided by pertinent metacognitive knowledge, instigate the metacognitive strategies of
surveying all that you have learned to see if it fits together into a coherent whole, if it seems plausible and consistent with your prior knowledge and expectation, and if it provides an avenue to the goal. (p. 909)

Difficulties on one or more of these points will consequently activate some metacognitive knowledge or experiences and the cycle will continue until the task is ended. The implications of this model for education pose questions concerning the value of cognitive monitoring in learning experiences. Campione (1987) supports the belief that low achievers are deficient in both knowledge and the ability to control that knowledge. If student learning is to be achieved, Campione asserts the need to inculcate metacognitive skills in addition to providing information.

Corno (1986) sees metacognitive components as a necessary but not sufficient condition for self-regulated learning in which a student actively acquires and transforms instructional materials. Corno characterized self-regulated learning (SRL) as a student's effort to deepen and manipulate an associate
network in the content areas and to monitor and improve upon that effort. These efforts are hypothesized to direct and control concentration during school learning tasks. This metacognitive control, from an early theory of volitional control, is labeled by six strategies: attention control, encoding control, information processing control, motivation control, emotion control, and environmental control. All of these are observed by Corno in a study on fifth grade students' verbalized thoughts. Corno suggests, in conclusion, that students who access and use these controls in school tasks can be expected to be more efficient, and perhaps more effective, learners.

Experiments have been conducted to test the effects of self-management on journal writing by college freshmen (Hull, 1981). The first experiment showed that goal-setting and self-monitoring would increase the number of lines and entries in weekly journals written for traditional freshmen composition classes. The second experiment replicated the first study using basic (remedial) writers and produced the same outcomes. In addition, the second study also
showed that extending the baseline rate did not increase writing output and the removal of the treatment resulted in the student reverting to their previous writing output. The study implicated that, in order for their writing behavior to change, students must actively monitor that behavior.

Teaching for Metacognition

Teaching for metacognition has resulted in changes in the methods traditionally used to teach. The blind instruction approach is abandoned and replaced with interactive teaching where the goal is to promote student understanding of the meaning of what is being taught and to help the student become more active in and responsible for the learning that occurs. The following studies illustrate the incorporation of metacognitive and contextual factors in learning.

Biggs (1986) seeks to enhance learning skills through strategic learning which he outlines as a metacognitively-oriented theory to relate personal relevance, context, task, and student resources. Biggs focuses on five motive-strategy packages called
approaches to learning: surface--minimal requirements through rote learning; deep--intrinsic interest through understanding; achieving--grade oriented through efficient comprehensive study; surface-achieving--achievement oriented but through extensive surface learning; and deep-achieving--intrinsic and grade oriented through meaning-based study. A regular study skills text was used as a background resource in addition to activities which included encouraging students to (a) reflect on their work (especially in subjects they found difficult), (b) keep diaries, (c) utilize self-testing or self-questioning, and (d) pair off with another student to monitor and discuss each other's progress. Two studies, one with college-level subjects and another with grade eleven students, resulted in a shift from surface-achieving to deep-achieving approaches and grades were significantly enhanced.

Reciprocal teaching, which promotes students to use methods for monitoring their understanding, was incorporated to teach comprehension skills to academically weak grade school children (Campione et al., 1989). Using a cooperative group format, this
form of reciprocal teaching featured guided practice in applying four concrete strategies: questioning, clarifying, summarizing, and predicting. Teachers initially modeled expert performance then transferred the work to the students. The reciprocal nature of the group procedures ensured student engagement. Positive results occurred when third grade minority students were encouraged to use this method to acquire coherent knowledge about biological themes concerning animal adaptation. On independent measures of comprehension, reciprocal teaching groups increased their performance by 32 percentage points. Twelve months later the effect of the instruction could be reproduced with retention test scores of 82% as compared to posttest scores of 85%.

Palincsar and Brown (1987) have shown success in improving reading comprehension by teaching summarizing (self-review), questioning, clarifying, and predicting. Their approach also uses modeling and interrogation as major components in teaching these self-instructional procedures.
Research on Metacognition

Pramling (1988) used qualitative methodology to study children's awareness of their own learning from their point of view. Conceptions of what was learned fell into two categories: learning of activities and learning as comprehension. Similarly, how was categorized as learning to know by either external influence or personal experience. Three teachers (three groups) worked for two to three weeks with the same content called "the shop." Two of the groups used the same structure which taught the content alternately from the customer's perspective and from the shop's perspective. One of these two also used metacognitive dialogue as an integral methodology. Interviews were conducted prior to the teaching segment, during the instruction, and again in six months. The teachers were not previously informed of the later interviews. The interview questions were variations of "what have you learned?" and "how did you learn it?" followed by "anything else?" Analysis of the first interview showed no significant differences in the three groups. Analysis of later interviews revealed the students having metacognitive
dialogue developed a greater awareness of their own learning and their difference from the other two groups increased during the six months before the final interview. It was not clear if the dialogue, or the dialogue combined with structure, accounts for the differences. The author suggests that her research be seen as exploratory, motivating a larger and more controlled future study where teaching is performed metacognitively, which is different from teaching strategies or facts.

The Higher Order Thinking Skills (HOTS) program, initiated to help students who have not learned to organize their thoughts or to generalize them, represents an example of teaching metacognitively (Pogrow, 1988). Computer software is employed to create the needed learning environment but the critical aspect is the intensive series of teacher-student interactions which follow. This Socratic environment is based on three basic techniques: (a) basic questioning, (b) improvisation techniques to change unanticipated answers to advantage, and (c) coaching techniques to promote independent and active learning. An example of the first is eliminating
hints from questions or asking students "why?" when simplistic answers are given. The second may be to ask students to explain the reasoning process behind unexpected answers, and the third might involve telling students you think they are smart enough to answer their own questions before walking away. Although HOTS teaches no subject area content, the results show dramatic success in improving problem-solving skills and stimulating achievement gains in basic skills.

In summation, the concept of metacognition generally focuses on two main, but non-distinct, areas: knowledge of cognition and control of cognition. Much of the current research on metacognition falls into the categories of memory development, reading, and special education. The study of metacognition argues for a change from the traditional, teacher-centered instruction to instruction which addresses cognition metacognitively. Examples of this new educational trend include reciprocal teaching, strategic learning, and using self-regulation in journal writing. In the studies cited, the role of reflective questioning was often
integral to the development of metacognitive skills. Researchers also posited the importance of metacognition in mathematics instruction which includes not only problem solving but all mathematical performance (Campione et al., 1989; Flavell, 1979; Garofalo & Lester, 1985; Long, 1986; Schoenfeld, 1987, 1989b; Weinert, 1987).

Metacognition and Mathematics

Lester & Garofalo (1982) found that elementary students do not routinely analyze problem information, monitor progress, or evaluate results. Schoenfeld (1985) asserts that even most college students have not developed many metacognitive skills.

Slife et al. (1985) established that certain aspects of metacognitive awareness can be distinguished both from general ability and from mathematical attainment. In this study, elementary students who were disabled in mathematics were matched with regular mathematics students based on IQ scores and performance on the same set of ten math problems. When compared on knowledge of their problem-solving skills (predicting their likely success rate on a
given set of computations) and on ability to monitor their problem-solving performance (identifying their correct and incorrect solutions), the learning disabled students were less skilled in each form of metacognition. The results seem to suggest that knowing how to solve a problem is different from knowing that one knows how to solve it.

In an effort to analyze metacognitive aspects of mathematics performance, Garofalo and Lester (1985) present a cognitive-metacognitive framework that is directly relevant to performance on a wide range of mathematical tasks. Comprised of four categories of activities--orientation, organization, execution, and verification--the framework specifies key points where metacognitive decisions are likely to influence cognitive actions. Their intent was to provide a guide for selecting appropriate research tasks that would provide information for delineating the role of metacognition in the learning of mathematics and implementing metacognitive aspects into mathematics instruction.
Teaching for Metacognition in Mathematics

Currently, many recommendations for mathematics teaching are addressing the development of metacognitive skills. Long (1986) supports the need for mathematics teachers to promote learning through metacognition by suggesting that teaching concentrate on a meaningful approach, making the connections between familiar knowledge and more advanced ideas explicit. She believes this will allow the student to transfer previously gained metacognitive knowledge to a new learning situation. Time should be allowed for reflection, whole class discussions, and for answering all questions carefully. Students should be encouraged to check their work often, perform inverse operations when possible, and identify strategies used repeatedly. Finally, teachers should provide short daily formative feedback to allow students the opportunity to clarify redeemable mistakes.

Birken (1986) suggests that students be taught to study mathematics within the context and content of the course. Two of her suggested strategies directly confront metacognitive issues: (a) students should identify their own problem-solving strategies and
listen to others, and (b) students should identify where their mathematics study skills break down and then develop their own coping strategies. Both suggestions seek to make the student more aware of capabilities as well as deficits.

Campione et al. (1989) have extended reciprocal teaching to mathematics instruction in a project which involves beginning algebra word problems and uses students who have the ability to perform algorithms correctly but are unable to give evidence of the conceptual understanding underlying their use. Using small cooperative groups, the teacher embodied expert modeling, scaffolding, and coaching in the methods utilized with a focus on externalizing strategies, monitoring progress, and imposing meaning. Successive chalkboards for (a) planning, (b) representing, and (c) doing were used by a person designated as the learning leader. This leader was to help the other students proceed systematically through producing an external record of the group's work that could be monitored, evaluated, and reflected upon. After 20 days of instruction on single-variable linear equations, two-variable linear equations, and monomial
by binomial equations, the experimental group outperformed the control group on both the target and the transfer problems. As in other studies, this example of reciprocal teaching initially featured the teacher modeling, and the student practicing metacognitive, self-regulatory skills as a strategy for learning. Students are then progressively made more responsible for selecting and monitoring the approaches they use.

This emphasis on teaching strategies that promote the development of metacognitive skills such as reflection about one's thinking and self-regulation was also cited in the use of computer programs. The design of instructional materials, no longer confined to procedural and declarative knowledge, was extended to insights into the cognitive processes that underlie the effective learning of complex topics.

Collins and Brown (1986) use the computer as a tool for focusing a student's attention directly on the thought processes being used. An example is a computer program which keeps a record of the student's attempts when solving an equation so the student can review previous work. After seeing which attempts
were unsuccessful, the student can use the insights from these reflections to provide the direction for future attempts.

Narode (1985) uses pair problem solving with college-level remedial mathematics students as a strategy for engaging students in mathematics while monitoring their thoughts. This method, a variation of group work, calls for one student to solve the problem while the second student listens carefully or asks questions until both clearly follow the solution. The problems used are meticulously selected to provide examples that are challenging but not frustrating. The students are then asked to alternate their roles. To initiate this method, the teacher models the role of a clinical interviewer so the student can learn what is expected.

Research on Metacognition in Mathematics

Similarly, interrogation has been used in research by Schoenfeld (1985). Schoenfeld suggests four classroom techniques that he uses to focus on metacognition in a mathematics problem-solving class. The first is the use of videotapes of students doing
problem solving for the purpose of promoting self-awareness through watching and discussing their behavior as well as the behavior of others. These activities produce a receptiveness to the interventionist techniques used later in the term. A second technique is the modeling of "expert performance" by the instructor. This performance illustrates the processes which yield the polished product or the solution of a problem. This modeling may include working through a problem step-by-step, including tentative explorations with evaluations throughout, and a "post-mortem" where the whole solution process is reviewed. The third technique is whole-class discussions with the teacher as a moderator—serving only as scribe and orchestrator. By asking for several proposed solutions, the class is forced to focus on control decisions. Later, the discussions are analyzed for the efficiency of the processes selected by the class. The fourth and last technique suggested is problem solving in small group settings. The teacher assumes the role of coach, moving from group to group as a problem-solving consultant. Groups understand that the "coach" may
ask, at any time, (a) if they can describe what they are doing? (b) if they can answer why they are doing it? and (c) do they know how it will help them? As group dynamics become established, the teacher's role is minimal with students monitoring their work without the intervention of the instructor.

In an effort to research changes in monitoring behavior as these techniques were used, Schoenfeld charted the time spent by his students on six stages of problem solving. The six stages identified for this study were: read, analyze, explore, plan, implement, and verify. At the end of the semester, these charts were compared to the problem-solving behavior of an expert and revealed the students no longer spent all of their time on their first guess but evaluated their progress and attempted alternate solutions. Although Schoenfeld does not believe the acquisition of metacognitive skills will guarantee success, he purports these skills will at least give access to it.

Bell (1991) is conducting a two-year research program on the metacognitive aspects of mathematical learning and teaching. The aims of the project are to
(a) identify and describe the levels of awareness on the part of pupils of their mathematical knowledge, and of their learning processes, (b) investigate relations between these levels of awareness and characteristics of the classroom environment and teaching style, (c) develop tactics by which awareness may be enhanced, and (d) evaluate the effects of such enhancement on the pupils' mathematical learning (p. 1). At present, researchers in this study are developing two kinds of materials. The first is evaluative tests and procedures for describing and quantifying pupils' awareness, and the second is "tactics" for classroom use to enhance pupils' awareness. Pilot tests include having students estimate their chance of answering a question successfully or having students indicate their level of confidence in their answers. Analysis of this data is presently underway (Alan Bell, personal interview, April 16, 1991).

A study of initial interviews, however, suggest that students do distinguish between being "able to perform" and "understanding." The meaning of such statements are now being probed by the research team
(Bell, 1991). Future work will address topics such as pupil-constructed tests, metaphors for learning, and concept maps as this study of pupils' awareness of learning continues.

In conclusion, current research in the area of metacognition and mathematics education supports the belief that metacognitive skills are needed, but not necessarily sufficient, condition for learning mathematics. The studies cited focus on promoting reflective thinking and rely heavily on reflective questioning to achieve this goal. Narode (1985) carries the interrogation a step further and suggests that expressive writing could be a vehicle for promoting this reflection.

Writing to Learn

Writing to learn is based on the theory that writing is powerful not only in presenting knowledge but also in producing knowledge (Connolly, 1989). Writing about a concept being taught is viewed as a process by which students can learn a content area. This process encourages the student to organize his thoughts, commit them to writing and, finally, to
analyze what has been said. In writing to learn, the emphasis shifts away from formal correctness toward context, meaning, and the process of writing.

Theoretical Underpinnings of Writing

Discussions of writing to learn commonly cite the theoretical works of Vygotsky (1962), Bruner (1960, 1972) and Britton et al. (1975) which focus on the role of language in the forming of meaning. Britton has examined language based on its functional use and, consequently, distinguishes between written language that is directed toward an audience, which he terms transactional writing, and language written for personal use, which he terms expressive. Britton sees expressive writing as being preliminary and exploratory writing that functions to express the writer's feelings and thoughts. This generation of exploratory writing, which is believed to be closest to the self, has value as a basis for writing to learn.

In her classic article, "Writing as a Mode of Learning," Emig (1977) initiates a case for writing as having a unique value for learning. She views writing
as the processing of information on Bruner's three levels of actuality: the enactive, the iconic, and the symbolic. Bell and Bell (1985) restates Emig's observation of writing by the following description of the three named levels: the motor level, the hand moving across the paper; the sensory level, the eye reading what has been written; and the analysis level, the mind processing what has been written.

Emig argues that writing parallels four powerful learning strategies: multi-representational and integral, connective, personally engaged, and those which generate self-provided feedback. All can be specifically achieved by writing. The multi-representational strategy has been addressed above. Writing is connective because it forces organization, grouping, synthesis, and analysis. Writing actively engages the student at a personal rhythm; that is, it is self-paced. Finally, the provision for self-provided feedback, which is particularly important to this inquiry, enables the learner to immediately review and evaluate the product of one's own thought. Emig's article very often provides the underpinnings
for the many articles and research on the use of writing in learning that have appeared in the last decade.

Writing in Content Areas

As early as the 1950s, support for writing to learn was emerging in post-war programs for returning GI's (Russell, 1987). Examples of these were the Functional Writing Program at Colgate (1949-1961) and the Prose Improvement Committee at the University of California at Berkeley (1950-1965). Both of these programs recognized the capacity of writing to improve learning but, nonetheless, these early beginnings of writing across the curriculum still died out after more than a decade of successful operation.

Since the 1950s, a considerable number of articles have appeared with anecdotal support for the use of writing to aid learning. Specifically, these uses include improvement of basic skills, reading-writing interaction, the study of introductory psychology, and the learning of mathematics as well as its traditional use in English classes. An article by Dittmer (1986) presents general guidelines for writing
assignments in content areas and specific information for the areas of mathematics, physics, accounting, and biology. Two extensive qualitative dissertation studies in the 1980s have also focused on writing to learn. Books are appearing in the science areas; Connolly and Vilardi (1989) edited *Writing to Learn in Mathematics and Science*, and Sterret (1990) edited *Using Writing to Teach Mathematics*. As evidenced by the number of publications devoted to the subject, writing in the mathematics classroom has growing support among educators.

Goodkin (1982), in her qualitative study of the "intellectual consequences of writing," provides a review of the theoretical support for the relationship of the writing process to the thinking/learning process, the established uses of writing, and the innovative writing strategies currently being implemented in the classroom. Her empirical study utilized ethnographic strategies to investigate case studies in four separate content areas: psychology, clinical nursing and dental assisting, entomology, and calculus and basic mathematics.
From analysis of the case studies, Goodkin classified student uses of writing as personal, curricular, and universal. The personal uses of writing include examples ranging from lists and notes to journal entries and functions in supporting intrapersonal relationships, summarizing, and exploring and controlling feelings. Curricular uses included writing assignments which helped the students establish or improve intellectual skills such as to summarize, focus, think through, clarify, infer and abstract. Finally, universal uses, which included flash-cards, brief in-class writings, reports and research papers, provided writing exercises for reflecting, organizing, comparing and contrasting, self-testing, and thinking analytically.

In general, personal, informal writing promoted introspection and self-discovery whereas the other writings supported intellectual accommodation. Goodkin felt the study established that writing fostered thinking and concentration and represented a personal search for meaning.

However, although many support writing to learn, others still question the relationship between writing
and learning. For example, Applebee (1984) questions what he calls the unexamined assumption of a relationship between writing and learning. Although the research evidence available is consistent with the ideas that writing promotes rational thought and a better understanding of the subject, Applebee feels it does not compel acceptance. He argues that "the research that does exist suggests a broad agenda for future work" (1984, p. 590).

The belief that writing promotes learning has theoretical and anecdotal support that dates back to the early 1950s. Theoretical arguments view writing as instrumental in personally developing meaning (Britton et al., 1975; Bruner, 1960; Vygotsky, 1962). The anecdotal support is evidenced by the plethora of journal articles expounding the benefits of using writing to promote learning in the content areas. Goodkin (1982), in her doctoral dissertation, provides additional support for the intellectual consequences of writing in general and in several specific content areas including the study of calculus and basic mathematics. A closer consideration of the use of writing in mathematics follows in the next section.
Writing in Mathematics

Rose (1989a), in her dissertation study, provides extensive information on how the use of expressive writing can benefit the student, the teacher, and student-teacher communication, and provides extensive information on the use of expressive writing for learning mathematics. Her conceptual analysis of expressive writing delineates benefits in three areas: students as writers, teachers as readers, and reader-writer interaction. The student, as writer, benefits in the affective domain by extroverting negative feelings about mathematics, in content through verbalizing experiences and impressions, in developing processes through greater awareness of the methods being used, in conceptual understanding by exploring their beliefs about mathematics personally. In general, growth is transferable to a variety of contexts through enhanced "critical thinking" and the adoption of a reflective attitude toward learning both in school and in social settings.

Teachers benefit from reading students' writings through the diagnostic information provided, course feedback, and data for instructional improvement.
When a student writes about mathematical concepts, understanding is revealed at a level not possible in the traditional classroom setting. Responding to student writings allows the teacher to individualize instruction and provide support as needed. The information provided by student writings allows the teacher to evaluate the effectiveness of instruction, the textbook, classroom procedures or management, and testing. Feedback may also provide surprising information about student errors or misconceptions or about the teachers themselves. Finally, the writing dialogue can afford a level of communication which establishes a better rapport between the teacher and the students, and a personalized environment for learning which is thought to be motivating for both the students and the teacher.

In addition, Rose (1989a) conducted an empirical study to further understand her conceptual analysis of the benefits of expressive writing presented earlier in this chapter. In this study, the writing activities, which included autobiographical writing, unstructured and structured dialogue journal entries written outside of class, and in-class focused
writings, were an integrated component of a regular mathematics course. In general, the students in this study wrote more often about their feelings about mathematics or the course and less often about course content, their ways of doing mathematics, or their conception of mathematics. As the study progressed, however, students broadened their range of topics and related numerous ways they perceived writing as beneficial to their learning of mathematics. In particular, students viewed unstructured journal writing as having the greatest potential for creating their own need and affording the opportunity to meet that need in the writers' own way and at their own pace. Restated, the majority of the students felt journal writing helped them to write specifically about what they did not understand and to generate questions that could be asked of the instructor. Further, as Rose points out, these entries can make the teacher aware of collective needs of the class and, in turn, can provide individual diagnosis and remediation.

Using qualitative methods, Rose examined the characteristics of students' journal writing and their
perceived benefits of these activities and found support for her conceptual framework. In her summary, Rose states students find journals most productive when they:

- write about what they do not understand;
- write about specific, current material;
- write in their own words;
- solve problems as far as they can go;
- ask questions;
- stay current and consistent with the writing;
- are honest;
- and, are open;
- writing will not work unless it is given a fair chance.

(p. 290)

Writing was summarized as being beneficial to the teacher as reader, because reading "facilitated individual diagnosis and evaluation; increased sensitivity to students; personalization; provided different points of view; provided short-term benefits for the course; provided long-term benefits for teaching" (p. 322).

For the student-teacher relationship, writing provided interaction which "promoted better one-to-one communication; provided better feedback from both parties; fostered open and comfortable relationships;
provided encouragement and stimulated motivation; fostered an involved and caring classroom" (p. 323).

**Research on Writing and Mathematics**

As cited earlier, Applebee (1984) has indicated there is little hard research to support the effects of writing to learn. In the area of mathematics, available research yields little deviation from this observation. In a study with students in a remedial mathematics course in an inner-city college, Pallmann (1982) found students benefitted affectively from writing about math by showing a significant difference in the retention and absentee rates of the class which used writing as an integral part of the instruction format. Statistical analysis of the pre- and posttests (California Achievement Tests) showed a higher average for the experimental group but the difference was not significant. The researcher interpreted these results as favoring the treatment group since the higher retention rate suggested the weaker students remained in the experimental class.

Weiss and Walters (1980) examined if subject-related writing tasks assigned in college courses
would affect amount and clarity of student learning, writing performance, and apprehension about writing. Four disciplines—research and statistics, reading, physical science, and educational psychology—were studied. Significant differences at the .01 level were found for clarity in learning with writing tasks. Amounts of learning were higher for the experimental classes but the difference was significant at the .05 level for only the statistics class. No significant differences were found for writing apprehension although decreases were shown in all four areas. Writing ability increased in all but the physical science class but no differences were significant. Weiss and Walters suggest additional research in other disciplines and at different levels are now indicated to support their findings.

Hirsh and King (1983) tested the effect of writing assignments as compared to traditional assignments on the performance of college students in an elementary algebra course. The writing assignments asked students to respond to conceptual questions whereas the traditional assignments included exercises on algebraic topics covered in class lectures. No
significant differences in achievement were found between the experimental and control groups. However, in this study, the writing was not an integral part of the class format as it was in the study by Pallmann (1982).

Bell and Bell (1985) studied the effect of expressive writing on problem-solving abilities in two ninth grade general math classes. In this study, students in the experimental section expressed in words whether or not they understood the material, and used paragraphs to verbally describe their numerical processes. After a four-week period, significant differences were found at the .01 level in favor of the experimental group. Again, a basic premise of this study is that the writing component must be perceived as integral to the teaching process.

Selfe, Peterson, and Nahrgang (1986) conducted a study with calculus classes and used journal assignments about mathematical concepts throughout the ten-week session. The students in this study perceived the writing as a positive addition to the class and some students reported using the journal to think and learn about mathematics. Although the
writing was an integral part of the teaching process, no significant differences in students' performance on content-area tests, students' attitudes towards mathematics, and students' writing apprehension were obtained in this study.

Others have stated that writing activities can help clarify attitudes that students have developed toward mathematics (Farkas, 1981). In an article on expressive writing, Farkas theorizes that students who use writing to express their feelings learn to cope with their emotions and have more energy to put into the academics and thus improve their work. In secondary algebra classes, Miller and England (1989) employed daily impromptu writing prompts which were placed in four major categories: contextual prompts, instructional prompts, reflective prompts, and other prompts. The Lewis Atkins Revised Mathematics Attitude Scale was administered to three experimental and four control groups. One of the Algebra II experimental groups showed a statistically significant improvement in attitude at the .01 level at the end of the year.
In the area of mathematics, numerous publications focus on student and teacher benefits of writing. Geeslin (1977) sees writing as a diagnostic tool for the instructor and a learning tool for the student. The amount of literature on writing and mathematics in the last ten years supports this premise; however, little hard data exists to further substantiate this claim. The research available has shown some significant results in the area of retention, attitude toward mathematics, problem solving abilities in a basic math class, and amount of learning in a statistics class. Again, as Applebee (1984) stated, much research remains to be done.

Using Writing to Promote Metacognition

Believing that metacognition is "facilitated through oral and written communication of think as-it-happens" (p.13), Narode (1985) requires students to write about their thoughts as they think them, to read them, and to reflect upon them. The length of these papers, sometimes called thought-process protocols, can range from 300 to 600 words and include pictures, charts, diagrams, and equations. Narode feels this
method to be useful in monitoring one's own thoughts, but he also argues the need for more research into the value of writing in order to identify the particular benefits.

Klein and Vukovich (1990) employ journal writing to promote metacognitive skills in remedial algebra classes. Topics used for journal entries include "My Math Past" which asked the student to focus on past experiences in mathematics and feelings of competence, "Test Preparation" which had a student identify specific learning strategies he possessed and also made the instructor aware of the student's needs, "Goal Setting" which sought to increase a student's self-monitoring ability, and "Time to Evaluate" which forced the student to make attributions to a recent test score. These journal entries, which focused on revealing student cognition, allowed the instructor to guide the student into more self-directed and controlled thinking.

Summary

This chapter provided support for the belief that metacognitive skills are a necessary but not
sufficient condition for learning mathematics. Research supports the belief that metacognitive skills are exhibited by the successful student or expert whereas the less successful learner does not exhibit these skills. A case has been made for the cognitive and affective benefits of using writing as a tool for learning in the mathematics classroom. Writing, because it promotes reflection and provides a record for review, is suggested by Narode (1985) and Klein and Vukovich (1991) as a viable means of promoting metacognitive skills. However, neither qualitative nor quantitative research is available to support this assertion.
CHAPTER THREE

METHODOLOGY

This study examined students’ awareness of both the accuracy of the method chosen to answer a mathematical question and the accuracy of the implementation of the chosen method. The findings include implications for identifying: (a) the metacognitive skill of self-awareness these students possess, (b) the relationship of this metacognitive skill to effective learning as evidenced by achievement scores on five quizzes, and (c) the role of the writing process in enhancing this self-awareness.

Method

Sample

The sample for this study was four sections of Developmental Algebra 092 at a four-year regional university in the Spring Semester, 1991. Students are placed in this course when their ACT math score is 17 or below and they score between 10 and 20 on a 25-item, in-house, content-based, introductory algebra test.
The four classes had initial enrollments of approximately 35 students and were taught by the same instructor, not the researcher. Two sections were taught on Tuesday and Thursday mornings for 75 minutes each and two sections were taught on Monday, Wednesday, and Friday mornings for 50 minutes each.

**Demographic Findings.** An informal survey was used to collect certain demographic information. Frequencies for math ACT [American College Testing] scores revealed that two-thirds of the group fell in a score span of 14-16 which is just below the 50th percentile nationwide. A salient finding was that 93% of the students participating in this study had completed at least an introductory algebra course during their college preparation. Approximately 48% had credit for three or more years of secondary mathematics. When asked to indicate their expectation of how well they thought they would do in this course, 17% responded "all right," 50% responded "not very well," and 32% responded "very poor." In general, these students were enrolled in a course repeating content they had been taught in high school and, yet, these students expected to do only moderately well.
Course Description. Developmental Algebra 092 (DVMA 092) is a three-hour, non-credit course focusing on introductory algebra concepts. These concepts include performing basic operations (on integers, rational numbers, polynomials, rational expressions, and radical expressions), solving linear equations in one or two variables, and solving linear inequalities in one variable.

The presentation of these concepts centers on pedagogy which promotes active class participation with responsibility for learning shared by the student and the instructor. Initial learning experiences utilize directed discovery methods and cooperative group learning. Since current research dictates the necessity of structured learning for low-achieving students, teacher-centered instruction is ultimately provided before closure on a concept.

The final grade for students in DVMA 092 is determined from the results of five 20-point announced quizzes, five 10-point unannounced quizzes, and four 100-point announced tests. Each quiz contains only new content, but all tests are cumulative. The 250-point cumulative final contains 25 content-based
questions. Developmental courses are taught on a pass-fail basis with a "P" assigned for satisfactory work and a "U" for unsatisfactory or failing work. To receive a passing grade, "P", the student must earn 560 of the possible 800 points with at least 150 points earned on the final. A total of 65 bonus points can be earned by utilizing the tutorial services for remediating errors on announced tests and quizzes. In essence, the format of the course places the emphasis on student ability at the end of the course with substantive support available throughout the course for those who maintain an active effort. Students must earn a satisfactory grade in this course in order to enroll in the next course.

Data Selection

Demographic Data

This study collected data reflecting certain demographic data, student perception of correct work, and achievement scores. The demographic data included each student's age, gender, ACT mathematics score, English placement, and background in college and high school mathematics coursework. Also, affective
information was collected in questionnaire form concerning the student's expectation for the course and attitude towards mathematics (See Appendix A).

**Achievement Data**

Achievement was defined as the points earned on each 20-point announced quiz and the scores on the pretest and posttest which were parallel forms of the fourth test in the course (See Appendix B). Each quiz contained 10 free-response questions based only on material introduced since the last evaluation. The fourth test in this course contained 20 free-response questions based on the course content.

**Metacognitive Data**

Data was collected on the student's awareness of the correctness of an answer on the above evaluations using two indicators. The first was the student's assessment of the correctness of the method chosen to answer a question; the second was the assessment of the correctness of the implementation of the chosen method. The student indicated whether he or she was
very certain, fairly certain, or not certain of the correctness for each assessment (See Appendix B).

Treatments

Writing Treatment

Procedure. The writing treatment consisted of using impromptu writing prompts (Miller & England, 1989) which were daily in-class expressive writings where the student responded for five minutes to a given question or statement. This time period was selected as a manageable period for the student, by limiting the time the student has to organize and present thoughts, and for the instructor, by limiting the amount of reading when reviewing student responses (Miller, 1991).

Instrument. The writing prompts consisted of statements or questions which directed students thinking toward a greater awareness of understanding of the mathematical concepts presented, the processes used with these concepts, and self-evaluation of the effectiveness of these processes as they reflected upon their homework and examinations (See Appendix C).
The following represent a sampling of the prompts used:

Look at the following problem. Is the problem worked correctly? Explain your reasoning. If the problem is incorrect, discuss where the mistake was made and why you think this may have happened.

Pick a problem from your quiz that you found the most difficult. Tell me what you find hard or confusing. Try to be as specific as possible.

Describe one topic covered in this chapter that has been the most difficult. Explain why you think you had difficulty understanding this. Do you understand it now? If yes, can you describe how you came to understand this concept?

On the test you have been returned, find the question that you understood the best. Tell me why you think you understood this concept so well.

Look over the items that you missed on your test. Can you tell me why you think you missed the problems that you did? Be as specific as you can.

When asked to square a binomial, many students say, ",(x + 5)^2 equals x^2 + 5^2". Is this correct? If not, explain how this binomial is squared.

The prompts were distributed daily on three-part paper (5" by 8") and collected at the end of the five-minute period. The instructor responded to the writings individually with comments encouraging further student reflection or providing direction for future study. For example, the instructor asked questions such as, "How will this help you?" or "Can
you tell me more?" Three-part paper, which generates two carbonless copies, allowed a copy of the student's written response with the instructor's comments to be returned to the student at the next class meeting, a second copy to be kept by the instructor for reflection, and the third copy to be kept by the researcher to monitor student participation.

**Lecture Treatment**

The lecture treatment consisted of including reflective questioning in the class lecture format. After being given a few minutes to think about their answers, students could respond to these questions verbally. The questions paralleled the reflective questioning included in the writing prompts—questions directing student thinking toward a greater awareness of conceptual understanding of content presented, the processes used with these concepts, and self-evaluation of the effectiveness of these processes. Examples are given in the following:

- Why is $x^2 + 9$ not factorable over the real numbers?
- Why did we agree to find the GCF first when factoring?
- Factor $8a(b + 6) - (b + 6)$. How would you explain why your answer is correct?
How do you know when an expression is completely factored? Explain your answer in detail.

Design

The four groups used in this study were four intact classes as described previously. All of these classes were taught by the same instructor, not the researcher. The classes were randomly assigned intact as Group A, B, C, or D.

Group D, with 34 students, was taught at 8:00 a.m. on Tuesdays and Thursdays and served as a control for the influence of the pretest. Only the demographic and achievement data outlined above was collected.

For Group C, which met at 9:30 a.m. on Tuesdays and Thursdays and contained 33 students, the information collected included the demographic and achievement data described above. In addition, on the pretest, the students were asked to indicate their perception of the correctness of their work as outlined in the above description. No other intervention occurred for Group C until the post-test when the students were again asked to indicate their perception of the correctness of their work.
The data collected from Group B, taught at 10:00 a.m. on Mondays, Wednesdays, and Fridays, included the demographic and achievement data described above. In addition, students in this group of 34 were also asked to indicate awareness of the correctness of their answers on all five quizzes, the pretest, and the posttest as for Group C. This group received the lecture treatment described in a later section.

Group A, taught at 11:00 a.m. on Mondays, Wednesdays, and Fridays, and also containing 34 students had data collected as described for Group B. In addition, this group performed daily impromptu writing by responding to a given prompt.

The chart in Figure 1 illustrates the data considered from each of the four groups.

<table>
<thead>
<tr>
<th>Group</th>
<th>Demo</th>
<th>Pretest</th>
<th>Quizzes</th>
<th>Writing</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>B</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>D</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

*Figure 1. Data considered from each group.*
Procedure

On the first day of class, the students in all four groups were asked to complete the informal survey. Groups A, B, and C also completed the single-form pre-test at this time. On the pretest, all students were asked to indicate their certainty of the correctness of the method chosen and its implementation when answering each question. Students were instructed that this test was to provide diagnostic information to the instructor concerning the background information of the students for the purpose of assisting the instructor in meeting the individual needs of the class. Students were allowed as much time as was needed; however, none of the groups exceeded thirty minutes.

For the first four of the five quizzes given during the semester, students in Groups A and B were asked to indicate their certainty of the correctness of the method chosen and its implementation. Since these groups met at consecutive class times, the same form of the quiz was used for both groups.

The same form of test four was given to all four groups at the end of the semester. It should be noted
that tests one through three, which were not a part of this study, used different forms of the test in each class to ensure test validity for test four where the same form was used for all four groups in the study. Test four was a cumulative test of the concepts taught in this course and students were also asked to indicate their certainty of the correctness of the method chosen and its implementation.

Evaluation of Data

Demographic Data

The informal survey provided demographic information about students' ACT scores, English placement, number of years of college preparatory mathematics courses, age, and gender. These data, together with the pretest, were used to establish group equivalency.

Metacognitive Data

For each question, the student's work was coded for correct method/incorrect method and for correct work/incorrect work by a developmental mathematics faculty member who was not the instructor in the study
nor the researcher. The method selected was considered correct if it represented a viable, but not necessarily efficient, method for reaching a correct solution. Otherwise, the method was coded as incorrect. Correctness for the implementation of the method was more easily coded, with an agreement reached to code a careless error as correct work. Explanations for coding of a sample of specific test questions are included in Appendix D.

The instructor in this study also independently coded the students' work and these results were compared with the original coding to determine reliability coefficients (See Appendix E).

In an effort to quantify awareness of correct work, numerical values of 2, 1, or 0 were assigned for each indicator as illustrated in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Indication</th>
<th>Very Certain</th>
<th>Fairly Certain</th>
<th>Not Certain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Incorrect</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
This method of scoring gave the highest value when performance matched perception—a student performing correctly and knowing the work was correct, and likewise, a student performing incorrectly and knowing the work was incorrect. The indication, "fairly certain," was ranked next. The lowest value was assigned to performance not in agreement with perception—students performing incorrectly and thinking the work was correct, and students working the problem correctly but having no confidence in their answer.

The numerical assignments were totaled for each quiz, the pretest, and the posttest to yield scores for awareness, or ability to assess correct work.

Data Analysis

To test the effects of treatment over time, an analysis of variance for repeated measures was used to determine if significant differences in self-assessment gains from quiz one to quiz four occurred between Group A and Group B.

Analysis of variance for repeated measures was used with the pre- and posttest awareness scores to
test the influence of the in-class expressive writings upon the student's ability to assess the correctness of work on the posttest. To test the influence of writing on achievement, an analysis of variance for repeated measures was also performed with achievement on the pre- and posttests.

Summary

This chapter described the sample, the writing treatment, the data collected, and the methods for analyzing the data. Chapter 4 will present the findings of the study, while Chapter 5 will summarize the study and its implications for further research.
CHAPTER 4

ANALYSIS OF THE DATA

The purpose of this study was to determine: (1) the ability of college introductory algebra students to assess the correctness of the method chosen and its implementation when answering questions on a quiz; (2) the effect of experience on this ability; (3) the effect of in-class expressive writing on this ability; and (4) the effect of in-class expressive writing on achievement on tests and quizzes.

Group Equivalency Testing

Analysis of variance was used to determine if the four groups were significantly different in pretest achievements scores, pretest awareness scores, math ACT subscores, English ACT subscores, and number of years of college preparatory mathematics courses. Results showed no significant differences in the four groups on any of these measures. Since no significant differences were found on the variables tested, it was determined the groups were equivalent at the beginning of this study. The results are given in Appendix F.
Effects of Pretesting

Group D, the group for which only a posttest was analyzed, was included to determine if the pre-test had an effect on the control group, Group C, in this study. An analysis of variance was performed on the posttest achievement scores and the posttest awareness scores. The results found for the posttest achievement scores were $F(2,97) = .21, p < .80, MS = 213.63$ and for the posttest awareness scores, $F(2,77) = 1.04, p < .36, MS = 209.01$. Since no significant differences were found on the variables tested, it was determined the pretest did not affect the control group, Group C. Group D was not used in the analysis of the data after this point.

Analyses and Results

Question One Analysis

The first research question examined was: When asked to do so, can students in a college-level introductory algebra class consistently assess the accuracy of the method chosen and of its implementation when determining an answer?
This question was addressed initially by analyzing the frequencies of the student assessment scores on the pretest and the posttest. As outlined in Chapter 3, the students had been asked to indicate their perception of: (1) the correctness of the method chosen to answer each question and (2) the correctness of its implementation. The pre- and posttest each had 20 items resulting in a total of 40 indicators for each test with the highest score possible being 80. On the pretest, 50% of the participants made an assessment score 26 or below while on the posttest, only 8.5% were in this range. Additionally, on the posttest, 31% of the participants scored 56 or above while only 12% of the participants on the pretest were above this score.

For the writing and the lecture groups, the students were also asked to indicate their awareness of the correctness of their work on the first four 10-item quizzes given during the course. From this information, the number of correct methods chosen and correct implementation was compared with the total number of methods chosen and methods implemented to determine a percentage for items correct. The results are given in Figure 3. Next, the percentage for the
instances students were "very certain" of correct work is given as perceived correct in Table 2.

Table 2

Student Perception of Correct Answers

<table>
<thead>
<tr>
<th>QUIZ</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items Correct</td>
<td>86.5%</td>
<td>88.2%</td>
<td>75.5%</td>
<td>74.7%</td>
</tr>
<tr>
<td>Those Perceived Correct</td>
<td>69.8%</td>
<td>69.8%</td>
<td>40.5%</td>
<td>33.3%</td>
</tr>
</tbody>
</table>

Considering quizzes one and two, in general, the rate for students to correctly choose methods and/or implementations and indicate they were certain of the correctness of their work was approximately 80%. However, for quizzes three and four, these percentages dropped to approximately 54% and 45%, respectively, although the percentage of items correctly answered only dropped between 10 and 15.

A further examination of the quiz responses was made to investigate the frequencies of all of the codings on the quizzes. A correct method or implementation was coded with "C" and incorrect work
with an "I." To code assessment of awareness, a "V" was used for "very certain" of correctness, an "F" for "fairly certain," and an "N" for "not certain." The possible codings were paired as follows: CV, CF, CN, IV, IF, and IN. When there were no indications of assessment, the second letter was left blank. For example, "CN" represents correct work where the student indicated "not certain," and "C" represents correct work with no certainty indicated. Figure 2 depicts the results of this analysis by a frequency bar graph.

![Frequency Bar Graph](image)

**Figure 2.** Frequencies of codings per student on quizzes.
This chart suggests a decline in the number of responses but consideration of student absenteeism accounts in part for this change. Further analysis of the data shows only 11% of the indicators were blank. An analysis of response patterns reveals only 11 (4.7%) of the quizzes had no indicators at all and another 10 (4.3%) of the quizzes showed the student stopped making indications midway through the quiz. The remaining missing indicators were evenly interspersed.

Additional analysis was made by comparing each achievement score with the corresponding self-awareness score for each of the four quizzes. Using ranges of high, average, and low for both achievement and self-awareness, cross-tabs were run. The ranges are defined in Table 3 and the results of the cross-tabs are illustrated in Figure 3.

Table 3

Ranges for Achievement and Awareness

<table>
<thead>
<tr>
<th></th>
<th>Achievement</th>
<th>Awareness</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>14 - 20</td>
<td>28 - 40</td>
</tr>
<tr>
<td>Average</td>
<td>8 - 13</td>
<td>15 - 27</td>
</tr>
<tr>
<td>Low</td>
<td>1 - 7</td>
<td>0 - 14</td>
</tr>
</tbody>
</table>
As the semester progressed, it is evident student ability to assess the correctness of answers declined over time. However, as the course content increased in complexity, the achievement scores remained more stable than the self-awareness scores. On the first two quizzes, 97% and 95% of the sample was categorized as having high achievement. This number was 76% on quiz 3 and dropped to 69% on quiz 4. Similar percentages for
high self-awareness on these quizzes were 86%, 80%, 44% and 38%, respectively. A comparison of awareness scores on Quiz 1 with awareness scores on Quiz 3 revealed a decrease for 69.4% of the students. Data suggest that the students in these introductory algebra classes did not consistently assess the accuracy of their work, nor did experience on quizzes over time increase this ability.

Analysis for Question Two

Question two asked the following: Does the ability to accurately assess the correctness of the method and its implementation increase over time and with experience?

This question was addressed by analyzing the awareness scores on four of the quizzes given during the semester. To determine student ability to assess correct work over time on quizzes, an analysis of variance for repeated measures was performed for Groups A and B over four times: Quiz 1, Quiz 2, Quiz 3, and Quiz 4. No significant differences were found, \( F(3, 198) = .52, p < .66, MS = 52.7. \)
Analysis for Question Three

The third research question asked: Does the intervention of daily in-class expressive writing or lecture containing reflective questioning affect the ability of students to assess the accuracy of the method chosen and its implementation when determining an answer?

To determine the effect of in-class expressive writing or reflective questioning on ability to assess correct work over time on pretest and posttest, an analysis of variance for repeated measures was performed on awareness scores for the Groups A, B, and C over two times: pretest and posttest. No significant differences were found between these groups, $F(2, 72) = .23, p < .79, MS = 311.5$. However, significant differences were found within the groups over time, $F(1, 72) = 57.52, p < .000, MS = 11356.34$. A significant difference was also found between the groups by time, $F(2, 72) = 5.22, p < .008, MS = 1030.72$. The cell means are given in Table 4.
Table 4

Six Cell Means for Awareness

<table>
<thead>
<tr>
<th>Group</th>
<th>Pretest</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Writing</td>
<td>22.852</td>
<td>50.222</td>
</tr>
<tr>
<td>Lecture</td>
<td>33.792</td>
<td>43.833</td>
</tr>
<tr>
<td>Control</td>
<td>30.833</td>
<td>45.708</td>
</tr>
</tbody>
</table>

Note. Writing n=27; Lecture n=24; Control n=24.

The relationship of these means are illustrated in the graph given in Figure 4.

Figure 4. Graph of the interaction of the cell means.
The graph indicates there is some interaction between the groups and the time of the tests. A oneway analysis of variance was performed on the differences between the pretest scores and the posttest scores. A significant difference was found between the groups, $F(2, 72) = 5.22, p < .008$, MS = 2061.43. A TukeyB post hoc multiple comparison test was performed and the results are illustrated in Table 5.

Table 5

Tukey’s Pairwise Comparisons

<table>
<thead>
<tr>
<th>Mean</th>
<th>Group</th>
<th>Lecture</th>
<th>Control</th>
<th>Writing</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.14</td>
<td>Lecture</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.86</td>
<td>Control</td>
<td></td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>27.37</td>
<td>Writing</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>

These results suggest the gains in awareness scores by the writing group are significantly greater than the gains in scores for the control group and the lecture group.

Analysis of Question Four

The fourth research question asked: Does the intervention of daily in-class expressive writing or
lecture containing reflective questioning affect the achievement of students? To determine the effect of writing or reflective questioning on achievement on the posttest, an analysis of variance for repeated measures was performed on achievement scores for the Groups A, B, and C over two times: Pretest and Posttest. No significant differences were found between these groups by time, \( F(2, 97) = .18, p < .83, \) MS = 79.42.

Summary

An analysis of the indications given for student perception of correct work revealed that college-level introductory algebra students declined over time in ability to assess correct work on quizzes which questioned new material only. This decline is suggested by the students' unwillingness to indicate confidence in their work. However, the ability to assess correct work again increased from the pretest to the posttest which contained questions on the content of the course.

Statistical analyses at the beginning of this study suggest the groups were initially equivalent on the measures named. After interventions of expressive
writing and reflective questioning, an analysis of variance for repeated measures was performed with measures of achievement and awareness on pretest and posttest. No significant differences in achievement were found for the three groups. The analysis did show a significant difference in the interaction between the groups. Post hoc comparisons were made to examine the differences in the pretest awareness scores and the posttest awareness scores. The writing group was found to be significantly different from the lecture and control groups in the ability to assess correctness of answers on tests.
CHAPTER 5
DISCUSSIONS, CONCLUSIONS, LIMITATIONS
AND IMPLICATIONS

The primary purpose of this study was to examine the influence of in-class expressive writing on student awareness of correct work. Through having students indicate the certainty of the correctness of their work on quizzes and tests and comparing this indication to the actual correctness, the researcher sought to validate a means by which awareness of correct work could be measured. As a result, the ability of students to assess the correctness of their work became a consideration. Finally, the study sought to examine the influence of writing upon student achievement as evidenced by scores on quizzes and tests. This chapter includes interpretations of the results of this study, conclusions formed on the basis of these results, limitations relative to this study, and implications for classroom practice and further research.

Discussion

Recalling that incorrect work for which students were "not certain" of their responses earned the maximum 2-point value along with correct work for which
students indicated "very certain" for their responses, it becomes conceivable that a pretest, which should contain much unknown content, could yield high awareness scores based on the first condition described. Likewise, the awareness scores could also be high for the posttest, which contained content that had become familiar, based on the second condition described. Predictably, the posttest achievement scores substantially increased from the pretest achievement scores. Analysis of the differences between the pre- and posttest awareness scores, however, showed considerable gain between these measurements, also.

An item analysis for the codings on the quizzes did not show the same increase in awareness scores. Quizzes, which tested only new material, remained relatively high in achievement, showing only a moderate decline for the third and fourth scores. The awareness scores, in comparison, were initially lower and then declined substantially for the third and fourth quizzes. On the first two quizzes, students were certain of correct work 80% of the time. On the remaining two quizzes, the percentage declined to 50.
Analysis of the responses for the quizzes also revealed a decline in the number of indications made by students. As the material increased in quantity and became more complex, achievement scores declined on quizzes and students were less willing, less able, or just less likely to indicate the certainty of their answers. Nonetheless, this reluctance, unwillingness or inability demonstrated on the quizzes decreased on the posttest as compared to the pretest. An effort to explain these data might question the sensitivity of the instrument in this study. This explanation would be consistent with this study’s theoretical framework which regards metacognition as an aid to learning new material.

However, other explanations are possible. These findings may suggest the framework for the study is inaccurate, and the development of metacognitive skills for a concept follows, rather than parallels, the development of the matching cognitive skills. This could be explained in reference to two theories. One theory posits that the rules students learn in algebra are not the rules they are taught. Kirshner (1989, 1987) argues that during the learning stage in the study of algebra, students are acquiring a system of implicit rules that underlies algebraic performance. These rules are believed to unconscious rules, rather like the rules of
grammar which a child acquires through interaction with a language community. Curricular rules may be only indirectly related to this implicit grammar. Alternately, Bereiter (1991) believes that experts are not using rules at all but are actually acquiring a connectionist pattern—a vast network of interrelated elements—that only approximates rule-based performance.

To explain the apparent phenomenon of introspection to rational or rule-based knowledge structures, Bereiter (1991) cites Harre's theory of the social nature of rationality. Harre' believes that when attempts are made to give a retrospective report on mental processes, instead what is given is a justification of actions based on the categories of phenomena sanctioned within the community of discourse (e.g. the rules presented by the teacher in a classroom). Thus personal rationality is an internalization of rules which occurs when the social process of justification is turned inward. This would explain why students metacognition was evidenced mainly following, not concomitant with the learning process.

Statistical analysis of the gains in awareness scores for the writing, lecture, and control groups found the gains for the writing group to be significantly different from the gains for the lecture and control groups. Since writing about mathematics is a learned
behavior and 14 weeks is a limited amount of time for effecting a change in writing about mathematics, this finding is strengthened by the limited length of the treatment. The lack of quantitative research about writing in the current literature could be explained as an artifact of length of treatment time required.

In this study, the use of writing had no significant effect on the achievement scores of these students. A discussion of these findings occurs in the limitations section of this chapter and in the section for implications for future study.

Conclusions

The following conclusions were formed after examination of the results obtained in this study:

(1) When students are not confident in their answers for quizzes on newly learned material, they are somewhat less likely to indicate the certainty of the correctness of their answers.

(2) As content increased in quantity and complexity, correct work and accurate assessment did not remain static. Although students maintained achievement, correct assessment of correct work declined.
(3) In-class expressive writing was found to have a positive effect on the metacognitive skill of self-assessment of correctness of work.

(4) The use of writing did not have a significant effect on the ability to answer questions on a test correctly.

Limitations of the Present Study
This section discusses limitations which affect the usefulness of this study to other academic researchers and to practitioners. The limitations are as follows:

Generalization of the Findings
The findings of this study are appropriate for introductory algebra students at the college level. The mean ACT mathematics score in geographic location for the sample used is lower than the national average; thus, similarity in the ACT math scores would contribute significantly to the validity of any generalizations.
**Instructor Preparation**

The instructor in this study participated in a preliminary study by the researcher for the purpose of becoming familiar with the use of writing prompts in the mathematics classroom. Outside readings on the use of writing were also part of the instructor's training. In addition, the researcher worked closely with the instructor in coaching the writing process in the classroom. This training became an influence on the findings within this study.

**Measure for Achievement**

Perhaps, the benefits of writing cannot be measured by achievement tests that do not delineate between performance based on rote versus conceptual knowledge. Achievement tests, at the introductory algebra level, may measure a student's ability to manipulate algebraic symbols without conceptual underpinnings. Developmental students often admit to working the problems over and over—a possible example of rote learning.
Implications

**Implications for Classroom Practice**

This study may stimulate the use of expressive writing as a technique for developing the metacognitive skill of self-awareness. Awareness during a mathematical task, in turn, supports the development of self-monitoring which implies a decision based on the awareness. This ability to manage one's learning is seen as an integral component in problem solving as well as all mathematical performance (Campione et al., 1989; Garofalo & Lester, 1985; Long, 1986; and Schoenfeld, 1987). For developmental mathematics students, this is evidenced in this researcher's finding that these students indicated they could work entire assignments incorrectly and not know this until the next class meeting (Allen, 1989).

The close monitoring provided by writing is needed to capture the growth in students' mathematical understanding not evident in the traditional data of objective achievement tests, classroom activities, and homework assignments. Miller (1991) views writing as a means of better assessing students' understanding. She asserts that giving students the opportunity to write
about concepts or algorithms provides a plethora of information about student understanding that might otherwise never be revealed. Thus, teachers should continue to explore writing in the mathematics classroom and collect case data and formal data on the effects of such writing for the purpose of gaining a better understanding of the learning process of different students.

**Implications for Future Study**

Although students were able to maintain performance, they were not able to maintain ability to self-monitor. As new material was introduced and became more difficult, students' certainty about their answers decreased. This suggests a need to extend this study to item analyses of each cumulative test as well as quizzes on new material to more closely monitor the development of awareness as evidenced by the willingness to indicate the certainty of answers. Should reflection be introduced after some evidence of conceptual understanding has occurred? This avenue of inquiry would begin to answer questions on the best time to initiate reflective questioning and/or writing.
As indicated earlier, attention must be given to the time frame that should be provided to allow students to develop their ability to perform reflective writing about mathematics. For writing to become the best indicator of conceptual knowledge, students need experience over time and more practice to allow for the development of the writing process. Replication of this study with high school mathematics students would allow a time frame of nine months in which to develop writing about mathematics.

Since efforts to capture the benefits of writing for increasing achievement have failed in several studies as indicated in Chapter 2, the concentration should be on the benefits of writing for promoting student understanding. At present, whether or not a student possesses the needed understanding is usually not evident until the study of intermediate or college level algebra. Blais (1988) describes the difference in learners who are novices as compared with experts by characterizing the experts as possessing the "essence" of a statement. As an example, Blais portrays "the essence of 2/7 + 3/7 as roughly two things plus three things, which are five things" (p. 624). Yet, this
question from a placement test for entering freshmen college students was answered incorrectly by 43% of the respondents. Perhaps a tool which would measure the presence of this essence would enable researchers to measure the effect of writing upon the learning of mathematics.

Summary

Writing in the mathematics classroom has previously received anecdotal support for its benefits to the learner and to the instructor. Limited quantitative benefits now exist in problem-solving ability (Bell & Bell, 1985) and in attitude toward mathematics (Miller and England, 1989). This study suggests additional quantitative support that writing is beneficial in promoting student ability to assess the correctness of work. If metacognitive skills are really a necessary condition for successful mathematics performance (Campione et al., 1989; Garofalo & Lester, 1985; Long, 1986; and Schoenfeld, 1987), the use of writing may provide the process for attaining these essential skills.
REFERENCES


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Watson, M. (1980). Writing has a place in a mathematics class. Mathematics Teacher, 73, 518-520.


APPENDICES
Appendix A

Informal Survey
Spring 1991
DVMA 092

Please circle the choice(s) that apply:

Indicate how well you expect you will do in this course:
1. Very well
2. Good
3. All right
4. Not very well
5. Very poor

Indicate how well you like math on the scale below:
(1 indicates least and 5 indicates greatest)

1  2  3  4  5

Circle the course(s) you completed in HIGH SCHOOL:
1. General Math
2. Algebra I
3. Geometry
4. Algebra II
5. Advanced Math
6. Trigonometry
7. Other____________________

Circle the course(s) taken in COLLEGE:
1. DVMA 90
2. DVMA 091
3. DVMA 092
4. Other____________________

Describe (HONESTLY) how you feel about math:

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
Appendix B

DVMA 92 Pre-test

Name____________________________ Sec______________

Show your work for each exercise in the space provided to the right of each exercise. Write your answer on the blank provided to the left of each question.

How certain are you that you have chosen the right approach for this problem? Mark [VC] for very certain, [FC] for fairly certain, or [NC] for not certain.

For the approach you have used, how certain are you that the problem is worked correctly? Mark [VC] for very certain, [FC] for fairly certain, or [NC] for not certain.

Simplify expressions 1 through 9 completely.

1. $\sqrt{48}$  
   [VC] [FC] [NC]

2. $\sqrt{49}$  (simplest radical form)
   [VC] [FC] [NC]

3. $(3x^{-2}y^{3})^{2}$ (leave no negative exponents)
   [VC] [FC] [NC]

4. $3x^{3} + 5x^{2} - (5x^{2} + 4x^{3}) - (4x^{2} - 3x^{2})$
   [VC] [FC] [NC]

5. $(4x - 1)^{2}$
   [VC] [FC] [NC]

6. $(2x - 1)(5x + 2)$
   [VC] [FC] [NC]

7. $\frac{-7}{4x^{2}} + \frac{11}{4x^{2}}$
   [VC] [FC] [NC]

8. $\frac{24b^{4}}{15xy^{3}} + \frac{16b}{45xy^{3}}$
   [VC] [FC] [NC]

9. $\frac{8x^{2} - 56x}{8x}$
   [VC] [FC] [NC]
Solve equations 10 through 13.

10 \[ 4(1 - 3(x + 2)) + 2x = 10 \]

11 \[ \frac{2}{3}x - 1 = \frac{1}{4} \]

12 \[ x^2 + 5x - 36 = 0 \]

13 \[ 6 - 4x^2 - 2x - 10 \]

14 Find the coordinate solution of \( 2x - 4y = 12 \) when \( x = -4 \)

15 Solve for \( y \): \( 12x - 6y = 36 \)

Factor exercises 16 through 18 completely.

16 \[ 3x^2 - 48 \]

17 \[ 2x^2 - 11x + 15 \]

18 \[ x^2 - 15x - 54 \]

Express the following situation as an algebraic equation and solve.

19 Three times a given number is equal to the sum of that number and 40. Find the number.

Evaluate the following expression for the given values.

20 \[ x^2 + y - (zx) \text{ if } x = -6, y = 24, z = -2 \]
Show your work for each exercise in the space provided to the right of each exercise. Write your answer on the blank provided to the left of each question.

How certain are you that you have chosen the right approach for this problem? Mark VC for very certain, FC for fairly certain, or NC for not certain.

For the approach you have used, how certain are you that the problem is worked correctly? Mark VC for very certain, FC for fairly certain, or NC for not certain.

Simplify expressions 1 through 9 completely.

1. \( -3^2 \)
2. \( \sqrt{\frac{24}{25}} \) (simplest radical form)
3. \((2x^{-3}y^3)^3\) (leave no negative exponents)
4. \(4x^3 - 5x^2 - (2x^2 + 3x^4) - (6x^2 - 2x^3)\)
5. \((3x - 2)^2\)
6. \((3x - 1)(4x + 2)\)
7. \(-\frac{8}{3x^3} + \frac{11}{3x^3}\)
8. \(-\frac{24b^3}{8xy^2} + \frac{9b^2}{32x^2y^2}\)
9. \(\frac{5y^3 - 45y}{5y}\)
Solve equations 10 through 13.

10 \[ 3(4 - 3(x + 2)) + 2x = 4 \]

11 \[ \frac{2}{3}x + 1 = \frac{1}{6} \]

12 \[ x^2 + 5x - 24 = 0 \]

13 \[ 5 - 3x \geq -x - 13 \]

14 Find the coordinate solution of \(2x - 3y = 12\) when \(x = -3\)

15 Solve for \(y\): \(6x - 3y = 15\)

Factor exercises 16 through 19 completely.

16 \[ 5x^2 - 20 \]

17 \[ 3x^2 - 13x + 12 \]

18 \[ x^2 + 15x - 54 \]

Express the following situation as an algebraic equation and solve.

19 Four times a given number is equal to the sum of that number and 30. Find the number.

Evaluate the following expression for the given values.

20 \( x + y^2 \div (zy) \) if \(x = 15, y = -3, z = -1\)
Show your work for each exercise in the space provided to the right of each exercise. Write your answer on the blank provided to the left of each question.

How certain are you that you have chosen the right approach for this problem? Mark VC for very certain, FC for fairly certain, or NC for not certain.

For the approach you have used, how certain are you that the problem is worked correctly? Mark VC for very certain, FC for fairly certain, or NC for not certain.

Simplify expressions 1 through 9 completely. (2 points each)

1. \( \frac{9}{6} \)

2. \(-7^2\)

3. \(-6 + 12 - 3 + 1\)

4. \(56 - 2(3 - 5) + 1\)

5. At daybreak the temperature was \(-4\)°. At noon it was 22°. What was the rise in temperature?

Write each phrase as an expression. (2 points each)

6. \(w\) less than 12

7. \(\frac{12}{x + y}\)

8. the difference between 5 times a number and 4

Evaluate each expression for \(a = 2\), \(b = -3\), and \(c = 2\) (2 points each)

9. \((a - b)^2\)

10. \(\frac{a^2 - b^2}{a - b}\)
Show your work in the space provided below each exercise. Write your answer on the blank provided to the left of each question.

How certain are you that you have chosen the right approach for this problem? Mark VC for very sure, FC for fairly sure, or NC for not sure.

For the approach you have used, how certain are you that the problem is worked correctly? Mark VC for very certain, FC for fairly certain, or NC for not certain.

Determine if the number given in the braces is a solution for the given equation. (YES or NO answer expected) (2 points)

\[ \text{3.1} \quad 2y + 8 = 3 + y; \quad \{-2\} \]

Solve each of the following equations for x. (2 points each)

\[ \text{3.1} \quad x - 7 = 3 \]

\[ \text{3.3} \quad 3x + 5 = 23 \]

\[ \text{3.2} \quad \frac{x}{4} = 12 \]

\[ \text{3.3} \quad -\frac{4x}{5} = 20 \]

\[ \text{3.2} \quad 2(x - 4) = 7x + 2 \]

\[ \text{3.3} \quad 4x + 1 - (3 - x) = 13 \]

\[ \text{3.3} \quad \frac{1}{8}x + 1 = \frac{1}{4}x - 2 \]

Solve each literal equation for the variable indicated. (2 points each)

\[ \text{3.4} \quad l = gr + t; \quad \text{solve for } g \]

\[ \text{3.4} \quad A = P(1 + r); \quad \text{solve for } P \]
Show your work in the space provided below each exercise. Write your answer on the blank provided to the left of each question.

How certain are you that you have chosen the right approach for this problem? Mark VC for very sure, FC for fairly sure, or NC for not sure.

For the approach you have used, how certain are you that the problem is worked correctly? Mark VC for very sure, FC for fairly sure, or NC for not sure.

Factor each of the following completely: (2 points each)

1. \(15x^2 - 10y + 30\)
2. \(18a - 27b\)
3. \(6x^2 - 9x^2\)

Supply the missing factor: (2 points each)

4. \(y(x+2) - 3(x+2) = (x+2\, \text{?})\)

Factor the following trinomials completely: (2 points each)

5. \(16x^2 - 49\)
6. \(a^2 - 9a + 18\)
7. \(2x^2 + 5x - 12\)
8. \(5x^2 + 20x + 20\)
9. \(x^2 - 5x - 6\)
10. \(6x^4 - 23x - 4\)
Show your work in the space provided below each exercise. Write your answer on the blank provided to the left of each question.

How certain are you that you have chosen the right approach for this problem? Mark VC for very sure, FC for fairly sure, or NC for not sure.

For the approach you have used, how certain are you that the problem is worked correctly? Mark VC for very sure, FC for fairly sure, or NC for not sure.

Determine the domain of the given expression: (2 points)

<table>
<thead>
<tr>
<th>Domain</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>VC</td>
<td>x ≠ 0</td>
</tr>
<tr>
<td>FC</td>
<td>x ≠ 0</td>
</tr>
<tr>
<td>NC</td>
<td>x ≠ 0</td>
</tr>
</tbody>
</table>

Simplify completely: (2 points each)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Simplified</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2x + 5}{3x + 2} )</td>
<td>( \frac{x}{3} )</td>
</tr>
<tr>
<td>( \frac{x^2 + 3x - 4}{x + 4} )</td>
<td>( x - 1 )</td>
</tr>
<tr>
<td>( \frac{8x^4 + 4x^2 - 12x}{4x} )</td>
<td>( 2x^2 - 3 )</td>
</tr>
<tr>
<td>( \frac{4x - 8}{4 - x^2} )</td>
<td>( 2 )</td>
</tr>
<tr>
<td>( \frac{x^2 - 1}{2x^2} \cdot \frac{12x^2}{x^2 - x - 2} )</td>
<td>( \frac{6}{x + 1} )</td>
</tr>
<tr>
<td>( \frac{x^2 - 4}{x + 2} \div \frac{x^2 + 4x + 4}{x^2 - 9} )</td>
<td>( \frac{(x + 2)(x - 3)}{x} )</td>
</tr>
<tr>
<td>( \frac{5}{6x^2} + \frac{3}{4x} - \frac{2}{3x} )</td>
<td>( \frac{14}{6x} )</td>
</tr>
<tr>
<td>( 2 - \frac{3}{x} )</td>
<td>( \frac{x - 3}{x} )</td>
</tr>
</tbody>
</table>

Solve the following fractional equation: (2 points)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x}{3x + 1} = \frac{11}{18} )</td>
<td>( x = \frac{11}{6} )</td>
</tr>
</tbody>
</table>

Express as a ratio in lowest terms: (2 points)

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{10}{15} )</td>
<td>( \frac{2}{3} )</td>
</tr>
</tbody>
</table>

On a recent exam, 15 students from a class of 35 failed. What is the ratio of the students passing to those who took the exam?
Appendix C

The prompts were structured to focus on metacognitive activities. And, although students were asked to review their work and go through processes that are cognitive, they were also asked to reflect upon and analyze these cognitive processes as they repeat them. The goal was to increase students' awareness of their cognition; therefore, although the differences between cognitive and metacognitive is a gray area, it may not be necessary to differentiate between them for the purposes of this study.

Prompt Development:

The structure of the prompts were consistent with current literature on the use of writing-to-learn, the coaching of writing, and the gathering of qualitative data. An example of the former would be to coax more detailed information by having students write to someone who is not knowledgeable about the subject, such as a seventh grader or a young neighbor. For the latter, the avoidance of dichotomous questions enhances the quantity and quality of the student's response. "How" and "why" are used to promote student reflection and "what" has been avoided when declarative answers
would not be desired. Content-oriented prompts may have asked students to think through the cognitive process first, but then the students were asked to describe their view of their cognition. The prompts focused on having students reflect on their understanding of a concept or a problem, ascertain the correctness of their work, or evaluate/monitor their study methods. The prompts have been field-tested and have produced satisfactory expressive writing responses. When necessary, the prompts were adjusted to avoid ambiguities or other difficulties.

The placement of the prompts paralleled the class schedule for topic presentation and for testing. Prompts that were content-oriented were usually presented the day after the concept was taught or, in two instances, on the day the concept was introduced. The placement of the remaining prompts was orchestrated by testing dates. Quizzes and tests became more time consuming as topics and/or concepts became more involved. Since students in previous studies had reacted negatively to writing before a test because of the time restraints, writing was not included before tests 2-4 and quizzes 4-5.
Writing Prompts (Spring 91):

Prompt 1 (January 14)
How do you feel about mathematics? Be as honest as you can.

Prompt 2 (January 16)
Describe a predominantly good or bad experience that you still remember clearly from a mathematics class.

Prompt 3 (January 18)
When you work a math problem, how do you determine if your answer is correct? When you think an answer is incorrect, what do you do?

Prompt 4 (January 21)
Look at problem #18 from your assignment on page 53. Do you think you worked this problem correctly? Explain in detail why you answered yes or no.

Prompt 5 (January 23)
Today you will take your first quiz in this course. How did you prepare for this quiz? Please give a detailed answer.

Prompt 6 (January 25)
Now that you have had your quiz returned, how useful was the way you prepared for the quiz? What could you do differently that would be helpful?

Prompt 7 (January 28)
Pick out a homework problem from page 85 that you thought looked difficult but you think you worked correctly. Explain why you think your work is correct. Pretend that you are writing for someone who does not understand algebra.

Prompt 8 (January 30)
What topic has been the most confusing so far in this course? Do you understand the concept now and, if so, how did you learn it?
Prompt 9 (February 1)
You are going to take your first test today. How was your preparation for this test different from how you prepared for your quiz? Why did you study the way that you did?

Prompt 10 (February 4)
What was the most difficult question on the test yesterday? Do you know why the question was so difficult? Please explain.

Prompt 11 (February 6)
What type of equation do you find hardest to solve? (Use page 143 to give an example.) Explain what you think makes this problem difficult for you.

Prompt 12 (February 8)
How do you know when you have solved an equation correctly?

Prompt 13 (February 15)
Did you study differently for the second quiz and, if so, how? If you did not change how you prepared for the quiz, explain why.

Prompt 14 (February 18)
Students often express concern that they cannot work with verbal statements in mathematics. Do you have this concern? How do you handle verbal problems? Explain your answer in detail.

Prompt 15 (February 20)
How do you compare solving inequalities with equations? How do you remember the differences in solving the two different types of problems?

Prompt 16 (February 22)
Tell me what you understand best about per cents? Tell me why you think you understand this so well.

No Prompt (February 25)
Test 2
Prompt 17 (February 27)
What was the hardest question on the test? Do you know how to find the answer now; how did you come to know it?

Prompt 18 (March 1)
How do you know if an expression has a common factor? Describe your decision-making in detail.

Prompt 19 (March 4)
Two methods have been taught for factoring trinomials. Which method works best for you? Describe your reasoning.

Prompt 20 (March 6)
What type of factoring was the most difficult for you? Explain why you think you have trouble with this problem.

Prompt 21 (March 8)
Give an example of the sum of two squares and the difference of two squares. Explain how you would determine if either or both of these can be factored.

Prompt 22 (March 11)
What preparation did you make for quiz 3 that you found to be the most beneficial? Be specific about how you think this preparation helped.

Prompt 23 (March 13)
Half of the semester has passed and you will take test 3 at the next class meeting. How do you feel writing about mathematics has helped you? Please, answer as honestly as you can without considering what I might want you to say.

No Prompt (March 15)
Test 3

Prompt 24 (March 18)
On the test that has been returned, find the question that you think you understood best. Tell me why you feel you understand this problem so well.
Prompt 25 (March 20)
We learned division by zero is not defined in mathematics. What does this statement mean to you?
How well do you understand this concept?

Prompt 26 (March 22)
How do you determine if a rational expression can be simplified? If an expression can be simplified, how would you do so?

Prompt 27 (April 1)
How do you know when two ratios are equal? Explain your answer as fully as possible.

Prompt 28 (April 3)
Have you had any difficulty with today's lesson? Can you identify the concept (or the problem) that is difficult for you?

Prompt 29 (April 5)
What has been the most confusing so far with algebraic fractions (Chapter 6)? Why do you think this has been a problem for you?

Prompt 30 (April 8)
What concept do you understand best about algebraic fractions (Chapter 6)? Why do you think this is easy for you?

Prompt 31 (April 10)
How would you go about showing that simplifying $1/2 X - 2 + 1/3 X$ is the same as or different from solving $1/2 X - 2 = 1/3 X$?
Why are these answers alike or different?

No Prompt (April 12)
Quiz 4

Prompt 32 (April 15)
Look over the items that you missed on the quiz. Can you tell me why you think you missed the problems that you did? Be as specific as you can.

Prompt 33 (April 17)
Discuss your understanding of solutions for a linear equation in two variables.
Prompt 34 (April 19)
You will take your last quiz at the next class meeting. How will you prepare for this quiz? How has your style of studying changed, if at all, to better prepare you for tests or quizzes?

No Prompt (April 22)
Quiz 5

Prompt 35 (April 24)
You have often been asked this semester to write about your perception of the correctness of your answers on a quiz or a test. How or what have you learned from these exercises? Please be as specific as you can.

Prompt 36 (April 26)
Have you had any difficulty with today's lesson? Can you identify the concept (or the problem) that is difficult for you?

Prompt 37 (April 29)
What is the most difficult concept in Chapter 9? Pick a problem from this chapter that you do not understand and work the problem as far as you can. On the side of your work, explain your work.

Prompt 38 (May 1)
Do you feel that writing in mathematics has helped you this semester? Explain your answer (and please be brutally honest). Should the use of writing be changed in any way?
Appendix D

In the semester prior to the study, the instructor of the experimental groups and the independent instructor did not code the students' work the same. Following each example is the agreement reached by these two persons and the researcher.

1. Evaluate \((a - b)^2\) for \(a = 2, b = -3,\) and \(c = 2\).

Students often knew how to replace the variables with the indicated value but could not implement the order of operations correctly. For example, the student would square the replacement for \(a\) and then square the replacement for \(b\). An agreement was reached to code the problem as correct method and incorrect implementation.

2. Evaluate \((a - b)^2\) for \(a = 2\) and \(b = -3\)

For this problem, the substitution in the numerator varied. Students sometimes wrote \(-3^2\) then \(-9\). It was decided to code the following examples as indicated below.

<table>
<thead>
<tr>
<th>Step 1:</th>
<th>3 - (-4)^2</th>
<th>3 - 4^2</th>
<th>3 - 4^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2:</td>
<td>3 + 4^2</td>
<td>3 - 16</td>
<td>3 + 16</td>
</tr>
</tbody>
</table>

Codings: CM, IW CM, CW CM, IW

(CM for correct method, CW for correct work, IW for incorrect work.)

3. Solve \(\frac{-4x = 20}{5}\)

Students sometimes began solving this equation by multiplying both sides of this equation by the number 5 rather than the more efficient method of multiplying by the reciprocal. This was considered a viable method and coded as correct method.

4. Factor \(2x^2 + 5x - 12\)

The correct answer is \((2x - 3)(x + 4)\) but the student may have answered \((2x + 3)(x - 4)\). It was decided that the student was aware of the correct method but the method was not implemented correctly.
Appendix E

The following table represents the reliability coefficients for the coding of the students' work on quizzes one, two, and three. These were determined by dividing the number of times the independent codings were not in agreement with the total number of codings made for that set of evaluations.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Coding Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz 1</td>
<td>1199/1220 = .983</td>
</tr>
<tr>
<td>Quiz 2</td>
<td>1238/1240 = .998</td>
</tr>
<tr>
<td>Quiz 3</td>
<td>1159/1160 = .999</td>
</tr>
<tr>
<td>Quiz 4</td>
<td>840/840 = 1.000</td>
</tr>
</tbody>
</table>
Appendix F

Group Equivalency Tests

A oneway analysis of variance was performed on the following measures: achievement pretest, awareness pretest, English ACT subscore, math ACT subscore, and number of years of high school preparatory mathematics. No significant differences between the four initial groups was found on any of these measures. The results of these tests follows.

Achievement Pretest: $F (2, 97) = .132, p < .88$
Awareness Pretest: $F (2, 89) = 2.80, p < .07$
English ACT Subscore: $F (3, 124) = .386, p < .76$
Math ACT Subscore: $F (3, 124) = .573, p < .63$
High School Preparation: $F (3, 124) = .986, p < .40$
VITA

Naomi Barbara Richardson Allen was born in New Orleans, Louisiana, on October 31, 1946, the daughter of Lloyd and Anne Richardson. Barbara lived and attended elementary and secondary school in Ponchatoula, Louisiana. After graduating as an honor student from Ponchatoula High, she attended Southeastern Louisiana University in Hammond, Louisiana where she earned a B. S. degree in Mathematics Education in 1968 and an M. Ed. in Mathematics Education in 1980.

In 1967, she married Melvin Darryl Allen and they are the proud parents of Paige Elizabeth, Melvin Darryl, Kathe Elizabeth, and Russell Gray. After fourteen years of teaching in the secondary schools and six years at the college level, she pursued the Doctor of Philosophy degree in Curriculum and Instruction with the emphasis on Mathematics Education which was awarded in December of 1991 at Louisiana State University.
DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: NAOMI (BARBARA) R. ALLEN

Major Field: EDUCATION

Title of Dissertation: A STUDY OF METACOGNITIVE SKILL AS INFLUENCED BY EXPRESSIVE WRITING IN COLLEGE INTRODUCTORY ALGEBRA CLASSES.

Approved:

[Signatures]

Major Professor and Chairman

[Signature]

Dean of the Graduate School

EXAMINING COMMITTEE:

Robert Perlew

[Signature]

[Signature]

Date of Examination: OCTOBER 30, 1991