Dynamics of vortical structures in a low-blowing-ratio pulsed transverse jet

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DYNAMICS OF VORTICAL STRUCTURES
IN A LOW-BLOWING-RATIO PULSED TRANSVERSE JET

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
In partial fulfillment of the
Requirements for the degree of
Master of Science in Mechanical Engineering

In

The Department of Mechanical Engineering

by

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December 2009
Acknowledgement

I would like to thank my advisor Dr. Dimitris Nikitopoulos who provides me support and advices in my research work during these two years.

I would like to thank the graduate students I worked with and helped me when I was facing difficulties in my research Sudheer Rani, Lem Wells, Jean Charles Dupupet and especially Guillaume Bidan.

I am very grateful to my friends Timur, Mahtab, Dave, Elham, Alborz and Somayeh with who I got wonderful moments in their company. I also would like to thank Ata and our ‘kitchen talks’ and in particular Sima and Gloria who are the best roommates someone can hope to have. A special thanks to Gloria, and the good times we got cooking.

Finally, I am very grateful to my little mum who always supports me in this adventure.
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Abstract

Large Eddy Simulation is used to study the interaction of a 35° inclined jet into a crossflow. Steady state cases, with a BR ranging from 0.150 to 1.2, are firstly examined to understand the dynamics of the flow field. Iso-surface of Laplacian pressure, vorticity contour and velocity fields highlight the presence of four main vortical structures: shear layer vortices, horse-shoes vortices, wake vortices and CRVP. Qualitative comparisons are performed between simulations and experiments.

The dynamics of the flow field is next characterized by pulsing the jet. The studied pulsed cases have same low BR and duty cycle respectively fixed at 0.150 and 50%. The presence of a vortex ring which evolves into a leading hairpin vortex is observed at the pulse. Good qualitative agreement is obtained between the numerical and experimental results.

Film cooling effectiveness, temperature contour and jet trajectory are extracted for both steady and pulsed cases. Overall, steady state cases provide better results in term of film cooling performance. POD is performed on steady and pulsed cases to obtain the dominant modes of the flow.
Chapter 1. Introduction

1.1. Background JICF

The injection of a fluid into a crossflow has many practical technical applications. Some examples are the flow issuing from the exhaust stacks of power plants, fumes from steam locomotives, fuel injection for burners, or thrust reversers for propulsive systems.

These past years, higher interest has been brought to jet in crossflow studies due to its numerous applications in environmental fields. In fact, the diffusion of pollutants into atmosphere from industrials chimneys, or the release of contaminated water into rivers obeys to the same phenomenon as a jet in crossflow (JICF).

One of the main applications of a jet in crossflow concerns fuel injection for combustion engines. Given our energetic dependence and our limited resources in term of energy, it is essential to optimize the mixing between air and fuel to reach higher or equivalent performance at lower energy cost. Higher energy efficiency would be beneficial in many areas, such as avionics and automobile industry with more fuel-efficient engines.

Another relevant application of jet in crossflow is the cooling of gas turbines.

1.2. Film Cooling

In gas turbines, higher is the temperature at the turbine inlet, higher efficiency will be reached. The increase of the turbine temperature has to be done by keeping in mind the limitation of the material. High temperature may damage the blades and therefore its performance. Many improvements have been realized on materials so that they support higher temperature. Over the past 20-30 years, alloy improvement, directional and single-crystal solidification have significantly contributed to reach higher temperature (Figure 1-1). The highest jump has been done with the introduction of coating system which has allowed an increase of temperature up to 100°C.

Since limitations in terms of materials have been reached, other ways to cool gas turbines have to be found. Two systems of cooling are predominant: internal and external cooling. The internal cooling
consists in the presence of channels inside the blades. Coolant fluid is passed into the channels to cool down the blades. The internal cooling is abundantly studied.

![Figure 1-1: Operational temperature of turbine components (Schulz et al, Aero. Sci. Techn. 7: 2003, p.73-80)](image)

In the case of the external cooling, small holes, where cool air passes through, compose the blade. For this technique, cool air is drawn from the compressor, pumped through internal passages in the blade and ejected through the holes at the blade surface. The blade is cooled as the air passes through the inner passages and impinges upon the internal surfaces. The ejected air, which is cooler, creates a thin protective boundary layer around the component.

However two main points have to be kept in mind as the external cooling applies. Firstly, the coolant air is taken from the compressor. More air pumped from the compressor for cooling purposes means less air available for the compressor and therefore a reduction of its performances. A trade-off between cooling and performance is necessary. Secondly, the ejection of the coolant air has to be performed without disturbing the primary flow. Any disturbances encounter by the main flow could decrease the performance of the turbine. Therefore it appeared the quantity of coolant air used to cool the blades down has to be maintained to a minimum level. Several possibilities are explored to reach this goal, as the use of inclined jet which may improve the blade area covered by coolant, or the use of pulsed coolant.
1.3. Literature Survey

During the past decades, jets in crossflow have been extensively studied. Up to date, most studies have been done on a vertical jet. It has been well established (Fric 1994, Kelso 1996) the interaction of a jet with a crossflow produces four main vortical structures: horse shoe vortices, shear layer vortices, wake vortices and counter rotating vortex pair (CRVP). Horse shoe vortices form on the windward side of the jet due to the blockage of the crossflow by the jet. Its legs extend around the jet exit. The wake vortices develop underneath the detached jet. Fric and Roshko (Fric 1994) reported wake vortices originate from crossflow boundary layer which has been entrained upwards due to ‘separation events’. In their simulation, Yuan and al. (Yuan 1999) have revealed the legs of the horse shoe vortices are lifted upwards by the crossflow motion and are incorporated into the wake vortex. In some cases, they may also merge with the CRVP, depending on the sign of the vorticity (Peterson 2004). The shear layer vortices form along the jet/crossflow interface on the lee-side and leading-edge of the jet exit. It has been shown the development of spanwise rollers on the upstream and /or downstream side of the jet exit is linked to the BR value and to the shape and orientation of the nozzle relative to the crossflow (New 2004). Broadly speaking, at low BR spanwise rollers at the trailing-edge of the jet appeared due to the dominance of the crossflow at the leading-edge. As the BR increases, the vertical jet momentum increases, as well as the penetration of the jet into the crossflow and therefore the jet/crossflow interaction intensifies. This
generates the shedding of spanwise rollers at the windward side of the jet. A trailing edge column of vorticity can be seen at relatively high BR. Concerning the CRVP, they result from the shearing of the jet by the crossflow on the lateral edges of the hole. As the CRVP are convected downstream, they exhibit the well-known kidney shape. This large-scale vortical structure is mainly responsible of the mixing of the crossflow with the jet.

Figure 1-3: Vortical structures in JICF (Frick and Roshko, 1994)

It has to be noted the four vortical structures mentioned above result from the interaction of a jet with a crossflow. However, their formation may slightly differ, or additional features may appear in function of the velocity ratio, aspect ratio, shape of the nozzle, jet velocity profile, or jet inclination. Several experiments (Haven 1997, New 2004, New 2006) have been performed to study the influence of these parameters. In general, the effects of these parameters are significant only in the near field and diminish in the far field.

If the coherent structures have been well identified experimentally, numerical simulation can help to better interpret the complexity of their interaction under experimental conditions. However, numerical simulations are relevant only if they are able to predict with accuracy the dynamics of the flow-field, therefore to solve the multi-scale vortical structures. To do so, several numerical models have been investigated. Initially, due to computational limitation, the primary approach consisted in solving the
Reynolds Averaged Navier Stokes (RANS) equation. In RANS model, the Navier Stokes equations are time-averaged, which results in the presence of an additional term: the turbulent Reynolds stress $-\rho \overline{u_i u_j}$, which is modeled to solve the flow field. Hoda and al (Hoda 2000) have reviewed the performance of seven two-equations models and compared them with the experimental results obtained by Ajersch at al (Ajersch 1997). The comparison between the numerical models and the experiment has been performed on a row of six square jets oriented normally with respect to the crossflow, with a velocity ratio of 0.5. The performance of seven numerical models has been investigated: a high-Re model, low-Re models (Launder-Sharma, Lam-Bremhorst and k-w models), nonlinear models (Mayong-Kasagi and Speziale models) and DNS based low-Re model. They concluded the two-equations models fail to solve the dynamics of the flow-field accurately. Overall the lateral shear stress $u''w''$ (responsible for the lateral mixing and spreading) is under-predict, while the vertical penetration is over-predicted. In the wake region of the coolant jet, the turbulent stress field is highly anisotropic and the unsteadiness of the jet-crossflow interaction influences the entrainment process. The use of an isotropic eddy-viscosity in the two-equation model explains the lack of similarities between the numerical predictions and the experiments.

The anisotropy of the turbulent stress field is incorporated in the Reynolds-Stress Transport (RST) model where each component of the Reynolds stresses is solved. Despite a better representation of the turbulent anisotropy, the RST predictions are not substantially better than the two-equation model ones (Acharya 2001).

The advent of faster computers with larger core memories has allowed pushing further the resolution of the flow field. New numerical models, like Large Eddy Simulation (LES) and Direct Numerical Simulation (DNS) appeared. In LES, the Navier Stokes equations are spatially filtered. The large eddies, whose the size is superior to the filter width are numerically resolved, while the small ones are modeled. This is in contrast with the RANS model where all eddies are modeled. Tyagi and Acharya (Tyagi 2003) performed comparisons between RANS and LES models with experimental results of Lavrich and Chiappetta (Lavrich 1990). The results have been obtained for a row of inclined cylindrical holes at
blowing ratio of 0.5 and 1. They concluded the RANS model fails to resolve the anisotropy and the dynamics of the flow-field. However, the LES predictions are in good agreement with the experiments; the large scale structures observed experimentally are reproduced by the LES model.

In DNS, the whole spectrum of turbulent scales is numerically resolved. If the DNS provides excellent comparison with the experiments (Muppidi 2007), its use is not widespread due to computational costs.

The purpose of the numerical simulation of a transverse jet is to identify the vortical structures, understands their formation process and better interpret the complexity of the interaction of a jet with a crossflow under experimental conditions. Several numerical studies have been realized. Most of them have been done on a 90° jet. Sau and Mahesh (Sau 2008) have studied the influence of the BR and stroke ratio on the flow structures generated by the interaction of a jet with a crossflow. The study has been done on a 90° circular jet using DNS for a range of velocity ratio and stoke ratio. Depending on these two parameters, they identified three distinct regimes. For blowing ratio below 2, whatever the stroke ratio, the jet shear layer rolls up on the trailing-edge of the jet, leading to the formation of a hairpin vortex downstream. At these given blowing ratio, the shedding of hairpin vortices intensifies with the increase of the stroke ratio. For blowing ratio higher than 2, two regimes have been characterized depending on the stroke ratio value. A vortex ring forms for low stroke ratio, while a vortex ring accompanied by a trailing column of vorticity is created at higher stroke ratio. They showed the threshold stroke ratio separating these two regimes decreases as the velocity ratio decreases. For high velocity ratio, the threshold stroke ratio approaches the ‘formation number’. Contrary to Sau and Mahesh, Yuan et al (Yuan 1999) do not observe vortex ring in their simulation. They performed LES simulation on a round jet issuing normally into a crossflow at BR 2 and 3.3. They identified the presence of ‘hanging vortices’ which form on the lateral edge of the jet exit due to the skewed mixing layer. As moving upwards and downstream, the hanging vortices encounter an adverse pressure gradient and breaks down. This generates a pair of weak CRVP aligned with the jet trajectory. Yuan et al do not assimilate the hanging vortices as vortex rings because a strong axial velocity is carried through the cores of the hanging vortices, but also because their
shedding is irregular downstream. They also distinguished other vortical structures as the spanwise rollers at the leading edge and lee-side of the jet, horse shoe vortices and wake vortices.

Guo and al. (Guo 2006) investigated the influence of the jet inclination on the flow field. The comparison carried on a 90° jet and on a 30° streamwise inclined jet at low blowing ratio 0.1 and 0.48. LES model has been used. In their simulation, Guo et al. do not mention the presence of hairpin vortices, contrary to Sau and Mahesh, but only a spanwise roll up on the trailing edge of the jet. This is due to the very low value of blowing ratio. As well, at these two given low blowing ratio, no horse shoe vortex is observed at the leading-edge of the jet because the crossflow does not undergo a blockage by the jet. In their work, Guo and al. mainly focused their attention on the primary large scale structure, the CRVP. They considered the CRVP is caused by ‘the shearing effect of the jet-crossflow interaction and the streamwise oriented vorticity contained in the near-wall layer of the jet and the crossflow’ (p 605). From their work, Guo and al. observed similar structures between a 90° and inclined jets: recirculation region, spanwise rollers at the leading-edge of the jet exit, CRVP.

Tyagi and Acharya (Tyagi 2003) also performed LES on a streamwise inclined 35° jet at low BR, 0.5 and 1. Contrary to Guo and al., they identified packets of hairpin vortices in the wake of the jet exit, but also in the far-field as they are convected downstream. Based on vorticity similarity, Tyagi and Acharya have shown the head of the hairpin vortex are associated with the strong spanwise vorticity from the spanwise rollers, its upright legs with the wake vortices and its horizontal ones with the CRVP. However, since the inclination of the jet reduces the pressure gradient upstream of the jet, no evidence of horse shoe vortex has been found. Since hairpin vortices are the primary large-scale coherent structures associated with a jet in crossflow, they ‘control’ the mixing and the entrainment of the crossflow with the coolant jet fluid. Tyagi and Acharya investigated the influence of these coherent structures on the cooling of the wall. They reported coolant jet fluid wraps around the head and the streamwise-oriented legs, while the crossflow fluid is entrained by the upright legs of the hairpin vortices, leading to the presence of crossflow fluid (hot spot) inside the arch of the hairpin.
Several experimental studies (Coulthard 2007, Ekkad 2006) focused on the quantification of the film cooling effectiveness by pulsing the jet. Coulthard et al. (2007) investigated the effect of jet pulsing on film cooling. Their experiments have been done on a single row of cylindrical holes inclined at 35° with respect to the surface. A range of blowing ratio (varying from 0.25 to 1.5), duty cycles and pulsing frequencies has been considered. They reported the cooling effectiveness in steady cases is a trade-off between the amount of coolant injected and the jet lift-off. As BR increases, higher amount of coolant flow is injected. However, the jet lift-off increases too. A steady BR of 0.5 provides the best result in term of cooling effectiveness. By pulsing the jet, their results showed high frequencies are overall detrimental to cooling effectiveness because it improves jet lift-off. At lower frequencies, pulsing has an opposite effect. The comparison between pulsing and steady cases at same BR mean demonstrates the highest effectiveness is achieved with continuous jet. However, pulsed jet improves the spanwise spreading.

The identification of the vortical structures can be accomplished through statistical analysis, like Proper Orthogonal Decomposition (POD). Meyer et al. (Meyer 2007) have applied the POD on a turbulent jet in crossflow at BR=1.3 and 3.3. The analysis has revealed the wake vortices and the jet shear layer vortices are the dominant vortical structure respectively at high and low BR. The presence of CRVP has also been identified. Graftieux et al (Graftieux 2001) have used the POD to separate from a highly turbulent swirling flow the turbulent fluctuations and the unsteady swirling motion. The main advantage of the POD consists in the identification of the main vortical structures with a very few modes. Therefore, the POD modes are widely used as a basis in the Galerkin projection to obtain low-dimensional model (Moehlis 2002, Smith 2005, Rowley 2003).

1.4. Motivations

Most of experimental and numerical studies of a jet with a crossflow have been done on a vertical jet. In this configuration, the vortical structures have been well defined. As mentioned earlier, these vortical structures are influenced by many parameters; one of them is the inclination of the jet relative to the crossflow. Very few studies have been performed with a streamwise inclined jet. In this present study, numerical simulation of a streamwise 35° inclined jet will be accomplished. The goal is to better
understand the dynamics and the interaction of the flow in this configuration. Simultaneously, the effect of the vortical structures on the cooling effectiveness will be investigated. To reach this point, steady and pulsed cases will be simulated. Since it has been shown RANS model does not capture with accuracy the complexity of the flow field, and DNS model is too computationally expensive, LES model will be used for the simulation. Besides, statistical analysis using proper orthogonal decomposition will be done on the numerical data.
Chapter 2. Notations and Definitions

2.1. Notations

BR  Blowing ratio
\(\text{BR}_m\)  Mean Blowing ratio
\(\text{BR}_l\)  Low Blowing ratio
\(\text{BR}_h\)  High Blowing ratio
\(\text{BR}_{pp}\)  Peak to peak Blowing ratio
DC  Duty cycle
\(D_j\)  Jet diameter
\(F_f\)  Forcing frequency
\(U_j\)  Jet velocity
\(U_\infty\)  Crossflow velocity
\(\bar{U}\)  Mean velocity
\(u'\)  Fluctuation velocity
TI  Turbulence intensity \(\left(\sqrt{\frac{\overline{u'^2}}{\bar{U}}}\right)\)

\(\text{Re}_j\)  Reynolds number based on jet diameter and jet velocity \(\left(\frac{U_j D_j}{\nu}\right)\)

\(\text{Re}\)  Reynolds number based on jet diameter and crossflow velocity \(\left(\frac{U_\infty D_j}{\nu}\right)\)

\(\rho_j\)  Jet density
\(\rho_\infty\)  Crossflow density
\(\delta\)  Boundary layer thickness at 99%
\(\eta\)  Film cooling effectiveness
\(\Delta P\)  Laplacian pressure

Subscripts

\(j\)  jet
\(\infty\)  Crossflow
rms  Root-mean square
mean  mean value

2.2. Definitions

2.2.1. Blowing Ratio

The blowing ratio (BR) is defined as the ratio of the jet to crossflow mass flow rate.

\[
BR = \frac{\rho_j U_j}{\rho_\infty U_\infty}
\]

Equation 2-1: Blowing ratio definition

In the simulation, the same fluid air is used for the jet and the crossflow. The blowing ratio becomes a ratio of velocity. In steady state cases, only one blowing ratio is necessary to define the flow condition.
Pulsed cases are characterized by two of the four blowing ratio: BR$_l$, BR$_h$, BR$_m$ or BR$_{pp}$. BR$_l$ is the blowing ratio in the low part of the cycle, BR$_h$ in the high part of the cycle, while BR$_m$ is defined as the average blowing ratio over a cycle and BR$_{pp}$ the difference between the high and the low blowing ratio. The following formulas emphasize the relations between these different blowing ratios and the duty cycle:

\[ BR_m = BR_h \ast DC + BR_l \ast (1 - DC) \]
\[ BR_m = BR_l + BR_{pp} \ast DC \]
\[ BR_m = BR_h - BR_{pp} \ast (1 - DC) \]

**BR$_{pp}$ = BR$_h$ - BR$_l$**

**Equation 2-2: Relations between the different parameters in pulsed cases**

From the duty cycle and two of the four blowing ratios, the configuration of the pulsed case is completely determined, as well as the two remaining blowing ratios.

![Diagram](image)

**Figure 2-1: Parameters definition in a pulsed case (Bidan 2008)**

**2.2.2. Duty Cycle**

The duty cycle (DC) is defined as the ratio of the time in the high part of the cycle over the time of the cycle. The graphics below gives a better interpretation of all the parameters encountered in a pulsed case.

In our present work, all pulsed cases have a duty cycle of 50%.
2.2.3. Forcing Frequency

The forcing frequency ($F_f$) is the frequency at which the jet flow is pulsed. In our simulations, forcing frequencies of 1Hz and 10Hz have been considered.
Chapter 3. Simulations

3.1. Geometry

The geometrical domain (figure 3-1) used for the numerical simulation is a model of the experimental set-up. It consists of a rectangular box representing a part of the test section of the wind tunnel used in experiment. The wind tunnel flow is parallel to the longest dimension of the box. A single elliptical hole with a minor axis Dj=1in, which is perpendicular to the flow direction is located on the bottom wall of the box. This hole is created by the intersection of a circular duct feeding the jet inclined at 35° with respect to the horizontal bottom surface of the box so that the major axis of the elliptical hole is parallel to the flow direction. The computational domain representing the part of the wind tunnel test section is 16Dj long (x-direction), 8Dj wide (y-direction) and 8Dj tall (z-direction). The duct feeding the jet is 7.5Dj in length and has the same geometry as in the experiment over that length. It is composed firstly by a square section side 0.7Dj and length 2Dj. The square section discharges into a circular duct through a smooth transition. This complex geometry for the jet has been chosen to reproduce in the most accurate way the experimental configuration. The jet exit center is located 4Dj downstream from the beginning of the box which represents the inlet of the computational domain.

![Figure 3-1: Geometry and boundary conditions applied for the simulation](image)

3.2. Mesh

The meshing of the geometry has been performed with the software Ansys ICEM by Guillaume Bidan. The computational mesh is structured and consists of approximately 1.6 million hexahedral
elements. Fine elements are used near the crossflow wall, jet wall and jet exit. Relatively coarse elements are employed far away from the jet exit and the walls.

The meshing of the jet has been the delicate part of this operation. To reduce the number of skewed elements in the cylindrical section of the jet, an ‘O-grid’ block has been employed. It arranges grid lines into an ‘O’ shape where a block corner lies on the continuous curved surface. Figure 3-2 shows respectively a cross-section of the mesh inside the square (left) and cylindrical (right) part of the jet. As it can be seen, the mesh of the square section of the jet is not exclusively composed of rectangular elements; it has been adapted to the use of an ‘O-grid’. Its meshing consists of an inner square of dimension half of the square section of the jet. 126 square elements of size 0.014Dj (edge length) result from the uniform meshing of the inner square. The corners of the inner square and the square section are linked to each other by radial edges containing 13 elements per edge. Therefore, the jet cross-section is composed of 2028 elements. The depth of the elements in the jet tube has been set up to 0.07Dj, except where the transition from square to cylindrical section occurs. A refine mesh has been used in depth 0.028Dj.

![Figure 3-2: Computational mesh inside the jet: square section (left), cylindrical section (right)](image)

Figure 3-3 represents the mesh of the cross-section at the jet exit (green mesh). The curved cells resulting from the meshing of the jet is considered at the wind tunnel wall (purple mesh) on a zone centered on the jet exit and which extends +/- 1Dj in the x-direction and +/-0.825Dj in the y-direction. In this region of high interest, a fine mesh is used to capture the interaction between the crossflow and the jet. The size element is 0.076Dj*0.055Dj*0.027Dj respectively in the x-, y-, and z-directions.
Since we don’t bring the same interest to the regions close to the wall and to the one far away from the wall, the computational domain is divided into several sub domains for meshing (figure 3-4). The z-direction is composed of three sub domains. The first sub domain which contains the wind tunnel wall is composed of fine elements ($\Delta z=0.027D_j$) near the wall to capture accurately the phenomena in the boundary layer. The mesh size increases linearly upwards at a rate of 1.05 till the mesh size is 0.114$D_j$ at $Z_j=2D_j$ approximately. A coarse mesh is used in the second sub domain which extends from $Z_j=2D_j$ to
Z_\text{j}=6D_\text{j}. In this zone the mesh size increases linearly from 0.12D_\text{j} to 0.155D_\text{j}. A uniform and coarse mesh (\Delta z=0.155D_\text{j}) is used in the third zone.

**Figure 3-4: Sub domains used for the meshing of the computational domain**

The y-direction is composed of 5 sub domains. A very fine mesh (\Delta y=0.055D_\text{j}) is used on the zone which extends +/-0.825D_\text{j} on the symmetry plane. As we get closer to the lateral walls, the mesh gets

**Figure 3-5: computational mesh of the wind tunnel inlet (yellow), lateral side of the wind tunnel (green)**
wider until it reaches a mesh size of 0.15Dj. The x-direction is composed of three sub domains. The first one extends from the wind tunnel inlet till 1Dj upstream of the center of the jet exit. The mesh size decreases linearly from ∆x=0.148Dj to 0.076Dj at a rate of 0.97. The sub domain centered on the jet exit is meshed uniformly with a mesh size of ∆x=0.076Dj. The last sub domain starts 1Dj downstream the jet exit until the outlet. As we go downstream, the mesh increases till ∆x=0.118Dj.

The quality of our mesh has been checked by using the determinant 3*3 criterion. No hexahedral element has a determinant lower to 0.7. 94% of the hexahedral elements have a determinant included between 0.9 and 1.

### 3.3. Grid Independency

A grid independency has been performed to determine the correct grid refinement level. Our reference grid contains 1.6 millions of cells. A scaling factor of 0.72 is applied in the three directions of our reference grid to get two coarser meshes. These two meshes contain 0.28 and 0.68 millions of cells. Figure 3-6 displays the evolution of the mean streamwise velocity at different planes for the three grids at BR=0.750.

![Figure 3-6: mean streamwise velocity scaled by the crossflow velocity at 4 different planes for the three grid levels.](image)
The main discrepancy between these three meshes is located between $Z_j=0.25$ and $Z_j=1$. It has been observed that the coarser meshes overestimate the flow variable compared to the reference grid. At the planes $X_j=1$ and $X_j=3$, the relative error between 1.58 millions of cells and 0.68 millions of cells is at maximum 1.8%, while the one between 0.68 millions of cells and 0.28 millions of cells is 2%. In the opposite, the relative error between our reference case and 0.68 millions of cells is higher than the relative error between 0.68 millions of cells and 0.28 millions of cells at $X_j=10$; 3.5% compared to 1.6%. With such a low value of the relative error at different given planes between the three meshes, the mesh with 1.68 millions of cells is considered to give a good representation of the flow field independently of the number of cells.

3.4. LES Theory

3.4.1. Theory

The numerical simulation has been performed using commercial software Fluent™. As mentioned earlier, Large Eddy Simulation (LES) model has been chosen to simulate the complexity of the dynamics of the flow field. Turbulent flows are characterized by a wide range of eddies of different size. Large eddies are flow dependent, their evolution is dictated by the geometry and the boundary conditions of the flow. They carry momentum, energy, mass and other passive scalars. On the opposite, small eddies tend to be more isotropic, they are less affected by the conditions of the flow; therefore they are easier to model. The theory behind LES consists in solving numerically the large eddies, while the small ones are modeled. By doing so, compared to RANS model, less turbulence is modeled, then the error introduced by modeling is reduced, a more accurate representation of the flow field is obtained. In addition, the unsteadiness of large eddies motion can be resolved.

3.4.2. Governing Equation in LES Model

For our study, an incompressible flow has been considered. The incompressible Navier-Stokes equations (equation 3-1) are spatially filtered in LES model. The filter width is function of the grid size. By spatially filtering the Navier-Stokes equations, eddies whose the size is smaller than the filter width
are removed and modeled by a sub-grid scale (SGS) model. On the opposite, larger eddies are directly solved numerically by the filtered transient Navier-Stokes equation. For the application of the LES theory, the velocity and pressure are decomposed as a resolved scale (denoted by an over bar) and a sub-grid scale (denoted by a prime) (equation 3-2).

\[
\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial u_i}{\partial x_j} \right)
\]

*Equation 3-1: Navier-Stokes equation*

\[u(x,t) = \bar{u}(x,t) + u'(x,t)\]

\[p(x) = \bar{p}(x) + p'(x)\]

*Equation 3-2: Decomposition of the velocity and pressure as a resolved and sub-grid scales*

The resolved scale refers to the filtered variable which is defined by equation 3-3. The filtered variable is the average of the variable over the computational cell. \(V\) represents the computational cell.

\[\bar{\phi}(x) = \frac{1}{V} \int_{V'} \phi(x')dx', \ x' \epsilon V\]

*Equation 3-3: Definition of the spatial filtering applied on a variable in LES*

The filtered Navier-Stokes equations are obtained by introducing the decomposition of the velocity and pressure into the Navier-Stokes equation (equation 3.1), and then filter them spatially. By applying the spatial filtering, the linear sub-grid scale terms are washed out.

\[
\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i u_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{\partial (u'_i u'_j)}{\partial x_j}
\]

*Equation 3-4: Navier-Stokes equations spatially filtered*

Due to the non linearity of the advective term, as the Navier-Stokes equations are spatially filtered an extra term (the left one in equation 3-4) appears. It characterizes the spatial filtering of the sub-grid scale terms and is named the sub-grid scale turbulent stress in the literature. The subscript ‘f’ refers to the spatial filtering.
\[
\frac{\partial (u_i' u_j')}{\partial x_j} = \frac{\partial (u_i u_j' - u_i' u_j)}{\partial x_j} = \frac{\partial \tau_{ij}}{\partial x_j}
\]

**Equation 3-5: Definition of the sub-grid scale turbulent stress**

The filtered transient Navier-Stokes equations are

\[
\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \bar{u}_i}{\partial x_j} \right) - \frac{\partial \tau_{ij}}{\partial x_j}
\]

**Equation 3-6: Filtered Navier-Stokes equations deduced from the LES theory**

The sub-grid scale turbulent stress is unknown and required modeling. In Fluent software, the turbulent stress is modeled using the Boussinesq approximation.

\[
\tau_{ij} - \frac{1}{3} \tau_{kk} \delta_{ij} = -2 \mu_t \bar{S}_{ij}
\]

**Equation 3-7: Boussinesq approximation**

From equation 3-7, \( \mu_t \) refers to the sub-grid scale turbulent viscosity, and \( \bar{S}_{ij} \) to the rate of strain tensor for the resolved scale (equation 3-8). The term \( \tau_{kk} \) emphasizes the isotropic part of the sub-grid scale which is not modeled, but added to the filtered static pressure term.

\[
\bar{S}_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)
\]

**Equation 3-8: rate of strain tensor for the resolved scale**

However, in LES model the sub-grid scale turbulent viscosity \( \mu_t \) is unknown and requires modeling.

Four sub-grid scale models are available in Fluent to model the sub-grid scale turbulent viscosity: Smagorinsky-Lilly, Dynamic Smagorinsky-Lilly, Wall-Adapting Local Eddy Viscosity (WALE) and Dynamic kinetic Energy sub-grid scale model. Considering our geometry and the configurations of our flow, the first two models have been considered. A comparison between these two models have been perform to evaluate their differences.
3.4.3. Sub-Grid Scale Model

The two following sub-grid scale models are very close to each other in term of theory. The underlying assumption behind these two models is the local equilibrium between the smallest resolved scales and the sub-grid scales. In turbulent flow, large eddies transfer energy to small eddies via vortex stretching; while the small eddies convert the kinetic energy into thermal energy via viscous dissipation. Therefore, Smagorinsky-Lilly model and Dynamics Smagorinsky-Lily model assume a local equilibrium between the transferred energy through the grid filter scale and the dissipation of kinetic energy at small sub-grid scales.

3.4.3.1. Smagorinsky-Lilly Model

The Smagorinsky-Lilly model has been introduced by Smagorinsky in 1963. For this given model, the turbulent viscosity (equation 3-9) is expressed as a function of the mixing length $L_s$ and the rate of strain tensor for the resolved scale.

$$
\mu_t = \rho L_s^2 \sqrt{2S_{ij}S_{ij}}
$$

Equation 3-9: Smagorinsky-Lilly model for the turbulent viscosity

The mixing length is computed respectively according to equation 3-10. It is a function of the Von Karman constant $k$, the distance to the closest wall $d$, the volume of the computational cell $V$ and the Smagorinsky constant $C_s$.

$$
L_s = \min\left( kd, C_s V^{\frac{1}{3}} \right)
$$

Equation 3-10: Definition of the mixing length

For homogeneous isotropic turbulence, a value of 0.17 has been obtained by Lilly for the Smagorinsky constant $C_s$. However, ‘this value was found to cause excessive damping of large-scale fluctuations in the presence of mean shear and in transitional flows as near solid boundary’ (Fluent manual section 12.9). For such geometry, a lower value of $C_s$ has to be used. As it is easily discernible, the main shortcoming in the Smagorinsky-Lilly model is the absence of universal value for $C_s$, which will
be applicable to all kind of flows. Nevertheless, it has been shown for a large variety of flow a value of 0.1 for Cs provided good results. Consequently, in our study, the default value of 0.1 will be applied.

3.4.3.2. Dynamics Smagorinsky-Lilly Model

The Dynamics Smagorinsky-Lilly model overcomes the shortcoming of the Smagorinsky-Lilly model by computing dynamically the Smagorinsky constant, Cs. This constant is calculated according to Germano et al. (1996) identity. It is determined locally from the velocity field which has been filtered on two different levels. A better guess for the Smagorinsky constant is provided at any points of the flow field.

3.4.3.3. Comparison Between Smagorinsky-Lilly and Dynamics Smagorinsky-Lilly Models

As mentioned earlier, these two models are suitable for our geometry and our flow conditions. To determine which of the following model will be used in our study, comparisons have been performed on a steady-state case at a blowing ratio of 0.400. Comparisons are realized on instantaneous and averaged variables. The averaged variables have been calculated from a sample of 150 data files. The time separating each data file is greater than the integral time scale. Therefore, two consecutive data files are uncorrelated.

The Smagorinsky-Lilly model exhibits higher normal velocity inside the jet tube than the Dynamics Smagorinsky-Lilly model (Figure 3-7). The maximum difference between these two models is in order of 3-4%, so negligible. From figure 3-6 which displays the comparison between these two models for the vorticity magnitude, no difference is seen. As well, no difference is noticed between the Smagorinsky-Lilly model and Dynamics Smagorinsky-Lilly model concerning the averaged streamwise velocity at the center plane of the jet in the vicinity of the jet exit. However, outside the jet exit, a maximum difference of 4% between the two models has been computed.
Figure 3-7: Comparison of the averaged normal velocity (right), averaged vorticity magnitude (left) inside the jet tube between the Smagorinsky-Lilly model (dashed line) and Dynamics Smagorinsky-Lilly model (solid line): at the jet exit (red), along the line perpendicular to the jet centerline located 0.65 Dj (green), 1Dj (blue) and 2 Dj (purple) downstream from the center of the jet exit.

Figure 3-8: Comparison of the averaged streamwise velocity between the Smagorinsky-Lilly model (dashed line) and Dynamics Smagorinsky-Lilly model (solid line): at the center plane along the lines Zj=0.001 (red), Zj=0.01 (green), Zj=0.1 (blue) and Zj=0.125 (purple)

The difference between the Dynamics Smagorinsky-Lilly and Smagorinsky-Lilly models has been computed for the averaged data on the whole flow field. From this operation, it results the difference between these two models is in order of magnitude of 10^{-7}, except for the velocity and vorticity variables. Figures 3-9 represents respectively the contour of the mean velocity magnitude (top) and mean spanwise vorticity (bottom) obtained from this difference. The highest difference between these two models is located in the recirculation region and at the interface between the jet and the crossflow fluids. At this given BR, the jet stays relatively close to the wall. The recirculation region extends in the stream wise
direction but not vertically. Then small eddies develop in this restricted area, which justifies the observation of the highest difference. The order of magnitude of the difference is $10^1$ and $10^1$ respectively for the velocity magnitude and the spanwise vorticity. However once scaled by the mean value of the Dynamics Smagorinsky-Lilly model, the errors are in order of 2% for both variables, which is insignificant.

![Image]

**Figure 3-9:** Contours of the mean velocity magnitude (top) and mean spanwise vorticity (bottom) obtained from the difference between the Dynamics Smagorinsky-Lilly and Smagorinsky-Lilly models

Figure 3-10 shows the isosurface of the Laplacian pressure at same time instant for both models. Qualitatively, both models show the same structures, a street of hairpin vortex. However, Dynamics Smagorinsky-Lilly model shows 8 well-formed hairpin vortices, while Smagorinsky-Lilly model represents only 7. Besides, in the far field of the flow, Dynamics Smagorinsky-Lilly model captures much more vortical structures. What it may be interpreted as legs of horse shoe vortices are present at the level of the fifth hairpin vortex, while this same configuration appears farther for the Smagorinsky-Lilly model, at the level of the sixth hairpin vortex.
From the averaged variables of the flow field, no difference appears between these two models. However, from the instantaneous plots, the Dynamics Smagorinsky-Lilly model captures more vortical structures than the Smagorinsky-Lilly model, in particular in the far-field of the flow. From these observations, and from the fact it is well established in the literature that the Dynamics Smagorinsky-Lilly model better handles transitional flows, this last model will be used in the present study. It has to be noted our simulations will run at transitional Reynolds number, which also explains our choice of using the Dynamics Smagorinsky-Lilly model.

3.5. Boundary Conditions

Figure 3-1 shows the boundary conditions applied during the simulation. Velocity profiles for the inlet are obtained from the experimental wind tunnel. The inlet velocity profile used for the crossflow is shown in figure 3-11 (left). The crossflow has a laminar boundary layer with a thickness $\delta_{99\%}=1.612 D_j$. At the inlet of the jet feeding duct a uniform velocity is defined so as to equal the volumetric flow rate of the experiment. In the pulsed case, this velocity is modulated by using the signal of the unsteady volumetric flow measurement from the experiment (figure 3-11 right).

As mentioned previously, the blowing ratio is defined as the ratio of the jet to crossflow mass flow rate. In the present simulation, the blowing ratio is calculated from the mass flow rate in the circular duct. Since the jet tube is composed of a square injection (with $A_{\text{square}}$, $U_{\text{square}}$ as the section and the velocity)
followed by a circular tube (with $A_{cyl}, U_{cyl}$ as the section and the velocity), the velocity at the inlet of the square section is defined by equation 3-11.

\[ BR = \frac{U_{cyl}}{U_{\infty}} \Rightarrow U_{square} = U_{\infty} BR \left( \frac{A_{cyl}}{A_{sq}} \right) \]

Equation 3-11: Definition of the velocity of the square section of the jet

To examine the effect of the coolant jet fluid on the main flow, the coolant jet is maintained at a constant temperature of 300K, while the crossflow is at 330K. The no-slip condition is employed along the wall of the jet and the wind tunnel. In reality, a row of several holes is used to cool the turbine blade surface down. To model the periodicity of the holes, as well as their interaction the periodic boundary condition applies along the lateral side of the box.

3.6. Procedure to Run LES

Before running LES, several steps have to be performed in order to obtain accurate, convergent and physical results. The flow is initialized by running a steady state laminar solution with the jet flow only, and then with the crossflow. Once the steady-state is reached, an unsteady laminar solution is used to bring unsteadiness in the flow for the RANS model which will allow speeding up its convergence. Turbulent effect is introduced in the simulation by running k-epsilon RANS model. To do so, turbulence at the jet inlet and wind tunnel inlet are defined. For the present study, a turbulent intensity of 1% has
been implemented at the wind tunnel inlet and 30% at the jet inlet. These values reflect the low turbulence of the wind tunnel free stream and the high fluctuation levels created by the seeding injection system in the jet. It has to be noted, even if the jet inlet conditions are taken from experimental data, these conditions may not give an accurate representation of the phenomena which take place inside the jet tube. In fact, complex interactions and structures result from the injection system which is not modeled. Running the RANS model has two purposes. First it provides a realistic initial field for the LES, and second it gives an estimation of the time step for unsteady LES. The time step is based on the smallest Kolmogorov time scale (equation 3-12) of our domain. For the present geometry, it has been found in the transition between the square injection and the circular tube, where the energy dissipation is the highest. We decided to base the time step on the Kolmogorov time scale because the Kolmogorov micro-scales represent the smallest scales of the turbulence.

\[ \tau = \sqrt{\frac{v}{\varepsilon}} \]

**Equation 3-12: Definition of the Kolmogorov time scale**

Once RANS model converges, LES model is activated. A second-order central-differencing scheme and a second-order implicit formulation are employed respectively for spatial and temporal discretizations. Since we are solving the incompressible Navier-Stokes equations at low velocity, the pressure-based solver has been selected instead of the density-based solver recommended for high speed compressible flows. The convergence criterion is reached when the residuals are below $10^{-5}$. To model fluctuating velocities at the inlet boundaries, the spectral synthesizer algorithm has been chosen. In this method, ‘fluctuating velocity components are computing by using a divergence-free velocity vector-field from the summation of Fourier vector harmonics’ (Fluent manual section 12-57). The number of Fourier harmonics is fixed at 100 by Fluent. Since Large Eddy simulation involves running a transient solution from some initial condition, the solution ran long enough to wash out the transients and to reach a statistically stable flow field for data acquisitions.
The simulations have been performed on LSU’s high performance cluster Pelican. This system is composed of 34 nodes with multi-processors. Thirty nodes are dedicated to parallel computing. Their distribution is as follow: 16 nodes have 16 processors and 14 nodes have 8 processors. Several tests have been performed to determine the number of processors which will provide results the fastest. The tests have been done on a steady-state case at BR=0.400 with the resolution of the energy equation. Figure 3-12 shows the time it takes to simulate 100 time steps (dt=0.6ms) in function of the number of processors used. It has to be mentioned all the tests have started at the same time instant for consistency in the comparison.

![Time vs number of processors](image.png)

**Figure 3-12: Time to perform 100 iterations in function of the number of processors**

As the number of processors increases, faster the simulations are. A configuration of 32 processors provides the fastest result, 70min. For our purpose, we have decided to perform simulations with 3 nodes and 8 processors per node (T=140min), which is not the fastest configuration. This choice has been influenced by two factors. Firstly, several simulations are run at the same time, so the use of 32 processors for each simulation would limit the number of simulation we can launch simultaneously. Secondly, the computational resources of Pelican are used by many other departments other than mechanical engineering; therefore all nodes are not available continuously.
3.7. Data Processing

The numerical data have been processed with the commercial software Tecplot 2006 or Tecplot 2009. The large-scale structures resulting from the interaction of a jet with a crossflow can be visualized with three different methods. In fact, the core of the large-scale structures is associated with strong vorticity, and therefore local pressure minima. Jeong and Hussain (Jeong 1994) have established three definitions which capture the pressure minima; they are based on the Laplacian pressure, Q-criterion or $\lambda_2$-criterion.

3.7.1. Laplacian of the Pressure

Since the vortex cores are associated to pressure minima, positive iso-surface of the Laplacian of pressure can be used to identify coherent structures.

3.7.2. Q-Criterion

For incompressible flows, the Laplacian pressure is directly related to the second invariant of the velocity gradient tensor $Q$, which is defined in equation 3-13. It depends on the strain rate tensor $S$ and rotation rate tensor $\Omega$. Appendix A presents the definition of the three invariants of the velocity gradient tensor.

$$Q = -\frac{1}{2} u_{ij} u_{j,i} = -\frac{1}{2} (S_{ij} S_{ji} + \Omega_{ij} \Omega_{ji})$$

Equation 3-13: Definition of the second invariant of the velocity gradient tensor $Q$

For incompressible flows, the relation between the Laplacian pressure and $Q$ is linear (equation 3-14). Positive iso-surface of $Q$ can be used to identify the vortical structures.

$$\Delta P = 2 \rho Q$$

Equation 3-14: relation between the Laplacian pressure and $Q$
3.7.3. Second Eigenvalue of the Tensor $S^2+\Omega^2$, $\lambda_2$

3.7.3.1. General Observations

The visualization of the coherent structures with iso-surface of Laplacian pressure or $Q$ is based on the identification of these structures by pressure minima. Jeong and Hussain have observed a pressure minimum may not always coincide with a vortex core and reciprocally. Considering a planar irrotational flow such as a sink or a source flow, there is a minimum pressure at the origin, but no swirl is involved in the flow. Then, an unsteady straining can create a pressure minimum without involving a vortical motion. Reciprocally, a pressure minimum can be eliminated in a flow with vortical motion by viscous effects. Considering Karman’s viscous pump, the centrifugal force is balanced by the viscous force. The swirling motion is induced by the rotating disk, whereas the pressure is constant in plane perpendicular to the vortex axis. There is no pressure minimum.

3.7.3.2. Identification of the Structures by $\lambda_2$

Jeong and Hussain established a new indicator for the detection of vortex core: the second eigenvalue of the tensor $S^2+\Omega^2$. This new criterion is not based on the identification of a pressure minimum; however they considered this observation as the starting point of their theory. Information about pressure extremum is included in the Hessian of the pressure ($P_{ij}$) which is obtained by taking the gradient of the Navier-Stokes equation (equation 3-15).

The Hessian of the pressure is symmetric. By using the decomposition of the velocity gradient tensor in function of the strain rate tensor and rotation rate tensor (equation 3-16), the acceleration gradient can be expressed as a symmetric and asymmetric part (equation 3-17).

The symmetric part of the gradient of the Navier-Stokes equation is expressed in equation 3-18.

$$ a_{i,j} = \frac{Du_{i,j}}{Dt} = -\frac{1}{\rho} P_{ij} + \nu u_{ij, kk} $$

Equation 3-15: Gradient of the Navier-Stokes equation
\[ u_{i,j} = S_{i,j} + \Omega_{i,j} \]

**Equation 3-16:** Decomposition of the velocity gradient tensor in function of the strain and rotation rate tensors

\[ a_{i,j} = \left( \frac{DS_{ij}}{Dt} + \Omega_{ik} \Omega_{kj} + S_{ik} S_{kj} \right) + \left( \frac{D\Omega_{ij}}{Dt} + \Omega_{ik} S_{kj} + S_{ik} \Omega_{kj} \right) \]

**Equation 3-17:** Decomposition of the acceleration gradient as a symmetric and asymmetric part

\[ \frac{DS_{ij}}{Dt} - \nu S_{ij,kk} + \Omega_{ik} \Omega_{kj} + S_{ik} S_{kj} = -\frac{1}{\rho} P_{ij} \]

**Equation 3-18:** Symmetric part of the gradient of the Navier-Stokes equation

To obtain a better indicator for the detection of vortex core, Jeong and Hussain discarded the contribution of unsteady irrotational straining (pressure minimum without vortex core) and viscous terms (vortical motion without pressure minimum) which are respectively present in the first and second terms of equation ?-6. Discarding these two effects, only the contribution of \( S^2 + \Omega^2 \) to the Hessian pressure is taken into account (equation 3--19).

\[ P_{ij} = -\rho(\Omega_{ik} \Omega_{kj} + S_{ik} S_{kj}) = -\rho(\Omega^2 + S^2) \]

**Equation 3-19:** Expression of Hessian pressure

The presence of a local pressure minimum in a plane requires \( P_{ij} \) to have two positive eigenvalues; therefore two negative eigenvalues for the tensor \( S^2 + \Omega^2 \). Since the tensor \( S^2 + \Omega^2 \) is symmetric, it has three real eigenvalues \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \). Jeong and Hussain define a vortex core as a region with \( \lambda_2 \leq 0 \).

**3.7.3.3. Relation Between Q and the Eigenvalues of \( S^2 + \Omega^2 \)**

The eigenvalues of the tensor \( S^2 + \Omega^2 \) can easily be related to the second invariant tensor \( Q \).

\[ Q = -\frac{1}{2} (S_{ij} S_{ji} + \Omega_{ij} \Omega_{ji}) = \frac{1}{2} (\|\Omega\|^2 - \|S\|^2) \]

**Equation 3-20:** Expression of the second invariant tensor \( Q \)
\[ \|\Omega\|^2 = tr(\Omega\Omega^T) \]
\[ \|S\|^2 = tr(SS^T) \]
\[ Q = -\frac{1}{2} tr(\Omega^2 + S^2) = -\frac{1}{2}(\lambda_1 + \lambda_2 + \lambda_3) \]

**Equation 3-21: Relation between Q and the eigenvalues of S^2+Ω^2**

### 3.7.4. Determination of the Visualization Method

From their work, Jeong and Hussain stated, in most cases, similar results are obtained for the identification of vortex cores by using the Q- or \( \lambda_2 \) criterion. Slight differences have been observed only for vortices with strong core dynamics; the vortex core boundary based on both definitions differs a little. It has to be noted Jeong and Hussain did not use iso-surface of Laplacian pressure to visualize the coherent structures. Therefore no comparison has been performed with it. However, since the Laplacian pressure is proportional to Q for incompressible flows (equation 3-14), it is legitimate to assume that the comparison between the Laplacian pressure and \( \lambda_2 \) leads to the same conclusion than the comparison between Q and \( \lambda_2 \). Overall, Jeong and Hussain concluded “\( \lambda_2 \)-definition represents the vortical structures more accurately than the Q-definition and the evolution of connected negative \( \lambda_2 \) domains appears more appropriate for studying evolutionary dynamics and related flows physics of vortices and coherent structures” (pp.91).

Figure 3-13 shows the vortical structures at the same time instant for BR=0.400 by using the iso-surface of the Laplacian pressure, Q and \( \lambda_2 \). Laplacian pressure is computed with the divergence of the pressure gradient, while Q is obtained by summing the eigenvalues of the tensor \( S^2 + \Omega^2 \). As it can be seen for the kind of flows we simulate, no difference is perceptible between these three methods. Taking into consideration Jeong and Hussain’s conclusions, we wanted to visualize the vortical structures by using the \( \lambda_2 \) criterion. However due to software limitations, it has been impossible to automate the calculation of \( \lambda_2 \). Therefore, the vortical structures will be visualized with iso-surface of the Laplacian pressure.
Figure 3-13: Iso surface of the Laplacian Pressure $\Delta P=2000\text{Pa.m}^{-2}$ (top), $Q=800\text{s}^{-2}$ (middle) and $\lambda_2=800\text{s}^{-2}$ (bottom)
Chapter 4. Experimental Apparatus and Procedures

As mentioned previously, the numerical simulations are realized in parallel to the experiments. The geometry used for the simulation is based on the experimental set-up. In this chapter, the experimental facilities, as well as the visualization and acquisitions methods are described.

It has to be noted all the pictures from the experiments have been taken by Guillaume Bidan and Lemuel Wells.

4.1. Facilities and Apparatus

4.1.1. Wind Tunnel

The experiments took place in the test section of the aerodynamic wind tunnel of Louisiana State University. The dimensions of this open loop wind tunnel are 10ft in length with an inlet cross-section of 3*2ft. To keep a constant pressure gradient inside the test section, and therefore a constant velocity, the side walls of the test section are angled with respect to the centerline. The test section is preceded by a contraction with an area ratio of 20. The contraction is composed of five conditioning screens in order to provide low turbulence inlet conditions (Figure 4-1). The flow in the wind tunnel is powered by a fan allowing a range of velocities going from 0 to 30m/s. To allow visualizations in planes parallel to the bottom wall and parallel to the jet symmetry plane, a set of transparent acrylic windows compose the roof of the test section and one side of the wall.

Figure 4-1: Schematic of the wind tunnel (Bidan 2008)
4.1.2. Jet

The jet is located 30in downstream of the test section and is mounted flush with the bottom of the wind tunnel. It is inclined at 35° with respect to the bottom of the wind tunnel and has a round section of 1in diameter. The jet is fed by dry air issued from a reservoir regulated at 20Psig. To perform pulsed experiments with variable blowing ratios, a set of three valves is present upstream of the jet (Figure 4-2).

This system is composed of a main and by-pass branch which comports a solenoid valve controlled by a computer. The main branch provides the desired low blowing ratio (solenoid closed), while the by-pass branch sets the high blowing ratio by adding air to the main branch (solenoid open). The flow from these two branches is monitored by the intermediate of metering valves. A flow-meter is placed downstream of the valves system. It allows controlling the experimental settings and obtaining time-resolved records of the blowing ratio during the experiments. Then, the flow-meter is connected to a stainless steel injection bloc with four injection ports, which is followed by a 6in square acrylic tube. Finally the square tube connects to a 5in stainless steel tube inserted in an acrylic block and attached to the bottom wall of the test section (Figure 4-3). For more information about the experimental set-up, the reader can refer to Lem (2010).

Figure 4-2: Valve setup controlling jet blowing ratio in steady state and pulsed conditions (Bidan 2008)
4.2. Visualizations

Images of the flow field are obtained by Laser sheet Mie scattering visualizations. The images are obtained from a digital camera which is synchronized with a laser. The camera and the laser are located perpendicularly to each other, allowing the camera to record images of the plane which is illuminated by the laser sheet. For our study, two planes have been considered: the center plane and a plane parallel to the wind tunnel wall and located 1/8in above the floor. To understand the mixing process between the jet and the crossflow, the jet is seeded with titanium dioxide (TiO$_2$). This component has been chosen as tracer due to the size of its particles; big enough to be illuminated by the laser, and detected by the camera sensor, and small enough to follow the path lines of the jet fluid. The titanium dioxide is generated from the reaction of water with titanium tetrachloride (TiCl$_4$). In addition water reacts with hydrogen chloride (HCl) to form hydrochloric acid.

\[
\text{TiCl}_4 + 2\text{H}_2\text{O} \rightarrow \text{TiO}_2 + 4\text{HCl}
\]

Equation 4-1: Reaction of TiCl$_4$ and H$_2$O

\[
\text{HCl} + \text{H}_2\text{O} \rightarrow \text{H}_3\text{O}^+ + \text{Cl}^-
\]

Equation 4-2: Reaction of HCl and H$_2$O

To prevent any adverse effect of the seeding system on the flow field, the reaction happens in the jet itself. The reaction takes place in the injection bloc where two injection ports are fed by water, and the two remaining ones by titanium tetrachloride. The two reagents are carried as vapor in pure nitrogen gas.
Then as the jet fluid discharges into the cross-flow, titanium dioxide is already formed. For the experiments, both fully reacted and reactive Mie scattering visualizations have been accomplished. Both reagents are used for reacted Mie scattering experiments, while the vapor is suppressed in reactive Mie scattering experiments so that the reaction takes place in the test section when the TiCl$_4$ enters in contact with ambient moisture.

4.3. Controls and Acquisitions

Hotwire anemometry and time resolved flow-rate records are the principal measurement methods used in experiments.

4.3.1. Flow-Meters

Two flow-meters are used in the experiments, one to control the main flow-rate inside the jet, and the second one to determine the seeding flow-rate. From the main flow-meter, flow-rate records are acquired on a sufficient long period of time to compute statistical independent averaged for steady state cases and phase averaged quantities for pulsed cases.

4.3.2. Hotwire Anemometry

Hotwire measurements allow obtaining accurate value of the velocity at any points of the flow field through a transverse system.

4.3.3. Images Acquisition

For steady-state cases, 3 series of 32 images are taken in order to assure statistical independence. For pulsed cases, the sampling differs in function of the forcing frequency. For a forcing frequency of 1Hz, 10 images are taken at 10 equally spaced phase locations within the cycle. For a forcing frequency of 10Hz, 10 images are taken at 50 equally spaced phase locations within the cycle. Images are then processed into Matlab$^{\text{TM}}$ to obtain either averaged or phase averaged images.

For more information about the measurement methods and the reduction of the experimental data, the reader is invited to refer to Bidan thesis (Bidan 2008).
Chapter 5. Results and Discussion

5.1. Steady-State Cases

Simulations with a constant BR have been performed to identify the vortical structures resulting from the interaction of a jet with a crossflow. Five cases with a BR of 0.150, 0.300, 0.400, 0.750, 1.00 and 1.2 have been investigated. The Reynolds number associated to each BR and based on the jet diameter and the velocity inside the cylindrical section of the jet is summarized in Table 5-1. The vortical structures identified in a vertical jet have been recognized in an inclined jet. It has to be noted this numerical work is done in parallel of experiments. Our initial goal was to verify the simulation provided a good representation of the dynamics of the flow field in order to better interpret the complexity of the jet/crossflow interaction under experimental conditions. However due to equipment failures, it has been impossible to validate quantitatively our simulated flow field. Only a qualitative comparison has been performed.

<table>
<thead>
<tr>
<th>BR</th>
<th>0.150</th>
<th>0.300</th>
<th>0.400</th>
<th>0.750</th>
<th>1</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re</td>
<td>417</td>
<td>834</td>
<td>1091</td>
<td>2087</td>
<td>2782</td>
<td>3338</td>
</tr>
</tbody>
</table>

Table 5-1: Reynolds number associated to each steady state case.

5.1.1. Velocity Profiles and vortical structures inside the jet tube

5.1.1.1. Velocity Profiles

Figure 5-1 (left) displays the mean velocity profile parallel to the flow direction along the S/Dj axis at a distance 4Dj upstream from the center of the jet exit. The S/Dj axis is perpendicular to the jet axis (figure 5-1 right). The mean variable has been computed from a set of 150 uncorrelated data files.

5.1.1.2. Vortical Structures Inside the Jet

The jet tube is the location where small vortical structures develop. The presence of small vortical structures inside the jet is directly related to the BR value. As the BR increases, the turbulence (or instability) inside the jet tube intensifies, which promotes the formation of vortical structures. At high BR, these structures are abundantly present and look like elongated vortices which are closely interlinked...
together (Figure 5-2). These elongated vortices extend chaotically in the flow direction and have a strong vortex core considering the threshold used for the Laplacian pressure. The transition from the square to the cylindrical section is favorable to the formation of these coherent structures. The transition is a backward step which creates a flow separation. Therefore, as the jet boundary layer from the square section discharges into the cylindrical section, the shearing effect at the interface between the jet boundary layer and the low velocity region increases.

Figure 5-1: Left: Mean velocity profile inside the jet tube for BR=1.2 (purple), BR=1 (yellow), BR=0.750 (cyan), BR=0.400 (blue), BR=0.300 (green) and BR=0.150 (red). Right: schema describing the localization where the velocity profiles have been taken.

Figure 5-2: Vortical structures inside the jet tube: iso-surface of ΔP=30 000Pa.m² at BR=1.

5.1.2. Attached Jet

Steady-state cases for the 35 degrees inclined jet are characterized by two regimes, attached jets at low BR and detached jets at high BR, with the presence of a transitional zone between these two regimes. Experimentally, the attached jet regime has been identified for BR lower than 0.400. Numerically, it has been impossible to simulate a refined range of BR. We have observed the jet is attached until BR=0.400.
Low BR produces attached jet downstream of the jet exit. The dominant vortical structures related to the jet/crossflow interaction are hairpin vortices resulting from the roll-up of the downstream jet shear layer. The walls of the jet are high vorticity regions. Negative spanwise vorticity is associated to the jet wall facing the crossflow and positive spanwise vorticity to the opposite wall (Figure 5-3). As the jet fluid exits the jet tube, it is deflected by the crossflow which has a higher momentum. The negative spanwise vorticity from the upstream jet shear layer interacts with the positive spanwise vorticity from the crossflow boundary layer. At these given BR, the negative spanwise vorticity of the upstream jet shear layer is either overcame or weakened by the opposite vorticity of the crossflow boundary layer as illustrated in Figure 5-3 which depicts the contour of the mean vorticity field at the leading-edge of the jet for BR=0.150, 0.300 and 0.400. The mean velocity vector field highlights that the shearing effect between the upper jet shear layer and the crossflow decreases as the jet fluid is convected downstream. This observation, as well as the absence of fluctuation in the spanwise vorticity along the upper shear layer at BR=0.400 (figure 5-4) supports the fact that no vortical structure forms along the upper shear layer. The merging between the upper and lower shear layers entrains a strengthening of the hairpin vortex.

![Figure 5-3: Contour of the mean spanwise vorticity superimposed with the mean velocity field at the leading edge of the jet exit (center plane) at BR=0.150, 0.300 and 0.400](image)
Figure 5-4: Contour of the fluctuation of the spanwise vorticity at the center plane at BR=0.400

Downstream of the jet exit, there is the recirculation region (Figure 5-5) formed of jet fluid and crossflow fluid which have been deflected from the lateral edges of the jet exit. Then, on the leeward edge of the jet exit, the lower jet shear layer discharges in the recirculation region where the velocity is low. Figure 5-5 shows the direction and magnitude of the mean velocity field at the trailing edge of the jet at BR=0.400 superimposed with the contour of the mean velocity magnitude. Since the direction of the streamwise velocity component between the jet fluid and the recirculation region is opposed, a high shear exists between these two zones. The intense shearing effect creates high spanwise vorticity, and therefore the roll-up of the lower shear layer. As the lower roll-up is convected downstream, it forms a hairpin vortex. Once a hairpin vortex is shed, another one starts to form, resulting in a hairpin vortex street. From Figure 5-6 and 5-7, it can be seen the legs of the hairpin vortices are interlocked between each other. The high pressure gradient upstream of the jet exit causes an important part of the jet fluid to leave from the lateral edges of the jet exit where the horizontal legs of the hairpin vortices start to form. The presence of hairpin vortices has also been identified in previous numerical works. Tyagi et al (2003) have performed LES on a 35° inclined jet at BR=0.5 and 1 corresponding to Reynolds number of 11000 and 22000. As in our work, they have stated the hairpin vortices result from the roll-up of the lower jet shear layer.

Figure 5-5: mean velocity vectors superimposed with the mean velocity field at BR=0.400 on the trailing edge of the jet exit
The formation of the hairpin vortices is quasi-periodic and BR-dependent. The increase in BR leads to an increase of the shedding frequency. Figure 5-6 represents the dominant vortical structures at three different low BR. Five, six and seven well-defined hairpin vortices are respectively visible at BR=0.150, 0.300 and 0.400. Figure 5-6 highlights a higher BR leads to the formation of a well defined hairpin vortex earlier in the flow field. As BR increases, the shearing effect along the lower shear layer intensifies; then the lower vortex forms earlier in the flow field. The first hairpin vortex is observed at Xj=5.5 at BR=0.150, Xj=2.8 at BR=0.300 and Xj=2.5 at BR=0.400. At such low BR, the hairpin vortex is convected far downstream until Xj=11, it does not break up into smaller vortices (figures 5-6 and 5-7 at BR=0.400). As it can be seen from the top visualization, the hairpin vortices lose their symmetry compared with the center plane as BR increases. As the hairpin vortices move in the streamwise direction, they get stronger and are lifted upwards due to their mutual induction (Figure 5-7). The vortical structures stretch horizontally since the head is entrained by the crossflow, therefore at a higher velocity than its legs. Their horizontal legs move closer to each other and upwards due to the mutual induction and interaction with their image vortices. The head of the hairpin vortex tilts as it propagates downstream and becomes normal to the crossflow direction due to its interaction with its image vortex. Once a hairpin vortex is shed; it can be decomposed in three parts: its head, its vertical and horizontal legs (Figure 5-8).

![Figure 5-6 cont.](image)
Figure 5-6: Top View: Iso-surface of Laplacian pressure of the instantaneous flow field at BR=0.150 ($\Delta P=520\text{Pa.m}^2$), BR=0.300 ($\Delta P=1500\text{Pa.m}^2$), and BR=0.400 ($\Delta P=2500\text{Pa.m}^2$)

Figure 5-7: Side View: Iso-surface of Laplacian pressure of the instantaneous flow field: at BR=0.150 ($\Delta P=520\text{Pa.m}^3$), BR=0.300 ($\Delta P=1500\text{Pa.m}^3$), and BR=0.400 ($\Delta P=2500\text{Pa.m}^3$)
In the attached jet regime, the horizontal legs of the hairpin vortices have opposite streamwise vorticity. They form a pair of counter rotating vortices. However this CRVP is different from what it is called CRVP in the literature; they result from a different mechanism which will be discussed in section 5-1-3. Figure 5-8 (left) presents a single hairpin vortex at BR=0.400. The hairpin vortex is visualized with iso-surface of Laplacian of the pressure, superimposed with the instantaneous velocity field on the left and the contour of the streamwise vorticity at the plane Xj=3.6 on the right. The contour of the streamwise vorticity at Xj=3.6 clearly shows that the horizontal legs of the hairpin vortex have opposite streamwise vorticity and form a pair of CRV whose their cores are centered along the horizontal legs.

![Figure 5-8: Detailed of the flow field in the vicinity of the horizontal legs of a hairpin vortex (BR=0.400). Iso-surface of ΔP=1500Pa.m².](image)

The CRVP structure is better seen on the average flow field. Figure 5-8 (right) represents the mean streamwise vorticity on various streamwise planes at BR=0.400. The contour of the mean streamwise vorticity shows a pair of symmetric CRV with opposite streamwise vorticity; positive for the right vortex and negative for the left vortex. As they are convected in the streamwise direction, the CRVP moves away from the wall and closer to each other’s due to their mutual induction and interaction with their image vortices. The vortical structure loses its strength as it moves downstream. The contour of the mean streamwise vorticity at the plane Xj=8 reveals the CRVP is not the dominant structure of the far field. In the attached jet regime, the jet does not penetrate deeply into the crossflow and the jet core remains close to the wall. As being convected downstream, the CRVP lose their strength and dissipate in the wall.
boundary layer. This is in contrast with the detached jet where the CRVP is the dominant vortical structures in the far-field on average.

![Streamwise planes colored by the mean streamwise vorticity at BR=0.400]

**Figure 5-9: Streamwise planes colored by the mean streamwise vorticity at BR=0.400**

Experimentally, the attached jet regime extends until BR=0.400 and is characterized by the formation of hairpin vortices. These vortical structures may originate from two distinct mechanisms. At very low BR (BR≤0.200) instability occurs in the upper shear layer from the jet/crossflow interaction, while the lower shear layer is stable. As the instability grows, it may roll-up to lead to the formation of a hairpin vortex once the upper and lower shear layers have merged. A visualization in Figure 5-10 of the flow field along the center plane at BR=0.200 illustrates the instability growing in the upper shear layer while the lower shear layer remains stable. The purple arrow points out the hairpin vortex shed after the merging of both shear layers. As the BR is increasing, the lower shear layer becomes unstable (figure 5-11) until it rolls-up and a hairpin vortex is shed. This configuration is well observed at BR=0.400 on figure 5-12. The green arrow points out the lower vortex and the instability growing in the lower shear layer. At such BR, the instability in the upper shear layer intensifies which may lead to the formation of a hairpin vortex (red arrow) before the merging between the lower and the upper shear layers. However, once both shear layers have merged, a larger hairpin vortex is formed.

As BR increases, the hairpin vortices form earlier in the flow field and their shedding frequency increases too as it has been observed in the simulation. The field of view covered by the experimental images taken in the X-Z plane is 1.5Dj and 5.75Dj respectively upstream and downstream the center of the jet exit. With such a field of view, no conclusion can be drawn concerning the behavior of the structures as they propagate downstream.
Figure 5-10: Fully reactive Mie scattering visualization at the center plane at BR=0.200 showing the formation of a hairpin vortex resulting from an instability in the upper shear layer

Figure 5-11: Fully reactive Mie scattering visualization at the plane Yj=0 at BR=0.300 showing the instability in the lower shear layer and the formation of hairpin vortex

Figure 5-12: Fully reactive Mie scattering visualization at the plane Yj=0 at BR=0.400 showing the hairpin vortices formed in the upper (red arrow), lower (green arrows) shear layers, after the merging of the shear layers (purple arrows)

The formation mechanism which leads to the hairpin vortices differs between the experiments and the simulations. This is explained by the fact that experiments provide a higher perturbed environment for the flow field to develop. Besides the absence of instability along the upper shear layer in the simulation may originate from the algorithm used to model the turbulent viscosity in Fluent.

On the top views, shadows representing the head of the hairpin vortices are observable. These shadows are located in front of the legs of the hairpin vortices since they are convected at a higher speed. The top view visualizations have been taken at a plane located 0.125Dj above the wind tunnel wall. This given plane cuts horizontally the legs of the hairpin vortices. The sense of rotation of the legs of the
hairpin (purple arrow) is visible on figure 5-13 and is consistent with the sense of rotation of the CRVP. The CRVP are better seen on figure 5-14 at BR=0.150. It represents Y-Z planes inclined at 30° with respect to the streamwise direction. The first plane is taken at the center of the jet exit and the last one at Xj=4. There is a step of 1Dj in the streamwise direction between each plane. Figure 5-14 highlights the slight upwards motion of the CRVP and their growth as they are convected downstream.

![Figure 5-13: Top View Visualization at BR=0.150 (left), BR=0.200 (middle) and BR=0.300 (right). The purple arrows point out the horizontal legs of the hairpin vortices.](image-url)
Along the jet body extend vortical structures (figure 5-13) which are intermittently trapped into the jet core. These vortical structures have also been noticed in the simulation and are referred as horse-shoe vortices in the literature. The horse-shoe vortex comes from the roll-up of the crossflow boundary layer as it is blocked by the jet fluid. Usually this vortical structure forms at the leading edge of the jet exit. At such low BR, the formation of the horse shoe vortex is pushed downstream, above the jet exit since the jet is highly bent by the crossflow. At BR=0.300, the horse-shoe vortex is at Xj=-0.47 (figure 5-15). As BR increases, the horse shoe vortex forms more upstream due to the higher blockage sustained by the crossflow and due to the high pressure zone which has moved upstream. The legs of the horse-shoe vortex extend downstream around the jet exit and are irregularly caught up by the jet body downstream (Figure 5-13). The streamwise vorticity carried by the structure opposes to the vorticity of the CRVP. As the reader looks at the flow field in the downstream direction, the left leg has a positive streamwise vorticity, while the right leg possesses a negative streamwise vorticity. The horse-shoe vortex contains both crossflow and jet fluid, since the legs transport seeded particles as seen in the top visualizations. In the attached jet regime, the horse-shoe vortices are stable.
5.1.3. Detached Jet

Numerically, the detached jet regime has been observed for a BR of 1 and 1.2, while the limit has been found at BR=0.800 experimentally. The main difference between the attached and detached jet regimes comes from the formation of both lower and upper vortices in the detached jet.

Horse-shoes vortices have been identified in the detached jet regime and result from the same mechanism as in the attached jet. Compared to the regime at low BR, the formation of this structure is promoting by two factors: a higher adverse pressure gradient upstream the jet exit, and an increase in the vertical momentum of the jet fluid which intensifies the blockage of the crossflow. The horse-shoe vortices form upstream the jet exit (Xj=-0.94) and are stable; they don’t oscillate. Figure 5-16 represents streamtraces at the upstream of the jet exit. The red line refers to the jet exit. On figure 5-16, the horse-shoe vortices contain crossflow fluid and jet fluid from the upstream of the jet. The legs of the horse-shoe vortices extend around the jet and are convected downstream. Cases at BR=1 and BR=1.2 illustrate two different behaviors of the horse-shoe vortices. At BR=1, only one horse-shoe vortex forms upstream of the jet exit. As the legs of the horse-shoe vortex extend around the jet and propagate in the streamwise direction, they are caught up by the jet body. Figure 5-17 illustrates the interaction between the jet body and the legs of the horse-shoe vortex with the typical ‘8’ shape which can be seen from the top view. Once the legs are trapped, they stay connected with the jet body as they are convected downstream. At
this given BR, no evidence has been found they are lifted upwards; they remain close to the wall at a distance \( Z_j = 0.1 \) above.

**Figure 5-16**: Streamtraces at the upstream side of the jet exit at \( BR = 1 \) which show horse-shoe vortex contains both jet and crossflow fluid

**Figure 5-17**: iso-surface of the Laplacian pressure \((\Delta P = 2000 \text{Pa.m}^2)\) colored by the streamwise vorticity and superimposed with streamtraces at \( BR = 1 \)

At \( BR = 1.2 \), the interaction between the jet/crossflow intensifies, two horse-shoe vortices are shed (figure 5-18 left). The main horse shoe vortex forms just upstream of the jet exit (\( X_j = -0.94 \)), while the second one is shed upstream of the first one (\( X_j = -1.1 \)). The main horse shoe vortex is bigger vertically in size and has stronger spanwise vorticity \((\Delta Z_j = 0.041, W_y = -221 \text{s}^{-1})\) than the second one \((\Delta Z_j = 0.01, W_y = -180 \text{s}^{-1})\). Both horse-shoes vortices are stable; they don’t oscillate. The shedding of the main horse-shoe vortex is regular while the formation of the second structure is random. As the horse-shoes vortices are convected downstream, their legs extend around the jet exit and interact with the jet body. The instantaneous streamlines plotted in figure 5-18 (right) show one leg of the horse-shoe which extends downstream and remains close to the wall while the second one is entrained into the recirculation region, lifted upwards and mixes with the jet body (figure 5-19). The upwards entrainment of the legs of the
horse-shoe vortex at BR=1.2 is possible because the jet core carries higher vertical velocity than at BR=1. Concerning the second horse-shoe vortex, as it extends downstream, it is dissipated since it carries lower vorticity.

Figure 5-18: Instantaneous streamtraces at BR=1.2: upstream the jet exit (left), in the far field (right)

Figure 5-19: Iso-surface of Laplacian pressure ($\Delta P=4000\text{Pa.m}^2$) at BR=1.2 overlaid with streamtraces which shows the entrainment of the legs of the horse-shoe vortex upwards

Experimentally, in the detached jet regime it has been observed the horse-shoe vortex forms upstream the jet exit. Its legs extend around the jet exit and are lifted upwards by the entrainment of the jet body motion. Figure 5-20 depicts a top visualization of the flow field at BR=0.800. The green arrows point to the legs of the horse-shoe vortex which extend downstream. The upwards entrainment of the structure is illustrated by figure 5-24 at BR=1.1. Behind the trailing-edge of the jet, vertical structures containing seeded particles appeared around $X_j=3$. These vertical structures represent the upwards lifting of the legs of the horse-shoe by the jet core in one hand, and the wake vortices in the other hand. This second point will be discussed later. At BR=1, the simulations have revealed the legs of the horse-shoe vortex remain close to the wall. This change of behavior between the simulations and the experiments explains by the difference of environment surrounding the jet body during the experiments and the
simulations. The motion of the legs are influenced by both the jet body and the perturbations present in the surrounding. The experiments provide a high disrupted environment which intensifies the upwards motion of the legs, while the simulations present a quiet environment for the development of the structure. This absence of high perturbation around the jet body justifies the legs remain close to the wall in the numerical simulations.

![Figure 5-20: Top visualization at BR=0.800 illustrating the position of the legs of the horse-shoe vortices in the far-field](image)

The detached jet is characterized by the formation of lower and upper vortices. These vortices have a random behavior as they are convected downstream; they may interact with each other once the upper and lower shear layer have merged to form a vortex ring, or they may evolve independently from each other as they move downstream. The upper and lower vortices have opposite streamwise vorticity. Figure 5-21 shows the shedding of a vortex ring from a lower and upper vortices once the upper and lower jet shear layers have merged. To make easier the localization of the vortices, the fluctuation of the spanwise vorticity along the center plane is also displayed at the same time instant. The black arrows in the bottom of figure 5-21 (A) point out two upper vortices. The shedding of upper vortices at high BR is observable due to two reasons. The BR increase leads firstly to an increase of the vorticity carried by the upper shear layer, and secondly the shearing effect along the jet/crossflow interface intensifies. These two mechanisms promote the roll-up of the upper shear layer, and consequently the formation of an upper
vortex. The presence of hairpin vortices resulting from the roll-up of the lower jet shear layer is also observed. However, these structures are located below the upper shear layer. Then, they are not clearly observable on the iso-surface of the Laplacian pressure, and are better well seen on the contour of the fluctuation of the spanwise vorticity. The lower vortices are seen on figure 5-21 (A) and are designated by the three black arrows on the top. As the upper vortices are convected downstream, they exhibit the shape of an inverse hairpin vortex (figure 5-21 B side view); the head is located upstream of its side arms which extend along the jet column. As the lower vortex is convected downstream, it lifts upwards and becomes spatially closer to the upper vortices. The shedding of the vortex ring is seen on figure 5-21 (C). The vortex ring forms once the upper and lower shear layers have merged. It results from the interaction of an upper and lower vortex which has been shed independently from each other in the near-field. As the vortex ring is convected downstream, this latter starts to break into smaller eddies (figure 5-21 D, E).

Figure 5-21cont.
Figure 5-21: Formation of a vortex ring issuing from the interaction of an upper and lower vortex at BR=1.2. Top: Iso-surface of $\Delta P=10000$Pa.m$^2$ colored by spanwise vorticity. Bottom: contour of the fluctuation of the spanwise vorticity at the center plane.

At BR=1, the shedding frequency of the upper vortices is lower than at BR=1.2, since the shearing effect along the jet/crossflow interface is less pronounced. The upper vortices are not as well formed as at higher BR, and the inverse hairpin shape is seen more occasionally. They dissipate earlier in the flow field than their counterparts at BR=1.2, therefore the formation of a vortex ring is more sporadic. However the shedding of a vortex ring still occurs once the upper and lower shear layers have merged.
When the upper and lower vortices evolve independently from each other as they are convected downstream, they break up into smaller eddies early in the flow field, respectively around Xj=1.9 for the upper vortices and Xj=3 for the lower vortices. Figure 5-22 displays a top and side views of the flow field at BR=1. The top view clearly reveals the early break-up of the vortices, while the side view illustrates the deep penetration of the vortical structures into the flow field due to high BR value.

![Image](image.png)

**Figure 5-22: Top and side views of iso-surface of ΔP (7500Pa.m²) at BR=1.**

Experimentally, it has been found the detached jet regime begins for a BR superior at 0.800. It is characterized by the formation of both upper and lower shear layer vortices. Figure 5-23, 5-24 and Figure 5-25 show the shedding frequency of the upper shear vortices is higher than the shedding frequency of the lower vortices; therefore all upper vortices cannot be part of a vortex ring. At BR=0.900 (Figure 5-23) the upper vortex starts to form at the leading-edge of the jet exit because the velocity associated to the jet is higher than the velocity of the crossflow, then the upper jet shear layer rolls-up in a counterclockwise direction. Further along the upper jet shear layer, the sign of the shear inverses because the crossflow velocity is higher than the jet velocity, then the upper jet shear layer rolls-up in a clockwise direction (pink arrow). At higher BR (Figure 5-25) the upper jet shear layer does not roll-up in the clockwise direction. Simultaneously at the trailing edge of the jet, the lower jet shear layer rolls-up and lower vortices are shed. Interaction between lower and upper vortices to form a vortex ring seems to occur. The
blue arrows emphasize the possible vortex rings shed before the merging between the lower and upper jet shear layers. Upper and lower vortices have a vorticity consistent to lead to the formation of a vortex ring, and both vortices have same size. The reader has to notice the experimental pictures are taken along the symmetric plane, therefore in two dimensions. Since the connection (lateral edges) between the upper and lower vortices is not seen, the existence of vortex ring is not certain. The orange arrows point out the independent lower or upper vortices. The far-field of the detached jet is dominated by vortical structures (green arrows) which look-like vortex rings. They may come from the interaction of independent upper and lower vortices once both shear layers have merged or from a vortex ring which has been formed before the merging between both shear layers.

Figure 5-23: Fully reactive Mie scattering visualization at the plane Yj=0 at BR=0.900

Figure 5-24: Fully reactive Mie scattering visualization at the plane Yj=0 at BR=1.1
As mentioned previously, experimentally the shedding frequency of the upper vortices is higher than the shedding frequency of the lower vortices (Figure 5-23 and 5-32). This explains by two main reasons. Firstly, the adverse pressure gradient at the upstream side of the jet exit intensifies the instability of the upstream shear layer and consequently the roll-up into vortices. In the opposite, the lower shear layer discharges into a low-pressure zone and faces a favorable pressure gradient which may reduce the instability of the shear layer and delay the formation of the lower vortices. In the simulation, none of this has been observed. In the contrary the lower and upper shear layer vortices seem to be shed at the same formation rate. However, since these vortices evolve spatially close to each other and are interlocked together, this conclusion cannot be confirmed with absolute certainty.

In the detached jet regime, the jet penetrates deeper into the crossflow. This is observable from the top view where the jet core is not visualized at $Z_j=0.125$, compared to the attached jet. The top visualization catches two longitudinal vortices (red arrows on figure 5-26) which form on the lateral edges of the jet exit and patches containing seeded particles which appear downstream. The origin of the patches will be discussed later.
The longitudinal vortices which develop along the lateral edges of the jet exit result from the roll-up of the jet shear layer along its lateral edges and the shearing of the crossflow boundary layer by the incoming jet. They form on average the CRVP in the far-field. The shearing of the crossflow by the jet is illustrated on Figure 5-27. The left column illustrates a general view of the flow field in the vicinity of the jet exit, while a zoom has been performed on the right column to better visualize the interaction between the jet and the crossflow. For consistency, the length of the velocity vectors has not been changed from one plane to another. At the plane Xj=-0.8, the crossflow boundary layer begins to react to the upcoming jet disruption by moving radially outwards the leading-edge of the jet. The strong vertical velocity component of the jet begins to shear the crossflow boundary layer fluid outside of the jet in the plane Xj=-0.6, creating streamwise vorticity consistent with the vorticity of the CRVP. By moving downstream, the shearing of the crossflow boundary layer is more pronounced because the jet fluid has a higher vertical velocity. At plane Xj=0.2, the roll-up of small vortices on both sides of the hole generates CRVP. The strength and the size of the vortices increase until the trailing-edge of the jet due an intensification of the shearing effect. The visualization of the streamwise vorticity in planes located inside the jet tube highlights the presence of the horse-shoe vortices whose their vorticity opposes to the CRVP vorticity. Beyond and below the horse-shoe vortices, a zone of similar vorticity as the CRVP develops. This streamwise vorticity arises from the shearing between the crossflow which is parallel to the wall and the crossflow which deflects around the jet. As the crossflow skirts around the jet, its spanwise velocity component varies in the vertical direction, which creates streamwise vorticity. Figure 5-28 depicts the
mean velocity field in this zone at the plane $X_j=0$. This zone of streamwise vorticity dies once the jet exit is passed because the crossflow velocity is once again parallel to the wall.

Figure 5-27 cont.
Figure 5-27: mean velocity field superimposed with the mean streamwise vorticity at Xj=-0.8, -0.6, -0.2, 0.2 and 0.6 at BR=1. The legend is indicated at the plane Xj=0.6.

Figure 5-28: Mean velocity field superimposed with mean streamwise vorticity at Xj=0, BR=1

As the vortical structures are convected downstream, the cores of the vortices are lifted upwards and move closer to each other due to the mutual induction and interaction with their image vortices as explained in the attached jet regime. Their propagation in the crossflow direction entrains a dissipation of the streamwise vorticity, causing the CRVP to grow in size and lose its strength, as it is illustrated on
figures 5-29 and 5-30. As well, for consistency the length of the velocity vector is unchanged from one plane to another.

Figure 5-29: mean velocity field superimposed with contour of the mean streamwise vorticity at various streamwise planes at BR=1. The legend is indicated at the plane Xj=1.

In the detached jet regime, the CRVP are the dominant vortical structure in the flow-field. Figure 5-30 represents the mean flow field at BR=1. By averaging the data, the hairpin vortices are washed out, while the CRVP extend downstream. Their upwards motion is observable on the side visualization.
Besides, the CRVP contain both jet and crossflow fluids. It is the main structure responsible of the mixing of the jet with a crossflow, and consequently has a detrimental effect on film cooling performance. This point is more detailed in section 5.3.

![Figure 5-30: mean streamwise vorticity at constant streamwise planes at BR=1](image)

![Figure 5-31: Iso-surface of ΔP (2500Pa.m²) of the mean flow field colored by the mean streamwise vorticity at BR=1](image)

The top views have revealed the presence of patches in the far-field. These patches are referred as wake vortices in the literature. As the BR increases, the jet penetrates deeper into the crossflow which makes easier the visualization of the wake vortices as they are convected downstream. From the top visualization (figure 5-26) the wake vortices seem to originate from vortical structures coming from the jet shear layer and which are convected downstream. This observation is in agreement with Tyagi and Acharya (Tyagi 2003) who have stated that the wall normal vorticity is associated to the upright legs of the hairpin vortices. Another origin to the wake vortices may be found from the side visualizations (figures 5-24 and 5-32) which show the presence of vertical vortical structures between the wall and the
jet body. Several studies have explained the presence of these structures. First of all, as it has been mentioned previously, in the detached jet regime, the legs of the horse-shoe vortices are entrained into the jet and reoriented in the vertical direction. This observation has been confirmed in our numerical work, but also by Yuan et al. (Yuan 1999) who have visualize the entrainment of the legs of the horse-shoe vortices in their simulation of a vertical jet. Therefore the wake vortices arise in one part from an interaction between the horse-shoe vortices and the jet core. In the opposite, Fric and Roshko stated the upright vortices originate from the crossflow boundary layer vorticity. If this hypothesis is feasible, it has to be superimposed by another mechanism which carries jet fluid into the wake vortices. In fact, the wake vortices are visible experimentally because they contained seeded particles, and consequently jet fluid.

![Image](image.png)

**Figure 5-32: Fully reactive Mie Scattering visualization at the plane Yj=0 at BR=1.6 showing the vertical vortical structures in the far field**

### 5.1.4. Transitional Zone

As mentioned previously, the limits of the transitional regime cannot be defined accurately in the simulation. It has been observed a steady-state case with a BR of 0.750 belongs to the transitional zone.

In our numerical study, the transitional regime is characterized by the shedding of upper and lower vortices. As mentioned in the detached jet regime, increasing the BR leads to a higher vorticity carried by the jet boundary layer along its wall and a superior shearing effect between the jet and the crossflow at the leading-edge of the jet exit. At BR=0.750, the formation of upper vortices along the upper shear layer has
been observed. However, this upper vortex does not associate with a lower vortex to create a vortex ring because the upper vortex is quickly dissipated into the flow field by the crossflow. Figure 5-33 presents the evolution of the upper vortex (signaled by black arrows) from its formation until its dissipation.

Figure 5-33: Time evolution of an upper vortex at BR=0.750. Top: iso-surface of $\Delta P$ (4000Pa.m$^2$) colored with spanwise vorticity. Bottom: Contour of spanwise vorticity at the center plane of the jet.

At $t=0$ms, the upper shear layer has rolled-up to form an upper vortex at $X_j=-0.2$. As it is convected downstream, the upper vortex does not link with a lower vortex to generate a vortex ring. This
observation is reinforced by looking at the contour of the spanwise vorticity at the center plane of the jet. In fact, the positive spanwise vorticity associated to the lower vortex is stronger than the negative spanwise vorticity of the upper vortex. Besides, the size of the lower vortex is bigger than the size of the upper vortex. At t=1.2ms, the upper vortex starts to lose its strength until being completely dissipated at t=1.8ms. Compared to the attached jet regime, the formation of upper vortices occurs for two reasons. The BR increase leads firstly to an intensification of the shearing effect between the crossflow and the jet at the leading-edge, and secondly pushes upstream the high pressure zone. Figure 5-34 represents the repartition of the pressure on the mean flow field at BR=0.400 and BR=0.750. The core of the high pressure zone is respectively located at Xj=-0.36, Zj=0.16 at BR=0.400 and Xj=-0.5, Zj=0.22 at BR=0.750. The upstream location of the high pressure zone at BR=0.750 promotes the transport of the upper vortices downstream.

Hairpin vortices compose the flow field in the transitional zone. They result from the roll-up of the lower shear layer as described in the attached jet regime. Figure 5-35 depicts a top and side views of the instantaneous flow field. The upwards motion of the hairpin vortices is well seen on the side view, while the top view shows hairpin vortices remain coherent until Xj=11. Compared to the attached jet regime, the motion of the hairpin vortices is more disorganized.
Experimentally, the transitional zone starts at BR=0.500 and contains features of both attached and detached jet regimes. Figure 5-36 represents a side view of the flow field at BR=0.700. On the trailing-edge of the jet, the roll-up of the lower shear layer leads to the formation of hairpin vortices. Green arrows point towards the three hairpin vortices which have been shed. The upper shear layer presents the behavior of both the attached and detached jet regimes. Red arrows on Figure 5-35 point out the instability which rolls-up along the upper shear layer. In this configuration, the hairpin vortex is formed before the merging between the lower and upper shear layers. The counterclockwise roll-up of the upper shear layer, characteristic of the detached jet, is noticeable and signaled by a blue arrow. However, as it has been observed in the simulation, the upper vortex is quickly dissipated and does not interact with a lower vortex. In the transition zone, the far-field is dominated by the hairpin vortices (purple arrows).
Figure 5-36: Fully reactive Mie Scattering along the center plane at BR=0.700. The red, blue, green and purple arrows point respectively out the instability in the upper shear layer, upper vortex, lower vortex and hairpin vortices shed downstream.

Figure 5-37: Left: reactive Mie Scattering at the plane Zj=0.125 at BR=0.700. Right: Reacted Mie Scattering of Y-Z planes inclined at 30° with respect to the spanwise direction at BR=0.600. The first plane is located at Xj=0. The last one at Xj=3. There is a step of 1Dj in the streamwise direction between each plane.
The top visualization on figure 5-37 (left) shows vortical structures which extend along the lateral edges of the jet exit. As in the detached jet, these structures initiate the formation of the CRVP. This is well-seen on the pictures of the cross-section (Figure 5-37 right). At Xj=1, the CRVP are already shed. As they are convected downstream, they lifted and become closer to each other due to their mutual induction and interaction with their image vortices. The same observation can be made from the simulation. Figure 5-38 presents the shearing of the crossflow by the jet as this latter is discharged into the flow field. As in the detached jet, the CRVP are the dominant vortical structure of the far-field (Figure 5-39).

![Figure 5-38 cont.](image-url)
5.2. Pulsed Cases

The second part of our study deals with pulsed cases using a predominantly square wave. In our numerical works, three pulsed cases have been studied. Table 5-2 summarizes the condition of each case in terms of blowing ratio, duty cycle and forcing frequency. The study of pulsed cases is extensive since each given configuration is determined by three parameters. We have restrained our work on the influence of the forcing frequency (cases 1 and 2), and BR$_h$ (cases 2 and 3) on the flow field. The three studied
pulsed cases have same low BR and duty cycle (DC). The velocity profiles used for our simulation come from the experiments where accurate measurements of time-resolved flow rate have been performed.

<table>
<thead>
<tr>
<th></th>
<th>BR_l</th>
<th>BR_h</th>
<th>BR_m</th>
<th>DC (%)</th>
<th>FF (Hz)</th>
<th>St</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0.150</td>
<td>0.750</td>
<td>0.400</td>
<td>50</td>
<td>1</td>
<td>0.016</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.150</td>
<td>0.750</td>
<td>0.400</td>
<td>50</td>
<td>10</td>
<td>0.16</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.150</td>
<td>0.450</td>
<td>0.300</td>
<td>50</td>
<td>10</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Table 5-2: Summarization of the conditions used in pulsed cases

5.2.1. Main Features of a Pulsed Case

A pulsed case can be decomposed in three sequences: the passage from low to high BR, from high to low BR and the plateaus at BR_l and BR_h. These three sequences are particularly well-seen at low frequency, and are well represented with case 1. The configuration of case 1 is \( BR_l = 0.150 \), \( BR_h = 0.750 \), \( DC = 50 \) and \( FF = 1 \)Hz. Figure 5-39 shows the associated time records. The reader has to notice, for clarity, only the part corresponding to the time of interest will be represented on the pictures illustrating this configuration.

![Figure 5-40: Time records associated to BR_l=0.150 BR_h=0.750 DC=50% and FF=1Hz](image)

The main difference with steady-state cases consists in the formation of a vortex ring as the jet switches from low to high BR. Figure 5-41 shows the formation and the time evolution of the vortical structures which are shed during this sequence. A low BR, the flow field is dominated by hairpin vortices with weak vorticity. This time instant is represented by figure 5-41 (A) where the threshold for the Laplacian of the pressure has been lower to 1000Pa.m^2 to visualize the vortical structures. The following
time-instant are represented with a threshold of 6000Pa.m² for the Laplacian of the pressure. By pulsing the jet, the upstream and downstream jet shear layers increase. The jet shear layer rolls up and forms a vortex ring (purple arrow) around the jet exit (Figure 5-40 B). Simultaneously, the pulse creates a higher adverse pressure gradient at the upstream side of the jet exit compared to the flow configuration at low BR; the upper shear layer is pushed upwards. Since the upper shear layer has a strong positive spanwise vorticity, it rolls up in a clockwise direction (black arrow) and forms a starting hairpin vortex. The BR increase leads to the formation of a horse shoe vortex (green arrow) upstream of the jet exit due to the blockage of the crossflow by the jet. The horse-shoe vortex detaches, and is entrained downstream by the upstream of the vortex ring.

![Diagram](image)

Figure 5-41 cont.
Figure 5-41: Evolution of the vortical structures during the pulse. Iso-surface of Laplacian pressure $\Delta P=6000\text{Pa.m}^2$, except for (A) where $\Delta P=1000\text{Pa.m}^2$. The black, purple and green arrows point respectively out the leading hairpin vortex, vortex ring and horse-shoe vortex.

Before reaching the stable $BR_h$ value, another significant pulse is perceptible on the time record. This oscillation comes from acoustic resonance in the jet supply system and is a characteristic present in the application environment (e.g. film cooling flows). The second pulse is also associated to the formation of a vortex ring and a horse-shoe vortex (Figure 5-41 D). As it is illustrated on figure 5-41 (E), the second horse-shoe vortex is transported downstream by the second vortex ring. However the vorticity associated to these structures is weaker than the one associated to their counterparts from the first pulse since the $BR$ peak to peak is lower. As the vortex ring is convected downstream, it is dissipated by the crossflow (Figure 5-41 F, G). Consequently, the second vortex ring evolves into a hairpin vortex. On figure 5-40 (F), the downstream of the second vortex ring is located just below the first hairpin vortex as represented in the locket. Simultaneously, the first vortex ring tilts downstream as it is convected. In fact, the upstream of the vortex ring is entrained by the crossflow, while its corresponding downstream roll-up is located in a low velocity zone; its motion is blocked by the starting hairpin vortex formed in front of it. The vortex ring and the starting hairpin vortex are connected together through their lateral arms as shown on figure 5-41 (E, F). Besides, the vortex ring deforms in the presence of the crossflow. The upstream of
the ring shrinks due to the strain applied by the crossflow which causes this latter to fold. Simultaneously, the downstream of the ring and the starting hairpin vortex interact and merge together to form a leading hairpin vortex, which is the dominant feature of the flow field. The leading hairpin vortex is different in size and in shape compared to the hairpin vortices which are formed at high BR (Figure 5-42). The legs of the leading hairpin vortex are stretched horizontally and vertically as the structure is convected downstream. This is the result of the mutual induction (Figure 5-41 H). At this time instant, the plateau at BR$_h$ is reached. With a duty cycle of 50% and a forcing frequency of 1Hz, the regime in the high part of the cycle has time to set up. The vortical structures observed at high BR are similar to the ones observed in steady-state case at equivalent BR. As seen earlier, it is characterized by the shedding of upper vortices which are quickly dissipated into the flow field and the shedding of hairpin vortices. A top and side views of the flow field at high BR is shown on figure 5-42. The black and blue arrows respectively refer to the leading hairpin vortex shed during the pulse, and the first hairpin vortex formed at BR$_h$.

![Figure 5-42: Hairpin vortices formed in the high part of the cycle. ΔP=6000Pa.m$^2$. The blue and black arrows respectively represent the first hairpin vortex shed at BR$_h$ and the big hairpin vortex resulting from the pulse.](image-url)
The side view clearly illustrates the upwards motion of the dominant hairpin vortex formed during the pulse. The upright legs of this structure, as well as those of the hairpin vortices can be associated to the wake vortices formed in the far-field in steady-state. Tyagi and Acharya (Tyagi 2003) stated the vorticity contained in the upright legs of the hairpin vortices are related to the vorticity of the wake vortices.

The second sequence of a pulse jet concerns the passage from high to low BR and is characterized by the ingestion of the crossflow into the jet feeding tube on the upstream side of the jet exit. The vertical and horizontal penetration of the crossflow depends on the sharpness of the decrease. In this configuration the drop occurs abruptly, therefore the crossflow penetration is deep. Figure 5-43 illustrates the crossflow penetration through the spanwise vorticity contour along the center plane. As well, the ingestion is accompanied by the transport downstream of the horse-shoe vortex. The iso-surface of the Laplacian of the pressure exhibits the vortical structure is located on the top of the jet exit. As the horse-shoe propagates downstream, it looks-like a hairpin vortex. The black arrow on figure 5-44 shows the time evolution of this horse-shoe vortex into a hairpin.

Figure 5-43: Ingestion of the crossflow into the jet feeding as the jet shut down. Top: iso-surface of Laplacian pressure ($\Delta P=6000\text{Pa.m}^2$) overlaid with spanwise velocity contour. Bottom: spanwise vorticity contour at the center plane.
The third and last sequence of a pulse case concerns the low part of the cycle. As mentioned previously for the high part of the cycle, in this given configuration, the regime at low BR has time to set up. The vortical structures observed at $BR_L$ are equivalent to the ones at same BR in steady-state; that is to say the shedding of hairpin vortices resulting from the roll-up of the lower shear layer. Figure 5-45 represents the flow field in the low part of the cycle.

A good qualitative agreement in term of vortical structures shed during the cycle is obtained between the simulations and the experiments. By pulsing the jet, the shedding of two vortex rings is observed in the experiment. A blue and red arrows point respectively out the first and second vortex rings on figure 5-46 (A). The convection of these two structures and their evolution into hairpin vortices is presented on figure 5-46 (B). The growth of the leading hairpin vortex is clearly visible on the experimental picture. It has to be noted the first and second hairpin vortices are located at the same streamwise position both in experiments and simulations. They are respectively 2.75Dj and 1.25Dj from the center of the jet exit. As
the plateau at high BR is reached, a hairpin vortex street is seen on the experimental picture (figure 5-46 C). The shadows in arch shape represent the head of hairpin vortices. Figure 5-46 (D) illustrates the ingestion of the crossflow into the jet feeding tube; no seeded particle is seen on the upstream side of the jet exit.

Figure 5-46: Comparison between experiments and simulations at BRl=0.150 BRh=0.750 DC=50% Ff=1Hz. Experiments: Fully reacted Mie Scattering visualization at plane Zj=0.125. Simulations: iso-surface of ΔP=5000Pa.m$^2$ colored by the temperature.
5.2.2. Influence of the Forcing Frequency on Pulsed Case

The influence of the forcing frequency on a pulsed case can be evaluated by comparing cases 1 and 2. Both cases have same low, high BR and DC respectively fixed at 0.150, 0.750 and 50%. Case 1 exhibits a forcing frequency 10 times lower than the forcing frequency of case 2 which is 10Hz. The time records associated to case 2 is presented on figure 5-47.

![Time Records associated to the pulsed case 2: BR_l=0.150 BR_h=0.750 DC=50% FF=10Hz](image)

**Figure 5-47: Time Records associated to the pulsed case 2: BR_l=0.150 BR_h=0.750 DC=50% FF=10Hz**

During the first sequence of a pulsed case, that is to say the passage from low to high BR, the vortical structures generated by the pulse are similar in both cases. Figure 5-48 illustrates the formation and time evolution of the vortical structures shed during the pulse. Figure 5-48 (A) depicts the flow field just before the pulse and emphasizes the presence of a horse-shoe vortex (green arrow), named as the first horse-shoe, riding on the top of the jet exit. This is a difference with case 1 where no horse shoe vortex on the top of the jet is seen before the pulse. This will be explained as the low BR will be reached. As the jet switches from high to low BR, the formation of a vortex ring (purple arrow) around the jet exit and a horse-shoe vortex upstream of the jet is observed on figure 5-48 (B). Simultaneously, the upper shear layer rolls-up in a clockwise direction to form a starting hairpin vortex (black arrow). As the first horse-shoe vortex is convected downstream, it merges with the starting hairpin vortex (figure 5-48 C.). The upwards and streamwise motion of the upstream of the first vortex ring causes the second horse shoe vortex to detach and propagates in the crossflow direction. As seen in case 1, the jet undergoes a second pulse due to acoustic oscillation present in the system. A second vortex ring and a third horse-shoe vortex form during the second pulse (figure 5-48 C). These two vortical structures evolve in the same manner as...
their counterparts at lower frequency; they are quickly dissipated since they carry lower vorticity. Therefore the second vortex ring transforms into a hairpin vortex. The locket on figure 5-48 (D) shows the downstream part of the second vortex ring; it is located just below the upstream of the first vortex ring. Once the third horse-shoe vortex is detached, a fourth one starts to form since the jet velocity is still high (figure 5-48 E).

Figure 5-48 cont.
Figure 5-48: Evolution of the vortical structures formed at $BR_v=0.150, BR_h=0.750, Dc=50\%$, $FF=10\text{Hz}$. Iso-surface of Laplacian pressure $\Delta P=7500\text{Pa.m}^2$ overlaid by contour of instantaneous spanwise vorticity.

At the same time, the first vortex ring tilts downstream (Figure 5-48 E) as it is convected. The strain applied by the crossflow to the upstream of the ring causes this latter to fold (Figure 5-48 D, E). As the starting hairpin vortex and the first vortex ring are convected downstream, these structures merge together to form a leading hairpin vortex. However, compared to low forcing frequency pulsed jet, the merging between these two vortical structures is initiated when the jet shuts down (figure 5-48 E), and is completed in the low part of the cycle (figure 5-48 F). Figures 5-48 E and 5-48 F highlight the high part of the cycle does not have time to set up due to the high forcing frequency. No hairpin vortex resulting from the traditional mechanism is formed at $BR_v$. Instead, the so-called high part of the cycle is dominated by the shedding of vortex ring and the merging between the first vortex ring and the starting hairpin vortex.

Figure 5-49: Ingestion of crossflow into the jet feeding tube as the jet shuts down. Right: Isosurface of $\Delta P=7500\text{Pa.m}^2$ superimposed with spanwise vorticity contour. Left: Spanwise vorticity contour along the center plane.
As the jet shuts down, the crossflow penetrates into the jet feeding tube. However the jet velocity drop occurs gradually compared to the first case. Then, the vertical and streamwise ingestion is less pronounced, as illustrated on figure 5-49. The passage from high to low BR is also accompanied by the fourth horse shoe vortex which passes above the jet exit. This horse-shoe vortex will evolve in the same manner as the first horse-shoe vortex in figure 5-48 (A). As in the high part of the cycle, the low part of the regime does not have time to establish. Only one hairpin vortex is formed on a regular way and is signaled by a blue arrow on figure 5-48 (F and G). Figure 5-48 (G) also reveals the evolution of the vortical structures in the flow field. The hairpin vortex resulting from the merging between the first vortex ring and the starting hairpin vortex is the dominant feature of the flow field. As it is convected downstream, its legs initially extend horizontally, which induce vertical vorticity and allow the legs to stretch vertically later in the flow field.

![Image](image_url)

**Figure 5-50:** Vortical structures in the far field. Black arrows point out to the hairpin vortex from the merging between the ring vortex 1 and the first hairpin and the purple arrow indicate the hairpin vortex from the second vortex ring. Iso-surface of Laplacian pressure $\Delta P=5500\text{Pa.m}^2$ superimposed with contour of streamwise vorticity

Figure 5-50 represents the dominant vortical structure in the far field. The side view reveals the flow field is dominated by leading hairpin vortices resulting from the merging between the first vortex ring and
the starting hairpin vortex. Black arrows signal these structures. The vertical stretching undergone by the leading hairpin vortex is visible by comparing figure 5-48 (G) and figure 5-50. The first leading hairpin vortex on figure 5-50 is the evolution, as it is convected downstream, of the one followed during the pulse. In one cycle, this structure extends vertically on a distance of one jet diameter. The flow field is also dominated by the hairpin vortex issuing from the second vortex ring. This structure is not subjected to a strong vertical stretching because it carries lower vorticity. As illustrated by the top and side views, it starts to break into smaller eddies in its motion. As observed in the first pulsed case, the legs of the leading hairpin vortex are almost vertical and can be assimilated to the wake vortices.

Figure 5-51cont.
Figure 5-51: Comparison between experiments and simulations at BR_c=0.150 BR_h=0.750 DC=50% F=10Hz. Experiments: Fully reacted Mie Scattering visualization at plane Z_j=0.125. Simulations: temperature contour at Z_j=0.125.

The comparison between the case 1 and 2 reveals the vortical structures associated to these two cases are equivalent during the pulse and as the jet shuts down. The first sequence is characterized by the
formation of a vortex ring and horse-shoe vortices at each pulse, and a starting hairpin vortex at the first pulse. In return, the second sequence distinguishes by the ingestion of crossflow into the jet feeding tube. The penetration is not as deep as in the first pulsed case because the velocity drop occurs gradually. Simultaneously, the transport of the horse-shoe on the top of the jet is observed. The main difference between these two cases consists in the absence of plateau at BR_h or BR_l in the second case. The high forcing frequency prevents the low and the high parts of the cycle to establish. Therefore, the flow field in case 2 is dominated by leading hairpin vortices which penetrate deeply into the flow field, while the flow field in case 1 is ruled by either strong hairpin vortices from the high part of the cycle or weak hairpins from the low part.

The comparison between experiments and simulations is presented on figure 5-51. Temperature contour at the plane Zj=0.125, plane at which top visualization have been taken, is displayed for the numerical results. The coolant fluid has a temperature of 300K, while the crossflow fluid is at 330K. The formation of the first vortex ring at the upstream side of the jet is signaled by a purple arrow on figure 5-51 (A). The convection of this structure as well as its upwards motion is seen on figure 5-51 (C). In fact, the upstream side of the vortex ring is represented by a shadow, which means this latter is not anymore in the plane Zj=0.125 but upwards. The formation of the second vortex ring is indicated by a cyan arrow on figure 5-51 (D). As they are convected downstream, the vortical structures growth in size (Figure 5-51 E). These same pictures highlight the direction of the streamwise vorticity of the legs of the previous leading hairpin vortex (red arrow) which is consistent with the CRVP. As the vortical structure is convected downstream, its legs get closer to each other by mutual induction and interaction. Besides, the ingestion of the crossflow fluid into the jet feeding tube is well seen on figure 5-51 (E). The difference around the jet exit between the experimental and numerical pictures on figure 5-51 (E) comes from the higher heat diffusivity than mass diffusivity associated to the seeded particles. A Prandtl number of 0.7 is used for the simulation, while the Schmidt number used for the experiments is between 100 and 1000. Overall, a good qualitative agreement is observed between the experimental and numerical results.
5.2.3. Influence of $BR_h$ on Pulsed Case

The influence of the high BR on pulsed cases can be evaluated by comparing pulsed cases 2 and 3. Both cases have same low, DC and forcing frequency respectively fixed at 0.150, 50% and 10Hz. The high BR in case 2 is 0.750 while it is set up at 0.450 in case 3. The time records associated to case 3 is shown on figure 5-52.

![Figure 5-52: Time records associated to BR_s=0.150 BR_h=0.750 DC=50% FF=10Hz](image)

The vortical structures formed during case 3 and their evolutions are described on figure 5-53. Similarly to the second pulsed case, the same vortical structures formed by pulsing the jet are noticeable; that is to say a vortex ring around the jet exit, a horse shoe vortex upstream of the jet and a starting hairpin vortex from the roll-up of the upper jet shear layer. These three structures are visible on figure 5-53 (A) and are respectively indicated by purple, green and black arrows. However, with a BR_s=0.450, the vorticity carried by the first vortex ring is lower than its counterparts at BR_s=0.750. As it is convected downstream, the upstream of the vortex ring interacts with the horse-shoe vortex formed upstream. The opposite vorticity of the horse-shoe vortex weakens the upstream of the vortex ring. This interaction is illustrated on figure 5-53 (B). Consequently, the first vortex ring evolves into a hairpin vortex. The corresponding downstream side of the first vortex ring is shown in the locket on figure 5-53 (B). The same figure highlights the second vortex ring and horse-shoe vortex formed by the second pulse. Compared to the two previous cases, the upstream of the second vortex ring is not convected downstream. It interacts with the second horse-shoe vortex until being vanished by this latter.
Figure 5-53: Evolution of the vortical structures formed at BR$_h$=0.150 BR$_n$=0.450 DC=50% FF=10Hz. Iso-surface of $\Delta P=5500$Pa.m$^2$ colored by spanwise vorticity. The black, purple, green and blue arrows respectively represent the leading hairpin vortex, the vortex ring, horse-shoe vortex and the hairpin vortex formed at low BR.

One of the main difference between a pulsed case with BR$_n$=0.450 and BR$_n$=0.750 concerns the interaction and evolution between the leading hairpin vortex and the first vortex ring. At BR$_n$=0.450, these two vertical structures do not interact with each other, and evolve downstream independently from each other. One of the reasons which may explain this change of behavior may be the recirculation region. The recirculation region associated to BR$_n$=0.450 extends farther downstream than the one at BR$_n$=0.750. Since the downstream of the first vortex ring is located in the recirculation region, while the
leading hairpin vortex is located above, the hairpin vortex is convected downstream at a higher velocity than the downstream of the vortex ring. Therefore, the probability of interaction between these two structures decreases. Each of these two structures evolves into a hairpin vortex as they are convected downstream (figure 5-53 C). Similarly to the second pulsed case, the high forcing frequency prevents the high part of the cycle to establish.

The passage from low to high BR is also identified by a slight ingestion of the crossflow into the jet feeding tube. Since the drop occurs gradually and the BR peak-to-peak is 0.300 compared to 0.600 in the two previous cases, the penetration is not as deep as previously (figure 5-54). The second horse-shoe vortex is transported downstream as the jet shuts down (figure 5-53 D), but this latter is quickly dissipated as it is convected downstream (figure 5-53 E). Similarly to the pulsed case at higher frequency, only one hairpin vortex resulting from the traditional mechanism forms in the low part of the cycle. It is signaled by a blue arrow on figure 5-53 (E).

Figure 5-54: Penetration of the crossflow into the jet tube as the jet shuts down in case 3. Left: iso-surface of ΔP=5500Pa.m² colored by spanwise vorticity. Right: spanwise vorticity along the center plane

Figure 5-53 (E) shows the leading hairpin vortex (black arrow) starts to dissipate as it is convected downstream. This is better well-seen on figure 5-55 which represents the vortical structures described previously but 0.1s later. The sup script on this figure refers to the pulse the structures have been formed. The sup script ‘1’ is related to the structures shed between t=0.01 and 0.21 (e.g. figure 5-53) and ‘2’ to the ones shed between 0.11 and 0.21. The leading hairpin vortex formed from the first pulse has completely disappeared from the top and side views. It broke into smaller eddies. Compared to case 2, the flow field
is dominated by coherent hairpin vortices. These latter do not penetrate as deep into the crossflow than the one formed at BR_h=0.750. This is explained by the fact the vortical structures carry lower velocity and vorticity, and therefore undergone a less intense stretching.

![Vortical structures in the far field](image)

**Figure 5-55:** Vortical structures in the far field. Isosurface of $\Delta P=5500\text{Pa.m}^2$ colored by streamwise vorticity. The superscript refers to the pulse the structures belong to.

### 5.3. Film Cooling Performance

One of the main applications of the discharge of a jet into a crossflow is film cooling. The resolution of the energy equation in the simulation allows to extract temperature distribution, film cooling effectiveness and understand the transport of the coolant fluid by the vortical structures.

#### 5.3.1. Steady-State Cases

**5.3.1.1. Film Cooling Effectiveness at the Wall**

Figure 5-56 presents the mean film cooling effectiveness averaged along the spanwise direction at the wall for steady state cases. The black vertical line represents the trailing edge of the jet exit. As BR increases, the mean film cooling effectiveness decreases. Figure 5-56 highlights the change of behavior between the attached and detached jet regimes. In the detached jet regime, the mean film cooling
effectiveness drops immediately downstream the jet exit. This is due to the high penetration of the coolant fluid into the crossflow and the presence of crossflow fluid trapped into the recirculation region. The transitional regime at \( BR = 0.750 \) also exhibits this drop but at a lower magnitude than in the detached jet. Conversely, the film cooling effectiveness decreases progressively, and almost linearly at \( BR = 0.150 \) as the streamwise distance from the jet exit increases in the attached jet. Besides, in the far field, the steady state case with a \( BR = 0.300 \) provides higher cooling effectiveness than at \( BR = 0.150 \). This is due to the smaller amount of coolant injected at \( BR = 0.150 \).

![Graph](image)

**Figure 5-56: mean film cooling effectiveness averaged along the spanwise direction at the surface of the wall for \( BR = 1.2 \) (red), \( BR = 1 \) (blue), \( BR = 0.750 \) (green), \( BR = 0.400 \) (purple), \( BR = 0.300 \) (orange) and \( BR = 0.150 \) (cyan). The black vertical line represents the trailing-edge of the jet exit.**

The lateral and longitudinal spreading of the coolant fluid in function of the BR at the wall and the plane \( Z_j = 0.125 \) is illustrated on figure 5-57. The lines mark the separation between the surfaces covered with a mean film cooling effectiveness superior and inferior at 0.5. Since the contour at high BR cannot be identified at the surface of the wall, cooling performance between the steady-state cases will be estimated at the plane \( Z_j = 0.125 \), plane at which the top visualizations have been taken in experiments. As BR increases, the area covered by low temperature is reduced. A drop in the longitudinal spreading appears between the attached and detached regimes. The lateral spreading gets wider as BR decreases. By looking only at the BR in the attached regimes, figure 5-57 spotlights the presence of an optimum BR. The longitudinal coverage at \( BR = 0.150 \) is lower than the one at \( BR = 0.300 \) and \( BR = 0.400 \), while the lateral spreading slightly varies between these three cases. If low BR is beneficial for film cooling...
performance because it keeps the coolant fluid close to the wall, a very low BR value may be detrimental because it prevents the coolant fluid to be convected far downstream due to the small injected amount.

Figure 5-57: Contour of the zone with a mean film cooling effectiveness equal to 0.5, at the wall (left), at Zj=0.125 (right) for BR=1.2 (red), BR=1 (blue), BR=0.750 (green), BR=0.400 (purple), BR=0.300 (orange) and BR=0.150 (cyan)

The coverage coefficient at BR=1 reveals high cooling effectiveness area is present along the spanwise edges of the jet at the plane Zj=0.125. This is associated with the formation of the CRVP in the detached jet regime. The contour of the mean cooling effectiveness at the wall is illustrated by figure 5-58. As previously observed, overall increasing BR leads to a decrease of the lateral spreading of the coolant fluid. However, BR=0.150 has a narrower spreading than BR=0.300. At high BR, the amount of coolant at the surface of the wall is negligible. Figure 5-59 shows the transport of the coolant by the vortical structures. It depicts the mean film cooling effectiveness at the plane Zj=0.125 overlaid with the contour of the jet exit. The attached jet shows the transport of coolant fluid by the horse-shoe vortices. As previously mentioned, as the horse-shoe vortex is formed upstream of the jet exit, jet fluid is caught in the vortical structure. The presence of horse-shoe vortex is beneficial to the effectiveness around the hole. In the detached jet, the transport of the coolant fluid by the horse-shoes vortices is seen at the wall (Figure 5-58). The zone where the highest cooling effectiveness is located is along the spanwise edge of the jet exit, where the CRVP are initiated. They contain both coolant and crossflow fluids.
Figure 5-58: Contour of the mean film cooling effectiveness at the surface of the wall. The black line represents the jet exit. The legend is indicated at BR=1.2.

Figure 5-59 cont.
Figure 5-59: Contour of the mean film cooling effectiveness at the plane Zj=0.125. The black line represents the jet exit. The legend is indicated at BR=1.2.

Figure 5-60 confirms the cooling effectiveness on a surface is dictated by the dynamics of the vortical structures. The attached jet regime is characterized by the formation of hairpin vortices. When a hairpin is shed, high cooling effectiveness is seen immediately downstream the jet exit. The instantaneous streamwise planes at BR=0.300 show the core of the hairpin vortices carried coolant fluid in the near field. The coolant fluid distribution is similar to a kidney shape, where the lowest temperature coincides with the core of the CRVP. As the hairpin vortex is convected downstream, its legs extend vertically. The velocity field induced by the upright legs and the head of the hairpin vortex entrains crossflow fluid into the arch of the hairpin vortex. This is shown by the temperature decrease inside the arch along the constant streamwise planes (figure 5-60 bottom). Conversely, the coolant fluid is transported by the horizontal legs of the hairpin vortices. At low BR, the horizontal legs remain relatively close to the wall until Xj=6 (figure 5-60 top); therefore crossflow fluid is not entrained beneath them and high cooling effectiveness is seen at the surface of the wall downstream the jet exit (figure 5-60 middle). Figure 5-60 (middle) presents the instantaneous temperature contour along the center plane. Low temperature zone is seen until Xj=6. Once this distance is passed, the slight upwards motion of the hairpin vortices leads to
the entrainment of crossflow fluid inside the arch, and consequently causes the cooling effectiveness to decrease at the wall. Figure 5-60 (middle) also reveals bulges in the coolant/crossflow interface corresponding to the head of the hairpin vortices.

**Figure 5-60: Entrainment of the crossflow fluid by the hairpin vortices in the attached jet regime.**
Top: iso-surface of ΔP=1500 Pa.m² colored by temperature. Middle: Temperature contour along the center plane. Bottom: Temperature contour at constant streamwise planes. The temperature legend is shown on the bottom figure. The three frames correspond to the same time instant and at BR=0.300

In the detached jet regime, the higher turbulence promotes the mixing between the jet and the crossflow fluid, which is detrimental for film cooling effectiveness. The higher penetration of the jet into the crossflow encourages the entrainment of crossflow fluid in the recirculation region. This is represented on figure 5-61 (middle) where zone of high temperature is seen immediately downstream the jet exit. It is also consistent with the drop of the mean film cooling effectiveness observed on figure 5-56.
Figure 5-61 (bottom) highlights the introduction early downstream of crossflow fluid into the jet body. At Xj=2.5, crossflow fluid already penetrates inside the jet body which exhibits a kidney-shape as in the attached jet. Once Xj=2.5 is passed, there is no more coolant fluid close to the wall. The zone of lower temperature moves upwards as it is convected downstream, and do not exhibit a compact shape (figure 5-61 bottom). This is the result of the intensification of the upwards motion of the vortical structures as they are convected downstream, but also their early break-up into smaller vortices in the flow field. These two mechanisms encourage the entrainment of crossflow fluid into the jet body.

Figure 5-61: Entrainment of the crossflow fluid by the vortical structures in the detached jet regime. Top: iso-surface of ΔP=7500 Pa.m² colored by temperature. Middle: Temperature contour along the center plane. Bottom: Temperature contour at constant streamwise planes. The temperature legend is shown on the bottom of the figure. The three frames correspond to the same time instant at BR=1.2.
5.3.1.2. Jet Trajectory

The mean jet trajectory of the six steady-state cases is presented on figures 5-62 and 5-63. In most of the studies, the jet trajectory is defined as the maximum concentration of a passive scalar along the center plane. No passive scalar has been introduced in our simulations. The mean jet trajectory is defined in two different ways: the streamline issuing from the center of the jet exit and the line of which the mean cooling effectiveness is 0.5 at the center plane. The second definition has been employed, among others, by Muppidi and Mahesh (Muppidi 2007 and Muppidi 2005) and Yuan et al. (Yuan 1999). Both definitions exhibit the same trend. The jet penetrates deeper into the crossflow as BR increases. However, the jet trajectory based on the streamline penetrates deeper into the crossflow than the jet trajectory based on the film cooling effectiveness value at same BR.

Figure 5-62: mean jet trajectory based on the streamline issuing from the center of the jet exit in steady-state for BR=1.2 (red), BR=1 (blue), BR=0.750 (green), BR=0.400 (purple), BR=0.300 (orange) and BR=0.150 (cyan).

Figure 5-63: mean jet trajectory based on 0.5 of the mean film cooling effectiveness in steady-state for BR=1.2 (red), BR=1 (blue), BR=0.750 (green), BR=0.400 (purple), BR=0.300 (orange) and BR=0.150 (cyan).
Figure 5-64 displays the mean jet trajectory based on the streamline issuing from the center of the jet exit and streamwise planes colored by the mean streamwise vorticity for BR=0.300, 0.750 and 1.2. The mean streamwise vorticity refers to the location of the CRVP. Whatever the regime considered, attached or detached jet, the CRVP are located below the mean jet trajectory. This observation is in agreement with Fearn and Weston (Fearn 1974) who stated a jet trajectory based on the streamline issuing from the center of the jet exit penetrates deeper into the crossflow than a trajectory based on vorticity or maximum concentration of a passive scalar. This is the result of the CRVP and low velocity region near the centerline, which entrain passive scalars into the center of the jet cross-section, and therefore in the low velocity region. Then, the fluid containing high concentration of scalar is present below the center streamline and not above. This is in agreement with the difference of jet penetration between the two definitions at equivalent BR.

![Figure 5-64: representation of the mean jet trajectory (black line) and the streamwise planes colored with the mean streamwise vorticity.](image)

### 5.1.1.3. Recirculation Region

Figure 5-65 displays the contour of the mean velocity superimposed with the vectors of the mean velocity for all BR. As BR is increasing, the recirculation region is decreasing in the streamwise direction, while it increases in the upwards direction due to the deeper penetration of the jet into the crossflow. The recirculation region is of importance for film cooling application since it contains both coolant and
crossflow fluids. At low BR, the recirculation region is mainly composed of coolant fluid as it can be seen from the temperature distribution along the center plane on figure 5-56, while at higher BR crossflow fluid is trapped in it, leading to a decrease of the coolant effectiveness.

Figure 5-65: Contour of the mean velocity magnitude superimposed with the mean velocity field

5.3.2. Pulsed Cases

5.3.2.1. Transport of the Coolant Fluid by the Vortical Structures

Steady-state cases have highlighted CRVP are the main vortical structures responsible of the mixing between the hot crossflow and coolant fluids. Hence, they control the cooling performance on the wall. We may believe pulsing the jet may increase cooling performance because the pulse will disrupt the formation of the CRVP.
As previously mentioned, pulsed cases are characterized by three sequences: the passage from high to low BR, from low to high BR and plateau at BR<sub>h</sub> and BR<sub<l</sub>. The structures generated by the two first sequences play an important role on the transport of the coolant fluid into the flow field. The passage from high to low BR distinguishes by the ingestion of the crossflow fluid into the jet feeding tube which results in a coverage break-up at the leading-edge of the jet due to the absence of jet fluid. Figures 5-66, 5-67 and 5-68 display the time evolution of the temperature contour along the center plane for the three pulsed cases. Iso-surface of the Laplacian of the pressure is also shown above to better understand which vortical structures transport the coolant fluid. The black arrows (figure 5-66, 5-67 and 5-68 A) refer to the coverage break-up. This phenomenon is accentuated at low forcing frequency. This is explained by the fact the transition from high to low BR is sharp at F<sub>f</sub>=1Hz whereas it occurs gradually at higher frequency. At such low frequency, the zone of high temperature resulting from the coverage break-up is convected downstream and then washed out by the low part of the cycle which has enough time to establish. In our configuration with a forcing frequency of 1Hz, this is hardly observable because BR<sub>l</sub> is very low (figure 5-66 B). In the low part of the cycle, the jet recovers. The temperature distribution along the center plane (figure 5-66 C) in the low part of the cycle emphasizes what it has been previously observed at low BR in steady state cases. Low BR is beneficial for cooling performance because it keeps the coolant fluid close to the wall, but simultaneously, the coolant fluid is not convected far downstream due to the small amount of coolant injected and its transport at such low velocity. On the opposite, the transient regime characterized by the coverage break-up is not washed out at higher frequency; it is convected downstream. Zones of high temperature appear in the flow field as seen on figure 5-67 (B) and 5-68 (C).

The passage from low to high BR identifies itself by the presence of two bulges carrying low temperature and which form at the leading edge of the jet exit (figures 5-66 D, 5-67 B, 5-68 C). The first and second bulges are respectively pointed out by a green and blue arrow. They originate from vortex rings which transport coolant fluid. At BR<sub>l</sub>=0.450, the formation of the second bulge is not as clear as in BR<sub>l</sub>=0.750 because the second vortex ring is hardly shed. For pulsed cases 1 and 3, the second bulge is
absorbed by the incoming flow (figures 5-66 F and 5-68 F), while for pulsed case 2, it remains distinct and separates from the first one as it is convected downstream (figures 5-67 D and 5-68 E). This change of behavior between these three pulsed cases can easily be justified. The discharge of the two bulges in case 1 is followed by the instauration of the high part of the regime. Then the second bulge merges with the incoming fluid which is transported at relatively high velocity. Only the first bulge persists as it is convected downstream and upwards by the leading hairpin vortex. In pulsed case 2, the second bulge does not propagate upwards because the second ring carries low vertical velocity; hence it remains relatively close to the wall and is absorbed by the fluid present close to the wall. In the opposite, the high velocity carried by the vortical structures formed during the pulses entrains the second puff upwards and downstream in pulsed case 3. In its transport, it remains distinct from the first bulge. The same behavior has been observed for the transport of the first bulge in the three pulsed cases. As it is convected downstream and upwards by the leading-hairpin vortex, it grows in size (figures 5-66 F, 5-67 E and 5-68 F). Then, the vortical structures formed during the pulse are detrimental for film cooling performance because the leading hairpin vortex and the first vortex ring carry coolant fluid away from the wall.

At low forcing frequency, the ejection of the bulges into the flow field is followed by the instauration of the high part of the cycle. The temperature distribution at BR_h is similar to the one observed at equivalent BR in steady-state case. By taking only into consideration cooling performance, pulsed cases at low forcing frequency are characterized by transitional regimes which are washed out once the plateau at BR_h and BR_l is reached. Conversely, pulsed cases with high forcing frequency distinguish by series of transient regime which is detrimental for cooling performance. Then, it is obvious the forcing frequency plays an important role on the coverage brought to the wall.

Figure 5-66 cont.
Figure 5-66: Time evolution of the temperature distribution at $BR_t=0.150$ $BR_h=0.750$ DC=$50\%$ $F_f=1$Hz. Top: iso-surface of $\Delta P=5000$Pa.m$^2$ colored by the temperature. Bottom: Temperature contour along the center plane. The black, green and blue arrows represent respectively the ingestion, the first and second bulges generated by the pulse.
Figure 5-67: Time evolution of the temperature distribution at BRL=0.150 BRH=0.750 DC=50% FF=10Hz. Top: iso-surface of ΔP=7500Pa.m² colored by the temperature. Bottom: Temperature contour along the center plane. The black, green and blue arrows represent respective
Figure 5-68: Time evolution of the temperature distribution at BRI=0.150 BRh=0.450 DC=50% Ff=10Hz. Top: iso-surface of ΔP=5000Pa.m2 colored by the temperature. Bottom: Temperature contour along the center plane. The black, green and blue arrows represent respective
5.3.2.2. Film Cooling Effectiveness at the Wall

Figure 5-69 shows the mean film cooling effectiveness averaged along the spanwise direction at the wall. Pulse case 2 which has the higher BR$_h$ and the higher forcing frequency presents the worst result in cooling effectiveness at the wall. This observation was expected since the high BR$_h$ leads to an entrainment of the coolant fluid away from the wall by the leading hairpin vortex, while the high forcing frequency prevents a film of coolant to settle at the wall. It is interesting to note pulse case 3 which has a lower BR$_h$ and a higher forcing frequency than case 1 provides better cooling performance than case 1. Then, a high forcing frequency pulse case can be adequate for cooling performance.

![Figure 5-69](image.png)

Figure 5-69: Left: mean film cooling effectiveness averaged along the spanwise direction at the surface of the wall. The black vertical line represents the trailing-edge of the jet exit. Right: Contour of the zone with a mean film cooling effectiveness equal to 0.5, at the wall: for BR$_l$=0.150 BR$_h$=0.450 DC=50% $F_f$=10Hz (cyan), BR$_l$=0.150 BR$_h$=0.750 DC=50% $F_f$=10Hz (red), BR$_l$=0.150 BR$_h$=0.450 DC=50% $F_f$=1Hz (blue).

Figure 5-69 represents the lines of the mean film cooling effectiveness at 0.5 at the wall. As expected the pulsed case with the lower forcing frequency shows a better coverage at the wall in the streamwise direction. This is explained by the presence of the plateau at BR$_l$ where coolant fluid is carried very close to the wall. However the spanwise coverage is much higher for the third pulsed case, which has the lowest BR$_h$ fixed at 0.450. In fact, the lateral spreading gets wider as BR decreases. This behavior has also been observed in steady-state cases. The contour of the mean film cooling effectiveness at the wall is depicted on figure 5-70 for the three pulsed cases.
5.3.2.3. Jet Trajectory

The mean jet trajectory between these three pulsed cases is presented on figure 5-71, and is defined as the streamline issuing from the center of the jet exit. Pulsed cases at BRₜ=0.750 penetrates deeper into the crossflow than pulsed case with BRₜ=0.450. The jet trajectory displayed by the two pulsed cases at same high BR remains relatively close to each other in the near field until Xj=6. Further downstream, pulsed case at Fₜ=10Hz has a mean jet trajectory which is slightly higher. This is justified by the fact that at high frequency, the jet trajectory remains globally high, even in the low part of the cycle since this latter has not enough time to establish compared to cases at low forcing frequency.

Figure 5-71: mean jet trajectory based on the streamline issuing from the center of the jet exit. for BRₜ=0.150 BRₜ=0.450 DC=50% Fₜ=10Hz (cyan), BRₜ=0.150 BRₜ=0.750 DC=50% Fₜ=10Hz (red), BRₜ=0.150 BRₜ=0.450 DC=50% Fₜ=1Hz (blue).
5.3.3. Comparison Steady-State/Pulsed Cases

5.3.3.1. Film Cooling Effectiveness at the Wall

Figure 5-72 compares the mean film cooling effectiveness averaged along the spanwise direction at the wall between pulsed cases at BR_{m}=0.400 (left) and BR_{m}=0.300 (right) and steady state cases at same BR. Steady-state cases provide higher film cooling effectiveness than pulsed cases, especially for pulsed cases at high forcing frequency. This difference in cooling effectiveness is reduced as the forcing frequency decreases, as seen on figure 5-72 for BR_{m}=0.400. The formation of a vortex ring and a starting hairpin vortex are detrimental for cooling effectiveness since the coolant fluid is transported away from the wall. In fact, it was expecting the tilting of the starting vortex brings additional coverage close to the wall, but its inclination is blocked by the formation of a hairpin vortex just downstream.

Figure 5-72: Comparison of the mean film cooling effectiveness at the wall between pulsed and steady state cases. Left: BR\textsubscript{m}=0.300, BR=0.300 (orange), BR\textsubscript{c}=0.150 BR\textsubscript{h}=0.450 DC=50% F\textsubscript{f}=10Hz (cyan). Right: BR\textsubscript{m}=0.400, BR=0.400 (purple), BR=0.750 (green), BR\textsubscript{c}=0.150 BR\textsubscript{h}=0.750 DC=50% F\textsubscript{f}=10Hz (red), BR\textsubscript{c}=0.150 BR\textsubscript{h}=0.450 DC=50% F\textsubscript{f}=1Hz (blue)

Figure 5-72 (right) also displays the comparison between steady-state and pulse cases at constant pressure supply, that is to say for a constant BR\textsubscript{h}=0.750. In the near field of the jet exit, until X\textsubscript{j}=4, pulsed cases provide better cooling effectiveness than the steady-state case. In fact, in steady-state, the jet lift-off as well as the entrainment of the crossflow fluid into the recirculation region explains the poor cooling performance at constant BR. Besides, in pulsed cases the passage at low BR, even at high forcing frequency, allows to settle a layer of coolant downstream the jet exit. In the far-field, the performance
between steady-state and pulsed case 1 are equivalent. Pulsed case at high forcing frequency shows lower cooling effectiveness than the steady BR case. In fact, at high forcing frequency, the vortical structures carrying coolant fluid penetrate deeper into the crossflow than at constant BR. Therefore, more coolant fluid is carried away from the wall.

Figure 5-73: Comparison of the contour of the mean film cooling effectiveness at 0.5 between steady and pulsed cases. Left: BRₘ=0.300, BR=0.300 (orange), BRₜ=0.150 BRₙ=0.450 DC=50% Fᵢ=10Hz (cyan). Right: BRₘ=0.400, BR=0.400 (purple), BR=0.750 (green), BRₜ=0.150 BRₙ=0.750 DC=50% Fᵢ=10Hz (red), BRₜ=0.150 BRₙ=0.450 DC=50% Fᵢ=1Hz (blue)

Figure 5-73 presents the contour of the mean film cooling effectiveness at η=0.5 for both pulsed and steady-state cases. It reveals steady-state cases have a wider lateral spreading than pulsed cases at equivalent mean BR.

5.3.3.2. Jet Penetration

Figure 5-74 displays the jet penetration for both steady and pulsed cases. The jet penetration is defined as the streamline issuing from the center of the jet exit. As it can be seen, the jet penetration varies slightly between steady-state and pulsed cases at same mean BR. This behavior may come from our definition of the mean jet trajectory. We were expecting pulsed cases to penetrate deeper into the crossflow since the vortical structures responsible for the lack of coolant fluid close to the wall and the poor coverage are also responsible for the deep penetration of the jet into the crossflow.
Figure 5.74: Comparison of the mean jet trajectory between steady-state and pulsed cases at constant BR<sub>m</sub>. Top: BR<sub>m</sub>=0.300, BR=0.300 (orange), BR<sub>i</sub>=0.150 BR<sub>i</sub>=0.450 DC=50% F<sub>f</sub>=10Hz (cyan). Bottom: BR<sub>m</sub>=0.400, BR=0.400 (purple), BR=0.750 (green), BR<sub>i</sub>=0.150 BR<sub>i</sub>=0.750 DC=50% F<sub>f</sub>=10Hz (red), BR<sub>i</sub>=0.150 BR<sub>i</sub>=0.450 DC=50% F<sub>f</sub>=1Hz (blue)

5.4. Proper Orthogonal Decomposition

To reach conclusions about the dynamics of the flow field, statistical analysis using proper orthogonal decomposition (POD) has been performed on the numerical data. It is well confirmed in the literature that the POD presents advantages compared to other statistical analysis. The POD provides a spatial orthonormal basis (referred as modes) spanning the entire data set. The number of modes composing the spatial basis is the same as the number of snapshots used for the POD. Each mode, on an energetic criterion, has a relative weight on the entire data set. Based on energetic criterion, the POD captures the most energetic structures of the flow field, and therefore the dominant structures with very few modes. A given snapshot can be reconstructed satisfactorily using a few numbers of modes. Besides, since the POD extracts the most energetic modes, these latter can be used in the Galerkin projection to obtain reduced order model of the data set. It has to be noted the POD can also be referred as Principal Component Analysis (PCA) or Karhunen-Loeve transform (KLT) in the literature.

5.4.1. Theory

The POD may be carried out using two methods: the direct method from Holmes (Holmes 1996) or the snapshot method introduced by Sirovich (Sirovich 1987). Both methods have been tested and no significant difference has been found, except the snapshot method is faster to compute and consume less memory. For our study, the results obtained by POD will be generated with the snapshot method since our number of grid points is 20 times larger than the number of snapshots. The POD may be applied on the instantaneous or fluctuating part of the velocity components. In the first case, the first POD mode will
generate the mean flow field and the next ones will concern the fluctuating part of the velocity components. To perform POD on the numerical data, snapshots of our flow field have been taken at regular time intervals. Using the POD, the velocity field which depends both on space and time can be expanded as a series of POD modes with a weight coefficient (named as time-dependent coefficients) associated to each mode (equation 5-1). The method to generate these two functions is the following.

Let’s consider the matrix $U$ (equation 5-2) whom the columns represent the $N$ snapshots and the rows the $M$ spatial positions. The matrix $U$ contains the three components of the velocity field and can be either composed of the instantaneous or the fluctuating velocity.

$$u(x, y, z, t) = \sum_{k=1}^{N} a_k(t) \phi_k^t(x, y, z)$$

**Equation 5-1: Decomposition of the velocity using POD**

The autocovariance matrix $C$ (equation 5-3) is computed. The resolution of the eigenvalues problem gives the time-dependent coefficients, which are the eigenvectors. Each eigenvector represent a time-dependent coefficient at a specific time instant. The eigenvalues are ordered in decreasing order. Each eigenvalue measures the relative energy of the flow field contained in the corresponding mode. The modes (equation 5-4) are derived using the time-dependent coefficients and the snapshots of the velocity field.

$$C = U^TU \implies C = ADA^{-1}$$

**Equation 5-3: Resolution of the eigenvalue problem on the autocovariance matrix**
\( \Psi = U.A \)

**Equation 5-4: Determination of the POD modes**

Since the eigenvalues have been ordered in decreasing order, the first POD mode (first column of \( \psi \)) represents the dominant mode of the flow field in terms of energy. To determine the relative weight of each mode on the flow field, each eigenvalue has been normalized by the sum of all the eigenvalues, which corresponds to the total energy of the flow field.

**5.4.2. Steady-State Cases**

The POD has been performed on BR=0.300, 0.400, 1 and 1.2. 150 snapshots have been used for the computation. The time interval between each snapshot is superior to the integral time scale; then two consecutive snapshots are uncorrelated. The analysis has been done on the velocity and vorticity flow field and at specific constant planes: \( X_j=1, 3, 6, Y_j=0, Z_j=0.5 \) and 1. The POD highlights the different vortical structures which are shed in the attached and detached jet. The attached jet is generally composed of two or three dominant modes emphasizing the well-defined hairpin vortices, while the detached jet is composed of several modes of same weight.

**5.4.2.1. Planes at Constant \( X_j \)**

In the attached jet regime, planes at \( X_j=3 \) and 6 clearly catch the main vortical structures, that is to say the CRVP. The POD modes of the streamwise vorticity at these two given planes are plotted on figures 5-75 and 5-76. Mode 1 and 2 are dominant; they contain respectively 29.3% and 25.2% at \( X_j=3 \) and 19.1% and 12% at \( X_j=6 \) of the energy. Modes 1 and 2 at these two given planes are symmetric and the sign of the variation of the POD modes is consistent with the vorticity associated to the CRVP. They are traces of CRVP. The higher modes lose the symmetry and carry less energy than the first two modes; they represent the smaller eddies which are shed in the flow field. Figure 5-77 displays the energy carried by each mode (either from the velocity or the vorticity field) in function of the mode number. The trend exhibits by the modes obtained from the velocity or vorticity field is the same. There are two dominant
modes at Xj=3, and Xj=6, while the higher ones are more negligible. This tendency is more pronounced at Xj=3 since the CRVP dissipate as they are convected downstream. Therefore the modes which identify them carry less energy at Xj=6 than at Xj=3.

Figure 5-75: Mean streamwise vorticity and POD modes of the streamwise component of the vorticity in the plane Xj=3 at BR=0.400

Figure 5-76 cont.
Figure 5-76: Mean streamwise vorticity and POD modes of the streamwise component of the vorticity in the plane Xj=6 at BR=0.400

Figure 5-77: Energy carried by each POD modes in function of the mode number at BR=0.400. Left: POD modes from the velocity field. Right: POD modes from the vorticity field

The POD mode at Xj=1 shows a different behavior (figure 5-78). The three dominant modes do not represent the CRVP. The absence of symmetry, as well as the wide spanwise distance between variations located in opposite spanwise direction supports this observation. The variations are situated at Yj=+/−0.8. Besides, the instantaneous iso-surface of Laplacian pressure at BR=0.400 plotted in figure 5-8 reveals the CRVP are not shed at Xj=1. In the attached jet regime, the horizontal legs of the hairpin vortices are assimilated on averaged to the CRVP. The latter are well-defined from Xj=1.5. The variation of the POD modes at Xj=1 represents the shear layer vortices. Modes 1, 2, 3 and especially mode 6 show the variations are positioned in a half-ellipse of which the center is Yj=0, Zj=0, the major horizontal axis is 0.7Dj and the minor vertical axis is 0.45Dj. This area matches with the area covered by the jet core at this plane.
Figure 5-78: Mean streamwise vorticity and POD modes of the streamwise component of the vorticity in the plane Xj=1 at BR=0.400

At higher BR, whatever the streamwise planes considered, there is no dominant mode (figure 5-79). In the opposite, there is a series of several modes of same weight which characterize the multitude of random vortical structures formed at higher BR. Figure 5-80 displays the contour of the first mode of the streamwise vorticity. Modes 1, 2, 3 and specially 6 highlight the variation of the modes are located around a circle of center Yj=0, Zj=0.4 and radius 0.4 at the plane Xj=1. This circle coincides with the jet body, as illustrated on figure 5-81. It represents the mean streamwise velocity superimposed with the velocity
vector of mode 6. The variations in the mode are clearly placed at the edge between the jet core and the crossflow. They represent the vortical structures which form at the jet/crossflow interface.

Figure 5-79: Energy carried by each POD modes from the vorticity field in function of the mode number at BR=1.2.

Figure 5-80: POD modes of the streamwise component of the vorticity in the plane Xj=1 at BR=1.2
Figure 5-81: Mean streamwise velocity superimposed with the velocity vector of POD mode 6

The contour of the POD modes at the planes Xj=3 and Xj=6 do not allow the identification of vortical structures (figure 5-82). This comes from the fact an insufficient number of snapshots has been used to compute the POD. A higher number of snapshots is necessary for turbulent flow.

Figure 5-82: POD modes of the streamwise component of the vorticity in the plane Xj=3 at BR=1.2
5.4.2.2. Center Plane

Figure 5-83 depicts the POD modes of the vertical component of the velocity along the center plane at BR=0.300. As at constant streamwise planes, two modes are dominant along the center plane in the attached jet. Mode 1 and 2 carry respectively 8.01% and 7.65% of the total energy. Their variations are located in the far field and have a round shape which represents the fluctuation associated to the head of the hairpin vortices. The second mode is equivalent to mode 1 with a shift of 0.5Dj modulo 2 in the streamwise direction. Two consecutive variations of same sign are separated by a distance of 2Dj, which is the averaged distance between two consecutive hairpin vortices. At BR=0.300, the hairpin vortices are convected downstream at a velocity of 0.13m.s^{-1}. The time separating two consecutive snapshots is 0.48s, which is the time taken by the hairpin vortices to cover 2.5Dj with a velocity of 0.13m.s^{-1}. Modes 1 and 2 represent the convection in the streamwise direction of the hairpin vortices. The presence of two similar modes has also been noticed by Meyer et al (2007). In their work, the two similar modes characterized the convection of the shear layer vortices downstream. All POD modes corresponding to the three components of the velocity reveal high variation in the far-field for their first POD modes (figure 5-84). The variations associated to the spanwise velocity are naturally lower in magnitude than the one associated to the streamwise and vertical velocity since the flow is symmetry. Variations in the near-field appear in higher mode, from mode 5, and the latter are lower in order of magnitude in the near-field than in the far-field at same POD mode (figure 5-83).

Figure 5-83 cont.
Figure 5-83: POD modes of the vertical component of the velocity along the center plane at BR=0.300.

Figure 5-84: Streamwise and spanwise component of mode 2 of the velocity along the center plane at BR=0.300

At BR=1, no mode is dominant in the center plane. The first mode carries 2.71% of the total energy while the tenth mode carries 1.61%. Figure 5-85 (right) represents the evolution of the energy carried by each mode in function of the mode number. Since the difference in term of energy between the first and tenth mode is negligible, all these modes have to be considered with the same attention. The variations of the POD modes are mainly located in the near-field and represent the intensification of the shear effect at the jet/crossflow interface (figure 5-86). It has been observed the variations of the POD modes representing the fluctuation along the upper shear layer are slightly higher than their counterparts representing the lower shear layer. The presence of variation in the far-field appears only from mode 8. However, these variations are only visible until Xj=6. Farther downstream they are negligible. This is consistent with the early break-up of the large scale vortical structures into smaller eddies observed in the detached jet. The shedding of the smaller eddies is random, and therefore the modes representing them carry fewer energy. The representation of the POD modes show the highest variations are located in the jet feeding tube which is a zone of high turbulence. No attention is brought to these variations since the shedding of these vortical structures is influenced by the jet geometry and the boundary conditions.
Figure 5-85: Energy carried by each POD modes from the velocity field in function of the mode number along the center plane at BR=0.300 (left), BR=1 (right).

Figure 5-86: POD modes of the streamwise component of the velocity along the center plane at BR=1.

5.4.2.3. Planes at Constant Zj

The POD have been performed on the planes Zj=0.5 and Zj=1. The two dominant modes at BR=0.300 along the center plane identify the hairpin vortices. The same conclusion can be reached from the first two modes at the plane Zj=0.5 and Zj=1. The variations of modes 1 and 2 show an alternation of positive and negative variation which is consistent with the variation observed along the center plane (figure 5-87). The distance between two consecutive variations of same sign is 2Dj and mode 2 is similar to mode 1 with a shift of 0.5Dj modulo 2 in the streamwise direction. At the plane Zj=0.5, the variations of mode 1 and 2 look like two rows of single spot. The two rows are symmetric along the center plane. They are traces of the upright legs of the hairpin vortices. In the opposite, at plane Zj=1, the variations are
represented by a row of single spot which reveal the fluctuation associated to the head of the hairpin vortices (figure 5-88). Besides, the variation appears only in the far-field where hairpin vortices have started their upwards motion. The higher modes at constant Zj plane exhibit stronger variation in the far-field than in the near-field. These latter may be interpreted as the interaction of the jet body with the crossflow boundary layer.

![Figure 5-87: POD modes of the vertical component of the velocity at the plane Zj=0.5 at BR=0.300.](image)

![Figure 5-88: POD modes of the vertical component of the velocity at the plane Zj=1 at BR=0.300.](image)

At BR=1, two modes are slightly dominant at Zj=0.5 while the plane Zj=1 is represented by a series of modes of same order (figure 5-89 right). Each of them contains around 2.5% of the total energy of the flow field. Modes 1 and 2 at Zj=0.5 constitute respectively 7.52% and 5.18% of the total energy. Their variations are located inside and along the area which is the orthogonal projection of the jet exit on this plane (figure 5-90). Higher modes show variations further downstream, until Xj=5. However as the representation of the modes along the center plane, no variation has been found far downstream where
smaller eddies are shed. From the variations of the POD modes at constant $Z_j$ plane, it is difficult to draw conclusions since the number of snapshots used for the POD is insufficient.

![Figure 5-89: Energy carried by each POD modes from the velocity field in function of the mode number along the planes $Z_j=0.5$ and $Z_j=1$ at $BR=0.300$ (left), $BR=1$ (right).](image)

![Figure 5-90: POD modes of the vertical component of the velocity at the plane $Z_j=0.5$ at $BR=1$.](image)

The observations draw from the representation of the POD modes at different $BR$ and planes can be summarized as follows: POD modes at low $BR$ is characterized by two dominant modes which identify the main vortical structures, the hairpin vortices. Mode 2 shows a flow structure displaced about $0.5D_j$ modulo 2 in the streamwise direction compared to mode 1. This characterizes the convection of the hairpin vortices in the streamwise direction. Cases at higher $BR$ are characterized by a series of modes of same importance; it illustrates the multitude of random vortical structures formed at higher $BR$. The variations of the modes are consistent with the presence of jet shear-layer vortices and small eddies in the far-field. However, more snapshots are necessary to identify the vortical structures at all the planes.
5.4.3. Pulsed Case

The POD has been performed on only one pulsed case of which the configuration is BR₁=0.150, BR₂=0.750, DC=50% and F₀=10Hz. As previously mentioned the snapshots used for the POD in steady state cases are uncorrelated. However, it is impossible to obtain uncorrelated snapshots for pulsed cases, especially for cases with a high forcing frequency. Therefore, applying the POD on pulsed case arise several questions such as the determination of the sampling frequency for the acquisition of the snapshots or the number of periods which should be used to compute the POD.

5.4.3.1. Determination of the Sampling Frequency

The method used by Siegel et al (Siegel 2005) has been employed for the determination of the sampling frequency. The method is the following: a signal containing five periods has been considered. The five periods are identical to each other; one period has been repeated five times. The time step of our period is dt=0.2ms, that is to say our 10Hz cycle contains 500 time steps. The POD has been computed on the five periods by taking different sampling frequency for the acquisition of the snapshots: 10dt, 20dt...90dt. The case with the lowest forcing frequency is considered as the reference case. The relative error between the mode 1 obtained from our reference case and the mode 1 obtained with a sampling frequency of 20dt is computed. This operation is repeated for all the sampling frequency and for the second mode. Figure 5-91 shows the results. A sampling frequency of 20, 30 and 40dt provide very good approximation of our reference case. At a sampling frequency of 40 dt, the relative error of mode 1 is slightly superior than the one with a sampling frequency of 20dt. The same observation can be made with the mode 2 and a sampling frequency of 30dt. Therefore, a sampling frequency of 20dt will be used to compute the POD on this given pulsed case.
Figure 5-91: Relative error between the mode 1 (or mode 2) obtained from our reference case and the mode 1 (or mode 2) obtained with a sampling frequency of 20dt (and higher)

5.4.3.2. Determination of the Number of Periods

Siegel et al (Siegel 2005) have stated for periodic flows, a minimum of one period is necessary to obtain satisfactory results with the POD. They name the POD as Short Time Proper Orthogonal Decomposition (DPOD) which refers to the use of only one period to compute the POD. Their conclusions have been verified with our flow field. The POD has been computed on cases with 0.5, 1 until 5 periods. The case with five periods is taken as reference. A sampling frequency of 20dt for the acquisition of the snapshots has been considered whatever the number of periods considered. The relative error of the mode 1 obtained from the reference case and the mode 1 obtained with 0.5 period has been computed. This operation is performed for all the different cases, and for the first two dominant modes. Figure 5-92 presents the results. The trend displayed by the first two dominant modes is similar. The relative error is negligible when an entire number of periods is considered for the POD (in order of 2%), while it is maximum for a non-finite number of periods. This is in agreement with Siegel results. Then, the SPOD will be applied on the pulsed case.
5.4.3.3. Results

The POD has been computed at the planes $X_j=3$, $Y_j=0$ and $Z_j=0.5$. The energy carried by each mode in function of the mode number is displayed on figure 5-93. For pulsed case at high forcing frequency, one or two modes are dominant. This trend is particularly well-seen along the center plane where the first mode represents 75.7% of the energy and the second mode only 7.91%. This trend was expected since this pulsed configuration is characterized by a succession of transient regime during which a leading hairpin vortex is shed. As it has been described in section 5.2, the leading hairpin vortex remains the main vortical structure as it is convected downstream. Then, this is consistent with the presence of a dominant mode which identifies this vortical structure.

Figure 5-93: Energy carried by each POD modes from the velocity field in function of the mode number along the planes $X_j=3$ (red), $Y_j=0$ (green) and $Z_j=0.5$ (blue) at $BR_s=0.150$ $BR_h=0.750$ $DC=50\%$ $F_f=10Hz$. 
Figure 5-94 displays the variations of the POD modes corresponding to the vertical component of the velocity along the center plane. All the POD modes, and in particular modes 1 and 2 show variation in the vicinity of the jet exit where the shedding of the vortex rings and the roll-up of the upper shear layer occur. Modes 3 and 7 present variations in the far-field. At same POD mode, variations in the near-field have a higher order of magnitude than the ones in the far-field. This is explained by the fact that as the vortical structure is convected downstream, even if it remains coherent, it dissipates.

**Figure 5-94**: POD modes of the vertical component of the velocity at the center plane at $BR_l=0.150$ $BR_r=0.750$ DC=50% $F_f=10Hz$.

The variations of the POD modes at the plane $Z_j=0.5$ are consistent with what it has been observed along the center plane. The first two modes show high variation in the area which is the orthogonal projection of the jet exit on this plane. Variations in the far-field are observed at higher modes.

**Figure 5-95 cont.**
Figure 5-95: POD modes of the vertical component of the velocity at the plane Zj=0.5 at BRl=0.150 BRh=0.750 DC=50% Ff=10Hz.

The contour of the POD modes representing the streamwise component of the vorticity at the plane Xj=3 is plotted on figure 5-96. No symmetry is visible in the contour whatever the modes considered. No feature of the flow field can be identified from the POD modes at the plane Xj=3.

Figure 5-96: POD modes of the streamwise component of the vorticity at the plane Xj=3 at BRl=0.150 BRh=0.750 DC=50% Ff=10Hz.
Chapter 6. Conclusion

Large Eddy Simulations have been performed on an inclined jet in crossflow. Steady-state cases, as well as pulsed cases have been investigated. Two regimes have been identified in steady-state: the attached and detached jet. The attached jet regime, at low BR is characterized by the formation of hairpin vortices resulting from the roll-up of the lower shear layer, while the detached jet differentiates by the shedding of upper and lower vortices. Both regimes reveal the presence of horse-shoes vortices upstream of the jet exit and CRVP. This latter has different origin in function of the regime considered. In the attached jet, the horizontal legs of the vortices are on averaged the CRVP, while in the detached jet, the CRVP are initiated by the roll-up of the jet shear layer on its lateral edges.

Pulsed cases study has revealed the presence of a vortex ring and starting hairpin vortex which are shed by pulsing the jet. As the vortical structures are convected downstream, they merge to form a leading hairpin vortex which becomes the dominant feature of the flow field. Pulsed cases at low forcing frequency are characterized by the instauration of steady-state regime at high and low BR punctuated by short transient regimes during the passage from low to high BR and inversely. In the contrary, pulsed cases at high forcing frequency differentiate by a series of transient regimes where the jet does not recover. Qualitative comparisons between the experiments and the simulations have been performed in both steady-state and pulsed cases. A good agreement has been observed.

Information about film cooling effectiveness have been extracted from the numerical data. The spanwise averaged film cooling effectiveness shows a drop between the attached and detached jet. This is the result of the transport of the coolant fluid by the vortical structures. At high BR, the upwards motion of the vortices entrain far from the wall the coolant fluid, while at low BR the horizontal legs of the hairpin vortices keep the coolant fluid close to the wall. Comparisons of cooling performance between pulsed and steady-state cases at same mean BR have been done. It reveals steady-state cases provide better cooling performance; hence the vortical structures generated by the pulse are detrimental to the cooling efficiency.
The application of the POD on steady cases has revealed the dominant features of the flow field, especially at low BR. In the attached jet, the two first modes identify the hairpin vortices, while the series of modes of same weight in the detached jet emphasizes the shedding of random vortical structures linked to high BR. However, the results obtained from the POD have to be considered in the view that not enough snapshots were at our disposition to obtain results independent of the number of snapshots.

Future work will consist in the use of the POD modes as a basis for the Galerkin projection in order to obtain reduced order model of the simulated flow field. Galerkin projection will also be carried out on experimental data thanks to accurate velocity measurements through Particle Image Velocimetry (PIV) visualizations. It will allow to get a low order model of the flow field in order to perform active control. Quantitative comparisons between the experiments and the simulations will be realized with the acquisition of the velocity field with the PIV.
References


Wells, Lemuel., 2010, MSc.

Appendix: Eigenvalues of the Velocity Gradient Tensor

The term general of the velocity gradient tensor is $u_{i,j}$. Equation A-1 details each component of the velocity gradient tensor.

$$\nabla \vec{u} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{pmatrix}$$

**Equation A-0-1: Definition of the velocity gradient tensor**

The dimension of the velocity gradient tensor is 3, therefore it has three eigenvalues. Its eigenvalues, called $\sigma$ are determined by equation A-2 and satisfies equation A-3.

$$\begin{vmatrix} \frac{\partial u}{\partial x} - \sigma & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} - \sigma & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} - \sigma \end{vmatrix} = 0$$

**Equation A-0-2: Determination of the eigenvalues of $\nabla \vec{u}$**

$$\sigma^3 - P\sigma^2 + Q\sigma - R = 0$$

**Equation A-0-3: Equation satisfied by the three eigenvalues of $\nabla \vec{u}$**

The first, second and third invariants of the velocity gradient tensor are respectively $P$, $Q$ and $R$.

$$P = u_{i,i}$$

$$Q = -\frac{1}{2}(u_{i,j}u_{j,i})$$

$$R = \det(u_{i,j})$$

**Equation A-0-4: Definition of the first, second and third invariant of $\nabla \vec{u}$**

In the case of an incompressible flow, the first invariant is null.
Vita

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