Numerical Study of Laterally Loaded Batter Pile Groups with the Application of Anisotropic Modified Cam-Clay Model

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NUMERICAL STUDY OF LATERALLY LOADED BATTER PILE GROUPS WITH THE APPLICATION OF ANISOTROPIC MODIFIED CAM-CLAY MODEL

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering

in

The Department of Civil and Environmental Engineering

by

Yida Zhang

B.S., Zhejiang University, 2010

December 2012
To my wife Yuxi, for her unwavering love and trust.
ACKNOWLEDGEMENTS

My deepest appreciation goes to my advisor Dr. Murad Abu-Farsakh for his invaluable guidance, inspiration and suggestions through this entire study. Dr. Murad’s broad knowledge on foundations, soil mechanics and numerical methods has influenced me very much, which directly lead to the accomplishment of this practical-motivated, theoretical-based and numerical-realized research. I am especially grateful for his endless tolerance and kindness in every situation occurred during this study. I will not achieve so many things without his trust on me.

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ABSTRACT

This study presents a series of numerical studies of laterally loaded batter pile groups based on data of the full-scale lateral load test on M19 eastbound pier foundation of the new I-10 Twin Span Bridge, Louisiana. The numerical studies include several continuum-based 3D finite element analyses on batter/vertical pile/pile groups and a FB-MultiPier analysis of the pile foundation. The Anisotropic Modified Cam Clay Model, has been implemented into UMAT and applied for describing clay behavior in all FE models. The explicit substepping scheme with modified Euler algorithm is selected to implement the model in ABAQUS software. The resultant UMAT shows good accuracy compared to the ABAQUS in-built Modified Cam Clay model. Also it exhibits wonderful computational stability and efficiency in the pile group analyses, which greatly accelerated the whole research processes. The results of FE analyses were compared with the measured field data from lateral load test and those predicted by FB-MultiPier. All of them showing good agreement on lateral deformation profiles and bending moment profiles. The comparison of the lateral deflection, bending moment, soil resistance and lateral/ vertical load distributions between different spacing batter/ vertical pile groups and single isolated pile illustrate that small spacing and the vertical piles will produce intensified group effect. The concept of “trapezoidal zone” is firstly proposed to explain the axial load distribution pattern of batter pile group foundation. An additional coupled pore fluid diffusion and stress analysis on a single pile model demonstrated that the resultant excessive pore pressure caused by lateral loads has limited influence on the result of FE analyses.
CHAPTER 1
INTRODUCTION

1.1 Pile Foundations

Piles are slender structural members that are often made of steel, concrete, timber, polymers or composite materials. They can be used in single or group to construct pile foundations. Several conditions that require the use of pile foundations include: 1) the upper soil layers are highly compressible and the use of shallow foundations cannot support the upper structure; 2) the structure are subjected to horizontal forces and 3) the presence of large amount of expansive and collapsible soils. Typically pile foundations can provide much higher vertical and lateral bearing capacities than shallow foundations, since they are able to transmit the vertical load to deeper stronger soil layers (or bedrock) as well as providing bending and lateral resistance to a horizontal loads.

Pile foundations are primarily designed for vertical support of structures, while specific consideration should be taken on their lateral response for supporting high buildings, bridges and offshore structures due to wind loads, frequent wave loads, possible huge lateral impact and earthquake loadings. According to Rao et al. (1998), lateral loads are usually in the order of 10-15% of the vertical loads in case of onshore structures and 25-30% in case of coastal and offshore structures. The lateral load capacity and maximum lateral deflection of the pile foundation are the
two most concerned aspects for engineers. It is believed that the later one is the major criterion on the design.

1.1.1 Laterally Loaded Single Pile

The resistance of a pile to a horizontal displacement is provided by its surrounding soils via two components: 1) the side resistance from friction and adhesion due to relative movement between soil and pile and 2) the normal stresses due to the difference between the soil passive pressure in front of the pile and the soil active pressure behind the soil (Figure 1.1a). The soil resistance to a certain portion of a pile is related with the lateral displacement of that portion, and their relations are always presented in the so-called p-y curves (Figure 1.1b).

The p-y curves are influenced by many factors such as soil types, soil properties, soil saturation and drained conditions, pile geometries and pile group interactions. Various p-y curves for sand and clay are discussed in Chapter 2.

1.1.2 Laterally Loaded Pile Groups

The p-y curves for group pile foundation can be different from those obtained in single pile tests due to the group interaction effect. In pile groups, each pile pushes against the soil behind it creating a shear zone which can be enlarged and overlapped with each other as the lateral load increases (Figure 1.2). Such overlapping will be amplified with the decreased pile spacing. The overlapping between two piles in the same row is called “edge effects” and the one between piles in different rows is known as “shadowing effects.” These group interaction effects result in reduced lateral resistance and thus produce larger bending moment and lateral displacement than
the single isolated pile for the same given average load per pile. To quantify the reduction of later resistance in group pile foundation, Brown et al. (1987) introduced a reduction factor or “P-multiplier ($P_m$)” concept to deduct the value of $p$ in the single pile p-y curves (Figure 1.3).

The lateral resistance of the piles within the group is a function of row location. It has been proved in many literatures that the lead row piles carried more load than the trailing row piles. Many researchers found that the last row carries the least load. However, some other researchers
(Rollins et al., 1998; Zhang et al., 1999; Rollins et al., 2003b) reported that the last row carries a slightly higher load than the preceding row.

Figure 1.2: Shadow and edge effects on a laterally loaded pile group (Walsh, 2005).

Figure 1.3: The concept of p-multiplier (Rollins et al. 1998).

Spacing also affects the lateral resistance of the pile groups. Rollins et al. (2003b) reported that the group spaced at 5.6D (D is the diameter of the pile) showed very little reduction on lateral
resistance. However, lateral resistance consistently decreases with closer spacing. The p-multipliers from past experiments by several researchers are summarized in Table 1.1.

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Soil Type</th>
<th>Row Spacing</th>
<th>p-multipliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brown et al. (1987)</td>
<td>Stiff clay</td>
<td>3D (30 mm deflection)</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3D (50 mm deflection)</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>Medium dense sand over clay</td>
<td>3D</td>
<td>0.8</td>
</tr>
<tr>
<td>Ruesta and Townsend (1997)</td>
<td>Sand over cemented sand</td>
<td>3D</td>
<td>0.8</td>
</tr>
<tr>
<td>Rollins et al. (1998)</td>
<td>Silts and soft clays</td>
<td>2.82D</td>
<td>0.6</td>
</tr>
<tr>
<td>Rollins et al. (2003a)</td>
<td>Silts and soft clays</td>
<td>5.65D (50 mm deflection)</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.65D (99 mm deflection)</td>
<td>0.90</td>
</tr>
<tr>
<td>Rollins et al. (2003b)</td>
<td>Medium stiff clays with sand</td>
<td>3D</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.3D</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.4D</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.6D</td>
<td>5.6</td>
</tr>
</tbody>
</table>

* D=diameter of the pile

1.1.3 Batter Piles

Batter piles or group batter pile foundations are often used in foundation design in order to achieve a higher lateral capacity of the foundation. According to their direction of inclination, batter piles are classified into positive batter piles which are inclined against the loading direction and negative batter piles which are battered toward the loading direction (Figure 1.4). It is reported (Prakash and Subramanyam, 1965; Ranjan et al., 1980) that negative batter piles show a definite
increase in lateral resistance over that of vertical piles while positive batter piles exhibit less lateral resistance than that of vertical piles. Lu (1981) supported these observations by explaining that the soil reaction at ground level is zero for a positive batter pile and maximum for a negative batter pile, indicating that the upper layer soil offer much higher lateral resistance to a negative batter pile than a positive batter pile. However, for the batter pile groups, such soil resistance distribution will also be influenced by the pile alignment. Tschebotarioff (1953) reported that the positive batter pile in the leading row of the group batter pile foundation took the most stress during their 1:10 scaled lateral load tests. Possible soil reaction distribution for a batter pile group is shown in Figure 1.5.

![Diagram of batter piles](https://via.placeholder.com/150)

**Figure 1.4:** Type of batter piles (after Prakash and Subramanyam, 1965)
1.2 Motivation, Objectives and Scope of the Work

Recently, a full-scale lateral load test was performed on the batter pile group foundation of the M19 eastbound pier of the new I-10 Twin Span Bridge over Lake Pontchartrain, Louisiana. The test provides complete subsurface information and large amount of measured data on pile deflection and moment distribution during the lateral loading, which allows one to evaluate current design methodology for laterally loaded piles, or to carry out comprehensive analyses of the performance of group batter piles subjected to lateral loads.

Currently, the most popular method in designing laterally loaded piles is the p-y method, which will be discussed in Chapter 2. FB-MultiPier, a nonlinear finite element method software
based on the concept of p-y curves, has been used during the design of the new I-10 Twin Span Bridge by LADOTD. Hence, the study of the full-scale lateral load test will provide a direct evaluation of the FB-MultiPier program and, of course, the p-y design method itself.

The most important objective of this research is to carry out a comprehensive three-dimensional continuum-based finite element analysis (FEA), which is considered to be the most advanced method in analyzing pile foundations, of the whole lateral load test mentioned earlier. Such FE analysis will provide much more details of the problem that p-y methods cannot provide, which helps the reader to better understand the response of the soil-pile system under lateral loads.

In order to achieve the objective, considering one FE model of the original batter pile group geometry is not enough. The author will study the same problem with a single vertical pile model, a vertical pile group model with the same spacing as the batter pile model, and two batter pile group model with larger and smaller spacings. These analyses will provide complete information of how the group effect and pile inclination will influence the lateral response of pile foundations.

Among all the factors that affect the quality and reliability of FEA, the selection of constitutive model for soil is critical. An advanced soil model, Anisotropic Modified Cam Clay (AMCC) Model, has been selected to describe the clay behavior in this study.

In addition, there is a section discussing the coupled pore water FE analysis. Since the test is performed on the foundation in the lake with clay-dominating subsurface, a correct assumption of drainage condition is very important in pertan high reliability of the FEA.
1.3 Structure of the Thesis

Chapter 2 includes a brief review of past lateral load tests on pile foundations and the description of the recent full-scale lateral load test on M19 pier of the new I-10 Twin Span Bridge. A detailed literature review covering current approaches with emphasis on finite element method on analyzing laterally loaded piles will also be presented in this chapter.

Chapter 3 starts with an overview of soil plasticity, followed by introducing Anisotropic Modified Cam Clay model. Its numerical implementation will also be discussed. Some numerical test results of the implemented model are presented at the end.

Chapter 4 presents the details of finite element models including FE mesh and the determination of various parameters for soil constitutive models. The coupled pore fluid diffusion and stress analysis in ABAQUS will also be introduced. In addition, the FB-MultiPier model is briefly described.

Chapter 5 presents the results of finite element analysis, including lateral deflection, bending moment, soil resistance, lateral and axial load distribution and back calculated p-y curves.

Chapter 6 presents summaries, conclusions and recommendations.
CHAPTER 2

LITERATURE REVIEW

2.1 The Full-Scale Lateral Load Test

Full-scale load tests are always desirable by researchers for providing direct field measurements that can be used to verify their models or to develop empirical methods. However, only a few full-scale lateral load tests on piles were reported in the literature due to their high cost and technical difficulties.

2.1.1 Summary of Past Full Scale Tests on Laterally Loaded Piles

The first full-scale lateral load test shown in the literatures is conducted by Feagin (1937) on timber and concrete piles. Matlock (1970) carried out a lateral load test on steel pipe piles of 12.75 diameters for both static and cyclic loading. Kim et al. (1976) conducted a full-scale test on batter pile group foundation and found out that battered piles provide more lateral resistance with less bending moment. Brown et al. (1987, 1988) conducted full-scale test on vertical pile groups on steel piles and generated the corresponding p-y curves. Later, Brown et al. (1988) introduced the p-multiplier concept to modify p-y curves of single piles to consider the reduced resistance of the group pile. Ruesta and Townsend (1997) carried out full-scale tests on reinforced concrete pile and noted that the outer piles took more loads than the inner pile of same row due to the smaller influence of shadowing effect they encountered than inner piles. Rollins et al. (1998) suggested various p-multipliers after conducting a full-scale test on vertical pile group in clay. They observed that the displacement of pile group is 2-2.5 times higher than the single
pile for the same average pile load. They also reported that the back row (trailing) carried somewhat higher loads than middle row, which completely conflicted with the conclusion of Brown et al. (1988) that the back rows resist lowest loads.

In the nearest decades, Huang et al. (2001) conducted a full-scale test on bored and driven precast pile groups to investigate the influences of installation procedure of pile in lateral soil resistance. He reported that the driven pile installation increased the group interaction by causing the soil to move laterally and hence becomes denser; while bored pile installation loosens the soil and hence decreases the group interaction. Rollins et al. (2003a, 2003b and 2005) carried out several full-scale lateral load tests of piles in clayey and sandy soils at various pile spacings. The major findings out of their study are: 1) the middle pile of the same row carries the smallest load in that row while the side piles carry 20-40% higher loads in the sandy soil, which agrees with other studies conducted in sands (Ruesta and Townsend. 1997; McVay et al., 1998) and conflict with some tests results in clays (e.g. Brown et al., 1987; Rollings et al., 1998); 2) The group effect becomes negligible when the spacing between rows increased to more than 6D, where D is the pile diameters; 3) Group pile has significantly higher bending moments than those in isolated single pile for a given load.

2.1.2 The Full-Scale Lateral Load Test at the New I-10 Twin Span Bridge

A new (5.4 mile long) I-10 Twin Span Bridge has been recently constructed over Lake Pontchartrain between New Orleans and Slidell, Louisiana, to replace the old bridge that was seriously damaged from the storm surge and water waves associated with Hurricane Katrina that
hit the southern part of Louisiana in August of 2005. The new bridge is located 300 ft east of the old bridge. It was built with an elevation of 30 ft, which is 21 ft higher than the old bridge, and an 80 ft high-rise section near Slidell to allow for marine traffic, making it less susceptible to high storm surge. The bridge consists of two parallel structures with three 12 ft travel lanes and two 12 ft shoulders on each side (a total of 60 ft wide).

The M19 pier is the second pier south of the marine traffic underpass. It supports 200 ft long steel girders in the north side and 135 ft long concrete girders in the south side. The foundations of M19 piers consist of 24 precast prestressed concrete (PPC) battered piles of 110 ft long and a 3 ft × 3 ft square section with an outer width of 36 inch and a circular void of 22.5 inch. All piles are inclined with a slope of 1:6; half of them are negative or reverse batter and the rest are positive or forward batter. The piles were spaced at 4.3 pile width in the direction of lateral loading and 2.5 pile width in the other direction. The average embedded length of the piles was 87 ft. The size of M19 pile cap (or footing) is 44 ft long × 42.5 ft wide × 7 ft deep. The water depth is 12 ft. Figure 2.1 presents a photo of the M19 eastbound and westbound piers site.

Subsurface exploration and testing programs were performed to characterize the subsurface soil conditions along the entire I-10 Twin Span Bridge, including the M19 eastbound pier location. This includes soil sampling, laboratory testing and in-situ standard penetration tests (SPT) and cone penetration tests (CPT). One soil boring was performed close to M19 pier to a depth of 200 ft and 48 Shelby tube samples were extracted from cohesive layers for laboratory testing, such as unconsolidated undrained (UU) triaxial tests. SPT were also conducted in sandy
layers. In addition, five CPT tests were conducted at M19 pier site down to a depth of 160 ft each, one CPT at the center of M19 pier foundation, and four CPTs at distances 5-10 ft out from the four corners of the pile cap. The purpose of the CPT was to define the subsurface soil profile and identify any variations across the site, and to locate the depth of the bearing sand layer to support the piles. The depth of sand layer at M19 pier was found to be at depths ranging from 100 ft to 110 ft below the water surface. However, the piles at M19 eastbound pier were tipped in stiff silty clay layer at an average depth of about 87 ft below the mudline.

Figure 2. 1: M19 east and west bound piers

The site investigation at the M19 pier site showed that the subsurface soil stratigraphy consists of a medium to stiff silty clay to clay soil with silt and sand pockets and seams down to 110 ft depth (the undrained shear strength, $S_u$, ranging from 0.28 tsf to 2.02 tsf) with a layer of medium dense sand located between 35 ft to 47 ft depth (the SPT-N values ranging from 16 to 22). Medium dense to very dense sand with interlayers of silty sand, clayey sand, and silty clay
soil was found between 110 ft and 160 ft depth (the SPT-N values ranging from 3 for loose clayey sand to 86 for very dense sand). The subsurface soil description from soil boring, profiles of $S_u$ and SPT-N values, profiles of cone tip resistance ($q_c$) and sleeve friction ($f_s$), and the CPT probabilistic region estimation soil classification are presented in Figure 2.2.

![Graph of soil properties](image)

**Figure 2.2:** Summary of in-situ exploration and testing of site M19 pier (Abu-Farsakh et al., 2011)

Eight selected piles were instrumented with Micro-Electro-Mechanical Sensor (MEMS) In-Place Inclinometers (IPIs) pile deflection profile and twelve selected piles were instrumented
with resistance type sister bar strain gauges. The locations of instrumented piles are presented in Figure 2.3. The IPI consists of a string of tilt sensors connected together and placed permanently in the pile through a PVC casing. Six IPI-MEMS sensors were installed in each of the eight piles at depths of 5, 15, 25, 35, 45, and 65 ft from the bottom level of the pile cap with the lowest one tied to an anchor point at the bottom of PVC casing at 85 ft from the bottom of the pile cap. Two pairs of strain gauges were installed in each of the 12 piles at locations of -16 ft and -21 ft from the pile top before pile cut off to measure the strain distribution. The profile of pile deformation due to applied lateral load can be calculated from the IPI data; and the bending moment and axial load of piles can be obtained from the strain gauges data.

![Figure 2.3: Layout of instrumented piles (Abu-Farsakh et al., 2011)](image)

The lateral load test was conducted by pulling the eastbound and west bound toward each other using high strength steel tendons run through two 4 in PVC pipes installed in both pile caps. For setup of lateral load test, the M19 eastbound pier was designed as dead end and the M19
west bound pier was designed as live end. The steel strands were first anchored at the dead-end side, and then were threaded one-by-one through the two 4 in PVC pipes from the dead-end at M19 eastbound pier toward hydraulic jack of the live-end at M19 westbound pier. Each steel tendon includes 19-0.62 in diameter strands of low relaxation, high yield strength steel ($E_s = 28,500$ ksi). The lateral load was applied using 600 ton jacks with piston-end facings to pull M19 eastbound and west bound pier toward each other using the steel tendons. Figure 2.4 presents the photos of M19 eastbound dead end and westbound piers live end design.

![Figure 2.4: a) Eastbound pier with steel strands anchored at the dead end side; b) Jacking system at the live-end of westbound pier (Abu-Farsakh et al., 2011)](image)

The designed sequence of lateral load test includes preloading each tendon to 300 kips, then loading, unloading and reloading. The design maximum applied load was 2000 kips. However, the test was unloaded earlier at a maximum applied load of 1870 kips when the stroke in one of the 600-ton jacks reached its maximum limit. The schematic diagram of lateral load setup at M19 pier is depicted in Figure 2.5 and the designed loading sequence is presented in Table 2.1.
Figure 2.5: Schematic diagram of lateral load test setup

Table 2.1: Designed loading time table

<table>
<thead>
<tr>
<th>No.</th>
<th>Lateral Loads Per Cable (kips)</th>
<th>Total Lateral Loads (kips)</th>
<th>Load Duration (min)</th>
<th>No.</th>
<th>Lateral Loads Per Cable (kips)</th>
<th>Total Lateral Loads (kips)</th>
<th>Load Duration (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preload</td>
<td>300</td>
<td>600</td>
<td>90</td>
<td>10</td>
<td>700</td>
<td>1400</td>
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<td>1700</td>
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<td>450</td>
<td>900</td>
<td>5</td>
<td>14</td>
<td>900</td>
<td>1800</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>500</td>
<td>1000</td>
<td>15</td>
<td>15</td>
<td>935</td>
<td>1870</td>
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<td>5</td>
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<td>800</td>
<td>5</td>
<td>16</td>
<td>800</td>
<td>1600</td>
<td>10</td>
</tr>
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<td>6</td>
<td>300</td>
<td>600</td>
<td>5</td>
<td>17</td>
<td>550</td>
<td>1100</td>
<td>10</td>
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<td>600</td>
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</tr>
<tr>
<td>8</td>
<td>500</td>
<td>1000</td>
<td>10</td>
<td>19</td>
<td>Strands Cut</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>600</td>
<td>1200</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.2 Research Methods on Laterally Loaded Piles

Due to different objectives of research studies, laterally loaded piles were investigated in various methods, which can be generally classified into two categories: simplified methods and continuum-based methods. The former emphasize on capturing the problem in the most effective way with maximum engineering applicability, while the later discuss the problem in a more
physical-sounded way which allows one to see how a particular factor affect the pile lateral behavior.

The simplified methods basically model the soil through a set of independent linear or nonlinear springs attached to the piles, which provide resistance to the lateral movement or vertical movement of piles. In this type of approach, which also known as the “p-y” or “t-z” methods (Matlock and Reese, 1960; Reese et al., 1974), the load displacement curves of springs are assumed to be known a priori. These curves are obtained from observations of past experiments results. Continuum-based methods treat the soils surrounding piles as elastic or elasto-plastic continuum media. It allows one to investigate how the lateral performance of the pile is influenced by many particular issues such as soil-pile interaction, soil inelastic behavior or pore water dissipation.

In the following sections, both the simplified methods and continuum-based methods will be discussed and the recent works on continuum-based finite element analysis of laterally loaded piles will be briefly summarized.

2.2.1 P-y Method

The well-known p-y method is proposed by McClelland and Focht (1956) and developed by Reese and his coworkers (Matlock and Reese 1960; Matlock 1970; Reese et al. 1974). This method can conveniently incorporate the effect of soil nonlinearity by indicating that the soil resistance $p$ is nonlinear function of local lateral deflection $y$. This method has become widely used for design with the development of computer techniques for solving nonlinear fourth-order
differential equations and the remote-reading strain gauges for obtaining p-y curves from field experiments.

Figure 2.6 illustrated the analytical framework of the laterally loaded pile problem for p-y analysis. Based on this model, the governing differential equations can be obtained by analyzing the equilibrium of momentum of an infinite small element of the pile (Figure 2.7):

Figure 2.6: Model of laterally loaded pile (Reese, 1997)

Figure 2.7: Element for beam-column (Hetenyi, 1946)
\[
\frac{dM}{dx} + P_x \frac{dy}{dx} - V_v = 0
\]  \hspace{1cm} (2.1)

or

\[
\frac{d^2M}{dx^2} + P_x \frac{d^2y}{dx^2} - \frac{dV_v}{dx} = 0
\]  \hspace{1cm} (2.2)

Substituting

\[
\frac{d^2M}{dx^2} = E_p I_p \frac{d^4y}{dx^4}
\]

\[
\frac{dV_v}{dx} = p
\]

\[
p = E_{py} y
\]

into Eq. (2.2), the governing differential equation take the form:

\[
E_p I_p \frac{d^4y}{dx^4} + P_x \frac{d^2y}{dx^2} + E_{py} y = 0
\]  \hspace{1cm} (2.3)

here \( P_x \) = axial load on pile; \( y \) = lateral deflection of pile at point \( x \); \( p \) = soil resistance per unit length along pile; \( E_p I_p \) = flexural rigidity of the pile; \( E_{py} \) = soil reaction.

Some typical p-y curves for sand and clay are presented in Figure 2.8. Figure 2.8a represents the p-y curve for soft clay, proposed by Matlock (1970). The initial portion can be established by using \( k_h \). The middle portion of the curve is described by the equation

\[
\frac{p}{P_u} = \left( \frac{y}{y_{50}} \right)^{1/3}
\]

where \( P \) is the lateral soil resistance per unit length; \( P_u \) is the ultimate lateral soil resistance per unite length; \( y \) is the lateral displacement; \( y_{50} \) represents the lateral displacement corresponding to one half of the ultimate lateral soil resistance and can be computed according to \( y_{50} = 2.5 B \varepsilon_{50} \).
Reese et al. (1975) conducted a series of lateral load tests on overconsolidated stiff clay and proposed the typical p-y curve for stiff clay (Figure 2.8b). Mathematical description of the curve is presented below:

a. The initial straight portion: \[ P = k_h z y \]

b. The first parabolic portion (a-b): \[ P = 0.5P_u \left( \frac{y}{y_{50}} \right)^{0.5} \]

c. The second parabolic portion (b-c): \[ P = 0.5P_u \left( \frac{y}{y_{50}} \right)^{0.5} - 0.0055P_u \left( \frac{y}{y_{50}} \right)^{0.25} \]

d. The third inclined straight line: \[ P = 0.5P_c (6A_s)^{0.5} - 0.411P_c - P_c \left( \frac{0.0625y}{y_{50}} \right) (y - 6A_s y_{50}) \]

where \( A_s \) stands for the area correction factor.

Figure 2.8c illustrated typical p-y curves for sand, which described as Reese model (Reese et al. 1974). Similarly, the initial straight line is \( P = k_h z y \); The slope of linear line between \( u \) and \( m \) is \( m = \frac{P_u - P_m}{y_u - y_m} \); The parabolic section (k-m) can be obtained from \( C' = \frac{1}{y_m^{1/n}} \), where \( n = \frac{P_m}{m y_m} \) and \( C' = \frac{P_m}{y_m^{1/n}} \).

Based on the p-y approaches, several computer programs such as COM624P (Wang and Reese, 1993), LPILE (Reese et al., 1997) and FB-MultiPier (University of Florida, 2000) are developed to facilitate the analysis and design of laterally load piles. COM624P and LPILE program are constructed by finite difference method. Nonlinear behavior of soils based on prior established p-y curves are implemented into the softwares. Piles are simulated as beams with lateral stiffness and typically concrete pile stiffness is assumed nonlinear whereas steel pile stiffness is assumed linear. One significant deficiency of LPILE and COM624P program is that they are unable to analyze pile group foundation.
The FB-MultiPier, developed by University of Florida, is programmed based on finite element method for carrying out more flexible analysis of piles in different configurations and alignments. The FB-MultiPier program couples the nonlinear structural finite element analysis and the nonlinear static soil models for axial, lateral and torsional soil behavior, thus being capable of analyzing complex structure-foundation systems. Due to these appealing characteristics of the FB-MultiPier, it will be adopted in this study to analyze the lateral load test.
on M19 foundation and the results will be compared to those produced by ABAQUS model and measured in the field.

The P-y methods successfully take the nonlinearity of soil property into account and are quite versatile in practical applications. However, the simplification of a three-dimensional pile-soil interaction by one-dimensional spring element is a major disadvantage of these methods (Ahmadi and Ahmari, 2009).

### 2.2.2 Elastic Analytical Approaches

Continuum-based analyses are attractive for their direct physical basis. Continuum-based methods treat the soils surrounding piles as elastic or elasto-plastic continuums. Solutions of this kind of analysis are attempted by many scholars both analytically and numerically (e.g. Poulos, 1971a; Sun, 1994; Basu and Salagado, 2007).

The analytical solutions of continuum-based method are of highly complexity, thus only some analytical or semi-analytical elastic solutions were achieved so far. The first systematic analysis based on the theory of elasticity was reported by Poulos (1971 a, b). In this approach, the soil is idealized to be an isotropic homogeneous elastic material with constant elastic parameters $E_s$ and $\nu_s$, which are unaffected by the presence of the pile, while the pile is assumed to be a thin rectangular vertical strip with constant flexibility $E_p l_p$. As a compromise of accuracy and computational cost, the pile is divided into 21 portions, and each portion is assumed to have a uniform horizontal stress $p$ distributed along width and length. The solution of the problem starts from equating soil displacements and pile displacements. The former is
evaluated from the Mindlin equation for horizontal displacement due to a horizontal load within a semi-infinite mass, and the latter is obtained from the equation of flexure of a thin strip. Two necessary boundary conditions for solving the equations are considered: a free-head pile and a fixed-head pile. The major obstacle of application of this theory is the determination of the in-situ soil modulus $E_s$. Poulos suggests that such modulus should be determined from a full-scale pile loading test.

Sun (1994) developed an elastic solution for cylindrical pile based on the modified Vlasov model for the static analysis of beams on elastic foundations. The soil and the pile are idealized similarly to those by Poulos (1971). Slippage and separation at the pile/soil interface are also not allowed in the analysis. Express soil displacements in cylindrical coordinate:

$$u = F(z)\phi(r)\cos(\theta)$$
$$v = -F(z)\phi(r)\sin(\theta)$$
$$w = 0$$

where $F(z)$ is the displacement along the pile axis; and $\phi(r)$ is a dimensionless function representing the variation of the soil displacement in the $r$ direction. Then governing equations can be derived by the principle of virtual work. Assuming $\bar{F} = F/H$ and $\bar{z} = z/H$, the governing equation can be expressed as

$$\frac{d^4\bar{F}}{d\bar{z}^4} - 2t \frac{d^2\bar{F}}{d\bar{z}^2} + k\bar{F} = 0$$  \hspace{1cm} (2.4)$$

or

$$r^2 \frac{d^2\phi}{dr^2} + r \frac{d\phi}{dr} \left( \frac{\gamma}{R} \right)^2 r^2 \phi = 0$$  \hspace{1cm} (2.5)$$
here $\gamma$ is a nondimensional parameter that is considered to be a principal parameter in this research. The determination of parameter $\gamma$ requires iterative technique that was discussed by Vallabhan and Das (1988). Solving the differential equations with different boundary conditions using the above procedures, the closed-form solutions of dimensionless displacement of vertical pile in homogenous medium, $\bar{F}(\bar{z})$, was expressed in equations. In this research, the effect of various factors to principal parameter $\gamma$ was studied and discussed, and a detailed computing procedure is outlined for design engineers.

Later on, Basu and Salagado (2007) extend Sun’s analysis to multi-layered elastic continua, which can take into account an arbitrary number of soil layers. The governing differential equations were obtained using the principle of minimum potential energy, and were solved analytically through the method of initial parameters. Similarly, the principal parameter $\gamma$ in Eq. (2.5) is determined through iterative technique.

The advantage of above analytical or semi-analytical approaches is that the soils are treated in a more physically founded and mathematically rigid way. The obvious disadvantages are that they over simplified the soil property and can hardly take into account the nonlinear soil behavior as well as the gapping and slippage effect of pile-soil interface. Therefore, many scholars yield to numerical methods including finite element method, finite difference method and boundary element method to extend their interpretation of the problem by investigating the influence of soil nonlinearity and interface effect to the lateral response of pile. Among them, the
finite element method is considered to be the most powerful method in analyzing the laterally loaded pile problems.

### 2.2.3 Finite Element Method Approaches

Several representative works on recent approaches by finite element method are presented herein. Muqtadir and Desai (1986) developed a 3D FEM model to simulate axially and laterally loaded steel batter pile groups. Soil was governed by nonlinear (hyperbolic) elastic model, and a special “thin layer element” was adopted to simulate the debonding and slippage of pile soil interface. The constitutive law at normal direction for such interface elements is similar to those of surrounding soil, whereas tangential behavior is calibrated by direct shear test of steel-soil interface. The research reveals that the inclusion of soil non-linearity and interface effects can significantly affect the behavior of pile group.

Trochanis et al. (1991) focused on investigating the behavior of single pile subjected to combinations of both lateral and axial loads with incorporation of nonlinear soil behavior and the soil-pile interaction effects by means of 3D finite element elastoplastic model which is built on ABAQUS 4.6. It is stressed that the vast amount of information obtained from the 3D FEM model can provide the reader much insight into the behavior of pile foundations, whereas such information could not be easily gained in field tests. The soil materials is idealized as Drucker-Prager elastoplastic continuum, and modeled by 3D quadratic isoparametric 27-node elements, and 2D quadratic 18-node interface elements governed by modified Coulomb’s friction theory were adopted to account for pile-soil separation and slippage. The model is verified by the
comparison of the vertical and horizontal soil surface displacements to those obtained by Poulos and Davis (1980) and the field data from a test conducted in Mexico City. Both of them show good agreements. This study showed that the pile-soil slippage could produce dominating influence of the vertical response of the pile, while pile-soil separation leads to significant change on the lateral behavior of the pile. Both cases were more or less affected by soil plasticity.

Figure 2.9 and Figure 2.10 illustrate the effect of pile-soil separation and soil yielding on the lateral response of a single pile and the soil displacements in the vicinity of the pile.

![Figure 2.9](image1.png) ![Figure 2.10](image2.png)

**Figure 2.9:** Effect of pile-soil separation and soil plasticity on lateral head load-deflection behavior of single pile (Trochanis et al., 1991)

Another systematic study of laterally loaded piles via 3D nonlinear FEA was carried out by Brown and Shie (1990, 1991), which put more emphasis on comparison with empirical design procedures. The FEM model was constructed on ABAQUS 4.7. Two types of soil, clay and sand, were considered in this model. The Von Mises yield surface with associate flow rule was used for clay and the extended Drucker-Prager model with nonassociated flow was adopted for sand. Gapping and slippage of pile-soil interface were also simulated by 18-node interface elements.
The development of this FEM model stressed on reveal some limitations of currently analytical techniques and design procedures rather than being applied to practical uses directly. The results of von Mises model were compared to those given by subgrade reaction approaches by comparing the p-y curves given by the FE model and those obtained from the COM624 output using soft clay criteria (as shown in Figure 2.11). The comparison of the two set of p-y curves shows that the finite element results gave greater soil stiffness near the ground surface than the design procedures’. The authors suggested the following explanations:

1) The loading path that the soil near ground surface experienced is more close to triaxial extension than triaxial compression that has been used to develop empirical p-y curves.

2) The Von Mises constitutive model is basically unable to simulate saturated clay subjected to undrained loading.

![Figure 2.10: Effects of pile-soil separation and soil plasticity on horizontal soil surface displacements away from pile loaded laterally (Trochanis et al., 1991)](image-url)
Yang and Jeremic (2002) developed a FE model based on the OpenSees finite element framework to study the soil layering effect on laterally loaded piles. The FE model was used to generate the p-y curves and were compared to those obtained from the centrifuge test and design method (LPILE program). The constitutive models adopted in this research were simple von Mises model for clay and Drucker-Prager with nonassociated flow rule for sand. Interface effects are taken into account by implementing interface elements. The influence of layered soil on pile lateral response was investigated in four cases: 1) uniform clay; 2) clay-sand-clay; 3) uniform sand; 4) sand-clay-sand. They investigated the distribution of plastic zone by visualizing the plastified Gauss points. It was illustrated that for unified clay soil the plastic zone propagates very deep while does not extend far horizontally (Figure 2.12a), whereas for uniform sand case the plastic zone was more likely to propagate toward the surface (Figure 2.12b). The main findings of their study were 1) When a sand layer is present within a clay deposit, the increase in
lateral pressure in clay near the interface is confined to a narrow zone, up to two times of pile width, therefore the layering effect in this case is not prominent; 2) When a clay layer is present within a sand deposit, the reduction in pressures spread well into the sand layer (up to four times of pile width). The layering effects are of more importance in this case since the disturbance zone is large and the pressure reduction is significant. Reduction factors were given in terms of charts of pressure reduction versus the distance from the interface.

![Figure 2.12: The plastic zones by different soils (Yang and Jeremic, 2002)](image)

Recently, Ahmadi and Ahmari (2009) studied Brown’s works and developed their own finite element model to further investigate the two proposed reasons that account for the disagreement of FEM and p-y methods on soil stiffness near ground surface. They analyzed the different stress paths behind and in front of the pile, and pointed out that the soil anisotropy effect was responsible for the discrepancy of shear strength values subjected to different stress paths. Thus, an isotropic von Mises constitutive law incorporation different back-calculated
shear strength values was used to account for anisotropy soil behavior. The results showed that the displacements decrease more rapidly in the horizontal direction than in the vertical direction, which was also obtained by Yang and Jeremic (2004). The predicted displacements showed good agreement with field data. Some discrepancy between the predicted and field measured bending moment was attributed to inaccuracies in simulation of the initial stress, constitutive law, and back calculation procedure.

Abbas et al. (2009) studied the consolidation effect of laterally loaded pile problems. A 2D finite element program was employed in their study and produced the development of pile lateral displacement, lateral soil pressure and soil friction stress with respect to time. Biot (1941) consolidation equations were implemented in incremental form at the element level, which can be expressed as:

$$
\begin{pmatrix}
[k_m] & [c_m] \\
[c_m]^T & -\theta \Delta t [k_c]
\end{pmatrix}
\begin{pmatrix}
\Delta u \\
\Delta u_w
\end{pmatrix}
= 
\begin{pmatrix}
\Delta f \\
\Delta t [k_c] [u_w]_0
\end{pmatrix}
$$

(2.6)

where $[k_m]$ and $[k_c]$ are the solid stiffness and fluid conductivity matrices; $[c_m]$ is the coupling matrix; $\theta$ is using to interpolate time; $\Delta t$ is the calculation time step; $\{\Delta f\}$ represents the external load vector which may itself be time dependent; $u$ & $u_w$ are the displacement and access pore water pressure, respectively. Their results showed that the pile in cohesionless soil has more resistance in the rapid loading and less resistance in the long-term loading, while the pile in cohesive soil had the opposite property.

Detailed information of soil constitutive models and simulation scopes of above mentioned FEM works are organized and compared in Table 2.2.
Table 2.2: The FE models on laterally loaded piles in literatures

<table>
<thead>
<tr>
<th>References</th>
<th>Pile/soil interface model</th>
<th>Drained/Undrained condition</th>
<th>Pore-water pressure analysis</th>
<th>Soil type</th>
<th>Constitutive modeling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trochanis et al., 1991</td>
<td>Coulomb Friction</td>
<td>Undrained</td>
<td>N</td>
<td>Clay/ sand</td>
<td>Drucker-Prager</td>
</tr>
<tr>
<td>Yang and Jeremic, 2002</td>
<td>Coulomb Friction</td>
<td>Not specified</td>
<td>N</td>
<td>Sand &amp; Clay</td>
<td>Von Mises/Drucker-Prager</td>
</tr>
<tr>
<td>Abbas et al., 2009</td>
<td>No interface elements</td>
<td>Biot Consolidation</td>
<td>N</td>
<td>Not Specified</td>
<td>Mohr-Coulomb</td>
</tr>
<tr>
<td>Ahmadi and Ahmari, 2009</td>
<td>Coulomb Friction</td>
<td>Undrained</td>
<td>N</td>
<td>Clay</td>
<td>Von Mises</td>
</tr>
</tbody>
</table>

In summary, using three dimensional FEA for laterally loaded pile problems has the following advantages as compared to other approaches:

1) It can simulate the soil surrounding piles as 3D continuums rather than simplify it as a series of one dimensional springs.

2) It is able to capture the different elasto-plastic behavior of clay and sand soils rather than assume them to be pure elastic media.

3) It is able to realistically capture some essential features of the lateral loading problems. For example, the pile-soil interface behavior can be simulated using interface elements, which allow separation and slippage of the interface while pile is subjected to large lateral loads.

However, it is noticed that most of the past and recent numerical approaches suffer the problem of over simplifying the subsurface soil conditions of the pile foundations (e.g.,
homogenous soil or maximum three layers of soil was considered). More importantly, nearly all of previous works adopted some simple constitutive laws (such as Mohr-Coulomb or Von Mises) to simulating natural soils, which has already been criticized to be inadequate (Brown and Shie, 1990) to represent the actual behavior of clay surrounding the laterally loaded piles.

In our case, the subsurface investigations of the M19 pile of 1-10 Twin Span Bridge indicate that the dominant of the soil layers consist of soft or stiff clays. Thus the selection of a proper constitutive model for clay will lead to a big difference of the reliability of the FEA results. As the great improvement of computer speed in the past decade, a FEA with improved constitutive models for soils are desirable to achieve more reliable finite element results.
CHAPTER 3
SOIL CONSTITUTIVE MODELING

3.1 A Brief Review on Soil Mechanics

3.1.1 Effective Stress and Consolidation Theories

The starting point of classical soil mechanics should be the principle of effective stress (Terzaghi, 1936), which can be illustrated by establishing equilibrium across a wavy plane joining interparticle contacts (Figure 3.1). The principle of effective stress can be expressed as

\[ \sigma' = \sigma - p_w \]  

(3.1)

where \( \sigma' \) is the effective stress, \( \sigma \) is the total stress and \( p_w \) is the pore water pressure.

Figure 3.1: Illustration of the principle of effective stress (Lambe & Whitman, 1979)

Such simple illustration highlights several basic principles that underlying classical soil mechanics (Gens, 2010): the material is multiphase; the microstructure is implicitly considered; a new variable, pore water pressure, is incorporated; and there is coupling between mechanical variables (stresses) and hydraulic variables (pore pressures).

One of the most critical issue in classical soil mechanics is to describe the consolidation behavior of soils. The pioneer work on one-dimensional consolidation theory was accomplished
by Terzaghi and Frolich (1936). The general three-dimensional consolidation theory was
established by Biot (1941). Assuming the soil to be isotropic and linear elastic, the consolidation
equation can be expressed as

\[
\frac{k}{\gamma_w m} \nabla^2 p_w = \frac{k}{\gamma_w m} \left( \frac{\partial^2 p_w}{\partial x^2} + \frac{\partial^2 p_w}{\partial y^2} + \frac{\partial^2 p_w}{\partial z^2} \right) = \frac{\partial p_w}{\partial t} - \frac{1}{3} \frac{\partial p_t}{\partial t}
\]  

(3.2)

where \( k \) is the permeability of the soil; \( \gamma_w \) is the unit weight of water; \( m \) is the inverse of the
bulk modulus; \( p_t \) is the total stress.

Biot’s consolidation equation is a result of the combination of a set of balance equations
(solid mass balance, water mass balance and equilibrium) and constitutive equations that
governing water flow and soil deformation. In fact the constitutive equations adopted in this
theory are very simple:

- Darcy’s law for water flow:

\[
q_w = -k \nabla (p_w + \gamma_w z)
\]

(3.3)

- Isotropic linear elasticity for soil skeleton:

\[
d\epsilon_v = m d\sigma' = \frac{1}{K} d\sigma'
\]

(3.4)

Obviously, the isotropic linear elasticity is inadequate in representing complex soil behavior.

The blank of soil elasto-plastic model has been well filled by the later development of critical
state soil mechanics (CSSM) (Roscoe et al., 1958; Roscoe and Burland, 1968; Schofield and
Wroth, 1968). CSSM provide a complete and unified framework that brought together many key
features of saturated soil behavior, such as shear and volumetric behavior, strength, dilatancy and
yielding, in a consistent manner. Based on the framework of CSSM, the Cam Clay model was
developed for mathematically describing the complex elastoplastic behavior of soils. Such model has been modified later (Roscoe and Schofield, 1963) into Modified Cam Clay Model (MCCM) by redefining the shape of the yield surface. The model itself is very attractive due to its simplicity; hence it has been widely used both in engineering practice and academic study. The MCCM also provide a foremost framework that can be extended to describe other features such as soil anisotropy (e.g. Ohta and Hata, 1971; Banerjee and Yousif, 1986; Dafalias, 1987), thermo behavior (e.g. Graham et al., 2001; Cui et al., 2000) and cyclic behavior (e.g. Dafalias and Hemann, 1980; Yu et al., 2006). Before extending our discussion to higher soil constitutive models, it is necessary to give a brief review of MCCM.

### 3.1.2 Critical State Soil Mechanics

One should firstly consider the volumetric behavior of clay under normal compression and unloading-reloading conditions, as shown in Figure 3.2a. It is often found that the linearity of normal compression lines and unloading-reloading lines in the compression plane is improved if data are plotted with a logarithmic scale for the mean stress axis. Mathematically, the normal compression line (ncl) takes the form:

$$ v = v_\lambda - \lambda \ln p' $$  \hspace{1cm} (3.5)

and the unloading-reloading line (url) can be expressed as:

$$ v^e = v_\kappa - \kappa \ln p' $$  \hspace{1cm} (3.6)

where $\lambda$ and $\kappa$ are slope of the two lines and $v_\lambda$ and $v_\kappa$ are the intercepts on the lines at $p' = 1$; $v$ is specific volume; $p'$ represents mean effective stress.
It is very important to notice that the unloading-reloading line actually represents a nonlinear elastic behavior (often referred as porous elastic), which differs from the linear elasticity of metal. Thus it is convenient to discuss clay’s elastic response in incremental form:

\[
\begin{bmatrix}
\frac{d\varepsilon_p}{d\varepsilon_q}
\end{bmatrix} = \begin{bmatrix}
1/K & 0 \\
0 & 1/3G
\end{bmatrix}\begin{bmatrix}
dp' \\
dq
\end{bmatrix}
\]  

where \( p' \) and \( q \) are effective mean stress and deviatoric stress; \( \varepsilon_p \) and \( \varepsilon_q \) are volumetric strain and deviatoric strain; \( K \) and \( G \) are tangent bulk modulus and shear modulus, respectively.

Obviously, the bulk modulus and the shear modulus of clay are dependent on current mean effective stress and specific volume, which can be obtained by taking derivative of both side of Eq. (3.6):

\[ K = \frac{vp'}{\kappa} \]  

A complete elastoplastic model needs a yield surface to separate the elastic and plastic regime. The yield surface of MCCM is an ellipse as illustrated in Figure 3.2b.

\[ \text{Figure 3.2: a) Normal compression line and unloading-reloading line in } \nu - \ln p' \text{ plane; b) Elliptical yield surface for Modified Cam Clay model in } p' - q \text{ plane.} \]
The corresponding yield function is expressed by

\[ f = q^2 - M^2 [p'(p'_0 - p')] = 0 \]  

(3.9)

where \( M \) is the slope of critical state line on \( p-q \) plane; \( p'_0 \) is the pre-consolidation pressure.

The shape and size of the yield surface is determined by both \( M \) and \( p'_0 \). However, only pre-consolidation pressure \( p'_0 \) controls the expansion (or shrinkage) of the yield surface, which means the yield surface can change size but keeps constant shape. In this way, the expansion of the yield surface and the hardening of the soil are linked with the normal compression of the soil.

The relation between \( p'_0 \) and specific volume \( v \) completely follows the normal consolidation line which can be expressed the same as Eq. (3.5). Rewriting Eq. (3.5) and Eq. (3.6) in incremental form and replace specific volume \( v \) by volumetric strain, one obtains the hardening law for MCCM:

\[
\begin{align*}
\frac{\partial p'_0}{\partial \varepsilon^p} &= \frac{v p'_0}{\lambda - \kappa} \\
\frac{\partial p'_0}{\partial \varepsilon^q} &= 0
\end{align*}
\]  

(3.10)

The formulation of MCCM has been completed now. The model can successively predict the response of soil samples in conventional triaxial drained and undrained compression tests. Besides, the model itself is simple, and the corresponding parameters \( \kappa, v, M \) and \( \lambda \) have direct physical meanings and can be easily obtained from triaxial or oedometer tests.
3.2 Anisotropic Modified Cam-Clay Model

3.2.1 Clay Anisotropy

Experimental results have shown that the yield surface for naturally deposited clay tends to align along the $K_0$ consolidation line (e.g., Graham et al., 1983). Banerjee et al. (1981) distinct such soil anisotropy into two types: the “inherent anisotropy” comes from the formation of the soil, and the “induced anisotropy” occurs during subsequent loading.

Stems from this finding, several researchers proposed anisotropic models based on extension of Modified Cam-Clay Model (Ohta and Hata, 1971; Banerjee and Yousif, 1986; Dafalias, 1987). Ohta and Hata (1971) presented an anisotropic model for normally consolidated clay. The yield surface can be inclined at the origin of the stress space at the before shearing but it does not include rotational hardening when subjected to consecutive shearing. Banerjee and Yousif (1986) developed an incremental plasticity theory to describe the mechanical behavior of anisotropically consolidated clays. The yield ellipse is allowed to initially align along the $K_0$ line and rotate, when it is subjected to shearing, following isotropic and anisotropic hardening law. Moreover, the concept of bounding surface is introduced in order to simulate the inelastic material behavior under monotonic and cyclic loading. Started with a different energy dissipation assumption, Dafalias (1987) developed an Anisotropic Modified Cam Clay Model (AMCCM) which extended the Modified Cam-Clay Model by introducing an anisotropic variable $\alpha$ and two anisotropic parameters $c$ and $x$. This model is attractive due to its simplicity and the effectiveness in capturing both “inherent” and “induced” anisotropic of clay. Hence, the AMCCM by Dafalias
(1987) will be implemented into UMAT in this study and then be assigned as the clay constitutive model in the FEA of a batter pile group foundation.

3.2.2 Formulation of AMCCM

In triaxial space, the formulation of the yield surface of traditional Modified Cam-Clay Model started from the plastic work dissipation assumption Eq. (3.11) proposed by Burland (1965):

\[ p d\varepsilon_p^p + q d\varepsilon_q^p = p \sqrt{(d\varepsilon_p^p)^2 + (M d\varepsilon_q^p)^2} \]  \hspace{1cm} (3.11)

Note that all the stresses we discussed in this chapter are effective stresses, so the prime used to denote “effective” is omitted in this section. Dafalias (1987) added a non-dimensional anisotropic variable \( \alpha \) into the plastic work dissipation assumption to account for the effect of internal residual stresses and the coupling of \( \varepsilon_p^p \) and \( \varepsilon_q^p \). Eq. (3.11) becomes:

\[ p d\varepsilon_p^p + q d\varepsilon_q^p = p \sqrt{(d\varepsilon_p^p)^2 + (M d\varepsilon_q^p)^2 + 2\alpha d\varepsilon_p^p d\varepsilon_q^p} \]  \hspace{1cm} (3.12)

Rearranging (3.12) and applying normality rule one obtains:

\[ \frac{dq}{dp} = \frac{M^2 - \eta^2}{2(\eta - \alpha)} \]  \hspace{1cm} (3.13)

Noticing \( q = \rho \eta \), we have \( dq = d\rho \eta + p d\eta \). Substituting \( dq \) in (3.13) and rearranging the terms:

\[ \frac{-dp}{p} = \frac{d\eta}{\eta + 2(\eta - \alpha)} \]  \hspace{1cm} (3.14)

Integrate both sides of (3.14):

\[ \frac{p}{p_0} = \frac{M^2 - \alpha^2}{M^2 - 2\alpha \eta + \eta^2} \]  \hspace{1cm} (3.15)

Replacing \( \eta \) with \( q/p \) and rearranging the terms, one obtains the yield function:

\[ f = p^2 - pp_0 + \frac{1}{M^2} (q^2 - 2\alpha pq + \alpha^2 pp_0) = 0 \]  \hspace{1cm} (3.16)
which is the identical to the yield function of AMCCM. Dafalias (1987) proposed an evolution
equation for anisotropic variable $\alpha$:

$$d\alpha = \langle \lambda \rangle \frac{1 + e_{in}}{\lambda - \kappa} \left| \frac{\partial f}{\partial p} \right| \frac{c}{p_0} (q - x\alpha)$$

(3.17)

where the $x$ and $c$ are two new model constants. The value of $x$ controls the ultimate level of
anisotropy and $c$ controls the pace at which such anisotropy develops. When setting $c=0$ and
initial $\alpha = 0$, the model degenerate into original Modified Cam-Clay Model.

An illustration of the yield surface is presented in Figure 3.3. The yield ellipse is rotated and
distorted comparing to the yield locus of MCCM. The degree of such rotation and distortion is
controlled by $\alpha$. Several characteristic points are shown in this figure. The normal of the ellipse
at points $C$ and $C'$ are along $q$-axis and also are the intersection point between yield locus and
critical state line. The normal of the ellipse at points $O$ and $A$ are along $p$-axis with point $A$
satisfying $\eta = q/p = q_0/p_0 = \alpha$. It should be noticed that the value $p_0$ is no more denoting the
intersection of yield locus and $p$-axis but is the projection of point $A$ on $p$-axis.

Equation (3.16) and (3.17) are discussed under the triaxial loading space. Generalization of
the above works into multiaxial stress space can be achieved by replacing the scalar $\alpha$ by a
second-order tensor $\alpha_{ij}$ and denoting:

$$\alpha = \sqrt{\frac{3}{2}} \alpha_{ij} \alpha_{ij}, \quad q = \sqrt{\frac{3}{2}} s_{ij} s_{ij}$$
Then the generalized AMCCM yield function and hardening law for $\alpha_{ij}$ is presented as follows (Dafalias, 1987):

$$f = p^2 - pp_0 + \frac{3}{2M^2}((s_{ij} - p\alpha_{ij})(s_{ij} - p\alpha_{ij}) + (p_0 - p)p\alpha_{ij}\alpha_{ij}) = 0$$ (3.18)

$$d\alpha_{ij} = (\lambda) \frac{1 + e_0}{\lambda - k} \left| tr \frac{\partial f}{\partial \sigma_{mn}} \right| \frac{c}{p_0} (s_{ij} - xp\alpha_{ij})$$ (3.19)

3.3 Implementation of AMCCM

3.3.1 The Structure of Nonlinear Finite Element Method

The displacement-based implicit nonlinear finite element method such as ABAQUS/STANDARD solver contains a Full/Quasi Newton iteration procedure to solve the global equilibrium equations. The solution scheme is schematically presented in Figure 3.4.
As summarized by Hashash and Whittle (1992), the implicit nonlinear finite element method contains a core procedure:

\[ K_{n+1}^{i-1} \Delta U_{n+1}^{i-1} = R_{n+1} - F_{n+1}^{i-1} \]  

\[ U_{n+1}^{i+1} = U_{n+1}^{i-1} - \Delta U_{n+1}^{i-1} \]

with initial conditions:

\[ K_0^{n+1} = K_n, F_0^{n+1} = F_n, U_0^{n+1} = U_n \]

where \( n \) is the increment step number; \( i \) is the iteration number of current step; \( K \) is the global tangent stiffness matrix; \( U \) is the nodal displacement vector, \( R \) is the vector of the applied nodal forces and \( F \) is the vector of nodal force due to stresses in the element. The term \( R_{n+1} - F_{n+1}^{i-1} \) at
the right hand side of Eq. (3.20) is often recognized as residual load vector. Every global Newton iteration start with a predicted incremental nodal displacement $\Delta U$. Such $\Delta U$ is passed into each element and processed with different interpolation methods (which depend on the type of the element) to obtain the incremental strains $\Delta \varepsilon$ at each Gauss point. Then the material constitutive equations will be integrated via some algorithms with respected to this strain increment $\Delta \varepsilon$ at each Gauss point. As a requirement of the global Newton iteration, the output of the material level calculation must contain the updated stress and the Jacobian matrix $J = \frac{\partial \Delta \sigma}{\partial \Delta \varepsilon}$. Finally the updated stress and Jacobian will be assembled into the global nodal force vector and global tangent stiffness matrix for the next iteration.

The user-subroutine UMAT is a user interface provided by ABAQUS that allows customers to use self-defined constitutive relations at material calculation level. In this research, the AMCCM will be implemented into the UMAT and then the subroutine will be numerically tested by a series of triaxial compression procedure under constant volume (CV) or constant pressure (CP) conditions.

**3.3.2 Integration of Elastoplastic Equations**

Numerically implementing the Cam Clay plasticity has been attempted by many researchers. Only several selected works are presented herein. Borja and Lee (1990) firstly implemented the Modified Cam-Clay Model under the framework of implicit return mapping algorithm with closest point projection for associative flow rule and central return mapping for non-associative flow rule. In the following work, Borja (1991) point out that an ‘average’ bulk modulus along
with an iterative scheme are required to calculate the nonlinear elastic behavior described in modified cam clay model. Hashash and Whittle (1992) also developed an integration algorithm for the Modified Cam-Clay Model using return mapping algorithm with general closest point projection algorithm. Both Borja and Lee (1990) and Hashash and Whittle (1992) emphasized the use of consistent tangent stiffness matrix (Simo and Taylor, 1985), which has been proved that it can guarantee a quadratic convergence for the global Newton iteration, in their works. Devi and Singh (2008) implemented the MCCM in an Object-Oriented FE system. Yamakawa et al. (2010) developed a return mapping algorithm of the subloading surface Cam-clay model applicable to large deformation analysis.

The use of implicit return mapping algorithm is conceptually appealing for its accuracy and unconditional stability. However, such method requires second order derivatives of the yield function, which will result in tedious Jacobian for a complex constitutive relation. One can see that just for Modified Cam-Clay model it becomes fairly complicated of the resultant consistent tangent matrix and the Jacobian for local Newton iteration. Noticing that the AMCCM has one more anisotropic variable $a_{ij}$ in its constitutive equations, it is desirable to adopt a more straightforward explicit integration algorithm to develop a sufficient robust and efficient code which can work with a 68,000-element FEA model. Sloan (1987) proposed an explicit substepping scheme for implementing general elastoplastic relations. Modified Euler algorithm is used in order to achieve a controlled error during the integration of the constitutive equations. This method has been concluded to have superior robustness and efficiency than implicit
achievements when applied to Modified Cam-Clay Model (Potts and Ganerdra, 1992; 1994). Sloan et al. (2001) refined this method with adding elastoplastic unloading algorithm and extending the applicability to Cam-Clay plasticity. In this work an explicit integration algorithm will be developed within the framework of substepping scheme for AMCCM.

### 3.3.3 Sub-Stepping Algorithm for AMCCM

The complete pseudo algorithm for implementing AMCCM can be found in Appendix I. In the following sections we will focus on discussing some key issues in realizing the AMCCM via explicit sub-stepping algorithm.

#### 3.3.3.1 Elastic Loading and Determination of Initial yielding

As introduced in section 3.1, clay exhibits nonlinear behavior even in elastic regime. Recall the tangent bulk modulus that relates infinite small stress and strain increments

\[ K = \frac{\delta p'}{\delta \varepsilon_p^e} = \frac{vp'}{\kappa} \]  

(3.22)

As indicated by Borja (1991) and Sloan et al. (2001), such tangent modulus cannot be directly utilized in numerically implementing the elastic response of MCCM since the calculation is over a finite time step \(\Delta t\) during which the stress and strain increments are so large that the nonlinearity of \(K\) cannot be ignored. Rearranging (3.22):

\[ \delta \varepsilon_p^e = \kappa \frac{\delta p'}{vp'} \]  

(3.23)

and integrating its both side over time span \((t_n, t_{n+1})\), one obtains:

\[ \Delta \varepsilon_p^e = \frac{\kappa}{v} \ln \frac{p'_{n+1}}{p'_n} \]  

(3.24)
Rearrange (3.24) equation and substitute $v = 1 + e$ where $e$ is the void ratio:

$$ p'_{n+1} = p'_{n} e^{\frac{1+e}{\kappa} \Delta \varepsilon^e} = p'_{n} + \left( -p'_{n} + p'_{n} e^{\frac{1+e}{\kappa} \Delta \varepsilon^e} \right) $$

$$ p'_{n+1} = p'_{n} + \left[ \frac{p'_{n}}{\Delta \varepsilon^e} \left( -1 + e^{\frac{1+e}{\kappa} \Delta \varepsilon^e} \right) \right] \Delta \varepsilon^e $$

(3.25)

Introducing secant bulk modulus:

$$ \bar{K} = \frac{p'_{n}}{\Delta \varepsilon^e} \left( e^{\frac{1+e}{\kappa} \Delta \varepsilon^e} - 1 \right) $$

(3.26)

then Eq. (3.26) becomes the familiar form as traditional linear elasticity:

$$ p'_{n+1} = p'_{n} + \bar{K} \Delta \varepsilon^e $$

(3.27)

This secant bulk modulus $\bar{K}$ actually represents the averaged changing of $K$ over $(t_n, t_{n+1})$. Assuming Poisson’s ratio $v$ to be a constant throughout the elastic regime, the secant shear modulus $\bar{\sigma}$ can be obtained using:

$$ \bar{\sigma} = \frac{3\bar{K}(1-2v)}{2(1+v)} $$

(3.28)

Keep in mind that both $\bar{K}$ and $\bar{\sigma}$ are functions of the void ratio $e$ and the stress state $\sigma$ at the beginning of the current step and the strain increment $\Delta \varepsilon$ of this step. Hence the assembled secant elastic stiffness matrix $\bar{C}$ is also a function of $e$, $\sigma$ and $\Delta \varepsilon$:

$$ \bar{C} = \bar{C}(e, \sigma, \Delta \varepsilon) = \begin{bmatrix} \bar{K} + \frac{4}{3} \bar{\sigma} & \bar{K} - \frac{2}{3} \bar{\sigma} & \bar{K} - \frac{2}{3} \bar{\sigma} \\ \bar{K} - \frac{2}{3} \bar{\sigma} & \bar{K} + \frac{4}{3} \bar{\sigma} & \bar{K} - \frac{2}{3} \bar{\sigma} \\ \bar{K} - \frac{2}{3} \bar{\sigma} & \bar{K} - \frac{2}{3} \bar{\sigma} & \bar{K} + \frac{4}{3} \bar{\sigma} \end{bmatrix} $$

(3.29)
3.3.3.2 Yield Surface Intersection

Stresses can be updated using the secant stiffness matrix $\bar{C}$ defined in Eq. (3.29) for a given strain increment $\Delta \varepsilon$ if the stress path is completely in the elastic realm. When the strain increments $\Delta \varepsilon$ causes current stress state exceeds the yield surface, it is necessary to find the intersection point of the stress path with the yield surface. Introducing a multiplier $\beta$ that controls the strain increments, such problem is equivalent to find a $\beta_{int}$ that satisfying:

$$f((\sigma + \beta_{int} \bar{C}: \Delta \varepsilon), \alpha, p_0) = 0 \quad (3.30)$$

For $\beta < \beta_{int}$ the stress state is inside the yield surface while for $\beta > \beta_{int}$ the stress state is outside the yield surface where integration algorithm should be applied. Since no yielding occurs within elastic regime, the anisotropic variable $\alpha$ and preconsolidation pressure $p_0$ will not develop and hence can be treated as constant. An algorithm for finding the intersection point based on the work of Sloan et al. (2001) is developed and presented in Appendix I. The difference here is that the method of false position is used for determining $\beta_{int}$ instead of the Pegasus algorithm.

3.3.3.3 Elastoplastic Loading

The constitutive equations of AMCCM can be summarized in tensor form (where a tensor is denoted in a bold letter instead of its components) as follows:

- Yield function:
  $$f = p^2 - pp_0 + \frac{3}{2M^2} ((s - p\alpha):(s - p\alpha) + (p_0 - p)p\alpha:\alpha) = 0 \quad (3.31a)$$

- Hardening laws:
\[
\begin{align*}
\delta \alpha &= (d\lambda) \frac{1 + e}{\lambda - \kappa} \left| tr \left( \frac{\partial f}{\partial \sigma} \right) p_0 \right| (s - x p \alpha) \quad (3.31b) \\
p_0 &= \frac{1 + e}{\lambda - \kappa} p_0 \delta \epsilon_p^p = (d\lambda) \frac{1 + e}{\lambda - \kappa} p_0 \left| tr \left( \frac{\partial f}{\partial \sigma} \right) \right| \\
\end{align*}
\]

- Flow rule:
\[
\delta \epsilon^p = (d\lambda) \frac{\partial f}{\partial \sigma} \quad (3.31d)
\]

A key point in explicitly implementing AMCCM is the derivation of the explicit expression for loading index \( d\lambda \). According to consistency condition:

\[
df(\sigma, \alpha, p_0) = \frac{\partial f}{\partial \sigma} : d\sigma + \frac{\partial f}{\partial \alpha} : d\alpha + \frac{\partial f}{\partial p_0} d p_0 = 0 \quad (3.32)
\]

Noticing \( d p_0 \) is a function of \( \delta \epsilon_p^p \), one can apply chain rule to the third term. Denoting \( B = \frac{\partial f}{\partial \sigma} \), Eq. (3.32) becomes:

\[
df = B : d\sigma + \frac{\partial f}{\partial \alpha} : d\alpha + \frac{\partial f}{\partial \epsilon_p^p} d\epsilon_p^p = 0 \quad (3.33)
\]

Applying additive decomposition of the incremental strain \( \delta \epsilon = \delta \epsilon - \delta \epsilon^p \) and flow rule, the constitutive equation can be rewrite as:

\[
\delta \sigma = \overline{C} : (\delta \epsilon - \delta \epsilon^p) = \overline{C} : \delta \epsilon - d\lambda \overline{C} : B \quad (3.34)
\]

Combining Eqs. (3.33) and (3.34), the loading index \( d\lambda \) can be derived as follows:

\[
d\lambda = \frac{B : \overline{C} : \delta \epsilon + \frac{\partial f}{\partial \alpha} : d\alpha}{B : \overline{C} : B - \frac{\partial f}{\partial \epsilon_p^p} tr B} \quad (3.35)
\]

One can notice the presence of the term \( d\alpha \) in the right hand side of Eq. (3.35). On the other hand, \( d\alpha \) also partially depend on the value of \( d\lambda \) according to Eq. (3.31b). Substituting Eq. (3.31b) into Eq. (3.35), the completely explicit expression of the loading index \( d\lambda \) is expressed as follows:
\[
d\lambda = \frac{B: \overline{C}: d\varepsilon}{B: \overline{C}: B - \frac{\partial f}{\partial \varepsilon_p^0} trB - \frac{1 + e_0^0}{\lambda - \kappa} |trB| \frac{c}{p_0} \frac{\partial f}{\partial \alpha}: (s - \nu \alpha)} 
\] (3.36)

The terms \( B, \frac{\partial f}{\partial \alpha} \) and \( \frac{\partial f}{\partial \varepsilon_p^0} \) are derived by Voyiadjis and Song (2000) and are rewritten in tensor forms here as follows:

\[
B = \left( \frac{2p - p_0}{3} \right) 1 + \frac{1}{2M^2} [(p_0 \alpha - 2s): \alpha] 1 + \frac{3}{M^2} [(s - p \alpha) - \frac{1}{3} tr(s - p \alpha) 1] \quad (3.37a)
\]
\[
\frac{\partial f}{\partial \alpha} = \frac{3}{M^2} (p p_0 \alpha - p s) \quad (3.37b)
\]
\[
\frac{\partial f}{\partial \varepsilon_p^0} = \frac{\partial f}{\partial p_0} \frac{\partial p_0}{\partial \varepsilon_p^0} = \left[ \left( -1 + \frac{3}{2M^2} \alpha: \alpha \right) \frac{1 + e}{\lambda - \kappa} p p_0 \right] \quad (3.37c)
\]

### 3.3.3.4 Yield Surface Correction

For explicit integration algorithms, the stresses may drift away from the yield surface at the end of each substep. This kind of drifting may be very small compared to the stress increment in that step but can accumulate to a large scale of error after thousands of steps. Sloan et al. (2001) proposed a combined consistent correction and normal correction scheme which provides an enhanced stability of the whole correction procedure. Such algorithm is adopted in the present work.

Firstly the uncorrected stresses and hardening parameters will be processed through a consistent correction scheme. The 1st Taylor polynomial of the yield function \( f \) about \( (\sigma, \alpha, p_0) \) is:

\[
f = f_0 + \frac{\partial f}{\partial \sigma} : \delta \sigma + \frac{\partial f}{\partial \alpha} : \delta \alpha + \frac{\partial f}{\partial p_0} \delta p_0 = 0 \quad (3.38)
\]

Here \( \delta \sigma, \delta \alpha \) and \( \delta p_0 \) will be viewed as a small correction to the current \( \sigma, \alpha \) and \( p_0 \). Such corrections make the change of stress and hardening parameters together while remaining
the total strain increment $d\varepsilon_{kl}$ unchanged, which is consistent with the philosophy of the
displacement finite element procedure (Potts and Gens, 1985). Assume a correction index $\delta \lambda_c$
that defined as:

$$
\delta \varepsilon^p = \delta \lambda_c \frac{\partial f}{\partial \sigma}
$$

(3.39)

Similar to Eq. (3.34) and noticing $\delta \varepsilon = 0$, one obtains the stress correction:

$$
\delta \sigma = -\delta \lambda_c \bar{C} : B
$$

(3.40)

The hardening parameter corrections can be simply obtained by replacing $d\lambda$ by $\delta \lambda_c$ in to
Eqs. (3.31b) and (3.31c):

$$
\delta \alpha = \delta \lambda_c \frac{1 + e}{\lambda - \kappa} \left| \text{tr} \frac{\partial f}{\partial \sigma} \right| \frac{c}{p_0} (s - xp\alpha)
$$

(3.41)

$$
\delta p_0 = \frac{1 + e}{\lambda - \kappa} p_0 \varepsilon_p \varepsilon_p = \delta \lambda_c \frac{1 + e}{\lambda - \kappa} p_0 \left| \text{tr} \frac{\partial f}{\partial \sigma} \right| (s - xp\alpha)
$$

(3.42)

Substituting (3.39)-(3.42) into (3.38), the expression for the correction index is obtained as:

$$
\delta \lambda_c = \frac{f_0}{B : \bar{C} : B - \frac{\partial f}{\partial \varepsilon_p} \text{tr} B - \frac{1 + e_0}{\lambda - \kappa} |\text{tr} B| \frac{c}{p_0} \frac{\partial f}{\partial \alpha} : (s - xp\alpha)}
$$

(3.43)

The correction of the stresses and hardening parameters can be proceeded using $\delta \sigma_{ij}$, $\delta p_0$
and $\delta \alpha_{ij}$ evaluated by $\delta \lambda_c$ from Eq. (3.43).

If the previous consistent correction scheme cannot achieve a convergence, such method
will be abandoned for one step and use the backup normal correction scheme (Sloan et al., 2001).
The hardening parameters $p_0$ and $\alpha$ are assumed to be constant and only stresses are corrected
back to the yield surface using formula:

$$
\delta \sigma = -\frac{f_0 B}{B : B}
$$

(3.44)
3.4 Numerical Testing

Firstly, the correctness of the implemented AMCCM will be tested by comparing the stress path, stress-strain curve and volumetric behavior with the corresponding curves produced by the inbuilt MCC model provided by ABAQUS. Then its capability of capturing soil anisotropy will be checked by comparing the yield locus of AMCCM with different anisotropic parameters.

Two types of triaxial tests are carried out under an axial strain of 10%. The CV test is aimed to simulate the undrained condition, while CP test is for drained condition. Table 3.1 summarized the parameters adopted in the numerical tests which are arbitrarily chosen from the common values of these parameters of medium clay.

<table>
<thead>
<tr>
<th>$e_{ini}$</th>
<th>$p_{0_{ini}}$ (psf)</th>
<th>$\kappa$</th>
<th>$\lambda$</th>
<th>$M$</th>
<th>$v$</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>100</td>
<td>0.05</td>
<td>0.2</td>
<td>0.9</td>
<td>0.2</td>
<td>1.33</td>
</tr>
</tbody>
</table>

3.4.1 Comparison with ABAQUS Inbuilt MCCM

Setting $a_{ini} = 0$ and $c = 0$, there is neither initial anisotropy nor evolution of $a$, hence AMCCM is reduced to classical MCCM (denote as UMAT-MCC). All tests will be run on the samples with two different initial mean stresses $p_{ini} = 30$ and $p_{ini} = 80$ in order to observe the yielding behavior from both wet side and dry side. The same thing will be done using ABAQUS inbuilt MCC model (denote as ABA-MCC) with identical material parameters. The stress paths of all the eight numerical tests are presented in Figures 3.5. The corresponding $q - \varepsilon_q$ curves and the $e - \varepsilon_q$ curves of the CP test are illustrated in Figure 3.6 and 3.7 respectively.
Figure 3.5: Stress paths predicted by the UMAT-MCC and ABA-MCCM

Figure 3.5 demonstrates a perfect match of the stress paths calculated by the UMAT and the inbuilt MCCM in all four cases. This figure is presented via MATLAB software and the yield ellipse is mathematically drawn according to the MCCM yield function Eq. (3.9) with input parameters $M=0.9$ and $p_0 = 100$. It can be seen that the sample entered the plastic zone exactly at the moment the stress paths touch the ellipse. The successive yield surfaces are expanding when the sample yields from the wet side, while they are shrinking when the specimen yields from dry side.

The stress strain curves given by UMAT-MCC and inbuilt ABA-MCCM shown in Figure 3.6 exhibit some discrepancies in elastic regime. The UMAT-MCC indicates a stiffer elastic response than the ABA-MCC and thus these curves achieve their peaks faster.
Constant Volume Tests

(a) Constant Volume Tests

Stress-strain curves predicted by the UMAT-MCC and ABA-MCC

(b) Constant Pressure Tests

Figure 3.6: Stress-strain curves predicted by the UMAT-MCC and ABA-MCC
One possible explanation is that these differences are due to the use of secant modulus in the elastic part calculations. According to ABAQUS Theory Manual 4.4.1 Porous elasticity, the shear modulus for porous material with zero tensile strength is:

\[ G = \frac{3(1 - 2\nu)(1 + e_0)}{2(1 + \nu)\kappa} p e^{\varepsilon_p^e} \]  

(35)

Comparing to Eq. (19), one can find the expression of bulk modulus adopted by ABAQUS is:

\[ K = \frac{1 + e_0}{\kappa} p e^{\varepsilon_p^e} \]  

(36)

which has been called “instantaneous” modulus in the Manual. This modulus clearly differs from the secant modulus adopted in this research. As we explained earlier the tangent bulk modulus \( K \) is a function of \( \nu, p' \) and \( \kappa \). With a constant \( \kappa \) and \( \nu \), increased \( p' \) will result in a stiffer modulus. Hence, the use of a secant modulus (averaged over the step) instead of the tangent modulus (evaluated at the beginning of that step) will produce a stiffer stress-strain response. After the sample starting plastic deformation, good agreements are observed at both peak stresses and stresses at critical state.

The volumetric behaviors predicted by both UMAT-MCC and ABA-MCC under the constant pressure tests show excellent agreement (Figure 3.7). It is observed that for lightly and heavily overconsolidated clay, the volumetric behavior in elastic regime is almost the same. However, as soon as the plastic strain develops, the former will continue shrinking in volume while the latter exhibit volumetric dilation.
3.4.2 Simulating Anisotropic Behavior

Both triaxial compression and extension tests under CV condition are performed to observe the anisotropic behavior. The use of CV condition is for the easiness of distinguishing the elastic and elastoplastic segments on \( p - q \) plane. The following cases are considered:

- No anisotropy: \( \alpha_{int} = 0, c = 0 \), CV compression;
- Only induced anisotropy: \( \alpha_{int} = 0, c = 1 \), CV compression;
- Both inherent and induced anisotropy: \( \alpha_{int} = 17^\circ, c = 1 \), CV compression+ extension.

Notice that the \( \alpha_{int} = \sqrt{\frac{3}{2}} \alpha_{int} : \alpha_{int} = 17^\circ \) is corresponding to a tensor:

\[
\alpha_{int} = \begin{bmatrix}
-0.1 & 0 & 0 \\
0 & -0.1 & 0 \\
0 & 0 & 0.2
\end{bmatrix}
\]
which is arbitrarily chosen for convenience. All the tests will be performed on the samples with two different initial mean stresses $p_{ini} = 30$ and $p_{ini} = 80$ in order to observe the yielding behavior for both wet side and dry side.

Figure 3.8 illustrates the stress paths of the samples yield from dry side and wet side with/without initial anisotropy and with/without induced anisotropy under triaxial compression/extension procedures. The figure is presented via MATLAB and the two yield surfaces are mathematically drawn according to the AMCCM yield function Eq. (3.18) with input parameters $M = 0.9$, $p_0 = 100$ and $\alpha_{ini} = 0$ or $\alpha_{ini} = 17^\circ$ respectively. Several conclusions can be made from Figure 3.8:

1) The samples under triaxial compression with initial anisotropy ($\alpha_{ini} = 17^\circ$) goes through a much longer elastic path before entering the plastic zone than the one with initial anisotropy ($\alpha_{ini} = 0^\circ$); while for those under triaxial extension the former will begin yielding much quicker than the latter. This effect is consistent with the concept of “inherent anisotropy”.

2) For the samples without inherent anisotropy effect ($\alpha_{ini} = 0$), the yield locus of the sample with $c=0$ only experience expanding/shrinking; while for those with $c=1$ this expanding/shrinking is also companied with an additional counter-clockwise rotation. Such rotation hardening is equivalent to the “induced anisotropy”.

3) As soon as the stress paths touch the analytically drawn ellipse, yielding occurs and they start to develop towards the CSL. This exact match between numerically obtained stress paths and analytically drawn yield locus indicates an outstanding accuracy of the code.
Figure 3.9 further displays such rotational hardening and the effect of inherent anisotropy in $q - \varepsilon_q$ plain. In conclusion, the validity of the AMCCM UMAT has been well proved through the series of numerical tests above. Setting the anisotropic factor $c=0$, the implemented AMCCM produces identical results as ABAQUS inbuilt MCC does except exhibiting a slight stiffer stress-strain response in elastic regime. CV tests running under different anisotropic parameter combinations illustrate that the implemented AMCCM can successively capture both inherent anisotropy and induced anisotropy of the soil. During the numerical verification, the authors noticed the efficiency and stability of the code is amazingly good, which greatly benefit the following FE analysis on group batter piles.

Figure 3. 8: Stress paths with different anisotropic parameters under CV tests
Figure 3.9: Stress-strain curves with different anisotropic parameters under CV tests
CHAPTER 4

FINITE ELEMENT ANALYSIS

4.1 Introduction

As the rapid development of computers, the application of numerical methods for solving geotechnical problems is becoming more recognized by both geotechnical researchers and other engineers. The numerical methods that are frequently used to study pile foundations includes finite difference method (FDM) (e.g. Fakharian et al., 2008; Huh et al., 2008), finite element method (FEM) (e.g. Muqtadir and Desai, 1986; Trochanis et al., 1991), boundary element method (BEM) (e.g. Filho, 2005) and discrete element methods (DEM) (e.g. Uchida and Kawabata, 2004; Lobo-Guerrero and Vallejo, 2007). Among those, the FEM have been regarded as the most versatile and widely used approach for analyzing boundary value geotechnical engineering problems (Carter et al., 2000).

As a numerical treatment, researchers firstly need to identify a mathematical description (such as a set of partial differential equations and boundary/initial conditions) of the problem that they are dealing with, and then develop their FEM codes to numerically solve these equations. Alternatively, there exist many commercial FEM packages (e.g. ANSYS, ABAQUS and PLAXIS) that can be directly used by geotechnical researchers. These commercial FEM softwares greatly promoted the application of FEM in geotechnical engineering and significantly facilitated the studies based on FE analysis. However, such advancement does not guarantee any enhanced reliability of the numerical results. In fact, the users of ANSYS or ABQUS, especially geotechnical engineers, should handle their FEA with great cautions since these softwares were developed for general-purpose analysis and design, which need a strong background on both FEM and soil mechanics to ensure the reliability of the outputs. As indicated by Potts and Zdravkovic
(1999), proficient experience with the finite element codes (i.e. understanding their capabilities and limitations) as well as understanding the relevant theories behind soil mechanics and soil constitutive models are the key issues in solving numerical problems accurately.

In this chapter, we will discuss in details the FE model used for the batter pile group foundation including its mesh, interface modeling, constitutive models for materials and the determination of the model parameters. The coupled pore fluid diffusion and stress analysis provided by ABAQUS will be discussed. A brief introduction of the FB-MultiPier model will also be presented since we intended to compare the result of FEM with the p-y approach.

4.2 The Finite Element Models

A three-dimensional finite element model with exact geometry of the M19 pier foundation was developed on ABAQUS. The lateral load was applied on the left side of pile cap and directed horizontally to the right. Only half of the foundation including 12 piles is simulated due to the symmetric nature of this problem. The size of the mesh was selected such that the length of soil media is 220 ft, which is 5 times the size of pile cap (44 ft), and the depth of the soil media is 165 ft, which is 1.7 times the embedded depth of the piles (97 ft). To assess the effect of boundary distance on the results of FE analysis, another model was developed using a mesh with 440 ft length (10 times of pile cap size) and the resulted lateral deformation of pile cap was about 2% different than the original model. This small discrepancy demonstrates that the boundary effect on the model is negligible. Thus, the FE model with 220 ft wide is adopted in this study for its economic computation cost. The mesh includes 12 piles of 110 ft long and 3 ft × 3 ft square section that are inclined with a slope of 1:6 and spaced exactly the same as the M19 pier foundation. The cap is located 12 ft above the mudline. The lateral load was applied on the left side of pile cap and
directed horizontally to the right (along +y direction). A uniformly distributed pressure load is applied to the loading positions as in the field test.

The final FE mesh (Figure 4.1a) consists of a total of 68,229 three-dimensional linear interpolation reduced integration solid elements (C3D8R). This type of elements is superior to the fully-integrated linear elements in representing more variations in bending and also avoided the “shear locking” problem that occurs during shearing. However, the linear reduced integration elements are allowing spurious singular modes (“hourglassing”) which can only be avoided by setting proper hourglassing stiffness by the users. A hourglassing stiffness of 1.0 is proved to be adequate for the FEA in this study.

The boundary conditions for the FE model include restraining the horizontal movement along the side boundaries \((u_x = 0, u_y = 0)\), restraining the vertical movement along the bottom boundary \((u_z = 0)\), set symmetric boundary \((u_x = 0, R_y = 0, R_z = 0)\), and applying a uniformly distributed hydrostatic pressure \((10\text{ ft water}=624\text{ psf})\) on the mud surface to simulate the effect of water above mudline.

The initial geostatic balance condition of layered soil is installed in the soil body using command “*initial conditions, type=geostatic” for each layer according to its total unit weight. Meanwhile, the gravity is applied to the whole model body with each soil layers assigned corresponding density. In this way the inner geostatic stress and the external gravity are able to reach a balance at the beginning of the analysis without any settlements (or with a neglectable amount of settlements due to the weight of pile group). The information of soil unit weight or density is directly obtained from the UU tests and the CPTs performed near M19 piers. It should be noticed that this procedure actually simulated the cast-in place types of pile since it assumes
initially undisturbed soil stress with the presence of piles. This is not the case for M19 foundation which is constructed with driven piles.

Trochanis et al. (1991) discussed the significance of incorporating slippage and separation in pile-soil interface in the FE simulation of laterally loaded piles. ABAQUS provides several contact formulations. Each formulation is based on a choice of a contact discretization, a tracking approach, and assignment of “master” and “slave” roles to the contact surfaces. The tracking approaches are to account for the relative motion of two interacting surfaces in mechanical contact simulations. Two options of tracking approaches are offered: finite sliding, which is the most general and allows any arbitrary motion of the surfaces, and small sliding assuming relatively little sliding of one surface along the other. ABAQUS applies conditional constraints at various locations on interacting surfaces to simulate contact conditions. The locations and conditions of these constraints depend on the contact discretization used in the overall contact formulation. Two discretization methods: “node-to-surface” and “surface-to-surface” are provided. Both contact discretization methods enforce the slave nodes not to penetrate into the master surface, however the node-to-surface contact discretization allows the master surface to penetrate into the slave surface while large undetected penetrations of master nodes into the slave surface do not occur with surface-to-surface contact discretization, since the surface-to-surface formulation enforces contact conditions in an average sense over regions nearby slave nodes rather than only at individual slave nodes. In this study, the surface-to-surface contact discretization with small-sliding tracking approach is adopted for the pile-soil interface modeling.

The mechanical contact property consists of two components: one normal to the surfaces and one tangential to the surfaces. The interface in the normal direction is assumed to be “hard” contact which minimizes the penetration of the slave surface into the master surface at the constraint
locations and does not allow the transfer of tensile stress across the interface, while the tangential interaction behavior is governed by Coulomb friction model which relates the normal force to its shear behavior. A friction coefficient of 0.424 has been assigned to the tangential behavior, which corresponds to an angle of interface friction $\delta = 23^\circ$ between the soil and the piles. Separation is allowed after contact and slippage can happen when the tangential stress reached certain limit.

One of the objectives of this research is to investigate the effect of group interaction and pile inclination. Such objective can be achieved by comparing the lateral deflection, bending moment and soil resistance profiles produced by FE models with different spacing and pile inclinations. Repeating the same techniques, four additional FE models with varied geometry features from the original group pile model are constructed. They are summarized in Table 4.1. The meshes of all the five FE models developed in this research are presented in Figure 4.1. The various colors in Figure 4.1b distinguish the soil stratification which will be discussed in next section. Figure 4.2 presents the pile plane view of the pile layout and the numbering of the piles.

<table>
<thead>
<tr>
<th>Table 4.1: Summary of all the FE models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Spacing Between</strong></td>
</tr>
<tr>
<td><strong>Rows</strong></td>
</tr>
<tr>
<td><strong>Spacing Between</strong></td>
</tr>
<tr>
<td><strong>Columns</strong></td>
</tr>
<tr>
<td><strong>Pile inclination</strong></td>
</tr>
<tr>
<td>Original Group Batter</td>
</tr>
<tr>
<td><strong>Pile Model</strong></td>
</tr>
<tr>
<td>$S_y = 13 \text{ ft} = 4.3D$</td>
</tr>
<tr>
<td>$S_x = 7.5 \text{ ft} = 2.5D$</td>
</tr>
<tr>
<td>$1/6$</td>
</tr>
<tr>
<td>Small-Spacing Group</td>
</tr>
<tr>
<td><strong>Batter Pile Model</strong></td>
</tr>
<tr>
<td>$S_y = 9 \text{ ft} = 3.0D$</td>
</tr>
<tr>
<td>$S_x = 4.5 \text{ ft} = 1.5D$</td>
</tr>
<tr>
<td>$1/6$</td>
</tr>
<tr>
<td>Large-Spacing Group</td>
</tr>
<tr>
<td><strong>Batter Pile Model</strong></td>
</tr>
<tr>
<td>$S_y = 18 \text{ ft} = 6.0D$</td>
</tr>
<tr>
<td>$S_x = 10.5 \text{ ft} = 3.5D$</td>
</tr>
<tr>
<td>$1/6$</td>
</tr>
<tr>
<td>Group Vertical Pile</td>
</tr>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td>$S_y = 13 \text{ ft} = 4.3D$</td>
</tr>
<tr>
<td>$S_x = 7.5 \text{ ft} = 2.5D$</td>
</tr>
<tr>
<td>Vertical</td>
</tr>
<tr>
<td>Single Vertical Pile</td>
</tr>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td>N/A</td>
</tr>
<tr>
<td>N/A</td>
</tr>
<tr>
<td>Vertical</td>
</tr>
</tbody>
</table>

*D = 3 ft is the pile diameter*
Figure 4.1: Finite element models

(a) Normal Spacing  (b) Single Vertical pile
(c) Small Spacing  (d) Large Spacing  (e) Group Vertical

Figure 4.2: Pile layout
4.3 Constitutive Models of Materials and Their Parameters

Subsurface condition of M19 pier foundation site is of high heterogeneity (as shown in Figure 2.2), so it is convenient to classify the soil into certain layers and then discuss their constitutive models. Combining the soils with similar properties, the subsurface soil can be divided into eight layers including two sand layers and six clay layers (Figure 4.2).

The unit weights and corresponding overburden pressure, the clays’ undrained shear strength $S_u$, and plasticity index $I_p$, and the sands’ friction angle $\phi$ are determined based on the overall evaluation of the five CPTs, the SPT and the UU tests and listed in Table 4.2.

Figure 4. 3: Soil classification for FE analysis
Table 4.2: Soil properties determined from in-situ and UU tests

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Depth Below Mudline</th>
<th>Total Unit Weight</th>
<th>$S_u$</th>
<th>Friction Angle $\phi$</th>
<th>$I_p$</th>
<th>$\sigma_0$</th>
<th>$\sigma'_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ft</td>
<td>pcf</td>
<td>psf</td>
<td>%</td>
<td>psf</td>
<td>psf</td>
<td>psf</td>
</tr>
<tr>
<td>Soft Clay</td>
<td>0-15</td>
<td>123</td>
<td>240</td>
<td>20</td>
<td>925.58</td>
<td>456.02</td>
<td></td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>15-25</td>
<td>119</td>
<td>1560</td>
<td>40</td>
<td>2446.05</td>
<td>1194.93</td>
<td></td>
</tr>
<tr>
<td>Medium Clay</td>
<td>25-38</td>
<td>108</td>
<td>1104</td>
<td>35</td>
<td>3742.78</td>
<td>1774.06</td>
<td></td>
</tr>
<tr>
<td>Medium Sand</td>
<td>38-49</td>
<td>120</td>
<td></td>
<td>35</td>
<td>5104.90</td>
<td>2387.38</td>
<td></td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>49-70</td>
<td>113</td>
<td>1533.6</td>
<td>30</td>
<td>6951.40</td>
<td>3235.48</td>
<td></td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>70-81</td>
<td>122</td>
<td>3162</td>
<td>35</td>
<td>8809.13</td>
<td>4094.81</td>
<td></td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>81-99</td>
<td>128</td>
<td>1796.4</td>
<td>35</td>
<td>9992.28</td>
<td>4685.16</td>
<td></td>
</tr>
<tr>
<td>Dense Sand</td>
<td>&gt;99</td>
<td>124</td>
<td></td>
<td>38</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.3.1 Sands

The sand soil layers are simulated using the Drucker-Prager (DP) model with non-associated flow rule. DP model had been used in many studies to describe the behavior of sand soils (e.g., Brown and Shie, 1990; Trochanis et al., 1991; Yang and Jeremic, 2002; Karthigeyan et al., 2006). The Drucker-Prager yield function incorporated the effect of hydrostatic stress, which is expressed as follows:

$$ F = t - p \cdot \tan \beta - d = 0 $$  \hspace{1cm} (4.1)

where

$$ t = \frac{1}{2} q \left( 1 + \frac{1}{K} - \left( 1 - \frac{1}{K} \right) \left( \frac{r}{q} \right)^3 \right) $$  \hspace{1cm} (4.2)

which describes the shape of the yield stress on the $\pi$-plane; $\beta$ describes the slope of the yield surface in the $p-t$ stress plane and is referred as the angle of friction; $d$ describe the cohesion of the material, $p$ is the mean stress, $q$ is the Mises equivalent stress which has the expression $q =$
\[ K = \frac{3}{2} s_{ij} s_{ij}; \]

\[ K \text{ is the ratio of the yield stress in triaxial tension to the yield stress in triaxial compression, and } r \text{ is the third invariant of deviatoric stress. The yield surface of Drucker-Prager Model on meridional plane is presented in Figure 4.3.} \]

Figure 4.4: Yield surface of Drucker-Prager model on meridional plane

It is worth mentioning that the angle of friction \( \beta \) and cohesion \( d \) in the Drucker-Prager criterion are different from the friction angle \( \phi \) and cohesion \( c \) in the Mohr-Coulomb criterion. They have following relations:

\[ \tan \beta = \frac{2 \sin \phi}{\sqrt{3(3 - \sin \phi)}}, \quad d = \frac{6c \cos \phi}{\sqrt{3(3 - \sin \phi)}} \quad (4.3) \]

The angle of friction \( \beta \) of sand layers was calculated from the estimated friction angle \( \phi \) from the corrected SPT-N values, and the cohesion \( d \) is artificially set to a small value to avoid convergence difficulties. The Young’s modulus of cohesionless soil was estimated from the corrected SPT-N value using the following formula proposed by Kulhawy and Mayne (1990):

\[ E_s = p_a \alpha N_{60} \quad (4.4) \]

where \( p_a \) is the atmospheric pressure, \( \alpha \) is 10 for clean normally consolidated sand and \( N_{60} \) is the corrected SPT-N value.
4.3.2 Clays

All the clay layers were simulated using the AMCCM that was implemented into ABAQUS through UMAT subroutine. The FE analysis is based on the assumption that all the clay layers are fully drained so no excessive pore water pressure exists (the validity of such assumption will be discussed in chapter 5). In other words, if we consider the whole soil body as the superposition of water and soil skeleton, the stresses contributed by water will be exactly the hydrostatic water pressure while the stresses offered by soil skeleton are under the governing of AMCCM or DP constitutive law. Hence the stresses given by ABAQUS must be processed with:

\[ \sigma' = \sigma - \gamma_w z \mathbf{1} \]  

before passing the stresses into AMCCM. After successful updating of the stresses and state variables, remember to recover the effective stress to total stress before passing back to ABAQUS:

\[ \sigma = \sigma_{updated} + \gamma_w z \mathbf{1} \]  

The Above treatment can guarantee the response of the whole system is driven by total stress while remain the implemented AMCCM only manipulate the effective stress. Keep in mind that the correctness of such handling is based on fully drained assumption.

The AMCCM has six material constants: \( \nu \) Poisson’s ratio; \( M \) slope of critical state line; \( \kappa \) slope of unloading-reloading line (or logarithmic bulk modulus); \( \lambda \) slope of normal compression line (or logarithmic hardening constant); and the anisotropic parameters \( c \) and \( x \). Also, it has three state variables (\( e \) void ratio, \( p_0' \) preconsolidation pressure and \( \alpha \) back stress), thus requires the users to provide three initial values of these state variables (\( e_{ini} \), \( p_0'_{ini} \) and \( \alpha_{ini} \)). These parameters will be either calculated from empirical correlations from CPT data or estimated based on past experiences of the normal range of the parameter for a certain type of clay.
The preconsolidation pressure $p'_0$ can be calculated from the OCR of soil which is defined as

$$OCR = \frac{p'_0}{\sigma'_{v0}}$$

(4.7)

where $\sigma'_{v0}$ is the effective overburden pressure of each layer, which can be simply calculated by averaging the overburden pressure at the top and bottom of that layer. The OCRs are obtained by $S_u/\sigma'_{v0}$ and plastic index $I_p$ via the diagram (Figure 4.5) proposed by Andresen et al. (1979).

![Figure 4.5](image)

**Figure 4.5**: Relationships between $S_u/\sigma'_{v0}$, OCR and $I_p$ based on correlations for Drammen clay (Andresen et al., 1979)

The initial void ratio $e_{int}$ of clay can be estimated from the typical range of soil void ratio suggested by Das (1990) (Table 4.3).

The logarithmic bulk modulus $\kappa$, Poisson’s ratio $\nu$ and logarithmic hardening constant $\lambda$ should be systematically calibrated via hydrostatic compression test and triaxial compression tests. Unfortunately, undisturbed soil samples were unavailable at the time of this research and also these parameters belong to the school of critical state soil mechanics that can hardly be correlated from CPT or SPT data. However, the in-situ tests provided some direct soil
strength information, which can help us make the proper judgments within the common range of these parameters. It will be helpful to make such judgments if one firstly obtains the Clay’s Young’s modulus $E_s$. According to U.S. Army Corps of Engineers Engineering Manual 1110-1-1904, the Young’s modulus is related with undrained shear strength $S_u$ by the following relation:

$$E_s = K_c S_u$$  \hspace{1cm} (4.8)

where $K_c$ is a function of OCR and plasticity index $I_p$ and can be determined according to Figure 4.6. Although the Young’s modulus cannot directly be used in AMCCM, they are good indicator of the relative elastic stiffness of all the six clay clays. According to Yu (2006), the slope of swelling line $\kappa$ for clay ranges from 0.01 to 0.06 and the Poisson’s ratio $\nu$ is among 0.15 and 0.35 for both clay and sand. The typical value of $\lambda$ for clay is in the range of 0.1-0.2. Combining all above information, $\kappa$, $\nu$ and $\lambda$ can be determined empirically.

<table>
<thead>
<tr>
<th>Table 4.3: Typical void ratio for some soils (Das, 1990)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of soil</strong></td>
</tr>
<tr>
<td>Loose uniform sand</td>
</tr>
<tr>
<td>Dense uniform sand</td>
</tr>
<tr>
<td>Loose angular-grained silty sand</td>
</tr>
<tr>
<td>Dense angular-grained silty sand</td>
</tr>
<tr>
<td>Stiff clay</td>
</tr>
<tr>
<td>Soft clay</td>
</tr>
<tr>
<td>Loess</td>
</tr>
<tr>
<td>Soft organic clay</td>
</tr>
<tr>
<td>Glacial till</td>
</tr>
</tbody>
</table>
The slope of critical state line $M$ is determined based on the back-calculation of undrained shear strength interpreted from CPT data. According to critical state soil mechanics, the relationship between clay undrained shear strength and Cam-clay parameters is expressed as follows (Wood, 1990):

$$\frac{S_u}{p'_i} = M \left(\frac{OCR}{r}\right)^{\Lambda\frac{\lambda - \kappa}{\lambda}}$$  \hspace{1cm} (4.9)

where $\Lambda = \frac{\lambda - \kappa}{\lambda}$; $p'_i$ is the initial effective mean stress; $r$ is the ratio of tip pressure to critical state pressure and for modified cam clay model $r = 2$. Using this equation, one can back-calculate the $M$ value with $S_u$, OCR, $\lambda$ and $\kappa$ that were determined earlier for each layer.

Some suggestions of anisotropic constants $c$ and $x$ from literatures can be used in this study. Wheeler (1997) suggested a value of 4/3 for $x$. In the same work by Wheeler (1997), a new parameter $\mu$ is introduced to describe the rate at which $\alpha$ approaching to its target value $\eta/x$:

$$\mu = \frac{3cx(1 + e_0)p'}{(\lambda - \kappa)p'_0}$$  \hspace{1cm} (4.10)
Wheeler (1997) suggested that in the absence of suitable experimental data, a value of 30 for $\mu$ can be considered for a typical value. Having each layer’s $p^', p_0^', x, e_0, \lambda$ and $\kappa$, one is can back calculate the $c$ parameter for each layer assuming $\mu = 30$.

As indicated by Graham et al. (1983), the yield surface for naturally deposited clay tends to align along the $K_0$ consolidation line, which means the initial inclination angle $\alpha_{ini}$ can be determined if one knows the $K_0$ of the in-situ soil. For each layer, $K_0$ is obtained according to the relationship between $f_s$, OCR and $K_0$ proposed by Masood and Mitchell (1993) (Figure 4.7). Then $\alpha_{ini}$ is set to be identical to the angle of the $K_0$ consolidation line on the $p-q$ plane, which can be determined as follows:

$$\tan \alpha_{ini} = \frac{q}{p} = \frac{\sigma_a - \sigma_r}{\frac{1}{3}(\sigma_a + 2\sigma_r)} = \frac{(1 - K_0)\sigma_a}{\frac{1}{3}(1 + 2K_0)\sigma_a} = \frac{3(1 - K_0)}{1 + 2K_0}$$ \hfill (4.11)

![Figure 4.7: $K_0$ as function of sleeve friction and overconsolidation ratio (Masood and Mitchell, 1993)](image)

Finally, the basic soil properties directly correlated from the in-situ tests and the material parameters used for AMCCM and Drucker-Prager model are summarized in Table 4.4.
<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Depth Below Mudline</th>
<th>$E_s$</th>
<th>$v$</th>
<th>OCR</th>
<th>$K_0$</th>
<th>AMCCM</th>
<th>Drucker-Prager</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ft</td>
<td>ksf</td>
<td></td>
<td>psf</td>
<td>degree</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soft Clay</td>
<td>0-15</td>
<td>71.7</td>
<td>0.30</td>
<td>3.2</td>
<td>0.9</td>
<td>1459.3</td>
<td>1.5</td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>15-25</td>
<td>1307.9</td>
<td>0.15</td>
<td>2.7</td>
<td>1.0</td>
<td>3226.3</td>
<td>1.2</td>
</tr>
<tr>
<td>Medium Clay</td>
<td>25-38</td>
<td>780.0</td>
<td>0.25</td>
<td>2.2</td>
<td>0.9</td>
<td>3902.9</td>
<td>1.3</td>
</tr>
<tr>
<td>Medium Sand</td>
<td>38-49</td>
<td>600.0</td>
<td>0.40</td>
<td>1.9</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>49-70</td>
<td>1307.9</td>
<td>0.20</td>
<td>1.3</td>
<td>0.8</td>
<td>4206.1</td>
<td>0.9</td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>70-81</td>
<td>2400.2</td>
<td>0.20</td>
<td>1.0</td>
<td>0.9</td>
<td>4094.8</td>
<td>0.9</td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>81-99</td>
<td>1620.0</td>
<td>0.20</td>
<td>1.0</td>
<td>0.8</td>
<td>4685.2</td>
<td>0.9</td>
</tr>
<tr>
<td>Dense Sand</td>
<td>&gt;99</td>
<td>1200.0</td>
<td>0.45</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.4: Parameters for AMCCM and DP model
4.3.3 Concretes

The piles and pile cap are made of concrete which is simulated using a linear elastic model in this study. The elastic Young’s modulus was estimated based on the average results of 28-day concrete compressive strength $f’_c$ on cylindrical specimens ($f’_c = 9064$ psi). Since the lateral load test was conducted six months after pile construction, the average compressive strength was increased by 20% to account for concrete curing and the prestressed confinement effect. The Young’s modulus of the concrete was then estimated using the following equation:

$$E = 57,000\sqrt{f_c}$$ (4.12)

The value of pile Young’s modulus was estimated to be $8.56 \times 10^8$ psf. A common value of 0.2 is assumed to be concrete’s Poisson’s ratio.

4.4 Coupled Pore Fluid Diffusion and Stress Analysis in ABAQUS

As mentioned earlier, we assume the soils to be under fully drained conditions when handling the pore water pressure problem. However, the adoption of such assumption will not be validated until one performs a coupled pore fluid diffusion analysis of this problem and compares its results with the non-coupled one. In this section, the coupled pore fluid diffusion and stress analysis in ABAQUS will be discussed, and then it will be used to carry out a series of drained, partially-drained and undrained analyses on the single pile model.

In ABAQUS, a coupled pore fluid diffusion/stress analysis is used to model single phase, partially or fully saturated fluid flow through porous media and can be performed in terms of either total pore pressure or excess pore pressure by including or excluding the pore fluid weight. In this
study we are focusing on the saturated fluid flow since all the soils are submerged under the lake’s water and are believed to be fully saturated.

ABAQUS can provide the solutions either in terms of total or “excess” pore fluid pressure. The excess pore fluid pressure at a point is the pore fluid pressure in excess of the hydrostatic pressure required to support the weight of pore fluid above the elevation of the material point. In ABAQUS the total pore pressure solutions are provided when the gravity distributed load is used to define the gravity load on the model, while excess pore pressure solutions are provided in all other cases; for example, when gravity loading is defined with body force distributed loads. Recalling the use of “gravity” type of load in our FE models, the pore fluid obtained from the FEA in this research is total pore fluid pressure.

The pore fluid flows are governed by Forchheimer’s law, which can be expressed as follows:

\[ f(1 + \beta\sqrt{v_w \cdot v_w}) = -\frac{k_s}{\gamma_w} \mathbf{k} \cdot \left( \frac{\partial u_w}{\partial x} - \rho_w \mathbf{g} \right) \]

where \( f = snv_w \) is the volumetric flow rate of wetting liquid per unit area of the porous medium; \( s = \frac{dv_w}{dv} \) is the fluid saturation (\( s=1 \) for a fully saturated medium); \( n \) is the porosity of the porous medium; \( v_w \) is the fluid velocity; \( \beta(e) \) is a “velocity coefficient” which may be depend on the void ratio of the material; \( k_s \) is the dependence of permeability on saturation of the wetting liquid such that \( k_s \) at \( s = 1.0 \); \( \rho_w, \gamma_w \) is the density of and specific unit weight the fluid respectively; \( \mathbf{k}(e, \theta, f_\beta) \) is the permeability of the fully saturated medium, which can be a function of void ratio \( e \), temperature \( \theta \) and field variables \( f_\beta \); \( \mathbf{g} \) is the gravitational acceleration;
The Forchheimer's law tells us that high flow velocities have the effect of reducing the effective permeability and, therefore, “choking” pore fluid flow. As the fluid flow velocity reduces, Forchheimer's law approximates the well-known Darcy's law.

In this research, a coupled pore fluid diffusion and stress analysis is carried out on single pile model. All the soil constitutive models and their parameters are the same as the uncoupled models. Clays and sands are assigned different permeability according to the typical permeability \( k \) for various soils (Table 4.5. Das, 1999). Times allowed for consolidation after each load increments is exactly following the time table of the full scale load test on M19 pier eastbound foundation, which is listed in Table 2.1. In the finite element model, the C3D8R elements are replaced by C3D8RP elements to perform the coupled analysis, since the latter one has an additional degree of freedom on pore water pressure. The upper surface of the soil bulk is assigned the boundary condition \( u_w = 0 \), which is equivalent to a fully drained condition at ground surface. Drainage from other part of the soil bodies including the pile-soil interface are not allowed.

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>( k ) (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium to coarse gravel</td>
<td>Greater than 10-1</td>
</tr>
<tr>
<td>Coarse to fine sand</td>
<td>10-1 to 10-3</td>
</tr>
<tr>
<td>Fine sand, silty sand</td>
<td>10-3 to 10-5</td>
</tr>
<tr>
<td>Silt clayey silt silty clay</td>
<td>10-4 to 10-6</td>
</tr>
<tr>
<td>Clays</td>
<td>10-7 or less</td>
</tr>
</tbody>
</table>
4.5 FB-MultiPier Analysis

The FB-MultiPier program is based on finite element approach and can be used to analyze the entire components of the bridge from bridge slab to soil layer. It uses the iterative solution technique to predict the lateral displacements. During iteration, the stiffness of soils and piles are calculated and eventually generated the stiffness matrix to predict the lateral displacement of piles as output. Then, this displacement is used to predict the internal forces of structure members. The advantages of FB-MultiPier for simulating laterally loaded piles is that it has an in-built library of soil p-y curves, and the group effect can be easily taken into account by user defined p–multipliers for pile group analysis. Some in-built p-y curves included in FB-MultiPier and their input parameters are listed in Table 4.6

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Soil stiffness</th>
<th>Soil location</th>
<th>Parameters</th>
<th>P-y curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>Loose-dense</td>
<td>Above groundwater table</td>
<td>$\phi, K_s, \gamma$</td>
<td>O’Neill and Murchison (1984)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Below groundwater table</td>
<td>$\phi, K_s, \gamma$</td>
<td>Reese et al. (1974)</td>
</tr>
<tr>
<td>Clay</td>
<td>Soft/medium stiff</td>
<td>Above groundwater table</td>
<td>$S_u, \varepsilon_{50}$</td>
<td>O’Neill and Gazioglu (1984)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Below groundwater table</td>
<td>$S_u, \varepsilon_{50}$</td>
<td>Matlock (1970)</td>
</tr>
<tr>
<td>Stiff</td>
<td></td>
<td>Above groundwater table</td>
<td>$S_u, \varepsilon_{50}$</td>
<td>Reese and Welch (1972)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Below groundwater table</td>
<td>$S_u, \varepsilon_{50}, K_s, \gamma$</td>
<td>Reese et al. (1975)</td>
</tr>
</tbody>
</table>

* $\phi$ is the internal friction angle; $K_s$ is subgrade modulus; $\gamma$ is the unit weight; $S_u$ is the undrained shear strength; $\varepsilon_{50}$ is the major principal strain at 50% soil strength.
The whole structure of M19 eastbound pier was modeled including 24 batter piles, 2-pier columns, shear wall, and a cantilever bent (Figure 4.8). Piles are modeled as three dimensional discrete elements. The nonlinear behavior of concrete material are modeled by using input or default stress-strain curves that are a function of compressive stress of concrete ($f_c$) and modulus of elasticity of concrete ($E_c$). The pile model is generated by inputting the same geometry properties as the original foundation. The fixed headed pile cap is modeled using nine nodded shell elements which is based on Mindlin’s theory that can consider the bending and shear deformations. The soils are classified into eight layers to be consistent with the FE models introduced earlier in this chapter. Each soil layer surrounding piles are modeled as an attached nonlinear spring that characterized by a proper selected of p-y curve. The input parameters ($\phi, K_s, \gamma$ for sand and $S_u, \varepsilon_{50}$ for clay) are determined using the UU tests or in-situ CPT and SPT tests results. The input parameters for the FB-MultiPier analyses are summarized in Table 4.7.

**Figure 4.8:** FB-MultiPier model for M19 pier
Table 4.7: Input parameters for FB-MultiPier Analysis

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Depth Below Mudline</th>
<th>lateral model (p-y curve)</th>
<th>$\gamma$</th>
<th>$Su$</th>
<th>$\phi$</th>
<th>$K_s$</th>
<th>$\varepsilon_{50}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ft</td>
<td>pcf</td>
<td>pcf</td>
<td>psf</td>
<td>pcf</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soft Clay</td>
<td>0-15</td>
<td>Clay (Soft &lt; Water)</td>
<td>123</td>
<td>240</td>
<td></td>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>15-25</td>
<td>Clay (Stiff &lt; Water)</td>
<td>119</td>
<td>1560</td>
<td>120</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>Medium Clay</td>
<td>25-38</td>
<td>Clay (Stiff &lt; Water)</td>
<td>108</td>
<td>1104</td>
<td>60</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>Medium Sand</td>
<td>38-49</td>
<td>Sand (Reese)</td>
<td>120</td>
<td>35</td>
<td>120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>49-70</td>
<td>Clay (Stiff &lt; Water)</td>
<td>113</td>
<td>1533.6</td>
<td>100</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>70-81</td>
<td>Clay (Stiff &lt; Water)</td>
<td>122</td>
<td>3162</td>
<td>150</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>81-99</td>
<td>Clay (Stiff &lt; Water)</td>
<td>128</td>
<td>1796.4</td>
<td>150</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>Dense Sand</td>
<td>&gt;99</td>
<td>Sand (Reese)</td>
<td>124</td>
<td>38</td>
<td>150</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 5
RESULTS AND DISCUSSION

5.1 Nomenclature

Before systematically presenting the FEA results, it is desirable to reintroduce the pile numbering and their position in the pile group (recall Figure 4.2) for the convenience of discussion in the following sections.

The nomenclature for the rows is the same as the one adopted by Zhang et al. (1999), where the 1st row is named as “lead row” and the 4th row is called “trail row”. The most outside column is called “side column”, followed by “middle column” and then the most inside one is named “inner column”. In the following sections many aspects will be compared between different rows (e.g., piles 2, 4, 5, 6) and different columns (e.g., piles 1, 2, 3). Among all, pile 2 is selected here to be the reference pile that connects the comparisons between rows and columns.

5.2 Coupled Pore Fluid Diffusion and Stress Analysis on Single Pile Model

First of all, it is necessary to examine the soundness of our fully drained assumption the fully drained assumption adopted in the construction of all the FE models, as introduced in section 4.3. A coupled pore fluid diffusion analysis is performed on a single pile model. In the coupled analysis, the time duration of each loading increment becomes very important since the consolidation process is involved. Such time durations are set to be the exactly the same as the time schedule followed in the full-scale lateral load test that carried on M19 east bound pier foundation. Unloading-reloading processes are not included in this analysis for the sake of
comparing with the previous uncoupled FE analysis. In this analysis, we neither assume its fully drained nor fully undrained condition, but set the upper surface of the soil body to be the drained boundary and let the whole soil body consolidate according to the testing time. Hence, this coupled pore fluid diffusion analysis can be regarded as “partially drained” analysis.

The lateral deflection and bending moment profiles obtained from the partially drained model and the previous fully drained model at 1870 kips lateral load are presented in Figure 5.1. It can be seen that the results from the partially drained analysis and the fully drained analysis are very close to each other with a maximum difference of about 3% in lateral deflection and 4% in bending moment. To investigate possible causes of such small discrepancy between the two draining conditions, we extracted the excessive pore water pressure developed along depth in front of the pile at the end of loading (when the steel strands are cut) and plot it with the total geostatic pressure in Figure 5.2. It can be seen that the major excessive pore water pressure is developed within the top 15ft of the soil, which is the soft clay soil layer. Such excessive pore pressure is really small compared to the total stress due to soil self-weight.

The excessive pore water pressure dissipation curve of the point at 3.75 ft, 11.25 ft and 21.65 ft below ground surface are plotted in Figure 5.3. It is observed that there is a bouncing up of the pore water pressure after each load increment applied, and then dissipation occurs during the resting period. In this four-hour loading procedure the excessive pore pressure is kept accumulating until the steel strands are cut. The overall excessive pore water pressure developed at 3.75 ft is lower than that at 11.25 ft since this point is very close to the ground surface which is
the drained boundary. The excessive pore pressure of soil at 21.65 ft depths is neglectable compared to the other two points.

Figure 5.1: Comparison between partially and the fully drained analysis
Figure 5.2: Excessive pore water pressure and total geostatic pressure along the path

Figure 5.3: Excessive pore water pressure dissipation curves of soil at various depths
Based on Figures 5.2 and 5.3, the small discrepancy of the lateral deflection and bending moment profile observed between partially drained and fully drained analysis in Figure 5.1 can be explained. The excessive pore water pressure are mainly concentrated near the ground surface which is the drainage boundary, hence the accumulated excessive pore water pressure after the four-hour loading procedure is still small compared to the initial geostatic pressure of the site.

In conclusion, although some variations (maximum 3% - 4%) are observed between the partially drained and fully drained analyses, the latter can still provide quite reliable results representing the lateral response of the piles in the full-scale lateral load test on M19 eastbound pier foundation while having a significant saving in the computational cost of the analysis.

5.3 Lateral Deformation

The lateral displacement contour of the whole soil body after applying a lateral load of 1870 kips is illustrated in Figure 5.4a. It can be seen that the pile cap and the surface soil close to piles have the largest displacement. The soils within ± 28 ft width and 29 ft depth from the center of pile location are mobilized from 0.21 in to 0.63 in due to the maximum applied lateral load of 1870 kips. Such influence is neglectable for the soil beyond this region. Figure 5.4b presents the displacement vectors for all nodes of the pile group at 1870 kips lateral load. A scale factor of 250 is adopted to visualize the lateral deflections of the piles. It is noticed that the toes of left side piles and right side piles are not in the same level. The negative battered piles (left side piles) are subjected to uplifting while the positive battered piles (right side piles) are subjected to down dragging, indicating a rotation of the entire pile group.
Figure 5.4: At a lateral load of 1870 kips, the a) contours of lateral displacement of the whole model and b) displacement vectors of the pile nodes on the deformed pile group.
As a result, the whole pile foundation exhibits a tendency of rising of the elevation due to the lateral load. This phenomenon is consistent with our experience: piles are easier to pull out than drive in, thus when rotation occurs the whole foundation are tend to be lift up. Figure 5.5 shows the distribution of void ratio of the first layer. As expected, the soil exhibit densification in front of the lead row and expansion behind the trail row.

**Figure 5.5:** Contour of the void ratio at first clay later after 1870kips lateral load

Figure 5.6 presents the profiles of lateral deflection generated from the FE model on batter pile group foundation under different lateral loads. The FE predicted maximum pile cap lateral deformation is 0.79 in. Due to the “fixed” condition of pile head (enforced by pile cap), the deflection curves always tend to keep perpendicular to bottom of the pile cap.
Figure 5.6: Lateral deflection under different lateral load

Figure 5.7 compares the FE predicted lateral deflection profiles obtained from FEM using AMCCM with those measured in the field using the IP inclinometers and those predicted by the FB-MultiPier analysis under lateral load 1870kips. Here the group factors of 0.9, 0.8, 0.8 and 0.7 for lead row to trail row were used in the FB-MultiPier analyses. The deflection profiles from the FEM show slightly larger deflection near the pile cap than the measured values. However, the overall deflection curves along the pile have very good match with the field measured data. The FB-MultiPier predicted pile head displacement is close to the measured value; however the deflection curve drifts away as the depth increasing along the pile. The “Stationary point” (the point with no lateral deformation) obtained from FB-MultiPier analysis is much shallower than
those given by field measurements and FEM. Such discrepancies were also reported by McVay et al. (2005) in their barge impact tests on a bridge pier at St. George Island Causeway. A possible explanation of such discrepancy could be due to the simple way that FB-MultiPier account for group interaction effects. In FB-MultiPier, the group effect is empirically considered using the p-multiplier, which is determined by comparing the single pile p-y curves with group pile p-y curves from field tests. The p-multiplier of the pile in the same row assumed to be constant along the pile depth regardless of the soil type. First of all, the unified p-multiplier for the pile representing average value of the different group interaction effects of various soil types. Secondly, such assumption neglected the possible variation of group interaction effect for the same soil at different depth. Thus, for the soil, the shallower layers may suffer more group interaction effect than the deeper layers due to larger mobilization of shallower soil and thus more shadowing effect. For the batter pile group cases, it becomes even more questionable to use a unified p-multiplier to reduce all the p-y curves along the entire pile depth, since the spacing of rows spacing vary dramatically from pile top to the pile toe. Take the M19 pier foundation as an example, the spacing between 2nd and 3rd rows is 4.5D (13.6ft) at ground level and becomes 15.4D (46.1ft) at pile toes. Obviously the pile-soil system near the ground surface will experience more pile-soil-pile interaction effects than at deeper depths. This statement will be further discussed later in this section.
The lateral deflection profiles of the piles aligned in the same row but different columns (pile 1, 2 and 3) and those in the same column but varied rows (pile 2, 4, 5 and 6) under 1870 kips lateral load are presented in Figures 5.8a and 5.8b, respectively. The lateral deflections of the piles near the pile cap are almost the same due to the confinement of the pile cap. However, some variation occurs at deeper portion of the pile and such difference reaches its maximum at
30 – 35 ft below pile cap, which is corresponding to the second soil layer (stiff clay). The piles located in the side column, lead row and trail row show larger bent than the other piles, indicating that these piles are encountered more soil resistance than the rest. This observation will be verified later in the soil resistance profile section.

![Figure 5.8: Lateral deflection profiles of piles in different location](image)

The Comparison of the lateral deflection profiles obtained from different FE models at a lateral load of 1870 kips is presented in Figure 5.9. In the figure GV represents group vertical pile foundation and GB stands for group batter pile foundation. All the comparison is based on the reference pile (pile 2). Many conclusions can be drawn from Figure 5.9:

1) All the pile groups regardless of their pile inclination or spacing show much higher lateral displacement than that of a single pile, which is consistent with most field or model tests.

2) For the batter pile groups, the small spacing model produces largest lateral deformation (0.94in) while large spacing model shows relatively small lateral deformation (0.71in).
3) The vertical pile group model exhibits significant large lateral deformation (1.22in), which is 54% greater than the batter pile group with the same spacing (0.79in) and 130% larger than the single isolated pile (0.53in).

![Figure 5.9: lateral deflection profiles obtained from different FE models](image)

Figure 5.9: lateral deflection profiles obtained from different FE models

Beside for above findings, Figure 5.9 supported the author’s discussion earlier in this chapter regarding the oversimplification of group effect using a unified p-multiplier for an entire pile. The “fixed point” for single vertical pile model is much shallower than the group piles, and is very close to the FB-Multiplier predicted depth. It suggests that when pile group is subjected to lateral loading, the soils near the ground surface are mobilized as a block, extending the soil’s influence zone due to lateral load to be deeper than the single pile case. This comparison
confirms the authors’ believe that the group effect has influence on both pile head movement and the deflection pattern along depth. The p-y curves of the soils located near ground level are subjected to greater reduction due to group interaction effects which can be incorporated with increased p-multipliers.

5.4 Contours of Stress\Strain Distribution

5.4.1 Mean Effective Stress & Volumetric Strain

The Contour of the mean effective stress (SDV10 in the UMAT) distribution for soil layer clays before and after lateral load is presented in Figure 5.10. The mean effective stress after geostatic process (Figure 5.10a) shows generally uniform distribution at the same elevation with only a little disturbance near the pile due to gravity of the pile group. Significant change of the mean effective stress field for the top three soil layers is observed after application of a lateral load of 1870 kips (Figure 5.10b). The soils behind the piles (on the left side of the pile in the figure) show a decrease of mean effective stress while the soils in front of the piles (on the right side of the pile in the figure) are subjected to increased mean effective stress. This finding indicates that the stress path of the soil near ground surface behind the pile is similar to that of triaxial extension test while the stress path of the soil in front of the pile is close to the one in triaxial compression test, which further supported the statement made earlier that the soil anisotropy could be a key issue in simulating the laterally loaded pile behaviors. The influence of lateral load to the mean effective stress distribution becomes less significant for the soil layers located at deeper depths.
The change in mean effective stress should be discussed together with the volumetric strain (SDV12 in the UMAT) distribution after loading (shown in Figure 5.11). Obviously the lateral load transmitted from the pile pushes the soil in front of the piles to a denser state and the gaps

Figure 5.10: Contour of the mean effective stress for clays a) before and b) after lateral loading
left behind the piles are instantaneously filled by soils and thus result in a relaxation of the soil behind the piles. Such densification effect is most significant for the soil in front of the lead row and the maximum relaxation occurs at the soils behind the trail row. It is worth to notice that the volumetric strain contour is very similar to the possible failure surfaces for batter piles described by Prakash and Subramanyam (1965), which is presented in Figure 5.12.

![Contour of volumetric strain after lateral loading](image)

**Figure 5.11:** Contour of volumetric strain after lateral loading

![Failure surface in positive and negative batter piles](image)

**Figure 5.12:** Failure surface in positive and negative batter piles (Prakash and Subramanyam, 1965)
5.4.2 Deviatoric Stress & Deviatoric Strain

The deviatoric stress \( q \) (also called “Mises stress” in ABAQUS) of piles at different columns is shown in Figure 5.13. Only the top halves of the piles are presented since the deviatoric stress variation of the bottom half are neglectable compares to the top portion. The yellow-red area indicates the largest concentration of the deviatoric stress. It is noticed that such high deviatoric stress area occurs at the cap-pile connection and 0-10ft and 24-35ft below pile cap. The piles located in the lead row show larger deviatoric stress at the cap-pile connection than those in the rest rows. Also, it is observed that the piles in the side column share more deviatoric stress than that of middle and inner column piles.

![Figure 5.13: Deviatoric stress of piles in different columns](image)

The contours of deviatoric stress of the first three soil layers are shown in Figure 5.14a. It is observed that a large fraction of the lateral load is absorbed by the second stiff clay layer, while the rest are mainly taken by the third medium clay layer. The first clay layer is so soft that only a small fraction of deviatoric stress is allocated in this layer. Such distribution pattern will be observed again when we discuss the soil resistance profiles along the piles in section 5.8. The
situation is reversed for the contours of deviatoric strain (Figure 5.14b) of the first three soil layers. For example, a large region of the first soft clay layer undergoes deviatoric strains while only a small area near the pile of the second stiff clay layer developed some deviatoric strains.

Extracting the second layer and readjust the limit ranges of the contour (Figure 5.15) allows one to overlook the deviatoric stress distribution at the cross-section that offer major lateral resistance. The deviatoric stress distribution pattern indicates that the soils in front of the lead row contributed much more soil resistance that those in between the piles or behind the trail row.

Figure 5.14: Contour of a) deviatoric stress and b) deviatoric strain of the first three soil layers
The shadowing\t\ledge\teffects caused by pile group interaction have been discussed in many literatures (e.g. Rollins et al., 1998; Zhang et al., 1999). However, no literature has illustrated how such effect influence the stress distribution of the soil under lateral loads. In this research, the shadow effect can be well visualized by comparing the plane view of deviatoric stress contours of the second stiff clay layer (which offer the major lateral resistance) of all the three varied-spacing group batter pile modes and the vertical group pile model, as shown in Figure 5.16. Notice that all the contours are plotted under the same color limit (from 300 psf to 2430 psf) for convenience of comparison.

Firstly, there is a significant increase of the deviatoric stress of the soil in front of the lead row for the vertical pile group (Figure 5.16b) comparing to the batter pile group (Figure 5.16a) at same spacing. The main reason is that major portion of the lateral load on pile cap is directly transferred to the soils for vertical piles, while for inclined piles part of the lateral load is transmitted into axial component of the piles and then digested by skin friction and toe resistance.
Hence, the deviatoric stress developed in the soil surrounding the vertical piles will be much larger than that of batter piles.

![Figure 5.16: Deviatoric stress distribution of various FE models](image)

Secondly, the deviatoric stress distribution patterns are very different for batter pile groups with various spacing. The negative batter piles of large spacing model (Figure 5.16c) produce relatively independent stress concentration zones under a lateral load of 1870 kips, while these isolated zones turn out to be a united region when comes to normal spacing model. Such region even extends into a larger area when the pile spacing further narrowed (Figure 5.16d). Obviously, this phenomenon is caused by increased overlapping of the piles’ influenced zones when pile
spacing getting closer. It is also interesting to notice that the maximum deviatoric stress in front of the lead row of large spaced piles is 1485 psf with a relatively spread out pattern, while for the normal spacing model it becomes 1924 psf and distributed more compactly. The small spacing model has the most concentrated stress zone with an average deviatoric stress of 2324psf located in front of the lead row. Still, such zone is formed due to the highly overlapped shadowing and edge effect caused by closely spaced piles.

5.5 Bending Moment

The piles bending moment profiles along depth were extracted from the FE model at different load increments and presented in Figure 5.17. Evidently, the maximum positive bending moments occur at pile head for all piles due to rigid cap-pile connection. The maximum negative bending moments occurs at approximately 28 ft below pile cap for all lateral loads.

As introduced in Chapter 2, two pairs of strain gauges were installed on each of the 12 selected piles at two different locations. The strain gauge data for each pair was used to calculate the bending moment and axial load. The moments can be calculated using the following equation:

$$M = \frac{EI(\varepsilon_t - \varepsilon_c)}{h}$$  \hspace{1cm} (5.1)

where $\varepsilon_t$ is the tensile strain; $\varepsilon_c$ is the compressive strain; $h$ is the horizontal distance between the two gauges; and $EI$ is the flexural stiffness of the pile.

Figures 5.18 present the comparison of bending moments generated from the FEM, strain gauges and the FB-MultiPier analyses at two different load increments (970 kips and 1745 kips).
Figure 5.17: Bending moment profiles under different lateral loads from FE analysis.
There is a missing of the second pair strain gauge data at pile 4 since one gauge was damaged during pile installation. The position of pile 8 is located in the trail row of the other half of the pile group foundation which is not simulated in the FE model. The counterpart to pile 8 is pile 1 in the FE simulated half, thus the FE and FB-MultiPier results shown in the bottom two figures of Figure 5.18 are actually the same and only the SG data changed.

![Figure 5.18: Bending moment profiles from the strain gauges, FEM and FB-MultiPier analyses](image-url)
Generally speaking, both FEM and the FB-MultiPier predicted bending moments agree well with those deduced from the strain gauges data. The point of zero bending moment (which is corresponding to the inflection point of the deflection curves) occurs at about 15-18ft below the bottom of the pile cap. For both case (970 kips and 1745 kips), the maximum positive moment predicted by the FEM are very close to those by FB-MultiPier; while the maximum negative bending moment obtained from FEM is 21%-24% lower than those predicted by the FB-MultiPier analysis. Besides, the location of maximum negative moment from the FEM occurs at slightly shallower depth (27.5 ft below pile cap) than the location from the FB-MultiPier analysis (31.5 ft below pile cap).

The bending moment profiles of the piles aligned in the same row but different columns (pile 1, 2 and 3) and those in the same column but different rows (pile 2, 4, 5 and 6) under 1870 kips lateral load are plotted in Figure 5.19a and 5.19b, respectively. A very important finding from this figure is that the pile located at side column (pile 1) shows 11.7% higher maximum positive bending moment (at pile head) and 18.0% higher maximum negative bending moment (at 27.5ft below pile cap) than those piles in inner column. Such significant variation of bending moment for the piles in the same row is normally neglected in practice. This phenomenon will be further discussed in section 5.8 when we comparing the soil resistance profiles of the piles in different columns. For the piles aligned in the same column, it is found that the piles in lead row and trail row take an averagely 7% more bending moment than those in the 2nd and 3rd rows, with a slightly higher bending moment of pile 6 than pile 2.
Figure 5.20 presents the different bending moment distribution of the reference pile (pile 2) obtained from all the five FE models described in Figure 4.1 at lateral load of 1870 kips. The figure showed that:

Figure 5.19: Bending moment profiles of piles in different location

Figure 5.20: Bending moment profiles from various FE models
1) The batter pile groups show very similar bending moment profile with the small spacing model produces a slightly larger bending moment compared to the normal spacing and large spacing models.

2) The batter pile groups have smaller (30%) bending moment than the vertical pile groups regardless of the spacing, but these are still significantly higher (60%) than the bending moment developed on a single isolated pile.

5.6 Distribution of Lateral Load

Researchers are very interested to know how the lateral load is distributed at each pile among the group pile foundation. Figure 5.21 illustrated the lateral load distribution pattern of all the piles in different rows and columns evaluated at pile head for various FE models. The lateral load distributions among piles in different columns (Figure 5.21a) show that the side column piles take the most lateral load followed by the middle column and then the inner column. Similar finding was also reported by Ruesta and Townsend (1997) in their lateral load test at Roosevelt Bridge. This variation exists in all pile group models and more significantly in small spacing group and less obvious in large spacing group, which indicates that the edge effect is intensified when spacing between columns is reduced.

For the piles in same column (Figure 5.21b), there is no surprise that the one located in lead row is subjected the highest lateral load, followed by the 2nd and 3rd rows. For the large spacing and normal spacing batter group models, the trail row takes the smallest portion of lateral load. This lateral load distribution pattern is well known by researchers and also obeys our knowledge.
Figure 5.21: lateral load distribution pattern evaluated at pile head for various FE models

(a) Piles in different columns

(b) Piles in different rows
on how the load is transferred for a deformable body. However, it is observed that the small spacing batter group and the vertical group model have slightly higher lateral load distributed on the trail row than that of the 3rd row or even the 2nd row.

This phenomenon is very interesting and there are some contradict observations to what reported in the literatures. Many researchers concluded that the lateral load distribution is rigidly decreasing from lead row to trail row (Brown et al., 1987; Ruesta and Townsend, 1997). However, according to the lateral load tests reported by other researchers (Rollins et al., 1998; Rollins et al., 2003b), trail row sometimes takes higher lateral load than 3rd or 2nd rows. In this study, we noticed that such effect occurred at small spacing batter group and vertical group models, which are believed to have the highest group effects among all the four models (as illustrated in Figure 5.16). It can be inferred that when piles comes closed to each other, the most influenced piles are those located in between lead and trail rows because they are suffering the shadowing effect of the rows behind them and meanwhile their supporting zone (the shadow they created) are disturbed by rows in front of them. On the contrary, the lead row and trail row are less influenced because the shadow created by the lead row remains integrity and the trial row is free from the shadow effect from other piles. Therefore when group interaction effect increases (due to closer spacing or changing of batter to vertical), the rows in between will suffer higher reduction of lateral strength while the lead and trail row are less affected. Based on this observation, a possible explanation of the phenomenon that the trail row takes higher lateral load than the 3rd or even the 2nd rows in high-group-effect pile foundations could be due to the
decreases of the lateral stiffness of the 2\textsuperscript{nd} and 3\textsuperscript{rd} row been faster than the lead and trail rows during development of group effect, which finally result in a higher load distribution on the trail row.

5.7 Evolution and Distribution of Axial Load

Batter pile group foundation is able to transfer part of the lateral load on pile cap to the axial load of the piles, which will increase the lateral capacity of the foundation. In the full-scale test, the changes in pile axial load can be derived from measurements of strain gauge data. Figures 5.22 compares the measured axial load increments for piles 1 and 7 with those obtained from the FE model. Good agreement is observed for the measured data and FE predicted curves. As expected, the negative batter pile (pile 1) is subjected to axial extension and the positive batter pile (pile 7) is subjected to axial compression. Both FEM and full-scale test results show that such increment or reduction in axial load is proportional to the increase of applied lateral load.

Figure 5.23 shows the axial load distribution pattern of all the piles in different rows for various FE models. Notice that the axial load shown in this figure are purely caused by applied lateral load at cap, which is a result of subtraction of the total axial load by the axial load after geostatic balance. The axial load developed on vertical pile groups are in accordance with our empirical judgments. The lead row is subjected to highest compressive axial load while the trail row takes the maximum axial traction load. The 2\textsuperscript{nd} row and 3\textsuperscript{rd} row are subjected to a small amount of axial compression and tension loads, respectively.
Figure 5.22: Increment of axial load from strain gauges and FEM
A dramatic difference of axial load distribution pattern among vertical pile group and batter pile groups is observed. The major axial load is no longer taken by the lead and trail rows but shifted onto the 2\textsuperscript{nd} and 3\textsuperscript{rd} row piles. One possible explanation is that the trapezoidal soil block formed between of 2\textsuperscript{nd} and 3\textsuperscript{rd} rows strengthened its resistance to rotation and then more axial load is shifted to the piles beside this block, as illustrated in Figure 5.24. One can find that for both vertical and batter pile group, there is a zone between 2\textsuperscript{nd} and 3\textsuperscript{rd} row where soil is subjected to rotational stress states. The resistance to such rotation comes from the soil-pile interface shear force developed along sides of the block and the normal force by the soil bed underneath the block. Obviously, the trapezoidal zone formed by the batter pile foundation is able to create higher rotation resistance by providing higher value of both components than the narrow rectangular zone of vertical pile foundation do, since the former has a broader side area and wider base zone. It is also noticed that the axial load developed in lead and trail rows are increased as the pile-pile spacing decreased, indicating that a stronger group effect will reduce the “trapezoidal zone effect” and thus shift more axial load to lead and trail row piles.

Summary of total axial load developed on both models at middle column is presented in Table 5.1. It shows that regardless of pile spacing, the 1:6 batter piles are able to transmit part of the horizontal load to their axial components, as expected.
Figure 5. 23: Axial load distribution pattern for piles in different rows from various FE models

<table>
<thead>
<tr>
<th></th>
<th>Trial row</th>
<th>3rd row</th>
<th>2nd row</th>
<th>Lead row</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large spacing-GB</td>
<td>28.60</td>
<td>156.26</td>
<td>-156.46</td>
<td>-27.96</td>
</tr>
<tr>
<td>Small spacing-GB</td>
<td>103.53</td>
<td>146.57</td>
<td>-147.23</td>
<td>-106.13</td>
</tr>
<tr>
<td>Normal spacing-GB</td>
<td>55.31</td>
<td>159.60</td>
<td>-160.20</td>
<td>-55.08</td>
</tr>
<tr>
<td>Normal spacing-GV</td>
<td>137.35</td>
<td>19.86</td>
<td>-20.61</td>
<td>-137.45</td>
</tr>
</tbody>
</table>

Figure 5. 24: The trapezoidal zone under batter pile group foundation
Table 5.1: Total axial load developed at middle column in different FE models at 1870 kips lateral load

<table>
<thead>
<tr>
<th></th>
<th>Normal spacing-GV</th>
<th>Normal spacing-GB</th>
<th>Small spacing-GB</th>
<th>Large spacing-GB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Axial Load at middle column (Kips)</td>
<td>315.27</td>
<td>430.19</td>
<td>503.46</td>
<td>369.28</td>
</tr>
<tr>
<td>Increased Percentage in Axial Component</td>
<td>Reference</td>
<td>36.5%</td>
<td>59.7%</td>
<td>17.1%</td>
</tr>
</tbody>
</table>

5.8 Soil Resistance Profiles

Figure 5.25 presents the soil resistance profiles obtained from normal spaced batter pile group model at different loads for all the piles. It can be seen that the first soft clay layer (0-15 ft below mudline or 12-27 ft below pile cap) provides a relatively small and uniform lateral resistance regardless of the depth at all levels of lateral load. Significant rise in the lateral resistance profile occurs when entered the second stiff clay layer (15-25 ft below mudline or 27-37 ft below pile cap). A peak of positive lateral resistance is achieved at 27.5 ft and a maximum negative lateral resistance occurs at 35.5 ft below pile cap for all the six piles under any lateral loads. Both peaks are within the second stiff clay layer. It is interesting to notice that the point 27.5 ft below the pile cap takes the maximum positive lateral resistance and the maximum negative bending moment at the same time. Soil resistance reduced to zero at 33 ft below cap. The third medium clay layer (25-38 ft below mudline or 37-50 ft below pile cap) are basically offering some negative lateral resistance to the piles. Under this layer, the soils resistance is negligible.
Figures 5.25: Soil resistance profiles at different lateral loads from FEM

Figures 5.26 present the soil resistance profiles for different piles under 1870 kips lateral load, which are also compared with the FB-MultiPier profiles. The Figure 26a shows that the side pile 1 encountered higher lateral resistance in the first soft clay layer than the piles located in the middle and inner columns do. It has also a slightly higher maximum positive soil
resistance at 27.5 ft depth than the middle and inner piles. The negative lateral resistance provided by soils below 33 ft shows little variance for the three piles.

![Soil resistance profiles for different piles from FEM and FB-MultiPier analysis](image)

**Figure 5.26:** Soil resistance profiles for different piles from FEM and FB-MultiPier analysis

Figure 5.26b shows that the piles located in the lead and trail rows have higher soil resistance in the first soft clay layer than the 2\textsuperscript{nd} and 3\textsuperscript{rd} row piles. The lead row has the highest maximum positive soil resistance (21.1 kips) at 27.5 ft depth, followed by the 2\textsuperscript{nd} row pile (19.5 kips) and the 3\textsuperscript{rd} row pile (18.4 kips). The trail row takes a slightly higher maximum resistance (18.8 kips) than the 3\textsuperscript{rd} row but still less than the 2\textsuperscript{nd} row and lead row. One major difference between the Figure 26a and 26b is that the negative lateral resistance below 33 ft has significant variation among different rows. The lead row takes the highest maximum negative resistance (-7.3 kips) followed by the trail row (-6.0 kips) and then the 2\textsuperscript{nd} and 3\textsuperscript{rd} rows (-4.0 and -3.9 kips respectively).
According to both figures, the soil resistances near the ground surface and the maximum positive lateral resistance predicted by FB-MultiPier are generally close to those generated by the FEM model. However, it predicts a deeper zero soil resistance point (43 ft below cap) which makes the fourth medium sand layer offer the major part of the negative lateral resistance.

The soil resistance profiles of the reference pile (pile 2) obtained from the different FE models are presented in Figure 5.27. Some very interesting facts can be observed:

1) All the three different spaced batter pile groups has similar soil resistance distribution pattern. Among them, the large spacing model encountered the highest soil resistance at all depth regardless of soft or stiff clay soil layer, followed by the normal spacing model and then the small spacing model. This phenomenon indicates: a) the group interaction effect will reduce the lateral resistance offered by soils; b) such reduction of resistance nearly covers the soil at all depth except those not influenced by the lateral load on pile cap; c) closely spaced pile group suffer more reduction in lateral soil resistance due to group interaction effect than the one that has larger spacing.

2) The comparison of the vertical pile group and the batter pile groups shows that the soil resistance offered by the former is higher than that of the latter. The maximum positive lateral resistance from the vertical group (26.4 kips) is 28.7% higher than that of the normal spacing batter group (18.8 kips).

3) It is also noticed that the vertical pile group exhibits extremely sharp fluctuation in soil resistance, while the single isolated vertical pile model produces a relatively smooth
distribution on the resistance. This phenomenon is very interesting since it shows that the group interaction not only affects the soil resistance but also influences its distribution pattern. For the easiness of discussion, we plot their soil resistance profile under different lateral load separately in Figure 5.28.

![Soil resistance profiles from various FE models](image)

**Figure 5.27:** Soil resistance profiles from various FE models

It can be seen that under all lateral loads the first soft clay later offers higher horizontal resistance to the single isolated pile than to the vertical group piles. Such stiffer response by soft layer directly reduced the responsibility of the second stiff clay layer and hence less lateral resistance is occurred in this layer compared to vertical group piles. An explanation of this could be that when isolated piles assembled into a group, their interaction (shadowing\edge effect)
between each other significantly reduced the strength/stiffness of the first layer, hence a large portion of lateral load have to be transmitted in to the second stiff clay layer, due to which a sharp increase in lateral resistance at the soft-stiff joint surface (27 ft below pile cap) is occurred.

**Figure 5.28:** Soil resistance profiles under different lateral loads from single pile and vertical pile group model

5.9 P-y Curves

As mentioned in chapter 2, the advantage of 3D finite element model is that it can consider the effect of pile geometry, group pile spacing, soil-pile interface and inelastic behavior of soil in a more physical sounded way. Such 3D FEM analysis can produce the p-y curves rather than assuming them as input parameters as the case for the FB-MultiPier. The p-y curves at selected depths can be deduced from the soil reaction p profiles obtained at different load levels and the corresponding lateral deformation y.
Observation of the soil resistance profiles of various models shows that the first soft clay layer and the second stiff clay layer provide the major part of positive lateral resistance. Hence, it is desirable to extract the p-y curves for both of them. The points locate at 16 ft and 28 ft below pile cap (or 4 ft and 16 ft below ground level) are selected to produce the p-y curves representing the first soft clay layer and the second stiff clay layer.

Figures 5.29 present the p-y curves obtained from the normal spacing group batter pile model for piles 2, 4, 5, and 6 at two different depths, 16 ft and 28 ft below pile cap, respectively. For comparison, the p-y curves generated by single pile model and those extracted from the FB-MultiPier for these two layers are also included in the figure.

For the p-y curves of piles in different rows of the batter group model, the one that belong to the lead row shows the highest stiffness at both depth, and that of 2nd and 3rd rows indicates a significant soften due to group effect. For both depths, the trail row pile produces higher stiffness p-y curves than the 2nd and 3rd rows but less stiff compare to the lead row. It is noticed that the p-y curves from the lead row and the trail row are very close for the point at 16 ft below pile cap but separated for deeper depth (28 ft below pile cap). Recall that for batter pile group, the pile spacing goes larger as the position going deeper, indicating a larger group effect near ground surface. Relating such fact to this case, it seems that larger group interaction effect will shift more responsibility of lateral resistance to the boundary piles such as lead row and trail row piles. Surprisingly, such conclusion was also obtained at section 5.6 when we comparing the lateral load distribution of different FE models.
Figure 5.29: P-y curves from FEM and FB-MultiPier analysis
As expected, the single pile has higher soil stiffness than the group piles at all depth due to the absence of group effect. Comparing the discrepancy between the single pile and the group pile p-y curves at 16 ft depth and 28 ft depth, it is again confirmed that the soils near ground level will experience higher pile group interaction effect than those in deeper location.

The FB-MultiPier p-y curves generally give larger soil resistance than the FEM generated p-y curves. At 16 ft, the resulted FB-MultiPier p-y curve suggests that the soil has entered a plastic range, which is not the case for the FEM predicted p-y curves. Interestingly, the tangent stiffness of single pile p-y curves are very close to the FB-MultiPier p-y curves at elastic stage.

The p-y curves of the reference pile (pile 2) that obtained from different FE models at both depths are presented in Figure 5.30. It further supported some conclusions that we made earlier in previous sections:

1) Large spacing batter group pile model has the least group interaction effect, while small spacing one has the most group effect. Normal spacing vertical group model also has high group interaction effect.

2) Soils near ground surface suffer higher group effect than those in deeper locations.
Figure 5.30: P-y curves from different FE models
5.10 P-Multipliers Obtained from FE Analyses

The p-multiplier for reducing the single pile p-y curve to the group pile p-y curve is a function of the location of the pile but is a constant over the entire depth of pile. According to the definition of P-multiplier, it should be determined by directly comparing the p-y curve obtained from single pile model and group pile model. However, such directly determined p-multiplier can be varied at different depth (e.g. 16 ft and 28 ft below pile cap). In practice, the p-multipliers are often determined by comparing the load-deflection curves of pile head of the group piles and that of the single pile, so that they can represent an average reduction of the piles over their entire depths.

The load-deflection curves for single pile and the piles 1 to 6 in different pile group model is presented in Figure 5.31. It can be seen that the single pile has the highest stiffness of the lateral responses, while due to group effect the piles in different locations in the batter pile group are more or less subjected to a reduction in lateral stiffness. By applying proper p-multipliers, the single pile load-deflection curve can be reduced to match those from batter pile group foundation. The back-calculated p-multipliers for different pile locations are summarized in Table 5.2. In the similar way, the calculated p-multipliers for the small-spacing GB model, large-spacing GB model and the group vertical pile model are also obtained and summarized in Table 5.2.
The comparison of the p-multipliers for the vertical pile group model and the batter pile model shows a significant reduction, indicating a better performance of batter pile group foundation than vertical pile group foundation when subjected to lateral loads. In all cases the lead row has the least reduction in soil reaction and such reduction will be intensified for the
trailing rows; while for those models suffer higher group effects, the trail row sometimes will take more load than the 3\textsuperscript{rd} rows. Side columns always have higher p-multipliers than the middle and inner columns.

**Table 5. 2: p-multipliers obtained from FE analysis**

<table>
<thead>
<tr>
<th>FE Model</th>
<th>Spacing Between Rows</th>
<th>Spacing Between Columns</th>
<th>p-multipliers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Lead Row</td>
</tr>
<tr>
<td>Normal-spacing GB</td>
<td>4.3D</td>
<td>2.5D</td>
<td>0.78</td>
</tr>
<tr>
<td>Small-Spacing GB</td>
<td>3.0D</td>
<td>1.5D</td>
<td>0.70</td>
</tr>
<tr>
<td>Large-Spacing GB</td>
<td>6.0D</td>
<td>3.5D</td>
<td>0.84</td>
</tr>
<tr>
<td>Normal-spacing GV</td>
<td>4.3D</td>
<td>2.5D</td>
<td>0.57</td>
</tr>
</tbody>
</table>

As discussed earlier, the above obtained p-multipliers representing an average reduction of the soil resistance of the pile over its depth. However, directly comparing the p-y curves at different depths for the normal spacing batter pile group model (Figure 5.29) allows one to evaluate the variation of p-multiplier for different soil layers. These directly obtained p-multipliers are summarized in Table 5.3. As the author expected, the shallower soils for batter pile group foundation will suffer much higher group interaction effect than deeper soils. Also one can observe that under high group interaction effect, the 2\textsuperscript{nd} and 3\textsuperscript{rd} row piles will have highest reduction in lateral reaction and shift more responsibilities to the boundary piles (the lead and trail row piles).
### Table 5.3: Variation of p-multipliers at different depth for normal spacing GB model

<table>
<thead>
<tr>
<th></th>
<th>Lead Row</th>
<th>2nd Row</th>
<th>3rd Row</th>
<th>Trail Row</th>
<th>Side Column</th>
<th>Inner Column</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average value</strong></td>
<td>0.78</td>
<td>0.74</td>
<td>0.67</td>
<td>0.59</td>
<td>0.73</td>
<td>0.55</td>
</tr>
<tr>
<td>(obtained from Load-deflection curves)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>16ft below pile cap</strong></td>
<td>0.29</td>
<td>0.14</td>
<td>0.14</td>
<td>0.28</td>
<td>0.44</td>
<td>0.26</td>
</tr>
<tr>
<td>(soft clay)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>28ft below pile cap</strong></td>
<td>0.63</td>
<td>0.52</td>
<td>0.48</td>
<td>0.55</td>
<td>0.95</td>
<td>0.34</td>
</tr>
<tr>
<td>(stiff clay)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 6
SUMMARY AND CONCLUSIONS

6.1 Summary

In this study, a series of finite element analyses are carried out based on the data obtained from the full-scale lateral load test on the batter pile group foundation of M19 eastbound pier of the new I-10 Twin Span Bridge over Lake Ponchartrain, Louisiana.

A finite element model with exactly the same pile geometry, inclination and pile to pile spacing as of the M19 eastbound foundation is constructed using the finite element analysis software ABAQUS. The subsurface exploration data (including five CPTs, a SPT and a soil boring test) from the same site of full-scale lateral load test project is directly used to interpret the subsurface condition of the M19 eastbound pier site. Soils are classified into eight layers including two sandy layers and six clayey layers. Different soil constitutive models are assigned to these layers, and the parameters associated with these models are correlated or estimated from laboratory and in-situ tests.

For the sake of investigating the effect of group interaction and pile inclination, four additional FE models (including a large spacing / small spacing batter pile group model, a normal spacing vertical pile group model and a single vertical pile model) are developed to perform the same lateral load tests. A coupled pore fluid diffusion and stress analysis on a single pile model is performed to estimate the influence of excessive pore water pressure on laterally loaded piles. To compare the continuum-based FEM results with the widely used p-y curve
methods, the FB-MultiPier software is used to perform a p-y curve based analysis of the full-scale lateral load tests.

All the results of FE and FB-MultiPier analysis are compared with the field measured data including lateral deflection profiles, bending moment profiles and axial load evolution curves. Besides, the lateral/axial load distribution, soil resistance profiles and the p-y curves obtained from these analyses are compared with each other and some interesting conclusions were obtained.

Another major contribution of this research is the use of a more advanced soil constitutive model for clay in the FE analysis of laterally loaded piles. Many researchers (e.g. Brown and Shie, 1990; Ahmadi and Ahmari, 2009) realized the importance of incorporating the anisotropic strength behavior of soils into numerical analysis of laterally loaded pile problems. In this research, the Anisotropic Modified Cam-clay Model (AMCCM) proposed by Dafalias (1987) is implemented in UMAT which is a user interface that provided by ABAQUS to allow the use of self-defined constitutive models in the FE analysis. Being aware that the UMAT is going to be used in a large 3D FE model, the algorithm that adopted to implement AMCCM needs to have high computational efficiency as well as a controllable accuracy. The explicit substepping scheme with modified Euler algorithm is selected to carry out such task since the combination of substepping and modified Euler algorithm provides a mean to control the error in explicit integrations. In the later FE analysis of the full-scale tests, the implementation of AMCCM
shows a wonderful computational stability and efficiency, which greatly accelerated the whole research process.

6.2 Conclusions

Some important conclusions obtained from the implementation of Anisotropic Modified Cam-clay model and the FE analysis of the laterally loaded batter pile groups are summarized below:

**AMCCM and its implementation**

1. The Anisotropic Modified Cam-clay model (Dafalias, 1987) is capable to capture both “inherent” and “induced” anisotropic of the clay behavior in a simple but effective formulation.

2. The explicit substepping scheme proposed by Sloan (1987) shows excellent stability, efficiency and accuracy in implementing complex elasto-plastic constitutive models. Such advantages greatly facilitated the application of advanced constitutive models in large 3D FE models.

**Coupled pore fluid diffusion and stress analysis**

3. The profiles of lateral deflection and bending moment from partially drained and fully drained analysis are very close to each other with a maximum difference of about 3% in lateral deflection and 4% in bending moment.
4. The effect of excessive pore pressure caused by lateral load on pile cap is really small compared to the total geostatic pressure due to soil self-weight. Therefore, the presence of pore water pressure has limited influence to the FE analysis results.

**Lateral deformations**

5. The soils within ± 28 ft width and 29 ft depth were mobilized ranges from 0.20 in to 0.65 in due to the applied lateral load. Such influence is neglectable for the soil beyond this region.

6. The lateral deflection profiles from the FEA show very good match with the field measured data. The one by FB-MultiPier has good prediction in pile cap movement, however, the “Stationary point” (the point with no lateral deformation) is much shallower than those given by the field measurements and the FEM. Such discrepancies were attributed to the simple way that the FB-MultiPier accounts for the group interaction effects.

7. The comparison between the five FE models that have various pile spacing and inclination shows that: 1) Pile groups have much higher lateral displacement than that of a single pile; 2) For the batter pile groups, small spacing model produces largest lateral deformation (0.94in) while large spacing model shows relative small lateral deformation (0.71 in); 3) Vertical pile group model exhibits significant large lateral deformation (1.22 in), which is 54% greater than the batter pile group with the same spacing (0.79 in) and 130 % larger than the single isolated pile (0.53in).


**Contours of stress\strain distribution**

8. The lateral load transmitted from the pile pushes the soil in front of the piles to a denser state and the gaps left behind the piles are instantaneously filled by soils and thus result in a relaxation of the soil behind the piles.

9. The second stiff clay layer takes major portion of the deviatoric stresses caused by lateral movement of piles, while the first soft clay layer accumulated highest deviatoric strains.

10. The comparison between the five FE models shows that: 1) There is a significant increase of the deviatoric stress of the soil in front of the lead row for the vertical pile group comparing to the batter pile group at same spacing. 2) Deviatoric stresses become more concentrated and intensified when pile spacing reduces due to stronger shadowing and edge effect between the piles.

**Profiles of bending moment**

11. Both the FEM and the FB-MultiPier predicted bending moments agree well with those deduced from the strain gauges data. The maximum positive moment predicted by the FEM are very close to those by FB-MultiPier; while the maximum negative bending moment from FEM is 21%-24% lower than those predicted by the FB-MultiPier. Besides, the location of maximum negative moment from the FEM occurs at slightly shallower depth (27.5 ft below pile cap) than the location obtained from the FB-MultiPier analysis (31.5 ft below pile cap).

12. The bending moment profiles of the piles aligned in different columns (pile 1, 2 and 3) show that the pile located at side column has 11.7% higher maximum positive bending moment (at
pile head) and 18.0% higher maximum negative bending moment (at 27.5 ft below pile cap) than those of the pile in inner column. For the piles aligned in different rows, it is found that piles in lead row and trail row have an averagely 7% more bending moment than those in 2\textsuperscript{nd} and 3\textsuperscript{rd} rows.

13. The comparison between the five FE models shows that: 1) Small spacing batter group model produces a slightly larger bending moment compared to the normal spacing and large spacing models; 2) All batter pile groups have smaller (30\%) bending moments than the vertical pile group regardless of the spacing; however, they are still significantly higher (60\%) than the bending moment developed on a single isolated pile.

\textit{Distribution of lateral load}

14. For the piles in different columns, the side column piles take the most lateral load followed by the middle column and then inner column. This variation exists in all pile group models and more significantly in small spacing group and less obvious in large spacing group, which indicates that the edge effect is intensified when spacing between columns reduced.

15. For the piles in different rows, the one located in the lead row has the highest lateral load, followed by the 2\textsuperscript{nd} and the 3\textsuperscript{rd} rows. For the large spacing and normal spacing batter group models, the trail row takes the smallest portion of lateral load; while for the small spacing batter group and vertical group model the trail row takes slightly higher lateral load than 3\textsuperscript{rd} row or even 2\textsuperscript{nd} row. The proposed explanation is that the increased group interaction effect
have more influence on softening the soil in between the rows and thus shift more lateral load to the lead and trail piles.

**Axial load evolution and distribution**

16. The results of axial load from FEA show good agreement with strain gauge data, showing that the negative batter pile is subjected to axial extension and the positive batter pile is subjected to axial compression.

17. For the vertical pile group model, more axial load is taken by the lead and trail rows; while more axial load are taken by the 2\textsuperscript{nd} and 3\textsuperscript{rd} rows piles in the batter pile group model. This phenomenon is attributed to the trapezoidal soil block formed in between of 2\textsuperscript{nd} and 3\textsuperscript{rd} rows, which strengthened this area’s resistance to rotation.

**Soil resistance profiles and p-y curves**

18. For the piles in different columns, the side piles encountered higher lateral resistances than the piles located in the middle and inner columns. For the piles in different rows, the piles located in the lead and trail rows have higher soil resistance than the 2\textsuperscript{nd} and 3\textsuperscript{rd} row piles.

19. The soil resistances by FB-MultiPier are generally close to those generated by the FEM model. However it predicts a deeper zero soil resistance point (43ft below cap).

20. The comparison between the five FE models shows that: 1) All the three different spaced batter pile groups has similar soil resistance distribution pattern; 2) The soil resistance by the vertical pile group is higher than that of the batter pile groups. The maximum positive lateral resistance from the vertical group (26.4 kips) is 28.7\% higher than that of the normal spacing
batter group (18.8 kips); 3) The vertical pile group exhibits extremely sharp fluctuation in soil resistance, while the single isolated vertical pile model produces a relatively smooth distribution on the resistance.

6.3 Recommendations for Future Works

From present work, it was found that piles in side columns take innegligible larger bending moments and lateral loads than those of the inner columns, which is also observed by Ruesta and Townsend (1997) in their full-scale lateral load test. However, in most literatures on laterally loaded pile groups (e.g. Brown et al., 1988; Rollins et al., 1998), such variations between columns are normally ignored and the only piles in different rows are treated with different p-multipliers. Therefore, it is suggested that researchers should pay more attention on lateral load distribution of piles in different columns in the future numerical studies or field tests on laterally loaded pile groups.

In this study, it was also observed that for some group pile models that the lateral load taken by lead row and trailing rows are descending, while in some other models show that the trail row takes larger lateral load than the 3rd or even the 2nd rows. Interestingly, the lateral load distribution patterns observed from many lateral load tests are also varied from each other. This phenomenon is attributed to the group interaction effect, which should be examined by future numerical analyses or field tests on pile groups with different spacing.

The concept of “Trapezoidal zone” for batter pile groups is firstly introduced in this study. The author believes this zone is responsible for the increased allocated axial load on the 2nd and
the 3rd row piles as compared to the lead and trail row piles. However, such axial load distribution pattern is fully obtained from the FE analysis without any support from field. It is suggested that the axial load distribution pattern in batter pile group foundation to be further investigated by conducting more field lateral load tests and/or numerical studies such as finite element analyses on batter pile group foundations to study the effect of trapezoidal zone to the distribution of axial load on batter piles.
REFERENCES


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Das, B. M. (1990), Principles of Foundation Engineering, 2nd ed. PWS-Kent, Boston, MA.


APPENDIX: PSEUDO ALGORITHM FOR IMPLEMENTING AMCCM

1) INPUT: $\sigma$, $\Delta \varepsilon$, $\alpha$, $e$, $p_0$, $FTOL$, $STOL$

2) Set $\beta_0 = \beta_1 = 0$

3) Calculate

$$\bar{C}_0 = \bar{C}(e, \sigma, \beta_0 \Delta \varepsilon)$$

$$\bar{C}_1 = \bar{C}(e, \sigma, \beta_1 \Delta \varepsilon)$$

$$\Delta \sigma_0 = \beta_0 \bar{C}_0 : \Delta \varepsilon$$

$$\Delta \sigma_1 = \beta_1 \bar{C}_1 : \Delta \varepsilon$$

$$f_0 = f(\sigma + \Delta \sigma_0, \alpha, p_0)$$

$$f_1 = f(\sigma + \Delta \sigma_1, \alpha, p_0)$$

4) If $f_1 \leq FTOL$, the stress path is within elastic regime. Update stress and void ratio:

$$\Delta \sigma = \Delta \sigma_1$$

$$\sigma_{updated} = \sigma + \Delta \sigma$$

$$e_{updated} = e - \kappa \frac{\Delta p'}{(1 + e)p'}$$

Else if $f_0 \leq -FTOL$ & $f_1 \geq FTOL$, the stress path is crossing yield locus. Perform steps 5 to 9 MAXITS times to determine $\beta_{int}$ and update stress and void ratio for the $\beta < \beta_{int}$ part.

Else, perform substepping algorithm for elastoplastic loading, goto step 11.

Method of False Position for Finding $\beta_{int}$

5) Calculate

$$\beta = \beta_1 - \frac{f_1(\beta_1 - \beta_0)}{f_1 - f_0}$$
\[ \bar{C} = \bar{C}(\varepsilon, \sigma, \beta \Delta \varepsilon) \]

\[ \Delta \sigma = \beta \bar{C} \Delta \varepsilon \]

\[ f = f(\sigma + \Delta \sigma, \alpha, p_0) \]

6) If \(|f| \leq FTOL\) then goto step 9

7) If \(f \cdot f_1 \leq FTOL\) then set \(\beta_0 = \beta_1, f_0 = f_1\);
Else, set \(\beta_1 = \beta, f_1 = f\).

8) Maximum iteration number has reached, **exit** with error message.

9) \(\beta_{int}\) has been found, update stress and void ratio:

\[ \sigma \leftarrow \sigma + \Delta \sigma \]
\[ e \leftarrow e - \kappa \frac{\Delta p'}{(1 + e)p'} \]

10) Set \(\Delta \varepsilon \leftarrow (1 - \beta) \Delta \varepsilon\), pass the current \(\Delta \varepsilon, \sigma\) and \(e\) into step 11 to perform substepping integration.

**Modified Euler Algorithm with Substepping for AMCCM**

11) Set \(t = 0, \Delta t = 1\). While \(t < 1\), Perform steps 12-25 MAXSTEP times.

12) For \(i = 1,2\), calculate

\[ \Delta \sigma_i^e = \Delta t \bar{C}_i \Delta \varepsilon \]
\[ \Delta \sigma_i = \Delta \sigma_i^e - \Delta \lambda_i \bar{C}_i \Delta \varepsilon B_i \]
\[ \Delta p_{0i} = \frac{1 + e}{\lambda - \kappa} p_{0i} \Delta \lambda_i tr B_i \]
\[ \Delta \alpha_i = \frac{1 + e}{\lambda - \kappa} \Delta \lambda_i |tr B_i| \frac{c}{p_0} (s_i - xp_i \alpha_i) \]

Where
\[ \bar{C}_i = \bar{C}(e, \sigma_i, \Delta t \Delta \varepsilon) \]
\[ B_i = B(\sigma_i, \alpha_i, p_{0i}) \]
\[ \Delta \lambda_i = \max\{0, \Delta \lambda(\sigma_i, \alpha_i, \Delta t \Delta \varepsilon, p_{0i})\} \]

And

\[ \sigma_1 = \sigma, \quad \alpha_1 = \alpha, \quad p_{01} = p_0 \]
\[ \sigma_2 = \sigma + \Delta \sigma_1, \quad \alpha_2 = \alpha + \Delta \alpha_1, \quad p_{02} = p_0 + \Delta p_{01} \]

The explicit expression of \( B(\sigma, \alpha, p_0) \) and \( \lambda(\sigma, \alpha, \Delta t \Delta \varepsilon, p_0) \) can be found in Eq. (27a) and (26).

13) Update the stresses and hardening parameters, the bar denotes that these quantities are not the final updated values but just stored temporarily.

\[ \bar{\sigma}_{\text{updated}} = \sigma + \frac{1}{2}(\Delta \sigma_1 + \Delta \sigma_2) \]
\[ \bar{\alpha}_{\text{updated}} = \alpha + \frac{1}{2}(\Delta \alpha_1 + \Delta \alpha_2) \]
\[ \bar{p}_{0\text{updated}} = p_0 + \frac{1}{2}(\Delta p_{01} + \Delta p_{02}) \]

14) Determine the relative error.

\[ R = \max\left\{ \frac{||\Delta \sigma_2 - \Delta \sigma_1||}{2||\bar{\sigma}_{\text{updated}}||}, \frac{||\Delta \alpha_2 - \Delta \alpha_1||}{2||\bar{\alpha}_{\text{updated}}||}, \frac{|\Delta p_{02} - \Delta p_{01}|}{2\bar{p}_{0\text{updated}}} \right\} \]

15) If \( R > STOL \) then this substep has failed. Reduce the size of current time step using the factor \( q \)

\[ q = \max\{0.9 \sqrt{STOL/R}, 0.1\} \]
\[ \Delta t \leftarrow \max\{q \Delta t, \Delta t_{\text{min}}\} \]

Then return to step 12.
16) This substep has succeeded. Permanently update the stresses and hardening parameters.

\[ \sigma_{\text{updated}} = \bar{\sigma}_{\text{updated}} \]
\[ \alpha_{\text{updated}} = \bar{\alpha}_{\text{updated}} \]
\[ p_{0_{\text{updated}}} = \bar{p}_{0_{\text{updated}}} \]

Update void ratio:

\[ e_{\text{updated}} = e - (\lambda - \kappa) \frac{\Delta p_0}{(1 + e)p_0} \]

**Yield Surface Correction Scheme for AMCCM**

17) If \( f \left( \sigma_{\text{updated}}, \alpha_{\text{updated}}, p_{0_{\text{updated}}} \right) > FTOL \) then perform step 18-21 MAXIT times. In the following equations the subscribe \( u \) and \( c \) denotes uncorrected and corrected quantities respectively.

18) Set

\[ f_u = f \left( \sigma_{\text{updated}}, \alpha_{\text{updated}}, p_{0_{\text{updated}}} \right), \]
\[ \sigma_u = \sigma_{\text{updated}} \]
\[ \alpha_u = \alpha_{\text{updated}} \]
\[ p_{0_u} = p_{0_{\text{updated}}} \]

Calculate \( \delta \lambda_c \) using Eq. (33).

19) Correct stresses and hardening parameters:

\[ \sigma_c = \sigma_u - \delta \lambda_c C_u B_u \]
\[ \alpha_c = \alpha_u + \delta \lambda_c \frac{1 + e}{\lambda - \kappa} \left| \text{tr} B_u \right| \frac{c}{p_0} (s_u - \chi p_u \alpha_u) \]
\[ p_{0_c} = p_{0_u} + \delta \lambda_c \frac{1 + e}{\lambda - \kappa} p_{0_u} (\text{tr} B_u) \]
20) If $|f(\sigma_c, \alpha_c, p_{0c})| > |f(\sigma_{updated}, \alpha_{updated}, p_{0updated})|$ then abandon previous correction and calculate:

$$\sigma_c = \sigma_u - \frac{f_0 B_u}{B_u \cdot B_u}$$

$$\alpha_c = \alpha_u$$

$$p_{0c} = p_{0u}$$

21) If $|f(\sigma_c, \alpha_c, p_{0c})| > FTOL$ then goto step 23;

Else, set $\sigma_u = \sigma_c$, $\alpha_u = \alpha_c$, $p_{0u} = p_{0c}$, GOTO step 18.

22) Convergence cannot be achieved after MAXIT iterations, exit with error message.

23) Converged, update stresses and hardening parameters

$$\sigma_{updated} = \sigma_c$$

$$\alpha_{updated} = \alpha_c$$

$$p_{0updated} = p_{0c}$$

*Estimation of Next Substep Size and Completion of the Total Step*

24) Estimate the size of the next step by using the following formula:

$$q = \min\{0.9\sqrt{STOL/R}, 1.1\}$$

Update pseudo time, ensure the next step size larger than the minimum step size and less than $1 - t$:

$$\Delta t \leftarrow \min\{\max(q\Delta t, \Delta t_{min}), 1 - t\}$$

$$t \leftarrow t + \Delta t$$

25) Integration of stresses at this whole step is successful. Pass the updated stresses, hardening
parameters and state variables back to ABAQUS. Set the Jacobian the same as the secant stiffness matrix of the latest substep, and pass it back to ABAQUS.
VITA

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