1975

Instrumental-Variable Estimation of Discrete-Time Model Parameters for Adaptive Control.

Armand Terrell Touchstone
Louisiana State University and Agricultural & Mechanical College

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A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirement for the degree of Doctor of Philosophy in The Department of Chemical Engineering

by

A. Terrell Touchstone
B.S., University of Alabama, 1969
M.S., University of Alabama, 1970

August, 1975
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ABSTRACT

A sequential instrumental-variable technique was studied for the estimation of the parameters of a discrete-time model of a linear dynamical system subject to disturbances and measurement noise. The method preserves the basic simplicity of the least-square estimation procedure, yet it does not require a priori information on the noise statistics. The approach is to approximate the optimal instrumental variables with the output of an auxiliary model that is run in parallel with the plant and utilizes current estimates of the parameters. The recursive algorithm is modified to permit tracking of parameter variation for a time variable system. The method is extended for use as a sequential linearization of a non-linear process and it is shown to apply to a simulated, jacketed chemical reactor.

The method was applied to an overdamped second-order example to demonstrate its performance in comparison to least-squares estimation and to study the selection of various "tuning" parameters to be used in its implementation. Principle among these parameters is the sampling interval which is shown to have a multicollinearity effect on the regression matrix of the difference equation model if the sampling rate is too rapid. Both the stochastic process and the deterministic process subject to measurement noise are considered. Applications include open- and closed-loop configurations and in the latter case it is shown that set point variation is necessary for estimation.
The culmination of the research is the use of the estimation technique in a self-adaptive control strategy. In such on-line applications, this straightforward, yet effective, method is able to function reliably on a minimum of statistical information.
CHAPTER I
INTRODUCTION

Dynamical systems representative of the process industry do not readily lend themselves to simple mathematical description. In contrast, many systems studied in fields like aerospace or electronic engineering are well understood and can be represented by low-order linear models. For this reason, control technologists in these areas have been free to concentrate on the development of control strategies. It is well known that the past fifteen years have wrought great advances in control theory for the process which is amenable to description. Consequently, there exists a world of space age technology yet to be tapped by the many fields of endeavor where the "black art" is still practiced. Before the chemical engineer can apply the sophisticated strategies of the control theorists he must be capable of describing his process mathematically.

Two approaches are available for the determination of a mathematical model. One is an analytic approach in which the engineer tries to incorporate all that is known about the process into equation form. Typically, the resulting expression is a high-order system of nonlinear differential equations. Since much of control theory is based on the linear process, it is generally necessary to linearize this result before it can be used. Alternately, it has been traditional in the process industry to describe a process using empirical methods. This approach simplifies the mathematics considerably and the answers
it provides are usually as close to the fact as those obtained via the former approach. The major limitation of the empirical method has been the inherent inflexibility. That is, a change in the operating conditions required that the problem be re-worked and, although it was frequently practiced, extrapolation was nothing short of guess work.

Today, with the aid of an on-line digital computer, it is now possible to use an empirical model to represent a non-linear plant in a sequential manner. Specifically, a low-order linear model is selected to represent the dynamic system. By use of the proper estimation algorithm, the computer is able to produce updated parameter estimates for the empirical model at each sample instant. With this quasi-linearization procedure it is now feasible for the chemical engineer to characterize the dynamics of his process sufficiently to be able to apply such advanced control techniques as the self-adaptive strategy.

Because the characteristics of many processes are time variant and/or non-linear, it is necessary to continuously monitor and adjust controller settings in order to maintain suitable control. In an effort to automate this procedure, the self-adaptive strategy employs an on-line identification method to derive a simple process model and this information is in turn used by an adaptive-controller to adjust itself to meet the changing plant dynamics. The heart of this strategy is on-line identification.

This research is a study of one method of process identification — namely, the instrumental-variable method. This method can be used to estimate model parameters from an array of linear algebraic equations involving these parameters, a set of observations, and a set of unob-
servable noise terms. The estimates are obtained by first multiplying the array by a rectangular matrix of "instruments" to yield a square invertible matrix which is then solved for the estimates.

This method is in fact one out of a host of methods that have been proposed. However, it is with the purpose that the identifier be included in an adaptive control configuration that the study is made. With this intention, the instrumental-variable method offers certain advantages. First, it is simple as it is based on the classical least-squares method. It can therefore be expressed in the recursive form of the least-squares estimator and accrue the advantages of real-time operation. It can be expressed in the dynamic form of the least-squares estimator and thereby realize the ability to track process parameter variations. Secondly, it carries a certain measure of sophistication without requiring statistics on the disturbances. That is, it will be shown to be able to estimate model parameters in the presence of measurement noise. It will be shown to be capable of approximating the dynamics of a non-linear process with a low-order linear model in a sequential manner.

The first portion of this research is an exhaustive study of the instrumental-variable identification method as it applies to the open-loop configuration. The presentation and development here are not unique but the computational results are both worthy and considerable. Both the static and dynamic algorithm are examined. The effect of measurement noise and plant disturbances are presented. A practical guide is developed whereby the estimation algorithm maybe "tuned". Principle here is the selection of the sample rate due to effects of
multicollinearity of the regression matrix. Finally, discussion is given to the effect of dead time.

Chapter III is concerned with identification of the process in the closed-loop system to include the self-adaptive configuration. The approach is to identify the process equation singularly from the simultaneous set in which it is embedded. It is shown that identification of the process in the feedback control loop must depend on set-point variation to be of practical use in on-line applications such as adaptive control.

Chapter IV is an application of the instrumental-variable method to a non-linear chemical reactor. This is specifically a study of the empirical approach to model determination and it is an attempt to get closer to application in the "real" world. Consideration is given both to open-loop and to closed-loop identification.
CHAPTER II

OPEN-LOOP IDENTIFICATION OF A NOISY PROCESS UTILIZING
A SEQUENTIAL INSTRUMENTAL-VARIABLE ESTIMATOR

Introduction

Stated simply, process identification is the problem of determining a description of the relationship between the input and output variables of the process. The fact that the unknown plant has been selected for study may suggest non-linear behavior, time-variable parameters, or simply a linear system operating in an extreme environment. Further, the plant may have a number of input and output variables in which case it is termed multivariable. It is important to make the distinction between the identification problem and the similar, but inverse, problem of output estimation. In estimation, the physical relationship between the input and output is given (known) and either a filtered or predictive estimate of the output is desired. The problem that is generally referred to as "optimal" requires a statistical description of all noise and disturbance processes introduced into the system. In this dissertation, the disturbance will always be an important unknown and consequent challenge. The "relationship" is commonly called the mathematical model of the process and, as suggested, may be utilized for the purpose of output estimation, filtration, prediction, or control.

More specifically, the following identification problem is considered. Given the discrete-data record of a sequence in inputs
\[ u(t); \ t=1,2,...,K \] or \[ u(t) \] and the corresponding, disturbance-corrupted outputs \[ y(t) \] of a dynamical system, determine the parameters of a suitable linear model to be fitted to the data record. The number of observations, \( K \), is larger than the number of parameters to be estimated.

To be of practical use in an adaptive control strategy, the problem as stated is too general and further restrictions are needed. A major requirement of adaptive philosophy is that identification be performed "on-line" and in the presence of the system's normal operating signals and disturbances. In fact, a worthwhile objective is the ability to accomplish the identification task using only the normal control inputs. An obvious restriction is that the computation time for identification be reasonably short compared to the sampling time required to maintain good control. It is further necessary that the computation time be short compared to the rate of variation of the process parameters. A restriction closely allied to short identification time is that data storage should be minimized. These requirements suggest a recursive technique which is capable of improving its estimates as more data is received and is able to detect parameter variation at the same time. The general approach is then a quasi-linearization procedure which yields a model of the plant at each sampling instant.

Identification methods can be broadly classified as either parametric or nonparametric methods. Techniques belonging to the former division are of interest here and they typically reduce the identification problem to parameter estimation once the form of the mathematical model is proposed. Parametric methods can be considerably
simplified if a priori information on the model order and dead time is available. In the absence of a priori information, assumptions should be made to avoid iterative solutions. In this connection, it is important to remember that parameter estimates are maintained at each sampling instant so even the simple first-order model should generally provide adequate structure for control purposes.

The procedure of parametric identification can be separated into two distinct stages. The algebraic equations relating the unknown model parameters, the observation set, and a set of unobservable noise terms are first established. It is then necessary to determine the model parameters by some suitable estimation technique. The theory in this chapter will be developed upon this framework and will set forth the nomenclature for the entire dissertation. A feature topic on multicollinearity effects of the difference equation model will be presented. Computational results and comparisons are given for a selected second-order plant with particular interest given to the effects of measurement noise on identifier performance and the selection of identifier tuning parameters.

**Specification of the Model**

The first step of the identification problem is the specification of an algebraic structure between the input and output variables. This model is postulated a priori and stems from the underlying theory governing the particular dynamic system under investigation. Because most adaptive control and optimization problems are time-varying and/or nonlinear, convenience has replaced the classical frequency-domain analysis with time-domain methods. It has become standard in time-
domain analysis to describe dynamic systems by first-order sets of simultaneous differential equations written in matrix format.

The Algebraic Structure

Consider the state-variable equation

\[ \dot{x} = F(t)x + G(t)u \]  

where

- \( x(t) \) is the \( n \)-vector of state variables,
- \( u(t) \) is the \( r \)-vector of controls,
- \( F(t) \) is an \( nxn \) matrix,
- \( G(t) \) is an \( nxr \) matrix.

If \( \gamma(t, t_0) \) is the state transition matrix, then the solution to Equation 1 is

\[ x(t) = \gamma(t, t_0)x_0 + \int_{t_0}^{t} \gamma(t, \tau)G(\tau)u(\tau)d\tau \]  

The discrete-time equivalent of Equation 2 for the case where the system is stationary is

\[ x_{t+1} = \gamma(T)x_t + \Gamma(T)u_t \]

where \( T \) is the sample interval \( [(t+1) - t] \) and is constant. For simplicity, it is assumed that \( \gamma \) is of maximum rank. This implies that \( \gamma \) is derived from a linear set of constant-coefficient differential equations, or if the model is obtained by linearizing about an operating point, it is assumed that the necessary conditions exist for the Jacobian matrix \( \gamma \) to be of rank \( n \). It is also assumed that the system is completely controllable and completely observable where the observability is given by

\[ y_t = H^T x_t \]

where \( y_t \) is an \( m \)-vector of observables.
Consideration is now restricted to the single-input, single-output model. In this case the vectors $y_t$ and $u_t$ become scalars. Further, it is convenient to make two transformations on the model of Equations 3 and 4. The first transforms Equations 3 and 4 into their canonical equivalents. The second transforms the canonical form into a difference equation, and vice versa. An expression for the relationship between the variables of the state and difference equations would require a lengthy development and for this reason the reader is referred to the monograph of Lee (1), pages 85-93, for the detailed transformations. The resulting difference equation can be written in terms of its disturbance-free output, $c_t$, to give

$$c_t = a_1 c_{t-1} + a_2 c_{t-2} + \ldots + a_p c_{t-p} + b_1 u_{t-M-1} + b_2 u_{t-M-2} + \ldots + b_q u_{t-M-q}$$

where $M$ represents the process dead time in integer multiples of the sample interval, $T$, and $p$ and $q$ are the number of output and input terms respectively. For clarification, the difference equation may also be expressed by its pulse transfer function (2)

$$H(z) = \frac{C(z)}{U(z)} = \frac{b_1 z^{-1} + b_2 z^{-2} + \ldots + b_q z^{-q}}{1 - a_1 z^{-1} - a_2 z^{-2} - \ldots - a_p z^{-p}} z^{-M}$$

Simplification of Equation 5 can be realized by utilizing the notation

$$c_t = A(z^{-1})c_{t} + B(z^{-1})u_t$$

where

$$A(z^{-1}) = a_1 z^{-1} + a_2 z^{-2} + \ldots + a_p z^{-p}$$

$$B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} + \ldots + b_q z^{-q}$$

and $z^{-1}$ is the backward shift or translation operator $z^{-1}c(t) = c(t-1)$. 
Finally, all model parameters can be combined yielding the vector equation
\[ c_t = \varphi x_{t-1} ; t = 1,2,\ldots,K \]  
(8)

where
\[ \varphi = [a_1 a_2 \ldots a_p b_1 b_2 \ldots b_q] \]  
(9)
\[ x^T_{t-1} = [c_{t-1} c_{t-2} \ldots c_{t-p} u_{t-M-1} u_{t-M-2} \ldots u_{t-M-q}] \]

The transformation to difference equation form was performed for two very important reasons. First of all, when applying regression methods to process identification, it is necessary to assume that all of the variables which appear in the model are available for measurement either directly or indirectly from sensor outputs (3). This is a problem with differential equation and state-variable equation models. Secondly, difference equation identification requires the estimation of only \( n \) parameters where \( n = (p + q) \) (1). On the other hand, estimation of the state-transition and driving matrix requires the estimation of \( (n^2 + nr) \) parameters. The reader is referred to Swanick and McCorkell (4) and the references at the end of that article for a complete series on regression methods applied to the identification of the state-variable model.

The Statistical Model

The task now is to make the mathematical model developed in the last section to fit within the realm of measurements and uncertainty. For the special case of Equation 8 in which the model is first-order, it is only required that two model parameters be estimated. Further, if \( c_t \) were accessible without error and the model fit the process exactly, then only two data sets (observations) would be required in order to determine the model parameters exactly. That is, two observations would give two equations and two unknowns. But in practice
there exist various amounts of uncertainty, hence the emergence of the problem which is to be discussed in this dissertation.

The remedy is to provide Equation 8 with a statistical degree of freedom. The two major sources of uncertainty in the equation are: (1) the inability of the model to describe the process and (2) the disturbances entering the process. The following equation-error model is therefore hypothesized to account for the uncertainties

\[ c_t = \phi c_{t-1} + \omega_{d,t}, \quad t = 1,2,\ldots,K \]  \hspace{1cm} (10)

where \( K \) is the number of observations.

Equation 10 is now extended to include the effect of measurement noise on the output (considered the dependent or endogeneous variable in regression analysis). Consider the following diagram:

Again, a set of \( K \) observations is given:

\[ \{u_t; \ t = 1,2,\ldots,K\} \]

\[ \{y_t; \ t = 1,2,\ldots,K\} \]

As before, the algebraic structure predicted by theory and adjusted for unmeasured disturbances and equation-uncertainty is given by Equation 10.
But now the true output $c_t$ is not accessible and only the noise corrupted measurement variable $y_t$ is available.

$$y_t = c_t + w_{n,t}$$

(11)

For illustration, consider the first-order, special case of Equation 10 with Equation 11 substituted for $c_t$.

$$(y_t - w_{n,t}) = a_1(y_{t-1} - w_{u,t-1}) + b_1u_{t-1} + w_{d,t}$$

or

$$y_t = a_1y_{t-1} + b_1u_{t-1} + (1 - a_1)w_{n,t} + w_{d,t}$$

(12)

For linear systems, the principle of superposition may be used to combine the effects of all unmeasured disturbances and measurement noise (and any other factors that produce a difference between the model output and the observed output) into the hypothetical equation-error term $w_t$ to give

$$y_t = a_1y_{t-1} + b_1u_{t-1} + w_t$$

(13)

The $w_t$ term represents a correlated disturbance in general and it is standard to interpret $w_t$ as the output of a filter which operates on "white" noise to generate the process disturbance. This relationship is given by

$$w_t = C(z^{-1})v_t$$

(14)

with

$$C(z^{-1}) = \frac{1 + R(z^{-1})}{1 + S(z^{-1})}$$

and $v_t$ is an independent random variable such that is has a zero mean and a constant variance, that is,

$$E[v_t] = 0; \quad t = 1,2,\ldots,K$$

(15a)

$$E[v_tv_t^T] = \sigma^2I; \quad t = 1,2,\ldots,K$$

(15b)

Elements in $S(z^{-1})$ represent autocorrelation terms while elements in $R(z^{-1})$ represent moving-average terms.
Using Equation 7, the generalized form of Equation 13 can be written

\[ y_t = A(z^{-1})y_t + B(z^{-1})u_t + C(z^{-1})v_t \]  \hspace{1cm} (16)

or

\[ [1 - A(z^{-1})]y_t = B(z^{-1})u_t + C(z^{-1})v_t \]

Finally, Equation 16 can be expressed in regression form by separating model parameters from the observations to yield

\[ y_t = \varphi = [a_1 \ a_2 \ \ldots \ a_p \ b_1 \ b_2 \ \ldots \ b_q] \]

\[ x^T_{t-1} = [y_{t-1} \ y_{t-2} \ \ldots \ y_{t-p} \ u_{t-M-1} \ u_{t-M-2} \ \ldots \ u_{t-M-q}] \]

**Parameter Estimation**

The second stage of the identification problem is the estimation of the parameters of the stochastic model, Equation 17. It is important to first discuss the desirable properties of the statistical estimator (5). An important property of estimates is that of being "unbiased" which refers to the first moment of the distribution of the estimating function. If \( f(x_1, x_2, \ldots, x_K) \) is an estimate of \( \varphi \), it is an unbiased estimate provided

\[ E[f(x_1, x_2, \ldots, x_K)] = \varphi \]

for any set of sample values \( x_1, x_2, \ldots, x_K \). Another criterion can be developed by considering the second moment of the distribution. The "best unbiased estimate" or "minimum-variance estimate" of \( \varphi \) is the one which has the property

\[ \sigma_f^2 = E[f(x_1, x_2, \ldots, x_K) - \varphi]^2 = \text{minimum} \]

For simplicity, the estimators to be considered are further restricted to those that are "linear" in the observations. The properties so far describe an estimator that is the "best linear unbiased estimate". For uncorrelated disturbances, the least-squares (LS) estimator would
possess these properties. In the case of correlated disturbances, the LS estimator would be non-minimum compared to the generalized least-squares (GLS) estimator which would now possess the properties above.

In practice it is difficult to obtain unbiased estimates of parameters, but in many cases it is possible to obtain estimates that have similar properties asymptotically as the sample size tends towards infinity. The estimate \( f(x_1, x_2, \ldots, x_K) \) is said to be a "consistent" estimate of \( \varphi \) if

\[
\lim_{K \to \infty} f(x_1, x_2, \ldots, x_K) = \varphi
\]

which says that the "limit in probability of \( f \) is \( \varphi \)."

The other asymptotic property of estimates is "efficiency". The estimate \( f \) is an efficient estimate of \( \varphi \) if, as the sample size increases towards infinity, the distribution of \( f \) tends towards the normal distribution with mean \( \varphi \), and its variance is less than any other estimate having a normal distribution as a limiting distribution. The joint criterion of consistency and efficiency is similar to that of a best unbiased estimate except that the efficiency criterion applies only in the limit as \( K \to \infty \) and it seeks minimum variance only among normally distributed estimates.

The Structural Relationship

It would at first seem proper to estimate the model parameters of Equation 17 by application of either ordinary least-squares (LS) or generalized least-squares (GLS). The choice would appear to depend upon the particular assumptions made on the disturbance term. The danger implicit in this procedure has been pointed out by Young (3). Equation 17 does not represent the simple regression model upon which the least-
Squares theory is based, but instead contains "errors in variables" within the $x_t$ vector and is known as a "structural model" (6).

The elements of the observation vector $x_t$ constitute what are called explanatory variables (independent variables) and application of LS requires that these variables be known exactly (deterministic) or at least be uncorrelated with the disturbance term if the explanatory variables are themselves random. That is, it must be assumed that

$$E[\omega_t x_t] = 0$$  \hspace{1cm} (18)

or the weaker condition

$$\lim_{K \to \infty} \omega_t x_t = 0$$  \hspace{1cm} (19)

for consistency, if not unbiasedness. Now, the vector $x_t$ contains passed values of the input and output of the process. These passed values are known as "lagged" variables. The passed values of the input are lagged values of the independent variable and cause no difficulty for open-loop identification. Lagged values of the output (dependent) variable, on the other hand, represent a really troublesome obstacle - namely, that the explanatory variables are correlated with the disturbance term and the assumptions of Equation 18 or 19 are no longer valid. Using the results of simple linear regression analysis in this case will result in estimates which are biased in finite samples to a degree dependent on the noise-to-signal ratio. If in addition, the disturbance term is autocorrelated, then the combination of lagged variables and autocorrelated disturbances result in the LS estimator being inconsistent (7). Hence, in any application of LS theory, it must be assumed that the conditions for the validity of the results are satisfied.
Survey of Estimation Methods

In the last section, the problems encountered in the application of classical regression to the structural model of Equation 17 were discussed. The technique of GLS requires complete specification of the disturbance for direct estimation of the model parameters. An on-line GLS method of estimating the coefficients of a signal processing filter to approximate the disturbance is presented by Hastings-James and Sage (8).

There are two main estimation methods which can be utilized with the structural model. One is the instrumental variable (IV) approach. The IV technique is the adopted method of this research and will be discussed at length in another section. The other technique is the method of maximum likelihood estimates. This method, originally developed for off-line computation, carries fairly strong assumptions about the covariance matrix of noise — namely, all the random variables involved are assumed to be Gaussian. Moreover, initial guesses of the parameters must be made close to the true values in order to guarantee convergence.

Two recent papers (9 and 10) deal specifically with the comparisons of the performance and computational expense of all the important on-line identification methods. Isermann et al (9) concluded that identification methods using the same a priori information of the process model results in about the same performance, if the "laws of good identification" are applied. They suggest that the key to selection of an identification method rest on such factors as the kind of input signal, the computational expense, the overall reliability and the necessary assumptions required.
**Least-Squares Estimator**

Replacement of the inaccessible true output, \( c_t \), of Equation 8 by the actual observation, \( y_t \), yields the equation

\[
y_t = \varphi x_{t-1}
\]  

with

\[
\varphi = [a_1 a_2 \ldots a_p b_1 b_2 \ldots b_q] \quad x_{t-1}^T = [y_{t-1} y_{t-2} \ldots y_{t-p} u_{t-M-1} u_{t-M-2} \ldots u_{t-M-q}]
\]

Letting Equation 20 represent the process model and taking the input and output sequences, \( \{u(t)\}_K \) and \( \{y(t)\}_K \), the generalized equation-error function, \( e_t \), at time \( t \) is defined as

\[
e_t = y_t - \xi_t = y_t \hat{\varphi} x_{t-1}
\]  

where \( \hat{\varphi} \) represents the parameter estimate and \( \xi_t = \hat{\varphi} x_{t-1} \) is the predicted output. This estimate can be obtained by minimizing some positive-definite criterion function in the generalized equation-error.

The method of least-squares requires that the estimate \( \hat{\varphi} \) be chosen which minimizes the least-squares criterion function

\[
J = \sum_{t=1}^{K} [y_t - \hat{\varphi}_K x_{t-1}]^2
\]

The minimization is performed by differentiating with respect to \( \hat{\varphi} \) and equating to zero

\[
[\sum_{t=1}^{K} y_t x_{t-1}^T] - \hat{\varphi}_K [\sum_{t=1}^{K} x_{t-1} x_{t-1}^T] = 0
\]

Solving Equation 23 for \( \hat{\varphi} \) yields the LS estimate

\[
\hat{\varphi}_K = [\sum_{t=1}^{K} y_t x_{t-1}^T][\sum_{t=1}^{K} x_{t-1} x_{t-1}^T]^{-1}
\]

Alternatively, since Equation 21 represents only one of a set of \( K \) equations, a generalized equation error vector can be constructed

\[
E_K = Y_K - \hat{\varphi}_K X_{K-1}
\]
where

\[ \begin{align*}
E_K &= [e_1 \ e_2 \ \ldots \ e_K] \\
Y_K &= [y_1 \ y_2 \ \ldots \ y_K] \\
X_{K-1} &= [x_0 \ x_1 \ \ldots \ x_{K-1}]
\end{align*} \]

Minimization of the least-squares criterion as before yields the vector form of the LS estimate,

\[ \hat{\phi}_K = Y_K X_{K-1}^T \left( X_{K-1} X_{K-1}^T \right)^{-1} \]  

(26)

By writing Equation 24 in partitioned form and introducing the matrix inversion lemma (1), the recursive least-squares estimate can be expressed as

\[ \begin{align*}
\hat{\phi}_{t+1} &= \hat{\phi}_t + (y_{t+1} - \hat{\phi}_t x_t) \left[ 1 + x_t^T P_{t-1} x_t \right]^{-1} x_t^T P_{t-1} \\
P_t &= P_{t-1} - P_{t-1} x_t \left[ 1 + x_t^T P_{t-1} x_t \right]^{-1} x_t^T P_{t-1}
\end{align*} \]  

(27)

(28)

with

\[ P_{t-1} = [X_{t-1} X_{t-1}^T]^{-1} \]  

(29)

Equations 27 and 28 simply perform a least-squares fit to the measured data and remain a deterministic estimation procedure in the sense that no assumptions have been made about the statistical nature of the estimates (1). The P matrix is simply a time-variable weighting matrix and cannot be interpreted as an error covariance matrix as in the analogous regression approach to parameter estimation.
If the least-squares residuals, \( e_t \), of Equation 21 are correlated, then the LS parameter estimates will be biased for the lagged variable model. This bias can be illustrated by returning to the stochastic model of Equation 17. Combining Equation 17 with \( K \) observations yields the vector equation

\[
Y_K = \phi_K X_{K-1} + W_K
\]

where \( Y_K \), \( \phi_K \), and \( X_{K-1} \) are the same as in Equation 25 and

\[
W_K = [w_1 \ w_2 \ldots \ w_K]
\]

Post multiplying Equation 28 by \( X_{K-1}^T \) yields

\[
y_K = Y_K X_{K-1}^T = \phi_K X_{K-1}^T X_{K-1}^T + W_K X_{K-1}^T
\]

The product \( [X_{K-1}^T X_{K-1}] \) is a square matrix and it is therefore possible to solve Equation 31 for \( \phi_K \)

\[
\phi_K = Y_K X_{K-1}^T [X_{K-1}^T X_{K-1}]^{-1} - W_K X_{K-1}^T [X_{K-1}^T X_{K-1}]^{-1}
\]

The first term on the right-hand side of Equation 32 is the LS estimate \( \hat{\phi}_K \). The bias in the LS estimate is thus given by

\[
\text{LS Bias} = \phi_K - \hat{\phi}_K = W_K X_{K-1}^T [X_{K-1}^T X_{K-1}]^{-1}
\]

If \( W_K \) is uncorrelated, the \( W_K \) and \( X_{K-1} \) are independent and the bias is zero. Unfortunately, if measurement noise is present, \( W_K \) is inherently correlated due to the lagged variables used as regressors. Hence the output elements, \( \{y_{t-1}, y_{t-2}, \ldots, y_{t-p}\} \), of \( X_{K-1} \) will be correlated with \( W_K \) and the bias will be nonzero.

In order to reduce the bias term to zero, it is necessary to reduce \( W_K \) to an uncorrelated sequence, so that the elements of \( X_{K-1} \), which contain the lagged values of \( y_t \), are independent of \( W_K \). This can be accomplished by an appropriate transformation of the data, \( \{u_t\}_K \) and \( \{y_t\}_K \).
Generalized Least-Squares Estimator

The method of generalized least-squares attempts to overcome the problem of bias introduced by correlated disturbances by transforming (or filtering) the original observations in order to obtain a new equation-error model with uncorrelated residuals. If all the disturbance statistics are known, then the filter parameters may be determined immediately and there is no problem. The practical problem exists with unknown noise statistics in which case the filter must be determined on-line as well.

This transformation may be realized by estimating the disturbance present in the plant output to be

$$\hat{\omega}_t \sim \mu_t = y_t - \hat{\phi}x_{t-1}$$

(34)

where reference is made to Figure 1. This approach is valid since in the limit, as $\hat{\varphi}$ approaches $\varphi$, the predicted error $\mu$ approaches the corrupting noise $w$.

The GLS method is an iterative procedure consisting of two basic steps. The first is the application of LS estimation on Equation 17. The second step requires that the residuals be calculated and analyzed by autoregression, assuming a model of the form

$$w_t = [1 - F(z^{-1})]^{-1} v_t$$

(35)

The assumption is therefore made that

$$[1 - F(z^{-1})] \sim \left\{\frac{[1 - A(z^{-1})][1 + R(z^{-1})]}{[1 + S(z^{-1})]}\right\}^{-1}$$

(36)

Equation 35 can be written in vector-regression form as

$$w_K = \Omega w_{K-1} + v_K$$

(37)
Figure 1: Diagram for generalized least-squares.
where
\[
\Omega = [f_1, f_2, \ldots, f_e]
\]

\[
\Pi_{K-1} = \begin{bmatrix}
  z^{-1}w & z^0w & \cdots & z^{-1}w \\
  z^{-2}w & \cdots & \cdots & \cdots \\
  \vdots & \cdots & \cdots & \cdots \\
  z^{-l}w & \cdots & \cdots & \cdots 
\end{bmatrix}
\]

\[
V_K = [v_1, v_2, \ldots, v_K]
\]

and \( l \) is the number of noise parameters, \( f_1 \), carried. Using \( w_t = \mu_t \)
the LS estimate of \( \Omega \) is given by

\[
\hat{\Omega} = \omega K \Pi_{K-1}^T \left[ \Pi_{K-1} \Pi_{K-1}^T \right]^{-1}
\]

This estimate is unbiased, as the elements of \( \Pi \) are independent of \( v \), the uncorrelated noise term.

This estimation of noise parameters allows the desired transformation of the data \( u_t \) and \( y_t \) to be made, which leads in the limit to an unbiased estimate of \( \varphi \). This can be seen by replacing the elements of \( u_t \) and \( y_t \) by

\[
u^F_t = [1 - F(z^{-1})]u_t
\]

\[
y^F_t = [1 - F(z^{-1})]y_t
\]

This reduces Equation 16 to

\[
[1 - A(z^{-1})]y^F_t = z^{-M} B(z^{-1})u^F_t + v_t
\]

Equation 40 leads to the vector equation

\[
Y^F_K = \varphi X^F_{K-1} + V_K
\]

where \( X^F_{K-1} \) is the regression matrix of filtered data. An unbiased estimate of \( \varphi \) is now available as

\[
\hat{\varphi} = Y^F_K X^F_{K-1}^T \left[ X^F_{K-1} X^F_{K-1}^T \right]^{-1}
\]
Thus, by obtaining a LS estimate of \( \varphi \) to start the procedure and iterating between the estimates of the disturbance and process parameters \( \hat{\Omega} \) and \( \hat{\varphi} \), using Equations 34, 38, 39 and 42, unbiased estimates of the system parameters can be obtained. The iteration is stopped when the sum of squares of the prediction errors reaches a minimum.

A completely recursive GLS algorithm can be derived which is equivalent to Equations 27 and 28 for LS estimation, except in the case of GLS estimation, the original observations are filtered and a set of equations similar to Equations 28 and 29 are appended to estimate the filter parameters (\( \theta \)).

**Instrumental Variable Estimator**

The difficulties encountered in obtaining an unbiased, minimum variance estimator are attributed to the correlation arising between \( w_t \) and \( x_{t-1} \) of the structural model of Equation 17. The instrumental variable (IV) approach to parameter estimation was first introduced in the early 1940's in an area of economics now known as econometrics (11). The method was specifically designed to deal with the errors-in-variables problem, i.e., the errors associated with the explanatory variables in regression calculations (6,12). The IV method is intended as a compromise between the range of techniques from largely deterministic procedures to sophisticated statistical methods based on the results of optimal estimation theory.

The first use of the IV approach in the field of process identification is generally attributed to Joseph et al (13). They suggested an IV procedure utilizing the input variable \( u_t \) as the instruments for identifying the parameters of a process described by a difference equation model. Andeen and Shipley (14) reported an essentially similar
Beginning with Equation 30, unbiased estimates can be obtained from postmultiplication by a transformation matrix, $Z^T_K$.

$$Y_k^T Z_{K-1}^T = \Phi_k X_{K-1}^T Z_{K-1}^T + W_k^T Z_{K-1}^T$$  \hspace{1cm} (43)

This transformation, or instrumental variable, matrix is chosen which satisfies

$$E[W_k^T Z_{K-1}^T] = 0$$  \hspace{1cm} (44)

$$E[X_{K-1}^T Z_{K-1}^T] \text{ nonsingular}$$

The elements of $Z_{K-1}^T$ are therefore chosen to be uncorrelated with the disturbance $W_{K-1}$, and at the same time, highly correlated with the regressors in $X_{K-1}$. Solving Equation 43, the IV estimate is given by

$$\hat{\gamma}_k = Y_k^T Z_{K-1}^T \left[X_{K-1}^T Z_{K-1}^T\right]^{-1}$$  \hspace{1cm} (45)
If the observation matrix, $X_{K-1}^T$, is used as the IV matrix, it can be seen from Equation 31 that LS estimation is a special case of the IV approach. Therefore, by proper choice of an IV matrix, it is possible to eliminate the bias due to the combination of noise and lagged variable regressors while preserving the simplicity of least-squares estimation.

Unfortunately, the universal law of the conservation of "advantage" is valid in this instance. The law states that to gain an advantage in one area requires a proportional disadvantage be established in another area. In this instance, the elimination of asymptotic bias, i.e. consistency, is accompanied by a certain loss of efficiency in the statistical sense. IV estimators lack efficiency because they make use of only a limited amount of a priori information. But the lack of a priori disturbance statistics was the major reason for selecting the IV approach. So efficiency has not really been lost - the problem requirement is simply more demanding. And as might be expected, the greater the correlation between the instrumental variable, $z_t$, and the noise-free signals, $c_t$, the smaller the estimation variance. Finally, a small bias will remain due to finite sample lengths.

Since a major advantage of the IV method is the preservation of the simple least-squares structure, the recursive or sequential form of Equation 45 follows directly from Equations 27-29. The recursive relations are

$$
\hat{\phi}_{t+1} = \hat{\phi}_t + (y_{t+1} - \hat{\phi}_t x_t)[1 + z_t^T p_{t-1} x_t]^{-1} z_t^T p_{t-1} \quad (46)
$$

$$
\begin{align*}
p_t &= p_{t-1} - p_{t-1} x_t [1 + z_t^T p_{t-1} x_t]^{-1} z_t^T p_{t-1} \quad (47) \\
p_{t-1} &= [x_{t-1}^T z_{t-1}^T]^{-1} \quad (48)
\end{align*}
$$
A diagram for the sequential IV estimator is shown in Figure 2 with
\[ d_{t-1} = [1 + z_t^T P_{t-1} x_t]. \]

Finally, Wong and Polak (16) and Rowe (18) have pointed out, in
regard to the selection of \( z_t \), that the direct GLS estimator represents
the optimal IV estimator. Referring to Figure 2, this implies the use
of \( c_{t+1} \) as the instrumental variable which in turn implies the pre-
cognitive solution to the problem by requiring complete a priori infor-
mation on the noise statistics. Utilization of the output of an
auxiliary model, as indicated, represents a close approximation to
\( c_{t+1} \). A by-product of this procedure is, of course, filtered estimates
of the system output.

**Multicollinearity (5)**

One of the basic assumptions underlying the application of least-
squares estimation to the difference equation model of Equation 30 is
that the observation matrix, \( X \), which is of order \([(p+q) \times K] \), has rank
\( p+q \). It is obvious that the LS estimator, \( \hat{\phi} = YX^T[XX^T]^{-1} \), requires
the inversion of the moment matrix \( [XX^T] \), which is impossible if the
rank of \( X \), and consequently the rank of \( [XX^T] \), is less than \( p+q \).
This situation represents extreme multicollinearity and exists when
some or all of the explanatory variables are perfectly collinear. The
situation of practical interest for open-loop estimation occurs when
the assumption of linear independence is only just satisfied. In this
case, some or all of the explanatory variables are highly but not per-
factly collinear. The moment matrix \( M_{XX} = [XX^T] = \sum_{t=0}^{K-1} x_t x_t^T \) is then
termed ill-conditioned.

In order to show the effects of multicollinearity on regression
estimates, consider the second-order model
Figure 2: Sequential IV parameter estimation.
\[ y_t = a_1 y_{t-1} + a_2 y_{t-2} + b_1 u_{t-1} + b_2 u_{t-2} + w_t \]  
(49)

The LS estimator is given by
\[ \hat{\phi} = \frac{M_{YX}}{M_{XX}} = y' x [x' x^{-1}] 
(50)

\[ M_{XX} = [x_0 \ x_1 \ \ldots \ x_{K-1}] \begin{bmatrix} x^T_0 \\ x^T_1 \\ \vdots \\ x^T_{K-1} \end{bmatrix} = 
\begin{bmatrix} y_0 & y_1 & \ldots & y_{K-1} \\ y_{t-1} & y_0 & \ldots & y_{K-2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{t-K+1} & y_{t-K+2} & \ldots & y_0 \end{bmatrix} 
\begin{bmatrix} y_0 & y_{t-1} & u_0 & u_{t-1} \\ y_1 & y_0 & u_1 & u_0 \\ \vdots & \vdots & \ddots & \vdots \\ y_{K-1} & y_{K-2} & u_{K-1} & u_{K-2} \end{bmatrix} = 
\begin{bmatrix} \sum_{t=0}^{K-1} y_t^2 & \sum_{t=0}^{K-1} y_t y_{t-1} & \sum_{t=0}^{K-1} y_t u_t & \sum_{t=0}^{K-1} y_t u_{t-1} \\ \sum_{t=0}^{K-1} y_{t-1} y_t & \sum_{t=0}^{K-1} y_{t-1}^2 & \sum_{t=0}^{K-1} y_{t-1} u_t & \sum_{t=0}^{K-1} y_{t-1} u_{t-1} \\ \sum_{t=0}^{K-1} u_t y_t & \sum_{t=0}^{K-1} u_t y_{t-1} & \sum_{t=0}^{K-1} u_t^2 & \sum_{t=0}^{K-1} u_t u_{t-1} \\ \sum_{t=0}^{K-1} u_{t-1} y_t & \sum_{t=0}^{K-1} u_{t-1} y_{t-1} & \sum_{t=0}^{K-1} u_{t-1} u_t & \sum_{t=0}^{K-1} u_{t-1}^2 \end{bmatrix} 
(51)

\[ M_{YX} = [y_1 \ y_2 \ \ldots \ y_K] \begin{bmatrix} x^T_0 \\ x^T_1 \\ \vdots \\ x^T_{K-1} \end{bmatrix} = 
\begin{bmatrix} y_0 & y_{t-1} & u_0 & u_{t-1} \\ y_1 & y_0 & u_1 & u_0 \\ \vdots & \vdots & \ddots & \vdots \\ y_{K-1} & y_{K-2} & u_{K-1} & u_{K-2} \end{bmatrix} \]
Each individual parameter estimate of Equation 50 can be expressed as

$$\hat{\phi}_i = \frac{4}{K} \sum_{t=1}^{K} |M_{XX}| \left| \prod_{j=1}^{i} y_{t,j} y_{t-1,j} \right| |M_{XX}|$$

where $|M_{XX}|$ is the determinant of $M_{XX}$ and $|M_{XX}|_{ji}$ is the $ji$ cofactor.

For illustration purposes, it is convenient to assume that all variables have zero mean and unit variance which is easily accomplished using the transformation (5)

$$y_{t-i} - \bar{y}_{t-1}, \quad u_{t-i} - \bar{u}_{t-1}, \quad i = 0, 1, 2, ...$$

where the bar indicates the sample mean and $S_x$ represents the standard deviation. Further, $r_{yt}$ is called the correlation coefficient and is defined as

$$r_{yt} = \frac{\sum_{t=1}^{K-1} y_{t-1} u_{t-1}}{K S_y S_{u_{t-1}}}$$

If perfect correlation exists between $y_t$ and $y_{t-1}$, then $r_{yt} y_{t-1}$ would equal one. If one the other hand, $y_t$ and $y_{t-1}$ are completely uncorrelated, then $r_{yt} y_{t-1} = 0$. Now, an alternate form of Equation 53 is possible by application of Cramer's rule to the normal equations. The estimate for the second parameter is given by
The other parameters are similarly obtained by appropriate replacement in the numerator determinant.

Now if \( y_t \) and \( y_{t-1} \) are perfectly correlated (i.e., \( r_{y_t y_{t-1}} = 1 \)), then the first two rows and columns of \( M_{XX} \) will be identical, and \( |M_{XX}| \) would vanish. Similarly, the first two rows and columns of the numerator term of Equation 55 would be the same and the determinant would be zero. Therefore, if any pair \( (x_i, t^*) \) of exogenous variables are perfectly correlated, then the estimate of \( \varphi \) becomes

\[
\varphi = 0
\]

The same indeterminate value would be obtained if any exact linear relationship existed among the \( x_i, t^* \).

The effect of perfect correlation on the weighting matrix, \( P_K \), can be seen by considering the following result from linear regression (1,5)

\[
\sigma^2_{\varphi} = \sigma^2 w P_K = \sigma^2 w [X_X^T X_X]^{-1} = \sigma^2 w [M_{XX}]^{-1}
\]

Equation 57 can be broken into its component parts to give

\[
\sigma^2_{\varphi} = \sigma^2 w \frac{|M_{XX}|_{ii}}{|M_{XX}|}
\]
This time the numerator does not vanish due to perfect correlation but the denominator is the same as before; hence, the variance of the estimate becomes infinite as the corresponding estimate becomes indeterminate.

\[ \sigma^2_q = \sigma^2 \frac{|X|}{W_{XX}} \rightarrow \infty \quad (59) \]

Again, the concern is not so much that of perfect collinearity, but the ill-conditioning which occurs when the correlation, \( r_{yt, u_t} \), between explanatory variables are high, but not unity. In this situation, numerical results for the parameter estimates are obtained, but these values must be properly interpreted.

The difference equation is particularly subject to multicollinearity and care must be exercised in the selection of the sample interval, \( T \). This is a case in which it is possible to sample too fast for good identification. Take for example, the second-order model of Equation 49. It is obvious that as the sample rate increases, the correlation between \( y_{t-1} \) and \( y_{t-2} \) will also increase leading to collinearity. This result has been established computationally and will be presented in a later section.

**Dynamic Estimation**

The parameter tracking problem is now considered where the process parameters change with time and the estimation algorithm is required to follow the variation. It is well known that LS and GLS estimation (and, by consequence, IV estimation) can be established as a special static form of Kalman filtering theory (19, 20). Paglin (21) and Lee (1), among others, have shown this relationship. This section extends the LS development of Lee to include IV estimation (17).
Memory Shaping

A decided advantage of the recursive relations of Equations 27 and 28 or Equations 46 and 47 over their nonrecursive counterparts is that it is no longer necessary to store and perform calculations on large quantities of data. The data is reduced at each sample instant and is accumulated with past reduced data in the considerably compact weighting-matrix, $P_K$. Hence the $P_K$ matrix represents a concise history of the observations. The weighting-matrix is therefore the key to parameter tracking.

Relations like Equations 46 and 47 represent finite-time averaging operations in which all data is weighted equally over the observation interval of $K$ samples. An important assumption implicit in this operation is that the unknown parameters are constant over the observation set (3). It then follows that in order to detect parameter variation, it is necessary to shorten the memory of the estimation procedure so that new observations are given more weight than outdated observations.

One approach to parameter tracking is obtained by weighting the data exponentially into the past to gradually remove information as it becomes obsolete. Examination of Equation 28 or 47 for the finite-time averaging case will show that $P_K$ is strictly a decreasing function of time. After a large number of samples (large $K$) have been collected, $P_K$ may diminish to the extent that new data no longer influences the estimates. That is, due to the averaging process, individual samples have less effect on the estimates as the number of samples, $K$, increases. The physical effect of exponential weighting the data is to set a lower bound on $P_K$ so that new data continues to influence the estimates via Equation 27 or 46 (22).
It is important to point out that this approach has the disadvantage that the effects of noise will also be detected and used to modify the estimates. It will therefore be necessary to assume that the parameter variations are larger than the residual fluctuations due to noise.

Modeling the Parameter Variations

An alternative to exponential weighting the data for parameter tracking is to describe the parametric variation by a suitable stochastic model, e.g. (1)

\[ \varphi_t^T = \gamma(T) \varphi_{t-1}^T + \Delta(T) \varphi_{t-1}^T \]  \hspace{1cm} (60)

where \( \gamma(T) \) is an assumed known \((n \times n)\) transition matrix, \( \Delta(T) \) is a known \((n \times n)\) input matrix, and \( q_{t-1} \) is a \((1 \times n)\) random disturbance vector with zero mean and covariance matrix \( \mathbb{E}[q_t^T q_{t}] = Q \delta_{ty} \) and \( \delta \) is the Kronecker delta function. The transition matrix \( \gamma(t) \) governs any deterministic parameter variation while \( q \) provides the statistical degree of freedom required to model random parameter variations between samples. Now, Equation 60 and Equation 17 can be combined to form a set which extrapolated [See Lee (1)] for the IV estimate yields

\[ \hat{\varphi}_{t+1} = \hat{\varphi}_{t+1/t} + (y_{t+1} - \hat{\varphi}_{t+1/t} x_t)(1 + z_t^T P_{t/t-1} x_t)^{-1} z_t^T P_{t/t-1} \] \hspace{1cm} (61)

\[ \hat{\varphi}_{t+1/t} = \gamma T^T \] \hspace{1cm} (62)

\[ P_{t/t-1} = \gamma T P_{t-1} \gamma + \Delta T \Delta \] \hspace{1cm} (63)

\[ P_t = P_{t/t-1} - P_{t/t-1} x_t(1 + z_{t}^T P_{t/t-1} x_{t})^{-1} z_{t}^T P_{t/t-1} \] \hspace{1cm} (64)

where \( \hat{\varphi}_{t+1/t} \) is the a priori update of \( \hat{\varphi}_t \) at time \( t+1 \), based on observations up to and including \( y_t \). \( P_{t/t-1} \) is a similar update of \( P_{t-1} \) and represents a \((n \times n)\) matrix which is determined experimentally.
Lee points out that the algorithm of Equations 61-64 are not optimal and the only justification is that "it works". It is interesting to note that this prediction-correction algorithm is a special form of the Kalman filter equations (19). However, there are two important differences between the IV algorithm and the Kalman equations with the result that the former becomes considerably more complicated. The IV algorithm would be identical to the Kalman filter if: (1) all of its noise terms were Gaussian and (2) all of the elements of X and Z were deterministic. The first difference is certainly within the realm of possible satisfaction but the second is caused by the lagged variables in the structural model. Hence, the set of Equations 61-64 can not be considered optimal. However, the Kalman form is preserved and "it works", as will be discussed below and demonstrated by experiment.

Actually, the dynamic IV algorithm is not particularly useful as it stands because it requires knowledge of the Y and Δ matrices. However, a useful special case results if Equation 60 is simplified by letting Y and Δ equal the identity matrix, I. The reduced parameter variation law is given by

\[ \hat{\phi}_t = \hat{\phi}_{t-1} + \phi_{t-1} \]  \hspace{1cm} (65)

The dynamic IV algorithm is correspondingly simplified to

\[ \hat{\phi}_{t+1} = \hat{\phi}_t + (y_{t+1} - \hat{\phi}_t x_t)[1 + z^T_{t} P_{t/t-1} x_t]^{-1} z^T_{t} P_{t/t-1} \]  \hspace{1cm} (66)

\[ P_{t/t-1} = P_{t-1} + D \]  \hspace{1cm} (67)

\[ P_t = P_{t/t-1} - P_{t/t-1} x_t [1 + z^T_{t} P_{t/t-1} x_t]^{-1} z^T_{t} P_{t/t-1} \]  \hspace{1cm} (68)

The only difference between this dynamic algorithm and the static scheme of Equations 46 and 47 is the addition of Equation 67. The D matrix
simply adds the necessary lower bound to \( P \) so that the algorithm can track parameter variations.

Although this is the same result obtained by exponential weighting the data, the dynamic approach has more inherent flexibility and is more easily implemented. Since \( D \) is a matrix it is possible to limit individual elements to different degrees. For example, if certain parameters are known to be time-invariant, then the corresponding elements of \( D \) would be zero. Further, the model approach allows any a priori information that is available to be used in the estimation algorithm. Young (17) illustrates this last point by including in the estimation additional measured data obtained from the process.

Young makes the assumption that the time variable parameter vector \( \varphi_t \) can be decomposed into the product of a known, nonsingular, time-variable transformation matrix \( \theta_t \) and an unknown, but fixed or slowly variable, basis parameter vector \( \varphi^* \). Thus,

\[
\varphi_t = \theta_t \varphi^*_t
\]  

(69)

\( \theta_t \) is a function of known or measurable variables that are associated with the changing environment which is affecting process parameters. Since \( \varphi^*_t \) is only slowly variable, it should be possible to model its behavior by a random walk model, i.e.

\[
\varphi^*_t = \varphi^*_{t-1} + q_{t-1}
\]  

(70)

Substitution of Equation 70 into Equation 69 yields with \( \varphi^*_{t-1} = \varphi^{-1} \varphi^*_t \) the following equation

\[
\varphi^*_t = [\theta \theta^{-1}] \varphi^*_t + \theta \varphi^*_{t-1}
\]  

(71)

It is obvious that Equation 71 is of the same form as Equation 60 and it can be used in the algorithm of Equations 61-64. If \( \theta_t \) provides a
suitable decomposition, then much of the rapid variation in $\varphi_t$ will be accounted for deterministically by the transition matrix, $\gamma = [\Theta_t \Theta_t^{-1}]$. Although Equation 71 is not specifically treated in this research, it was included for completeness because it represents an extension of the basic IV method to practical applications in which related problem variables (which are not specifically included in the model) can be incorporated to improve the estimation.

**Computational Results**

An important observation to be made from the theory developed in this chapter is that IV estimation is a general computational procedure which includes both OLS and GLS as special cases. These special cases represent both ends of the IV spectrum with GLS being the optimal procedure. OLS, which is the simplest case, is necessarily a special case of the GLS procedure where the latter reduces to OLS for the process which has uncorrelated disturbances with no measurement noise. The choice as to the proper IV procedure selected in a given application depends upon the amount of a priori information available on the noise statistics. If, for example, all the noise statistics are known then GLS is the best-linear-unbiased-estimator (5). In the practical problem, the statistics of the disturbance are not known and the OLS approach fails in the presence of correlated disturbances and measurement noise so that a variable which approximates the optimal must be adopted for the instrument. In the computational results to follow, this instrumental variable is chosen to be the output of a model which uses the current estimate of the parameters, $\hat{\varphi}$, being estimated.

In this sense, the auxiliary model approach to IV estimation represents an extension of the LS procedure. The former has consid-
erably more flexibility and may be "tuned" in a given situation so that the instrumental variable closely approximates the optimal. On the other hand, if the parameters used in the tuning process are not chosen properly, the inherent stability of the LS method may be lost. The objective of this computational study will be to examine the effect of measurement noise on the performance of the auxiliary-model IV estimator and the selection of parameters necessary to tune the algorithm. In each study, the method will be compared with the results of OLS. This comparison has become standard practice in the literature where it at once affords a reference upon which to judge the relative performance of separately reported estimation methods. At the same time, it is interesting to see the improvement that is possible with the IV method in those cases in which the LS estimate is severely biased.

A survey of computational results discussed in the literature for IV estimation contains applications of the method to a variety of systems. Wong and Polak (16) used a difference equation model to identify a single-input-single-output fourth-order process. Isermann et al (9) compared the performance of IV estimation with five other methods for three separate systems: an underdamped second-order system, a second-order nonminimum phase system and a third-order low pass delay process. Rowe (18) extended the method to the multivariable case and included four example systems: first-order scalar, second-order scalar, a fourth-order two-input-two-output system and a third-order two-output autoregressive, moving-average process. Young (17) illustrated the dynamic IV algorithm with an underdamped second-order system utilizing a differential equation model. Young reported equipment problems that prevented him from truly testing the auxiliary model approach except
for the "optimum" case where the parameters of the auxiliary model were varied in accordance with the actual process parameter variation.

**Second-Order Example**

A single-input, single-output process described by a linear differential equation was made the object of study for the investigation. The process

\[ 2 \frac{d^2 c}{dt^2} + 3 \frac{dc}{dt} + c = u(t) \]  

represents an overdamped second-order system with the transfer function

\[ G(s) = \frac{1}{(2s + 1)(s + 1)} \]  

The time constants and gain are thus

\[ \tau_1 = 2 \]
\[ \tau_2 = 1 \]
\[ k = 1 \]

The equivalent difference equation parameters, \( \varphi \), of Equation 9 are

\[ a_1 = 0.97441 \]
\[ a_2 = -0.22313 \]
\[ b_1 = 0.15482 \]
\[ b_2 = 0.09390 \]

for the sample interval \( T = 1 \).

The results in this chapter were obtained utilizing Equation 72 as the simulated plant. Notice that this system is deterministic because it is representative of Equation 8 rather than Equation 10. The more general, stochastic plant will also be considered in a later section. The observed output \( y_c \) was obtained by adding measurement noise to the true output \( c \) according to Equation 11.
**Input Signal**

The system was forced with discrete white noise, e.g., a pseudo-random-binary-sequence (PRBS), with a clock interval $\lambda$ equal to the sampling interval $T$. The autocorrelation function for the maximum length PRBS is

$$\varphi_f(\tau) = \begin{cases} 
  u^2[1 - (1 + \frac{1}{\rho}) \frac{1}{\lambda} \tau] & 0 \leq |\tau| \leq \lambda \\
  -\frac{u^2}{\rho} & \lambda < |\tau| \leq (\rho - 1)\lambda 
\end{cases}$$

The period, $\rho$, of the PRBS is

$$\rho = 2^j - 1; \ j = 1, 2, \ldots.$$  \hspace{1cm} (75)

The effective frequency band covered by a maximum length PRBS is from $f = \frac{1}{\rho\lambda}$ to $\frac{1}{3\lambda}$ cps. A corrected PRBS is utilized where a d.c. bias is added to the signal yielding $\varphi_u^\beta(\tau) = 0, \lambda \leq |\tau| \leq (\rho - 1)\lambda$. The bias, $\beta$, is

$$\beta = \frac{u}{\rho} \sqrt{1 + \rho} - 1$$

The input signal is limited by

$$-\frac{\theta}{2} \leq u(t) \leq \frac{\theta}{2} \text{ with } \theta = 1.0$$

See Eveleigh (23) and Davies (24).

The PRBS starter sequence was not varied between runs so that the input would be reproducible and thereby facilitate the forming of conclusions.
Discrete White Noise

The discrete white noise input $v(t)$ was generated by using sub­routines RANDU and GAUSS as described in the "IBM System/360 Scientific Subroutine Package". Subroutine RANDU computes uniformly distributed random real numbers $0 \leq x(t) \leq 1$. Subroutine GAUSS utilizes the uniformly distributed numbers to compute a normally distributed random number with a given mean and standard deviation.

Comparison

A summary of the effect of measurement noise on estimator performance is given in Figures 3 and 4. The bias reduction that can be realized with the IV method in the presence of noise is considerable. Each curve of the two figures represents the time history of the estimates of parameter $\alpha_1$ of the second-order example for a selected noise-to-signal ratio, $\delta$, defined as

$$\delta = \frac{\text{r.m.s. value of the noise, } w(t)}{\text{r.m.s. value of the signal, } c(t)}$$

In this case, as well as all cases to follow in this chapter, the initial parameter guesses, $\hat{\phi}_0$, are zero.

A series of long-sample comparisons between LS and IV estimation are given in Figures 5-7 for selected noise-to-signal ratios. The criterion used throughout the present chapter and Chapter III for evaluating estimation performance is the "estimation error fraction" defined as

$$\epsilon = \frac{||\varphi - \hat{\varphi}_t||}{||\varphi - \hat{\varphi}_0||}$$

(78)

where $\varphi$ is the true system parameter vector and $\hat{\varphi}_t$ and $\hat{\varphi}_0$ are the estimates at time $t$ and $0$. Notice that it is only possible to define $\epsilon$ for
Figure 3: Bias in parameter $a_1$ for LS estimation for various levels of uncorrelated measurement noise.
Figure 4: Bias in parameter $a_1$ for IV estimation for various levels of uncorrelated measurement noise.
Figure 5: Long-sample comparison of noise-free LS and IV estimation.
Figure 6: Long-sample comparison of LS and IV estimation with $\delta = 0.0655$. 
Figure 7: Long-sample comparison of LS and IV estimation with $\delta = 0.521$. 
the deterministic plant since only then is it possible to obtain the true system parameters. The scalar \( \epsilon \) represents the normalized vector norm of the estimation error in the parameter space. The lowest curve on each graph represents the best convergence to the true parameter values, \( \varphi \).

The IV estimator is seen to outperform the LS estimator, as predicted, for all noise-to-signal ratios except the case \( \delta = 0 \), the noise-free system, where the two methods are equivalent. It is well known that the LS estimator is best for this case of all the estimators in the class of linear unbiased estimators.

Selecting a Sample Rate

The important first step in the "tuning" procedure of a regression estimation technique employing a difference equation model is the selection of an appropriate sample rate. It must be appropriate in the sense that the effects of multicollinearity are minimized. This effect of the sample rate on the estimates is illustrated in Figure 8. Here the estimation error fraction is plotted against the observation period. The sample interval, \( T \), is the graphical parameter and is varied from 0.05 to 4.0 where \( T \) is in some suitable time units, say seconds. It is clear from Figure 8 that the estimates rapidly deteriorate as the sample rate is increased due to the correlation arising between \( y_{t-1} \) and \( y_{t-2} \) of Equation 17. Figure 9, on the other hand, shows a decay of estimator performance due to the sample rate becoming too small and resulting in a loss of dynamic information. Figure 10 is a summary of Figures 8 and 9 where the minimum number of samples required to stay within a given estimation error fraction is plotted as the ordinate.
Figure 8: Effect of the sample interval $T$ on noise-free IS estimation.
Figure 9: Effect of the sample interval $T$ on noise-free LS estimation.
Figure 10. Effect of the sample size on speed of convergence.
Fortunately, the optimum sample rate is not a single value but a range which appears as band on the time spectrum from roughly $T = 0.5$ to $T = 1.25$. In addition, excellent results ($\epsilon$ less than 0.005 after 250 samples) are still obtained within the band of $T = 0.25$ to $T = 4.0$. Figure 11 illustrates the effect of the sample rate on the variance of the estimate $\hat{a}_1$. These results were presented for LS estimation, since it is best for $\delta = 0$, but the results for IV estimation are equivalent.

A study was made to determine how the optimum sample-interval band depended upon the ratio of the two time constants. The results presented in the above paragraph had a time constant ratio of $\tau_2/\tau_1 = 0.5$. The cases were obtained with $\tau_1$ maintained at 2.0 and $\tau_2$ varied to give the selected ratios. The following results are reported for the sample interval range for which $\epsilon$ is less than 0.005 after 250 samples.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Range of T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.2 to 1.5</td>
</tr>
<tr>
<td>0.50</td>
<td>0.25 to 4.0</td>
</tr>
<tr>
<td>0.75</td>
<td>0.25 to 4.0</td>
</tr>
<tr>
<td>1.00</td>
<td>0.30 to 3.5</td>
</tr>
</tbody>
</table>

The conclusion is that the sample time selection is not sensitive to the time constant ratio. This result is significant because it will facilitate the estimation of a process whose time constants are changing. Parameter tracking results for this instance will be given in a section to follow.

**Effect of the Variation of $P_0$**

To this point, the method of initializing the recursive IV estimator of Equations 46 and 47 has not been discussed. From Figure 2 it can be seen that it is necessary to specify the initial weighting
Figure 11: Effect of the sample rate on variance of parameter $a_4$ for selected sample periods.
matrix, \( P_0 \), and the initial parameter estimate, \( \hat{\varphi}_0 \). \( P_0 \) can be obtained from Equation 29 by inverting after the first \( n \) observations. Alternatively, in his chapter on identification using LS estimation, Lee (1) suggested a mathematical trick: that the computation process be initiated after the first \( n \) observations by assuming

\[
\hat{\varphi}_0 = 0
\]

\[
P_0 = \begin{bmatrix}
\alpha & \\
& \alpha & \\
& & \ddots & \\
& & & \alpha
\end{bmatrix} = \alpha I
\]

where \( \alpha \) is very large. This assumption implies the addition of a set of \( n \) equations near zero. In practice, with \( P_0 \) set to \( \alpha I \) where \( \alpha \) is large, it does not matter what value is used for \( \hat{\varphi}_0 \) because the algorithm's "memory" is so short that it ignores the initial parameter estimate. Hence, any value may be used and zero is as good as any other. On the other hand, this suggests that if a reasonably good guess can be made of \( \hat{\varphi}_0 \), then this confidence should be reflected in \( P_0 \) by lowering the value of \( \alpha \). The effect of noise on the selection of \( \alpha \) was studied for IV estimation. The results and comparative results for LS estimation are shown in Figures 12-16.

The results show that the convergence properties for LS estimation are not significantly affected by \( \alpha \) as long as it is reasonably large for a given noise-to-signal ratio. For example, Figure 13 shows only slight improvement for increases made in \( \alpha \), over 100. The only adverse effect of an increase in \( \alpha \) is increased oscillations within the first several samples. In contrast, the effect of \( \alpha \) on IV estimations is more pronounced and less predictable as is evident in Figure 14. In
Figure 12: Effect of the variation of $\alpha$ on noise-free LS & IV estimation.
Figure 13: Effect of the variation of $\alpha$ on LS estimation with uncorrelated measurement noise, $\delta = 0.059$. 
Figure 14: Effect of the variation of $\alpha$ on IV estimation with uncorrelated measurement noise, $\delta = 0.059$. 
Figure 15: Effect of the variation of \( \alpha \) on LS estimation with uncorrelated measurement noise, \( \delta = 0.468 \).
Figure 16: Effect of the variation of $\alpha$ on IV estimation with uncorrelated measurement noise, $\delta = 0.488$. 
fact, if \( \alpha \) is chosen to be too high for a given \( \delta \), the IV method may not converge at all. However, it is also evident that if \( \alpha \) is chosen properly, the results of IV estimation are much improved over the LS method when measurement noise is present.

**Parameter Tracking**

The dynamic IV algorithm of Equations 66-68 was studied to determine the parameter tracking ability of a nonstationary system. The second-order example of the previous sections was initiated and the gain, \( k \), was doubled during a 1500 sample observation period. The algorithm was therefore required to follow two separate step changes in two of the parameters of the example. The first step exercises the initial "locking" of the algorithm and the second step, initiated at the 500th sample, requires the algorithm to track an abrupt change in two of the plant parameters. The initial estimate of the parameter vector \( \varphi \) was taken to be zero.

The two difference equation parameters affected by the process gain are parameters \( b_1 \) and \( b_2 \) which are directly proportional to \( k \). The matrix \( D \) of Equation 67 was chosen to be the diagonal matrix

\[
D = \begin{bmatrix}
0 & 0 \\
0 & d \\
d & d
\end{bmatrix}
\]

where \( d \) was varied in the study to demonstrate its effect on tracking. With \( d = 0 \) the dynamic algorithm reduces to the static form of Equations 46 and 47.

Figures 17 and 18 show the excellent tracking performance of the IV method for random measurement noise with a standard deviation, \( \sigma_v \), equal to 0.0125 which corresponds to a noise-to-signal ratio of 0.0655.
Figure 17: IV tracking estimates of parameter $b_1$ for selected values of $d$ with $\sigma_v = 0.0125$. 
Figure 18: Effect of the variation of $d$ on IV parameter tracking with $\sigma_v = 0.0125$. 
for the first 500 samples. Again, the parameter \( d \) is indicative of the algorithm's memory as is demonstrated in both figures. In practice, \( d \) would be selected experimentally on the basis of desired speed of response and the opposing smoothness. For the low level of noise introduced here, the results of the dynamic LS estimator are approximately the same.

Figures 19-22 illustrate the dynamic algorithm operation at a noise level of \( \sigma_v = 0.025 \). Figure 19 shows increased erratic behavior of the estimates due to the increase in noise level. The combination of Figures 20 and 21 show the estimation error for four separate values of \( d \). Figure 22 shows a comparative plot for LS tracking. The effect of increased noise is further illustrated in Figures 23-25. It is clear that the erratic behavior of the estimates is increased in proportion to the increase in measurement noise level.

A second experiment was conducted in order to determine the parameter tracking performance of the IV estimator for a second-order system in which a step change was made in its time constant ratio during the course of the estimation. At the 500th sample, the plant's time constants were changed from \( \tau_1 = 2.0 \) and \( \tau_2 = 1.0 \) to the set \( \tau_1 = \tau_2 = 2.0 \). In this case, all four system parameters change so that the necessary D matrix is now \( dI_4 \), where \( I_4 \) is a (4x4) identity matrix. The results are shown in Figure 26 for the case of \( \sigma_v = 0.025 \). Similar results were obtained when the original system was switched to the system with time constants \( \tau_1 = 2.0 \) and \( \tau_2 = 0.5 \). Again, it is clear that the time constant ratio of a second-order system has little influence on the performance of the IV estimator.
Figure 19: IV tracking estimates of parameter $b_1$ for selected values of $d$ with $\sigma_v = 0.025$. 
Figure 20: Effect of the variation of $d$ on IV parameter tracking with $\sigma_v = 0.025$. 
Figure 21: Effect of the variation of $d$ on IV parameter tracking with $\sigma_v = 0.025$. 
Figure 22: Effect of the variation of d on LS parameter tracking with $\sigma_v = 0.025$. 

}$d = 0$

}$d = 10^{-3}$

Number of Samples

$\epsilon$

$0,001$

$0,01$

$0,1$

$1,0$
Figure 23: IV tracking estimates of parameter $b_1$ for selected values of $d$ with $\sigma_v = 0.05$. 
Figure 24: Effect of the variation of $d$ on IV parameter tracking with $\sigma_v = 0.05$. 
Figure 25: IV tracking estimates of parameter $b_1$ for selected values of $d$ with $\sigma_v = 0.1$. 
Figure 26: IV estimator performance for a step change in the system's time constant ratio with $\sigma_v = 0.025$. 
Dead Time

In the introduction to this chapter, it was stated that the method presented could be considerably simplified if a priori information on the model order and dead time is available. An assumption on the model order presents no problem in the control application since it is perfectly acceptable to use the second-order model as a universal empirical equation to which all plants can be approximated. The dead time, however, deserves more consideration as it is entirely likely that it may change during the operation of some processes.

If the dead time is unknown or is expected to change during process operation then it will be necessary to identify the system rather than simply estimate its parameters. The procedure becomes an iterative one in which a number of values of the transportation lag are selected and used to determine estimates on the same set of data. The value of the dead time that results in the lowest variance in the parameters would be selected and the dead time is thus identified. This procedure however, precludes the use of the dynamic algorithm as the weighting matrix (which contains the variance information) is artificially maintained at the lower bound matrix D.

The consideration to this point has inferred the use of a dead time which is an integral multiple of the sampling interval, T. It is, of course, possible to identify and/or estimate a dead time which is not a simple multiple of the sample time via the modified z-transform (25). This has the effect of adding additional lagged values of the input $u_t$ to the model. However, the use of the modified z-transform model is unnecessary if the dead time is known a priori because, as has been shown, the designer has considerable freedom to choose the sample inter-
val so that the dead time can be made a multiple of $T$. To include dead
time in this manner has little effect on the estimation performance as
can be seen in Figure 27.

**Plant Disturbances**

To this point, consideration has been limited to a deterministic
system disturbed only by measurement noise. More specifically, $w_{d,t}$
in Equation 12 has been considered to be zero and attention has been
focused on the $(1 - a_1 z^{-1})w_{n,t}$ term which represents the autocorrela-
tion caused by measurement noise. It will be of interest in this sec-
tion to drop the measurement noise term and examine the single effect
of plant disturbances. In this instance, the second-order example is
no longer deterministic for it is now considered to be stochastic.

For the case where the plant disturbances of Equation 17 are white
noise, the estimation problem fulfills all of the assumptions of the
LS estimator in the probability limit and that estimator is therefore
the optimal estimator. Figure 28 shows the effect of the noise-to-
signal ratio on optimal estimation and illustrates an important point.
Although the LS estimator is not biased in this case, as less of the
output $y_t$ is explained by the explanatory variable $u_t$, then the vari-
ances of the estimates increase and the estimator performance decreases.
This point will be especially important for closed-loop estimation to
be discussed in the next chapter. The results for the IV estimator in
this case are not significantly different.

Figures 29 and 30 illustrate the effect of autocorrelated plant
disturbances for LS and IV estimation. From Equation 14, the autocor-
relation factor, $s_1$, is a measure of the correlation from one sample
to the next according to the expression
Figure 27: Effect of dead time on IV estimation, noise-free.
Figure 28: Effect of plant disturbances on IS estimation.
Figure 29: Effect of correlation of plant disturbances on LS estimation with $\sigma_{vp} = 0.025$. 

- $s_1 = 0.95$
- $s_1 = 0.5$
- $s_1 = 0$
Figure 30: Effect of correlation of plant disturbances on IV estimation with $\sigma_{vp} = 0.025$. 
The combination of autocorrelated plant disturbances and lagged values of the input and output cause the LS estimator to become inconsistent (7) and this adverse effect is shown in Figure 29. On the other hand, Figure 30 is evidence that the IV estimator is capable of maintaining consistent estimates because it can effectively correct the problem associated with the correlated errors contained in the lagged variables. However, the IV method will not be efficient in this instance because it has dealt only with one of the difficulties and has not corrected the autocorrelation of the disturbance.

\[
\dot{w}_t = -g_i w_{t-1} + v_t
\]  

\[ (79) \]

Summary

A sequential technique for estimating the parameters of a difference equation model of a process subject to noise has been presented for the open-loop configuration. The technique requires no assumptions to be made on the noise statistics and preserves much of the simplicity of least-squares estimation while dealing effectively with measurement noise.

The method was applied to a deterministic second-order example to illustrate its performance in comparison to LS estimation and to study the selection of various parameters to be used in its implementation. Demonstration was given on the parameter tracking capability of the dynamic form of the IV estimator. The dynamic algorithm is important for use in an adaptive control strategy to be discussed in the following chapter.
LITERATURE CITED


CHAPTER III
CLOSER-LOOP IDENTIFICATION OF A NOISY PROCESS UTILIZING
A SEQUENTIAL INSTRUMENTAL-VARIABLE ESTIMATOR

Introduction

The problem objective in the preceding chapter was to estimate the model parameters for a relationship which was postulated to explain observed behavior between a single input and its corresponding output. An underlying assumption of single-equation estimation is that the process has been sufficiently isolated so that the conditions for the validity of the results are satisfied. For this reason, the parameter estimation was performed open-loop so that the process could be freely and independently forced from without. It was also necessary to maintain the constancy of any extraneous variables known to influence the output. The disadvantages of this procedure are well-known so that the effort in the present chapter will be to explore a procedure whereby estimation may be performed while the process is in its normal configuration. Hence, for closed-loop estimation, the interest is still centered on a single equation, but now consideration must be given to the autonomous system of relations in which the equation is embedded.

The development here for the process control application follows the presentation of Klein (1) on simultaneous-equation systems where the latter development applies to the study of economic relationships. It is necessary to first define several terms required for the establishment of a simultaneous equation model.
Definitions

Endogenous variables are dependent variables whose values are determined by the simultaneous interaction of the relations in the model. These variables have a probability distribution whose parameters are elements of the system being estimated. That is,
\[ E[y_t v_t] \neq 0 \]
where \( y_t \) is an endogenous variable and \( v_t \) is a random disturbance. Variables that have an effect on the system but are not affected by the system are called exogenous or independent variables. Exogenous variables, \( x_t \), may be fixed variates or random variables but in either case, the condition
\[ E[x_t v_t] = 0 \]
is imposed for linear systems. An exogenous variable is predetermined, but lagged values of endogenous variables are also predetermined. Here the condition
\[ E[y_{t-1} v_t] = 0 \]
applies provided the errors \( v_t \) are mutually independent, i.e.,
\[ E[v_t v_{t'}] = 0 \quad t \neq t' \]

Simultaneous-Equation Model

The general model is given by
\[ A_0 Y_t + A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + BX_{t-1} = V_t \]
where
\[ A_j = \begin{bmatrix} \alpha_{11} & \ldots & \alpha_{1s} \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \ddots \\ \alpha_{s1} & \ldots & \alpha_{ss} \end{bmatrix} \]
are (sx$s)$ coefficient matrices for the endogenous variable vectors

\[ Y_t^{T-j} = [y_1, y_2, \ldots, y_s]_{t-j}, \]

\[
B = \begin{bmatrix}
\beta_{11} & \cdots & \beta_{1m} \\
\vdots & \ddots & \vdots \\
\vdots & & \ddots \\
\beta_{s1} & \cdots & \beta_{sm}
\end{bmatrix}
\]

is an (sx$s\times m$) coefficient matrix for the exogenous variable vector

\[ X_t^{T-1} = [x_1, x_2, \ldots, x_m]_{t-1}, \quad V_t^{T} = [v_1, v_2, \ldots, v_s]_{t} \] is a vector of random variables and $s$ and $m$ are the number of dependent and independent variables respectively. The $i$th equation of this linear dynamic system is

\[
\sum_{j=1}^{s} \alpha_{ij} y_{j,t} + \sum_{k=1}^{m} \sum_{j=1}^{s} \alpha_{ij,k} y_{j,t-k} + \sum_{j=1}^{s} \beta_{ij} x_{j,t-k} = v_{i,t} \tag{2}
\]

If there is no serial dependence among the elements of $x_t$, i.e.,

\[ E[v_{i,t}, v_{j,t}] = 0 \quad t \neq t' \]

the $y_{t-1}$ can be treated like other predetermined variables for large samples. Hence, with no serial dependence, the lagged $Y_{t-1}$ vectors of Equation 1 may be included in the $X_{t-1}$ vector and Equation 1 simplifies to

\[ AY_t + BX_{t-1} = V_t \tag{3} \]

In practice, the $A$ and $B$ matrices are often sparse and identification requires specific a priori information in order to distinguish one equation from the other.

Criteria for Identification

The $i$th equation of the simultaneous system of Equation 3 is

\[
\sum_{j=1}^{s} \alpha_{ij} y_{j,t} + \sum_{j=1}^{s} \beta_{ij} x_{j,t-1} = v_{i,t} \tag{4}
\]
Because of the presence of zero coefficients and other linear restrictions on the $\alpha_{ij}$ and $\beta_{ij}$, the usual case is to have many of the terms in Equation 4 absent. This, in essence, makes it possible to identify the $i$th equation for if it is impossible to reproduce the statistical form of the $i$th equation by linear combination of some or all of the equations of the system, then the $i$th equation is said to be identified.

By reproduction of statistical form, it is intended to mean the derivation of an equation expressing the same stochastic linear relation among the same variables that appear with non-zero coefficients and having the same linear restrictions that occur among the parameters.

The purpose of this and several sections to follow is to determine the possibility of separating the process model equation from the simultaneous set in which it is embedded and estimating its parameters, i.e., identifying it using a single-equation estimator. Primarily, interest concerns the estimation of a second-order model of a plant in a feedback-control loop. Where it is possible to do so, the parallel development of the general simultaneous set will be included for completeness.

**Feedback Control Loop**

Consider the process control diagram in Figure 1 and the process detail in Figure 2. Figure 1 illustrates the closed-loop identification problem. Figure 2 shows the relationship of the theoretical disturbance $w$ and the actual estimation errors. From Chapter II, the statistical model for the process $G(z)$ is

$$y_t = \varphi x_{t-1} + w_t$$

with

$$\varphi = [a_1 \ a_2 \ \cdots \ a_p \ b_1 \ b_2 \ \cdots \ b_q]$$

$$x^T_{t-1} = [y_{t-1} y_{t-2} \ \cdots \ y_{t-p} u_{t-M-1} u_{t-M-2} \ \cdots \ u_{t-M-q}]$$
Figure 1: Feedback control loop.
Figure 2: Process detail.
where $\varphi$ is the vector of model parameters, $y_t$ is the noise corrupted output and $x_{t-1}$ is the vector of passed inputs and outputs. The exact transfer function for the controller is defined as

$$u_t = \sum_{j=1}^{p} h_j u_{t-j} + \sum_{j=0}^{q} q_j y_{t-j}$$

or

$$u_t = \frac{\sum_{j=0}^{q} g_j z^{-j}}{1 - \sum_{j=1}^{p} h_j z^{-j}} y_t$$

where the $g_j$ are the controller input parameters and $h_j$ are the controller output parameters.

**Second-Order Example**

A specific example will now be selected for study from the general set of Equations 5 and 6. The "velocity" form of the PID control law is

$$u_t - u_{t-1} = K_c [e_t - e_{t-1} + \frac{T_d}{T_i} e_t - \frac{T_d}{T} (y_t - 2y_{t-1} + y_{t-2})]$$

where $K_c$ is the controller gain, $T_d$ is the derivative time, $T_i$ is the integral time, and $e_t$ is the feedback error, $(r_t - y_t)$, with $r_t$ being the set point. If the set point is considered to be constant and equal to zero, Equation 7 may be re-arranged to give the following transfer function for the PID control algorithm

$$u_t = \frac{g_0 + g_1 z^{-1} + g_2 z^{-2}}{1 + h_1 z^{-1}} y_t$$

with

$$g_0 = -K_c (1 + \frac{T_d}{T_i} + \frac{T_d}{T})$$
$$g_1 = K_c (1 + 2\frac{T_d}{T})$$
$$g_2 = -K_c \frac{T_d}{T}$$
$$h_1 = -1$$
From the general process of Equation 5, the second-order model is selected to give

\[ y_t = a_1 y_{t-1} + a_2 y_{t-2} + b_1 u_{t-1} + b_2 u_{t-2} + w_t \]  

(9)

Equations 8 and 9 are rearranged and combined to form the following equation set

\[
\begin{align*}
 y_t & = -a_1 y_{t-1} - b_1 u_{t-1} - a_2 y_{t-2} - b_2 u_{t-2} + w_t \\
 -g_0 y_t + u_t - g_1 y_{t-1} - u_{t-1} - g_2 y_{t-2} & = 0
\end{align*}
\]

(10)

From Figure 2, for the case of negligible measurement noise with random, disturbance and modeling errors, i.e., the theoretical disturbance \( w_t \) is random, the lagged values of the process output may be combined with the exogenous variables (1) to give

\[
\begin{bmatrix}
1 & 0 \\
g_0 & 1
\end{bmatrix}
\begin{bmatrix}
y_t \\
u_t
\end{bmatrix}
+ \begin{bmatrix}
a_1 & b_1 & a_2 & b_2 \\
-g_1 & -1 & -g_2 & 0
\end{bmatrix}
\begin{bmatrix}
y_{t-1} \\
u_{t-1} \\
y_{t-2} \\
u_{t-2}
\end{bmatrix}
= \begin{bmatrix}
w_t \\
0
\end{bmatrix}
\]

(11)

Equation 11 is now in the simultaneous-equation model form and the variables of Equation 3 will be taken to represent the respective matrices and vectors of Equation 11.

The simultaneous-equation model will now be transformed into its "reduced form". This is an alternate way of writing a system so that each endogenous variable is expressed as a function of predetermined variables alone. Multiplication of Equation 3 by \( A^{-1} \) and rearrangement gives

\[
Y_t = -A^{-1}B X_{t-1} + A^{-1}Y_t
\]

\[
= \Pi X_{t-1} + W_t
\]

(12)
where $\Pi$ is the reduced-form matrix of model parameters and $W_t$ is the reduced-form matrix of errors.

Now, in terms of the second-order example

$$A^{-1} = \begin{bmatrix} 1 & 0 \\ g_0 & 1 \end{bmatrix}$$ (13)

$$W_t = A^{-1}V_t = \begin{bmatrix} w_t \\ g_0w_t \end{bmatrix}$$ (14)

$$\Pi = -A^{-1}B = \begin{bmatrix} 1 & 0 & a_1 b_1 & a_2 b_2 \\ g_0 & 1 & g_1 & g_2 & 0 \end{bmatrix}$$ (15)

$$= \begin{bmatrix} a_1 & b_1 & a_2 & b_2 \\ (g_o a_1 + g_1) & (g_o b_1 + 1) & (g_o a_2 + g_2) & g_o b_2 \end{bmatrix}$$

The reduced equations are therefore

$$\begin{bmatrix} y_t \\ u_t \end{bmatrix} = \begin{bmatrix} a_1 y_{t-1} + b_1 u_{t-1} + a_2 y_{t-2} + b_2 u_{t-2} \\ (g_o a_1 + g_1) y_{t-1} + (g_o b_1 + 1) u_{t-1} + (g_o a_2 + g_2) y_{t-2} + (g_o b_2) u_{t-2} \end{bmatrix} + \begin{bmatrix} w_t \\ g_0w_t \end{bmatrix}$$ (16)

or alternately

$$\begin{bmatrix} y_t \\ u_t \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ g_1 + g_o a_1 \end{bmatrix} \begin{bmatrix} y_{t-1} \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ g_2 + g_o a_2 \end{bmatrix} \begin{bmatrix} y_{t-2} \end{bmatrix} + \begin{bmatrix} w_t \\ g_0w_t \end{bmatrix}$$

**Identifiability**

The connection between the structural and the reduced form coefficients is

$$\Pi = -A^{-1}B$$ (17)

or $$A\Pi + B = 0$$
which may be written

\[ \Pi = 0 \]  

(18)

where \[ \Gamma = [A \ B] \] and \[ \Omega = \begin{bmatrix} \Pi \\ I \end{bmatrix} \]

\( \Gamma \) is a \([s \times (s+m)]\) matrix of all structural coefficients and \( \Omega \) is a \([(s \times m) \times m] \) matrix with rank \( m \) containing reduced-form coefficients. For the second-order example from Equation 10

\[
\Gamma = \begin{bmatrix}
1 & 0 & -a_1 & -b_1 & -a_2 & -b_2 \\
-g_0 & 1 & -g_1 & -1 & -g_2 & 0
\end{bmatrix}
\]  

(19)

A necessary and sufficient condition for the identification (1) of the \( i \)th equation in the system

\[ AY_t + BX_{t-1} = V_t \]

is

\[ \text{rank } \Gamma \Theta_i = s - 1 \]  

(20)

where \( \Theta_i \) is a matrix of known coefficients with \( s + m \) rows and as many columns as are necessary to express all the restrictions. In this example, for the first equation

\[
\Theta_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]  

(21)

expresses the restriction from Equation 10 that \( \alpha_{12} = 0 \). Therefore

\[
\Gamma \Theta_1 = \begin{bmatrix} 1 & 0 & -a_1 & -b_1 & -a_2 & -b_2 \\
-g_0 & 1 & -g_1 & -1 & -g_2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]  

(22)

Since the rank \((\Gamma \Theta_1) = 1 = s - 1\), the first equation is identifiable.

The second equation of Equation 10 contains the restrictions \( \alpha_{22} = -\beta_{22} \) and \( \beta_{24} = 0 \) which give
\[ \theta_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \] 

(23)

Once again

\[ \Gamma \theta_2 = \begin{bmatrix} 1 & 0 & -a_1 & -b_1 & -1 & -a_2 \\ -g_0 & 1 & -g_1 & -1 & -g_2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -b_1 & -b_2 \\ 0 & 0 \end{bmatrix} \] 

(24)

Since the rank \((\Gamma \theta_2) = 1 = s - 1\), the second equation is identifiable.

It must be added that this result was to be expected since the first equation was identifiable and there are only two. Further, there is no interest in estimating the parameters of the second equation since they are known.

**Single Equation Estimation**

The results of the last section showed that it is theoretically possible to separately identify the component equations of the simultaneous set which describes the single-input-single-output feedback control loop. Note that the identification criterion was not burdened with considerations germane to the parameter estimation itself. It will be shown that the closed-loop configuration causes extreme linear dependence among regressors with the consequential result that estimation with the methods discussed in Chapter II is impossible except for an important special case.

Multicollinearity was discussed at length in Chapter II where the interest centered around the correlation which arises between the dependent variable, \(y_t\), and its lagged values. It was demonstrated that
with proper selection of the sample interval the effects of multicollinearity could be minimized. However, the introduction of the feedback path causes correlation to arise between the dependent output variable, \( y_t \), and heretofore "independent" input variable, \( u_t \). Upon examination of Equation 10, it is clear that the set contains the endogenous (dependent) variables \( y_t \) and \( u_t \) and their lagged values. Equation 10 contains no exogenous (independent) variables. Only on the strict condition that the theoretical disturbance, \( w_t \), is random can the predetermined variables of Equation 10 be considered to be exogenous (1). The correlation between these predetermined variables and their effect on the estimation of the first equation of Equation 10 will now be examined in detail.

**Closed-Loop Multicollinearity**

An estimator employing the regressive technique requires the inversion of the moment matrix \( [X_{K-1} Z^T] \) where \( X_{K-1} \) in this instance refers to the \((n \times K)\) observation matrix of instrumental variables. The inversion is of course impossible if the rank of \( X_{K-1} \), and consequently the rank of \( [X_{K-1} Z^T] \), is less than \( n \).

For the second-order example

\[
X_{K-1}^T = \begin{bmatrix}
y_0 & y_{-1} & u_0 & u_{-1} \\
y_1 & y_0 & u_1 & u_0 \\
\vdots & \vdots & \vdots & \vdots \\
y_t & y_{t-1} & u_t & u_{t-1} \\
\vdots & \vdots & \vdots & \vdots \\
y_{K-1} & y_{K-2} & u_{K-1} & u_{K-2}
\end{bmatrix}
\]

(25)

The data are assumed to have been taken over a sufficiently "active" period so that the \( y_t \) and \( y_{t-1} \) and \( u_t \) and \( u_{t-1} \) columns are linearly independent through proper selection of the sample rate.
Consider now the three common control laws

\begin{align*}
\text{PID: } u_t &= u_{t-1} + g_0 y_t + g_1 y_{t-1} + g_2 y_{t-2} \\
\text{PI: } u_t &= u_{t-1} + g_0 y_t + g_1 y_{t-1} \\
\text{P: } u_t &= u_{t-1} + g_0 y_t - g_0 y_{t-1}
\end{align*}

(26) (27) (28)

It is obvious from Equations 27 and 28 that utilization of either the PI or the P controller will result in exact linear dependence among the columns of $X_{K-1}^T$ regardless of any particular values which may be assigned to $g_0$ and $g_1$. Inspection of Equation 26 will show that the possibility exists that the $y_{t-2}$ term in the PID control law will prevent exact linear dependency among the vectors of the $X_{K-1}^T$. However, it is suggested in this case that the linear dependence will still be considerable - though not exact - so that the $X_{K-1}^T$ matrix will be ill-conditioned at best. For reasons to be given shortly, it was not possible to confirm this experimentally.

A little reflection will reveal that the situation cannot be improved if an increase in the order of the model is made or should the modified z-transform of the second-order process be utilized. With either option, the PID controller will result in exact multicollinearity by the addition of $y_{t-1}$, $i = 2,3,...$, terms to the model. On the other hand, the possibility does exist for a linear independent $X_{K-1}^T$ matrix by the use of a first-order model and either a PI or PID controller (2). This situation is seen to provide more promise than that to be foreseen by utilization of the second-order model. In the case of the former, it is thought that the addition of integral and/or derivative action would be a stronger deterrent to linear dependency than the simple addition of derivative action in the case of the second-order model.
Although, this discussion is unusually interesting and none the less significant, it is purely academic because of the assumption stated immediately after Equation 25. The assumption was made that the system under investigation was in a state of dynamic change and it must certainly be agreed that this is necessary for identification. However, the simultaneous set represented in Equation 10 contains no vehicle by which it can be forced, i.e., the system has no independent variable.

The predetermined variables of Equation 11 cannot be used as exogenous variables in the practical case for the following reason. In order to obtain Equation 11, it was necessary to assume that the disturbance \( w_t \) is serial uncorrelated. Clearly this is an unfruitful situation, for if the system is driven only by random disturbances, then the noise and signal are necessarily of the same order of magnitude and, as was pointed out in the last chapter, it is not computationally possible to estimate model parameters if the explanatory variables do not play the major role in "explaining" the behavior of the system's output.

**Assurance of Linear Independence**

In order to establish linear independence of the \( X^T_{K-1} \) matrix, it is necessary to retain the set-point terms that were earlier dropped from the control algorithm given in Equation 7. This time the "velocity" form of the three controllers are explicitly expressed as

**PID:** 
\[
\begin{align*}
  u_t &= u_{t-1} + K_c \left[ (r_t - r_{t-1}) + \frac{T}{T_i} (r_t - y_t) - (1 + \frac{T_d}{T}) y_t ight] \\
  &\quad + \left( 1 + \frac{2T_d}{T} \right) y_{t-1} - \frac{T_d}{T} y_{t-2} \\
  &\quad + \left( 1 + \frac{2T_d}{T} \right) y_{t-1} - \frac{T_d}{T} y_{t-2} \\
\end{align*}
\]

**PI:** 
\[
\begin{align*}
  u_t &= u_{t-1} + K_c \left[ (r_t - r_{t-1}) + \frac{T}{T_i} (r_t - y_t) - y_t + y_{t-1} \right] \\
\end{align*}
\]

**P:** 
\[
\begin{align*}
  u_t &= u_{t-1} + K_c \left[ (r_t - r_{t-1}) - y_t + y_{t-1} \right] \\
\end{align*}
\]
It is at once evident, upon condition, that the presence of the set point terms in each of the control algorithms will make it possible for the vectors of $X^T_{K-1}$ to be linearly independent. The condition is that the set-point must change during the observation period. A constant set-point (i.e., $r_t = r_{t-1}$) will reduce to the linear dependent case discussed above. On the other hand, experimental results suggest that reasonable changes in the set-point were shown to give closed-loop estimation results comparable to those obtained by open-loop testing.

**Computational Results**

The second-order process described in Chapter II was used to illustrate closed-loop identification. The difference equation parameters to be identified are:

- $a_1 = .97441$
- $a_2 = -.22313$
- $b_1 = .15482$
- $b_2 = .09390$
- $T = 1.0$

A digital PID algorithm was first selected for the controller. The controller was tuned to give a minimum integral of time multiplied by the absolute value of the error (ITAE) response to a step change in the set point (3). The controller parameters for the 1 unit sampling interval are:

- $K_c = 1.776$
- $T_i = 3.400$
- $T_d = .3906$

The set point function consisted of a series of step changes as shown in Figure 3. $\lambda$ is the set-point switching interval in an integral
Figure 3: Square-wave set point function.
number of sampling instants. Figure 4 shows the effect of this parameter on the estimation error fraction $\epsilon$ for closed-loop IV identification. The series was run without disturbances. Notice the stair-step character of the curves which is a result of the improvement of the estimates each time a new set point change is received.

Figure 5 illustrates the effect of measurement noise on the estimation for $\lambda = 10$. The noise supplied is zero-mean, random, and its magnitude is given by the noise-to-signal ratio, $\delta$. The results here compare with the results of open-loop identification in the last chapter.

The dynamic form of the IV estimator is demonstrated in Figures 6 and 7. The algorithm was required to track the parameter changes which occurred when the process gain was doubled on the 500th sample instant. The standard deviation of the noise, $\sigma_v$, is equal to 0.025 in Figure 6 and is equal to 0.1 in Figure 7. Two values of the lower bound element, $d$, are given in each figure. The $d = 0$ curve represents the finite-time averaging process. Again, the results are excellent and compare with the results of open-loop estimation.

Adaptive Control

The culmination of this research on the IV estimation algorithm is for implementation in an adaptive control strategy as shown in Figure 8. To illustrate this control-loop configuration, the second-order example was used for the unknown process and it was identified utilizing the second-order model as before. But in this instance, the advantage of the on-line estimator's ability to provide current updates of the model parameters can be realized by incorporating a control law which uses this information to select its own parameters. This procedure is generally referred to as controller synthesis and some commonly
Figure 4: Effect of the set point switching interval on noise-free IV estimation for a FID controller.
Figure 5: Effect of measurement noise on IV estimation for a PID controller, $\lambda = 10$. 
Figure 6: Effect of the variation of the lower bound element \( d \) on IV parameter tracking, \( \sigma_v = 0.025 \).
Figure 7: Effect of the variation of the lower bound element $d$ on IV parameter tracking, $\sigma_\nu = 0.1$. 
Figure 8: Adaptive control configuration.
used control algorithms of this type are the deadbeat, Dahlin, and Kalman algorithms (3).

For illustration, the Dahlin method was selected and for the case of a second-order model with zero dead-time the control law is

$$D(z) = \frac{g_0 + g_1 z^{-1} + g_2 z^{-2}}{1 - h_1 z^{-1} - h_2 z^{-2}}$$

where $g_0 = \frac{1}{b_1} Q$, $g_1 = \frac{-a_1}{b_1} Q$, $g_2 = \frac{-a_2}{b_1} Q$, $h_1 = \frac{b_1 - b_2}{b_1}$, and $h_2 = \frac{b_2}{b_1}$

with $Q = (1 - e^{-T/\rho})$ and $\rho$ is the time constant of the desired closed-loop system response. The constant $\rho$ is effectively a tuning parameter and is selected by the designer. When a second-order model is utilized, the Dahlin control law yields a PID controller. For the example plant with $\rho = 2$ and $T = 1$, the corresponding PID parameters are:

- $K_c = 1.34227$
- $T_i = 2.12347$
- $T_d = 0.42247$

Figure 9 illustrates the dynamic IV estimator performance in the adaptive environment with the Dahlin control algorithm for the case where the process gain was doubled at the 200th sample and the output was subject to measurement noise, $\sigma_v = 0.025$. As the estimator identifies the new gain, the Dahlin algorithm compensates by halving the controller
Figure 9: Parameter tracking for adaptive control employing the Dahlin controller. Gain doubled at 200.
gain. Clearly, Figure 9 is evidence enough that the IV estimator is most appropriate for the adaptive configuration.

Summary

The parameter estimation technique presented in the last chapter has been extended in the present to the closed-loop configuration. A general development has been given whereby determination can be made of the possibility of identifying a single-equation from the simultaneous set in which it is embedded.

The most important consideration pertaining to single-equation parameter estimation dealt with the existence of an independent forcing function through which the system could be excited. The question of using predetermined variables (i.e., lagged values of the input and output variables) for this purpose was examined and it was shown to be infeasible due to linear dependence of the observation matrix and the inability of estimating a system driven only by random disturbances on its output. In order to solve the problem, it was shown that a set point variation could assure linear independence and excite the system above the level of the disturbances so that estimation could be performed. Under this condition, the computational results show that closed-loop estimation is equally as good as open-loop estimation. This was further demonstrated by presenting closed-loop results of the dynamic IV algorithm.

Finally, the overall objective of this work was realized with the implementation of the sequential IV estimator into an adaptive control configuration. It remains to show that the method will work for the non-linear process.
LITERATURE CITED


CHAPTER IV

AN APPLICATION OF THE SEQUENTIAL INSTRUMENTAL-VARIABLE
ESTIMATOR TO A NON-LINEAR CHEMICAL REACTOR

Introduction

While Chapters II and III have dealt with the performance of the instrumental-variable estimator for the process which is contaminated with measurement noise or has time-variable parameters, the present chapter will deal specifically with an application of the method to a non-linear process. It is important to determine if the IV estimation algorithm can approximate non-linear dynamics by sequentially estimating the parameters of a simple model. Should this procedure prove successful, it would provide a determination to be made on the current linearized characteristics of the plant. This information could in turn be utilized by an adaptive controller to assure that the plant-controller system is functioning in some "best" sense regardless of the non-linearities of the plant.

In this chapter, a jacketed, backmix chemical reactor is adapted for use from an example presented by Chiu (1). The static form of the IV estimation algorithm is applied to the reactor in order to determine the parameters of a second-order model. Results are presented for both the open- and closed-loop configurations.

A Water-cooled Chemical Reactor

Consider the stirred-tank chemical reactor of Figure 1. The contents of the reactor are assumed to be perfectly mixed, and the reaction
Figure 1: The water-cooled chemical reactor.
The reaction is occurring at the rate

\[ R_A = k_o e^{-a (T_R + 460)/C_A^2} \quad (1) \]

where

- \( R_A \) = lbs of A decomposing per min per cubic foot of reacting mixture
- \( k_o \) = Arrhenius rate constant, ft\(^3\)/lb-min
- \( a \) = Arrhenius temperature constant, °R
- \( T_R \) = temperature of the reacting mixture, °F
- \( C_A \) = concentration of A in the reacting mixture, lbs/ft\(^3\)

The reactants enter the reactor at concentration \( C_{A_0} \) (lbs/ft\(^3\)), temperature \( T_{A_0} \) (°F) and at the rate of \( F \) (lbs/min). The products are removed at the rate \( F \), concentration \( C_A \) and temperature \( T_R \). The reactor volume is \( V \) (ft\(^3\)) and the reacting mixture has a density \( \rho \) (lbs/ft\(^3\)) and specific heat \( C_p \) (Btu/lb-°F).

The reaction is exothermic with \( \Delta H \) Btu of heat being generated for each lb of A that reacts. Hence, in order to control the reactor, cooling water is supplied at the variable rate \( W_c \) (lbs/min) and temperature \( T_w \) (°F) and leaves the jacket at temperature \( T_c \) (°F). The heat capacity of the wall between the reactor and the jacket is combined with that of the water in the jacket and is denoted by \( M_c \) (Btu/°F). The specific heat of water is taken to be unity and heat losses to the surroundings are negligible. The overall heat transfer coefficient between the reactor contents and the jacket is assumed constant at \( U \) (Btu/min-ft\(^2\)-°F). The heat transfer area of the jacket is \( A \) (ft\(^2\)).
The equations describing this system are derived by making an unsteady-state mass balance on reactant A, an energy balance on the reacting mixture, and an energy balance on the jacket. The resulting equations are given in Table 1 and a summary of the simplifying assumptions are listed in Table 2. The specific values for the parameters and initial steady-state conditions are given in Table 3.

**Non-Lineairties**

The purpose for selecting the jacketed chemical reactor for study is because it exhibits strong non-linear behavior. The primary interest is to determine if the IV estimation algorithm presented in Chapter II can estimate the parameters of a second-order model of a non-linear system. It is therefore important to look briefly at some of the non-linearities present in the example.

The objective is to derive the relationship between the cooling water rate $W_C$ (the control variable) and the reactor temperature $T_R$ (the controlled variable). The easiest way to observe non-linear behavior is to inspect the process reaction curves for several size step changes in the water rate $W_C$. Figure 2 shows these step responses for $\Delta W_C$ of -100, -200, and -300 lbs/min. The process gains are observed to be -0.0128, -0.0140, and -0.0153 °F-min/lb, respectively. The corresponding step responses for $\Delta W_C$ values of 100, 200, and 300 lbs/min are shown in Figure 3. First, notice that the response from a step up in cooling water rate is considerably different from the corresponding step down. The respective gains from Figure 3 are -0.0113, -0.0106, and -0.0100 °F-min/lb. As expected, the most non-linear behavior is exhibited by the larger step responses. That is, the difference in gain between the up and down steps of $W_C$ is 0.0053 °F-min/lb for $|\Delta W_C|$
Table 1

Process Equations

Mass Balance on Reactant A

\[
\frac{d}{dt} C_A = \frac{F}{V} (C_{A_0} - C_A) - k C_A^2 \quad (2)
\]

\[
k = k_0 e^{-\frac{a}{(T_R + 460)}}
\]

Energy Balance on Reacting Mixture

\[
\frac{d}{dt} T_R = \frac{F}{V} (T_{O} - T_R) - \frac{UA}{\rho V C_p} (T_R - T_c) + \frac{\Delta H}{\rho C_p} k C_A^2 \quad (3)
\]

Energy Balance on Cooling Water

\[
\frac{d}{dt} T_c = \frac{UA}{M_c} (T_R - T_c) - \frac{W}{M_c} (T_c - T_w) \quad (4)
\]
Table 2
Summary of Assumptions

Mass Balance on Reactant A
1. The feed and product rates are equal and constant.
2. The density \( \rho \) of the reacting mixture is constant.
3. The contents of the reactor are perfectly mixed so that the concentration of the reactor contents and product are the same.

Energy Balance on Reacting Mixture
1. The specific heat \( C_p \) of the reacting mixture is constant.
2. The heat of reaction \( \Delta H \) is constant.
3. The contents of the reactor are perfectly mixed so that the temperature of the reactor contents and product at the same.

Energy Balance on Cooling Water
1. The heat loss to the surroundings is negligible.
2. The contents of the reactor are perfectly mixed so that the reactor temperature is uniform.
3. The holdup reactor volume is constant.
4. The overall heat transfer coefficient between the reactor contents and the jacket is constant.
5. The specific heat \( C_p \) of the reacting mixture is constant.
### Table 3

**Initial Conditions and System Parameters**

**Initial Conditions**

\[ C_A = 3.6 \text{ lbs/ft}^3 \]
\[ T_R = 190^\circ \text{F} \]
\[ T_C = 120^\circ \text{F} \]

**System Parameters**

\[ C_{Ao} = 9 \text{ lbs/ft}^3 \]
\[ W_c = 1074 \text{ lbs/min} \]
\[ F = 16.7 \text{ ft}^3/\text{min} \]
\[ T_w = 80^\circ \text{F} \]
\[ \Delta H = 870 \text{ Btu/lb} \]
\[ C_p = 0.9 \text{ Btu/lb}^\circ \text{F} \]
\[ V = 250 \text{ ft}^3 \]
\[ U_A = 600 \text{ Btu/min}^\circ \text{F} \]
\[ T_o = 150^\circ \text{F} \]
\[ a = 2560^\circ \text{R} \]
\[ \rho = 60 \text{ lbs/ft}^3 \]
\[ k_o = 1.43 \text{ ft}^3/\text{lb-min} \]
\[ M_c = 6000 \text{ Btu}^\circ \text{F} \]
Figure 2: Process reaction curves for step down in cooling water rate $W_c$ (lbs/min).
Figure 3: Process reaction curves for step up in cooling water rate $w_c$ (lbs/min).
= 300 lbs/min. These tests were conducted from a steady-state reference of \( T_R = 190°F \). Figure 4 shows the effect of changing the reference to \( T_R = 187°F \) for a cooling-water rate change of \( \Delta W_c = -300 \). The process gain is now \(-0.00986°F\)-min/lb compared to the \(-0.0153\) obtained at the \( T_R = 190°F \) level.

**Second-Order Model**

The second-order difference equation model to be used to represent the dynamic relationship of the reactor is expressed as

\[
T_{R,t} = a_1 T_{R,t-1} + a_2 T_{R,t-2} + b_1 W_{c,t-M-1} + b_2 W_{c,t-M-2} + C \quad (5)
\]

As before, the coefficients \( a_1, a_2, b_1, \) and \( b_2 \) are the model parameters to be determined and \( M \) is the process dead time. The new variable \( C \) is a constant term which is included in Equation 5 to represent the intercept which is in general non-zero. At steady state Equation 5 may be written

\[
\bar{T}_R = a_1 \bar{T}_R + a_2 \bar{T}_R + b_1 \bar{W}_c + b_2 \bar{W}_c + C \quad (6)
\]

where the bar indicates the steady-state values of \( T_R \) and \( W_c \). Subtraction of Equation 6 from 5 and substitution of the numerical steady-state values yields

\[
(T_{R,t} - 190) = a_1 (T_{R,t-1} - 190) + a_2 (T_{R,t-2} - 190) + b_1 (W_{c,t-M-1} - 1074) + b_2 (W_{c,t-M-2} - 1074) \quad (7)
\]

or more simply

\[
y_t = a_1 y_{t-1} + a_2 y_{t-2} + b_1 u_{t-M-1} + b_2 u_{t-M-2} + C \quad (8)
\]

where \( y_t \) and \( u_t \) are the deviation variables for the output and input variables respectively and \( C \) is an intercept or bias term which is reintroduced into Equation 8 to allow for a non-zero mean for either the
Figure 4: Process reaction curve from a reference of $T_R = 187 \, ^\circ F$.
input or disturbance function. In general, best results are obtained when $c$ is small.

Since the second-order model is selected to be used as an empirical representation of the reactor, the only other a priori information necessary is the system dead time $M$. From Figures 2 and 3 it can be seen that a suitable choice of $M$ would be 1 minute. If $M$ is constrained to be an integral multiple of the sample interval, then the largest possible choice for the sampling interval would be 1 minute. Note, that for reasons given in Chapter II, it is important for estimation that the sampling interval, $T$, not be too small. If the sampling interval is chosen to be larger than 1 minute, it will then be necessary to ignore the dead time and use $M = 0$. Hence, it therefore seems reasonable to use $T = M = 1$ from inspection of the process reaction curve.

Performance Criterion

For the deterministic second-order example utilized in the previous chapters, it was convenient to base the estimator performance on the true parameter values of the process. This was possible because the model fit the process exactly. However, in the present concern, there exists no "true" parameter values upon which to base an on-line performance criterion. The only on-line alternative is to use the error between the plant output and the output of the auxiliary model used in the parameter estimation, but this criterion breaks down for two reasons. First, the magnitude of the error would depend on the amplitude of the output deviations and, secondly, the auxiliary model is "free-running" so that it usually takes some time for poor initializations to disappear. It therefore becomes necessary to judge the estimator performance off-line.
The performance criterion procedure adopted is to compare the process reaction curve of the reactor with that computed from the second-order model using the model parameters determined after a fixed sample period. The process reaction curve selected is arbitrarily chosen to be the step response of the model to an input change of $|\Delta W_c| = 300$ lbs/min and a reference reactor temperature of $190^\circ F$. This step response is the most non-linear of the ones shown in Figure 2. The sample period is taken to be 250 minutes.

The input test signal has a profound influence upon the performance criterion, so it is relevant to discuss the forcing function in this section. A square-wave test signal is utilized and the form for open-loop estimation is shown in Figure 5. The amplitude of the wave is $X$ lbs/min and $\lambda$ is the switching interval in multiples of the sample interval, $T$. It is interesting to point out that a test signal must make negative step changes as well as positive changes. Since the reactor's step response changes with direction, the estimation algorithm will attempt to fit a second-order model between the up-step and the down. This is to be seen more clearly in Figure 6 where both the step-up (in temperature) response to a $\Delta W_c = -300$ lbs/min change in input and a step-down response to a $\Delta W_c = 300$ lbs/min change are plotted in the absolute value of the deviation variable $y_t$. By using the absolute value, the step-down response appears folded over on the time axis. The estimation algorithm will be averaging these two responses so that the model step response will be expected to lie within the envelope created by the curves in Figure 6. In addition, Figure 6 also shows the response of the analytically linearized reactor via the Jacobian Matrix. The linearized transfer function is
Figure 5: Square-wave forcing function.
Figure 6: Process reaction curve envelope.
\[ \frac{T_R(s)}{W_c(s)} = \frac{-0.01228(3.741s + 1)}{(10.80s + 1)(4.114s + 1)(3.488s + 1)} \] (9)

**Programming Particulars**

The jacketed chemical reactor of Equations 2-4 was simulated digitally in a Fortran IV subroutine (see Appendix A). The stepwise Euler technique was used for the integration with a step size of 0.05 minutes. Equation 8 was used as the model and the static instrumental-variable estimator of Chapter II was implemented. In each study, the initial estimates of the parameters were taken to be zero and the initial diagonal of the weighting matrix was 100.

**Open-Loop Estimation**

Open-loop tests were undertaken in order to determine the performance of the sequential IV estimator for determining the second-order model of the chemical reactor. The major factor found to influence estimator performance was the input switching interval, \( \lambda \). Several model step responses for selected switching intervals are shown in Figure 7. In general, the larger the switching interval used, the better the estimation. The curve for \( \lambda = 50 \) is judged to be close to the correct response since it lies near the center of the response envelope. The lower curve of the envelope and the \( \lambda = 20 \) response curve nearly overlap. The response for \( \lambda = 10 \) is seen to be of the correct form but, like the \( \lambda = 20 \) curve, the steady state gain is underestimated. It is suggested that, since the forcing function is periodic and the reactor is known to be a low-pass filter, the longer switching intervals are preferred for more low frequency content. From Figure 2, an approximate first-order time constant would be in the neighborhood of 18 min-
Figure 7: Effect of the switching interval of the square-wave input on open-loop IV estimation.
utes. This corresponds to a corner frequency of \(1/18\) radians per minute and is much lower than the \(2\pi/20\) radians per minute of the \(\lambda = 10\) square-wave.

On the other hand, it is apparent from Figure 8 that the size of the input amplitude has little effect on the estimation performance. It is interesting to examine the exceedingly small span utilized on the output variable under these laboratory conditions. Although the actual step size is twice the amplitude \(X\), the maximum deviation from the mean for the case where \(X = 5\) lbs/min is observed from the data to be 0.0625 °F and the differences between successive \(y_t\) values is less than 0.005 °F. Clearly, this precision is not possible in practice, but it is comforting to know that this is not a limitation of the estimator.

Closed-Loop Estimation

Closed-loop estimation studies were performed for the simple feedback control loop with a PID controller and for the adaptive control configuration employing a Dahlin synthesis controller. No problems were experienced in either implementation, but since it is necessary to give the Dahlin controller reasonable first estimates of the parameters, only the non-adaptive results are presented.

The reactor in the closed-loop configuration was forced with a square-wave test signal similar to the open-loop forcing function of Figure 5. In this case the amplitude becomes the deviation of the set point which is assigned to the variable \(R\).

The effect of the set point switching interval, \(\lambda\), is shown in Figure 9. As in the case of the open-loop studies, the conclusion is that the estimation algorithm prefers the low frequency forcing function.

The tests in Figure 9 were conducted using a set point amplitude of
Figure 8: Effect of the input amplitude of the square-wave input on open-loop estimation.
Figure 9: Effect of the set point switching interval on closed-loop IV estimation.
R = 2°F. Figure 10 illustrates the effect of the set point amplitude on the estimation performance. As shown, the strength of the input signal is more important for closed-loop estimation but remains to be very reasonable in the practical sense. It may be noticed that the estimated response for the closed-loop system in Figure 10 has a smaller gain than the reaction curve for the open-loop model in Figure 8. The reason is that the PID controller behaves differently depending upon the direction of the set point change. The controller parameters were obtained from Chiu (1) and represent the optimum ITAE tuning parameters for a set point change made from $T_R = 190$ to $185°F$ with a sample time of 1 minute. The parameters are

$$K_c = -890 \text{ lbs/min}^°\text{F}$$

$$T_1 = 27.1 \text{ min}$$

$$T_d = 1.5 \text{ min}$$

Since the controller "weights" the averaging process of the estimation algorithm, it is therefore seen that the reactor model will not necessarily fall within the center of the reactor envelope.

Aperiodic Input

It is of considerable interest to investigate the use of an aperiodic forcing function for the closed-loop configuration because in many cases set point changes do not occur often enough in the course of normal process operation. In this regulatory situation, it is desirable to hold process upsets to a minimum. At the same time, it must be realized that the process must be driven enough to excite all nodes of the system.

The model step responses presented in the last several sections were obtained using parameter estimates after the 250th sample. This
Figure 10: Effect of the set point amplitude on closed-loop IV estimation.
procedure was arbitrary and was done for comparative purposes. However, a re-examination of the results for cases for which $\lambda$ was greater than 50 showed that the estimates obtained after the second switch interval were essentially the same as those estimates computed 500 or 1000 samples later. This suggests that estimation could be accomplished by a series of two pulses after which the set point is returned to its normal level of operation. This special test function is illustrated in Figure 11. The parameter time-series estimate of $b_1$ is included to demonstrate the stability of the estimation algorithm after the input excitation has ceased.

**Dynamic Estimation**

For the linear example studied in Chapters II and III, it was shown the dynamic estimator offered an improved measure of flexibility by allowing the estimation algorithm to track parameter variation. In the case of the non-linear reactor, care must be exercised in the selection of the elements of the lower bound matrix $D$ to insure that the desired results are obtained. Specifically, by introduction of $d$ elements, the memory of the algorithm is shortened accordingly and the estimator can be made to track either of the parameter sets associated with the two limits of the reactor envelope. This tracking may be more desirable for a particular application than the average result obtained with $d = 0$. This point is illustrated in Figures 12 and 13 for the PID controller and in Figures 14 and 15 for the adaptive strategy employing the Dahlin control algorithm. These diagrams also show the effect of a step disturbance added at the 200th sample instant.

Figure 12 shows the effect of the aperiodic test input on the estimation of parameter $b_1$ which is proportional to the process gain. The
Figure 11: Time series showing test signal and parameter $b_1$ to illustrate the IV algorithm stability after an aperiodic input.
Figure 12: Time series showing the effect of an aperiodic set point function and a step disturbance on the dynamic IV estimation of parameter $b_1$ for the FID controller.
Figure 13: Control action for the PID controller.
Figure 14: Time series showing the effect of an aperiodic set point function and a step disturbance on the dynamic IV estimation of parameter $b_1$ for the adaptive configuration.
Figure 15: Control action for the adaptive configuration employing the Dahlin control law.
initial parameters for these figures were obtained from the results of the experiment performed in Figure 11. In the present case, \( \alpha \) was chosen to be zero which indicates that the parameters are known to be accurate. For the static estimator (i.e., \( D = 0 \)) \( \alpha = 0 \) freezes the estimates regardless of any system inputs or disturbances. However, with nonzero elements in \( D \), the algorithm is free to track even with \( \alpha = 0 \). The diagonal elements of \( D \) were chosen to be the corresponding final values of the weighting matrix from the experiment of Figure 11. Following the aperiodic input, the set point is first increased by \( 2^\circ F \) and the controller responds by reducing the cooling water rate \( u_t \). The estimator correctly predicts that the process gain has increased and this is reflected by the increase in absolute value of the parameter \( b_1 \). In other words, the estimator was initialized with the "average" model and correctly tracked the upper limit of the reactor envelope from the new information received. The other two steps of the aperiodic input follow the same reasoning. The initial values of the controller settings for the adaptive and the non-adaptive controllers were the same. These values were obtained from the Dahlin control law using the initial parameter set. These settings were

\[
\begin{align*}
K_c &= -632.6 \text{ lbs/min}^\circ F \\
T_i &= 20.9 \text{ min} \\
T_d &= 1.76 \text{ min}
\end{align*}
\]

The desire to maintain a stable estimation algorithm in the face of large and abrupt load changes is generally in opposition with the desire to track system parameters. This subject is introduced here to point out the consideration that must be taken for the selection of the \( D \) matrix. At the 200th sample, the flow of reactants to the reactor
was changed from \( F = 16.7 \) to \( F = 15.0 \text{ ft}^3/\text{min} \). Figures 12-15 illustrate this disturbance and its effects for the adaptive and the non-adaptive systems. Although the load does not cause the output to deviate in a large measure in Figures 12 and 14, Figures 13 and 14 show that the same amount of control action is required to offset the load disturbance as was required to compensate for the change in set point of -2°F. The important result of this experiment is that the dynamic estimation algorithm remains stable for the adaptive configuration even with abrupt load changes. It is conceded that much work remains to be done in this area, but at the same time, it is felt that results presented in this dissertation indicate that the dynamic IV estimation approach holds considerable promise.

**Summary**

This chapter has dealt with the all important feature of an on-line estimation method applied in the process industry - that it be capable of representing non-linear dynamics with low-order linear models. An application to a jacketed chemical reactor was presented with a discussion on the specific non-linear behavior. A criterion was given by which the performance of the estimation algorithm could be judged and demonstration was given of that performance for both the open- and closed-loop configurations. The open-loop estimation performance was seen to be relatively independent of the amplitude of the forcing function. The closed-loop performance was shown to be more sensitive to input amplitude, but for this example application, the difference was not significant. Finally, evidence was given that the closed-loop reactor system could be estimated by use of an aperiodic test pulse used to minimize disturbance to the system.
LITERATURE CITED

CHAPTER V
CONCLUSIONS

The contribution of this research is not considered to be a result in itself, but a link, for much work remains. Ultimately, it must be demonstrated that the identification method studied here can be utilized in an adaptive control strategy to effectively improve the control of an existing process. Further, this improvement must be sufficient to cover the necessary increased complexity and expense involved in the implementation. However, the importance of bridging the gap between the theorist and the practitioner cannot be minimized. The abiding conclusion of this work is that the evidence presented here indicates that it is feasible to employ the adaptive control approach outlined in this dissertation to an existing physical control problem.

This work offers the major computational studies and conclusions necessary for implementation of the instrumental-variable estimation algorithm into a practical self-adaptive control system. The work here is intended to serve as a practical guide for the selection of various "tuning" parameters of the estimation algorithm to include the selection of the sampling rate. It was shown that multicollinearity within the regression matrix can cause estimator performance to deteriorate if sampling is too frequent. It was shown that the time constant ratio of an overdamped second-order process did not significantly affect estimation. Information was given for the selection of the initial weighting matrix and the lower bound matrix for parameter tracking. This was generally referred to as memory shaping and represents an important
feature of the estimation algorithm if it is used in an adaptive loop. The important effects of measurement noise on estimator performance were exhaustively covered and should provide a valuable reference source for actual implementation. The dead time was shown to present no difficulty to the method as long as it could be specified a priori and expressed as an integer multiple of the sample interval. This last difficulty can be minimized since the designer is relatively free to choose the sample interval. Finally, parallel results of the ordinary least-squares method were presented so that informative comparisons can be drawn about the IV approach relative to other presentations.

It was shown in Chapter III that an examination of assumptions necessary for closed-loop estimation provided some interesting conclusions. Two considerations contributed to the judgement that a process could not be estimated with predetermined variables generated solely by random plant disturbances. The first established that due to linear dependency within the closed-loop regression matrix the only possible model-controller combinations were the second-order model and PID controller, the first-order model and PI controller, and the first-order model and PID controller. Doubt was expressed that the first of these combinations would be practical. Secondly and precluding the use of experiments to examine the first difficulty, expression was made that it is not practically feasible to estimate the difference equation of a system driven only by its random disturbances. Hence, in order to estimate the closed-loop process it was shown that set-point variation is required. With the introduction of this truly independent variable the computational results show excellent IV estimator performance for both the static and the dynamic cases.
Results in Chapter IV demonstrated that IV estimation can indeed be used to numerically linearize a non-linear process in an on-line situation. These results for the chemical reactor are certainly encouraging and will serve to convince the practitioner more than any other single conclusion.

Perhaps this research has brought adaptive control much closer to the chemical engineer - perhaps not. Perhaps it is not the answer to problems peculiar to the process industry. On the other hand, it is interesting to note that if the adaptive control law and estimation algorithm are taken together, that is, if the digital computer itself is considered to be the controller, then the simple controller-process loop remains. This suggests that something is missing from the conventional PID controller. This also suggests that the complete self-adaptive algorithm may be reduced to such a degree that it is itself considered a conventional controller.
<table>
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<th><strong>English</strong></th>
<th><strong>Definition</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(sx$s$) coefficient matrix</td>
</tr>
<tr>
<td>A($z^{-1}$)</td>
<td>linear function of $a$ parameters</td>
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<tr>
<td>B</td>
<td>(sx$m$) coefficient matrix</td>
</tr>
<tr>
<td>B($z^{-1}$)</td>
<td>linear function of $b$ parameters</td>
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<tr>
<td>C</td>
<td>difference equation intercept or bias term</td>
</tr>
<tr>
<td>C($z^{-1}$)</td>
<td>disturbance transfer function</td>
</tr>
<tr>
<td>D</td>
<td>(nx$n$) lower bound matrix</td>
</tr>
<tr>
<td>D($z^{-1}$)</td>
<td>controller transfer function</td>
</tr>
<tr>
<td>E</td>
<td>expected value operator</td>
</tr>
<tr>
<td>$E_K$</td>
<td>(lx$K$) generalized equation error vector</td>
</tr>
<tr>
<td>F($z^{-1}$)</td>
<td>linear function of $f$ parameters</td>
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<tr>
<td>F($t$)</td>
<td>(nx$n$) state coefficient matrix</td>
</tr>
<tr>
<td>G</td>
<td>general transfer function</td>
</tr>
<tr>
<td>G($t$)</td>
<td>(nx$r$) state driving matrix</td>
</tr>
<tr>
<td>H</td>
<td>(mx$n$) observability matrix</td>
</tr>
<tr>
<td>H($s$)</td>
<td>hold element</td>
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<tr>
<td>HG($z^{-1}$)</td>
<td>pulse transfer function</td>
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<td>I</td>
<td>identity matrix</td>
</tr>
<tr>
<td>K</td>
<td>total number of observations</td>
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<tr>
<td>$K_c$</td>
<td>controller gain</td>
</tr>
<tr>
<td>M</td>
<td>dead time</td>
</tr>
<tr>
<td>$M_{XX}$</td>
<td>moment matrix</td>
</tr>
<tr>
<td>N</td>
<td>auxiliary model update delay</td>
</tr>
</tbody>
</table>
null matrix

(weighting matrix)
covariance

(linear function of moving average terms)

standard deviation

(linear function of autocorrelation terms)

sample interval

(controller integral time)

(controller integral time)

(random noise vector sequence)

(vector of disturbances)

(vector of exogenous variables)

(observation matrix)

(vector of endogenous variables)

(vector of outputs)

(instrumental-variable matrix)

(model output parameters)

(model input parameters)

(true system output)

element of lower bound matrix

generalized equation error function; feedback error, \((r_t - y_t)\)

noise parameters

(controller input parameters)

(controller output parameters)

(index)

(index)
k  |  gain; index
m  |  number of observation variables; number of exogenous variables
n  |  system order \((p + q)\)
p  |  number of output parameters
p' |  number of controller output parameters
q  |  number of input parameters
q' |  number of controller input parameters
Tq  |  \((n \times l)\) random disturbance vector
r  |  number of control variables
rt |  set point
rxx|  correlation coefficient
s  |  number of endogenous variables; Laplace transform variable
sl  |  autocorrelation factor
t  |  time index
ut  |  control variable
u(t) |  \((r \times l)\) control variable
vt  |  white noise
wt  |  combined or theoretical disturbance term
xt  |  \((n \times l)\) vector of passed inputs and outputs; exogenous variable
x(t) |  \((n \times l)\) state vector
yt  |  disturbance-corrupted output; endogenous variable
y(t) |  \((m \times l)\) vector of observables
zt  |  \((n \times l)\) instrumental variable vector
Greeks

\( \Gamma \) (nxr) driving matrix; \([A \ B]\)

\( \Delta \) (nxn) parameter input matrix

\( \Theta \) (nxn) transformation matrix

\( \Pi \) reduced form model parameters, \(-A^{-1}B\)

\( \Pi_{K-1} \) (lxK) predicted error matrix

\( \Sigma \) summation operator

\( \Psi \) phi

\( \Psi \) (nxn) parameter transition matrix

\( \Omega \) (lxl) vector of noise parameters; \([\Pi \ I]^T\)

\( \Theta \) initial diagonal element of \( P_0 \)

\( \Theta_{ij} \) element of \( A \)

\( \phi \) PRBS bias

\( \phi_{ij} \) element of \( B \)

\( \Psi \) (nxn) state transition matrix

\( \delta \) noise-to-signal ratio; Kronecker delta function

\( \epsilon \) estimation error fraction

\( \xi_t \) auxiliary model output

\( \theta \) magnitude of input signal; restriction matrix

\( l \) number of noise parameters

\( \lambda \) square-wave switching interval; PRBS clock interval

\( \mu \) prediction error

\( \rho \) PRBS period; Dählin tuning constant

\( \sigma \) standard deviation

\( \tau \) time constant

\( \varphi \) (lxn) vector of model parameters
--- THIS IS THE MAIN PROGRAM USED FOR INITIALIZATION AND CONTROL
AND FOR THE I/O TASKS.

--- PURPOSE OF MAIN SUBROUTINES
SUB STID IS THE SEQUENTIAL ESTIMATION ALGORITHM
SUB ST1TE CONTAINS THE SIMULATED PROCESS - THERE EXIST two
SUBROUTINES BY THAT NAME (ONE IS FOR THE OVERDAMPED 2ND
ORDER EXAMPLE AND ONE IS FOR THE CHEMICAL REACTOR)
SUB STSAM IS THE AUXILIARY MODEL
SUB SLMCT CONTAINS THE CONTROLLER - THERE ARE two SUBROUTINES
BY THAT NAME (ONE IS THE PID CONTROLLER AND THE OTHER IS
THE UMILIN CONTROLLER)
SUB BMLOG IS USED TO GENERATE THE MAX LENGTH SEQUENCE.

--- INPUT PARAMETERS
DUM1-DUM7 = DUMMY PARAMETERS WHICH SPECIFY THE SIMULATED PROCESS
FOR THE 2ND ORDER OVERDAMPED
AK = PROCESS GAIN
TH1 = INVERSE OF TIME CONSTANT
TH2 = INVERSE OF SECOND TIME CONSTANT
REF = INITIAL VALUE OF OUTPUT C AND SET POINT R.
SDUM1-SDUM4 = DUMMY PARAMETERS WHICH SPECIFY STEP CHANGES TO BE
MADE IN THE SIMULATED PLANT PARAMETERS AT TIME KSWITCH
ALFO = INITIAL PARAMETER ESTIMATES, USUALLY ZERO
D = THE 5 DIAGONAL ELEMENTS OF THE LOWER BOUND MATRIX FOR THE
DYNAMIC ALGORITHM FOR PARAMETER TRACKING
CDUM1-CDUM4 = DUMMY PARAMETERS FOR THE CONTROLLER SPECIFICATION
CK = CONTROLLER GAIN
TI = INTEGRAL TIME
TD = DERIVATIVE TIME
uin = INITIAL VALUE OF THE CONTROLLER OUTPUT
KPLT = PLOT OPTION
KPRT = PRINT OPTION
IPDF = PLOTTER OUTPUT SELECT
VMA = VERTICAL MAX FOR PLOTTER
VMM = VERTICAL MIN
HMA = HORIZONTAL MAX
HMM = HORIZONTAL MIN
V1IC = NUMBER OF TIC MARKS ON VERTICAL AXIS
HTIC = FOR AXIS TIC MARKS
MCY = NUMBER OF DELAY UNITS TAKEN TO UPDATE AUX. MODEL
IODEL = SET POINT SWITCHING INTERVAL
IODEL = SWITCHING INTERVAL FOR DISTURBANCES
ILCP = OPTION TO SELECT LS OR IV ESTIMATION
JCNTR = OPTION TO SELECT DAHLIN CONTROLLER
M = LEAD TIME AS AN INTEGRAL NUMBER OF SAMPLE UNITS SPT
KCP = SELECTS OPEN OR CLOSED LOOP ACTION
H = INITIAL VALUE OF WEIGHTING MATRIX DIAGONAL
SPT = SAMPLE INTERVAL
SD = STANDARD DEVIATION OF MEASUREMENT NOISE
F1 = AUTOCORRELATION FACTOR OF MEASUREMENT NOISE
RCIS = DISPLACEMENT OF SET POINT SQUARE WAVE
YUC = DUMMY VARIABLE FOR GENERAL USE
AEG = AMPLITUDE OF BINARY GENERATOR
SCP = STANDARD DEVIATION OF PLANT DISTURBANCES
FIP = AUTOCORRELATION OF PLANT DISTURBANCES

EQUIVALENCES:

EQUIVALENCE (DUM1,AK), (DUM2,THT1), (DUM3,THT2), (DUM4,DC,REF)
EQUIVALENCE (SDUM1,SAK), (SDUM2,SHT1), (SDUM3,SHT2)
EQUIVALENCE (CDUM1,CK), (CDUM2,TI), (CDUM3,TJ), (CDUM4,UN)

INTEGER TITLE
DIMENSION U(5), TITLE(2C), 2L(5), YL(5), CSTCHE(2500)
DIMENSION YUM1(5), ZUM1(5), PO(5,5), PM1(5,5), ALF(5), ANALF(5), AD(5)
DIMENSION ALF0(5), AOD(5)

DATA IV, LS, IDL, ASTOP/ 'IV ', 'LS ', 'OPEN', 'STOP'/
DATA FNC, NUC, NUC, MGC/ 10000, 2.2, 0.6/
DATA IPLT, IPRT, N1CC0, N2/ 'PLT', 'PRT', 'D', 'C'/
DATA N1:1/N4/

DATA UIN, REF, RKG/ 0, 0, 0.1/

CALL IDNT( '1303, 818C2, ATT' )
CALL PLCT( C, -30., -3)
CALL PLUT( 0., 0.5, -3)

N = NC + NU + 1

READ INPUT DATA

**************************************************************************
100 J   READ(5,10) TITLE
     FORMAT(2,A4)
     IF(TITLE.EQ.'ASTOP') STOP
10    FORMAT(A4,A4,12,A4,A4,2X,F10.0)
303C   FORMAT(HF10.0)
30    FORMAT(10X,/F10.4)
333   FORMAT(1014)
44    FORMAT(A3,A3,A3,A3,S14,11,7F10.0)
C     READ(5,10) SDUM1,SDUM2,SDUM3,SDUM4
     READ(5,20) ALFO
     READ(5,10) D
     READ(5,10) CDUM1,CDUM2,CDUM3,CDUM4
     READ(5,30) SDUM1,CDUM2,CDUM3,CDUM4,SDUM5,SDUM6,SDUM7
CONTINUE
     READ(5,4) KPLT,KPHY,IPDP,VMA,VMI,HMA,HMI,VTIC,HTIC
     READ(5,3030) SD,F1,RDI5,Y0DD,ABG,SUP,F1P
2     CONTINUE
     READ(5,18) IPD,JCNT,M,KDP,H,SPT
C
     KMAX = HMA
     KSATCH = SDUM4
     IF(KPLT.NE.KPLT) GO TO 87
     IF(NE.EQ.N2) GO TO 84
     SET UP GRAPH DISPLAY
     CALL PLOT(8.5,0..2)
     CALL PLOT(8.5,11..2)
     CALL PLOT(0..11..2)
     CALL PLOT(0..0..2)
C
     CALL PLOT(2..4,3..5,-3)
C
     CALL PLOT(5..0,0..2)
     CALL PLOT(5..0,6..2)
     CALL PLOT(0..6..2)
     CALL PLOT(0..0..2)
C
     FX = S/(HMA-HMI)
     FY = S/(VMA-VMI)
     ZEX = A*FX
     ZEY = A*FY
     IF(VMI.GE.0) ZEY = 0
     CALL PLOT(ZEX,ZEY,3)
C PRINT \:JOJT DATA
C***************************************************************************
C
47 WRITE(6,39) TITLE
35 FORMAT(1HM1, 30X, 2CA4/)
WRITE(6,61) IOPT,KOP
61 FORMAT(1X,A4,1H METHOD, 5X, A4, 6H LOOP )
WRITE(6,30) VMA,VM1,HMA,HMI, H,SPT
WRITE(6,30) DUM1,DUM2,DUM3,DUM4,DUM5,DUM6,DUM7
WRITE(6,30) SDUM1,SDUM2,SDUM3,SDUM4
WRITE(6,30) ALFD
WRITE(6,30) DUM1,DUM2,DUM3,DUM4
WRITE(6,30) SD,F1,RO15,YODD,ABG,SDP,1P
IF (SD.EQ.0.CC) WRITE(6,36)
IF (F1.EQ.0.CC) WRITE(6,37)
36 FORMAT(5X,18HNOISIE)
37 FORMAT(5X,18HMUNCURRELATED NOISL )
WRITE(6,90) DMLEY, IRDEL, RO15,YODC,1DDEL,SDP,F1P
90 FORMAT(12X4HD = 11.4/9X,7HMDLY = 11/8X,7HIRDEL = 14/12X4HR =
2*E11,5/9X,7HYODD = E11,5/8X4HR1DDEL = 14/11X5HSPP = E11,5*11X,
) 5HP1P = ,*11,5*/
C C INITIALIZE
C***************************************************************************
C
K = 0
C
DU 7 I = 1,N
ALF(1) = ALF0(1)
YU(I) = 0
ZU(I) = 0
YUM1(I) = 0
ZUM1(I) = 0
DC 7 J = 1,N
PO(J,1) = C*
IF (J.EQ.1) PO(J,1) = 0
CONTINUE:
7 C
PM1(1,1) = B
C
YU(S) = RGK
ZU(S) = RGK
YUM1(S) = RGK
ZUM1(S) = RGK
C
CALL SETPTF(SPT,AK,THT1,THT2,DO,M,SD,F1,ANALF,SDP,F1P)
CALL SETAN( ALFO(1),ALFO(2),ALFO(3),ALFO(4),ALFO(5),K,UIN,REF)

CALL PRTPTE
CALL SETCNT(CK,TK,TC,SPT,REF)
CALL PRTCNT
WRITE(6,CC)

C

U = U1
X = C
Y = REF
Z = ALF
YU(3) = U
ZU(3) = U
YUMI(3) = U
YUMI(4) = U
DISTUR = YUWW

C

SERR = *
SGER = *
CDEL = IDDEL
RDEL = IDDEL

C

CALL ACMMTX( ANALF, ALFO, AOD, 1, N, -1)
CALL MPYMTX( AOD, AOD, AODNM, 1, N, 1)
IF(AOC(1).LT.0.01) CALL MPYMTX( ANALF, ANALF, AODNM, 1, N, 1)

C

PRDBG = 2**PRBG - 1
BIAS = -1.0/PRDBG + SORT((1.0+PRDBG)/(PRDHG*PRDBG))

C

WRITE(6,20) K,U,Y,C,Z,ALF

C

CALCULATE K=1 CONDITIONS(T-1)

C

************************************************************

K = 1
WK = K

C

X0 = X
WDSO = *
CALL PTF(U,X0,C,WDSO)
YUMI(2) = C
CSTORE(K) = C

C

#SG = C
CALL MEASUR(Y,WSG)
YUMI(1) = Y
YU(2) = Y
CALL AUXMOL(U, X, Z)
ZUM1(1) = Z
ZU(2) = Z
C
R = X
CALL GUWL(N, Y, C)
IF (KOP*Q*Q1OU) U = C*
U1 = U + X
C
IF (M * CE * 1) GO TO 390
YUM1(3) = U + X
ZUM1(3) = U + X
390
YU(4) = YUM1(3)
ZU(4) = ZUM1(3)
IF (M * KE * 1) GO TO 39
YU(3) = U + X
ZU(3) = U + X
GO TO 34
C
CALL AUXMOL(U, X, Z)
ZU(1) = Z
C
CALL AUXMOL(U, X, Z)
ZU(1) = Z
C
R = X + DIS
CALL FTFT(U, X, C, WDSQ)
CSTOR(I) = C
C
CALL MEASUR(Y, WDSQ)
YU(1) = Y
C
CALL AUXMOL(U, X, Z)
ZU(1) = Z
C
R = X + DIS
CALL CONTRL(R, Y, C)
IF (KOP*Q*Q1OL) U = C*
CALL EMSG(MLS, FGB, K)
ECS = MLS
X = AUC*(BG + BIAS)
U2 = U + X
C
IF (M * CE * 1) GO TO 35
YU(3) = U + X
ZU(3) = U + X
GO TO 3,
C
A2
CONTINUE
IF (10P.EQ.IV) CALL SETID (NC,NU,M,YU,ZU,YM1,ZUN1,PO,D,ALF,PM1,U2,
2 U1)
IF (10P.EQ.ILS) CALL SETID (NC,NU,M,YU,YU1,YM1,YM1,PO,C,ALF,PM1,U2,
2 U1)
IC = 2
C
RECURSIVE CALCULATIONS
******************************************************************************
S1
K = K + 1
IF (K.EQ.KSWTCH) CALL SETPTF (SP1,SAK,STHT1,STHT2,ALFC(5),N,SD,F1,
2 ANALF,SDP,FIP)
IF (K.EQ.KSWTCH) CALL PHITOF
RK = K
IC = IC + 1
C
CALL AUXWQ(U,X,Z)
IF (FLCAT(K/IDDEL).E0.(RK/IDDEL)) DISTUR = -DISTUR
XO = X + DISTUR
CALL PTF(U,XO,C,DSQ)
CSTORE(K) = C
CALL MEASURE(Y,WSQ)
C
IF (FLUAT(K/IRDEL).EQ.(RK/IRDEL)) RDIS = -RDIS
R = REF + RDIS
CALL CONTROL(R,Y,U)
C
IF (KOP.0.0.10L) U = 0.
CALL DMSTG(MLS,BG,K)
BG = MLS
X = ASEG(BG + UIAS)
C
IF (10P.EQ.LS) CALL ID(U,X,Y,Y,ALF,PM1)
IF (10P.EQ.IV) CALL ID(U,X,Y,Z,ALF,PM1)
ARGK = ALFC(5)*RGK
CALL UPADM(ALFC(1),ALFC(2),ALFC(3),ALFC(4),ARGK)
C
OUTPUT CALCULATIONS
******************************************************************************
39 CALL ACMTX(ANALF,ALF,AD,1,N,-1)
CALL WPYMIX(AD,AO,ADNOM,1,N,1)
SAD = S.WT(ADNRM/ADNOM)

C
ERR = AE5(C-Z)
SERR = SERR + ERR
SQR = SQF0 + ERR*ERR
EPSRM = SQER/RK
ERW = SFRP/RK

C
IF(K,L.E.10) GO TO 46
IF(KPHT,NE,IPRT AND 0,FLOAT(K/10),NE,(RK/10,1)) GO TO 89
WRITE(6,20) K, U,Y,Z,ALF(1),ALF(2),ALF(3),ALF(4),ALF(5),
2 PM1(1,1),SAD*X
FORMAT(1X,13,11E11,4,F7,3)
89 IF(SAC,NE,J,1) ALSAD = ALOG1C(SAD)
C
C PLCT OUTPUT
C***************************************************************************
IF(KPLT,NE,1PLT) GO TO 111
IF(IPCP,NE,0) YOUT = FY*(ALSAD - VMI)
IF(IPCP,NE,1) YOUT = FY*(U - VMI)
IF(IPCP,NE,2) YOUT = FY*(Y - VMI)
IF(IPCP,NE,3) YOUT = FY*(C - VMI)
IF(IPCP,NE,4) YOUT = FY*(Z - VMI)
IF(PM1(1,1),NE,0) PMIL = ALOG10(ABS(PM1(1,1)))
IF(IPCP,NE,5) YOUT = FY*(PMIL - VMI)
IF(IPCP,NE,6) YOUT = FY*(ALF(IPCP-5) - VMI)
FXRK = FX*(RK - PM1)
CALL PLCT(FXRK,YOUT,2)

C
111 IF(K=2)42,49
49 IF(IC,EO,1ICU) GO TO 47
50 IF(K,EO,KMAX) GO TO 52
GO TO 51
C
47 CALL PRINTX(PM1,N,N)
WRITE(6,60)
FORMAT(1/2X,1H,K,6X, 1HU, 9X,1HY,10X,1HC,10X,1HZ,9X,2HA1,
2 9X,2HA2, 9X,2MB1, 9X,2MB2, 8X,3HDO, 9X,3HPM1,7X,2HM6)
IC = C
GO TO 5:
C
STATISTICS ON C AND W
***************************************************************************
52 CONTINUE
WCRMS = SORT(WOSQRK)
\[ \text{rms} = \sqrt{\frac{\text{sum}}{K}} \]

\[ \text{sum} = 0 \]
\[ \text{do } i = 1, k \]
\[ \text{sum} = \text{sum} + \text{cstore}(i) \]
\[ \text{cmean} = \frac{\text{sum}}{K} \]
\[ \text{sum} = 0 \]
\[ \text{do } i = 1, k \]
\[ \text{sum} = \text{sum} + (\text{cstore}(i) - \text{cmean})^2 \]
\[ \text{crms} = \sqrt{\frac{\text{sum}}{K}} \]
\[ \text{wcrms} = \frac{\text{crms}}{\text{crms}} \]
\[ \text{write}(6, 83) \quad \text{rms}, \text{crms}, \text{cmean}, \text{wcrms}, \text{rms}, \text{rms} \]
\[ \text{format}(//4x11) \quad \text{rms} = \text{e12.3}, 4x12 \text{ signal rms} = \text{e12.3} / \]
\[ 2 \quad 4x13 \text{ signal mean} = \text{e12.3}, 4x2 \text{ rmsnoise tc signal ratio} = \text{e12.3} / \]
\[ 3 \quad 4x5 \text{ nrms} = \text{e11.4}, 4x7 \text{ hesquadrms} = \text{e11.4} \]

**PROGRAM CONTROL**

**----------------------------------------**

\[ \text{read}(5, 46) \quad \text{ne} \]
\[ \text{format}(//a1) \]
\[ \text{if}(\text{ne}.eq.2) \quad \text{go to 2} \]
\[ \text{call plot}(8, -3.5, -3) \]
\[ \text{if}(\text{ne}.eq.1000) \quad \text{go to 1000} \]
\[ \text{if}(\text{ne}.eq.201) \quad \text{go to 201} \]
\[ \text{call plot}(0, 0, 955) \]
\[ \text{end} \]
SUBROUTINE SETID(NC, NU, YM, ZU, YUM1, ZUM1, PM2, PD, ALF, PM1, U2, U1)

PURPOSE TO IDENTIFY PULSE-TRANSFER-FUNCTION MODEL PARAMETERS

NC = NUMBER OF LAGGED OUTPUT VARIABLES
NU = NUMBER OF LAGGED INPLT VARIABLES
M = MODEL DEAD TIME IN A INTEGRAL = OF SPT
UP1 = CONTROL VARIABLE T+1
XP1 = TEST INPUT T+1
YP1 = MEASUREMENT OF OUTPUT VARIABLE T+1
ZP1 = AUXILIARY MODEL OUTPUT T+1
YU = ((NC+NU+1)*1) EXPLANATORY VECTOR T
ZU = ((NC+NU+1)*1) INSTRUMENTAL VARIABLE T
PM1 = (NC+NU+1)**2 COVARIANCE MATRIX T-1
PM2 = (NC+NU+1)**2 COVARIANCE MATRIX T-2
ALF = (1*(NC+NU+1)) VECTOR OF MODEL PARAMETERS T+1

DIMENSION ALF(5), PM1(5,5), PM2(5,5), YU(5), YUM1(5), ZU(5), ZUM1(5)
DIMENSION AM(5), PM(5,5), PV(5,5), ZTPM1(5), UD(20), PM1Y(5), PM2Y(5)
DIMENSION PD(5)

KC = 1
N = NC + NU + 1
M1 = M + 1
NM1 = N - 1
NCPI = NC + 1
CALL PRMTX(PM2, NC)
DO 702 1 = 1, 20

702 UD(1) = YUM1(1)
UD(M1) = U2
IF(M.CT.0) UD(M) = U1
WRITE(6, 500) YUM1
WRITE(6, 500) YU

CALL MPYMTX(PM2, YUM1, PM2Y, NC, NU, 1)
CALL MPYMTX(ZUM1, PM2Y, CM2, 1, NU, 1)
PM2 = 1/(1. + CM2)

CALCULATE 'g'(T-1)
IFLAG = 1
GO TO 21
IFLAG = 0
RETURN

ENTRY I(U1P1, X1P1, YP1, ZP1, ALF, PM1)
KC = KC + 1
UX = UP1 + XP1

CALL MPYMTX(PM1, YU, DM1, N, N, 1)
CALL MPYMTX(ZU, DM1, N, N, 1)

X = 1/(1 + DM1)
IF(KC > N) GO TO 501
CALL PRMTX(PM1, N, N)
WRITE(6, 501) YU
WRITE(6, 500) ZU, CM1

500 FORMAT(10X, E12.4)

UPDATE ALPHA(T+1) WITH Y(T+1), YL(T), ZL(T), AND P(T-1)

501 CALL MPYMTX(ZU, DM1, N, N)
CALL MPYMTX(ALF, YU, ALFY, 1, N, 1)
AV = CM1*(YP1 - ALFY)
CALL MPYSCA(AV, ZTPM1, AM, 1, N)
CALL ADMTX(ALF, AM, ALF, 1, N, 1)

UPDATE YU(T-1) WITH YL(T)

502 CALL DFNNMTX(YU, YUM1, N, 1)
CALL CFNMNTX(ZU, ZUM1, N, 1)

UPDATE P(T-2), YU(T-1), P(T-2) AND P(T-2)

DM2 = DM1
CALL CFNMNTX(PM1, PM2, N, 1)
CALL DFNMNTX(PM1, PM2, N, N)

UPDATE P(T-1) WITH YU(T-1)
CALL MPYMTX(PM2Y,ZUM1,PMV,N1,N)
CALL MPYMTX(PY,PM2,PMV,N1,N)
CALL MPYMTX(UM2,PM2,PMV,N1,N)
CALL MPYMTX(PM2,PM1,PMV,N1,1)
DO 90 I = 1,N
40 PM1(I,1) = PM1(I,1) + UD(I)
IF( IFLAG.EQ.1 ) GO TO 21
C
FORM YU(T) AND ZU(T) WITH Y(T+1), U(T-M+1) AND Z(T+1)

C

O 1 I = 1,NM1
K = NCP1 - I
IF(I.LT.NC) YU(K) = YU(K-1)
IF(I.LT.NC) ZU(K) = ZU(K-1)
K = N - I
IF(I.LT.NU) YU(K) = YU(K-1)
IF(I.LT.NU) ZU(K) = ZU(K-1)
1 CONTINUE
YU(1) = YP1
ZU(1) = ZP1
C
IF(M.EQ.C) GO TO 27
DO 2 I = 1,M
2 U(1) = UD(1+I)
UD(M1) = UX
26 YU(NCP1) = UD(1)
ZU(NCP1) = UD(1)
GO TO 43
C
27 UD(1) = UX
GO TO 26
C
43 CONTINUE
RETURN
END
SUBROUTINE SETPTF(SPT,AK,THT1,THT2,UN,W,SD,F1,ANALF,SCC,F1P)

SECOND ORDER OVERDAMPED SYSTEM

\[ A = \frac{1}{(S + THT1)(S + THT2)} \]

SPT = SAMPLE TIME
AK = PROCESS GAIN
THT1 = TIME CONSTANT 1
THT2 = TIME CONSTANT 2
DC = STEADY STATE VALUE OF C FOR ZERO C
M = PLANT DEAD TIME IN INTEGER MULTIPLES OF SPT
SD = STANDARD DEVIATION OF RANDOM NOISE
F1 = CORRELATED DISTURBANCE COEFFICIENT
C = SYSTEM OUTPUT * T
Y = M-ADULT SYSTEM OUTPUT * T
UM1 = SYSTEM INPUT * T-1
XM1 = TEST INPUT * T-1
ANALF = ANALYTICAL PLANT COEFFICIENTS
SC = SUM OF SQUARES OF NOISE W

DIMENSION U(10),ANALF(5)

E1 = EXP(-THT1*SPT)
E2 = EXP(-THT2*SPT)
A1 = E1 + E2
A2 = -E1*E2

IF(THT1.EQ.THT2) GO TO 100
PK = AK/(THT1*THT2*(THT1-THT2))
B1 = PK*(THT1*(1.*-E2)-THT2*(1.*-E1))
P2 = PK*(E2*THT2*((1.*-E1)-E1*THT1*(1.*-E2))
GO TO 101

100
PK = AK/(THT1*THT2)
B1 = PK*(1.*-E1*(1.*+THT1*SPT))
B2 = PK*E1*(E1 - 1.*+THT1*SPT)

101
ANALF(1) = A1
ANALF(2) = A2
ANALF(3) = B1
ANALF(4) = B2

D = CD*(1.*-A1-A2)
ANALF(5) = C
CM2 = C;
CM1 = D1;

156
DC 30 I = 1, 10
.U(1) = .**.0
W1 = W+1
W2 = W+2
W3 = W+3

C
IX = 77
CALL GAUSS(I, SD, 0, V)
WM1 = V
IXP = 7.869
CALL GAUSS(I, SD, V, WCIC)
WCM1 = WCIC
RETURN

C
ENTRY PTF(WM1, XM1, C, WSQ)
UX = WM1 + XM1
DO 20 I = 1, M1
20 U(M3-I) = U(M2-I)
U(1) = UX
CALL GAUSS(I, SD, C, WD)
WD = F1*WD11 + WD
WM1 = WD
WSC = WSQ + WD

C
CM2 = CM1
CM1 = C
RETURN

C
ENTRY M-ASU*(Y, *SQ)
CALL GAUSS(I, SD, V, W)

C
W = F1*WM1 + V
WM1 = W
WSC = W, W*

C
Y = C + W
RETURN

C
ENTRY P-PTF
WRITE(6, 10) SPT, AK, THT1, THT2, DO, W, SD, F1, A1, A2, B1, B2, D, CM2, U(M2)
10 FORMAT(105HSP = E12.5/11X*AK = E12.5/9X6HTHT1 = E12.5/9X6HTHT2
20 E12.5/11X4HDO = E12.5/12X3HM = E12.5/12/11X4HSD = E12.5/
111X4HF1 = E12.5/
4 11X4HA1 = E12.5/11X4HA2 = E12.5/11X4HA1 = E12.5/11X4HA2 = E12.5/
5 12X3HD = E12.5/11X4HD0 = E12.5/11X4HD0 = E12.5/
SUBROUTINE SETAM(A1, A2, B1, B2, D, M, UM, Z)

C
C AUXILIARY MODEL
C
C
DIMENSION UM(10)
ZM2 = ZL
ZM1 = ZC
DO 20 I = 1, 10

30 U(I) = LD
M1 = M+1
M2 = M+2
M3 = M+3
RETURN

C
C ENTRY UPDAM(A1, A2, B1, B2, D)
RETURN

C
C ENTRY AUXM0L(UM1, XM1, Z)
UX = UM1 + XM1
DO 20 I = 1, M1

20 U(M3-I) = U(M2-I)
U(1) = UX
Z = A1*ZM1 + A2*ZM2 + B1*U(M1) + B2*U(M2) + D
ZM2 = ZM1
ZM1 = Z
RETURN
END
SUBROUTINE SETCNT(CK, TI, TD, SPT, Y(0))

PIC CONTROLLER

ck = steady state gain
Ti = integral time
td = derivative time
SPT = sample time
w = set point
Y = measured system output

if(TI.EQ.0.) TI = 99999999.
ST1 = SPT/TI
TDS = Ti/SPT
TII = 1. + ST1
TID1 = TDS + TII

RM1 = YC
YM1 = YC
YM2 = YC
RETURN

ENTRY CONTRL(R,Y,U)
DU = CK*TIII*R - RM1 - TID1*Y + (1.*2.*TDS)*YM1 - TDS*YM2)
U = U + DU
YM2 = YM1
YM1 = Y
RM1 = R
RETURN

ENTRY PHTCNT
*WHITE(6,10) SPT, TI, TD, CK
10 FORMAT(/10X$HSPT = ,E12.5/11X$HT1 = ,E12.5/11X$HTD = ,E12.5/11X$HCK
2 = E12.5/)
RETURN
END
**SUBROUTINE AMLSGLR,M,IDX**

**PURPOSE TO GENERATE A PRACTICAL MAXIMUM LENGTH SEQUENCE**

LR = GENERATOR OUTPUT (+1 OR -1)

M = NUMBER OF STAGES (3 TO 10)

PERIOD OF MAX. LENGTH SEQUENCE = (2**M - 1)

**DIMENSION L(10),LO(10)**

DATA LD/0,1,0,1,0,0,0,0,1,0/

IF(IDX.GT.2) GO TO 2

DO 30 I = 1,10

L(I) = LD(I)

30 CONTINUE

IF(M.EQ.8) GO TO 60

IF(M.EQ.9) LR = XOR(L(4),L(9))

IF(M.EQ.5) LR = XOR(L(2),L(M))

IF(M.EQ.10) LR = XOR(L(3),L(M))

IF(M.NE.5.OR.M.NE.10) LR = XOR(L(1),L(M))

MM1 = M-1

DO 10 I = 1,MM1

J = M-1

10 L(J+1) = L(J)

L(1) = LR

LR = -1 + 2*LR

RETURN

60 LR = XOR(L(6),L(7))

LR = XOR(LR,L(2))

LM = XOR(LR,L(1))

END
FUNCTION XOR(M,N)
  IF(M = N) XOR = 0
  IF(M NE N) XOR = 1
  RETURN XOR
END
SUBROUTINE GAUSS(IX,S,H,M,V)
A = 0.
DO 50 I = 1,12
CALL HANDU(IX,Y,Y)
IX = IY
A = A + Y
V = (A - 6.) * S + AM
RETURN
END
SUBROUTINE RANCUI(IX, IY, YFL)

IV = IX*65539
IF(IY)5,6,6
IY = IY*2147483647+1
YFL = IY
YFL = YFL*1.0E-9
RETURN
END
SUBROUTINE RDMTX(A,NR,NC)
C PURPOSE TO READ FROM CARDS BY READING (BE10.0) FIELDS
DIMENSION A(NR,NC)
READ(5,1)((A(I,J),J=1,NC),I=1,NP)
1 FORMAT(6E10.0)
RETURN
ENTRY RDMTX(A,NR,NC)
C PURPOSE TO PRINT THE ELEMENTS OF MATRIX A BY ROWS
WRITE(6,2)(J,J=1,NC)
2 FORMAT(/*10COL *1,2X,10(110,*2x))
DO 3 J=1,NR
3 WRITE(6,4)I,(A(I,J),J=1,NC)
4 FORMAT(*ORIGIN,13,1CE12.5/(7X,10,12.5))
RETURN
END
SUBROUTINE DFNMIX(A,B,NR,NC)

PURPOSE TO DEFINE MATRIX B AS MATRIX A OF NR ROWS AND NC COLUMNS

DIMENSION A(NR,NC),B(NR,NC)
DO 1 I=1,NR
  DO 1 J=1,NC
    B(I,J)=A(I,J)
  1 CONTINUE
RETURN
END
SUBROUTINE ADMX(A,B,C,NR,NC,IFG)

PURPOSE TO COMPUTE MATRIX C OF NR ROWS AND NC COLUMNS

IF (IFG .LT. C) G0T0 10

DO 1 J=1,NC

1 C(I,J) = A(I,J) + B(I,J)

RETURN

IF (IFG .GE. C) G0T0 20

DO 2 J=1,NC

2 C(I,J) = A(I,J) - B(I,J)

RETURN

END
SUBROUTINE MPYMTX(A, B, C, NR, M, NC)

PURPOSE TO MULTIPLY MATRIX A(NR, M) TIMES MATRIX B(M, NC)

PRODUCT MATRIX C(NR, NC)

DIMENSION A(NR, M), B(M, NC), C(NR, NC)

DO 1 I = 1, NR
    DO 1 J = 1, NC
        SUM = 0.
        DO 2 K = 1, M
            SUM = SUM + A(I, K)*B(K, J)
        2 C(I, J) = SUM
    1 RETURN

C

SUBROUTINE MPYSCA(FACTOR, A, B, NR, NC)

PURPOSE TO FORM MATRIX B OF NR ROWS AND NC COLUMNS BY MULTIPLYING
MATRIX A BY A SCALAR FACTOR

DIMENSION A(NR,NC), B(NR,NC)
DO 1 I=1, NR
DO 1 J=1, NC
1 B(I,J) = FACTOR * A(I,J)
RETURN
END
SUBROUTINE SETPTF(SPT, TRI, FI, AMC, DC, M, SD, F1, AL, SF, FIP)
C
C CHEMICAL REACTOR ------- TO BE SUBSTITUTED FOR THE SECOND-ORDER
C SYSTEM SUBROUTINE WHEN REACTOR IS WANTED.
C
C DIMENSION ANALF(5)
DATA (Chi, Fh, A, CP, UA, AKO) /87, 250., 60., 2560., 3, 630., 1.43/
DATA CT/0.057/
DATA CAMI, TDI, TWI/9., 150., 8C./
C
F = FI
IF(SOP .NE. C) RETURN
TR = TRI
CAO = CAMI
TC = TDI
T = TWI
C
EXPTRM = EXP(-A/(TR+46C.))
RCp = RP*CP
CA1 = (-F/V + SQRT((F/V)*(F/V) + 4.*((F/V)*CA*AKO*EXPTRM)))/(2.*AKO*EXPTRM)
CA = CA1
CKEXP = CA*AKO*EXPTRM
TCI = TR-((V*CP/CA)*((F/V)*(TC-TR) + DH*KE*PCA/RCP)
TC = TCI
WC = UA*(TC-TCI)/(TC - T)
WC = WC
WRITE(6,SO0) CA, TC, WC
S00 FORMAT(12X,'CA = ',F7.4,3X,'TC = ',F7.3,3X,'WC = ',F7.2)
C
IX = 7/
CALL CAUS((IX,SD,0.,VVT)
WM1 = XM1
RETURN
C**********************************************
ENTRY PIF(UM1, XM1, C, WSQ)
WC = UM1 + XM1 + WM1
T = 0.
DC 10 1 = 1, 201
T = T + DT
IF(T.GT.SPT) GO TO 20
FOV = F/V
ET = EXP(-A/(TR+46C.))*CA*CA
CA = CA + CT*(FOV*(CAO - CA) - AKO*ET)
TR = TR + DT*(FOV*(TO - TR) - (CA/(V*RHO*CP)))*(TR-TC) + (DH/(RHO*
2 (*P)*AK(J*ET)
    TC = TC + LT*(((UA/AMC) / TR-TC) - (WC/AMC) * (TC - TA))
10 CONTINUE
20 C = TR - TH
    RETURN

C************************************************************
ENTRY MFASUM(Y,WSQ)
    CALL CAUSS(I X,5D,C,W,VT)
    # = F**M1 + WVT
    WSQ = WSQ + ***
    Y = C + #
    RETURN

C******************************************************************************
ENTRY PHTPF
    WRITE(6,3C)SPT,M,SD,F1
30 FORMAT(10X9H3P T,E12.3/12X7HM =,12/11X4HSD = ,E12.5/11X4HF1 = ,
2 E12.5/)
    RETURN
    END
VITA

A. Terrell Touchstone is the son of Mr. and Mrs. Armand A. Touchstone of Huntsville, Alabama. He was born July 22, 1946 in Laurel, Mississippi, but he considers himself a native of Huntsville, Alabama where he graduated from Huntsville High School in June, 1964.

Beginning in the fall of 1964, he attended the University of Alabama, and received his Bachelor of Science Degree in Chemical Engineering from that institution in January 1969. He was awarded a Master of Science Degree in Chemical Engineering in May 1970. From June 1970 to August 1972, he was an officer in the United States Army stationed at Redstone Arsenal, Alabama where he served as an engineer in the Research, Development, Engineering and Missile Systems Laboratory of the U. S. Army Missile Command.

On January 25, 1969 he married the former Diane Larkins of Decatur, Alabama. They are the parents of one daughter - Jennifer Anne, age 4.

He is presently a candidate for the degree of Doctor of Philosophy in Chemical Engineering.
EXAMINATION AND THESIS REPORT

Candidate: A. Terrell Touchstone

Major Field: Chemical Engineering

Title of Thesis: Instrumental-Variable Estimation of Discrete-Time Model Parameters for Adaptive Control

Approved.

[Signatures]

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

July 15, 1975