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A Metaheuristic-Based Simulation Optimization Framework For Supply Chain Inventory Management Under Uncertainty

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A METAHEURISTIC-BASED SIMULATION OPTIMIZATION FRAMEWORK FOR SUPPLY CHAIN INVENTORY MANAGEMENT UNDER UNCERTAINTY

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
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by

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ABSTRACT

The need for inventory control models for practical real-world applications is growing with the global expansion of supply chains. The widely used traditional optimization procedures usually require an explicit mathematical model formulated based on some assumptions. The validity of such models and approaches for real world applications depend greatly upon whether the assumptions made match closely with the reality. The use of meta-heuristics, as opposed to a traditional method, does not require such assumptions and has allowed more realistic modeling of the inventory control system and its solution.

In this dissertation, a metaheuristic-based simulation optimization framework is developed for supply chain inventory management under uncertainty. In the proposed framework, any effective meta-heuristic can be employed to serve as the optimizer to intelligently search the solution space, using an appropriate simulation inventory model as the evaluation module. To be realistic and practical, the proposed framework supports inventory decision-making under supply-side and demand-side uncertainty in a supply chain. The supply-side uncertainty specifically considered includes quality imperfection. As far as demand-side uncertainty is concerned, the new framework does not make any assumption on demand distribution and can process any demand time series. This salient feature enables users to have the flexibility to evaluate data of practical relevance. In addition, other realistic factors, such as capacity constraints, limited shelf life of products and type-compatible substitutions are also considered and studied by the new framework.
The proposed framework has been applied to single-vendor multi-buyer supply chains with the single vendor facing the direct impact of quality deviation and capacity constraint from its supplier and the buyers facing demand uncertainty. In addition, it has been extended to the supply chain inventory management of highly perishable products. Blood products with limited shelf life and ABO compatibility have been examined in detail. It is expected that the proposed framework can be easily adapted to different supply chain systems, including healthcare organizations.

Computational results have shown that the proposed framework can effectively assess the impacts of different realistic factors on the performance of a supply chain from different angles, and to determine the optimal inventory policies accordingly.
CHAPTER 1 INTRODUCTION

1.1 Supply chain inventory optimization

A supply chain (SC) can be expressed as the sum of parties involved in fulfilling a customer demand (Chopra and Meindl, 2007). A supply chain comprises a network of partnering companies (or business units in the same company) that belongs to an industry vertically embedded in a business environment. The purpose of supply chain design and analysis is to maximize an organization’s profit or minimize the cost in the process of providing the product to the customer. This research focus on inventory management, a very important aspect of the overall supply chain management. It plays a key role in the functioning of industrial and commercial enterprises. A desirable inventory control is one that will guarantee a satisfactory service level without keeping too much unnecessary stocks that are costly and difficult to handle.

However, almost every inventory system contains uncertainty. There are various possible uncertainties such as uncertainty in demand, in lead time, in costs and in supplied quality and capacity. Supply chains are subjected to internal uncertainties resulting from the interaction between firms within the supply chain and to external uncertainties that are faced by all supply chain networks in the industry. Dealing with uncertainty is an important issue in the supply chain modeling and in analysis of supply chain behavior and performance. Inventory management decisions have to take these uncertainties into consideration to be realistic. Furthermore, it is very important to capture the dynamic behavior of most real-world applications. The term "supply chain uncertainty" is often used interchangeably in practice with the term "supply-chain
risk” (Peck 2006, Ritchie and Brindley 2007) and there is an increasing global attention in supply chain risk management in recent years. It is very important because it has a great influence on the stability of dynamic cooperation among SC partners and it is directly related to the performance of the SC inventory management as a whole. A robust decision-making model is the cornerstone for the efficiency of supply chain management. According to Friesz (2011), financial viability and financial sustainability of any supply chain are dependent upon an integrated web of the means of supply of factor inputs, the inventory of those factors, the inventory of finished goods, and the distribution of finished goods.

In this research, both demand-side uncertainty and supply-side uncertainty that are pervading in industry are investigated:

- Demand fluctuation
- Uncertain product quality

Besides, other realistic factors studied in this research include:

- Limited capacity constraints
- Limited life time of highly perishable products
- ABO/Rh(D) compatibility of blood products

1.2 Justification

Evaluation and optimization of inventory policy is a typical inventory research problem. Various inventory models have been developed for tackling this problem. However, even though almost all companies and enterprises are increasingly trying to apply scientific methods for better
handling their inventories, the use of those methods is often limited to some basic tools such as
the computation of economic order quantities and rough approximations of safety stock. The
wide use of more elaborate and appropriate methods for inventory management in practice is
hindered by the following most notable reasons:

The traditional approaches on inventory control mostly are based on the assumption that
lead time demand follows a certain type of distribution. This is often not the case in real practice.
It is not unexpected that real world data simply does not fit perfectly to demand distributions
assumed by those models. Using those traditional approaches as approximations in cases where
real world data significantly deviates from the assumed distribution can lead to very
unsatisfactory results.

The problem gets more complicated when dealing with stochastic demand. It is found
through previous literature that stochastic theoretical analysis leads to tractable expressions only
under specific assumption. Furthermore, most theoretical models assume that the demand data
are independently and identically distributed (i.i.d.) while some real-world data may be
auto-correlated. Failing to account for the auto-correlation between demand data can also cause
serious inaccuracy in calculating inventory level.

Since the feasibility of a theoretical model depends on whether or not it is mathematically
tractable for the subject cost function under the distribution of lead-time demand, it is almost
impossible to develop a theoretical model that covers more than one type of distribution. Besides,
when considering real world application, “in a context where the optimization must be carried
out relatively frequently for many thousands of items the computational effort can be regarded as
too heavy”, quoted from Axsäter (2006).

On the other hand, uncertainty affects the supply as well. Supply uncertainty, a particularly
important aspect of supply chain risk should also receive a great deal of attention in order for any
inventory decision making tools to be practical. “Supplier reliability” may refer to a number of
attributes ranging from the availability of the responding resource to fulfill orders, to variability
in delivery lead time as well as the quality of each replenishment order. Inventory managers are
often faced with the challenge of incorporating the issue of supplier’s reliability into their
management decisions since it is negatively impacted by a variety of real-world factors such as
material shortages, capacity constraints, and equipment deviations. Silver (1981) appears to be
one of the authors in the early days to indicate the need for models dealing with supply
uncertainty. Moinzadeh and Nahmias (1988) also discuss the importance of incorporating this
type of uncertainty in inventory models.

Based on those observations mentioned above, an innovative global computational
framework is proposed in this Ph.D. research for the optimization of inventory management
policies under uncertainty. The new global computational framework comprises of two major
components: a meta-heuristic optimizer and an event-driven simulation model (without the need
of any explicit mathematical function). This new framework, hence, can be more specifically
called a Metaheuristic-based Simulation Optimization (MSO) approach. The proposed
framework of optimizing inventory will be developed for the optimization of supply chain
inventory models. The proposed framework once developed will offer unique features that existing theoretical/analytical models are lack of. It is, to the best of my knowledge, the first quantitative multi-period operational level supply chain model/system that deals with both supply-side and demand-side uncertainties. In addition, it is extended to optimize the supply chain for highly perishable products and blood products with ABO compatibility. It is a totally distribution-free intelligent search technique. It will be able to find near-optimal inventory policies regardless the demand distribution type and the dependence among data points. The cost function is not explicit and does not need to be convex and mathematically tractable. This makes the proposed framework advantageous in that it can handle various types of real world demand data without the need to derive new models.

1.3 Project Goal

This study is intended to establish a metaheuristic-based simulation optimization (MSO) framework for the evaluation and optimization of supply chain inventory policies under customer and supplier uncertainties. The MSO framework is designed to be able to handle a wide scope of demand patterns including deterministic, stochastic, and time series. The main motivation for this work is to offer a new tool for advancing research in supply chain inventory management and to bridge the gap between theory and practice so as to deliver a user-friendly and flexible computational approach for rationalizing the considered inventory control system. The proposed approach unifies many theoretical models into one and is expected to be a totally new
high-performance approach for inventory optimization and hopefully will revolutionize the field of inventory research.

Fig. 1.1 illustrates the outline of this dissertation study. In the first phase, the research topic is explored and the related literature is reviewed. The results of the first phase are documented in Chapters 1 and 2. The main part of this dissertation study is roughly divided into five subparts, as indicated in Fig. 1.1. Chapter 3 gives a general overview of the proposed MSO framework. Chapter 4 emphasizes on the development of an up-to-date hybrid meta-heuristic optimizer for the proposed framework. Chapter 5 presents the application of the proposed framework to the optimization of a capacitated supply chain inventory problem and its managerial implications. Chapter 6 further extends the application of the proposed framework to the optimization of a supply chain system with imperfect quality products. A new bounding techniques for determining the solution quality is proposed by integrating the MSO approach with sample average approximation method (Kleywegt et al., 2001). Chapter 7 utilized the newly developed combined procedure to the optimization of highly perishable production and inventory supply chains. Chapter 8 further examines the blood supply chain with ABO compatibility. Lastly, we conclude this dissertation with possible future studies in Chapter 9.

By fulfilling the above mentioned tasks and objectives, this dissertation research is expected to offer the following key contributions:

(1) Due to the fact that the MSO inventory optimization framework relies basically on simulation optimization approach and the dimension of problem can grow rapidly as the supply
chain network expends, the first contribution is to develop a highly effective and efficient meta-heuristic algorithm for the overall optimization framework. The hybrid meta-heuristic algorithm developed is called “DE-HS-HJ” because it is comprised of two cooperative meta-heuristic algorithms (Differential evolution and Harmony search) and one effective local search (LS) method (Hooke and Jeeves direct search method). The best timing of when to apply the LS method is carefully compared and selected from 18 specific local search application strategies. (Duan et al., 2013)

(2) The second contribution is to develop a multi-period operational level supply chain inventory decision support system in consideration of suppliers’ capacity constraints. The MSO framework is first applied to deterministic capacitated inventory systems facing various demand patterns. The capacity constraint comes from the supplier’s side and four levels of capacity tightness are investigated through ten different demand patterns, under centralized and decentralized control. The input demand data can be either the raw demand data collected from sales or predicted demand data generated from any demand forecasting method. Based on the realization of the input data, the proposed framework is capable of determining near-optimal replenishment policies for each player under each subject setting. The main contribution of this study is three-fold: 1) to be the first that develops both centralized and decentralized models for a capacitated inventory system focusing on the retailers' side; 2) to investigate the interactions between capacity constraint, control strategy, and demand; and 3) to evaluate the performance of
the supply chain in more details from different angles to provide some useful managerial insights. (Duan and Liao, 2013a)

(3) The third contribution lies in extending the MSO framework to stochastic settings. A new methodology, stochastic MSO, that integrates the deterministic MSO framework with sample average approximation method (Kleywegt et al., 2001) has been developed. The combined procedure is demonstrated to find near-optimal inventory policies for an integrated SC model in consideration of quality imperfection in the products supplied. The quality level of each lot from the supplier is assumed to follow some discrete/continuous distribution. Portion of each lot that meets the specification is used to fulfill customer demand directly; the remaining defective portion is remanufactured at additional cost and delivered in the next lead time period. The consideration of supplier quality imperfection makes the subject supply chain inventory problem a stochastic optimization problem. The combined procedure helps to locate a high quality near-optimal solution and the solution quality is bounded by constructing confidence intervals on the corresponding “optimality gap”. (Duan and Liao, under review)

(4) The next contribution comes from tackling highly perishable production and inventory supply chains by proposing new age-based replenishment policies. The stochastic MSO framework is utilized to determine the optimal production and inventory levels for a platelets supply chain as a numerical example. Platelets (PLTs) are highly perishable, usually with a restricted shelf life of 5 days. Moreover, demand for PLTs is quite unpredictable and fluctuating. The efficient use of PLTs is critical to healthcare services. Supply of PLTs needs to be closely
matched with the stochastic transfusion demand by accounting for the numerous possible inventory states (number and age distribution of stocked items). The SC system considered in the numerical example consists of a blood center supplying PLTs to several hospital blood banks (with one major medical center and smaller peripheral hospitals). While the supply is voluntary and costly, and shortage may put lives at risk, the inventory objective is to minimize the expected system outdate rate under a predetermined maximal allowable shortage level. The new Old Inventory Ratio (OIR) policy is compared with two existing order-up-to policies: one is the order-up-to policy without age consideration; the other one is the "EWA" policy developed by Broekmeulen and van Donselaar (2009). The three policies are compared under both decentralized and centralized controls for different levels of the fill-rate constraint. The computational results show that adopting centralized control over the whole platelet supply chain greatly helps reducing the system expected outdate rate from 19.6% down to 1.04% on average while keeping sufficiently high fill rate at each entity. The two policies considering item ages are generally better than the policy without age consideration under both control strategies. This is particularly true for decentralized control. The new OIR policy is recommended because it is the best among all three and consistently yields good results in all cases studied. (Duan and Liao, 2013b)

(5) Last but not least, the proposed framework is used to optimize blood supply chain inventory management with ABO blood group compatibility. In clinical practice, red blood cells (RBCs) have a maximal shelf life (MSL) of 42 days. Recent studies suggest managing RBC
inventory with a more restrictive MSL (Frank et al., 2013 and Koch et al., 2008). To this end, the proposed SO framework is incorporated with a new metaheuristic optimization algorithm, TA-TS, to identify near-optimal inventory policies more quickly (in reasonably acceptable computational time). The inventory objective is to minimize the expected system outdate rate under a predetermined maximal allowable shortage level. The new SO framework is shown to offer a better tradeoff between solution accuracy and computational expenses. The efficiency of the proposed framework is evaluated in detail for RBC units with shortened shelf lives of 7, 14 and 21 days. The OIR policy in Duan and Liao (2013b) is used to control the freshness of the entire inventory. Three different operating scenarios are investigated: 1) no ABO compatible substitution; 2) ABO compatible substitution only at hospital; 3) ABO compatible substitution at both hospital and blood center. The proposed framework is capable of identifying near-optimal solution for each of these scenarios and quantify the potential savings offered by ABO compatible substitution. Allowing ABO/Rh(D)-compatible blood substitution will help to reduce the system-wide outdate at least by 16% even under the most restrictive MSL. For more relaxing MSL of 14 days and 21 days, the proposed framework is capable of keeping the highest system-wide outdate rate at as little as 2%. (Duan and Liao, 1st revision)
Figure 1.1 Outline of the dissertation research
CHAPTER 2 LITERATURE REVIEW

This chapter reports on the general literature review that has been carried out to explore the research topic. The scientific literature on determining optimal inventory management policy is huge and it is impossible to provide a comprehensive overview of all literature. Therefore, we will focus on those publications motivated by practical applications and are directly related to this dissertation topic.

2.1 Traditional approach for supply chain inventory management

When it comes to supply chain inventory management, two types of optimization have been defined: local optimization and global optimization. Local optimization of each member does not guarantee optimization of the total supply chain. In global supply chain optimization, all members of a supply chain attempt to maximize a chain’s revenue or minimize the cost of a chain in close collaboration with each other. Global optimization is very complex because of the large numbers of the supply chain’s members and difficulties in their relationships.

Co-ordination models have been presented to optimize the benefits of all the members and alignment of decisions between entities of a supply chain. Those models focus on search for efficient replenishment policies for all locations in the system. Goyal (1977) is credited to be the first researcher to describe integrated models of single-vendor single-buyer. Goyal (1977) proposed a joint expected lot size model to minimize total relevant costs, which is compared with total costs incurred if vendor and buyer acts independently. Many researchers followed this
research direction and use analytical modeling technique in their studies, such as Hill (1997), Chen (1999), Axsäter and Zhang (1999), Hoque and Goyal (2000), Hill and Omar (2006).

Since the dissertation work will finally move up to single vendor multiple buyer system, an overview of single-vendor multi-buyer coordinated inventory analytical models is reported in this section. Axsäter (2000) considered continuous review installation stock (R, Q) policies for one central warehouse and N-retailers system under independent compound Poisson demand. He presented a method for exact evaluation of control policies that provides the complete probability distributions of the retailer inventory levels. Axsäter (2001) considered the similar 2-echelon distribution inventory system but suggest and evaluate an approximate method for optimization of a two-echelon inventory system. Zhao et al. (2002) presents a study on the impact of forecasting method selection on the value of information sharing in a supply chain with one capacitated supplier and multiple retailers. They use a computer simulation model to examine the effect of five demand forecasting methods on the supply chain performance under four types of demand patterns and three level of supplier capacity tightness. Axsäter (2003) assumes that the system is controlled by continuous review installation stock (R,Q) policies with given batch quantities and presents a simple technique for approximate optimization of the reorder points.

Due to the importance of these systems pervading the business world, many researchers have studied their operating characteristics under a variety of conditions and assumptions. Zavanella and Zanoni (2009) consider a one-vendor multi-buyer system under consignment stock case. Geng et al., (2010) consider a single-distributor multi-retailer inventory system under six
operating scenarios for information sharing. Five scenarios are in decentralized control and the other one is in centralized control. Approximate dynamic programming procedures are developed to obtain system performances for scenarios in decentralized control and a stochastic dynamic programming approach is used for the scenario in centralized control. System performances are investigated and compared for different operating scenarios using the centralized control as a baseline.

The major limitation of the traditional analytical modeling technique lies in that the operating characteristics of a supply chain can only be studied under certain conditions and assumptions. This is mainly due to the complexity of the model will grow rapidly with the increase of the dimensions of the problem state. Most researchers have tried to set some constraints so that the objective function of the model becomes mathematically trackable and can be solved by some mathematical approaches like the lagrangian, the derivative, or markov chains methods. However, since most integrated supply chain model is integer-nonlinear in nature, reaching an analytical solution (if any) to the considered problem is difficult. In addition, efficient treatment of integer nonlinear optimization is one of the most difficult problems in practical optimization. This explains why more and more recent SC research utilizes simulation approach as an alternative to evaluate given solutions/policies (Fu and Healy, 1997, Bollapragada et al., 1998, Bashyam and Fu 1998) and realize robustness studies (Tee and Rossetti 2002).

Tee and Rossetti (2002) studied the one warehouse multiple retailer system employing (R, Q) inventory policies using simulation. The simulation model was developed to explore the
robustness of a standard model specifically the models discussed in Axsäter (2000) under conditions that violate the model’s fundamental modeling assumptions. Their testing of the model indicated that care must be taken when applying the standard model to situations that violate its fundamental assumption. Practitioners should better understand those assumptions of these models so that they can determine when or when not to apply those models in practice.

As stated earlier, most analytical models have the overall drawback that they can only represent very limited practical situations. Alternatively, many previous supply chain research uses simulation to verify the results obtained from exact/approximate analytical modeling; or to quantify bullwhip effect in certain cases, as in Dejonckheere et al., (2003, 2004), Chandra and Grabis (2005), Lau et al., (2008), Yu et al., (2010) and Chaharsooghi and Heydari (2010).

Simulation generates numerical and logical models based on real-world problems, and imitates different scenarios using computers to find solutions for problems. For complex problems with high risks or that are impossible for real-world testing, computer simulation technology provides an effective tool to help plan for solving, analyzing and evaluating different alternatives. However, simulation method itself is not an optimization method. All possible solutions need to be fed into the simulation system to compute the best parameters setting and resources combination. Not only is this method very time-consuming and ineffective, but it is also costly. Moreover, brute force cannot be applied to realistic problems; for example, in practice the inventory systems control thousands of Stock Keeping Units (SKUs), so the optimal control levels must be estimated for all these SKUs: this is called ‘the curse of dimensionality’.
Furthermore, the inventory system may be only a subsystem of a multi-echelon inventory system (including central warehouses), a production-inventory system (some SKUs are not purchased but are manufactured by the same company), a supply chain, etc. Therefore this dissertation research is meant to illustrate how in practice optimization of realistic inventory systems may be done.

2.2 Simulation Optimization

In recent years, many researchers and practitioners have purposed that by integrating optimization algorithms, computer simulation can determine the best combination more rapidly and more efficiently. Simulation optimization has attracted considerable attention of both academia and industry. To be more specifically, it is since Fu (2002) that the use of combining simulation model with optimization algorithms has caught wide attention. This integration allows a more advanced searching process for the best combination of decision variables based on the output of a simulation model of the system. In those related studies, meta-heuristics serve as a powerful optimizer due to the ease of their adaptation in problem domains and it can intelligently generate trial solutions. The research technique is generalized as “simulation optimization”.

In the literature of simulation optimization applied to supply chain problems, several techniques have been proposed to solve inventory management problems. Bashyam and Fu (1998) is the first application of the feasible directions idea to carry out constrained optimization via simulation for inventory problems. They proposed a gradient-based simulation optimization method to optimize service level constrained (s, S) systems that allow orders across in time. They
combined Perturbation Analysis with Feasible Directions method in the optimization process. Lopez-Garcia and Posada-Bolivar (1999) develop a simulation system called Stochastic Inventory System Simulator (SISS) and optimize five stochastic inventory models with a wide range of probability distributions for demand and lead time by using tabu search.

Köchel and Nieländer (2005) proposed the simulation optimization approach where a simulator is combined with an appropriate optimization tool to study multi-location serial inventory models. Their study is concentrated on continuous-review, (R,Q) policies. They used KaSimIR as the simulator and rely on genetic algorithms for optimization. In the numerical study, an echelon system composed of five serial stages under decentralized and centralized control is studied. The objective is to find optimal (R,Q) policies for all stages with respect to minimize the total costs per year. Some conclusions are drawn from their testing results. They point out that it will be interesting to investigate the influence of various model parameters on performance measures and to find conditions under which a given policy class dominates other ones. Kleijnen and Wan (2007) illustrates the optimization of a simulated (s, S) inventory system with stochastic demand and lead time. Results are reported for three optimization methods, Bashyam and Fu (1998), Angün et al., (2009) and OptQuest method. Their results show that OptQuest give the best estimate of the true optimum, followed by Bashyam and Fu (1998). Angün et al., (2009)’s solution is relatively far away from the true optimum.

Alrefaei and Diabat (2009) use simulated annealing based simulation-optimization method for solving multi-objective problem and is applied to a (s, S) inventory model in which
minimizing multi-objective function (average ordering cost, the average holding cost average shortage cost) is desired. Ramaekers (2009) use simulation-optimization methods to determine the best strategy in combining inventory decision making and demand forecasting when demand is intermittent. Three simulation-optimization methods (Taguchi’s method, Response Surface Methodology and Tabu Search) are used and compared in her PhD dissertation to determine the optimal among two distinguished options for optimal strategies. Li et al., (2009) develop a hybrid cell evaluated genetic algorithm for optimizing the production planning and control policies in the dedicated remanufacturing system with simulation. The simulation model is designed with a prioritized stochastic batch arrival mechanism, taking into account for the special characteristics of the dedicated remanufacturing. Keskin et al., (2010) utilized a simulation-optimization approach to analyze a generalized vendor selection problem that integrates vendor selection and inventory replenishment decision of a firm. In their work, they assume that each plant follows a continuous review (R, Q) policy. The goal is to select the best set of vendors along with the corresponding EOQ at each plant so that the system-wide total cost is minimized. They build a discrete-event simulation model that works collaboratively with a scatter search-based metaheuristic to search the best solution. The impact of different network sizes on cost and computational time are examined. The impact of disruptions and problem parameters on the vendor selection process and the model performance are further discussed.

Köchel and Thiem (2011) combined a single-warehouse multi-retailer simulator with both TA and PSO optimizers. In their considered problem, a central warehouse (which has ample
amount of products but limited transportation resources) supplies products to M retailers. The objective is maximization of the total profit over the planning period. In their numerical example, all retailers use an \((s, nQ)\) ordering policy. They show that both method yield good optimization results for several examples of a 5-retailer system investigated according to their reordering strategy and transportation resources. Some conclusions are derived on the optimal structure of a single-warehouse, multi-retailer system.

In summary, simulation optimization methods have proved to be useful for analysis of different supply chain configurations and/or inventory management systems under uncertainty. Most of those studies are stochastic, which include Köchel and Nieländer (2005), Kleijnen and Wan (2007), Alrefaei and Diabat (2009), Ramaekers (2009), Keskin et al. (2010), and Köchel and Thiem (2011). Other studies, such as Dong and Leung (2009), Liao and Chang (2010), are deterministic. In this dissertation research, both deterministic and stochastic models are considered. A new MSO framework is proposed to find (near)-optimal replenishment policies for all members of the supply chain under different configuration. Also noticed is that how fast can a simulation optimization procedure reach the (near)-optimal solution will be the major concern when the model gets bigger and complicated that is mathematically intractable. In order to maintain high efficiency, the inventory management simulation model is combined with a hybrid metaheuristic recently developed (Duan et al., 2013). By enhancing its performance with a local searcher, the hybrid metaheuristic is expected to speed up the whole optimization procedure.
2.3 Hybrid meta-heuristics

Hybrid meta-heuristics are algorithms that do not purely follow the concept of one single traditional meta-heuristic, rather they combine various algorithmic ideas, sometimes from outside of the meta-heuristic field. The hybridizations of different algorithmic concepts is usually motivated by the desire to obtain better performing systems that exploit and unite advantages of the individual pure strategies, i.e. such hybrids are believed to benefit from synergy. The key is to maintain a good balance between exploitation and exploration during search.

Talbi (2002) develops taxonomy of hybrid meta-heuristics in an attempt to provide a common terminology and classification mechanisms. By combing various view points, Raidl (2006) groups hybrids of meta-heuristics according to several criteria, which include algorithms used, level of hybridization, order of execution, and control strategy. Numerous hybrid meta-heuristics have been developed and applied to both combinatorial and continuous optimization problems.

A very common way of designing hybrid meta-heuristic is based on a diversification phase followed by intensification. Both the diversification and intensification phases could be computationally expensive. To decrease the computational cost, it is desirable to stop the diversification phase earlier in order to start the intensification phase sooner. However, switching to intensification too early increases the possibility of being trapped in local minima. Determining the optimal switching time between the diversification phase and the intensification phase is thus very important for any hybrid meta-heuristic algorithm. To the best of my
knowledge no study has been carried out to date to systematically evaluate and compare the entire spectrum of different strategies for applying a local search (LS) method.

To fill in this gap, in this dissertation work, the first task is set out to carry out such a comparative study of 18 local search application strategies incorporated into a hybrid meta-heuristic. All different local search strategies are tested on 19 engineering optimization problems. Results obtained in this study are compared based on six performance indices, covering both accuracy and efficiency. The objective is to identify the most effective and efficient way of applying a local search method embedded in a hybrid algorithm. The hybrid meta-heuristic algorithm employed in this study is one that is recently developed, as a further improvement of Liao (2010). The basic building block of the hybrid include two cooperative meta-heuristic algorithms, i.e., differential evolution (DE) and harmony search (HS), and a local search method, i.e. Hooke and Jeeves direct search (HJ). Population-based metaheuristics such as differential evolution (DE) and harmony search (HS) are stochastic in nature. They are very good at diversification and identifying promising searching areas (in order to jump out from those local optima). The combined use of different search mechanism further enhances the robustness of global search with respect to changes in the problem instances. Similar observations were reported in other studies (Crainic et al., 2004, Pelta et al., 2006, and Cadenas et al., 2009).

The HJ local searcher, on the other hand, is a highly efficient deterministic direct search method superior in intensification. It enhances the hybrid’s ability to further exploit into those promising areas by a “direct search”. The phase “direct search” by Hooke and Jeeves (1961) is
used to describe sequential examination of trial solutions involving comparison of each trial solution with the “best” obtained up to that time together with a strategy for determining (based on the objective function) what the next trial solution will be. It uses the steepest ascent pivot rule in which it performs a coordinate search for each dimension of the trial solution so that it deeply exploits all the neighborhood of the trial solution so that the nearest local optimum within that area is guaranteed. Therefore, the new hybrid achieves a better balance between diversification and intensification which will enhance its ability to find the optimum more efficiently. Nevertheless, the issue of local search application is particularly important when the local-search method is of high computational cost, as the Hooke & Jeeves used here.

In light of the No Free Lunch (NFL) theorem for Optimization (Wolpert and Macready 1997), it is important to emphasize that the focus of this study is on constrained mixed integer optimization problems in engineering domain. Those optimization problems normally have mixed (e.g., continuous and discrete) design variables, nonlinear objective functions and nonlinear constraints. The optimal local search application strategy identified in this study is shown to be effective in this specific domain. However, one should not generalize those results to problems from other domains without double checking.

2.4 Supply chain inventory management under different realistic factors

The following review will report specific literature dealing with different realistic factors considered in this dissertation work.
2.4.1 Demand uncertainty

When dealing with inventory models, the probability distribution of demand is often used to capture the demand uncertainty and is an important characteristic in inventory management. Hence, we first proceed with the literature review on this topic.

In practice the demand during a certain time is nearly always a nonnegative integer, i.e., it is a discrete stochastic variable. (Exceptions may occur when we deal with products like oil measured in weights or volumes.) When the demand is reasonably low, it is natural to use a discrete demand model, which resembles the real demand. If the demand is relatively high, it is common practice to use a continuous demand model as an approximation.

Most textbooks assume that demand for an item is formed from a large number of smaller demands from individual customers. According to the central limit theorem, it is common to assume that the resulting demand is continuous and follows a Normal distribution. For fast-moving items, a Normal distribution is appropriate. Silver and Peterson (1985) recommend the Normal distribution for items with average lead time demand larger than ten. However, later research showed that using the Normal distribution for a demand distribution is questionable because (a) the distribution is defined both on the positive and negative axes, which implies that a negative demand may be generated at random and the negative demand situation is very rare in practice (it occurs only when return is larger than sale); (b) it is completely symmetrical, which is rare in real practice. While the Normal distribution could be approximately correct in many cases,
conceptually it is not. When of practical relevance, one should look for a distribution that is defined only for nonnegative values and allows for some skewness and kurtosis.

To rectify the above mentioned shortcomings of Normal distribution, many researchers later recommended Gamma distribution for fast-moving items due to the fact that it is defined only on nonegative values and, by changing its parameters, it ranges from a monotonic decreasing function, through unimodal distributions skewed to the right, to Normal distributions. The Gamma distribution is attractive because of the ease with which it can deal with fixed lead times and because of how the situations can be extended to probabilistic distributions of lead times (Burgin 1975). When the magnitude of the demand is large and the distribution is neither symmetric nor too erratic, the usual alternative to the normal distribution is the gamma distribution (Silver and Robb, 2008).

For slow-moving items, Silver and Peterson (1985) propose the Laplace or Poisson distributions. The Poisson distribution has been found to provide a reasonable fit when the demand is very low (only a few items yearly). It is typically applied to the case when demand is intermittent but unitary. The typical assumption is that it follows a Poisson distribution and equivalently the time between the occurrences of a demand is an exponentially distributed random variable. In such cases, the usual inventory control policies are of base stock type. When demand is generally non-unitary, a more appropriate demand distribution is to use both distributions for the demand occurrence and for the demand size. It is obvious that lots of models have been developed using a Poisson distribution for the demand occurrence. When the demand
size is characterized by an arbitrary probability distribution and the demand occurrence process follows a Poisson process, the total demand can be described by a compound Poisson distribution (Adelson, 1966). This means that the customer demands arrive according to a Poisson process where the quantity demanded by each customer is an independently stochastic variable.

The compound Poisson distribution is obviously the generalization of the simple Poisson distribution. At each time point, the compound Poisson distribution has batches of demand rather than single unit demand, but the distribution of time interval of demand occurrence is identical between batches for compound Poisson and between unit demands for Poisson. In both case the time between arrivals distribution is exponential. In Feeney and Sherbrooke (1966), the salient features of compound Poisson distribution have been identified:

1. Any compound Poisson distribution has a variance that exceeds or equal to its mean. When the variance equals to its mean, the compound Poisson simply reduced to Poisson.
2. The compound Poisson distributions are the most general class of “memory-less” discrete distributions, i.e., the demand size occurring in any time period has no effect on the probabilities of demand in any other over-lapping time period.
3. One of the reasons many researchers are interested in generalizing the assumption of Poisson demand to compound Poisson demand is that most real-world demand data usually produce variances that exceeds the means. Furthermore the physical model of customers who can order more than one unit appears to be a reasonable description of many supply processes.
In this dissertation research, a new MSO framework is proposed to handle demand uncertainty. The proposed framework integrates a simulation-based evaluation module with a hybrid metaheuristic, which does not need any prior knowledge of the demand distribution. Most real-world demand data does not fit well to any assumed distribution and some may even be auto-correlated. Therefore, this advantage of the proposed framework is substantial because real-world data are handled more properly and the optimized results are more reliable. By circumvent the need for putting any assumption on the demand distribution and demand parameter fitting, the proposed framework is capable of covering more than one type of distribution. It extends the scope of inventory data a single approach can handle and fits the very need of most industrial practice.

2.4.2 Capacitated supply chain inventory management under both decentralized and centralized control strategy

In today’s global economy, the competition is among supply chains, instead of individual companies. Researchers have been devising various coordination mechanisms for moving relatively inefficient decentralized supply chains toward more efficient centralized supply chains.

Several studies have been devoted to comparing the performance of a centralized SC inventory system with a decentralized one. These studies include Saharidis et al., (2009), Leung (2010), Geng et al., (2010), Ye and Xu (2010), and Baboli et al., (2011). However, all these studies assume infinite capacity and the effects of capacity constraint are ignored. The same is true for the majority of other supply chain inventory optimization studies, except a few described in the next paragraph.
Based on game theory, Mahajan et al. (2002) analyzed a supply chain consisted of an uncapacitated/capacitated supplier distributing two independent products through multiple retailers. Jemai and Karaesmen (2007) investigated a two-stage supply chain consisted of a capacitated supplier and a retailer in the framework of Nash game. Sitompul et al. (2008) formulated the safety stock placement problem for an n-stage capacitated serial supply chain as a shortest path problem and proposed a solution procedure with the objective to maintain the required overall service level at the lowest cost. Karaman and Altiok (2009) developed a model based on decomposition approximations to study an n-stage serial supply chain consisted of a supplier, a plant with finite production rate, a distribution center, and a retailer. Their model was solved by an iterative optimization procedure with the objective to minimize total system cost. Toktaş-Palut and Ülengin (2011) modeled a 2-stage supply chain consisted of multiple suppliers and a manufacturer with limited production capacities as a queuing system. They developed both decentralized and centralized models and examined three different transfer payment contracts for the coordination of the supply chain. To the best of my knowledge, the study of Toktaş-Palut and Ülengin (2011) is the only one that considers both decentralized and centralized capacitated supply chains and their study focuses on the supplier side.

Chapter 5 examined a capacitated inventory system considers a two-tier supply chain consisted of a distributor distributing a single product to multiple retailers for meeting end customer demands. Both decentralized and centralized models are developed with finite planning horizon. A periodic review replenishment policy, specifically \((s, S)\), is adopted by both the
distributor and retailers. The distributor orders from a capacitated supplier and the retailers face varying deterministic customer demands. Unfulfilled demand is assumed lost to resemble the case in fast-moving retail market; customers who do not find what they want most likely will shop somewhere else. Both decentralized and centralized models are simulated for performance evaluation; and the optimal replenishment policies for all members of the supply chain are determined.

2.4.3 Inventory management with uncertain product quality

In the early literature of inventory models, such as classical economic production/order quantity (EPQ/EOQ) models, the items produced/received are implicitly assumed to be with perfect quality. However, it may not always be the case. Due to imperfect production process, natural disasters, damage or breakage in transit, or for many other reasons, the lot sizes produced/received may contain some defective items. Fortunately, some researchers have thought about it and introduced the concept of the defective percentage into the different inventory models. Research on the management of the inventory system with imperfect quality becomes more and more important for the enterprises in the real-life situations.

Early researchers working along this line include Porteus (1986) and Rosenblatt and Lee (1986), who introduce the concept of defective items in economic order quantity (EOQ) models and discuss the relationship between quality imperfection and lot sizing. Later, Paknejad et al. (1995) propose an EOQ model dealing with stochastic demand and the number of defective items in each lot is considered as a random variable. The defective items in each lot are returned
to the supplier for rework and delivered in the next batch. Salameh and Jaber (2000) extend the traditional EPQ/EOQ inventory model by accounting for imperfect quality items. They assume that each lot contains a random fraction of defectives with known probability density function and the defective items are sold at a discounted price prior to receiving the next lot. They derive an optimal order quantity without shortage that maximizes the retailer's profit. Hayek and Salameh (2001) assume that the defective percentage is uniformly distributed, and the shortages are permitted. The imperfect items can be remanufactured into the good-quality ones. Chiu (2003) extend Hayek and Salameh (2001)'s model and assume that not all products can be reprocessed into good-quality ones. Therefore part of imperfect items has to be sold at a discounted price. Balkhi (2004) consider a production lot size inventory model with deteriorated and imperfect products taking into account inflation and time value of money. By assuming that production, demand, and deterioration rate are known, continuous, and differential functions of time, they perform a mathematical analysis of the total relevant cost of the inventory model and come up with a closed-form solution. Papachristos and Konstantaras (2006) extend the model of Salamah and Jaber (2000) and investigate the disposal time of the imperfect items. They point out that the sufficient conditions given in Salamah and Jaber (2000) for ensuring no shortages may not really prevent their occurrences, and argue that models with items of random proportional imperfect quality are stochastic and shall be treated as models of random yield. Wee et al. (2007) also extend the model of Salamah and Jaber (2000) to consider permissible shortages, which are fully backordered. Mondal et al. (2009) investigate a finite replenishment
inventory model of a single product with imperfect production process. The unit cost function is formulated by incorporating factors such as raw material, labor and replenishment rate. Lin (2010) investigates an integrated periodic review inventory model of a single supplier and a single retailer. The percentage of defective items in each supplier's delivery to the retailer is fixed. The objective is to determine the optimal review period, the optimal backorder price discount, the optimal lead time and the optimal number of shipments so as to minimize the joint expected annual total cost. Barzoki et al.(2011) link the EPQ model with work in process and imperfect item production for a manufacturing system. Barzoki et al.(2011) basically extend Boucher (1984)'s Group Technology Order Quantity model of work in process by taking account of both reworkable and non-reworkable imperfect items. Sarkar (2012) discussed the effect of machine reliability on the economic manufacturing quantity model for an imperfect production process. Without exception, all of the above-mentioned analytical models focus primarily on extending the classical EOQ/EPQ models with certain assumptions in order to make them mathematically tractable. Khan et al., (2011) present a comprehensive review of the extensions of modified EOQ/EPQ model for imperfect quality items. One of their major comments is that more research effort shall be devoted to a supply chain setting closer to practical scenarios. Research that addresses these issues may lead to modeling insights that can strengthen coordination between all members of a supply chain.
2.4.4 Inventory theory on perishable products

Inventory management for perishable products has been given much attention in the literature due to its prevalent existence in the industry. Inventory theory on perishables has been discussed extensively, as reported in the review articles by Nahmias (1982), Raafat (1991), Goyal and Giri (2001), Karaesman et al., (2011) and Bakker et al., (2012). In the existing perishable inventory management literature, research has been done assuming single/multi echelon, fixed or random shelf life, periodic or continuous review, different distributions of the demand process, lost sales or backorder, etc. Modeling method and solution approaches include exact solution, heuristics/approximations, markovian model, dynamic programming, etc. The model investigated in this research can be characterized as a multi-echelon periodic-review inventory problem for fixed shelf life perishable goods with stochastic demand. The framework present in this work falls under the category of heuristics/approximations.

Nahmias (1982) provides the first comprehensive survey of early works under periodic review. Under periodic review, if items cannot be retained in stock for more than one period, the problem reduces to the well-known newsvendor problem. Van Zyl (1964) first analyzes this type of perishable inventory problem where the shelf life is exactly two periods and showed the existence of an optimal order-up-to policy. Nahmias and Pierskalla (1973) also consider the two-time-period perishing problem with a different cost structure for both lost sales and complete backorder cases. The convexity of the corresponding cost function was characterized. Extending this early model to the general $m$-period model is far more complex due to the required
multi-dimensional state space. Early pioneers in this line of research include Fries (1975) and Nahmias (1975). The main difficulty of mathematical modeling lies in that, when demand is uncertain and the product shelf life exceeds one period, it is no longer possible to obtain a replenishment ordering policy so that there is no perishing. The problem state vector must include the stock level of each possible age category. Due to the complicated nature of the problem, it is unlikely to find optimal ordering policies for general perishable inventory models with stochastic demand. Therefore, later efforts have been largely focused on finding approximations of the true optimal policy (Chazan and Gal 1977; Cohen 1976; Nahmias 1976 a, b; Nandakumar and Morton 1993).

Raafat (1991) reviews continuous inventory models of perishables until 1991. Goyal and Giri (2001) review perishable inventory literature between early 1990s and 2000, in which they classify literature based on fixed/random shelf life, deterministic/stochastic demand, incorporating more realistic conditions such as delay in payments, price discount, time value of money, etc. Karaesman et al. (2011) review the supply chain management literature of perishable products having fixed or random shelf lives, especially of food supply chains. Due to the nature of perishable food, there is a relatively short length of time during which the food is fit for sale and consumption. For example, fresh vegetables and fruit will be spoiled in a few days. This study specifically focuses on tackling this challenging problem. The methodology to be presented is generally applicable for supply chains of perishable products with a short shelf life.
Bakker et al. (2012) give a comprehensive review of models for inventory control with perishable items that have been published since the review of Goyal and Giri (2001). It has been suggested in Bakker et al. (2012) that a stronger focus on stochastic modeling of perishable inventory is needed in order to better represent inventory control practice. At the same time, multi-echelon perishable inventory system is gaining importance due to the need for supply chain integration in today's competitive environment. Therefore, the multi-echelon stochastic problem tackled in this study is of highly practical relevance. The age-based policy proposed in this study is also promising as shown in Tekin et al. (2001) that policy including age information has clear advantage over policies without age considerations.

2.4.5 Inventory management of blood products

In this dissertation, the proposed formulation has been tested on blood supply chains. Therefore, it is beneficial to review literature relevant to blood management.

A variety of methods for improving the management of perishable inventory system have been studied and some are related to blood, which is a precious perishable product. Comprehensive reviews of the research in perishable inventory (Nahmias 1982, Prastacos 1984) indicate that numerous theoretical models have been developed. The previous studies primarily implement dynamic programming models, simulation, regression, queuing and Markov chain analyses to examine various policies at a blood bank. The earliest review specifically dealing with blood inventory management is due to Prastacos (1984). Followed by Pierskalla (2005), who discusses blood inventory and supply chain management. The production and inventory
management of blood products have received growing attention in both the traditional operations research (OR) literature (usually in the form of analytical models) and in the area of transfusion medicine and blood management (usually in the form of case studies). The most up-to-date review of the advances made in the field blood products management is presented by Belien and Force (2012). They distinguish blood products into different components such as red blood cells (RBCs), plasma and blood platelets (PLTs). Different components have different shelf lives and their distinguishing enables one to apply different management strategies to the specific product and adapt different uses of those products to the specific needs of a patient. Belien and Force (2012) pointed out that the number of papers on PLTs is substantially smaller than the number of papers on other blood products such as red blood cells and whole blood. Most research done on PLTs is qualitative in nature, i.e., case studies. Platelets are very important component of today's therapies including those related to bone marrow transplants, chemotherapy, radiation treatment and organ transplants. Unlike whole blood cells, platelets (PLTs) have a very short shelf life, typically 5 days, and sometimes extendable to 7 days. The very short shelf life of PLTs makes their production and inventory management a challenging task. This may explain why the number of papers on PLTs is substantially fewer than papers in other fields such as red blood cells and whole blood.

The limited studies on PLTs have been devoted to two topics, i.e., the hospital blood bank transfusion operation and the regional blood center production operation. At the hospital level the majority of the research has been directed toward setting standard hospital inventory stock levels
of blood products (Jennings 1975, Cohen and Pierskalla, 1979a, Sirelson and Brodheim 1991, Jagannathan and Sen 1991). The only exception is Sirelson and Brodheim (1991), which tackles specifically the inventory control of PLTs. Other research includes the presentation of a real clinical program to anticipate inventory needs based on current patient census and usage (Fuller et al., 2011). The most recent quantitative analytical model developed to optimize inventory policy for platelets is Zhou et al., (2011), in which they consider the inventory replenishment of transfusable platelets with a mere three-day life span. Their studies are based on a single hospital setting and they develop an optimal inventory policy for PLTs with dual modes of replenishments, one regular order every two days and one expedited order in between if necessary.

At the regional blood center level, although operational functions are more complex than that of a hospital blood bank, the primary concerns remain the same, i.e., forecasting demand and supply, and determining PLTs production and inventory levels. Most research includes the development of information systems and decision support systems to better manage the information related to the distribution of blood groups among the donors and recipients and the inventory levels of each blood type (Kros and Pang 2004, Pitocco and Sexton 2005, Li et al., 2008). All of them are case studies. The most recent quantitative work on PLTs production inventory problem at a blood center is due to Haijema et al., (2007, 2009), in which they propose a combination of dynamic programming and simulation approach and apply it to explore the "near optimal" inventory policies for PLTs. They show that "near optimal" solutions can be found in the class of simple order-up-to rules. Dijk et al., (2009) provides a description of the
quantitative method proposed by Haijema et al., (2007, 2009) for readers with little OR background, followed by an editorial comment of Blake (2009). Haijema et al., (2007, 2009) and van Dijk et al. (2009) assume that the age distribution of stock can be ignored while Blake (2009) shows that it cannot always be ignored when placing an order. Ghandforoush and Sen (2010) incorporate integer nonlinear programming into their decision support system for a regional blood center to produce a superior production plan and mobile assignment schedule.

To the best of my knowledge, there are only three studies on platelet supply chain management. Blake et al. (2003) present a dynamic programming formulation for solving an instance of the platelet inventory problem for an environment involving a single supplier of platelets and a single consumer. They implement a dynamic programming model for both the supplier and the consumer to identify optimal local ordering policies. These policies are then tested via a simulation model to identify good practical policies that minimize the overall outdate rates within the system as a whole. Blake et al. note the potential for developing a dynamic programming approach for developing optimal joint supplier/consumer policies, but state that even for a restricted problem environment the “curse of dimensionality” prevents the implementation of their model. The only other two more recent studies that consider PLTs supply chains are Fontaine et al., (2009) and Mustafee et al., (2009). However, both of them are case studies rather than quantitative models.

In summary, most studies on PLTs production and inventory management in the literature deal merely with single-echelon inventory problems, with only three considering the entire
supply chain. Among the three, only one is a quantitative study and the dynamic programming approach taken is difficult to solve for a large scale program. Clearly, there is a need for more quantitative studies on PLTs supply chain management. In this dissertation study, the application of the proposed MSO framework to PLTs inventory management is intended exactly to fill in this void.

On the other side, studies on RBCs usually do not include the effect of blood substitution. There are 8 major blood types, varying from 37.4% (O+) to 0.6% (AB-) in the US population. An efficient blood management applicable to the reality has to be examined for multiple blood products, since it involves 8 blood types and Rhesus grouping, all of which have to be simultaneously controlled. Another interesting issue in the blood transfusion process is the blood substitution. Ideally, a patient should be transfused with the same blood group as his/her own, but this is not always possible. When a patient's blood group at the time of request is unavailable, a compatible blood group has to be provided. For example, patients with Group A+ can receive blood from A-, O-, or O+.

Research effort devoted to consider multiple ABO/Rh(D)-compatible blood products is significantly limited compared to the large body of research in the single unified blood product. The process of testing a sample of a patient's blood against units of blood from inventory to ensure compatibility is called cross-matching. Early attempts to include cross-matching in blood inventory modeling have been focused on the study of assigned sub-inventories (Cohen and Pierskalla, 1979a, Cohen, Pierskalla and Sassetti 1983, Jagannathan and Sen 1991). The reason
for keeping assigned sub-inventories lies in the use of the antiglobulin cross-match as the method for pre-transfusion compatibility testing, as well as in the fact that although many surgical reserves are requested on a precautionary basis and are not eventually used; this outcome cannot be anticipated for a particular patient. An assigned sub-inventory consisting of cross-matched blood units that are kept reserved for a particular patient for a number of days, and that are returned to the inventory if left unused at the end of the period. All those previous research studies the cross-match release period (the maximum number of days that cross-matched units spend in the assigned sub-inventory), the transfusion to cross-match ratio (the proportion of cross-matched blood is eventually transfused into the patient). Those analyses were usually performed for one single blood type and the interaction between all possible substitutions is not included. All those previous studies have shown that the optimization of the inventory for each single blood type can be achieved by issuing blood according to a FIFO (first in, first out) policy and by reducing as much as possible the cross-match to transfusion ratio and the cross-match release period.

In a recent study of Pereira (2005), it is shown that blood inventory performance could be further improved by keeping the transfusion to cross-match ratio very close to one and the cross-match release period very close to zero, that is, by managing surgical reserves without assigned sub-inventories. This can be achieved by using the type and screen procedure (T&S) in preference to the antiglobulin cross-match in pre-transfusion compatibility testing. Reports on outcomes from institutions that have consummated such substitution seem to confirm the
beneficial effect on blood inventory performance (Feng and Ng, 1991, Georgsen and Kristensen 1998). Over the past two decades the T&S procedure has gained increasing acceptability as a safe and convenient alternative to the antiglobulin cross-match (Georgsen and Kristensen 1998). In the T&S policy the blood bank does not assign specific RBC units to particular patients, provided that the prospective recipient has no clinically significant unexpected antibody. If transfusion proves necessary, the blood bank issues any RBC unit from the ABO/Rh(D)-compatible inventory with only electronic or abbreviated serological cross-match.

The T&S procedure allows hospital blood banks to meet the requests for surgical reserves without assigned sub-inventory. The widespread acceptance of this policy makes it necessary to reconsider previous models on the management of RBC inventories, as most such models were just focused on the optimization of variables related to the assigned, cross-matched sub-inventory. Most importantly, it allows us to shift the research focus to the study of all the 8 blood groups and the interaction between all possible substitutions.

Fortunately, there are two recent publications that have considered the inventory management of RBC by ABO/Ph(D) type. Katsaliaki and Brailsford (2007) provided a case study for managing the blood inventory system in a UK hospital where the supplier was a regional blood center. They tested the effects of nine policies using a discrete-event simulation and offered some recommendations tailored to the system. They implied that such a simulation could be used as a decision support tool to investigate different procedures and policies. Fontaine et al., (2010) evaluated several red blood cell maximum shelf lives (7, 14, 21, 28, 35, 42 days) and their
impacts on RBC availability and outdate rate, using Excel VBA (Visual Basic for Applications) simulation. They concluded that changing the maximum shelf life for RBC units would require novel approaches to RBC inventory management to meet hospital demands with acceptable outdate rates.

Both studies are based on simulation. Simulation alone boils down to "what-if" scenario analysis, evaluating the effectiveness of a set of policies. However, it is not an approach that can gear itself to obtain a near-optimal policy. In this dissertation, the proposed simulation optimization framework is finally tailored to optimize the replenishment policies of all blood types under their ABO/Rh(D) compatibility.

2.5 Concluding remarks

To the best of my knowledge, a quantitative multi-period SC inventory decision support system that deals with various operational deviation is lacking. Realizing deviations are much more common than those relatively less frequent disruptions as explained in Gaonkar and Viswanadham (2007), it is set out to develop a quantitative multi-period supply chain inventory decision support system for potential use at the operational level to address not only uncertainty (such as supplier quality imperfection and customer demand fluctuation) but also some realistic issues (such as capacity constraints, limited shelf life of products and type-compatible substitutions). The ultimate goal of the dissertation research is to develop a system that is practical and realistic, not relying on any unrealistic assumption. In addition, it is desirable for the system to be able to offer not just any solution, but near-optimal solution if not true optimum.
The cores of the framework include a simulation model and a metaheuristic optimizer. Since demand is always uncertain in the real world, to be realistic and practical the system is desirable to be able to handle any demand. To this end, theoretical mathematical approaches that make any assumption on demand are inadequate. The effectiveness of analytical methods is generally limited to small systems or restricted cases because as the models get bigger and complicated they will quickly become mathematically intractable. Using simulation to model supply chains operation under deviations can provide a more realistic view by accounting for the natural variations that occur in the various processes within the supply-chain, which could not be captured analytically. Unfortunately, most existing simulation software programs have limited optimization capability and the details of its inner working are often hidden and proprietarily protected.

The reason for choosing a meta-heuristic approach as the optimization module in our work is its proven effectiveness and efficiency in solving various optimization problems such as travelling salesman (Dorigo, 1997a,b), assignment problem (Costa and Hertz, 1997; Stützle and Hoos, 2000; Wagner, 2000), vehicle routing (Dorigo, 1999; Reimann et al., 2004), feature selection (Liao, 2009), and project scheduling (Duan and Liao, 2010), to name just a few. Moreover, unlike theoretical approaches meta-heuristic approaches are more promising to be able to work with more complex supply chain models for studying the interactions between numerous system parameters. It has advantage in handling non-differentiable, non-convex multi-modal, stochastic problems because they utilize no gradient information, ideal to find
near-optimal solutions for NP-hard combinatorial optimization problems. The integration of computer simulation and optimization allows a more advanced searching process for the best combination of decision variables based on the output of a simulation model of the system. Therefore, the proposed framework is developed based on meta-heuristic together with tailored evaluation modules behaving like simulation, explained more in detailed in chapter 3.
CHAPTER 3 PROPOSED APPROACH

Supply chain inventory management in general has been a topic of interest to many researchers as well as practitioners. Numerous studies have been carried out in particular concerning the optimization of supply chain inventory/replenishment polices. From the solution methodology viewpoint, both analytical approaches such as Leung (2010) and (meta-) heuristic approaches such as Yang et al. (2012) have been used to optimize supply chain inventory/replenishment policies. Both approaches have their own pros and cons. Generally speaking, an analytical approach is able to guarantee global optimal solution if the mathematical model is solvable. However, the model is often based on a number of restrictive assumptions. For inventory problems, each analytical model assumes either constant demand or stochastic demand following a specific distribution. The applicability of the model thus depends upon the validity of the assumption. On the other hand, (meta-) heuristic approaches do not guarantee finding the global solution. Nevertheless, often (meta-) heuristic approaches are able to find the global optimum or near-optimal solutions and they are able to solve mathematical models with fewer and more realistic assumptions, non-differential, nonconvex, and do not have explicit form. In particular, (meta-) heuristic approaches can work with simulation models to mimic a more complicated system while analytical approaches cannot. The area of simulation optimization relies more on powerful metaheuristics than traditional optimization methods (Fu, 2002).

In this chapter, the general overview of the MSO approach proposed in this dissertation is described as follows.
The general MSO framework outlined in Fig. 3.1 consists of a metaheuristic optimizer and a tailed supply chain simulation model. The integrated framework works iteratively to find near-optimal solutions for the considered supply chain inventory problem. The process is initiated by inputting an initial guess of trial solutions. The population of trial solutions is sent to the simulation model. Running the simulation model generates their corresponding output performance measure, which are fed into the hybrid metaheuristic. The quality of the output guides the metaheuristic in the selection of new input solutions, based on the intelligent searching mechanism of the metaheuristic. The process is repeated until the maximum number of iterations is reached or no further improvement can be found. At the end of the search, the best solution found so far will be reported as the (near)optimal solution to the problem.
The metaheuristic optimizer designed for this MSO framework follows the principle of separating the method from the simulation model. In such a context, the optimization routine is defined outside the complex simulation system. Therefore, the evaluator (i.e. the simulation model) can change and evolve to incorporate additional elements of the complex system, while the optimization routines remain the same. Hence, there is a complete separation between the model that simulates the system and the procedure that is used to solve optimization problems defined within this model.

As stated in previous chapter, the uncertainties and complexities modeled by the simulation are often such that the analyst has no idea about the shape of the response surface. Generally, there exists no closed-form mathematical expression to represent the space, and there is no way to gauge whether the searching space is smooth, discontinuous, etc. While this is enough to make most traditional optimization algorithms fail, metaheuristic, such as Genetic Algorithm, Ant Colony Optimization, overcome this challenge by making use of adaptive memory techniques and population sampling methods that allow the search to be conducted on a wide area of solution space, without getting stuck in local optima.

The metaheuristic-based simulation optimization framework is also very flexible in terms of the performance measures the decision-maker wishes to evaluate. Provided that a feasible solution exists, metaheuristic ideally carries out an intelligent search where the successively generated candidate solutions produce varying evaluations, not all of them improving, but which over time provide a highly efficient trajectory to the globally best solutions. The process
continues until an appropriate termination criterion is satisfied (usually based on the user’s preference for the amount of time devoted to the search).

In this dissertation, a newly developed hybrid metaheuristic is integrated into the MSO framework as the optimizer. A comparative study is performed to identify the most effective and efficient way of applying local search within the hybrid metaheuristic. It is expected to enhance the searching ability of the optimizer since the simulation model may get very complicated and it needs to be run under a large number of experimental conditions.

Table 3.1 gives a summary of the inventory problems studied in this dissertation research.

Table 3.1 Summary of the SC inventory problems studied

<table>
<thead>
<tr>
<th>Inventory problem</th>
<th>Practical factors considered</th>
<th>Inventory policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitated SC under decentralized and centralized control</td>
<td>Supplier's capacity constraint</td>
<td>(s, S) policy</td>
</tr>
<tr>
<td>Integrated SC facing quality imperfection</td>
<td>Quality imperfection in the products supplied</td>
<td>(s, S) policy</td>
</tr>
<tr>
<td>Highly perishable platelets SC under decentralized and centralized control</td>
<td>Restricted shelf life in days</td>
<td>Order-up-to policy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>OIR Policy</td>
</tr>
<tr>
<td>Blood SC with ABO compatibility</td>
<td>Shortened shelf lives</td>
<td>OIR policy</td>
</tr>
<tr>
<td></td>
<td>ABO/Rh(D) compatibility</td>
<td></td>
</tr>
</tbody>
</table>

The metaheuristic optimizer and its best local search application strategy will be discussed in detail in chapter 4. The resulting hybrid is chosen to as the optimizer in the MSO framework. In chapter 5-8, the successful application of the proposed MSO framework to different supply
chain inventory problems will be demonstrated in detail. In summary, the differences between the proposed MSO approach and theoretical/analytical approaches are summarized in Table 3.2.

Table 3.2 Comparison between the proposed approach and traditional analytical approaches

<table>
<thead>
<tr>
<th></th>
<th>Analytical approach</th>
<th>MSO approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumptions on demand distribution</td>
<td>Required</td>
<td>None</td>
</tr>
<tr>
<td>Guarantee on optimality</td>
<td>Yes (on restricted cases)</td>
<td>No (usually near-optimal)</td>
</tr>
<tr>
<td>Explicit mathematical models (cost equations)</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Solution methods</td>
<td>Exact, approximation, heuristic, meta-heuristic</td>
<td>Any meta-heuristic with a specially designed simulation model</td>
</tr>
<tr>
<td>Demand data that can be handled</td>
<td>Only those satisfy the assumption</td>
<td>Any</td>
</tr>
<tr>
<td>Adaptability to practical situations</td>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>
CHAPTER 4 HYBRID METAHEURISTIC DEVELOPMENT AND IMPROVEMENT

4.1 Introduction

Since the research goal is to develop a MSO framework to supply chain inventory optimization, it is anticipated that the dimension of problem will grow rapidly as the network expands. As a result, the ability of finding the optimal solution to a large extent depends on the searching ability of the optimization algorithm. Metaheuristic is chosen as the optimizer in the framework. The term metaheuristic, first introduced by Glover (1986), generally refers to any optimization methods that implements certain strategies for searching the space to find the global optima. Although there is no theoretical guarantee that they can always find the global optima, it has been reported that metaheuristic algorithms are able to produce very good results, as reported in research and in practice. Furthermore, metaheuristics are known to have the advantage to deal with optimization problems with non-convex and non-differentiable functions.

In order to fully utilize those advantages offered by meta-heuristic, it is of interest to do a comparative study and select a highly efficient metaheuristic algorithm for the supply chain model. The key to achieving high performance for any metaheuristic algorithm is to maintain a good balance between exploitation and exploration during search. Hybrid metaheuristics is believed to a very promising alternative. Hybrid metaheuristics are algorithms that do not purely follow the concept of one single traditional metaheuristic, rather they combine various algorithmic ideas, sometimes from outside of the metaheuristic field. The hybridizations of different algorithmic concepts is usually motivated by the desire to obtain better performing
systems that exploit and unite advantages of the individual pure strategies, i.e. such hybrids are believed to benefit from synergy. A very common way of hybridization is based on a diversification phase followed by intensification. Both phases could be computationally expensive. To decrease the computational cost, it is desirable to stop the diversification phase earlier in order to start the intensification phase sooner. However, switching to intensification too early increases the possibility of being trapped in local minima. Determining the optimal switching time between the diversification phase and the intensification phase is thus very important for any hybrid metaheuristic algorithm. In order to enhance the optimization ability of the MSO framework, I decided to carry out a comparative study with the objective to identify the most effective and efficient way of applying a local search method embedded in a hybrid algorithm. The hybrid optimization algorithm once identified will serve as the powerful optimizer in the supply chain model to facilitate the searching process.

4.2 Method

The hybrid meta-heuristic algorithm employed in this study is one that is recently developed as a further improvement of Liao (2010). The hybrid is comprised of two cooperative metaheuristic algorithms and one local search method. The reason for using two cooperative metaheuristic algorithms is because they outperform one alone. In this regard, our hybrid differs from most existing hybrid algorithms based only on one metaheuristic (details are in Duan et al. 2013, Yi et al., 2013). Figure 4.1 depicts the overall flowchart of the “DE-HS-HJ” metaheuristic. Because of space limitation, it is not possible to draw all 18 local search application strategies
investigated. Moreover, including all of them into one flowchart will be very confusing and messy. Hence, only two strategies are presented, i.e., applying to every solution with probability (LS 6) and applying it as an additional neighborhood solution generation operator (LS 8). Blocks in dash are for LS 6 and Block in double dotted line is for LS 8. Figure 4.1 can be easily modified to show other LS application strategies to be presented in section 4.2.4.

4.2.1 Differential Evolution

The underlying relationship between optimization and biological evolution led to the development of an important paradigm of computational intelligence—the evolutionary computing techniques for performing very complex search and optimization. Differential evolution (DE) has emerged as a very competitive form of evolutionary computing. Das and Suganthan (2001) identify DE as one of the most powerful stochastic real-parameter optimization algorithms in current use. DE was originally proposed by Storn and Price (1995, 1997). It is a reliable and versatile function optimizer. DE, like most popular evolutionary algorithms (EAs), is a population-based optimization tool. DE, unlike most other EAs, generates offspring by perturbing solutions with a scaled difference of two randomly selected population vectors, instead of recombining the solutions by means of a probability function. When it comes to selection, DE employs a steady state logic which allows replacement of an individual only if the offspring outperforms its corresponding parent. The DE algorithm is described here with three evolution operators: mutation, crossover and selection.
Figure 4.1 Flow chart of the “DE-HS-HJ” meta-heuristic
For each target vector \( x_i(t) \) at generation \( t \), \( DE \) generates a new mutated vector, \( v_i(t) \), by adding the weighted difference between two population vectors, \( x_{i_1}(t) \) and \( x_{i_2}(t) \), to a third vector, \( x_{i_3}(t) \). These three vectors are randomly chosen and must be different from the target vector. Therefore, \( i_1 \neq i_2 \neq i_3 \neq i \). The new mutated vector is then calculated as

\[
v_i(t) = x_{i_3}(t) + F(x_{i_1}(t) - x_{i_2}(t)).
\]

(4.1)

\( F \) is a scale factor used to control the amplification of the differential variation and the value of \( F \) is often set to be between 0 and 2, i.e. \( F \in (0,2) \). Then the target vector, \( x_i(t) \) is combined with elements from the mutated vector, \( v_i(t) \), using the binomial crossover,

\[
\mu_j(t) = \begin{cases} 
v_j(t) & \text{if } \text{rand}(j) \leq CR \text{ or } j = j_{rand} \\
 x_j(t) & \text{otherwise}
\end{cases}
\]

(4.2)

where \( j = 1, \ldots, D \) refers to the number of dimension of a single chromosome, \( \text{rand}(j) \) is the \( j \)th component of a \( D \)-dimensional uniform random number \( \in [0,1] \), and \( j_{rand} \in [1, 2, \ldots, D] \) is a randomly chosen index from the set \( \{1, \ldots, D\} \) to ensure that at least one mutated dimensional value is used in the trial vector. \( CR \) is the crossover rate. Finally, \( DE \) implements a common selection procedure. The generated trial vector, \( \mu_i(t) \), replaces \( x_i(t) \) only if its fitness is better than the target vector.

This study employs the modified differential evolution (labeled MDE') algorithm as in Liao (2010) that has been shown effective in solving constrained mixed integer engineering design problems. It comprises of three components: the MDE algorithm proposed by Angira and Babu
(2006) with some modifications, Deb’s constraint handling method (2000), and a generalized discrete variable handling method to deal with mixed integer problems (Schmidt and Thierauf 2005). For the MDE’ algorithm, when dealing with variable bound violation, both ‘forced bound’ and ‘method without forcing the bound’ are used together with equal probability. In the original MDE, they are used individually and independently. Another modification involves the use of rounding operation rather than the truncation operation. For the detailed pseudo code of the MDE’ algorithm, readers are referred to Liao (2010).

4.2.2 Harmony search

Harmony search is chosen as the second metaheuristic to cooperate with MDE’ based on past experience in obtaining better performance from such a hybrid than from combing MDE’ with other algorithms. Harmony search is a relatively new metaheurisitic, proposed by Geem et al. (2001). This meta-heuristic algorithm is derived from an artificial phenomenon found in musical performance, mimicking the process of search for better harmony. The main idea is to treat the optimization algorithm seeking a global optimum determined by objective function as the musical performance seeking a fantastic harmony determined by aesthetic estimation. The musician’s improvisations are analogous to explore and exploit search operators in optimization schemes. The harmony memory is initially populated with randomly generated solutions sorted by their objective function values. Next, a new trial harmony is improvised from the harmony memory using a stochastic random search mechanism based on the harmony considering rate (HMCR) and the pitch adjusting rate (PAR).
This study employs the improved harmony search (IHS) proposed by Mahdavi et al. (2007) by adapting two parameters: pitch adjusting rate, \( PAR \) and band-width, \( bw \). For each improvisation step \( t \), \( PAR \) and \( bw \) are determined as follows:

\[
PAR(t) = \frac{PAR_{min} + PAR_{max} - PAR_{min}}{NI} \cdot t 
\]

\[
bw(t) = bw_{max} \cdot \exp(t \cdot c) 
\]

\[
c = \frac{\ln(bw_{min} / bw_{max})}{NI} 
\]

In the above equations, \( NI \), \( PAR_{min} \), \( PAR_{max} \), \( bw_{min} \) and \( bw_{max} \) denote number of improvisations, minimum/maximum pitch adjusting rate/bandwidth, respectively. In the MDE’-IHS hybrid, the current number of evaluations is used as improvisation step \( t \). The detailed pseudo code of the MDE’–IHS hybrid algorithm can be found in Liao (2010).

4.2.3 Hooke and Jeeves direct search method

The Hooke and Jeeves (HJ) direct search method was first developed by Hooke and Jeeves (1961). It is a classical and very powerful local descent algorithm, making no use of the objective function derivatives. Population-based metaheuristics such as differential evolution (DE) and harmony search (HS) used in this study are very good at exploring the search space and identifying promising searching areas. However, they are usually not very good in refining solutions. The HJ local searcher collaborates with the cooperative hybrid by enhancing the algorithm’s ability of exploiting further into those promising areas and further refining those solutions. The phase “direct search” by Hooke and Jeeves (1961) is used to describe sequential
examination of trial solutions involving comparison of each trial solution with the “best” obtained up to that time together with a strategy for determining (based on the objective function) what the next trial solution will be. It uses the steepest ascent pivot rule in which it deeply exploits all the neighborhood of a considered solution so that the nearest local optimum within that area is guaranteed. By maintaining the merits of each algorithm, the new hybrid achieves a better balance between exploration and exploitation. It is expected to find the global optimum more efficiently. In this study, the Hooke and Jeeves local search is applied according to various application strategies to be detailed in Section 4. The detailed pseudo code of the HJ direct search method is presented as follows.

Hooke and Jeeves local search
% beat_patn is the pattern search indicator to show whether a pattern search succeeds (1) or not (0)
% beat_explr is the exploration search indicator to show whether an exploration search succeeds (1) or not (0)
% $\lambda$ is the adaptive step size parameter, initially set as 10% of the domain range
% $g_{\text{obj}}$ is the global optimum
% $\epsilon$ is the acceptable error

% Main Function
(1) Let the candidate solution obtained from MDE’-IHS be $x_{\text{base}}$;
(2) Evaluate $x_{\text{base}}$ and store its objective function value in $f_{\text{base}}$, its constraint violation in $g_{\text{base}}$
(3) While current number of cycles < maximum number of cycles or $f_{\text{base}} > g_{\text{obj}} + \epsilon$
(4) \[ [\text{beat_explr}, x_{\text{base}}, x_{\text{explr}}] = \text{Subfunction}_\text{ExplSrhp}(x_{\text{base}}, \lambda) \]
(5) If $\text{beat_explr} = 1$,
(6) \[ [x_{\text{base}}] = \text{Subfunction}_\text{PatnSrhp}(x_{\text{base}}, x_{\text{explr}}, \lambda) \]
(7) Otherwise
(8) reduce $\lambda$;
(9) end
(10) handle discrete numbers
(11) increment cycle number by one;
(12) end while
(13) Output final x_base

% Subfunction_ExplSrh: Exploration search
(1) set beat_explr=0;
(2) set x_explr and x_temp as x_base;
(3) for i=1: dimensions of x_explr
(4) x_explr(i)=x_base(i)+\lambda(i);
(5) repair boundary of x_explr if needed;
(6) evaluate new x_explr
(7) if x_explr is superior to x_temp
(8) set beat_explr=1;
(9) x_temp(i)=x_explr(i); update f_temp and g_temp accordingly
(10) else
(11) x_explr(i)=x_base(i)-\lambda(i);
(12) repair boundary of x_explr if needed;
(13) evaluate new x_explr
(14) if x_explr is superior to x_temp
(15) set beat_explr=1;
(16) x_temp(i)=x_explr(i); update f_temp and g_temp
(17) end
(18) end
(19) end

% Subfunction_PatnSrh: Pattern search.
(1) initialize beat_patn=0;
(2) x_patn_start=2*x_explr-x_base;
(3) handle discrete numbers and boundary repair
(4) evaluate x_patn_start and store its f_patn_start and g_patn_start
(5) set x_patn and x_temp as x_patn_start;
(6) for i=1:dimensions of x_patn
(7) x_patn(i)=x_patn_start(i)+\lambda(i);
(8) repair boundary of x_patn
(9) evaluate x_patn
(10) if x_patn is superior to x_temp
(11) beat_patn=1;
(12) replace x_temp(i) as x_patn(i);
(13) else
(14) x_patn(i)=x_patn_start(i)-\lambda(i);
(15) repair boundary of x_patn
(16) evaluate x_patn
if x_patn is superior to x_temp
    beat_patn=1;
    replace x_temp(i) as x_patn(i);
end

if beat_patn==1
if x_patn is superior to x_explr
    replace x_base as x_explr; replace x_explr as x_patn;
    \[x_base\]=Subfunction_PatnSrh (x_base, x_explr, \lambda);
else
    if x_patn_start is superior to x_explr
        replace x_base as x_patn_start;
        beat_patn=0;
    else
        replace x_base as x_explr;
        beat_patn=0;
    end
end
else
    replace x_base as x_explr;
    beat_patn=0;
end

4.2.4 Different Local Search Application Strategies

In order to investigate the effect of different local search application strategies, the HJ local search method is chosen because it performs better than random walk with direction exploitation (RWDE) used in Liao (2010). The HJ local search can be integrated with the MDE'-IHS cooperative meta-heuristic in many different ways. The criterion used to stop a run is either achieving the global optimum (or the known best solution) within $10^{-6}$ error or exceeding the pre-specified maximum number of evaluations. The maximum number of evaluations is varied form problem to problem based on its difficulty, but is kept the same for all LS strategies for any
particular problem to ensure fair comparison. As reviewed in Section 2, roughly 11 different local search application strategies can be distinguished. For this study, 18 specific local search application strategies were implemented and evaluated. Each of them is described in more details below.

• **LS1**: Applying local search to an improved solution or the best solution in each population periodically or with probability (Chiou and Wang 1999; Chelouah and Siarry 2003; Victoire and Jeyakumar 2004; Liu et al. 2005; Gudla and Ganguli 2005; "The staged pipelining hybrid" of Barbosa et al. 2005; "Scheme 2" of Petalas et al. 2007; Pedamallu and Ozdamar 2008). For ease of presentation later, we label this strategy “NUS”, as short for Newly Updated Solution. The probability is set equaling to 0.1 in our study.

• **LS2**: Applying it to the updated current global best solution (called EveryUpdatedGbest). Similar work include the “Scheme 1” in Petalas et al. 2007; Salhi and Queen 2004; Sevkli and Sevilgen 2008; Mashinchi et al. 2011.

• **LS3**: Applying it to the updated current global best solution with probability (called GBP). This strategy can be considered as a generalization of LS2 in the sense that LS3 becomes LS2 when the probability is one. Wei and Zhao (2005) used this technique in their hybrid with simplex search and called it as "Simplex search in the best promising zone." The probability is set equaling to 0.1 in our study.

• **LS4**: Applying it to niches of solutions obtained in every iteration. This strategy is labeled “NOS” as short for Niches of Solutions. It is well known that the clustering process enables
the creation of clusters that hopefully correspond to relevant regions of attraction. Local-search procedures can then be started once in every such region. Similar works include the "Simplex search in potential niches" technique in Wei and Zhao (2005) and Martínez-Estudillo et al. (2006). In each iteration the entire population is clustered to form niches according to some clustering criterion. Fuzzy-C-Means is applied in this study to generate those niches (Bezdek et al. 1984; Pal and Bezdek 1995). Note that the number of clusters has to be pre-specified and five are used in this study for all tested problems. The local search is applied to the best individual in each niche, hoping to locate the global optimum in the promising zone more quickly and reliably. It should be pointed out that, in this particular method, determining the number of best individuals to consider as well as the number of clusters is non-trivial. These two parameters are usually carefully selected for each particular problem in order to achieve best performance.

- LS5: Applying it to a selected number of top ranked solutions. This strategy is labeled “TRS” as short for Top Ranked Solutions. In an N-dimensional problem, the population size of this hybrid approach is set to 2N. The entire population is sorted according to their fitness values. The best N solutions are fed into the local search algorithm to improve their fitness values. The N worst solutions are improved by the DE+HS method. Then they are joined together to update the population. The whole searching process stops until the termination condition is met. Fan et al. (2006) use the similar strategy.
• LS6: Applying it to each solution with probability. This strategy is labeled “ESP” as short for Every Solution with Probability. A trial solution will undergo the local search operation if randomly selected based on a pre-specified percentage, \( p \) (set equaling to 0.1 in our study). LS6 can be considered as a generalization of LS 18 in the sense that LS6 becomes LS18 when the probability is one. Related works please refer to Liao (2010); Nguyen and Yao (2008).

• LS7: Applying it to each solution on condition. Similar technique is found in Wang et al. (2005) and Miettinen et al. (2006). This study follows that of Wang et al. (2005) and calls this strategy “Dradius”. In this strategy, LS is applied to a candidate solution when its fitness is within the diversion radius (abbreviated as Dradius). The diversion radius is set as 0.1 in this study.

• LS8: Applying it as an additional neighborhood solution generation operator. This strategy is labeled ‘NNG’, as short for New Neighborhood Generator. The HJ local search method is applied as if it is a standard neighborhood solution generator. It is used to produce a new solution candidate to compete with other trial individuals generated by MDE’ and IHS. This strategy is defined in Barbosa et al. (2005) as “the additional-operator type hybrid”.

• LS9: Applying it to the global best solution on condition (Gimmler et al. 2006; Vasant and Barsoum 2009, 2010; Vasant 2010, 2013). This strategy, which was used in Gimmler et al. (2006), is labeled “Ddomain.” Once the convergence criterion is met, the HJ local search is applied to the global best solution obtained so far. In particular, the convergence criterion is
met if $\frac{d_{\text{max}}}{D_{\text{domain}}} < 0.001$, where $d_{\text{max}}$ is the maximum Euclidean distance to the best solution found so far over all current solutions and $D_{\text{domain}}$ is the diameter of the search space, mathematically $D_{\text{domain}} = \sqrt{n(b_u - b_l)^2}$, where $b_u$ and $b_l$ are the upper and lower bound of the search space.

- **LS10-17**: Combining the best strategy above with one of the others. This study will only consider combining two strategies together. Based on the test results to be presented in the next section, the claimed best solo LS application strategy is combined with the rest strategies one by one. Only the best solo is combined with others because they are more promising to produce better results given that considering all possible combinations is too time-consuming. Interested readers can refer to Hedar and Fukushima (2004, 2006), “Scheme 3” of Petalas et al. (2007) and Fesanghary et al. (2008) for more details on how to design a hybrid local search strategy. Those strategies are labeled as one plus the other reflecting specific combinations used accordingly.

- **LS18**: Applying it to each solution unconditionally. This strategy is labeled “All”, indicating that all trial solutions are subject to local search. Related work can be found in Baskar et al. (2006); Schmidt and Thierauf (2005); Hwang and He (2006); Alikhani et al. (2009) and Csébfalvi (2009). This strategy of hybridization is to perform local optimization for every single potential solution. Thus whenever the algorithm finds a potential solution, it performs intensification on it and update the corresponding result. In other words, the HJ local search is applied to every trial candidate generated by both MDE’ and IHS in this study.
4.3 Test Problems and Results

The performances of all the local search application strategies as described in 4.2.4 were verified experimentally using a set of 19 constrained mixed integer optimization problems. All programs were coded in Matlab and all executions were made on a HP Pavilion a4317c with AMD Athlon™ II × 2@ 2.70 GHz.

4.3.1 Test problems

The 18 versions of hybrid metaheuristic algorithms implemented with all 18 different local search application strategies were tested with 19 problems; most of them have been used in the literature except the last one. For example, functions 1-14 were used in Liao (2010) and readers are referred to the Appendix of Liao (2010) for the detailed list. Those functions have also been used by others before Liao (2010). For example, Functions 1–3 were used in Mathur et al. (2000) whereas functions 4–9 were used in Angira and Babu (2006), all in the field of process synthesis and design. Functions 10–12 were used in Yokota et al. (1996), in the field of system reliability. Function 13 was used in Kitayama and Yasuda (2006), in the field of pressure vessel design. Function 14 was used in Lee et al (2007), in the field of manufacturing process design. Functions 15-18 were taken from Chen (2006), in the field of system reliability redundancy allocation problems. Function 19 is a supply chain consignment model; its details will be presented in a separate paper in the near future. Since all those functions have been used in the literature, the best solution for each function is known.
Our test results will show that existing methods such as the latest work by Liao (2010) can be further improved. In other words, a number of versions of algorithms implemented in this study can generate more accurate results in less time without being trapped in local minima. For fair comparison of different local search application strategies, we fixed all parameter settings for all test cases. Those settings were found to work well based on our previous research. Table 4.1 summarizes the settings of all relevant algorithmic parameters (organized into groups related to MDE’, HIS and HJ, respectively). However, the maximal number of function evaluations, $MaxFE$, is varied from problem to problem depending upon its difficulty.

Table 4.1 Global algorithmic parameters for MDE’, IHS and HJ

<table>
<thead>
<tr>
<th>MDE’</th>
<th>IHS</th>
<th>HJ local search</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population Size</td>
<td>Harmony memory size</td>
<td>Stepsize</td>
</tr>
<tr>
<td>Scale Factor</td>
<td>Harmony considering rate</td>
<td>Maximum number of cycle</td>
</tr>
<tr>
<td>Crossover Rate</td>
<td>Number of improvisations</td>
<td>$\lambda$ =0.1</td>
</tr>
<tr>
<td></td>
<td>Minimum pitch adjusting rate</td>
<td>$bw_{min}$=1× 10^{-5}</td>
</tr>
<tr>
<td></td>
<td>Maximum pitch adjusting rate</td>
<td>$bw_{max}$=4</td>
</tr>
<tr>
<td></td>
<td>Minimum bandwidth</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Maximum bandwidth</td>
<td></td>
</tr>
<tr>
<td>$NP=10\times$Dimensions of the problem</td>
<td>$HMS=10\times$Dimensions of the problem</td>
<td>$M =10$</td>
</tr>
<tr>
<td>$F=0.5$</td>
<td>$HMCR=0.99$</td>
<td></td>
</tr>
<tr>
<td>$CR=0.95$</td>
<td>$NI= MaxFE$</td>
<td></td>
</tr>
<tr>
<td>$PAR_{min}=0.45$</td>
<td>$PAR_{max}=0.99$</td>
<td></td>
</tr>
<tr>
<td>$bw_{min}=1\times 10^{-5}$</td>
<td>$bw_{max}=4$</td>
<td></td>
</tr>
</tbody>
</table>

4.3.2 Performance measures

For each test problem, 30 runs were made and the mean and standard deviation of objective values, elapsed CPU times, and number of function evaluations taken were recorded. A run is
declared successful if the global optimum or known best solution was found within $10^{-6}$ error.

We adopt the following performance measures, following Nguyen and Yao (2008).

1. Success rate, $SR = rs/rT$, where $rs$ and $rT$ are number of successful runs and total number of runs, respectively. A run is declared successful if it finds the global optimal solution within $10^{-6}$ error within the specified $maxFE$. Obviously, the higher the success rate is the better.

2. Normalized number of evaluations used in solving a function ($NFS$). There are various ways for comparing different optimization methods in terms of computational cost. The most common approach is to count the number of function evaluations required to find the global optimum (or known best) solution. Basically we are interested in finding the global optimum of a problem with the least number of function evaluations possible. Since the maximal number of evaluations is varied from problem to problem, for fair comparison the mean of evaluations for each strategy is normalized by the maximal mean within the problem.

3. Success performance, $FES = (meanNFS*rT)/rs$, where $meanNFS$ denotes the average number of normalized evaluations used in solving the function. Success performance, specified in Suganthan et. al. (2005), combines number of evaluations and success rate together to measure the overall performance of a certain method. The aim is to find a method which takes least possible number of evaluations to achieve high success rate. The lower the success performance value is the better.

4. Number of functions successfully solved in all runs. This measure shows how many functions that an algorithm can successfully solve within the maximal number of evaluations
in all runs. Obviously the more problems an algorithm is able to solve in all runs, the better it is. However, focusing only on successful runs may not completely reflect how well an algorithm performs compared to others. As an alternative, one can use the next measure “performance rank”.

5. Performance rank. Performance rank represents the rank of an algorithm in solving a certain function based on the mean best value found within the pre-specified maximal number of evaluations. The algorithm with the best function value found will be ranked first and so on. We compare the averaged performance rank that each algorithm earns in solving all tested functions. The lower the performance rank is the better.

6. Normalized CPU time. Another interesting measure is the computational cost of each algorithm, which can be evaluated using CPU time. The normalized CPU time for each strategy will be listed and compared. Since the maximal number of evaluations (equivalent to maximal CPU time) is varied from problem to problem, for fair comparison, the mean of CPU time for each strategy is normalized by the maximal mean for the problem. The smaller the normalized average CPU time is the better.

4.3.3 Test results

The test results are organized into two major tables. Table 4.2 includes three indices measuring the effectiveness of each local search application strategy. On the other hand, Table 4.3 includes the other three indices measuring the efficiency of each strategy.
In Table 4.2, the mean and standard deviation of success rate/performance rank for each local search application strategy are listed. The last column of Table 4.2 shows how many functions out of 19 were successfully solved by each strategy in all 30 runs. For each performance measure, the best three strategies are highlighted in bold. It can be observed that ESP is the best local search application strategy with 93.86% average success rate. Based on this result, it was decided to combine this most promising strategy, i.e., ESP, with other strategies together; in so doing, eight combinations of local search strategies were constructed (LS10-17). Focusing on success rate, 5 strategies score higher than 90%. Among them, the NUS+ESP combination is the best with 94.22% average success rate. Surprisingly, ESP is only slightly better than applying LS unconditionally (93.86% vs. 93.69%), which to some extent implies that the percentage of individuals subject to LS matters not much. Both strategies start exploration right from the beginning. As far as performance rank is concerned, the best three strategies in descending order are All, NUS+ESP and Dradius+ESP, respectively. The same three methods excel in performance rank also outperform all others in the number of problems successfully solved in all 30 runs, i.e., 13 out of 19 problems. Applying LS to all means every new trail solution has been exploited to its best possible value. The global optimum is reported to be found more likely using this local search strategy based on its performance rank (1st place) and number of problems successfully solved (1st place, tie with other two).

Considering all three indices together, the NUS+ESP application strategy is a very effective and reliable one because it is in the top three for all three indices. Though ESP has the highest
success rate, it solved only 11 functions in all 30 runs. Though Dradius solved 12 functions, its success rate, however, is only 85.44%. Considering the fact that the two methods which solved 11 functions both have a success rate above 90%, it seems to imply that the effectiveness of Dradius is to some extent problem-dependent and not as stable and effective as ESP. The reason is that ESP has a very balanced percent success rate over all problems and it outperforms Dradius in terms of percent success rate. This fact is also reflected from their corresponding standard deviation values. The standard deviation of Dradius more than doubles that of ESP. A detailed table of percent success rate (Table 4.4) for each strategy on all 19 functions individually will be further discussed in the next section.

Table 4.3 summarizes the normalized average number of evaluations, normalized average elapsed CPU time and success performance of each local search application strategy. The best three among each category are shown in bold. In terms of success performance, EUGbest+ESP is the best, followed by ESP and NUS+ESP. When considering the normalized average number of evaluations taken in solving all problems, EUGbest+ESP is again the best, followed by ESP and Ddomain+ESP. As far as normalized average CPU time is concerned, ESP is the best followed by EUGbest+ESP and NUS+ESP. The above results indicate that ESP is overall a very efficient strategy because the best three in Table 4.3 are either ESP itself or combinations of ESP with others.
<table>
<thead>
<tr>
<th>Method</th>
<th>Success Rate</th>
<th>Performance Rank</th>
<th># of function solved in all runs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
<td>Mean</td>
</tr>
<tr>
<td>1. NUS</td>
<td>0.8315</td>
<td>0.2595</td>
<td>8.7895</td>
</tr>
<tr>
<td>2. EveryUpdatedGbest</td>
<td>0.7842</td>
<td>0.2891</td>
<td>11.5263</td>
</tr>
<tr>
<td>3. GBP</td>
<td>0.5964</td>
<td>0.3423</td>
<td>15.6842</td>
</tr>
<tr>
<td>4. NOS</td>
<td>0.5333</td>
<td>0.4214</td>
<td>12.9474</td>
</tr>
<tr>
<td>5. TRS</td>
<td>0.6526</td>
<td>0.4296</td>
<td>9.1053</td>
</tr>
<tr>
<td>6. ESP</td>
<td><strong>0.9386</strong></td>
<td>0.1288</td>
<td>6.8947</td>
</tr>
<tr>
<td>7. Dradius</td>
<td>0.8544</td>
<td>0.2815</td>
<td>7.2632</td>
</tr>
<tr>
<td>8. NNG</td>
<td>0.7211</td>
<td>0.3795</td>
<td>8.1053</td>
</tr>
<tr>
<td>9. Ddomain</td>
<td>0.6597</td>
<td>0.3998</td>
<td>13.5789</td>
</tr>
<tr>
<td>10. NUS+ESP</td>
<td><strong>0.9422</strong></td>
<td>0.1337</td>
<td><strong>4.7895</strong></td>
</tr>
<tr>
<td>11. Dradius+ESP</td>
<td>0.9123</td>
<td>0.1816</td>
<td><strong>5.5263</strong></td>
</tr>
<tr>
<td>12. EUGbest+ESP</td>
<td>0.9263</td>
<td>0.1496</td>
<td>7.6842</td>
</tr>
<tr>
<td>13. NNG+ESP</td>
<td>0.8509</td>
<td>0.2356</td>
<td>6.7368</td>
</tr>
<tr>
<td>14. TRS+ESP</td>
<td>0.6667</td>
<td>0.4225</td>
<td>9.1053</td>
</tr>
<tr>
<td>15. NOS+ESP</td>
<td>0.7176</td>
<td>0.3394</td>
<td>9.8421</td>
</tr>
<tr>
<td>16. GBP+ESP</td>
<td>0.8368</td>
<td>0.2627</td>
<td>9.8421</td>
</tr>
<tr>
<td>17. Ddomain+ESP</td>
<td>0.8649</td>
<td>0.2314</td>
<td>8.7895</td>
</tr>
<tr>
<td>18. All</td>
<td><strong>0.9369</strong></td>
<td>0.1865</td>
<td><strong>4.3684</strong></td>
</tr>
</tbody>
</table>

*Numbers on the upper right of each heading indicate the corresponding performance measure explained in section 4.3.2*

In summary, the two best local search application strategies are ESP and NUS+ESP because they are both effective and efficient. They all do not use much time to compute and return good results with average success rate above 90%. The NUS+ESP strategy is a perfect example to show how a combination of two local search application strategies might be more effective than using one alone, though at the cost of losing some efficiency. The next two best local search application strategies are EUGbest+ESP and All. Note that although the efficiency of All is not in the top three (in the middle of the pack), it is however very effective (in third place in terms of percent success). On the other hand, although the effectiveness of EUGbest+ESP is not in the top
three (in fourth place in terms of percent success), it is however very efficient (the best in number of evaluations and success performance). When EveryUpdatedGbest is used alone, its performance is not satisfying probably because this strategy focuses too much on exploitation. When it is combined with ESP, the performance of the combined strategy greatly improves because some randomness is introduced into the search process through ESP. It achieved a success rate of 92.63% on average and the whole process for searching the optimum is shortened.

It is very important to include both diversification and intensification into a metaheuristic algorithm design. A good optimization algorithm always seeks to achieve a desirable balance between these two factors. Other factors of interest include the complexity of the optimization problem itself. It should be pointed out that the testing functions used in this study are relatively easy to program. Nevertheless, practitioners should be alerted of the fact that some real-world problem may not be easily expressed into mathematical form and simulating the process might become necessary. In those cases, evaluation/simulation of the problem may dominate the CPU time. In this case, finding the near optimal solution with the fewest number of evaluations is desirable. In order to make a wise choice, practitioner should be well informed of what specific problem they are dealing with and what modeling method is used to formulate the problem, etc.
Table 4.3 Comparison on efficiency in 19 multimodal functions

<table>
<thead>
<tr>
<th>Method</th>
<th>Normalized Number of Evaluations$^2$</th>
<th>Normalized CPU Time$^6$</th>
<th>Success Performance$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
<td>Mean</td>
</tr>
<tr>
<td>1. NUP</td>
<td>0.367</td>
<td>0.219</td>
<td>0.280</td>
</tr>
<tr>
<td>2. EveryUpdatedGbest</td>
<td>0.484</td>
<td>0.354</td>
<td>0.580</td>
</tr>
<tr>
<td>3. GBP</td>
<td>0.500</td>
<td>0.369</td>
<td>0.533</td>
</tr>
<tr>
<td>4. NOS</td>
<td>0.690</td>
<td>0.305</td>
<td>0.487</td>
</tr>
<tr>
<td>5. TRS</td>
<td>0.623</td>
<td>0.341</td>
<td>0.272</td>
</tr>
<tr>
<td>6. ESP</td>
<td><strong>0.243</strong></td>
<td>0.155</td>
<td><strong>0.158</strong></td>
</tr>
<tr>
<td>7. Dradius</td>
<td>0.444</td>
<td>0.313</td>
<td>0.374</td>
</tr>
<tr>
<td>8. NNG</td>
<td>0.696</td>
<td>0.261</td>
<td>0.383</td>
</tr>
<tr>
<td>9. Ddomain</td>
<td>0.487</td>
<td>0.392</td>
<td>0.439</td>
</tr>
<tr>
<td>10. NUS+ESP</td>
<td>0.335</td>
<td>0.176</td>
<td><strong>0.188</strong></td>
</tr>
<tr>
<td>11. Dradius+ESP</td>
<td>0.423</td>
<td>0.314</td>
<td>0.225</td>
</tr>
<tr>
<td>12. EUGbest+ESP</td>
<td><strong>0.237</strong></td>
<td>0.161</td>
<td><strong>0.162</strong></td>
</tr>
<tr>
<td>13. NNG+ESP</td>
<td>0.586</td>
<td>0.256</td>
<td>0.302</td>
</tr>
<tr>
<td>14. TRS+ESP</td>
<td>0.662</td>
<td>0.342</td>
<td>0.296</td>
</tr>
<tr>
<td>15. NOS+ESP</td>
<td>0.620</td>
<td>0.291</td>
<td>0.405</td>
</tr>
<tr>
<td>16. GBP+ESP</td>
<td>0.339</td>
<td>0.271</td>
<td>0.198</td>
</tr>
<tr>
<td>17. Ddomain+ESP</td>
<td><strong>0.334</strong></td>
<td>0.257</td>
<td>0.197</td>
</tr>
<tr>
<td>18. All</td>
<td>0.593</td>
<td>0.340</td>
<td>0.306</td>
</tr>
</tbody>
</table>

*Numbers on the upper right of each heading indicate the corresponding performance measure explained in section 4.3.2

4.4 Discussion

The test results presented in Section 4.3 provide us an overall picture of the performance of different LS application strategies. However, it does not show us detailed information about how each strategy performs on each function individually. To this end, the following tables are prepared for further discussion. Table 4.4 gives the success rate of each strategy in solving each problem. The results indicate that Problems 13 and 15 are the most difficult two among all. Problem 13 has an average success rate of 28.52% while problem 15 has an average success rate
of 29.27% over all 18 strategies. On the other hand, Problems 9 and 14 are the easiest two. All strategies successfully solved them in all runs. Robustness of a method is of practical interest because the global optimum of most real-world problems is unknown. A more robust strategy will always be preferred because it can handle problems of varying difficulty. The dispersion measures of success rates such as variation, standard deviation, and range all provide some indication of the robustness of a strategy. Accordingly, NOS (LS 4), TRS (LS 5), NNG (LS 8), Ddomain (LS 9), and TRS+ESP (14) are examples of non-robust strategies that perform well on certain functions while perform badly on others. Note that they all have a range value of 1 (the largest possible), as shown in the last row of Table 4.4. The three best strategies mentioned in the previous section all have small standard deviation and range values, implying that they will more likely be able to solve different types of problems equally well.

To test whether the differences in success rates, CPU times and number of evaluations are significant, ANOVA analyses were carried out. Subsequently, the overall performance of different LS application strategies were analyzed by using the multiple comparison procedure “Tukey’s HSD (Honestly Significant Difference)” (1953) implemented in the statistical package MINITAB. Lastly, we further analyzed all 18 LS application strategies by using another multiple comparison procedure referred to as “Hsu’s multiple comparisons with the best (Hsu’s MCB)” (1981), also available in MINITAB. “Hsu’s MCB” performs comparisons between each sample mean and the “best” of all the other means, aiming to find whether a global best strategy exists or not.
Table 4.4 Success rate of 19 problems for all 18 local search application strategies

<table>
<thead>
<tr>
<th></th>
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<tbody>
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<td>1</td>
<td>50000</td>
<td>0.933</td>
<td>0.8</td>
<td>0.133</td>
<td>0.233</td>
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<td>0.933</td>
<td>1</td>
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<td>0.933</td>
<td>0.967</td>
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<td>0.833</td>
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<td>0.767</td>
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<td>0.733</td>
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<td>0.867</td>
<td>0.6</td>
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<td>0.633</td>
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<td>1</td>
<td>1</td>
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<td>0.967</td>
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<td>1</td>
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<tr>
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<td>0.667</td>
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<td>0.7</td>
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<td>0.367</td>
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<td>0.133</td>
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<td>0.967</td>
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<td>1</td>
<td>0.833</td>
<td>0.167</td>
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</tr>
</tbody>
</table>


Range 0.867 0.9333 0.9333 1 1 0.5 0.933 1 1 0.5 0.633 0.533 0.8 1 0.933 0.8 0.733 0.8
In the Hsu’s MCB procedure, the critical value is obtained from Dunnett’s distribution (1955). The confidence interval (CI) are calculated as
\[ \min(\bar{Y}_i - \bar{Y}_j - CV \sqrt{\frac{2MSE}{n}}, 0), \max(\bar{Y}_i - \bar{Y}_j + CV \sqrt{\frac{2MSE}{n}}, 0) ] \]
in which \( \bar{Y}_i \) is the mean achieved by each LS application strategy, \( \bar{Y}_j \) is the best mean achieved such that \( i \neq j \), \( CV \) is the corresponding one-sided Dunnett critical value, \( MSE \) is the mean square error and \( n \) is the number of functions tested (Minitab technical support document). In both tests, the family error rate is 5% and the computed intervals are 98% confidence intervals. The results of all of the above-mentioned analyses are presented below. The \( p \)-value of 0.000 in Report 4.1 indicates that the mean success rate for at least one LS strategy is significantly different from those of the others. Based on the Tukey’s method, the confidence intervals for all possible pairwise comparisons are computed (not shown to save space). The results indicate that GBP is significantly inferior to the any strategy in the group \{ESP, NUS+ESP, All\} and NOS is significantly inferior to any strategy in the set \{ESP, NUS+ESP, Dradius+ESP, EUGbest+ESP, All\}.

We further analyzed the overall performance of LS application strategies by Hsu’s MCB. As explained before, “Hsu’s MCB” compares each sample mean with the “best” of all other means, attempting to identify which strategy is the best. For each comparison, Hsu’s MCB computes a confidence interval. A statistically significant difference can only be observed between corresponding means if one of the endpoints of the interval is zero. The results, computed by setting the default options of statistical package MINITAB, are depicted in Report 4.1. Since zero
is not one of the endpoints in all other confidence intervals, hence the NUS+ESP strategy, that has the highest success rate, cannot be claimed as a globally best strategy in statistical sense. However, NUS+ESP is significantly better than any strategy in the set \{GBP, NOS, TRS, Ddomain, TRS+ESP\}.

**Report 4.1 ANOVA report of percent success rates among different LS Strategies**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor</td>
<td>17</td>
<td>5.3610</td>
<td>0.3154</td>
<td>3.53</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
<td>324</td>
<td>28.9738</td>
<td>0.0894</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>341</td>
<td>34.3348</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S = 0.2990</td>
<td>R-Sq = 15.61%</td>
<td>R-Sq(adj) = 11.19%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hsu's MCB

Family error rate = 0.05

One-sided Dunnett Critical value = 2.60

Intervals for level mean minus largest of other level means

<table>
<thead>
<tr>
<th>Level</th>
<th>Lower</th>
<th>Center</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUS</td>
<td>-0.3629</td>
<td>-0.1106</td>
<td>0.1416</td>
</tr>
<tr>
<td>EveryUpdatedGbest</td>
<td>-0.4102</td>
<td>-0.1579</td>
<td>0.0944</td>
</tr>
<tr>
<td>GBP</td>
<td>-0.5980</td>
<td>-0.3457</td>
<td>0.0000</td>
</tr>
<tr>
<td>NOS</td>
<td>-0.6611</td>
<td>-0.4088</td>
<td>0.0000</td>
</tr>
<tr>
<td>TRS</td>
<td>-0.5418</td>
<td>-0.2895</td>
<td>0.0000</td>
</tr>
<tr>
<td>ESP</td>
<td>-0.2558</td>
<td>-0.0035</td>
<td>0.2488</td>
</tr>
<tr>
<td>Dradius</td>
<td>-0.3400</td>
<td>-0.0877</td>
<td>0.1645</td>
</tr>
<tr>
<td>NNG</td>
<td>-0.4734</td>
<td>-0.2211</td>
<td>0.0312</td>
</tr>
<tr>
<td>Ddomain</td>
<td>-0.5348</td>
<td>-0.2825</td>
<td>0.0000</td>
</tr>
<tr>
<td>NUS+ESP</td>
<td>-0.2488</td>
<td>0.0035</td>
<td>0.2558</td>
</tr>
<tr>
<td>Dradius+ESP</td>
<td>-0.2822</td>
<td>-0.0299</td>
<td>0.2224</td>
</tr>
<tr>
<td>EUGbtest+ESP</td>
<td>-0.2681</td>
<td>-0.0158</td>
<td>0.2364</td>
</tr>
<tr>
<td>NNG+ESP</td>
<td>-0.3435</td>
<td>-0.0913</td>
<td>0.1610</td>
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<tr>
<td>TRS+ESP</td>
<td>-0.5277</td>
<td>-0.2754</td>
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<tr>
<td>NOS+ESP</td>
<td>-0.4768</td>
<td>-0.2245</td>
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<td>GBP+ESP</td>
<td>-0.3576</td>
<td>-0.1053</td>
<td>0.1470</td>
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<tr>
<td>Ddomain+ESP</td>
<td>-0.3295</td>
<td>-0.0773</td>
<td>0.1750</td>
</tr>
<tr>
<td>All</td>
<td>-0.2575</td>
<td>-0.0053</td>
<td>0.2470</td>
</tr>
</tbody>
</table>
Report 4.2 gives the analyses results of CPU times. The $p$-value of 0.000 indicates that the mean CPU time for at least one LS application strategy is significantly different from the others.

Report 4.2 ANOVA report of normalized CPU times among different LS Strategies

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor</td>
<td>17</td>
<td>5.36</td>
<td>0.3154</td>
<td>5.07</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
<td>324</td>
<td>20.17</td>
<td>0.0622</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>341</td>
<td>25.53</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$S = 0.2495$  $R$-Sq = 21.00%  $R$-Sq(adj) = 16.86%

Hsu's MCB

Family error rate = 0.05

One-sided Dunnett Critical value = 2.60

Intervals for level mean minus smallest of other level means

<table>
<thead>
<tr>
<th>Level</th>
<th>Lower</th>
<th>Center</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUS</td>
<td>-0.088</td>
<td>0.1226</td>
<td>0.3331</td>
</tr>
<tr>
<td>EveryUpdatedGbest</td>
<td>0.000</td>
<td>0.4222</td>
<td>0.6327</td>
</tr>
<tr>
<td>GBP</td>
<td>0.000</td>
<td>0.3758</td>
<td>0.5862</td>
</tr>
<tr>
<td>NOS</td>
<td>0.000</td>
<td>0.3295</td>
<td>0.5400</td>
</tr>
<tr>
<td>TRS</td>
<td>-0.096</td>
<td>0.1142</td>
<td>0.3247</td>
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<tr>
<td>ESP</td>
<td>-0.214</td>
<td>-0.0043</td>
<td>0.2061</td>
</tr>
<tr>
<td>Dradius</td>
<td>0.000</td>
<td>0.2161</td>
<td>0.4265</td>
</tr>
<tr>
<td>NNG</td>
<td>0.000</td>
<td>0.2256</td>
<td>0.4360</td>
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<tr>
<td>Ddomain</td>
<td>0.000</td>
<td>0.2812</td>
<td>0.4916</td>
</tr>
<tr>
<td>NUS+ESP</td>
<td>-0.180</td>
<td>0.0300</td>
<td>0.2405</td>
</tr>
<tr>
<td>Dradius+ESP</td>
<td>-0.143</td>
<td>0.0676</td>
<td>0.2781</td>
</tr>
<tr>
<td>EUGbest+ESP</td>
<td>-0.206</td>
<td>0.0043</td>
<td>0.2148</td>
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<tr>
<td>NNG+ESP</td>
<td>-0.066</td>
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</tr>
<tr>
<td>TRS+ESP</td>
<td>-0.072</td>
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<td>0.3488</td>
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<tr>
<td>NOS+ESP</td>
<td>0.000</td>
<td>0.2478</td>
<td>0.4583</td>
</tr>
<tr>
<td>GBP+ESP</td>
<td>-0.170</td>
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<tr>
<td>Ddomain+ESP</td>
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<td>0.0392</td>
<td>0.2496</td>
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<tr>
<td>All</td>
<td>-0.062</td>
<td>0.1483</td>
<td>0.3587</td>
</tr>
</tbody>
</table>

According to the confidence intervals for all-possible pairwise comparisons computed, the Tukey results indicate that NUS is significantly superior to EveryUpdatedGbest while
EveryUpdatedGbest is significantly inferior to any strategy in the set \{TRS, ESP, NUS+ESP, Dradius+ESP, EUGb\textit{e}st+ESP, TRS+ESP, GBP+ESP, Ddomain+ESP\}. GBP is significantly inferior to any strategy in the set \{ESP, NUS+ESP, Dradius+ESP, EUGb\textit{e}st+ESP, GBP+ESP, Ddomain+ESP\} and NOS is significant inferior to any strategy in the set \{ESP, NUS+ESP, EUGb\textit{e}st+ESP, GBP+ESP, Ddomain+ESP\}. Report 4.2 also shows the Hsu’s MCB results.

Recall that ESP has the lowest mean CPU time among all strategies. Since zero is not one of the endpoints in all confidence intervals, ESP cannot be claimed as the global best in statistical sense. However, ESP is significantly faster than any strategy in the set \{EveryUpdatedGbest, Gbest\textit{w}ithP, NOS, Dradius, NNG, Ddomain, NOS+ESP\}.

Similarly, the p-value (=0.000) in Report 4.3 indicates that mean number of evaluations for at least one LS application strategy is significantly different from the others. The Tukey results indicate that (i) NOS is significantly inferior to any strategy in the set \{ESP, NUS+ESP, EUGb\textit{e}st+ESP, GBP+ESP, Ddomain+ESP\}, (ii) TRS is significantly inferior to any strategy in the set \{ESP, EUGb\textit{e}st+ESP\}, (iii) ESP is significantly superior to any strategy in the set \{NNG, NNG+ESP, TRS+ESP, NOS+ESP, All\}, (iv) NNG is significantly inferior to any strategy in the set \{NUS+ESP, EUGb\textit{e}st+ESP, GBP+ESP, Ddomain+ESP\}, and (v) EUGb\textit{e}st+ESP is significantly superior to any strategy in the set \{NNG+ESP, TRS+ESP, NOS+ESP, All\}.

Report 4.3 also shows the Hsu’s MCB results. EUGb\textit{e}st+ESP cannot be claimed as the global best; however, it requires significantly fewer number of evaluations than any strategy in the set \{GBP, NOS, TRS, NNG, Ddomain, NNG+ESP, TRS+ESP, NOS+ESP, All\}. 


Report 4.3 ANOVA report of normalized number of evaluations among different LS Strategies

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor</td>
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<td>0.000</td>
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<td>Error</td>
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<td>27.7316</td>
<td>0.0856</td>
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</tr>
<tr>
<td>Total</td>
<td>341</td>
<td>35.1768</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ S = 0.2926 \quad \text{R-Sq} = 21.17\% \quad \text{R-Sq(adj)} = 17.03\% \]

Hsu's MCB
Family error rate = 0.05
One-sided Dunnett Critical value = 2.60
Intervals for level mean minus smallest of other level means

<table>
<thead>
<tr>
<th>Level</th>
<th>Lower</th>
<th>Center</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUS</td>
<td>-0.1173</td>
<td>0.1295</td>
<td>0.3763</td>
</tr>
<tr>
<td>EveryUpdatedGb</td>
<td>-0.1005</td>
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<td>0.3931</td>
</tr>
<tr>
<td>GBP</td>
<td>0.0000</td>
<td>0.2626</td>
<td>0.5094</td>
</tr>
<tr>
<td>NOS</td>
<td>0.0000</td>
<td>0.4527</td>
<td>0.6995</td>
</tr>
<tr>
<td>TRS</td>
<td>0.0000</td>
<td>0.3859</td>
<td>0.6327</td>
</tr>
<tr>
<td>ESP</td>
<td>-0.2413</td>
<td>0.0055</td>
<td>0.2523</td>
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<td>Dradius</td>
<td>-0.0401</td>
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<td>0.4535</td>
</tr>
<tr>
<td>NNG</td>
<td>0.0000</td>
<td>0.4582</td>
<td>0.7050</td>
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<tr>
<td>Ddomain</td>
<td>0.0000</td>
<td>0.2493</td>
<td>0.4961</td>
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<tr>
<td>NUS+ESP</td>
<td>-0.1491</td>
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<td>0.3446</td>
</tr>
<tr>
<td>Dradius+ESP</td>
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<td>0.4325</td>
</tr>
<tr>
<td>EUGbest+ESP</td>
<td>-0.2523</td>
<td>-0.0055</td>
<td>0.2413</td>
</tr>
<tr>
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<td>0.5955</td>
</tr>
<tr>
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<td>0.4243</td>
<td>0.6712</td>
</tr>
<tr>
<td>NOS+ESP</td>
<td>0.0000</td>
<td>0.3829</td>
<td>0.6297</td>
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<tr>
<td>GBP+ESP</td>
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<td>0.3483</td>
</tr>
<tr>
<td>Ddomain+ESP</td>
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<td>0.3433</td>
</tr>
<tr>
<td>All</td>
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<td>0.3554</td>
<td>0.6022</td>
</tr>
</tbody>
</table>

Next, we turn our attention to the searching pattern of each LS application strategy. Our hybrid metaheuristic is designed to track the average, the best solution of the population and their corresponding number of evaluations over time. In the following, the convergence profiles of the average best solution over all 30 runs are presented. Strictly speaking, convergence profiles of
the average of the average objective value over all 30 runs can also be shown. Such profiles, however, are not shown because of the difficulty in plotting them all in one figure where some strategies have infinite values. The objective value of an infeasible solution is set to be infinite in this study in using the Deb’s constraint handling method (2000) and the average is rendered to be infinite if any strategy does not find a feasible solution in any one run. The purpose of this analysis is to visually show the difference in searching patterns between different LS application strategies. Firstly, the starting point of each profile marks the time when the solution becomes feasible. Secondly, the final magnitude of the convergence profile is an indirect relative indicator of the % successful rate. The lower the final magnitude is, the higher the successful rate of the average best solution over all runs.

The convergence profiles of the average best solution over all 30 runs of each LS application strategy for two selected problems (1 and 13) are shown in Figs. 4.2 and 4.3, respectively. The 18 LS application strategies are distinguished by different colors and format in order to make it as clear as possible. Problem 1 is moderately difficult in terms of average successful rate while Problem 13 is the most difficult. They are selected to show how the search patterns of different LS application strategies vary. Note that Problem 1 has a global optimum of 87.5. Judging from the convergence profiles of Problem 1, most strategies perform the search very efficiently because their objective values quickly reduce to the minimum. However, a few strategies perform rather poorly on this problem. The performance of LS 3 is the worst in this regard; its profile is relatively flat and stays high throughout the search and does not converge to the
minimum. Notice that the successful rate of LS 3 to Problem 1 is only 0.1333, which is the lowest among all. This explains why its profile is far away from the others. Besides LS 3, both LS 4 and LS 15 found feasible solution fairly late and their convergence-profiles are also very flat and do not go as low as other strategies do.

![Convergence rates of the average best solution of problem #1 over 30 runs](image)

Figure 4.2 Convergence rates of the average best solution of problem #1 over 30 runs

Similarly, Fig. 4.3 shows the corresponding convergence profile of problem 13. Note that Problem 13 has a global optimum of 5850.77. It can be seen from Fig. 4.3 that LS 3 is the worst in terms of average best solution over all 30 runs. The convergence profile is very flat and does not converge at a low value. Note that LS 4 does not find a feasible solution until very late in the search. Also note that none of those convergence profiles attain the real global optimum of 5850.77. This is because none of those LS application strategies has been 100% successful in finding the optimum.
4.5 Summary and Conclusions

This chapter has presented a comprehensive comparative study of 18 local search application strategies incorporated into a newly developed hybrid metaheuristic (Duan et al., 2013, Yi, et al., 2013). All local search application strategies were tested with 18 benchmark functions plus one additional supply chain consignment problem using the same set of algorithmic parameters for fair comparison. According to the test results, ESP is the best single local search application strategy in terms of successful rate. Integrating it with NUS further improve the overall effectiveness. In particular, the NUS+ESP strategy outperforms other strategies in all three accuracy-related indices (i.e., success rate, performance rank and number of
functions solved in all 30 runs). In terms of efficiency, EUGbest+ESP is the best because it outperforms others in all three efficiency-related indices (i.e., normalized number of evaluations, normalized CPU time and success performance). Moreover, it is able to achieve pretty high level of accuracy as well (specifically, the fourth place in success rate). In summary, this study identifies the best three LS strategy is ESP, NUS+ESP and EUGbest+ESP. Noticed that ESP takes the least CPU time among all (most efficient) at the same high accuracy is warranted, it is decided to use ESP as the local search strategy in the following supply chain problem.
CHAPTER 5 OPTIMIZATION OF DETERMINISTIC SUPPLY CHAIN INVENTORY PROBLEM AND ITS MANAGERIAL IMPLICATIONS

5.1 Introduction

In this chapter, the simulation optimization framework based on the newly developed hybrid metaheuristic is applied to capacitated inventory systems facing various demand patterns. The input demand data can be either the raw sales data or predicted demand data generated from any forecasting method. Based on the deterministic customer demands, the optimal replenishment policies of capacitated supply chains (SC) operating under different control strategies (decentralized vs. centralized) are determined and insights useful to management are discussed. The overall methodology can be called as a Deterministic Metaheuristic-based Simulation Optimization (DMSO) framework. In the numerical example, a two-tier supply chain consisted of a distributor distributing a single product to multiple retailers is considered. Four levels of supplier’s capacity tightness are investigated through ten different demand patterns, under centralized and decentralized control. A periodic review replenishment policy, specifically \((s, S)\), is adopted by both the distributor and retailers. The distributor orders from a capacitated supplier and the retailers face varying deterministic customer demands. Unfulfilled demand is assumed lost to resemble the case in fast-moving retail market; customers who do not find what they want most likely will shop somewhere else. Both decentralized and centralized models are simulated for performance evaluation; and the optimal replenishment policies for all members of the supply chain are determined by the DMSO framework.
The developed framework does not require any unrealistic assumption on demand. Ten different demand patterns are tested in this study. The developed framework allows to (1) determine near-optimal replenishment policies for each SC member under both decentralized and centralized control strategies; (2) to examine and quantify the effect of different demand patterns, supplier’s capacity constraint and control strategies (centralized/decentralized) on the unit inventory cost; (3) to investigate the changes in various internal performance measures such as ordering patterns, cost distribution and internal service level, and (4) to develop an incentive mechanism for coordinating a decentralized supply chain. The majority of analytical models can only provide general information on system performance such as the overall system profit/cost based on restrictive assumptions. The proposed framework is advantageous over analytical models in its ability to produce additional detailed information to enable deeper investigation of the dynamics of internal system operations, thus gaining some useful insights for inventory managers to make better and more informative decisions. In summary, the main contribution of this study is three-fold: 1) to be the first that develops both centralized and decentralized models for a capacitated inventory system focusing on the retailers' side; 2) to investigate the interactions between capacity constraint, control strategy, and demand; and 3) to evaluate the performance of the supply chain in more details from different angles to provide some useful managerial insights.
5.2 The proposed supply chain inventory optimization framework

The proposed framework is designed to find sufficiently good solutions for a decentralized or centralized supply chain inventory problem. For clarity, a brief definition is given below.

**Decentralized control.** All players of the SC make their replenishment decisions based on their local information independently. Players in the SC are treated as individual company in which they aim to minimize their own inventory cost regardless of the system cost.

**Centralized control.** Centralized decisions are made to minimize the overall SC inventory cost. It can be applied when the distributor and all retailers belong to the same enterprise. It can also be adopted for special configurations, e.g., vendor-managed inventory (VMI).

The overall structure of the proposed framework is depicted in Fig. 5.1. The hybrid meta-heuristic generates a trial solution and supplies this candidate solution to the SC simulation model to evaluate its objective function value. Based on the result provided by the evaluation module, the meta-heuristic optimizer then generates a new input set according to its intelligent searching mechanism. This cycle is repeated until a near-optimum solution is obtained. Such a simulation optimization formulation allows more realistic modeling of the inventory control system.

Generally speaking, a supply chain inventory model can be adapted to different network configuration, replenishment policies, control strategies, etc. In this study, inventory models are tailored to a single-distributor multi-retailer supply chain system. The retailers can be identical or non-identical. It is assumed that all supply chain members adopt the \((s, S)\) inventory policy; but
other policies can be easily implemented. The distributor faces the possibility of insufficient capacity from its supplier. The retailers, on the other hand, are faced with deterministic demand with varying fluctuating patterns. The inventory systems are reviewed periodically over a finite planning horizon.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>System</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand data</td>
<td>Meta-heuristic optimizer</td>
<td>Near-optimal policy</td>
</tr>
<tr>
<td>Capacity constraint</td>
<td>Generate a candidate solution</td>
<td>Inventory cost</td>
</tr>
<tr>
<td>Control strategy</td>
<td>Performance analysis</td>
<td>Ordering pattern</td>
</tr>
<tr>
<td>Allocation rule</td>
<td>SC simulation model</td>
<td>Cost distribution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Internal Service level</td>
</tr>
</tbody>
</table>

Figure 5.1 The overall structure of the proposed framework

5.2.1 Mathematical formulation of both decentralized and centralized capacitated supply chain inventory models

The mathematical formulation of the supply chain inventory model in consideration of capacity constraint is described below. The notations used in the mathematical formulation of the inventory system are first introduced as detailed in Table 5.1.
Table 5.1 Notations used in the mathematical formulation of the inventory system

<table>
<thead>
<tr>
<th>Subscripts</th>
<th>Notations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>period $t$ in the planning horizon, $t=1, 2, \ldots, T$</td>
</tr>
<tr>
<td>$i$</td>
<td>Supply chain player/entity, $i=0, \ldots, M$</td>
</tr>
</tbody>
</table>

$D_i^t$ Demand at entity $i$ for period $t$

$ax_i^t$ Available on-hand inventory for entity $i$ at the beginning of period $t$

$x_i^t$ Remaining on-hand inventory (after fulfilling customer demand) for entity $i$ at the end of period $t$

$y_i^t$ Target inventory order-up-to level for entity $i$ at time period $t$

$z_i^t$ Actual inventory order-up-to level for entity $i$ at time period $t$

$U$ Supplier capacity

$K_i$ Fixed ordering cost for entity $i$

$h_i$ Holding cost per unit per period for entity $i$

$p_i$ Lost-sale penalty cost per unit for entity $i$

$L_i$ Lead time for entity $i$

Given system state $X^0 = \{x_0^0, x_1^0, x_2^0, \ldots, x_M^0\}$ as the initial inventory level at the beginning of the first period $(t = 0)$, for each time period, $t=1, 2, \ldots, T$, do the following:

At the beginning of period $t$, retailer $i$ observes its customer demand $D_i^t$ and try to satisfy $D_i^t$ with its available on-hand inventory, $ax_i^t$. Any shortfall occurred at period $t$ will be considered as lost sales to retailers with a penalty cost, $p_i$. Any item held at stock at period $t$ will be charged with inventory holding cost, $h_i$. Let $G_i(ax_i^t)$ be the summation of the holding and penalty costs at entity $i$ for period $t$ with on-hand inventory level $ax_i^t$, we have

$$G_i(ax_i^t) = h_i(ax_i^t - D_i^t)^+ + p_i(D_i^t - ax_i^t)^+ \quad (5.1)$$

where $x^+ = \max\{x, 0\}$
Next, retailer $i$ checks its remaining inventory level $x_i'$ and decides to bring its inventory level to $y_i'$. Let $z_i'$ denote the actual inventory level at retailer $i$ after receiving items from the distributor in consideration of capacity constraint. Obviously, $z_i' \leq y_i'$.

The retailers’ orders become demand to the distributor. Two cases can be distinguished in fulfilling the demand by the distributor:

1. If the available inventory at the distributor is abundant, i.e., $ax_0' \geq \sum_{i=1}^{M} (y_i' - x_i')$, all retailers’ orders are fully satisfied.

2. Otherwise, the orders are satisfied according to the ranking of retailers, in descending order of importance with $i=1$ indicating the most important.

Mathematically,

$$z_i' = \begin{cases} 
  y_i', & ax_0' \geq \sum_{i=1}^{M} (y_i' - x_i') \\
  x_i' + \min\{y_i' - x_i', ax_0'\}, & ax_0' < \sum_{i=1}^{M} (y_i' - x_i') \text{ and } i=1 \\
  x_i' + \min\{y_i' - x_i', (ax_0' - \sum_{j=1}^{i-1} (y_j' - x_j'))^+\}, & \text{otherwise}
\end{cases} \quad (5.2)$$

Eq. (2) is the ranking allocation rule employed in this study but it is possible to implement other rules to adapt to different cases. The effect of different allocation rules are discussed in Section 4.2. In addition, it is assumed that the shortfall of each retailer’s ordering quantity will be met by obtaining the shortfall from an “alternative” source but at a higher cost. The distributor will be responsible for the penalty cost to this shortfall. In this way, the supply to each retailer’s
ordering quantity is guaranteed. The penalty cost to the distributor represents the additional cost to the distributor for obtaining items from alternative suppliers, which is presented as \( p_0 \).

Next the distributor observes its inventory level \( x_0 \) and decides to bring its inventory level to \( y'_0 \). Note that the distributor orders from a supplier which has limited capacity. This implies that the distributor’s order might not be fully satisfied. Distributor’s ordering items cannot exceed the supplier’s capacity, \( U_0 \). That is,

\[
y'_0 - x'_0 \leq U_0 \quad \text{for any } t = 1, ..., T \tag{5.3}
\]

and

\[
y'_0 \geq x'_0 \tag{5.4}
\]

All orders placed to upstream are due after certain lead time. As a result, the available inventory for each entity used to satisfy its down-stream customer at the beginning of period \( t \), \( ax'_i \), is given by

\[
ax'_i = \begin{cases} 
x'_i, & t \leq L_i \\
x'_i + (y'_{t-L_i} - x'_{t-L_i}), & t > L_i
\end{cases} \quad i = 0, ..., M. \tag{5.5}
\]

The inventory level at the end of period \( t \) to determine the order quantity for entity \( i \) is calculated as follows:

\[
x'_i = (ax'_i - D'_i)^+ \quad i = 0, ..., M. \tag{5.6}
\]

where \( D'_0 = \sum_{i=1}^{M} (y'_i - x'_i) \)

The target inventory order-up-to levels, \( y'_i \), \( \forall i, t \), are determined accordingly to the subject inventory policy used in the system. In this study, we adopt the widely used \((s, S)\) policy. When
the inventory position (inventory on hand plus outstanding orders) declines to or below $s$, a number of product units are ordered such that the resulting inventory position is raised to $S$. The decision variables thus are $s_i$, $S_i$, $i = 0, 1, \ldots, M$. For the $(s, S)$ policy, the inventory order-up-to levels are determined as follows:

$$y_i^t = \begin{cases} x_i^t, & \text{if } x_i^t + \sum_{j=t-L_t+1}^{t-1} (y_j^t - x_j^t) > s_i \\ S_i - \sum_{j=t-L_t+1}^{t-1} (y_j^t - x_j^t), & \text{else} \end{cases}$$  \hspace{1cm} (5.7)

Denote by $v_i^t(x)$ the total cost of entity $i$ with inventory level $x$ at period $t$. We have:

$$v_i^t(x_i^t) = K_i \delta(y_i^t - x_i^t) + G_i(\alpha x_i^t) \quad i = 0, \ldots, M, \ t = 1, \ldots, T$$  \hspace{1cm} (5.8)

where $\delta(x)$ is an indicator function as follows:

$$\delta(x) = \begin{cases} 1, & x > 0 \\ 0, & x \leq 0 \end{cases}$$  \hspace{1cm} (5.9)

Given the initial system state, the objective function for both decentralized and centralized control, over the planning horizon is determined as follows:

- For decentralized control case, each entity minimizes its own inventory cost over the entire planning horizon, and the summation of the overall cost is computed. In other words,

$$TC_{\text{decentralized}} = \sum_{i=0}^{M} \min\{\sum_{t=1}^{T} v_i^t(x_i^t)\}$$  \hspace{1cm} (5.10)

- For centralized control case, the objective is to minimize the system inventory cost over the entire planning horizon. In other words,
\[ TC_{centralized} = \min \left\{ \sum_{i=1}^{T} \left[ \sum_{i=0}^{M} v_i^j (x_i^j) \right] \right\} \] (5.11)

Unfortunately, closed-form solutions cannot be obtained for the above models and iterative algorithms have to be used to search for near-optimal solutions. In this study, the models are solved by a hybrid metaheuristic optimization algorithm (Duan et al., 2013).

5.2.2. The hybrid meta-heuristic algorithm

The hybrid metaheuristic algorithm (Duan et al., 2013) used to solve both the decentralized and centralized capacitated supply chain inventory models comprises of two cooperative metaheuristic algorithms, i.e., differential evolution (DE) and harmony search (HS), and one local search (LS) method, i.e., Hooke and Jeeves (HJ) direct search. Population-based metaheuristics such as differential evolution (DE) and harmony search (HS) are very good at exploring and identifying promising searching areas (in order to escape from those local optima). The HJ local searcher collaborates with the cooperative hybrid by enhancing the algorithm’s ability to further exploit those promising local areas. It uses the steepest ascent pivot rule in which it deeply exploits all the neighborhood of a considered solution so that the best solution within that area is guaranteed. Therefore, the new hybrid achieves a better balance between exploration and exploitation. Interested readers are referred to (Duan et al., 2013) for a detailed explanation of the hybrid metaheuristic algorithm and its key components (DE, HS and HJ). The following pseudo-code shows how the hybrid metaheuristic is embedded into the proposed framework and how it iteratively improves the quality of solution.
The hybrid metaheuristic optimization procedure

1. Specify DE-related parameters, i.e., \( F \), \( CR \) and \( MaxFE \). Specify HS-related parameters, i.e., \( NI, HMCR, PAR_{min}, PAR_{max}, bw_{min} \) and \( bw_{max} \). Specify HJ related parameters, i.e., \( \lambda \), \( M \) and \( p \). Set the population size of both DE and HS to be equal, i.e., \( NP=HMS \).
2. Randomly generate the initial population of solutions
3. Evaluate the initial population by the Inventory model.
4. While current number of evaluation < \( MaxFE \)
   (5) For each target vector (=1, 2, ..., \( NP \))
   (6) Generate a new trial vector according to DE algorithm
   (7) Construct a new solution according to the HS algorithm
   (8) If the new solutions generated by DE and HS are randomly selected (with \( p=10\%)\),
   (9) Apply HJ local search
   (10) End if
   (11) Evaluate each new solution by the Inventory model;
   (12) Replace the target vector with the new trial vector for the DE algorithm there is an improvement, and update the harmony memory
   (13) Update the best solution found so far
   (14) End for
   (15) End while
(16) Output the best result

5.3 Computational Experiments

To ease analysis without loss of generalization, the considered supply chain is assumed to consist of one distributor and 2 retailers facing identical end customer demand. In the numerical experiments, each deterministic but fluctuating 52-point demand series for each retailer, \( D_i^t \), \( i=1,2, \ t=1,\ldots,52 \), were generated from a known demand distribution. But as pointed out before, the framework is designed to be able to handle any raw demand data series that is of practical relevance. In practice, company’s information systems that keep historical or predicted sales are commonly the source of the input raw data, reflecting the market. The proposed framework is applied to obtain near-optimum solutions for ten different demand patterns under four levels of capacity constraint and two different control strategies: decentralized and centralized.
Specifically, the ten demand series were realized from: Auto-correlated (with $\mu=60.83$, $\sigma=13.39$), Constant (50, 0), Cyclic (50.5, 49.98), Exponential (53.42, 48.89), Gamma (51.56, 22.29), Laplace (49.69, 8.88), Lumpy (35.27, 47.56), Normal (52.67, 18.62), Poisson (59.12, 8.58), and Uniform (47.33, 31.69). 52 points were chosen to simulate weekly demand data over one year period.

The four levels of capacity constraint are $CT=\infty$ (unlimited), 1.33, 1.18 and 1.05. The capacity tightness (CT), proposed by Zhao et al. (2002), refers to how tight the supplier’s production capacity is relative to the demand. Once the demand information for all retailers over the planning horizon is gathered, the grand total capacity needed is calculated (equal to the total demand of all retailers over all the periods, i.e., $D_{total} = \sum_{i=1}^{2} \sum_{t=1}^{52} D_i^t$). In order to generate the available capacity for each period, the CT parameter is used to indicate the ratio of the total available capacity to the total capacity needed. The total capacity available for all periods is equal to the total demand multiplied by the CT factor. It is assumed that this total capacity available is evenly distributed over all simulation periods. The higher the CT value, the higher the capacity is to process orders (in other words, looser constraint).

The lead time to replenish an order is considered constant to avoid the possibility of ordering cross-over caused by the variable lead time. However, the lead time for each player can be different. In this study, the lead time is assumed 2 periods for all members. The on-hand inventory of all nodes in the SC is initialized to be 50 at the beginning of period 1. For all scenarios tested, other inventory-related parameters such as ordering cost per order, holding cost
per unit item and unit time, lost sale cost per unit item, remanufacturing cost per unit item are also fixed at $100, $2, $20 and $10, respectively.

For fair comparison, all parameter settings for all test cases are fixed. The fixed algorithmic parameter values in DE include population size $NP=10 \times \text{Dimensions of the problem}$, $F = 0.5$, and $CR= 0.95$; For HS the parameters include harmony memory size $HMS=10 \times \text{Dimensions of the problem}$, $HMCR=0.99$, $NI=\text{MaxFE}$, $PAR_{\text{min}}=0.45$, $PAR_{\text{max}}=0.99$, $bw_{\text{min}}=1 \times 10^{-5}$, and $bw_{\text{max}}=4$; For HJ local search the parameters include the probability to perform the search $p=0.1$, adaptive step size, $\lambda = 0.1$ and maximum number of cycle, $M=10$. Those parameter values were chosen based on their good performance based on our previous experience.

The best solution obtained over 30 runs is taken as the (near)optimal solution of a given problem with each run stops once it reaches the maximal number of iterations, which is set at $\text{MaxFE}=50000$ based on our preliminary study. All programs were coded in Matlab and all executions were made on a HP Pavilion a4317c with AMD Athlon™ II $\times 2@ 2.70$ GHz.

5.3.1 Verification of the proposed framework

To verify the proposed framework, the optimal solutions $(s_i^*, S_i^*)$ were estimated by brute-force simulation over the $(s_i, S_i)$, $i=0,1,2$, search space, which is a widely-accepted verification method (Angün et al., 2009, Bashyam and Fu 1998, Kleijnen and Wan 2007, Zhang et. al., 2007, just to name a few). To show whether the near-optimal solutions obtained by the proposed framework are closed to the true optimal values estimated through brute force, the
optimality gap, defined as \(\frac{(TC - TC^*)}{TC^*} \times 100\%\), is computed. In addition to the optimality gap, the efficiency is also discussed.

The verification is carried out on three selected demand types: normal, uniform and lumpy demand under centralized control with CT=1.33. The domain for \(s_i\) values are defined as zero to maximum lead time demand while the domain for \(S_i\) values are defined as zero to doubled maximum lead time demand. Note that, for a problem stance of dimension \(K\) and each dimension has \(N\) possible values, a naïve exhaustive search needs \(N^K\) evaluations. The possible problem space grows exponentially with numbers of decision variables, which cause the problem of “curse of dimensionality”.

For normal demand case, the grid was initialized with step size of 100, in the search space defined by \(0 \leq s_i \leq 219\) and \(0 \leq S_i \leq 438\) \((s_i \leq S_i, i = 1,2); 0 \leq s_0 \leq 438\) and \(0 \leq S_0 \leq 876\) \((s_0 \leq S_0)\), which equivalent to 34,560 combinations of candidate solutions. The best combinations among them lie inside the subarea defined by \(100 \leq s_i \leq 200\) and \(100 \leq S_i \leq 200\) \((s_i \leq S_i, i = 1,2); 200 \leq s_0 \leq 300\) and \(200 \leq S_0 \leq 300\) \((s_0 \leq S_0)\). Focusing on this subarea with a step size of 50, 729 combinations of solutions were tested. The result leads to a subarea defined by \(100 \leq s_i \leq 150\) and \(100 \leq S_i \leq 150\) \((s_i \leq S_i, i = 1,2); 200 \leq s_0 \leq 300\) and \(200 \leq S_0 \leq 300\) \((s_0 \leq S_0)\). Continuing searching with a step size of 10, 156,816 solution combinations were tested and the best solution lies in the subarea defined by \(100 \leq s_i \leq 120\) and \(100 \leq S_i \leq 130\) \((s_i \leq S_i, i = 1,2); 200 \leq s_0 \leq 230\) and \(200 \leq S_0 \leq 250\) \((s_0 \leq S_0)\). Continuing searching with a step size of 5, 94,325 solutions were tested and the best solution lies in the subarea defined by \(100 \leq s_i \leq 110\) and \(120 \leq S_i \leq 125\).
\[ s_i \leq S_i, i = 1,2; \quad 200 \leq s_0 \leq 210 \quad \text{and} \quad 240 \leq S_0 \leq 250 \quad (s_0 \leq S_0). \]

Testing 527,076 solutions in this subarea, with step size of one, lead to the final optimal solution.

Similar procedures are applied to uniform and lumpy demand patterns. The results of this verification are given in Table 5.2. The optimality gaps defined by \((TC-TC^*)/TC^* \times 100\%\) and the CPU time (in seconds) are recorded in Table 5.3. As can be seen, the solutions found by the proposed framework are all very close to optimality. The proposed framework achieves the maximal optimality gap as small as 0.2\%, and it is highly efficient (about three orders of magnitude faster).

Table 5.2 Verification results

<table>
<thead>
<tr>
<th></th>
<th>The proposed framework</th>
<th>Brute Force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>((s_0, s_1, s_2; S_0, S_1, S_2))</td>
<td>((s_0^<em>, s_1^</em>, s_2^<em>; S_0^</em>, S_1^<em>, S_2^</em>))</td>
</tr>
<tr>
<td>Normal</td>
<td>(233, 108, 107; 245, 123, 122)</td>
<td>55,784</td>
</tr>
<tr>
<td>Uniform</td>
<td>(214, 100, 102; 339, 137, 139)</td>
<td>61,488</td>
</tr>
<tr>
<td>Lumpy</td>
<td>(139, 17, 16; 256, 94, 94)</td>
<td>63,364</td>
</tr>
</tbody>
</table>

Table 5.3 Solution quality for the verification problems

<table>
<thead>
<tr>
<th></th>
<th>The proposed framework</th>
<th>Brute Force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\text{Gap (%)}})</td>
<td>(\text{CPU (s)})</td>
</tr>
<tr>
<td>Normal</td>
<td>0.0143</td>
<td>59.5344</td>
</tr>
<tr>
<td>Uniform</td>
<td>0.1988</td>
<td>59.0672</td>
</tr>
<tr>
<td>Lumpy</td>
<td>0.1169</td>
<td>59.3734</td>
</tr>
</tbody>
</table>
5.3.2 Analysis of results

In this subsection, the results obtained from the computational experiments in studying the effect of different demand patterns, capacity constraints and control strategies on the supply chain unit inventory cost are reported and analyzed. By generating weekly demand series from a specified demand distribution, there is no way to guarantee that the total demand units over the planning horizon for different demand distribution are exactly the same. For fair comparison, it was decided to divide the total SC inventory cost by the corresponding total demand units over the planning horizon for each demand series. In other words, the comparison is based on cost per unit.

The first analysis carried out was ANOVA to determine the significance of main effects and interaction effects on total supply chain system cost per unit. There are totally 80 possible combinations (10 demand patterns x 4 capacity levels x 2 control strategies). The results indicate that the main effects are all significant at p<0.001. For interaction effect, the interaction between demand and capacity (F-value=6.72, P-value=0.000), between demand and control strategy (F-value=84.3, P-value=0.000) are significant at p<0.001. The system unit cost is affected by the difference of demand patterns the most (F-value=974.23, P-value=0.000), followed by different control strategies (F-value=276.01, P-value=0.000), and then capacity tightness (F-value=22.93, P-value=0.000).

The main effect plot for each factor is shown in Fig. 5.2. Fig. 5.2(a) indicates that higher unit system costs come from relatively more unstable demand patterns with higher standard
deviations, i.e., lumpy, exponential and uniform. Demands such as Gamma, Normal, Laplace and Poisson with lower standard deviations all have moderate unit system cost. Constant demand incurs low unit system cost, which is easy to understand because of its zero variance. Cyclic demand pattern seems to be an exception. Even though Cyclic demand has the highest standard deviation (49.98); it surprisingly has low unit system cost. The reason is that it has a regular cyclic pattern and once it is detected, optimization can be carried out to deal with it as easy as in Constant patterns. Autocorrelated demand also achieves a relatively low unit cost, despite of its relatively high standard deviation of 13.39, which is higher than the standard deviation of Laplace and Poisson. The reason is that there is a dependency in the Autocorrelated demand, making it easier to adapt a replenishment policy to better handle it. In summary, in order to make better decision practitioners are suggested to examine at least the following three characteristics in the demand data: the standard deviation of the data; the cyclic pattern of the data and the autocorrelation among the data.

The main effects of capacity tightness and control strategy are plotted in Fig. 5.2(b) and 5.2(c), respectively. Generally speaking, tighter capacity constraint leads to higher unit system cost and the rate of increase is almost linear. Consistent with previous studies in the open literature, centralized control has a lower unit system cost than decentralized control; the difference is approximately 11% on average. Hence, the total cost difference will be substantial if the demand volume is high.
Figure 5.2 Main effect plot for each considered factor
(a) Main effect plot of demand for unit cost
(b) Main effect plot of capacity for unit cost
(c) Main effect plot of control for unit cost
Fig. 5.3 shows the three two-factor interaction plots. The interaction between demand and capacity, between demand and control strategy are significant at p<0.001, as shown in Fig. 5.3(a) and 5.3(b), respectively. It implies that both the effect of capacity constraint and the effect of control strategy on the unit system inventory cost depend on the demand pattern. Lumpy, Exponential, Uniform, Autocorrelated and Cyclic demand patterns seems to be affected by capacity constraint a lot (the slope of their corresponding profile are relatively steep). For Lumpy and Cyclic demand patterns, the unit-cost increase due to the imposing of capacity constraint appears to be most significant. To further investigate the cause of such significant cost increase, ordering patterns for Lumpy demand with centralized control under CT=∞ and CT=1.33 are examined in more details. (Ordering patterns for other demands can also be generated in a similar fashion but it is omitted here to avoid redundancy and also to save space).

As shown in Fig. 5.4, the ordering patterns for the SC system for Lumpy demand change significantly from CT=∞ and CT=1.33. Operating under no capacity constraint (Fig. 5.4(a)), the system minimizes its total cost by ordering several big lots less frequently. Retailer 1 and Retailer 2 take turns to order from the distributor. The lot size from Retailer 1 is slightly smaller than Retailer 2 because Retailer 1 orders 7 times while Retailer 2 orders 6 times over the planning horizon (recall that both retailers are assumed to have the same demand). The system intelligently balances the orders from each retailer and the availability of the supplier and come up with a solution to minimize the system cost.
Figure 5.3 Interaction plot for any two of the considered factors
Operating under capacity constraint (Fig. 5.4(b)), the ordering patterns for the whole system are significantly changed. Each player tends to order smaller quantity more frequently. The ordering quantity of distributor reduces greatly from 365 to 93 because of the constraint and it has to order every period after period 32 when the system is experiencing a sudden increase of its demand volume. The ordering size from retailers also reduced, from almost 200 per order to less than 100 per order with increased ordering frequency.

Figure 5.4 Ordering patterns for Lumpy demand with centralized control

Based on the results presented above, it is shown that the proposed framework clearly is capable of providing detailed analyses internal to the system if necessary. The standard
deviations of Lumpy and Cyclic demand patterns are the highest among all. Therefore, by imposing capacity constraint greatly changes the ordering patterns and increase the unit system cost. For demand patterns with relative small standard deviations, the changes in ordering patterns are not as significant. It is thus concluded that capacity constraint poses a more serious threat to unstable demand patterns (those with high standard deviation) than to stable demand patterns. Without proper optimization of the replenishment policies, the system will end up with unnecessary cost increase. Taking Lumpy demand as an example, by plugging the solution found with unlimited capacity constraint into the case with capacity constraint, say CT=1.33, it will result in a 31.27% cost increase per unit.

From Fig. 5.3(c), it can be concluded that centralized control generally are better than decentralized control, regardless of the level of capacity constraint. The unit system costs achieved by centralized control are all lower than those of decentralized control. However, the magnitude of the savings differs from one demand pattern to another as shown in Fig. 5.3(b). It is observed that adopting centralized control is more likely to lead to substantial savings for demand patterns such as Lumpy, Exponential, Gamma and Laplace. For regular demand patterns such as Constant and Cyclic, the savings are relatively less. Note again that the comparison is performed based on unit system cost. For those demand patterns with relatively insignificant savings in terms of unit system cost, the savings are expected to become substantial if the demand volume is large. The results clearly suggest the benefit of using centralized control over
decentralized control. Independent companies should look for ways to collaborate with each other to get close to centralized control as possible.

5.4 Discussion

5.4.1 Cost distribution and coordination of distributed SC

In this section, the Normal demand is used as an example to discuss possible coordination mechanisms so that every player in the SC will be better off in order to realize the benefit of centralized control. Normal demand is chosen because it is typically assumed in most inventory control literature and commonly used in practice, mainly due to the Central Limit Theorem.

The proposed framework enables us to record cost of each individual SC member, as shown in Table 5.4. This provides a clear picture of the cost distribution among SC members under decentralized versus centralized control. Table 5.4 records the total cost for each SC member over the entire planning horizon instead of unit cost, aiming to show the magnitude more clearly. The percentage savings (last column of Table 5.4) realized by centralized control under all 4 levels of capacity constraint are shown and should remain the same as if calculated based on unit cost. It can be seen that centralized control overall result in lower system cost and the percentage savings range from 2.4% to 5.1%. According to Fig. 5.3(b), Normal demand is one of those demand patterns that make the least savings. The benefit will be even higher for other demand patterns.

Centralized control strategy is clearly better than decentralized control. However, centralized control might not be appropriate for every supply chain. In such case, it is essential to put in
place some coordination mechanism so that conflicting interests between individual companies can be smoothed over so that the entire supply chain behave as close to centralized control as possible. Retailers probably are not willing to adopt centralized control because they have to bear more cost than what they do in decentralized control (see Table 5.4). The results indicate that centralized control favors the distributor at the expense of the retailers. To entice the retailers to bear a higher inventory cost, it is recommended that the distributor must offer some incentive in the form of a fixed fee to each retailer up front. The fixed fee amount shall be determined in such a way that each retailer only pay at most as much as they do under decentralized control. The goal is to motivate the retailer to accept centralized control so that the whole system is coordinated and the overall system cost is reduced. It is worth mentioning that Chen et al. (2001) discussed discount pricing as the main mechanism to coordinate a one-supplier, multi-retailer supply chain, assuming ample supply without any capacity constraint.

The “After Incentive” column of Table 5.4 shows the base-line case in which each retailer does not suffer any cost increase by accepting centralized control and the overall system cost is the same as the amount in centralized control without incentive. This is not a coincidence. Let $A_i$ be the cost for SC member $i$ under centralized control while $B_i$ the cost for SC member $i$ under decentralized control ($i = 0$ stands for the distributor, and $i = 1, 2, \ldots, M$ denote retailers). Regardless of capacity constraint, the following relationship holds: $A_i > B_i$, for $i = 1, 2, \ldots, M$ and $A_0 < B_0$. 

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Table 5.4 Implementation of centralized control for normal demand pattern as an example

<table>
<thead>
<tr>
<th>Capacity Tightness</th>
<th>Cost Components</th>
<th>Centralized</th>
<th>Decentralized</th>
<th>After Incentive</th>
<th>Δ*</th>
<th>%Δ**</th>
</tr>
</thead>
<tbody>
<tr>
<td>CT=∞</td>
<td>Retailer 1's total</td>
<td>15666</td>
<td>14978</td>
<td>14978</td>
<td>688</td>
<td>4.593</td>
</tr>
<tr>
<td></td>
<td>Retailer 2's total</td>
<td>15762</td>
<td>14978</td>
<td>14978</td>
<td>784</td>
<td>5.234</td>
</tr>
<tr>
<td></td>
<td>Distributor's total</td>
<td>24324</td>
<td>27680</td>
<td>25796</td>
<td>-3356</td>
<td>-12.124</td>
</tr>
<tr>
<td></td>
<td>System total</td>
<td><strong>55752</strong></td>
<td><strong>57636</strong></td>
<td><strong>55752</strong></td>
<td>-1884</td>
<td>-3.269</td>
</tr>
<tr>
<td>CT=1.33</td>
<td>Retailer 1's total</td>
<td>16130</td>
<td>14978</td>
<td>14978</td>
<td>1152</td>
<td>7.691</td>
</tr>
<tr>
<td></td>
<td>Retailer 2's total</td>
<td>16206</td>
<td>14978</td>
<td>14978</td>
<td>1228</td>
<td>8.200</td>
</tr>
<tr>
<td></td>
<td>Distributor's total</td>
<td>23448</td>
<td>27688</td>
<td>25828</td>
<td>-4240</td>
<td>-15.313</td>
</tr>
<tr>
<td></td>
<td>System total</td>
<td><strong>55784</strong></td>
<td><strong>57644</strong></td>
<td><strong>55784</strong></td>
<td>-1860</td>
<td>-3.227</td>
</tr>
<tr>
<td>CT=1.18</td>
<td>Retailer 1's total</td>
<td>15126</td>
<td>14978</td>
<td>14978</td>
<td>148</td>
<td>0.988</td>
</tr>
<tr>
<td></td>
<td>Retailer 2's total</td>
<td>15850</td>
<td>14978</td>
<td>14978</td>
<td>872</td>
<td>5.822</td>
</tr>
<tr>
<td></td>
<td>Distributor's total</td>
<td>24814</td>
<td>27226</td>
<td>25834</td>
<td>-2412</td>
<td>-8.859</td>
</tr>
<tr>
<td></td>
<td>System total</td>
<td><strong>55790</strong></td>
<td><strong>57182</strong></td>
<td><strong>55790</strong></td>
<td>-1392</td>
<td>-2.434</td>
</tr>
<tr>
<td>CT=1.05</td>
<td>Retailer 1's total</td>
<td>16130</td>
<td>14978</td>
<td>14978</td>
<td>1152</td>
<td>7.691</td>
</tr>
<tr>
<td></td>
<td>Retailer 2's total</td>
<td>16304</td>
<td>14978</td>
<td>14978</td>
<td>1326</td>
<td>8.853</td>
</tr>
<tr>
<td></td>
<td>Distributor's total</td>
<td>23370</td>
<td>28836</td>
<td>25848</td>
<td>-5466</td>
<td>-18.955</td>
</tr>
<tr>
<td></td>
<td>System total</td>
<td><strong>55804</strong></td>
<td><strong>58792</strong></td>
<td><strong>55804</strong></td>
<td>-2988</td>
<td>-5.082</td>
</tr>
</tbody>
</table>

*Δ*= Cost of Centralized - Cost of Decentralized

**%Δ=(Cost of Centralized - Cost of Decentralized)/Cost of Decentralized *100%
After offering fixed fee incentive to each retailer so that it bears the same cost as in decentralized control, the distributor needs to adjust its cost from \( A_0 \) to \( A_0 + \sum_{i=1}^{M} (A_i - B_i) \). It is trivial to show that the overall system cost after the implementation of fixed fee incentive is still \( \sum_{i=0}^{M} A_i \) as that in centralized control (Table 5.5). For this base-line coordination case, the cost of each retailer is not worsened while the distributor still can have some savings over decentralized control because \( A_0 + \sum_{i=1}^{M} (A_i - B_i) - B_0 = \sum_{i=0}^{M} A_i - \sum_{i=0}^{M} B_i < 0 \).

However, retailers might not be satisfied with the base-line coordination and demand the distributor to share more of its savings with them. Theoretically speaking, the total incentive amount can be any within the following interval \( \left[ \sum_{i=1}^{M} A_i - \sum_{i=1}^{M} B_i, B_0 - A_0 \right] \). The lower boundary is when retailers bear costs as much as in decentralized control; while the upper boundary is when the distributor bears cost as much as in decentralized control. This fixed fee coordination mechanism works only when the total saving at the distributor \( B_0 - A_0 \) is larger than the total fixed fee needed to cover the cost increases at all retailers, i.e. \( B_0 - A_0 > \sum_{i=1}^{M} A_i - \sum_{i=1}^{M} B_i \), or equivalently \( \sum_{i=0}^{M} A_i < \sum_{i=0}^{M} B_i \), which is usually the case because centralized control generally leads to lower system cost than decentralized control. Although the data shown above was obtained for a single-distributor two-retailer system, the same results are expected for the general single-distributor multi-retailer case, per the relationships given above.
Table 5.5 The relationship of cost components between centralized and decentralized control

<table>
<thead>
<tr>
<th></th>
<th>Centralized</th>
<th>Decentralized</th>
<th>After Incentive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer (i)'s total ((i=1,2,...,M))</td>
<td>(A_i)  &gt;</td>
<td>(B_i)  =</td>
<td>(B_i)</td>
</tr>
<tr>
<td>Distributor's total</td>
<td>(A_0)  &lt;</td>
<td>(B_0)  &gt;</td>
<td>(A_0 + \sum_{i=1}^{M} (A_i - B_i))</td>
</tr>
<tr>
<td>System total</td>
<td>(\sum_{i=0}^{M} A_i)  &lt;</td>
<td>(\sum_{i=0}^{M} B_i)  &gt;</td>
<td>(\sum_{i=0}^{M} A_i)</td>
</tr>
</tbody>
</table>

5.4.2 Effect of allocation rules

One might argue that the default ranking allocation rule used in this study is not realistic. Always preferring certain retailer over others in a SC would surely make those retailers treated with low priorities unsatisfied all the time. This section intends to address this concern. Two other more balanced allocation rules (Geng et al., 2010) are introduced for comparison with the default ranking allocation rule. The two rules are namely, batch-size allocation rule and proportional allocation rule. For sake of completeness, both allocation rules and the corresponding mathematical formulas are presented below.

The batch-size allocation rule works as follows:

1. If the available inventory at the distributor is abundant, i.e., \(a x_0^i \geq \sum_{i=1}^{M} (y_i^j - x_j^i)\), then all retailers’ orders are fully satisfied.

2. Otherwise, retailer's order is satisfied according to their batch sizes, from the largest to the smallest. If more than one retailer ordered the same quantity (a tie), the distributor satisfies them according to their sequence in entity numbers.
Equation 12 given below mathematically captures the batch-size allocation rule.

\[
\begin{align*}
  z_i^t &= \begin{cases} 
  y_i^t & \text{if } ax_0^t \geq \sum_{i=1}^{M} (y_i^t - x_i^t) \\
  x_i^t + \min [y_i^t - x_i^t, (ax_0^t - \sum_{j \in J_i(y_i^t - y_j^t)} (y_j^t - x_j^t))] & \text{otherwise}
  \end{cases}
\end{align*}
\] (5.12)

In Eq. (12), \( J_i(y) \) is the set of retailers who are prioritized higher than retailer \( i \) according to the batch size if retailer \( i \) orders a quantity \( y \). In other words, retailer \( j \) in \( J_i(y) \) either orders a quantity greater than \( y \) or orders \( y \) but \( j < i \).

The proportional allocation rule works as follows:

1. If the on-hand inventory at the distributor is abundant, i.e., \( ax_0^t \geq \sum_{i=1}^{M} (y_i^t - x_i^t) \), then all retailers’ orders are fully satisfied.

2. Otherwise, retailer's order is satisfied according to the proportion of their individual order size to the total order quantity. The distributor allocates its inventory to retailers according to the proportion, with the amount being truncated to its nearest lower integer. If there are \( n \) remainder items (residual), randomly choose \( n \) retailers from those who have placed orders, and then allocate one unit of goods to each of them.

Equation (13) given below mathematically captures the proportional allocation rule.

\[
\begin{align*}
  z_i^t &= \begin{cases} 
  y_i^t & \text{if } ax_0^t \geq \sum_{i=1}^{M} (y_i^t - x_i^t) \\
  x_i^t + \frac{(y_i^t - x_i^t)ax_0^t}{\sum_{j=1}^{M} (y_j^t - x_j^t)} + \epsilon_i^t & \text{otherwise}
  \end{cases}
\end{align*}
\] (5.13)
In Eq. (13), $\varepsilon'_i$ represents a possible residual. If retailer $i$ receives one unit from the remainder items, then $\varepsilon'_i=1$ otherwise $\varepsilon'_i=0$.

The distributor’s service level to each retailer is called internal service level because it is internal to the SC network versus external service level to the end customer. Note that with the “alternative” distributor, orders from the retailers will be 100% satisfied but with higher cost. The internal service level represents the actual percentage of quantity that the main distributor is able to provide before resorting to the “alternative” distributor. Following the previous subsection, the detailed results for Normal demand pattern is illustrated here. Table 5.6 shows how the main supplier satisfies each retailer under different allocation rules. It shows that generally there is a decrease in internal service level as the capacity constraint gets tighter, on either only one retailer or both retailers depending upon the allocation rule. Furthermore, centralized control generally achieves higher internal service levels than decentralized control in all cases. Under the ranking allocation rule, Retailer 2 may experience lower internal service level than Retailer 1 because its orders are not handled as a high priority by the distributor. Generally, the service level gap between Retailer 1 and Retailer 2 widens as tighter capacity constraints are involved in the supply chain under the ranking allocation rule. The reason is that under tighter capacity constraint, the total available stock at the distributor is reduced and Retailer 1 and Retailer 2 have to compete for limited items. Under the ranking allocation rule, orders from Retailer 1 are always met first because of its assumed higher rank.
On the other hand, batch-size allocation rule is a dynamic rule in which the priority ranks might switch places between retailers according to their batch-sizes of orders. In Table 5.6, the internal service levels of using batch-size allocation rule do not differ from those of the ranking allocation rule because both retailer order the same quantity and whenever a tie exists, the distributor satisfies them according to their sequence in entity numbers, which works equivalently as ranking allocation rule in this case. Under the proportional allocation rule, each retailer will be fulfilled more or less according to the proportion of their individual order size to the total order quantity the distributor received. The corresponding ranges are 0.96~0.95 for both retailers under centralized control and 0.95~0.93 for both retailers under decentralized control. Typically, both batch-size and proportional allocation rules lead to more balanced internal service levels between the two retailers. Regardless the allocation rule, centralized control achieves the same or higher internal service levels than decentralized control.

Generally speaking, allocation rule do not have effect on the overall output (in terms of total cost) of the system as a whole. However, it will have a direct impact on the internal service level to each retailer. In practice, choosing the appropriate allocation rule is also a very important aspect in SC design since it has a direct effect on whether the collaboration between each player in the SC will be strengthened or not. Some supply chain may not use one single allocation rule alone as the supply chain expands. In this case, categorize its retailers and apply a different allocation rule to each category or combined use of multiple allocation rules may become necessary.
Table 5.6 Internal service level achieved by each retailer for normal demand pattern

<table>
<thead>
<tr>
<th></th>
<th>Centralized Control</th>
<th>Decentralized Control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ranking</td>
<td>Batch size</td>
</tr>
<tr>
<td></td>
<td>Retailer 1</td>
<td>Retailer 2</td>
</tr>
<tr>
<td>CT=∞</td>
<td>0.97</td>
<td>0.95</td>
</tr>
<tr>
<td>CT=1.33</td>
<td>0.97</td>
<td>0.95</td>
</tr>
<tr>
<td>CT=1.18</td>
<td>0.97</td>
<td>0.94</td>
</tr>
<tr>
<td>CT=1.05</td>
<td>0.97</td>
<td>0.93</td>
</tr>
</tbody>
</table>

5.5 Conclusion

This chapter has presented the DMSO framework used to determine the near-optimal supply chain replenishment policies under various demands and control strategies. The framework includes an optimization module based on a newly developed hybrid metaheuristic algorithm and an evaluation module built on a SC inventory simulation model. In this chapter, a capacitated single-distributor-multi-retailer supply chain system was examined in detail. To the best of our knowledge, this is the first study of both decentralized and centralized controls performed in the simulation-based optimization setting for a capacitated inventory system. Other than its ability to find near-optimal solutions, the DMSO framework can track the system from different perspective (such as ordering patterns, cost distribution over the system, distributor's internal
service level to each retailer) to help inventory manager make better and more informative decisions.

Ten different demand patterns, four levels of capacity constraint and two different control strategies were tested. It was found that capacity constraint might lead to significant change in ordering patterns for demand with high variations. Without proper optimization of the replenishment policies, the entire supply chain system will experience unnecessary cost increase. Adopting centralized control is beneficial, especially for those supply chains facing unstable demand patterns. For decentralized supply chains, it is feasible to coordinate the whole system by designing an incentive mechanism which will benefit all members in the system. Distributor's allocation rule do not have an effect on the overall output of the SC as a whole but it will have a direct impact on the satisfaction level to each retailer and indirectly affect their willingness to collaborate within the chain.

In summary, it should be pointed out that the SC model can be generalized to other demand patterns, cost functions as well as the replenishment policies, under the consideration of supplier’s capacity constraint. The proposed framework is not only a great vehicle for researchers to investigate interesting research issues, but also an excellent tool for practitioners with broad applicability. In next chapter, the application of the proposed framework to stochastic settings is discussed and an enhanced framework is developed to find (near)optimal inventory policies for an integrated SC model in consideration of quality imperfection.
6.1 Introduction

In this chapter, the proposed MSO framework is further extended to stochastic setting. The new Stochastic Metaheuristic-based Simulation Optimization (SMSO) methodology integrates the DMSO with sample average approximation method (Kleywegt et al., 2001) with the aim to find (near)optimal inventory policies for an integrated SC model in consideration of quality imperfection.

In most of the early literature dealing with inventory problems, the possibility of defective items is always ignored. However, defective items, and hence random yield, are common in real-world practice. Quality deviations from specifications are something that almost all inventory systems have to deal with on a daily basis. As indicated in Fitzgerald (2005), lack of proper product monitoring and/or damage in transit create supply quality associated problems, leading to the reduction of the actual portion of each delivery that can be used directly to satisfy customer demand. Consequently, these defective items will influence the on-hand inventory level, the frequency of orders and finally the service level. In order to reflect the real business environment, more and more researchers are developing inventory models in consideration of quality-related issues. In the most up-to-date literature review presented by Khan et al., (2011), they comment that more research effort shall be devoted to a supply chain setting closer to
practical scenarios. Research that addresses these issues may lead to modeling insights that can strengthen coordination between all members of a supply chain.

The approach proposed in this chapter is intended exactly to bridge the gap. It is commonly accepted that modern global business competition is no longer between individual firms, but among supply chains (SCs). The rapid change in modern business environment has clearly outpaced the development of mathematically tractable supply chain inventory models for real world applications. Simulation optimization, as reviewed by Fu (2002) and Tekin and Sabuncuoglu (2004), serves as a relatively new alternative technique that has attracted attention in recent years. In this chapter, the new metaheuristic-based simulation optimization methodology that integrates metaheuristic with sample average approximation method is developed with the aim to find (near)optimal inventory policies for an integrated SC model in consideration of quality imperfection in the products supplied. The quality imperfection/deviation is modeled as a discrete or continuous distribution function. Only a portion of each lot can be used directly to fulfill customer demand. The remaining defective portion is returned to the supplier for remanufacturing at an additional cost and will be delivered in the following lead time period. Therefore, the consideration of supplier quality imperfection makes the subject supply chain inventory problem a stochastic discrete optimization one. In order to approximate the expected total inventory-related cost, sufficient samples of quality deviations are generated and then the corresponding sample average function is optimized by the newly developed hybrid meta-heuristic algorithm. The proposed methodology and its individual
components are presented. Numerical results of a single-distributor-multiple-retailer supply chain system with each adopting the \((s, S)\) replenishment policy indicate that the proposed methodology is capable of obtaining high quality supply chain inventory policies with percentage optimality gap within 0.01%.

**6.2 Mathematical model for the considered SC problem**

An integrated supply chain system composed of single-distributor and multiple-retailer is considered. The retailers can be identical or non-identical. Such a supply chain system is common for major retailers such as Walmart and Lowe’s. It is assumed that all parties in the supply chain adopt a periodic review \((s, S)\) inventory policy; but other policies can also be easily implemented. The distributor faces the possibility of product defects from its supplier. The retailers, on the other hand, are faced with the uncertainty from customer demand. Our model is capable of taking any demand streams directly, without any demand distribution assumption. This study considers a single product with the possibility of defective items in a delivered lot.

The mathematical formulation of the integrated supply chain inventory model in consideration of imperfect items is described below. Let entity \(i=0\) stand for the distributor, and entities \(i=1, 2, \ldots, M\) denote retailers, respectively. Given the initial system state \(X^0 = \{x_0^0, x_1^0, x_2^0, \ldots, x_M^0\}\) as the initial inventory level at each supply chain node at the beginning of the first period \((t = 0)\), for each time period, \(t=1, 2, \ldots, T\), do the following:

Subscripts

\(t\) period \(t\) in the planning horizon, \(t=1, 2, \ldots, T\)

\(i\) Supply chain entity, \(i=0, \ldots, M\)
Notations

\( D_i \) Demand at entity \( i \) for period \( t \)

\( Q_i \) Quality imperfection of supplier, represented as proportion of items meeting the specification at period \( t \), a random variable

\( K_i \) Fixed ordering cost for entity \( i \)

\( h_i \) Holding cost per unit per period for entity \( i \)

\( p_i \) Lost-sale penalty cost per unit for entity \( i \)

\( R_i \) Remanufacturing cost per unit for supplier

\( L_i \) Lead time for entity \( i \)

At the beginning of period \( t \), retailer \( i \) observes its customer demand \( D_i \) and try to satisfy \( D_i \) with its available on-hand inventory, \( ax_i \). Any shortfall occurred at period \( t \) will be considered as lost sales to buyers with a penalty cost, \( p_i \). Any item held at stock at period \( t \) will be charged with inventory holding cost, \( h_i \). Let \( G_i(ax_i) \) be the summation of the holding and penalty costs at entity \( i \) for period \( t \) with on-hand inventory level \( ax_i \), we will have

\[
G_i(ax_i) = h_i(ax_i - D_i)^+ + p_i(D_i - ax_i)^+ \tag{6.1}
\]

where \( x^+ = \max\{x,0\} \)

Next, retailer \( i \) checks its remaining inventory level \( x_i \) and decides to bring its inventory level to \( y_i \). Let \( z_i \) denote the actual inventory level at retailer \( i \) after receiving items from the distributor. Obviously, \( z_i \leq y_i \) is always true as explained below.

The retailers’ orders become demand to the distributor. Two cases can be distinguished in fulfilling the demand by the distributor:
1. If the available inventory at the distributor is abundant, i.e., $ax_0^t \geq \sum_{i=1}^{M} (y_i^t - x_i^t)$, all retailers’ orders are fully satisfied.

2. Otherwise, the orders are satisfied according to the ranking of retailers, in descending order of importance with $i=1$ indicating the most important.

Mathematically,

$$
z_i^t = \begin{cases} 
y_i^t, & ax_0^t \geq \sum_{i=1}^{M} (y_i^t - x_i^t) \\
x_i^t + \min\{y_i^t - x_i^t, ax_0^t\}, & ax_0^t < \sum_{i=1}^{M} (y_i^t - x_i^t) \text{ and } i = 1 \\
x_i^t + \min\{y_i^t - x_i^t, (ax_0^t - \sum_{j=1}^{i-1} (y_j^t - x_j^t))^+\}, & \text{otherwise}
\end{cases}
\tag{6.2}
$$

Eq. (2) is the ranking allocation rule employed in this study but it is possible to implement other rules to adapt to different cases. In addition, it is assumed that the shortfall of each buyer’s ordering quantity will be met by obtaining the shortfall from an “alternative” source but at higher cost. The distributor will be responsible for the penalty cost to this shortfall. In this way, the supply to each retailer’s ordering quantity is guaranteed. The penalty cost to the distributor represents the additional cost to the distributor for obtaining items from alternative suppliers, which is denoted as $p_0$.

Next the distributor observes its inventory level $x_0^t$ and decides to bring its inventory level to $y_0^t$. Note that the distributor orders from a supplier that might deliver defective items, implying that the distributor’s order might not be fully satisfied. The supplier’s quality deviation is included in the model following that of Mahnam et al. (2009). The non-defective items of each
lot is treated as a random variable, \( Q_t \), which follows some probability distribution \( P \). Let \( Q = \{Q_1, Q_2, \ldots, Q_T\} \) be one realization of the random variable over \( T \) periods and \( Q_t \) states the proportion of each order at period \( t \) that can be immediately used to fulfill demand. Therefore, for each lot ordered from the supplier, only \( \left\lfloor Q_t (y'_0 - x'_0) \right\rfloor \) number of items can be used directly. The remaining \( (1-Q_t)(y'_0 - x'_0) \) units failing to meet the product specification will be returned to the supplier for remanufacturing at an additional cost, \( R_0 \), and will be delivered in the following lead time period. Obviously, \( y'_0 \geq x'_0 \).

All orders placed to upstream are due after some constant lead time. As a result, the available inventory for each entity used to satisfy its down-stream customer at the beginning of period \( t \), \( ax'_i \), is given by

\[
ax'_i = \begin{cases} 
    x'^{-1}_i, & t \leq L_i \\
    x'^{-1}_i + (y'_i - x'^{-1}_i), & t > L_i
\end{cases}, \quad i = 1, \ldots, M. \tag{6.3}
\]

\[
ax'_0 = \begin{cases} 
    x'^{-1}_0, & t \leq L_0 \\
    x'^{-1}_0 + Q_{t-L_0} (y'^{-1}_{i-L_0} - x'^{-1}_{i-L_0}), & L_0 < t \leq 2L_0 \\
    x'^{-1}_0 + Q_{t-L_0} (y'^{-1}_{i-L_0} - x'^{-1}_{i-L_0}) + (1-Q_{t-2L_0})(y'^{-1}_{i-2L_0} - x'^{-1}_{i-2L_0}), & t > 2L_0
\end{cases} \tag{6.4}
\]

The inventory level of period \( t \) to determine the order quantity for entity \( i \) is as follows:

\[
x'_i = (ax'_i - D'_i)^+ \quad i = 0, \ldots, M. \tag{6.5}
\]

where

\[
D'_0 = \sum_{i=1}^M (y'_i - x'_i)
\]

The inventory order-up-to levels, \( y'_i, \forall i, t \), are determined accordingly to the subject inventory policy used in the system. This study adopts the widely used \((s, S)\) policy. When the inventory position (inventory on hand plus outstanding orders) declines to or below \( s \), a number
of product units are ordered such that the resulting inventory position is raised to \( S \). The decision variables thus are \( s_i, S_i, i = 0, 1, \ldots, M \). For the \((s, S)\) policy, the inventory order-up-to levels are determined as follows:

\[
y'_i = \begin{cases} 
    x'_i, & \text{if } x'_i + \sum_{j=-L_i+1}^{t-1} (y'_j - x'_j) > s_i \\
    S_i - \sum_{j=-L_i+1}^{t-1} (y'_j - x'_j), & \text{else}
\end{cases}
\]  

(6.6)

The objective is to minimize the total supply chain inventory cost over the entire finite planning horizon. The objective function is described next. Let \( \delta(x) \) be an indicator function as follows:

\[
\delta(x) = \begin{cases} 
    1, & x > 0 \\
    0, & x \leq 0
\end{cases}
\]  

(6.7)

Let \( v'_i(x'_{t_i}) \) denote the total cost of entity \( i \) with inventory level \( x' \) at period \( t \), then the following equations hold:

\[
v'_i(x'_{t_i}) = K_i \delta(y'_i - x'_i) + G_i(ax'_i) \quad i = 1, \ldots, M, \ t = 1, \ldots, T
\]  

(6.8)

and

\[
v'_0(x'_0) = K_0 \delta(y'_0 - x'_0) + G_0(ax'_0) + R_0 \left[ (1 - Q_t) (y'_0 - x'_0) \right], \quad t = 1, \ldots, T
\]  

(6.9)

Given the initial system state, the total system cost \( G \) over the finite time horizon is

\[
G = \sum_{t=1}^{T} \left[ \sum_{i=0}^{M} v'_i(x'_{t_i}) \right].
\]  

(6.10)
Note that the function $G$ implicitly contains a random vector, $Q$, generated from the probability distribution $P$. Let $X$ represent the set of decision variables, i.e., $(s_i, S_i)$ policies for each entity, $i=0,…,M$. The optimization problem considered in this study is therefore of the form

$$\min_{X \in S} \{ f(X) := E_p[G(X, Q)] \}$$  \hspace{1cm} (6.11)

In Eq. (11), $S$ is a finite discrete set of possible $(s_i, S_i)$ policies, $i=0,…,M$. $E_p[G(X, Q)] = \int G(X, Q) P(dQ)$ is the corresponding expected value function. Clearly, the objective function cannot be solved analytically, but has to be measured or estimated. Given certain demand stream and inventory policies, $(s_i, S_i)$ values, the function can be easily computed for a given realization of $Q$. Also note that, the set of feasible solutions $(s_i, S_i)$ policy for each entity in the SC, although finite, can be extremely large. The size of the feasible set grows exponentially with the number of variables and the number of periods in the planning horizon. Therefore, it is not possible to use the enumeration approach for its solution.

### 6.3 Stochastic metaheuristic-based simulation optimization methodology

The proposed methodology integrates a metaheuristic-based simulation optimization framework with a sample average approximation method. The general metaheuristic-based simulation optimization framework is depicted in Fig. 6.1. Theoretically, any effective metaheuristic algorithm can be used here. Nevertheless, the meta-heuristic employed here is one of the most recent hybrid metaheuristics called “DE-HS-HJ” (Duan et al. 2013). The hybrid metaheuristic comprises of two cooperative metaheuristic algorithms, i.e., differential evolution (DE) and harmony search (HS), with each enhanced by a local search (LS) method, i.e., Hooke
and Jeeves (HJ) direct search. The HJ local search is applied to every new solution generated by DE and HS with a specified probability, $p=0.1$. This local search application strategy has been shown in Duan et al. (2013) to be the best in terms of success rate. For more detailed information on the hybrid meta-heuristic DE-HS-HJ, interested readers are referred to Duan et al. (2013).

The general simulation optimization framework outlined in Fig. 6.1 is used to find near-optimal solutions for the stochastic supply chain inventory problem. The process is initiated by inputting an initial guess of trial solutions. The population of trial solutions is sent to the simulation model. Running the simulation model generates their corresponding output performance measure, which are fed into the hybrid metaheuristic. The quality of the output guides the metaheuristic in the selection of new input solutions, based on the intelligent searching mechanism of the metaheuristic. The process is repeated until the maximum number of iterations is reached or no further improvement can be found.

In order to obtain a high quality solution for the considered stochastic discrete optimization problem, an effective procedure is needed to select the best, if not really optimal, solution with high probability. To this end, the work of Kleywegt et al. (2001), in which they originally developed an approximation method based on the Monte Carlo simulation approach, is adapted and integrated into the metaheuristic-based simulation optimization framework. The basic idea is that a set of random samples is generated and the expected value function is approximated by the corresponding sample average function. The obtained sample average problem is optimized by the hybrid meta-heuristic and the procedure is repeated until the stopping criterion is satisfied.
Let \( Q^1, Q^2, \ldots, Q^j, \ldots, Q^N \) be a set of independent and identically distributed (i.i.d.) random samples of \( N \) realizations of the random vector \( Q \). Equation (11) is approximately solved by

\[
\min_{X \in S} \hat{f}_N(X)
\]

(6.12)

and

\[
\hat{f}_N(X) := \frac{1}{N} \sum_{j=1}^{N} G(X, Q^j)
\]

(6.13)

Note that \( E[\hat{f}_N(X)] = f(X) \). Unless the random vector \( Q \) has a small number of possible realizations (scenarios), it is usually impossible to solve the problem exactly. The proposed methodology is primarily concerned with solving the sample average approximating problem by meta-heuristic and bounding the solution quality using the so-called optimality gap.

Let \( z^* \) and \( \hat{z}_N \) denote the optimal values of the respective problems,

\[
z^* := \min_{X \in S} f(X) \text{ and } \hat{z}_N := \min_{X \in S} \hat{f}_N(X)
\]

(6.14)

It has been proven in Kelywegt et al. (2001) that (i) \( \hat{z}_N \rightarrow z^* \) convergence with probability one (w.p.1) as \( N \rightarrow \infty \) and (ii) the estimator \( \hat{z}_N \) has a negative bias because

\[
E[\hat{z}_N] \leq E\{\min_{X \in S} \hat{f}_N(X)\} \leq \min_{X \in S} E[\hat{f}_N(X)] = z^* \]

(Norkin et al.(1998), Mak et al.(1999) and Kelywegt et al. (2001)). The idea is to use the metaheuristic-based simulation optimization framework to approximate \( \hat{z}_N \).
Figure 6.1 The proposed stochastic metaheuristic-based simulation optimization framework
The corresponding pseudo-code of the metaheuristic-based simulation optimization framework is given as follows:

Algorithm 6.1: Metaheuristic-based simulation optimization algorithm
Specify meta-heuristic-related parameters, number of maximum evaluations, \( Maxnfe \); population size, \( NP \).
(1) Randomly generate the initial population of solutions, \( X^0 \).
(2) Draw \( N \) samples of quality realizations.
(3) Evaluate the initial set of solutions through the simulation model and determine the objective value of the sample average problem of Eq. (13) for each solution. Let current number of evaluation, \( nfe=NP \). Determine the best objective value, \( f^{\text{best}} \), and the best solution, \( X^{\text{best}} \), of the initial population.
(4) While \( nfe<Maxnfe \) or the improvement of \( f^{\text{best}} > 0.1 \), do
(5) For each target solution in the population (=1, 2, ..., \( NP \))
(6) Generate a trial vector according to the DE algorithm;
(7) Construct a new solution according to the HS algorithm;
(8) If the new solutions are randomly selected (with a specified percentage, \( p \))
(9) Apply the HJ local search to converge the solution to its nearest local optimum under the \( N \) quality samples;
(10) Increment \( nfe \) accordingly;
(11) Else
(12) Evaluate the new solutions found by both algorithms through the simulation model and determine their corresponding objective values of the sample average problem of Eq. (6.13);
(13) Increment \( nfe \) by two, i.e., \( nfe=nfe+2 \).
(14) End if
(15) Replace the target solution with the new trail solution for the DE algorithm and HS algorithm, respectively, if there is an improvement.
(16) Update the best solution among the whole population, \( X^{\text{best}} \), found so far
(17) End For
(18) End While
(19) Output \( X^{\text{best}} \)

To obtain more than one replicate, the simulation optimization framework is run \( L \) times such that \( L \) is statistically sufficient(>30). In each run \( l, l=1,\ldots,L \), a near-optimal solution \( \hat{X}_N^l \) is obtained. Note that meta-heuristic cannot guarantee finding the true optimum 100%. Therefore,
all candidate solutions \( \hat{X} = \{ \hat{X}_N^1, ..., \hat{X}_N^L \} \) found by Algorithm 1 are first subjected to a screening procedure to get rid of significantly inferior solutions. The screening procedure is based on Hsu’s multiple comparisons with the best procedure (Hsu’s MCB, 1981). To determine whether the resultant subset size is large enough, the \( \% \) difference in overall average expected cost is tracked and compared with a specified cutoff value. Let the current solution number to be \( C \), the “\( \% \) difference” values is calculated as

\[
\left\{ \frac{1}{C} \sum_{k=1}^{C} \hat{f}_N(\hat{X}_N^k) \right\} / \left\{ \frac{1}{C-1} \sum_{k=1}^{C-1} \hat{f}_N(\hat{X}_N^k) \right\} - 1 \right\} \times 100\%.
\] (6.15)

After the screening procedure and the subset size is determined large enough, the subset \( \hat{X}' = \{ \hat{X}_N^1, ..., \hat{X}_N^K \} \), \( (K \leq L) \) solutions that are not significantly inferior to the “best” is chosen to estimate the true optimum \( z^* \) as

\[
z_N^K = \frac{1}{K} \sum_{k=1}^{K} \hat{f}_N(\hat{X}_N^k),
\] (6.16)

where \( \hat{X}_N^k \) denotes the \( k \)th solution in the subset \( \hat{X}' \). Note that \( E[z_N^K] = E[\hat{z}_N] \), and hence the estimator \( z_N^K \) has the same negative bias as \( \hat{z}_N \). The subset solution \( \hat{X}' \) found by the hybrid meta-heuristic probably contains true (near)optimal solution for the stochastic supply chain inventory problem. For each candidate solution \( \hat{X}_N^k \), the corresponding expected cost of operating the inventory system, \( f(\hat{X}_N^k) \), is computed via the sample average estimator

\[
\hat{f}_N(\hat{X}_N^k) = \frac{1}{N'} \sum_{j=1}^{N'} G(\hat{X}_N^k, Q_j).
\] (6.17)

In Eq. (17), it is suggested to use a larger sample size of quality realizations such that \( N' > N \) to obtain an accurate estimate \( \hat{f}_N(\hat{X}_N^k) \) to the objective value \( f(\hat{X}_N^k) \) of solution \( \hat{X}_N^k \).
It has been shown in Mak et al. (1999) and Kelywegt et al. (2001) that it is an unbiased estimator of the true cost of the solution $\hat{X}_N^{ik}$, i.e.,

\[
E[\hat{f}_{N'}(\hat{X}_N^{ik})] \geq z^*
\]

(6.18)

and $\hat{f}_{N'}(\hat{X}_N^{ik}) \rightarrow f(\hat{X}_N^{ik})$ convergence with probability one (w.p.1) as $N'\rightarrow \infty$. A measure of the accuracy of $\hat{f}_{N'}(\hat{X}_N^{ik})$ of $f(\hat{X}_N^{ik})$ is given by the corresponding sample variance, $S^2_N(\hat{X}_N^{ik})/N'$, which can be calculated from the same sample of size $N'$.

Of course, the chosen $N'$ involves a trade-off between computational effort and accuracy measured by $S^2_N(\hat{X}_N^{ik})/N'$. Typically, estimating the value of function (6.17) for a candidate solution $\hat{X}_N^{ik}$ by the sample average estimator $\hat{f}_{N'}(\hat{X}_N^{ik})$ requires much less computational effort than trying to minimize the sample average approximation problem (function 6.12). Thus, it is feasible to use a significantly large sample size $N'$ to estimate the true cost of operating the supply chain inventory system with the solution $\hat{X}_N^{ik}$. In this study, we select to follow Rinott’s indifference zone approach (1978) and choose the second stage sample size $N'$ as follows:

\[
N' = \arg \max_{i=1,...,k} \{N'_k\}
\]

(6.19)

\[
N'_k = \max\{N, \left[\frac{h}{\delta} S^2(\hat{X}_N^{ik})\right]\},
\]

(6.20)

where $\left[ y \right]$ is the smallest integer that is greater than or equals to $y$; $N'_k$ is the lower bound of the sample size needed for evaluating solution $\hat{X}_N^{ik}$; $S^2(\hat{X}_N^{ik})$ is the corresponding sample variance of solution $\hat{X}_N^{ik}$; $\delta$ is the indifference zone determined by the decision maker; $h$ can be obtained from table of Wilcox (1984).

To determine the performance bound of each solution found by our simulation optimization framework, the optimality gap $f(\hat{X}_N^{ik}) - z^*$ for a candidate solution $\hat{X}_N^{ik}$ is computed. Unfortunately, the very reason for us to develop the new framework implies that both terms of
the optimality gap are hard to compute exactly. Recall that \( E[\hat{z}_N] \leq z^* \leq E[\hat{f}_{N^*}(\hat{X}_{ik})] \) and \( E[\pi_{ik}^N] = E[\hat{z}_N] \). Hence, the optimality gap \( f(\hat{X}_{ik}) - z^* \) is estimated by \( \hat{f}_{N^*}(\hat{X}_{ik}) - \pi_{ik}^N \), at the solution \( \hat{X}_{ik} \) and it can be obtained that

\[
E[\hat{f}_{N^*}(\hat{X}_{ik}) - \pi_{ik}^N] = f(\hat{X}_{ik}) - E[\hat{z}_N] \geq f(\hat{X}_{ik}) - z^*.
\] (6.21)

It follows that on average the above estimator overestimates the optimality gap \( f(\hat{X}_{ik}) - z^* \). It has been shown in Norkin et al. (1998), Mak et al. (1999) and Kleywegt et al. (2001) that the bias \( z^* - E[\hat{z}_N] \) is monotonically decreasing with the sample size \( N \).

If the \( K \) quality realizations of size \( N \) and the evaluation samples of size \( N' \) are independent. The variance of the optimality gap estimator \( \hat{f}_{N^*}(\hat{X}_{ik}) - \pi_{ik}^N \) can be estimated by the sum of \( S_{N^*}^2(\hat{X}_{ik}) / N' \) and \( S_K^2 / K \). Specifically, the variance of \( \hat{f}_{N^*}(\hat{X}_{ik}) \) is estimated by

\[
S_{N^*}^2(\hat{X}_{ik}) / N' = \frac{1}{N'(N'-1)} \sum_{j=1}^{N'} (G(\hat{X}_{ik}, Q^j) - \hat{f}_{N^*}(\hat{X}_{ik}))^2.
\] (6.22)

The variance of \( \pi_{ik}^N \) is estimated by

\[
S_K^2 / K = \frac{1}{K(K-1)} \sum_{k=1}^{K} (\hat{X}_{ik} - \pi_{ik}^N)^2.
\] (6.23)

Following the Central Limit Theorem, for sufficiently large \( N \) and \( N' \), the accuracy of an optimality gap estimator can be taken into account by multiplying a \( z_\alpha \) of its estimated standard deviation to the gap estimator. Note that \( z_\alpha = \Phi^{-1}(1-\alpha) \), \( \Phi(z) \) is the cumulative distribution function of the standard normal distribution. The approximate (1-2\( \alpha \))-level confidence interval (CI) for the optimality gap for a given near-optimal solution \( \hat{X}_{ik} \) found by the hybrid meta-heuristic is given by

\[
[0, \hat{f}_{N^*}(\hat{X}_{ik}) - \pi_{ik}^N + z_\alpha \left( \frac{S_{N^*}^2(\hat{X}_{ik})}{N'} + \frac{S_K^2}{K} \right)^{1/2}]
\] (6.24)
Due to the possibility of sampling error, \( \hat{f}_N(\hat{X}^{ik}_N) < z_N^k \) might actually be observed. Hence, it is recommended to use the following more conservative confidence interval

\[
[0, \left[\hat{f}_N(\hat{X}^{ik}_N) - z_N^k\right]^+ + z_\alpha \left(\frac{S_N^2(\hat{X}^{ik}_N)}{N'} + \frac{S_k^2}{K}\right)^{1/2}], \tag{6.25}
\]

Where \([x]^+ = \max\{x,0\}\)

Select the solution that has the smallest optimality gap and label it as \( \hat{X}^* \). In other words, the proposed methodology guarantees with probability at least equal to \((1-2\alpha)\) that the chosen solution has the best objective value \( f(\hat{X}^*) \) over all candidate solutions \( \hat{X}^i \) and the estimated error of its optimality gap is at most equal to the upper endpoint of Eq.(6.25). Algorithm 2 summarizes the proposed methodology.

Algorithm 6.2: The proposed methodology to obtain a high quality solution

1. Choose initial sample size \( N \) for quality imperfection realizations, a decision rule for determining the number of meta-heuristic runs \( L \) with the sample size \( N \)
2. For \( l=1,\ldots,L \), do
   1.1. Generate a set of quality samples of size \( N \) and try to solve the sample average approximation problem (function 12) using Algorithm 1 and record the objective value \( \hat{z}_N^l \) and suboptimal solution \( \hat{X}_N^l \).
   1.2. Perform the screening procedure of Hsu’s MCB to get rid of solutions that are significantly inferior to the best solution and obtain the subset \( \hat{X}^l \)
   1.3. Test whether the subset size is large enough by tracking whether % difference in overall average expected cost is less than a specified cutoff value. Increase \( L \) if not.
3. Using the subset \( \hat{X}^l = \{\hat{X}^l_N, \ldots, \hat{X}^{ik}_N, \ldots, \hat{X}^{iK}_N\} \) to calculate \( \hat{z}_N^k \) (function 16)
4. Choose a sufficiently larger sample size \( N' > N \)
   For each candidate solution \( \hat{X}^{ik}_N \), \( k=1,\ldots,K \), do
   Estimate \( f(\hat{X}^{ik}_N) \) (the expected cost of operating the inventory system with the solution \( \hat{X}^{ik}_N \)) via the sample average estimator \( \hat{f}_N(\hat{X}^{ik}_N) \) (function 17)
(5) Estimate the optimality gap and construct the confidence interval by Eq. (25)

(6) Choose the solution among all candidate solutions $\hat{X}'$ which has the smallest optimality gap as the best solution, $\hat{X}^*$. 

### 6.4 Verification of the proposed methodology

Before proceeding to solve the supply chain inventory optimization problem with imperfect quality, the proposed methodology is first tested with a typical M/M/1 queuing system (Eq. 6.26) as that used in Alrefaei and Alawneh (2005). The objective is

$$\min_{\mu \in (3,6)} \{ (\mu - 4)^2 + \omega(\mu) \},$$

(6.26)

Where $\omega(\mu)$ is the average system time per customer, given that the service rate is $\mu$ customers per hour and the inter-arrival rate is $\lambda$ customers per hour with $\mu \in (3,6)$ and $\lambda = 3$. The service rate $\mu$ is to be decided so that the cost function $(\mu - 4)^2 + \omega(\mu)$ is minimized.

Theoretically the average system time for the considered M/M/1 queuing system can be expressed as $\frac{1}{\mu - \lambda}$ and the analytical optimal solution is $\mu = 4.3$ with an optimal value of 0.859.

Treating the above problem as a stochastic optimization problem for which $\omega(\mu)$ is estimated by a queuing simulation model, the proposed methodology is applied to determine the optimal solution that minimizes the objective value of the cost function. Following Alrefaei and Alawneh, the real valued decision variable is discretized by considering the feasible set of solutions, $\theta = \{3.0 + 0.01i, i = 1,...,300\}$. Using 200 realizations of $\mu$ to simulate the system for sufficiently long time period (150,000 customer arrivals), the optimal solution found by the proposed methodology is 4.3 with an objective value of 0.8591. It has been shown in Alrefaei and
Alawneh (2005) that the cost function is quite flat around the optimal solution and therefore successfully identifying the true optimum is not trivial.

Figure 6.2 shows a typical convergence profile of cost function values in solving the M/M/1 queuing system. The dash-double-dots (red) line represents the profile for the average objective value achieved by the entire population and the solid (blue) line represents the best objective value achieved. It can be observed that the hybrid meta-heuristic converges to the true optimum rather quickly within 250 evaluations. This result verifies that the proposed methodology is effective in finding the optimal solution.

6.5 Computational results

As an illustration, a single-item SC inventory system with single distributor and three retailers is considered. One salient feature of our supply chain inventory model is that any demand stream can be handled. Those demand series do not have to follow any particular
distribution/assumption. However, no demand uncertainty is considered in this study. The distributor must confront the quality uncertainty from its main supplier and fill up the shortfall caused by those defects from another more expensive supplier. The quality level of delivered lots is assumed to follow a simple discrete distribution with three possible events: worst=0.8, most likely=0.9 and best=1. Quality level of 0.8 in the worst event means that 80% of the supplied items conformed to the specification and can be used to fulfill customer demand directly. The probability of these three events is assumed to be $P(0.8)=0.1$, $P(0.9)=0.8$, $P(1)=0.1$. The aim is to optimize the $(s_i, S_i)$ policy for each entity in the SC so that the total expected inventory-related cost over the finite planning horizon is minimized. Because the quality uncertainty exists in the SC system, the optimization problem is of the form shown in Eq.(6.11). The proposed methodology is applied to find the set of candidate solutions by using Algorithm 1 and to select a high quality (near)optimal solution by following Algorithm 2.

The parameter setting for Algorithm 1 (organized by the three components of the hybrid meta-heuristic) is given in Table 6.1. Those parameter values were chosen based on their good performance from our previous experience. The maximal number of function evaluations, $MaxFE=50000$, was used as the stopping criterion. The on-hand inventory in the SC was initialized to be 200 for retailers and 400 for distributor at the beginning of period 0. Inventory-related parameters such as lead time, ordering cost per order, holding cost per unit item and unit time, lost sale cost per unit item, remanufacturing cost per unit item, were fixed at 2 periods, $100, $2, $20 and $10, respectively.
Table 6.1 Parameter settings used in the hybrid “DE-HS-HJ” meta-heuristic

<table>
<thead>
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<td><strong>Scale Factor</strong></td>
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<td><strong>Crossover Rate</strong></td>
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<td><strong>Harmony memory size</strong></td>
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<td><em>λ</em>=0.1</td>
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<td></td>
<td><strong>Maximum number of cycle</strong></td>
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<td><em>M</em>=10</td>
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<tr>
<td></td>
<td><strong>Probability to perform HJ</strong></td>
</tr>
<tr>
<td></td>
<td>*p=0.1</td>
</tr>
</tbody>
</table>

The planning horizon was assumed one year with 52 weekly demand data and all three retailers had the same demand stream generated from a Gamma distribution with mean and standard deviation equals to 49 and 4.9, respectively. Figure 6.3 shows the demand stream considered in this numerical example. The random quality vector was realized 400 times (*N*=400) and consistently used in all 50 replicates of meta-heuristic runs (*L*=50). The lower and upper bounds of possible $s_i$ values for each supply chain entity were set as the minimum weekly demand and the maximum lead-time demand, respectively; the lower and upper bounds of possible $S_i$ values for each supply chain entity were set as the minimum weekly demand and the doubled maximum lead-time demand, respectively. There were totally 8 variables to be optimized (4 entities in the SC system and each has 2 values to be optimized). All programs were
coded in Matlab and all executions were made on a HP Pavilion a4317c with AMD Athlon™ II × 2@ 2.70 GHz.

Figure 6.3 Data plot of the demand stream faced by all three retailers

At the end of Step 2 of Algorithm 2, 50 candidate solutions $\hat{X}$ and their estimated expected costs for all 400 realizations of the quality vector were obtained. The screening procedure was subsequently applied to get rid of solutions that are significantly inferior to the best solution. The Hsu’s MCB test result is given in Report 6.1.

The confidence intervals (CI) of Hsu’s MCB procedure were calculated as

$$[\min(\bar{y}_i - \bar{Y}_j - CV \sqrt{\frac{2MSE}{N}}, 0), \max(\bar{y}_i - \bar{Y}_j + CV \sqrt{\frac{2MSE}{N}}, 0)],$$

in which $\bar{y}_i$ is the mean achieved by each solution, $\bar{Y}_j$ is the best mean achieved such that $i \neq j$, $CV$ is the Dunnett’s critical value (1955), $MSE$ is the mean square error and $N$ is the number of quality realizations. A statistically significant difference is concluded if the interval has zero as an end point. In the screening procedure, the family error rate is selected to be 0.1. The test results in Report 6.1 indicate that 15 solutions among the total 50 are significantly inferior to the best and shall be removed from further analyses.
### ANOVA results when testing population means

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<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
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<td>9990887860</td>
<td>203895671</td>
<td>255.09</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
<td>19950</td>
<td>15946398784</td>
<td>799318</td>
<td></td>
<td></td>
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<tr>
<td>Total</td>
<td>19999</td>
<td>25937286644</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S = 894.0  R-Sq = 38.52%  R-Sq(adj) = 38.37%

---

### Hsu's MCB (Multiple Comparisons with the Best)

**Family error rate = 0.1**

**Critical value = 2.59**

**Intervals for level mean minus smallest of other level means**

| Level | Lower | Center | Upper | --+---------+---------+---------+-------|
|-------|-------|--------|-------|-----------------|-----------------|-----------------|--------|
| 6556.32 | -124.9 | 38.7 | 202.2 | (---*) |
| 6560.48 | -85.7 | 77.9 | 241.4 | (**) |
| 6562.085 | -129.1 | 34.5 | 198.0 | (-*--*) |
| 6563.39 | -57.8 | 105.8 | 269.3 | (-***) |
| 6591.88 | -99.3 | 64.3 | 227.8 | (-*) |
| 6527.63 | -166.8 | -3.2 | 160.3 | (-***) |
| 6549.885 | -41.3 | 122.3 | 285.8 | (-*--*) |
| 6584.765 | 0.0 | 320.1 | 483.7 | (-----*--) |
| 6505.36 | -85.8 | 77.7 | 241.3 | (**) |
| 6566.44 | -23.7 | 139.8 | 303.4 | (---*) |
| 6560.37 | -80.8 | 82.7 | 246.3 | (-*--*) |
| 6558.855 | -102.2 | 61.3 | 224.9 | (-*) |
| 6553.87 | -160.3 | 3.2 | 166.8 | (---*) |
| 6566.935 | -24.2 | 139.3 | 302.9 | (-*--*) |
| 6570.605 | 0.0 | 179.0 | 342.5 | (---*) |
| 6557.495 | -116.7 | 46.9 | 210.4 | (---*) |
| 6554.705 | -150.5 | 13.1 | 176.6 | (---*) |
| 6554.555 | -146.6 | 16.9 | 180.5 | (---*) |
| 6559.84 | -31.3 | 132.2 | 295.8 | (--*-*) |
| 6566.02 | -125.2 | 38.4 | 201.9 | (**) |
| 6554.425 | -116.8 | 46.8 | 210.3 | (-*) |
| 6560.74 | -84.4 | 79.1 | 242.7 | (--) |
| 6555.195 | -136.0 | 27.6 | 191.1 | (---*) |
| 6579.66 | 0.0 | 265.0 | 428.6 | (----*) |
| 6559.035 | -96.1 | 67.4 | 231.0 | (-*) |
| 6561.205 | -60.0 | 103.6 | 267.1 | (---*) |
| 6570.89 | 0.0 | 182.3 | 345.8 | (---*) |
| 6570.145 | 0.0 | 263.8 | 427.4 | (----*) |
| 6594.825 | 0.0 | 417.2 | 580.7 | (-----*) |
| 6564.42 | -46.8 | 116.8 | 280.3 | (--*) |
| 6561.75 | -72.4 | 91.1 | 254.7 | (---*) |
| 6557.03 | -118.1 | 45.4 | 208.9 | (---*) |
| 6575.21 | -116.0 | 47.6 | 211.1 | (---*) |
| 6556.59 | -127.6 | 36.0 | 199.5 | (---*) |
| 6556.3 | -134.9 | 28.7 | 192.2 | (---*) |
| 6562.35 | -61.8 | 101.7 | 265.3 | (---*) |
| 6557.76 | -120.4 | 43.1 | 206.7 | (---*) |
| 6556.135 | -115.0 | 48.5 | 212.1 | (---*) |
| 6555.43 | -136.9 | 26.7 | 190.2 | (---*) |
| 6572.145 | -119.0 | 44.5 | 208.1 | (---*) |
| 6573.031 | -118.1 | 45.4 | 208.9 | (---*) |
| 6609.795 | 0.0 | 562.2 | 725.7 | (-----*--*) |
| 6629.63 | 0.0 | 766.0 | 929.3 | (-----*) |
| 6745.435 | 0.0 | 2117.8 | 2281.4 | (---------*) |
| 6724.515 | 0.0 | 2196.9 | 2360.4 | (---------*) |
| 6762.33 | 0.0 | 2124.7 | 2288.2 | (---------*) |
| 6760.225 | 0.0 | 2152.6 | 2316.1 | (---------*) |
| 6746.06 | 0.0 | 1958.4 | 2122.0 | (---------*) |
| 6761.83 | 0.0 | 2094.2 | 2257.7 | (---------*) |
| 6769.95 | 0.0 | 2166.3 | 2329.9 | (---------*) |

---

0  700  1400  2100

134
The remaining 35 solutions constitute the subset of $\hat{X}$ and their sample average function values (Eq. 6.13) are recorded in Table 6.2. Whether the subset of $\hat{X}$ is large enough is determined by tracking the percentage difference of the average expected cost according to Eq. (6.15), as listed in the last column of Table 6.2.

Table 6.2 The remaining 35 solutions and their sample average objective values

<table>
<thead>
<tr>
<th>Solution #, $k$</th>
<th>( \hat{f}<em>{400}(\hat{X}^k</em>{400}) )</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65566.32</td>
<td></td>
</tr>
<tr>
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<td>-0.012103719</td>
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<tr>
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<td>-0.016313261</td>
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<td>65649.885</td>
<td>0.014976905</td>
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<td>65605.36</td>
<td>0.002745642</td>
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<tr>
<td>9</td>
<td>65667.44</td>
<td>0.012651508</td>
</tr>
<tr>
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<tr>
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<tr>
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<td>-0.008883218</td>
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<tr>
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<td>65666.935</td>
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</tr>
<tr>
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Table 6.2 continued

<table>
<thead>
<tr>
<th>Solution #, k</th>
<th>$\hat{f}<em>{400}(\hat{x}</em>{400}^{ik})$</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
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<td>65629.35</td>
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<td>65570.76</td>
<td>-0.001135474</td>
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<td>65576.135</td>
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<td>33</td>
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<td>65572.145</td>
<td>-0.000864036</td>
</tr>
<tr>
<td>35</td>
<td>65573.03</td>
<td>-0.000776119</td>
</tr>
</tbody>
</table>

Mean $\bar{z}_N^K$          65590.33814

Variance $S_K^2$          1493.181349

The % difference values in Table 6.2 are plotted in Figure 6.4 for better visualization. The % difference greatly fluctuates when the subset size of indifferent solutions is small and gradually stabilizes as the subset size increases. Using 0.0008% as the cutoff, a subset size of 35 solutions is determined large enough, meaning that additional replications are not likely to worth the effort. Based on these 35 solutions, $\hat{z}_N^K$ was computed ($N=400$ and $K=35$ in this case) to estimate the true $z^*$ of the problem and the associated variance was computed by $S_K^2$.

![Figure 6.4 Plot of the percentage difference of the average expected cost values](image)

Each of these 35 solutions was further evaluated using a much larger sample size $N'$ to estimate the expected cost of operating the supply chain inventory system. Two cases of sample
size \( N' = 10,000 \) and \( N' = 100,000 \) were tested. Note that both sample sizes are sufficiently larger than the bound provided by Rinott (1978). Tables 6.3 and 6.4 record the corresponding results. One interesting finding is that the estimated variance for each candidate solution do not appear to increase much as more samples are drawn, which implies that the performance of each solution has been stabilized. It can be observed that both cases lead to the same best solution \#32: \( s = [78, 77, 89, 252]; \ S = [152, 152, 152, 313] \), which means that the near-optimal \((s_i, S_i)\) policy for the three retailers and the distributor are \((78, 152), (77, 152), (89, 152)\) and \((252, 313)\), respectively. Besides Solution 32, actually a large portion of all 35 solutions (around 43%) is expected to perform well. Their optimality gaps are all very small. Note that the optimality gaps are absolute values which are almost negligible relative to the system expected cost. For those high quality solutions, the percentage optimality gaps are all within 0.01%. However, for some significantly inferior solutions such as Solutions 9, 13, 15 and 28, their optimality gaps stay relatively large even with increased sample size \( N' \). Generally speaking, increasing the confidence level tends to expand the CI a little bit, which means the margin of error will be increased in order to increase the confidence level.

Last but not least, it should be noted that the accuracy of the optimality gap estimators is related to the number of replications \( K \) and sample sizes \( N \) and \( N' \). For small \( K, N \) and \( N' \), the gap estimator may be large even for a near-optimal solution, \( \hat{X}^* \). According to Kleywegt et al (2001), the upper boundary of the optimality gap estimator can be divided into components as follows:
\[
\hat{f}_N(\hat{X}^*) - z_N^K + z_{\alpha}\left(\frac{S_{N'}^2(\hat{X}^*)}{N'} + \frac{S_K^2}{K}\right)^{1/2} \\
= (\hat{f}_N(\hat{X}^*) - f(\hat{X}^*)) + (f(\hat{X}^*) - z^*) + (z^* - z_N^K) + z_{\alpha}\left(\frac{S_{N'}^2(\hat{X}^*)}{N'} + \frac{S_K^2}{K}\right)^{1/2}
\]

(6.27)

The first term on the right-hand side of the above equation has an expected value zero if \(N' \to \infty\); The second term is the true optimality gap; the third term has a positive expected value that is decreasing as \(N \to \infty\); the last term is the measure of accuracy, which decreases with increased number of replications \(K\) and the sample size \(N'\). Even with an optimal solution \(\hat{X}^*\), i.e., \(f(\hat{X}^*) - z^* = 0\), the value of other three terms can be large if \(K\), \(N\) and \(N'\) are small, Therefore, it is recommended to use a sufficient large number of \(K\), \(N\) and \(N'\) to achieve the desired level of accuracy.

6.6 Discussion

It is desirable to determine the effect of supplier quality imperfection on supply chain inventory policies and cost. To this end, the proposed simulation optimization framework was used to optimize the policies under perfect quality by setting \(Q_i = 1\), \(t = 1\), \(\ldots\), \(T\). In addition, detailed analyses including cost components, ordering patterns, and service level were also carried out.

For the perfect quality case, the solution found by the is \(s = [72, 91, 77, 218]\) and \(S = [152, 152, 152, 306]\) with a total supply chain cost of $57,900, which means that the \((s_i, S_i)\) policy for the three retailers and the distributor are (72, 152), (91, 152), (77, 152) and (218, 306), respectively.
Table 6.3 Test results of candidate solution based on $N' = 10,000$ quality scenarios

<table>
<thead>
<tr>
<th>NO.</th>
<th>Candidate solution</th>
<th>Estimated expected cost ($N' = 10,000$)</th>
<th>Estimated variance ($N' = 10,000$)</th>
<th>90% CI</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>s=[93 75 86 250] S=[154 151 152 311]</td>
<td>65604.7566</td>
<td>874408.564</td>
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<td>s=[74 89 79 289] S=[152 153 153 313]</td>
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<tr>
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<td>900763.9207</td>
<td>15.93126676</td>
<td>18.9832752</td>
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<tr>
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<tr>
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<td>18.40164219</td>
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<tr>
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<td>15.44314495</td>
<td>18.40164219</td>
</tr>
<tr>
<td>17</td>
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<td>871610.3464</td>
<td>15.68176112</td>
<td>18.68597099</td>
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<tr>
<td>18</td>
<td>s=[78 77 89 252] S=[152 152 152 311]</td>
<td>65573.0162</td>
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<td>15.68176112</td>
<td>18.68597099</td>
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<td>15.68176112</td>
<td>18.68597099</td>
</tr>
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Table 6.3 continued

<table>
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<tr>
<th>NO.</th>
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<th>Estimated variance (( N' = 10,000 ))</th>
<th>Optimality gap</th>
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</tr>
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<td>16.42611596</td>
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<tr>
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<td>900763.9207</td>
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</tr>
<tr>
<td>27</td>
<td>([86 \ 86 \ 72 \ 219] \ S=[153 \ 152 \ 152 \ 311] )</td>
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<td>876155.4118</td>
<td>15.72091987</td>
</tr>
<tr>
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</tr>
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Table 6.4 Test results of candidate solution based on \( N' = 100,000 \) quality scenarios

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<tr>
<th>NO.</th>
<th>Candidate solution</th>
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<th>Estimated variance (( N' = 100,000 ))</th>
<th>Optimality gap</th>
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140
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<td>NO.</td>
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<td>Estimated variance ($N'=100,000$)</td>
<td>Optimality gap 90% CI</td>
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<td>878485.5057</td>
<td>5.819576843</td>
<td>6.934453517</td>
<td></td>
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Recall the best solution in the quality imperfection case (solution #32 in Table 6.4), $s=[78 77 89 252]$ and $S=[152 152 152 313]$, with a total expected cost of $65,588.6. It is clear that quality imperfection leads to a cost increase of $7,688.6, equivalent to a 13.28\% increase. Figure 6.5 indicates that the major cost increase comes from the additional remanufacturing cost, accounting for 89\% of the total cost increase. Note that the lost-sale cost and the holding cost in the imperfect quality case are also slightly higher than the perfect quality case, mainly at the burden of the distributor. The ordering costs in both cases remain the same, to be further discussed together with the ordering patterns given below.

![Cost partition comparison](image)

Figure 6.5 Cost partition comparison (imperfect quality case vs. perfect quality case).

The ordering patterns achieved by both solutions are very similar, as shown in Fig. 6.6-6.7. The ordering patterns for retailers are identical, resulting in the same high level of service rate of 0.986. The ordering patterns for the distributor are slightly different but with the same frequency, ordering every lead time period.
For the perfect quality case, ordering every lead time period keeps the inventory moving fast, old stock are used up by the end of current lead time period while new items are filled in on a continuous basis. For the imperfect quality case, ordering every lead time period helps to minimize the possibility of shortage caused by quality imperfection on the distributor's stock. The assumption is that those imperfect items will be remanufactured and delivered in the next lead time period. When the ordering frequency matches with the lead time, the amount
remanufactured in the last lead time period perfectly serves to make up for the shortage caused by the defects in the current lot.

The distributor's service level to each retailer is called internal service level in the sequel because it is internal to the supply chain network in contrast to external service level to the end customer. The internal service levels of the distributor to each retailer for both cases have been investigated. Both cases lead to solutions of ordering frequency matching with the lead time period and the internal service levels are all able to keep at sufficiently high levels even in the imperfect quality case. In both cases, retailer 1 and 2 are able to be satisfied 100% from the major distributor. The internal service levels for retailer 3 experience 1% decline, from 97% of perfect quality case to 96% of imperfect quality case, respectively. This result is due to the ranking allocation employed in this study, which prefers retailer 1 and 2 over retailer 3. Essentially, the optimized policies indicate that most of the shortage caused by quality imperfection is covered by remanufacturing. Therefore, lost-sale cost at the distributor's side is not significantly increased.

One interesting finding is that the (near)optimum solutions found for both cases do not differ much. This leads us to suspect that the solution for the perfect quality case serves as a good initial solution to perform search on the solution for the imperfect quality case to take advantage of the fact that finding a (near)optimum solution in the deterministic perfect quality case requires much less computational effort than the stochastic imperfect quality cases. Taking the solution for the perfect quality case as a solution for the quality imperfection case and testing it with the
100,000 scenarios as those used to obtain results given in Table 6.4, the optimality gap was found
to be 101.106469986 and 103.6591884211, for the 90% CI and 95% CI, respectively. Nevertheless, a more comprehensive study is needed to verify this idea.

It is worthy to point out that the proposed approach requires a considerable amount of computer time as most simulation-optimization-based approaches do. For example, a typical run of algorithm 1 alone for the stochastic inventory problem of the single-distributor-three-retailer network considered in this study takes approximately 25,000 seconds CPU time. This is the main reason that prevents us from testing more complicated supply chain settings. This is also one of the major barriers in future simulation optimization applications since a model has to be run under a large number of experimental conditions. One possible way to alleviate this problem is to use parallel processors or the utilities of distributed computing. Another possibility to reduce the computational burden is to use statistical methods such as factor screening techniques to reduce the search space. Clearly, more research needs to be conducted in these areas to improve the efficiency of simulation optimization systems.

6.7 Conclusion

This chapter has presented a new stochastic simulation optimization methodology that integrates a metaheuristic-based simulation optimization framework with a modified sample average approximation method for optimizing integrated supply chain inventory systems with supplier quality imperfection. Theoretically any metaheuristic algorithm can be used; however, one of the most recent hybrid metaheuristics is employed here. Considering defective items in
delivered lots are more truthful to the real world. In order to handle the stochastic effect of quality imperfection and to obtain high quality solution with high probability, a sample average approximation method modified from Kleywegt et al. (2001) serves as an integrated part of the proposed methodology. Using the proposed methodology, the (near)optimal inventory policies for each entity within the supply chain can be estimated and the optimality gap can be quantified.

The proposed methodology was illustrated with an example to find the optimal \((s, S)\) policies for an integrated supply chain system composed of a single-distributor multiple-retailer system with the supplier product quality modeled as a three-point discrete distribution function. The results shows that the proposed procedure is capable of finding high quality solutions with the percentage optimality gaps are all within 0.01%. The implication of quality imperfection on supply chain inventory policies and costs is also discussed in light of the perfect quality case.

The effectiveness and efficiency of the proposed methodology rely on the good choices of number of replications \(K\) and sample sizes \(N\) and \(N'\). Generally speaking they have to be fairly large for the optimality gap estimator to indicate that the solution is good. Unfortunately, this also implies lengthy computational time. Therefore, a possible topic for future research is developing approaches that offer better tradeoff between problem solvability, solution accuracy and computational expenses. Taking the solution for the perfect quality case as an initial solution will also be explored further in the future.
CHAPTER 7 SUPPLY CHAIN INVENTORY OPTIMIZATION WITH HIGHLY PERISHABLE PRODUCTS

7.1 Introduction

In this chapter, the stochastic metaheuristic-based simulation optimization (SMSO) framework is further extended to the inventory management of highly perishable products.

The strategic importance of perishable goods in food, chemical, pharmaceutical and healthcare industries cannot be emphasized more. The sale of perishable products makes up over 50% of the 550 billion U.S. retail grocery industry (Ferguson and Ketzenberg 2006). For supermarkets, perishables are the driving force behind the industry's profitability. Customer's choice is significantly influenced by the assortment and shelf availability of fresh food. The increasing food prices and at the same time billions of dollars' worth of food expiring every month (Grocery Manufacturer Association 2008) raise public concerns. Perishables loss at grocery retailers can be as high as 15% due to damage and spoilage (Ferguson and Ketzenberg 2006). On the other hand, supermarkets lose revenue when products are not available on the shelf. Gruen et al., (2002) report that the worldwide average out-of-stock rate is 8.3%, in the U.S. is 7.9%. The financial consequences for retailers and grocery producers/manufacturers are severe. For healthcare practice, the problem gets more serious since almost all blood products have limited shelf lives. While the supply is voluntary and costly, shortage of blood products may put life at risk. Blood banks and hospitals have strived to minimize shortages and outdates of all their blood products. In general, inventory control of perishables is very challenging. Major
challenges come from uncertain demand, limited shelf life, and high customer service level requirements. A close match between supply and demand is of the essence.

In this chapter, we tackle this problem by suggesting a simulation optimization formulation of supply chain inventory system for highly perishable products. In addition, a new age-based replenishment policy that accounts for not only the stock levels but also the age distribution of the items in stock is proposed. This newly developed policy is named as "Old Inventory Ratio" policy (OIR) and it uses only partial age information of the stock and therefore is easy to implement in practice.

The effectiveness of the proposed policy is evaluated in detail for a platelet supply chain. We chose platelets (PLTs) mainly due to its critical medical importance. Among those various blood products, PLTs is the one with the shortest shelf life, typically 5 days, and sometimes extendable to 7 days. PLTs are a very important component of today's therapies including those related to bone marrow transplants, chemotherapy, radiation treatment and organ transplants. The very short shelf life of PLTs makes their production and inventory management a really challenging task. In 2008, 12.7% of the platelets produced were outdated, slightly more than those outdated in 2006 (10.9%), according to the 2009 National Blood Collection and Utilization Survey Report.

In the numerical example, we focus on the production and inventory management of PLTs for a supply chain consisting of a blood center supplying primarily one major medical center and a number of smaller peripheral hospitals. On the supply side, PLTs collections and/or productions
are done by the blood center. The unit production cost of apheresis PLTs, which includes collection, testing, processing, and distribution costs, is high (approximately $500 according to Fontaine et al., 2009). For any blood center, PLTs production volumes must be set carefully to prevent large numbers of outdated units without risking a major shortage. At the hospitals' side, they paid even more with an average of $538.56 in 2008 in comparison with $525.05 in 2006, a significant increase of 2.6% (p<0.005), according to the 2009 National Blood Collection and Utilization Survey Report. The critical issue to decision makers at hospitals is how to manage PLTs inventory in a manner that minimizes outdates while satisfying demand. The inventory objective is set to minimize the system outdate rate under a pre-specified fill-rate constraint. This is achieved by an efficient SMOS framework that optimizes the production and inventory of platelets from blood centers to hospitals. One of the critical elements of the SMOS framework is the embedded non-convex constrained stochastic optimization model. For comparison, we study both decentralized and centralized control and three replenishment policies, one without age consideration and the other two with age consideration, under three different fill-rate constraints.

While the focus is on PLTs, the age-based policy of the proposed structure would be applicable to and promising in other perishable inventory settings (with a short shelf life) as well. Hopefully it would initiate further research in this direction. The main contributions of this chapter include:

- developing a quantitative formulation of the highly perishable supply chain inventory model with production/delivery lead time
proposing a simulation optimization approach based on a hybrid metaheuristic algorithm to find near-optimal order-up-to policies for the entire supply chain (SC) system

being the first to develop both centralized and decentralized models for the highly perishable supply chain and quantify the potential savings of centralized control

developing a new replenishment policy (OIR) that bases reordering decisions not only on the stock levels but also on the age of the stock. It is shown that OIR helps improving the SC performance.

7.2 Modeling framework

This section presents the highly perishable supply chain problem, its mathematical formulation, and the proposed approach for its solution.

7.2.1 The perishable supply chain and model formulation

I consider a class of order-up-to policies for a single-vendor multi-buyer supply chain. The product that the SC is handling has a short shelf life. Typical examples include fresh vegetables/fruit, dairy/meat products, blood products, etc. The formulation presented here is designed specifically as a PLTs production-inventory model. The PLT product is chosen here because: 1) it is of great medical importance to save lives; 2) substantially less research has been done in this field. It should be pointed out that the overall modeling structure holds for all kinds of perishable products with short shelf lives.

Considering stocking hospital blood banks with units of PLTs based on scheduled daily deliveries from a regional blood center (the delivery schedule for other products that are not as
critical as PLTs may not need to be daily if the transportation expense is relatively high). The order-up-to level at each hospital can vary day to day over the seven days of the week. The production schedule is needed at the blood center to replenish its inventory to a set of order-up-to levels; but production only occurs during the weekdays. The performance measures of interest are "shortage rate" (the fraction of demand that cannot be satisfied immediately from stock on hand) and "outdate rate" (the proportion of total units within the SC system that are spoiled after their usable shelf lives). The objective is to minimize the outdate rate under a pre-specified maximum allowable shortage constraint. The PLTs supply chain includes a regional blood center and several hospitals; their production and inventory replenishment operations are detailed below.

- The regional blood center

At each period (typically a day), the inventory state (total number of on-hand inventory and their age distribution) is observed. The decision maker determines the number of platelets to be produced during the day, before the demand is observed. In practice, the restrictions with respect to the production capacity and storage capacity are usually negligible except the fact that there is no production on Saturday and Sunday. The process of producing platelets will take one day to complete so they will not be available for distribution until the next day. For each day, the blood center receives orders from the hospitals in the supply chain. The blood center satisfies each hospital's order from its available stock, following the First-In-First-Out (FIFO) policy. If orders cannot be fulfilled from the available stock, hospitals reach out to other regional blood centers to
expedite delivery of additional suppliers, following the real-world practice between the Stanford blood center and the Stanford University Medical Center as described in Fontaine et al., (2009). Under this situation, a shortage for the blood center is registered. To replenish its inventory, the blood center has to decide how many platelets to produce each day.

- The hospitals

Demand at each hospital is assumed to be stochastic and follows a known distribution (i.e., Poisson). There is no prior information about the future demand other than the statistical information of the demand distribution. However, it is worthwhile to note that our framework can handle any demand series and is not restricted to Poisson distributed demand. The hospital's inventory structure is similar to the blood center with one major difference. Unlike the blood center, the units received by the hospital may or may not always be of the same age upon arrival. This is because the mismatch between the supply at the blood center and the demands at the hospitals. The blood center may produce more PLT units than the order quantities issued by the hospitals. As a result, it is common for hospitals to receive products that have already lost one or more days of shelf lives upon receipt. Furthermore, because of the variations in the collection and production schedules at the blood center (demand is experienced seven days per week while platelet collection and testing take place only during the weekdays), the ages of arriving units vary from day-to-day. For instance, platelets produced on Friday typically will supply the demand for the next three days (weekend and Monday). Therefore, the platelets received by the hospitals on Mondays normally have lost at least three days of shelf lives.
The mathematical formulation of the platelet SC model to capture the above described operations is described below. Let entity \( i = 0 \) stand for the blood center, and entities \( i = 1, 2, \ldots, N \) denote hospitals, (with 1 representing the main hospital), respectively. The transfusion demand at each hospital is assumed to be stochastic and follows a certain distribution. Use \( P = \{ P_1, \ldots, P_i, \ldots, P_N \} \) to represent their corresponding probability density function (pdf). Let matrix \( D^k = \{ D_1^t, \ldots, D_i^t, \ldots, D_N^t \} \) be one realization of the stochastic demands faced by \( N \) hospitals, \( k = 1, \ldots, K \), and \( D_i^t, t = 1, \ldots, T \) is the demand series from the \( i \)th hospital over the \( T \)-period planning horizon. In order to estimate the expected outdating rate and shortage rate, the sample size, \( K \), needs to be statistically sufficient. The following notations are used in formulating the model for one realization of the demand matrix.

**Notations**

- \( D_i^t \): Demand from entity \( i \) for period \( t \)
- \( Q_i^t \): Production/Ordering quantity from entity \( i \) for period \( t \)
- \( S_i^t \): Number of shortage for entity \( i \) at period \( t \)
- \( O_i^t \): Number of outdating for entity \( i \) at period \( t \)
Obviously, determining the inventory level for perishables is more complicated than for other non-perishables. One should account for not only the total inventory level but also the age distribution of the inventory. It is obvious that the set of possible inventory states is immense. The number of states easily becomes prohibitively large for computation. For example, a PLT ordering problem with 10 possible inventory levels for each age category with a 5-day PLT shelf life has $10^5$ potential inventory states on each day, not to mention adding the consideration of number of days in the planning horizon, the number of entities within the SC, and the possible order sizes.

Order-up-to policies are suggested in Haijema et. al., (2007, 2009) as good approximation of optimal policies because the set of optimal policies has a fairly complicated structure impractical to implement. Hence, we decide to investigate two modified order-up-to policies, using the original order-up-to policy as the benchmark. One is the new age-based policy that uses only partial age information of the stock. The other one is taken from the existing literature. In the following, the common part of the three policies is described and the variations among those policies are explained in Section 3.2.

To simplify the presentation, matrix $x = (x'_0,\ldots,x'_t,\ldots,x'_N)$ is used to represent the inventory state for all entities at period $t$. Each $x'_i$ is a vector, recording the inventory state for entity $i$ at time period $t$. As shown in Fig. 7.1, the inventory state is updated through $t=1,\ldots,T$. A natural way to describe the inventory state is to categorize the PLTs in stock according to their age.
Using $M$ to represent the maximal residual shelf life allowed (For PLTs, $M=5$), the inventory state for entity $i$ at time period $t$ can thus be denoted by

$$x_i^t = (x_{i,1}^t, ..., x_{i,r}^t, ..., x_{i,M}^t)$$

where $x_{i,r}^t$ represents the number of PLTs with a residual shelf life of $r$ days before outdating. For example, $x_{i,1}^t$ records the number of items that will be expired in one day; therefore, it indicates the oldest item(s) on stock. The transition of inventory states from one day to the next is determined jointly by demand, production/ordering quantity, and platelet replenishment policy. For each day, the inventory state is first updated after satisfying the demand during the day (to use the oldest units first), next removing units that are outdated, then decreasing the residual shelf life of all remaining units by 1 day, and finally adding the newly received units.

Given the initial system state $X^1 = \{x_0^1, x_1^1, x_2^1, ..., x_n^1\}$, i.e. the initial inventory level at each supply chain entity at the beginning of the first period ($t = 1$), the shortage quantity during the $t^{th}$ day for entity $i$ with inventory state $x_i^t$ can be computed by

$$S_i^t (x_i^t) = (D_i^t - \sum_r x_{i,r}^t)^+ \text{ (shortage quantity) \quad t = 1, ..., T}$$

(7.1)

where $x^+ = \max\{x, 0\}$ and $D_i^0 = \sum_{j=1}^N q_{ij}^0$

The platelet demand considered in this study is mainly for traumatology and general surgery. Platelets of any age up to its maximal shelf life (5 days) can be used because those patients receiving transfusion can quickly resume their own platelet production and circulation. In current practice, the platelet demand is usually met by issuing the oldest stocks in the pool first. Early studies have established that a first-in-first-out (FIFO) management strategy is the optimal
issuing policy for perishable products in order to reduce the number of leap-frogged units. Leapfrogging refers to the use of a fresher, non-expiring unit when an older, expiring unit is available. Therefore, the inventory state, \( x'_t \), is updated by removing those consumed items from their corresponding residual shelf life.

\[
x'_{i,r}(t) = \begin{cases} 
(x'_{i,r} - D'_t)^+, & \text{if } r = 1 \\
(x'_{i,r} - (D'_t - \sum_{j=1}^{r-1} x'_{i,j})^+)^+, & \text{if } r > 1 
\end{cases}
\]  

(7.2)

Based on the updated inventory state, the outdating quantity for each period can be calculated by

\[
O'_t(x'_t) = \max\{x'_{i,t}, 0\} \quad \text{(outdating quantity)} \quad t = 1, \ldots, T
\]  

(7.3)

Then all remaining units decrease their corresponding residual shelf lives by 1 day and the inventory state is last updated by receiving new replenishment. After the replenishment, the resulting inventory state will be the beginning inventory state for the next period, \( t+1 \).

The shortage rate for entity \( i \), \( SR_i \), based on the \( k^{th} \) demand realization, \( D^k \), over the planning horizon is given by

\[
SR_i = \frac{\sum_{t=1}^{T} S'_i(x'_t)}{\sum_{t=1}^{T} D'_t}, \quad i = 0, \ldots, N
\]  

(7.4)

The outdating rate for entity \( i \), \( OR_i \), based on the \( k^{th} \) demand realization, \( D^k \), over the planning horizon is given by
To investigate the difference in the expected system outdate rate between decentralized control and centralized control, two different objective functions are formulated below.

**Decentralized control.** All entities of the SC make their replenishment/production decisions based on their local information independently. In order words, each entity in the SC is treated as an individual company aiming to minimize its own outdate rate with no consideration of the system-wide outdate rate. For decentralized control, the objective function (i.e. system outdate rate) is in the following form:

\[
F_{\text{decentralized}} = \sum_{i=0}^{N} \min\{E_p(OR_i)\}
\]

subject to \( \max\{E_p(SR_i)\} \leq 1 - \phi \quad \text{for } i = 0, \ldots, N \)

\(E_p\) is the corresponding expected value function with respect to the stochastic demand distribution \(P\).

**Centralized control.** Centralized decisions are made to directly minimize the system-wide outdate rate. For centralized control, the objective function is of the form:

\[
F_{\text{centralized}} = \min\{\sum_{i=0}^{N} E_p(OR_i)\}
\]

subject to \( \max\{E_p(SR_i)\} \leq 1 - \phi \quad \text{for } i = 0, \ldots, N \)
In both cases, $\phi$ is the fill rate service measure (fraction of demand that can be satisfied immediately from the stock on hand). The objective function, the expected outdate rate, needs to be optimized subject to a predetermined minimum fill rate requirement.

7.2.2 Considered replenishment policies

In addition to the traditional order-up-to policy, two additional replenishment policies for perishable PLTs are investigated in this study; each of which takes into account the age of the platelets held at the blood center. Due to new information technologies like radio frequency identification (RFID), it is now economically feasible to track this type of information. The three replenishment policies are described in more details below.

- Policy 1: Order-up-to policy without age consideration

Each player in the SC follows an order-up-to policy to determine the amount to order (at each hospital) and the amount to produce (at the blood center) each day. Each hospital replenishes its inventory according to its target order-up-to level for each of the seven days of the week. By the end of each day, a replenishment decision is made at each hospital without prior knowledge of the demand for the next day, other than its statistical demand distribution. If the inventory position is less than the target level, an order is placed to bring the inventory position up to the target level. The blood center produces only during the weekdays and no production occurs on Saturday and Sunday. The items produced during the day become available in the early morning of the next day.
Therefore, the policy works as follows:

\[
\text{if } IP_i^t < SS_i^t \quad \text{then } \quad Q_i^t = SS_i^t - IP_i^t \quad (7.8)
\]

where \( IP_i^t \) is the inventory position at entity \( i \) on day \( t \) and \( SS_i^t \) is the target order-up-to level for entity \( i \) on day \( t \). The order-up-to level can vary from day to day.

- Policy 2: the EWA policy

The second policy studied is called the EWA policy introduced by Broekmeulen and van Donselaar (2009). This policy works similar to the base order-up-to policy except that the inventory position is corrected for the estimated amount of outdating and an order is placed if this revised inventory position drops below the target order-up-to level. We use this estimated outdating amount to determine the production volume at the blood center only because it is the source where all those fresh platelets are produced. As long as the blood center can keep producing fresh items every day, each hospital will receive fresh items as well. In other words, more fresh platelets will be circulated within the supply chain; hence the overall inventory system is moving fast to avoid outdate. Since the demand faced by each hospital is assumed to follow a Poisson distribution and the demand is independent of each other, aggregated demand faced by the blood center can be approximated by Poisson as well, with the mean equaling to the sum of the hospital means for each day of the week. With the known inventory state at the current period, the possible number of outdating with residual shelf lives from 1 to 5 days at the blood center can be determined as follows:

\[
\hat{O}_{0,r}^t = \max \{x_{0,r}^t - \hat{D}_{0,r}^t, 0\} \quad (r = 1, \ldots, 5) \quad (7.9)
\]
\( \hat{O}_{0,r} \) is the estimated number of outdates with residual shelf life of \( r \) days on day \( t \); \( x_{0,r} \) is the on-hand inventory on day \( t \) of items with residual shelf life of \( r \) days. \( \hat{D}_{0,r} \) is the cumulative estimate demand for the next \( r \) days on day \( t \). The estimated total amount of outdating on day \( t \) based on the current inventory state is calculated as the sum of estimated outdates with all residual shelf lives. Hence, the order-up-to policy at the blood center is:

\[
\sum \sum = + = 5 \ 1 \ 5 \ 1 \ 331 \ 145 + - = - 145 5 149 345 145 348 366 145 358 364 142 364 369 142 r r t r t t t t r t r O \ P S S Q \text{then SS OIP if } \]

\[
(7.10)
\]

where \( IP_0^t \) is the inventory position of the blood center on day \( t \) and \( SS_0^t \) is the target order-up-to level of the blood center on day \( t \).

• Policy 3: the Old Inventory Ratio (OIR) policy

This is an age-based policy newly proposed in this study, named as OIR. On top of the base order-up-to policy, the production decision at the blood center is also based on an old inventory ratio (the proportion of "old" items to the total items on hand) at that period. In the following numerical example of platelets with 5 days of shelf life, we define items with residual shelf lives of 1 and 2 days as "old". It should be pointed out that the definition of old items can also be subject to optimization with respect to the length of its shelf life if it is applied to general perishable cases. In the OIR Policy, the production quantity is first determined according to the original order-up-to level which only accounts for the number of items on stock. Next, the proportion of "old" items to the total items on hand is calculated. If this proportion exceed a certain threshold level, \( \delta \), (this threshold percentage is optimized together with the order-up-to levels), an additional replenishment is triggered to account for the possible outdating caused by
those "old" items. The size of additional replenishment equals to the total number of "old" items.

Similar to Policy 2, this modification applies only to the blood center:

First,

\[
\text{if } IP_0^t < SS_0^t \quad \text{then } \quad Q_0^t = SS_0^t - IP_0^t
\]  

(7.11)

Secondly,

\[
\text{if } \frac{\sum_{r=1}^{2} x_{0,r}^t}{\sum_{r=1}^{5} x_{0,r}^t} \geq \delta \quad \text{then } \quad Q_0^t = Q_0^t + \sum_{r=1}^{2} x_{0,r}^t
\]  

(7.12)

where \(\sum_{r=1}^{2} x_{0,r}^t\) is the number of "old" items on stock and \(\sum_{r=1}^{5} x_{0,r}^t\) is the total number of items on stock.

7.2.3 The simulation optimization approach

To solve the problem formulated in the previous section, the SMOS framework with sample average approximation is modified for finding those "near-optimal" order-up-to policies, as shown in Fig. 7.1. Specifically, the framework is based on a metaheuristic optimizer together with a SC simulation model tailored to the formulated problem. The metaheuristic optimizer used here is a hybrid metaheuristic proposed in Duan et. al., 2013 and Yi et al., 2013. It comprises of two cooperative metaheuristic algorithms, i.e., DE and HS, and HJ direct search. DE and HS generally try to capture a global picture of the search space. During the search process they successively focus on more promising regions of the search space. HJ local search refines the solutions and try to identify the best solutions in these high quality regions.
In order to estimate the expected system outdate/shortage rate, the sample average approximation method of Kleywegt et al., (2001) originally developed for Monte Carlo simulation, is adapted and integrated into the simulation optimization framework. The basic idea is that random samples of the stochastic demand data at each hospital are generated and their expected value functions are approximated by the corresponding sample average function. The obtained sample average problem is optimized by the hybrid metaheuristic.

Let \( D^1, D^2, \ldots, D^k, \ldots, D^K \) be the set of \( K \) realizations of independent and identically distributed (i.i.d.) random samples of demand data. The expected outdate rate at each entity, \( E_p(OR_i) \), is approximated as \( \frac{1}{K} \sum_{k=1}^{K} (OR_i \mid D^k) \), where \( OR_i \mid D^k \) is the calculated outdate rate based on the \( k^{th} \) demand realization. For the same token, the expected shortage rate at each entity, \( E_p(SR_i) \), is approximated as \( \frac{1}{K} \sum_{k=1}^{K} (SR_i \mid D^k) \), where \( SR_i \mid D^k \) is the calculated shortage rate based on the \( k^{th} \) demand realization. The reasons are \( E[\frac{1}{K} \sum_{k=1}^{K} (OR_i \mid D^k)] \to E_p(OR_i) \) and \( E[\frac{1}{K} \sum_{k=1}^{K} (SR_i \mid D^k)] \to E_p(SR_i) \), as \( K \to \infty \).

The hybrid metaheuristic algorithm generates a population of trial solutions and supplies those candidate solutions to the SC simulation model. The simulation model is used for estimating (through statistical sampling) expected performance measures (objective function value). Because demands at hospitals are assumed stochastic, the problem falls into the category of constrained stochastic optimization problems. The fill-rate constraint in the problem is
handled based on the parameter-less constraint handling method proposed by Deb (2000). Solutions with the associated expected shortage rate smaller or equal to $1-\phi$ are considered feasible; otherwise, infeasible, where $\phi$ is the fill rate requirement. The Deb's constraint handling method (2000) works like a tournament selection operator, where two solutions are compared at a time, and the following criteria are always enforced:

1. Any feasible solution is preferred to any infeasible solution.

2. Between two feasible solutions, the one having a better objective function value (lower outdate rate here) is preferred.

3. Between two infeasible solutions, the one having a smaller constraint violation is preferred.

Selection in metaheuristics is needed to rank all trial solutions according to their fitness, so that the whole searching process will be biased towards high quality solutions. The superiority of this method lies in that penalty parameters are not needed because in any of the above three scenarios, solutions are never compared in terms of both the objective values and the amounts of constraint violation together. This method avoids the difficulty in choosing a good penalty coefficient and therefore it is adopted here as a reliable and efficient constraint handling method. The metaheuristic optimizer works independently of the simulation model. Therefore, the simulation model can change and evolve to incorporate additional elements of the complex SC system, while the optimization routines remain the same. As shown in Fig. 7.1, simulation and optimization are two separate modules and interact with each other. The simulation model
represents the SC system, which evaluates the fitness of the suggested candidate solutions. The metaheuristic optimizer uses the outputs from the simulator and decides on a new population of candidate solutions as input to the simulator. It is used to solve the optimization problem defined within the model.

The search strategies of different metaheuristics are highly dependent on the philosophy of the metaheuristic itself. DE and HS generally fall into the category of evolution computing strategies, in which new solutions are generated by the application of mutation and crossover operators and the individuals for the next generation are selected from the union of old population and the offspring population. HJ consists of a sequence of neighborhood exploitation moves about a based point which, if successful, are followed by pattern moves to determine the next best search direction. It is used to refine the solutions generated by DE and HS. Interested readers are referred to (Duan et al., 2013) for a detailed explanation of the hybrid metaheuristic. Provided that a feasible solution exists, the metaheuristic optimization procedure tries to learn the relationships between decision variables. Within the whole simulation optimization loop, it tries to iteratively improve the quality of the best solution in the population, which over time provide a highly efficient trajectory to the global best. The proposed simulation optimization approach can operate under a stochastic setting that defies analytical tractability and its excellent solution ability enables more realistic modeling of the production and inventory control system, as demonstrated in the subject study.
Figure 7.1 Outline of the proposed simulation optimization framework
7.3 Computational experiments and results

As an illustration, a supply chain consisting of one regional blood center to replenish PLTs to three hospitals is considered. The demand for each entity in the SC is assumed to follow a Poisson distribution with varying daily means ($\lambda$) and they are independent of each other. This assumption is justified by Haijema et al., (2007, 2009). The computational experiments are based on the following data:

- Poisson demand from Hospital 1 with daily means: 29 32 33 31 33 22 20
- Poisson demand from Hospital 2 with daily means: 12 12 15 10 11 6 5
- Poisson demand from Hospital 3 with daily means: 5 7 7 6 5 1 1
- Replenishment at all 3 hospitals: seven days a week
- Production at the blood center: Monday through Friday at daytime
- Maximal shelf life of platelets: 5 days

7.3.1 Experimental details

For each hospital, the order-up-to level can be different for every day of the week. For the blood center, the order-up-to production volume can also be different for each workday. Therefore, the total number of variables to be optimized is 26 ($3 \times 7 + 5$) for Policy 1 and Policy 2; and 27 (with one more variable, $\delta$, ranging from 0 to 1 to represent the threshold proportion) for Policy 3. In order to facilitate the searching procedure for the optimal order-up-to levels, we employ a method to roughly set up the lower bound and upper bound of each decision variable, as suggested by Blake et al., (2010). We set the lower bound as the minimum order-up-to level,
$S_{\text{min}}$, (the minimum amount of stock that must be on hand after replenishment). It represents the smallest integer such that the probability of demand less than or equal to this value is $\phi$, where $\phi$ is the target minimum fill rate requirement. Since Poisson distributed demand is assumed, $S_{\text{min}}$, can be determined as

\[ S_{\text{min}} = F^{-1}(\lambda, \phi) \]  

(7.13)

where $F^{-1}(\lambda, \phi)$ be the inverse of the Poisson cumulative probability function (cpf) with mean equal to $\lambda$. This function returns the smallest value $S_{\text{min}}$ such that $F(S_{\text{min}}, \lambda) \geq \phi$.

Similarly, the upper bound (the maximum amount of stock that can be on hand after replenishment) should be set in a way that the resulting inventory level will not lead to too much unnecessary outdating. In this study, we start with a target upper limit of the probability of outdating, $\beta$. Order-up-to levels that will lead to probability of outdating exceed $\beta$ are not included in the searching process. For an item arriving on the $l^{th}$ day with a residual shelf life of $r_i$ days, clearly, it will be spoiled after $r_i$ days. We need to estimate the demand over its residual shelf life to determine whether it will be used to fulfill the customer demand. If it is not used over its residual shelf life, it is wasted and is considered as an outdated unit. Since demand for each day is assumed independently and identically Poisson distributed, the demand over the residual shelf life of an item (arriving on the $l^{th}$ day with a residual shelf life of $r_i$ days) can be estimated by the cumulative demand over the $r_i$ days, $CD(l, r_i)$. $CD(l, r_i) = \sum_{t=i+1}^{l+r_i} D'$, in which $D'$ is the demand on day $t$. Therefore, the upper bound for the order-up-to level is:
\[
S_{\text{max}} = \max_y \left\{ \sum_{x \in \mathbb{N}} (y - x) p(x, CD(l, r_j)) \right\} \leq \beta 
\]

where \( p(x, \mu) \) is the probability mass function of a Poisson random variable with a mean of \( \mu \), evaluated at \( x \). \( \beta \) can be set as the outdate rate in the current practice. That is, the optimized order-up-to policies should improve over that used in the current practice. In this study, we set \( \beta = 0.05 \).

We consider both decentralized control and centralized control. For each control strategy, the three replenishment policies described earlier are tested under three levels of shortage constraints: the minimum fill rate at each entity cannot be less than 0.99, 0.995 and 0.999, corresponding to a shortage rate of 0.01, 0.005 and 0.001, respectively. The optimization is performed based on a one year planning horizon (52 weeks). The stochastic yearly demand faced by each hospital is realized 30 times with varying daily average on each day of the week. That is, each of the 30 realizations includes 364 daily demand data points (7 days \( \times \) 52 weeks). Based on those realizations, the metaheuristic optimizer runs for 30 times (with maximum evaluation number of 10,000 per run) to search for the (near) optimal solutions for each problem setting. After the metaheuristic optimization, the identified (near) optimal solution is further simulated for 10,000 weeks in order to estimate its long-run performance in practice. All programs were coded in Matlab and run using a laptop equipped with a Intel® Core™ i5-2450M CPU @ 2.50 GHz. The simulated results are first checked using the Anderson-Darling normality test to confirm that all
data follow normal distribution and then the corresponding 95% confidence intervals (CI) are constructed. The detailed computational results are presented next.

7.3.2 Computational results and analyses

7.3.2.1. ANOVA

First, analysis of variances (ANOVA) was carried out to determine the significance of main effects and interaction effects on the system-wide outdate rate. The optimized solutions found in both decentralized and centralized controls are further evaluated for 10,000 weeks. The system-wide outdate rate is calculated according to Eq. (7.6) for decentralized control and Eq. (7.7) for centralized control. There are totally 18 possible combinations (3 policies × 3 levels of shortage constraints × 2 control strategies) of computational experiments. For each combination, 30 replications were made to capture the randomness effect due to the stochastic demand. The results indicate that all the main effects and all the 2-factor and 3-factor interaction effects are significant at p<0.001. The outdate rate is affected the most by the control strategy (F-value= 9259901.95, P-value=0.000), followed by replenishment policy (F-value= 1408178.37, P-value=0.000), and then fill rate constraint tightness (F-value= 542302, P-value=0.000). For 2-factor interaction effects, the interaction between policy and control strategy is the most significant (F-value=1343347.67, P-value=0.000), followed by the interaction between fill rate constraint tightness and control strategy (F-value=330005.13, P-value=0.000), and then the interaction between policy and fill rate constraint tightness (F-value= 101353.06, P-value=0.000). The 3-factor interaction is also significant (F-value=112159.55, P-value=0.000).
The main effect plot for each factor is shown in Fig. 7.2. Fig. 7.2 indicates that the new OIR Policy overall performs the best among the three (the average system outdate rate by adopting Policy 1, Policy 2 and Policy 3 is 16.55%, 10.33% and 4.01%, respectively). Generally speaking, a tighter fill-rate constraint leads to a higher system outdate rate and the rate of increase is almost linear. Centralized control substantially reduces the system outdate rate, from 19.6% in decentralized control to approximately 1.04%, on average.

Figure 7.3 shows the three two-factor interaction plots. From the interaction plot between policy and fillrate constraint, it is found that Policy 2 performs nearly as good as Policy 3 at low level of fillrate constraint=0.99 (2.54% vs 2.36%). But as the fillrate constraint gets tighter, Policy 2 leads to a much more significant increase in the system outdate rate than Policy 3. (The reason is explained in more details in Section 7.3.2.2). In all three levels of fillrate constraints, Policy 3 is able to keep the system outdate rate at relatively low values with only slight increase. From the interaction plot between control strategy and fillrate constraint, it is observed that the effect of different fillrate constraints on the outdate rate increase is much less in centralized control than in decentralized control. When the fillrate constraint gets tighter from 0.99 to 0.995 in centralized control, the average increase in outdate rate is only 0.3% (from 0.3% to 0.6%). However in decentralized control, as fillrate constraint is tightened from fillrate=0.99 to fillrate=0.995 the average increase in the system outdate rate is as large as 9% (from 12% to 21%). For all three fillrate levels considered, centralized control effectively keeps the increase in system outdate rate at a much slower pace than decentralized control.
Figure 7.2 Main effect plot for system outdate rate

Figure 7.3 Interaction plot for system outdate rate
From the policy-control interaction plot, it is found that accounting for age consideration is especially helpful in decentralized control strategy. Policy 2 and Policy 3 both involve age consideration and both have achieved much lower system outdate rates than Policy 1, which does not account for age consideration of the stock. The average system outdate-rates achieved by Policy 1, Policy 2, and Policy 3 in decentralized control are 31.8%, 19.9% and 7.05%, respectively. Therefore, using Policy 3 in decentralized control is recommended to minimize the system outdate rate. On the other hand, the differences among the three policies in centralized control are much less than those differences in decentralized control. That is because centralized control is designed to minimize the system-wide outdate rate by adjusting the replenishment/production decision for each entity in the SC. Since it is assumed that inventory gets replenished every day (except for Saturday and Sunday for the blood center). If properly optimized, the base order-up-to policy (with no age consideration) is good enough to synchronize the system in a way that keeps items on stock at each entity fresh and ready to be used. Therefore, outdated platelets are greatly reduced.

7.3.2.2. Decentralized control

Table 7.1 summarizes the results of shortage rate and outdate rate in 95% confidence intervals (CIs) for each entity in a decentralized supply chain under different replenishment policies and fillrate constraints. Policy 3 turns out to the best alternative with the lowest outdate rate in decentralized control. Policy 1 results in a huge amount of outdates at hospitals. That is because in decentralized control, the blood center minimizes own outdate rate, not the system-wide
outdate rate. As the result, the blood center ends up with a production schedule of large volume on certain days of the week and sporadic volume in between to make up any possible shortages. From the viewpoint of the blood center, it does not hurt to do so because the outdate rate at the blood center can still be kept at a low level. The consequence is that the downstream hospitals end up with receiving a lot of old items with residual lifetime of only one or two days. If those items are not consumed immediately, they become outdated quickly. In other words, under this circumstance, a large portion of items transferred from the blood center are too old to be really useful to fulfill patients' demand. Hence, the outdate rates at hospitals increase dramatically.

Figure 7.4 shows the expected age distribution of on-hand inventory at each hospital received from the blood center under decentralized control at fillrate=0.99 (similar patterns are observed in other fillrate requirements but omitted to save space). Note the higher percentage of on-hand inventory with shorter residual life for Policy 1, most notably at Hospital 1, the main hospital. By accounting for age as done in Policy 2 and Policy 3, more fresh items are received by each hospital, leading to a shift of distribution to the right (with higher percentage for items with longer residual life) and more symmetrical age distribution of inventory state. This helps cut down the number of outdates at hospital because fresh items will stay longer before they get expired. Generally, Policy 3 is the most effective in keeping items on stock fresh for longer duration while holding the least number of items on average.

According to Table 7.1, Policy 2 is not as effective as Policy 3 in controlling the system outdate rate as the fillrate constraint gets tighter. This is probably because the difficulty in
accurately estimating the possible outdates at each time period, since the patients' demand is fluctuating and hard to predict accurately. In Policy 2, the estimated amount of outdating is used to correct the inventory position based on which the corresponding order-up-to level is determined. Note in Fig. 7.4 that Policy 2 generally leads to a slightly higher inventory level in decentralized control than Policy 3. Those additional items can be deemed as safety stock to buffer any possible shortage caused by the expiration of old items. However, if those additional items are not consumed through the SC channels in time, they increase the risk of having more outdates. This might occur if the actual demand is lower than those anticipated. Therefore, any mismatch between supply and demand will increase the possibility of having more outdates than Policy 3. In decentralized control, this problem becomes more acute as fillrate constraint gets tighter.

On the other hand, Policy 3 adds another dimension of flexibility to get around this problem by introducing an additional decision variable, the threshold proportion. This variable measures the freshness of the items on stock. This type of detailed information at the item-level can be collected through auto-ID technology such as RFID with necessary sensors. By taking account for not only the number of items on stock but also their corresponding age information, inventory managers will be able to make informative replenishment decisions. The OIR policy proposed here is shown to work sufficiently well under decentralized control. The new OIR policy is recommended because it is the best among all three, consistently yielding good results in all three cases studied.
Table 7.1 Results of decentralized control (shortage and outdate rates are listed as 95% CI)

<table>
<thead>
<tr>
<th>Policy 1: Order-up-to without age consideration</th>
<th>fillrate=0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>[hospital 1]</td>
<td>[hospital 2]</td>
</tr>
<tr>
<td>shortage 0.006383</td>
<td>0.002115</td>
</tr>
<tr>
<td>[0.14506]</td>
<td>[0.060412]</td>
</tr>
<tr>
<td>outdate 0.14539</td>
<td>0.060746</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>fillrate=0.995</th>
</tr>
</thead>
<tbody>
<tr>
<td>[hospital 1]</td>
</tr>
<tr>
<td>shortage 0.00497</td>
</tr>
<tr>
<td>[0.14746]</td>
</tr>
<tr>
<td>outdate 0.14779</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>fillrate=0.999</th>
</tr>
</thead>
<tbody>
<tr>
<td>[hospital 1]</td>
</tr>
<tr>
<td>shortage 0.0001013</td>
</tr>
<tr>
<td>[0.089916]</td>
</tr>
<tr>
<td>outdate 0.084523</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Policy 2: EWA policy</th>
<th>fillrate=0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>[hospital 1]</td>
<td>[hospital 2]</td>
</tr>
<tr>
<td>shortage 0.00004882</td>
<td>0.001474</td>
</tr>
<tr>
<td>[0.14746]</td>
<td>[0.070871]</td>
</tr>
<tr>
<td>outdate 0.14779</td>
<td>0.071343</td>
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</tbody>
</table>

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
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<td>[hospital 1]</td>
</tr>
<tr>
<td>shortage 0.0000932</td>
</tr>
<tr>
<td>[0.089916]</td>
</tr>
<tr>
<td>outdate 0.084523</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>fillrate=0.999</th>
</tr>
</thead>
<tbody>
<tr>
<td>[hospital 1]</td>
</tr>
<tr>
<td>shortage 0.000009</td>
</tr>
<tr>
<td>[0.089916]</td>
</tr>
<tr>
<td>outdate 0.084523</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Policy 3: OIR policy</th>
<th>fillrate=0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>[hospital 1]</td>
<td>[hospital 2]</td>
</tr>
<tr>
<td>shortage 0.000842</td>
<td>0.002208</td>
</tr>
<tr>
<td>[0.14746]</td>
<td>[0.070871]</td>
</tr>
<tr>
<td>outdate 0.12223</td>
<td>0.059505</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>fillrate=0.995</th>
</tr>
</thead>
<tbody>
<tr>
<td>[hospital 1]</td>
</tr>
<tr>
<td>shortage 0.000006</td>
</tr>
<tr>
<td>[0.089916]</td>
</tr>
<tr>
<td>outdate 0.094112</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>fillrate=0.999</th>
</tr>
</thead>
<tbody>
<tr>
<td>[hospital 1]</td>
</tr>
<tr>
<td>shortage 0.000006</td>
</tr>
<tr>
<td>[0.089916]</td>
</tr>
<tr>
<td>outdate 0.094112</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Policy 4: OIR policy</th>
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</tr>
</thead>
<tbody>
<tr>
<td>[hospital 1]</td>
<td>[hospital 2]</td>
</tr>
<tr>
<td>shortage 0.000842</td>
<td>0.002208</td>
</tr>
<tr>
<td>[0.14746]</td>
<td>[0.070871]</td>
</tr>
<tr>
<td>outdate 0.12223</td>
<td>0.059505</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>fillrate=0.995</th>
</tr>
</thead>
<tbody>
<tr>
<td>[hospital 1]</td>
</tr>
<tr>
<td>shortage 0.000006</td>
</tr>
<tr>
<td>[0.089916]</td>
</tr>
<tr>
<td>outdate 0.094112</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>fillrate=0.999</th>
</tr>
</thead>
<tbody>
<tr>
<td>[hospital 1]</td>
</tr>
<tr>
<td>shortage 0.000006</td>
</tr>
<tr>
<td>[0.089916]</td>
</tr>
<tr>
<td>outdate 0.094112</td>
</tr>
</tbody>
</table>
Figure 7.4 Expected age distribution of on-hand inventory at each hospital under decentralized control with fillrate=0.99
7.3.2.3. Centralized control

As shown in the interaction plot, the differences among different replenishment policies in centralized control are much smaller than those differences in decentralized control. In centralized control, all entities in the SC aim to minimize the system-wide outdate rate under a specified fillrate requirement. The (near) optimal solutions for each policy found by the hybrid meta-heuristic all serve the same purpose, achieving a low system outdate of 1.04% on average (0.34% on fillrate=0.99, 0.6% on fillrate=0.995, 2.17% on fillrate=0.999). As in the decentralized control analysis, we also examine the expected age distribution of on-hand inventory at each hospital. Even with the tightest fillrate requirement (=0.999), centralized control leads to more fresh items on stock for all three policies. Comparing Figs. 7.4 and 7.5, it is not difficult to observe a significant shift of more old items with shorter residual life in decentralized control to more fresh items with longer residual life in centralized control. Because items on stock are kept fresh to better meet patient's demand under centralized control, the system outdate rates are greatly reduced.

Table 7.2 shows the 95% CIs of shortage and outdate rates for each policy under centralized control. Comparing the CIs between Table 7.1 and 7.2, the results clearly suggest the substantial benefit of using centralized control over decentralized control. The outdate rates at all entities of the PLTs SC are greatly reduced, especially at the hospitals. At the same time, centralized control is able to keep the shortage rate at either the blood center or the hospitals below the required constraint in all cases.
Figure 7.5 Expected age distribution of on-hand inventory at each hospital under centralized control with fillrate=0.999
Table 7.2 Results of centralized control (shortage and outdate rates are listed as 95% CI)

<table>
<thead>
<tr>
<th>Policy 1: Order-up-to without age consideration</th>
<th>fillrate=0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>hospital 1</td>
<td>hospital 2</td>
</tr>
<tr>
<td>shortage</td>
<td>[0.000008, 0.000111]</td>
</tr>
<tr>
<td>outdate</td>
<td>[0.000195]</td>
</tr>
<tr>
<td>Fillrate</td>
<td>0.99</td>
</tr>
<tr>
<td>hospital 1</td>
<td>hospital 2</td>
</tr>
<tr>
<td>shortage</td>
<td>[0.000005, 0.000057]</td>
</tr>
<tr>
<td>outdate</td>
<td>[0.000263]</td>
</tr>
<tr>
<td>Fillrate</td>
<td>0.995</td>
</tr>
<tr>
<td>hospital 1</td>
<td>hospital 2</td>
</tr>
<tr>
<td>shortage</td>
<td>[0.000008, 0.0000259]</td>
</tr>
<tr>
<td>outdate</td>
<td>[0.000166]</td>
</tr>
<tr>
<td>Fillrate</td>
<td>0.999</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Policy 2: EWA policy</th>
<th>fillrate=0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>hospital 1</td>
<td>hospital 2</td>
</tr>
<tr>
<td>shortage</td>
<td>[0.000008, 0.000009]</td>
</tr>
<tr>
<td>outdate</td>
<td>[0.000307]</td>
</tr>
<tr>
<td>Fillrate</td>
<td>0.995</td>
</tr>
<tr>
<td>hospital 1</td>
<td>hospital 2</td>
</tr>
<tr>
<td>shortage</td>
<td>[0.0000242, 0.0000259]</td>
</tr>
<tr>
<td>outdate</td>
<td>[0.000539]</td>
</tr>
<tr>
<td>Fillrate</td>
<td>0.999</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Policy 3: OIR policy</th>
<th>fillrate=0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>hospital 1</td>
<td>hospital 2</td>
</tr>
<tr>
<td>shortage</td>
<td>[0.0000055, 0.00000562]</td>
</tr>
<tr>
<td>outdate</td>
<td>[0.0001479]</td>
</tr>
<tr>
<td>Fillrate</td>
<td>0.995</td>
</tr>
</tbody>
</table>

180
Table 7.2 confirms that adopting centralized control will help significantly improve the PLTs SC performance. The message is that hospitals and blood center should look for ways to collaborate with each other in a manner as close to centralized control as possible.

Next, we are interested in knowing whether there is a statistical difference in employing different policies under centralized control. An ANOVA analysis was first carried out, followed by the multiple comparison procedure "Tukey's HSD (Honestly Significant Difference)" (Tukey, 1953) implemented in MINITAB. The corresponding results are given in Report 7.1.

The ANOVA report with p-value equaling to 0.009 indicates that at least one policy is significantly different from the others at $\alpha = 0.01$ in centralized control. Based on the Tukey’s method, the confidence intervals for all possible pairwise comparisons are computed with individual confidence level = 98.00% (not shown to save space). According to grouping information using the Tukey's Method, Policy 1 is significantly different from Policy 2. However, there is insufficient statistical evidence to conclude that the difference between Policy 2 and Policy 3 is significant. The “Hsu’s multiple comparisons with the best (Hsu’s MCB)” (Hsu, 1981), also available in MINITAB, was used to verify the results. “Hsu’s MCB” performs comparisons between each sample mean and the “best” of all the other means, aiming to find whether a global best strategy exists or not. In Hsu's MCB, the family error rate is 5%. Report 7.2 shows the result of Hsu's MCB.
Report 7.1 Report of different policies under centralized control using Tukey’s HSD

One-way ANOVA: Outdate Rate versus Policy

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Policy</td>
<td>2</td>
<td>0.0006989</td>
<td>0.0003495</td>
<td>4.76</td>
<td>0.009</td>
</tr>
<tr>
<td>Error</td>
<td>267</td>
<td>0.0196036</td>
<td>0.0000734</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>269</td>
<td>0.0203025</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S = 0.008569  R-Sq = 3.44%  R-Sq(adj) = 2.72%

Individual 95% CIs For Mean Based on Pooled StDev

<table>
<thead>
<tr>
<th>Level</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>--------------------</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>0.012587</td>
<td>0.011911</td>
<td>(--*------)</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>0.008863</td>
<td>0.005724</td>
<td>(--*------)</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>0.009609</td>
<td>0.006755</td>
<td>(--*------)</td>
</tr>
</tbody>
</table>

Means that do not share a letter are significantly different.

From the confidence intervals shown in Report 7.2, it is confirmed that Policy 1 is inferior to Policy 2 under centralized control. The lower limit of the CI for the mean difference between Policy 1 and Policy 2 is zero, indicating that the differences are significant. On the other hand, the mean difference between Policy 2 and Policy 3 is concluded insignificant. Therefore, it is
generally recommended to use policies with age consideration (Policy 2 and Policy 3) over the one without age consideration (Policy 1) in centralized control. It should be pointed out that the magnitude of difference between considering and not considering age distribution is much less in centralized control (the corresponding F-value is only 4.76). This is because replenishments are assumed to be fulfilled every day as in most practice. If each entity in the SC collaborates well as in centralized control, each supply chain entity gets to refresh its inventory daily. On-hand stocks at all the entities in the SC are kept moving and fresh all the time even with the base order-up-to policy.

Report 7.2 Report of different policies under centralized control using Hsu's MCB

| Hsu's MCB (Multiple Comparisons with the Best) |
| Family error rate = 0.05 |
| Critical value = 1.92 |

<p>| Intervals for level mean minus smallest of other level means |</p>
<table>
<thead>
<tr>
<th>Level</th>
<th>Lower</th>
<th>Center</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000000</td>
<td>0.003724</td>
<td>0.006172</td>
</tr>
<tr>
<td>2</td>
<td>-0.003193</td>
<td>-0.000746</td>
<td>0.001702</td>
</tr>
<tr>
<td>3</td>
<td>-0.001702</td>
<td>0.000746</td>
<td>0.003193</td>
</tr>
</tbody>
</table>

Policy 2 is originally proposed by Broekmeulen and van Donselaar (2009) for a retail process of perishable products at a single store. In this study, we have implemented it into a supply chain setting. The interesting finding is that Policy 2 works well in a supply chain setting for centralized control, actually equally well as Policy 3 statistically. This is probably because centralized control requires synchronization/collaboration between entities of the SC to minimize the system-wide outdate rate. The supply and demand are matched more closely than that in
decentralized control. Hence, the problem associated with the mismatch is minimized; the estimation of possible outdates can be done more accurately. Policy 2 is expected to be a good alternative to offer superior performance in centralized control.

7.4 Discussion

7.4.1 Internal fillrate

The blood center's fill rate to each hospital is called internal fill rate because it is internal to the SC network versus external fill rate to the end customer. In practical sense, a shortage of PLTs requires a hospital to expedite PLTs from another source (alternative blood center), often at a higher cost. Note that with the “alternative” blood center, orders from the hospitals will be 100% satisfied. The internal fill rate represents the actual percentage of quantity that the main blood center is able to provide before resorting to the “alternative” blood center. The internal shortage values presented in Table 7.3 actually is to partition the shortage rate at the main blood central to each hospitals. Table 7.3 shows how the blood center satisfies each hospital for all cases studied, listing in 95% CIs. In this study, we employ ranking allocation rule in which Hospital 1 is considered the most important and the stock at the blood center will be used to fulfill orders from Hospital 1 first, and so on and so forth. Note that the demand from the three hospitals are intentionally set according to this allocation rule with Hospital 1 accounts for 66% of the total PLTs usage while Hospital 2 and Hospital 3 account for 23.4% and 10.6%, respectively. The largest order from Hospital 1 is always fulfilled first. This is the case when a regional blood center supplies primarily to one major hospital and several small hospitals, as illustrated in
Fontaine et al., (2009). Because of the ranking allocation rule, it is easy to understand that in Table 7.3, the internal fill rate achieved by the blood center to Hospital 1 is always the highest, followed by internal fill rate to Hospital 2 and Hospital 3, respectively. In all cases studied, the shortage caused by the main blood center is almost negligible (0~0.6%).

Note that in some cases centralized control has higher internal shortage than decentralized control. Centralized control should not be regarded as inferior to decentralized control in those cases. It is misleading to interpret shortage values alone, without referring to the outdate values. Actually, in order to achieve those relatively low internal shortages, decentralized control results in a huge amount of outdates. Recall that a maximal shortage rate is set as the constraint; hence, any solution meeting the constraint is considered as feasible. Achieving smaller shortage values too far away from the predetermined constraint is counter-productive because it reduces the flexibility of controlling the other side of the tradeoff (i.e., outdate rate). In other word, centralized control should be regarded as superior to decentralized control instead because it achieves a better tradeoff within the allowable shortage constraint in all cases.

7.4.2 Effect of different network configuration

This section reports the results obtained from testing two more instances:

(a) a less complex single-vendor-single-buyer configuration;

(b) a more complex single-vendor-six-buyer configuration.
Table 7.3 Internal shortage (95% CIs) caused by the main blood center to each hospital

<table>
<thead>
<tr>
<th>Policy 1: Order-up-to without age consideration</th>
</tr>
</thead>
<tbody>
<tr>
<td>fillrate=0.99</td>
</tr>
<tr>
<td>hospital 1</td>
</tr>
<tr>
<td>decentralized</td>
</tr>
<tr>
<td>centralized</td>
</tr>
<tr>
<td>fillrate=0.995</td>
</tr>
<tr>
<td>hospital 1</td>
</tr>
<tr>
<td>decentralized</td>
</tr>
<tr>
<td>centralized</td>
</tr>
<tr>
<td>fillrate=0.999</td>
</tr>
<tr>
<td>hospital 1</td>
</tr>
<tr>
<td>decentralized</td>
</tr>
<tr>
<td>centralized</td>
</tr>
<tr>
<td>Policy 2: EWA policy</td>
</tr>
<tr>
<td>fillrate=0.99</td>
</tr>
<tr>
<td>hospital 1</td>
</tr>
<tr>
<td>decentralized</td>
</tr>
<tr>
<td>centralized</td>
</tr>
<tr>
<td>fillrate=0.995</td>
</tr>
<tr>
<td>hospital 1</td>
</tr>
<tr>
<td>decentralized</td>
</tr>
<tr>
<td>centralized</td>
</tr>
<tr>
<td>fillrate=0.999</td>
</tr>
<tr>
<td>hospital 1</td>
</tr>
<tr>
<td>decentralized</td>
</tr>
<tr>
<td>centralized</td>
</tr>
<tr>
<td>Policy 3: OIR policy</td>
</tr>
<tr>
<td>fillrate=0.99</td>
</tr>
<tr>
<td>hospital 1</td>
</tr>
<tr>
<td>decentralized</td>
</tr>
<tr>
<td>centralized</td>
</tr>
<tr>
<td>fillrate=0.995</td>
</tr>
<tr>
<td>hospital 1</td>
</tr>
<tr>
<td>decentralized</td>
</tr>
<tr>
<td>centralized</td>
</tr>
<tr>
<td>fillrate=0.999</td>
</tr>
<tr>
<td>hospital 1</td>
</tr>
<tr>
<td>decentralized</td>
</tr>
<tr>
<td>centralized</td>
</tr>
</tbody>
</table>
Table 7.4 Testing results of different network configuration

<table>
<thead>
<tr>
<th>Control strategy</th>
<th>Instance</th>
<th>Policy</th>
<th>blood center</th>
<th>hospital 1</th>
<th>hospital 2</th>
<th>hospital 3</th>
<th>hospital 4</th>
<th>hospital 5</th>
<th>hospital 6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.000012, 0.000136]</td>
<td>[0.00006, 0.00007]</td>
<td>[0.000075, 0.000085]</td>
<td>[0.002491, 0.002565]</td>
<td>[0.00221, 0.002325]</td>
<td>[0.002377, 0.002437]</td>
</tr>
<tr>
<td>Centralized</td>
<td>1</td>
<td>0</td>
<td></td>
<td>[0.007747, 0.00794]</td>
<td>[0.001536, 0.001605]</td>
<td>[0.002071, 0.00217]</td>
<td>[0.035069, 0.035379]</td>
<td>[0.06148, 0.062199]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td></td>
<td>[0.007018, 0.007176]</td>
<td>[0.003663, 0.003767]</td>
<td>[0.002372, 0.002434]</td>
<td>[0.014265, 0.014538]</td>
<td>[0.037492, 0.038026]</td>
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</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td></td>
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<td>[0.003001, 0.003089]</td>
<td>[0.002735, 0.002829]</td>
<td>[0.014634, 0.014927]</td>
<td>[0.03898, 0.039459]</td>
<td></td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>[0.074761, 0.075129]</td>
<td>[0.073297, 0.073684]</td>
<td>[0.020822, 0.021051]</td>
<td>[0.027562, 0.027781]</td>
<td>[0.07793, 0.078321]</td>
<td>[0.017418, 0.017651]</td>
</tr>
<tr>
<td>Decentralized</td>
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<td>0</td>
<td></td>
<td>[0.007793, 0.0078321]</td>
<td>[0.017418, 0.017651]</td>
<td>[0.037515, 0.037805]</td>
<td>[0.12062, 0.12116]</td>
<td>[0.12947, 0.13019]</td>
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<tr>
<td></td>
<td>2</td>
<td>0</td>
<td></td>
<td>[0.083911, 0.084405]</td>
<td>[0.011638, 0.011822]</td>
<td>[0.018645, 0.018875]</td>
<td>[0.088712, 0.089282]</td>
<td>[0.09129, 0.091946]</td>
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<td></td>
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<td></td>
<td>[0.05958, 0.06017]</td>
<td>[0.009593, 0.009782]</td>
<td>[0.011781, 0.012012]</td>
<td>[0.039537, 0.040064]</td>
<td>[0.060667, 0.06125]</td>
<td></td>
</tr>
</tbody>
</table>

*(a) single-vendor-single-buyer; (b) single-vendor-six-buyer
The daily demand (from Monday to Sunday) for each instance is: (a) 29 32 33 31 33 22 20; (b) 29 32 33 31 33 22 20; 22 22 23 21 24 19 18; 19 18 19 17 19 15 15; 12 12 15 10 11 6 5; 5 7 7 6 5 1 1; 4 4 4 4 4 1 1. For both instances, the planning horizon is kept the same as the first instance, i.e. 52 weeks. As the network expands, prolonged CPU time is expected mainly because of the increasing number of decision variables. Therefore, the maximum evaluation number is increased to 30,000 per run for instance (b). Our preliminary study shows that the average CPU time increases almost linearly as more numbers of entities (hospitals) are involved. To deal with more complex networks, advanced computing technology such as distributed computing and parallel computing are needed. Exploring that will be our future study.

As an illustration, the fill-rate constraint of 0.999 is chosen for both instances and their results are compared with that of the single-vendor-three-buyer configuration. Fill rate of 0.999 is the tightest constraint considered in this study and it is the most difficult for optimization. For each instance, we consider both centralized control and decentralized control. Table 7.4 records the 95% CI of outdate rate for each entity in the SC. Overall, the main conclusion obtained from the first instance (single-vendor-three-buyer) does not change. New finding is that the main source of outdate-rate increase in the SC system comes from relatively slow-moving demand, as in hospitals 5 and 6 of instance (b). That is because low volume of demand slow down the circulation of items in stock. Inventory tends to stay in stock for a longer time period, which is especially challenging for minimizing outdating. For both instances, policy 3 continuous to provide superior results for both centralized and decentralized control.
7.5 Conclusion

This chapter has presented a novel modeling and solution framework to optimize replenishment policies for highly perishable supply chains. The framework is based on a metaheuristic simulation optimization methodology that is set to minimize the expected system outdate rate under a predetermined fill-rate constraint. Some interesting insights are gained from applying this methodology to platelets SCs, with one blood center and several hospitals. Three instances with one, three and six hospitals have been tested. A new age-based replenishment policy called the Old Inventory Ratio (OIR) policy is proposed and it is compared with other two order-up-to policies in the literature. The general conclusion is that policies accounting for age distribution of stocks are superior to the policy without age consideration; and this result is particularly true for a decentralized supply chain.

Adopting centralized control over the whole PLTs supply chain greatly helps reducing the system expected outdate rate from 19.6% to 1.04% on average. Centralized control coordinates the SC very well so that the demand and the supply are matched closely. Even employing a policy without age consideration in centralized control can achieve a much lower system outdate rate than those policies with age consideration in decentralized control.

The new age-based OIR policy consistently provides good results in all cases studied. The advantages of OIR policy can be summarized as: 1) achieving the same performance with less information about the different ages in the inventory; 2) offering superior performance in both centralized or decentralized control (It may not be easy to implement centralized control in some
organizations due to the conflicting interest.); 3) ease of implementation in the real-world practice. Warehouse manager can easily count the number of "old" items in stock and calculate the corresponding ratio, instead of a sophisticated computation of estimated outdates as dictated in the EWA policy. Therefore, the OIR Policy is worthwhile to be communicated to the interest readers in academics and industries alike.

Possible topics for future studies include allowing transshipments among hospitals, considering different blood types and cross-matching, etc.
CHAPTER 8 OPTIMIZATION OF BLOOD SUPPLY CHAIN WITH SHORTENED SHELF LIVES AND ABO COMPATIBILITY

8.1 Introduction

Unlike many other substitutable resources, blood is a resource that has unpredictable supply and demand and when demand is not met, lives may be lost. Thus, maintaining adequate inventory to fill demand is of critical importance. Historically, red blood cells (RBCs) account for the largest proportion of blood transfusion (Prastacos 1984). Approximately 15 million RBC units are transfused annually in the United States, according to the 2009 national blood collection and utilization survey report (Washington, DC: Department of Health and Human Services) (2009). RBCs are useful in many scenarios such as for patients with chronic anemia, kidney failure, gastrointestinal bleeding, acute blood loss from trauma, and surgery. Thus, the wide range of usages for RBCs makes it a critical blood product to analyze.

In current practice, the Food and Drug Administration (FDA) permits red blood cells to be stored for as long as six weeks (42 days). However, according to a recent study, researchers from The Johns Hopkins Medical Institutions (Frank et al., 2013) found that red blood cells stored any longer than three weeks (21 days) lose the flexibility they need to get through tiny capillaries throughout the body. And if they can't travel through the capillaries, then the red blood cells aren't able to bring oxygen to needed places in the body. Furthermore, the loss of flexibility in the red blood cells that are more than three weeks old is permanent even though they are back to the patients' body. The study follows previous research on potential harms of using older blood. A past study in the New England Journal of Medicine (Koch et al., 2008) showed that death risk
doubled for people if they got blood that had been stored for longer than two weeks (14 days) during cardiac surgery, compared with those who received blood that had only been in storage for 10 days. Longer storage times may also be associated with an increased risk for a number of deleterious events including pneumonia, longer hospital stay, longer ventilation hours, organ failure, and increased infectious complications in cancer patients due to an immunomodulatory effect of blood transfusion. Other special group of patients such as children, infants and neonates also usually require fresher RBC transfusion (Sayers et al., 2011).

In addition, two large trials are currently being conducted to determine the validity of choosing fresher blood in some patients (Steiner and Stowell 2009). If the current trials should confirm that there are complications attributable to the transfusion of older RBC, then it would justify shortening the current permissible RBC shelf life, implying that current blood inventory management technique may need to be revisited. The challenge is that RBCs would become a more perishable product requiring a more scrutinized inventory management strategy similar to that for platelets (Fontaine et al., 2009). Managing such an inventory would imply a greater risk of increased RBC outdates, and novel approaches to RBC inventory management may be necessary to meet patient needs in this setting. Based on our successful experience on managing highly perishable platelets (Duan and Liao, 2013b), we extend our simulation optimization approach to RBC inventory management with ABO/Rh(D) substitution possibilities. Simulation is chosen as the most appropriate modeling method because of the following reasons: demand is stochastic; the system is complex (multiple products and the possibility of ABO/Rh(D)
compatible substitution); and the processes and outcomes (such as shortages and outdates) are
time driven as discrete events. This complex stochastic supply chain perishable inventory
problem is intractable by analytic techniques (Cohen and Pierskalla 1979b). Based on the output
of the simulation model, optimization is used to direct the search to the best combination of
decision variables.

This study aims to develop such a new simulation optimization approach, especially taking
into account the substitution possibilities with blood types and the age of the blood. No one has
implemented any simulation optimization approach that considers both blood type and age to
analyze RBCs inventory management. Therefore, the limited current research together with the
recent calls for shortening RBC storage lifetime and the lack of simulation optimization
framework provides an opportunity for new research exploration. To avoid high computational
time and to ease analyses, the study focuses on a single-hospital single-blood-center supply chain
system. The inventory target is to maintain an adequate RBC inventory to minimize the number
of outdated units without risking a major shortage, which may be accomplished by optimizing
the replenishment policies for both the hospital and the blood center to improve the RBC supply
chain performance. Additionally, the presence of 8 different types of RBC units with various
substitution possibilities adds complexities to the management decision-making when requesting
and transfusing RBCs. Learning how to better manage RBC inventory under those substitution
possibilities is an important endeavor for blood centers and hospitals.
The main contributions of this paper include:

- developing a quantitative formulation of RBC supply chain inventory model for multiple blood types with substitution possibilities and quantifying its potential savings,
- providing a potential tool not only capable of modeling the product flow in the entire supply chain but also able to search for the near optimal replenishment polices,
- proposing a simulation optimization approach based on a new hybrid metaheuristic algorithm (TA-TS) to find near-optimal replenishment policies for both hospital and blood center,
- being the first inventory optimization model that considers RBC supply chain with shortened permissible shelf life. To be more specific, we examine the inventory management of RBCs with maximal shelf life shorten to 21, 14 and 7 days.

The remainder of this chapter is organized as follows. Section 8.2 describes the proposed SC system and the simulation optimization methodology in detail. Section 8.3 compares the testing results of two different ABO/Rh(D) compatible substitution scenarios with the base scenario without any substitution, under three different MSLs (=7, 14 and 21 days). Next, further analyses detailed to each blood type are presented. Finally, the paper is concluded.

8.2 Modeling and Solution Framework

Blood is a living tissue of unique medical value to the human body. There are 8 major blood types, varying from 37.4% (O+) to 0.6% (AB-) in the US population. An efficient blood management applicable to the reality has to be examined for multiple blood products, since it
involves 8 blood types and Rhesus grouping, all of which have to be simultaneously controlled. Another interesting issue in the blood transfusion process is the blood substitution. Ideally, a patient should be transfused with the same blood group as his/her own, but this is not always possible. When a patient's blood group at the time of request is unavailable, a compatible blood group has to be provided. For example, patients with Group A+ can receive blood from A-, O-, or O+. Table 8.1 shows all the possible combinations and also the distribution of blood groups in the US Population.

Table 8.1 Blood type distribution and all possible ABO/Rh(D) compatible substitutions.

<table>
<thead>
<tr>
<th>Donors</th>
<th>US (%)</th>
<th>O+</th>
<th>O-</th>
<th>A+</th>
<th>A-</th>
<th>B+</th>
<th>B-</th>
<th>AB+</th>
<th>AB-</th>
</tr>
</thead>
<tbody>
<tr>
<td>O+</td>
<td>37.4</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>O-</td>
<td>6.6</td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>A+</td>
<td>35.7</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>A-</td>
<td>6.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>B+</td>
<td>8.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>B-</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>*</td>
<td>*</td>
</tr>
<tr>
<td>AB+</td>
<td>3.4</td>
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<td></td>
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<tr>
<td>AB-</td>
<td>0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

* denotes a possible substitution

The newly developed hybrid metaheuristic generates one trial solution at a time and fed into the ABO/Rh(D) RBC simulation model. The simulation model is used for estimating the expected performance measures. The hybrid metaheuristic iteratively improves the quality of the solution, which over time provides a highly efficient trajectory to the global best. The best solution found eventually can be considered as the near-optimal solution to the problem. Such a
formulation provides a quantitative tool not only capable of modeling the product flow in the entire chain but also able to search for the near-optimal solution for each blood type. The application of the TA-TS hybrid to the ABO/Rh(D) RBC problem is discussed in detail as follows.

8.2.1 The ABO/Rh(D) RBC inventory problem and model formulation

This chapter considers stocking a hospital blood bank with units of RBCs based on scheduled daily deliveries from a regional blood center. Instead of treating RBC units as one unified product, the study considers all 8 ABO/Rh(D) types and their compatibility. The order-up-to level for each blood type at the hospital can vary day to day over the seven days of the week. The production schedule is needed at the blood center to supply all 8 blood types to the hospital. Similarly, the blood center replenishes its inventory to a set of order-up-to levels; but production occurs only during the weekdays. As far as performance measures are concerned, the focus is on the tradeoff between "shortage rate" (the fraction of demand that cannot be satisfied immediately from stock on hand) and "outdate rate" (the proportion of total units within the SC system that are spoiled after their usable shelf lives). The reason is because RBCs are so precious that other costs are dwarfed by outdate and shortage costs. In addition, these two important costs in the RBC problem are intangible and hard to quantify. To avoid the difficulty in setting costs, it is chosen to measure them in relative percentage rates. The objective is to minimize the outdate rate under a pre-specified maximum allowable shortage constraint. The current model considers a two-echelon RBC supply chain with ABO/Rh(D) compatible substitution. The demand is first
satisfied by the exact ABO/Rh(D) match. If such an exact match is not available, a ABO/Rh(D) compatible match is then seek by following the ABO and Rh(D) compatible substitution shown in Table 8.1.

The production and inventory replenishment operations at both the blood center and the hospital are detailed below.

- The regional blood center

At each period (typically a day), the inventory state for each blood type (total number of on-hand inventory and their age distribution) is observed. The number of RBCs to be produced for each blood group is determined before the demand is observed. In practice, the restrictions with respect to the production capacity and storage capacity are usually negligible except the fact that there is no production on Saturday and Sunday. The process of producing RBC units will take one day to complete so they will not be available for distribution until the next day. For each day, the blood center receives orders from the hospital. The blood center satisfies the hospital's order from its available stock, following the First-In-First-Out (FIFO) policy. If orders cannot be fulfilled from the available stock, hospitals reach out to other regional blood centers to expedite delivery from additional suppliers, following the real-world practice between the Stanford blood center and the Stanford University Medical Center as described in Fontaine et al., (2009). Under this situation, a shortage for the blood center is registered. To replenish its inventory, the blood center has to decide how many RBCs to produce each day.
• The hospital

The hospital is facing the stochastic demand for 8 different blood groups. Demand at the hospital is assumed to follow a known distribution (i.e., Poisson). There is no prior information about the future demand other than the statistical information of the demand distribution. However, it is worthwhile to note that our framework can handle any demand series and is not restricted to Poisson distributed demand. Unlike the blood center, the units received by the hospital may or may not always be of the same age upon arrival. This is because the mismatch between the supply at the blood center and the demands at the hospitals. The blood center may produce more RBC units than the order quantities issued by the hospital. As a result, it is common for the hospital to receive products that have already lost one or more days of shelf lives upon receipt. Furthermore, because of the variations in the collection and production schedules at the blood center (demand is experienced seven days per week while RBC collection and testing take place only during the weekdays), the ages of arriving units vary from day-to-day. For instance, RBCs produced on Friday typically will supply the demand for the next three days (weekend and Monday).

The mathematical formulation of the RBC supply chain with ABO/Rh(D) compatible substitution model described above is discussed below. Let entity $i$ stands for different blood groups, i.e., entities $i = 1, 2, ..., 8$ denote blood groups from O+ to AB- at the hospital; entities $i=9, 10,...,16$ denote the same 8 blood groups at the blood center, respectively. The transfusion demand for each blood group at the hospital is assumed to be stochastic and follows a certain
distribution. Use $P = \{P_1, \ldots, P_i, \ldots, P_8\}$ to represent their corresponding probability density function (pdf). Let matrix $D^i = \{D^i_1, \ldots, D^i_t, \ldots, D^i_T\}$ be one realization of the 8 stochastic demands faced by the hospital, $k = 1, \ldots, K$, and $D^i_t, t=1, \ldots, T$ is the demand series for blood group $i$ over the $T$-period planning horizon. In order to estimate the expected outdating rate and shortage rate, the sample size, $K$, needs to be statistically sufficient ($K \geq 30$). The following notations are used in formulating the model for one realization of the demand matrix.

Subscripts

$t$ Period $t$ in the planning horizon, $t=1, 2, \ldots, T$

$i$ Blood group entity, $i=1, \ldots, 16$

$r$ Residual shelf life of item on stock, $r=1, 2, \ldots, M$

Notations

$D^i_t$ Demand for blood type $i$ for period $t$, $i=1, \ldots, 8$

$Q^i_t$ Production/Ordering quantity from blood type $i$ for period $t$

$S^i_t$ Number of shortage for blood type $i$ at period $t$

$O^i_t$ Number of outdating for blood type $i$ at period $t$

$MS^i_t$ Scheduled ABO/Rh(D) compatible matching quantity at period $t$ for blood type $i$

$MA^i_t$ Actual ABO/Rh(D) compatible matching quantity at period $t$ for blood type $i$

$QS^i_t$ Quantity of blood type $i$ serving as a ABO/Rh(D) compatible substitution at period $t$

$x^i_t$ Inventory state for blood type $i$ at period $t$
To simplify the presentation, matrix $x' = \{x'_1, ..., x'_{16}\}$ is used to represent the inventory state for all blood groups at period $t$. Each $x'_i$ is a vector, recording the inventory state for blood group $i$ at time period $t$. The inventory state is updated through $t=1, ..., T$. A natural way to describe the inventory state is to categorize the RBCs in stock according to their age. Using $M$ to represent the maximal residual shelf life allowed, the inventory state for blood type $i$ at time period $t$ can thus be denoted by

$$x'_i = (x'_{i,1}, ..., x'_{i,r}, ..., x'_{i,M})$$

where $x'_{i,r}$ represents the number of RBCs with a residual shelf life of $r$ days before outdated. For example, $x'_{i,1}$ records the number of items that will be expired in one day; therefore, it indicates the oldest item(s) on stock. The transition of inventory states from one day to the next is determined jointly by demand, production/ordering quantity, and replenishment policy. For each day, the inventory state is first updated after satisfying the demand during the day (to use the oldest units first), next removing units that are outdated, then decreasing the residual shelf life of all remaining units by 1 day, and finally adding the newly received units.

Given the initial system state $X^1 = \{x^1_1, x^1_2, ..., x^1_{16}\}$, i.e. the initial inventory level at the beginning of the first period ($t = 1$), the amount of compatible match (blood substitution) during the $t^{th}$ day for blood type $i$ with inventory state $x'_i$ can be computed by

$$MS^t_i(x'_i) = (D^t_i - \sum_r x'_{i,r})^+ \quad t = 1, ..., T$$

(8.1)

where $x^+ = \max\{x, 0\}$ and $D^t_i = Q^t_{i-8}$, for $i = 9, ..., 16$.
In the case that \( MS_i'(x_i') > 0 \), then an attempt is made to satisfy the excessive demand with possible blood substitution according to Table 8.1.

For each blood type, its substitution is selected according to the US blood percentage distribution (second column in Table 8.1). That is, the blood group with larger percentage distribution in US population has a higher possibility of being selected as a compatible blood group.

\[
P_i(j) = \begin{cases} 
\frac{PD_j}{\sum_{j \in BS_i} PD_j}, & \text{if } j \in BS_i \\
0, & \text{else}
\end{cases}
\]  

\( P_i(j) \) denotes the probability of selecting \( j \) as a substitution for blood group \( i \). \( BS_i \) represents all the possible compatible blood groups for type \( i \) according to Table 8.1; \( PD_j \) is the population distribution for blood type \( j \) in the U.S. The substitution goes on until all successive demands are satisfied or the on-hand inventories of all possible compatible blood groups are used up. The total amount of substitution for blood type \( i \) fulfilled by other compatible blood type(s) are then denoted as \( MA_i'(x_i') \), which represents the quantity actually ABO/Rh(D) compatibly matched. Therefore, the shortage quantity during the \( t^{th} \) day for blood type \( i \) with inventory state \( x_i' \) can be computed by

\[
S_i'(x_i') = (MS_i'(x_i') - MA_i'(x_i'))^+ \quad t = 1, ..., T
\]

The RBC demand for each blood group is met by issuing the oldest stocks in the pool first. Early studies have established that a first-in-first-out (FIFO) management strategy is the optimal issuing policy for perishable products in order to reduce the number of leap-frogged units.
Leapfrogging refers to the use of a fresher, non-expiring unit when an older, expiring unit is available. Therefore, the inventory state, \( x'_t \), is updated by removing those consumed items from their corresponding residual shelf life.

\[
x'_{t,r} = \begin{cases} 
  (x'_{t,r} - D'_t - QS'_i)^+, & \text{if } r = 1 \\
  (x'_{t,r} - (D'_t + QS'_i - \sum_{j=1}^{r} x'_{t,j})^+), & \text{if } r > 1 
\end{cases} \tag{8.4}
\]

At each time period, the inventory for blood type \( i \) is not only consumed by its own demand but also by the substitution requirement from other compatible blood types. \( QS'_i \) represents the total quantity of blood type \( i \) consumed as a compatible substitution for other blood types at time period \( t \). Based on the updated inventory state, the outdating quantity for each period can be calculated by

\[
O'_t(x'_t) = \max \{x'_t, 0\} \quad t = 1, \ldots, T \tag{8.5}
\]

Then all remaining units decrease their corresponding residual shelf lives by 1 day and the inventory state is lastly updated by receiving new replenishment. After the replenishment, the resulting inventory state will be the beginning inventory state for the next period, \( t+1 \).

The relative shortage rate for blood group \( i \), \( SR_i \), based on the \( k^{th} \) demand realization, \( D^k \), over the planning horizon is given by

\[
SR_i = \frac{\sum_{t=1}^{T} S'_t(x'_t)}{\sum_{t=1}^{T} D'_t}, \quad i = 1, \ldots, 16 \tag{8.6}
\]

The relative outdating rate for blood group \( i \), \( OR_i \), based on the \( k^{th} \) demand realization, \( D^k \), over the planning horizon is given by

202
The blood center and the hospital works jointly to minimize the system-wide outdate rate. The system-wide outdate rate is defined here as the summation of the relative outdate rate for each blood type. Using this objective function, it is possible to partition the system-wide outdate rate according to each blood type and measure the outdate rate for each specific blood type. That is, how much percentage has been spoiled relative to the corresponding order size for that blood type. It should be pointed out that the system-wide outdate rate defined here generally overestimates the system outdates, if one only cares about the ratio between the total outdating units to the total ordering size of all blood types.

The objective function for this study is of the form:

\[
Min \{ F := \sum_{i=1}^{16} E_{\rho}(OR_i) \} \tag{8.8}
\]

subject to \( \max \{ E_{\rho}(SR_i) \} \leq 1 - \phi \quad \text{for } i = 1, \ldots, 16 \)

\( E_{\rho} \) is the expected value function with respect to the stochastic demand distribution \( P \).

\( \phi \) is the fill rate service measure (fraction of demand that can be satisfied immediately from the stock on hand). The objective function, the expected summation of the relative outdate rates for all 8 blood types, needs to be optimized subject to a predetermined minimum fill rate requirement (or equivalently maximal shortage rate requirement).
8.2.2 Replenishment policies

This study employs an age-based policy newly proposed by Duan and Liao (2013b), namely the old inventory ratio (OIR) policy. On top of the base order-up-to policy, the production decision at the blood center is also based on an old inventory ratio (the proportion of "old" items to the total items on hand) at that period. In the following numerical example of RBCs, items with residual shelf lives of 1, 2 and 3 days are defined as "old". This definition was found to work well with respect to the considered MSLs. Nevertheless, it should be pointed out that the definition of old items could also be subject to optimization with respect to the length of its shelf life if it is applied to other general cases. In the OIR Policy, the production quantity is first determined according to the original order-up-to level which only accounts for the number of items on stock without age consideration. Next, the proportion of "old" items to the total items on hand is calculated. If this proportion exceed a certain threshold level, \( \delta \), (this threshold percentage is optimized together with the order-up-to levels), an additional replenishment is triggered to account for the possible outdating caused by those "old" items. The size of additional replenishment equals to the total number of "old" items.

First,

\[
if \quad IP_i^t < SS_i^t \quad then \quad Q_i^t = SS_i^t - IP_i^t, \quad (8.9)
\]

Secondly,

\[
if \quad \frac{\sum_{r=1}^{3} x_{i,r}^t}{\sum_{r=1}^{M} x_{i,r}^t} \geq \delta \quad then \quad Q_i^t = Q_i^t + \sum_{r=1}^{3} x_{i,r}^t \quad (8.10)
\]
where $\sum_{r=1}^{3} x_{i,r}'$ is the number of "old" items on stock and $\sum_{r=1}^{M} x_{i,r}'$ is the total number of items on stock. This modification of order-up-to levels only applies to the blood center; that is, $i=9,...,16$. This is because the blood center is the source where all those fresh RBCs are produced. As long as the blood center can keep producing fresh units every day, the hospital will receive fresh items as well. As a result, more fresh RBCs will be circulated within the system; hence the overall inventory supply chain system is moving fast to avoid outdated.

8.2.3 The simulation optimization approach

To solve the problem formulated in the previous sub-section, a simulation optimization approach with sample average approximation is employed for finding those "near-optimal" order-up-to policies. This framework has been shown successfully applied to highly perishable platelets supply chain in Duan and Liao (2013b). This framework is deployed with a totally new hybrid metaheuristic, tailored for this study. Figure 8.1 shows the simulation optimization framework with ABO/Rh(D) compatibility. The new hybrid, called TA-TS, is shown to perform the search more efficiently than that of Duan et al., (2013) and Yi et al., (2013), thanks to its trajectory-based mechanism. Trajectory-based search utilizes single point search and this single point is intelligently directed by the new hybrid to produce a highly efficient trajectory to the global best over time. Compared to the population-based search as presented in Duan et al., (2013) and Yi et al., (2013), the new TA-TS hybrid requires much less computational effort to locate the near-optimal solution. To deal with the stochastic demand data for each blood type at the hospital side, random samples are generated.
Figure 8.1 Overall structure of the simulation optimization framework with ABO/Rh(D) compatibility

Initial guess → TA-TS metaheuristic optimizer → Suggest candidate solution(s) → ABO type-specific match → ABO type-compatible match → Keep updating through \( t = 1, \ldots, T \) → Record type-specific match and type-compatible match into their corresponding matrix → Load demand \( D^1_{(O^+, \ldots, AB^-)} \) → \( D^2_{(O^+, \ldots, AB^-)} \) → \( D^K_{(O^+, \ldots, AB^-)} \) → infeasible → Shortage \( > 1 - \phi \) → infeasible → feasiible → Shortage \( \leq 1 - \phi \) → Final best solution → Outdate

Inventory State

<table>
<thead>
<tr>
<th>O+</th>
<th>( x^t_{1,1} )</th>
<th>( x^t_{1,2} )</th>
<th>( \ldots )</th>
<th>( x^t_{1,r} )</th>
<th>( \ldots )</th>
<th>( x^t_{1,M} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \ldots )</td>
<td>( \vdots )</td>
<td>( \ldots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>AB-</td>
<td>( x^t_{1,1} )</td>
<td>( x^t_{1,1} )</td>
<td>( \ldots )</td>
<td>( x^t_{1,r} )</td>
<td>( \ldots )</td>
<td>( x^t_{1,M} )</td>
</tr>
</tbody>
</table>

Figure 8.1 Overall structure of the simulation optimization framework with ABO/Rh(D) compatibility
Let \( D^1, D^2, \ldots, D^k, \ldots, D^K \) be the set of \( K \) realizations of independent and identically distributed (i.i.d.) random samples of demand data. The expected outdate rate at each entity.

\[
E_p(OR_i), \text{ is approximated as } \frac{1}{K} \sum_{k=1}^{K} (OR_i \mid D^k), \text{ where } OR_i \mid D^k \text{ is the calculated outdate rate based on the } k^{th} \text{ demand realization.}
\]

For the same token, the expected shortage rate at each entity.

\[
E_p(SR_i), \text{ is approximated as } \frac{1}{K} \sum_{k=1}^{K} (SR_i \mid D^k), \text{ where } SR_i \mid D^k \text{ is the calculated shortage rate based on the } k^{th} \text{ demand realization.}
\]

The reasons are \( E[\frac{1}{K} \sum_{k=1}^{K} (OR_i \mid D^k)] \to E_p(OR_i) \) and \( E[\frac{1}{K} \sum_{k=1}^{K} (SR_i \mid D^k)] \to E_p(SR_i) \), as \( K \to \infty \). The expected value functions are approximated by the corresponding sample average function (Kleywegt et al., 2001). The obtained sample average problem is optimized by the TA-TS hybrid metaheuristic.

Threshold accepting (TA) was initially proposed by Dueck and Scheuer (1990). It is very similar to simulated annealing (SA) and can be considered as a modification of it (Metropolis et al., 1953, Kirkpatrick et al., 1983). The essential difference between SA and TA lies in the acceptance rule. TA accepts every new configuration which is not much worse than the old one (within a threshold), while SA accepts worse solutions only with rather small probabilities. An apparent advantage of TA is its greater simplicity. In TA, it is not necessary to compute acceptance probabilities or to generate random numbers, which leads to generally faster execution time and less computational effort. Compared to SA, there are still very few TA applications reported in the literature. The version used in this study is Hu et al., (1995), which is
a modified version of TA with a non-monotonic self-tuning threshold schedule with the hope to improve the algorithm's performance.

Tabu search (TS), initially suggested by Glover (1986, 1989, 1990), is the combination of local search with short-term memories, called tabu lists, which record the recent searching history and is used to prevent cycling back to previously visited solutions. The size of a Tabu list is one of TS parameters needed to be determined. It should not be too small to take full advantage of the memory, resulting in the solution space cannot be sufficiently searched. On the other hand, it cannot be too large to keep many former moves from being revisited. This may also prevent promising areas from being explored further, which results in final solutions with low precision. The Tabu restriction is often counterbalanced with an aspiration criterion, which gives an opportunity to override the Tabu status, providing the move a second chance to be qualified as admissible. The aspiration criterion used here is to allow a tabooed move, if it results in a solution with an objective value better that of the current best-known solution (Glover and Laguna 1997). Together, they provide a mechanism to escape from local minima.

By considering historical information during the search process, TS has been reported to have a more flexible and effective search behavior than other stochastic methods. In the TA-TS hybrid, TS explores the search space of feasible solutions by a sequence of moves (Glover and Laguna 1997). A move is an operation that changes the current solution to another solution. TS starts from a randomly generated solution. A set of neighbor solutions, \( N(x) \), are constructed by modifying the current solution, \( x \). Each neighborhood solution, \( i \), is generated by first
determining a random direction and then applying the step size, \( \Delta r_i \), taking the format of a sine function as follows:

\[
\Delta r_i = \frac{1}{2} \left(1 + \sin \frac{i \theta \pi}{|N(x)|} \right)
\]  

(8.11)

\( i \) is the index of the neighbor solution; \( |N(x)| \) is the total number of neighbor solutions generated at each iteration; \( \theta \) is used to control the oscillation period of \( \Delta r_i \). \( \theta \) is taken as 4.0001 in this study (instead of an integer) to avoid the possibility of \( \Delta r_i \) being zero, resulting in no modification of the current solution.

In order to improve the performance, the step size is made adaptable during the search as shown below:

\[
\Delta r_i = \frac{1}{2} \left(1 + \sin \frac{i \theta \pi}{|N(x)|} \right) \times \left(1 - \frac{nfe}{\max nfe} \right)
\]  

(8.12)

After constructing a new trial solution, it goes through a Tabu function embedded to check whether the new trial solution is in the tabooed area. In continuous space, the probability of visiting the same solution twice is very small. Although Tabu lists only record specific solutions, the entire area surrounding each solution is classified as Tabu. For each solution in the Tabu list, a tabooed area is defined as \([x_i - \text{perc}_i, x_i + \text{perc}_i]\) in each dimension. The Tabu area is empirically classified as 20% of the search range (center at the current solution) along each dimension and decreases proportionally as the number of function evaluations, \( nfe \), increases, i.e., \( \text{perc}_i = 20\% (1 - nfe/\max nfe) \text{DR}_i \), where \( \max nfe \) is the maximal number of function evaluations specified to stop a run and \( \text{DR}_i \) is the domain range for dimension \( i \).
If the best neighbor solution has a better objective function value than the best solution ever seen, its Tabu property can be invalidated by the improved-best aspiration criterion (Glover and Laguna 1997). Otherwise, if the best neighbor is not better than the current solution, it is tabooed and added to the Tabu list. The Tabu property remains active throughout a time period, called the Tabu tenure. As new solutions are added to the Tabu list, older solutions will be released from the bottom. Thus, TS makes decisions for future searches based on up-to-date information. Memory, implemented with Tabu lists, is employed to escape from local optima and to prevent cycling. In this study, the Tabu tenure is dynamically changed throughout the search. It has been suggested that the adoption of dynamic Tabu tenure usually obtains better results (Grabowski and Wodecki 2004). The use of an adaptive memory enables TS to "learn" and creates a more flexible and effective search strategy to complement the "memoryless" methods, TA. For each new trial solution generated by TS, the new hybrid accept it according to a threshold, \( t_k \). If the difference between the new solution and the current best solution is within the threshold, the hybrid accepts it as the new current solution. In this way, the newly developed hybrid follows the TA scheme to accept every new configuration which is not much worse than the old one. The threshold itself is adaptive along with the number of function evaluations during the whole searching process, i.e., \( t_{k+1} = t_k (1 - nfe \text{ / max} nfe) \). For any reduction in threshold, a new Tabu tenure is generated from a uniform distribution between two pre-determined values.
The pseudo-code of the newly developed TA-TS hybrid is presented as follows:

Initialize TA-TS parameters: \( I_{\text{max}}, t_k, \min_T, \max_T, \max_{\text{fne}}, \text{NumRun} \), where \( I_{\text{max}} \) denotes the number of iterations the search proceeds with at a particular threshold, \( t_k \); \( t_k \) represents the threshold scheme; \( \min_T \) and \( \max_T \) are the minimum and maximum Tabu tenure, respectively; \( \max_{\text{fne}} \) denotes the maximum number of evaluations at each run; \( \text{NumRun} \) denotes the total number of metaheuristic runs.

For \( r=1: \text{NumRun} \)

Generate an initial solution \( x^c \) and let it be \( x^* \)

While \( nfe<\max_{\text{fne}} \)

For \( i=1 \) to \( I_{\text{max}} \)

Generate a trial solution \( x^n \in N(x^c) \) according to Eq. (8.11)

Handle bound violations by either taking the bound violated or a new random value with equal probability

Evaluate the trial solution

Check whether the trial solution is tabooed

Check the improved-best aspiration criterion

If the trial solution is feasible

Compute \( \Delta = f(x^n) - f(x^c) \)

If \( \Delta < t_k \)

Set \( x^c = x^n \)
If \( f(x^c) < f(x^e) \)

Set \( x^* = x^n \)

End If

End If

Else

If \( g(x^c) > 0 \) (current best solution is also not feasible)

Compute \( \Delta = g(x^n) - g(x^c) \)

If \( \Delta < t_k \)

Set \( x^e = x^n \)

End If

End If

End If

End If

\( i = i + 1 \)

Update the Tabu list

End For

Update the threshold \( t_k = t_k (1 - nfe/\max nfe) \)

Update the Tabu tenure = \( \min T + U(0, \max T - \min T) \)

End While

End For

Output the best result
The TA-TS hybrid generates the trial solution one at a time and feed it into the ABO/Rh(D) RBC simulation model. The simulation model is used for estimating (through statistical sampling) the expected performance measures (objective function value). Because demand for each blood type is assumed stochastic, the problem falls into the category of constrained stochastic optimization problems. The fill-rate constraint in the problem is handled based on the parameter-less constraint handling method proposed by Deb (2000). Solutions with the associated expected shortage rate smaller or equal to 1-\(\phi\) are considered feasible; otherwise, infeasible, where \(\phi\) is the fill rate requirement.

Simulation and optimization interact with each other. The simulation model evaluates the fitness of any candidate solution. The metaheuristic optimizer uses the outputs from the simulator to derive a new candidate solution as input to the simulator. This cycle goes on until a near-optimum solution is obtained. The iterative nature of this simulation optimization approach is relatively simple to implement since the model and the optimizer work independently to each other. Once a model is built, iterative experimentations can be conducted within the meta-heuristic framework. On the other hand, the widely used traditional OR optimization procedures—(non)linear programming and (mixed) integer programming—require an explicit mathematical formulation. Such a formulation is generally impossible for the type of supply chain problems that arise in practical applications. The use of meta-heuristics, as opposed to a traditional OR method, has allowed more realistic modeling of the inventory control system.
Furthermore, the newly developed TA-TS hybrid helps to improve the efficiency of the optimization routine, which allows locating a near-optimal solution in less computing time.

8.3 Computational experiments and results

The total weekly demand is classified by blood type. Demand of each blood type faced by the hospital are assumed to be Poisson with varying daily means ($\lambda$) and they are independent to each other. This assumption has been justified by Haijema et al., (2007, 2009). The demand distribution by type is based on the general population distribution of blood types. The assumption here is that the people demanding RBC represent a sample of the entire US population such that their demand distribution by type is in consistent with the distribution of the entire US. The value in the bracket indicates the corresponding percentage of each blood type in US population, based on the statistics published on Stanford University School of Medicine website. The daily demand volume ($\lambda$) of each blood type is selected to reflect such a distribution. The computational experiments are based on the following daily Poisson mean demand data:

O+ (37.40%): 37 30 28 22 33 4 4

O- (6.6%): 7 5 5 4 5 1 1

A+ (35.7%): 36 29 27 21 32 4 4

A- (6.3%): 6 5 5 4 6 0 0

B+ (8.5%): 8 7 6 5 8 1 1

B- (1.5%): 2 1 1 1 2 0 0
AB+ (3.4%): 3 3 3 2 3 0 0

AB- (0.6%): 1 0 0 1 1 0 0

Replenishment at the hospital: seven days a week

Production at the blood center: Monday through Friday at daytime

Based on the age limits for RBCs suggested in recent studies, three levels of RBC Maximal Shelf Life (MSL), i.e., 7 days, 14 days and 21 days, are studied.

For each level of MSL, three different scenarios are considered.

Scenario 1- suggests no blood substitution at either the hospital or blood center. This scenario will serve as the base case to quantify the potential savings by allowing blood substitution.

Scenario 2- includes ABO/Rh(D) compatible substitution only at the hospital.

Scenario 3- includes ABO/Rh(D) compatible substitution at both the hospital and blood center.

8.3.1 Experimental details

For the hospital, the order-up-to level can be different for every day of the week. For the blood center, the order-up-to production volume can also be different for each workday. Therefore, the total number of variables to be optimized is 96 (8×7+8×5). Clearly, the RBC supply chain problem to be optimized is a high dimensional stochastic optimization problem. In order to keep the total number of decision variables to be as smaller as possible to reduce the computational burden, the threshold level, \( \delta \), in the OIR policy for each blood type is fixed at 0.5. To facilitate the search for optimal order-up-to levels, we employ a method to roughly
establish the lower bound and upper bound of each decision variable, as suggested by Blake et al., (2010). Accordingly, the lower bound is set as the minimum order-up-to level, $S_{min}$, (the minimum amount of stock that must be on hand after replenishment). It represents the smallest integer such that the probability of demand less than or equal to this value is $\phi$, where $\phi$ is the target minimum fill rate requirement. Since Poisson distributed demand is assumed, $S_{min}$, can be determined as

$$S_{min} = F^{-1}(\lambda, \phi)$$

where $F^{-1}(\lambda, \phi)$ is the inverse of the Poisson cumulative probability function (cpf) with mean equal to $\lambda$. This function returns the smallest value $S_{min}$ such that $F(S_{min}, \lambda) \geq \phi$.

Similarly, the upper bound (the maximum amount of stock that can be held on hand after replenishment) should be set in a way that the resulting inventory level will not lead to too much unnecessary outdating. For a unit arriving on the $l^{th}$ day with a residual shelf life of $r_i$ days, clearly, it will be spoiled after $r_i$ days. It is necessary to estimate the demand over its residual shelf life to determine whether it will be used to fulfill the customer demand. If it is not used over its residual shelf life, it is wasted and is considered as an outdated unit. Since demand for each day is assumed independently and identically Poisson distributed, the demand over the residual shelf life of the unit (arriving on the $l^{th}$ day with a residual shelf life of $r_i$ days) can be estimated by the cumulative demand over the $r_i$ days, $CD(l, r_i)$.

$$CD(l, r_i) = \sum_{t=l+1}^{l+r_i} (D'_j + D'_i), \quad j \in CM_i,$$

in which $D'_j$ is the demand on day $t$ for this specific type $i$ while $D'_i$ is the all possible compatible-matching demand for type $i$ on day $t$. $CM_i$ represents
the all other compatible-matching blood types whose demand can be satisfied by type \( i \) according to Table 8.1. Therefore, the upper bound of the order-up-to level is:

\[
S_{max} = \max_y \left\{ \sum_{i=0}^{y} (y-x) p(x, CD(l, r_i)) \right\} \leq \beta
\] (8.14)

where \( p(x, \mu) \) is the probability mass function of a Poisson random variable with a mean of \( \mu \), evaluated at \( x \). \( \beta \) can be set as the outdate rate in the current practice. In this study, \( \beta = 0.05 \) is set.

Three different scenarios are considered for possible blood substitution. For each scenario, three levels of maximal shelf lives are further considered: 7, 14 and 21 days, respectively. The minimum fill rate at each side cannot be less than 0.99, corresponding to a shortage rate of 0.01. The optimization is performed based on a one-year planning horizon (52 weeks). The stochastic yearly demand for each blood type, faced by the hospital, is realized 30 times with varying daily average on each day of the week. That is, for each blood type, its demand data is realized 30 times. Each one of the 30 realizations includes 364 daily demand data points (7 days \( \times \) 52 weeks). Based on those realizations, the TA-TS metaheuristic optimizer runs for 30 times (with 10,000 maximum number of evaluations per run) to search for the near-optimal solutions for each problem setting. Once identified, the near-optimal solution is further simulated for 10,000 weeks in order to estimate its long-run performance in practice. All programs were coded in Matlab and run using a laptop equipped with an Intel\textsuperscript{\textregistered} Core\textsuperscript{TM} i5-2450M CPU @ 2.50 GHz. The simulated results are first checked using the Anderson-Darling normality test to confirm that all
data follow normal distribution and then the corresponding 95% confidence intervals (CI) are constructed. The computational results are presented and analyzed next.

8.3.2 Computational results and analyses

8.3.2.1 Verification of the new TA-TS hybrid

Before proceeding to using the new TA-TS hybrid to optimize the replenishment policies for all testing cases, the new hybrid is first compared with the DE-HS-HJ hybrid. The effectiveness of the DE-HS-HJ hybrid has been thoroughly demonstrated in 19 engineering design problems (Duan et al., 2013) and in studying capacitated supply chains (Duan and Liao, 2013a) and highly perishable supply chain inventory optimization (Duan and Liao 2013b).

We first test both algorithms with some high-dimensional benchmark problems. Those problems are selected from Yao et. al., (1999) and each problem has 100 decision variables. The benchmark problems are listed in Table 8.2:

Table 8.2 Benchmark problems.

<table>
<thead>
<tr>
<th>Function</th>
<th>Dimensions (n)</th>
<th>Searching Space (S)</th>
<th>Global optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1(x_i) = \sum_{i=1}^{100} x_i^2 )</td>
<td>100</td>
<td>([-50,50]^n]</td>
<td>0</td>
</tr>
<tr>
<td>( f_2(x_i) = \sum_{i=1}^{100}</td>
<td>x_i</td>
<td>+ \prod_{i=1}^{100}</td>
<td>x_i</td>
</tr>
<tr>
<td>( f_3(x_i) = \sum_{i=1}^{99} [100(x_{i+1} - x_i)^2 + (x_i - 1)^2] )</td>
<td>100</td>
<td>([-30,30]^n]</td>
<td>0</td>
</tr>
<tr>
<td>( f_4(x_i) = \max_i {</td>
<td>x_i</td>
<td>, 1 \leq i \leq 100} )</td>
<td>100</td>
</tr>
</tbody>
</table>
To locate the optimal solution for each function, both metaheuristic algorithms are run 30 times. For each run, the algorithm stops either the global optimum is found within an allowable threshold ($\varepsilon = 1 \times 10^{-6}$) or the maximum evaluation number ($\text{maxnfe} = 5 \times 10^6$) is reached. The best solution found by both hybrid algorithms and their corresponding CPU times are recorded in Table 8.3.

Table 8.3 Testing results for both hybrid metaheuristics.

<table>
<thead>
<tr>
<th></th>
<th>Best solution found</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1$ $f_2$ $f_3$ $f_4$</td>
<td>$f_1$ $f_2$ $f_3$ $f_4$</td>
</tr>
<tr>
<td>TA-TS</td>
<td>0 0 0 6</td>
<td>5.14 5.32 31.4 3461.02</td>
</tr>
<tr>
<td>DE-HS-HJ</td>
<td>0 0 0 11</td>
<td>9.67 9.95 56.81 5182.31</td>
</tr>
</tbody>
</table>

From Table 8.3, it is shown that TA-TS is capable of obtaining very high quality solutions with generally less time than DE-HS-HJ, especially for function 4. Function 4 can be regarded as a relatively difficult problem in which both algorithms failed to find the global optimal in the allowable number of evaluations. However, the solution obtained by TA-TS is better. From this preliminary testing, it can be concluded that the overall performance of TA-TS is at least as good as DE-HS-HJ. Most importantly, it is faster as expected.

Next, both the TA-TS hybrid and DE-HS-HJ hybrid were tested on one problem instance for the considered RBC supply chain inventory replenishment problem. It would be very time-consuming to test more problem instances considered in this study. Because the stochastic demands for 8 blood types are considered, to estimate the expected system performance it is
needed to take at least 30 realizations for each blood type demand. The above-described evaluation process then has to be repeated inside the metaheuristic algorithm for 30 runs (30 realizations $\times$ 30 runs). This significantly increases the computational effort. It is impractical to run the program forever in search for really accurate solutions. After a careful tradeoff between accuracy and efficiency, it is elected to produce a solution of a guaranteed quality with an affordable computing effort. The problem instance tested by both hybrid meta-heuristics is scenario 3 with MSL equaling to 7 days. After 10,000 evaluations, the best solutions reported by both algorithms lead to approximately 31% total system-wise outdate rate. Because stochastic demand is considered, the final results are expressed in the 95% confidence interval. To test whether there is a statistical difference in the solutions found by both algorithms. An ANOVA analysis was first carried out, followed by the multiple comparison procedure "Tukey's HSD (Honestly Significant Difference)" (Tukey, 1953) implemented in MINITAB. The corresponding results are given in Report 8.1.

The results verify that the proposed methodology is as effective as the proven DE-HS-HJ hybrid in finding good quality solutions. In addition, the new TA-TS hybrid is capable of locating a near-optimal solution in much less computing time (the mean CPU time per metaheuristic run is 11588.7s vs. 15197.3s in DE-HS-HJ hybrid). For high-dimensional optimization problem, the new TA-TS hybrid is expected to perform the search more efficiently.
Report 8.1 Test results of both hybrid metaheuristics on the considered RBC problem (scenario 3 with MSL=7 days)

One-way ANOVA: DE-HS-HJ, TA-TS

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor</td>
<td>1</td>
<td>0.0000007</td>
<td>0.0000007</td>
<td>0.06</td>
<td>0.813</td>
</tr>
<tr>
<td>Error</td>
<td>58</td>
<td>0.0006996</td>
<td>0.0000121</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>59</td>
<td>0.0007003</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S = 0.003473  R-Sq = 0.10%  R-Sq(adj) = 0.00%

Level     N     Mean    StDev
DE-HS-HJ  30  0.31243 0.00375
TA-TS     30  0.31222 0.00317

Individual 95% CIs For Mean Based on Pooled StDev

| Level     | --------+---------+---------+---------+--
|-----------|---------|---------|---------|--
| DE-HS-HJ  | (-----------------*-----------------)
| TA-TS     | (-----------------*-----------------)
|           | --------+---------+---------+---------+--
|           | 0.31150 0.31220 0.31290 0.31360

Pooled StDev = 0.00347

Grouping Information Using Tukey Method

<table>
<thead>
<tr>
<th>N</th>
<th>Mean</th>
<th>Grouping</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE-HS-HJ</td>
<td>30 0.312428</td>
<td>A</td>
</tr>
<tr>
<td>TA-TS</td>
<td>30 0.312215</td>
<td>A</td>
</tr>
</tbody>
</table>

Means that do not share a letter are significantly different.

Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons
Individual confidence level = 95.00%

DE-HS-HJ subtracted from:

<table>
<thead>
<tr>
<th>Lower</th>
<th>Center</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>TA-TS</td>
<td>-0.002008</td>
<td>0.001582</td>
</tr>
<tr>
<td></td>
<td>-0.000213</td>
<td>0.001582</td>
</tr>
</tbody>
</table>

| TA-TS     | (-----------------*-----------------)
|           | --------+---------+---------+---------+--
|           | -0.0020 0.0010 0.0000 0.0010

221
Because generally in high-dimensional problems, more emphasize should be put on diversification to explore the overall landscape of the searching space as much as possible. The new TA-TS hybrid is a trajectory-based method enhanced with Tabu memory to prevent cycling and it can move rapidly from one location to another without being trapped into local optima because of its threshold acceptance mechanism. Over time it is capable of producing a highly efficient trajectory to a near-optimal solution. The old DE-HS-HJ is a population-based method, which shows more strength in low-dimensional optimization problems. The Hooke and Jeeves local search uses the steepest ascent pivot rule in which it performs a coordinate search for each dimension of a solution to deeply exploit and identify the nearest local optimum. Comparing to the new TA-TS hybrid, it focuses more on exploitation locally. When it comes to high-dimensional problem, such a strategy becomes inefficient since it spends too much computational cost on a small area. Therefore, the new TA-TS hybrid is developed to deal with the considered high-dimensional problem because it involves 96 variables in total.

8.3.2.2 ANOVA results

Three scenarios were tested and each was tested with different MSLs (7, 14 and 21 days). In other words, there are nine problem instances. For each problem instance, the new TA-TS hybrid coupled with the simulation model, is used to identify the corresponding near-optimal solution. Each near-optimal solution is then further simulated for 10,000 weeks to estimate its long-run performance and their corresponding 95% confidence intervals (CIs) are constructed.
The first analysis carried out was ANOVA to determine the significance of main effects and interaction effects on system-wide outdate rate. There are totally 9 possible combinations (3 scenarios × 3 levels of MSLs). For each combination, 30 replications were made to study the randomness caused by the stochastic patient demand. The results in Report 8.2 indicate that the main effects and the interaction effects are all significant (p-value approximately equals to 0.000). The outdate rate is affected by the difference in MSL the most (F-value=1271407.13, P-value=0.000), followed by different scenarios (F-value= 230416.86, P-value=0.000). For interaction effect, the interaction between MSL and different scenarios is also significant (F-value=72977.16, P-value=0.000).

**Report 8.2 Summary of ANOVA results**

Analysis of Variance for System Outdate, using Adjusted SS for Tests

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution</td>
<td>2</td>
<td>2.8415</td>
<td>2.8415</td>
<td>1.4208</td>
<td>230416.86</td>
<td>0.000</td>
</tr>
<tr>
<td>MSL</td>
<td>2</td>
<td>15.6790</td>
<td>15.6790</td>
<td>7.8395</td>
<td>1271407.13</td>
<td>0.000</td>
</tr>
<tr>
<td>Substitution*MSL</td>
<td>4</td>
<td>1.7999</td>
<td>1.7999</td>
<td>0.4500</td>
<td>72977.16</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
<td>261</td>
<td>0.0016</td>
<td>0.0016</td>
<td>0.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>269</td>
<td>20.3220</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S = 0.00248314  R-Sq = 99.99%  R-Sq(adj) = 99.99%

The main-effect plot for each factor is shown in Fig. 8.2. Figure 8.2 indicates that by shortening the MSL for BRCs, it does have a detrimental impact on the overall system outdate rates without substitution. Furthermore, the relationship is clearly nonlinear. Shortening the MSL from 21 days to 14 days, the system outdate-rate increases from 1.8% to 7.5%; but shortening the
MSL from 14 days to 7 days, the system outdate-rate shoots up to 55.6%. Managing RBCs with MSL of only 7 days is especially difficult and it will be shown in the following that allowing ABO/Rh(D) compatible substitution helps reduce the associated outdates. The system outdate rates of scenario 3 (allowing blood substitution at both the hospital and blood center) are overall the lowest among the three scenarios (the average system outdate rate by scenario 1, 2 and 3 are 35.25%, 19.19% and 10.49%, respectively).

Figure 8.2 Main effect plot of each factor on the system outdate rate

Generally speaking, allowing ABO/Rh(D) compatible substitution provides a lot more flexibility for the hospital and blood center to adjust their on-hand inventory for better utilization. It prevents a lot of unnecessary spoils and cut the system outdate rate down by half and more. To some extent, allowing ABO/Rh(D) compatible substitution offers each supply chain entity a second chance to better match the demand with supply.
Figure 8.3 shows the two-factor interaction plots. From the interaction plot between MSLs and different scenarios of substitution, it is found that at MSL=21 days, the differences between different scenarios are relatively small. The maximal outdate rate occurs in scenario 1, which is around 3.7%, following by 1.7% in scenario 2 and 0.005% in scenario 3, respectively. As the MSL reduces more dramatically, more significant difference between the three scenarios are observed. At the MSL=14 days level, scenario 1 has led to a 19.7% system outdate rate, while scenario 2 and 3 are still able to keep the system outdate rate at low levels (2% and 0.2%, respectively). When it comes to MSL=7 days level, the differences are further enlarged. The system outdate rate for scenario 1 is as high as 82%, comparing to the system outdate rate of 53.1% and 31.2% for scenario 2 and 3, respectively. As mentioned earlier, the system outdate rate is defined here as the summation of the relative outdate rate for each blood type. Mathematically, the relative outdate rate is calculated as the outdating units for the considered blood type divided by the total ordering units for the considered blood type. Therefore, it may appear that the system outdate rate is unreasonably high but actually it only accounts for a very small portion of the total ordering size for all blood types. If the ratio between the total outdating units to the total ordering size for all blood types is examined, then the ratio is much smaller. Actually, the ratios are all kept within 3% for all problem instances tested.

From the interaction plot between MSLs and different scenarios, it is found that ABO/Rh(D) compatible substitution offers much lower system outdate rate. In the next subsection, we detail
further analyses to each blood type and examine how blood substitution affects coordinating the product flow between different blood types.

![Interaction Plot for System Outdate](image.png)

**Figure 8.3 Interaction plot of the system outdate rate**

8.3.2.3 Detailed analyses of each blood type

Tables 8.6 and 8.7 detail the 95% CIs of the relative outdate rate for each blood type at the hospital and blood center, respectively, when the fillrate constraint for each blood type is kept at least as high as 0.99. All the values recorded in both tables are already converted to percentage values. It is clear that outdate rates at the blood center is almost negligible comparing to those at the hospital. That means the coordination between the hospital and blood center is good. The main problem lies in the hospital when facing the stochastic demand. On the hospital side, it is found that the major outdates come from those slow-moving demand types, such as B- and AB-.

These two blood types account only for 1.5% and 0.6% in the US population. It is a difficult task
to replenish the corresponding inventory of these blood types with sparse demand volume. The problem becomes more acute when the MSL is short. As MSL is shortened to 7 days, items held on stock are running higher risk of being spoiled. Without the ABO/Rh(D) compatible substitution, excessive amount of type-specific products have to be in stock to maintain sufficient level of fillrate. As a result, the outdate rate increases dramatically for scenario 1. Scenarios 2 and 3 to some extent ease the problem of the possible over-stocking for certain blood type, particularly AB-, as a result of substitution. Demands for slow-moving blood products such as B- and AB- can resort to other ABO/Rh(D) compatible match blood types such as O- and A- when their own stocks are low or empty. This arrangement maximizes the utilization of all blood types. Furthermore, if substitutions are allowed at both the hospital and the blood center as in scenario 3, each supply chain entity enjoys more flexibility.

There is no doubt that efficient inventory management is critical. Making blood-type substitutions whenever possible is an important behavior for optimal replenishment and transfusion decisions. Additionally, understanding the different value of holding each blood type in inventory should help the blood center and hospital more efficiently fulfill demand and hold blood for future use. To this end, it is desirable to examine the ABO/Rh(D) compatible match, detailed to each blood type. Fig 8.4-8.7 shows the corresponding ABO/Rh(D) compatible matching bar charts for each blood type in scenario 3. Each percentage value shown in those figures is calculated as the ratio between the cross-matched amount of this specific blood type to the total cross-matched amount of all blood types.
### Table 8.4 95% CIs of outdate rates detailed to each blood type (on the hospital side)

<table>
<thead>
<tr>
<th>Blood Type</th>
<th>without substitution</th>
<th>with substitution at hospital</th>
<th>with substitution at hospital and blood center</th>
</tr>
</thead>
<tbody>
<tr>
<td>US (%)</td>
<td>7 Days</td>
<td>14 Days</td>
<td>21 Days</td>
</tr>
<tr>
<td><strong>O+</strong></td>
<td>0.0032±0.0005</td>
<td>0.006±0.0002</td>
<td>0</td>
</tr>
<tr>
<td><strong>O-</strong></td>
<td>0.313±0.0064</td>
<td>0.0004±0.0002</td>
<td>0</td>
</tr>
<tr>
<td><strong>A+</strong></td>
<td>0.0003±0.0001</td>
<td>0.0001±0.0001</td>
<td>0</td>
</tr>
<tr>
<td><strong>A-</strong></td>
<td>3.734±0.0245</td>
<td>0.0019±0.0006</td>
<td>0</td>
</tr>
<tr>
<td><strong>B+</strong></td>
<td>0.893±0.0109</td>
<td>0.0005±0.0004</td>
<td>0</td>
</tr>
<tr>
<td><strong>B-</strong></td>
<td>0.0018±0.0004</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>AB+</strong></td>
<td>0.0012±0.0004</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>AB-</strong></td>
<td>0.0032±0.0007</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 8.5 95% CIs of outdate rates detailed to each blood type (on the blood center side)

<table>
<thead>
<tr>
<th>Blood Type</th>
<th>without substitution</th>
<th>with substitution at hospital</th>
<th>with substitution at hospital and blood center</th>
</tr>
</thead>
<tbody>
<tr>
<td>US (%)</td>
<td>7 Days</td>
<td>14 Days</td>
<td>21 Days</td>
</tr>
<tr>
<td><strong>O+</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>O-</strong></td>
<td>0.0018±0.0004</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>A+</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>A-</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>B+</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>B-</strong></td>
<td>0.864±0.0107</td>
<td>0.0028±0.0008</td>
<td>0</td>
</tr>
<tr>
<td><strong>AB+</strong></td>
<td>0.0012±0.0004</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>AB-</strong></td>
<td>0.0032±0.0007</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Therefore, the percentage values shown in Figs. 8.4 and 8.6 are summed up to 1 for each MSL and the same applies to Figs. 8.5 and 8.7. For example, the percentage values of all blood types represented by those blue bars (MSL=7 days) in Figs. 8.4 and 8.6 are summed up to 1. The bar charts for scenario 2 are very similar to those of scenario 3; therefore are omitted to avoid redundancy. In scenario 3, blood substitutions are allowed at both the hospital and blood center. We have prepared the figures from two different points of view, as "donor" and "receiver". Figures 8.4 and 8.6 are "donor" figures, which show the percentage distribution of different blood types in terms of using them as substitutions to other blood types. Figures 8.5 and 8.7 are "receiver" figures, which show the percentage distribution of different blood types in terms of matching other compatible blood types to those specific blood types.

It is surprisingly found that a near-optimal replenishment decision does not lead to a large portion of ABO/Rh(D) compatible matching. As shown in Figs. 8.4-8.7, the overall percentages of ABO/Rh(D) compatible matching among all 8 blood types are kept at low levels: 3.75%, 3.23% and 2.31% for MSLs equaling to 7, 14 and 21 days, respectively. The corresponding statistics as shown in the legend is very much in line with the clinical practice. That is, ABO/Rh(D) compatible substitution should be kept at a low level and should be allowed only when absolutely necessary. Some patients and procedures are especially sensitive to proper matching of donor and patient blood type. For instance, pregnant women and newborns are more sensitive and often must only receive blood of their own type. Additionally, certain procedures such as open heart surgery require an exact blood type match. Therefore, blood substitution should occur
as less often as possible. With respect to the overall blood supply 3-4% should be a reasonable range. It is shown that the solutions provided by the simulation optimization approach is capable of alleviating system-wide outdates without relying too heavily on blood substitution.

When looking into Fig. 8.4 more closely, it is found that in general, fast-moving blood types (such as O+, O-, A+, A-) account for the most proportion in terms of serving as ABO/Rh(D) compatible donors. In particular, the O- blood type continues to be the most popular donor in all three levels of MSL. This is due to the fact that O- is the universal red cell donor. It can contribute to substitute any other blood types whenever it is necessary. O+ also has very high utilization as a blood donor (even higher than O- when MSL=14 and 21 days). The reason is because O+ is the type that enjoys the highest percentage in US population distribution. The population pool of O+ is the biggest while it can serve as a donor for a lot of blood types: A+, B+, AB. Ample supply of O+ offers more flexibility than other blood types in terms of blood substitution. The conclusion is that in the two blood substitution scenarios, there is a clear trend of increased use of group O blood. As the MSL decreases, they are needed more frequently as donors for other ABO/Rh(D) compatible blood types.

On the other hand, Figure 8.5 shows the "receiver" percentages. It is found that in general, relatively slow-moving blood types (such as A-, B+, AB+, AB-) account for the most proportion as ABO/Rh(D) compatible receivers. The low volume of demand slows down the circulation of type-specific product in stock. Hence, inventory tends to stay in stock for a longer time period, which is especially challenging for minimizing outdates. Receiving blood-type substitutions
frees up the inventory of those blood types because type-specific products do not have to be over-stocked to prevent possible shortfall. Their demand volume is compounded with the demand volume of those compatible blood types in a way that helps to speed up inventory circulation. Blood types such as B+ and AB+ appear to enjoy the ABO/Rh(D) compatible match the most, probably because of their wide possible choices as a receiver. On the blood center side, it is shown that the type-compatible matching happens much less frequently than the hospital side. This makes sense because blood center is the source where all blood products are produced. It is desirable to provide the right product types, as many as possible. Type-compatible substitution happens only in the case of immediate shortage in supply.

Fig. 8.4-8.7 show scenario 3 as the last column in Table 8.4 and 8.5. Scenario 3, allowing type-compatible substitution at both hospital and blood center, has been shown in Table 8.6 and 8.7 to further reduce the overall system outdates. A good solution if optimized properly can achieve lower system outdate rate without relying too heavily on ABO/Rh(D) compatible matching. The proposed SO framework is shown to work efficiently to identify such a near-optimal solution.

In conclusion, inventory management stress associated with the shortened MSL for RBCs results in the provision of many type-compatible, as opposed to type-specific, matches. It is desirable for optimization models to capture those ABO/Rh(D) compatible substitution possibilities to reflect the real-world practice.
Figure 8.4 ABO/Rh(D) compatible match donor by blood type at hospital

Figure 8.5 ABO/Rh(D) compatible match receiver by blood type at hospital
Next, it is of interest to know the ordering size for each blood type to gain a deeper understanding of how RBC products with different blood types interact with each other. Table 8.8 shows the ordering size of each blood type at the hospital, proportional to the percentage value. Each percentage value recorded in Table 8.6 is calculated as the ratio between the ordering size of the specific blood type to the total ordering size of all blood types. It is a measure of how
much weight each specific blood type is taken with respect to the overall ordering size. Only the ordering size percentage for the hospital is shown here mainly due to the following two reasons: (1) it is most relevant to the end customer (patients) and it directly impacts how the overall supply chain satisfies its customers; (2) very similar patterns are observed at the blood center and basically they provide the same information. For scenario 1, it is found that that the overall percentage for each blood type is very close to the US population percentage, as shown in the second column of Table 8.1. This is not a coincidence. Remember that the demand volume tested in this study is proportional to the percentage in the US population, which is believed representative to the real practice. Each blood type is independent of each other in scenario 1 because there is not any blood substitution. In order to satisfy its corresponding demand, the ordering size distribution has to be equivalent to the population distribution. When blood substitution is allowed as in scenarios 2 and 3, some deviations are evident, mainly in a large increase of the ordering size for O-. The ordering size for O- is more than doubled in both cases as a universal red cell donor. With blood substitution alternatives, the hospital tends to alleviate outdates of slow-moving blood types (B+, B-, AB+, AB-) by substituting those types with other ABO/Rh(D) compatible matches, especially O-. This conclusion is consistent with our observations in Fig 8.4-8.7. Very similar patterns are observed in the ordering size percentage at the blood center, showing that the overall system is well coordinated. The detailed information is omitted here as previously explained.
Table 8.7 gives the mean age of on-hand inventory per blood type on the hospital side. Only the values at the hospital are provided because they show the cumulative effect of products flowing from the blood center. Ample supply of young blood products at the hospital will eventually assure sufficient supply to its end customer with minimal outdates. The values reported in Table 8.7 are in days and the mean age of on-hand inventory is calculated as

$$\sum_{r=1}^{MSL} \sum_{r} \overline{x}_{i,r} \times (MSL + 1 - r), i = 1, ..., 8,$$

in which $\overline{x}_{i,r}$ is the mean number of stock units for blood type $i$ with a residual shelf life of $r$ days. Mathematically,

$$\overline{x}_{i,r} = \frac{1}{K} \sum_{k=1}^{K} \left[ \frac{1}{T} \sum_{t=1}^{T} x_{i,r}^t \right]$$

($T = 7$ days per week $\times 10,000$ weeks, $K = 30$ realizations). From Table 8.9, it is observed that the two blood substitution scenarios 2 and 3 tend to offer younger on-hand inventory than the alternative without blood substitution (i.e., scenario 1). Please refer to the last row of the table, in which the grand average age of all 8 types is calculated and reported for different MSLs. Table 8.7 confirms that the minimization of system-wide outdate in blood substitution scenarios is achieved by preventing the aging mainly in slow-moving blood types. In particular, the average age of on-hand inventory for slow-moving blood types such as B- and AB- is reduced greatly in both scenarios 2 and 3, leading to a much less outdate rates than scenario 1. In this way, the overall system-wide outdate rate is reduced.

Last but not least, the average number of on-hand inventory per blood type at the hospital is investigated. The values reported in Table 8.8 are in units and each is calculated by taking the average of the inventory states over 2,100,000 days.
(7 days per week × 10,000 weeks × 30 realizations). Using the results given in Table 8.7 and Table 8.8 together, it is possible to have a whole picture of the inventory state for each blood type, their age, and the number of units in stock. It is found that in general, there are fewer total inventory units (as reported in the last row) kept on stock for the two blood substitution scenarios: 2 and 3. This is probably the reason why they are able to keep a relative low outdate rate because the inventory is moving fast and getting refreshed more frequently. Both scenarios 2 and 3 tend to stock less units of slow-moving blood types such as B-, AB+, AB- and stock more of O- in case of possible type-compatible substitutions. In conclusion, the optimized solutions found in scenarios 2 and 3 reduce the inventory that one needs to keep on hand, which in turns contribute to lower blood outdate rates.

Scenarios 2 and 3 are designed to reflect different clinical practices. Some organization may only allow blood substitution to occur at the hospital. Some may allow it to occur at both the hospital and the blood center. This study investigates both operating situations and their potential savings. In both scenarios, the proposed simulation optimization approach is capable of providing a near-optimal solution that leads to a lower system outdate rate. Allowing ABO/Rh(D)-compatible blood substitutions helps reduce the system-wide outdate at least by 16% (scenario 2) under the most restrictive MSL, i.e., 7 days. For more relaxing MSL as 14 days and 21 days, the proposed framework is capable of keeping the system-wide outdate rate at most at 2%.
Table 8.6 Percentage order size by blood type on the hospital side for each instance

<table>
<thead>
<tr>
<th>US (%)</th>
<th>Blood Type</th>
<th>7 Days</th>
<th>14 Days</th>
<th>21 Days</th>
<th>7 Days</th>
<th>14 Days</th>
<th>21 Days</th>
<th>7 Days</th>
<th>14 Days</th>
<th>21 Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.4</td>
<td>O+</td>
<td>37.03</td>
<td>37.14</td>
<td>37.18</td>
<td>37.74</td>
<td>37.11</td>
<td>37.60</td>
<td>38.19</td>
<td>40.34</td>
<td>39.36</td>
</tr>
<tr>
<td>6.6</td>
<td>O-</td>
<td>6.44</td>
<td>6.55</td>
<td>6.58</td>
<td>17.60</td>
<td>17.18</td>
<td>17.82</td>
<td>17.92</td>
<td>18.83</td>
<td>18.80</td>
</tr>
<tr>
<td>35.7</td>
<td>A+</td>
<td>35.44</td>
<td>35.97</td>
<td>35.98</td>
<td>37.64</td>
<td>37.52</td>
<td>36.99</td>
<td>35.65</td>
<td>34.29</td>
<td>35.30</td>
</tr>
<tr>
<td>6.3</td>
<td>A-</td>
<td>6.25</td>
<td>6.11</td>
<td>6.12</td>
<td>1.98</td>
<td>2.72</td>
<td>1.40</td>
<td>3.13</td>
<td>1.70</td>
<td>1.81</td>
</tr>
<tr>
<td>8.5</td>
<td>B+</td>
<td>8.41</td>
<td>8.46</td>
<td>8.47</td>
<td>3.49</td>
<td>3.61</td>
<td>3.71</td>
<td>3.16</td>
<td>2.65</td>
<td>2.64</td>
</tr>
<tr>
<td>1.5</td>
<td>B-</td>
<td>2.06</td>
<td>1.66</td>
<td>1.64</td>
<td>0.63</td>
<td>0.64</td>
<td>0.74</td>
<td>0.44</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>3.4</td>
<td>AB+</td>
<td>3.43</td>
<td>3.26</td>
<td>3.29</td>
<td>0.65</td>
<td>0.86</td>
<td>1.51</td>
<td>1.23</td>
<td>1.47</td>
<td>1.37</td>
</tr>
<tr>
<td>0.6</td>
<td>AB-</td>
<td>0.95</td>
<td>0.85</td>
<td>0.73</td>
<td>0.28</td>
<td>0.36</td>
<td>0.23</td>
<td>0.28</td>
<td>0.27</td>
<td>0.27</td>
</tr>
</tbody>
</table>
Table 8.7 Mean age of inventory-on-hand per blood type on the hospital side

<table>
<thead>
<tr>
<th>US (%)</th>
<th>Blood Type</th>
<th>without substitution</th>
<th>with substitution at hospital</th>
<th>with substitution at both hospital and blood center</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.4</td>
<td>O+</td>
<td>3.76  6.66  8.55</td>
<td>3.57  9.21  13.17</td>
<td>3.87  8.58  8.58</td>
</tr>
<tr>
<td>6.6</td>
<td>O-</td>
<td>2.46  4.77  5.66</td>
<td>2.92  6.08  9.82</td>
<td>4.47  6.75  6.77</td>
</tr>
<tr>
<td>35.7</td>
<td>A+</td>
<td>4.51  8.05  8.19</td>
<td>3.16  7.70  10.75</td>
<td>3.80  7.04  9.04</td>
</tr>
<tr>
<td>6.3</td>
<td>A-</td>
<td>3.71  5.25  8.30</td>
<td>3.86  2.40  4.30</td>
<td>4.89  6.72  8.72</td>
</tr>
<tr>
<td>8.5</td>
<td>B+</td>
<td>3.35  5.45  6.56</td>
<td>3.47  4.27  3.70</td>
<td>4.52  5.55  5.55</td>
</tr>
<tr>
<td>1.5</td>
<td>B-</td>
<td>6.00  5.65  7.21</td>
<td>4.26  4.87  3.21</td>
<td>2.13  4.65  4.64</td>
</tr>
<tr>
<td>3.4</td>
<td>AB+</td>
<td>5.33  4.49  6.41</td>
<td>3.91  3.45  2.57</td>
<td>3.88  4.78  5.78</td>
</tr>
<tr>
<td>0.6</td>
<td>AB-</td>
<td>4.50  7.40  9.11</td>
<td>4.21  5.83  11.77</td>
<td>3.94  3.58  3.65</td>
</tr>
<tr>
<td></td>
<td>Grand mean age</td>
<td>4.20  5.97  7.50</td>
<td>3.67  5.47  7.41</td>
<td>3.94  5.96  6.59</td>
</tr>
</tbody>
</table>

Table 8.8 Average number of on-hand-inventory per blood type on the hospital side

<table>
<thead>
<tr>
<th>US (%)</th>
<th>Blood Type</th>
<th>without substitution</th>
<th>with substitution at hospital</th>
<th>with substitution both at hospital and blood center</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.4</td>
<td>O+</td>
<td>105.59  126.28  160.65</td>
<td>99.14  116.80  116.26</td>
<td>95.68  105.44  105.44</td>
</tr>
<tr>
<td>6.6</td>
<td>O-</td>
<td>13.02  16.81  11.73</td>
<td>30.07  55.75  58.44</td>
<td>29.28  49.73  49.76</td>
</tr>
<tr>
<td>35.7</td>
<td>A+</td>
<td>74.67  93.86  100.85</td>
<td>56.43  55.48  57.41</td>
<td>96.56  40.48  40.49</td>
</tr>
<tr>
<td>6.3</td>
<td>A-</td>
<td>14.12  19.78  14.59</td>
<td>12.19  9.64  4.30</td>
<td>2.38   1.50  1.50</td>
</tr>
<tr>
<td>8.5</td>
<td>B+</td>
<td>17.81  27.18  17.57</td>
<td>16.25  8.56  12.03</td>
<td>3.18   2.04  2.04</td>
</tr>
<tr>
<td>1.5</td>
<td>B-</td>
<td>5.73   5.47  5.34</td>
<td>4.79   2.66  2.47</td>
<td>0.46   0.56  0.56</td>
</tr>
<tr>
<td>3.4</td>
<td>AB+</td>
<td>9.11   10.21  8.83</td>
<td>7.18   2.96  4.55</td>
<td>1.30   0.47  0.47</td>
</tr>
<tr>
<td>0.6</td>
<td>AB-</td>
<td>3.97   3.49  3.91</td>
<td>0.99   1.99  2.00</td>
<td>0.74   0.28  0.28</td>
</tr>
<tr>
<td></td>
<td>Total number of units</td>
<td>244.02  303.08  323.46</td>
<td>227.05  253.84  257.47</td>
<td>229.58  200.51  200.55</td>
</tr>
</tbody>
</table>
8.4 Conclusion

This chapter has presented a study carried out to optimize the RBC order-up-to policies for a single-hospital single-blood center supply chain system in consideration of products of 8 different blood groups and their compatible substitutions. The limited current research together with the recent advances in shortening RBC storage lifetime presents an opportunity for new research exploration. The research experience obtained in managing the highly perishable platelets supply chain problem (Duan and Liao 2013b) inspires us to build a simulation optimization (SO) framework to tackle the subject RBC problem. This study is intended to extend this line of research by proposing a simulation optimization (SO) approach, which includes a simulation model formulation coupled with a totally new metaheuristic optimization algorithm, TA-TS, to find high quality solutions to the problem with affordable computational time.

One critical aspect of this study is ABO/Rh(D)-compatible blood substitution. Although demands are characterized by the blood type of the recipients, the requests may be filled with other blood types more readily available in the inventory. Although not possible in all scenarios, blood type substitution offers incredible alternatives to fulfill demand when supply of a specific blood is constricted.

Two blood substitution scenarios were tested and their performances were compared with that of the base scenario without blood substitution. All three scenarios were tested with three levels of maximal shelf lives: 21, 14 and 7 days. The simulation optimization approach is used to
identify a solution that will lead to a lowest possible system-wide outdate rate under a specified fillrate requirement. Then the best solution identified is simulated for a sufficient long period to estimate the long-run effect of shortening the MSL of blood units on RBC inventory.

Using such a formulation and simulation optimization approach has enabled us to identify the potential challenge in managing RBC inventory with a shortened MSL. It generally results in an increase in the RBC outdate rates associated with those slow-moving blood types. To maximize the utilization of RBC units, allowing ABO/Rh(D)-compatible blood substitution is shown to reduce the system-wide outdate by at least 16% (scenario 2). An optimization model for blood management should account for the possible type-compatible substitutions for the sake of practical relevance. In general, allowing substitution can frees up a unit of one type that may become outdate if go unused and lead to many substitution possibilities that may allow what previously would be characterized as unmet demand to be fulfilled. The SO approach shown in this study speeds up the circulation of all blood types, maximizes their utilizations, and as a result, leads to lower outdate rates. Thus, the blood substitution analysis as shown here is a new and important facet to study when considering options to mitigate shortages and outdates. Information obtained from this study should be helpful to policy makers if a change in transfusion practice becomes necessary.
CHAPTER 9 SUMMARY AND CONCLUSION

As an alternative to traditional analytical approach to the SC inventory optimization problems, a totally new metaheuristic-based simulation optimization (MSO) approach is proposed in this dissertation. It contains a hybrid meta-heuristic optimizer and an evaluation module which simulates SCs operations. This dissertation study explores how a new metaheuristic-based approach is significantly expanding the power of simulation optimization for supply chain inventory management. The proposed MSO framework is leading to new opportunities to solve problems more efficiently. Specially, in applications involving supply chain uncertainty, the MSO approach surpasses the capabilities of other methods, not only in solvability, but also in their interpretability and practicality.

Many real world systems are too complex to be modeled analytically. Discrete event simulation has long been a useful tool for evaluating the performance of such systems. However, a simple evaluation of performance is often insufficient. A more advanced searching process for the best combination of decision variables based on the output of a simulation model of the system is needed. The proposed MSO approach is a promising tool that provides near-optimal solutions to important practical inventory problems previously beyond reach. As the major component in the MSO framework, a hybrid metaheuristic is tailored and enhanced by a comparative study to speed up convergence to the solution more rapidly and more efficiently.

The meta-heuristic generates a candidate solution and supplies this candidate solution to the SC evaluation module. The evaluation module is tailored to each considered inventory problem
to simulate the inventory movement within the system. It calculates the corresponding objective function value and evaluates how good a solution is for the considered system. Based on the feedback from the evaluation module, the hybrid meta-heuristic optimizer iteratively improves the quality of the solution according to its searching mechanism. The hybrid meta-heuristic optimizer performs an intelligent search on the problem space. This cycle goes on and on until a near-optimal solution is obtained (usually based on the user’s preference for the amount of computational time devoted to the search).

The proposed MSO approach has been applied to three distinct cases:

1. Capacitated SCs under various demands for both decentralized and centralized control (Chapter 5);
2. Integrated SCs with uncertain product quality (Chapter 6);
3. Highly perishable SCs with new age-based policy (Chapter 7);
4. Blood SCs with shortened shelf lives and ABO compatibility (Chapter 8).

These four cases represent common operational situations that most modern SCs have to face on a daily basis. By studying these four cases, it is concluded that the new MSO approach is capable of adapting into different practical situations and providing high quality solution. Considering the practical need of industry for which a comprehensive model is desirable to be embedded in decision support system to assist in inventory management, it is concluded that the robustness of the MSO approach is high for real world implementation.
For example, the MSO is capable of incorporating the stochastic character of the demand in the real world without making any assumptions on its distribution, which makes it applicable to a wide range of demand data. This is exactly the main motivation behind developing such a MSO approach: to bridge the existing gap between theory and practice so as to deliver a user-friendly and flexible framework for the management of various inventories in a business environment. From this standpoint, it is believed that the proposed MSO approach offers a promising alternative to existing inventory control methods. It can handle a wide scope of problems, especially those of practical relevance. By embedding the MSO framework into the decision support system, a company will be able to gain fact-based insight and keep the inventory at a level that best tailored to their needs. Surplus stock will be cut down and the associated costs will be reduced significantly without impacting on its customer service level.

In principle, the inventory simulation model can be tailored to any company-dependent form. The uncertainties and complexities modeled by the simulation are often such that the analyst has no idea about the shape of the solution space. There exists no closed-form mathematical expression to represent the space, and there is no way to gauge whether the region being searched is smooth, discontinuous, etc. While this is enough to make most traditional optimization algorithms fail, metaheuristic optimization approach, such as the one developed in this dissertation research, overcomes this challenge by making use of adaptive memory techniques and population sampling methods that allow the search to be conducted on a wide area of the solution space, without getting stuck in local optima.
The MSO framework creates a tool for decision maker that is effective (the optimization algorithm guides the search for good solutions without the need to enumerate all possibilities), inexpensive and non-disruptive (solutions can be evaluated without the need to stop the normal operation of the business, as opposed to pilot projects which can also be quite expensive).

This dissertation work illustrate how simulation optimization technique can serve as a useful tool in decision support system. Most real-world systems are so complex that computing values of performance measures and finding optimal decision variables analytically is very hard and sometimes impossible. The need for dynamic and stochastic models to analyze complex systems is clearly expected to increase the interest for simulation optimization in the future. A simulation model is typically a descriptive model of the system, i.e., it describes the behavior of the system under consideration, and help us to understand the dynamics and complex interactions among the elements of the system. In the Operational Research community, simulation has been critiqued for a lack of optimization capability. Simulation results are typically based on runs with a set of scenarios, rather than an optimum solution to the problem as usually guaranteed by the prescriptive or normative models (such as (non)-linear programming, dynamic programming). By combining optimization algorithms with simulation systems, it is expected to not only cover its major limitation, but also open a new line of research possibilities and their application areas.

Possible topics for future studies from the methodology point of view can be to develop approaches that offer better tradeoff between problem solvability, solution accuracy and computational expenses. The general simulation optimization framework is expected to provide
solutions to various problems. Those problems vary in terms of the number and structure (i.e., discrete or continuous, quantitative or qualitative) of decision variables, and the shape of the response function (uni-modal or multi-modal). There is no global method that can solve all problems. Research efforts can be devoted to developing more robust techniques that can handle as wide a scope of problems as possible.

At the same time, there is also a need for specific methods that can work efficiently under certain circumstances (Fu et al., 2000). This is especially true when simulation optimization has to be performed relatively frequently to solve daily operational problems. It has to be pointed out that simulation optimization still requires a considerable amount of computer time. This will be the major obstacle for future simulation optimization applications. Possible direction of future studies is to reduce the computational burden by parallel or distributed computing. The other possibility is to narrow down the searching space by statistical screening techniques. Clearly, more research needs to be done in these areas to improve the efficiency of simulation optimization systems.
REFERENCES


Sayers, M., Centilli, J., Sutor, L. J., 2011. Implications for management of a community blood program inventory if the red blood cell shelf life is shortened. *Transfusion* 51 Supplement 241A-241A.


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VITA

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