Modeling the Crude Oil Scheduling Problem with Integration with Lower Level Production Optimization and Uncertainty

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MODELING THE CRUDE OIL SCHEDULING PROBLEM WITH INTEGRATION WITH LOWER LEVEL PRODUCTION OPTIMIZATION AND UNCERTAINTY

A Thesis
Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Science

in

The Cain Department of Chemical Engineering

by
Jorge Asis Charbel Chebeir
B.S. Universidad Nacional del Sur, 2006
May 2015
This thesis is dedicated to my wife Patricia Leon and my parents for their love, endless support, and encouragement.
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<td>( v = 1, \ldots, NV )</td>
<td>Vessels</td>
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<td>( i = 1, \ldots, NS )</td>
<td>Storage tanks</td>
</tr>
<tr>
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### Parameters

<table>
<thead>
<tr>
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<tr>
<td>( cap )</td>
<td>Maximum capacity of each tank</td>
</tr>
<tr>
<td>( S_k )</td>
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<td>( a_{min_{j,k}} )</td>
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<td>( C_u )</td>
<td>Unloading cost per unit time interval</td>
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<td>( C_{sw} )</td>
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<tr>
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<td>( F_{v,i,max} )</td>
<td>Maximum flow of crude transferred from vessel ( v ) to storage tank ( i )</td>
</tr>
</tbody>
</table>
\[ F_{i,n,\text{max}} \] maximum flow of crude transferred from storage tank \( i \) to blending tank \( n \)

\[ F_{n,k,\text{max}} \] maximum flow of crude transferred from blending tank \( i \) to distillation column \( k \)

\[ F_{n,k,\text{min}} \] minimum flow of crude transferred from blending tank \( i \) to distillation column \( k \)

\[ V_{V_v,j,t=1} \] initial volume of crude oil \( j \) in vessel \( v \)

\[ V_{S_{i,j},t=1} \] initial volume of crude oil \( j \) in storage tank \( i \)

\[ V_{B_{n,j},t=1} \] initial volume of crude oil \( j \) in blending tank \( n \)

\[ \text{TVA}_v \] vessel \( v \) arrival time at docking station

\[ NPU \] number of time intervals to unload the cargo of each vessel

\[ M \] Big M method constant

**Binary Variables**

\[ V_{SCB_{V_v,i,t}} \] equal to 1 if a connection between vessel \( v \) and storage tank \( i \) is established during interval time \( t \), otherwise 0

\[ S_{BCB_{V_{i,n},t}} \] equal to 1 if a connection between storage tank \( i \) and blending tank \( n \) is established during time interval \( t \), otherwise 0

\[ B_{DCB_{V_{n,k},t}} \] equal to 1 if a connection between blending tank \( n \) and distillation column \( k \) is established during time interval \( t \), otherwise 0

\[ V_{SCB_{E_{V_v,i},t}} \] equal to 1 if a connection between vessel \( v \) and storage tank \( i \) is established at the beginning of time interval \( t \), otherwise 0

\[ S_{BCB_{E_{V_{i,n},t}} \] equal to 1 if a connection between storage tank \( i \) and blending tank \( n \) is established at the beginning of time interval \( t \), otherwise 0

\[ B_{DCB_{E_{V_{n,k},t}} \] equal to 1 if a connection between blending tank \( n \) and distillation column \( k \) is established at the beginning of the time interval \( t \), otherwise 0

\[ Z_{n,n_p,k,t} \] equal to 1 if a transition from a crude oil mixture in blending tank \( n \) to a crude oil mixture in blending tank \( n_p \) is done during time interval \( t \) in distillation column \( k \), otherwise 0

\[ U_{IBV_{V_v,t}} \] equal to 1 if unloading of vessel \( v \) during time interval \( t \) initiates, otherwise 0
\( VLBV_{v,t} \) equal to 1 if vessel v leaves the docking station during time interval t, otherwise 0

\( STTBV_{i,j,t} \) equal to 1 if a crude oil type j is store in the storage tank i during time interval t, otherwise 0

Continuous Variables

- \( TUIB_v \): unloading initiation time of vessel v
- \( TVL_v \): unloading completion and departure of vessel v
- \( VV_{v,j,t} \): volume of crude oil j in vessel v during time interval t
- \( VS_{i,j,t} \): volume of crude oil j in storage tank i during time interval t
- \( VB_{n,j,t} \): volume of crude oil j in blending tank n during time interval t
- \( FVS_{v,i,j,t} \): volumetric flow rate of crude oil j from vessel v to storage tank i during time interval t
- \( FSB_{i,n,j,t} \): volumetric flow rate of crude oil j from storage tank i to bending tank n during time interval t
- \( FBD_{n,k,j,t} \): volumetric flow rate of crude oil j from blending tank n to distillation column k during time interval t
- \( \lambda \): total satisfaction variable
ABSTRACT

This research is focused on the modeling and optimization of the crude oil scheduling problem in order to generate the most appropriate schedule for the unloading, charging, blending, and movement of crude oil in a refinery, which means obtaining the schedule that generates the lowest costs. Uncertainty, which is often present in these types of optimization problems, is also analyzed and taken into account for the resolution of crude oil scheduling problem.

A comprehensive novel model is proposed to describe the upper level crude oil scheduling problem, generate an optimal solution for the mentioned problem, and allow integration with the lower level production optimization problem of a refinery. This integration is possible due to the consideration of total flows of the different types of crude oil instead of flows of a particular key component in the crude oil to linearize the upper level problem and generate a less complex model. The proposed approach incorporates all the logistical costs including the sea waiting, unloading and inventory costs together with the costs associated with the transfer of crude oil from one to another entity. Moreover, this model also offers the possibility of considering multiple tank types including storage and blending tanks throughout the supply chain and the incorporation of the capability of storing more than one crude oil type in the storage tanks during the schedule horizon. A comparative analysis is performed against other models proposed and preliminary results of integration with a lower operational level are provided.

In order to take into account the possibility of uncertainty or fuzziness in the scheduling problem, for the first time an approach is proposed to face the resolution of this problem in order to obtain a more realistic scheduling of the allocations of crude oil. Fuzzy linear programming theory is used here to represent this uncertainty in order to find an optimal solution that takes into account the lack of precise information on the part of the decision maker without losing the
linearity of the original system. Uncertainty in the minimum demand to be satisfied in the distillation unit according to the necessities of the market and the lack of precise information about certain costs involved in the operations throughout the supply chain are separately considered. Among the different approaches utilized in fuzzy linear programming, the flexible programming or Zimmermann method and its extension to fuzziness in objective functions are implemented. A comparison between the two cases studied and the crisp model is performed with the aim of determining the effect of these uncertainties in the schedule of the crude oils movements between the different entities in the supply chain and the total cost generated.
1. INTRODUCTION

Crude oil refining is an extremely competitive business where the continuous improvement of the profitability has an essential role for the success of a company performing in this field. The current picture of the refining industry is characterized by stiff competition, stricter environmental regulations, heavier, sourer, costly crude oils, uncertainty in crude oil prices, and uncertainty in the availability of the crude oil.\(^1\) Of course, these factors have an important role in the margins of profitability obtained in this business throughout a predefined planning horizon. In order to maintain those margins as high as possible, refineries have to explore different types of potential costs savings strategies. Among the cost saving strategies that can be implemented, the appropriate planning and scheduling of the crude oil may have an essential role in maintaining the competitiveness of a refining company.

The global supply chain of the crude oil is depicted in Figure 1. The crude oil exploration is at the highest level of the chain. Vessels transport the crude oil from the oilfields to the terminals, which are connected to the refineries through a pipeline network. Decisions at this level incorporate transportation modes and supply planning and scheduling. Products generated at the refineries are then sent to distribution centers. Crude oil and products up to this level are often transported through pipelines. From this level on, products can be transported either through pipelines or trucks, depending on consumer demands. In some cases, products are also transported through vessels or by train.\(^2\) The generation of a model that takes into account each of the decision levels of the supply chain in order to obtain an optimal scheduling of the crude oil involves a tremendous complexity. Because of that, several approaches circumvent the problem to certain decision levels. Many of those approaches include the generation of models for the decision level
that involves the scheduling of the unloading, blending, charging and movements of crude oil in the refinery.

In industry, scheduling is necessary to define which products are to be produced, which products are to be consumed at each time instance over a given period; to determine sequencing of products on each unit, to decide when to perform changeovers; to predict the inventory levels of different products, raw material arrival, and final product dispatch in order to meet the product demands. Scheduling the different allocations of crude oil in a refinery is not a simple task, and schedulers often have to use their own planning, analytical, problem solving, and decision making skills together with other tools to determine the optimal allocations of crude oil throughout the supply chain. Historically, the scheduling of the crude unloading, blending and charging has been done manually by schedulers. However, manual scheduling is a time consuming task and normally does not determine the optimal schedule. Obviously, millions of dollars are involved in an imprecise schedule, which can be determining for the short or/and large term success of a
refining company. Thus, manual scheduling is gradually leaving room to more sophisticated approaches to face the determination of optimal schedules.

During the last decades, the scheduling of the unloading, charging, blending, and movements of crude oil in a refinery, also known as crude oil scheduling problem (CSP), has been investigated in several papers in order to develop the most appropriate model to describe this problem and determine the optimal solution for different case studies. The methods developed for the resolution of this problem can be classified into two types, exact and heuristic. The exact method can also be defined as discrete or continuous depending on the type of time representation utilized. A more detailed explanation about the models based on these two time representations is given in Chapter 1. The heuristic method can be a powerful tool when implementing any exact approach involves an enormous and impractical use of computational time. Therefore, the selection of the method to be used is normally dependent of the specific case analyzed.

Among the different types of models based on the exact approach for the resolution of the CSP, it is worth to mention the mixed integer linear programming (MILP) models. These models are characterized by combining discrete and continuous variables without losing the linearity on their mathematical formulation. A drawback of this and other linear models is that the generation of more detailed schedules often requires a continuous time representation that usually includes nonlinear equations. Clearly, this represents an inherent limitation to describe certain operations in a refinery and generate a more accurate scheduling of the crude oil. Hence, there is plenty of room for improvement in these types of models in order to generate a more appropriate description of the CSP and obtain a more precise solution.

The objective of this research is to formulate and implement a novel MILP model to describe the short-term CSP of a single refinery. The approach implemented has to maintain certain
simplicity of modeling and resolution without losing the necessary realisms to represent the mentioned problem. In other words, it is required the development of a model capable of being adaptable to different situations and able of including different conditions. Furthermore, the model has to be able of being integrated with other decision levels such as the lower level production optimization of the refinery, which includes an important number of nonlinear equations as well as incorporate uncertainties always present in this type of problems.

Summarizing, the proposed model and its variations will be compared with other previous models at different conditions including variations of initial conditions and/or parameters and incorporation of uncertainty. In Chapter 2 the proposed model will be introduced describing the CSP to obtain the optimal schedules for a number of case studies analyzed. The different results obtained using the proposed model, compared to existing models, are included and explained in Chapter 3. In Chapter 4 extensions and modifications will be incorporated in the deterministic model described in Chapter 2 to incorporate uncertainty or fuzziness in constraints and parameters. In Chapter 5 the results of the model including uncertainty in the short-term CSP are provided and compared with the ones obtained with the crisp or deterministic model. Finally, Chapter 6 contains the general conclusions of this research as well as some further directions.
2. MODELING THE CRUDE OIL SCHEDULING PROBLEM

2.1 Literature Review

Over the recent decades, a large number of models have been developed to describe the problem of scheduling the allocations of crude in a refinery. Depending on whether the events of the schedule can only take place at some predefined time points, or can occur at any point in time during the time horizon, models can be classified into discrete and continuous time formulations.\(^7\) Of course, the implementation of each of these approaches for the resolution of the CSP is subject to the complexity involved, the degree of precision required, and computational efficiency sought for these types of problems. Thus, the method utilized is highly dependent of each case study analyzed.

In the discrete model the scheduling horizon is partitioned into time intervals of equal size, and binary variables are used to indicate if an action starts or terminates in the beginning of the associated time interval.\(^5\) This method offers a simple representation of small crude oil scheduling problems, but its performance is limited by the computational effort involved in the resolution of more complex and larger problems. This approach is implemented by Lee et al.\(^8\) to develop a MILP model for short term refinery scheduling problem with inventory management decisions. The model considers the logistical costs of the crude oil loading and unloading schedule including inventory, changeover, sea-waiting and unloading costs. In this model the nonlinearities are avoided through the utilization of linear terms based on a key component in the crude oil. An important outcome of this model is the presence of a tradeoff between sea waiting and inventory costs. Hamisu, et al.\(^9\) incorporate certain modifications in this MILP model, including the possibility of a penalty for demand violation among others, to reflect in a more realistic way the scheduling problem. Yüzgeç et al.\(^10\) present a different work based on the implementation of a
model predictive control (MPC) strategy to solve the short-term scheduling problem. This strategy is effective in reducing the operating cost over the schedule horizon. Another interesting approach is the one implemented by Saharidis et al.⁵, who develop a discrete model based on the total amount of the different types of crude oil instead of a specific component to generate a linear system. Moreover, they implement the model in an event-base mode in order to reduce the computational time required to solve these type of optimization problems. This model has an enormous potential for the integration with lower level problems, but it does not take into consideration all the possible costs involved in the loading and unloading processes and has the limitation of considering only one type of tank throughout the supply chain. Moreover, this model requires the implementation of event-case discrete time representation with the aim of avoiding the excessive time required for its resolution when the classical time representation is used for more extensive planning horizons. Therefore, it can be observed that the models based on discrete time representation have an enormous potential for the description of the CSP, but they still present certain limitations such as their applicability for large problems.

The methods based on continuous time representation have the main advantage of a less complex resolution due to a lower number of binary variables involved in the mathematical formulation. Furthermore, the models based on this type of time representation have the advantage of a more precise solution. However, one of the most important disadvantages of these types of continuous methods is the inclusion of nonlinear constraints in their formulations. Among the different continuous approaches implemented, it is important to remark the work developed by Pinto et al.¹¹ to overcome the computational limitations of discrete models and find the most appropriate schedule for the unlading, charging blending and movements of crude oil. To circumvent the problem, they generate a model with variable length time slots, which represents
the crude oil receiving operations as well as periods between two receiving tasks. A similar approach is applied by Joly et al.\textsuperscript{12} to develop their continuous time model. This model has the capacity of generating a short-term schedule, spanning a horizon of approximately one week and taking into account volume and quality constraints as well as operational rules. Jia et al.\textsuperscript{13} present a continuous time formulation that results in fewer variables and constraints and can be efficiently solved using available MILP solvers. Furman et al.\textsuperscript{14} also propose a continuous time model with the potential of reducing the complexity of the movements of crude oil between tanks in a refinery. They generate a model with the capability of decreasing the number of binary variables, which provides a significant reduction in combinatorial complexity. This capability is also presented in the event-based continuous time formulation developed by Li et al.\textsuperscript{15}, where several realistic features such as multiparcel vessels, single-parcel vessels, crude blending, brine settling, and crude segregation among others are also incorporated in order to describe in a more accurate way the scheduling problem. Another interesting continuous approach is proposed in the work developed by Jia and Ierapetritou\textsuperscript{16}, where the entire refinery system is divided into three problems. The first problem involves the crude-oil unloading, mixing and inventory control, the second problem consists of the production unit scheduling which includes both fractionation and reaction processes, and the third problem depicts the finished product blending and shipping end of the refinery. An important advantage of this model is the capability to solve large scale problems in an efficient manner.

Most of the models developed for the CSP, whether discrete or continuous, have the aim of finding the optimal schedule in order to obtain the minimum cost due to the operations involved in transferring crude from vessels to tanks, internal transfers among tanks, and charges to the crude oil distillation units. This represents a single level optimization problem where other costs related
to the refinery operation are not taken into consideration. The development of a model, with the capability of integrating different optimization problems’ levels may have a substantial role in the generation of a more intelligent schedule for the movements of crude oil in a refinery. An integration of optimization and simulation models is presented by Abraham and Rao\textsuperscript{17} for the lube oil section using a reactive scheduling algorithm. Mouret et al.\textsuperscript{18} propose a model for the integration of the refinery planning and crude oil scheduling through the utilization of a Lagrangian decomposition approach. Basically, Lagrange multipliers are used to link and communicate economic information between the two subsystems. Another interesting approach is presented by Geraili and co-workers\textsuperscript{19, 20}, who develop an optimization framework that integrates different levels of decisions for a biorefinery through the utilization of yields as linking parameters. A metaheuristic optimization is implemented for the operational level and then the effect of its nonlinearities is incorporated in the upper level in order to obtain an optimal global solution. Robertson et al.\textsuperscript{21} develop one of the first approaches to find a more comprehensive model for the loading and unloading crude oil problem, which integrates the upper level problem of the unloading, charging, blending, and movements of crude oil with the operational level of the refinery. In this model, the nonlinearities of the lower level are circumvent through the generation of a linear function for the cost involved in the process in terms of the total flow of each crude oil type. Then, the integration of the two problems becomes effective by incorporating this linear cost function into the total cost determined in the upper level. Although significant possibilities offered by this comprehensive model, it does not take into account certain substantial logistical costs such as the sea waiting and unloading costs. Besides, only one type of tank is considered to store the different crudes and prepare the blends throughout the supply chain. Thus, the integration between
the CSP and other decision levels is possible, but it still has room for more development and improvement.

The MILP model for the CSP proposed in this research utilizes the traditional constraints and equations required by this type of optimization problems as well as the concepts that are necessary to generate a more comprehensive approach that has the possibility to integrate problems of different levels. The model proposed for the upper level scheduling problem not only maintains the same capability of being integrated with a lower production level but also incorporates all the mentioned costs including the sea waiting costs, unloading costs, inventory costs, and the costs related to the movements of crude oil from one to another entity. Another important advantage of this approach is the possibility of considering multiple tank types throughout the supply chain (storage and blending tanks). Furthermore, this approach generates a more flexible and realistic description where the storage tanks are not limited to store only one type of crude in the entire schedule horizon. Thus, the aims of the model is to represent in an accurate way the CSP through the incorporation of all the costs involved in the problem of study and maintain the potential to be integrated with the lower operational level.

2.2 Model Formulation

The system analyzed is a refinery composed of different entities including the vessels that unload crude oil from different sources in the docking station or harbor of the refinery, tanks that store different types of crude oil, blending tanks where the appropriate mixtures or blends are generated in order to accomplish the required specifications of the distillation units, distillation columns where different cuts are separated, and required pipeline networks to connect each of these different entities through the supply chain (Figure 2).
The premium goal of the model is to impart a greater degree of realism to the actual representation of the crude oil scheduling problem. In order to reach this goal, it is necessary to obey certain operations rules that govern a process. The operating rules considered in this MILP model are the following: 1) each vessel arrives and leaves the dock only one time in the entire schedule horizon, 2) the vessel has to arrive before the unloading starts, 3) the vessel has to leave after unloading finishes, 4) the vessel has to leave the dock after its arrival, 5) the vessel cannot arrive until the previous leaves, 6) a storage tank cannot charge a blending tank and be charged by a vessel at the same time, 7) a blending tank can only be charged by only one storage tank, 8) a blending tank cannot charge a distillation column and be charged by a storage tank at the same time, 9) a blending tank can only charge one distillation column in each interval of time, 10) a distillation column can only be charged by only one blending tank, 11) there is no possibility of mixing in a storage tank in each interval of time, and 12) certain number of intervals is required to
unload each vessel. These operation rules are reflected in the model through the different equations and constraints in the mathematical formulation.

The development of a model may involve an extremely high number of unknowns and equations, which makes essential the proposition of certain assumptions or suppositions that facilitate the resolution of such complex system. The MILP model proposed for the CSP includes the following assumptions: 1) one docking station is considered for unloading of crude oil, 2) the crude oil is unloaded in a predetermined schedule, 3) the amount of crude oil in the lines is neglected, 4) the changeover time is neglected comparing to the entire schedule horizon, 5) there is perfect mixing in the blending tanks, 6) the schedule horizon is discretized in time intervals of identical size. Of course, these assumptions have a direct impact on the problem complexity, because the size of the problem is dramatically reduced. Therefore, the overall model turns into a much less complex system to be solved.

The proposed MILP model for the CSP incorporates certain equations to describe the operation rules mentioned previously.

\[
\sum_{t} UIBV_{v,t} = 1, \forall v \tag{1}
\]

\[
\sum_{t} VLBV_{v,t} = 1, \forall v \tag{2}
\]

\[
TVA_{v} \leq TUIB_{v}, \ \forall v \tag{3}
\]

\[
TUIB_{v+1} \geq TVL_{v} + 1, \ \forall v \tag{4}
\]

\[
\sum_{v} VSCBV_{v,i,t} + \sum_{n} SBCBV_{i,n,t} \leq 1 \ \forall i, t \tag{5}
\]

\[
\sum_{i} SBCBV_{i,n,t} + \sum_{k} BDCBV_{n,k,t} \leq 1 \ \forall n, t \tag{6}
\]
\[ \sum_n BDCBV_{n,k,t} = 1 \quad \forall \, k, t \quad (7) \]

\[ \sum_i VSCBV_{v,i,t} \leq \sum_{t' \leq t} UIBV_{v,t'} \quad \forall \, v, t \quad (8) \]

\[ \sum_i VSCBV_{v,i,t} \leq \sum_{t' \geq t} VLBV_{v,t'} \quad \forall \, v, t \quad (9) \]

Equations (1) and (2) ensures that the unloading and leaving of each vessel occurs only once throughout the schedule horizon. Equation (3) defines that the arrival to the docking station of each vessel takes place before the unloading of its cargo. Equation (4) denotes that the unloading of the cargo of a new vessel has to occur after the previous vessel leaves the docking station. Equations (5) and (6) establish that any of the storage or blending tanks cannot be loaded and unloaded at the same time. Equation (7) ensures that the distillation column is charged by only one blending tank in each interval of time. Eq. (8) states that each vessel can be connected to no more than one storage tank in each interval, and this connection can only be established after unloading initialization has begun. Clearly, if unloading initiation had not occurred, the right hand side of the constraint would be zero and the vessel would not be allowed to connect to any tank. Otherwise, the right hand side would achieve its maximum value of 1 and the vessel could connect to only one storage tank. In a similar fashion, Eq. (9) states that the unloading of a vessel occurs before it leaves the dock. This is performed by setting that the sum of all the vessel tank connections to be less than the sum of all vessels leaving decision variables for the rest of the time horizon.

In order to determine the time that each vessel \( v \) initiates the unloading of its cargo, \( TUIB_v \), and the time that each vessel leaves the docking station, \( TVL_v \), it is necessary to include the following equations in the model:

\[ TUIB_v = \sum_t t \times UIBV_{v,t} \quad \forall \, v \]

(10)
\[ TVL_v = \sum_t t \cdot VLBV_{v,t} \quad \forall \ v \]  
\[ TVL_v - TUI_B \geq NPU \quad \forall \ v \]  

Equations (10) and (11) determine the unloading initiation and leaving times by summing the product of each interval of time and the respective binary variable of each event \((UBV_{v,t} \text{ for the time the vessel unload and } VLBV_{v,t} \text{ for the time the vessel leaves})\). Then, the number of periods or intervals of time required for each vessel to unload the cargo is established by equation (12). Depending of the base case analyzed, the number of intervals required to unload could change.

To model no blending in the storage tank \(i\) that contains crude oil type \(j\) at the time interval \(t\), an additional binary variable \(STTBV_{i,j,t}\) is created. Then, an equation is imposed not allowing multiple crude oil types simultaneously in the same storage tank:

\[ \sum_j STTBV_{i,j,t} \leq 1 \quad \forall \ i, t \]  

Equation (13) divides the capacity of the storage tanks into crude types. Then, it is possible to establish that the amount of the crude type \(j\) selected for the tank \(i\) is less than this capacity. On the other hand, for all crudes not selected for the tank, their amount is zero. Clearly, the effect of Equation (13) is observed in Equation (14).

\[ \sum_j VB_{n,j,t} \leq \text{cap} \quad \forall \ n, t \]
Equation (15) describes the capacity constraint for the blending tank, where it is considered the summation of all the crudes type $j$ in the blending tank $n$ at the time interval $t$. The model proposed requires the additional constraints (16) and (17) for the blending tanks in order to restrict the amount of each crude type $j$ that is charged in each distillation column $k$ through the utilization of the Big M method. This method establishes that these constraints are only imposed if a blending tank $n$ is connected to a distillation column $k$. Comparing the crude oil $j$ in each tank with the total amount allowed of this specific crude, it is possible to limit the charges of each crude unloaded to the distillation column. The amounts allowed are the total crude amount in each tank multiplied by the upper and lower percentages $amin_{j,k}$ and $amax_{j,k}$. These parameters represent these minimum and maximum percentages of crude type $j$. The difference between the crude $j$ in the tank and the minimum amount of crude $j'$ allowed in the tank should always be greater than or equal to zero if there is a connection between the blending tank and the distillation unit. If there is no connection between the blending tank and distillation unit, the right hand sides of the constraints take the values of $M$ and $–M$. Because the value of $M$ is extremely large, these constraints become totally irrelevant. This concept is also implemented by Saharidis et al. for the case when the crude oil blend is generated in a tank instead of a manifold. However, their model does not establish any type of bounds for the flow of each type of crude oil unloaded to the distillation column, and it is only focused on maintaining the maximum and minimum percentages in the blending tanks. Equation (18) also restricts the amounts unloaded to the distillation column $k$ through the
establishment of bounds for the flows of each type of crude oil $j$. These bounds utilize the same upper and lower percentages mentioned previously. Thus, the amount of each type of crude oil is bounded for both the blending tank and the charge of crude unloaded from that tank.

The maximum and minimum amounts allowed of each crude oil can be also defined for each blending tank. This is possible through the consideration of a maximum and minimum percentage of each crude type $j$ for each blending tank $n$ and distillation column $k$, $am_{ax}^{j,n,k}$ and $amin^{j,n,k}$. This allows us to define multiple maximum and minimum amounts of each crude oil type (multiple ranges instead of only one) that distillation columns may process in each interval of time. Then, the equations (19), (20), and (21) are slightly modified as follows:

\[
VB_{n,j,t} - \sum_{j^{'}} VB_{n,j^{'},t} * am_{ax}^{j,n,k} \leq M * (1 - \sum_{k} BDCBV_{n,k,t}) \quad \forall n, j, t
\]  

(19)

\[
VB_{n,j,t} - \sum_{j^{'}} VB_{n,j^{'},t} * amin^{j,n,k} \geq -M * (1 - \sum_{k} BDCBV_{n,k,t}) \quad \forall n, j, t
\]  

(20)

\[
\sum_{j} FBD_{n,k,j,t} * amin^{j,n,k} \leq FBD_{n,k,j,t} \leq \sum_{j} FBD_{n,k,j,t} * am_{ax}^{j,n,k} \quad \forall n, j, t
\]  

(21)

It is important to mention the advantages and disadvantages of using each approach regarding the crude oil percentage range for the distillation column. A single percentage range of each crude oil type generates a less complex system. The use of this approach reduces the number of equations and variables involved in the model. Moreover, this approach facilitates the integration with the lower production level. Robertson et al.\textsuperscript{21} have demonstrated that it is possible to integrate the CSP with the operational level through the utilization of a single crude oil percentage range for the distillation column. This single percentage range is also considered for the flows in the lower layer in order to generate a linear operational cost, which is incorporated in the total cost function to be minimized in the upper level. On the other hand, multiple percentage ranges allows representing more complex situations, especially the case where each blending tank
has its own percentage range of crude oil. In this case, the percentages ranges for each crude oil type do not only depend on the distillation column but also in the blending tanks. Consequently, a single range is not enough to describe blending tanks with different specifications. Although the capacity to represent more complex problems, it is worth to mention that the incorporation of multiple percentage ranges of crude oil could increase the complexity to develop an integration with other levels. Clearly, the use of a single or multiple ranges can have advantages and disadvantages, and it is necessary to consider the limitations of each specific approach in order to implement the most appropriate for each case studied.

As a general concept, the implementation of these constraints, whether using one or other approach, allows us to generate a more flexible model comparing to other models developed in previous works. In the proposed approach, there is no restriction to maintain the percentage of crude oils in the blending tanks during the entire schedule horizon but only when there is a connection between a determined blending tank and distillation column. Consequently, it may be possible to find certain blending tanks out of specification when they are not connected to a distillation column in some intervals of the schedule horizon.

The model also involves other physically limiting constraints such as the amount of crude oil that the pipelines can contain when there is a connection between a vessel and a storage tank, a storage tanks and a blending tank, or a blending tank and a distillation column. This is expressed by the following set of equations:

\[ \sum_j FVS_{v,i,j,t} \leq VSCBV_{v,i,t} * F_{v,i,max} \quad \forall \ v,i,t \]  \hspace{1cm} (22)

\[ \sum_j FSB_{i,n,j,t} \leq SBCBV_{i,n,t} * F_{i,n,max} \quad \forall \ i,n,t \]  \hspace{1cm} (23)
Equation (22) states the maximum amount of crude oil that can be sent from a vessel \( v \) to a storage tank \( n \) in each interval of time \( t \). Equation (23) states the maximum amount of crude oil that can be sent from a storage tank \( i \) to a blending tank \( n \) in each interval of time \( t \). Finally, equation (24) defines the maximum and minimum amounts of crude oil that can be sent from a blending tank to a column distillation. In this case there is also a minimum rate in order to guarantee a feed of crude oil to the distillation column in each interval of the schedule horizon. In other words, the distillation column demand of crude oil varies throughout the schedule horizon, but it is ensured that certain charge is always unloaded in each interval. Therefore, it is avoided the possibility of not charging of crude oil in the distillation unit during any of the intervals of the schedule horizon.

To describe the inventory profiles of the vessels \( v \), storage tanks \( i \), and blending tanks \( n \) of each crude type \( j \) at each time interval \( t \), \( \text{VV}_{v,j,t}, \text{VS}_{i,j,t}, \text{and} \ \text{VB}_{n,j,t} \), the following equations are required:

\[
\text{VV}_{v,j,t} = \text{VV}_{v,j,t=1} - \sum_{t \leq t'} \sum_{i} FV_{S,v,i,j,t} \quad \forall \ v, j, t \tag{25}
\]

\[
\text{VS}_{i,j,t} = \text{VS}_{i,j,t=1} + \sum_{t \leq t'} \sum_{v} FV_{S,v,i,j,t'} - \sum_{t \leq t'} \sum_{n} FS_{B,i,n,j,t'} \quad \forall \ i, j, t \tag{26}
\]

\[
\text{VB}_{n,j,t} = \text{VB}_{n,j,t=1} + \sum_{t \leq t'} \sum_{i} FS_{B,i,n,j,t'} - \sum_{t \leq t'} \sum_{k} FBD_{n,k,j,t'} \quad \forall \ n, j, t \tag{27}
\]

Equation (25) states that the volume of a vessel \( v \) with a cargo of crude type \( j \) at the time \( t \) is equal to its initial volume, \( \text{VV}_{v,j,t=1} \), minus the total amount of crude that is sent to the storage tanks \( i \) until that time interval \( t \). Equation (26) states that the volume of a crude type \( j \) in a storage tank \( i \) at the time \( t \) is equal to its initial volume, \( \text{VS}_{i,j,t=1} \), plus the total amount of crude that is received from the vessels \( v \) minus the total amount of crude that is sent to the blending tanks \( n \).
until that time interval $t$. Equation (27) states that the volume of a crude type $j$ in a blending tank $n$ at the time $t$ is equal to its initial volume, $V_{B_v,j,t=1}$, plus the total amount of crude that is received from the storage tanks $i$ minus the total amount of crude that is sent to the distillation columns $k$ until that time interval $t$.

It is necessary to ensure that all the inventory of the vessels is unloaded to the storage tanks in the schedule horizon. This is accomplished through the implementation of an overall material balance where the control volume not only takes into account the entities but also all the time periods or intervals that all the crude type $j$ in vessels $v$ must be unloaded. Thus, it is ensured that each vessel leaves the dock station completely empty. This material balance is described as follows:

$$\sum_i \sum_{t} F_{V,S_v,i,j,t} = V_{V_v,j,t=1} \quad \forall v, j \quad (28)$$

The demand could be expressed as the amount of crude oil required for each distillation column at each interval of time $t$. Nonetheless, this approach requires detailed information of the demand of each column and the implementation of certain extra assumptions in the model. For example, it is required the amounts of crude oil in each time interval, or it has to be assumed certain rate to be charged in the distillation column. A less complex way to avoid the implementation of these type of considerations and circumvent this difficulty is defining the total demand for all the crude oil type $j$ and blending tanks $n$ over the entire schedule horizon. Thus, it is only required to satisfy a total demand without considering the particular amounts of crude oil sent in each time interval $t$. The total demand is represented by the following equation:

$$\sum_n \sum_j \sum_{t} F_{B,D_n,k,j,t} = S_k \quad \forall k \quad (29)$$
Equation (29) together with equation (24) may generate a more flexible schedule where the demand is not fixed in each interval and fluctuates throughout the entire schedule horizon. The aim is to satisfy the total demand at the end of the loading and unloading process in the refinery.

The objective function to be minimized is represented by the summation of all the logistical costs including the unloading, sea waiting, inventory, and the cost related to the transfers of crude oil between entities along the supply chain. The equations that describe each of the logistical costs are given as follows:

\[ \text{Unloading Costs} = C_u \sum_v (TVL_v - TUIB_v) \]  
(30)

\[ \text{Sea Waiting Costs} = C_{sw} \sum_v (TUIB_v - TVA_v) \]  
(31)

\[ \text{Inventory Costs} \]

\[ = C_{\text{invst}} \sum_i \sum_t \left( \frac{\sum_j VS_{i,j,t} + \sum_j VS_{i,j,t-1}}{2} \right) + C_{\text{invst}} \]

\[ \times \sum_i \sum_t \left( \frac{\sum_j VB_{i,j,t} + \sum_j VB_{i,j,t-1}}{2} \right) \]  
(32)

It is worth to mention that two types of approaches are considered to represent the costs involved in the movements of crude oil between entities. One of the approaches is to consider the transition from one to another blend, given by Equation (34) when the blending tanks are connected to the distillation columns. Clearly, the resulting cost does not take into account the possibility of other costs related to the movements of crude between other entities. In other words, the costs involved in the connections between the vessels and the storage tanks and the connections between the storage tanks and the blending tanks are neglected. This changeover cost, expressed by equation (33), is implemented in other models such as the one developed by Lee et al.\(^8\) The idea of including this cost in the proposed model is to avoid discrepancies in the determination of the
total cost when confronting with other previous models. Consequently, the differences between the results obtained will be the consequence among the alternative approaches and considerations included in each model with no influence of the method implemented to determine the cost involved in transferring the crude oil. The following equations describe the changeover cost.

\[
\text{Changeover Costs} = C_{\text{cover}} \times \sum_n \sum_{n_t} \sum_k \sum_t Z_{n,n_p,k,t} 
\]  

(33)

where

\[
Z_{n,n_p,k,t} \geq BDCBV_{n_p,k,t} + BDCBV_{n,k,t-1} - 1
\]  

(34)

The binary variable \(Z_{n,n_p,k,t}\) represents the switch from a crude oil mixture in blending tank \(n\) to a crude oil mixture in the blending tank \(n_p\). This way to determine the total cost is going to be used throughout when this model is contrasted with other models.

Another way to consider the cost involved in the operations to transfer crude oil between entities is taking into consideration each setup established to load and unload all the entities in the supply chain. In other words, the cost of the setups between the vessels and the storage tanks, the storage tanks and the blending tanks, and the blending tanks and the distillation columns. This cost is expected not only to be higher but also more accurate because it considers all the operations involved in the movements of crude oil. In order to make possible the utilization of this type of cost, certain constraints need to be included in the model to guarantee that the setup cost is charged at the beginning of each loading or unloading period. In the proposed model, the implementation of these constraints is extended to the different types of tanks. These constraints describe the relationships between the setup established binary variables, \(VSCEBV_{v,i,t}\), \(SBCEBV_{i,n,t}\), and \(BDCEBV_{n,k,t}\), and the connection established binary variables, \(VSCBV_{v,i,t}\), \(SBCBV_{i,n,t}\), and \(BDCBV_{n,k,t}\).
Equations (35), (36) and (37) state that if the setup established binary variable is zero, then nothing occurs and previous time period connection binary variable is equal to the current time period connection established binary variable. On the other hand, if a setup is made, the previous connection established variable must have been zero and the current must be one. Then, it is possible to define a setup cost as follows:

$$\text{Setup costs} = C_{\text{setup}} \left( \sum_i \sum_n \sum_t SBCBV_{i,n,t} + \sum_v \sum_i \sum_t VSCBV_{v,i,t} + \sum_n \sum_k \sum_t BDCBV_{n,k,t} \right)$$ (38)

where the setup cost per connection established between two entities, $C_{\text{setup}}$, may be different for each connection in some cases.

Equation (38) establishes that the cost of transferring crude oil between entities depend on the number of setups established between the vessels and the storage tanks, the storage tanks and the blending tanks, and the blending tanks and the distillation columns. Establishing a setup represents a time consuming process that includes certain costly operations such as configuring the pipeline network, filling pipelines with crude oil, sampling crude for chemical analysis, measuring of crude oil stock, starting loading/loading, and stopping loading/unloading. Consequently, it is expected, like in the case of changeover cost, that this specific cost will have a notorious weight and will dominate over the other logistical costs.

The objective function to be minimized is the total cost, which is determined through the summation of all the logistical costs as follows:
Total Logistical Costs 

\[ Total \ Logistical \ Costs = UnloadingCosts + SeaWaitingCosts + InventoryCosts \]

\[ + \ ChangeoverCosts \]

or

\[ Total \ Logistical \ Costs = UnloadingCosts + SeaWaitingCosts + InventoryCosts \]

\[ + \ SetupCosts \]

The only difference between equations (39) and (40) is the utilization of the changeover or setup cost. As noted previously, these costs have much more weight in the total cost determined than the other logistical costs. Clearly, any type of difference or variation due to the use of one or the other may have a significant impact in the total costs determined by the model.

2.3 Case Studies Definition

In order to demonstrate the utility of our proposed model, a petroleum refinery is considered for each case study. Particularly, the first case study analyzed in this chapter (Case Study 1) consists of two vessels, two storage tanks, two blending tanks, and only one distillation column. It is essential to define certain conditions for the base case studied such as the times that each vessel arrives, the initial volumes present in the vessels and different tanks of the supply chain, and the amounts of each types of crude oil in each entity. Vessels 1 and 2 arrive at the time intervals 1 and 5, respectively, within a schedule horizon composed of eight time intervals (each interval represents a day). Those vessels contain 1000000 bbl of Crude Oil Type 0 and 1, respectively, when each of them arrives at the docking station. Storage Tanks 1 and 2 initially have 250000 bbl of crude Type 0 and 750000 bbl of crude oil of Type 1, respectively. Finally, Blending Tanks 1 and Blending Tank 2 initially contain 500000 bbl of two different blends constituted by
the crude oils Type 0 and 1. This base case is basically the same analyzed by Lee et al.\textsuperscript{8}, allowing us to perform a comparative analysis for the CSP.

It is important to mention that the model developed by Lee et al.\textsuperscript{8} also considers the concentration of the key component sulfur in each type of crude oil (0.1 and 0.6) and a composition range of the same component in the blending tanks (0.015-0.025 and 0.045-0.055) among the other initial conditions mentioned above. These initial conditions cannot be directly implemented in the proposed model because it is based on the total flows of each type of crude oil throughout the supply chain without taking into consideration any type of concentration or flow of a specific key component in the crude oils or blends. However, it is possible to consider the initial conditions involving the key component by assigning its concentrations to each of the crude oil type involved in the proposed model (in other words, the concentration of sulfur, the key component, in Crude oil Type 0 and Crude Oil Type 1 are 0.1 and 0.6, respectively). This is the first step to adequate the proposed model in order to include the same initial condition. In this regard, the initial concentrations in the blending tanks are represented by the average of the sulfur concentration range in those tanks. These initial concentrations are used to determine the initial amounts of each crude oil in the blending tanks utilized by the proposed model. This is done through the implementation of a material balance using the concentrations of sulfur in each crude oil type and the average concentrations in the blending tanks. Consequently, the total amounts of each crude oil type are used to represent the concentrations of the sulfur in the blending tanks. It should be mentioned that the implementation of this transformation is required just to perform the comparison between the models. Table 7 (see Appendix A) depicts the initial conditions and parameters of utilized to perform the comparison between models in Case Study 1.
After performing the mentioned comparison between the two models, certain variations are introduced including the replacement and/or incorporation of certain equations. This is explained in more detail in the Chapter 3 (see section 3.3). In this regard, a second case study is presented (Case Study 2), based on the use of a single percentage ranges for the concentrations of each crude oil type, as well as an alternative approach to determine the total cost based on the inclusion of the setup cost instead of the changeover cost. Case Study 2 presents the same network configuration and most of the conditions and parameters of Case Study 1 (Table 7). Among the conditions and parameters that differ from the ones presented in the mentioned table, it is worth to mention the initial crude oil amounts of the blending tanks, and the concentrations of crude oils required by the distillation column. Although the concept of the setup cost is different from the one of the changeover cost, its base value is exactly the same for this case study (US$50000 per connection established). The initial amounts of the blending tanks are of 167500 bbl of Crude Oil Type 0 and 332500 bbl of Crude Oil Type 1. The distillation column concentrations required are 0.335-0.390 of Crude Oil Type 0 and 0.610-0.665 of Crude Oil Type 1 when any of the blending tanks is connected to the distillation column in each tank.

To demonstrate the capability of integration of the proposed model with other decision levels, a third case study is discussed (Case Study 3). Again, the supply chain network is the same of the previous two cases, but the parameters and conditions are based on the ones implemented by Robertson et al.21 This allows the utilization of their results of the operational level to develop a first attempt of integration with the proposed model. The model for the upper level is implemented again with a single percentage range for each crude oil type and the costs associated to each setup between entities. Regarding the setup costs, an important difference from the second base case is not only the value used for the setup costs but also the implementation of different
setup costs for the different connections throughout the supply chain. Basically, different weight to the connections between entities are given, which means that the setup costs between the blending tanks and the distillation columns are different from the other setup costs. This may be more adequate than considering the same setup cost for all the connections. Clearly, the connections between entities differ in complexity and number of operations involved, which has a direct impact in their costs. This is also different from the base case used by Robertson et al.²¹, where only one type of setup cost was implemented (corresponding to one type of tank). Table 8 illustrates the initial conditions and the different parameters for Case Study 3.
3. RESULTS FOR CRUDE OIL SCHEDULING PROBLEM MODEL

3.1 Computational Resolution

Several modeling systems can be used to solve this types of optimization models. In the case of the MILP analyzed in Chapter 2, the modeling system GAMS with the CPLEX solver is utilized in a CPU with a processor Intel Core 7, 16GB of RAM Memory, and a 64-bit Operating System. The resolution of the model taking into account the Case Study 1, which includes different concentrations of crude oil and the changeover costs, involves 572 single equations, 514 single variables, and 198 discrete variables. Secondly, the model is developed in GAMS using the Case Study 2 and including the required variations. The variations include the utilization of a single range of concentrations and the setup cost instead of the changeover costs among the others mentioned in Chapter 2. Considering these modifications, the number of single equations, single variables and discrete variables increase. Now, the model is constituted by 644 single equations, 568 single variables, and 252 discrete variables. Finally, for the integrated model, where is considered the Case Study 3, the model with the same variations implemented in Case Study 2 is developed in GAMS. The only difference is the inclusion of the linearized operation cost in the objective function, which in effect represents the integration with the lower operational level. The resolution of this model involves 438 single equations, 383 single variables, and 168 discrete variables. Of course, one of the main points that explains the lower size of the problems is the utilization of a less extended planning horizon compares to the other two case studies.

3.2 Case Study 1: Comparative Analysis

Table 1 illustrates the results obtained through the implementation of the two models. Comparing the results, it is possible to highlight certain aspects of the different costs. First, the results of the MILP models reflect exactly the same cost related to the time that the vessel has to
wait in the sea until be able to unload its cargo and the time required to discharge the crude oil in the storage tanks. Second, we can notice a difference in the inventory cost determined by the two models. Basically, the proposed model predicts a higher cost related to the time that the crude oils remain in the tanks. This will be analyzed in more detail for each particular entity of the supply chain later. Third, the changeover cost obtained with the two different models is exactly the same. Cleary, both approaches predict the same amount of transitions from one mixture of crude to another. In other words, the number of switches from one blending tank to another is the exactly the same for both models.

Overall, the total cost obtained through the implementation of the two models is similar since the changeover cost has a dominant weight over the other costs and the small discrepancy is generated by the difference in the inventory cost, which has a much lower impact in the total cost. This can be reversed through the relaxation of certain constraints in the proposed model to allow some simultaneous operations, which generates an even closer model to the one developed by Lee et al.\textsuperscript{8} Through the replacement of constraints given by Equations (5) and (6) by ones that permit simultaneous loading and unloading of the storage tanks and loading of the blending tanks from more than one storage tank (see Equations 103, 104, 105, 106, and 107 in Appendix B), we can lower the inventories throughout the different entities in the supply chain. This implies the possibility of reducing even more the costs determined by the proposed model, which are also included in Table 1 (column 4). Although this variation is not considered for the comparison with the Lee et al.\textsuperscript{8} model, it shows the capability of the proposed model to be adapted to new conditions and/or restrictions.
The schedule of the vessels in each interval of the time horizon is described by Figure 3. As mentioned above, the same unloading schedule is generated for the vessels in both models with the only difference being where each charge is directed. Because the unloading process of each vessel is not restricted to only one of the storage tanks in the proposed model, it is possible that a vessel switches from one tank to another during the unloading process. The mixture of crude oils is prevented by constraints described in Chapter 2. Consequently, the storage tanks can only store one crude oil type in each interval, but it is not necessary the same crude oil in the entire schedule horizon.

Table 1. Comparison between Lee et al.\textsuperscript{8} MILP Model and Proposed MILP Model

<table>
<thead>
<tr>
<th>Costs Involved (k$)</th>
<th>Lee et al.\textsuperscript{8} MILP Model</th>
<th>Proposed MILP Model</th>
<th>Proposed MILP Model with less restrictive constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unloading + Sea Waiting Cost</td>
<td>52</td>
<td>52</td>
<td>52</td>
</tr>
<tr>
<td>Inventory Cost</td>
<td>65.667</td>
<td>69.750</td>
<td>62.75</td>
</tr>
<tr>
<td>Changeover Cost</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Total Cost</td>
<td>217.667</td>
<td>221.750</td>
<td>214.75</td>
</tr>
</tbody>
</table>

![Figure 3: Unloading schedule of Vessel 1 (blue) and Vessel 2 (orange) for Proposed MILP Model](image-url)
The loading and unloading process of Storage Tank 1 is described by Figure 4, where is possible to highlight certain differences between the schedules of the MILP models. The proposed model predicts a single switch from Crude Oil Type 0 to Crude Oil Type 1 for this tank while the previous model does not predict the storing of more than one crude oil type over the entire horizon for the base case analyzed. Clearly, in the proposed model both vessels take part in the loading process of Storage Tank 1. Because the mixture of two crude oils is not allowed, this is done during the time intervals that the storage tank is fully empty and the switch to the other crude oil type can be implemented. Another important point to remark about the differences between the two models is regarding the levels of crude oil in the tank. The level of crude oil in Storage Tank 1 by the proposed model is lower than the one predicted by the previous model. Since variations in the amounts of crude oil stored in the storage tank have a direct effect in the inventory cost and considering the lower level predicted for this tank, the higher inventory cost obtained with the proposed model has to be related to a high level of crude in any of the other entities. In conclusion, the difference between the two models regarding Storage Tank 1 does not only reside in the types of crude oil stored in the tank but also in the total amount of those crudes.

Figure 4: Charging Schedule of Storage Tank 1 for both models
Figure 5 describes the variation of volume in Storage Tank 2, where some differences can be observed when comparing the schedules generated by the two models. The number of transitions from one crude oil type to another in the proposed model is even higher for this particular tank. The initial charge of Crude Oil Type 1 is totally unloaded during the first days, and the tank is loaded again with Crude Oil Type 0 provided by Vessel 1. Finally, this crude oil is discharged, and then Vessel 2 takes part in the process of reloading Storage Tank 2 with Crude Oil Type 1 again. Finally, the tank send its entire cargo of Crude Oil Type 1 to the distillation unit in the last day of the schedule horizon. Basically, the type of crude oil in the tank is switched two times in the same schedule horizon while the previous model predicts the storing of the same crude oil type during the entire schedule. Regarding the crude oil profiles, both models present similar levels until the beginning of the fourth day. Nonetheless, an important difference exists from the day 4 until the last day of the schedule horizon. The proposed model predicts a switch to Crude Oil Type 0, which involves an increase of the level during the fourth day and a decrease of the level during the fifth day. The other switch is produced at the beginning of the seventh day with an increase of the level and then, a decrease during the last day. Although the level of the proposed model is higher in certain periods, this could be compensated by the higher level predicted by the Lee et al. model for the last periods of the schedule horizon. Clearly, the levels predicted for this tank is not as determining the difference observed in the inventory costs. As in the case of Storage Tank 1, the proposed model and the previous model does not only differs in the types of crude oil stored but also in the profiles predicted for this specific tank.
The variation of level of each type of crude oil in Blending Tank 1 throughout the schedule horizon is depicted by Figure 6. There is a decrease of each crude oil type until the second day due to the movement of this crude oil blend to the distillation column. Then, the blending tank is loaded again with Crude Oil Type 0 and Crude Oil Type 1 until the fifth day with some periods of level stability for both crude oils. It is clear that during those days the two crude oils in the blending tank maintain a level outside of the specific concentrations percentages (blend is out of specification), and there is not possibility to feed the distillation unit. This deviation from the specification lasts three days, meaning that most of the time this tank is in the appropriate range of concentrations for each crude oil and feeds the distillation column. Finally, the level of the two crude oils in the blending tank diminishes constantly from the day 5 until the last day of the schedule horizon, which is the time period that the blending tank unload the required blend to the distillation unit.
Figure 6: Charging Schedule of Blending Tank 1

Figure 7 shows the variation of the level of each crude oil in Blending Tank 2 through the schedule horizon. Until the second day there is an increase of the amount of Crude Oil Type 1 while Crude Oil Type 0 stays stable. Then, from the second to the fifth day there is a constant decrease of both crude oil types until completely discharge the tank. During this period of three days the blend generated in Blending Tank 2 has the minimum percentage of Crude Oil Type 0 and maximum percentage of Crude Oil Type 1. Although the blend is in the limit of concentration, it still satisfies the requirements of the distillation unit and is unloaded. The level of Crude Oil Type 0 is recovered during the fifth day, but the amount of Crude Oil Type 1 remains insufficient for three more days. This fact makes impossible the unloading of the blend during that period of time, and it remains in the blending tank. The recovering of Crude Oil Type 1 only starts during the last day of the schedule horizon without achieving the necessary level to make possible the feed of the distillation unit. This specific tank only takes part of the unloading process for three days; however two of these three days represent the highest charges of crude oil in the distillation unit.
Figure 7: Charging Schedule of Blending Tank 2

Figure 8 illustrates the level profiles for the blending tanks of both models. Comparing the profiles generated we can see again some differences in the total amount of crude oil stored by Blending Tank 2 (Figure 8b) in each interval of the schedule horizon. During the first days the inventory profiles generated by both models are almost the same. However, from the fifth to the last day of the schedule horizon the proposed model predicts the presence of a much higher level of total crude oil in this tank (higher inventory costs). Similarly, higher level of crude oil in Blending Tank 1 (Figure 8a) can also be observed although in a less scale. These higher levels in the blending tanks are responsible for the higher inventory cost predicted by the proposed model. A possible explanation for this difference may reside in some extra operation rules that prohibit certain dangerous maneuvers. Among the different operation rules mentioned in Section 3, the impossibility to load and unload a storage tank at the same time and load a blending tank from different storage tanks simultaneously may have a significant role in the level profiles observed in the different tanks. To satisfy the total demand of the distillation unit at the end of the schedule horizon with a lower number of operations available, significant amounts of crude oil are
transferred from the storage tanks to the blending tanks to compensate this limitation. As mentioned before, a blending tank in the proposed model does have the flexibility of storing crude oil blends out of specification when it is not connected to the distillation column. Consequently, it can receive as much as possible of the storage tanks inventory to compensate the lack of operations and satisfy the total demand. This could clarify not only the high level of crude oil in this type of tanks but also the presence of blends out of specification in some intervals (see Figures 6 and 7). Thus, the model compensates the inability of performing more operations simultaneously through an increase of the inventory in the blending tanks.

![Figure 8: Total Crude Oil Inventory Profiles of Blending Tanks for Lee et al.\textsuperscript{8} MILP Model (blue) and Proposed MILP Model (red). (A) Inventory Profiles of Blending Tanks 1. (B) Inventory Profiles of Blending Tank 2](image)

Although the capability of the proposed model of absorbing part of the limitations imposed by the operation constraints 5 and 6 through the possibility of generating blends out of specification, the model still generates a high inventory level that impacts negatively in the total cost. As observed previously, a relaxation of these constraints leads to a lower inventory level among the entities in the supply chain, which has a direct impact in the total cost obtained. Obviously, these modifications lead to a totally new schedule for the case studied, which is not
shown in this research. However, this relaxation assumes a condition of operation that is not always possible in a refinery. Consequently, the utilization of the original constraints, even generating a higher penalty, leads to more conservative (safer) operations in the plant and are the most appropriate.

There is a variation in the charges of each crude oil type to the distillation column in the planning horizon (Figure 9). During the first day the column receives a blend where the major component is Crude Oil Type 0 from Blending Tank 1. During the second, third, and fourth days the proportion of crude oils is inverted, and Crude Oil Type 1 turns into the largest component charged to the distillation unit. This is the period that Blending Tank 2 is satisfying the required specifications of concentrations and feeding the distillation unit. During the fifth day and throughout the rest of the schedule horizon, Blending Tank 1 feeds the distillation unit. In this period, the charge is only significant during the day 5. The same pattern of feed is observed during the days 6, 7, and 8, however, the charges during those days are not as important as in other intervals. Clearly the pattern of unloading is not maintained constant, which is related to the fact that the demand of the distillation column is not the same in each interval and fluctuates.

Figure 9: Charges of each crude oil type in the distillation column for Proposed MILP Model
Analyzing the charges to the distillation columns of both MILP models (Figure 10), we can see that the distribution of charges of crude oil is similar, except during the intervals 4 and 5. Remembering that the concentration of each type of crude oil is related to the concentration of sulfur, the concentrations predicted by our model is totally different for interval 4. Crude oil is unloaded from Blending Tank 2 instead of Blending Tank 1. In other words, a higher amount of Crude Oil Type 1 than Crude Oil Type 0 is predicted, which means that the concentration of sulfur is in the range 0.045-0.055 (see explanation of the relation between crude oil flows and concentration of sulfur in Chapter 2). This represents one of the main differences between the two models. The situation in interval 5 is completely different, but the difference does not reside in the concentration and is only related to the total amount sent to the distillation column. The proposed model generates a much higher charge of total crude oil to the distillation column in this specific time interval. Therefore, relevant discrepancies in the charges to the distillation column can be observed only in two intervals thus confirming what was mentioned before; the total amounts that are unloaded from each blending tank in the schedule horizon are comparable.

Figure 10: Charges of total crude oil in the distillation column for Lee et al.\textsuperscript{8} MILP Model (right column in each time interval) and Proposed MILP Model (left column in each time interval). Black bars represent the charges of crude from Blending Tank 1. Red bars are the charges of crude from Blending Tank 2
3.3 Case Study 2: Use of a Single Percentage Range and Setup Costs

Another important feature of the proposed model is the possibility of considering a single percentage range of each crude oil type in the blending tanks and the utilization of a setup cost instead of the changeover cost. In order to consider this feature, equations (16), (17), (18), (35), (36), (37), (38), and (40) instead of (19), (20), (21), (33), (34), and (39) are now included in the model.

The base case is slightly modified in order to implement the mentioned modifications in the model for the new approach. None of the percentages ranges utilized in the previous section could be implemented for the single range because one of the crude oil would have an extremely low demand and the other an extremely high demand in the entire schedule horizon. So, the same percentage ranges utilized by Robertson et al.\textsuperscript{21}, which represent a less extreme situation for the demands, are selected. A minimum ($a_{\text{min}ij,k}$) and maximum ($a_{\text{max}ij,k}$) percentages of 0.335 and 0.390 for Crude Oil Type 0 and a minimum ($a_{\text{min}ij,k}$) and maximum ($a_{\text{max}ij,k}$) percentages of 0.610 and 0.665 for Crude Oil Type 1 are considered respectively. The rest of the conditions for the case study are described in the previous chapter.

Table 2 shows the results obtained through the implementation of the model with the mentioned modifications. Clearly, higher unloading + sea waiting as well as inventory costs are obtained when comparing to Case Study 1. As it will be explained later, the demand of each type of crude oil from the distillation column, which is the consequence of the single percentage ranges selected, play a significant role in the vessels’ unloading and level profiles observed principally in the storage tanks.
Table 2. Cost obtained for Proposed MILP Model

<table>
<thead>
<tr>
<th>Costs Involved (k$)</th>
<th>Proposed MILP Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unloading + Sea Waiting Cost</td>
<td>60</td>
</tr>
<tr>
<td>Inventory Cost</td>
<td>93.37</td>
</tr>
<tr>
<td>Setup Cost</td>
<td>450</td>
</tr>
<tr>
<td>Total Cost</td>
<td>603.37</td>
</tr>
</tbody>
</table>

Besides the importance of the deviations in certain logistical costs such as the sea waiting + unloading and inventory costs, the main difference in the total cost obtained is associated to the higher setup cost generated by the proposed model. This is comprehensible since the inclusion of the setup cost instead of changeover cost, which penalizes each connection, has an important impact in the results obtained but yet again providing a more realistic total cost.

The unloading schedule of the vessels involved in the discharge process of the different crude oils is described in the Figure 11. Vessel 1 starts to unload the third day, remains the fourth day in the dock station without unloading its cargo and begins to unload in the fifth day until the rest of the cargo of Crude Oil Type 0 is discharged. The day that Vessel 1 stays without loading Storage Tank 1 is penalized as another unloading day. This is a direct consequence of the lower demand of Crude Oil Type 0 in the distillation column, which leads to higher levels of this crude throughout the different entities of the supply chain. In some entities the level could even reach the maximum capacity if the entire cargo of the vessel was unloaded. Thus, the combination of a low demand and high level of Crude Oil Type 0 can explain this extra day of the vessel in the harbor. In the case of Vessel 2, the unloading of Crude Oil Type 1 to Storage tank 2 starts the seventh day and finishes in the last day. We can see that both vessels spend the same amount of time in the sea before starting the unloading process and they have the same relevance in the sea waiting + unloading cost regarding to the time that they have to wait in the sea. Clearly, the
unloading schedule of the first vessel has a larger weight in the sea waiting + unloading cost determined.

![Figure 11: Unloading schedule for Vessel 1 (Blue) and Vessel 2 (orange) for Proposed MILP Model](image)

Figures 12 and 13 depict the variation of level of the storage tanks throughout the schedule horizon. In the case of Storage Tank 1, the profile spends most of the time in stable levels (without loading or unloading of cargo), and there are no switches from one type of crude oil to another in any time interval. In other words, Storage Tank 1 maintains the same type of crude oil during the entire schedule horizon. A possible explanation to this fact is the demand of Crude Oil Type 0. In fact, observing the percentage range for the crude oil used, it is clear that this type of crude is required in a lower proportion by the distillation column. Furthermore, the tank is never completely emptied in any of the intervals of the schedule horizon, which confirms the impossibility to switch to another crude oil type.

For Storage Tank 2 the level of Crude Oil Type 1 decreases during the second, third, and sixth days (tank feeds the blending tanks). After achieving the minimum level at the end of the sixth day, the tank is loaded again until the day 8 with the same type of crude. Again, no switches
of crude oils are implemented in any of the intervals of the schedule horizon. An interesting point to remark is that Storage Tank 2 spends more intervals of time connected to the blending tanks than Storage Tank 1. This is again related to the higher demand of Crude Oil Type 1 from the distillation column throughout the schedule horizon. Clearly, the demand of each type of crude oil plays a significant role in the level profiles observed in each storage tank.

Figure 12: Charging Schedule of Storage Tank 1 for Proposed MILP Model

Figure 13: Charging Schedule of Storage Tank 2 for Proposed MILP Model
The variation of inventory in the blending tanks throughout the time horizon is shown in the Figures 14 and 15. There is a decrease of the level of Crude Oil Type 0 and Crude Oil Type 1 in Blending Tank 2 during the first day. Blending Tank 1 maintains a stable level, which reveals that it is not loaded from any of the storage tanks or unloaded to the distillation unit in that period of time. From the second until the sixth day, the level of Blending Tank 1 decreases constantly to satisfy the demand of the distillation column. However, Blending Tank 2 is loaded again during that period, which allows to generate the appropriate blend (with the correct percentages of each crude oil) to be unloaded from the sixth day until the last day of the schedule horizon while the other tank is maintained with only one type of crude oil (out of specification). Clearly, the participation of Blending Tank 2 is more relevant in the satisfaction of the total demand of the distillation column.

Figure 14: Charging Schedule of Blending Tank 1 for Proposed MILP Model
Figure 15: Charging Schedule of Blending Tank 2 for Proposed MILP Model

Regarding the distribution of charges to the distillation column in Figure 16, during days 1, 2, 6, and 7 the blending tanks unload the highest charges of crude oil. Of these major charges, three are unloaded from Blending Tank 2 (days 1, 6, and 7), and only one corresponds to Blending Tank 1 (day 2). The other charges (during the days 3, 4, 5, and 8) are comparatively much lower than the others, thus confirming the substantial role of Blending Tank 2 in the process of feeding the distillation column. Another important aspect to be mentioned is the percentages of each crude oil in each charge. It is clear that the proportions are always in the same range throughout the schedule horizon because a single minimum and maximum percentage for each crude oil is used (one lower than the other), which eliminates the possibility of any type of inversion in the proportions of the crude oils that are sent to the distillation column. Consequently, the importance of the participation of Blending Tank 2 in the feeding of the distillation unit and the uniform proportion of the crude oils in each charge are the most substantial aspects noted in Figure 15.
Figure 16: Charges of each crude oil type in the distillation column for Proposed MILP Model

To describe more clearly the distribution of the charges, Figure 17 shows the flows to the distillation column in terms of total crude oil and the corresponding blending tank. It is again possible to see again the main role of Blending Tank 2 in the charging process of the distillation column.

Figure 17: Charges of total crude oil in the distillation column for Proposed MILP Model. Black bars represent the charges of crude from Blending Tank 1. Red bars are the charges of crude from Blending Tank 2
3.4 Case Study 3: Integration with Operational Level

The integration of the proposed MILP model with the lower operational level is performed following the same approach used by Robertson et al.\textsuperscript{21} The operational level is linearized through the implementation of multiple regressions around single percentage ranges of each crude oil type. Then, the linear function obtained is embedded in the objective function of the upper level. A main difference between the model proposed in this chapter and the one developed previously is the configuration of tanks considered throughout the supply chain. The previous model only considers storage tanks and a manifold to perform the blending of the crude oils, but our model takes into account not only storage tanks but also blending tanks. This could have an important impact in the inventory level and number of setups determined in each model, which has a direct impact in the total cost obtained. It is also worth to mention another difference related to the size of the intervals implemented in each model. The proposed model is based on days (large intervals) and the previous one on hours (small intervals). A large interval offers a low computational effort, but also a lower resolution for the schedule obtained. On the other hand, a small interval offers a higher resolution related to the schedule obtained, but it implies an enormous computational effort to find the definitive solution. Due to the higher complexity of the proposed model for the upper level, it is preferred to maintain the interval size based on days. Of course, this difference can also have certain impact in the schedule determined by each model.

As a first attempt to analyze the capability of integration of the proposed model, the results obtained by Robertson et al.\textsuperscript{21} for the lower operational level are utilized to generate the linear expression of costs in terms of the flows of each crude oil. The final aim is to observe whether the proposed model, when integrated and under the same conditions, offers the same capability of
generating a more intelligent schedule of movements of crude oil with a direct impact in the total cost.

The expression of the linear function for the operational level is thus given as follows:

\[
\text{Operational Costs} = \sum_{j} \sum_{k} \sum_{t} \sum_{n,k,j,t} C_j \times FBD_{n,k,j,t} + A
\]  \hspace{1cm} (41)

where \(C_j\) is the cost coefficient of each crude oil \(j\) and \(A\) is a shifting factor that adjusts the total value to the correct order of magnitude.

After applying the multiple regression, the cost coefficients for each crude oil type and the shifting factor obtained are:

\(C_0 = -5.635\) for Crude Oil Type 0

\(C_1 = 1.963\) for Crude Oil Type 1

\(A = 3610000\) shifting factor corrected

It is worth to mention that the flows are affected by a factor to express the values of flow in m\(^3\) per day instead of m\(^3\) per hour when the operational cost function is incorporated in the upper level.

Finally, the objective function to be minimized in the globally integrated model is now given as follows:

\[
\text{Total Cost} = \text{Logistical Costs} + \text{Operational Costs}
\]  \hspace{1cm} (42)

Table 3 depicts the results obtained with and without integration between the CSP and the lower level production optimization when is implemented the proposed MILP model and the one developed by Robertson et al.\(^{21}\) for the upper level.
Table 3. Comparison between Proposed MILP Model and Robertson et al.\textsuperscript{21} Model with integration and without integration with the production level

<table>
<thead>
<tr>
<th>Objective</th>
<th>Proposed MILP Model</th>
<th>Robertson et al.\textsuperscript{21} Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Logistical Cost (K$)</td>
<td>Total Cost (K$)</td>
</tr>
<tr>
<td>Min(logistical cost)</td>
<td>154.25</td>
<td>441.21</td>
</tr>
<tr>
<td>Min(total cost)</td>
<td>157.16</td>
<td>267.49</td>
</tr>
<tr>
<td>Difference</td>
<td>-2.91</td>
<td>173.72</td>
</tr>
</tbody>
</table>

Observing the results obtained through the integration of the different levels, clearly there is an interesting trade-off between the logistical and the total cost. Certain increase in the logistical cost is produced while the total cost, which involves the combination of logistical and operational costs, suffers an important decrease. Basically, the optimizer accommodates the movements of crude oils in the upper level in such way that generates the appropriate charges of each crude oil in the distillation column throughout the schedule horizon. This implies a slightly penalization in the logistical costs due to an increase of the inventory in the tanks, but an enormous reduction of the costs associated to the operation. Of course, the decrease of operational costs is reflected in the lower total cost determined. This substantial reduction of costs is related to a strong linear relationship between operational costs and the amount of Crude Oil Type 0 (the lightest of the crude oils) processed in the distillation unit, which is observed in the negatively high value determined for its cost coefficient. Robertson et al.\textsuperscript{21} explain how the energy costs involved in the operation of furnaces, condensers, and columns among other units decrease when a higher proportion of the lighter crude is fed in the distillation unit. Clearly, the higher quality of this crude has an important role in the decrease of the energy consumption to generate the different types of cuts required by the market. Moreover, this quality has also certain positive impact in the amount of undesirable products or pollutants generated, which also has associated certain cost for the
operation. Thus, the type of crude oil feed in the distillation unit has an essential role in the efficiency of the refinery, which also has an important influence in the generation of the most appropriate schedule when the model is globally integrated. This is also noticed in Figure 18, which depicts the charges of Crude Oil Type 0 in the distillation column in each interval.

![Figure 18: Charges of Crude Oil Type 0 in the distillation column for Proposed MILP Model non-integrated and integrated with the production level](image)

The amount of this crude charged in the first interval is higher for the non-integrated model, but the situation is inverted in the last period of time. The last amount of Crude Oil Type 0 that is sent to the distillation unit is slightly higher, but enough to generate an enormous impact in the total cost when the two layers are integrated.

A comparison with the approach developed by Robertson et al.\textsuperscript{21} confirms that the integration of the proposed model with the production level produces an even larger decrease of the total costs. As mentioned previously, the proposed configuration of entities in the supply chain, which involves different types of tanks, produce a significant improvement in the total cost. Most of the deviations generated due to the new schedule proposed (taking into account the production costs), are absorbed by a slight correction of the level in the tanks. This higher level in the tanks is
a penalty generated by the new schedule that can be easily paid by the model if it is compared to the enormous improvement offered in terms of operational and total costs. In Table 4 can be observed the variation of the inventory cost due to the correction in the levels of the tanks to generate the most appropriate schedule for the case studied.

Table 4: Logistical costs for Proposed MILP Model non-integrated and integrated with the production level

<table>
<thead>
<tr>
<th>Logistical Costs (K$)</th>
<th>Non-Integrated Model</th>
<th>Integrated Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea Waiting + Unloading</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Inventory</td>
<td>33.34</td>
<td>36.16</td>
</tr>
<tr>
<td>Setup</td>
<td>105</td>
<td>105</td>
</tr>
<tr>
<td>Total</td>
<td>154.34</td>
<td>157.16</td>
</tr>
</tbody>
</table>

3.5 Summary

Throughout this chapter the proposed MILP model for the CSP was analyzed and confronted with a previous model to determine its capabilities for the same case study. It was also analyzed the possibility of relaxing certain operation rules to observe its effect over the total cost obtained. Modifications were proposed including a single percentage range of concentrations of the crude oils and the incorporation of all the possible costs involved in the establishment of connections among others. After implementing the mentioned variations in the model, the total cost increased as a result of a setup cost that penalizes each connection throughout the supply chain. Finally, the proposed model with the single percentage range and setup costs was used in a first attempt of integration with a lower operational level. A positive effect of that integration was observed in the total cost obtained due to an inherent trade-off between the logistical and the total costs. A significant decrease of the total costs was obtained through a slight increase of the inventory of the different entities in the supply chain. The model accommodated the inventories in
the upper level in order to favor a higher feed of the lightest crude oil, which involved less operational costs.
4. MODELING THE PRESENCE OF UNCERTAINTY OR FUZZINESS IN THE CRUDE OIL SCHEDULING PROBLEM

4.1. Literature Review

The determination of the most appropriate model to represent the unloading, charging, blending, and movements of crude oil in a refinery involves an extremely complicated challenge. A refinery is a system constituted of docks, pipelines, a series of tanks to store the different crude oils and prepare the different types of blends, distillation units, production units, blenders and tanks to store the raw materials and final products. Of course, the number and complexity of the operations required to perform the movements of material between the different units involved in these types of systems are colossal. Thus, any attempt to describe these operations represents an arduous and complicated task.

There is no model that can cover all the variations of such complex problem of generating the optimal schedule for the allocations of crude oil in a refinery. Moreover, in most of the cases the applicability of a model is restricted to certain specific conditions or case studies. Among the different models developed to describe the CSP, a particular characteristic in most of them is the utilization of a deterministic approach. This means that there is no uncertainty present in the different equations and constraints that describe the problem of study, which offers the possibility of a more elegant and less complex resolution. In this group of deterministic models are included all those mentioned in Chapter 2. Therefore, it is necessary to expand the formulation of our deterministic model to represent the inherent imprecision reflected in the absence of sharp boundaries or exactness in certain data.

Constraints or goals may not be defined properly due to ill-defined and subjective requirements based on human judgments or preferences. A way to face the uncertainty present in an optimization problem is through the implementation of the stochastic approach. Diwekar and
Rubin\textsuperscript{23} incorporate the stochastic capability to the Aspen chemical process simulator to evaluate the performance of a chemical plant with the presence of uncertainty. Pistikopoulos and Ierapetritou\textsuperscript{24} utilize a stochastic approach for optimal process design involving ill-defined parameters. The problem is formulated as a two-stage stochastic model where the process uncertain parameters are described through continuous probability distribution functions. Acevedo and Pistikopoulos\textsuperscript{25} present a mixed integer stochastic optimization based algorithms and computational studies for the solution of process synthesis involving uncertainty. They also implement an optimization framework based on a two-stage stochastic formulation for the resolution of the problem with uncertain parameters. Geraili and co-workers\textsuperscript{26} utilize the stochastic approach for the optimization of biorefineries involving uncertainty in prices and demands of products. In this work is applied a distributed strategy composed of different layers including strategic optimization, risk management, detailed mechanistic modelling, and operational level optimization. A multi-objective stochastic optimization approach is utilized to incorporate the trade-offs between the expected cost and the financial risk involved in the process and then the process is simulated in Aspen Plus.

Although it is possible to treat this vagueness of information through the implementation of stochastic approaches, the fuzzy concepts introduced firstly by Zadeh\textsuperscript{27} offer another powerful way to deal with this type of problems without the necessity of statistical data. Zimmermann\textsuperscript{28} present an interesting approach to solve fuzzy linear problems with soft constraints, which means that the uncertainty is located on the right hand side of certain constraints of the system analyzed. This uncertainty is described by a fuzzy set with an interval support and a membership function. This treatment of linear fuzzy systems, often called flexible programming, is also discussed by Rommelfanger\textsuperscript{29}, who explains the implementation of this fuzzy linear programming method.
among others to solve different types of optimization problems. This flexible programming approach was expanded by Rommelfanger et al.\textsuperscript{30} to make possible the resolution of linear fuzzy problems with the uncertainty presented in the coefficients of the objective functions. Chanas et al.\textsuperscript{31} propose another approach to deal with fuzzy linear problems based on the application of parametric techniques. Another interesting approach is the one presented by Julien\textsuperscript{22}, who develop a possibilistic programming method to solve fuzzy linear problems where the uncertainty is also located in the parameters of the constraints, which means that both sides of the constraints have certain imprecision. The solution represent an extension to the one determined by Buckley\textsuperscript{32}, who also propose a simple and useful method to solve linear programming problems with uncertain parameters. Liu and Sahinidis\textsuperscript{33} implement fuzzy linear programing to model the uncertainty in a typical problem of process planning. They implement both the flexible and possibilistic approaches to solve a long-range problem for a chemical process involving a network of chemical processes. This represent an interesting utilization of the fuzzy linear programming concepts to real life problems.

Although the variety of implementations of the fuzzy approach to solve different types of problems in presence of uncertainty, the first and only treatment of the CSP considering the possibility of fuzziness in some constraints is performed by Cao et al.\textsuperscript{34} They implemented chance constrained fuzzy programming to eliminate the fuzziness of the system, and the crisp equivalents of the fuzzy chance constraints are utilized to solve two different cases. A different approach is presented in this chapter to eliminate the uncertainty of the constraints. The model for the CSP implemented is based on the one described in Chapter 2, with a single percentage of the crude oils and the consideration of the setup costs. It is worth to mention that the only difference with the mentioned model is the incorporation of a maximum and minimum demand instead of the
utilization of a fixed value for the entire schedule horizon. Obviously, this offers a higher flexibility to the model in order to determine the most appropriate schedule horizon. Two specific cases of fuzziness in the system are considered including fuzziness in the minimum demand to be satisfied in the distillation unit as well as fuzziness present in certain costs involved in the required maneuvers to allocate the different types of crude oil throughout the supply chain. The flexible programming method mentioned previously is used to describe these two cases of uncertainty, which allows maintaining the linearity and simple resolution of the system. For each case, it will be analyzed the differences between the schedule of the crude oil allocations and the total costs determined in each fuzzy case and the ones obtained through the utilization of the crisp or deterministic model. The aim is to perform a study of the impact of the uncertainty in the decision making process of a scheduler or decision maker in a refinery.

4.2. Background Model for the Crude Oil Scheduling Problem with Uncertainty

The system analyzed is a supply chain composed of different entities including the vessels that transport crude oil from different sources to the docking station of the refinery, the tanks destined to store the different types of crude oil, the tanks where the different blends are generated according to the requirements of the distillation unit, and finally the complex distillation system to produce the cuts required by the local or international market. Of course, all these entities are connected through the pipeline network, which involves complex and often costly operations. This system is the as the one described by Figure 2.

Recalling the general mathematical formulation described in Chapter 2, with the considerations mentioned previously, we have the following:

\[ \sum_{t} UIBV_{v,t} = 1, \forall v \]  

(43)
\[ \sum_{t} VLBV_{v,t} = 1, \quad \forall v \quad (44) \]

\[ TVA_{v} \leq TUIB_{v}, \quad \forall v \quad (45) \]

\[ TUIB_{v+1} \geq TVL_{v} + 1, \quad \forall v \quad (46) \]

\[ \sum_{i} VSCBV_{v,i,t} \leq \sum_{t' \leq t} UIBV_{v,t'}, \quad \forall v,t \quad (47) \]

\[ \sum_{i} VSCBV_{v,i,t} \leq \sum_{t' \geq t} VLBV_{v,t'}, \quad \forall v,t \quad (48) \]

\[ TUIB_{v} = \sum_{t} t \cdot UIBV_{v,t}, \quad \forall v \quad (49) \]

\[ TVL_{v} = \sum_{t} t \cdot VLBV_{v,t}, \quad \forall v \quad (50) \]

\[ TVL_{v} - TUIB_{v} \geq NPU \quad \forall v \quad (51) \]

\[ \sum_{v} VSCBV_{v,i,t} + \sum_{n} SBCBV_{i,n,t} \leq 1 \quad \forall i,t \quad (52) \]

\[ \sum_{i} SBCBV_{i,n,t} + \sum_{k} BDCBV_{n,k,t} \leq 1 \quad \forall n,t \quad (53) \]

\[ \sum_{n} BDCBV_{n,k,t} = 1 \quad \forall k,t \quad (54) \]

\[ \sum_{j} STTBV_{i,j,t} \leq 1 \quad \forall i,t \quad (55) \]

\[ VS_{i,j,t} \leq STTBV_{i,j,t} \cdot \text{cap} \quad \forall i,j,t \quad (56) \]

\[ \sum_{j} VB_{n,j,t} \leq \text{cap} \quad \forall n,t \quad (57) \]
\[ VB_{n,j,t} - \sum_{j'} VB_{n,j',t} \cdot \text{amax}_{j,k} \leq M \cdot (1 - \sum_{k} D_{BDBV_{n,k,t}}) \quad \forall n, j, t \]  
(58)

\[ VB_{n,j,t} - \sum_{j'} VB_{n,j',t} \cdot \text{amin}_{j,k} \leq -M \cdot (1 - \sum_{k} D_{BDBV_{n,k,t}}) \quad \forall n, j, t \]  
(59)

\[ \sum_{j} FBD_{n,k,j,t} \cdot \text{amin}_{j,k} \leq FBD_{n,k,j,t} \leq \sum_{j} FBD_{n,k,j,t} \cdot \text{amax}_{j,k} \quad \forall n, j, t \]  
(60)

\[ \sum_{j} FVS_{v,i,j,t} \leq VSCBV_{v,i,t} \cdot F_{v,i,max} \quad \forall v, i, t \]  
(61)

\[ \sum_{j} FSB_{i,n,j,t} \leq SBCBV_{i,n,t} \cdot F_{i,n,max} \quad \forall i, n, t \]  
(62)

\[ BDBV_{n,k,t} \cdot F_{n,k,min} \leq \sum_{j} FBD_{n,k,j,t} \leq BDBV_{n,k,t} \cdot F_{n,k,max} \quad \forall n, k, t \]  
(63)

\[ W_{v,j,t} = V_{Vv,j,t=1} - \sum_{tzt'} \sum_{i} FVS_{v,i,j,t'} \quad \forall v, j, t \]  
(64)

\[ S_{i,j,t} = S_{i,j,t=1} + \sum_{tzt'} \sum_{v} FVS_{v,i,j,t'} - \sum_{tzt'} \sum_{n} FSB_{i,n,j,t'} \quad \forall i, j, t \]  
(65)

\[ VB_{n,j,t} = VB_{n,j,t=1} + \sum_{tzt'} \sum_{l} FSB_{i,n,j,t'} - \sum_{tzt'} \sum_{k} FBD_{n,k,j,t'} \quad \forall n, j, t \]  
(66)

\[ \sum_{i} \sum_{t} FVS_{v,i,j,t} = W_{v,j,t=1} \quad \forall v, j \]  
(67)

\[ VSCBV_{v,i,t-1} + VSCBV_{v,i,t} \geq VSCBV_{v,i,t} \quad \forall v, i, t \]  
(68)

\[ SBCBV_{i,n,t-1} + SBCBV_{i,n,t} \geq SBCBV_{i,n,t} \quad \forall i, n, t \]  
(69)

\[ BDBV_{n,k,t-1} + BDBV_{n,k,t} \geq BDBV_{n,k,t} \quad \forall n, k, t \]  
(70)

\[ \sum_{n} \sum_{j} \sum_{t} FBD_{n,k,j,t} \geq D_{k}^{L} \quad \forall k \]  
(71)

\[ \sum_{n} \sum_{j} \sum_{t} FBD_{n,k,j,t} \leq D_{k}^{U} \quad \forall k \]  
(72)
As mentioned above, minimum and maximum demands are considered in this model, which is observed through the inclusion of Equations (71) and (72). A minimum demand to be satisfied in the distillation unit is related to the requirements of the market. Clearly, the distillation unit has to be capable of minimally generating the required amount of the different cuts of crude oil. A maximum demand to be satisfied is associated to the capacity of the distillation unit and the appropriate operation condition. Normally, the condition of maximum production of a unit is below the design capacity. The aim of the maximum demand is to describe that operation condition where the unit can operate without any type of inconvenience.

Finally, the objective function to be minimized is the total cost, which is determined through the summation of all the logistical costs as follows:

\[

total\ costs = \text{unloading\ costs} + \text{sea\ waiting\ costs} + \text{inventory\ costs} + \text{setup\ costs}
\]

\[
= C_u \sum_v (TVL_v - TUIB_v) + C_{sw} \sum_v (TUIB_v - TVA_v) + C_{invst} \\
+ \sum_n \sum_{i,t} \frac{(\sum_j VS_{i,j,t} + \sum_j VS_{i,j,t-1})}{2} + C_{invbt} \\
+ \sum_n \sum_{i,t} \frac{(\sum_j VB_{i,j,t} + \sum_j VB_{i,j,t-1})}{2} + C_{Setup1} \\
+ \left( \sum_i \sum_n \sum_{i,t} SBCEBV_{i,n,t} + \sum_v \sum_i \sum_{i,t} VSCEBV_{v,i,t} \right) + C_{Setup2} \\
+ \sum_n \sum_k \sum_{i,t} BDCEBV_{n,k,t}
\]

Equation (73) includes the cost of unloading the cargo of each vessel, the cost related to the time that each vessel has to remain in the sea before entering in the docking station to unload, the cost of maintaining the crude oil in each of the entities of the supply chain, and the cost of establishing each connection between entities. It is worth to remark that not only the inventory
cost varies from an entity to other ($C_{invst}$ and $C_{invbtt}$) but also the cost per connection established between two entities ($C_{Setup1}$ and $C_{Setup2}$). This is related to the complexity involved in each unit and the connections established between them.

4.3 Flexible Programming Approach for the Treatment of the Uncertainty

The linear model presented in the previous section, as other models developed for the treatment of the CSP, is completely deterministic. This means that there is no room for any type of uncertainty, which makes the resolution of the system simple. However, in real life systems there is a lack of precise information that often makes unable the assumption of a perfectly crisp model. Coefficients or parameters in some constraints cannot be defined with accuracy, which often leads to the selection of levels instead of an exact value. This means that the scheduler or decision maker is allowed to select subjectively certain aspiration levels to restrict the inherent uncertainty of the system. The utilization of these levels to circumvent the problem of the presence of fuzziness in the system reflects the vagueness of the decision maker. As mentioned previously, it is considered two different cases where certain fuzziness can be present in some parameters or coefficients of the MILP model given in the previous section. First, the possibility of uncertainty in the minimum demand to be satisfied in the distillation unit, which could be the consequence of the absence of precise information about the demand of the different types of crude oil cuts required by the local or international market. For example, let suppose that there is a lack of precise statistical data of a specific market, this could expose the decision maker to the situation of selecting at least a range or levels for this parameter. Of course, the level of knowledge or experience of the decision maker will lead to a smaller range or interval of uncertainty. Second, the case when the knowledge of certain costs involved in the allocations of the different types of crude oil is not accurate. Let suppose that the market is located in a country with certain economic
instability that precludes the determination of exact costs, then the decision maker may have to take the decision of selecting certain levels of costs to circumvent this problem.

The flexible programming approach is implemented for the treatment of the different fuzzy constraints mentioned above. In order to facilitate the understanding of the concepts applied in the problem of study, the different fuzzy cases will be presented and solved for a basic model. In other words, the most important concepts could be depicted without generating an overly extensive mathematical development.

4.3.1 Fuzzy Minimum Demand Constraint

The first case analyzed is the possibility of fuzziness in the minimum demand constraint of the distillation unit. In order to consider this variation in the MILP model presented in Section 2, the equation (26) is replaced by the following one:

\[ \sum_n \sum_j \sum_t FBD_{n,k,j,t} \geq D^L_k \forall k \]

(74)

For this specific case, the mathematical formulation of the model to be solved can be generally expressed as follows:

\[ z(x) = c_1x_1 + c_2x_2 + \cdots + c_n x_n \rightarrow \text{to be minimized} \]

(75)

Subject to

\[ g_i(x) = a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \geq b_i \quad \forall i = 1, \ldots, m_1 \]

(76)

\[ g_i(x) = a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \geq B_i \quad \forall i = 1 + m_1, \ldots, m \]

(77)

\[ x_1, x_2, \ldots, x_n \geq 0 \]

(78)
Even though our particular case involves only one fuzzy constraint, it will be maintained the general expression for a system with $m_1$ fuzzy constraints and $m$ crisp constraints to generate a general resolution for this types of fuzzy optimization problems. Equation (75) describes the objective function to be minimized by the optimizer. Equation (76) and (77) represent all possible fuzzy and crisp constraints of the system, respectively. Finally, equation (78) represents a crisp constraint related to the variables of the system, for example in our model the material flows between the entities have physical limitations associated to the impossibility of taking values lower than 0.

According to Zimmermann\textsuperscript{28}, this fuzzy system can be expressed through the following analogous system:

$$
c_1x_1 + c_2x_2 + \cdots + c_nx_n \leq z_0
$$

subject again to Equations (76), (77), and (78) mentioned above.

In this case the decision maker accepts or tolerates certain violation of the constraints. In other words, as expressed by Delgado et al.\textsuperscript{35}, the decision maker permits the constraints to be satisfied “as well as possible”. Then, the uncertainty $\tilde{\Xi}$ in the constraint can be described with a fuzzy set and a support interval $[b_i - \Delta b_i, b_i]$, $\Delta b_i \geq 0$, and a membership function. According to Liu and Sahinidis\textsuperscript{33}, a linear membership function not only provides an easy way to handle fuzzy programming but also has very good properties in terms of the quality of the solution. Thus, a linear increasing membership function is utilized to represent the individual satisfaction of the decision maker in relation to the constraint.

$$
u_i(g_i) = \begin{cases} 
0 & \text{if } g_i < b_i - \Delta b_i \\
\frac{g_i - b_i}{\Delta b_i} + 1 & \text{if } b_i - \Delta b_i \leq g_i \leq b_i \\
1 & \text{if } g_i > b_i 
\end{cases}
$$
Graphically expressed as follows:

![Figure 19: Linear membership function utilized for the representation of the fuzziness in the constraint](image)

Following the procedure described by Zimmermann\textsuperscript{28}, the right-hand side of the objective function, which is also considered uncertain, is described through a fuzzy set, a support interval and a membership function. In this case, the selection of the upper and lower levels is not left in the hands of the decision maker. The bounds of the interval are obtained through the resolution of the original system, but considering each of the extremes of the support interval selected by the decision maker for each fuzzy constraint. This is summarized as $z_0^- = \{\min z(x) \text{ s.t. } g_i \geq b_i - \Delta b_i\}$ and $z_0^+ = \{\min z(x) \text{ s.t. } g_i \geq b_i\}$. Then, the membership function for the objective function is given as follows:

$$u_0(z) = \begin{cases} 
1 & \text{if } z < z_0^- \\
\frac{z - z_0^-}{z_0^+ - z_0^-} & \text{if } z_0^- \leq z \leq z_0^+ \\
0 & \text{if } z > z_0^+ 
\end{cases}$$

(81)
Graphically given as follows:

Figure 20: Linear membership function utilized for the representation of the fuzziness in the objective function

The necessity of a compromise solution is expressed through the utilization of the minimum operator as follows:

$$\lambda(x) = \min(u_0, u_1, ..., u_m)$$  \hspace{1cm} (82)

The parameter \( \lambda \) can be described as the total satisfaction of the decision maker.\(^{29}\)

Although certain empirical researches demonstrated that using the minimum operator could be too pessimist for the resolution of these types of problems, the mathematical simplicity involved in this approach is often preferred.

Finally, the resolution of the original fuzzy system is equivalent to the resolution of the following system

$$\lambda \rightarrow \text{to be maximized}$$  \hspace{1cm} (83)

Subject to

$$\lambda \leq u_x = \frac{z^+_0 - z}{z^+_0 - z^-_0}$$  \hspace{1cm} (84)
\[ \lambda \leq u_i = \frac{g_i - b_i}{\Delta b_i} + 1 \quad \forall \ i = 1, ..., m_1 \]  

(85)

where \( \lambda \in (0, 1] \) and \( x_1, x_2, ..., x_n \geq 0 \)

The rest of the crisp constraints are included in this system, which implies that the resolution of the new system generated involves only one more constraint and variable than the original one.

4.3.2 Fuzzy Cost Coefficients of Objective Function

Another possible case to be analyzed is the existence of uncertainty in the costs related to the different operations involved in the allocations of crude oil throughout the supply chain. More specifically, the uncertainty in the inventory cost coefficients of the objective function. This function, which includes certain fuzzy coefficients, is now expressed as follows:

**Total costs = unloading cost + sea waiting cost + inventory cost + setup cost**

\[ = C_u \sum_v (TVL_v - TUIB_v) + C_{sw} \sum_v (TUIB_v - TVA_v) + \bar{C}_{invst} \]

\[ \times \sum_i \sum_t (\sum_j VS_{i,j,t} + \sum_j VS_{i,j,t-1}) + \bar{C}_{invbt} \]

\[ \times \sum_n \sum_t (\sum_j VB_{i,j,t} + \sum_j VB_{i,j,t-1}) + C_{setup1} \]

\[ \times \left( \sum_i \sum_n \sum_t SBCEBV_{i,n,t} + \sum_v \sum_l \sum_t VSCEBV_{v,l,t} \right) + C_{setup2} \]

\[ \times \sum_n \sum_k \sum_t BDCEBV_{n,k,t} \]

Clearly, the presence of fuzziness in the inventory cost coefficients, \( \bar{C}_{invst} \), and \( \bar{C}_{invbt} \), also makes uncertain the objective function to be minimized. Again the decision maker have to take
the decision of selecting certain levels of aspiration for these coefficients in order to describe the mentioned fuzziness.

In the same fashion that in Section 4.3.1, a general system is used to describe the proposed case in order to highlight the concepts taken into account to solve the optimization problem:

\[ z(x) = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + \ldots + \tilde{c}_n x_n \]  \hspace{1cm} (87)

where \( \tilde{c}_1 = [c^L_1, c^U_1] \), \( \tilde{c}_2 = [c^L_2, c^U_2] \), \ldots, \( \tilde{c}_n = [c^L_n, c^U_n] \)

Subject to

\[ a_{i_1} x_1 + a_{i_2} x_2 + \ldots + a_{i_n} x_n \leq b_i \quad \forall i = 1, \ldots, m \]  \hspace{1cm} (88)

\[ x_1, x_2, \ldots, x_n \geq 0 \]  \hspace{1cm} (89)

Rommelfanger et al.\(^{30}\) proposed an approach to reduce the infinitely many objective functions

\[ z(x) = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + \ldots + \tilde{c}_n x_n = \tilde{c} x \rightarrow \text{minimization} \]  \hspace{1cm} (90)

by extreme positioning to the two extreme objective functions \( z_{\text{min}}(x) \rightarrow \text{minimization} \) and \( z_{\text{max}}(x) \rightarrow \text{minimization} \) and finally finding the solution of the following vector optimization system:

\[
\begin{pmatrix} z_{\text{min}}(x) \\ z_{\text{max}}(x) \end{pmatrix} = \begin{pmatrix} c^L x \\ c^U x \end{pmatrix} \rightarrow \text{minimization} \]  \hspace{1cm} (91)

In order to obtain a compromise solution of the mentioned system, it is extended the concept of flexible programming introduced by Zimmermann\(^{28}\) to the resolution of the fuzzy coefficients in the objective function. Following the procedure described by Rommelfanger et al.\(^{30}\), it is first minimized the objective functions for the case of the two extremes, \( z_{\text{min}}^* \) and \( z_{\text{max}}^* \), through the solution of the conventional linear programming problem.
\[ z^*_\text{min} = z_{\text{min}}(x^*_{\text{min}}) = \min_{x \in X} z_{\text{min}}(x) \quad (92) \]

\[ z^*_\text{max} = z_{\text{max}}(x^*_{\text{max}}) = \min_{x \in X} z_{\text{max}}(x) \quad (93) \]

The solution vectors of the system that most diverge are also determined as follows:

\[ \bar{z}_{\text{min}} = z_{\text{min}}(x^*_{\text{max}}) \quad (94) \]

\[ \bar{z}_{\text{max}} = z_{\text{max}}(x^*_{\text{min}}) \quad (95) \]

Then, the decision maker will be able to accept a solution \( x \) which has the properties \( z_{\text{min}}(x) \leq \bar{z}_{\text{min}} \) and \( z_{\text{max}}(x) \leq \bar{z}_{\text{max}} \). Thus, it can be expressed the objective through the utilization of the membership functions \( u_{z_{\text{min}}}(x) \) and \( u_{z_{\text{max}}}(x) \), which reflect the decision maker satisfaction with the attained objective values. As in Section 4.3.1, linear membership functions are selected to maintain the linearity of the system and facilitate its resolution

\[ u_{z_k}(x) = \begin{cases} \frac{\bar{z}_k - z_k(x)}{\bar{z}_k - z_k^*} & \text{if } z_k^* \leq z_k(x) \leq \bar{z}_k \quad k = \text{min, max} \\ 0 & \text{otherwise} \end{cases} \quad (96) \]

Finally, the problem involves the resolution of the following fuzzy vector optimization system:

\[ \begin{pmatrix} u_{z_{\text{min}}}(x) \\ u_{z_{\text{max}}}(x) \end{pmatrix} \rightarrow \text{minimization} \quad (97) \]

This system is subject to the same constraints (88) and (89) of the original system presented in this section and has the same complete solution. The compromise solution of the optimization system presented above is solved again through the utilization of the minimum operator.

\[ \lambda(x) = \min(u_{z_{\text{min}}}, u_{z_{\text{max}}}) \quad (98) \]

As mentioned in the previous section, the variable \( \lambda \) can be interpreted as the total satisfaction on the part of the decision maker, which intention is to improve both objectives as well as possible.
Finally, the equivalent system to be solved is given as

\[ \lambda \rightarrow \text{to be maximized} \quad (99) \]

Subject to

\[ \lambda \leq \frac{\bar{z}_{\text{min}} - z(x)}{\bar{z}_{\text{min}} - z^*} \quad (100) \]

\[ \lambda \leq \frac{\bar{z}_{\text{max}} - z(x)}{\bar{z}_{\text{max}} - z^*} \quad (101) \]

\[ a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \leq b_i \quad \forall \ i = 1, \ldots, m \quad (102) \]

where \( \lambda \in [0,1] \) and \( x_1, x_2, \ldots, x_n \geq 0 \)

This system, as the previous one obtained in Section 4.3.1, is linear and can be solved through the implementation of any of the methods used for the treatment of this type of optimization problems.

4.4 Case Study Definition

The base case implemented in this chapter considers two vessels with a cargo of 90000 m\(^3\) of crude oil each one. Vessel 1 arrives the first day with a cargo of Crude Oil Type 0, while Vessel 2 arrives the fifth day with a cargo of Crude Oil Type 1. Two storage tanks with an initial amount of each crude oil type receive the cargo of the vessels and unload the required charges to the blending tanks. The initial volumes of these tanks are 9400 m\(^3\) of Crude Oil Type 0 in Storage Tank 1 and 42000 m\(^3\) of Crude Oil Type 1 in Storage Tank 2. The two blending tanks, which receive the different crude oil types from the storage tanks and prepare the required blends to be charged in the distillation unit, also have an initial inventory of crude oils (blends). Both Blending Tanks 1 and 2 contain a mixture of 13400 m\(^3\) of Crude Oil Type 0 and 22600 m\(^3\) of Crude Oil Type 1. The charges of the different blends to the distillation unit have to respect certain range percentages of concentration due to certain quality requirements. These percentages are 0.335 to
0.390 for Crude Oil Type 0 and 0.610 to 0.665 for Crude Oil Type 1. These conditions are similar to the ones utilized in the Case Studies 2 and 3 described in Chapter 2. In the first case study is considered the possibility of certain violation of the minimum demand to be satisfied which value, $\Delta D_k$, is 10000 m³. In the second case study, where the uncertainty is located in the objective function, is considered that the support intervals (levels of aspiration) for the inventory cost coefficients, $[C_{invkt}^L, C_{invkt}^U]$ and $[C_{invt}^L, C_{invt}^U]$, are [0.0114, 0.514] and [0.0303, 0.703], respectively. Besides the variations mentioned for each case regarding the location of the fuzziness in the model, the rest of the conditions and parameters are basically the same. More detail about the different model conditions and parameters are depicted in Appendix A (Table 9).
5. RESULTS FOR CRUDE OIL SCHEDULING PROBLEM WITH UNCERTAINTY

5.1 Computational Resolution

For the resolution of the crisp or deterministic model and the modified optimization systems for the different cases of fuzziness analyzed, the modeling system GAMS with CPLEX solver is used in a CPU with a processor Intel Core 7, 16GB of RAM Memory, and a 64-bit Operating System. The resolution of the case of uncertainty in the minimum demand constraint requires the resolution of the crisp model for the lower and upper bounds of the support interval. Each of the systems to be solved involves 644 single equations, 567 single variables, and 252 discrete variables. After solving each of these crisp systems, their solutions are used to solve the desfuzzified model, which includes 647 single equations, 569 single variables, and 252 discrete variables. For the case of fuzziness in the inventory cost coefficients, the optimization system includes the resolution of the crisp model a couple of times (lower and upper bounds of the coefficients) to obtain the conventional or optimal solution and the most divergent values of the objective function. Each of the systems solved includes 645 single equations, 568 single variables, and 252 discrete variables. Then, the results are utilized for the resolution of the desfuzzified model, a system constituted of 650 single equations, 571 single variables, and 252 discrete variables.

5.2 Case Study 1: Comparative Analysis between Crisp Model and Model with Fuzzy Minimum Demand Constraint

The consequence of considering a fuzzy minimum demand constraint can be observed in the different costs obtained (Table 5). Of course, these costs have a correlation with the new schedule generated under the new conditions of fuzziness analyzed. It is worth to mention the important decrease seen in the inventory and setup costs, which has a major consequence in the reduction of the total cost when comparing to the crisp model. The only cost that is increase is the
one corresponding to the time the vessels remain in the sea, which is included in the sea waiting + unloading costs. Because the weight of this cost is low, its increase is preferred and has a minimum effect in the total cost comparing to the substantial effects of the others costs. The maximum value obtained for the total satisfaction parameter $\lambda$ is 0.312.

Table 5: Comparison between crisp or deterministic model and model with fuzzy minimum demand constraint

<table>
<thead>
<tr>
<th>Costs Involved (k$)</th>
<th>Crisp Model</th>
<th>Fuzzy Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unloading + Sea Waiting</td>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>Inventory</td>
<td>58.27</td>
<td>48.02</td>
</tr>
<tr>
<td>Setup</td>
<td>95</td>
<td>70</td>
</tr>
<tr>
<td>Total</td>
<td>169.28</td>
<td>154.02</td>
</tr>
</tbody>
</table>

The flexible programming approach implemented in this chapter to describe the fuzziness of the system allows certain degree of violation of the demand constraint. This violation of the constraint implies the possibility that the optimizer takes into account even lower values of demand to find the most adequate schedule for the movements of crude oil. This is confirmed by the total amount that is charged in the distillation unit during the schedule horizon. At the end of the schedule, the total amount of crude oil received by the distillation unit is 148852.5 m$^3$ for the crisp model while 140000 m$^3$ for the fuzzy model. Of course, a determination of a lower demand has a significant impact in the different allocations of the crude oil throughout the supply chain.

Figure 21 shows the unloading schedule of the vessels for both the crisp and fuzzy model. In the case of Vessel 1, the cargo of Crude Oil Type 0 (blue) is retained for an extra day instead of being unloaded since the first day like in the crisp model. This extra day of the cargo in the sea allows the reduction of the time that is maintained in Storage Tank 1. In the case of Vessel 2, which has a cargo of Crude Oil Type 1 (orange), the sea waiting time is even higher than the one of Vessel 1 and the same vessel of the crisp model, which allows a longer period of time of low
inventory level in Storage Tank 2. Because the costs involved in maintaining the crude oils in the storage tanks are high, it is cheaper to retain the cargos of crude oil in the vessels for longer periods.

Figures 22 and 23 depict the inventory levels in the storage tanks for the deterministic and fuzzy models. For Storage Tank 1, the effect of retarding the unloading of Crude Oil Type 0 from Vessel 1 is reflected in the decrease of inventory observed during the first interval. The rest of the level profile is exactly the same one observed in the deterministic model. For Storage Tank 2, the decrease of level due to the delayed unloading of the second vessel is appreciated from the beginning of the third day until the seventh day. Although the proposed fuzzy approach does not predict a fully empty Storage Tank 2 like in the crisp model, a longer period of low inventory level can be seen throughout the schedule horizon. In both tanks can be observed that the determination of a lower demand generates a schedule where the inventory levels are in average lower than the ones obtained for the same tanks in the crisp model. Thus, this leads to the lower inventory costs depicted by Table 5.

Figures 24 and 25 show the inventory profiles of the blending tanks determined by the crisp and fuzzy models throughout the schedule horizon. An earlier unloading of Blending Tank 1 is seen in the fuzzy model while the level of Blending Tank 2 is maintained constant during that first day. Although the combination of connections between blending tanks and distillation unit is different from the one of the deterministic model, a similar result is observed in the amount of crude oil unloaded to the distillation unit during that first time interval. The remaining crude oil of Blending Tank 1 is transferred in the subsequent two days, and then it is maintained empty during the rest of the schedule. This is similar in both the deterministic and fuzzy models. Blending Tank 2 is loaded by the storage tanks during the second and third days, generating a much higher inventory level than the one obtained by the crisp model during the same days. The combination
of this higher level together with the prediction of lower requirements of crude oil in the distillation unit allows the accommodation of charges in such way that generates a lower number of connections between the blending tanks and the distillation unit for the case of the fuzzy model and an extremely high charge of crude oil in one of the time intervals. These connections involve highly complicated operations, which implies a higher cost. Clearly, a lower number of setup connections represent a substantial decrease in the costs obtained (see Table 5).

Figure 26 describes the charges of crude oil blends that are sent to the distillation unit from the different blending tanks. As mentioned before, there is an important charge during the first day, which is almost the same for both models, but then the fuzzy model only predicts one more and even more important charge in the fifth day instead of two main more charges like in the crisp model. This higher charge is the consequence of the mentioned combination of higher inventory level in Blending Tank 2 and a reduction in the number of connections. Of course, all of this is again possible due to a lower demand predicted for the distillation unit. Therefore, it is possible to say that the fuzzy approach implemented to describe the uncertainty in the minimum demand constraint affects the demand to be satisfied in the distillation unit and consequently, there is a direct impact in the schedule determined for the different allocations and inventories and the subsequent costs obtained.

Figure 21: Comparative of vessels unloading between crisp model (left) and model with fuzzy minimum demand constraint (right)
Figure 22: Comparative of Storage Tank 1 inventory between crisp model (left) and model with fuzzy minimum demand constraint (right)

Figure 23: Comparative of Storage Tank 2 inventory between crisp model (left) and model with fuzzy minimum demand constraint (right)

Figure 24: Comparative of Blending Tank 1 inventory between crisp model (left) and model with fuzzy minimum demand constraint (right)
5.3 Case Study 2: Comparative Analysis between Crisp Model and Model with Fuzzy Inventory Cost

In the specific case analyzed in this section, the coefficients related to the inventory costs of the different tanks throughout the supply chain are considered fuzzy. In other words, the fuzziness or uncertainty is now located in the objective function of the model. After implementing the flexible programming approach to desfuzzify and solve the optimization problem (see Section 4.3.2), the schedule for the movements of crude oil and inventory in each tank will represent a compromise solution.
The costs obtained through the implementation of the crisp model, considering the lower and upper bounds or aspiration levels of the inventory costs parameters, and finally the compromise system are shown in Table 6. For the optimized fuzzy model the total satisfaction variable obtained is $\lambda = 0.335$.

Table 6: Comparison between the crisp model costs, maximum and minimum optimized costs, and maximum and minimum cost optimized simultaneously

<table>
<thead>
<tr>
<th>Costs Involved (k$)</th>
<th>Crisp Model</th>
<th>Fuzzy Model</th>
<th>Optimized Fuzzy Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum Level</td>
<td>Maximum Level</td>
<td>Minimum Level</td>
</tr>
<tr>
<td>Unloading + Sea Waiting</td>
<td>16</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>Inventory</td>
<td>58.27</td>
<td>27.1</td>
<td>57.97</td>
</tr>
<tr>
<td>Setup</td>
<td>95</td>
<td>100</td>
<td>95</td>
</tr>
<tr>
<td>Total</td>
<td>169.28</td>
<td>143.10</td>
<td>188.97</td>
</tr>
</tbody>
</table>

As observed in the results obtained, the approach used does not provide an exact value of the total cost for the schedule of movements and inventory predicted. Because the schedule is obtained through the simultaneous optimization of the maximum and minimum objective functions (see explanation in Section 4.3.2), the total costs depicted in the third and fourth columns of the table above could be understood as the optimal values of the aspiration levels for the total costs when certain fuzziness is introduced in the inventory cost coefficients.

Observing the different plots generated by the crisp model and the fuzzy model obtained through the implementation of the flexible programming approach extended to fuzzy objective functions, there are some main points that can be highlighted. Although there is no difference between the two models in the unloading schedule of Crude Oil Type 0 (blue) from Vessel 1, the sea waiting time of Vessel 2 is higher in the fuzzy model (Figure 27). Clearly, the optimizer tries again to retain for a higher amount of time the cargo of Crude Oil Type 1 (orange) to maintain as
low as possible the inventory level in Storage Tank 2 (Figure 29). Different is the case of Storage Tank 1, where the optimizer tries to maintain its inventory low by moving its entire cargo of Crude Oil Type 0 to Blending Tank 2 and 1 during the third and fourth days, respectively (Figure 28). Thus, it is possible to see again in this case that the inventory levels of the storage tanks are lower than the ones obtained in the crisp model and even lower than the ones of Section 5.2. The possibility of having higher inventory cost coefficients may be compensated through the unloading of higher amounts of crude oil from the storage tanks to the blending tanks. As mentioned before, the weight of the inventory costs in the storage tanks is higher than the one of the blending tanks, so it is preferred to maintain as much as possible of the crude oil in the blending tanks to avoid the negative effect of a possible increase of the costs coefficients. Having lower inventory cost coefficients does not impact negatively in the total cost, so it is no expected that the optimizer tries to compensate any type of reduction in those coefficients.

Figures 30 and 31 show the inventory levels of the blending tanks throughout the schedule horizon. Both models present the same profile for Blending Tank 1 during the first two days. However, the inventory is much higher in the fuzzy model during the rest of the schedule due to the movement of the entire cargo of Crude Oil Type 0 from Storage Tank 1. Different is the case of Blending Tank 2, which inventory profile is the same that the one obtained in the crisp model. This is because the demand now is not fuzzy anymore, and the optimizer does not have the possibility of searching for other solutions at lower values of demand. The impossibility of violation of the minimum demand constraint restricts the amount of crude oil that is sent to the distillation unit to the same one obtained in the deterministic model. Thus, Blending Tank 2, which is connected most of the time to the distillation unit, does not present any type of deviation in its inventory profile from the one obtained with the crisp model. This is also confirmed by Figure 32,
where are shown the different charges to the distillation unit. Those charges are the same to the ones determined in the deterministic model. Clearly, the uncertainty in the inventory costs does not have any type of effect in the demand to be satisfied in the distillation unit at the end of the schedule horizon, so the distribution of charges remains the same. As mentioned previously, the main effect of the fuzziness in the inventory cost coefficients is observed in the distribution of the allocations between the vessels and storage tanks and between the storage and blending tanks and the resulting inventories.

Figure 27: Comparative of vessels unloading between crisp model (left) and model with fuzzy inventory cost coefficients in objective function (right)

Figure 28: Comparative of Storage Tank 1 inventory between crisp model (left) and model with fuzzy inventory cost coefficients in objective function (right)
Figure 29: Comparative of Storage Tank 2 inventory between crisp model (left) and model with fuzzy inventory cost coefficients in objective function (right)

Figure 30: Comparative of Blending Tank 1 inventory between crisp model (left) and model with fuzzy inventory cost coefficients in objective function (right)

Figure 31: Comparative of Blending Tank 2 inventory between crisp model (left) and model with fuzzy inventory cost coefficients in objective function (right)
Figure 32: Comparative of column distillation charges between crisp model (left) and model with fuzzy inventory cost coefficients in objective function (right)

5.4 Summary

Along this chapter a model was utilized to represent the CSP problem with the possibility of certain uncertainties in some constraints and coefficients. Basically, fuzziness was considered in the minimum demand to be satisfied in the distillation unit, which was described through the utilization of the flexible programming approach in order to modify the original optimization model without affecting the linearity of the system. The possibility of certain violation of the minimum demand constraint allowed the prediction of a lower demand to be satisfied in the distillation column, which had a direct effect in the schedule and subsequent lower costs obtained. The case of fuzziness in certain cost coefficients of the objective function was also analyzed in this chapter. The flexible programming approach was extended to this specific problem, and the optimization model was modified again in order to include the description of the fuzziness in the objective function and maintain the linear formulation of the system at the same time. The presence of uncertainty in the inventory cost coefficients tried to be compensated by the optimizer through the reduction of the inventory levels of the storage tanks, which was related to the higher weight of its costs compared to the ones of the blending tanks in the determination of the total cost. In
both cases, the consideration of the uncertainty allowed the determination of a more intelligent schedule for the allocations of crude oil throughout the supply chain analyzed.
6. CONCLUSIONS AND FUTURE WORK

Nowadays, a company performing is such competitive business as the crude oil refining faces several challenges to maintain their margins of profitability as high as possible. Among the different strategies utilized to increase those margins, the adequate planning and scheduling of the operations involved in the movements of crude oil throughout a refinery may have an essential role as a tool to maximize the benefits of this business. An efficient scheduling of all the parts of an enterprise can eliminate the waste related to unnecessary inventory of material, operations, and generation of products out of specification among others. Thus, scheduling of the unloading, charging, blending, and movements of the crude oils in a refinery is a key to economic gain or loss.

The development of an accurate model to describe the process of unloading and loading of crude oil represents an essential tool for schedulers and decision makers in charge of generating the most appropriate schedule for the plant. Avoiding the development of manual schedules, they are able to determine a more optimal solution to the CSP through the utilization of any of the programs available in the market for the resolution of the appropriate model. Clearly, a model to represent the CSP and obtain the optimal schedule of crude oil is fundamental for the success of any company performing in the crude oil refining business.

Throughout this research a MILP model is proposed with the aim of offering a more comprehensive representation of the short-term CSP and obtain a more adequate schedule of the crude oil in a refinery. The time is discretized in equal time intervals where mass balances, rules operations, and different types of constraints are implemented. Any possible type of nonlinearities are avoided through the utilization of the total flows of each crude oil involved in the supply chain in order to generate a completely linear model. In addition to the possibility of including all of the
costs related to the operations of the upper CSP, the mathematical formulation of the model allows its integration with other decision levels such as the production level of the refinery. Therefore, the model offers a more realistic representation of the CSP when compared with previous models developed for the same problem.

Real life problems involve the presence of inherent uncertainty in data, so it is often looked for a way to describe that fuzziness in the different models formulated. After implementing the MILP model to describe the CSP, a more realistic approach is sought through the consideration of uncertainty or fuzziness in certain constraints and parameters of the model. Basically, the minimum demand constraint and the inventory cost coefficients, analyzed separately, are assumed fuzzy. The description of that fuzziness in the MILP model is accomplished through the utilization of the flexible programming approach, which allows to maintain the linearity of the mathematical formulation. A more realistic and intelligent schedule is obtained for each case analyzed when compared to the crisp or deterministic model.

Future works can involve further analysis of the flexibility of the MILP model proposed through the utilization of other case studies and conditions. In the case of these type of optimization problems many time the applicability and the results obtained are base case dependent, so it could be an interesting approach to study the effects of considering other conditions to see in what extent the model is affected and how the results vary. Additionally, more studies can be performed for the integration of the higher level CSP with the lower level nonlinear production optimization problem through the implementation of a different approach including an iterative strategy for its resolution. For the case that uncertainty or fuzziness is included in the MILP model, the utilization of other approaches for the description of that fuzziness can represent another interesting study. The selection of other approaches to face the cases of fuzziness presented in this research can be
significant to obtain the most adequate mathematical formulation for the allocations of the crude oil. Parametric and possibilistic programming among other approaches can be utilized, and the results analyzed and compared with the aim of finding the best description of the fuzziness for the CSP.
REFERENCES


### APPENDIX A: CONDITIONS AND PARAMETERS FOR CASE STUDIES

Table 7: Different conditions and parameters considered in Case Studies 1 and 2

<table>
<thead>
<tr>
<th>Conditions and parameters</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time horizon</td>
<td>8 days</td>
</tr>
<tr>
<td>Time interval</td>
<td>1 day</td>
</tr>
</tbody>
</table>
| Arrival times of vessels  | Vessel 1: 1\(^{st}\) day  
                           | Vessel 2: 5\(^{th}\) day |
| Crude oil types           | Type 0  
                           | Type 1  |
| Sea waiting cost          | 5000 [US$/day] |
| Unloading cost            | 8000 [US$/day] |
| Inventory cost for storage tanks | 0.008 [US$/((day bbl))] |
| Inventory cost for blending tanks | 0.005 [US$/((day bbl))] |
| Changeover cost           | 50000 [US$] (per switch of crude oil blend) |
| Number of vessels         | 2       |
| Number of storage tanks   | 2       |
| Number of blending tanks  | 2       |
| Number of distillation columns | 1      |
| Initial crude amount in each vessel | Vessel 1: 1000000 bbl Crude Oil Type 0  
                           | Vessel 2: 1000000 bbl Crude Oil Type 1 |
| Initial crude amount in each storage tank | Storage Tank 1: 250000 bbl Crude Oil Type 0  
                           | Storage Tank 2: 750000 bbl Crude Oil Type 1 |
| Initial crude amount in each blending tank | Blending Tank 1: 400000 bbl Crude Oil Type 0, 100000 bbl Crude Oil Type 1  
                           | Blending Tank 2: 100000 bbl Crude Oil Type 0, 400000 bbl Crude Oil Type 1 |
| Capacity of each storage and blending tanks | 1000000 bbl |
| Maximum flow from vessels, storage tanks, and blending tanks | 500000 bbl/day |
| Minimum flow from blending tank | 5000 bbl/day |
| Distillation column total demand | 2000000 bbl |
| Distillation column concentrations required | Connected to Blending Tank 1: 0.7-0.9 Crude Oil Type 0  
                           | Connected to Blending Tank 2: 0.1-0.3 Crude Oil Type 0 |
Table 8: Different conditions and parameters considered in Case Study 3

<table>
<thead>
<tr>
<th>Conditions and Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time horizon</td>
<td>5 days</td>
</tr>
<tr>
<td>Time interval</td>
<td>1 day</td>
</tr>
</tbody>
</table>
| Arrival times of vessels  | Vessel 1: 1<sup>st</sup> day  
                           | Vessel 2: 3<sup>rd</sup> day |
| Crude oil types           | Type 0  
                           | Type 1  |
| Sea waiting cost          | 5000 [US$/day] |
| Unloading cost            | 8000 [US$/day] |
| Inventory cost of storage tanks | 0.05 [US$//(day m<sup>3</sup>)] |
| Inventory cost of blending tanks | 0.03 [US$//(day m<sup>3</sup>)] |
| Setup cost of connection between blending tanks - distillation columns | 25000 [US$] (per switch) |
| Setup cost of connection between vessels - storage tanks | 5000 [US$] (per switch) |
| Setup cost of connection between storage tanks - blending tanks | 5000 [US$] (per switch) |
| Number of vessels         | 2      |
| Number of storage tanks   | 2      |
| Number of blending tanks  | 2      |
| Number of distillation columns | 1 |
| Initial crude amount in each vessel | Vessel 1: 90000 m<sup>3</sup> Crude Oil Type 0  
                           | Vessel 2: 90000 m<sup>3</sup> Crude Oil Type 1 |
| Initial crude amount in each storage tank | Storage Tank 1: 9400 m<sup>3</sup> Crude Oil Type 0  
                           | Storage Tank 2: 42000 m<sup>3</sup> Crude Oil Type 1 |
| Initial crude amount in each blending tank | Blending Tank 1: 6700 m<sup>3</sup> Crude Oil Type 0, 13300 m<sup>3</sup> Crude Oil Type 1  
                           | Blending Tank 2: 6700 m<sup>3</sup> Crude Oil Type 0, 13300 m<sup>3</sup> Crude Oil Type 2 |
| Capacity of each storage and blending tanks | 100000 m<sup>3</sup> |
| Maximum flow from vessels, storage tanks, and blending tanks | 120000 m<sup>3</sup>/day |
| Minimum flow from blending tank | 1200 m<sup>3</sup>/day |
| Column distillation minimum demand | 146880 m<sup>3</sup> |
| Column distillation concentrations required | 0.335-0.665 Crude Oil Type 0 |
Table 9: Different conditions and parameters considered in Case Study 1 and 2 of Crude Oil Scheduling Problem Model with Uncertainty

<table>
<thead>
<tr>
<th>Conditions and Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time horizon</td>
<td>8 days</td>
</tr>
<tr>
<td>Time interval</td>
<td>1 day</td>
</tr>
</tbody>
</table>
| Arrival times of vessels  | Vessel 1: 1\textsuperscript{st} day  
|                           | Vessel 2: 5\textsuperscript{th} day |
| Crude oil types           | Type 0  
|                           | Type 1 |
| Sea waiting cost          | 5000 [US$/day]|
| Unloading cost            | 8000 [US$/day]|
| Inventory cost of storage tanks | 0.0503 [US$/m\textsuperscript{3}/day] |
| Lower bound of inventory cost of storage tanks for Case of Study 2 | 0.0303 [US$/m\textsuperscript{3}/day] |
| Upper bound of inventory cost of storage tanks for Case of Study 2 | 0.0703 [US$/m\textsuperscript{3}/day] |
| Inventory cost for blending tanks | 0.0314 [US$/m\textsuperscript{3}/day] |
| Lower bound of inventory cost of blending tanks for Case of Study 2 | 0.0114 [US$/m\textsuperscript{3}/day] |
| Upper bound of inventory cost of blending tanks for Case of Study 2 | 0.0514 [US$/m\textsuperscript{3}/day] |
| Setup cost of connection between blending tanks - distillation columns | 25000 [US$] (per switch of crude oil blend) |
| Setup cost of connection between vessels - storage tanks | 5000 [US$] (per switch of crude oil blend) |
| Setup cost of connection between storage tanks - blending tanks | 5000 [US$] (per switch of crude oil blend) |
| Number of vessels         | 2     |
| Number of storage tanks   | 2     |
| Number of blending tanks  | 2     |
| Number of distillation columns | 1     |
| Initial crude amount in each vessel | Vessel 1: 90000 m\textsuperscript{3} Crude Oil Type 0  
|                           | Vessel 2: 90000 m\textsuperscript{3} Crude Oil Type 1 |
| Initial crude amount in each storage tank | Storage Tank 1: 9400 m\textsuperscript{3} Crude Oil Type 0  
|                           | Storage Tank 2: 42000 m\textsuperscript{3} Crude Oil Type 1 |
| Initial crude amount in each blending tank | Blending Tank 1: 13400 m\textsuperscript{3} Crude Oil Type 0, 26600 m\textsuperscript{3} Crude Oil Type 1  
<p>|                           | Blending Tank 2: 13400 m\textsuperscript{3} Crude Oil Type 0, 26600 m\textsuperscript{3} Crude Oil Type 2 |
| Capacity of each storage and blending tanks | 100000 m\textsuperscript{3} |</p>
<table>
<thead>
<tr>
<th>Conditions and Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum amount flow from vessels, storage tanks,</td>
<td>120000 m³/day</td>
</tr>
<tr>
<td>and blending tanks</td>
<td></td>
</tr>
<tr>
<td>Minimum amount flow from blending tank</td>
<td>1200 m³/day</td>
</tr>
<tr>
<td>Column distillation minimum demand</td>
<td>146880 m³</td>
</tr>
<tr>
<td>Column distillation maximum demand</td>
<td>186880 m³</td>
</tr>
<tr>
<td>Minimum demand constraint violation for Case of Study 1</td>
<td>10000 m³</td>
</tr>
<tr>
<td>Column distillation concentrations required</td>
<td>0.335-0.390 Crude Oil Type 0</td>
</tr>
<tr>
<td></td>
<td>0.610-0.665 Crude Oil Type 1</td>
</tr>
</tbody>
</table>
APPENDIX B: EQUATIONS REQUIRED BY LESS RESTRICTIVE MODEL

\[
\sum_v VSCBV_{v,i,t} + \sum_n SBCBV_{i,n,t} \leq 2 \quad \forall \ i, t
\]  \quad (103)

\[
\sum_v VSCBV_{v,i,t} \leq 1 \quad \forall \ i, t
\]  \quad (104)

\[
\sum_n SBCBV_{i,n,t} \leq 1 \quad \forall \ i, t
\]  \quad (105)

\[
\sum_i SBCBV_{i,n,t} + 2 \sum_k BDCBV_{n,k,t} \leq 2 \quad \forall \ n, t
\]  \quad (106)

\[
\sum_k BDCBV_{n,k,t} \leq 1 \quad \forall \ n, t
\]  \quad (107)
VITA

Jorge Chebeir was born in Viedma, Argentina and received his Bachelor degree in Chemical Engineering from Universidad Nacional del Sur in 2006. Thereafter, he performed as a Process Engineer in the Engineering Department of the company Solvay INDUPA. The work consisted of developing the basic engineering required by the plants of the company and the cost estimation of different projects. Then, he worked as a Production Engineer in the Chlorine-Soda plant of the same company, where his responsibilities included the development of conceptual and production engineering. After performing by two years in that company, he moved to Honeywell SAIC and performed as an Application Engineer. During the four years in the company, his work was based on the steady state and dynamic simulation of processes of several refineries through the utilization of Honeywell’s software. Finally, he performed as a Process Engineer for two years in the company Techint Engineering and Construction. This last period of his professional career was important to deepen his knowledge of basic and conceptual engineering as well as other essential concepts related to the chemical engineering science. As his interest in this science grew, he made the decision to enter in a master program. After receiving the scholarship from Fulbright Commission and the Government of the City of Buenos Aires, he entered in the Cain Department of Chemical Engineering at Louisiana State University. He is currently a candidate for the Master of Science in Chemical Engineering, to be awarded in May 2015.