The Economics of J. M. Keynes.

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THE ECONOMICS OF J. M. KEYNES

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Economics

by

John Steele Henderson
M. A., University of North Carolina, 1943
August, 1945
"Nor from eternal granite shall I hew
One fixed form, that ageless asons seeing,
Shall in its image find my life anew." (B.H.)

On examining a new and revolutionary composition, a reader's mind turns naturally to considerations of ultimate truth. Has the creator of this work thrown up an imperishable monument for all posterity to admire; or is the composition destined to travel the road to ultimate oblivion? Some intellectual creations derive authority from their very form and structure; others from a suggestive power, operating over the minds of others. Indeed, Marshall once defined a classical author as one who, "... either by the form or matter of his words or deeds ... has stated or indicated architectonic ideas in thought or sentiment, which are in some degree his own and which, once created, can never die, but are an existing yeast ceaselessly working in the cosmos."¹

What claim has the General Theory to such a position? It is much too early to say, but let these comments be considered. The ideas of Mr. Keynes, and with it the thought processes of numerous economists, have undergone a profound transformation in the past twenty years. In his "early period," represented by a Tract on Monetary Reform Mr. Keynes was operating within the traditional boundaries of economics. Yet even then we were warned as to

the limitations of the long run approach in economics – a portent of the changes to come.

Then, after a substantial interval, there emerged the impressive Treatise on Money. While the traditional monetary theory was held to have a useful subordinate position, the center of attention was focussed on new and important "fundamental equations." These were held to explain the fluctuations of the economic system around a norm. Still no suggestion was thrown out that we have here a potential successor to orthodox economic theory. Meanwhile, under the influence of telling criticism, the inscrutable mind of Mr. Keynes was undergoing further mysterious changes.

Then, almost without warning, the General Theory burst upon a surprised circle of economists. These poor unfortunates who crouched safely within the broad arms of the Treatise were rudely forsaken. The Treatise it turned out was simply "a natural evolution in a line of thought" which Mr. Keynes had been pursuing for several years. And at that time Mr. Keynes had not yet sufficiently rid himself of an unfortunate adherence to traditional methods. Thus, the implication goes, let us surrender to the inevitable and enjoy the full fruits of the Keynesian evolutionary process.

Lest our capitulation be too swift, a moment of reflection may be desirable. What assurance have we that the General Theory is not itself but "a natural evolution in a line of thought?" To this question no dogmatic answer may be given. Certainly, Mr.
Keynes' thoughts are coming of age; his critics have found no disastrous weaknesses, such as those which permeated the Treatise.¹

Yet, withal, caution is required. From one so delightfully changeable as Mr. Keynes, many profound and fruitful ideas have sprung. But these ideas are not inevitably of that architectonic quality essential to rocklike durability. Let us, then, mix admiration with caution. But whatever the future of the General Theory per se, we can be absolutely certain that Mr. Keynes has profoundly affected the ideas of present-day thinkers. And, by the same token, it is certain that his work will pass into the future, either as an independent work of discovery, or as part of the great current of economic thought.

¹ The Treatise, along with its definitions and its dynamic approach, has been utterly abandoned. The General Theory, despite certain weaknesses, remains substantially intact.
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ABSTRACT

The purpose of this study was to investigate the theoretical and practical aspects of Mr. Keynes' *General Theory of Employment, Interest, and Money*. From a theoretical point of view, Mr. Keynes' most important contribution is his emphasis on the general interdependence of the parts of the economic system. His interdependence analysis is of a particular kind, entitled "the aggregative equilibrium type." It is so called because the system deals with certain quantities which apply to the economy as a whole. Thus, such magnitudes as total income, consumption, saving, and investment come up for discussion. Keynes deals with a system which strives towards a "quasi-dynamic" equilibrium. The analysis is dynamic only because it deals with an economy in which a steady flow of investment occurs.

The points of chief interest in Keynesian analysis arise out of the monetary equilibrium. According to this conception income, saving, investment, consumption, the rate of interest, and the quantity of money interact in such a way as to produce an equilibrium situation. There are several forces which govern the equilibrium. The "propensity to consume," a functional relationship between the level of consumption and the level of income, serves to determine consumption. The marginal efficiency of capital gives a relationship between...
the rate of earnings of capital, on the one hand, and the level of investment and the level of consumption, on the other. This relationship serves to determine the level of investment, given the rate of interest and the level of consumption. The preference of the public for liquidity, its demand for cash, is said to depend on the level of income and the rate of interest. If the supply of money is given, the rate of interest will always move so as to equate the supply of and the demand for cash. The multiplier relationship states the division of income between consumption and investment. Finally, the identity of saving and investment, by definition, rounds out the list of forces governing the monetary equilibrium.

Mr. Keynes brings the level of employment into the picture by means of his aggregate supply function. This transformed version of marginal productivity doctrine gives a relationship between the level of money income and the quantity of employment.

In applying the completed system to practical problems Mr. Keynes is concerned with the problem of effective demand. Effective demand is the sum of consumption and investment. As Mr. Keynes sees it, the weakness of effective demand occasions an under-employment equilibrium. Governmental action designed to stimulate consumption and investment is his answer to the problem. The implementation of such policies leads to progressive taxation designed to increase consumption and governmental deficit spending designed to increase investment. Arguments were advanced by the present writer to show that such policies may eventually negate themselves. Furthermore,
the problem of secular stagnation is demonstrated by Keynes only for the case of fixed techniques or tastes; under conditions in which these factors are changing rapidly a full employment equilibrium is probable.
CHAPTER I

METHODOLOGICAL ASPECTS OF THE KEYNESIAN SYSTEM

INTRODUCTION

In the controversies between the Neo-Keynesian and the Anti-Keynesian schools, one may single out a characteristic situation. The antagonists square off around some particular point of theory. When the battle is over, the antagonists are generally willing to admit that the source of the dispute is a matter of definition or of methodology. Rarely, has the outcome of the discussion revealed a breach of logic on either side. Thus the characteristic error is misdirected criticism; the disputants find that the issues are factitious, that, granting their opponents' assumptions, definitions, and method, no dissent is possible.

Accordingly, it seems necessary to preface a detailed analysis of the General Theory by a discussion of methodology. In this way, we can point out the uses of this theory, what it can hope to explain; for every theory is inexorably bound by the laws of its own methodology. It may not transgress these bounds without risk of methodological inconsistency. Thus its potentialities are forever limited by the very nature of the processes necessary to bring it into existence.

An evaluation of the significance of any single phenomenon in the universe presupposes a precise knowledge of the surrounding
phenomena. Any obscurity regarding the rapport of the chosen phenomenon with surrounding circumstances, will lead to a misunderstanding of its significance. Let us consider, then, for a space, the general framework of economic reference.

Economics may be considered as having three facets, the scientific, the artistic, and the ethical. In its first capacity, economics is a body of propositions devoid of empirical content.¹ Thus, a proposition of pure theory will assert, "If such and such be the case, then so and so will be the consequence." But this is not the whole of the scientific process; a further step is necessary. The economist, as a scientist, is under obligation to demonstrate the empirical content of his hypothetical proposition. Accordingly, he must adduce evidence sufficient to demonstrate that the assumptions which he has chosen are realized in a wide variety of instances.

Thus economics may be defined as "the science which studies human behavior as a relationship between ends and scarce means which have alternative uses."² As a science, economics has no concern with the choice of ends; rather, scientific theories of the subject are based upon ends commonly pursued by members of society.

Although some writers deal with economics purely as a science, others pursue an artistic course, while a further group points out a relationship between economics and ethics. Let us


deal with the latter functions for a space. First of all, what particular meanings should we attach to these terms "art" and "ethics?" An art may be defined as a "systematic application of knowledge or skill in effecting a desired result."$^1$ Thus, if the abolition of the trade cycle be selected out as the desired end, an investigation into the possible means of carrying out this end may be conducted. Such an investigation is to be classified as art; here the end precedes and determines the choice of means.$^2$ It must not be supposed that this sort of investigation is less taxing than scientific research. Quite the contrary, the imposition of the fixed end imposes a considerable strain on the powers of the investigator. In consequence, the end result is often inconsiderable.

Finally, ethics is sometimes considered to have a close connection with economics. As Knight puts it, "the consideration of wants by the person who is comparing them for the guidance of his conduct and hence, of course, for the scientific student...

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$^1$ Webster's New International Dictionary, see art.

$^2$ Thus, J. M. Keynes in his Scope and Method of Political Economy (2nd ed., New York: MacMillan, 1897), p. 32, says, "Then we leave the enquiry into the veritable order of economic phenomena, their coexistences and sequences, under existing or assumed conditions, we still have to take account of a further subdivision of some significance. There is, on the one hand, the investigation of economic ideals and the determination of a standard by reference to which the social worth of economic activities and conditions may be judged; and there is also the investigation of economic rules, i.e., the determination of maxims or precepts by obedience to which given ends may be best attained." Thus, Economics as science, ethics, and art falls within the purview of the Economist, if this analysis be accepted.
thus inevitably gravitates into a criticism of standards."¹

Ethics deals with the choice among ends; and wants, which are the ends to be achieved, receive examination by the economist, as ethical analyst.

It is not one of the purposes of this paper to investigate the validity of the alliance of economics as science, art and ethics. But it is important to differentiate the three functions in our minds, because they are often confused, to the detriment of the investigation in question.

In the main, the General Theory is concerned with the formulation of a new scientific approach to economics. Although Keynes has subjoined as a sort of epilogue, some notes bearing on "the social philosophy towards which the General Theory might lead," his explicit purpose is theoretical and scientific. "The object of our analysis is . . . to provide ourselves with an organized and orderly method of thinking out particular problems."²

In effect, he wishes to set up certain tools of analysis capable of application to the real world. It must not be supposed, however, that his followers have exercised similar restraint: in the Neo-Keynesian school there are to be found practitioners of art and ethics. Where the theory itself is not at fault, its applications may be questioned; where science is neutral in its view of the social process, art and ethics are partisan. Let the


theory itself, then, be differentiated from efforts to use it for the purposes of art or ethics.¹

POSITION OF KEYNES' THEORY AMONG OTHER ECONOMIC ANALYSES

A TAXONOMIC APPROACH

In order to elucidate the precise relationship of the General Theory to other bodies of economic thought, we shall subjoin and analyze a classification of the methods of economic analysis.

| Inductive                          | Historical                  |
|                                   | Statistical                  |
| Method of Specific Experience     | Partial                     |
|                                  | Static (Equilibrium)         |
|                                  | Partial                     |
| Math., or Quasi-Math.             | General                     |
| Deductive                         | Quasi-Dynamic               |
| Literary Logic                    | Partial                     |
|                                  | Aggregative (Keynes' General Theory) |
|                                  | Equilibrium                 |
|                                  | Partial                     |
|                                  | General                     |
|                                  | Dynamic                     |
|                                  | Equilibrium                 |
|                                  | Partial                     |
|                                  | General                     |
|                                  | Process                     |
|                                  | Analysis                    |
|                                  | Disequilibrium              |
|                                  | Other                       |
|                                  | Systems                     |

¹ Consider the investigation involved in the article by Oscar Lange, "The Rate of Interest and the optimum Propensity to Consume," Economica, V (1938), N.S. After restating in an elegant form the conditions of monetary equilibrium, following the General Theory, Dr. Lange seeks to solve a particular problem with the aid of this apparatus. He formulates his end, the maximization of the rate of investment per unit of time, and then proceeds to determine the conditions under which this end will be fulfilled. Thus he asserts, "The general theory of interest outlined in this paper enables us to solve this problem and to determine the optimum propensity to consume which maximizes investment." P. 24. This is art, not science.
Any classification is, by its very nature, arbitrary; therefore, as the occasion arises, it behooves us to define these categories carefully, in order to avoid the risk of misunderstanding.

Induction may be defined as the attempt to reach generalizations concerning phenomena in the real world by a systematic study of facts. Deduction, on the other hand, is the systematic study of the results of given postulates. Any theoretical study is deductive, since theory is simply a systematization of the interrelations of deductive thinking. And the General Theory, as its name implies, falls within this grouping. Deductive economic theory has a limiting property, internal to it as a form of reasoning: conclusions derived from deductive theory can express no more than the interrelations between, and the content of, the hypotheses on which it is based. In the past economists were content to found their systems on postulates far removed from reality.\(^1\) In consequence, economics has not, up to the present time, satisfied that goal of science, "the technique of prediction" for the purpose of control.\(^2\)

At the present time economists are wont to adopt a pragmatic attitude towards truth: the truth or worth of a theory is coming to be judged, not by theoretical elegance and elaboration, but by correspondence with reality, and usefulness for purposes of prediction. Under the influence of such considerations economists are beginning to manifest increasing dissatisfaction with unreal theoretical constructions. As one writer

\(^1\) The Classical stationary state springs to mind at once, as well as Robinson Crusoe economics, and barter assumptions.

puts it, the "development of economics under the impulse of theoretical construction, for its own sake, along paths which become ever more remote from the real world, is best described by the term 'theoretic blight.' From the point of view of one who desires to use economics for the discovery of concrete truth, theories built on incredible postulates seem a parasitic growth."

Under the influence of such considerations, economists have latterly been concerned to revise their assumptions to meet the stern test of reality. The General Theory of Mr. Keynes is one manifestation of this tendency. (And if Mr. Keynes' thoughts are, in any sense, revolutionary, it is because he has started with a new packet of assumptions. As he himself states the matter: "if orthodox economics is at fault, the error is not to be found in the superstructure, which has been erected with great care for logical consistency, but in a lack of clearness and generality in the premisses. Thus I cannot achieve my object of persuading economists to re-examine critically certain of their basic assumptions except by a highly abstract argument and also by much controversy." Mr. Keynes appears to feel that his assumptions are of such a nature as to warrant dubbing his theoretical construction the General Theory. This point will receive examination at a later stage.


2 G. T., p. v., preface. This is distinctly reminiscent of Veblen.
Use of Mathematics

In spinning out his logical constructions Mr. Keynes resorts to a method hateful to some, and good to others. I refer to his use of mathematics. While the English Classical and Neo-Classical schools made sparing use of mathematics, it formed the basic logical method of the general equilibrium approach sponsored by Walras and Pareto. Thus writers on the continent have been accustomed to working out their theories primarily in mathematical terms.

In England, and to some extent in America, the famous passage of Marshall advising caution in the use of mathematics has exercised considerable influence. That master asserted, "The chief use of pure mathematics in economic questions seems to be in helping a person to write down quickly, shortly, and exactly, some of his thoughts for his own use: and to make sure that he has enough, and only enough, premises for his conclusion (i.e., that his equations are neither more nor less in number than his unknowns)."

Since Marshall's day, it has come to be recognized that this account of the matter is incomplete. Let us review shortly the possibilities of the method.

The advantages of the method: (1) The idea of mutual interdependence comes to supplant the concept of cause and effect. Such analysis runs in terms of a mutual interaction of margins,

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rather than in terms of one force determining another. Thus, with Ricardo, it was cost of production that determined value, given the utility of the object. Later analysis, under mathematical influences, relates value to an interaction of demand and cost at the margin. (2) It is possible to handle a large number of variables without danger of slipping into logical error. While problems in one and two variables can be treated conveniently without algebra, difficulties arise when the number of variables is multiplied. Thus, increase the number of variables to four or five or \( n \), and the resources of ordinary logic, and intuition will no longer suffice. Something more is needed; and that may be filled by algebra.\(^1\) (3) The method carries with it the necessity of rendering a strict account of assumptions, variables and hypotheses (functional relationships) connecting the variables. In short, this approach makes for logical consistency. (4) Finally, problems of change may be treated with greater firmness and accuracy. More than this, it is frequently possible to derive rules which express the direction of change in a given situation. And the principle of Occam's Razor is ever at work here. What science requires is a theory which will account for the observed results with a minimum of assumptions. For the

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\(^1\) As J. R. Hicks puts it, "When the verbal (or geometrical) argument is conclusive, it gains nothing from being put in another form. What can be gained, however, is the assurance that our argument is completely general; that that which has been proved in the text for two, or three, or four commodities, is true for \( n \) commodities." *Value and Capital* (Oxford: Clarendon Press, 1939), Mathematical Appendix, p. 303. This is the minimum value which Dr. Hicks expects to derive from the use of algebra.
fewer the assumptions with which it is necessary to deal, the
easier will be the problem of applying the theory to the real
world. In respect to problems of change, mathematics becomes
particularly useful, in that it can determine the minimum number
of conditions which will suffice to ensure a given result.\footnote{Consider, for example, the secondary conditions for a maximum, as applied to an economic problem. One of the simplest economic problems is to determine the effect of a rise in price on the quantity offered by an individual firm under competition. The secondary conditions for a maximum, the stability conditions, yield the information that the marginal cost curve must be positively sloped. Therefore, the quantity supplied must increase.}

Over and against these advantages are to be set several
disadvantages which may be said to attach to the use of the method:
(1) The lack of intelligibility of mathematics occasions distress
among those who are not acquainted with it. While this obscurity
limits the size of the audience, it has no effect whatsoever
on the validity of the conclusions derived by the use of this
method. In reading Keynes the discriminating reader will notice
that the core of his argument is always presented in non-mathematical
form. (2) In using mathematics there is an almost irresistible
temptation to deal with "tractable" assumptions, whether or not
the assumptions correspond with reality. Marshall expressed
himself as follows on this matter: "many important considerations,
especially those connected with the manifold influences of the
element of time, do not lend themselves easily to mathematical
expression: they must either be omitted altogether, or clipped
and pruned till they resemble the conventional birds and animals
of decorative art. And hence arises a tendency towards assigning
wrong proportions to economic forces; those elements being most emphasized which lend themselves most easily to analytical methods.\textsuperscript{1}

Later, in a critical section of this paper, we shall have occasion to evaluate the Keynesian system according to its success in reaping the advantages of the method, without falling a prey to its characteristic defects.

**Static, Quasi-Dynamic, and Dynamic Systems**

Fundamentally, economic theory represents an attempt to ascertain the nature of those forces determining the temporal pattern of economic magnitudes.\textsuperscript{2} The timing of events assumes importance because the economist is concerned with prediction. Should the economist be able to predict the course of events under given circumstances, he should be invested with the power to influence present events with a view to eliciting a desired future result. Prediction for the purpose of control - this is the Mecca of the economist. In projecting himself into the field of temporal relationships the economist inevitably becomes concerned with dynamics. For a dynamic theory is one which deals with a system of dated magnitudes and flows, whose values vary with the passage of time.

While the Neo-Classical economists\textsuperscript{3} were concerned to push

\footnotesize
\begin{itemize}
  \item\textsuperscript{1} Marshall, *op. cit.*, p. 850.
  \item\textsuperscript{2} As Erik Lundberg puts it, "The ultimate purpose of all economic theory is to analyze changes in economic life with respect to time." *Studies in the Theory of Economic Expansion* (London: P. S. King, 1937), p. 1.
  \item\textsuperscript{3} This term is quite arbitrary. I use it to mean the great theorists following Ricardo who developed the partial and general equilibrium theories, namely, such men as Marshall, Walras, Pareto, Wicksell, and Edgeworth.
\end{itemize}
their studies towards reality, their energies were largely absorbed in laying out the boundaries of the stationary state. Yet it must not be supposed that their work was fruitless because it did not deal successfully with economic development. Quite the contrary, the conditions of stationary equilibrium provide a norm with which to compare a condition of continuous evolution.¹

In the stationary state the general conditions of production, distribution and exchange would be always the same. There would be continuous movement in this system, but movement of a perfectly repetitive sort.² Every flow, whether of productive services, or of economic goods, would persist at a constant rate.³

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¹ As Erik Lindahl points out, nevertheless, "Static theory also has for an object economic developments taking place in time, only the variables studied do not change their values with the lapse of time. The corresponding time curves have thus the nature of straight lines parallel with the time axis ... a community that is thus characterized by a repetition of the same economic processes is called a stationary community." Studies in the Theory of Money and Capital (London: George Unwin, 1929), p. 32.

² That Marshall was concerned with the problem of dynamics is illustrated in a passage in his Principles: "... this volume is concerned with normal conditions; and these are sometimes described as Statical. But in the opinion of the present writer the problem of normal value belongs to economic Dynamics: partly because Statics is really but a branch of Dynamics, and partly because all suggestions as to economic rest, of which the hypothesis of a Stationary state is the chief, are merely provisional, used only to illustrate particular steps in the argument, and to be thrown aside when that is done." Op. cit., p. 366 n. Of course, Marshall's analysis of change may be classed as comparative statics.

³ As V. Pareto put it in his discussion of economic equilibrium, "When it is clearly understood that the phenomenon studied is a continuous phenomenon, we may, without inconvenience, avoid the burdening of the exposition of the theory, involved in repeating constantly: 'in a unit of time.' When we shall speak, for example, of the exchange of 10 kilograms of iron against one kilogram of silver, it will be necessary to understand 'which is accomplished in a unit of time' and that we do not speak of an isolated exchange, but of a repeated exchange." Manuel D'Economie Politique (2nd ed.; Paris: Giard, 1927), p. 148. The translation is free.
Whereas the population would be constant both as to numbers and age structure, each individual would pass through the cycle of youth, middle age, and senility. Likewise, the magnitude of resources, and their distribution among the members of society would always have the same aspect. And these resources would always be used in the same way, for no innovations or inventions would occur to the members of this self-satisfied society. Tastes would remain unchanged in such wise that the demand for each particular commodity or personal service would be ever the same. No monetary perturbation would disturb the even surface of the economic sea. Further, the legal, moral, and social system, within which economic forces operate, would be fixed.¹

Thus a static theory is one which deals with an undated system of magnitudes and flows, whose values are so equilibrated as to maintain a constant level in time. The supply and demand schedules are invariant through time and by the same token all prices are constant. No new saving or investment occurs. Prices are equal to cost of production.

Some writers even suggest that the static state is the one towards which society is actually evolving at any moment. Thus J. B. Clark asserts, "The static state which has here been pictured is the one toward which society is at every instant tending, under the influence of competition. The static system of groups and sub-groups should, then, be thought of as an ideal

¹ See Knight, op. cit., pp. 173-74. Also see a volume by the same author, Risk, Uncertainty and Profit (New York: Houghton Mifflin, 1921), pp. 145-47.
arrangement, projecting itself through the disturbed and changing

group system of actual society just as the imaginary level sur-
face of the sea projects itself through the waves." ¹

However, Clark defines this condition more accurately
later: "the static state "... would be reached, if we were
to paralyze the dynamic forces all at once and wait long enough
for the slowest static adjustments to be made." ²

What is precisely certain is that the static state is not
the accurate description of a process of change. For the rates
of change of the factors held constant in the static state
take on independent significance. It is of no use to stop the
process of evolution and determine the static equilibrium. Such
information, while yielding a useful norm or point of departure,
cannot inform us as to the level of economic activity attained
by a society in process of change. That level of activity depends
on the rate of evolution. Surely, only those theories which
take account of the rate of evolution can attain the desired
end of analysis, prediction for the purpose of control.

In static theory the analyst provisionally paralyzes
certain forces in order to simplify the investigation. Tastes,
resources, population, productive equipment, techniques, monetary
conditions, social institutions, all are fixed by assumption.
In passing to problems of dynamics, this artificial paralysis

¹ J. B. Clark, Distribution of Wealth (New York: Mac-
² Ibid., p. 429.
is released. Some of these factors are allowed to vary, and the analysis then hinges on the incorporation of the new variable elements into the theoretical system. Elements formerly exogenous, or external to the system, now become endogenous forces and therefore form an integral part of that system. Furthermore, all economic magnitudes, in a fully dynamic system, must be dated.\footnote{Hicks says, "I call Economic Statics those parts of economic theory where we do not trouble about dating; Economic Dynamics those parts where every quantity must be dated." \textit{Op. cit.}, p. 116.} More than this – the problem of planning economic activity is related to magnitudes of past, present and future.\footnote{Some definitions of dynamics, alternative to the one given earlier, have great prestige. Following Frisch, Paul Samuelson asserts that: "A system is dynamical if its behaviour over time is determined by functional equations in which 'variables at different points of time' are involved in an essential way. Dynamics, Statics, and the Stationary State," \textit{Review of Economic Statistics}, XIV (1943), p. 59. Then Lindahl asserts that, "the object (of theory) is to determine certain variables as functions of time (or time curves) with the help of equations, based on what is known as the initial values of these variables and the conditions which determine their fluctuations. A theory of this type must be called dynamic." \textit{Op. cit.}, p. 31.}

In dealing with economic development, the problem of the analyst is to determine the factors which he will allow to vary. Virtually all analysts of economic development admit the growth of productive equipment into their systems. In particular, a positive level of saving and investment is common to most such theories.\footnote{Consider, for example, J. A. Schumpeter, \textit{The Theory of Economic Development} (Cambridge: Harvard University Press, 1934). Also see Lundberg, \textit{op. cit.}}
And it must be admitted that a society characterized by new saving and investment cannot be described as stationary. For in this instance productive resources, which are constant under stationary conditions, are in process of change. Yet this growth in productive equipment does not fulfill completely the conditions of a dynamic system; dating of economic magnitudes is also necessary.

For our purposes a category somewhere between "static" and "dynamic" is required. In such an analysis, the theorist would abate the rigor of his assumptions, while leaving undated the relevant magnitudes. A formal definition of one such type of system might be put as follows. A Quasi-Dynamic Theory is one which deals with an undated system of magnitudes and flows, whose values are so equilibrated as to maintain a constant level in a short period of time, but wherein one or more of the flows is inconsistent with the ultimate maintenance of these values. Such a system is one wherein positive investment takes place. Under these conditions society's productive equipment will grow, the forms of the short-term production functions will change, real income will increase, and so on. Yet a short period equilibrium may be said to exist at a moment of time, or during a short interval of time. And we may gain some insight into dynamic processes by comparing a succession of these short period equilibria.¹

¹ This should not be confused with the type of analysis known as comparative statics. Strictly speaking, this method compares the stationary equilibrium which would be reached if one of the underlying factors were allowed to vary with the original equilibrium. Thus, if a society were in a stationary condition, and technical improvements were made, positive investment would occur for a time, until the flow of investment reduced the net productivity of new capital to the rate of interest prevailing at zero net saving. The conditions of this new stationary equilibrium may be compared with those of the old. See the discussion of these matters in Schumpeter, op. cit.
Keynes' *General Theory* falls into this category. In his construction positive investment occurs continuously. Furthermore, monetary conditions are not specified as constant in the sense common to stationary analysis. And the level of money income is not therefore fixed in advance. Yet his system does not take account of dating explicitly, despite the persistent references to anticipations. And it must, accordingly, be excluded from the class of dynamic theories.\(^1\)

**Equilibrium Method**

In the Keynesian system, the concept of inter-temporal interdependence is not formally treated, despite the constant references to anticipations. At best, the anticipations are those which look forward to the current market period. Yet the state of equilibrium reached cannot persist beyond the current market period, in all probability, because of internal forces which would tend to disrupt a recurrence of this pattern.

The variability of forces considered bears on the problem of equilibrium in the Keynesian system. In many senses, the problem of the choice of an equilibrium or disequilibrium system is inseparable from the choice of static or dynamic analysis.\(^2\) Those

\(^1\) Another example of Quasi-Dynamic Theory is to be found in Walras', *Eléments d'Economie Pure*. Here he presents a theory of capital accumulation, formally integrated into his general equilibrium theory. But his system is undated. It ignores the interdependence of economic magnitudes in time. It is quasi-dynamic.

\(^2\) Robbins, *op. cit.*, pp. 67-68 says, "Instead of dividing our central body of analysis into a theory of production and a theory of distribution, we have a theory of equilibrium, a theory of comparative statics, and a theory of dynamic change." This statement implies, if it does not state explicitly, that equilibrium theory is essentially static; and that the best the equilibrium theory can hope to do is to show variations as between one static state and another.
writers who deal with the stationary state find it convenient to adopt an equilibrium method. In fact the two points of view become virtually identical, part and parcel of one another.\(^1\)

In his analysis of stationary conditions Pareto says, "economic equilibrium is that state which would maintain itself indefinitely if there were no change in the conditions in which it is seen (to be).\(^2\) What is this but the stationary state — a condition in which economic magnitudes and flows maintain themselves indefinitely without tendency toward change?

In some senses, of course, the equilibrium method whereby stationary conditions are analyzed is the ultimate one. For the only ultimate ends which may be attained in reality are those worked out when all change is suspended. "It is the nature of every change in the universe known to science to have 'final' results under given conditions, and the description of the change is incomplete if it stops short of the statement of these ultimate tendencies."\(^3\)

But "given conditions" must be interpreted to mean stationary conditions; for a stationary society is the end reached when the conditions of economic life assume a purely repetitive pattern. Clearly, "given conditions" may not be assumed to include fixed

\(^1\) Lundberg, theorist of economic development that he is, puts the matter as follows: "The stationary state as such does not exist in modern economic life and is, per se, of no interest to us. The possibility of its existence is to be looked upon wholly as a by-product derived from the equilibrium systems. These systems aim to determine the conditions for a specific situation or state in such a way that this status will be maintained only if these conditions are fulfilled." \textit{Op. cit.}, p. 2.


\(^3\) Knight, \textit{Risk, Uncertainty and Profit}, p. 17.
conditions about rates of change. For in this event we encounter
the possibility of a system which is so constructed as to oscillate
in such a perfectly irregular way that it never reaches a definable
"equilibrium." Stationary equilibrium analysis presupposes the
impounding of those forces which would lead to a disruption of
equilibrium; the system is bound by the laws of its own existence.

Our inquiry, thus, turns on the possibility of including
within an equilibrium analysis certain rates of change which are in-
compatible with a stationary condition. One such possibility is an
investigation of the effects of a discontinuous change in the under-
lying stationary conditions. When this change is introduced the
system is dynamic; it is in process of change; it strives toward an
equilibrium.¹

To illustrate, a firm is in full equilibrium when the price
of the final product is equal to average cost. Under these circum-
stances, the short run marginal cost curve cuts the short run
average cost curve at the same output that the long run marginal
cost curve cuts the long run average cost curve. Then price equals
short run marginal cost equals short run average cost equals long
run marginal cost equals long run average cost. Such is the condition
of stationary equilibrium. But
we may displace the firm from this
position and show how it approaches
equilibrium through time. In the
short market period (Marshall's "day")

¹ As Robbins puts it, "... we may ... endeavor to trace out
the path actually followed by different parts of a system if a state
price is governed by demand and dealers' stocks; in the short
run normal period price is governed by the condition that the
firm should match short run marginal cost and price; in the long
run price is governed by the stationary conditions indicated
above.

In essence, this time analysis is the description of the
means whereby an industry works back towards full or stationary
equilibrium when displaced from it by an external disturbance.
In some sense it analyzes change. Yet, given the disturbing
condition, the system will work itself down to a new final equilib-
rium. And the analysis does not treat of the nature of those
forces which will cause perpetual change, or a moving equilibrium.
In a sense, the short run cost conditions are moving equilibria;
but it must be remembered that they do not fully satisfy the
rationale which underlies the system of which they are a part.
This rationale is the competitive instinct animating the members
of society. Each several member seeks to maximize profit, and,
in the end, these members collectively eliminate it. Thus, the
system completes itself only in the obliteration of profit for
each individual firm, and therefore in the cessation of the urge
towards change.

What Keynes has done with equilibrium analysis is something
more ambitious.¹ He has incorporated a flow into his system which

¹ Dr. Hicks puts it somewhat overenthusiastically, as follows,
"Ordinary (static) economic theory, so the old argument went, explains
to us the working of the economic system in 'normal' conditions. Booms
and slumps, however, are deviations from this norm and are thus to be
explained by some disturbing cause . . . The present theory breaks away
from the whole of this range of ideas . . . The changing, progressing,
fluctuating economy has to be studied on its own, and cannot be referred
to the norm of a static state." "Mr. Keynes' General Theory of Employment,"
Economic Journal, XLVI (1936), p. 239.
is inconsistent with static analysis, namely, the rate of saving and investment. Yet this flow is consistent in a short period of time with a stable set of values for the economic magnitudes considered. In this way, one magnitude which is constant under stationary conditions is treated as a flow.

The General Theory is not concerned to elucidate the path by which the economic system approaches a distant goal. Attention is directed towards the present. Reality is sought in the transient, evanescent equilibrium which ever yields its place to a successor. For, "... this long run is a misleading guide to current affairs. In the long run we are all dead. Economists set themselves too easy, too useless a task, if in tempestuous seasons they can only tell us that when the storm is long past the ocean is flat again."\(^1\)

Yet a serious gap in the General Theory is the absence of a positive analysis of the forces which maintain his system in a state of moving equilibrium. The persistence of the moving equilibrium demands a continuous, positive flow of investment; but this is only possible so long as the net efficiency of some uses of capital exceeds the rate of interest. In the absence of technological improvements, a continuous fall in the rate of interest will set it. The end of this process is reached only in the stationary state.\(^2\)

Apparently, Keynes feels that he escapes this dilemma by focusing attention on that short period during which the rate of

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\(^2\) Keynes explicitly instructs us that techniques of production are fixed in his system. *G. T.*, p. 245.
interest will reach a temporary equilibrium. From a long run point of view, however, some account is required of the means whereby the flow of investment is maintained. In brief, a description of technological change is in order. Failing this, the system is in danger of collapsing of its own weight. At any rate the theory, in its unmodified form, embodies stationary implications which Mr. Keynes would certainly disavow. Herein is to be found a distinct hiatus in the Keynesian structure.

One may launch a more vigorous methodological attack on the Keynesian system by means of a general critique of the equilibrium method. And in place of the equilibrium method we may substitute a "disequilibrium method." The starting point is the beginning a short interval; let us call it a "day." During the "day" certain market transactions will be effected. On the morning of the first day price fixing occurs; and if the prices are adjusted by noon, producers and consumers will formulate their plans for the rest of the day in accordance with these prices and the surrounding circumstances. Producers will resolve upon their prospective use of factors and their production, while consumers will decide upon their purchase plans for the day. During the afternoon these plans will be carried through as far as possible.

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1 For instance, Lundberg says, "The conditions of equilibrium may only be intended to characterize a given situation, in order to show the possibility of its existence. To establish the possibility of a general equilibrium by no means proves the existence of a tendency toward it in an actual economic development." Op. cit., p. 27.

2 The discussion which follows is simply a reworking of Lindahl's exposition. Op. cit., pp. 62 ff. The above analysis may also be used to describe the way in which supply and demand work towards equilibrium. The difference is that this method does not suppose that the equilibrium will ever be achieved. The attainment of equilibrium would simply represent a special case.
At the end of the day each individual seeks to evaluate the results of his experiences. Traders and manufacturers consider their stocks and orders with a view to determining whether they are greater or less than expected. Some manufacturers will have over-estimated the demand; others will have underestimated it. And so with dealers.

In accordance with the divergence of reality from anticipations, each business entity will revise its plans for the next day. Some will raise prices, others will lower them. Some dealers will increase orders, others will diminish them. And so for the following days.

By a systematic analysis along these lines a genuine theory of economic development may be evolved. And such a system may be thought to surpass the equilibrium theory in its approach to reality. But in actuality the two systems are not competitive; for other equilibrium systems exist which are dated and which do take account of change between periods.¹

Hicks, for example, treats a system in the process of moving equilibrium, wherein the link between the present and the future is to be found in anticipations. It is equilibrium theories such as these which should be compared with the disequilibrium method.

All that the Keynesian system can hope to do is to explain a given amount of employment and real income. As such it is not adapted very well to indicating the cumulative movement of a system under the stress of internal forces. Let these limitations be clearly recognized.

¹ The most masterly and complete formal treatment along these lines which I have encountered is in Lindahl, op. cit. Here is a system in dynamic equilibrium; it is dated and includes a rather complete account of capital accumulation. As a purely formal exercise in logic it is a masterpiece, carrying to completion the work of the Swedish master, Vicksell.
They are inseparable from the nature of the approach which Keynes has adopted. Within its inherent limitations, this system will prove most useful in analyzing certain types of tendencies in the economic system. A case in point is the rationalization of the existence of unemployment, or a "sub-normal" level of real income.  

In his analysis Keynes employs an equilibrium system of the aggregative type; in so doing he seeks to project onto our horizon a few easily pictured relationships connecting some highly significant variables. These variable magnitudes are ultimately derived from a consideration of small economic units; values which hold for these micro-economic units are summed horizontally through society to derive the aggregates which hold for the economy in its entirety. In most cases, the relationships between the magnitudes, on the micro-economic level, are presumed to apply also to the whole economy; in short, the relevant functions are assumed to remain invariant under the process of summation and combination.

The end result of this process of summation is a small group of magnitudes, relating to the whole economy, connected by an equal number of relationships. This approach yields a macroscopic

1 An alternative possibility, by means of which the Keynesian system might be adapted to dynamic use, is the pursuit of Schumpeter's line of approach. The effects of discontinuous changes in techniques, and "innovations," might be studied. And the striving of the system for a new equilibrium under these circumstances might be worked out. As in Schumpeter's system, the adaptation process might be shown to imply a cyclical type of fluctuation.

2 As Hicks puts it, "A great part of Mr. Keynes' work may be regarded as an endeavour to cut through the tangle (of economic relationships), by grouping complex factors together in bundles. This process is one of drastic simplification, but it is necessary if the theory is to become an instrument of practical thought." "Mr. Keynes' General Theory of Employment," p. 239.
view of economic affairs; events are viewed as the result of broad forces running through the whole economy, and determining the general course of its affairs. Thus, the method has regard for the general interdependence of magnitudes which dominate and inform the economic system - magnitudes such as income, saving, investment, and the like.

This explicit treatment of general economic interdependence is commendable; many theories of the trade cycle merely treat special features of the economic system which are presumed to induce instability. The residual relationships are not dealt with explicitly; the reader is left to supply the deficiency from general economic theory. In avoiding this characteristic vice, Keynes has rendered a valuable service to monetary theory.1

Yet the aggregative method also has its pitfalls. "So long as aggregates, even restricted in scope are used, there is always the danger that the internal structure of these aggregates (in other words, the relationships between their subdivisions) may prove to be significant; this would force the economist to split up the aggregates so far undivided and to try to construct his system in terms of subdivisions of these aggregates."2 In such case, an unsupplemented use of the aggregative method yields unsatisfactory results. The smooth surface of its equilibrium

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conceals internal stresses which lead to instability, and a disrup­tion of the supposed equilibrium.

Inevitably, analytical precision is sacrificed for the sake of simplicity. Should a more detailed analysis be required, the economist should recede to the micro-economic units underlying the broad aggregates. Analysis of the relation of each of these small units to the general economic pattern yields a very exact formal picture of the economic process. However, the very intricacy of the network of relationships virtually precludes a fruitful study of the movements of an economic system.¹

Let the truth be faced. The aggregative equilibrium method sacrifices theoretical precision with a view to simplicity and workability. Holding these limitations clearly in mind, we may yet derive some instructive lessons from this sort of theory.

In summary, the General Theory is a deductive, quasi-mathematical, quasi-dynamic, equilibrium system of the aggregative type. It is not essentially a theory of economic fluctuations. A theory of the trade cycle is necessarily fully dynamic, dealing explicitly with variations in important economic magnitudes. Keynes, on the contrary, treats the temporary equilibrium of these magnitudes.

¹ Hicks' brilliant study, Value and Capital, seems to belie this generalization. In fact, however, his fundamental method is based on the fact that goods which behave in the same fashion may be treated as a single good. If this line of attack can be followed out, we may one day wake up and discover that the microscopic-general equilibrium approach is the most fruitful method of all.
Nevertheless, the system is dynamic in that productive equipment is growing continually. And this, in turn, entails the emergence of equilibria, shifting from period to period. Each equilibrium can persist but for a short interval, and then must yield its place to a new.

Finally, the equilibrium portrayed is one of the aggregative type. Such a method deals with a limited number of macroeconomic magnitudes, connected by an equal number of relationships. By thus grouping strategic factors in the economic system, Mr. Keynes achieves a system marked by simplicity and elegance. However, a corresponding weakness is thus incorporated into the framework of his system; for it is subject to the danger of illicitly representing a disequilibrium situation as a position of stable equilibrium. Such are the chief methodological aspects of the General Theory.
CHAPTER II

Aggregative Equilibrium and General Equilibrium

In evolving a critique of the General Theory a fundamental point of reference is required. Oftentimes, the economist is able to demonstrate weakness in a system while powerless to suggest the remedy. In this chapter an alternative approach to problems of interdependence is outlined - to the end that a specific alternative to the theory criticized will be available. This alternative system is commonly known as "the General Equilibrium theory."

And the lessons learned in this discussion will be applied immediately to that summary of Keynesian thought, known as "the Principle of Effective Demand."

General Equilibrium

"Thou canst not stir a flower without troubling of a star" (F.T.)

Perhaps the most universal and powerful theoretical tool which neo-classical economics bequeathed to the present generation of economists is the theory of general economic equilibrium.¹ This system of thought depicts that all pervading network of relationships

¹ For practical purposes, no doubt, partial equilibrium theory is more effective. However, the latter system may be considered simply as a special case of the theory of general equilibrium. What partial equilibrium theory strives toward is the accurate analysis of cases which are considered to be of the greatest practical value. The above statement, then, is not to be interpreted as a judgment of general worth, but of theoretical fulness and consistency.
which holds together the component parts of the economic system
by those "bonds, which though strong as steel, are light as air."

Where this school found its point of departure was in that
series of exchange relationships which signalizes the economic
activities of man in production, distribution and consumption. The
exchange of goods and services, then, is the subject matter; and the
interaction of forces which reach a focus in these exchange relation­
ships constitutes the form of analysis.

Under the methods of general equilibrium the fundamental
variables whose values seek an equilibrium are these: (1) the
quantities of the productive services, and their prices: (2) the
quantities of the several goods produced, and their prices. Given
the problem to be solved, the equilibrium of these variable magni­
tudes, our attention turns, thereupon, to those forces which may
yield the solution. And this analysis finds the origin of these
forces in those micro-economic units, the household and the firm.
Each such unit forever seeks to derive the maximum net advantage
from a given situation: pursuit of this aim by the several micro-
economic units imposes certain conditions upon each in severalty,
and, ultimately, upon the entire economy.

In the short run, given the supplies of the productive factors,
the conditions thus imposed may be stated as follows: (1) prices
of final goods and services are set at such levels that the
individual households, in achieving equal increments of utility from
equal increments of expenditure in any direction, collectively exhaust
the supplies of each good produced: (2) prices of productive services
are set at such levels that individual firms, in achieving equal
increments of value product from equal increments of expenditure in any direction, collectively produce the supplies thus demanded and collectively exhaust the supplies of each productive service. In the long run, further variables are determined by further conditions.

To be more explicit, the pursuit of the first economizing condition by the individual household gives rise to a series of individual demand curves. By summing the individual demand curves for each particular product, the several industry demand curves may be derived. On similar principles, the industry supply curves of final products, and the demand curves for productive services are derived from the individual curves fulfilling the second condition.

Then by supply and demand analysis the desired prices and quantities are determined. However, the several prices are related by reason of the necessity for proportioning outlay on the aforementioned principles. Thus we have a series of relationships whereby the demands or supplies of micro-economic units are related to the industry. Finally, a series of relationships between these industries, as determined by the interconnections of supply and demand schedules, will also exist. In this way, the micro-economic units are closely related to the several industries which are themselves so connected as to determine the contour of the total situation. In the last analysis, then, the system links together the parts of the economic structure - even as the bloodstream brings every member of the human body into relation with every other member.

1 Knight, Ethics of Competition, pp. 175-76.
Certain advantages may be said to attach to the use of the concept of general equilibrium. Clearly, the results derived spring directly from human volition; for the economizing activities of micro-economic units give rise to those individual supply and demand curves which form the basis of the system. In dealing with the motives of individual economic units, we avoid an excessively mechanical or deterministic approach to economic problems. And this danger is considerable; indeed, the desire for scientific precision often leads investigators to slur over those root principles of human volition which activate and inform the economic system.

Furthermore, the analysis of rational rules of action for the individual leads to a consideration of the stability of his maximum situation. And this is the basis for rules determining the forms of the individual supply and demand schedules near the equilibrium values. Such rules may often be transferred to an analysis of the industry. In this way, the direction of many changes can be predicted on the industry level. Moreover, similar sorts of goods may often be grouped together, and perturbations affecting them treated in a way common to industry analysis. In this way, rules are derived which may be used to forecast the effects of given changes.

Finally, analysis may be directed to any desired level - the inter-industry level, the particular industry, or the micro-economic unit. Flexibility is the keynote of general equilibrium.

In setting up his theoretical construction Keynes drops this theoretical complexity in favor of a simpler approach: a Keynesian
might say, a more **straightforward** approach. Under his system attention is concentrated on such questions as total employment, output as a whole, the quantity of money. Several problems immediately present themselves: First, can the quantities under consideration actually be summated; and, if a technique of summation is found, the issue is transferred to the economic significance of the summation. In essence, this is equivalent to questioning whether the relevant functions may be combined to produce a single function, applying to society. Second, is there, in reality, any significance in an equilibrium of aggregate magnitudes? For, there exists the possibility of the disequilibrium of smaller economic units within the aggregative equilibrium. Then, the aggregative "equilibrium" turns out not to have been stable, after all, and will move so as to take up a new and stable position. In short, this is equivalent to saying that the necessary and sufficient conditions of equilibrium may not be determined without an investigation of the structure which underlies the aggregates.

The question may then occur to the dispassionate reader: why bother with aggregates at all? To this question there is no really adequate answer.¹

From a purely theoretical viewpoint, the method of aggregative equilibrium betrays a certain crudity. For example, consider

¹ As Schumpeter says, "It is obvious that this kind of equilibrium is compatible with most violent disequilibria in every other sense. And these disequilibria will assert themselves by changing the given situation including the aggregative quantities themselves." *Business Cycles*, I, p. 43.
the monetary equilibrium implied in the equality of saving and investment. In his *Treatise on Money* Mr. Keynes set out to define an equilibrium by certain "fundamental equations." His analysis went about as follows. Income is to be defined as payments to factors of production, inclusive of "normal" factor profits. Saving is the excess of income over consumption, and investment is unconsumed output. In equilibrium, saving is equal to investment; but an excess of investment over saving implies the emergence of profit, production expands, and the system will move until saving once more reaches equality with investment. A similar excess of saving over investment implies a contraction of activity until equilibrium is restored.

For some time this neat and pleasantly simple explanation of economic change passed muster. But Hawtrey pointed out the following relationship: on the above definitions, the difference between saving and investment is the profit, and cannot therefore be a cause of profit or economic expansion. In this way, the tautological character, the lack of real enlightening analysis in such a condition, was demonstrated.\(^1\)

This danger of oversimplification occurs wherever economic theory deals with a few fundamental variables, defined in peculiar

\(^1\) Furthermore, as Lutz has pointed out in a lucid and penetrating article, "Since S (saving) and I (investment) may be equal in the depression, it is evident that their equality cannot provide a norm of monetary policy, at least not by itself. We can use it solely as an instrument of analysis." "The Outcome of the Saving-Investment Discussion," *Quarterly Journal of Economics*, LIII (1938), p. 598. This point was founded on the Robertsonian definitions of saving and investment. Keynes' definitions were designed to avoid this by defining income as inclusive of "normal" profits. But again the criterion of normality is undefined, and this omission leaves the analysis in the air.
ways, and related by simple conditions. Unfortunately, economic relationships are not simple; and any attempt to make them appear so is subject to the danger that the analysis will lose touch with the very object of analysis.\(^1\) While the body of analysis in the General Theory is considerably more complex than that of the Treatise, the danger persists, even after expansion of the number of variables and conditions. In short, the solution to the problems analyzed by aggregative equilibrium may lie in the very structure of those aggregates.

How, then, may this system be defended? A justification of the study of aggregative equilibrium might proceed on the following lines. In the first place, the structural changes encountered in reality may be small in a short period of time. Thus it may well be that those changes which do occur in structure will disrupt the equilibrium in a long, but not in a short interval. Secondly, in these instances in which the aggregate functions may be correctly derived from the corresponding micro-economic functions, a genuine summation of a workable character may be achieved. By restricting attention to those magnitudes and functions which may be thus summed, a partial aggregative equilibrium may be achieved— that is, an equilibrium applying to some sector of the entire economy.

\(^1\) Dr. Schumpeter's comment on the General Theory states this danger as positively true of that analysis: "Ricardian as the book is in spirit and intent, so is it in workmanship. There is the same technique of skirting problems by artificial definitions which, tied up with highly specialized assumptions, produce paradoxical-looking tautologies, and of constructing special cases which in the author's own mind and in his exposition are invested with a treacherous generality." "Review of General Theory," Journal of the American Statistical Ass'n., XXXI (1936), p. 792.
Granting that the micro-economic units and the industries are not seriously out of equilibrium in short periods, many instructive lessons may be derived from a study of aggregative equilibrium. First, it is possible to group factors in such a way as to concentrate on the basically important factors. As Hicks puts it, "I must confess that, as I have worked with Mr. Keynes' book, I have been amazed at the way he manages ... to cut through the tangle of difficulties that beset him, and to go straight for the really important things."¹

Secondly, providing that techniques for aggregating the chosen magnitudes are available, the statistical problem is likely to be manageable. Since the functions to be derived are limited in number, and the interrelations between functions simple, elementary statistical techniques will often produce satisfactory results. As compared with the problem of checking statistically the theory of general equilibrium, the analysis involved in verifying the General Theory is absurdly simple. Furthermore, prognostication becomes feasible, when statistical analysis can verify or disprove past predictions.²

Thirdly, the simplicity of the system recommends it to the legislator. The very compactness of the theory, the simplicity of the relationships assumed, and the easy results derived with a minimum of argument - these are qualities well adapted to appeal to the legislator. And this means that the General Theory may serve a

¹ Value and Capital, p. 4.

useful purpose merely by inculcating a few elementary truths in the capricious legislative mind.\footnote{On occasion Keynes is not averse to dealing out a few stiff cracks on the knuckles to laymen who presume to discuss economic matters. When Sir Harry Goschen of the National Provincial Bank protested against too much discussion of inflation and deflation, and advocated letting "matters take their natural course," Keynes remarked tartly, "Is it more appropriate to smile or to rage at these artless sentiments? Best of all, perhaps, just to leave Sir Harry to take his natural course." \textit{Fortune}, XXIX, (1944), p. 253.} Indeed, however perfect the theory of general equilibrium, this system can scarcely hope to compete in attractiveness with Keynesian analysis.

In passing to that summary of Keynesian economics found in "The Principle of Effective Demand," we encounter the problem of basic units. These basic units constitute the building material out of which the system is constructed; accordingly, their analysis constitutes the overture to the investigation which follows.

Certain Basic Units and Assumptions

As Keynes puts it, "In dealing with the theory of employment I propose . . . to make use of only two fundamental units of quantity, quantities of money value and quantities of employment."\footnote{G. T., p. 41.} Clearly, sums of money are reducible to homogeneous units and may therefore be summed directly; but it is not immediately evident how the quantities of the several grades of labor are to be made homogeneous, to the end that they may be summed. The solution to this problem may be given as follows. Quantities of labor of varying
grades may be summed by regarding a unit of a given grade of labor as a multiple of a unit of unskilled labor. This multiple depends upon "the wage unit," the price of a unit of this grade of unskilled labor, and the price of the grade of labor in question. Suppose the price of unskilled labor to be $1 an hour; then a unit of another grade, receiving $2 an hour, represents two such units in the Keynesian schematism. At first blush, no difficulty is to be found here; the actual difficulty is to be found in the summation of certain functions involving diverse grades of labor.

A certain time unit must be chosen to satisfy a basic condition of the Keynesian doctrine. Then Mr. Keynes says, "We take as given . . . the existing quality and quantity of available equipment." He must base this assumption on a particular sort of time period. Indeed, is it possible for the capital equipment of society to be fixed while increasing continuously through the investment process? Only in an indivisible instant of time! Once admit the passage of a fleeting instant and capital equipment will have been enlarged.

Thus, it will be necessary to choose a small slice of time, so proportioned that the change in capital equipment during the interval may be neglected.

1 Ibid., p. 245.

2 This sort of time period is often called an "operational time period." The period is to be differentiated from a "clock time period," in that it does not represent the passage of a given fixed interval. Rather, the interval which elapses may vary as between two or more of such periods.

An operational time period is an interval so adjusted as to satisfy some condition. For example, Marshall's long period is an interval of time sufficiently long to allow the factors of production to become fully adjusted to the demand. Capital becomes a fully variable factor, and the quantity of capital appliances is so adjusted
In general the existing quantity of capital equipment is large relative to the increment which can be produced in a year. Accordingly, this interval need not be chosen very small to satisfy the given condition. Yet it is clear that the necessity of utilizing an "operational time period" imposes a certain restriction on the argument; and it is not clear that Mr. Keynes has understood this.

Our digression completed, we return to the fundamental purpose of this chapter, the interpretation and methodological criticism of Keynes' basic approach. In some senses, the epitome of the aggregative method is to be found in Keynes' "Principle of Effective Demand." Our procedure is a detailed analysis, followed by some methodological reflections.

The Basic Theorem

In a preliminary chapter, entitled the "Principle of Effective Demand" Keynes advances a sweeping simplification of the concept of economic interdependence. Here he seeks to summarize the great relations of production and consumption by means of the

that the supply price of capital is exactly equal to the sum of the discounted returns. And other factors, fixed in the short run, become fully variable and shall upon may be adjusted to the demand. See, Margot, Theory of Prices, II (New York: Prentice Hall, 1942), ch. VII.

Pigou makes this point in his review, "Mr. Keynes' General Theory," Economics, III (1936), p. 122. Theoretically, the fixity of capital equipment can only be perfectly satisfied at a point of time. Consequently, the system is merely adapted to analyze conditions at one given point of time. Now if capital equipment were incorporated in the system as a variable, and a new condition advanced to determine this magnitude, the theoretical perfection could be maintained. From a practical point of view, however, the growth of capital equipment will not affect significantly the relevant functions in a short period of time.
"Aggregate Demand Function" and the "Aggregate Supply Function.

Imposing by reason of a certain grand simplicity, the apparatus deserves our careful attention. In Keynes' words, "Let \( Z \) be the aggregate supply price of the output from employing \( N \) men, the relationship between \( Z \) and \( N \) being written \( Z = \phi(N) \), which can be called the Aggregate Supply Function. Similarly, let \( D \) be the proceeds which entrepreneurs expect to receive from the employment of \( N \) men, the relationship between \( D \) and \( N \) being written \( D = \int(N) \), which can be called the Aggregate Demand Function.\(^1\)

\( Z \) and \( D \) are total money sums related to actual units of employment. To illustrate how these relations are derived suppose that the marginal value productivity schedule of labor were taken as the firm's schedule of demand prices for labor, given the price of the product. If this price varies, then the quantity of product produced will undergo an induced variation. And by connecting points representing the money demand corresponding to a given level of employment, a sort of supply curve may be derived, which is a variation of the type employed by Auspitz and Lieben.

In constructing this curve, it is appropriate to start from the total product schedule of the firm connecting the quantity of labor with total product. Then, multiplying total product by price, we have a curve relating total revenue to the quantity of labor. Let it be called the total revenue curve. Clearly, the entrepreneur will wish to maximize the excess of total revenue over total cost; and the necessary condition for this maximum is the well-known

\[^1\text{G. T., p. 25.}\]
relation - the wage of labor equals marginal value product.

Graphically, this implies finding points on the total cost curve and the total revenue curve, common to a given level of employment, where the slopes of these curves are equal. At a level of employment, 0 L, this condition is satisfied. But this is not very helpful, because we require a supply curve connecting total revenue and the quantity of labor. So far there is a point, not a curve.

What is required is a series of upward shifts in the total revenue curve while the total cost curve remains in a fixed position; and these upward shifts need be derived from a rise in price; for the total product curve is assumed fixed. By finding the equilibrium point for each such total revenue curve and connecting the points we derive the desired relation, the firms Z curve.
In general, the Z curve will not start from the origin, because the equilibrium conditions state that the firm must be operating within the stage of diminishing returns, meaning here - diminishing marginal physical product of labor. Accordingly, the point of origin of the Z curve cannot lie to the left of D the point of diminishing returns on the TR curve.

A tabular approach may serve to dispel any lingering doubts which the reader may entertain.

<table>
<thead>
<tr>
<th>Units of Labor</th>
<th>Marginal Wage Cost (in Dollars)</th>
<th>Marginal Physical Product</th>
<th>Total Product</th>
<th>Price (in Dollars)</th>
<th>Marginal Value Produce (in Dollars)</th>
<th>Z (Total Revenue) (in Dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.046</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5.00</td>
<td>50</td>
<td>50</td>
<td>0.046</td>
<td>2.30</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>5.00</td>
<td>100</td>
<td>150</td>
<td>0.046</td>
<td>4.60</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>5.00</td>
<td>120</td>
<td>270</td>
<td>0.046</td>
<td>5.52</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>5.00</td>
<td>110</td>
<td>380</td>
<td>0.046</td>
<td>5.00</td>
<td>17.48</td>
</tr>
<tr>
<td>25</td>
<td>5.00</td>
<td>70</td>
<td>450</td>
<td>0.070</td>
<td>5.00</td>
<td>31.50</td>
</tr>
<tr>
<td>30</td>
<td>5.00</td>
<td>60</td>
<td>510</td>
<td>0.083</td>
<td>5.00</td>
<td>42.33</td>
</tr>
<tr>
<td>35</td>
<td>5.00</td>
<td>50</td>
<td>560</td>
<td>0.100</td>
<td>5.00</td>
<td>56.00</td>
</tr>
<tr>
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<td>5.00</td>
<td>40</td>
<td>606</td>
<td>0.125</td>
<td>5.00</td>
<td>75.00</td>
</tr>
<tr>
<td>45</td>
<td>5.00</td>
<td>30</td>
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<td>0.167</td>
<td>5.00</td>
<td>105.21</td>
</tr>
<tr>
<td>50</td>
<td>5.00</td>
<td>20</td>
<td>650</td>
<td>0.250</td>
<td>5.00</td>
<td>162.50</td>
</tr>
</tbody>
</table>

In equilibrium, marginal wage cost is equal to marginal value product. Under perfect competition, the marginal wage cost for the individual firm is constant, by reason of constant factor prices. So, in the above table, the firm reaches a stable equilibrium at a level of employment wherein 20 workers are hired, when the price of the product is $.046. If the price is successively raised, the level of employment at which marginal wage cost and marginal value product will be equated must likewise be raised. In
essence, every new price means a new equilibrium.¹

What the preceding table comes to is this: the several
prices listed will equate marginal wage cost and marginal value
product at these several levels of employment. Then, multiplying
these prices by the corresponding total outputs, we secure the
total revenue which will just induce the firm to hire the correspond­
ing number of workers.

To proceed from the individual firm to the industry is a big
step; but, assuming symmetry among the individual firms and assum­
ing that they all employ the same single grade of labor, no un­
manageable problem will arise. And the result will be a macro­
scopic supply curve relating total proceeds (money demand) to
employment. This is the Keynesian Z or aggregate supply curve,²
\[ Z = \phi (s). \]

¹ Keynes makes it a rule to consider labor as the only
variable factor of production in the short run. However, he does,
at a later point offer some points relating to raw materials as
a variable element which changes with the level of employment.
In the above analysis, this interpretation is not followed out;
but the argument can be adjusted to take care of this circum­
stance by adding in a marginal raw materials cost varying with
the level of employment.

Since labor is treated as the only variable factor,
the law of marginal productivity need be applied only to labor,
and this will serve as an explanation of the forces whereby the
entrepreneur achieves the maximum net profit, and thus hits upon
the production of the most profitable output.

The above condition regarding the equality of marginal
value product and marginal wage cost is commonly stated in a little
different form, when the application is made to competition. Then
it is usually stated that the wage is equal to the price of the
final product times marginal physical product. This is the condi­
tion above.

² To illustrate what will happen when the individual functions
are summed, consider the following logic. \[ \sum_{i=1}^{s} Z = \sum_{i=1}^{s} F_{i}(X) = sF(X), \]
under symmetry assumptions, where \( s \) is the number of firms. Now
consider the polynomial \( aX^n + bX^{n-1} + \ldots + C \), which may be
The interpretation of the "Aggregate Demand Function" is less complex. At the outset it is necessary, in this analysis, to consider the income of society. On the above analysis, the "level of proceeds" received by entrepreneurs is related to the level of employment by the "aggregate supply function." This "level of proceeds" is nothing else than the expenditure of society, or its income. The level of income, then, is related to employment by this function.

It may be stated as a fundamental principle that the expenditure of society on consumption varies with the level of income. Since consumption expenditure varies with the level of income, and since income varies with the level of employment, the consumption expenditure may be represented as varying directly with the level of employment. Careful consideration of one point is needful here: this aggregate consumption demand is not equal to income, in general. At low levels of income, consumption may exceed income, whereas at high levels of income, the reverse will be true. At any rate, this aggregate consumption demand may be represented as a variable function of employment. Let it be written $D_1 = \Upsilon(N)$.\[1\]

\[1\] Several matters may be considered here. First, it may not be self-evident to some readers that consumption may exceed income. But reflection reveals that depreciation allowances may be consumed, and consumption increased above income by this means. The algebra may be expressed as follows. $Y = Z = \Phi(N)$, where $Y$ represents income. Then the so-called propensity to consume gives expenditure on consumption as a function of income: $C = \Psi(Y)$, where $C$ represents expenditure on consumption. After eliminating the variable $Y$ between the two equations we secure the function $D_1 = C = \Upsilon(N)$. The volume of investment, another form of expenditure written $D_2$, must be added to $D_1$ to secure total expenditure. Let $D_2$ be given. Then $D_1 + D_2 = D = \Phi(N)$.\[2\]
To derive the total expenditure of society a sum, $D_2$, representing expenditure on investment, needs to be superadded to $D_1$, expenditure on consumption. This sum, $D_2$, is supposed given; its value is determined by forces which are not analyzed until later.

The final equilibrium, then, may be represented by an equation $D_1 + D_2 = D = Z$, representing the equality of proceeds necessary to support a given level of employment and the aggregate expenditure associated with that level of employment. Thus, in equilibrium, $\phi(N) = f(N)$. Graphically, the situation may be built up as follows. In the accompanying diagram, the segment TR, representing aggregate demand at employment OR, is derived by adding the fixed sum RS, representing investment, to the quantity VR, representing the level of consumption corresponding to employment OR. Thus $RS + RV = RT$.

At the point E, where the two curves cross, an equilibrium is reached. Here the gap between income and expenditure or consumption (the vertical distance between the Z curve and the $D_1$ curve) is exactly equal to the quantity of investment (the vertical
distance between the D and the D_1 curves. From this simple analysis Keynes concludes that "the volume of employment is given by the point of intersection between the aggregate demand function and the aggregate supply function."\(^1\)

Here it will be appropriate to sound a note of caution. We must remember, the curves themselves are complex phenomena. As Edgeworth said of his foreign trade curves, "A movement along a supply- and demand curve . . . should be considered as attended by rearrangements of internal trade; as the movements of the hand of a clock correspond to considerable unseen movements of the machinery."\(^2\)

In a sense, this diagram spells out the fundamental message of the General Theory.\(^3\) That message is: the economic system does not automatically create its own demand. If that were the case, the Z curve would represent an indeterminate path along which the system might oscillate, or settle, in some indeterminate fashion.

In reality, however, as production is expanded without limit, a gap between production and consumption develops which drives the system back to that equilibrium in which they are equal. There is only one such point. It lies at E; here the difference between income and consumption is exactly equal to expenditure or

\(^1\) G. T., p. 25.


\(^3\) We may as well summarize the algebra: three equations determine D_1, Z, and N, while D_2 is given. They are (1) \(Z = \Phi(\mu)\), (2) \(D_1 = \Phi(Z)\), and (3) \(D_1 + D_2 = Z\). By elimination of \(Z\) from (2) we secure the desired functions whose equality is represented by (3).
investment. At a higher level of employment, the gap between income and consumption is greater than investment, demand falls short of the expected proceeds which will induce entrepreneurs to maintain that level of employment; and the quantity of employment is driven downward.

At a lower level of employment, the gap between income and consumption is less than investment, demand oversteps the expected proceeds which will induce entrepreneurs to maintain that level of employment; and the quantity of employment is driven upward. Thus a unique level of employment is the outcome of a given situation. Say's law falls by the wayside.

As a summary of the fundamental point implied in the General Theory, this account is no doubt desirable. It elucidates at least one fundamental point - namely, that supply cannot be said to produce its own demand. Here is a simple technique for showing the falsity or limited significance of this famous proposition.

Mr. Robertson asserts that "there is a verbal obscurity in Mr. Keynes' exposition of his central apparatus which may have troubled others than myself."¹ Inasmuch as Mr. Keynes does not explain the derivation of his aggregate supply function, this is something of an understatement. However, Mr. Robertson appears to sense the presence of an actual hiatus in the apparatus. Here is his logic:

Suppose that entrepreneurs increase employment and output beyond the equilibrium point; income will increase, and consumers

¹ Robertson, Essays in Monetary Theory (London: P.S. King, 1940), p. 114.
will spend a part of their additional income on consumer's goods. This increment of receipts, while less than the increment in the aggregate supply price, \( Z \), is still greater than the increment in factor cost. Thus the increment in employment would yield a surplus, would therefore be profitable and desirable.

At first glance this argument is quite plausible. However, if our interpretation of the aggregate supply function be correct, this criticism is unfounded, and based on a logical error. The aggregate supply price is that curve connecting receipts and employment, along which the marginal profit is zero.

What may seem paradoxical to the reader is that the \( Z \) curve represents the locus of points at which the entrepreneur is just induced to maintain that level of employment. Entrepreneurs will never accept less than the sum given on the \( Z \) curve, at a given level of employment. So Mr. Robertson's argument clearly does not apply here. This may be shown more clearly by reverting to the derivation of the supply curve.

At a level of "proceeds" \( OZ_1 \) entrepreneurs will find it profitable to hire \( ON_1 \) workers. At this level of employment marginal value product equals marginal wage cost - a condition evidenced by the equal slopes of \( TC \) and \( TR_1 \) at \( E_1 \). In order to
induce entrepreneurs to expand output, the price must rise at a
given level of output, thus raising the $TR_1$ curve to $TR_2$. In the
absence of such a rise in "proceeds," the entrepreneur would have
no inducement to expand employment. Thus a movement from $E_1$ to
$E_2$ is necessary to induce the entrepreneur to expand output; in
so doing, if the increased demand is in evidence, he will procure
an increment of profit. Yet at each several point on the curve
no marginal profit exists, given the level of demand. So that
when Mr. Robertson asserts that the increment in income, arising
out of added employment, while less than the increment in the
supply price, $Z$, might exceed the increment in factor cost, he
is on the wrong track. The increment in $Z$ is the minimum incre­
ment in receipts required to cause the expansion; and the $Z$
curve is derived from a consideration of the process whereby
the entrepreneur weighs the increment in factor cost against the
increment in receipts.

Whereas this supply and demand apparatus appears to be
sufficiently well constructed to withstand an assault on its
logic, its methodological weakness is quite evident. It must not
be supposed that the intricate workings of economic interdependence
can be effectively summarized by the intersection of an "aggregate
demand function" and an "aggregate supply function." Can the
gravitational equilibrium of the solar system be represented by
the intersection of two curves? Hardly! Furthermore, the
statistical techniques involved in aggregation may well drain the
apparatus of any precise economic significance. If this apparatus
is to be useful, the uses must follow a recognition of its limitations.
The major advantage of the apparatus lies in its simplicity; it has heuristic value, in that it indicates by simple means the importance to society of effective demand. But let it be recognized that any system of thought which thus simplifies the economic process involves enormous oversimplification. Having absorbed the simple lesson taught by the apparatus, we may then recognize that the very lesson taught is inadequately portrayed.

Once the mind has become accustomed to the complex interplay of forces represented by general equilibrium analysis, simpler analysis seems unsafe. Yet, unsafe or not, such simpler methods can be used to point out certain fundamental truths which a more complex apparatus can scarcely portray. Such is the value of this "principle of effective demand" - a fundamental truth, simply demonstrated. While the proof is hardly rigorous, it is convincing. And such proofs have their place in economics; no harm can result so long as the reader remains aware of the stringent assumptions on which the analysis is constructed.

1 In his review of the General Theory Schumpeter had this to say of the aggregate demand and supply apparatus: "... the old supply and demand apparatus renders its very limited service, only if applied to individual commodities, strictly speaking to individual commodities of relatively small importance, and that it either loses or changes its meaning if applied to comprehensive social aggregates ... Mr. Keynes speaks of aggregate demand in the one case and aggregate supply in the other and makes them yield a unique 'point of intersection.' There is as little justification for this extension of the 'Marshallian Cross' as there is for its application to the case of money, which has remained a besetting sin of the Cambridge group to this day." "Review of General Theory," p. 793. Our critique is less severe than Dr. Schumpeter's, because we see in the little apparatus an heuristic value, for littler minds than that possessed by Dr. Schumpeter.
CHAPTER III

Monetary Equilibrium—Saving and Investment

In the work which Mr. Keynes accomplished before the publication of the General Theory he showed himself to be a specialist in the field of money. Insofar as his ideas touched upon the theory of economic fluctuations they were suffused with monetary implications. In the General Theory, however, Mr. Keynes began to veer towards the study of the short period equilibrium of the economic system taken as a whole. In his own words: "When I began to write my Treatise on Money I was still moving along the traditional lines of regarding the influence of money as something, so to speak, separate from the general theory of supply and demand... This book, on the other hand, has evolved into what is primarily a study of the forces which determine changes in the scale of output and employment as a whole; and, whilst it is found that money enters into the economic scheme in an essential and peculiar manner, technical monetary detail falls into the background."

Apart from any claim to originality or successful pioneering in this field, his effort must be pronounced commendable. Prior to the publication of this work a widespread belief existed to the effect that fluctuations in economic activity could be fruitfully analyzed from an exclusively monetary point of view, or

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1 G. T., preface, pp. vi, vii.
from a point of view emphasizing "real" phenomena. That
distinguished writer, Mr. Hawtrey, even uttered the unfortunate
words - the trade cycle is "a purely monetary phenomenon."\(^1\)
Such a pronouncement probably exerted an undesirable effect,
especially when quoted out of context to readers unfamiliar with
the greater part of his work. More important than such individual
pronouncements was the lack of an explicit apparatus designed to
analyse the interrelations of "monetary" and "real" phenomena.

True, such a general system was to be found in Walras' labious, but monumental work, *Élémnts d’Économie Pure*. Yet
this work was written in French; furthermore, the ideas were
entrenched behind a formidable array of mathematical obstacles.
It is not surprising, therefore, that Walras' influence extended
only to advanced circles outside the continent. Accordingly,
Mr. Keynes' renewed emphasis on the synthesis of monetary theory
and the theory of supply and demand probably exerted a salutary
effect in Anglo-American circles.\(^2\)

\(^1\) *Monetary Reconstruction* (2nd ed.; London: Longmans
Green, 1936), p. 132.

\(^2\) It would be quite futile to enter into an extended
discussion of Mr. Keynes' claims to priority in this connection.
Mr. Marget has analyzed this point exhaustively in his *Theory
of Prices*, Vol. II. See "output" in the index. Regardless of
emphasis directed to such a synthesis in earlier writers, the
fact is that an explicit analysis of output was generally lacking
in earlier writings on money; or if there were an analysis of
output, then the monetary aspects were generally slurred over.
Thus, Marshall's *Principles* is constructed on the assumption "that
all values are expressed in terms of money of fixed purchasing
power," p. 593. On the other hand, both Hawtrey and Robertson
have been explicitly concerned, in the main, with monetary
fluctuations. What is important in this connection is not a
series of references to the interrelations of "real" and "monetary"
phenomena, but an apparatus indicating formal and substantive
unity in subject matter. This deficiency Keynes has attempted to
supply.
Where Mr. Keynes is particularly successful, nevertheless, is in his summary of monetary forces. Here his powers of analysis reach their zenith, and it is with this part of his analysis that we shall be primarily concerned in the following chapters. Before thus particularizing the analysis it will be useful to summarize succinctly those fundamental relationships which make up his system of thought.

Variables

(1) Money income = Y
(2) Saving = S
(3) Investment = I
(4) Consumption = C
(5) Rate of interest = i
(6) Employment = N
(7) Level of output = O

Equations

(1) Y = C + I
(2) C = f(Y, i)
(3) M = L(Y, i)
(4) I = I(C, i)
(5) S = I
(6) Y = E(N)
(7) O = F(N)

Given

Quantity of Money = M

An elementary proposition of mathematics states that the unique solution of a given number of variables, requires an equal number of hypotheses (functional relationships) connecting the variables. In order to determine the seven variables indicated above, seven relationships are required; these necessary relationships are stated above in the form of equations. In Keynesian analysis the quantity of money is taken as given - it is treated as a constant. Should the quantity of money be treated as a variable, it would be necessary to introduce an additional equation to determine it.
Within this overall structure there is a sub-system which may be entitled "the monetary equilibrium." Consider the first five variables and equations. Each of these variables — money income, saving, investment, consumption, and the rate of interest — has a monetary connotation. And their values will be determined by the first five equations. It is this sub-system which economists have found most fruitful for purposes of analysis; and, accordingly, it will occupy the major share of our attention.

Definitions - Income, Saving, and Investment

"When I use a word," Humpty-Dumpty said, "it means just what I choose it to mean — neither more nor less." (Through the Looking Glass)

First to claim our attention are the much controverted definitions of income, saving, and investment. The failure of opposing schools to comprehend each other's definitions has led to a wordy war out of which much sterile controversy has arisen. In passing over these matters, however, some matters of substance may be brought to light by a consideration of the saving-investment controversy.

1 J. M. Keynes says justly: "If some economists waste time by treating problems of definition in too great detail, others waste more time in verbal disputes unrecognized as such. Failing to give precision to their own use of terms, and failing also to appreciate the sense in which the same terms are used by other writers, they fall easily a prey to the fallacy of ignoratio elenchii. Much controversy in economics might be avoided by a clear understanding of the senses in which words are used, and the relation of the different meanings one to another." Scope and Method of Political Economy, pp. 141-52.
Fundamental in these matters is the definition of income.\(^1\) Keynes' position may be summarized as follows. During a given period of time an entrepreneur will have sold finished output to consumers or other entrepreneurs for a sum, \(A\). In order to arrive at a definition of entrepreneur's income it is necessary to make certain deductions. One such deduction is the entrepreneur's factor cost, \(F\). Another relates to certain avoidable sacrifices in the use of equipment which are summarized by the term "user cost."\(^2\) Let \(G\) represent the actual value of the capital equipment at the end of the unit period, including the value of stocks of finished and unfinished goods. This value, \(G\), is the result of the entrepreneur's having actually depreciated that equipment through use, and on the other hand, having maintained it during the period. Now if the equipment had not been used, and an optimum sum, \(B'\), had been spent on it during the period, this capital equipment would have had a value, \(G'\), at the end of the period.

Clearly, the quantity, \(G' - B'\), is the net value of equipment which the entrepreneur might expect to have at the end of the period, if he did not use it to produce the output, \(A\). The quantity, \(G - (G' - B')\), represents the excess (or deficiency) in

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1 Dr. Viner says this of Mr. Keynes' use of words in the General Theory: "no old term for an old concept is used where a new one can be coined, and if old terms are used new meanings are generally assigned to them." Mr. Keynes on the Causes of Unemployment. Quarterly Journal of Economics, LI (1936-37), p. 147.

2 G. T., p. 53.
the actual value of equipment over its value if left unutilized, but at the same time maintained. This sum measures a change in capital values through investment. Accordingly, the quantity, \( A_1 - G - (G' - B') \), where \( A_1 \) represents purchases from other entrepreneurs, measures the excess of purchases from other entrepreneur over investment in equipment, during the period. It is known as "user cost," and measures "what has been sacrificed (one way or another) to produce A."^1

Entrepreneur's income is equal to \( A - (U + F) \), the excess of sales over user plus factor cost; but factor cost is the income of the rest of society. Hence \( A - (U + F) + F = A - U \) is the income of society as a whole. A further principle is needed to complete the picture, namely, the consideration of "an involuntary loss or gain in the value of . . . capital equipment."^2 While this quantity, \( V \), is suffered involuntarily, it is not unforeseen, in general. It includes such insurable misfortunes as "a change in market values, wastage by obsolescence or the mere passage of time, or destruction by catastrophe."^3 Subtracting \( V \) from gross income, \( A - U \), we secure net income, \( A - U - V \). This calculation does not include an adjustment for "unforeseen changes in

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1 *Ibid.*, p. 55. What may not be self-evident to the initiate is the position of the sacrifice involved in the wastage of machinery. Now that we refer to the term, \( G - (G' - B') \), as investment, this element seems to have disappeared. But reflect that a reduction in \( G \) in consequence of wear and tear will reduce the above term, investment. Then less is subtracted from \( A \) in the total expression for user cost, and so user cost is greater by this amount. The riddle is solved.


market values, exceptional obsolescence or destruction by
catastrophe which is both involuntary and ... unforeseen."¹

These windfall losses are disregarded in reckoning income.

That it is possible to draw a precise line of demarcation
between involuntary, but foreseeable losses, V, and involuntary,
and unforeseeable losses seems doubtful. Accordingly, a certain
vagueness persists in this concept of net income.

Following directly on the heels of the definition of income
are the definitions of saving and investment. Saving is defined
as the excess of income over consumption. Consumption is the
excess of total sales over sales from one entrepreneur to
another, i.e. A - A₁. Income minus consumption, then, equals
A - U - V - (A - A₁) = A₁ - U - V. Investment is "the current
addition to the value of the capital equipment which has
resulted from the productive activity of the period."² It is
thus "that part of the income of the period which has not passed
into consumption," or A₁ - U - V.³

Accordingly, savings equals investment, identically:
A₁ - U - V = A₁ - U - V. The two terms are the same magnitude,
when taken in the aggregate. And here is to be found a fruitful
source of controversy.

¹ Ibid., p. 57.
² Ibid., p. 62.
³ Ibid., p. 62.
Mr. Hawtrey asserts that "in fact saving and investment are defined to be identical; they are two different names for the same thing. That is so because income is so defined as to be identical with output." And he goes on to say that the word "saving" could be substituted for "investment" in any context in which the latter is used.

Mr. Keynes, however, denies the validity of this comment for several reasons. In the first place, he asserts that "it is only aggregate saving and investment which are equal." Moreover, "acts of saving and investment are frequently or usually performed by different people." Thus, since an individual may save more or less than he invests, the distinction continues to apply to individuals. Finally, "aggregate saving and aggregate investment, in the senses in which I have defined them are necessarily equal in the same way in which the aggregate purchases of anything are equal to the aggregate sales. But this does not mean that 'buying' and 'selling' are identical terms, and that the laws of supply and demand are meaningless."

The controversy between Mr. Hawtrey and Mr. Keynes need not be pursued here; Mr. Hawtrey admits that the identification of the two terms can be carried out only in the aggregate, but

1 Capital and Employment (London: Longmans Green, 1937), p. 174
3 Ibid., p. 249.
4 Ibid., p. 249.
denies that he ever intended the criticism to be taken in any other way. 1 But he does take exception to Mr. Keynes' analogy between the equality of saving and investment, on the one hand, and that of supply and demand, on the other. In fact, he asserts: "Purchases and sales are also 'different aspects of the same thing.' And surely, if demand were defined to mean purchases and supply to mean sales, any proposition about economic forces tending to make supply and demand equal, or about their equality being a condition of equilibrium, or indeed a condition of anything whatsoever, would be nonsense."2

In short, an exchange is defined to be a condition in which purchases and sales are identically equal. And, to be sure they are always equal after the exchange. But this does not mean that economic theory defines supply and demand so that they are identically equal in the aggregate. It is the function of the marketing process to make them equal; only in retrospect are they equivalent, and only then do supply and demand become purchases and sales. Accordingly, we are compelled to agree with Mr. Hawtrey that the analogy to a market completely breaks down. As Keynes defines saving and investment, there is no interaction whereby the magnitudes are equated in equilibrium. On Keynes' definitions they are identical at all times, in the aggregate.3

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2 Ibid., p. 436.
3 Mr. Robertson has this to say on the matter, "Mr. Keynes' critics have not charged him, as he seems to think, with portraying the processes of saving and investing as identical. They have merely
In tracing a path through the jungle of controversy surrounding these definitions we need to take account of some abstract considerations. In the analysis of terminology it is necessary to differentiate between those terms having a unique signification to all men and those terms which are tied to a particular set of ideas. As yet, economic science has not progressed to that point whereat economists agree on the meaning of basic aggregative concepts such as income, saving, investment and the quantity of money. As a rule, each theorist pitches upon the set of definitions which best suits his purpose. And Mr. Keynes provides no exception to this rule.

The General Theory is so framed as to provide an explanation for a given level of income and employment; no attempt is made to put forward an explicit analysis of the continuous change of all economic magnitudes in time - his system is quasi-dynamic. Clearly, the pursuit of this form of analysis does not require those definitions of terms which exhibit change - at least, the need is not so great as it is in the science of economic dynamics. Accordingly, Mr. Keynes has not merely chosen to define his terms in such a way that they do not facilitate an analysis of change;

maintained that he has so framed his definitions that amount saved and amount invested are identical, that it therefore makes no sense even to inquire what the force is which ' ensures equality' between them: and that since the identity holds whether money income is constant or changing, and, if it is changing, whether real income is changing proportionately, or less than proportionately, or not at all, this way of putting things does not seem to be a very suitable instrument for the analysis of economic change." "Alternative Theories of the Rate of Interest," Economic Journal, XLVII (1937), pp. 428-29.

1 Consider Mr. Hicks' words: "we shall be well advised to eschew income and saving in economic dynamics. They are bad tools which break in our hands." Value and Capital, p. 177. While this opinion may be extreme, it indicates that care is required in the approach to these basic aggregative concepts.
rather they seem to defy such an analysis, on superficial inspection.

What is particularly confusing in the Keynesian terminology is the failure to differentiate between the results of distinct and separate processes. Where terms are so defined that actions taken by different groups lead to identical results, a paradoxical tone is imparted to the analysis. The mind of the reader seeks after the interaction of economic magnitudes, not their equality by definition.

Perhaps this may be illustrated most clearly by the definition of income. This magnitude is defined to be the value of output, or the expenditure of society in purchasing output. Clearly, the receipt of funds, as income, and their expenditure on output are differentiable processes. Yet the results of the two processes are assumed to be identical in the aggregate, leading to the famous identity of saving and investment.

\[
\text{Income} = \text{value of output} = \text{consumption} + \text{investment}
\]

\[
\text{Saving} = \text{income} - \text{consumption}
\]

\[
\text{Therefore saving} = \text{investment}^1
\]

What logic lies behind the identification of the two magnitudes, income and expenditure on output? Essentially, this - the expenditure of society is destined to be received by the several producers, and these receipts will be resolved eventually into payments to the factors of production. Eventually, not immediately! Herein lies the key to the situation. From a methodological point

---

1 Keynes, *op. cit.*, p. 63.
of view the problem of definition may be attacked from two
directions - one taking account of time, and one neglecting this
factor. Keynes adopts the latter procedure: his characteristic
method is to consider magnitudes at a moment of time, in such a
way that these magnitudes are all referred to this moment.

Perhaps it is not immediately evident to everyone that a
magnitude, say a sum of money, may shift its classification with
the passage of time. For instance, a consumer may allocate the
sum of $5 to affect the purchase of a pair of shoes on the morn-
ing of a given day - call it 1. As yet, the sum is merely a planned
expenditure. On day 2 the consumer acquires the pair of shoes and
a retailer receives the $5. On day 2 this $5 may be classified
as expenditure on consumption. But that sum also represents the
receipts of a retailer. On day 3 the retailer reorders a pair of
shoes and this day witnesses its production and the payment into
income of the sum of $5. That sum which represented a quantity
of expenditure in day 2 has become a quantity of income in day 3.

On day 2 it is doubtless possible to predict that the
receipt of $5 by the retailer will give rise to a payment of
income on day 3. Accordingly, this $5 may be counted as income
in advance of the actual disbursement of the sum as income; but
it is clear that the step thus taken in so designating dealers'
or producers' receipts is somewhat arbitrary. Yet this practice
lies at the basis of the Keynesian system.¹

¹ Analytically, the point may be put as follows. Let \( E(T) \)
represent expenditure = value of output, as a function of time;
and let \( Y(T) \) represent income as a function of time. Now \( E = Y \),
but only after a time lag of length \( ST \). Thus, \( E(T) = Y(T + ST) \).
In effect, the definitions which Keynes employs are so constructed as to telescope economic processes in time. Whereas the generation of income is separated temporally from expenditure out of income, the two magnitudes involved are identified. And this has the effect of describing economic processes as occurring without time lag. The simplification involved in describing the generation of income and expenditure out of income as simultaneous processes is perhaps not more unrealistic than the assumption of perfect competition. Under certain assumptions expenditure will always generate income and so may be described as the latter for particular purposes. Where the analyst does not attempt to describe change, but seeks rather to describe a given situation, the approach has a limited usefulness.

Graphically, the situation may be described thus in a dynamic situation: whereas the \( E \) (expenditure) curve and the \( Y \) (income) curve possess the same conformation with respect to time, the \( E \) curve lies to the left of the \( Y \) curve, preceding the latter in time. In effect, the Keynesian definitions ignore this condition, and refer the two curves to the same moment of time.

\[
E(T) = Y(T) + ST Y_t(T) + \frac{ST^2}{2} Y_{tt}(T) + \ldots
\]

If the interval \( ST \) is so small that the squares and higher powers of \( ST \) may be neglected, then \( E(T) = Y(T) + ST Y_t(T) \). Clearly, if \( Y_t(T) = 0 \), that is, if income remains constant through time, then \( E(T) = Y(T) \) and no difficulty is to be found. But if expenditure is changing, then \( E(T) - Y(T) = ST Y_t(T) \). The difference will be equal to the increment of expenditure which occurs during the lag. Any investigation which involves the change of \( Y \) and \( E \) must take account of this lag in time, if it is to be realistic.
Actually, if the two magnitudes are considered at time $OR$, it will be seen that expenditure has the value $RM$, while income has the value $RN$. Where the economic system is in equilibrium in the sense that the rates of flow of income and expenditure are constant and equal, both curves are lines parallel to the axis of time; thus they coincide in such a situation, as shown in the accompanying diagram. Seen from a static or quasi-dynamic, that is an essentially timeless, point of view, it may then be considered inexpedient to differentiate between the two.

\[ Y = E \]

Suppose, however, that a movement from one equilibrium to another is being considered; the time sequence of affairs once more assumes significance. During the transition the movement of income to the new equilibrium, through time, may not coincide with the movement of expenditure. In fact, the several paths followed may have a distinct affinity with a dynamic situation, as the accompanying diagram suggests. In the interval lapsing between the two equilibria the value of income will first fall short of, then exceed, and then equal the value of expenditure.
If the Keynesian system is to prove useful, it must surely be able to give a fairly accurate account of what happens in the time interval elapsing between one short period equilibrium and the next. The fundamental fault in Mr. Keynes' definitions is that they do not facilitate an understanding of the temporal relationships between economic magnitudes when the system is in limbo between two equilibria. All we can see is the old equilibrium and the new. The intervening processes are seen "darkly, as through a glass." But Economics is concerned with prediction and the tracing out of processes in time. Change cannot be fruitfully analyzed by a purely retrospective analysis, which sees only the end result. "Economists set themselves too easy, too useless a task if in tempestuous seasons they can only tell us that when the storm is long past the ocean is flat again."¹

For comparative purposes it will be desirable to consider definitions of saving and investment which have a prospective tinge. By examining alternative definitions and the relationships existing between these and Keynes' definitions, it may be possible to arrive at some conclusions useful in analyzing the General Theory.

D. H. Robertson has detailed definitions which indicate a possible difference between saving and investment. His analysis proceeds from the definition of a "day" - a short but indivisible unit of time so brief that the income received in one "day" cannot

¹ Keynes, Monetary Reform, p. 83. Probably it is an exaggeration to say that the definitions can only give us information as to the flatness, i.e., the equilibrium of the economic sea. No doubt the system can also give us some information about the height of this flat sea at any given time.
be spent until the next. In actuality, the period may be a week or more; for the "day" is an operational time period, an interval of variable length so adjusted as to satisfy a given condition. Thus the actual length of the day depends on the habits of the community with respect to methods of payment.

The earned income of one day, meaning generally the expenditure of society on investment and consumption, having been received on the given day, will become "disposable" income on the morrow. The savings of a given day are defined as disposable income minus the consumption of that day. On the other hand, investment represents expenditure on new capital goods during the day. Since investment may be financed out of bank credit or the cash balances of entrepreneurs, investment may exceed saving. Since the expenditure of one day, the earned income of that day, becomes disposable income tomorrow, this excess of investment over saving implies a rise in today's earned income and of tomorrow's disposable income. Thus when Mr. Robertson speaks of an excess of I over S, he means the same thing that Keynes does when the latter speaks of a rise in income. This Mr. Keynes admits: "When Mr. Robertson says that there is an excess of saving over investment, he means literally the same thing as I mean when I say that income is falling, and the excess of saving in his sense is exactly equal to the decline

1 Essays in Monetary Theory (London: P. S. King, 1940), p. 65.

2 $S_t = Ydt - Ct$ where the subscript t refers to a given period, and d indicates disposable income. Since Ydt is given and Ct is a free variable, $S_t$ will be known when Ct is fixed; $S_t$ is a residual depending on Ct. But it is also a free variable; and there is no a priori reason to suppose that it will be equal to Ydt - Ct.
of income in my sense.  

All reaction times of the individuals concerned must be shorter than the "day," otherwise a discrepancy between I and S in one period might show up several periods later. Thus, if investment exceeds saving on day 1, and the earned income of that day rises by the difference, this rise in earned income will be reflected in a rise in disposable income the following period, under Robertson's assumptions. But suppose that the rise in earned income were reflected in an increase in disposable income two periods later, in some sectors of the economy. Then the neatness of the system disappears. Accordingly, the reaction times are all assumed to take effect within the day.  

Beneath the surface of this controversy, then, two of the principals have come to an agreement in substance, though not in words. In order to point out how these two sets of definitions work out, it will be appropriate to work out a few examples.  

1 G. T., p. 78. And he adds: "Thus Mr. Robertson's method might be regarded as an alternative attempt to mine (being perhaps, a first approximation to it)." The conceited man!  

2 Another simplification is that the disposable income is always set equal to the earned income of the preceding period. This need not be true in actuality. If expenditure (earned income) has been rising in successive periods, producers may anticipate a further rise in expenditure, and thus produce goods to meet the anticipated rise in demand. This will generate incomes over and above those necessary to sustain the level of the preceding period. Clearly, Robertson's definitions are not the last word. They do, however, possess the quality desired by analysts of change - an explicit relationship to time.  

3 The algebra of this famous identity, mentioned above, may be shown as follows. If \( Y_t \) represents earned income, then \( Y_{t+1} = Y_{t+1} = C_t + I_t + \ldots \). Therefore,
Suppose the community commences in period one with an income of 100; of which 70 is consumed and 30 is saved and invested. In period two, however, consumers decide to save an additional 10 units, while investors continue to plan an investment of 30 units. Will this lead to an excess of saving over investment? Not on Keynes' definitions. The additional savings will cause a fall of 10 units in income, because consumption has diminished to 60, while investment outlay by producers remains at 30. Thus consumption will fall to 60, savings will remain at 30. The only effect of the attempted saving has been to drive down income.

In tabular form this may be summarized as follows:

<table>
<thead>
<tr>
<th>Period I</th>
<th>Period II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y = 100</td>
<td>Y = 90</td>
</tr>
<tr>
<td>C = 70</td>
<td>C = 60</td>
</tr>
<tr>
<td>S = 30</td>
<td>S = 30</td>
</tr>
<tr>
<td>I = 30</td>
<td>I = 30</td>
</tr>
</tbody>
</table>

On Robertson's definitions, the process might be described as follows: the increase in savings to 40 units, while investment

\[ I_{t+1} - S_{t+1} = Y_{t+1} - Y_t = Y_{t+1} - Y_{t+1} - S_{t+1} \]


It should be clear that the relationship between the rise of income and the disparity between saving and investment is one of definition. The difference between saving and investment is the rise in income.

1 Such a result may be calculated to cause surprise, if the reader has not yet come to accept the inevitable. And we must consider the words of Mr. Lerner, who says the misunderstanding of the Keynesian dogma often arises from "the failure to realize that the proposition \( S = I \) is only an analytical proposition, and not about the real world at all." "Saving Equals Investment," Quarterly Journal of Economics, LIII (1938), p. 305.
remains constant at 30 units, leads to an excess of saving over investment. The 10 units difference results in a decline of disposable income to 90 in the following period. In tabular form:

<table>
<thead>
<tr>
<th>Period I</th>
<th>Period II</th>
<th>Period III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_d = 100$</td>
<td>$Y_d = 100$</td>
<td>$Y_d = 90$</td>
</tr>
<tr>
<td>$C = 70$</td>
<td>$C = 60$</td>
<td>$C = 60$</td>
</tr>
<tr>
<td>$S = 30$</td>
<td>$S = 40$</td>
<td>$S = 40$</td>
</tr>
<tr>
<td>$I = 30$</td>
<td>$I = 30$</td>
<td>$I = 30$</td>
</tr>
</tbody>
</table>

If consumers are considered at the beginning of period three, their situation is no better than at the beginning of period two: whereas their savings at the end of period two are up by ten, their income is down ten units at the outset of period three.

Thus their cash resources are identical for the moment with the situation as it existed at the commencement of period two. Clearly, the two methods can be used to analyze the same processes.

A further school of economists, the Stockholm school, employs a terminology making distinctions useful for our purposes. This group finds it convenient to contrast two points of view, the prospective and the retrospective. On the one hand, individuals draw up plans at a given moment which are destined to be executed during the ensuing period. The planned or anticipated magnitudes

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2 The period must be chosen in such a way that plans do not change during the period. Cf. Lutz, "The Outcome of the Saving-Investment Discussion," Quarterly Journal of Economics, LIII (53), p. 604.
under consideration are — planned saving, consumption and invest-
ment, and anticipated income. This anticipated income we inter-
preset to mean disposable income, or the earned income of the
preceding period, in the Robertsonian sense.\footnote{1}

Producers and consumers attempt to carry through these
plans during the period, but find that the results differ from
those anticipated. Thus, the process of exchange results in a
series of retrospective or realised magnitudes — earned income,
realised consumption, realised investment, and realised savings.
The differences between the planned and realized magnitudes are
denominated undesigned or unexpected saving, and so on.

Planned saving may differ from planned investment, but as
these plans are carried out, reactions occur which drive back
the realised magnitudes into equality.\footnote{2} Suppose, for example,
that during period one the community is in equilibrium with
an income planned and realised of 100, consumption of 70, saving
and investment of 30. With a disposable income of 100 in period

\footnote{1} This is necessary in order to avoid a certain indeter-
minacy in the argument. Planned or anticipated income, being
a prospective concept, does not form a very convenient tool with
which to handle processes actually under way. For example, it
is scarcely possible to save out of anticipated income.

Furthermore, the schedules with which Professor Ohlin
identifies the anticipated or planned magnitudes renders them
indeterminate, since the schedules represent a whole series of
alternative plans. Accordingly, this usage is not adopted here.

\footnote{2} As Haberler points out: "What actually happens, if
planned saving and investment differ, is assumed by way of illustra-
tion . . . No particular process is required to make S and I equal
\textit{ex post}. All sorts of reactions are possible, but, whatever actually
happens, they must be equal because the terms are chosen in such a
two consumers plan to save 20 and consume 80, while investment plans are unchanged at 30. What will be the result? Retail sales will rise by 10, but since the period is too short to replace the whole extra quantity sold, stock are depleted by 5. Thus, an unexpected disinvestment of 5 occurs. Reorders of 5 being effected within the period and this sum being passed out to wage earners, the latter find themselves with an increment of earned income, or undesigned saving. Thus realized income is up to 105, being composed of realized consumption of 80, and realized investment of 25. Likewise, realized saving is up to 25. The realized investment is resolvable into planned investment of 30 and undesigned disinvestment of 5, representing depletion of stocks; realized saving of 25 is resolvable into planned saving of 20 and undesigned saving of 5.1

These results may be summarized in a table, wherein the subscript r denotes "realized;" u, undesigned or unexpected; and p, planned. Realized income is the same as earned income in Robertson's terminology; and planned income is to be interpreted here as disposable income.2

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1 The term "undesigned" was introduced by Hawtrey who used it to denote these adventitious changes in dealers stocks. Cf. Capital and Employment, passim.

2 The algebra may be summarized as follows. (1) \( I_p = C_p + I_p \), (2) \( Y_r = C_r + I_r = C_r + S_r \). Therefore \( I_r = S_r \). In Ohlin's terminology, the one adopted here, realized saving and investment are defined to be identically equal, even as with Keynes. Other relations are: (3) \( I_r = I_p + I_u \) (2) \( S_r = S_p + S_u \). Therefore, \( I_p + I_u = S_p + S_u \). Since planned consumption is assumed always to be realized \( CP = C_r \).
These results are perfectly consonant with those of Keynes. In every case, however, the latter's magnitudes are to be identified with those realized at the end of the period. Which set of definitions, then, is actually the most appropriate? A definite answer cannot be given here, for the interpretation depends to a great extent on the interpretation of other parts of his system. But this, at least may be said. The retrospective viewpoint leads to a certain tautological quality in the system. Inasmuch as Keynes defines saving and investment to be identical in the aggregate he cannot, at the same time, regard their equality as a condition of equilibrium. Nevertheless, the condition is imposed on the functional relationships which make up his system.

If some change is introduced into the system, saving and investment will remain equal, but some of the functional relationships will undergo an abnormal change, necessary to fulfill the condition. A latent disequilibrium will exist, below the smooth surface of the equilibrium. For example, in the preceding example, an undesigned decrement in stocks has occurred

Then \( Y_r - Y_p \equiv C_r + I_r - (C_p + S_p) \equiv I_r - S_p \equiv I_p + I_u - S_p \equiv S_u \)

in virtue of equations (1), (2), (3), and (4). All these results follow in perfect tautological fashion from the definitions. Cf. Lange, Loc. Cit.
which requires to be eliminated before "full" equilibrium can come to pass. Yet the apparatus which Keynes employs does not explicitly reflect the lack of equilibrium.

Perusal of the General Theory reveals the belief of the author that a process is necessary to equate saving and investment. Yet it is fruitless to speak of the process of equating two magnitudes so defined as to be identical (in the aggregate). Accordingly, it seems advisable to release the restriction placed upon the analysis by the artificial definitions of income, saving and investment.

The following procedure might fit the requirements of a quasi-dynamic equilibrium system. Admit a formal distinction between income and expenditure. Define income to be equal to consumption plus saving. Define expenditure to be equal to consumption plus investment. The equilibrium of such a system will be reached when income is equal to expenditure. This will occur when saving equals investment. Simply by making the formal distinction between income and expenditure the system may be rescued from the danger of sterility.¹

¹ Anyone familiar with the works of Mr. Hawtrey will see that this suggestion is consonant with his teachings. The monetary equilibrium of such a system may be represented by six equations, comparable to the first five equations of the Keynesian system, Infra p. 50.

Equations

<table>
<thead>
<tr>
<th>Equations</th>
<th>Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y = C + S )</td>
<td>Y = income</td>
</tr>
<tr>
<td>( E = C + I )</td>
<td>E = expenditure (on output)</td>
</tr>
<tr>
<td>( I = I_k (C, i) + I_b (C, i) )</td>
<td>I = investment</td>
</tr>
<tr>
<td>( C = C (Y, i) )</td>
<td>C = consumption</td>
</tr>
<tr>
<td>( M = L (Y, i) )</td>
<td>M = quantity of money</td>
</tr>
<tr>
<td>( E = Y )</td>
<td>i = rate of interest</td>
</tr>
</tbody>
</table>
Under these definitions, a disparity between saving and investment would be reflected in a corresponding disparity between income and expenditure. An excess of saving over investment may be entitled hoarding, and an excess of investment over saving, dishoarding. This is not intended to indicate that an excess of saving over investment results in an actual increase in the money supply, or that a deficiency of saving with respect to investment leads to a depletion of the money supply. The concept of hoarding is used here to denote a flow concept, not the addition to or subtraction from a stock.

Under this set of definitions, the position of equilibrium of money flows is (partially) reached when saving equals investment, and income equals expenditure. The equality of these two pairs of magnitudes becomes a condition of equilibrium, as opposed to identity by definition. And this approach has the

Several comments are in order. First, the subscripts k and s indicate investment considered as capital outlay and investment in dealers stocks, respectively. Second, it will be seen that the system has no defined relationship to time. But this is not essential to a quasi-dynamic system. Such a system does not take explicit account of time. What is required of such a system is not that it date its magnitudes, but that the relationships be stated in such a way that they can register differences between magnitudes which diverge in the interval between the departure from one equilibrium and the arrival at another. To put the matter another way, the system should be designed to register internal disequilibrium quite clearly. In the above system it may be seen that equation (6) holds only in equilibrium. Now the equations (1) and (2) are identities, holding at all points. The result of (1) and (2) is the following: \( E - Y = I - S \). In a disequilibrium situation, saving is unequal to investment, while in equilibrium they are thus equal: the two equalities express the same equilibrium. The virtue of this approach is that the disequilibrium is registered by a divergence between I and S. This is a most convenient way of looking at things.
advantage that a disequilibrium situation is registered very simply by a divergence between saving and investment. Since an excess of (say) investment over saving is clearly inflationary, the usage is convenient. It indicates that the disequilibrium sets up forces which tend to restore equilibrium. In the case above, an excess of investment over saving would give rise to an increase in the level of income, and so on until the equality of saving and investment income and expenditure had been attained once more.

Two types of equilibrium may be considered, that holding in the short period, and that holding in the long run. In the short period, investment may be interpreted in such a way as to take account of undesigned variations in stocks. Thus investment may be broken up into two items - outlay on instruments of production, and expenditure on stocks. The latter item is subject to considerable variation in the short period, depending on the level of expenditure; if consumption should increase suddenly, stocks would undergo a change, restricted more or less by dealers who might resort to price variations. In the short period, the dealer will be prone to adjust prices slowly, and stocks may then change considerably, under such influences. In the long period, the dealer will wish to reach a definite optimum situation correlated with such variables as the level of consumption; accordingly variations in stocks will be a less important factor in long period adjustments.
CHAPTER IV

THE PROPENSITY TO CONSUME

AND THE MULTIPLIER

Perhaps no point in Keynesian analysis has received more widespread attention than the concepts - the propensity to consume and the multiplier. So neat is the analysis, so precise the results achieved - that numerous economists of a high intellectual caliber have entered the Keynesian fold. Behind the simple facade, let it be said, there lurk complications. In part, our exposition will be devoted to setting out the theory in a simple form, and, in part, to an explicit consideration of some of these complications.

In his original formulation Mr. Keynes describes the propensity to consume, as a relationship connecting the level of aggregate consumption, measured in wage units, to the level of income, measured in wage units. In the surrounding textual material he has hedged around this relationship with a seemingly exhaustive list of qualifications. The analogy to Ricardo’s formidable list of simplifying assumptions, qualifying his labor theory of value, is striking.

After setting out this relationship, he qualifies it as follows: "The amount that the community spends on consumption obviously depends (i) partly on the amount of its income, (ii) partly on the other objective attendant circumstances, and (iii) partly on the subjective needs and the psychological propensities
and habits of the individuals composing it and the principles on which the income is divided among them (which may suffer modification as output is increased)." Among the objective circumstances is "the rate of time - discounting." After a discussion of this matter Mr. Keynes concludes "The short period influence of the rate of interest on individual spending out of a given income is secondary and relatively unimportant, except, perhaps, where unusually large changes are in question." Whereas Mr. Keynes deems it appropriate to exclude the rate of interest from the list of variables formally treated as determining consumption, writers under his influence take an opposite course. Despite the existing uncertainty as to just how consumption is affected by changes in the rate of interest, it seems wise to go over Mr. Keynes' head and write it in as a second variable affecting consumption. In our notation $C_w = C_w (X_w, i)$.  

1 G. I., pp. 90-91. The six objective factors affecting the propensity to consume are (1) A change in the wage-unit (2) A change in the difference between income and net income (3) Indemnity in capital - values not allowed for in calculating net income (4) Changes in the rate of time - discounting (5) Changes in fiscal policy (6) Changes in expectations of the relation between the present and the future level of income. Pp. 91-96. There is also a group of subjective factors affecting the motives for saving.

2 Ibid., p. 94. Writers taking this view include Mr. J. R. Hicks, "Mr. Keynes and the Classics, Economica, V (1937); Mr. Oskar Lange, "The Rate of Interest and the Optimum Propensity to Consume," Economica, V, (1938) N.S.; Beael Timlin, Keynesian Economics (Toronto: University of Toronto Press, 1942); Franco Modigliani, "Liquidity Preference, Interest and Money," Economica, XII (1941); others can be cited. All these writers include the rate of interest as a variable formally affecting $C_w$. 
In order to understand what the "propensity to consume" can do for analysis, it will be necessary to subject it to some destructive criticism. The reader will be poorly repaid from a reading of this material if he should gain the impression that this "propensity to consume" is a precise functional relationship, founded on reasoning as firm as that which underlies the theory of demand. It is not, and few words are required to demonstrate this. In the first place, Mr. Keynes proceeds from a macro-economic point of view whereby he treats the consumption of an entire economy as a function of its level of income; but we shall proceed from a micro-economic point of view. Only in this way can we hope to justify this "portmanteau function."^1

Given the rate of interest, it seems indisputable, on a priori grounds, that the consumption of an individual household will vary with the level of its income. Granted that this relationship holds for each individual household (that is, for each consuming unit), how are these individual relationships to be summed? First, it is quite clear that, if we know all of the individual propensities to consume, we may sum the level of consumption pertaining to each household, in such a way that aggregate consumption is shown to be dependent on the aggregate income, as it is distributed among the members of society.\(^2\) Does it require

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1 Mr. Keynes' phrase.

2 Thus if \( C_{w1} = C_{w1} (Y_{w1}) \) represents the consumption function of the \( i \)th household, then

\[
\sum_{i=1}^{n} C_{w1} (Y_{w1}) = C_{w1} (Y_{w1}) + C_{w2} (Y_{w2}) + \ldots + C_{wn} (Y_{wn}) =
\]

\[
Pw (Y_{w1}, Y_{w2}, \ldots, Y_{wn}) = C_w (Y, D) \text{ where } D \text{ represents the principles on which the income is divided. Notes on Jocker Lange's Seminar, 1942.} \]
demonstration that a given aggregate income would give rise to different levels of consumption, if the income were to be redistributed? Presumably, the greater the concentration of aggregate income in the hands of a few, the lower the corresponding level of consumption. 1

If the concept, the propensity to consume, is to be treated with precision, it is necessary to connect the level of each household's income to the level of aggregate income. If such a set of relationships were to exist, then it would be legitimate to proceed to the connection between aggregate consumption and aggregate income by the following logic: (1) individual consumption varies with individual income (2) individual income varies (on some known principle) with the level of aggregate income (3) the consumption of the individual consuming units may be summed; and since the consumption of each part of aggregate consumption varies with the

1 The idea of the propensity to consume is not new. Hobson spoke of a "golden mean" between spending and saving. Malthus was similarly concerned with the "effective demand." These men did not refer to the propensity to consume as a functional tendency connecting the level of consumption and the level of income. This is where Keynes contributed a more positive idea.

The idea that the average propensity to consume declines with the level of income is to be found in the Brookings study; Leven, Moulton, and Warburton, America's Capacity to Consume (Washington: Brookings, 1934), ch. VIII. This information does not bear on the psychological law, but on Mr. Keynes' obiter dictum that the proportion of income saved tends to increase with the level of income.

For a study which indicates that the distribution of income between spending and saving is of great importance see, H. L. McCraken, Value Theory and Business Cycles (2nd ed.; New York: McGraw-Hill, 1936). See particularly pp. 248-49 where the effects of a maladjustment between spending and saving are discussed. Such studies as those mentioned indicated the importance of the propensity to consume before Mr. Keynes' arrival on the scene.
incomes of each of the individual units, and since these several incomes vary with aggregate income, aggregate consumption may be represented as a function of aggregate income. In short, the distribution of income must be shown to be a function of the level of aggregate income.\(^1\)

We need not gratuitously assume that Mr. Keynes neglects this consideration. Quite the contrary, he explicitly points out that this circumstance - the distribution of income - affects the propensity to consume.\(^2\)

But he assumes that this condition will not disrupt the relationship posited. What this process of assumption cannot abolish is the existence of a lurking skepticism in our minds concerning the stability of this function. Mr. Keynes has not adduced satisfactory a priori reasons for believing that the distribution of income is also a unique function of aggregate income; yet this relationship is vital to the construction. Accordingly, there is reason to agree with Dr. Gilboy that Mr. Keynes has "stated in the propensity to consume a statistical and not a psychological tendency or law."\(^3\) What is required, then,

\[ \text{If } I_{W1} = \chi_1(Y), I_{W2} = \chi_2(Y), \ldots, \text{ then } C_W = F_W(\chi_1(Y), \chi_2(Y), \ldots) \]
\[ \neq C_W(Y), \text{ or in alternative form, } D = f(Y), \text{ then } C_W = \phi_W(Y, D) \]
\[ = \phi_W(Y, f(Y)) = C_W(Y). \text{ Infra, p. 25, note 1.} \]

\(^1\) G. I., p. 91.

is inductive evidence indicating the existence of the assumed relationship. Here, it may be seen, the macro-economic approach slips away from a deductive foundation to reliance on statistically observable relationships. Thus, Mr. Keynes' formulation, if it is to be considered valid, rests to a far greater extent than does the economics of pricing, on an empirical foundation which does not yet exist, in the main.

Furthermore, the propensity to consume actually depends on the entire price structure. An individual household, if it is to achieve its optimum situation, must weigh the advantages of consumption in any given direction against (say) the advantage of saving. In the optimum position the net advantage of employing a marginal unit of money for purchasing goods in any direction cannot exceed the net advantage to be derived from saving that unit. If, following Walras, we treat the reciprocal of the rate of interest, \( \frac{1}{i} \), as the price of a unit of perpetual net revenue, then all prices must be in equal proportion to their marginal utilities. Clearly, this proposition may be translated into price ratios and marginal rates of substitution, following the modern practice, so that the condition relates to the equality of price ratios and marginal rates of substitution as between any two goods (including a unit of perpetual net revenue). In order to know how much people will rationally save, we should then need to know the price structure.

It is precisely this necessity which Keynes seeks to avoid; for the problem of determining the price structure is formidable. Accordingly, a further assumption is necessary - that the price
structure is uniquely correlated with the level of income, measured in wage units. Perhaps this tendency also holds true in an actual situation; but, again, no theoretical reason is advanced. We have another postulate, which, as it is not derived from any set of deductions from rational human action, must be derived from empirical investigations. Again the analysis is seen to recede from deduction to an empirical basis which does not yet exist.

It would be dangerous to suppose that we are dealing with a theoretical tool possessing the analytical precision and solid deductive basis of demand and supply theory. Actually, we are dealing with a theory based on rough approximations, designed to yield maximum simplicity. If the analysis recedes from the complex interrelations of reality in the interests of simplicity, it must pay the price of that simplicity. We cannot "have our cake and eat it too."

A point which has also been the source of some conjecture is the relationship of this "propensity" to time. Is the schedule relating aggregate consumption to aggregate income to be interpreted in an ex-ante sense? In short, is it anticipated consumption which is thus related to anticipated income? Or is it realized consumption which is related to realized income? Mr. Keynes gives no definite answer to this question.

1 Hansen says that there are three possible interpretations of the propensity to consume (1) Ratio of anticipated consumption to anticipated income (2) Ratio of planned consumption of current period to realized income of the preceding period (3) Ratio of realized consumption to realized income. "Mr. Keynes on Underemployment Equilibrium," Journal of Political Economy, XLIV (1936), pp. 672-73.
The problem which the Keynesian theory faces here bears a close relationship to a similar problem in the theory of supply and demand. By analogy to the latter may we not say that the propensity is the schedule relating prospective levels of consumption to prospective levels of income? Undoubtedly this concept suffers from a certain vagueness; and the source of this vagueness is to be found in a corresponding lack of clarity with respect to the actual working out of the equilibrium of which this propensity forms a necessary part.

A quasi-dynamic equilibrium system will require a certain period of time necessary to effectuate the steps leading to equilibrium. Under the assumption that the conditions of equilibrium can be reached in a period of time shorter than that during which the stock of capital may be assumed constant, there is no reason why the propensity to consume cannot be treated as an ex-ante concept. During the time period in question the several individuals, consuming at a rate determined by the level of income developing, will so affect the situation by thus consuming that the situation will move towards an equilibrium. If this interpretation is to be carried out logically, it must rest on the formal distinction, advanced above, between income and expenditure.

If we attempt to identify income, considered as consumption plus saving, with expenditure, considered as consumption plus investment, then it is very difficult to treat consumption in the ex-ante sense. In the first place, when the relationships are defined in this way the system will not attain a position of stable
equilibrium until the propensity to consume has reached its "normal value." This may be illustrated, as follows. Suppose that, by governmental action, an additional quantity of credit is created and handed to some one who uses it to purchase a piece of capital equipment. At the moment the person spends the sum, in Keynesian terminology, it becomes income. Suppose that the economic system had previously been in equilibrium with an income of 100, consumption of 80, saving and investment respectively of 20. Now investment has risen to 30, and income has risen to 110. Since the end of the period is fixed (by assumption) at the moment when the piece of capital equipment is purchased, then consumption is by definition 80; the extra sum cannot be spent until the next clock time period. Accordingly, the propensity to consume has undergone an abnormal change; for the level of consumption has remained fixed at 80, while the level of income has increased to 110. If the formal distinction between income and expenditure were to be adhered to, this embarrassment would not occur. The extra sum would simply represent expenditure until the money was analyzed by retailers, and until they had varied their orders to manufacturers, and until the latter had increased their production in such wise that the level of income had risen.1

It would seem that the Keynesian propensity to consume is normal only in equilibrium; for only then is it true that income is equal to expenditure without time lag. If the propensity to consume is only normal in equilibrium, then it cannot help us to

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1 Robertson, Essays in Monetary Theory (London: P. S. King, 1940), p. 118.
find equilibrium without reinterpretation. In the opinion of the present writer, this difficulty may be avoided by affecting a distinction between income and expenditure. Under this set of definitions the propensity to consume retains its normal value through all clock-time periods in which final equilibrium is not achieved. Interpreted thus, in an ex-ante fashion, this propensity becomes an unambiguous tool for the analysis of equilibrium.

As a preliminary to a detailed analysis of the properties of the propensity to consume, it will be useful to consider the basic diagram which is to be employed in the following analysis. In this diagram expenditure is plotted on the vertical axis against income on the horizontal axis. A straight line is drawn from the origin at a 45° angle, bisecting the two axes. Along this line income equals expenditure.

Expenditure is made up of two elements, expenditure on consumption and expenditure on investment. Consumption is assumed to vary with income in a fashion determined by the propensity to consume. Thus the curve connecting the several levels of consumption to the corresponding levels of income is the graphical representation of the propensity to consume. Investment may be treated in two ways; its value may be treated as a constant for some purposes, or it may be assumed to vary with income. In this construction, as a first approximation, it is assumed to remain unaffected by the level of income. By adding a constant amount to the C (consumption) curve, we secure the C + I curve; this construction corresponds closely to the one used in Chapter II, the only difference being that income is
plotted along the horizontal axis instead of the level of employment.

At a level of income OM, consumption is MQ and investment is MN. Extend MQ to P so that OP is equal to MN. Then the ordinate OP equals total expenditure corresponding to a level of income OM. In this way trace out the E (expenditure) curve.

At R the E (expenditure) curve is cut by the line representing equality of income and expenditure. Here is the position of equilibrium whereat income equals expenditure. In this construction, the level of saving corresponding to a given level of income is represented by the height of the line OS above the C curve. It will be noted that in equilibrium the level of saving equals the level of investment; the distance between the C and C + I curve (investment), equals the distance between the C and the OS curve (saving).¹

¹ The equilibrium thus set forth may be represented by the equations: (1) $C_w = C_w (Y_w, i)$ (2) $C_w + I_w = Y_w$. If $I_w$ is given and $i$ is fixed, we have two equations with which to determine the two unknowns, $C_w$ and $Y_w$. Saving may also be determined from the relation (3) $C_w + S_w = Y_w$.

I do not know who originated this diagram. Timlin - Keynesian Economics, University of Toronto Press, Toronto, 1942 - uses similar diagrams for the representation of the consumption function and the 45° line along which income equals consumption. Fellner - "Period Analysis and Timeless Equilibrium," Quarterly Journal of Economics, LVIII (1944), No. 2, pp. 315 ff. - employs this diagram.
Three concepts relating to the propensity to consume are found to be useful in Keynesian analysis - the total, average, and the marginal propensities to consume. The (total) propensity to consume is the basic function which relates total consumption to the level of income. The average propensity to consume, $\frac{C_y}{Y}$, represents the proportion between consumption and income. Presumably, the average propensity is always positive, since consumption and income are always positive. But, as to size, it may be greater than, equal to, or less than unity. If $\frac{C_y}{Y}$ is greater than 1, this implies that consumption exceeds income. This assumption is registered in the diagram by having the C curve cut the line OT from the left. At all points to the left of U, consumers effect negative savings, whereas to the right they effect positive savings. Thus at income $OM = RM$ consumption is $SM$; the difference between $SM$ and $RM$, namely $SR$, represents dissaving. Consumers are able to effect this by drawing down balances, or negotiating loans and spending the proceeds thereof. But at income $OM'$ consumers' income is $OM' = R'M'$, while consumption is $S'M'$; the difference between $S'M'$ and $R'M'$, namely $S'R'$, represents saving.

Inspection reveals that the ratio $\frac{SM}{OM}$ is greater than $\frac{S'M'}{OM'}$. 
Thus the average propensity to consume is represented as diminishing with the increase in the level of income. This is reasonable on a priori grounds, but is not part of basic Keynesian doctrine.

A somewhat more complex notion is the marginal propensity to consume. This propensity, like the average propensity to consume, takes a value determined by the level of income, and varies continuously with the latter. This relationship expresses the ratio of the rise in consumption to a rise of $1. Thus, if income increases by $1 and consumption rises $0.80, then the marginal propensity is $\frac{8}{10}$ or $0.8$, at that particular level of income. In mathematical notation it is written $\frac{dC}{dY}$.

Graphically, this propensity appears as the slope of the tangent to the consumption curve; and this slope varies continuously with the level of income in such wise that a given propensity to consume is associated with a certain level of income (the rate of interest remaining constant). The marginal propensity to consume at income $OY_1$ is the ratio $\frac{N'P'}{N'P}$ and is associated uniquely with the level of income $OY_1$.

The value happens to be $3/4$.

On the other hand, the average propensity to consume is $\frac{N'P'}{Y_1}$ and is associated uniquely with the level of income $OY_1$.

This value happens to be $\frac{3}{1} = 3$.

As the level of income rises to $OY_2$, a change occurs in both these propensities. Now the marginal propensity to consume is $\frac{M'P'}{M'P}$ or $\frac{2}{5}$.
the average propensity to consume is \( \frac{MY_2}{0Y_2} \) or \( \frac{9}{10} \). Thus the values of both the average and the marginal propensities are represented as diminishing with the increase in the level of income. Neither of these assumptions is necessary to the Keynesian doctrine; but they seem, a priori, to be reasonable.

There is a relationship between the average and the marginal propensities. So long as the marginal propensity to consume is less than the average propensity, the latter must diminish. Since we represent the marginal propensity as diminishing from the point of zero income, the average propensity must likewise diminish from the point of zero income.

The meaning of these concepts may be restated as follows. At an income \( 0Y_2 \), an accession of \$1 of income to consumers will occasion an added expenditure or consumption of \( \frac{2}{5} \) of this amount, or 40%. On the other hand, at an income of \( 0Y_2 \) total consumption is \( \frac{9}{10} \) of total income \( (0Y_1) \). Thus, if income is 100, consumption will be 90, and the average propensity to consume \( \frac{9}{10} \). All this is based on the assumption of a given level of the rate of interest.

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1 The average propensity is, of course, \( \frac{C}{Y} \), while the corresponding marginal propensity is \( \frac{dC}{dY} \). If \( C(Y, i) \) is a known function, then the values may be obtained without reference to tangents.

It should be noted that, if we start from the individual propensities to consume, the M.P. to C. must be written

\[
\sum_{i=1}^{n} \frac{dC_{yi}}{dY_i} \frac{dY_i}{Y} = \frac{dC_{y1}}{dY_1} \frac{dY_1}{Y_1} + \frac{dC_{y2}}{dY_2} \frac{dY_2}{Y_2} + \ldots + \frac{dC_{yn}}{dY_n} \frac{dY_n}{Y_n} .
\]

Thus the social propensity to consume is the sum of the individual propensities weighted by the change in individual incomes in response to a change in the social income. Stachle, "Short Period Variations in the Distribution of Incomes," *Review of Economic Statistics*, XIX (1937), p. 132.
The basic postulate relating to consumption is that the marginal propensity to consume is positive and less than unity throughout the range of values of income which are considered. As Keynes puts it, "the fundamental psychological law, upon which we are entitled to depend with great confidence both a priori from our knowledge of human nature and from the detailed facts of experience, is that men are disposed, as a rule and on the average, to increase their consumption as their income increases, but not by as much as the increase in their income."¹

In a later statement he qualifies his position, saying: "My theory itself does not require my so-called psychological law as a premise. What the theory shows is that the psychological law is not fulfilled, then we have a condition of complete instability."²

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¹ C. J., p. 96.


A diagram will show this clearly. If the marginal propensity to consume is greater than unity, the slope of the \( C + I \) curve will be greater than that of the income equals expenditure line. Thus:

This criterion is a necessary and sufficient condition for stability in the elements considered, only if investment is invariant under changes in \( Y \). If \( I_w = I_w (X_w) \), the criterion of stability is: \( \frac{d(C_w + I_w) - Y_w}{dY_w} < 0 \)

i.e. \( \frac{dC_w}{dY_w} + \frac{dI_w}{dY_w} < 1 \).
The latter statement is cautious and quite acceptable; it is good practice in economics to build up rules derived from stability conditions. And this interpretation appears preferable to one based on some "psychological law." In fact, there is no such law; or, if it does exist, it cannot truthfully be described as a psychological law; for the propensity to consume depends in a perfectly formal way on the redistribution of income which occurs as income undergoes a variation (Infra. p. 25, note 1). Actually the available statistical evidence indicates that some relation connecting aggregate consumption and income probably exists, and that its value is consistently less than unity, as Keynes believes.

The Multiplier

"If you can look into the seeds of time, and say which seeds will grow and which will not." (Macbeth).

In this concept we find one of Mr. Keynes' most elusive contributions. In a general form, this idea is familiar to any student of the trade cycle; but in Mr. Keynes' hands the concept has taken on a more precise significance. Its general import may be set out as follows. Suppose that society is in equilibrium with saving and investment proceeding at a given rate. Then some cause may occasion an increment in the rate of investment. The consequence is that an added sum of money is paid out (say) to wage earners as income; the level of income has already risen; but this is not all. As consumers, these wage earners spend their money, thus raising the level of consumption; but before they
spend the money, a part leaks out into saving. This diminished sum, passing into consumption, stimulates additional production, this time in the field of consumers' goods. The additional activity generates new incomes, part of which leaks out into savings, but part of which stimulates consumption further, and so on. The eventual outcome of this process (if the advanced level of investment is maintained) is that incomes rise to a level higher than the original by several times the amount of the new investment. The factor which sets a limit to the rise in income is the leakage of incomes into saving. And the limit is reached when the level of income has risen to that point at which additional savings out of additional income are being effected at the same rate as the new investment. Thus the stimulating effect of new investment is being offset by the rise in savings which exactly equals it. In time, the process might proceed as indicated in the accompanying diagram.

The multiplier may be said to have two interpretations - a tautological one and a non-tautological one. The tautological one is "the logical theory of the multiplier, which holds good continuously, without time lag, at all moments of time."¹ In formal terms, the

¹ G. I., p. 122.
multiplier is the ratio between the increment of income and the increment in investment. It is derived from the relation -

\[ Y_w = C_w + I_w \]

\[ \frac{\Delta Y_w}{\Delta I_w} = \frac{\Delta Y_w}{\Delta Y_w - \Delta C_w} \]

dividing numerator and denominator of the right hand side by \( \Delta Y_w \); we secure \( \frac{\Delta Y_w}{\Delta I_w} = \frac{1}{1 - \frac{\Delta C_w}{\Delta Y_w}} \). Given Mr. Keynes' definitions, this result follows in a perfectly tautological fashion. Several comments may be passed on this version of the multiplier. This result follows directly from the relation - income equals consumption plus investment. If this relation holds true, then the ratio of an increment in income to an increment in investment is one over one minus the ratio of the increment of consumption to the increment of income.

Stated in this way the multiplier is free of any relationship to the marginal propensity to consume. That is, we are free to

\[ \Delta Y_w \equiv \Delta C_w + \Delta I_w \] (and therefore \( Y_w \equiv C_w + I_w \)) in a different form; thus, \( \frac{\Delta Y_w}{\Delta I_w} = \frac{1}{\Delta Y_w - \Delta C_w} = \frac{\Delta Y_w}{\Delta Y_w - \Delta C_w} \). Cross-multiplying, we see that \( \Delta Y_w (\Delta Y_w - \Delta C_w) = \Delta Y_w \cdot \Delta I_w \) or \( \Delta Y_w \equiv \Delta C_w + \Delta I_w \).

For an excellent discussion of this as well as of other matters concerning the multiplier see Haberler, Prosperity and Depression (Geneva: League of Nations, 1941).
interpret the ratio \( \frac{\Delta C}{\Delta Y} \) in any way that we choose. One such interpretation is to substitute in its place the marginal propensity to consume, but this is unnecessary.

Suppose we start with the community in equilibrium at the beginning of a clock time period, call it 1. During this period income is 100, saving 30 and investment 30, consumption 70. During period two consumers continue to spend 70, while investment rises to 40. Income then rises to 110, but before the extra 10 has had a chance to be spent the period closes;

\[
\frac{\Delta Y}{\Delta I} = \frac{1}{1 - \frac{\Delta C}{\Delta Y}} = \frac{1}{1 - 0} = 1.
\]

Now it is perfectly clear that the marginal propensity to consume is never zero in the formal sense; but we chose the clock time period in such a way that the ratio \( \frac{\Delta C}{\Delta Y} \) was zero. Could it be clearer that "the logical multiplier" does not depend on any formal functional relationship at all, but on an identity between three free variables, one of which \( \Delta I \) is fixed? When stated in this way, the multiplier suffers from excessive generality. That is, we have three increments in three free variables (one of which is given) and only one relationship connecting them. Thus, the multiplier is capable of interpretation in almost any way that is suitable to the fancy.

In part, this ambiguity derives from the identification of income and expenditure on output. It would not be possible to derive any such loose relationship between income and investment, if expenditure were defined to be investment plus consumption. A further property of this multiplier may be noted: "It holds good
continuously, without time lag, at all moments of time" whether
or not the system is in equilibrium. As later analysis will
show, this relation holds at any intermediate point between one
equilibrium and another. Thus it is not bound by the functional
relationships which must be satisfied in equilibrium. This lack
of restriction in the variables involved implies a certain
ambiguity - the relationship in this form is indeterminate.¹

There remains the non-tautological interpretation of the
multiplier. In this form the multiplier is the measure of a
relative change in investment and the consequent change in income,
as between two positions of equilibrium; it reveals what will
happen to the equilibrium level of income following an autonomous
increase in the rate of investment, assuming that the rate of
interest is held constant throughout.² In this construction,
the ratio $\frac{C_W}{Y_W}$ is to be interpreted in the schedule sense as the

$\frac{\delta I_w}{\delta I_w} (1 - \frac{\delta C_w}{\delta Y_w}) = 1 \quad \frac{\delta I_w}{\delta Y_w} = \frac{1}{1 - \frac{\delta C_w}{\delta Y_w}}$. And this

may also be shown to be the value resulting from the introduction
of a change in $I_w$ into the equations of monetary equilibrium -
(1) - (5) above, when $i$ is held constant.

¹ For a criticism of the multiplier in terms of its
retrospective quality, see Saulnier, Contemporary Monetary Theory
(New York: Columbia University Press, 1938). Saulnier says,
"Keynes determines the value of the multiplier after the effect on
income has been produced. This sort of procedure does not explain
why the assumed change did occur, yet this is the problem which
calls for explanation." Pp. 335-36.

² The multiplier described here may be derived by substituting
in the equation $Y_W = I_w + C_w$ the relation $C_W = C_W (Y_W, I)$. Differentiate with respect to $Y_W$,
$\frac{\delta Y_W}{\delta I_w} = 1 + \frac{\delta C_W}{\delta Y_W} \cdot \frac{\delta Y_W}{\delta I_w}$

$\frac{\delta I_w}{\delta I_w} (1 - \frac{\delta C_w}{\delta Y_w}) = 1 \quad \frac{\delta I_w}{\delta Y_w} = \frac{1}{1 - \frac{\delta C_w}{\delta Y_w}}$. And this

may also be shown to be the value resulting from the introduction
of a change in $I_w$ into the equations of monetary equilibrium -
(1) - (5) above, when $i$ is held constant.
"normal" value of the propensity to consume; as such, the propensity to consume reflects the unambiguous value \( \frac{\partial Y_w}{Y_w} \) which it must achieve when it has settled into its equilibrium position. And it is this interpretation whereof Mr. Keynes speaks when he says: "The novelty in my treatment of saving and investment consists, not in my maintaining their necessary aggregate equality, but in the proposition that it is, not the rate of interest, but the level of incomes which ensures this equality." Of course, taken literally this is a misstatement. The level of incomes is not needed to ensure equality between \( A' - U - V \) and \( I' - U - V \). What he should have said is: the level of incomes ensures equality between saving, in the schedule sense, and the level of investment in the schedule sense. This saving, in the schedule sense is income minus consumption, in the schedule sense. And the normal value of this saving is reached only when the equilibrium is worked out. By reason of Keynes' artificial definitions, this paradoxical interpretation is made necessary; its meaning will appear in the subsequent discussion. In order to comprehend the

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2 Haberler, op. cit., p. 193. "It is misleading to say that income must change, in order to secure the equality of \( S \) and \( I \). Whatever the level of income may be, \( S \) and \( I \) must be equal, because they are made so by definition. The change of level of income comes in as a condition only because Mr. Keynes takes the 'multiplier' — and 'the marginal propensity to consume' — as a constant quantity . . . Income must change, not because it is necessary to ensure the equality between \( S \) and \( I \), but because we have assumed it by assuming the multiplier. We have here an example of a confusion between a terminological relationship between symbols . . . and an empirical relationship between conceptually independent magnitudes."
process which gets underway when investment changes, we shall have occasion, from time to time, to resort to our distinction between income and expenditure in order to clarify this discussion.

In our detailed exposition of this doctrine, it will be convenient to discuss first the non-tautological interpretation. Having fixed this in our minds, it will then be appropriate to show how the tautological interpretation accompanies the working out of the specific unambiguous value arrived at from considerations of equilibrium. Diagrammatically, the investment multiplier appears as the ratio of the increment of income, and disparity between two equilibrium values of the same, and the increment of investment which occasions this variation of equilibrium. In our diagram we plot only the income = expenditure line, and the expenditure curve. Along one expenditure \((C + I)\) curve, the value of investment is assumed to remain constant. Accordingly, the increase in \(E=(C + I)\) along one such curve, as income increases, is the result entirely of the rise in consumption.

In this diagram the level of investment which has a constant value along \(E_1\) is raised uniformly by a value \(S_I\), causing the \(E_1\) curve to shift upward to the position represented by \(E_2\). Whereas the old equilibrium was achieved at \(C\), the new equilibrium...
of income and expenditure is achieved at $T$. The rise in income is $QR = TR$. The rise in consumption is $SR$, and the increase in investment is (approximately) $TS$. Then

$$\frac{Y}{I} = \frac{QR}{TS} = \frac{TR}{SR} = \frac{1}{\frac{TR}{TR} - \frac{SR}{TR}} = \frac{1}{1 - \frac{SR}{TR}}.$$  

But

$$\frac{SR}{QR} = \frac{C}{Y},$$  

or the marginal propensity to consume. Hence, we have a graphical demonstration that the multiplier, $K = \frac{1}{1 - \text{M.P.C.}}$.

In the above diagram $Y = 7$, $C = 4$, and $I = 3$. $K = \frac{Y}{I} = \frac{7}{3}$

$$= \frac{1}{1 - \frac{4}{7}} = \frac{1}{\frac{3}{7}} = \frac{7}{3}. $$  

Thus, we see that when the marginal propensity to consume is $4/7$, the multiplier is $7/3$, or the reciprocal of the marginal propensity to save.$^1$

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$^1$ An alternative geometrical interpretation may be set out by means of an interpretation of equilibrium as the interaction of saving and investment. Here the formal distinction between $E$ and $Y$ must be adhered to. According to this interpretation, we plot saving and investment along the vertical axis and income along the horizontal axis. As a first approximation, the value of investment is assumed to remain unaffected by income.

\[ K = \frac{\Delta Y}{\Delta C + \Delta S} = \frac{\Delta C}{\Delta Y} + \frac{\Delta S}{\Delta Y} = \frac{\Delta S}{\Delta Y} = 1 - \frac{\Delta C}{\Delta Y}. \]

Thus, in the above diagram, we arrive at the result that the multiplier $K$ is the reciprocal of the marginal propensity to save. This saving-investment diagram may be found in Kaldor, "A Model of the Trade Cycle," *Economic Journal* L (1940), pp. 70 ff.
In the diagrammatic analysis alone, which treats the multiplier as an aspect of a shift from one equilibrium to another, it is implicitly assumed that the change is the outcome of a rise in the flow of investment. An alternative explanation is possible. The multiplier may be interpreted as the result of a discontinuous act of investment, occurring at a point of time. If the effects of this injection are traced out, they may be compared with those of a rise in the flow of investment.¹

According to this analysis a quantity I of new investment occurs in period 1, and, if we follow Robertsonian time periods, this sum becomes disposable income the following period. Of this sum ∆I, a part, ∆I . p, will be spent, and this leads to secondary income of ∆I . p, leading to tertiary spending of ∆I . p² and so on. If the series ∆I + ∆I . p + ∆I . p² + ... + ∆Ipⁿ is summed, we derive the value ∆I(1 / (1 - p)) after n time periods, where n is infinite. The term p may be interpreted as the marginal propensity to consume, and so we arrive at our original formula:

\[ \frac{\Delta Y}{\Delta I} = \frac{1}{1 - \frac{\Delta C}{\Delta Y}}. \]

A point of considerable interest is raised by this procedure. If the number of periods is arbitrarily chosen small, then the formula for such a progression becomes ∆Y / ∆I = (1 - pⁿ). It is obvious that this expression depends for its value on n, the number of time periods.

periods elapsing after the original injection of investment. This expression has been called the "truncated multiplier." ¹

By means of an example we may show the relationship between the "truncated multiplier" and the "instantaneous" or tautological multiplier of Keynes. In the following table, \( Y_r \) and \( S_u \) indicate the increments of realized income, and unexpected (or undesigned) savings, respectively. Also \( Y_d \), \( C_r \), and \( S_p \) represent the increments of disposable income (in the Robertsonian sense), realized consumption, and planned saving, respectively, while \( K \) is the investment multiplier. In the 6th column, we have the cumulative increments in consumption divided by the cumulative increments in income. The latter magnitude is registered in the eighth column. It may be noted that the increment in realized income is the increment in

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>( Y_r = S_u )</th>
<th>( Y_d )</th>
<th>( C_r )</th>
<th>( S_p )</th>
<th>( \sum C_r )</th>
<th>( \frac{\sum C_r}{\sum Y_r} )</th>
<th>( K )</th>
<th>( \text{CUMULATIVE INCOME} )</th>
</tr>
</thead>
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<tr>
<td>1</td>
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<td>0</td>
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<td>2.44</td>
<td>244</td>
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<td>2.59</td>
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<td>32.77</td>
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<td>20.97</td>
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<td>8.60</td>
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<td>14.53</td>
<td>5.43</td>
<td>553.43</td>
</tr>
</tbody>
</table>

A number of points come to light from examination of this table. First, let it be said that the table is constructed on the basis of the Robertsonian day which may be identified with these periods. The disturbance herein analyzed arises from an increment of investment amounting to 100 which occurs in period 1. This sum, while it is earned in period 1, cannot be spent until period 2. Thus, for the time being, the propensity to consume out of the added 100 is zero. Clearly the Keynesian multiplier is one. For investment has risen by 100, and so has realized income. In period 2 the income of 100 may be spent, and as the psychological marginal propensity to consume, \( \frac{dC}{dY} \), is .80, consumers plan, and carry out savings of 20 in the Robertsonian sense. But the remainder, or 80, is spent on consumption, and the result is an increment of 80 in realized income. Now the cumulative increment in realized income is 180, while 80 has been consumed; hence the marginal propensity to consume is 80/180 or .44. And the multiplier is seen to be 1.8.

Tracing out the process we can see that the multiplier gradually rises towards its "full" value of 5, which is the limiting value reached after an infinite number of periods.

---

1 The table and accompanying discussion were suggested by an article of E. S. Shaw, "A Note on the Multiplier," Review of Economic Studies, VI (1938), pp. 61 ff.

2 Thus, \( \frac{1}{1 - .44} = \frac{1}{1 - .00} = 1 \)

3 Again, \( \frac{1}{1 - .44} = \frac{1}{.56} = 1.8 \)
It is 1.82 after 15 periods. Note also that the value of the marginal propensity to consume in the ex-post or tautological sense rises gradually to the value realized in the ex-ante or psychological sense. By period 15 the tautological (ex-post) value has risen to .79, while the psychological (ex-ante) value is .80, as it has been throughout.

The value of the multiplier which is worked out before the "full" value of five is reached, is the instantaneous multiplier in the Keynesian sense, or the "truncated multiplier" in the sense used above. Now the Keynesian instantaneous multiplier is not determinate; it yields no definite answer, but a range of values from one to five. That is so because the formula itself is indeterminate; one equation connecting two free variables, $a$ and $b$, is insufficient to determine both. The truncated multiplier will give us the answer, for any period, however.

For example, \( \frac{1 - p^n}{1 - p} \) in period three turns out to be \( \frac{1 - (0.8)^3}{1 - (0.8)} \)

= 2.44, or the value of the multiplier. Such an interpretation may have considerable value, since, in planning public works, it is necessary to estimate not only the ultimate value of \( x \), but the value as of a particular time.

One point which deserves attention is the relation of saving to investment. In period 1, investment rises by 10%, and so does unexpected saving; therefore, saving ex-post is equal to investment. In period 2 no new investment occurs, but the value of planned or ex-ante saving rises to 20 and the unexpected (ex-post) saving drops to 80. So total saving still equals investment, but now the planned portion of the former has risen. The successive periods are worked
by a cumulation of planned saving until the sum approaches, as a limit, the value of the original investment, namely, 100. At the same time, the unexpected (ex-post) saving dwindles until it reaches zero, at the limit. Now in any intermediate period, the cumulative planned saving plus ex-post (unexpected) saving equal investment. Thus, in Keynesian terminology, saving equals investment at all times, while in Robertsonian terminology, investment exceeds saving until cumulative saving out of disposable income equals investment.

It can easily be shown that the limit of the planned savings after n periods must equal investment. If an increment of investment, call it I, occurs in period one, planned saving will form in succeeding periods the series, \((I - I \cdot p + (I \cdot p - I \cdot p^2)
\)
\(\vdots + (I \cdot p^{n-1} - I \cdot p^n)\), where \(p\) is the propensity to consume. Since the intermediate terms cancel, the series reduces to \(I - I \cdot p^n\), which approaches \(I\) in value, as \(n\) approaches infinity.

Clearly, the process whereby the sum of the successive increments of planned savings comes to equal the increment of investment requires for its completion the passage of many periods. In any intermediate period, the disparity between planned saving and investment must be financed from some source; the alternatives are the creation of new money by the banking system or the utilization of idle balances. The former possibility is the simpler, since it is to be presumed that idle balances would not be attracted without a rise in the rate of interest. And this development would transgress one of the assumptions on
which the analysis is founded. Evidently, then, the difference between investment and planned saving may be financed by new money; but as planned savings accumulate, this new money may be cancelled. The limit to this process is reached in the complete cancellation of the increment of new money, whose original value may be assumed to have been equal to the increment of investment.

This approach to the multiplier has its counterpart in an ingenious diagram devised by Mr. Fellner. In this diagram disposable income is measured along the horizontal axis, and expenditure on output (or earned income) is measured along the vertical axis. As usual, the equilibrium position is to be found at the point where income equals expenditure, where the \( Y_d = Y_e \) line cuts the \( E (C+I) \) curve at \( Q \).

In period one equilibrium prevails with the system settled at an income of \( OY_1 \). During period two an autonomous increase in investment of \( QP = a \) occurs, leading to an excess income of that amount. In period three this disposable income, \( OY_1 + aP = OY_2 \), gives rise to expenditure, \( Y_2S \). At \( S \) we have a level of expenditure \( SY_2 = KY_3 \), giving rise to a level of disposable income \( OY_3 \) in the

---

following period. Now \( OY_3 - OY_1 = Y_1 Y_3 = QL = ML = SP = b \)
is the excess income thus generated in the second period over
and above the sum generated in the first period, \( a \). But \( b =
a (b) \). The system has then moved to \( N \). The disposable income
\( OY_3 \), then gives rise to an expenditur \( Y_3N \). The difference
\( Y_3N - Y_3 L = Y_3 N - Y_1 \) \( Q = LN \), is the excess of earned income
in this period over the original income, \( OY_1 \). Now \( LN = QL \cdot LN \),
\[ \text{where } QL = b = a (b); \text{ and } \frac{LN}{QL} = b \text{ by similar triangles}. \] Hence
\[ LN = a (b) \cdot (b) = a (b)^2. \] Now the sum of these excess incomes
\[ a + a (b) + a (b)^2 + ... = a \left( \frac{1}{1 - b} \right), \] where \( b \) represents
the propensity to consume. Thus, \( \frac{\Delta Y}{\Delta I} = \frac{1}{1 - \frac{4C}{\Delta Y}} \), or the usual
multiplier. Here, we trace out the effects of a single injection,
or dose of investment.

An alternative interpretation of this approach may be
carried out, whereby a series of equal increments of investment
are effected in each successive period. And the results which
this analysis achieves are similar to those indicated by our original
procedure whereby the Keynesian system shifts from one equilibrium
to another. In the end, then, this analysis yields results identical
with the flow analysis treated in the diagram on p. 96, and the
surrounding textual material. What is interesting in such analysis
is the process whereby the end result is achieved.
## The Serial Injection Multiplier

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<tr>
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<td>800</td>
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<td>1878.8</td>
<td>900</td>
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<td>2.49</td>
<td>2875.5</td>
<td>1300</td>
<td>2.21</td>
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</table>
In the accompanying table, the Robertsonian day is again chosen as the time period. And the earned income of one period is the disposable income of the next. In each period an increment of 100 units of investment occurs. The progress of a given 100 units of investment insofar as it gives rise to consumption, may be traced by the figures included within the diagonal lines.\footnote{This analysis follows Machlup, "Period Analysis and Multiplier Theory," Quarterly Journal of Economics, LIV (1939), pp. 1-28.}

Thus, with an assumed propensity to consume of .60, the original sum of 100 gives rise to 60 induced units of consumption in the next period, and a rise in earned income of that amount. This 60 becomes disposable income the following period, and gives rise to 36 units of consumption, and so on. By following the diagonal line, we may follow the progress of any single injection of investment.

Since a new injection of 100 occurs in each period, the induced increments of income begin to overlap until they form a corresponding series of increments. From the nature of the assumptions, these overlapping increments, seen by scanning the figures included between two horizontal lines, form a series identical with the diagonal series. Furthermore, the multiplier thus derived, yields a value exactly equal to the multiplier secured from a single injection. But in this instance, the quantity multiplied is the current increment in investment. Accordingly, the current level of income will rise by the increment in current investment times the multiplier.

Several interesting phenomena are revealed by the table. As we have seen in our analysis of the single-injection multiplier,
this (serial-injection) multiplier rises gradually towards a full level of 2.5, destined to be reached with a (psychological) marginal propensity to consume of .60. And the value of this serial-injection multiplier, at any intermediate period, may be calculated by the identical method used in connection with the single-injection type. The formula is thus

$$\frac{\Delta Y}{\Delta I} = \frac{(1 - p^n)}{1 - p}$$

where \( p \) is the marginal propensity to consume.

Another interesting relation is revealed: period savings (in the Robertsonian sense) gradually rise as income increases; for period savings form a constant proportion of a rising income. Yet investment exceeds planned saving in every period, with the gap steadily narrowing. Of course, the planned saving of any period plus the unexpected saving of that period, the rise in earned income, always equals the investment of the period. Since planned savings alone are available for investment, the difference between planned savings and investment must be supplied out of the creation of new credit, or by borrowing from idle balances in such wise that the rate of interest does not change. By the thirteenth period, it will be noted, current planned savings are virtually equal to current investment; thus it is the sum of the foregoing series of differences between planned savings and investment which must be financed by the banking system, so long as the higher level of activity persists.

An average multiplier, here entitled \( M \), may also be noted. The average multiplier is the ratio of the cumulative increments in income to cumulative increments in investment. In a sense, this average multiplier may be even more significant for policy
than the period multiplier. When the legislator looks at the effects of public spending, he is likely to desire a measure of the ratio of the increment in total income to the increment in total spending, as measured from a certain point of time. It will be seen that the average multiplier rises more slowly than the period multiplier, K.

Following Robertson, these results may be summarized algebraically in the adjoining table.¹

One point of interest is that the rise in new money must be exactly equal to the rise in the level of income. Now it may be claimed that this increment in money might cause the interest rate to rise, to fall, or to remain constant, when taken in conjunction with the new level of activity. Yet it is the constancy in the level of interest rates on which the formula is based; for an alteration in interest rates would disturb investment in a way not considered by the formulas advanced. Accordingly, this interpretation may be considered as a sort of median hypothesis, achieved when income velocity is equal to one. Actually, sufficient new money must be injected to cause interest rates to remain constant, when taken in conjunction with the new level of activity.

Thus far, in our discussion, we have dealt only with the simple investment multiplier which expresses the effect of an autonomous increase in investment on income via consumption. Later work on the multiplier has resulted in so-called compound multipliers; such multipliers deal explicitly with the induced

¹ Essays in Monetary Theory, p. 119. He does not derive the average multiplier.
<table>
<thead>
<tr>
<th>Period</th>
<th>Investment</th>
<th>Earned Income</th>
<th>Planned Saving</th>
<th>New Money</th>
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<td>I</td>
<td>(1 - P)I</td>
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<td>I</td>
<td>(1 + P)I</td>
<td>(1 - P)I</td>
<td>P I</td>
</tr>
<tr>
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<td>I</td>
<td>(1 + P + P^2)I</td>
<td>(1 - P^2)I</td>
<td>P^2 I</td>
</tr>
<tr>
<td>4</td>
<td>I</td>
<td>(1 + P + P^2 + P^3)I</td>
<td>(1 - P^3)I</td>
<td>P^3 I</td>
</tr>
</tbody>
</table>

Approaching, as the number of periods becomes infinite, ______

\[ N \rightarrow \infty \]

\[ \frac{1}{1 - P} \]

1. Excess of investment over planned saving, sum of column (2) - column (4) \( I + P \cdot I + P^2 \cdot I + \ldots + P^n \cdot I = \frac{I}{1 - P} \).

2. If the injections of investment are unequal, the total rise in incomes may be calculated as follows:

\[
\begin{align*}
\text{Period} & \quad \text{Investment} & \quad \text{Then we have at period } n \\
1 & \quad I_1 & \quad I_n \\
2 & \quad I_2 & \quad I_1 + I_1 P + I_1 P^2 + \ldots + I_1 P^n \\
& \quad \vdots \\
N & \quad I_n & \quad I_2 + I_2 P + \ldots + I_2 P^{n-1}
\end{align*}
\]

\[
\sum_{i=1}^{n} \Delta Y_i = \sum_{i=1}^{n} \Delta I_i \cdot \frac{(1 - P^n + 1 - 1)}{1 - P}
\]

The average multiplier will be:

\[
\sum_{i=1}^{n} \frac{\Delta Y_i}{\Delta I_i} = \sum_{i=1}^{n} \frac{\Delta I_i (1 - P^n + 1 - 1)}{\Delta I_i}
\]
effects of increased investment on income via consumption and investment. It will be appropriate to review the simple multipliers in such wise that the relationship to compound multipliers will be intelligible.

In the following analysis I is the rate of investment, C is the rate of consumption, and Y the rate of income per unit of time. We may write \( I = I(Y) \), \( C = C(Y) \). By differentiating the relation \( Y = C + I \) with respect to \( I \), thus treating it as a free variable, we secure \( \frac{dY}{dI} = \frac{dC}{dY} \frac{dY}{dI} + 1 \). \( \frac{dY}{dI} = \frac{1}{1 - \frac{dC}{dY}} \).

Similarly, treating \( C \) as a free variable, we derive the relation:

\[
\frac{dY}{dC} = 1 + \frac{dI}{dC} \frac{dY}{dC} \quad \Rightarrow \quad \frac{dY}{dC} = \frac{1}{1 - \frac{dC}{dY}}.
\]

Now (1) and (2) may be derived by the Kahn-Clark method of leakages, as sums of infinite geometric progressions:

Here we write \( C' = \frac{dC}{dY} \) and \( I' = \frac{dI}{dY} \)

(1)

\[
\frac{dY}{dI} = 1 + C' + (C')^2 + \ldots = \frac{1}{1 - C'}
\]

(2)

\[
\frac{dY}{dC} = 1 + I' + (I')^2 + \ldots = \frac{1}{1 - I'}
\]

The results are identical with (1) and (2) above.

1 Rather than attempt to pick up the various multipliers scattered around in the literature, we have elected to follow an excellent synthesis of the subject by Lange. Lange derives his results by the leakages method alone, save with respect to results (1) and (2) below. He does not work out the identity of (1) and (2) with (3) and (4). Neither does he derive results (8) and (9). One of his multipliers, furthermore, is incorrect, but this is not analyzed here by reason of limitations of space. Our notation follows Lange's closely. "The Theory of the Multiplier, Econometrica, XII (1943), pp. 227-45.

2 The result (1)' holds provided \( C' < 1 \); and (2)' holds provided \( I' < 1 \). We may assume that \( C' < 1 \) is an empirically
Now we may show how the two methods may be used to
derive compound multipliers. Whereas the above multipliers
deal explicitly only with the induced effects on consumption or
investment separately, of a rise in investment or consumption,
respectively, compound multipliers deal explicitly with both
sets of cross relationships. In the following discussion, the
subscripts f and i denote whether the variable is free or bound
by a functional relationship.

Write (a) \( Y = C_f + C_i + I_f + I_i \) where 
\[ C_i = C_i (Y) \] and 
\[ I_i = I_i (Y) \] and where 
\( C_f \) and \( I_f \) are free
variables.

Now differentiate (a) with respect to \( I_f \), holding \( C_f \)
constant.

\[ \frac{dY}{dI_f} = \frac{dC_i}{dY} \frac{dY}{dI_f} + 1 + \frac{dI_i}{dY} \frac{dY}{dI_f} \]

(3) \[ \frac{dY}{dI_f} = \frac{1}{1 - \left( \frac{dC_i}{dY} \frac{dI_i}{dY} \right)} \]. This is the compound
investment multiplier.

established fact. Stability in the saving-investment market
requires that \( C' + I' < 1 \). It follows from these two conditions
that \( I' < 1 \). The stability condition mentioned is often represented
by a diagram. The S curve intersects the I curve from below.

\[ S, I \]
\[ 0 \]
\[ Y \]
Now differentiate (a) with respect to $C_f$, holding $I_f$ constant. 

\[
\frac{dY}{dC_f} = 1 + \frac{dC}{dY} + \frac{dI}{dY} \frac{dY}{dC_f}.
\]

(4) \[
\frac{dY}{dC_f} = \frac{1}{1 - (\frac{dC}{dY} + \frac{dI}{dY})}. \quad \text{This is the compound consumption multiplier.} \quad \text{Both (3) and (4) may be derived by the method of leakages. Thus, let } C_i' \text{ represent } \frac{dC_i}{dY} \text{ and } I_i' \text{ represent } \frac{dI}{dY}, \text{ and let } dI_f \text{ be an autonomous increment of investment; This leads to an equal direct increase in income and an induced rise in expenditure (and income) of } (C_i' + I_i') dI_f. \text{ The result is a further induced increase in income } (C_i' + I_i') (C_i' + I_i') dI_f \text{ and so on. The total increase in income is thus,}
\]

\[
dY = (1 + (C_i' + I_i')) + (C_i' + I_i')^2 + \ldots \) dI_f.
\]

(3) \[
\frac{dY}{dC_f} = \frac{dI_f}{1 - (C_i' + I_i')}. \quad \text{or (3) above.}
\]

Similarly, let $C_f$ be an autonomous increase in consumption. This leads to an equal direct increase in income and an induced rise in expenditure (and income) of $(C_i' + I_i') dC_f$. The result is a further induced increase in income $(C_i' + I_i') (C_i' + I_i') dC_f$ and so on. The total increase in income is thus,

\[
dY = (1 + (C_i' + I_i')) + (C_i' + I_i')^2 + \ldots \) dC_f.
\]

(4) \[
\frac{dY}{dC_f} = \frac{dC_f}{1 - (C_i' + I_i')}. \quad \text{or (4) above.}
\]

It is clear that an autonomous increase in expenditure of any kind, whether of investment or consumption, must lead to the same

---

1 The stability conditions for (3) and (4) are $C_i' + I_i' < 1$. This multiplier seems to have been recognized first by J. M. Keynes, *Investment and Business Cycles* (New York: McGraw-Hill, 1941), p. 196.
results. What may not be self-evident, is that (1) and (2) alone express results identical with one another and with (3) and (4). In effect, the reason why (1) and (2) come out differently is that there is a concealed element in the elements $dC$ and $dI$. In short, there are several concealed factors here. Let us uncover them.

Expression (1) may be written $dY = \frac{dI}{1 - C'}$. Here $dI$ must be assumed to include, not only the original autonomous increase in investment, but the induced secondary effects. Thus,

$$dY = (1 + I_1' + I_1' (C' + I_1')) + I_1' (C' + I_1')^2 + \ldots \frac{dI'}{1 - C'}$$

$$= \frac{1}{1 - C'} \cdot (1 + \left(\frac{I_1'}{1 - (I_1' + C')}\right) \frac{dI'}{1 - C'} = \frac{1 - I_1' - C' + I_1'}{1 - (I_1' + C')} dI'$$

$$= \frac{dI'}{1 - (I_1' + C')}.$$ Hence (1) reduces to (3).

Similarly (2) may be written $dY = \frac{dC}{1 - I'}$. Here $dC$ must be assumed to include, not only the original autonomous increase in investment, but the induced secondary effects. Thus,

$$dY = (1 + C_1' + C_1' (I_1' + C_1')) + C_1' (I_1' + C_1')^2 + \ldots \frac{dC}{1 - I_1}$$

$$= \frac{dC}{1 - I_1} \cdot \left(1 + \frac{C_1'}{1 - (I_1' + C_1')}\right) = \frac{dC}{1 - I_1} \frac{1 - I_1' - C_1' + C_1'}{1 - (I_1 + C_1)}$$

$$= \frac{dC}{1 - (I_1' + C_1')}.$$ Hence (2) reduces to (4).

In all probability the formulas (1) and (2) are not very useful, especially (2). If we include in the numerators of (1) and (2) the induced, as well as the original increments of $I$ and $C$, respectively, then these formulas will yield correct results. But perhaps it is easier to deal with these secondary effects explicitly by means of the compound multiplier. Following Lange, the identical
results (3) and (4) may be written:

\( \frac{dY}{dE} = \frac{1}{1 - \phi} \) \hspace{1cm} \text{in virtue of}

(a) \( \psi \cdot E' (Y) = G' + I' \)

(b) \( \frac{dY}{dI} \equiv \frac{dY}{dG} \) from (3) and (4).

We may call \( \psi \) the marginal propensity to spend, and 
\( (1 - \phi) \) the marginal propensity to hoard; the latter name
derives from the fact that the term refers to the excess of planned income over planned expenditure.

We may now turn to the effects of spending of a country engaged in international trade. The algebraic relations may be set down, as follows:

\begin{align*}
\text{Equations} & \quad \text{Unknowns} \\
(1) & \quad Y = E_1 + R \quad (1) \quad Y = \text{income} \\
(2) & \quad E = E_1 + E_2 \quad (2) \quad E = \text{total spending} \\
(3) & \quad E_1 = E_1 (Y) \quad (3) \quad E_1 = \text{internal spending} \\
(4) & \quad E_2 = E_2 (Y) \quad (4) \quad E_2 = \text{external spending} \\
(5) & \quad R = R (E_2) \quad (5) \quad R = \text{External receipts - Other system's propensity to spend back.}
\end{align*}

It will be noted that income is regarded as the sum of internal spending and external receipts (foreign expenditures in the home country). Both internal and external spending are treated as dependent upon income. And external receipts are related to external or foreign spending by the function entitled "the propensity to spend back."

The internal spending multiplier, which calculates the effect on income of an autonomous increase in internal spending, may be obtained by the usual methods.
Differentiating,

\[(4) \quad Y = E_{1f} + E_{1i} + R\]

with respect to \( E_{1f} \) where the subscripts \( f \) and \( i \) refer to free and induced internal spending, respectively, we secure:

\[
\frac{dY}{dE_{1f}} = 1 + E_{1i}' \quad \frac{dY}{dE_{1f}} + R' \quad E_2' \quad \frac{dY}{dE_{1f}}
\]

\[(6) \quad \frac{dY}{dE_{1f}} = \frac{1}{1 - (E_{1i}' + R' \quad E_2')}.\]

We may secure the same result by the leakages method.

Thus, if \( dE_1 \) is an autonomous increment in the rate of internal spending, the system's income increases by \( dE_1 \). Out of this only \( E_1' \) \( dE_1 \) is spent internally; \( E_2' \) \( dE_1 \) is spent externally and of this \( R' \quad E_2' \) \( dE_1 \) is returned to the system.

Accordingly, the initial increment, \( dE_1 \), in the system's income leads to further income within the country of \((E_1' + R' \quad E_2') \) \( dE_1 \). From this \( E_1' \) \( dE_1 \) is spent internally and \( R' \quad E_2' \) \( dE_1 \) is returned to the system. Thus the income of the system experiences a tertiary increase of \((E_1' + R' \quad E_2')^2 \) \( dE_1 \). And so for the later repercussions. The total increase in the system's income is therefore:

\[dY = (1 + (E_1' + R' \quad E_2')) + (E_1' + R' \quad E_2')^2 + \ldots \quad dE_1,\]

and (6)' \[\frac{dY}{dE_1} = \frac{1}{1 - (E_1' + R' \quad E_2')}\] or (6) above.\(^1\)

The same result as (6) above will evidently be obtained if an increase in external receipts occurs. Thus, differentiating \( (B) \) where the subscripts have the usual meaning:

\[1 \quad \text{The stability condition for (6)', and, in fact, for the remaining multipliers, is } E_1' + R' \quad E_2' < 1.\]
The same result would be obtained by the leakages method.

Suppose that an increase in external spending should occur. What will be the effect on income of a change in external spending? This case must be treated with some care. We have the relation to be differentiated —

(C) \( Y = E_1 + R \)

and we have the relation

(D) \( E_2 = E_2f + E_{21} \)

First, differentiate (D) with respect to \( E_{2f} \) and secure:

(E) \( \frac{dE_2}{dE_{2f}} = 1 + E_{21} \frac{dY}{dE_{2f}} \)

Now differentiate (C) with respect to \( E_{2f} \) and secure

\[
\frac{dY}{dE_{2f}} = E_1' \frac{dY}{dE_{2f}} + \frac{dR}{dE_2} \frac{dE_2}{dE_{2f}}
\]

Substituting from (E), we secure —

\[
= E_1' \frac{dY}{dE_{2f}} + R' \left( 1 + E_{21}' \frac{dY}{dE_{2f}} \right) = R' + (E_1' + R' E_{21}') \frac{dY}{dE_{2f}}
\]

(8) \( \frac{dY}{dE_{2f}} = \frac{R'}{1 - (E_1' + R' E_{21}')} \)

This result may be derived by the leakages method. An autonomous increment in external spending \( dE_2 \) leads to a direct increase in external receipts, \( R'dE_2 \), leading to a similar increase in income. This leads to internal spending \( R'E_1' \ dE_2 \) and external spending \( R' E_{21}' \ dE_2 \) of which \( R'^2 E_{21}' \ dE_2 \) returns to the system.

The secondary increase in income is thus \( R' (E_1' + R' E_{21}') \ dE_2 \).

This in turn leads to further internal spending of
\[ R^1 E_1' (E_1' + R^1 E_2') \, dE_2 \] and external spending of
\[ R^1 E_2' (E_1' + R^1 E_2') \, dE_2 \] of which \( R^1 E_2' \) returns to the system. The tertiary increase in income is thus
\[ R^1 (E_1' + R^1 E_2')^2. \] The total increase in income is therefore
\[ dy = R^1 + R^1 (E_1' + R^1 E_2') + R^1 (E_1' + R^1 E_2')^2 + \ldots \) \, dE_2

\[ (8') \frac{dy}{dE_2} = \frac{R^1}{1 - (E_1' + R^1 E_2')} \quad \text{or (8) above.} \]

Now the effect of internal spending on external spending may also be calculated by the relation: \( E_2 = E_2 (Y) \):

\[ (A') \frac{dE_2}{dE_1} = E_2' \frac{dy}{dE_1} \]. Substituting (6) above in this expression, we secure --

\[ (8) \frac{dE_2}{dE_1} = \frac{E_2'}{1 - (E_1' + R^1 E_2')} \].

This may also be calculated directly by the leakages method. An increment \( dE_1 \) in external spending leads to an increase in income of \( dE_1 \) and leads to external spending \( E_2' \) \, dE_1. The system's income increases further by \( (E_1' + R^1 E_2') \) \, dE_1, leading to external spending of \( E_2' (E_1' + R^1 E_2') \) \, dE_1. The income of the system rises by \( (E_1' + R^1 E_2')^2 \) \, dE_1 leading to further external spending of \( E_2' (E_1' + R^1 E_2')^2 \) \, dE_1. And so for later repercussions. The total increase in external spending is, accordingly --

\[ dE_2 = E_2' (1 + (E_1' + R^1 E_2') + (E_1' + R^1 E_2')^2 + \ldots) \, dE_1 \]

\[ \frac{dE_2}{dE_1} = \frac{E_2'}{1 - (E_1' + R^1 E_2')} = E_2' \frac{dy}{dE_1} \quad \text{or (8) above.} \]

By a similar process of logic we may derive the effect on internal spending of a change in external spending. Using \( E_1 = E_1 (Y) \) we also get

\[ (9) \frac{dE_1}{dE_2} = E_1' \, (Y) \frac{dy}{dE_2} = \frac{E_1' \, (Y) \cdot R^1}{1 - (E_1' + R^1 E_2')} \]
This may be calculated by the leakages method, as follows. An increment \( \Delta E_2 \) in external spending will lead to external receipts \( R' \Delta E_2 \). This, in turn, leads to a corresponding rise in income, and an increase in internal spending of \( R' \Delta E_1 \). External spending will rise by \( R' \Delta E_2 \), leading to further external receipts of \( R'^2 \Delta E_2 \). Thus income will rise by

\[
R' (E_1' + R' E_2') \Delta E_2 \quad \text{and internal spending by} \quad R' (E_1' + R' E_2') \Delta E_2.
\]

External spending will rise to \( R'^2 E_2' \) and income to \( R' (E_1' + R' E_2')^2 \). This will lead to a tertiary increase in internal spending of \( R' (E_1' + R' E_2')^2 \Delta E_2 \), and so on.

The total increase in internal spending is thus

\[
\Delta E_1 = (R' E_1' + R' E_1') + R' E_1' (E_1' + R' E_2')^2 + \ldots \Delta E_2.
\]

\[
\therefore (9) \quad \frac{\Delta E_1}{\Delta E_2} = \frac{R' E_1'}{1 - (E_1' + R' E_2')} = E_1' \frac{dY}{dE_2} \text{ or (9) above.}
\]

An interesting interpretation of this apparatus is possible. These multipliers may be interpreted as applying to the relations between the private economy and the treasury. Thus \( Y \) may be interpreted as aggregate private income, \( E_1 \) as aggregate private spending, \( E_2 \) as the treasury's taxation and borrowing out of private income, and \( R \) represents the spending of the treasury. In this way all possible cross effects as between treasury spending and private spending may be worked out.

Summary

The propensity to consume, in the strict sense, is that functional relationship between income and consumption which determines total consumption, when the level of income is given. Various other factors, including the rate of interest and the distribution of
income, may affect the propensity to consume. The rate of interest is assumed by Keynes to produce a small effect on consumption out of a given income, and the distribution of income is assumed to remain constant or to vary with the level of income. Consequently, Keynes does not write these two factors in as independent variables affecting consumption.

Besides that propensity to consume which determines the total volume of consumption, there are the average and marginal propensities to consume. The average propensity to consume is the ratio between consumption and income. The marginal propensity to consume is the ratio between an increment of consumption and an increment of income. Both the average and the marginal propensities depend on the level of income. Furthermore, there is a "fundamental psychological law" which states that an increment in income will cause a somewhat smaller increment of consumption. In short, the marginal propensity to consume is taken as less than unity.

The multiplier, defined as one over one minus the marginal propensity to consume, was first introduced into economic analysis by Mr. Kahn. Previously, many writers had considered the effects of fluctuations in the level of investment. Until the development of the multiplier, however, no one had attempted a precise measurement of the relative change of income and investment. It is this precision which marks off the multiplier from earlier efforts in the field.

According to Mr. Keynes, the multiplier is the ratio between an increment of income and the increment of investment to which it may be attributed. The injection of investment is accompanied
by that variation in the money supply which leaves the rate of interest constant. An increment of investment expenditure represents payments to the factors of production, and consequently passes into income. At this point the multiplier is equal to one, because the increase in the level of income is equal to the increment of investment. The repercussions do not cease here. The additional income is partially spent by consumers, and partially saved. The quantity that is saved drops out of the active circulation and ceases to stimulate further rounds of income and expenditure. When all the original investment thus leaks out into saving, the stimulus will be exhausted, for savings constitute a sort of trap for the active circulation of money.

The additional quantity consumed out of the original increase in income leads to additional production and the payment of a corresponding sum of money into income. Of this sum, a part will be spent and a part saved. At every round of income, a further part leaks into saving, the circulation of income, and consumption going around and around, until the original injection of investment into the money stream has entirely leaked out into savings. The simplest approach to the multiplier expresses it in terms of the marginal propensity to save, which is the proportion of the increment of income which is unspent, or one minus the marginal propensity to consume. Suppose that the government spends $100 on road building. If the marginal propensity to save is 1/5, how much will income have to rise before sufficient additional savings are created to equal the increment in investment? To put it another way, how much income will have
to be created before the investment all leaks out into savings? In this case the answer is $500; when income has risen by this amount, 1/5 of this additional income, or $100, will have been saved. And this means that the additional sum injected into the money stream has been withdrawn. In every case, the multiplier is the reciprocal of the marginal propensity to save. In the above case it is 5. If the marginal propensity to save is 1/4, the multiplier will be 4, if 1/3, it will be 3, and so on.

Certain ambiguities crop up in the interpretation of the multiplier. It is necessary to draw a distinction between the effects of a single injection of investment on income, on the one hand, and the effects of a series of such injections, on the other. These two multipliers require somewhat different treatment, although they are fundamentally similar. Probably, the multiplier fundamental to the Keynesian analysis is the one which treats of a series of injections, that is, an increase in the rate of investment.

Furthermore, it is necessary to distinguish between the instantaneous, or tautological multiplier, and the non-tautological multiplier. The latter is an unambiguous concept which is framed in terms of a stable psychological propensity to consume; it is a measure of the ratio of the increment of income to the increment of investment as between two equilibria of saving and investment. The tautological multiplier, on the other hand, yields a range of values between 1 and the limiting value expressed by the non-tautological multiplier. It depends on a correspondingly variable marginal propensity to consume. This form of the multiplier is
indeterminate, until some added information fixes the value of the propensity to consume in its non-psychological sense. Those multipliers which are worked out in terms of time periods can determine the intermediate values, and do not render necessary resort to a variable propensity to consume. Simply by taking account of the number of time periods which have elapsed between the original investment and the date in question, the appropriate multiplier can be computed.
Notes on the Propensity to Consume

I

Some controversy has arisen concerning the average and the marginal propensities to consume. G. R. Holden, in a discussion of Mr. Keynes' theory drew some conclusions which the latter regarded as incorrect, i.e., inapplicable to his theory.¹ Holden asserted that his interpretation of Mr. Keynes was to the effect that the assumption, \( \frac{C_w}{Y_w} < 1 \), was necessary to the General Theory; also that \( \frac{d^2 C_w}{d Y_w^2} < 0 \). Mr. Keynes expressly denies that \( \frac{C_w}{Y_w} \) need be less than 1, that he ever said so, or that it is necessary to his theory. Mr. Keynes seems to have the stronger position here. As for the possibility that \( \frac{d^2 C_w}{d Y_w^2} < 0 \), Mr. Keynes appears to have little to say on this matter. It should be noted that the second derivative of \( C_w \) has nothing to do with Mr. Keynes' obiter dictum (G.T., p. 97) that \( \frac{d}{d Y_w} \left( \frac{C_w}{Y_w} \right) < 0 \).

That is, \( \frac{d}{d Y_w} \left( \frac{C_w}{Y_w} \right) = Y_w \frac{d C_w}{d Y_w} - C_w \). The sign of this expression is plus or minus according as \( \frac{d C_w}{d Y_w} \) is greater or less than \( \frac{C_w}{Y_w} \).

Mr. Keynes' "fundamental psychological law" is that $\frac{dC}{dW}$, the marginal propensity to consume, is less than unity. Statistical studies tend to indicate that this propensity is less than unity. For example, a study of Richard and W. M. Stone indicates marginal propensities to consume in different countries (U.S.A. (1929), Japan (1926-27), Germany (1926-27) for various income classes. Only in farm families with incomes of $0 to $500 in the United States was the marginal propensity to consume greater than unity. All other income classes in all the countries studied exhibited a marginal propensity to consume of less than unity. Other studies show similar results.\(^1\)

A more fundamental point has been raised by Dr. Staehle. His study of the propensity to consume among wage earners in Germany casts doubt on the hypothesis of a unique relationship between consumption and income. Whereas the correlation between income and consumption is negligible, consideration of distribution effects along with income yields statistically significant results. As Dr. Staehle treated the problem, the distribution effect, as measured by a coefficient $B$, was treated as an independent variable. Such a treatment is inconsistent with the basic hypothesis of the

General Theory, according to which distribution effects must be treated either as negligible, or as varying with the level of income. By reason of the restrictions on the data, and the failure of the author to check the results by treating (in some way) distribution effects as a function of income, this investigation cannot be accepted as definitive.

While Mr. Keynes considers \( \frac{dcw}{dt} \) < 1, as a "psychological law," he speculates about other properties of the consumption function. His obiter dictum runs: "These reasons will lead, as a rule, to a greater proportion of income being saved as real income increases."\(^1\) The available statistical evidence supports this view. For example, studies by Mendershausen and Gilboy indicate that the marginal propensity to save is greater than the average propensity to save, or that the income elasticity of saving is greater than unity, within the ranges studied.

The income elasticity of saving is \( \frac{ds}{dy} \cdot \frac{Y}{S} > 1 \). If this be true, \( \frac{ds}{dy} > \frac{S}{Y} \). We also find that \( \frac{d}{dy} \left( \frac{s}{Y} \right) = \frac{Y \cdot \frac{ds}{dy} - S}{Y^2} > 0 \), if \( \frac{ds}{dy} > \frac{S}{Y} \). So Keynes' statement holds true so long as the marginal propensity to save is greater than the average propensity to save, irrespective of whether the elasticity is increasing or decreasing.

Now if the elasticity of consumption is less than unity, the elasticity of saving is greater than unity. This (perhaps)

\(^1\) C. T., p. 97.
If \( \frac{dC}{dY} < \frac{C}{Y} \) and \( e_C = \frac{dC}{dY} \cdot \frac{Y}{C} < 1 \), then the numerator of \( e_s \) is greater than the denominator, and \( e_s > 1 \). Therefore, if \( e_C < 1 \), \( e_s > 1 \); if \( e_C > 1 \), \( e_s < 1 \). Therefore, when Mrs. Gilboy says: "For all except farm communities the income-expenditure elasticity is usually just under unity, 0.8 or 0.9," she also asserts that the elasticity of savings is greater than unity, barring some statistical quirk or other. Both of these studies indicate also that the income elasticity of savings tends to diminish as income rises.

On the whole, then, both Keynes' psychological law, \( \frac{dC}{dYw} < 1 \), and his obiter dictum, \( \frac{d}{dYw} \left( \frac{Sw}{Yw} \right) < 0 \), turn out well, when considered against the background of the facts. The latter tendency, however, appears to be less fundamental in nature.
CHAPTER V
THE RATE OF INTEREST AND THE
MARGINAL EFFICIENCY OF CAPITAL

I
The First Account.

One of the most controversial of Mr. Keynes' contributions to economic science is his theory of the rate of interest. Not the least confusing thing about the new theory is its complete lack of continuity with received doctrine. And, indeed, it is even today difficult to find any sort of consistency as between the "classical" approach, the Keynesian doctrine, and a third analysis which may be synonymous with the first, the loanable funds theory.¹

The "classical" theory, while it deals with monetary phenomena, seeks to trace these matters back to a source resting

¹ The words "classical economist" have always denoted to me Ricardo, John Stuart Mill, Malthus (in his rent and population theories), and perhaps Smith. The term "neo-classical economist" denoted to me particularly close followers of Ricardo during the late nineteenth century, of whom Marshall is the outstanding example. But when the word "classical" is applied without restriction to almost any thinker of note, the term begins to lose any meaning which it might otherwise have had. As Mr. Robertson says, "It does not seem to me likely to be helpful to label Professor Pigou as a 'classical economist' - still less to clap that label opprobrii causa on to the vacuous countenance of some composite Aunt Sally of uncertain age." "Alternative Theories of the Rate of Interest," Economic Journal, XLVII (1937), p. 436. I am also at sea when it comes to selecting out the "classical" theory of interest. In general, I take it to mean the Marshallian, or Fisherian; for it is my impression that the two men stress roughly the same forces and methods.

in certain real forces. Thus, Fisher's theory combines the elements of time preference, governing the supply of savings coming into the market, and productivity (rate of return over cost), governing the demand. Marshall, too, favors the division of causal elements into two categories, "waiting" which governs supply, and "productiveness" which governs demand.

The Keynesian liquidity preference theory, on the contrary, is predominantly a monetary theory. It seeks its rationale in the behavior of the holder of resources, and traces the causes of interest back to the desire for cash. At first glance there appears to be little connection between this sort of theory and one founded on "waiting" and "productiveness"; it will be a part of our task to show that some connection does exist.

Sooner or later, the discreet reader of the General Theory is likely to be faced with a puzzling dilemma. Mr. Keynes asserts forcibly, and with a suitable marshalling of evidence, that the "classical" theory is characterized by circular reasoning. Furthermore, the reader is told, the General Theory of interest is not thus circular, and he would do well to adopt it. At this point the reader is likely to be in some distress. Unable to see any connection between Mr. Keynes' doctrine and the "classical" one, and perhaps feeling that the elements of "waiting" and

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"productiveness" are not void of influence on the rate of interest, the reader may be forced into an agnostic point of view. Failing this, he may be induced to adopt the one viewpoint or the other, and assume an intransigent attitude towards the opposing school. In this discussion of the problem we cannot hope to bridge the gap between opposing schools, nor yet to suggest the one true path. In part, the trouble lies in the fact that, while Mr. Keynes appears to have hit upon some sort of truth, he has not succeeded in baring its underlying rationale to the average reader. Such a difficulty inheres in the macro-economic approach. Mr. Keynes provides no account of the means whereby the individual achieves his optimum situation, while taking account of liquidity preference. Until such an account appears, successfully rationalizing liquidity preference, much of the theoretical foundation will remain obscure. And, lacking such an account, it is most difficult to connect Mr. Keynes' theory with the pricing system.

On one point, at least some agreement does exist, and that relates to the definition of interest. Mr. Keynes defines the rate of interest as "The inverse proportion between a sum of money and what can be obtained for parting with control over the money in exchange for a debt for a stated period of time." Is this not substantially identical with the Marshallian version whereby interest is defined as "The payment made by a borrower for the use of a loan for, say a year ... expressed as a ratio

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1 G. T., p. 167.
A debt is actually a loan; the former is seen from the viewpoint of the borrower, the latter from the viewpoint of the lender. If all views could be as easily harmonized as these, a happy state of agreement would soon exist.

The gulf is soon exposed. As Mr. Keynes sees it, "The rate of interest is not the 'price' which brings into equilibrium the demand for resources to invest with the readiness to abstain from present consumption. It is the 'price' which equilibrates the desire to hold wealth in the form of cash with the available quantity of cash." In strictly Keynesian terminology, "the rate of interest is the reward for parting with liquidity for a specified period of time." Parting with "cash" is taken to be equivalent to parting with liquidity. Since, the rate of interest is treated as the price regulating the exchange of "cash" for debts, it is necessary to differentiate these two terms. On this point, Mr. Keynes says, "we can draw the line between 'money' and 'debts' at whatever point is most convenient for handling a particular problem. For example, we can treat as money any command over general purchasing power which the owner has not parted with for a period in excess of three months,

2 ibid., p. 167.
3 Ibid., p. 167.
and as debt what cannot be recovered for a longer period than this; or we can substitute for 'three months' one month or three days or three hours or any other period . . . as a rule, I shall . . . assume that money is co-extensive with bank deposits.\(^1\)

Clearly, the distinction between cash and debts is quite loosely drawn.\(^2\) In the ensuing discussion, we shall assume that "cash" is to be taken as bank deposits.

In the General Theory Mr. Keynes points out two main sources of demand for cash - money to use, and money to hold. According to this account the demand for money, as divided into these two sources, depends on the level of income and the rate of interest. \(M_1\), the demand for money to use, = \(L_1 (Y)\), while \(M_2\), the demand for money to hold = \(L_2 (i)\). Thus

\[ M = M_1 + M_2 = L_1 (Y) + L_2 (i) = L (Y, i). \]

According to Mr. Keynes' first account, the demand for money to use is derived from three sources - the income-motive, the business-motive, and the precautionary motive, all of which are supposed to depend on the level of income. The income motive arises from the need to bridge the gap between income and expenditure and depends on the level of income and the interval between its receipt and disbursement. The business-motive arises.

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\(^1\) Ibid., p. 167. Italics under "any other period" are mine.

\(^2\) Mr. Shaw comments on this, "It is so difficult to handle the problem of relative interest rates by this method that its exponents are reduced to defining 'cash' as a fearful mixture of money and short (?) securities."

from the need for cash to bridge the interval between the
time of incurring costs and the receipt of sale-proceeds.
This motive, depends on the value of output, and hence upon
income. The precautionary-motive occasions the holding of
cash to make provision for unforeseen or unforeseeable circum-
stances requiring sudden expenditure. The sum of these three
elements constitutes the demand for money to use and is denoted
by the equation \( M_1 = L_q \) (1).

A second motive governing the desire to hold cash is the
speculative motive. The necessary condition for this source of
demand for cash "is the existence of uncertainty as to the
future of the rate of interest." Essentially, the logic is
this. Given the money rate to be paid annually on a bond, its
capitalized value will fluctuate inversely with the rate of
interest. If the rate of interest on newly issued bonds rises,
the investor takes a loss; if it falls, he takes a profit. And
this uncertainty gives rise to a guessing game. In this game
the accumulated interest to be derived from investing funds
in bonds is weighed against the capital loss arising out of a
possible rise in the rate of interest. Of course, if an
individual entertains an undoubting expectation that the rate
of interest will remain constant or fall, there is nothing to
deter his purchase of bonds. Conversely, given such an expecta-
tion there is no inducement to hold cash. But if this individual
should feel that the advantage of holding cash outweighs the

1 G. T., p. 168.
advantages of holding bonds— that is, if the expected net
depreciation in the value of a bond exceeds the accumulated
value of the interest charges during the interval in question
— then he will hold cash. This is much the same thing as
saying that, given the present rate on a bond today, there is
a limiting future rate at time \( t \), which will result in a zero
net yield on the bond. If this rate relationship is confidently
expected, there is no advantage in trading cash for bonds.

This aspect of liquidity preference has been clearly
analyzed by Mr. R. W. Goodwin.\(^1\) In this treatment, let \( C_0 \) and
\( C_n \) represent the value of the bond at times \( 0 \) and \( n \), respectively;
let \( a \) represent the annual yield in dollars, and \( i_0 \) and \( i_n \) the
rates of interest existing and expected at times zero and \( n \),
respectively. The break-even point of the investor occurs
when the bond yields no net return. This is defined as the point
where the loss on capital account equals accumulated interest
over the interval, \( 0 - n \); in equation form, (1) \( C_0 - C_n \)
\[ = a + a (1 + i_0) + \ldots + a (1 + i_0)^{n-1}. \]
Two further relations
are needed, linking the capital value of the bond to the rate
of interest. These are given by the familiar capitalization
formula. Thus, (2) \( C_0 = \frac{a}{1 - i_0} \) (3) \( C_n = \frac{a}{1 - i_n} \). Substituting (2)
and (3) in (1), we secure,
\[ \frac{a}{1 - i_0} - \frac{a}{1 - i_n} = a + a (1 + i_0) + \ldots + a (1 + i_0)^{n-1} \]
or by the formula for a geometric progression,
\[ \frac{1}{i_0} - \frac{1}{i_n} = (1 + i_0)^n - (1 + i_0)^{n-1} \).

\(^1\) "Keynesian and other Interest Theories," Review of Economic
Therefore, \( i_n - i_o = i_n \left( (1 + i_o)^n - 1 \right) \) or (4) \( i_n(2 - (1 + i_o)^n) = i_o \), subtracting \( i_o(2 - (1 + i_o)^n) \) from both sides, and factoring we secure, \( (i_n - i_o)(2 - (1 + i_o)^n) = i_o(1 + i_o)^n - i_o \), therefore

(5) \( i_n - i_o = \frac{i_o(1 + i_o)^n - i_o}{2 - (1 + i_o)^n} \)

The equation (4) gives in a perfectly general form the relationship between the present rate of interest, \( i_o \), and that future rate, \( i_n \), which would render the holding of cash equally as profitable as holding a bond. Equation (5) gives us the relationship between the present rate of interest, \( i_o \), and that increment in the rate, \( i_n - i_o \), which would render the holding of cash equally as profitable as holding a bond.

The plot of (4) yields an indifference curve system representing those relations between present and future rates, at various times, which yield no net return from investment.

The present rate, \( i_o \), is plotted along the horizontal axis, while the expected future rate, \( i_n \), is plotted along the vertical axis. A 45° line, bisecting the axes, indicates the locus of points at which present rates are equal to expected future rates.

The "investment indifference curves" for the several time periods indicate a state of no-gain-from-investment. If the rate of interest is 2.5% today on perpetual bonds (consols), and if the investor expects confidently that a rate of 2.88% will prevail five years from now, it will not pay him to buy a bond and hold it for five years. For the fall in the price of the bond will exactly offset the interest, compounded annually at 2.5%. Accordingly, the investor would be balanced in a state of perfect indecision between
holding bonds and holding cash. Actually, of course, superior convenience would dictate the holding of cash; but let us provisionally neglect this consideration. At other present rates there are corresponding future rates for such five year investments in perpetual bonds at which a corresponding state of indifference would possess the investor. The locus of such points is the lower investment indifference curve depicted in the diagram.

All of these indifference curves have a positive slope greater than unity; all lie above the 45° line for the reason that a rise in the expected future rate entails a loss to be set off against interest payments. And it follows from the nature of the construction that any point lying above the relevant investment indifference curve represents a state in which the holding of cash is more advantageous than holding the relevant bond. In terms of our example, if the future rate on perpetual bonds yielding 2.5% currently were expected to be (say) 2.90% five years hence, investment in such bonds would entail a slight loss in money. On the contrary were this rate expected to be 2.85% five years hence, investment in such bonds would yield a slight net return.

Consequently, states of expectation which place future rates above the relevant indifference curves lead to the desire to hold cash for speculative reasons. This is precisely the sort of thing Mr. Keynes seeks to explain when he asserts that the necessary condition for the speculative demand for cash "is the existence of uncertainty as to the future of the rate of interest."

But perhaps we should say, in this connection, that it is the
certainty that interest rates will rise to such a degree as to wipe out the accumulated interest on investment.

An alternative construction employed by Goodwin sets the expected rise in the rate of interest, plotted on the vertical axis, against the present rate, plotted on the horizontal axis. The curves corresponding to each time period depict that rise in the present rate, as related to the present rate, which would result in a state of no-gain from investment. In terms of our example, the perpetual bond, currently bearing 2.5% interest, yields no net return if the rate rises .38%, five years hence. Again, a greater rise would entail a net loss, a lesser rise would entail a net gain.

A highly significant feature of the liquidity doctrine is clearly revealed by the alternative construction pictured in Figure II. At low rates of interest the effects of a given rise in the interest rate on the desire to hold cash (or conversely to acquire bonds) is much stronger than at high rates. Consider the twenty year curve. At a rate of 1 3/4%, a rise of approximately 3/4% will serve to equalize the advantages of holding cash or bonds. But at 2 1/2%, a rise of nearly 4% is required to render equally desirable the holding of cash or bonds. Thus, we come to the important conclusion that an expected rise of the interest rate will exert a far more powerful impetus to the holding of cash at low rates of interest than at high. Accordingly, the lower the rate of interest, the more powerful is the force inducing investors to hold cash. Even if investors do not anticipate a definite increase in the rate, when it stands at a low level, the
knowledge that such a rise, even of moderate size, would wipe out their gains if it did occur, serves as a stimulus to the holding of cash. The lower the present rate, then, the greater the desire to hold cash for "precautionary" purposes.

It may be seen that the character of expectations bears on the demand for money. In this analysis we depart from a world wherein rationality has full sway towards one in which anticipations assume a dominant role. It would be comfortable to suppose that the results of economic action focus the activities of men guided purely by thoughts of self-interest, based upon solid facts. Unfortunately, this is not the case. The most we can say is that rational action, when it does occur, is guided by anticipations, uncertainties, and hopes. Nowhere does this guiding force of anticipations appear more clearly than in the desire for liquidity. Mr. Keynes has hit on something important here, but the approach is subject to qualifications. Mr. Hawtrey cautions us that, "... however important the part played by expectations may be, it is not to be inferred that they can or should be given precise quantitative measurement. Any forecast of a future economic quantity is likely to be not merely vague and approximate, but actually incomplete. The expectation often relates only to an upper or a lower limit, or it is contingent upon factors of which no forecast at all is made."¹

Clearly, expectations are not capable of measurement in any precise sense. Anticipations as to the magnitude of a future

economic quantity extend over a range within which one or another value is anticipated as more or less probable. Perhaps the future development of tools of thought constructed around anticipations is contingent upon the development of greater simplicity and precision in the relation of this range of anticipated values to the theory of probability. Meanwhile, we must observe some caution in dealing with theories resting thus on anticipations. So, perhaps, it is wise to follow the theory of liquidity preference with some reservations on this score.

Let us provisionally summarize the account given in the General Theory before plunging into the latest developments.

The demand for cash may be divided into two elements – cash held by reason of the transactions' motive and that held by reason of the speculative motive. The speculative demand for cash, that is, for money to hold, is said to depend solely on the rate of interest. The transactions demand for cash, that is, for money to use, is said to depend solely on the level of income. Provisionally, the supply of cash may be treated as a constant. The equilibrium between the supply of cash and the demand for cash may be represented by the equation, \[ M = L_1(Y) + L_2(1) \].

Given the level of economic activity, represented by \( Y \), it is the

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rate of interest which equates the supply of and the demand for cash. Diagrammatically, this equilibrium may be pictured as follows.

At a level of income, \( Y \), the demand for cash dictated by the transactions motive assumes the value \( OS \). And this demand for cash is not affected (directly) by changes in the rate of interest. This demand is represented by the vertical line, \( RS \). The speculative demand for cash is represented by the distance between the curve \( PL \) and the line, \( RS \). The sum of the two demands is represented by the curve \( PL \). Equilibrium prevails at the point \( E \) where the total demand curve for cash, \( PL \), cuts the line \( M \), representing the available quantity of cash.

The speculative demand for cash varies inversely with the rate of interest; for the lower the rate of interest, the greater the loss to which the bondholder is exposed from a given rise in the rate of interest. Accordingly, the lower the rate, the keener is the desire to hold cash to avoid such unpleasant contingencies.

Thus fundamental tendency is reinforced by another. When
the rate falls extremely low, the probabilities become greater that the rate is destined to rise. Reversing the law of gravity, we say that "what goes down must come up." This assumes that some "normal" rate exists to which the market rate returns when it deviates therefrom. The elliptical nature of this argument derives from the condition that no explanation is offered concerning the "normal" rate. Consequently, this argument cannot be considered as independent of the more fundamental forces causing the inverse relationships between the demand for cash and the rate of interest.¹

While the graph indicates that the rate of interest is determined by the demand for cash taken in conjunction with the supply, it still may not be entirely self-evident that the desire to hold cash can affect, much less determine, the rate of interest. To satisfy ourselves as to the validity of this approach, let us work out an example. Select as "the" rate of interest the rate prevailing in the market for long-term bonds, a gilt-edged type of security. Suppose that the quantity of money is increased by action of the monetary authority. The representative individual will allocate his cash as between various alternatives. Since the level of economic activity is unchanged, he needs no more cash for transactions purposes. Given the rate of interest, he will not want any more cash for speculative

¹ For an elaborate, but not entirely satisfying account of these and other matters see Malin, Keynesian Economics (Toronto: University of Toronto Press, 1942), ch.'s V and VI.
purposes. Hence he will be induced to purchase bonds, as a suitable alternative. The pressure of demanders bidding for bonds, will force up the quotations. Since the amount of return written on the face of the bonds cannot undergo change, the rise in quotations implies a fall in the rate of return. The rate of interest has fallen.

This account of the matter may be seen to be indirect. Rather than focus attention directly on the supply of and the demand for bonds, Keynes has elected to follow a less direct route. But we need not suppose that this indirect approach need involve any circularity. And it has the value that it directs attention specifically to the monetary factors affecting the rate of interest.

The speculative demand for cash is the source of much that is peculiar to Keynesian doctrine. A special property of the speculative demand for cash is deserving of some attention. At low rates of interest, the speculative demand for cash becomes highly elastic. And this attribute of the speculative demand for cash is possibly the most important feature of Keynesian interest doctrine. Previously, monetary theorists had stressed the possibility of the manipulation of the quantity of money in such a way as to govern the rate of interest. Given control over the rate of interest the monetary authority could hope to mitigate cyclical fluctuations. In recent years, however, increasing scepticism has been manifested by monetary theorists as to the feasibility of monetary stimulation of the economic system at the bottom of a depression. Here is a rationalization of this belief.
When the supply of cash is pushed to the point where the elastic part of the speculative curve is reached, control over the interest rate slips away from the monetary authority. Within this range a given increase in the supply of money produces a very slight decline in the rate of interest. Thus the speculative demand becomes almost insatiable, and capable of absorbing large quantities of additional cash. Thus, the demand for cash for speculative purposes is the villain of the piece, robbing the monetary authority of power over the rate of interest and, hence, over the activity of the economic system. And this leads to a series of considerations concerning policy.

Failing the power to control the rate of interest in a downward direction, what means are available to the government to combat the ravages of a deep depression? Other means must be found, and some of them involve direct control over investment by the government. The multiplier, too, appears as a deus ex machina to save the day. Policy and theory are seen to be closely related in Mr. Keynes' world.

II

The Revised Account

Under the influence of criticism Keynes has offered some substantial amendments or additions to his theory. The occasion for this revision of his theories was an extensive controversy between several proponents of the so-called "loanable-funds" doctrine and Mr. Keynes, himself. These theorists, notably Mr. Robertson and Mr. Ohlin, sketched out an alternative theory, based on the supply of and demand for loanable funds. Seeking
Keynes elaborated on a new concept, the "demand for finance," which he considers to be the "coping-stone" of his theory.¹

The demand for "finance" is brought into being by the time lag between decisions to invest and the execution of these decisions. If an entrepreneur contemplates an act of investment, he will customarily lay up a store of cash to bridge the time interval between planning and execution of the decision. In short, the entrepreneur's receipts per unit of time, before the investment, exceed his expenditures per unit of time, and the consequence is a temporary increase in the cash balance of the entrepreneur. During the period of investment, the rate of receipts, as supplemented by cash, may be maintained above the former level of receipts, but will now fall short of the level of expenditure and balances begin to fall from their temporary high. And this fall continues until the investment is completed; borrowings are cut off, receipts once more equal expenditures, and balances maintain a steady level. This level of balances is presumably about equal to the former one. Diagrammatically, this process appears as follows.

¹ See the extensive controversy in the Economic Journal, XLVII (1937), and XLVIII (1938), between Messrs. Keynes, Ohlin, Robertson, and Hawtrey.
Rates of receipts and expenditure are plotted along the vertical axis and time along the horizontal axis. At time $T_1$ the entrepreneur, foreseeing investment to take place at $T_2$, begins to borrow from the banks, and thus his cash balance begins to rise, because his rate of receipts, including loans, exceeds his rate of expenditure. At time $T_2$ investment begins, and the rate of expenditure rises until it reaches the rate of receipts at time $T_3$. At this point balances have reached their maximum. Beyond $E$, the rate of expenditure rises for a time and then declines, while the rate of receipts declines continuously. Between $T_3$ and $T_4$ the rate of expenditures exceeds the rate of receipts, and balances are being drawn down. At $T_4$ the two rates are equal and balances then remain constant.

The rise in money balances between $T_1$ and $T_3$ is represented by the curved area $ME$. The decline in money balances between $T_3$ and $T_4$ equals $ME$. If $ME$ exceeds $NE$ then balances will have risen by time, $T_4$. If, on the other hand, $NE$ is greater than $ME$, then balances will have fallen by time, $T_4$.

What is important, however, is the sum of all demands for finance by entrepreneurs. Keynes says on this matter: "If investment is proceeding at a steady rate, the finance (or the commitments to finance) can be supplied from a revolving fund of a more or less constant amount, one entrepreneur having his finance replenished for the purpose of a projected investment as another exhausts his on paying for his completed investment. But if decisions to invest are (e.g.) increasing, the extra
finance involved will constitute an additional demand for money.\(^1\)

We may illustrate the meaning of this statement by an extension of the diagram below.

The receipts of the first entrepreneur (or entrepreneurial group) begins to rise in time at \(M\), following the path \(MRV\). The occasion for this rise in receipts is a provision for future investment, the proceeds to accomplish which are being built up in advance. Investment begins at \(N\) and follows the path \(NRS\). So long as the line \(MRV\) lies above \(NRS\), the rate of receipts exceeds the rate of expenditures and this group "absorbs" balances. At \(R\) expenditure begins to exceed receipts and balances are being released by this group. But another group is beginning to expand receipts in anticipation of future investment. The expenditure curve of this group is \(PQT\), while its receipts curve is \(PTU\). At \(P\) receipts begin to exceed expenditures, so this group is currently absorbing balances, while the other group is releasing them. If the rate of release effected by the one group is identical with the rate of absorption of the other, the requirements for finance are constant. But if one group is absorbing balances faster than the other releases them,

\(^1\) "Alternative Theories of the Rate of Interest, Economic Journal, XLVII (1937), pp. 247-48."
requirements for finance and increasing and the rate of interest must rise.

If the condition is satisfied that the one group is releasing balances at the same rate that the other group is absorbing them, at time OP no additional finance is required beyond this time. The requirements for finance are represented by MNR; they represent the demand for funds caused by the lag between investment decisions and the execution of these decisions.

So far, so good. Our next question relates the means of acquiring finance. Mr. Keynes says:

"How is it supplied? The entrepreneur when he decides to invest has to be satisfied on two points: firstly, that he can obtain sufficient short-term finance during the period of producing the investment; and secondly, that he can eventually fund his obligations by a long term issue on satisfactory conditions. Occasionally, he may be in a position to use his own resources or to make the long term issue at once; but this makes no difference to the amount of 'finance . . .' but only to the channel through which it reaches the entrepreneur and to the probability that some part of it may be found by the release of cash on the part of himself or the rest of the public. Thus it is convenient to regard the two-fold process as the characteristic one."

1 Messrs. Robertson and Shaw have subjected this concept to some searching criticism. And, indeed, Mr. Keynes' account of the process whereby the finance is made available to entrepreneurs is obscure, and none too consistent. In my opinion, the process of the releases and absorptions of cash (Hawtrey's terms) is to be considered as occurring continuously in time. And in equilibrium the process may be conceived of as occurring frictionlessly and without time lag, as explained above.


What Keynes seeks to describe here is the channelization of funds flowing into investment. The above technical detail is of little moment for his theory; the ramifications of the investment process are merely seen to be more complex. Perhaps the process might be described with reference to a series of containers connected by pipes. The fluid in the containers represents the level of balances. Entrepreneurs have a group of such containers filled with a certain volume of fluid, distributed in a certain way. Savers also are possessed of a certain series of containers filled with a certain volume of fluid, distributed in a certain way. Between these two groups of containers rests a large container. Fluid is constantly flowing from each particular savers’ container to the large container. Thence the fluid flows in a particular way to each of the investors’ containers. Then the fluid flows from the investors’ containers into a large pipe whence it is forced back to the containers of the savers. The aggregate volume of fluid is constant.

In equilibrium, the aggregate flow of fluid from the savers’ containers into the large container is equal to the reverse flow from investors through the large pipe to the savers. Now it is true, the level of fluid in the several investors’ containers will vary. For, at times the flow from the central container into an investor’s individual container will be so rapid as to raise the level of fluid in the latter. But, at the same time the level in all the other investors’ containers will be diminished by a like amount as less flows into the others from the central container.
In our analogy, of course, the level of fluid represents the level of balances, and a change in the level represents a difference between inflow (borrowings) and outflow (investment). It should be sufficiently clear that this concept of finance begins to approach fairly close to dynamic theory. Certainly the rates of flow in time take on great significance. Can it be more self-evident that it is utterly fruitless to attempt to divorce the concepts of flows and stocks? For stocks of money are a function of the differences between inflows and outflows of money. As well divorce flows and stocks as cause and effect.

The revised view of Mr. Keynes may be summarized as follows. We now have three sources of demand for cash arising from (1) the business motive (2) the precautionary motive (3) finance. Mr. Keynes never indicates whether this new demand depends on the rate of interest, the level of income or both. At least, if he does, the indication is obscure. I assume, however, that he considers the demand for finance to be purely a function of the rate of interest. \[ M = M_1 + M_2 + M_3 = L_1 (Y) + L_2 (I) + L_3 (I), \] where \( L_3 \) is the demand for finance, and \( L_1 \) and \( L_2 \) are the business and precautionary demands, respectively.

![Diagram](image-url)
The rate of interest is set at the rate which equates the demand for cash from these sources with the available supply.¹

III

Criticism

We enter the troubled waters of controversy with considerable trepidation. Messrs. Robertson and Shaw have launched a series of attacks against Mr. Keynes' position. These two men, proponents of the loanable funds doctrine, have advanced criticisms of a telling but occasionally obscure nature. The general effect of reading their strictures on the Keynesian doctrine is a confused state of mind concerning the latter. Accordingly, these criticisms appear to offer little assistance in reconciling these two theories of interest, the liquidity preference theory and the loanable funds doctrine. In order to understand what is in the minds of these men it is necessary to understand their theory of interest. Substantially, their doctrine says that the rate of interest is determined by the supply of new credit during an interval of time and the demand for additional credit during that interval. The supply of credit is based substantially on the supply of savings but may be supplemented by the creation of new funds or the reduction of idle balances ("dishoardings"). The demand for loanable funds is derived principally from the increment of new investment, but may be supplemented by consumption demand and so on. This concept focuses attention directly on the flow of loanable

¹ For this diagram see Shaw, op. cit., p. 838.
funds supplied and demanded. There is no place here for the stock of money. It is not needed in the analysis.

Perhaps it will be wise to indicate the substantial identity of this view with the Keynesian doctrine. They explain, that is, the same conditions in different ways.

Suppose that we select a time period during which the stock of money turns over once against the available output. The level of income and expenditure, during the period, then, is equal to the quantity of money. If we wish, we may think of the market as divided into three parts, the market for money, for consumption, and for savings. In the period of unit velocity the turnover of money is equal to the sum of exchanges in the consumption market and the savings market. Now if the consumption market is in equilibrium, and the money market is in equilibrium, then the savings market must be in equilibrium. On the other hand, if the consumption market and the savings markets are in equilibrium, then the money market must be in equilibrium. Whereas Mr. Keynes directs attention to the former alternative, Messrs. Robertson, Ohlin et al. select the latter courses. To "make assurance doubly sure" let us consider a diagram.

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The supply of goods is held to be a demand for money, and the demand for goods a supply of money. Since the commodity market is held to be in equilibrium, this supply—demand for money is represented by the vertical line, AM. Then there is a reservation demand for money which varies inversely with the rate of interest. Since this demand is automatically supplied, this supply—demand for cash can be represented by the horizontal difference between, BM and AM. Now the claims market is an additional source of demand and supply: the supply of claims represents a demand for money, while the demand for claims represents a supply of money. The lower the rate of interest, the greater is the demand for claims, and the less is the supply. Adding in this demand and supply, separately, to the BM curve, we secure the total supply curve £G and the total demand curve DF. It is only when the demand for claims equals the supply that the demand for money equals the supply.  

It should be clear that we are dealing with flow and not stock concepts here. Keynes is concerned to show that the stock of money is exactly demanded at a certain rate of interest. The above analysis shows that the flow of money demanded is equal to the flow of money supplied. But it should be clear that when the flow demanded is equal to the flow supplied, the stock demanded is also equal to the stock supplied. For an excess of the money...

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flow demanded over that supplied, by society, implies an attempt to increase the stock of money held. When, however, the two flows are equalised, the stock of money is exactly demanded a number of times, depending on the number of periods. In our case, it is demanded exactly once, since our operational time period is one during which the stock of active money turns over once against the available supply of goods and claims.

In terms of General Equilibrium analysis the argument runs as follows. "Suppose that there are \( n \) kinds of exchangeable goods and services; then there are in all \( n \) prices to be determined. For among the 'goods' must be reckoned that good which is taken as a standard of value (money). This leaves us \( n - 1 \) prices of the other goods and services in terms of the standard, and one rate of interest . . . . This makes \( n \) prices in all. To determine the \( n \) prices, we have \( n - 1 \) equations of supply and demand for the \( n - 1 \) commodities (excluding money), one equation of supply and demand for loans, and one for money. This makes \( n + 1 \) in all. However . . . one of these equations follows from the rest. This leaves us \( n \) equations to determine the \( n \) prices."\(^1\)

Which equation should be eliminated? Actually, it does not matter. Consider the problem first from the side of money. A person's desire to effect net hoarding is expressed by his attempt to sell more goods and claims than he buys. And the net hoarding of the individual is precisely equal to the difference between purchases and sales. Now net hoarding for the community

is the sum of all the differences between the purchases and sales of all the goods. In equilibrium, however, all these other supplies and demands are in equilibrium. The condition that net hoarding be zero for the community is automatically fulfilled by the equilibrium of the n - 1 goods other than money, and a corresponding equilibrium in the market for loans. And the condition that net hoarding be zero is the same as the condition that the supply of money be equal to the demand.

It would seem therefore that the equation to eliminate would be the equation expressing equality of supply and demand for money. And if this equation be eliminated, the equation expressing the supply of and demand for loans may be retained to determine the rate of interest. But suppose we eliminated the equation of supply and demand for loans. If planned hoarding is zero, and aggregate income equals expenditure, while other supplies and demands are equal, then equality of supply and demand for loans follows from these conditions. Now we have the equation of supply of and demand for money to determine the rate of interest. This seems to be the logical basis of Keynes' method. Saving and investment are automatically equal.

But would it be possible to retain both equations? Can we have our cake and eat it too? Yes, both equations could be retained, and another equation eliminated. For example, suppose we elect to express values in wage units, it would then be logical to eliminate the equation for unskilled labor. And this method does not appear to be inconsistent with the Keynesian system in any way.
We have indicated as far as possible that the loanable funds and the liquidity preference doctrines come to the same thing. Unfortunately, Mr. Keynes has seen fit to cling so tightly to his terminological garments that he is unwilling to admit the possibility that the loanable funds theory can express the same relationships as his form of analysis. Thus, he says, "The investment market can become congested through shortage of cash. It can never become congested through shortage of saving." ¹

This is an unusually dogmatic statement. And it can only rest on Mr. Keynes' peculiar definitions. And at this point, the reader may be inclined to agree that:

"Words are like leaves; and where they most abound
Much fruit of sense beneath is rarely found." (Pope)

Of course, ex-post savings are always exactly equal to ex-post investment in Mr. Keynes' system. And this condition may result from simultaneous equilibrium in the consumption and money markets. But suppose we admit conditions in which the claims (saving-investment) market is not in equilibrium. Assuming the consumption market is in equilibrium it follows that when the money market is in disequilibrium the market for loans is also in disequilibrium. For in the period of unit velocity the demand for money is equal to income. The money is held (say) until the end of the period, and is then used in the final instant to turn over against goods and claims. If the supplies and demands of

all consumption goods are equalized, the inequality of supply and demand in the market for loans implies an attempt by society to accomplish net hoarding. Supply of and demand for money would then be unequal. It would appear that Mr. Keynes' dogmatism on this point involves logical error.

Messrs. Robertson and Shaw have subjected Mr. Keynes' doctrine to a painstaking examination. And, indeed, it must be conceded, Mr. Keynes' exposition presents a somewhat disordered picture. But, if the argument in section II be accepted, it would appear that the above point is the only one on which Keynes is seriously in error. And this mistake appears to arise mainly out of an excessive preference for his own approach.

A further source of controversy lies in a question of methodological consistency. Now you can slice a cake into wedges, or you can slice it into squares. But you cannot, at once, do both. Economists do well, likewise to adopt a single method of slicing the theoretical cake. Thus, while Robertson agrees that the method of supply and demand for money leads ultimately to the same results as the loanable funds doctrine, he senses a methodological inconsistency in the new concept, the demand for finance. As he puts it: "In attempting to graft on to his old static apparatus such concepts as 'the flow of new finance' and the 'supply of liquidity' he seems to me to be engaged in breeding a monstrous hybrid between the two methods of approach."¹

I find myself out of sympathy with this remark. The concept of a revolving fund must include an account of the flows causing the revolution. We need not suppose that because Mr. Keynes employs the concept of a fund of cash that he must ignore flows. It is precisely changes in the relationship between inflows and outflows which express changed demands for finance.

Is Mr. Keynes guilty of methodological inconsistency in using time rates of flow? Not if these time rates all refer to the same moment of time and he gives no indication that he uses another method. And all static theory, as well as Quasi-Dynamic Theory, implicitly involves rates of flow per unit of time. Such theories simply do not take account explicitly of the inter-temporal relationship of economic events.

Furthermore, may we inquire, are Messrs. Robertson and Ohlin immune from this charge of methodological inconsistency? Mr. Robertson, for example, employs a dynamic, disequilibrium method known as process analysis, involving discontinuous change. Yet in his interest theory he employs continuous supply and demand curves to depict a (presumably) dynamic, equilibrium situation. It would appear that Mr. Robertson in looking for a mote in Mr. Keynes' eye has overlooked the beam lodged in his own.1

The Marginal Efficiency of Capital

It seems perfectly obvious to most economists that there is an intimate relationship between the rate of interest and the yield of capital goods. In fact, Fisher's theory of interest

1 See Mr. Robertson's account in "Mr. Keynes and the Rate of Interest," in Essays in Monetary Theory (London: P.S. King, 1940).
pictures the rate of interest as the outcome of a process whereby the "rate of return over cost" of a capital good is equated to the marginal rate of "time preference." But we have already seen that Mr. Keynes believes the rate of interest to be the "price which equilibrates the desire to hold wealth in the form of cash with the available quantity of cash." That may puzzle the reader is the precise relationship of the rate of interest to the yield of capital. This is one of the problems which we will explore in this section.

Mr. Keynes' term for the yield of capital is the "marginal efficiency of capital." And he defines this magnitude "as being equal to that rate of discount which would make the present value of the series of annuities given by the returns expected from the capital-asset during its life just equal to its supply price."¹ Thus, if \( P \) represents the price of the capital good and \( Q_r \) the expected return in the \( r \)th year,

\[
P = \frac{Q_1}{1+i} + \frac{Q_2}{(1+i)^2} + \cdots + \frac{Q_r}{(1+i)^r}, \text{ in equilibrium.}
\]

So far Mr. Keynes is working with conventional ideas. But the distinctive properties of his system come to light with a consideration of the variables which are held to determine the marginal efficiency of capital. Conventional theory proceeds from a micro-scopical point of view. The individual producing unit is thought of as weighing the proportional return from a marginal unit of investment in capital against the rate of interest. For an individual the problem is simple - it is just that of maximizing

¹ G. T., p. 135.
the present value of his expected returns, and is achieved by weighing the rate of interest against the prospective net returns from acquiring capital goods. For society, however, the problem is considerably more complex. The individual producing unit has to consider only the quantity of capital in estimating diminishing returns from investment. But society has to consider also the rate of investment. As the rate of investment in a particular type of capital increases, "The prospective yield will fall as the supply of that type of capital is increased . . . partly because, as a rule, pressure on the facilities for producing that type of capital will cause its supply price to increase."\(^1\)

From the viewpoint of a progressive society it is the marginal efficiency of capital corresponding to a given rate of investment that is supremely important. Nevertheless, the marginal efficiency of capital corresponding to a given rate of investment will decline as the stock of capital in the community undergoes an increase. There are thus two sources of diminishing marginal efficiency: one, an increase in the stock of capital; two, an increase in the rate of investment. We may provisionally write the marginal efficiency of capital, \(i\), as

\[ i = \phi(K, I) \]

where \(K\) is the stock of capital, and \(I\) is the rate of interest.

This complex notion may be represented by a three-dimensional diagram.\(^2\)

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\(^1\) G. T., p. 136.

\(^2\) See Lerner, The Economics of Control (New York: MacMillan, 1944), ch. 25. The following account also owes something to some unpublished notes of Lange in his 1942 seminar.
If the quantity of capital is considered as constant, say OK₁, then we get a certain slice of the surface, K₁, i₁, ST. This represents the marginal efficiency of capital at varying rates of investment at a stock of capital OK₁. This schedule shows up as K₁ in diagram II. This is the approach which Keynes habitually uses, and is based on the assumption of constancy in the stock of capital. If the quantity of capital increases in the long run this schedule shifts downward; these curves, corresponding to an increased capital stock are represented by K₂ and K₃ in diagram II. They are secured by taking slices out of the surface, parallel to K₁T but farther from the origin.

If the slices are taken in the other direction, we get the conventional marginal productivity of capital curve, relating rate of return to quantity of capital. At a rate of investment OI₁, we secure the slice, I₁ i₁ UV. This is represented by the curve I₁ in diagram III. And this is the relationship between the rate of return and quantity of capital. As the rate of investment is increased, and as we take slices farther out from the origin but parallel to I₁UV, the curves will fall. Thus we find that I₂ and I₃, corresponding to increasing rate of investment, fall below I₁. This arises from the increasing price of capital goods as the rate of investment increases.

Static theory was concerned with a state in which the rate of investment is zero, that is, in which the stock of capital remains constant. In diagram I this is represented by the curve KD₂W, a curve depicting the stationary marginal productivity of
capital. This curve shows the effect on the marginal productivity of capital if, the quantity of other factors remaining constant, society temporarily increases investment and then relapses into a static condition. This relationship is proper to comparative statics.

It may be seen that Keynes obtains two sorts of schedules (1) rate of return to quantity of capital, which we shall entitle hereafter, the marginal productivity of capital, (2) rate of return to rate of investment, which we shall entitle hereafter, the marginal efficiency of investment. The first sort of schedules are represented in diagram III, the second in diagram II. Since Keynes is concerned primarily with short run analysis, it is rate of return to rate of investment, or the marginal efficiency of investment, with which he deals.

One additional complication needs attention. In the equation \( i = \phi(K, I) \) we find a macroscopic approach to the efficiency of capital. And, as usual, the function depends not merely on the values of the variables but on certain further implicit assumptions. Thus the marginal efficiency of capital depends, not merely on the quantity of capital, but on its distribution among productive units. A further complication arising from the macroscopic approach lies in the effect of consumption on this function. The value of the expected future returns earned by a capital good depends directly on the level of consumption. The higher the level of consumption (and therefore the level of prices), the greater will be the value of the discounted future returns. The reason that this variable
did not appear in the classical account was that it was taken care of by the expected prices which were multiplied by the expected physical yields in each of the future years. For the individual firm these prices were constant under competitive assumptions. The macroscopic analysis cannot thus treat the demands for goods as constant and thus C must be introduced as a variable into the investment function. It may, therefore be written

\[ i = \phi(C, X, I). \]

The next topic deals with the way in which the marginal efficiency of capital is related to the rate of interest. From an individual point of view the problem is to equate the marginal productivity of the last dollar invested in capital to the rate of interest. Here only quantity of capital is relevant. But as all individual producing units seek after this equality, and order capital goods in pursuit of this goal, a certain marginal efficiency of investment, makes its appearance. If for the individual producing unit, the marginal productivity of capital is less than the rate of interest, it will continue to order capital goods. The result is a rise in the rate of investment and a fall in the marginal efficiency of investment until the latter equals the rate of interest. And this will be the equilibrium rate of investment in the short period.

This apparatus can also be used to analyze long period or intermediate period theory. Suppose that we start at a period

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1 In our algebraic summaries this equation has been written, \( I = \phi(C, i) \). The stock of capital was not considered, because it may be treated as a constant in short run problems.
when the stock of capital is OK₀. The rate of interest is i₀, and remains fixed at that level throughout. Under these conditions the rate of investment is increased to the point where the marginal efficiency of investment is equal to the rate of interest at P₁. At P₁ it is clear that the stationary marginal productivity, corresponding to capital OK₀, exceeds the marginal efficiency of investment. It will therefore pay, in succeeding periods to push investment until the stationary marginal productivity equals the rate of interest. Further, at P₁ the two curves representing (1) marginal productivity of capital (2) marginal efficiency of investment are equal.

As investment continues the stock of capital will be increasing. After a certain length of time the growth will become perceptible, and the marginal efficiency of investment curve will shift downward. In our diagram the slice through the surface must be taken at OK₁. The rate of investment must now be contracted, but the marginal efficiency of investment still exceeds stationary marginal productivity. Equilibrium at capital stock OK₁ is found at P₂. This process will continue following the line SP₁P₂T until the rate of investment finally approaches zero. At zero rate of investment the rate of interest will equal the stationary marginal productivity of capital which is equal to the marginal efficiency of investment. Contraction in the rate of investment no longer yields a differential in marginal efficiency of investment over stationary marginal productivity; the force of expansion is exhausted and we find ourselves in the stationary state.

We now come to a point of crucial importance. Just what
is the relationship of the marginal efficiency of capital to the rate of interest? This much is clear: "the rate of investment will be pushed to the point on the investment-demand schedule where the marginal efficiency of capital if general is equal to the market rate of interest."\(^1\)

The question remains - can a variation in the marginal efficiency of capital affect the rate of interest? In short, is the marginal efficiency of capital a determinant of the rate of interest, or not? In answer to this Mr. Keynes says, "I fully agree that the increased demand for money resulting from an increase in activity has a backwash which tends to raise the rate of interest."\(^2\)

This "backwash" works itself out as follows: "If a decline in investment leads to a decline in output as a whole, this may result . . . in a reduction of the amount of money required for the active circulation, which will release a larger quantity of money for the inactive circulation, which will satisfy the propensity to hoard at a lower level of the rate of interest."\(^3\)

Today, or at any rate in 1938, Mr. Keynes would probably say that a decline in the marginal efficiency of capital would reduce investment, which would reduce the demand for finance and thus reduce the rate of interest. The revised account gives a closer connection between the marginal efficiency of capital and the

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1 G. T., p. 137.


3 Ibid., p. 218.
rate of interest. We pass over with a blush the statement:
"The schedule of the marginal efficiency of capital may be said to govern the terms on which loanable funds are demanded for the purpose of new investment; whilst the rate of interest governs the terms on which funds are being currently supplied."

Summary

In Keynesian analysis the rate of interest is the price paid for surrendering liquidity, that is, cash, for a specified period of time. Furthermore, the rate of interest is thought to be the price which brings into equality the demand for and the supply of cash. Ordinarily, the supply of cash is treated as a constant, although this assumption is merely provisional. The demand for cash is represented as varying inversely with the rate of interest; for the higher the rate of interest, the less is the inducement to hold cash and the greater the inducement to spend or hold securities.

On first examination of the doctrine, it is not entirely evident how the demand for and the supply of cash can affect the rate of interest. Suppose "the" rate of interest is taken to be the percentage rate earned on newly floated bonds in the open market. If the supply of cash is increased, the demand for cash remaining constant at the existing rate of interest, the holders of this new cash will wish to allocate it to more attractive alternatives. Thus the new money may be used to purchase securities, and this will bid up the quotations on the
newly issued bonds; since the annual return written on the face of the bond remains fixed, the rise in quotations implies a fall in the percentage rate. The rate of interest has fallen.

The latest version of the doctrine includes a new element, the demand for "finance." This added element of demand is the advance provision made by entrepreneurs in anticipation of newly planned investment. Together with the demands for cash arising out of the business and precautionary motives, a total demand curve for cash is derived, which is set equal to the supply of cash on principles traced out previously.

In the foregoing pages it has been shown that this approach to the theory of interest is consonant with the more conventional approach represented by the loanable funds approach. This new theory does not represent a great advance in technique, but is merely a revised way of looking at the same fundamental relationships. A study of its elements has the effect, moreover, of forcing the student to review some very fundamental relationships in the theory of money and to hold these firmly in mind at all times.

The marginal efficiency of capital is that rate of discount which will equate the price of a newly produced capital instrument with the sum of its expected future returns. The marginal efficiency of capital depends on the rate of investment, the level of consumption and the stock of capital. For short period problems the stock of capital may be taken as a constant, and only the former two quantities need be taken as variable.

The rate of investment is always pushed to the point at which the marginal efficiency of investment is equal to the rate
of interest. This relationship, of course, affects the rate of interest by varying the demand for finance according to the current relationship between the rate of interest and the marginal efficiency of capital.

Formally, the amended Keynesian doctrine is satisfactory. Certainly, it may be interpreted in a way which shows it to be an internally consistent theory. Yet, this approach has several disadvantages compared with those theories which stress supply and demand for loanable funds as the determinants of the rate of interest. In the first place, the latter theory seems more direct and straightforward, while Keynes' approach appears to be a little backhanded. Secondly, the loanable funds approach can easily be generalized to deal with a complex of rates. It is hard to see how it would be possible to do this with Keynesian doctrine.

The Keynesian doctrine has one shining advantage. It shows in a direct fashion how the quantity of money can affect the rate of interest and thereby economic activity in general. The choice between the two methods of approach is one of convenience. Personally, for the simplified sort of analysis which Keynes seeks to set forth, it seems to me to be ideal. If, on the other hand, the more intricate ramifications of pricing are the subject of analysis, the alternative method would appear to be possessed of the decisive advantage of superior flexibility.
Notes on Finance

For an individual firm which is not borrowing or lending,
\[
\frac{dBr}{dt} = \frac{dB_0}{dt} + \frac{dBr}{dt} - \frac{dS}{dt},
\]
where \( B_0 \) represents operating expenditures; \( E_0 \), investment expenditures; \( R_0 \) operating receipts; and \( \frac{dS}{dt} \) the firm's rate of accumulation of balances. For an individual consuming unit which is not borrowing or lending
\[
\frac{dB_i}{dt} = \frac{dBr}{dt} - \frac{dC}{dt} = \frac{dS}{dt}.
\]
Suppose that the individual firms are in equilibrium in the sense that their time rates of operating receipts are equal to their time rates of operating expenditures. In the absence of borrowing, the expression \( \frac{dBr}{dt} \) is the rate of decumulation of balances, experienced by the sth firm. If savers are accumulating balances as fast as firms are decumulating them, the release of balances by savers to investing firms at this time rate would leave balances unchanged. This is the fundamental condition - saving equals investment - under conditions of constant money supply. Summing over all units holding balances, we have,
\[
\sum_{j=1}^{n} \frac{dB_j}{dt} = \sum_{s=1}^{m} \frac{dE_{is}}{dt} - \sum_{r=1}^{m} \frac{dS_r}{dt} = 0 \text{ where } j = r + s.
\]
If a quantity of new money is released into the system over a period, \( \frac{dB_i}{dt} \) will be positive. Thus,
\[
\int_{t}^{T} \sum_{j=1}^{n} \frac{dB_j}{dt} \, dt = \int_{t}^{T} \left\{ \sum_{s=1}^{n} \frac{dE_{is}}{dt} - \sum_{r=1}^{m} \frac{dS_r}{dt} \right\} \, dt
\]
represents the quantity of money released, and the aggregate excess of investment
over saving during the period, assuming equilibrium in the consumption market.

Let us now assume explicitly that borrowing and lending do occur. Let $L$ represent lending. The finance requirements of an individual firm initiating an act of investment may be written:

$$
\int_{t}^{T} \frac{d F_a}{dt} \, dt = \int_{t}^{T} \left\{ \left( \frac{\Delta F_a}{dt} + \frac{\Delta L}{dt} \right) - \left( \frac{\Delta S}{dt} + \frac{\Delta I}{dt} \right) \right\} \, dt.
$$

If the interval $t - T$ extends over the whole plan of investment, the integral may be zero. But there will be an interval, $t - t + \alpha t$, during which the integral is positive, and an interval, $t + \alpha t - T$, during which it is negative.

The condition of constancy in finance may be written,

$$
\sum_{s=1}^{n} \frac{d F_a}{dt} = 0. \text{ The amount of finance required over the period during which the equilibrium condition holds is } \int_{t}^{T} \sum_{s=1}^{n} \frac{d F_a}{dt} \, dt = K. \text{ One firm releases finance as fast as another requires it. Under these conditions the demand for finance is constant.}
$$

The general conditions of equilibrium are:

1. \( \sum_{s=1}^{n} \frac{d F_a}{dt} = 0. \)

2. \( \sum_{s=1}^{n} \frac{d F_a}{dt} = \sum_{s=1}^{n} \frac{\Delta S}{dt} = \sum_{r=1}^{m} \frac{\Delta C_r}{dt} \)

3. \( \sum_{s=1}^{n} \frac{d I}{dt} = \sum_{s=1}^{n} \frac{\Delta I}{dt} = \sum_{r=1}^{m} \frac{\Delta S}{dt} \).

The equilibrium will be stable if the condition (2) applies to the individual firms, as well as to the aggregates.
CHAPTER VI

SOME PROPERTIES OF THE GENERAL THEORY

I

The System Rounded Out

In the General Theory Mr. Keynes sought a broader canvas on which to depict his conception of the economic system. And in the new view "technical monetary detail falls into the background," yielding its place to a general analysis of the entire economic system. In the main, this broadening of perspective is admirable, although the revised emphasis seems destined to impose an almost intolerable strain upon the intellects of some specialists.

However, there may be such a thing as diminishing returns in response to widening the field of perspective. In dealing with monetary phenomena the fertile, if capricious, mind of Mr. Keynes is everywhere at work, pouring forth a stream of interesting suggestions, some possessing a form conducive to further analysis. But when Mr. Keynes comes to deal with the problems of production and pricing, his powers of analysis diminish, with the result that his view of the economic system becomes vague and indistinct at this margin. Since criticism must be proportioned to the importance of the contribution scrutinized, this aspect of his work must pass with a fairly sketchy examination.

In dealing with the theory of production Mr. Keynes resorts
to simplification once more, but with less happy results.

By means previously discussed Mr. Keynes transforms the familiar theorem - the wage of labor equals marginal value product - into an Auspitz and Lieben supply curve. In this form, the relationship states a connection between the level of aggregate receipts and the offer of employment - \( Y = F(N) \), where \( N \) is the level of employment. While there is no valid objection to the use of such a supply curve, when applied to a single industry functioning under conditions of competition, Mr. Keynes' variant is subject to suspicion. In the first place, the relationship connects total revenue and the level of employment. The level of employment, if it is to be a meaningful quantity, must represent a homogeneous mass of labor, all of the same grade. By treating a unit of a skilled grade of labor as a multiple of a unit of unskilled labor, the multiple being determined by the ratio of the prices, this reduction process may be carried out. This method, which assumes the existence of a stable structure of money wages, is still questionable after a hundred and fifty years. Keynes is no more convincing on this point than was Ricardo.

Furthermore, is it expedient to neglect variable agents other than labor, even in the short period? Here it is necessary to make a point which is painfully obvious to the marginal productivity theorist. A factor of production whose total supply is fixed, may often appear as a variable magnitude to an individual productive unit; for the firm may effectuate an increment in its supply of the particular factor by a small
The competitive bidding of the several firms leads to that equilibrium price which distributes the total supply of the factor in question among the several demanders. Accordingly, the fixity in the total supply of the factor is irrelevant from the viewpoint of the individual firm. Land may therefore be considered as variable to the individual productive unit in the short run. Consequently, it seems inexpedient to work with a theory of production, based on the assumption that land does not enter into the structure of production as a variable element.

Of course, it may be claimed that a productive agent cannot be treated as variable in the short run, if time is required to adjust the structure of production to an added supply of it. And land may be thought to have this property when considered in its capacity as situs. But when we contemplate the potential variability of raw materials which are the products of land, this point loses its validity. Furthermore, in agriculture, the supply of land may be adjusted in the short run without disrupting the structure of production. Accordingly, it seems dangerous to omit land from the list of factors which the entrepreneur is at liberty to vary in the short run.

In seeking thus to reduce the number of variable factors of production Mr. Keynes appears to be taking a step which is likely to vitiate his analysis of production. It would not be surprising to find a theory unsatisfactory which deals with a single variable where a structure of variables requires attention. And we shall find it so in his analysis of wage reductions.
A further unsatisfactory aspect of Mr. Keynes' approach to the theory of production lies in his treatment of the marginal productivity doctrine. Sophisticated theorists have long taken into account complications such as "limitational" factors; productive agents of this type must be varied in exact proportion to the level of output. When such complications are introduced, some knotty problems make their appearance. And it must not be supposed that a monetary theorist can settle these questions in a few words. Yet consider the following words of Keynes:

"Even if we assume that the marginal cost of purchases from other firms involved in selling an additional unit of output has to be deducted from the sales proceeds per unit to give us what we mean by our firm's supply price, we still have to allow for the marginal disinvestment in the firm's own equipment involved in producing the marginal output." ¹

It is probably quite evident that the deduction of a certain set of magnitudes from price before setting it equal to marginal factor cost, has the same effect as adding these magnitudes to marginal factor cost before setting it equal to price. And if it is required thus to add some additional element, then this element cannot be considered as an independent variable, for it clearly depends on the level of output or the quantity of some other productive agent. To place raw materials in this category is to take a questionable step. In a wide variety of instances it may be possible to substitute labor for raw materials

¹ G. T., p. 67.
in the production of a given output. Such a condition implies that the application of a greater quantity of labor to a given flow of raw materials reduces wastage in the latter and gives rise to a greater volume of production per unit of time. If this condition holds true within the neighborhood of the equilibrium, then the quantity of raw material and other factors may be treated as independent variables. If we provisionally treat production as a timeless process, then the marginal cost of production is to be sought at many margins, in the following way. The marginal cost of producing a unit of output is the ratio of the cost of a unit of the factor to its marginal physical product. Thus,

\[ M.C. = \frac{P_a}{M.P.a} = \frac{P_b}{M.P.b} = \cdots = \frac{P_r}{M.P.r} = \cdots \]

If raw material is a substitutable factor, then it may be treated like the others. And there will be no need for additions or subtractions in calculating marginal cost. There will be a margin at which the price of the final product, assuming competition, equals the marginal cost of producing a unit of product with the assistance of added raw material. Mr. Keynes has stated an empirically doubtful proposition which throws over marginal productivity theory.

The above matter is illustrative of the sort of difficulties in which Mr. Keynes becomes involved when he tries to dispose of the problem of production. In this well-worked field his famed powers fail to produce their usual results. While Mr. Keynes has tried, very properly to integrate the theory of production with monetary theory, his effort cannot be considered
satisfactory. Yet in all fairness, we should not censure him for the attempt:

"A man's reach must exceed his grasp
Or what's a heaven for." (Browning)

Mr. Keynes' adventures in the field of production theory may be concluded with a consideration of his doctrine of wage reductions. Let us approach this topic in three stages: (1) a summary of his system; (2) a critique of his definition of unemployment; (3) a critique of his analysis of wage reductions.

The General Theory is designed to determine the level of employment, the level of income, "the" rate of interest, the volume of saving, investment, and consumption. Among the relationships which serve to determine these variables are (a) the multiplier, which states the relationship between the rate of increase of income with respect to an increase of investment, on the one hand, and the marginal propensity to consume, on the other; otherwise expressed, this theorem states that income equals the sum of investment plus consumption; (b) the famed identity of saving and investment (by definition); (c) the propensity to consume, which relates consumption to the level of income and the rate of interest; (d) the schedule of the marginal efficiency of capital, which relates the level of investment to the level of consumption and the rate of interest; (5) the liquidity preference function which relates the supply of money to the three sources of demand for money which are, taken together, functions either of the rate of interest or of the level of income; (6) the employment function which gives the
relation between the level of income, measured in wage units, and the level of employment. This is the aggregate supply function of chapter II. Given these six relations, we can determine the variables in question. Now the first five relations between them determine all the variables except the level of employment. The sixth determines this variable.

Let us consider now Mr. Keynes' definition of unemployment. "Men are involuntarily unemployed if, in the event of a small rise in the price of wage-goods relatively to the money-wage, both the aggregate supply of labor willing to work for the current money wage and the aggregate demand for it at that wage would be greater than the existing volume of employment."¹

Taken literally, the situation described can be represented by the following figure. In it the real wage rate is

![Diagram]

plotted along the vertical axis and the level of employment along the horizontal axis. In this situation a fall in the real wage rate (a rise in the price of wage-goods relatively to the money wage) would cause the offer of employment and the demand for employment to increase. But is there anything involuntary about the unemployment existing at Q? This represents

¹ G. T., p. 15.
a point on the supply curve of workers and is, to that extent
at least, a point of acquiescence. Consequently, here Mr.
Keynes does not appear to draw a fruitful distinction.

At another place, Mr. Keynes sketches out another criterion.
At the risk of misinterpretation I shall restate and discuss it.
There is just one point on the aggregate demand function at
which it intersects the aggregate supply function. Only by chance
will the point correspond to full employment. Yet any attempt
on the part of entrepreneurs to expand employment beyond this
point cannot persist; for an increment of income paid to workers
will be only partly consumed— the remainder will be saved.
And this saving will not flow back to purchase the output thus
produced. Entrepreneurs will incur losses and employment will
be contracted to the old level. Consequently, involuntary unemploy­
ment will exist whenever the aggregate demand function cuts the
aggregate supply function at a point below that which corresponds
to full employment.

The distinction between "voluntary" and "involuntary"
unemployment may be a matter of definition. But if these terms
are to have meaning, we must be able to attach a precise signifi­
cance to them. "Involuntary" unemployment would seem to denote
a state in which some workers cannot find jobs despite reasonable
efforts in that direction. Assume away for the moment all fric­
tional unemployment arising out of imperfect knowledge or lack
of mobility. What resistances bar the way to full employment?
On the one hand, there is the lack of effective demand for the
services of labor, reflecting the lack of effective demand in
general; and, on the other hand, there is a price resistance to
the offer of labor. Of course, the lack of demand for labor is completely involuntary, so far as laborers are concerned. But the only way in which their volition may be exercised lies in the offering of their services at a higher or lower price.

If laborers were to offer no resistance to the working of demand— that is, if they threw their services on the market for what they would bring— then unemployment would be entirely involuntary. For any decline in the demand for labor would lead to a progressive fall in wages, continuing until full employment was reached. Now, it is true that such a fall in wages might give rise to a completely unstable situation in which wages, prices, and incomes chased one another down the scale towards zero. But this is irrelevant so far as the volition of the worker is concerned.

Any supply schedule for labor, other than one represented by a vertical line, corresponding to full employment, is one in which some degree of unemployment would be tolerated by laborers at some prices. Since the lack of effective demand for labor is always involuntary, so far as labor is concerned, this factor cannot be made the basis of the distinction. To be more explicit, the weapon of the laborer in combating unemployment is a variation in the rate of wages. It is only when this method fails that unemployment can be entitled "involuntary." Consequently, the crucial point is the volition of the worker, as expressed through the wage rate. It is only when a reduction in the wage rate, sanctioned by laborers, fails to produce the required increment in employment, that a failure of demand can be said to
induce involuntary unemployment. Consequently, the argument of Keynes referring to the principle of effective demand appears to be misdirected.

The foregoing discussion has produced no tool which may be used to forward a study of wage reductions. This is unfortunate, for the distinction drawn between voluntary and involuntary unemployment seemed promising. Nevertheless, if the analysis of unemployment stands on its own feet, the failure of the distinction will be felt but little.

Mr. Keynes asserts at the outset of his argument: "A reduction in money wages is quite capable in certain circumstances of affording a stimulus to output as the classical theory supposes. My difference from this theory is primarily a difference of analysis." And this is literally what Mr. Keynes means. Unless the change in money wages favorably affects one of the six fundamental relations of his system, employment will not increase.

His argument may be paraphrased as follows. Suppose that a general cut in wages is announced. Entrepreneurs, expecting perhaps that the level of demand will remain unchanged, expand output in the belief that such action will enlarge their profits. In producing the enlarged output, entrepreneurs hire more men and pay out a greater wages bill, and incomes rise. Consumers, however, do not spend the whole of this increment of income, but save a certain proportion. A part of the expenditure laid out

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1 *C. T.*, p. 257.
by entrepreneurs in producing the increment of output will fail to return in the form of a flow of demand. And entrepreneurs will incur losses to the extent of this gap between the increment of expenditure and the consequent increment of consumption. Only if a rise in the rate of investment occurs, will the gap be filled, and losses avoided. This can only happen if the rate of interest falls, or if the marginal efficiency of capital rises. Of course, if the marginal propensity to consume happens to be equal to unity, the additional sums paid into income will be returned to producers in toto. But under these circumstances no point of stable equilibrium would exist for the economic system.

Keynes continues his argument, saying: "The reduction in money wages will have no lasting tendency to increase employment except by virtue of its repercussions either on the propensity to consume for the community as a whole, or on the schedule of marginal efficiencies of capital, or on the rate of interest." And after an extended discussion of the possible effects of changes in these three factors, concludes: "It is on the effect of a falling wage- and price-level on the demand for money that those who believe in the self-adjusting quality of the economic system must rest their argument."

The effects run as follows: a reduction in money wages, accompanied by a fall in prices and in money incomes will reduce

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1 G. I., p. 262.
2 Ibid., p. 266.
the demand for cash in the active circulation. This will shift the demand curve for cash to the left, lower the rate of interest and cause investment to increase. The increased investment will fill the gap between the increment of income occasioned by the increment of employment and the consequent increment in consumption. It will therefore be possible to sustain the advanced level of employment, if things work out in this way. This result cannot be counted upon, if the level of income is low and the elastic part of the speculative demand curve is in force, because in this event a slight reduction in the rate of interest will cause a considerable increase in the demand for cash. Thus, the cash released from the active circulation will be absorbed by the inactive circulation with very little reduction in the rate of interest. Furthermore, it should be evident that, interpreted in this way, "A flexible wage policy and a flexible money policy come, analytically, to the same thing." Both may improve the situation by lowering the rate of interest.

As far as it goes, this discussion is satisfactory. But, in reality, the analysis indicates the limitations of the Keynesian apparatus, not of the policy of wage reductions. First, let us consider a possibility within the framework of the General Theory that Mr. Keynes has not contemplated. This consideration bears on the reaction of increased incomes, via consumption, on the level of investment. Suppose that a wage reduction stimulates entrepreneurs to increase employment, resulting in the

1 C. T., p. 267.
payment of a certain increment of income to the factors of production; this added income will be consumed only in part. However, the rise in consumption may stimulate investment to such an extent that the gap between income and consumption will be filled, so that sufficient expenditure will flow back to producers to induce them to sustain the higher level of employment.

If the reaction of consumption on investment is considerable, then the system will be unstable in an upward direction. Of course, this is what is wanted, if the desideratum is increased employment. Later, if the reaction of consumption on investment diminishes in strength, the system may become stable at a higher level of employment. This situation may be illustrated by a diagram.

Suppose saving is in stable equilibrium with investment at an income $OY_1$. At this point a rise in the level of income will cause saving to increase faster than investment. A wage cut occurs and entrepreneurs, sensing an opportunity to increase their profits may increase employment and therefore income. In this situation, the question arises whether income will rise to $OY_2$ before entrepreneurial expectations are disappointed. Assume that the increase in the level of income required to reach $OY_2$ is sufficiently
small, and the lag between entrepreneurial actions and the registration of events sufficiently long that the jump is accomplished. If income \( OY_2 \) is attained, the situation becomes unstable in an upward direction, and the system will move to \( C \), provided the temporarily disappointed expectations do not cause a reversal of the movement. And this need not occur, for the good news may reach entrepreneurs before they decide to turn back.

On these assumptions the system will move to \( C \), which represents a position of stable equilibrium.

In the early stages of this development the added flow of investment might be derived from increments of working capital. Later on, investment in fixed capital would step in to take the place of this temporary investment process.

A further circumstance may take effect in this situation. Suppose the fall in the rate of wages releases cash from the active circulation and so acts to reduce the rate of interest. As the rate of interest falls, the investment curve will shift upward and the saving curve in the opposite direction, giving rise to the situation shown in the diagram.
Under these circumstances there is no need to "get over the hump" from A to B, and the system will automatically move up to C. Thus, if we put the two effects together, the fall in the rate of interest having lessened the hump to be surmounted, and the wage reduction having imparted the initial stimulus, the system will move to the higher level of activity.

A situation such as we have described might materialize at a time when the economic system was depressed. Such being the case a general cut in wages, shrewdly timed, might occasion the revival desired. Of course, in this situation, any action designed to induce entrepreneurs to expand output might bring about the same result. We merely wish to show that circumstances may arise in which a policy of wage reductions might produce the desired result. Mr. Keynes' conclusions lack the unshakable validity that he assumes them to have. Although we have analyzed a case in which the stability conditions in the investment market may not be fulfilled, a case which might seem to be a mere curiosum to a Keynesian, the instance has more general significance. Cyclical conditions are not fruitfully analyzed by elaborating the stability conditions of a system, and declaring that these conditions must always be fulfilled. From the viewpoint of business cycle analysis the very point of interest is the situation in which the economic system is unstable. Taken in this sense, our case assumes greater importance.¹

In our discussion we have thus far merely drawn attention

to the repercussions of a wage cut in a potentially unstable situation. More generally, a reduction in wages relative to other factor prices may be expected to increase employment. If this be true, why does Mr. Keynes ignore the case? The answer is simple. Since there is only one variable factor of production in Mr. Keynes' system, there is a matter incapable of analysis by his system.

In the following discussion a greater simplicity will be achieved, if we assume the production of a constant real income (output). That is, we assume that a certain output is going to be produced - an output determined by the principle of effective demand. Our purpose is to investigate the effects of a reduction in wages on the employment of labor required to produce this fixed output. Under these assumptions the principle of effective demand cannot be used to undermine our argument; for no additional real income is generated by the cut in wages, under the assumed conditions.

Consider the problem of a firm in producing a given output at minimum cost with the aid of two factors, land and labor. Suppose the current output is fixed at 1000 units. There will be a certain number of combinations of units of land and labor which will produce this output, and the locus of these points is the "indifference product curve," $P_1$. 
The problem is to produce the given output at minimum cost. This is always achieved when the indifference product curve, $P_1$, touches the lowest possible indifference line of cost. Since the indifference product curve is convex to the origin, this is necessarily achieved when this curve touches the indifference line of cost which is nearest the origin. This optimum position is found at $E$. At this point the funds laid out on production purchase $OK$ units of land plus $ON$ units of labor. And the ratio of the prices of labor to land is as $MK$ to $ME$, and this ratio is equal to the marginal rate of substitution of land for labor along $P_1$ at $E$. Suppose the price of labor falls, the price of land remaining constant.

The amount of land which has a value of labor falls from $MK$ to $MK'$ and the total cost of production in terms of land falls from $OK$ to $OK'$. But since $KIV$ does not touch the curve $P_1$, costs can be reduced to $OR$ by shifting the indifference line of cost, $KIV$, downward to the position represented by the line $RS$. The line $RS$ is a lower indifference line of cost; it represents the locus of points connecting quantities of land and labor having a given cost, and this cost is lower than that associated with $KIV$.

At the new equilibrium point more labor, to the amount $NN'$, is employed, and less land, to the amount of $MK'$. This analysis is based on the assumption that the price of labor has fallen relatively to that of land. If a wage reduction is to result in a substitution of labor for other factors, the prices of these other factors must be held constant or reduced in a less
proportion. The use of the several factors depends on price ratios, and if the price ratios turn in favor of labor, the latter will tend to be substituted for other factors. Perhaps, then, it is better to speak of "relative wage reductions" rather than "wage reductions."

We have seen that Mr. Keynes is not quite at home in the field of production theory. There his analysis departs from marginal productivity doctrine; yet the reason assigned for this latest bit of unorthodoxy is open to question. And the shaky foundation of Mr. Keynes' production theory becomes more evident when we consider his doctrine of wage reductions. At this point his analysis appears to be based on the assumption that there is only one factor of production. A theory which departs to so great an extent from reality can hardly be expected to yield trustworthy conclusions. Furthermore, even on Mr. Keynes' own assumption that labor is the only factor, a wage reduction may serve the purpose of pushing the system towards a point at which it is unstable in an upward direction.

If we depart from Mr. Keynes' own battleground, the field of quasi-dynamic theory, it is most unlikely that his conclusions will be found to be accurate. The introduction of lags of various kinds would open up all sorts of possibilities which are not comprehended by the Keynesian apparatus. We have chosen, in the present discussion, to indicate two important exceptions to his analysis which can be derived from quasi-dynamic theory. Thus, it is clear that even on his own home ground Mr. Keynes' conclusions are subject to doubt.
Having rounded out our discussion of Mr. Keynes' views, it now behooves us to investigate more closely the properties of his system. Perhaps the most convenient way that we can forward this aim is by the method of contrast. And this carries us into a comparison of Mr. Keynes and the classical economists.

II

Keynes and the Classical Economists

In drawing comparisons between Mr. Keynes and the "classical" economists we take the dubious step of comparing him with a "composite aunt Sally of uncertain age." And, in fact, no such "classical" system as we intend to discuss has ever been written out in detail. But some such set of ideas does seem to have possessed the minds of the English Classical School.

In summarizing the classical view we leave out of account any consideration of the pricing system, and reduce that system of thought to Keynesian terms. Since the classical theory was founded on the pricing system, this discussion hardly gives an adequate rendition of its theoretical method. However, since our purpose is to inquire into the nature of Keynesian economics, with a view to reaching a fuller understanding of its properties, such comparisons may not be amiss.

The classical view may be summarized by the following equations and unknowns.

**Equations**

1. \( M = K \cdot Y \)
2. \( I = I(Y, i) \)
3. \( S = S(Y, i) \)
4. \( Y \equiv F \cdot O \)
5. \( O = f(M) \)
6. \( M = f'(W) \)
7. \( \frac{W}{P} = E(W) \)

**Unknowns**

1. \( O = \) Volume of Output
2. \( I = \) Investment
3. \( S = \) Saving
4. \( Y = \) Income
5. \( i = \) Rate of Interest
6. \( P = \) Price Level
7. \( N = \) Level of Employment

**Givens**

1. \( M = \) Quantity of Money
2. \( W = \) Level of Wages
3. \( k = \) Proportion of Income Held as Balances

One of the most important properties of this system derives from equation (1), the Cambridge Quantity Equation. If the quantity of money, \( M \), and the proportion of income held as balances, \( k \), are assumed given, then the level of money income is determined by equation (1) alone. And since the quantity equation fixes the level of income, it is really unnecessary to insert income in the saving-investment equations. And this means that saving and investment, no matter what their variation, cannot affect the level of money income, so long as \( k \) and \( M \) are fixed. In short, the problem of effective demand does not arise at all. Total demand is always equal to income in equilibrium, and this income is assumed constant. Therefore the demand tends to be fixed also.
The classical monetary equilibrium can be represented accurately by two diagrams. The first is the familiar saving-investment diagram, wherein interest is the price which equates the rate of saving and investment. The second represents the several possible levels of income by a series of rectangular hyperbolas. Given the quantity of money and $1/k$ (income velocity), the level of income is determined.

In equations (7) and (8) we have the demand and supply functions respectively for labor. In equation (6) we have the function relating output to the level of employment. Equations (1), (5) through (8), taken together, determine the level of employment. Eliminating $P$ between (5), (7), and (8), we secure the equations, (a) $W = f'(N) \cdot Y/O$, (b) $W = E(N) \cdot Y/O$, (c) $M = k \cdot Y$, and (d) $O = f(N)$. These four equations serve to determine $W$, $N$, $Y$, $O$. Thus the level of employment is determined independently of saving and investment. Furthermore, since the problem of effective demand does not exist, such a non-existent problem cannot affect the level of employment. That is, income is the reflection of the state of demand; and since income is fixed independently by the quantity equation, there is no such interaction of income and employment as we find expressed in Mr. Keynes'
Principle of Effective Demand.

The equilibrium in the labor market may be represented partially by a pair of supply and demand curves. Here the demand for and the supply of labor are represented as dependent upon the real wage. These curves will shift if the level of income changes. The same is true of the saving-investment curves. The conditions in both markets are affected by changes in the level of income.

This apparatus may be used to treat trade fluctuations by introducing some change into the system. If the investment schedule shifts to the right, the rate of interest will rise, and saving will increase. As the level of saving increases, the level of investment will rise correspondingly. A redistribution of employment as between consumers' goods industries and investment goods industries may occur, but the aggregate will be unaffected. Furthermore, the level of income will remain unchanged throughout. Fluctuations in saving and investment are incapable of affecting the level of income directly.

A more promising avenue of approach to fluctuations lies in the money equation. Suppose the quantity of money increases. In order to maintain $k$ at the usual level people will increase their expenditure until the level of income rises proportionately to the rise in $M$. The saving and investment schedules will shift
to the right; more is saved and invested. Meanwhile, the rise in the level of income and prices occasions an increase in the demand for labor, and thereby an increase in employment. The same general results will follow from a reduction in k.

As far as it goes, this analysis is satisfactory. But the system is weak at one point. The problem of effective demand is assumed away; for the relationships are stated in such a way that income is always fixed. This approach, therefore does not admit of full interdependence between the saving-investment market, the level of income, and the labor market.

Mr. Keynes' view may be expressed by eliminating equations (1) and (8). Equation (1) is replaced by the liquidity preference equation, \( M = L(Y, i) \). We must now take account of equation (8). Mr. Keynes is not explicit on his views regarding the nature of the supply curve for labor. Perhaps it is safest to treat the money wage as a constant, at least up to the point of full employment. One further change, the substitution of the equation, \( S = I \), for equation (4) rounds out the system. This identity, it will be remembered, follows from Keynes' definitions of saving and investment, both of which are defined as income minus consumption.

The substitution of the liquidity preference equation for the Cambridge Quantity Equation is the source of some significant substantive changes in the system. No longer is the level of income determined independently of the saving-investment market. The four equations, (1) \( M = L(Y, i) \), (2) \( I = I(Y, i) \),
(3) \( S = S(Y, i) = Y - C(Y, i) \), (4) \( I = S \), are required to determine \( Y, i, I, \) and \( S \). These equations are, in revised form, the monetary equilibrium to which we have constantly adverted in our previous analysis. The level of money income is determined from these four equations taken together. A change in the propensity to save, for example, will affect both the level of income and the rate of interest, reacting on the quantity of money and the level of investment. The money equation and the saving-investment system are interdependent. The monetary equilibrium may conveniently be represented by a single diagram.

The saving-investment equation gives us a relationship, IS between the rate of interest and the level of income. Here is the logic. The higher the rate of interest, the lower is the rate of investment, and the higher is the rate of saving. Accordingly, there will be a virtual tendency for saving to exceed investment. In order to fulfill the equation of saving and investment at a "normal" value of the propensity to consume, the level of income must fall; the decline in the level of income will reduce saving more than investment and restore the equality of saving and investment. Accordingly, a rise in the rate of
interest is associated with a fall in the level of income, along the IS curve. That curve is negatively sloped.¹

The money equation, $M = L(Y, i)$, gives us the L (liquidity preference) curve, which is positively sloped. An increase in the level of income leads to an increased demand for money to use; in order to fulfill the equation of demand and supply of money at the higher level of income, the rate of interest will have to be raised. For the increased rate of interest will reduce the demand for money to hold, and the equation of supply and demand will reassert itself.²

The monetary equilibrium is determined by the intersection of these two curves. Whereas the classical theory determines the rate of interest and the level of income in severality, the Keynesian theory determines them jointly in a way indicating their interdependence. Thus the General Theory shows a certain advance in technique and substantial content over the classical theory, in this respect at least.

The properties of the L (liquidity preference) curve may be noted. At low rates of interest the L curve is almost

¹ $I(Y, i) - S(Y, i) = 0$, $\frac{di}{dY} = -\frac{(I_Y - S_Y)}{(I_1 - S_1)}$. The denominator may be supposed negative, since a lowering of the rate of interest diminishes investment but increases saving. The numerator may be supposed negative, since a rise in the level of income presumably raises investment less than saving. If the term $I_Y - S_Y < 0$, as it may conceivably be, then the IS curve will be positively sloped.

² $M = L(Y, i)$, $\frac{di}{dY} = \frac{-I_Y}{I_1}$. Since $I_Y > 0$, $I_1 < 0$, $\frac{di}{dY} > 0$. 
horizontal. A small rise in the rate of interest causes a considerable decline in the demand for money to hold. Accordingly, a large rise in the level of income is necessary to increase the demand for cash in the active circulation to a corresponding extent, so as to equate the demand and supply of cash. At high rates of interest, a given increment in the level of income causes, perhaps, a moderate increase in the demand for cash in order to use; a very large rise in the rate of interest is required to affect a corresponding diminution in the quantity of cash demanded in order to hold. For, at high rates of interest the demand for cash to hold is insensitive to changes in the rate. Accordingly, the L curve becomes almost vertical at high rates of interest.$^1$

If the inducement to invest is strong and the system is operating at a high level of income, the IS curve is pushed to the right and the inelastic part of the L curve is called into affect. Under these circumstances an increase in the inducement to invest or a diminution in the propensity to save, which shifts the IS curve to the right, takes effect chiefly in a rise in the rate of interest; little effect is produced on the level of income. Here the results of the analysis are similar to those of the classical model; for in that system the changes in question do not affect the level of income.

$^1$ \( M = L(Y, i), \ \frac{dL}{dY} = -\frac{L_Y}{L_i} \). At low rates of interest \( L_i \to -\infty \) and \( \frac{dL}{dY} \to 0 \). At high rates of interest \( L_i \to 0 \) and \( \frac{dL}{dY} \to -\infty \).
If, on the contrary, the inducement to invest is weak and the system is operating at a low level of income, the IS curve is pushed to the left and the elastic part of the L curve is called into effect. Under these conditions an increase in the inducement to invest or a reduction in the propensity to save, which shifts the IS curve to the right, takes effect chiefly in a rise in the level of income. Here we find ourselves at the opposite pole from the classical analysis.

We have already seen that the demand for money is linked to the saving-investment market in the General Theory. Likewise, the level of employment is related to the money market and the saving-investment market. The "real" part of the Keynesian system, up to the neighborhood of full employment, may be represented by the equations: (a) \( W = f'(N) \), (b) \( O = f(N) \), (c) \( Y = P \cdot O \). The level of income relates employment to the money market and the saving-investment market. Any perturbation taking effect in an increase in the level of income will tend to raise the demand for labor. That is, we know from the equation, \( Y = P \cdot O \), that a rise in the level of income will result either in an increase in price or output, or both. If price increases, the money wage constant, then the real wage declines and employment increases in response. Furthermore, an increased output will require additional employment.

If money wages are reduced, the real wage drops, reducing marginal wage cost which causes the price level to decline, and this, in turn, reduces incomes, releasing cash from the active circulation. If the moderately elastic part of the speculative
demand curve for cash is in effect, the rate of interest falls appreciably, the level of investment is raised, and a compensating increase in the level of income is set up. This increase in the level of income takes effect partially in a rise in output, partially in a rise in price. Presumably, both these reactions will increase the quantity of labor needed; employment will rise.

This set of reactions is worked out more directly when the quantity of money is increased. The level of income is raised, because the added money supply reduces the rate of interest, stimulates investment, raises income, and so on. The two variations have roughly the same effects and the same limitations as regards policy. The level of income will not rise appreciably if the elastic part of the speculative demand curve for cash is in effect; for the release of cash will produce little effect on the rate of interest under these circumstances, and therefore will not affect investment and income significantly.¹

The General Theory may be generalized by the addition of a definite supply function for money. This will permit us to treat the quantity of money as a variable. Thus, if the quantity of money supplied varies directly with the rate of interest and the level of income, the shape of the L curve will undergo a transformation. The L curve will remain highly elastic at low rates of interest, but it will no longer become highly inelastic.

¹ The reservations expressed in the earlier part of the chapter on the policy of wage reductions, apply with undiminished force.
at high rates. As the rate of interest rises to a high level, an increase in the rate of interest will lower the demand for cash very slightly; but, such an increase will augment the supply of cash appreciably. The excess of supply over demand will be significant; and the rise in the level of income necessary to produce a compensating increase in the demand for cash will then have to be at least moderately great. Accordingly, the L curve will have more nearly the following conformation.

Since the L curve no longer approaches perpendicularity at a high level of income, the generalized Keynesian analysis slips away almost completely from the classical model. A rightward shift in the IS curve always produces an appreciable increment in the level of income; whereas a change in the saving-investment system never affects the level of income in our classical model, such a change always affects the level of income appreciably in the generalized Keynesian system.

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1 \( L(Y, i) - M(Y, i) = 0 \), where \( X(Y, i) \) is the supply function. \( \frac{dI}{dY} = -\frac{L_Y - M_Y}{L_Y - M_Y} \). Now \( L_Y - M_Y < 0 \), for \( L_Y < 0 \) and \( M_Y > 0 \). Now \( \frac{dI}{dY} \) is not small. Perhaps we may assume that the effect of a rise of income on the supply of money is weak. In this event \( L_Y - M_Y > 0 \). Accordingly, \( \frac{dI}{dY} \) is not small.

\( L \rightarrow -\infty \), \( \frac{dI}{dY} \rightarrow 0 \), but if \( L_Y \rightarrow 0 \), while \( M_Y \) is a number of sen. size \( \frac{dI}{dY} \) is not small.
In the foregoing discussion we have compared the General Theory with a model system which is supposed to represent the thoughts of some "classical" economist, our "Aunt Sally of uncertain age." And the model described is perhaps the sort of system Ricardo or John Stuart Mill might have had in mind. Since these men were not concerned, in the main, with problems of general interdependence, it is hardly fair to them to draw comparisons wherein the criteria of judgment are drawn from this field of theory. Yet that is exactly what we have done, perhaps with the view of giving Mr. Keynes' theory a good showing before subjecting it to a final critical scrutiny.

Our results may be stated provisionally, in the following fashion. The General Theory is a system expressing a higher degree of interdependence in its several parts than is our classical model. Indeed, the money market, the saving-investment market, and the labor market are represented as interdependent in the General Theory. In the classical model the level of income is fixed independently by the money equation. The consequence of this condition is that a perturbation in the saving-investment market cannot affect the level of income, and thereby the demand for money, the level of employment, or the rate of interest. By similar logic the labor market cannot exert influences on other sectors of the economy by modifying the level of income. Clearly, the classical model set forth in the chapter exhibits a lesser degree of interdependence than the General Theory. And, by the same token, it is less perfectly adapted to analyzing the economics of the short period; for short period economics rightfully
deals with changeable and variable elements, featuring interdependence.

In a subsequent chapter, we shall summarise the case for and against the General Theory in terms less flattering to Mr. Keynes. And that is so because our classical model will have changed. Since Keynes is an interdependence theorist, it seems only fair to compare him to "classical" authors who have seriously treated this aspect of economics. The above model is a counterpart to the thinking of men whose main energies were channeled into the field of partial equilibrium theory, and who customarily treated such topics as value and distribution. Consequently, the comparison must be considered to be more useful as a method of exhibiting the characteristics of the General Theory than a valid comparison of Keynesian and classical economics in general.

And, in fact, our purpose was precisely to bring out in sharper focus the special properties of the General Theory.
In the foregoing body of analysis we have confined ourselves to setting out and examining critically the properties of the General Theory. In a sense this inquiry has dealt with the structural properties of the system. In order to round out our analysis, a further step is required, the investigation of perturbations set up by disturbances within the system. The sort of analysis with which we shall be concerned has no dynamic character; for it is not concerned essentially with the timing of events. Neither are the changes under consideration of a type which might be called self-perpetuating or disequilibrating. In the main, we shall be concerned with shifts in the schedules of the system, and our analysis will deal with the repercussions of such changes.

Static equilibrium theory may be adapted to the study of change. If a variation is introduced into the analytical system, the prevailing equilibrium will be disrupted, and the variables of the system must be adjusted to reestablish equilibrium. The study of such changes is commonly entitled "comparative statics." The present analysis of quasi-dynamic equilibrium has its counterpart in comparative statics. By analogy may we not say that this study deals with comparative quasi-dynamics. If some attention is directed here to the actual timing of events, that is because
we have elected to make our analysis as realistic as possible.

In our discussion we shall deal with a variant of the Keynesian monetary equilibrium. As previous discussion has indicated, it is the monetary part of the Keynesian system which carries with it the greatest promise of development. Accordingly, the perturbations considered are monetary in nature. And perhaps it is better thus to restrict the analysis rather than to analyze and draw conclusions of a doubtful character. A further restriction is imposed on the analysis by a lack of consideration for the conditions of international trade. Our system is closed.

The equations and unknowns of the quasi-dynamic monetary equilibrium to be analyzed may be set out as follows:

**Equations**

1. \( Y = C + S \)
2. \( E = C + I \)
3. \( E = Y \)
4. \( I = I(C, i) \)
5. \( C = C(Y, i) \)
6. \( M = M_1(Y, i) + M_2(Y, i) \)
7. \( K = K(Y) \)

**Unknowns**

1. \( Y \) = money income
2. \( E \) = money expenditure
3. \( C \) = money consumption
4. \( S \) = money savings
5. \( I \) = money investment
6. \( i \) = rate of interest
7. \( K \) = liquidity ratio, reciprocal of income velocity

\( M \) = quantity of money, a parameter.¹

In carrying out our analysis we require certain assumptions about the nature of the functions in question, particularly touching

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¹ See the mathematical appendix for the basis of the following discussion.
on the response of the several functions to change. These assumptions may be set forth as follows: (a) an increase in consumption occasions a rise in the level of investment, but a rise which is less than the increment of consumption, (b) a rise in the interest rate causes the level of investment to decline, (c) a rise in the level of income increases consumption, but not by so much as the rise in the level of income, (d) a rise in the rate of interest diminishes consumption, (e) an increase in the level of income increases the demand for cash, (f) an increase in the rate of interest lowers the demand for money.

Given these assumptions, it will prove no great task to ascertain the laws of change characterizing the system. Or rather, we have assumed the laws by assuming the conditions above, and our investigation will merely display more clearly the results of these assumptions. A more subtle approach to the problem might be worked out by a consideration of stability conditions. But this investigation belongs, perhaps, to a higher branch of the subject. Our approach will, at least, have the merit of simplicity.

In this analysis, we shall assume that some parameter, or independent variable, acts upon one of the several relevant functions, causing it to shift. The effect of such a change will be studied insofar as it affects three significant variables of the system - the level of income, the rate of interest, and the liquidity ratio, $K = \frac{M}{Y}$ representing the proportion of cash to income. As a first approximation, we shall deal with the liquidity ratio, $\frac{M}{Y}$, representing the proportion of the total
supply of cash to income. And this involves a study of the
income velocity of the entire stock of money, or the number of
times in a unit period the total stock turns over against income.
Later, we shall consider the special liquidity ratio, $Ml$, representing the proportion of the active stock of cash to income.
This involves a consideration of the income velocity of the active stock of money, or the number of times in a unit period the active stock turns over against income.

I
The Marginal Efficiency of Capital

The perturbation which has received the lion's share of attention in Keynesian analysis is a variation in the level of investment. As Keynes himself deals with the problem by the famed multiplier analysis, this perturbation finds expression merely in a change in the level of income. This approach neglects the repercussions of increased investment on the rate of interest, for in Keynesian multiplier theory the latter variable is fixed. What we require at this point is a more general analysis.

Suppose that a new invention occurs, causing the schedule of the marginal efficiency of capital to shift to the right. The immediate effect of the change is to increase the flow of investment executed at the existing rate of interest. As expenditure increases, and these sums pass into the hands of manufacturers, income will rise. As income rises, expenditure will be further stimulated by the consequent increase in consumption. Not only
is the direct increase in consumption felt, but the repercussion of increased consumption on the level of investment, and so income and expenditure will rise. As the level of income rises, the demand for money will increase. Since the supply of money is fixed, the increased demand for money will occasion a tightness in the money market, and the rate of interest will rise. This rise in the rate of interest will check expenditure. In the first place, the rise in the level of investment will be checked, for this level is governed by the condition that the marginal efficiency of capital be pushed to equality with the rate of interest. Since the rate has risen, the level of investment cannot rise to so great an extent. Furthermore, the rise in the rate of interest may check consumption, since the prospect of investing in securities is now more attractive. And this check to consumption will likewise check investment, since the schedule of the marginal efficiency of capital depends on the level of consumption, and a reduction in consumption will tend to lower this schedule and so check investment.

If the quantity of money is appropriately increased, by action of the monetary authority, concomitantly with the increase in the level of investment, the increased demand for cash can be met without a rise in the rate of interest. Under these circumstances the entire burden of the readjustment will be thrown on the level of income. This amounts to saying that a shortage of cash never arises, because it is supplied as soon as it is felt. Thus, the rate of interest need not rise. Accordingly,
under these circumstances, the level of income must rise sufficiently
to equate income and expenditure. Under such pure multiplier theory,
the rise in the level of income must be greater than is the case
when the shortage of cash is not thus supplied. The level of
expenditure is not checked by a rise in the rate of interest;
consequently income and expenditure reach equality at a higher
level.

Returning now to our analysis, consider the effect of the
variation on the liquidity ratio, K. Since the level of income
has risen, while the stock of cash has remained fixed, the proportion
between the stock of cash and the level of income has fallen, and
thereby, the liquidity ratio. This implies that the stock of
money is turning over more rapidly against income; velocity is on
the increase.

II
Propensity to Consume

Suppose that consumers foresee a future rise in the flow of
incomes. Guided by this anticipation of an enlarged income they
will desire to spend more keenly. The propensity to consume will
increase; at the given rate of interest, and at the level of income
then prevailing, consumption will increase. This increment in con-
sumption will stimulate investment, and the level of expenditure
will therefore experience a twofold rise. From this point onward,
the general character of the readjustment is similar to that produced
by a shift in the marginal efficiency of capital. As expenditure
rises, income rises, consumption is stimulated and this further
raises the level of investment. Expenditure thus increases further,
and with it the level of income. However, this increase in the level of income occasions an increased demand for cash which raises the rate of interest. The rise in the rate of interest checks the rise in the level of investment; it also discourages consumption which in turn checks the increase in the level of investment. Thus the increase in expenditure is checked, and a new equilibrium of income and expenditure is established.

The increased income, taken in conjunction with the given supply of cash, implies a lower liquidity ratio, \( \frac{M}{Y} \). And it also implies that the given stock of money turns over more often against the enlarged income. Income velocity experiences an increase.

III

The Demand for Money

Suppose that a pessimistic atmosphere pervades the money market. The liquidity preference curve shifts to the right. The increased demand for cash, taken in conjunction with the given supply, occasions a stringency in the money market, and the rate of interest rises. The rise in the rate of interest diminishes expenditure in three ways. First, it discourages investment; second, it tends to reduce consumption; third, the reduction in consumption reduces investment still further. Accordingly expenditure tends to fall below income, and a gap appears between income and expenditure. In order for the gap to be filled, it is necessary for income to fall. This is true because a fall in the level of income is supposed to induce a
less gradual fall in expenditure. Then as income falls, the
gap between income and expenditure is narrowed. The stringency
in the money market which expressed itself in a rise in the
rate of interest is further relieved by the fall in the level
of income which lessens the demand for cash. Thus, the rise
in the interest rate sets up repercussions in the level of
income and expenditure which help to relieve the stringency.
When the level of income and expenditure is reduced sufficiently,
and the rate of interest has risen, equilibrium will once more
prevail in both sectors of the economy.

The increased demand for money has elevated the rate of
interest and depressed the level of income. The quantity of
cash has remained fixed. In consequence the liquidity ratio,
\( M \), has increased. This implies, in turn, that the income velocity
of money has diminished. The given stock of cash turns over
more slowly against a diminished level of income.

IV

The Quantity of Money

An increase in the quantity of money relieves the pressure
in the money market. The rate of interest is therefore reduced.
As the rate of interest is reduced, expenditure tends to increase
for several reasons. Not only is the level of investment elevated
as entrepreneurs seek to push the marginal efficiency of capital
to equality with the rate of interest, but the level of consump­
tion will rise. Securities are now less attractive relative to
consumption and more expenditure is directed towards the latter
channel. In consequence of the increased consumption, investment will rise further. Expenditure has thus risen above income. In order to redress this difference it is necessary for income to rise; for the increased income will stimulate a less increase in expenditure and the gap will tend to be narrowed. Thus, the increment of expenditure generates additional income, and this income generates additional expenditure, but to a less extent. So the increased income which is generated tends to set up forces which remove the cause of the disturbance in this sector, namely the gap between expenditure and income. Furthermore, the rise in the level of income tends to increase the demand for cash, and to absorb a part of the newly created cash without a further fall in the rate of interest. The rate of interest must fall and the level of income must rise until the joint equilibrium is reestablished.

It is uncertain whether the liquidity ratio will rise or fall. The quantity of money has increased and so has the level of income. The situation may be pictured more clearly by a diagram. This diagram displays the rectangular hyperbolas which depict the several levels of income corresponding to combinations of the quantity of money and income velocity (the reciprocal of the liquidity ratio).
Given the level of income, \( Y_1 \), and the quantity of money, \( M_1 \), the income velocity of money will be \( V_1 = \frac{1}{K_1} \). In the Keynesian scheme velocity is determined as a sort of after-effect of the system. The same holds true of the liquidity ratio. That is, the demand for money is treated as the demand for a stock of cash, and the liquidity ratio is determined by the working out of the level of income, since the stock of cash is assumed given.

Thus \( K = \frac{M}{Y} \), and given the stock of cash, \( M_1 \), is known when the level of income is determined. In short, \( K \) and its reciprocal \( V \) are treated as independent variables determined by the relations:

\[
K = \frac{M}{Y}, \quad V = \frac{1}{K}.
\]

If the quantity of money increases from \( M_1 \) to \( M_2 \), the level of income will increase in a determinate fashion. Thus, as the quantity of money changes from \( M_1 \) to \( M_2 \), the level of income rises from \( Y_1 \) to \( Y_2 \), and income velocity rises to \( V_2 \). This variation may be thought of as occurring in two stages. First, the quantity of money rises to \( M_2 \), the level of income constant, and we move from \( P_1 \) to \( P_2 \). Income velocity drops. Then income changes, and we move from \( P_2 \) to \( P_3 \) on the new income curve, while the quantity of money remains constant. Income velocity rises. The outcome in this instance is a rise of income velocity. But if income does not rise appreciably in response to an increase in the quantity of money (i.e., if the system's elasticity of income in response to an increase in money supply is low), then velocity may fall and \( K \) increase.
Up to the present point we have analyzed the liquidity ratio, $\frac{M}{Y}$, the ratio of the total quantity of money to the level of income, and its reciprocal the income velocity of the entire stock of money. The effects of the given changes on income velocity may perhaps be studied more fruitfully by comparing the stock of active money with the level of income. The ratio $\frac{M_1}{Y}$ of active money to income may be entitled the active liquidity ratio, and its reciprocal, $\frac{Y}{M_1}$, the income velocity of active money. The distinction between active money and idle money is conceptually an absolute one. The stock of money may be thought of as divided into two parts, an active part which continually turns over against income, and an idle part, which is continually at rest. This money at rest corresponds to the minimum balance of a depositor which is never touched, and which consequently never turns over against income. The division into these two elements is determined by the demand. Thus the total demand for money $M(Y, i)$ is composed of two sets of schedules, $M_1(Y, i)$, the demand for active money and $M_2(Y, i)$, the demand for idle money.

The change in analytical point of view does not alter the results of the analysis. It simply directs attention to changes in the active liquidity ratio, $\frac{M_1}{Y}$, and its reciprocal, $\frac{Y}{M_1}$, the income velocity of active money. From this viewpoint we derive the same results with respect to the four perturbations studied, insofar as they affect the level of income and the rate of interest. The only change is the consideration of the different type of
velocity, and liquidity ratio.

Suppose that the schedule of the marginal efficiency of capital shifts to the right. As we have explained above, the rate of interest and the level of income experience an increase. The increased rate of interest tends to diminish the demand for active money, while the level of income is temporarily held constant. Thus the active liquidity ratio tends to decline by reason of the interest effect. The level of income then rises, the rate of interest remaining fixed. If the rise in the level of income lowers the active liquidity ratio, then the income effect and the interest effect work in the same direction and the liquidity ratio must fall. But the (average) liquidity ratio, \( \frac{M_1}{Y} \), will fall only if the marginal liquidity ratio, \( \frac{dM_1}{dY} \), expressing the ratio of the increment of active money demanded to an increment of income, is less than the average, \( \frac{M_1}{Y} \). This need not be the case. And if the opposite is true, then the income effect will produce an offsetting rise in the (average) liquidity ratio, and the impact of the increment of investment on the liquidity ratio will depend on the relative strength of the two forces.

Another way of putting the matter is to state it in terms of elasticities. The income effect will produce a decline in the average liquidity ratio, if the income elasticity of demand for active money is less than unity; but if the elasticity is greater than unity, it will produce a rise. In the latter case, the income effect works against the interest effect.
The results of an increase in the propensity to consume are roughly the same. A rise will occur in the rate of interest and the level of income. The same criteria will determine whether the active liquidity ratio will rise or fall.

The results of an increase in the quantity of money are a rise in the level of income and a fall in the rate of interest. The fall in the rate of interest tends to increase the quantity of cash held in the active circulation, the level of income remaining temporarily constant. This tends to raise the active liquidity ratio. As the level of income rises, the (average) liquidity ratio will certainly rise if the marginal liquidity ratio exceeds the average. This will happen if the income elasticity of demand for active balances is greater than unity. But if this is not true, and the marginal ratio is less than the average, then the income effect will tend to diminish the average ratio. In this event, the income and interest effects work in the opposite directions, and the result depends on the relative strength of the two tendencies.

V

Idle Balances

One last perturbation may be considered, an increase in the demand for idle balances. As the demand for idle balances increases, the rate of interest rises, expenditure is checked, and the level of income falls. The reduction in the level of income tends to reduce expenditure in less proportion and the equality
of income and expenditure is reestablished. Also the fall in the
level of income reduces the demand for balances and relieves the
stringency in the money market. Now the rise in the rate of
interest tends to diminish the quantity of active balances, the
level of income being held temporarily constant. Accordingly,
the interest effect tends to diminish the liquidity ratio. The
level of income then falls and this variation, too, will occasion
a reduction in the (average) liquidity ratio, if the marginal
liquidity ratio is less than the average. In this event, the
(average) liquidity ratio, \( \frac{M_t}{Y} \), will certainly fall. But if the
marginal ratio, \( \frac{dM_t}{dY} \), is greater than the average, \( \frac{M_t}{Y} \), then
the average ratio will tend to rise. In this case the interest
effect and the income effect work in opposite directions. The
outcome will then depend on the relative strength of the two forces.

In this analysis, we have seen how shifts in the relevant
schedules tend to produce fluctuations in certain of the fundamental
variables of the system. Can such analysis be used to study the
actual working of the economic system? Perhaps there is no definite
answer to this question. Certainly, the study of such perturbations
is helpful to the thinking of the economist. The analysis of the
conditions under which the economic system will move from one
equilibrium to another tends to clarify thinking and to assist the
economist in formulating policy. In all probability, a quasi-
dynamic analysis of this sort is too crude to provide reliable
criteria for a prediction of future changes. It would seem that
the main value of the study lies in the restriction of the problem
to certain simple specified forces. Thus limited, the problem becomes manageable; and the economist is at least able to think the problem through and check the results against intuition and common sense.
CHAPTER VIII

REVIEW AND CRITICISM

In this chapter an attempt will be made to bring together the ideas treated in the preceding pages. We shall proceed from questions of methodology to matters of substance, and finally to the criticism of the General Theory.

The General Theory is a quasi-dynamic equilibrium system of the aggregative type. Its subject matter is a progressive economy in which capital is constantly accumulating. The recognition accorded this state of affairs imparts a dynamic tinge to the analysis, for the flow of capital tends to lower the schedule of the marginal efficiency of capital. This condition makes it necessary to select for consideration a time interval so short that the stock of capital will not change sufficiently to shift the schedule of the marginal efficiency of capital. Despite the implicit importance of time, the analysis is not fully dynamic, for it does not deal with a system of dated magnitudes and flows whose values vary with the passage of time. Rather, it is a quasi-dynamic system - one which deals with an undated system of magnitudes and flows, whose values are so equilibrated as to maintain a constant level in a short period of time, but where one or more of the flows is inconsistent with the ultimate maintenance of these values.
While the **General Theory** is quasi-dynamic, it may also be described as an equilibrium system of the aggregative type. Keynes deals with a few broad aggregates, such as income, saving, and investment, which work through the whole structure of society. Certain forces are said to be at work which insure the temporary equilibrium of these quantities. Thus, the system pictures the simultaneous interaction of these forces as giving rise to a short period equilibrium of the several magnitudes and flows. Each equilibrium is transient, giving rise to another with the passage of time and the accumulation of capital.

A macro-economic approach yields far greater simplicity than a system which deals with individual units. It is a tractable sort of method, and one that can be manipulated with ease. Sometimes its principles can even be explained to the public, and this is all to the good. Yet, beneath the broad surface of the aggregative magnitudes unanalyzed forces are at work, and it may very well be expedient to adopt a way of viewing the facts which brings these submerged forces to light. In short, the study of the micro-economic magnitudes underlying the aggregates may prove ultimately to be the most fruitful way of looking at things.

From a philosophical point of view the choice of method depends on the reader's view of causation. Those who lean toward a deterministic, and mechanistic view of causation will prefer a macroscopic approach; for such a method stresses the influence which the social aggregate exerts on the causation of events. Those who prefer to believe that social forces are the
outcome of a myriad of individual decisions will prefer the micro-scopic approach, for such a method stresses the influence which the individual exerts on the social organism. And yet, limited aggregates are capable of representation on this view. The price making forces, whereby individual actions give rise to a price which conditions these very actions, are clearly depicted. Such a view avoids unilluminating paradoxes designed to show that human volition does not fulfill itself. This line of thought leads to the use of a micro-economic interdependence analysis of the Walrasian type. Such a choice is by no means inevitable. Both methods may be used to good effect; but it seems to me to be highly dangerous to expect more than limited results from an aggregative approach.

That further property of Keynesian analysis, its quasi-dynamic equilibrium aspect, is also worthy of note. Unquestionably, this approach yields the maximum simplicity and elegance, while approximating reasonably the conditions of a progressive economy. Despite the considerable simplification of analysis resulting from elimination of the time factor, a manifest disadvantage attaches to this practice. All economic events take place in time, and a description of these events which omits reference to the temporal order of things must be based on a simplifying assumption. This assumption is that the economic forces existent at any moment give rise to an immediate equilibrium, whereupon economic magnitudes and flows become invariant through time. While the system may move from one such set of constant equilibrium magnitudes and flows to another, the analysis provides no explicit
means of analyzing the movement. So the characteristic behavior of an economic system in a state of dynamic flux is not subjected to examination; yet the process by means of which one economic state gives rise to another is perhaps the fundamental problem of economics. Not only do we require connectedness between economic forces at a moment of time, but also at different points of time so that the contour of economic evolution is seen to be part of a single interrelated process. This quasi-dynamic equilibrium theory cannot do.

If the quasi-dynamic equilibrium depicted could explain accurately the economic situation at a given instant, it would possess marked usefulness. Yet it is not certain that the method can accomplish even this. Under those circumstances in which economic magnitudes and flows are conditioned by a rate of growth, the quasi-dynamic equilibrium method requires considerable modification, for influences of this type are not incorporated into the structure of such a system. If Mr. Keynes and his disciples held this point firmly in mind, it would present no great barrier to a suitable use of the General Theory. Unfortunately, Keynesians are wont to apply the doctrine without adjustment for dynamic influences. Whereas this gives rise to a state of mind in which economic events are forced into an artificial mold of theory, the appropriate course lies in another direction. Theories should take their form from the contour of reality. The failure to realize this principle leads to a misdirected use of theory, a failing which seems to be especially prevalent among Keynesians, including the master himself.
Since the limitations of the Keynesian method stand clearly outlined, it is now appropriate to consider the substance of the *General Theory*. The *General Theory* seeks to determine those magnitudes and flows which are deemed to be of vital importance to the functioning of the economy, namely, income, consumption, saving, investment, employment, and the rate of interest. All of these, save the rate of interest, may be measured in terms of the wage unit, the price of a unit of unskilled labor.

Perhaps the easiest way to approach the Keynesian system is to think of it as composed of a set of forces leading to a monetary equilibrium, on the one hand, and another set of related forces determining the level of employment, on the other. The monetary equilibrium whereby the values of income, consumption, saving, investment, and the rate of interest are determined, is attained by the establishment of two interdependent equilibria. One is the equilibrium of income and effective demand, the other, the equality of supply and demand for cash. The final adjustment in the monetary sector requires that these equilibria be achieved simultaneously; and the total equilibrium requires that the employment determining forces reach an adjustment with the monetary equilibrium.

A principle factor affecting the monetary equilibrium is effective demand. In Keynesian analysis, effective demand is defined as the quantity of money laid out by society to purchase the real income produced; it is composed of two elements, investment and consumption. Income, on the other hand, constitutes the sum of payments advanced to the factors of production for their services.
The recipients of this income allocate it among the several available uses. One such use is expenditure on consumption, an outlay whose magnitude is determined by the propensity to consume. This constitutes the first element in effective demand. A further source of effective demand is to be found in the outlay of entrepreneurs who desire to purchase capital goods which may be used to assist in further production. This magnitude is governed by the marginal efficiency of capital, the rate of interest, and the level of consumption. The effective demand, consisting of investment plus consumption, is conditioned by and, in turn, conditions the level of income.

It should be clearly understood that the effective demand, constituting the total outlay on the current output, becomes the receipts of entrepreneurs. And the income of society is made up of the outlays of the entrepreneurs. Consequently, an excess of effective demand over income implies an excess of entrepreneurial receipts over outlays. Under such conditions production tends to be accelerated, additional factors are hired, and income rises. This increased level of income stimulates consumption, raising the marginal efficiency of capital, which tends to raise the level of investment and thus to further increase effective demand. Equilibrium is attained when the level of income has risen to such an extent that it is once more equal to effective demand. And this equality is attained because the excess of effective demand over income raises income; and it also increases effective demand, but not by so much as the rise in income. Hence the equality of income and effective
demand is accomplished.

So far the rate of interest has been treated as constant, in order to simplify the discussion. Keynes thinks of the rate of interest as being "determined" by the supply of and the demand for cash. The desire of a person for cash is entitled his "liquidity preference," and varies in intensity with the rate of interest and the level of income. Assume, as a first approximation, that the quantity of money is given. If the level of income is likewise given, the rate of interest must be so adjusted as to equate the supply of and the demand for cash. Under these conditions a sort of temporary equilibrium between the demand and supply will exist, subject to disruption by a change in the level of income.

The co-existence of the equilibria described is not the condition of final adjustment in the monetary sector. A final account takes into consideration the effect of the rate of interest on the relation between income and expenditure, on the one hand, and the effect of the level of income on the demand for money, on the other. Suppose that the conditional equilibrium of the supply of and demand for cash is in force, while effective demand exceeds income. The excess effective demand gives rise to an increment in entrepreneurial receipts, giving rise, in turn, to an increased level of activity and income. The increased income will give rise to a lesser increase in consumption, thus narrowing the gap between effective demand and income. Furthermore, the increased income raises the demand for cash, and the rate of interest must be raised to re-establish the equality of supply
and demand for cash. The rise in the rate of interest tends
to check effective demand and thus to restore complete equality
between income and effective demand. For the level of investment
is always adjusted to that point at which the marginal efficiency
of capital is equal to the rate of interest. And this rise in
the rate of interest renders necessary a reduction in the level
of investment, thus lowering the level of effective demand.
Accordingly, the equality of income and effective demand tends
to be restored, while a similar equality between the supply of
and the demand for cash is in effect. The monetary equilibrium
is complete.

Among the variables determined by the monetary equilibrium
is the level of income. Since this income equals the expenditure
of society on output, this quantity equals the receipts of entrepre-
neneurs. On principles explained in the second chapter, the
level of income or entrepreneurial receipts is correlated with
the level of employment. Consequently, when the level of income
is determined by the monetary equilibrium, the level of employ-
ment takes its value from this relationship. Accordingly, the
level of employment adjusts itself to the level of income deter-
mained from the monetary equilibrium. Total equilibrium has been
reached.

The present writer finds it impossible to discuss in short
compass the establishment of the Keynesian equilibrium when income
is defined to be equal to the value of output (effective demand).
Accordingly, the distinction between income and effective demand,
which is identical with the distinction proposed in Chapter III
between income and expenditure, is followed throughout the
above discussion. To do otherwise would involve a discussion
framed in such terms as "a temporary distortion of the propensity
to consume." And this sort of analysis appears to be both para-
doxical and unilluminating. Accordingly, the above discussion
avoids the confusion which results from Mr. Keynes' own defini-
tions and presents his theory in a light far more favorable than
the circumstances warrant. The choice is one of necessity.

A general consideration and criticism of the Keynesian
doctrine is now in order. These criticisms will be summarized
under particularized headings.

The Principle of Effective Demand

Among the new concepts advanced by Mr. Keynes one stands
out as preeminent, the Principle of Effective Demand. According
to this rule, the level of economic activity is always pushed
to the point at which the effective demand is equal to the level
of income generated by that activity. Any attempt to further
extend output or employment will cause income to exceed effective
demand, entrepreneurial outlays will exceed receipts, and losses
will be incurred. The level of activity will then fall. And
this leads to the important conclusion that the economic system
is not self adjusting in the sense that it automatically gives
rise to the maximum level of output and employment.

This principle undoubtedly contains an important element
of truth. It is particularly descriptive of a society in which
techniques and tastes are static. If, however, we assume that
techniques of production are constantly undergoing change, the character of the analysis requires revision. A changing state of technique implies a favorable shift in the schedule of the marginal efficiency of capital in such wise that a pressure in the investment market is ever present. Such a pressure expresses an inducement to invest which is continually reinforced by any postponement of activity. The longer investment is postponed, the more techniques are revised and the higher is the marginal efficiency of capital. Barring some peculiar maladjustment in the propensity to consume, such a situation gives rise to a steady and well sustained level of investment; this force, operating through the multiplier, gives rise, in turn, to a high level of consumption. Under such circumstances the economic system is continually being pushed towards full employment. The only change introduced into the analysis is the assumption that the schedule of the marginal efficiency of capital is shifting upwards with the passage of time, ceteris paribus.

Once introduce this dynamic assumption and the pessimistic conclusion that the economic system does not necessarily give rise to full employment undergoes modification. Now we must say that there is a continuous pressure towards full employment which grows in strength with the passage of time. Thus the tone of the conclusions drawn from the principle depends upon the assumptions relating to the progress of techniques, and the habits and tastes of the population. Pessimism is justified only if the rate of change of these factors is slow. Then the marginal efficiency of capital does not improve very rapidly in the absence of investment. In
this instance the application of the principle of effective demand leads to pessimistic conclusions. Since Keynes assumes the limiting case, unchanging techniques, it is not surprising to find him reaching the conclusion that "... the effective demand associated with full employment is a special case, only realized when the propensity to consume and the inducement to invest stand in a particular relationship to one another."¹

The steady change in techniques posited above, however, tends to strengthen the inducement to invest. If any untoward circumstance hinders investment, the marginal efficiency of capital will, under this dynamic influence, tend to rise. And the steady tendency will be to push upwards the inducement to invest until it stands in that "particular relationship" to the propensity to consume, corresponding to full employment.

Let three cases be differentiated. First, there is the case in which techniques are being modified at a rate such that the marginal efficiency of capital will remain unchanged at the given rate of investment. In the second case, the rate of modification of techniques occasions a rise in the marginal efficiency of capital, at the given rate of investment. In the third case, the rate of modification of techniques, occasions a fall in the marginal efficiency of capital at the given rate of investment. Only in the first and third cases do the conclusions of Mr. Keynes hold. And this condition, particularly the third case, accurately describes a condition of secular stagnation. The General Theory is the economics of

¹ G. T., p. 28.
secular stagnation, and not a general theory at all.

The Monetary Equilibrium and Paradoxes

Among the contributions of Mr. Keynes is his concept of a monetary equilibrium whereby income, consumption, saving, investment, and the rate of interest find their levels. This is a valuable idea, particularly since the magnitudes under consideration are empirically significant. Furthermore, the relationships between these magnitudes are so stated as to throw into clear relief the interdependence which characterizes them in reality. And the abolition of the concept of a constant money income gives rise to some fruitful new methods of analysis. All this is good.

However, Mr. Keynes seems to delight in the creation of paradoxes. He frames his approach to problems in such a way that queer results pop up in the most peculiar places. Doubtless, his purpose is to emphasize the distinctive qualities of his approach, but the results are often confusing. One such problem arises with respect to saving and investment.

Mr. Keynes' definitions of saving and investment are so framed as to render these magnitudes identical in the aggregate; both are set equal to income minus consumption. Now it is my privilege to call every man in the world "George." But in conversation the usage becomes confusing, if the identity or the actions of the men in question assume importance. So it is with saving and investment. Only one word need be used to denote the excess of income over consumption, and it would appear that the generally accepted word is "saving." What then becomes of "investment?"
It would appear that there is no longer any need for this word, when it is used to apply to an aggregate. Or the word "investment" may be adopted and "saving" dropped, if the reader prefers.

If the above definitions are employed, a disturbance in the system expresses itself, not in a divergence between saving and investment, rather in a temporary distortion of the propensity to consume or the inducement to invest. This is an inconvenient and paradoxical mode of expression, and tends to vitiate Mr. Keynes' analysis at every turn. Accordingly, it seems appropriate to select definitions which admit of differences between the two magnitudes.

To carry out the conceptual distinction between saving and investment it is necessary to differentiate between income and expenditure. Income, which constitutes the payments made to the factors of production, may be defined to be equal to consumption plus saving. Expenditure, which is the sum of money laid out by society to purchase the current output, may be set equal to consumption plus investment. There is no reason to believe that income and expenditure will be equal to one another at all moments of time, as Mr. Keynes supposes. The excess of income over expenditure, at any moment, is identically equal, on the above definitions, to the excess of saving over investment. The equality of income and expenditure, saving and investment, is treated as a condition of equilibrium, not an identity. By the same token, it is possible to describe the movement of the economic system in terms of an excess of investment over saving which sets in motion forces leading to a new equilibrium.
Problems of Aggregation

A third important criticism returns to questions of method. The concepts, the propensity to consume, the schedule of the marginal efficiency of capital, and the schedule of liquidity preference are said to depend on the level of social income or consumption. In reality, these aggregative functions are derived by summing the corresponding individual functions. And these are dependent directly on the level of the individual incomes. If the level of social income is uniquely correlated with the division of that income among the members of society, all is well. But no a priori reason is advanced to support this view. If the functions in question actually exist they do so by virtue of a statistical, and not a deductive law. Supply and demand theory never encounters any such difficulty. In this field of theory, the rules derived depend merely on the assumption that the individual household pursues its own self-interest. A strong argument can be presented for the existence of this condition on a priori grounds. Thus a certain consistency pervades the analysis. It is the lack of such consistency which is subject to criticism in the Keynesian approach. The melange of a priori and statistical methods employed in the latter analysis falls short of a desirable standard of theoretical consistency.

The Multiplier

The version of the multiplier which is subject to criticism is the "instantaneous" or "tautological" multiplier which holds
good, "without time lag at all moments of time." The difficulties arise out of the fact that it takes time for income to rise following an increment of investment.

The "tautological" multiplier applies throughout the entire variation of income, from the moment when the increment of income is equal to the increment of investment to the moment when the stimulus is entirely worked out. And this means that the "tautological" multiplier defines a whole range of values. Thus the formula is indeterminate in this version, and it depends upon a correspondingly ambiguous interpretation of the propensity to consume. If Mr. Keynes, with his quasi-dynamic technique, had confined himself to the eventual result, no such difficulty would have arisen. There is a limiting value to the multiplier which is determined by the psychological propensity to consume. This is the only unambiguous multiplier proper to a quasi-dynamic approach.

For the rest, the parts of his monetary equilibrium seem well constructed. Both the theory of interest and the concept of the marginal efficiency of capital appear to be soundly based. In some cases Mr. Keynes' language is obscure, but the substance of his analysis seems to be solid.

Theory of Production

When attention is directed to the theory of production, Mr. Keynes does not fare so well. He chooses to treat labor as the

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1 G. T., p. 122.
only variable factor of production. Furthermore, he uses the questionable technique of reducing the several grades of labor to units of unskilled labor. This method relies on the existence of a structure of money wages which remains unchanged despite disturbances in other sectors of the economy. Even if this unlikely condition be accepted, the conceptual paralysis of other variable agents of production remains to be considered. It seems quite probable that certain grades of land and some forms of circulating capital are variable in the short run. Accordingly, it is inexpedient to omit them from a list of the variables influencing decisions to produce.

Perhaps connected with Keynes' treatment of land is his handling of marginal productivity theory. Rather than provide a separate margin of variation for the raw materials used in production, he elects to include their expenses, conceptually, by making a suitable addition to marginal factor cost. The sum of marginal factor and raw material cost is supposed to be set equal to price by the entrepreneur. While this analysis is appropriate to special cases of "limitational" factors, it seems unnecessary thus to restrict marginal productivity theory in general. In a wide variety of instances it may be possible to substitute other factors for the raw materials used in production, within a limited range. In the absence of more convincing argument, it seems appropriate to reject Mr. Keynes' strictures on this point.

So questionable is Mr. Keynes' treatment of the theory of production that it seems unwise to accept his version of the
integration of monetary and production theory. His major contri-
bution is the doctrine of monetary equilibrium which, rightly
interpreted, is one of the most convenient bits of apparatus
of the last decade.

General Evaluation

Any attempt to evaluate what the Keynesian system of
thought can do for analysis involves a further question, namely,
what is the purpose of economic theory? In one sense it is surely
to discover the truth about the economic system. From a more
practical point of view it is the truth directed towards the good
of man which is the quassitum of the economist: "know the truth
and the truth shall make you free." Yet all truth is merely
that way of looking at events which imparts to them a certain
order and rationale. And the economist judges the truthfulness
of a theory accordingly as it seems to weave the myriad happenings
of economic events into a meaningful pattern. Thus the General
Theory is a way of looking at the facts. And if this system has
a single great advantage, it consists in the ease with which it
may be manipulated. If the economic system is really governed
in a broad way by the magnitudes and relationships treated in
Keynesian analysis, then the truth is not so mysterious. The
number of relationships involved is small, their interaction
simple; and the precision with which the results of change may
be predicted is great. Yet this view, with its entrancing sim-
plcity, may divert our attention from more subtle, but ultimately
more fruitful, ways of connecting events.
From a purely theoretical viewpoint the **General Theory** can hardly bear comparison with the impressive structure of modern general equilibrium economics. The picture which the latter theory presents of the subtle interrelations between the component parts of the economic system is wholly admirable. Yet the difficulties of applying this admirable theory in practice are, at present, almost insuperable. From a practical point of view, the tractable, manageable **General Theory** is an ideal guide to policy. Yet this bright new tool of analysis may prove treacherous in the hands of those who are ignorant of its deficiencies. Used with a proper understanding of its limitations the theory should prove eminently useful; carelessly applied it is capable of doing great harm.

"A little knowledge is a dangerous thing
Drink deep or touch not the Illyrian spring." (Pope)
CHAPTER IX

THE GENERAL THEORY AND POLICY

The pure theorist, safely ensconced in his ivory tower, need take no account of the welfare of that struggling humanity whose actions he seeks to interpret. His purpose may be merely to render an account of the multifarious activities which mark the economic life of man. If called upon to solve some pressing economic problem, such a theorist might feel as Hamlet, when the latter said:

"The world is out of joint, oh cursed spite That I was ever born to set it right."

Mr. Keynes certainly could not be classed among the rapidly thinning ranks of such recondite economists. At every point in the exposition of his theory the discriminating reader can perceive the practical problems with which Mr. Keynes was wrestling when he composed the work. Furthermore, his comments on the practical working of the modern capitalistic economy mark the thinking of a man who has pondered the underlying rationale of economic activity.

Mr. Keynes is the practical theorist, par excellence. He has designed an apparatus which seems particularly well adapted to the formulation of policy by reason of its simplicity and tractability. When Mr. Keynes turns to matters of policy, however, he encounters that general difficulty which appears to plague economic analysis at every turn. When a theoretical system
founded on certain assumptions is applied in practice, it often turns out that the analytical tools created are not adapted to the problem which calls for solution. If Mr. Keynes' theory is adapted to solve any practical question, what is it? It would appear to be the problem of secular stagnation. The world of the General Theory is one in which techniques of production and habits of consumption are fixed. Under such conditions the economic system may have sufficient force stored up within it to remain dynamic for a considerable time, if the marginal efficiency of capital is high. Nevertheless, the inevitable tendency of a continued flow of investment is to lower the marginal efficiency of capital until it is equal to (stationary) marginal productivity. Under such circumstances, investment can progress no further and society has reached the stationary state.

Mr. Keynes, however, does not place any limitation on his theory. In fact he asserts, "Since we claim to have shown ... what determines the volume of employment at any time ... our theory must be capable of explaining the phenomena of the Trade Cycle." This statement is incorrect. It does not follow that because Mr. Keynes' apparatus is capable of treating the problem of employment in a stagnating economy it is also capable of dealing with the dynamic forces of the business cycle. Consequently, in using his theory in practice it cannot be too strongly emphasized that it cannot, in its present form, explain a business cycle. Undoubtedly, it can throw some light on perturbations occurring

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1 G. I., p. 313.
within the framework of the economic system, but such perturba-
tions do not constitute a business cycle. They merely indicate
the behavior of the economy in passing from one condition of
equilibrium to another.

The gulf which separates Mr. Keynes from those who advocate
a more conservative policy may be traced to a divergence of funda-
mental assumptions. Until the recent great depression economists
had little reason to suppose that the level of investment
opportunities would diminish seriously with the passage of time.
Indeed, the area of entrepreneurial activity was being so rapidly
extended as to occasion a heavy pressure in the capital market
at all times. The characteristic difficulty was an insufficiency
of savings. In part, this shortage arose from the opening up
of new territories, and the expansion of population; and in part
from a revolution in techniques, and in the habits and customs
of the people. So great was the need for capital that the social
code sanctioned that inequality in the distribution of wealth and
income which furthered the progress of saving. While consumption
might have been considered to be the ultimate end of all economic
activity, parsimony was glorified as a bourgeois virtue.

What Mr. Keynes feels is that the age of expansion is
finished; that there will be no great new territories to open
up, and that such progress of techniques and alterations of
tastes as are likely to occur will produce little effect on the
inducement to invest. Consider some of the fruits of Mr. Keynes'
stationary assumptions, as interpreted through the medium of the
*General Theory*. First, he proceeds from a time honored postulate
"Consumption, to repeat the obvious, is the sole end and object of all economic activity." Even if this statement be accepted as true, and it is not at all certain it is, further interpretation is necessary. There is the question of when income is to be enjoyed, now or at some future date. To say that consumption is the end of all economic activity hides the less obvious fact that consumption is a matter to be planned and patterned in time. In short, for a rational individual the discounted future satisfactions which may be derived from an expected income stream must be at a maximum. The attainment of this maximum implies a suitably patterned plan of saving. Mr. Keynes' rule does not mean, then, that present consumption is the end of all economic activity.

Aggregate demand is derived from present consumption or from present provision for future consumption (investment). Now the level of economic activity cannot be pushed beyond that point at which the level of income is equal to the effective demand. For the effective demand constitutes the receipts of entrepreneurs, and income constitutes their expenditure on the factors of production.

Yet the inducement to spend so as to provide for present or future consumption rises less rapidly than income. The higher is the level of income, the greater the extent to which people are willing to refrain from present consumption in order to provide for the future. The strengthened willingness to

1 G. T., p. 104.
refrain from present consumption existing at the higher level of activity can only be satisfied if entrepreneurs feel that additional present expenditure is necessary to provide capital equipment for an increased future level of consumption. If, however, entrepreneurial anticipations are unchanged, there is no reason to suppose that they will thus increase investment. A reduction in the effective demand relative to income implies a reduction in entrepreneurial receipts relative to expenditures, and hardly furnishes a reason for expanding output. The very thriftiness of society, its desire to forego present pleasure in favor of future enjoyments, is a hindrance to the fulfillment of a high level of economic activity.

Reflections such as these furnish Mr. Keynes with a philosophical reason for a revised view of the social advantages of saving. No longer is the prudent saver the hero of the piece, who by his heroic abstinence makes possible a permanent increase in the capital equipment and consequently in the well being of society. What modern society requires is the strengthening of effective demand, not that weakening brought about by excessive saving. Thus, "... in contemporary conditions the growth of wealth so far from being dependent on the abstinence of the rich, as is commonly supposed, is more likely to be impeded by it."

The nature of the problem points the way to the solution. If the members of society persist in a course of action which

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1 G. T., p. 373. The discussion which follows is a conflation of accounts scattered throughout the book, especially in Chapter XXIV.
weakens the effective demand and depresses the economic system, the government must set in motion opposing forces. Whereas the invisible hand, representing the collective private actions of the members of society, pushes the economic system toward an equilibrium, the upshot of the hidden mechanism need not be a state of full utilization of resources. Since the myriad of private forces giving rise to this unfortunate situation cannot immediately be transformed into socially appropriate actions in a democracy, it follows that society must counteract this situation by government intervention. The anomaly of a situation in which private and public action strive against one another is surely curious. It is not an optimum situation most certainly, for in this more favorable state of affairs private motives and the public will, expressed through governmental action, work in the same direction. Barring the transformation of those private habits and customs which give rise to results inimical to the public weal, it is incumbent upon the government to take some action to restore the situation.

Since effective demand is insufficient, the task of the state is to fill the gap between the level of income and the effective demand corresponding to full utilization of resources. Effective demand has two components, investment and consumption. Accordingly, the remedial action of the state must serve to increase one or both of these magnitudes. The usual course of action would be such a reduction in the rate of interest as would assure a high level of investment and a strong effective demand. Unfortunately, this scheme has its limitations, for the
inducement to invest arises from a rate of interest which is appropriately low with respect to the marginal efficiency of capital. The rate of interest may be lowered by that increase in the money supply which will satisfy the liquidity preference of the public at a lower rate of interest. When, however, the rate of interest reaches a low figure, the liquidity preference schedule becomes highly elastic, and the monetary authority loses control over the rate of interest. Unless the public can somehow be induced to revise its preference for liquidity, there will be no hope of reducing the rate of interest very much below a certain minimum figure. More drastic methods will be required to increase effective demand.

Perhaps the most obvious method is a policy designed to increase the propensity to consume. A course of governmental action so designed as to alter the distribution of income would tend to increase the propensity to consume. For it is well known that the higher income groups save a considerably larger proportion of their incomes than do those with lower incomes. Since saving is no longer considered to be a social virtue, no rupture of the moral code which binds the government is involved.

That further constituent of effective demand, investment expenditure, is not subject to effective control via the interest rate, because the latter magnitude has a lower limiting value. More direct methods are therefore required. Direct social control over investment would suffice to create that level of effective demand necessary to effectuate a condition of full employment in the form of a depressed marginal efficiency of capital, and
consequently, in a low rate of interest. When private investment is insufficient to push up the level of activity to the point which the governmental authority deems appropriate, public investment in approved and socially valuable projects will fill the gap. Since investment outlets are severely limited in a society characterized by a minimum of dynamic forces, this course of action would rapidly reduce the marginal efficiency of capital to a low figure. This policy would result eventually in the "euthanasia of the rentier" (the rentier class would gradually disappear); for with a rate of interest verging on zero the owners of capital would be unable to live in idleness from the mere circumstance of this ownership.

Assume for the moment that the assumptions of Mr. Keynes are realized in an actual situation. Will the remedies proposed by Mr. Keynes have the appropriate effect? There are circumstances which might conspire to negate a policy designed to push upward the level of investment and consumption, simultaneously.¹ If the method of public investment, financed by governmental borrowing, is carried out vigorously, it implies a steady rise in the national debt. Under some circumstances the level of private investment might be considered to be a function of the size of the current governmental deficit or of the total national debt. The greater the size of the current deficit or of the total debt the greater would be the anticipation of enlarged future taxation, the less also the confidence in the future stability

of a government so managed that the debt mounts continuously
and without limit. Accordingly, the level of private investment
would tend to vary inversely with the level of public investment.
Likewise, the continuance of unlimited governmental expenditure
might induce that loss of confidence in investors which would
cause an increase in the liquidity preference of consumers. If
this raised the minimum rate of interest, the results might be
important. Otherwise the government could inflate the money supply
in such a way as to satisfy the demand for money at the existing
rate of interest.

Another set of repercussions might offset measures designed
to raise the propensity to consume. Suppose that the taxation
policy were so modified as to cause a redistribution of incomes.
This would lead to the heavier taxation of those classes who are
closely tied up with the initiation of investment activities.
Should this occasion a certain permanent depression of expecta-
tions, the schedule of the marginal efficiency of capital might
shift to the left. As the level of consumption rose, the level
of investment would decline, pari passu.

Policies designed to increase the propensity to consume
may defeat themselves by causing a corresponding reduction in the
inducement to invest. Governmental compensatory spending designed
to increase the level of investment may also be offset by a com-
mensating reaction in private investment. Consequently, it is
dangerous to proceed on the assumption that a course of action
taken by the government will have only those favorable reactions
which are planned. The unfavorable repercussions should be taken into account, as well. It is a matter of opinion whether the plan of action advocated by Mr. Keynes will yield the desired results.

Suppose we call into question Mr. Keynes' fundamental assumption regarding the constancy or slow alteration of techniques and tastes. His recommendations, however appropriate they may be for a quasi-dynamic society, would then be wide of the mark. For a fully dynamic society, whose tastes and techniques of production are in continuous and rapid flux, requires a removal of those structural maladjustments which inhibit progress. Under such circumstances the need is not for socially controlled investment, but for that set of policies which would facilitate and make way for the forces of private enterprise. Probably there is no such capitalistic society in existence today. However, the United States may be marked by a rate of dynamics so rapid that a course of action taking effect in a moderate alteration in the propensity to consume would clear the way for something like full employment without much governmental investment. This is merely a supposition, but it is hard to bring oneself to believe that the United States has already reached a state where dynamic forces are so inconsequential as to cause the level of spontaneous private investment to become inadequate to the social need.

In the end, suppositions come up for discussion. There is some truth in the view which sees the diminution in population growth and territorial expansion as the cause of a weakening in the forces of consumption and investment. Whether techniques
and tastes will, in the future, change with sufficient rapidity to occasion that level of private investment required to maintain full employment is a matter of opinion. So it appears to me the recommendations of Mr. Keynes regarding policy are of questionable worth. They do not follow inevitably from his theory, given his assumptions; neither are his assumptions beyond cavil. In general, it would appear that Mr. Keynes has underestimated the strength of dynamic forces working in the present day economy. Likewise, he has underestimated the harmful effects of governmental intervention on the level of effective demand. In treating the effects of governmental spending Mr. Keynes is apt to treat investment as an independent variable. This approach overlooks the fact that private investment, at least, is a dependent variable which may be adversely affected by changes in governmental policy. It is the neglect of factors such as these which vitiate the results of Mr. Keynes' excursion into the field of policy.

If Mr. Keynes has a characteristic strength it is his ability to reduce the interrelations of a complex group of forces to the simplest possible terms. Yet this strength is transformed into a weakness at crucial times, for it appears occasionally that his analysis is not sufficiently general to account for the myriad complexities of the real world. Yet no man can be "all things to all men." If Mr. Keynes has failed to produce the masterpiece of formal logic which a Walras or a Marshall could create, he has not failed to enrich and stimulate the thinking of economists for many years to come.
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This appendix is designed to set down the results of Chapter VII in mathematical form.

The approach used here starts with three equations (1) the equation expressing the equilibrium of income and expenditure, (2) the equation expressing the equality of supply and demand for money, (3) the Cambridge quantity equation where \( K \) is treated as a variable. The change in the independent variables \( a, b, \) and \( c \) serve to shift the functions in question. \( M \) is treated as a constant - \( Y, C, K \) and \( i \) as variables.

(1) \( Y - C (Y, i, a) - I (C, i b) = 0 \)
(2) \( M (Y, i, C) - M = 0 \)
(3) \( K \cdot Y - M = 0 \)
(4) \( C = C (Y, i, a) \)

Differentiating with respect to \( a \) we secure

\[
(1 - C_y - I_C C_y) \frac{\partial Y}{\partial a} (C_1 + I_C C_1 + I_1) \frac{\partial i}{\partial a} = C_a + I_C C_a
\]

\[
\frac{M_y}{\partial a} + \frac{M_i}{\partial a} = 0
\]

\[
K \frac{\partial Y}{\partial a} + Y \frac{\partial K}{\partial a} = 0
\]

(a) \[
\frac{\partial Y}{\partial a} = \frac{C_a + I_C C_a}{(1 - C_y - I_C C_y) - (C_1 + I_C C_1 + I_1) \frac{\partial i}{\partial Y}}
\]

(b) \[
\frac{\partial i}{\partial a} = \frac{C_a + I_C C_a}{(1 - C_y - I_C C_y) \frac{\partial Y}{\partial i} - (C_1 + I_C C_1 + I_1)}
\]
\( \frac{\partial K}{\partial a} = -\frac{K}{Y} \frac{C_C + I_C C_a}{(1 - C_Y - I_C C_Y) - (C_1 + I_C C_1 + I_1) \left( \frac{\partial I}{\partial Y} \right)_M} \)

\[ = -\frac{K}{Y} \frac{\partial I}{\partial a} \text{ when } \left( \frac{\partial I}{\partial Y} \right)_M = -\frac{M_y}{M_1} \]

\[ \left( \frac{\partial Y}{\partial I} \right)_M = -\frac{M_1}{M_y} \]

If \( b \) varies, the results are identical with those stated above with the exception that \( I_b \) is substituted for \( C_a + I_C C_a \).

Also note that \( \frac{\partial K}{\partial Y} = -\frac{K}{Y} \) when the equation (3) is differentiated.

If \( C \) varies, using the same methods, we derive the results:

\( \frac{\partial Y}{\partial c} = \frac{-M_a}{M_1 \left( \frac{\partial I}{\partial Y} \right)_{e,y} + M_y} \)

\( \frac{\partial I}{\partial c} = -\frac{M_a}{M_1 + M_y \left( \frac{\partial I}{\partial Y} \right)_{e,y}} \)

\( \frac{\partial K}{\partial c} = -\frac{K}{Y} \frac{\partial Y}{\partial C} \cdot M_a, \text{ where } \left( \frac{\partial Y}{\partial I} \right)_{e,y} = \frac{C_1 + I_C C_1 + I_1}{1 - C_Y - I_C C_Y} \)

\[ \left( \frac{\partial I}{\partial Y} \right)_{e,y} = \frac{1 - C_Y - I_C C_Y}{C_1 + I_C C_1 + I_1} \]

If \( y \) varies we derive the results,

\( \frac{\partial Y}{\partial M} = \frac{1}{M_y + M_1 \left( \frac{\partial I}{\partial Y} \right)_{y,e}} \)

\( \frac{\partial I}{\partial M} = \frac{1}{M_1 + M_y \left( \frac{\partial I}{\partial Y} \right)_{y,e}} \)

\( \frac{\partial K}{\partial M} = -\frac{K}{Y} \frac{\partial Y}{\partial M} = \left( \frac{\partial K}{\partial Y} \right) \left( \frac{\partial Y}{\partial M} \right) \text{ where } \left( \frac{\partial I}{\partial Y} \right)_{e,y} \text{ and } \left( \frac{\partial I}{\partial I} \right)_{e,y} \text{ have the same significance as before.} \)
If, following the treatment in the text, we treat the demand for money as consisting of two parts — the demand for active balances, and the demand for idle balances, the equation set-up becomes:

1. \( Y - C (Y, i, a) - I(C, i, B) = 0 \)
2. \( M(Y, i, C) - M = 0 \)
3. \( M_1 Y - M_1 (Y, i, C) = 0 \)
4. \( C = C(Y, i, a) \)
5. \( M(Y, i, C) = M_1(Y, i) + M_2(Y, i, C) \)

Differentiating with respect to \( a, b, \) and \( c, \) respectively, we derive identical results for \( \frac{\partial X}{\partial a}, \frac{\partial X}{\partial b}, \frac{\partial X}{\partial c}, \frac{\partial Y}{\partial a}, \frac{\partial Y}{\partial b}, \frac{\partial Y}{\partial c}, \) and \( \frac{\partial a}{\partial c} \). The new results are:

\[ \frac{\partial X}{\partial a} \]
\[ \frac{\partial X}{\partial b} \]
\[ \frac{\partial X}{\partial c} \]
\[ \frac{\partial Y}{\partial a} \]
\[ \frac{\partial Y}{\partial b} \]
\[ \frac{\partial Y}{\partial c} \]

\( \frac{\partial a}{\partial c} \). The new results are:

\( \frac{\partial X}{\partial a} \) = \( \frac{\partial X}{\partial b} \) = \( \frac{\partial X}{\partial c} \) = \( \frac{\partial Y}{\partial a} \) = \( \frac{\partial Y}{\partial b} \) = \( \frac{\partial Y}{\partial c} \)

\( \frac{\partial a}{\partial c} \) = \( \frac{\partial a}{\partial b} \) = \( \frac{\partial a}{\partial c} \) = \( \frac{\partial a}{\partial c} \)

The derivatives \( \frac{\partial a}{\partial c} \) etc. are derived from results (a), (b), (c), (d), (e), (f), and surrounding material.
VITA

I, John Steele Henderson, was born on March 3, 1919, in Chapel Hill, North Carolina. In 1935 I graduated from high school in Chapel Hill and entered the University of North Carolina. I received the Bachelor of Arts degree from the University of North Carolina in 1939 and the Master of Arts degree in 1943. For the past two years I have been working toward a doctorate in economics at Louisiana State University.
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August 3, 1945