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A BEHAVIOR ANALYTIC APPROACH TO DEVELOPING MATHEMATICS SKILLS IN SECOND GRADERS: A PRECISION TEACHING INTERVENTION

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
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in partial fulfillment of the
requirement for the degree of
Doctor of Philosophy
In
The Department of Psychology

by

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ABSTRACT
This study examined the relative effects of two rates of responding on student endurance, accuracy and retention of basic math skills. Three cohorts, each consisting of four students, participated in the study and received training on two separate math skills. One skill was trained to a rate of responding consistent with a fluency standard and the other skill was trained to a rate of responding consistent with accurate but not fluent performance. The main finding underscored by the present study is that successful learning is enhanced if precipitated by fluent rates of responding. The data presented in each cohort demonstrated that training skills to fluency standards resulted in gains of student endurance, retention and student accuracy on basic math skills. For students in the endurance cohort, the most important finding was that students responding fluently remained engaged in the activity longer than the students who were not fluent. The most important finding for the generalization cohort was that students performed faster, with greater accuracy and persisted (i.e., less distractible) on new material for longer intervals of time if they were trained to levels of fluency at the onset. The main finding of the retention cohort was that in three out of four cases, skills trained to less than fluent levels of responding yielded diminishing student performance over time.
CHAPTER I
INTRODUCTION AND REVIEW OF LITERATURE

The majority of students who are referred for learning problems present deficits in reading; however a significant number of referred students often possess concomitant impairments in math (McLeod & Armstrong, 1982, Rivera, 1997, Skinner & Schock, 1998). Indicators of the prevalence of math deficits among our elementary-aged students are presented in the Digest of Educational Statistics, presented in the National Assessment of Educational Progress (NAEP). As noted in the NAEP Digest of Educational Statistics (Table 394) American fourth graders achieved only 58.4% correct on general math skills assessments. In 1996, 64% of fourth graders were performing at a basic skill level (e.g., partial mastery of the four basic arithmetic operations), and only 21% of fourth graders were rated as performing at a proficient level (e.g., demonstrated mastery of the four basic operations). As additional evidence for the growing concern regarding student math performance, the NAEP reported that the percentage of eighth graders who can add, subtract, multiply, divide and solve one-step word problems (i.e., skills expected to be mastered within the second and third grade curriculum) was 79%.

In addition to reporting on the status of student math performances, NAEP investigated the impact of deficient math knowledge within the business sector. Reportedly, skill deficiencies among workers impact productivity and offset earnings by requiring that companies offer remedial training classes for its employees. Several major corporations note that they are unable to fill 1 in 10 positions due to insufficient entry level skills of applicants. NAEP further reported that being able to demonstrate specific
math competencies would boost an individuals' earning potential. Workers who tested in the top quartile of math skill on NAEP earn 37% more than those students in the lower quartiles (The Condition of Education, 1998).

Fundamental to the view that our students perform poorly on math-related tasks are assumptions about the most effective means to teach our students. One current view on math instruction involves understanding how accurately students process information and apply strategies to solve problems.

**Accuracy Based Instruction**

**Cognitive strategy instruction.** Cognitive Strategy Instruction (CSI) (Goldman, & Hasselbring, 1997, Montegue, 1997) is a method to teach students how to select and apply cognitive strategies for successful task completion. CSI, like other cognitive methods involves (a) accurately defining the problem, (b) selecting lower-order procedures, (c) forming a strategy, (d) developing a mental representation of the problem and the steps to solve it, (e) applying the strategy, (f) monitoring task performance, and (g) evaluating the strategy (Montegue, 1997, Naglieri & Gottling, 1997, Thornton, Langrall, & Jones, 1997).

Studies demonstrating the effectiveness of CSI have begun to emerge in the learning disabilities literature in the last ten years. Montegue (1997) reported on a series of studies conducted with learning disabled students who after being taught CSI, improved their accuracy scores on tasks of problem solving to the level of non-disabled peers.
Naglieri and Gottling, (1997) taught 12 elementary-age students to use procedures consistent with CSI and improved accuracy scores for all but one student. In their study, Naglieri and Gottling reported that the students were each provided with math worksheets, and were instructed to work without stopping for ten minutes. Following the first 10-minute work period the examiners and students talked for a period of 10-minutes, about a topic unrelated to the worksheets during baseline, and about CSI procedures during the intervention phase. During the intervention phase the examiners prompted students to verbalized their methods and discussed which strategies worked well and which strategies did not work well. A third ten-minute work period followed the discussion period. Data analysis revealed that student accuracy improved for most students during the intervention phase. In some cases, students tripled their accuracy scores over baseline performances.

As an increased number of school districts embrace the National Council of Teachers of Mathematics’ (NCTM) suggestions for changes in instructional format, advocacy for application of CSI increases. In addition to conducting CSI within the classroom, more and more school districts are ending math-related drill and practice activities after January in grade three, and after October in grades four and five (Vann, 1995). Instead, students use the CSI protocol to understand math concepts and calculators to optimize instructional time. Students who are weak in the four basic operations receive remediation via computer-presented packages (Vann, 1995). According to the Louisiana State Department of Education: Louisiana Content Standards,
"The basic facts of addition, subtraction, multiplication, and division are important; however, technology, specifically the development of calculators, allows all students to expand and extend much of traditional school mathematics far beyond the basic math facts and repeated drill and practice. Students should concentrate on understanding ideas, reasoning, solving problems, communicating, and making connections within mathematics and between mathematics and its growing applications in other fields.” (p.6)

Louisiana’s position that math instruction should emphasizes accurate problem solving strategies over rote drill and practice exercises is a reflection of standards defining the national K-12 curriculum. The Arizona State Department of Education: Mathematics Standards outline the goals for elementary-aged students including the expectation that students in grades K-3 will become accurate in the basic math processes and proficient with the use of a calculator (e.g., objective 1M-F7). Standards for the four basic operations (without the use of a calculator) was defined as “accurate and consistent solving of computational problems” (Mathematics Standards p.3).

CSI represents a systematic approach to meet the standards set forth by the NCTM and Goals-2000. Teachers who incorporate CSI in their daily lessons enable students to construct mathematical knowledge, improve communication of math concepts, and improve real-world problem-solving abilities (Goldman, & Hasselbring, 1997, Montegue, 1997, Naglieri, & Gottling, 1997). Another well established teaching strategy involves raising students to levels of fluency, minimizing reliance on cognitive processing and calculators by facilitating levels of automatic responding. The importance and advantage of fluency-based instruction are discussed next.
Fluency-Based Instruction

Recent advances in the science of human behavior, specifically applied behavior analysis (Baer, Wolf & Risley, 1968), hold promise for students who encounter academic learning problems. The application of fluency-based instruction to remediate math deficits addresses not only how a student performs a skill but also how fast a student performs a skill. Recent investigations of fluency-based instruction include rate of response and performance monitoring programs (Coulter, 1985, Daly, Witt, Martens & Dool, 1997, Gresham & Lambros, 1998).

Variables related to student behavior have recently been addressed as not failure to learn a new skill but as failure to respond at an adequate rate (Belfiore, Lee, Vargas & Skinner, 1997, Howell & Lorson-Howell, 1995, Skinner, 1989, Skinner, Fletcher & Henington, 1996). The conceptualization of student academic success as one of rate of response rather than accuracy has several advantages. Viewing student performance in terms of rate of response provides qualitatively more information regarding student learning than statements of accuracy. Rate of response provides, in addition to how many trials were correct or incorrect, child-performance data on how fast the child worked and the latency between attempted trials (Miller, Hall & Heward, 1995, Skinner, 1998, White, 1984). Accuracy statements only offer feedback in the form of number of correct responses and number of incorrect responses for an exercise.

Several studies have reported on the advantages of rapid responding on developing decoding or word recognition skills (Breznitz, 1997, Tan & Nicholson, 1997). Tan and Nicholson (1997) reported that children who possess slower decoding skills will
produce deficient passage comprehension scores. However, if students were trained to a level of fluent responding (e.g., read one word per second), the results indicated higher comprehension scores compared to a control group of untrained students. Tan and Nicholson provided additional evidence that these gains may be achieved by training either individual students or groups of students. Interestingly, students who were accurate on word-lists but were not trained to fluency standards presented more errors when reading the same target words in short passages. Fluent students read word lists and short passages with equally superior speed and accuracy.

Breznitz (1997) reported that accelerated reading rates increased overall reading effectiveness in special and general education students. Slower or inefficient decoding processes occupied excessive amounts of cognitive resources, thus limiting the readers' comprehension of the passage. Breznitz reported that speed of visual word recognition is a powerful predictor of later reading ability. Support for this supposition is based on the findings that dyslexic students can substantially improve their reading performance following accelerated reading trials.

Ivarie (1986) provided additional evidence for the benefits of achieving fluency rather than accuracy standards for academic functioning. Ivarie investigated the effects of two rates of responding (fluency or accuracy) on a recall and written expression task. The results of the study suggested that students who performed at the higher rate of responding were superior in recall and writing compared to students responding at the accuracy rate.
In an early study on the effects of speed of student performance, Van Houten and Thomas (1976) reported that students could enhance their rate of responding on math tasks with no adverse effects to accuracy. When timed, students in the Van Houten and Thomas study performed at increased rates of responding on basic subtraction and addition probes. Following the conclusion of the study, under conditions of no explicit timing, the teacher reported that several students persisted in an accelerated rate of responding. In addition, student ancillary strategies, such as counting on fingers, using scrap paper or using beads, were not employed as frequently.

Miller et al., (1995) describe the effects of explicit timing and feedback on student rate of response with general and special education first graders. Initially, during a ten-minute work-session, the first graders' rate of responding on single digit math probes equaled 4.5 responses per minute. When Miller et al. modified the length of the work-session to reflect one minute timed trials with immediate feedback, student rate of responding increased to between 13-17 problems per minute. In addition, follow up probes five weeks after the conclusion of the study revealed continued acceleration of student rate of responding on timed trials.

Other Benefits of Increased Rate of Responding

Teachers can train basic math skills to a specific rate of responding or performance standard that makes future applications more likely (Haughton, 1972, Peters, Lloyd, Hasselbring, Goin, Bransford & Stein, 1987). A study by Belfiore, et al., (1997) demonstrated that a student who performed at a certain rate on single digit multiplication problems was more likely to attempt a multiple digit multiplication
problem. Belfiore, et al., provided two special education students with the option to select one of two types of worksheets: worksheet A. consisted of a series of multi-digit problems and worksheet B. consisted of a series of three single digit problems followed by one multi-digit problem. The outcome of the study indicated that both students selected the latter worksheet and both students were more likely to initiate the multi-digit problem sooner (e.g., will shorter latency) after completing a series of single digit problems.

In addition to making future application of skills more likely, there is evidence that as the execution of fundamental math skills increase in speed and accuracy, the acquisition of higher level skills is enhanced (Binder, 1996, Lindsley, 1991, White, 1986). In a series of studies comparing effective instructional methods (e.g., teaching by means of rote and drill or teaching rules), Van Houten (1993) demonstrated that once the rule was mastered, students substantially increased their correct rate of responding on single digit subtraction problems. Students, who in some cases showed no gains in rate of responding using a rote or drill method, completed problems 2-6 times faster with the method of teaching rules. In addition, several students were able to generalize a specific subtraction rule to untrained problems and maintain levels of responding higher than recorded during baseline. One of the conclusions drawn from Van Houten’s study was that the use of rules, when applied in conjunction with sufficient levels of responding, aid in generalizing and maintaining learned skills.

In summary, increasing rate of responding to levels of fluency improves how precisely teachers can relate information on student functioning across academic
domains. The studies reported here supported the conclusion that as students improve their rates of responding, execution of basic skills required less cognitive processing (i.e., processing is automatic). Raising student rates of responding to levels of fluency increases student access to reinforcement, making future applications more likely and improving accessibility to skills that are more complex.

The last 25 years have witnessed the development of empirically sound fluency-based teaching activities to improve academic deficits in elementary aged students. Three such interventions, Direct Instruction (Engelmann, & Carnine, 1991), flashcard strategies and Precision Teaching (Lindsley, 1995) will be discussed next.

Fluency Based Interventions

Direct instruction. Direct instruction (DI) evolved from the work of Sigfried Engelmann, and Douglas Carnine, whose goal was to provide teachers with a technology-based approach, encompassing what to teach and how to teach it (Becker, 1992). DI establishes a minimum sequence of teaching steps that will ensure student mastery of the curriculum for each student. DI is often defined as one or more trained teachers following scripted lesson plans within small groups of actively responding students (Becker, 1992, Martens, et al., in press). Other central features of DI include programmed monitoring of student performance, providing sufficient time for student engagement and clear and immediate feedback to student responses (Becker, 1992, Kameenui & Carnine, 1998).

There is extensive empirical support for DI within several sub-populations in American classrooms. Perhaps the largest population of children to benefit from DI
include those who participated in federally funded enrichment programs in the late 1960's and early 1970's (Becker, 1992, Mercer & Mercer, 1993). Projects Headstart, and Follow-Through availed thousands of disadvantaged children nationwide to trained DI teachers and the application of effective teaching methods. Analysis of the federal program data indicated DI was superior to several other research-based strategies in raising the performance level for at-risk children (Becker, 1992, Engelmann & Carnine, 1991). In addition to the positive outcomes for disadvantaged children in this country, researchers assessed DI's effectiveness with non-English speaking populations on several continents. One longitudinal study conducted in Asia produced positive academic gains for students receiving DI and increased teacher satisfaction for behavioral interventions (Nakano, Kageyama & Kinoshita, 1993). A second teacher-directed activity that holds great promise for low achieving students is the use of flashcards.

**Flashcard practice.** Flashcard practice has been utilized as a teaching tool across academic skills (Tan, & Nicholson, 1997, Van Houten, 1993, Van Houten, & Rolider, 1989). In the area of reading, Tan and Nicholson (1997) reported that students who received single word flashcard drills significantly outperformed students who did not receive training on all measures of reading comprehension.

Van Houten and Rolider (1989) investigated the impact of several variables on the acquisition of math facts using flashcards. An investigation of the efficacy of flashcard practice on learning rates (e.g., trials to criterion) was conducted in an alternating treatments design, where either several independent flashcard variables or one combined treatment package was applied to the students. Variables applied individually included...
(a) re-presenting missed facts after one trial, (b) error-contingent reprimands and, (c) sessions where the student and tutor were seated knee to knee. The final assessment of learning rates included a combination of all three variables into one treatment package. Van Houten and Rolider (1989) reported that students reached criterion levels in fewer trials with the combined treatment package than with any of the individual variables. As demonstrated in previous research, flashcard practice has been effective at improving the performance of low achieving students. A final teacher-directed activity that holds great promise for low achieving students is Precision Teaching.

**Precision teaching.** Ogden Lindsley is generally regarded as the engineer of precision teaching (PT). Lindsley, a student of B.F. Skinner, sought to apply Skinner's experimental analysis protocol to the needs of children and teachers in the classroom (Lindsley, 1991, McDade & Goggins, 1993, West & Young 1992, White, 1986). Lindsley's original intent was to develop a method to monitor student academic behavior and at the same time, offer teachers a protocol for database decisions. The precision in PT refers to the changes made to instructional strategies based on continuous and frequent monitoring, and analysis of student performance (Binder, 1996, White, 1984). Therefore, PT is not so much a method of instruction but it is a method of evaluating the effects of instructional efforts (Lindsley, 1991, West, 1995). As originally predicated, PT rests on the following seven principles:

1. Student academic performance is utilized as a guide for teachers to develop and refine instructional strategies. This principle is referred to as “the student knows best” (Lindsley, 1991).
2. Direct and continuous measurement of behavior. The focus on directly observable behavior includes daily assessment of student performance recorded on teacher-made probes.

3. The metric of measurement is rate of response. Precision teachers use rate of response to determine the progress of a student through an instructional program.

4. Outcome measures of student performances are represented graphically. Evaluation of student progress is enhanced (i.e., decisions regarding student performance are made in an efficient and accurate fashion) through the use of a graph or chart.

5. An emphasis on a behavior analytic view of student behavior. Student strengths and weaknesses on any task can be lawfully determined based on a functional understanding of the student's response.

6. Continuous monitoring of the impact of the environment (e.g., instructional activities) on student performance. Teachers can only modify a program of instruction if they have data to support hypotheses of progress or non-progress.

7. Develop functional methods to build appropriate behavior rather than primarily focusing on which student behavior to eliminate. The student is provided with instructional methods that foster generative effects rather than methods that only respond to the students' skill deficit (Johnson & Layng, 1992).

**Precision Teaching Interventions**

Precision teaching has been investigated in several formats, including a computer-mediated program (McDade & Goggins, 1993, Yaber & Malott, 1993). The advantage to computer-based precision teaching is that this format offers efficient on-line data
collection, analysis and decision making for individual and groups of students. For example, McDade and Goggins (1993) reported that using computerized precision teaching with college students produced positive effects on grades in composition, algebra and statistics courses. Additionally, course completion rates increased by 30 percent following the implementation of the precision teaching study.

Teachers have also applied precision teaching to secondary populations and have shown precision teaching methods to be beneficial with students with mild handicapping conditions. Byrnes, Macfarlane, Young, and West (1990) reported using precision teaching in conjunction with a flashcard protocol to increase minimum competency test scores of four students in a special education classroom. In addition to achieving their main objective, (increasing the student’s test performance), Byrnes, et., al. noted that their procedure generated meaningful data on the effectiveness of instruction as directly related to student performance. This kind of immediate feedback for teachers is important for monitoring student performance and maintaining the highest level of effective teaching (White & Haring, 1980). It is generally the unfortunate circumstance that by the time a teacher realizes that the instructional approach has failed, so has the child (White, 1986).

Perhaps where precision teaching has had the greatest impact is with the elementary school-aged at-risk population. Precision teaching has been cited in the literature as an effective approach to addressing both problematic student behavior (Young, West, Heward & Whitney, 1986, Schoen & Jones, 1993) and student academic

In their case study of a student who chronically called out in class Schoen and Jones (1993) taught precision teaching strategies to a classroom teacher resulting in a dramatic reduction in the level of student inappropriate behavior. Interestingly, this study provided an example of the pliability of precision teaching; the intervention for calling out was modified on three occasions during the course of the study. Prior to each modification, the teacher was able to make informed decisions regarding the best path for treatment due to the explicit components of precision teaching (Schoen & Jones, 1993).

Binder et., al. (1995) reported using precision teaching to build fluent levels of responding is a necessary condition to reduce student off-task behavior. Nonfluent performance of a first grade sample of students was associated with less attentive behavior as well as increased error rates on math and reading tasks. As demonstrated by Binder et., al. (1995), students who wrote digits 0-9 at fluent rates (e.g., 70 digits per minute) were less distracted and maintained their performance levels when the interval of time was extended to a 16-minute probe. Students who were nonfluent exhibited higher rates of off-task behavior during the shorter probes and failed to continue writing altogether during the extended probe. The general finding was that students will exhibit more on-task behavior and endure on academic tasks for longer intervals, only after fluent rates of responding are achieved.

Young, et al., (1985) conducted a series of studies to assess the effect of fluent reading skills with students in a special education classroom. Outcome data from the
Young, et al., studies indicated that students with disabilities achieved fluent levels of responding on word recognition tasks. Further, rates of responding on skills trained to a fluency criteria maintained their levels of performance, in comparison to skills trained only to an accuracy criteria, whose levels of performance decelerated once the intervention was discontinued.

**Retention, Endurance, Application Performance Standards**

As a student of Lindsley, Haughton, (1972) furthered the developing field of precision teaching by looking not at what the teacher was doing but at what the student was doing. Specifically, Haughton endeavored to assess how fast and how accurately a student needed to work (i.e., at what performance standard) to make future learning more accessible (Binder, 1996, Haughton, 1972). Haughton is credited with the development that certain performance standards may ensure development of component skills, and he coined the acronym REAPS (retention, endurance, application performance standards), thus challenging researchers to determine the response levels for school-aged children across all academic learning tasks. What follows is a brief description of each component of REAPS.

**Retention.** Retention refers to the student's ability to maintain a composite skill overtime. Empirical analyses of variables that contribute to student retention indicate that a student who responds more rapidly possesses greater retention (Howell & Lorson-Howell, 1995).
Endurance. Endurance refers to the student’s ability to perform or attend to a task for extended periods of time. Binder, et al., (1995), reported that as student fluency increased so did the student’s endurance to complete a timed task, even in the presence of distractible stimuli.

Application. Application refers to the integration of component responses into composite response classes. Application is achieved when a student can recognize not only how the skill is applied, but when the skill is required and not required (Johnson & Layng, 1992). Application is achieved when the student actively employs the skills taught under novel or untrained conditions.

Performance standards. Performance standards refer to minimal criteria that ensure the task can be performed after periods of no practice or opportunity, under distractible conditions and within novel contexts.

Performance standards are commonly referred to as “aims” and are expressed as ranges of student functioning on a particular task (Evans & Evans 1985, Lindsley, 1991). For example, student aims on single-digit addition problems can range from 40 digits correct per minute (Koorland, Keel & Uberhorst, 1995, Miller & Heward, 1992, Shapiro, 1996) to 125 digits correct per minute (Wood, Burke, Kunzelmann & Koenig, 1978). Given that performance standards differ across skills and across children, several authors caution that a student should not be considered fluent when a certain number of responses per minute are emitted but when that student can utilize a skill for their personal and academic benefit (Evans & Evans, 1985, Mercer, Mercer & Evans, 1982, Van Houten, 1979).
Unfortunately, the empirical literature base for REAPS is not well disseminated. The majority of data to support this precision teaching convention is drawn from special interest group meetings and data sharing newsletters. While the REAPS outcome data exist in relative obscurity, the utility of the premise of REAPS extends across direct instruction, curriculum based assessment and cognitive strategies instruction. Hence, a substantial body of evidence exists for the positive outcomes of REAPS in the form or structure of direct instruction or precision teaching interventions.

Statement of the Problem

Despite a large volume of literature on effective teaching strategies in mathematics, American students post mediocre scores on national and international assessments of math skills. The review of the literature reveals that two methods, fluency-based and accuracy-based instruction emerge as the dominant math strategies used by classroom teachers. In this study, fluency-based instruction involved using flashcards with a re-teaching strategy to increase a student's accurate rate of responding to a level which makes future learning more likely. Accuracy-based instruction included using flashcards without a fluency contingency. Accuracy training continued until students achieved a level of skill mastery or greater than 20 digits correct.

The purpose of this study was to examine the extent to which training children to a fluency standard would facilitate acquisition of math facts and offer additional benefits to children experiencing math deficits. This was accomplished by evaluating student endurance, generalization and retention of math skills, which were trained to a fluency
standard, and comparing those results to student endurance, generalization and retention of math skills, which were performed accurately but not fluently.

An evaluation of rate of responding on student performance of two math skills was conducted across three cohorts of four students each. Within each cohort, student skills were trained to one of the two pre-determined fluency standards (i.e., fluent or accurate but not fluent) via the systematic application of a treatment package consisting of flashcard presentations and repeated time trials. It was expected that training to a fluency standard would be the most effective method of maintaining a learned skill over time, improving student endurance, and facilitating student generalization of what was learned.
CHAPTER II
METHODOLOGY

Participants and Setting

A total of twelve second-grade students were nominated and participated in the study. Student demographic data collected included: (a) gender, (b) race, (c) age at the time of the study and (d) math performance average at the start of the study (see Table 1).

Table 1. Student demographic data

<table>
<thead>
<tr>
<th>Name</th>
<th>Gender</th>
<th>Race</th>
<th>Age</th>
<th>Math Average</th>
<th>Fluent Skill</th>
<th>Accurate Skill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kris</td>
<td>F</td>
<td>C</td>
<td>7-4</td>
<td>79</td>
<td>Subtraction</td>
<td>Time</td>
</tr>
<tr>
<td>Tay</td>
<td>M</td>
<td>NA</td>
<td>7-7</td>
<td>80</td>
<td>Time</td>
<td>Subtraction</td>
</tr>
<tr>
<td>Eric</td>
<td>M</td>
<td>C</td>
<td>7-5</td>
<td>78</td>
<td>Subtraction</td>
<td>Time</td>
</tr>
<tr>
<td>Jess</td>
<td>F</td>
<td>C</td>
<td>7-10</td>
<td>83</td>
<td>Time</td>
<td>Subtraction</td>
</tr>
<tr>
<td>Mic</td>
<td>F</td>
<td>A</td>
<td>7-10</td>
<td>79</td>
<td>Subtraction</td>
<td>Time</td>
</tr>
<tr>
<td>Aaron</td>
<td>M</td>
<td>C</td>
<td>7-9</td>
<td>80</td>
<td>Time</td>
<td>Subtraction</td>
</tr>
<tr>
<td>Albert</td>
<td>M</td>
<td>H</td>
<td>7-11</td>
<td>77</td>
<td>Subtraction</td>
<td>Time</td>
</tr>
<tr>
<td>Craig</td>
<td>M</td>
<td>C</td>
<td>7-10</td>
<td>77</td>
<td>Time</td>
<td>Subtraction</td>
</tr>
<tr>
<td>Grant</td>
<td>M</td>
<td>C</td>
<td>7-9</td>
<td>80</td>
<td>Time</td>
<td>Subtraction</td>
</tr>
<tr>
<td>Elle</td>
<td>F</td>
<td>C</td>
<td>7-8</td>
<td>79</td>
<td>Subtraction</td>
<td>Time</td>
</tr>
<tr>
<td>Alex</td>
<td>M</td>
<td>C</td>
<td>7-10</td>
<td>80</td>
<td>Time</td>
<td>Subtraction</td>
</tr>
<tr>
<td>Jeremy</td>
<td>M</td>
<td>C</td>
<td>7-11</td>
<td>77</td>
<td>Subtraction</td>
<td>Time</td>
</tr>
</tbody>
</table>

Note. C = Caucasian, A = Asian, NA = Native American, H = Hispanic
Teacher demographic data collected included: (a) gender, (b) race and (c) number of years teaching (see Table 2). Participation by the students and the teacher was voluntary and each participant provided verbal and written assent prior to the beginning of the study.

Table 2. Teacher demographic data

<table>
<thead>
<tr>
<th>Name</th>
<th>Gender</th>
<th>Race</th>
<th>Years Teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mrs. J. Prior</td>
<td>Female</td>
<td>C</td>
<td>11</td>
</tr>
<tr>
<td>Ms. J. Novy</td>
<td>Female</td>
<td>C</td>
<td>13</td>
</tr>
</tbody>
</table>

Response Measurement and Reliability

The study yielded data on student rate of responding (e.g., fluency or accuracy) on two grade appropriate math skills. Primary dependent variables included student endurance, expressed as total intervals engaged; student generalization, expressed as mean digits correct per session on untrained math problems; and student retention expressed as mean digits correct per session following a pre-determined period of no practice. Measurement of student performance was conducted by review of permanent products. Measurement of student endurance was conducted through direct observation of the students.

Math fluency. Math fluency has been operationalized as rate of accurate responding (Binder, 1996, Miller & Heward, 1992, Skinner & Schock, 1997). In this study, fluency was defined as student performance of basic arithmetic operations (e.g., subtraction and telling time), in a see-to-write channel, at a rate of no less than 55 digits correct per minute (Mercer, Mercer & Evans, 1982, Miller & Heward, 1992). The
fluency criterion of 55 digits correct per minute was based on previous studies in the precision teaching literature.

**Accuracy.** Accuracy was defined as percent correct. Accuracy was assessed by scoring student responses and dividing total number of problems correctly performed by the total number of problems attempted. To be considered accurate, the student had to perform at a level of 90 percent correct or higher. To be considered accurate but not fluent, the speed with which students performed the math skills could not exceed 40 digits correct per minute. The accuracy criterion was based on data outlined in the curriculum-based assessment literature.

**Endurance.** Endurance refers to the student’s ability to solve math problems over longer intervals of time, to persist under novel circumstances, or to persist in the presence of distracting stimuli (Binder, et al., 1995, Miller, et al., 1995). Endurance was measured by recording the frequency of 10-second whole intervals engaged prior to an established termination criterion on a consultant designed observation form (Appendix A). The termination criterion was defined as any three nonconsecutive 10-second intervals of off-task behavior within one minute.

**Generalization.** Generalization was defined as the students’ performance on untrained math problems in a see to write channel. Generalization was assessed by providing each student with a novel but related set of math problems and recording student rate of responding and student endurance.
Retention. Retention was defined as student performance (e.g., accuracy and digits correct per minute) on trained math skills after a pre-determined period of no practice. Student retention for solving math problems was assessed following a period of one week, post training. During the week following the end of training, the student did not interact with the consultant.

Materials

The consultant furnished the materials required for implementation of the study. Materials included (a) subtraction facts flashcards (minuends to ten), (b) time fact flashcards, (c) a stopwatch, (d) a math-facts grid (Appendix B), and (e) all pre-treatment assessment probes (Appendix C), including those used to assess generalization, in-class and exit probes. Pre-treatment probes, consisted of a consultant prepared 10-page packet of vertically and horizontally presented single and double-digit math problems, time-fact problems and one-step word problems. Math problems included on the pre-treatment probes were selected from mid-unit and unit tests taken from the students' second grade workbooks. All in-class and exit probes consisted of drill and practice worksheets (e.g., Basic Skill Builders) (Appendix D) including single and double-digit problems or worksheets containing photocopied analog clock faces (Appendix E). All packets used during each probe session contained more problems than could be completed during the time allotted. This procedure controlled for the slowest and the fastest students to have the same opportunity to demonstrate their speed and accuracy. A standard micro-cassette player and earphone were used throughout the study to cue the consultant to ten-second intervals.
Experimental Design

A multiple baseline design (Kazdin, 1982) was used to demonstrate the effects of the independent variable on student endurance, student-generalization and student retention across basic math skills. The multiple baseline design permits the demonstration that a behavior changes as a function of the independent variable and does not change when experimental manipulations are not in effect. The multiple baseline design removes the necessity to withdraw or reverse a treatment to demonstrate experimental control (Kazdin, 1982).

The present study applied a multiple baseline design in which each baseline phase is followed by the training phase and then a post-training baseline phase is introduced to evaluate the effects of the independent variable on three dimensions of student behavior (i.e., endurance, generalization and retention will be altered). The present study assessed the functional relationship between different rates of responding and their effects on student performance across three cohorts.

For subjects in the endurance cohort, to assess the effects of fluency on student endurance, math skills were trained to either a fluency standard or an accuracy standard. Students were considered fluent when they responded, under timed conditions, at a rate of 55 digits correct per minute or higher. Students were considered accurate when they responded, under timed conditions, at a rate consistent with mastery of a skill (e.g., 20-40 digits per minute) and with 90% correct responding or higher. Math skills were counter balanced across subjects to enhance interpretation of the impact of the treatment.
For subjects in the generalization cohort, once training raised student rate of responding to the prescribed standards, the effects of fluency on student performance was extended to include untrained math problems under normal classroom conditions.

The retention cohort continued to examine the relationship between fluency and student performance with four new subjects. The purpose of the retention cohort was to examine retention of math skills. Subjects in the retention cohort three however, were exposed to a period of no practice after which student performance was reassessed and student rate of responding was measured.

**Baseline.** Prior to any training, probes across skills were conducted in the student's classroom during the regularly scheduled math lesson. During baseline, the consultant provided probe packets of same skill worksheets (e.g., Skill Builders) to each student. Worksheets within each packet were sequenced in random order before each new administration. The consultant prompted each student to either begin or stop working and provided no additional feedback on student performance. Following each baseline session, the consultant collected the student math probes and directed the students to return to their seats.

**Training.** Following baseline, student skills were counterbalanced and assigned to either training to fluent rates of responding or training to accurate (e.g., not fluent) rates of responding. For each student, skills were trained to fluent and accurate levels in an analogue setting.
**Fluency training.** Fluency training was modeled after a flashcard protocol delineated by Van Houten and Rolider (1989) and consisted of (a) consultant and the student seated, facing each other; (b) re-teaching nonfluent facts after one interspersed fact and, (c) verbal rehearsal of non-fluent facts. At the completion of the training session, an exit probe (identical to probes given during baseline in level of difficulty and administration procedures) was administered to assess the effects of the independent variable.

**Accuracy training.** For each student, one math skill was trained to an accuracy standard without a fluency criterion. Accuracy training followed the same flashcard protocol delineated for fluency training with the omission of the two second fluency rule. Following the flashcard drill, students were prompted to complete an exit probe. Accuracy training continued until students achieved probe accuracy scores equal to or greater than 90% correct.

To control for exposure to either the treatment or opportunities to practice each skill, students received an equal number of probe sessions for skills trained to an accuracy standard and for skills raised to a fluency standard.

**Post-training.** To assess the effects of the independent variable on student performance, student rate of response, endurance, generalization and retention were assessed under a post-training baseline condition (i.e., within the classroom and without flashcards or feedback). The consultant provided probe packets to each student with the directions pertaining to when to start or stop working.
Endurance cohort. To test for student endurance, conditions were re-implemented where the consultant provided probe packets of same skill worksheets and recorded student behavior. The consultant prompted each student to either begin or stop working and provided no additional feedback on student performance. Following each probe session, the consultant collected the student math probes and directed the students to return to their seats.

Generalization cohort. To test for generalization, the consultant administered generalization probes to the students within the classroom setting. Generalization probes consisted of a 10-page packet of vertically and horizontally presented single and double-digit math problems, time-fact problems and one-step word problems (Appendix C).

Retention cohort. Following a period where training and probe sessions were not available, a retention phase was initiated where in-class-probes were administered to students to assess levels of retention of previously trained skills. During the test for retention, procedures were identical to baseline in administration and consultant recording of student performance.

Procedure

Following a pre-treatment phase, where nominated students completed in-class probes, each student received baseline probes across all math skills. Baseline probes provided data on student rate of correct responding and student endurance. Following stable responding during baseline, student skills were exposed to the treatment (e.g., a flashcard-based fluency intervention) in a multiple baseline format. For each student, rate of responding on one skill was raised to a fluency standard and a second skill was
raised to an accuracy standard. Once stable patterns of responding were achieved, treatment ended and student endurance, generalization and retention for each skill was assessed using in-class-probes under conditions identical to baseline. The generalization phase differed in that student performance on probes administered during pre-treatment assessments were compared with student performance on probes administered during generalization.

Pre-treatment assessment. To determine eligibility for the study, a pre-treatment assessment of student rate of responding on basic subtraction or analog time problems was conducted in two phases with each student. The first phase began with a teacher nomination and consultant validation of math deficit. Second grade teachers nominated students who were experiencing difficulty completing math computation assignments at a rate consistent with their peers. Teachers nominated students who demonstrated problems with math and did not nominate students who demonstrated deficits in academic skill areas in addition to math (e.g., reading, language, or written expression). Further, only students who returned the signed permission form (Appendix G), were permitted to participate in the study.

To validate that each student presented deficits in responding, the consultant used flashcards to measure each student’s level of fluency for the skills being studied. In an analogue setting, the pre-treatment assessment began with a verbal prompt from the consultant for the student to solve the problem on the flashcard as soon as it was presented. Simultaneously, the consultant started a timer to aid in determining if the student answered each flashcard problem within a two-second interval.
(Hasselbring, et al., 1987). Following the student's response, the flashcards were placed into three stacks: fluent (correct response within two seconds) accurate, (correct response within three seconds or more), and inaccurate. At the completion of the series of flashcards, the consultant entered the fluency data into a math-facts grid (see Appendix B). Each student was presented with three series of flashcards for each math skill. The student's median performance for each skill was used to determine how close the student was to the fluency standard and to aid in selection of skills for the study. Any student who exhibited frustrational-level performances during the assessment (e.g., accuracy below 60%) was excluded from the study. The skills used in the study were selected based on the following criteria: (a) skills which students were not performing at fluent levels, (b) skills which were represented in the second grade math curriculum, (c) skills which were pre-requisite for third grade, and (d) skills which were endorsed by the student's teacher as an area of weakness.

During the second phase of the pre-training assessment, nominated students were given a probe consisting of a 10-page worksheet packet containing one-step word problems, time fact problems and single and double digit computation problems (see Appendix C). Students were prompted to work on the packet without stopping for a period of ten minutes. During the ten-minute work period, student fluency was recorded by marking a photocopy of the students' packet at the end of every sixth 10-second interval. The notation reflected student rate of response and was converted into digits correct per minute during scoring. Student endurance was recorded on an observation
form (see Appendix A) during the ten-minute work period. Following this work period student packets were collected and scored for accuracy.

**Baseline.** During baseline, in-class-probes were conducted in the student’s classroom during the regularly scheduled math lesson. At the designated time the consultant entered the classroom and moved toward the math center. At that time, the classroom teacher directed the student to sit at the math center with a pencil. Once seated the consultant presented the student with a math probe and the instructions to either begin or stop working. The consultant provided no further instruction or encouragement during the session.

During baseline, math fluency was recorded by marking a photocopy of the student’s probe at the end of every sixth 10-second interval as prompted by a micro-cassette recorder (the consultant had four seconds to execute this recording procedure). The notation on the consultant’s photocopy provided a measure of the number of problems completed by the student per minute. During data analysis, rate of response was converted to digits correct per minute for the session. Math accuracy was recorded by scoring the student’s probe and computing a mean percent correct for the session.

Student endurance for solving basic math problems during baseline was recorded on an observation form (see Appendix A) as the total number of whole intervals the student worked without stopping. A period of thirty seconds or longer was set as a termination criteria in that any three nonconsecutive intervals of off-task behavior within one minute indicated the ceiling for endurance recording. It was hypothesized that a student who was asked to perform a task that he/she was not fluent in would persist at the
task for less time than a student who was fluent (Binder, et al., 1995, Miller, et al., 1995). To assess the effects of fluency on endurance, the total number of ten-second intervals prior to the termination criteria during baseline was compared to the total number of intervals prior to the termination criteria following the treatment phase.

During baseline, probes were completed when the students worked for at least six minutes. If the student remained engaged beyond the six-minute mark, sessions were extended as long as the student was working and did not meet the termination criteria. Following each session, probes were collected by the consultant and stored in a student folder for scoring.

**Fluency training.** Training took place in a resource room in the school’s library. During fluency training, the teacher prompted the student to accompany the consultant to the library. Once in the library the consultant sat knee to knee with the student and instructed the student to answer a single digit math problem presented on a flashcard. The student was instructed to respond as soon as each card was presented. Simultaneous with the presentation of the first flashcard, the consultant started a timer to assess the fluency of the students’ response. If the student responded fluently (e.g., correct and within two seconds), the flashcard was placed in a stack in front of the consultant. If the student responded non-fluently (e.g., correct but not within two seconds), or if the student responded incorrectly, the consultant provided a re-teaching strategy. The re-teaching strategy consisted of (a) the consultant reading the problem and answer to the student, (b) placing the flashcard behind the next flashcard in the series, and (c) when the student was re-presented with the missed flashcard, the consultant prompted the student to read aloud
the entire problem and provide the answer. If the student provided the answer within two seconds of reading the problem, the card was placed in the stack in front of the consultant. If the student hesitated for three or more seconds or provided an incorrect response, the re-teaching procedure was initiated until the student achieved a fluent response.

Following one complete series of flashcards the student was administered an exit-probe. Exit probes were used to assess the degree of fluency achieved as a function of the independent variable. Exit probes were identical to probes administered during baseline in the number of worksheets (10) and degree of difficulty. Fluency of student responding was measured by marking the consultant’s photocopy of the exit probe, at one-minute intervals. The purpose of the exit probe was to assess the effects of training on fluency; hence, student endurance was not assessed. At the completion of the exit probe, each student was prompted to put his or her pencil down and the probe was collected and stored in a student file. The consultant provided no verbal instruction or encouragement to the student during the session, or feedback on accuracy at the completion of the probe.

A second session consisting of a flashcard drill and exit probe was initiated following a brief rest period. The second training session was conducted in an identical manner to the session described above. Following the completion of the second exit probe, the student was escorted back to the classroom. Fluency training ended when the student demonstrated a problem-solving rate of 55 digits-per-minute on the treated skill.
**Accuracy training.** Skills that were trained to an accuracy standard were probed to ensure levels of responding met or exceeded 90% correct. Accuracy training took place in a resource room in the school library. Accuracy training consisted of flashcard trials with the omission of a fluency contingent re-teaching strategy. Unlike re-teaching during fluency training, during accuracy training re-teaching was provided only if the student responded incorrectly. Re-teaching was not provided for non-fluent responses. Following one complete series of flashcards, the student was given an exit probe to complete. Upon completion of the exit probe the consultant repeated the flashcard trial and then administered the second exit probe. Accuracy training continued until the student indicated performances equal to or greater than 90% correct.

Skills trained to an accuracy standard were evaluated for student percent correct. Student rate of responding (converted to digits correct per minute) was collected during exit probe administration.

**Endurance.** During the test for endurance, student on-task behavior (e.g., endurance) and rate of responding was assessed using in-class-probes. Probes administered were identical to baseline probes described earlier.

**Generalization.** During the test for generalization, in-class-probes were administered to the students following training to determine the effects of the interventions on untrained math problems. Generalization probes consisted of a 10-page packet of vertically and horizontally presented single and double-digit math problems, time-fact problems and one-step word problems. Generalization probes were conducted...
and analyzed in an identical manner to the probes administered during the pre-training assessment.

**Retention.** The test for retention of math skills that were trained to either a fluency or accuracy standard was conducted following a period of no training or opportunity to respond to in-class probes. After skills were trained to either fluent or accurate standards, students were told that the probe sessions would be discontinued for one week. On the sixth school day post-training, the consultant returned to the student’s class and administered probes to test for retention of math skills. During the test for retention, probe procedures were identical to baseline in administration and consultant recording of student performance.

**Reliability of Procedures and Observations**

To insure that all experimental procedures were implemented with fidelity and that all observations were conducted reliably, several strategies were implemented to assess reliability and fidelity.

**Consultant fidelity to experimental procedures.** To insure the consultant conducted the intervention correctly, a trained independent observer used a copy of the intervention steps (see Appendix F) and recorded whether or not the consultant completed each intervention step during in-class and exit probe sessions. Procedural fidelity was computed by dividing the number of steps executed by the total number of intervention steps and then multiplying by 100. Procedural fidelity for all 12 cases ranged from 94% to 100%.
Behavioral observations. Behavioral observations for student endurance were recorded on consultant-designed observation forms using a whole interval scoring procedure. Observers included the consultant and two undergraduate research assistants. The consultant served as the primary observer. Training of the two research assistants was conducted in a second grade classroom prior to the start of the study. Training was completed when at least 90% agreement for student endurance was attained for two consecutive ten-minute observations. A student was recorded to be engaged only if he or she worked without stopping during a ten second interval. Any observed off-task behavior during the ten-second interval resulted in a designation of non-engaged. Data were converted to total intervals engaged prior to the termination criterion for student endurance. A second observer simultaneously, but independently, recorded data during 25% of all sessions.

Interobserver Agreement

Two measures of interobserver agreement were calculated: (1) recording of digits correct based on student performance on in-class, exit and generalization probes and, (2) the recording of student endurance based on whole interval observations taken during probe sessions. A second observer was trained by the consultant until 90% agreement or better was obtained with regard to the recording of digits correct prior to the onset of the study.

When probe sessions were observed, interobserver agreement was calculated for digits correct by dividing the total number of agreements for digits correct per minute by the number of agreements plus disagreements multiplied by 100. Interobserver
agreement for student endurance was calculated using the same method: total intervals per session were tallied and agreements on engaged behavior were divided by agreements plus disagreements multiplied by 100. Two observers independently and simultaneously recorded student performances (digits correct per minute) for 40% of the sessions for all twelve cases. Across all sessions where reliability was assessed, interobserver agreement for student performance ranged from 89% to 100% (M = 96%). Interobserver agreement for student endurance ranged from 84% to 96% (M = 90%).
CHAPTER III
RESULTS

The 12 students in this study were grouped into three cohorts. Each cohort consisted of four students and answered one of the following experimental questions: (1) To what extent does either a fluency standard rate of responding or accuracy standard rate of responding result in gains of student endurance? (2) Which, if either, rate of responding results in higher scores on untrained math problems? (3) To what extent do skills trained to either fluency standards or accuracy standards maintain over time?

Endurance Cohort

The primary dependent variable for the endurance cohort was student endurance expressed as total intervals engaged prior to an established termination criteria. Of secondary interest was student rate of responding on skills raised to either fluency or accuracy standards.

Student endurance. Figure 1 represents the mean number of intervals engaged across baseline and post-training phases on skills trained to a fluency standard. These data indicate that student endurance on skills trained to a fluency standard increased for each student in the endurance cohort. During baseline, student endurance on skills trained to a fluency standard ranged from 2 – 29 intervals (M = 8). Student endurance during post-training ranged from 33 – 60 intervals (M = 52). The average increase of student endurance for students in cohort one was 44 intervals or a benefit of 7.3 minutes of working without interruption. Students trained to an accuracy standard quit working on average, six minutes sooner than students trained to a fluency standard.
Figure 1: Fluency Standard

Figure 2: Accuracy Standard

Figure 2 represents the mean number of intervals engaged across baseline and post-training phases on skills trained to an accuracy standard. These data indicate slight
improvements in student endurance from baseline to post-training. On skills trained to an accuracy standard, student endurance during baseline ranged from 0-15 intervals (M = 5). Student endurance during post-training ranged from 0-25 intervals (M = 12). The average increase of student endurance on skills trained to an accuracy standard for students was 7 intervals or an increase of on-task behavior of one minute and ten seconds.

Figure 3 displays rate of responding across three phases for students included in cohort one. Student rate of responding for skills trained to a fluency standard and skills trained to an accuracy standard is graphed as mean digits correct for each session.

Kris. During baseline, Kris’ rate of responding on the skill trained to a fluency standard ranged from 3 – 15 digits correct per minute (M = 9). Kris’ rate of responding on the skill trained to an accuracy standard ranged from 7 – 13 digits correct per minute (M = 9). During the training phase, Kris responded well to the fluency training, achieving a stable criteria of 55 digits correct per minute or better in 9 sessions (M = 48). Kris’ performance on the skill trained to an accuracy standard indicated rate of responding within acceptable limits (M = 29). The data on Kris’ post-training rates indicated an initial drop in rate of responding on day one of the post-training phase, followed by increased rates over four subsequent sessions. During post-training, Kris’ rate of responding on the skill trained to a fluency standard (M = 49) indicated a much higher performance compared to baseline rates (M = 9).
Figure 3. Rate of Response
During post-training, Kris’ rate of responding on the skill trained to an accuracy standard ($M = 27$) was within acceptable limits.

**Tay.** During baseline, Tay’s rate of responding on the skill trained to a fluency standard ranged from 12 – 19 digits correct per minute ($M = 16$). Rate of responding during baseline on the skill trained to an accuracy standard indicated a decelerating data path with a range of 3 – 15 digits correct per minute ($M = 8$). Tay reached the training criterion for the fluent skill in 13 sessions ($M = 40$). During training, Tay’s performance on the skill trained to an accuracy standard indicated increases over baseline rates, but remained within acceptable limits ($M = 18$). During post-training, Tay’s rate of responding decelerated on the skill trained to a fluency standard during the initial session but returned to training levels during sessions 19 through 22 ($M = 52$). Tay’s performance on the skill trained to an accuracy standard indicated a modest increase in responding ($M = 24$).

**Eric.** The data during baseline for Eric indicated a variable rate of responding for both skills. The skill trained to a fluency standard ranged from 15 – 29 digits correct per minute ($M = 24$). Eric’s performance on the skill trained to an accuracy standard ranged from 12 – 23 digits correct per minute ($M = 18$). During the training phase, Eric’s rate of responding on the skill trained to a fluency standard indicated an ascending trend until the criteria for fluent responding was achieved by the ninth session ($M = 47$). Eric’s performance on the skill trained to an accuracy standard indicated stable levels of responding following an initial increase observed during training ($M = 26$). Consistent with his peers in cohort one, during post-training Eric’s rate of responding on the skill
trained to a fluency standard indicated a drop in rate for session one. Eric’s rate of responding returned to levels observed during training for the remainder of the post-training sessions ($M = 54$). During post-training, Eric’s rate of responding on the skill trained to an accuracy standard indicated a decelerating trend for the first three post-training sessions and then stable low rates of responding for the remaining sessions ($M = 24$).

Jess. During baseline Jess’ rate of responding on the skill trained to a fluency standard ranged from 5 - 14 digits correct per minute ($M = 9$). Jess’ rate of responding on the skill trained to an accuracy standard ranged from 11 - 24 digits correct per minute ($M = 15$). During the training phase Jess responded well to the treatment, reaching the fluency criterion in 8 sessions ($M = 49$). Jess’ performance on the skill trained to an accuracy standard increased slightly over baseline ($M = 22$), but was within acceptable limits. Jess’ performance following training indicated stable high rates of responding on the skill trained to a fluency standard ($M = 54$), and low rates of responding on the skill trained to an accuracy standard ($M = 20$).

The endurance cohort measured the relative impact of student rate of responding on student endurance to complete two grade appropriate math skills. It was observed that each student presented low levels of endurance across skills during baseline. Subsequent to training the math skills to one of two levels of responding, student endurance increased for each skill trained to a fluency standard and remained within baseline limits for each skill trained to an accuracy standard. Correspondingly, the speed with which each student solved grade appropriate math problems was observed to either accelerate or
maintain on skills trained to fluent standards rather than nonfluent standards. The relevance of the results from the endurance cohort to student performance suggests that student behavior, specifically student endurance and rate of responding, may be altered as a function of which rate of response the student is trained to.

In order to systematically examine the relationship, if any, between rate of responding and student performance it was necessary to assess whether student fluency generates meaningful outcomes on untrained math problems.

**Generalization Cohort**

Four students participated in the generalization cohort that assessed which, if either, rate of responding resulted in higher scores on novel math problems expressed as percent correct responding. The evaluation of student performance was completed by first assessing student percent correct responding and student rate of responding using pre-training probes, then raising student rate of responding to either fluency or accuracy standards using in-class and exit (skill builder) probes. A final assessment of student rate of responding and student percent correct on novel problems was implemented by reintroducing pre-treatment (i.e., generalization) probes. Of secondary interest, student endurance was recorded and analyzed as intervals engaged prior to an established termination criteria to assess student endurance.

**Percent correct responding.** Figure 4 displays the mean percentage of correct responding for each student in the generalization cohort during the pre and post training phases. These data indicate that mean levels of correct responding increased for skills
trained to a fluency standard across all students in cohort two. All four students increased both skills by at least 25% over pre-training levels.

![Graph](image_url)

Figure 4. Percent Correct Responding on a Fluent Skill

Mic’s results on the skill trained to a fluency standard suggested an increase in correct responding from pre-training ($M = 66\%$) to post-training ($M = 99\%$). Aaron’s performance on the skill trained to a fluency standard improved from pre-training ($M = 65\%$) to post-training ($M = 97\%$). Albert’s results indicated an increase in correct responding for the skill trained to a fluency standard from pre-training ($M = 64\%$) to post-training ($M = 99\%$). Craig’s performance on the skill trained to a fluency standard also increased from pre-training ($M = 59\%$) to post-training ($M = 97\%$).

Figure 5 displays the mean percent correct responding on skills trained to an accuracy standard for students in the generalization cohort. These data indicate that mean levels of correct responding also increased for skills trained to an accuracy standard.
across all four students. Mic’s results on the skill trained to an accuracy standard indicated an increase in correct responding from pre-training ($M = 63\%$) to post-training ($M = 91\%$). Aaron’s performance on the skill trained to an accuracy standard indicated an increase in correct responding from pre-training ($M = 66\%$) to post-training ($M = 93\%$). Albert’s performance on the skill trained to an accuracy standard indicated modest gains during post-training relative to the gains demonstrated by his peers. Albert’s results indicated an increase in correct responding from pre-training ($M = 64\%$) to post-training ($M = 79\%$). Craig’s performance on the skill trained to an accuracy standard increased from pre-training ($M = 69\%$) to post-training ($M = 95\%$).

**Rate of response.** Figure 6 represents student rate of responding across four phases included in the generalization cohort. Student rate of responding on skills trained to a fluency standard and skills trained to an accuracy standard is graphed as mean digits correct per minute for each session. Student rate of responding during phase two
Figure 6: Rate of Response
and three (e.g. baseline and training) was measured using Skill Builder Probes, and was collapsed to demonstrate treatment integrity (see figure seven for analysis of baseline and training phases).

**Mic.** During pre-training, Mic’s rate of responding on the skill trained to a fluency standard ranged from 21 – 28 digits correct per minute (M = 24). Mic’s rate of responding on the skill trained to an accuracy standard ranged from 17 – 26 digits correct per minute (M = 20). During the baseline and training phases, Mic achieved performance averages of 26 and 62 digits correct per minute respectively, for the skill trained to a fluency standard, and averages of 21 and 29 digits correct per minute respectively for the skill trained to an accuracy standard. During post-training, Mic’s rate of responding on both skills indicated a decrease in rate from previous training averages; however, the skill trained to a fluency standard indicated gains in rate of responding over pre-treatment levels (M = 38). The skill trained to an accuracy standard maintained pre-training rates of responding during the post-training phase (M = 22).

**Aaron.** Aaron’s rate of responding during pre-training ranged from 20 – 25 digits correct per minute (M = 23) for the skill trained to a fluency standard. During pre-training, Aaron’s rate of responding on the skill trained to an accuracy standard ranged from 15 – 22 digits correct per minute (M = 18). Aaron achieved baseline and training averages of 17 and 55 digits correct for the skill trained to a fluency standard and 23 and 27 digits correct for the skill trained to an accuracy standard. During post-training, Aaron’s rate of responding on the skill trained to a fluency standard indicated higher rates of responding compared to pre-training levels (M = 34). Aaron’s rate of responding, on
the skill trained to an accuracy standard, indicated a similar pattern to Aaron’s performance on the skill trained to fluency standard (Aaron’s rate of responding decelerated over three sessions but returned to pre-treatment levels during the last two sessions, \( M = 19 \)).

**Albert.** The data for Albert indicated the skill trained to a fluency standard ranged from 17 – 21 digits correct per minute, \((M = 19)\); and the skill trained to an accuracy standard ranged from 4 – 11 digits correct per minute \((M = 7)\). Albert’s baseline and training averages indicated 19 and 54 digits correct for the skill trained to a fluency standard, and 15 and 21 digits correct for the skill trained to an accuracy standard. During post-training Albert’s performance on the skill trained to a fluency standard indicated variable performance during the first three sessions of post-training, and then a gradually ascending rate of responding for the remaining sessions \((M = 20)\). Albert’s post-training performance on the skill trained to an accuracy standard indicated similar levels of responding \((M = 8)\).

**Craig.** During pre-training, Craig’s rate of responding on the skill trained to a fluency standard ranged 16 – 24 digits correct per minute \((M = 19)\). Craig’s rate of responding on the skill trained to an accuracy standard, during pre-training ranged 17 – 21 digits correct per minute \((M = 19)\). During the baseline and training phases, Craig achieved performance averages of 16 and 58 digits correct per minute respectively, for the skill trained to a fluency standard, and performance averages of 19 and 23 digits correct per minute respectively for the skill trained to an accuracy standard. During post-training, Craig’s rate of responding on the skill trained to a fluency standard decelerated...
at the onset of the phase, and then stabilized for the remainder of the sessions ($M = 34$).

The skill trained to an accuracy standard indicated variable performance and overall rates of responding consistent with pre-training levels ($M = 17$).

Figure 7 displays rate of responding across the baseline and training phases for students included in the generalization cohort. Student performance was measured using Skill Builder Probes and are presented to demonstrate fidelity with the training protocol. Student rate of responding for skills trained to a fluency standard and skills trained to an accuracy standard is graphed as mean digits correct for each session.

During baseline, Mic’s rate of responding on the skill trained to a fluency standard ranged from 17 - 27 digits correct per minute ($M = 22$). Mic’s rate of responding on the skill trained to an accuracy standard ranged from 23 - 30 digits correct per minute ($M = 26$). During the training phase, Mic responded well to the fluency training, achieving a stable criteria of 55 digits correct per minute or better immediately following training ($M = 60$). Mic’s performance on the skill trained to an accuracy standard indicated rate of responding within acceptable limits ($M = 29$).

During baseline, Aaron’s rate of responding on the skill trained to a fluency standard ranged from 16 - 19 digits correct per minute ($M = 18$). Rate of responding during baseline on the skill trained to an accuracy standard ranged from 21 - 27 digits correct per minute ($M = 24$). Aaron reached the training criterion for the fluent skill in 12 sessions ($M = 55$). During training, Aaron’s performance on the skill trained to an accuracy standard indicated stable rates of responding ($M = 27$).
Figure 7: Rate of Response

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The data during baseline for Albert indicated a stable rate of responding for both skills. The skill trained to a fluency standard ranged from 17 – 20 digits correct per minute (M = 19). Albert's performance on the skill trained to an accuracy standard ranged from 12 – 20 digits correct per minute (M = 15). During the training phase, Albert's rate of responding on the skill trained to a fluency standard ascended until the criteria for fluent responding was achieved by the ninth session (M = 54). Albert's performance on the skill trained to an accuracy standard indicated levels of responding within acceptable limits during training (M = 21).

During baseline Craig's rate of responding on the skill trained to a fluency standard ranged from 16 – 22 digits correct per minute (M = 19). Craig's rate of responding on the skill trained to an accuracy standard ranged from 15 – 22 digits correct per minute (M = 16). During the training phase Craig responded well to the treatment, reaching the fluency criterion in 8 sessions (M = 58). Craig's performance on the skill trained to an accuracy standard increased slightly over baseline (M = 23), but was within

Student endurance. Figure 8 represents student endurance across the pre and post-training phases for skills trained to a fluency standard. Student endurance is graphed as the frequency of whole intervals engaged prior to an established termination criteria.

During pre-training, Mic's endurance ranged from 10 – 20 intervals (M = 14) for the skill trained to a fluency standard. During post-training, Mic's endurance ranged from 13 – 60 intervals (M = 42) for the skill trained to a fluency standard. During pre-training, Aaron's endurance ranged from 1 – 5 intervals (M = 3) for the skill trained to a fluency standard. During post-training, Aaron's endurance ranged from 7 – 60 intervals
(\(M = 36\)) for the skill trained to a fluency standard. During pre-training, Albert’s endurance ranged from 0–1 intervals (\(M = 1\)) for the skill trained to a fluency standard. During post-training, Albert’s endurance ranged from 6–60 intervals (\(M = 44\)) for the skill trained to a fluency standard. During pre-training, Craig’s endurance ranged from 2–6 intervals (\(M = 4\)) for the skill trained to a fluency standard. During post-training, Craig’s endurance ranged from 16–58 intervals (\(M = 36\)) for the skill trained to a fluency standard.

\[\text{Figure 8. Student Endurance on a Fluent Skill}\]

Figure 9 represents student endurance across the pre and post-training phases for skills trained to an accuracy standard. During pre-training, Mic’s endurance ranged from 6–18 intervals (\(M = 12\)) for the skill trained to an accuracy standard. Mic’s endurance decreased and remained low during post-training (\(M = 10\)).

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Aaron’s endurance during pre-training ranged from 1-9 intervals ($M = 5$). During post-training, Aaron’s endurance ranged from 4-42 intervals ($M = 17$). Albert’s endurance during pre-training ranged from 1-6 intervals ($M = 4$). Albert’s endurance during post-training remained low ($M = 4$). During pre-training Craig’s endurance ranged from 1-4 intervals ($M = 3$). During post-training, Craig’s endurance ranged from 1-9 intervals ($M = 5$).

The generalization cohort assessed the relative benefits of two rates of responding on novel math problems. Student’s in the generalization cohort consistently indicated higher rates of responding, greater accuracy and greater on-task performances on skills trained to a fluency standard rather than an accuracy standard.
The retention cohort continued the examination of rate of responding and student performance. Students in the retention cohort received the same in-class probes administered during baseline phases in the previous studies; however, student performance (e.g., rate of responding, percent correct responding and student endurance) was assessed following a period of five school days where opportunities to practice the math skills were withheld to the greatest extent possible.

Retention Cohort

The primary dependent variables for students in the retention cohort included student endurance, expressed as whole intervals engaged prior to a termination criteria; student rate of responding expressed as mean digits correct per session; and overall student performance, expressed as mean percent correct responding for each phase of the study.

Student endurance. Figure 10 represents the mean number of whole intervals engaged across baseline and post-training phases on skills trained to a fluency standard. These data indicated student endurance increased for each student in cohort three. During baseline, student endurance on skills trained to a fluency standard ranged from 0 intervals engaged to 3 intervals engaged (M = 1). Considerably higher levels of engaged behavior were recorded during post-training; student endurance ranged from a low of 2 intervals to a high of 48 intervals engaged (M = 33). The average increase of student endurance for students was 31.25 intervals or a benefit of 5.2 minutes of working without interruption.
Figure 11 represents the mean number of intervals engaged across baseline and post-training phases on skills trained to an accuracy standard. These data indicate minimal improvements in student endurance from baseline to post-training sessions. On skills trained to an accuracy standard student endurance during baseline ranged from 1 interval engaged to 10 intervals engaged ($M = 2$). Consistent levels of engaged behavior were recorded during post-training; student endurance ranged from a low of 1 interval to a high of 40 intervals engaged ($M = 10$). The average increase of student endurance on skills trained to an accuracy standard for students was 9 intervals or an increase of on-task behavior of one minute and thirty seconds.
Rate of response. Figure 12 displays rate of responding across three phases for students in the retention cohort. Student rate of responding for skills trained to fluency standards and skills trained to accuracy standards is graphed as mean digits correct for each session.

Grant. During baseline, Grant’s rate of responding on the skill trained to a fluency standard ranged from 11 – 16 digits correct per minute (M = 13). Grant demonstrated similar levels of responding on the skill trained to an accuracy standard (range 7 – 16 digits correct per minute, M = 11). During training, Grant’s rate of responding on the skill trained to a fluency standard indicated a steady ascent until reaching the fluency criteria by the eleventh session (M = 40). A slight increase in rate of
responding on the skill trained to an accuracy standard was recorded during the training
phase (M = 16). Following the period of no-practice, Grant’s rate of responding on the
skill trained to a fluency standard indicated initial variability in responding across three
sessions, and then leveled off at rates consistent with what was recorded during training
(M = 48). Grant’s rate of responding on the skill trained to an accuracy standard
indicated performances consistent with baseline and training observations (M = 17).

Elle. During baseline, Elle’s performance yielded low rates of responding (M = 10
digits correct per minute for each skill). Correspondingly, during training Elle’s
performance on both skills increased with the introduction of the independent variable.
During training, Elle’s rate of responding on the skill trained to a fluency standard
produced a variable data path until the fluency criterion was achieved by the twelfth
session (M = 42). Elle’s performance on the skill trained to an accuracy standard also
increased during this phase, but maintained acceptable levels of responding (M = 21).
During the first post-training session Elle’s performance on the skill trained to a fluency
standard decreased in rate of responding. Subsequent sessions indicated Elle’s rate of
responding returned to levels observed during training (M = 47). During the post-
training phase, Elle’s rate of responding on the skill trained to an accuracy standard
indicated variable performance resulting in a decrease in mean digits correct per minute
(M = 17).

Alex. On the skill trained to a fluency standard, Alex’s baseline rate of
responding ranged from one to four digits correct per minute (M = 2). However, baseline
rates of responding on the skill trained to an accuracy standard ranged from 18 to 43
digits correct per minute ($M = 27$). During the training phase, Alex responded well to fluency training achieving a stable rate of responding of 55 digits correct per minute or better within 8 sessions ($M = 50$). During training, Alex’s performance on the skill trained to an accuracy standard indicated mean rates consistent with baseline observations ($M = 27$). Following the period of no practice, Alex’s responding on the skill trained to a fluency standard indicated accelerated rates for the first five sessions. Alex’s rate of responding then deceased by almost 18 digits per minute to a rate that was consistent with observations collected during the training phase ($M = 61$). On the skill trained to an accuracy standard, Alex demonstrated a decrease in rate following the period of no-practice ($M = 17$).

Jeremy. During baseline, Jeremy’s rate of responding on the skill trained to a fluency standard ranged from 10 to 19 digits correct per minute ($M = 14$). Jeremy’s rate of responding on the skill trained to an accuracy standard ranged from 1 to 11 digits correct per minute ($M = 4$). During the training phase Jeremy’s rate of responding increased for both skills. Jeremy’s rate of responding on the skill trained to a fluency standard reached the fluency criterion within ten sessions ($M = 47$). Jeremy’s rate of responding on the skill trained to an accuracy standard remained within acceptable limits ($M = 23$). Following the period of no practice Jeremy’s performance on the skill trained to a fluency standard indicated an initial drop in rate of responding but recovered to levels consistent with those recorded during the training phase ($M = 51$). A decrease in rate of responding was recorded for the skill trained to an accuracy standard ($M = 17$).
Figure 12: Rate of Response
Percent correct responding. Figure 13 represents the mean percent correct responding for the skills trained to a fluency standard for all students in the retention cohort. These data indicate that mean levels of correct responding increased for skills trained to a fluency standard across all students in the retention cohort.

Grant’s results on the skill trained to a fluency standard suggested an increase in correct responding across phases from baseline (M = 78%) to training (M = 97%) to post-training (M = 98%). Elle’s performance on the skill trained to a fluency standard also improved from baseline (M = 77%) to training (M = 92%) to post-training (M = 99%). On the skill trained to a fluency standard, Alex’s performance indicated an increase in correct responding from baseline (M = 26%) to training (M = 92%) to post-training (M = 97%). Jeremy’s results also indicated increased correct responding from baseline to post-training. Jeremy’s performance on the skill trained to a fluency standard increased from baseline (M = 94%) to training (M = 97%) to post-training (M = 98%).

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Figure 14 represents the mean percent correct responding on skills trained to an accuracy standard in the retention cohort. The data on student performance suggested that across three phases completed in the retention cohort, student performance declined for three out of the four students during the final phase of the study.

Grant's percent correct responding on the skill trained to an accuracy standard indicated stable performances across the three phases (baseline $M = 90\%$, training $M = 97\%$, post-training $M = 96\%$). Elle's performance on the skill trained to an accuracy standard indicated improvement from baseline ($M = 66\%$), to training ($M = 96\%$) and then decelerated during post-training ($M = 76\%$). Alex's performance on the skill trained to an accuracy standard indicated a similar pattern of responding (e.g., baseline: $M = 89\%$; training: $M = 97\%$; post-training: $M = 84\%$). Jeremy's performance on the skill trained to an accuracy standard increased from baseline ($M = 57\%$) to training ($M = 93\%$), but then decreased during post-training ($M = 80\%$).
CHAPTER IV
DISCUSSION

The purpose of the present study was to systematically examine the relative impact of two rates of responding on student performance. Results, while preliminary, indicated that a small difference in the training protocol (re-teaching strategy with a fluency contingency) was sufficient to impact rate of responding for 12 students across two skills. In addition, students were more likely to perform at higher levels, and maintain their speed of work completion, if first trained to a fluent rate of responding. In addition to increased speed of completing basic math facts, students who performed at a rate consistent with fluency exhibited greater levels of endurance when compared to peers performing at a rate consistent with accurate but not fluent levels.

**Endurance Cohort**

The most important implication of the endurance cohort was that when a student performs fluently, he or she may be more likely to remain engaged in that task for longer periods of time. This finding supports previous research indicating that teachers may increase active learning time (Gettinger, 1995) by first raising student responses to fluent levels before expecting them to complete assignments (Binder et al., 1995, Skinner et al., 1996).

The findings of the endurance cohort also indicated that when a student is trained to a fluent level of responding, the student continues to accelerate or complete math problems faster than peers trained to an accurate level of responding. Van Houten, and
of functioning on math problems, persisted longer in the task, performed with greater accuracy, and improved the speed with which they performed the math tasks. This finding is consistent with some of the theories of instruction stating that students must master a specific rate of responding before they can successfully progress through a curriculum. Engelmann (1997) related that performing at any level less than fluency is "mislearning" (p. 178), and increases the likelihood that the student will experience failure within the curriculum.

**Generalization Cohort**

The most important finding of the generalization cohort was that students completed novel math problems with greater speed and accuracy if they achieve a level of fluent functioning prior to beginning the problems. This finding is important because it implies that students who perform at fluent rates of responding progress through novel learning experiences with fewer errors compared with non-fluent peers. Further, student performance as measured by percent correct responding and student endurance indicated greater gains on skills trained to a fluency standard. Increases in percent correct were also noted for skills trained to an accuracy standard, however the gains were lower. Students in cohort two exhibited greater levels of endurance on skills trained to a fluency standard. Students gained over five minutes of active learning time once trained to fluency standards.

The primary difference between the endurance and generalization cohorts was the use of untrained or novel math problems with students in cohort two. The implication of the findings from cohort two for the classroom teacher are that students will perform
faster, with greater accuracy and will persist (i.e., less distractible) on new material for longer intervals of time if they are initially trained to a fluency standard on related materials. These findings support the concepts of generative learning (Johnson & Layng, 1992) or response adduction (Binder, 1996). One possible reason for improved student performance on skills trained to a fluency standard is that the student has learned the strategies to successfully solve the novel problem and the student does not have to relearn the details common within the exercise (Engelmann, 1997). Hence, without the necessity of relearning, the student is able to accelerate through the exercise.

Taken together, results from the endurance and generalization cohorts suggest that student performances can be enhanced using a fluency training protocol. However with cohort one and cohort two, measurements of student performance were collected immediately following the application of the independent variable. As noted by Johnson and Layng (1992), “to show true mastery (i.e. fluent responding) the student must demonstrate the skill after a period of no practice” (p. 1476). Hence, the student’s skills in the retention cohort were trained to either fluency or accuracy standards and then received a period of no practice for one week.

Retention Cohort

The most important finding from the retention cohort was the impact training to a fluency standard had on student endurance, speed and accuracy. Skills trained to a fluency standard resulted in greater levels of student endurance, (e.g., mean increase of active learning time over five minutes). Only marginal improvements in student endurance were recorded on skills trained to an accuracy standard. Similarly, unless the
skill was trained to fluent levels of responding prior to the period of no practice, three out of four students in cohort three exhibited a deceleration in rate of responding.

Student performance on skills trained to a fluency standard offer preliminary evidence that student gains maintained after the period of no practice. In three out of four cases, skills trained to an accuracy standard suggested lower student performance over time. In some cases the decrease in student performance was minimal; however, other student performances dropped sharply, as much as 30 percentage points, following a period of no practice. The importance of setting a fluency standard is becoming clearer given typical curriculum experiences currently in practice: students receive instruction in lesson X, then receive instruction in lesson Y but not X, and finally the students are tested on both lessons in the form of a unit test or comprehensive exam. Student performance data from cohort three suggest that raising student responding to fluent levels may serve as a proactive teaching strategy to ensure satisfactory work during later exercises.

Taken together, the results of the three cohorts add to the limited literature base supporting the retention-endurance-application-performance-standards model or REAPS (Haughton, 1972). As noted by Haughton (1972), specific rates of responding would make similar rates more likely after periods of no practice. This study demonstrated the impact of the “R” in REAPS (i.e., retention) in the analysis of the retention cohort. The endurance component of REAPS or the “E” was illustrated in the Endurance cohort by evaluating student mean intervals engaged for each rate of responding. Haughton’s Application criteria (“A”) was manifest in the Generalization cohort of this study.
Students who achieved fluent rates of responding also scored higher on untrained math problems.

The basic assumption underscored by REAPS and the present study is that successful learning is enhanced if precipitated by fluent rates of responding. The present study also adds to the collection of empirical analyses (e.g., Kelly, 1996, Miller & Heward, 1992, Skinner, 1998) reviewing student failure as not so much a failure to learn but a failure to respond at an adequate rate.

In addition, the results of the present study offer some preliminary evidence that practices frequently criticized as overly dependent on a drill and practice method serve as useful instructional tools in today’s classrooms. Further, the implication of acquiring skill levels that enable “accurate and consistent solving of computational problems” (Arizona State Department of Education) was shown to be not as advantageous to overall student performances (e.g., less time on task, lower accuracy scores, lower rates of responding). The present study, however, is not without its limitations or concerns. These limitations will be presented next.

Limitations of the Present Study

A general limitation of this study includes the confound that student whom were trained to fluency received more opportunities to respond as a function of re-teaching (e.g., slow responses during fluency training resulted in re-teaching whereas slow responses during accuracy training did not). Consequently, a treatment confound exists for all 12 subjects in the study. A second general limitation was that consideration of instructional match (antecedent and consequent factors present in the classroom) was not
addressed for any cohort. Any consideration of strategies to remediate math deficits must take into consideration the antecedent and consequence (e.g., instruction, modeling, opportunities to respond, reinforcement schedules) conditions within the setting.

A specific methodological limitation of the present study was the length of the experimental phases for each student. By having equal phase lengths for the final phase of the endurance and generalization cohorts, one can not rule out the possibility that a variable unrelated to the study exerted uniform effects over student performance. Evidence for potential uniform influence may be interpreted for each student's performance following the training phase; when returning to the classroom, students show sharp declines in rate of responding in at least one skill. Possible reasons for the change in rate of responding is the re-introduction to the classroom where availability of distractible stimuli are greater than in the preceding training phase. Interestingly, only the skills trained to a fluency standard returned to training or near training levels of responding following 1 to 3 sessions in the classroom. Similar methodological concerns included the retention cohort's student samples on pre-treatment probes. Student performances were restricted to three probe sessions to reduce the likelihood of practice effects or familiarity. An alternative to restricting the range of student exposure might include increasing the number of available stimulus probes.

A final limitation of the study has to do with the degree of math deficits presented by the students in the study. Pre-study math averages and teacher endorsements indicated that each student was experiencing problems with their present second-grade curriculum. It is possible however, that variables including attention to task, motivation, and
reinforcement histories influenced pre-study math averages. Future studies will need to strengthen the data included herein by modifying screening procedures to assess student performances prior to the study.

Implications of the Present Study

The data presented in each cohort demonstrated that training skills to fluency standards resulted in gains of student endurance, rate of responding and student accuracy on basic math skills. The results of this study must be interpreted cautiously as the format of the contribute to the literature on precision teaching by adding data to the systematic approach of teaching math concepts. Further, with the growing referral rate of student who present attention difficulties, this study underscored and supported the need for additional research on the relationship between rate of responding and student endurance.

A second implication of the present study was if program specific rates of responding, students will match our expectations. Further, if we program high rates of responding, the student will match that expectation even under novel conditions. Correspondingly, if we program lower rates of responding, students will demonstrate minimal gains in rate and student endurance.

In sum, this study provides preliminary support for the benefit of training students to a fluency standard over an accuracy standard. The data presented also support a growing literature addressing methods of remediating math deficits in school-age children. Direct and systematic replication of the methodology is needed before conclusions regarding efficacy may be drawn.
REFERENCES


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APPENDIX A: OBSERVATION FORM

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APPENDIX B: MATH FACTS GRID

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FLUENT FACTS – SUBTRACTION: ________________________________

FLUENT FACTS – TIME: (HOUR) ________________________________
The families arrived at the park at 9:00.

Write the time each family left the park.

1. Ann’s family stayed for 1 hour and 30 minutes.

2. John’s family stayed for 2 hours.

3. Tim’s family stayed for 4 hours.

4. Cindy’s family stayed for 3 hours.

5. Joan’s family stayed for 1 hour.
APPENDIX E. ANALOG CLOCK FACES.

Write the times.

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
APPENDIX F. CONSULTANT INTERVENTION STEPS

In-Class Probes
1. arrive to class prior to math lessons
2. provide student with probe
3. provide instructions
4. prompt student to begin
5. begin stopwatch
6. record whole interval observations
7. mark photocopy of probe at minute intervals
8. prompt student to stop working
9. collect student probe

Exit Probes
1. arrive to class prior to math lesson
2. escort student to library
3. sit knee to knee
4. present first flashcard
5. begin stopwatch
6. place cards met with fluent responses on table
7. recite nonfluent attempts and provide answer
8. insert missed fact after next flashcard
9. prompt student to recite re-presented fact and answer
10. present student with exit probe
11. provide instructions
12. begin stopwatch
13. mark copy of probe at minute intervals
14. prompt student to stop working
15. collect probe
16. repeat steps 3-15
17. escort student back to classroom

Accuracy Training
1. arrive to class prior to math lessons
2. escort student to library
3. provide student with probe
4. provide instructions
5. prompt student to begin
6. begin stopwatch
7. record whole interval observations
8. mark copy of probe at minute intervals
9. prompt student to stop working
10. collect probe
11. escort student back to class
Dear parents,

My name is Bruce P. Mortenson and I am the school psychologist at Kyrene de la Colina. I am in the process of completing my dissertation and I would like to invite your child _____________ to participate in a Math Facts Program. The Math Facts Program is a chance for your child to receive additional direct instruction in the basic math facts (e.g., addition, subtraction and telling time skills). My goal is to raise each child’s skill level to what is considered “fluent.” Fluent skills are automatic skills. For adults, we do not have to think about how we write our names or add basic facts; hence, we are fluent and these skills require very little cognitive energy. By being able to function at fluent levels, we can allocate our concentration to more complex activities (e.g., performing tougher algorithms). If our students are laboring over basic facts, (solving takes longer than 1-2 seconds), then their success in later skills may be acquired at a slower pace. However if you are answering math questions at a rate of 50 per minute then later math processes (e.g., multiplication) are more easily mastered.

The Math Facts Program is designed to help students increase their rates of functioning. The program will consist of using skill-builder worksheets (not unlike the MAD MINUTE worksheets), flashcards, and a timer. Daily sessions will be conducted in the back of your child’s classroom or at a table in the library for a period of ten to twenty minutes. Sessions will be conducted at a time that minimizes any disruption to the classroom routine and would not take the place of the curriculum or replace special activities.
If you would like your child to participate in the Math Fact's program please sign the back of this form and return it to either your child's teacher or my office. I would be happy to answer any questions that you have or talk to you further about the benefits of fluent performance and student success.

Thank you for your time.

Sincerely,

Bruce P. Mortenson
783-2685

I give permission for my student ________________________________ to participate in the Math Facts Program at Kyrene de la Colina Elementary.

_________________________  ____________
Teacher's Signature          Date

I give permission for my child ________________________________ to participate in the Math Facts Program at Kyrene de la Colina Elementary.

_________________________  ____________
Parent's Signature          Date

Please return this form to Mr. Mortenson
VITA

Bruce P. Mortenson is originally from Woodhaven New York. After dropping out of Franklin K. Lane High School in 1984, Bruce earned his General Equivalency Diploma while simultaneously attending a community college in the Catskills region of New York State. With renewed appreciation for higher education, Bruce proceeded to earn one bachelor’s degree, two master’s degrees, one Psychology Specialist Degree and finally the degree of Doctor of Philosophy from Louisiana State University. Bruce is currently a member of the Towson University faculty in the department of psychology.
DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: Bruce Pierce Mortenson
Major Field: Psychology

Title of Dissertation: A Behavior Analytic Approach to Developing Mathematic Skills in Second Graders: A Precision Teaching Intervention

EXAMINING COMMITTEE:

Date of Examination: December 1, 1999

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