Three Essays on Political Economy and International Trade.

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THREE ESSAYS ON POLITICAL ECONOMY AND INTERNATIONAL TRADE

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in
The Department of Economics

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ABSTRACT

In the standard common agency model of politics, the interest groups always lobby a single policy-making entity for policy favor. To deal with this unreality, I step in some issues about the trade policy making that is entangled with the multiplicity of public decision-makers. My study cooperates multi-agent and the common agency model to analyze the trade policy making under the political system that is controlled by a number of lawmakers. The analysis points out that the committee of symmetric lawmakers behaves like a single government but the equilibrium policy is not efficient. Because the presence of multiple players on the both sides, lobbies and lawmakers, creates the strategic externalities among lawmakers, it is impossible for all the players to achieve the optimum of their sum bliss. Moreover, focusing on the phenomena of the bipartisan corruption in the United States, I examine the fund-raising race between two parties in the bipartisan political system. If collecting political money implies that the party sells its service to the rent seekers, the party’s popularity will fade as its fund raising activity is more vigorous than its opponent’s is. The interaction between two political parties is modeled as a differential game. The results show that the subgame perfect equilibrium considerably lifts political money collecting activity compared with time consistent. This partially explains why the political parties ignore the public’s criticism on their soft money collecting activities and engage in an ever-escalating fund raising race. Finally, I apply the common agency model and Nash bargaining process to analyze the immigration policy of a small country. Our results show that, under incomplete political economy, the political equilibrium is to set up quantitative restrictions on the inflow of foreign labors. If the marginal cost of deterring illegal immigrants can be covered by the marginal benefit of allowing legal immigrants for entry, setting an optimal border control
to deter the illegal immigrants from entry and simultaneously allowing a certain level of legal foreign labor for entry is the political equilibrium. The bargaining power of government does not affect the immigration policy, but in the long run it does.
CHAPTER 1. INTRODUCTION

1.1 Influence-Driven Contribution Model

In recent years there have been considerable works devoted to developing a positive theory of policy-making, whose emphasis is on modeling the political process by which public policy is made and examining the policy that emerges as the equilibrium of this process. The research in this field provides a number of political economy approaches on its focus, such as majority voting (Mayer 1984), lobbying (Hillman 1982 and 1988) and electoral competition (Magee, Brock and Young 1989). The modern democracies have two noticeable features: the political campaign contributions serve an important role in enhancing the politicians' chance of being elected or re-elected, and the activities of rent seeking have a prominent part for inducing the government to create market inefficiencies and distortions. However, most of the political economy approaches can not link satisfactorily the motive for campaign contribution to the activities of rent seeking. An exception is the influence-drive contribution model. It is beyond the approach that individual contribution is to aim on electoral outcome, while argues that political contributions are designed to influence the choice of policy than to influence election outcome. Since the individual political contribution has marginal effect on the election outcomes of modern democracies, the model of influence-drive contribution has more solid theoretic ground than the others do in building up a positive theory of policy making.

In the particularly appealing and useful model of Grossman and Helpman (1994), they portray a policy maker, who takes bid on trade policy from organized interested group. The relationship between government and lobby groups is modeled as a common agency of politics. The basic idea of this model is based on the observation that in the
modern democracies the opportunistic politicians have nothing to sell but their policies, and meanwhile a politician's professional and personal success is often tied to financial contribution.

In my dissertation, I engage in three works based on the structure and ideas of Grossman and Helpman (1994). First, I extend their structure of common agency model to deal the multi-principal, multi-agent problem that arises in the political economy of trade policy. Second, I expand their idea, which the politicians' success are tied to financial contribution under the modern democracy, to expose the phenomenon that the political parties ignore the fierce public criticism of collecting soft money and still chase it. In the final part, I directly apply their structure to analyze the issues that are concerning the immigration policy of a small country.

1.2 An Extension of Common Agency Model to a Multi-Agent, Multi-Principal Problem

The common agency model of politics, as developed by Grossman and Helpman (1994), produces some keen insights in the topic of special-interest group politics. It is well suited to analyze the structure of economic policy across a set of industries and to examine the choice between various policy instruments (Dixit 1996, Dixit, Grossman and Helpman 1998, and Aidt 1998). In these models, the interest groups always lobby a single policy-making entity for policy favors. However, in reality, there hardly exists such a simple political system. The modern democracies are often involved legislative activities, which are related to the allocation of policy jurisdiction across legislators serving as ministers or committee chairs; in other worlds, the public policies are made by a number of public decision-makers.
In the second chapter "Lobby and Policy Decision-Makers: An Application to Trade Policies", I analyze some issues about the trade-policy-makings that are entangled with the multiplicity of public decision-makers. Based on some recent works on multiple agents (McAfee and Schwart 1994, Segal 1999, and Prat and Rustichini 1999), the study is cooperated multi-agent with common agency to analyze trade policy-making under the political system that is controlled by a number of powerful legislators or ministers. The political process is featured by a sequence of games in which each lobby is connected with a number of public decision-makers. My results show that the committee of symmetric lawmakers behaves like a single political entity but the equilibrium policies are not efficient. Because the presence of multiple players on the both sides (lobbies and lawmakers) creates a strategic externality, it is impossible for all the players to achieve the optimum of their sum bliss.

1.3 Modeling the Fund Raising Race between Two Competing Political Parties

To collect political money to finance the electoral expenditure, the incumbent politician, portrayed as in Grossman and Helpman (1994), takes the strategy, "seeking rents by creating rents". In fact, no matter which party is in rule, it seems unavoidable that the politicians from political party collect soft money to finance campaigns. However, the interest groups may influence the government’s policy making via the political contribution, and the corruption emerges in the interactions between politicians and special interests. Recently, the general public has fiercely criticized that the soft money is a main access for interest groups to corrupting the politicians from the Republican and Democrat. But, in the bipartisan system of the United States of America, the both parties ignore the criticisms and compete the soft money to finance the campaigns like an arm race. The political parties fund raising race have been the public
issue for a decade, but it is rare for the current economists to investigate the pervasive bipartisan corruption in a democracy, which the popularity and political contribution have impacts on the politicians' chance of being elected or re-elected.

In the third chapter "The Dynamics of Political Fund Raising Race: A Differential Game", I take a close look at the phenomenon that two political parties engage in fund raising race in spite of the fact, which collecting political moneys hurts political parties' popularity. The basic model we adopt is that of Feichtinger and Wirl (1994) in which the incumbent party has to trade off between popularity and political money collection. In my model, if a party's fund-raising activity is more vigorous than its opponent is, its popularity gets damaged. The intertemporal interactions between the two competing political parties are modeled as a differential game. My main conclusion is that: (a) If the competing parties are very similar to each other, they trash their popularity, and (b) the fund raising activity is considerably higher under subgame perfect equilibrium than under time consistent. These provide a partial explanation of why the political parties ignore the fierce public criticism of their soft money collections and continue to involve themselves in fund-raising race.

1.4 An Application of Common Agency Model to Immigration Policy

One often observes a host country permitting some legal entries of foreign labors and simultaneously deterring the illegal entries of foreign labors. This observation gives rise to two related puzzles. **Puzzle 1:** An optimal policy for a small country concerning labor mobility is to let foreign labor in until the marginal labor product is driven down to the foreign wage. That implies that all foreign labor should be legally admitted for a national optimum. **Puzzle 2:** If for some reason illegal entry should be deterred, why allow for legal entry of foreign labors. The existing models cannot account for the
simultaneous existence of legal and illegal immigrants and hence cannot answer these
two puzzles. They either do not distinguish two types of labor or assume that legal labor
is non-existent or outside the model (for example, Ethier 1986, Bond and Chen 1987, and
Djajic 1997).

In the final chapter "Immigration, Border Control and the Relative Bargaining
Power of Government under Lobbying Process", I incorporate the common agency mode
of Grossman and Helpman (1994) into the framework of Ethier (1986) to explain the
puzzles mentioned in the previous paragraph. Since the model's basic setup about the
political economy is based on the framework of Grossman and Helpman (1994), it
inherits an extreme characteristic: the government has monopoly power in relation to
multi-lobby. To relieve this uncomfortable feature, I further expand my one sector setting
to multi-sector and portray the government-lobby negotiation as a Nash bargaining
process. My results show that: If the production factors are not all presented by the
lobbies, the political equilibrium is to set up a quantitative restriction on the inflow of
foreign labors. If the marginal cost of deterring illegal immigrants can be covered by the
marginal benefit of allowing legal immigrant for entry, deterring illegal entry of foreign
labors to allow a certain level of foreign labors for legal entry is the host country's
political equilibrium. Furthermore, I show that in the short run the relative bargaining
power of the government only affects the distribution of the surplus, derived from the
political process, between the government and the lobby; however, it affects the
immigration policy of host country in the long run.
CHAPTER 2. LOBBYING AND POLICY DECISION-MAKERS:
AN APPLICATION TO TRADE POLICIES

2.1 Introduction

In the standard common-agency model of politics, the interest groups always lobby a single policy-making entity for policy favors. However, in reality, there hardly exists such a simple political system. In modern democracies, they involve legislative activities, which are often related to the allocation of policy jurisdiction across legislators serving as ministers or committee chairs; in other words, the policy decisions are made by a number of public decision-makers. There are scarce works that try to integrate multi-decision-makers into the lobbying process.

An Exception is the work Helpman and Persson (1998). However, their assumption that each public decision-maker of the political system is associated with a particular interest group, harks back to the original common agency model, and rendering it unable to analyze multi-principal, multi-agent interactions. In this paper, we extend some recent work on multiple agents (McAfee and Schwartz 1994, Segal 1999, Prat and Rustichini 1999) to the common agency setting and to analyze policy-making under the political systems that are controlled by a number of powerful legislators or ministers.

In our model, we assume that some of the groups in society are organized and able to make implicit offers to influence the trade policies. The decisions of public policies are brought up under a US-style congressional system or a European-style parliamentary system by combining every individual policy proposal of lawmakers into the final one.

Under the collectivized decision-making process, for every public issue, there is a weight corresponding to the policy proposal of each individual lawmaker. The weight is
exogenous and reflects the seniority of individual lawmaker in political system. However, to simplify our analysis, we assume that the lawmakers equally share the power of deciding the public policies.

Given the specific rules of political system decision-making, the political contributions are made strategically to influence the design of individual lawmaker's policy proposals and the collective policy decisions in the political system. Moreover, we assume that every individual lawmaker is concerned with the weighed sum of the aggregate social welfare and the total contribution he receives. Hence there are externalities among individual lawmakers because each lawmaker's utility depends not only on his own policy proposal but also on the other lawmakers' proposal. In this process, a set of lawmakers makes the decisions that affect the payoffs of a set of interest groups, while the lobbies can influence the decision of lawmakers by means of monetary inducements.

In what follows, I assume that a number of lawmakers collectively decide the trade policy in a small country. The economy is assumed to have the Ricardo-Viner structure. In Section 2.2, we portray an economy that the trade policy is a result of linear convex combination of the lawmakers' policy proposals. We consider that an interest group lobbies a trade policy committee for favors. We model the relationship between the committee of trade policies and the interest group as a game played through agents. Specifically, this game includes a principal and a set of agents. Under the assumption that the lawmakers are symmetric, we confirm that each lawmaker who holds passive belief will behave as if it were maximizing a social welfare that weighs the bliss of interest group more heavily. Moreover, we also show that because of the strategic externalities,
the domestic prices depart from free trade more under the equilibrium policy proposal profiles than under the efficient policy proposal profiles.

In Section 2.3, we introduce multi-agent into our model. Since each lawmaker holds passive beliefs, she or he is isolated as a common agency in the lobbying process. Thus, the entangled interactions between multi-principal and multi-agent are resolved by imposing constraint on the agents' beliefs. It is different from the work of Grossman and Helpman (1996), in which they imposed some specific beliefs to the principals. The results of our analysis reinforce the result of Grossman and Helpman (1994).

Overall, the analysis points out that the committee of symmetric lawmakers behaves like a single government but the equilibrium policies are not efficient. Because the presence of multiple players on the both sides creates a strategic externality, it is impossible for all the players to achieve the optimal of the sum bliss of interest groups and lawmakers. Moreover, the strategic externality also makes the interior solution hard to find.

2.2 The Model

Consider a small economy that the populations size N. In this economy each individual has a utility function

\[ u(h) = h_0 + \sum_{i=1}^{n} u_i(h_i) \] (2.1)

where \( h_i \) is the consumption of product \( i \) and \( u_i \) is an increase concave function. He spends his income to consume \( n+1 \) kind of goods. Good 0 is produced with labor alone and serve as numeraire, whose price is equal to unity. Each non-numeraire sector \( i \) uses labor and the sector specific capital to produce good \( i \). With the wage rate fixed at one,
the aggregate reward to the specific factor used in produce good \(i\) depends on the domestic price of that good, \(p_i\). This reward is denoted by \(\pi_i(p_i)\).

The consumer surplus from good \(i\) for an individual is \(S_i = u_i(d_i(p_i)) - p_i d_i\), where \(d_i(p_i)\) is an individual's demand for good \(i\). The net import demand function is denoted by \(m_i = N d_i(p_i) - y_i(p_i)\), where \(y_i(p_i)\) is the domestic output of good \(i\). The net revenue from sector \(i\)'s tariffs or subsidies that is expressed on a per-capita basis, is given by \(r_i(p_i) = (p_i - p_i^*)[d_i(p_i) - y_i(p_i)]/N\) where \(p_i^*\) is the world price for goods \(i\).

Consequently, we can define ad valorem trade taxes or subsidies to be \(t_i = (p_i - p_i^*)/p_i^*\).

Moreover, suppose that the government redistributes the tariff revenue equally to every individual in a lump-sum fashion.

We can express the joint welfare, gross of the contributions, as

\[
W_i = q_i + \pi_i + \alpha_i N \left( \sum_{i=1}^{n} S_i(p_i) + \sum_{i=1}^{n} r_i(p_i) \right)
\]

where \(p\) is a vector, \((p_1, p_2, \ldots, p_n)\). The first term on the right hand side \(q_i\) is the total labor supply of the specific input used in industry \(i\), and the last term represent their share in tariff rebates and in consumer surplus, in which \(\alpha_i\) is the fraction of the population that own factor \(i\). The aggregate gross welfare \(W\) equals aggregate income plus trade tax revenues and total consumer surplus; that is,

\[
W(p) = \sum_{i=1}^{n} q_i + \sum_{i=1}^{n} \pi_i(p) + N \left( \sum_{i=1}^{n} S_i(p_i) + \sum_{i=1}^{n} r_i(p_i) \right)
\]

Now, consider an economy in which the professional and personal success of a politician under the institutions of representative democracy is tied to the financial contributions of some special interests. We assume that in some exogenous set of sectors,
denoted \( L \), the specific-factor owners have overcome the free-rider problems and been able to organize themselves into lobby groups. More specifically, the set of interest groups is denoted as \( L = \{1, 2, \ldots, I\} \).

Moreover, we assume that the interest groups can make deals with a set of lawmakers, denoted \( Z = \{1, 2, \ldots, Z\} \). To simplify our analysis, we assume that the domestic price of product in sector \( i \) is decided by a linear combination of the individual lawmakers' policy proposals. In this simple linear process, the lawmaker \( j \)'s tariff proposal about sector \( i \) is denoted by \( P_{ij} \in \Lambda_{ij} \) where \( \Lambda_{ij} \) is a continuous compact subset of \( \mathbb{R} \), real number; that is \( \Lambda_{ij} = [P_{ij1}, P_{ij2}] \). Let the vector \( P_i = (P_{i1}, P_{i2}, \ldots, P_{iu}) \in \Lambda_{i1} \times \ldots \times \Lambda_{iu} \) denote the lawmakers' policy proposal profiles concerning the domestic price of sector \( i \)'s product. Let \( \Lambda_i = \prod_{j=1}^{J} \Lambda_{ij} \) and \( P = (P_1, \ldots, P_u) \). Moreover, let \( A' = \prod_{i=1}^{I} A_{ij} \). Let \( P_j = (P_{j1}, \ldots, P_{ju}) \in A' \) denote the lawmaker \( j \)'s policy proposal vector.

Next, we assume that each lawmaker \( j \) has equal power on the setup of domestic prices, and thus his tariff proposal for sector \( i \), \( P_{ij} \) is weighed by \( 1/Z \). Then, we can write the domestic price in sector \( i \) to be

\[
p_i = \frac{1}{Z} \sum_{j=1}^{Z} P_{ij}.
\]

The sector \( i \) lobby raises money from its members to influence the policy outcome \( p \). Its political contributions depend on the policy proposal vectors proposed by individual lawmakers. Unlike the standard common agency model, the interest groups in our model may lobby more than one lawmaker. Let \( C_i(P_{ij}, \ldots, P_{iu}) \) be the contribution schedule offered by lobby \( i \) to lawmaker \( j \). The joint welfare of lobby group \( i \)'s member can be expressed as
\[ V_i = W_i - \sum_{j=1}^{r} C_{ij}. \]  

(2.2)

Each lobby maximizes the total welfare of its member via their contribution schedules. Each lawmaker cares about the total level of political contributions and the well being of the general public. As in Grossman and Helpman 1994, we choose a linear form for the lawmaker \( j \)'s objective function, namely

\[ G_j = \sum_{i=1}^{r} C_{ij} + a_j W(P) \]  

(2.3)

where \( a_j \) is a parameter that represents the marginal rate of substitution between welfare and contribution.

The non-cooperative game takes place in two stages: first the lobbies simultaneously pick out their political contribution schedules, and each lobby \( i \) makes each lawmaker \( j \) an offer \( C_{ij} \) which is privately observed by the lawmaker. Lawmaker \( j \) observes his own contribution schedule vector \( C_j = \{C_{ij}\}_{i \in L} \) offered by the lobbies and forms beliefs \( \eta(C_j) \) about the contribution schedules made to the other lawmakers. In the second stage, the lawmakers propose policy proposals simultaneously.

We focus on the Perfect-Bayesian Equilibrium, in which each lawmaker sets a policy proposal vector to maximize his objective \( G_j \) contingent on given contribution schedules and his beliefs. Each lobby \( i \) sets up a set of contribution schedule vectors to maximize the joint welfare of its members, taking the other lobbies' schedules as given.

2.2.1 One Interest Group and Lawmakers

We start our analysis by studying the case in which there is a unique interest group \( g \) and \( z \) number of individual lawmakers \( z \). Here, the game is similar to the unobservable game of McAfee and Schwartz 1994. The lobbies offer political
contributions to the lawmakers simultaneously and secretly, and the lawmakers never learn the others' deals. Each lawmaker $j$'s object function is $C_{\text{t}} + a_j W(P)$ and the lobby group $g$'s joint welfare is $W_g - \sum_{j=1}^{m} C_{\text{t}}$.

In this two-stage game, we focus on the pure-strategy, perfect Bayesian-Nash equilibrium. When a lawmaker receives an off-equilibrium-path offer, arbitrary belief can be assign to the unobservable offers made to others. Then, the multiplicity of equilibria appears. To construct an accurate and complete prediction, we need to pin the belief on out-equilibrium-path on some certain domain. Following the suggestion of McAfee and Schwartz, we assume that the lawmakers interpret any unexpected offers as trembles and believe that they are uncorrelated. In other word, each lawmaker hold so called passive beliefs: when a lawmaker receives a political contribution different from what is expected in the candidate equilibrium, he believes that the other lawmakers face their equilibrium political contributions.

Consider the lobby $g$'s incentive to deviate from an equilibrium outcome $\{C_{\text{t}}\}_{g \in Z}, \{P_i\}_{i \in \{1, \ldots, n\}}$. Since the lobby group can offer zero contribution to any lawmaker whose policy proposal is free trade for all sectors, we can focus on the deviations in which all lawmakers accept their offers. Holding passive beliefs, each lawmaker $j$ makes policy proposal $P' = \{P_{i\neq j}, P_j\}$ and accepts political contribution $C_{\text{t}}$ if and only if $C_{\text{t}} + a_j W(P', \tilde{P}^{-j}) \geq a_j W(p^*, \tilde{P}^{-j})$ where $\tilde{P}^{-j}$ is the others' policy proposal vector. The lobby $g$'s optimal deviation should maximize the welfare of its member subject to these participation constraints:

$$\text{Max}_{P_{i\neq j}, \ldots, P_j} W_g(P) - \sum_{j=1}^{s} C_{\text{t}}(P')$$
subject to \( C_g + a_j \mathcal{W}(P^j, \tilde{P}^{-j}) \geq a_j \mathcal{W}(P^*, \tilde{P}^{-j}) \) for all \( j \in \mathbb{Z} \).

\( \{ \{ \tilde{C}_g \} \}_{j \in \mathbb{Z}}, \{ \tilde{P}_i \}_{i \in \{1, ..., n \}} \) is an equilibrium outcome if and only if the lobby does not want to deviate from it; that is, it solves this program.

Given any \( P \), the lobby \( g \) will set its political contributions so that the cost of implementing this policy proposal is the minimum. Consequently all the participation constraints must bind. We can use the binding constraints to express the political contributions in the lawmakers' utility, and substitute them into the objective function. Since \( a_j \mathcal{W}(P^*, \tilde{P}^{-j}) \) is constant in this program, the policy proposal vector \( \{ \tilde{P}_i \}_{i \in \{1, ..., n \}} \) can be sustained in equilibrium if and only if the vector satisfies the following condition:

\[
\{ \tilde{P}_i \}_{i \in \{1, ..., n \}} \in \arg \max_{P \in \Lambda_{ij} \times \Lambda_a} W_g(P) + \sum_{j=1}^i a_j \mathcal{W}(P^j, \tilde{P}^{-j}).
\] (2.4)

Let \( \Omega \) denote the set in which every policy proposal profile satisfies (2.4) and is an interior solution. When \( \Lambda_{ij} \) is an interval and \( W_g(P) + \sum_{j=1}^i a_j \mathcal{W}(P^j, \tilde{P}^{-j}) \) is continuous in \((P^j, \tilde{P}^{-j}) \) and quasi-concave in \( P \), the set for (2.4) is not empty (see the proof in the Appendix 2 of Segal 1999). The program of (2.4) is equivalent to the following condition:

\[
\{ \tilde{P}_i \}_{i \in \{1, ..., n \}} \in \arg \max_{P \in \Lambda_{ij} \times \Lambda_a} W_g(P^j, \tilde{P}^{-j}) + a_j \mathcal{W}(P^j, \tilde{P}^{-j})
\]

for all \( j \in \mathbb{Z} \). As a result, the program (2.4) says that, in equilibrium, the lobby's political contribution will induce each lawmaker \( j \), who holds passive belief, to behave as if he were maximizing a social welfare function that weighs the interest group's welfare more comparing with those not so represented.
We can interpret the program (2.4) as each lawmaker $j$ puts weight $1 + a_j$ on sector $g$ and $a_j$ on the others. The results show that each lawmaker's optimizing behavior is similar to that of the single policy-maker in Grossman and Helpman (1994).

Now, we can portray the interior equilibrium trade policy supported by the differentiable contribution schedules. Given the differentiability and the fact that the equilibrium is in the interior, we have the first order conditions for (2.4):

$$\nabla W_g(P) + \sum_{j=1}^{I} a_j \nabla W(P', \bar{P}^{-j}) = 0 \quad (2.5)$$

where the operator $\nabla$ applied to a function denotes the gradient vector of the partial derivatives of the function with respect to the vector argument that appear as the subscript of the operator.

From Eq. (2.5), we can evaluate the effect of each lawmaker's marginal policy proposal change on the various groups' welfare by separately calculating $\nabla W_g(P)$ and $\nabla W(P', \bar{P}^{-j})$ and combining them. For interest group $g$ we have

$$\frac{\partial W}{\partial P_g} = \frac{1}{z} [(1 - \alpha_g) y_g(p_g) + \alpha_g (p_g - p_g^*) m_g^*(p_g)] \quad (2.6)$$

where $m_g^*(p_g) \equiv Nd_g(p_g) - y_g(p_g)$ is the net import demand function. Eq. (2.6) states that an increase in the lawmaker $j$'s domestic price proposal of sector $g$ above the free trade level can increase the interest group $g$'s welfare. However, each lawmaker $j$'s marginal policy proposal only has a partial effect on the domestic price of sector $g$ that depends on his power in political system, and hence the marginal effect is discounted by $1/z$. Moreover, for the aggregate welfare, we have

$$\frac{\partial W}{\partial P_g} = \frac{1}{z} [(p_g - p_g^*) m_g^*(p_g)]. \quad (2.7)$$
Eq. (2.7) shows that the marginal deadweight loss, which is caused by each lawmaker’s policy proposal change, increases as the economy is more distorted.

Plugging (2.6) and (2.7) into (2.5), we have:

\[(1 - \alpha_y) y_i(p_x) + \alpha_y (p_x - \hat{p}_x) m'_y(p_x) + a_i (p_x - \hat{p}_x) m'_x(p_x) = 0. \quad (2.8a)\]

For \(i \neq g\), we have

\[-a_i y_i(p_i) + \alpha_i (p_i - \hat{p}_i) m'_i(p_i) + a_i ((p_i - \hat{p}_i) m'_i(p_i) = 0. \quad (2.8b)\]

Eqs. (2.8) tell us that if \(a_j\) is not identical for all \(j\), there does not exist a pure strategy interior Bayesian Nash equilibrium. However, if \(a_j\) is equal to a constant \(a\) for all \(j \in Z\), the first order condition of (2.4) with respect to \(P_g\) will be identical. Therefore, we assume that every lawmaker is symmetrical for the rest of our analysis.

Since the lawmakers have the same objective function, the policies of the political system are the same as that of a single policy-making entity. Furthermore, in equilibrium, the trade policy satisfies

\[
\frac{\hat{\gamma}_i}{1 + \hat{\gamma}_i} = \frac{I_i - \hat{\alpha}_i \hat{\gamma}_i}{\alpha_i + \hat{\gamma}_i} \quad (2.9)
\]

where if \(i = g\), \(I_i = 1\); otherwise, \(I_i = 0\). In (9) \(\hat{\gamma}_i = y_i(\hat{p}_i)/m_i(\hat{p}_i)\) the equilibrium ratio of domestic output to import and \(\hat{\gamma}_i = m'_i(\hat{p}_i) \hat{p}_i/m_i(\hat{p}_i)\) is the elasticity of import demand or export supply. Eq. (2.9) is identical to the equation for the equilibrium tariff rate found in Proposition 1 of Grossman and Helpman (1994).

Dixit et al. (1996) argue that if the government’s objective weighs positively the well being of all members in society, then the efficiency for the government and lobbies can be achieved under the lobbying process. However, in this secret game, the efficiency that the relevant parties or players succeed in maximizing their joint surplus is not
accomplished under the multi-lawmaker political process. Adding up the objectives of lawmakers and interest group, we have the joint surplus of the lawmakers and interest groups, \( W_g(P) + \sum_{j=1}^{i} aW(P) \). Let \( \tilde{A} \) denote the set of lawmakers' policy proposal maximizing the total surplus of identical lawmakers and interest group:

\[
\tilde{A} = \arg \max_{P \in \tilde{H}_1 \times \tilde{H}_2} W_g(P) + \sum_{j=1}^{i} aW(P).
\]  

Moreover, let \( \Psi \) be the set in which every policy proposal profile satisfies (10) and is an interior solution. Let \( Q = \{ (1/z \sum_{j=1}^{i} P_j, \ldots, 1/z \sum_{j=1}^{i} P_j), P \in \Psi \} \) denote the set of interior efficient domestic prices.

The first-order condition of (2.10) can be express as: for \( i = g \)

\[
(1-\alpha_z) y_z(p_z) + \alpha_z(p_z - p_z^*) m_z'(p_z) + za(p_z - p_z^*) m_z'(p_z) = 0
\]

and for \( i \neq g \)

\[
-\alpha_i y_i(p_i) + \alpha_i (p_i - p_i^*) m_i'(p_i) + za(p_i - p_i^*) m_i'(p_i) = 0.
\]

Then, we can calculate the interior efficient trade policy

\[
\frac{t_i^o}{1 + t_i^o} = \frac{I_i - \alpha_i s_i^o}{za + \alpha_i e_i^o}
\]  

where \((s_i^o, e_i^o)\) has the same definition as \((s, e)\).

Since each lawmaker's objective function has a weight on the aggregate welfare of the general public, his policy proposal change causes externalities on other lawmakers' objective function. Moreover, the game is secretly played by players in the model, and hence the lobby can not publicly commit to her bilateral offers. As a result, the externalities will be presented at an efficient policy proposal profile, and the lobby \( g \) that
can not commit to compensate lawmakers for these externalities has an incentive to
deviate from this policy proposal profile.

Each lawmaker's policy proposal has partial impact on the trade policy, which
affects every sector in this economy. The externalities among lawmakers are presented if
each lawmaker puts forward a policy proposal, which has the same distortion direction as
the others' proposals do. Specifically, for the sector \( g \) each lawmaker proposes a positive
tax proposal (tariff) for the import, while for sector \( i \neq g \) each lawmaker proposes a
negative tax (subsidy) for the import. In other world, the externalities among lawmakers
are caused by the lawmakers' policy proposals, which have multi-dimensional. In Segal
(1999) every agent's trade profile is one-dimensional; however, in our model the policy
profiles of lawmakers are a multi-dimensional. Therefore, giving a little modification of
the term in Segal (1999) seems necessary. Let \( \Lambda^*_y = [\bar{p}_y, \check{p}_y] \) for \( i \neq g \) and \( \Lambda^*_y = [p_i^*, \check{p}_y] \)
for \( i = g \). From Eqs. (2.9) and (2.11), the lawmaker \( j \)'s equilibrium trade policy proposal
\( \hat{P}_j \) and efficient trade policy proposal \( \check{P}_j \) for sector \( i \) belong to \( \Lambda^*_y \).

Definition. The externality on efficient policy proposal profile are negative in absolute
value if for all \( P \in \Psi \) and each lawmaker \( j \), \( aW(P^j, P^{\cdot j}) \) is non-increasing in the
absolute value of \( P_k - p^*_i \in \Lambda^*_k - p^*_i \) for all \( k \neq j \) and all \( i \in \{1, \ldots, n\} \).

Since we assume the country is small enough not to affect the world price,
increasing the distortion of domestic price decreases the aggregate welfare no matter it is
tariff or subsidy. Consequently, that the externalities on efficient policy proposal profile
are negative in absolute value can be applied to our model. Let denote the set of interior
equilibrium domestic prices. Let \( E = \{(1/z \sum_{j=1}^{r} P_{j}, ..., 1/z \sum_{j=1}^{r} P_{j}) ; P \in \Omega \} \) denote the set of interior equilibrium domestic prices.

One would like to speculate that with the negative externalities on efficient policy proposal profile, the equilibrium tariffs and subsidies are higher than the efficient levels. Of course, this could be shown by comparing the first order conditions of (2.4) and (2.10). However, the set \( E \) and \( Q \) may not be single-point set. Suggested by Topkis 1998, using the strong set order to do the comparison between two sets may be more natural than using the first-order conditions. The definition of strong set order is that: namely, for two sets \( A \) and \( B \), we will say that \( A \leq B \) if whenever \( c \in A, d \in B \) and \( c \leq d \), we must also have \( c \in B \) and \( d \in A \). Let \(|E|\) denote the set that is constituted by the points, whose every coordinate is the absolute value of the coordinate of the points from \( E \).

**Proposition 1.** If each lawmaker is symmetric, \(|E - p^*| \cup |Q - p^*| \geq |Q - p^*|\).

**Proof.** Suppose that \(|\bar{p} - p^*| \in |Q - p^*| \) and \(|\bar{p} - p^*| \in |E - p^*| \cup |Q - p^*|\), and that \(((\bar{p}_{g}, ..., \bar{p}_{g})) \in \Omega\), with \(|\bar{p}_{i} - p_i^*| = |1/z \sum_{j=1}^{r} \bar{p}_{j} - p_i^*| \leq |\bar{p}_{i} - p_i^*|\). Since \(|\bar{p} - p^*| \in |E - p^*| \cup |Q - p^*|\) trivially, the strong set order comparison only require that \(|\bar{p} - p^*| \in |Q - p^*|\).

Since \( \bar{p}_{q}, \bar{p}_{q} \) belong to \( \Lambda_{q} = [P_{q}, P_{q}^*] \) for \( i \neq g \) and \( \bar{p}_{q}, \bar{p}_{q} \) belong to \( \Lambda_{q} = [P_{q}^*, P_{q}] \) for \( i = g \), there exists \( \bar{p}_{i} \in \Lambda_{i}^* \times \cdots \times \Lambda_{i}^* \) such that \(|\bar{p}_{i} - p_i^*| = |1/z \sum_{j=1}^{r} \bar{p}_{i} - p_i^*| \geq |\bar{p}_{i} - p_i^*|\). Then, we can write
The first inequality obtains from the equilibrium condition (2.4), the second from the fact that the externalities on efficient policy proposal profile are negative in absolute value, and the last equality is from the assumption that the domestic price is equal to a linear combination of lawmakers' policy proposals. Consequently, \( \tilde{p} \in \bar{Q} \), which implies

\[
\left| \tilde{p} - p^* \right| \leq \left| Q - p^* \right|.
\]

Proposition 1 tells us the direction of distortion that the domestic prices are more away from free trade when the equilibrium policy proposal profiles are deployed than when the efficient policy proposal profiles are unfolded. In Grossman and Helpman 1994, the equilibrium supported by truthful strategies is efficient in viewing the joint surplus of all active players in their model. Since the government is the exclusive agent, there is no externality for its policy.

However, the equilibrium supported by the passive beliefs is not efficient, and the deal between the interest group and each lawmaker is secret. The inefficiencies emerge since the interest group cannot publicly commit its bilateral offers and also is not able to compensate the lawmakers for the externalities imposed on them. Given the efficient policy proposal profiles, the interest group has incentives to renegotiate with some lawmakers once the others accept their efficient policy proposal profiles. The equilibrium policy proposal profiles are the outcome after the interest group deliberates the profitability of multi-agent deviations.
The deals between interest group and lawmakers are secretly initiated, and each lawmaker holds passive belief when receives an off-equilibrium-path offer, so there is no interaction among lawmakers. The interest group can take advantage of the lawmakers’ failure to coordinate their policy profiles, and capture all the surplus of lobbying process. The political money contributed by interest group is no more than what keeps lawmakers’ utilities in the status quo, in which the lawmakers bear no political pressure from the interest group. Each inactive lawmaker can not get compensated about the externalities caused by the secret bilateral trades between interest group and the other lawmakers, and meanwhile the externalities produce inefficiencies.

In next subsection, we analyze the situation in which the lawmakers can coordinate their policy proposals to forming while facing the interest group. The analysis can help us illustrate how the lawmakers’ internalization of externalities can eliminate the inefficiency.

2.2.2 The Efficiency and the Coordination among Lawmakers

Since each lawmaker’s policy proposal has impact on other lawmakers’ utilities, the externalities among lawmakers arise. Moreover the lawmakers’ passive beliefs about off-equilibrium-path offer and secret bilateral trades between interest group and lawmakers enable the interest group to extract the surplus of lobbying process without compensating lawmakers for the externalities. Pondering the adverse position dealing with interest group and the externalities, the lawmakers collude with each other to design the trade policy.

Facing the lawmakers’ collusion, the interest group \( g \) donates political contribution contingent on the policy vector proposed by the coalition of lawmakers. Let
C(p) be the contribution schedule offered by interest group j. The joint welfare of lobby group g’s member can be express as

$$V_t = W_t - C(p).$$

Here, we assume that the coalition of lawmakers equally distributes the political contribution to its members. The lawmaker j’s objective function is

$$G_j = \frac{1}{z} C(p) + aW(p).$$

The coalition of lawmakers has the objective function

$$G = C(p) + zaW(p).$$

The coalition of lawmakers accepts political contribution C if and only if

$$C + zaW(p) \geq zaW(p^*).$$

The lobby g’s optimal inducement to the government’s decision about trade policy should maximize the welfare of its member subject to the participation constraint:

$$\max_p W_t(p) - C(p)$$

subject to $C + zaW(p) \geq zaW(p^*)$.

$(\hat{C}, \hat{p})$ is an equilibrium outcome if and only if it solve this program.

Since lobby g will set its political contribution so that the cost of implementing any trade policy is the minimum, the participation constraint must bind. Here, $zaW(p^*)$ is constant, so the trade policy vector $\hat{p}$ can be sustain in equilibrium if and only if the vector satisfies the following condition:

$$\hat{p} \in \arg \max_p W_t(p) + zaW(p).$$

The first order condition of interior solution for (2.12) is the same as that for (2.10). So, when the lawmakers can coordinate their policy proposals, each lawmaker’s welfare...
change, caused by the trade policy change, is compensated. The lawmakers' collusion
does not only enable the lawmakers to improve their welfare, but also nullify the
externalities produced by the secret bilateral trade between interest group and lawmakers.
After the lawmakers internalize their trade policy proposals, the interest group must
compensate every lawmaker's welfare change caused by the policy change. The interest
group can not impose externalities in the lawmakers via the secret deals, and hence the
efficiency comes out from the lobbying process.

2.3 Several Interest Groups and Lawmakers

We have so far examined the situation in which a single interest group lobbies a
number of symmetrical individual lawmakers. The simplification helps us avoid some
complications, which are involved several interest groups with opposing interests.
However, it misses something important in an economy in which a number of organized
interest groups design contributions to influence the lawmakers' choice about the trade
policy. Since there are more than one interest group in the economy, none of the interest
group possesses the privilege of exclusively dealing with the lawmakers. Thus the
interest groups may not extract the entire surplus from its political relationship with the
lawmakers. On the other hand, even though the deals between individual lawmaker the
interest groups are secreted, he can still make use of the rivalry among interest groups to
capture the surplus of lobbing process conditional on his belief of the other lawmakers'
deals. Also, the interactions among interest groups can evoke them to communicate with
each other. The equilibrium under the scenario of multi-principal, multi-agent must face
the test that is triggered by the lobbies' incentive to communicate among them with the
intention of arranging a stable, mutually preferable joint deviation.
To consider this realism, we assume that there are more than one interest group in 
this economy; that is, \( L = \{1, \ldots, l\} \), where \( l \geq 2 \). As in the previous subsection, the game is 
secretly played by the player in the model and the lawmakers' beliefs for out-of 
equilibrium paths are passive. Moreover, we relax the notion of truthfulness in Bernheim 
and Whinston 1986 and follow Prat and Rustichini 1999 to give a weaker condition: \( \{C_{ij}\} \) 
is weakly truthful relative to \( \tilde{P} \), if for every \( j \in Z \) and \( P \in A \),
\[
W_i(\tilde{P}) - \sum_{j=1}^{l} C_{ij}(\tilde{P}^j) \geq W_i(P) - \sum_{j=1}^{l} C_{ij}(P^j).
\]
This condition is similar to that of truthful contribution schedules in that the weakly 
truthful contribution schedules keep the offers on the out-of-equilibrium policy proposals 
high enough.

Given the passive beliefs, the lawmaker \( j \) proposes policy proposal \( P' \) and accepts 
the political contributions \( \sum_{j=1}^{l} \tilde{C}_{ij}(P^j) \) if and only if
\[
\sum_{i=1}^{l} \tilde{C}_{ij}(P^j) + aW(P^j, \tilde{P}^j) \geq \sum_{i=1}^{l} \tilde{C}_{ij}(P^j) + aW(P^j, \tilde{P}^j)
\] (2.13)
for any \( \tilde{P}^j \in A_j \times \ldots \times A_w \). If the set \( \{P^j, \{\tilde{C}_{ij}(P^j)\}_{i=1}^{l}\}_{j=Z} \) satisfies (2.13), it also satisfies 
the pairwise-proofness of McAfee and Schwartz 1994. Holding his belief that the offers 
made to others are the equilibrium ones, each lawmaker \( j \) chooses a policy proposal to 
maximize his objective function. As a result we find that lawmaker \( j \)'s policy proposal 
\( \tilde{P}^j \) is an interior solution in equilibrium if and only if the following condition is satisfied:
\[
\tilde{P}^j \in \arg \max_{P^j \in A_j \times \ldots \times A_w} \sum_{i=1}^{l} \tilde{C}_{ij}(P^j) + aW(P^j, \tilde{P}^j).
\] (2.14)
Under passive beliefs the lawmaker \( j \) maximizes of his objective function, and the first-
order condition of (2.14) is:
Here, \( \hat{P}^{-j} \) is taken as being constant in program (2.14).

An interest group \( k \) takes the contribution schedules \( \{\hat{C}_q\}_{q \in \mathcal{Z}} \) of all the other interest groups \( i \neq k \) as given. Let \( \{P^j, \hat{P}^{-j}\} \) be the policy proposal profile when all lawmakers play the equilibrium policy proposals but lawmaker \( j \) deviate to \( P' \). The interest group \( k \) knows that if it does not lobby, the lawmaker \( j \) will achieve his political welfare

\[
G_j^{*k} = \max_{P^j \in \mathcal{P} \setminus \hat{P}^{-j}} \left( \sum_{q \in \mathcal{Z}} \hat{C}_q(P^j) + aW(P^j, \hat{P}^{-j}) \right);
\]

that is, the lawmaker \( j \) will choose a policy proposal vector that maximizes his objective function disregarding the interest group \( k \)'s preference. Consequently, if the interest group \( k \) wishes to affect the policy outcome, it needs to offer a contribution schedule that provides the lawmakers with at least \( \sum_{j \in \mathcal{Z}} G_j^{*k} \). In other word, the interest group \( k \) can draw any policy proposal profile \( P \in \mathcal{P} \times \ldots \times \mathcal{P} \) from the lawmakers provided it promises a contribution schedule that satisfies

\[
\hat{C}_q(P^j) \geq G_j^{*k} - \left( \sum_{q \in \mathcal{Z}} \hat{C}_q(P^j) + aW(P^j, \hat{P}^{-j}) \right)
\]

to each lawmaker \( j \) for his policy proposal \( P' \). This is a standard participation constraint in the principal-agent problem. This must hold with equality because the interest group \( k \) does not want to give the lawmakers more than what is necessary to bring out a policy change. The interest group \( k \) will not choose \( \hat{P} \) unless

\[
\hat{P} \in \arg \max_{P \in \mathcal{P} \times \ldots \times \mathcal{P}} W_k(P) - \left( \sum_{j \in \mathcal{Z}} (G_j^{*k} - \left( \sum_{q \in \mathcal{Z}} \hat{C}_q(P^j) + aW(P^j, \hat{P}^{-j}) \right)) \right).
\]
Since $G_t^{k}$ can be treated as a constant in this program, the policy proposal $\tilde{P}$ must fulfill the first-order condition

$$\nabla W_k(P) + \sum_{j \in \mathbb{Z}} \left( \sum_{i \in k} \nabla C_{ij}(P^i) + \alpha \nabla W(P^j, \tilde{P}^{-j}) \right) = 0. \quad (2.17)$$

Combined together, (2.15) and (2.17) imply

$$\nabla W_k(P) = \sum_{j \in \mathbb{Z}} \nabla C_{ij}(P^i), \quad (2.18)$$

for all $k \in L$. In fact, (2.18) is equivalent to

$$\frac{\partial W_k}{\partial P_j} = \frac{\partial C_{ij}}{\partial P_j}$$

for all $k \in L$, where $i \in \{1, ..., m\}$ and $j \in \mathbb{Z}$. Eq. (2.18) tells us that the contribution schedules of each interest group to lawmakers are all locally and conditionally truthful around $\tilde{P}$; that is, each interest group's marginal contributions for changing each lawmaker's policy proposal must equal the marginal benefits. Because each lawmaker holds passive beliefs, in equilibrium, every interest group can make bilateral offer with him quite independently of other lawmakers. Substituting (2.18) and (2.15) into (2.17), we find that

$$\sum_{k \in L} \nabla W_k(P) + \sum_{j \in \mathbb{Z}} \alpha \nabla W(P^j, \tilde{P}^{-j}) = 0. \quad (2.19)$$

Eq. (2.19) portrays the lawmakers' equilibrium policy proposals supported by the differentiable contribution functions and therefore the equilibrium domestic prices. Following the work of Bernheim and Whinston 1986, we adopt the notion of truthfulness to define a conditionally truthful contribution schedule, which reflects the interest group's true preference conditional on the individual lawmaker's beliefs for out-of-equilibrium paths. Each interest group pays lawmaker $j$ for his policy proposal $P'$ the excess of
interest group's gross welfare at \((P^i,P^j)\) relative to some base level of welfare. As in Grossman and Helpman, we take the form of a conditionally truthful contribution function to be

\[
C^T_{ij}(P^i, P^{-i}, B_{ij}) = \max[0, W_i(P^i) - \sum C_{ih}(P^h) - B_{ij}].
\] (2.20)

All equilibria supported by the conditionally truthful strategies are named conditionally truthful Nash equilibrium. From Eq. (2.16), we have that

\[
W_j(\hat{P}^i, \hat{P}^{-i}) + \sum \left( \sum \hat{C}_j(P^h) + aW(\hat{P}^h, \hat{P}^{-h}) + \sum \hat{C}_j(P^i) + aW(\hat{P}^i, \hat{P}^{-i}) \right)
\geq W_j(P^i, \hat{P}^{-i}) + \sum \left( \sum \hat{C}_j(P^i) + aW(\hat{P}^h, \hat{P}^{-h}) \right) + \sum \hat{C}_j(P^i) + aW(P^i, \hat{P}^{-i})
\] (2.21)

for all \(P \in \Lambda\). If the contribution functions are conditionally truthful, then from the definition (2.20),

\[
\hat{C}_j(P^i) = W_i(\hat{P}^i, \hat{P}^{-i}) - \sum_{h \in Z, h \neq i} C_{ih}(\hat{P}^h) - \hat{B}_j,
\] (2.22a)

here \(\hat{B}_j\) is the equilibrium net benefit to the interest group \(k\), and

\[
\hat{C}_j(P^i) \geq W_i(P^i, \hat{P}^{-i}) - \sum_{h \in Z, h \neq i} C_{ih}(\hat{P}^h) - \hat{B}_j
\] (2.22b)

for all \(i \in L\), all \(j \in Z\), and all \(P^i \in \Lambda^i\).

Summing Eqs. (2.22) for \(i \in L\) except \(k\), and then substituting the sum into Eq. (2.21), we have

\[
\sum_{i \in L} W_i(\hat{P}^i, \hat{P}^{-i}) + aW(\hat{P}^i, \hat{P}^{-i}) \geq \sum_{i \in L} W_i(P^i, \hat{P}^{-i}) + aW(P^i, \hat{P}^{-i}).
\]

Consequently, we can conclude that the lawmaker \(f\)'s equilibrium policy proposal vector of conditionally truthful Nash equilibrium, \(\hat{P}^i\), satisfies the following condition:

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\[
\hat{P}^j \in \arg \max_{P^j \in \Lambda_Y} \sum_{i \in I} W_i(P^j, \hat{P}^{-j}) + aW_i(P^j, \hat{P}^{-j}),
\]
for all \( j \in Z \).

Equation (2.23) says that, in equilibrium, the conditionally truthful contribution schedules induce each lawmaker, who believes the others will propose equilibrium policy proposals, to behave as if he were maximizing a social welfare function that weights different members of society differently. This equation is very similar to the one in Grossman and Helpman 1994; it brings out the behavior of each individual lawmaker under the interest groups' conditionally truthful strategies. Comparing Eq. (2.23) with Eq. (2.4) shows that there exists consistency between the setup of a single interest group and that of the multi-interest groups. In addition, the first-order condition from (2.23) is the same as Eq. (2.19). When the individual lawmaker hold passive belief and all the other interest group are following conditionally truthful strategies, the interest groups treat every lawmaker as an independent common agency.

The calculation of the equilibrium domestic prices is the same as that in the previous subsection, and hence we can directly plug Eqs. (2.6) and (2.7) into Eq. (2.19) to get the equilibrium tariffs or subsidies. Specifically, we have

**Proposition 2.** In the political system in which the lawmakers are symmetrical, if the contribution schedules are differentiable around the equilibrium point, and if the equilibrium are located on \( \Lambda_1 \times \ldots \times \Lambda_n \), then the trade tariff taxes and subsidies satisfy

\[
\frac{\hat{t}_i}{1 + \hat{t}_i} = \frac{I_i - \alpha_L \hat{s}_i}{\alpha + \alpha_L \hat{s}_i},
\]

where \( \alpha_L = \sum_{i \in I} \alpha_i \).
Proposition 2 has the same structure of the equilibrium tariffs and subsidies as the one in Proposition 1 of Grossman and Helpman 1994. Since each lawmaker holds passive beliefs, in equilibrium, the interest groups can treat him like a common agency. Moreover, the lawmakers are symmetrical, and hence they weight aggregate welfare in the same way and share the power equally on forming the public policies. Consequently, the policy outcomes under the linear process of lawmakers' policy proposals are the same as the outcome under a single policy-maker. Even so, the assumption that the lawmakers are identical and hold passive beliefs helps us to illustrate the interactions among interest groups and lawmakers.

Suppose that efficiency means maximizing the joint surplus of the whole active players in the model. The efficient policy proposal vector \( \{ \bar{P}_i \}_{i \in \{1, \ldots, n\}} \) must satisfy the following condition:

\[
\{ \bar{P}_i \}_{i \in \{1, \ldots, n\}} \in \arg \max_{P \in \Lambda} \sum_{i \in L} W_i(P) + zaW(P). \tag{2.24}
\]

Plugging (2.6) and (2.7) into the first order condition of (2.24), we have the efficient trade taxes and subsidies that satisfy

\[
\frac{\bar{t}_i}{1 + \bar{t}_i} = \frac{I_i - \alpha_s \bar{s}_i}{za + \alpha_z \bar{e}_i}.
\]

We can use the information revealed from the efficient taxes (or subsidies) and equilibrium trade taxes (or subsidies) to tell the direction of distortions caused by the strategic externalities. Using the strong set order to make the comparison of the efficient domestic prices with the equilibrium domestic prices, we can show that the domestic prices depart from free trade more under the equilibrium policy proposal profiles than under the efficient policy proposal profiles.
Now, we apply the algorithm of Lemma 1 in Prat and Rustichini 1999 to discuss the property of each lawmaker's political contribution. If an interest group wants to induce lawmakers to choose the equilibrium policy $\tilde{P}$ and he tries to minimize his political contribution $\sum_{j \in Z} C_q(\tilde{P}^j)$, then its optimal political contribution schedule $\{\tilde{C}_q\}_{j \in Z}$ must satisfy the following condition:

$$
\{\tilde{C}_q\}_{j \in Z} \in \min_{\{C_q\}_{j \in Z}} \sum_{j \in Z} C_q(\tilde{P}^j)
$$

subject to $C_q(\tilde{P}^j) + \sum_{q \neq j} \tilde{C}_q(\tilde{P}^q) + aW(\tilde{P}) \geq C_q(P^j) + \sum_{q \neq j} \tilde{C}_q(P^q) + aW(P)$

for all $j \in Z$ and all $P \in \Lambda_1 \times \ldots \times \Lambda_n$. (2.25)

However, the solution for program (2.25) must satisfy the following condition:

$$
\sum_{j \in Z} \tilde{C}_q(\tilde{P}^j) = \sum_{j \in Z} \{G_j^{-i} - \sum_{i \in k} \tilde{C}_q(P^j) + aW(P^j, \tilde{P}^{-i})\},
$$

where

$$
\tilde{C}_q(\tilde{P}^j) = G_j^{-i} - \sum_{i \in k} \tilde{C}_q(P^j) + aW(P^j, \tilde{P}^{-i}),
$$

and

$$
\tilde{C}_q(P^j) \leq G_j^{-i} - \sum_{i \in k} \tilde{C}_q(P^j) + aW(P^j, \tilde{P}^{-i})
$$

for all $j \in Z$ and all $P \in \Lambda_1 \times \ldots \times \Lambda_n$. Thus

$$
\sum_{j \in Z} \tilde{C}_q(\tilde{P}^j) + aW(\tilde{P}^j, \tilde{P}^{-i}) = \max_{P^j \in \Lambda^j} \sum_{i \in k} \tilde{C}_q(P^j) + aW(P^j, \tilde{P}^{-i}).
$$

Actually, this condition is derived from the participation constraint. All the participation constraints must bind; otherwise the interest groups can profitably deviate by increasing
political contribution for some lawmakers. The interest group which maximizes the welfare of its members, must reduce its cost of implementing the equilibrium domestic prices as small as possible, and hence the participation constraint is squeezed until binds.

2.4 Conclusion

This paper extends the Grossman and Helpman (1994) model of common agency to the multi-agent case. We assume that public policies are determined under a committee system that reflects every individual lawmaker's policy proposal and we also compare the results with those from the single-agent model.

Our results suggest that if the lawmakers in a committee are symmetric and hold passive belief, in equilibrium, the lobbies' truthful political contributions will induce every lawmaker to behave as if he were maximizing a social welfare function that weighs the interest groups' welfare more heavily.

Moreover, since the lawmakers are symmetric, they weigh aggregate welfare in the same way and share the power equally on forming public policies. Consequently, the policy outcomes under the linear political process are the same as those under a single-agent model. This reinforces the results in Grossman and Helpman 1994.

Even so, the equilibrium policies are not efficient. Since each lawmaker is partially benevolent, his policy proposal causes externalities on the other lawmakers' objective functions. Moreover, the game is secretly played by players in the model, and hence the lobbies can not publicly commit to her or his bilateral offers. Therefore, the domestic prices depart from free trade more under the equilibrium policy proposals than under the efficient policy proposals.
CHAPTER 3. THE DYNAMICS OF POLITICAL FUND RAISING RACE: 
A DIFFERENTIAL GAME

3.1 Introduction

Under the institution of representative democracy, the politician's professional and personal success is often tied to financial contribution. No matter which party is in rule, it seems unavoidable that the politicians from political parties collect money to finance campaigns. However, politicians have nothing to sell but their policies as what is portrayed in Grossman and Helpman (1994). They betray the public’s trust to sell public policies for personal gain at the general publics’ cost.

Since political contribution has a marginal effect on the election outcome, the interest groups design their contributions to the politicians to influence the choice of policies rather than to influence election outcomes. For example, a lobbyist from gambling industry, Fahrenkopf, in an interview with CNN (1997), simply said, "You don't buy a vote. And, what money may buy you in Washington, to be very honest and frank with you, is money may give you a hearing. It may give you the opportunity." In other words, the political contributions may create accesses to influence government's policymaking. Democracy itself does not seem a sufficient guaranty against the corruption in which the politicians are seeking rents by creating rents.

But some economists from Chicago School claim that the political competition will ensure politician to avoid the sneaky inefficient policies of redistribution, which might cost him to be voted out of office. In particular, Becker (1976) and Wittman (1989) question how voters can be persistently fooled even they remain rationally ignorant. Friedrich (1972) also claims that there is an inverse relation between corruption and popularity. In the United States, one of the political contributions, soft money, is derived
so much criticism that the general public has been crying out campaign reform for a long time. Actually, the soft money is a loosely regulated political contribution, which may be given in any amount, but only to political parties, not to individual candidates. However, it is possible that the parties' members manipulate the money to aid their candidates' various expenses. Thus, soft money becomes an access of interest groups to corrupt politicians.

The poll (1997) showed that Americans by a 63-24 percent margin support banning soft money. Nevertheless, the politicians from political parties still expand their collections of political contributions. In 1994, the political parties of the United States raised a total of $101.7 million, of which only about $18.8 million was raised by the congressional and senatorial campaign committees. In 1998, soft-money fund raised more than doubled to $224.4 million, and the congressional and senatorial campaign committees raised $92 million of that (see Miller and Sifry 1999). In addition, the McCain-Feinolgold campaign-finance legislation is killed by the both political parties, Republican and Democrat. In the first glance, the political parties seem to only care about the collections of political contribution, even though there is a trade-off between parties' popularity and the collection of soft money under fierce parties' competitions. The two political parties' chase for soft money is like an ever-escalating arms race. The fund raising race between the parties has been a hot public issue; however, there have been few researches to examine why the parties are enthusiastic about collecting soft money while have not much reaction to the public's criticism.

Although there are some theoretical works on the corruption in some less developed countries' governments, it is rare for the current economists to investigate the pervasive parties' corruption in a modern democracy. Lui (1986) considers the impact of
exogenous corruption deterrence and views anti-corruption campaigns as an effort to shift from an unfavorable to a favorable equilibrium. Cadot (1987) considers the government’s corruption as a gamble played by officials in a static framework. Within an intertemporal framework, Feichtinger and Wirl (1994) argue that the trade-off between popularity and corruption may lead to complex patterns in which for a dictatorship government popularity is less important and cyclical, while for a democracy high popularity is sufficient for stable regimes. None of these works touches the issue of bipartisan corruption in a democracy.

In this chapter, focusing on the phenomena of political fund raising race between two parties, we take a close look at the issue that the bipartisan corruption is the center. The basic model we adopt is that of Feichtinger and Wirl (1994), in which the incumbent political party has to trade off between popularity and money collection. Our main conclusions are that: (a) If the competing parties are very similar to each other, they trash the popularity. (b) The fund raising activity is considerably higher under subgame perfect equilibrium than under time consistent.

The remaining of this chapter is organized as the following: Section 3.2 introduces a model of the two competing political parties that attempt to derive bliss from popularity and money collection. Section 3.3 analyzes the time consistent strategies. Section 3.4 studies the closed loop strategies and compares it with the open loop strategies.

3.2 The Model

Here, extending the work of Feichtinger and Wirl (1994), we consider two rivalry parties, which engage in soft money collecting activities and compete political power in every dimension. Every individual party as a rational player maximizes intertemporal
benefits from popularity and collecting soft-money. The amount of soft money collected by each party implies its corruption depth; that is, more soft money the party collects, the deeper the party is corrupted. The soft moneys can finance the party’s campaign expenditure, but hurt its popularity if the party’s fund raising activity is more vigorous than its opponent’s is.

The party A's popularity stock is measured by scale $p(t)$. The popularity per se provides additional benefits to political party, and the discretionary power increases if the people trust the party. $x(t)$ is the political money rate collected by party A, and $y(t)$ is the political money rate collected by party B. If party A increases its soft money collection, its popularity stock will decrease. However, if party B increases its soft money collection, party A's popularity will be promoted. The following differential equation captures the competing parties' dynamic responses to party A's popularity:

$$p(t) = -x(t) + y(t)$$  \hspace{1cm} (3.1)

where $p(0) = p_0$ is given. Moreover the higher the value of $p(t)$, the more the party A benefits. But, the less popular the party A is, the more the party B gains. The both parties appreciate political contributions.

The function $W^A(p)$ captures party A's bliss from its popularity; however, $W^B(p)$ is the bliss of party B's popularity net from the pressure that party A's popularity exerts over party B's popularity. Here, how the popularity affects the parties' bliss depends on some exogenous factors, for example general public's perception about the deals between the interest groups and political party, and the culture of economy. $U^A(x)$ is the bliss that party A can get from political contribution $x$. $U^B(y)$ is the bliss that party A can get from political contribution $y$. These two type of bliss are separable.
\[ W^i(.) + U^i(.) \text{ where } i = A \text{ or } B. \] To ensure there exists an interior solution, we assume that \( U \) and \( W \) have the usual properties: for party A, \( dU^A/dx > 0, \ d^2U^A/d^2x < 0, \ dW^A/dp > 0 \) and \( d^2W^A/d^2p < 0; \) for party B, \( dU^B/dx > 0, \ d^2U^B/d^2y < 0, \ dW^B/dp < 0 \) and \( d^2W^B/d^2p < 0. \)

The objective of the parties is to maximize the present value of their bliss from collecting political contribution and accumulating the parties' popularity. The both parties discount their future utility by a discount rate \( r. \) We therefore get the following differential game:

**Party A**

\[
\max_{x(t)} J^A = \int_0^\infty e^{-rt} [U^A(x(t)) + W^A(p(t))] dt \quad (3.2a)
\]

**Party B**

\[
\max_{y(t)} J^B = \int_0^\infty e^{-rt} [U^B(y(t)) + W^B(p(t))] dt. \quad (3.2b)
\]

Thus, a pair of dynamic optimization problems subject to Eq. (3.1) describes the fund raising race between the both parties. The control variables are the amount of parties' campaign fund collection rates \( x(t) \) and \( y(t); \) the state variable is party A's stock of popularity \( p(t). \) To simplify our analysis, we confine this differential game to linear-quadratic game for which the equation of motion is linear in the state and control variables, and the objective functions are quadratic in the state and control variables.

In our further analysis, we use the following specification of the appearing functions: for party A

\[
U^A(x) = c_1x + 1/2c_2x^2, \quad W^A(p) = a_0 + a_1p + 1/2a_2p^2,
\]

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and for party B

\[ U^B(y) = d_1 y + \frac{1}{2} d_2 y^2, \quad W^B(p) = b_0 + b_1 p + \frac{1}{2} b_2 p^2 \]

with \( c_1 > 0, \ c_2 < 0, \ a_2 < 0, \ d_1 > 0, \ d_2 < 0, \ b_1 < 0, \ b_2 < 0. \) Since party A's popularity has opposite impact on its opponent, \( a_i \) and \( b_i \) are of opposite sign. The coefficients \( a_2, \ b_2, \ c_2 \) and \( d_2 \) are negative to ensure the concavity of the both parties' objective functions.

For the case in which \( a_i = -b_i, \ a_2 = b_2 \) and \( c_i = d_i, \ i \in \{A, B\}, \) we call it quasi-symmetric; furthermore, if the condition \( a_0 = b_0 \) is included, we call it symmetric.

### 3.3 Open Loop Solution

Proceeding in a conventional manner, we define the current value Hamiltonian and current costate variables: For party A, the current value Hamiltonian \( H^A \) and costate variable \( \lambda \) fit in the function form

\[ H^A = U^A + W^A + \lambda (y - x). \]  

(3.3a)

The Hamilton maximizing condition

\[ x^* = \arg \max_x H^A \]  

(3.3b)

reduces from interior solution to

\[ H^*_x = dU^A / dx - \lambda = c_1 + c_2 x - \lambda = 0 \]  

(3.3c)

and it is assumed that such interior solution exists. Intertemporal optimality requires that the marginal utility from fund raising equals the positive shadow price of popularity. The following differential equation determines the evolution of the costate \( \lambda \)

\[ \dot{\lambda} = r\lambda - dW^A / dp = r\lambda - a_1 - a_2 p. \]  

(3.3d)
The necessary optimality conditions in Eqs. (3.3b)-(3.3d) are sufficient if additionally the limiting transversality conditions are satisfied,

$$\lim_{t \to \infty} e^{-\eta} \lambda(t)x(t) = 0.$$  (3.3e)

For party B, the current value Hamiltonian $H^B$ and costate variable $\mu$ fit in the following function,

$$H^B = U^B + W^B + \mu(y - x).$$  (3.4a)

The necessary optimality conditions for maximizing $H^B$ are

$$H_y^B = dU^B/dy + \mu = d_1 + d_2y + \mu = 0,$$  (3.4b)

$$\mu = r\mu - dW^B/dy = r\mu - b_1 - b_2p,$$  (3.4c)

and the transversality condition is

$$\lim_{t \to \infty} e^{-\eta} \mu(t)y(t) = 0.$$  (3.4d)

The Hamiltonian maximizing conditions for interior solution and the evolution equations of costates allow us to replace the costates. Using the differentiation of (3.3c) and (3.3b) with respect to time to eliminate the costates leads to the following system of linear differential equation in state and control variables

$$\dot{x} = r(x + \frac{c_1}{c_2}) - (a_1 + a_2p)/c_2,$$  (3.5a)

$$\dot{y} = r(y + \frac{d_1}{d_2}) + (b_1 + b_2p)/d_2,$$  (3.5b)

$$\dot{p} = y - x.$$  (3.5c)

The matrix gives the Jacobian of this dynamic system, which is evaluated at the equilibrium
The eigenvalues \( \{ e, i = 1 \text{ to } 3 \} \) of the Jacobian (3.6) are crucial to characterize the local dynamics of the linear system that approximates the Eqs. (3.3). They can be explicitly computed:

\[
\begin{bmatrix}
  r & 0 & -\frac{a_2}{c_2} \\
  0 & r & \frac{b_2}{d_2} \\
  -1 & 1 & 0
\end{bmatrix}
\]

(3.6)

The eigenvalues \( \{ e, i = 1 \text{ to } 3 \} \) of the Jacobian (3.6) are crucial to characterize the local dynamics of the linear system that approximates the Eqs. (3.3). They can be explicitly computed:

\[
e_1 = r, \quad (3.7a)
\]

\[
e_2^1 = \frac{1}{2}[r \pm \sqrt{r^2 + 4(a_2d_2 + b_2c_2)/c_2d_2}]. \quad (3.7b)
\]

Since \((a_2d_2 + b_2c_2)/c_2d_2 > 0\) all the eigenvalues are real and one and only one is negative. Furthermore, we can compute the stationary solution of system (3.7), which determines the long run outcome of campaign fund raising and its impact on their reputation:

\[
x_\omega^0 = \frac{-[r(c_2b_2 + d_2a_2) - (b_2a_2 - a_2b_1)]}{r(c_2b_2 + d_2a_2)} = y_\omega^0, \quad (3.8a)
\]

\[
p_\omega^0 = \frac{[r(c_2d_2 - d_1c_2) - (d_1a_1 - b_1c_2)]}{c_2b_2 + a_2d_2}. \quad (3.8b)
\]

Because the objective function and the state transition equation are concave in state and control variables, the open loop equilibrium of fund raising activity is uniquely determined. By the properties of the eigenvalues of (3.7), the associated stationary state (3.8) is a saddle point. Moreover, since the intertemporal fund raising activities are linear function of the state \( p \), we claim the following proposition:
Proposition 1. Party A's fund raising activities increases with respect to the state of its own popularity, but party B's fund raising activities decline with respect to the state of its opponent's popularity.

Proof: One can compute the fund raising activities in this form:

\[ x^0 = x^0_a + f_1(p - p^0_a), \]  
\[ y^0 = y^0_a + g_1(p - p^0_b), \]

where

\[ f_1 = -\frac{d_1a_2}{b_2c_2} \sqrt{r^2 + 4\left(1 + \frac{d_1a_2}{b_2c_2}\right) \frac{b_2}{d_2}} > 0 \]

and

\[ g_1 = \frac{r - \sqrt{r^2 + 4\left(1 + \frac{d_1a_2}{b_2c_2}\right) \frac{b_2}{d_2}}}{2\left(1 + \frac{d_1a_2}{b_2c_2}\right)} < 0. \]

The detail derivation of \( f_1 \) and \( g_1 \) is shown in the Appendix. Since \( f_1 > 0 \) and \( g_1 < 0 \) the proposition is confirmed. ■

This proposition reveals that: Since party A's money collection has negative impact on its own popularity, there is a trade off between its money collection and its popularity. When party A has more popularity, it holds more stakes in collecting political money. Therefore, the higher the party A's popularity is, the more money it collects. The contrary applies to party B. The higher its opponent party's popularity is, the more harm there is for party B. Consequently, the lower party A's popularity, the more campaign money the party B dares to collect.
For the case in which the party A's popularity linearly benefit itself and harm its opponent, we have $a_2 = 0$ and $b_2 = 0$, and hence $f_t = 0$ and $g_1 = 0$. So, from equation (3.9), we know that in this case the both parties make constant campaign fund raising.

From Eqs. (3.8), the quasi- symmetric equilibrium can be simply written as:

$$x^0 = y^0 = -\left(\frac{c_1 + a_1}{c_2} \right),$$  \tag{3.10a}$$

$$p^0 = 0.$$  \tag{3.10b}$$

In order to guarantee the existence of positive fund raising activities $x^0$ and $y^0$ given by (3.10), we have to assume that

$$c_1 > a_1/r.$$ 

This inequality tells us that the positive impact of political money on the parties' bliss is greater than the positive impact of popularity on the parties' bliss. Since popularity is a state variable and directly affects the parties' utilities, the discount rate $r$ enters this inequality via the transition equation.

Eqs. (3.10) reveal that the smaller discount rate and the larger concavity of its bliss function with respect to campaign fund raising lower campaign fund raising activities. The constant terms of marginal bliss represent the direct benefit or harm of the change in fund-raising activity or popularity. So, the higher the constant terms of marginal bliss with respect to fund raising activity, the more vigorous the parties' fund raising activities are. Moreover, since there is trade-off between party A's popularity and fund raising activity, the larger constant terms of marginal bliss with respect to popularity lower the parties' fund raising activities.
The steady state of the dynamic, open loop and symmetric equilibrium (3.10a) can be computed in the following heuristic way (this interpretation follows the algorithm in the chapter 13 of Funderberg and Tirole 1990): Suppose that the parties are in a steady state at open loop level \((x^o, y^o)\), and let party A, say, increase its rate of fund raising-activity by 1 during a period of time \(dt\), and then revert to its previous rate of fund-raising activity once time \(dt\) has elapsed. The revenue of lifting fund-raising activity during \(dt\) is \((c_1 + c_2 x^o)dt\). If this lifting fund-raising activity does not affect party B’s fund-raising activity (this is the open-loop assumption underlying the steady-state Nash levels), the extra cost for party A is

\[
\int_0^a (a_1 dt) e^{-\eta s} ds = \frac{a_1 dt}{r},
\]

(3.11)
since party 1’s marginal cost is \(a_1 - a_2 (-x^o + y^o)\) and in the steady state \(x^o = y^o\). At the Nash level, one must have

\[
x^0 = y^0 = \left(\frac{c_1}{c_2} - \frac{a_1}{rc_2}\right).
\]

3.4 Subgame Perfect Equilibrium

The open loop trajectories of the previous section are time consistent. They are completely prespecified; in other words, the both parties completely ignore the information about their opponent’s popularity that is revealed over time. Time consistent requires that the strategies need only be the best responses in the subgame starting from the nodes on equilibrium path, instead of all subgames. However, closed loop trajectories can be revised after it starts; that is, they are perfect equilibrium strategy which is the best response in all subgames. Moreover, the closed loop equilibrium is always strongly time
consistent which essentially is subgame perfect (Basar and Olsder 1995). Under this interpretation, time consistent is less stringent condition than perfect.

Now we move on to derive a stationary feedback solution for the differential game. We assume that the two parties choose the values of their controls at time $t$ depending on the value of the state variable at $t$. As we want to calculate stationary closed loop Nash solution of the game, we have to find the continuously differentiable value functions $Q(p)$ of party A and $S(p)$ of party B, which satisfy the following Hamilton-Jacobi-Bellman equations

$$rQ(p) = U^A(x) + W^A(p) + Q'(p)(y - x), \quad (3.12a)$$
$$rS(p) = U^B(y) + W^B(p) + S'(p)(y - x), \quad (3.12b)$$

$$x(p) = \arg \max_x U^A(x) + W^A(p) + Q(p)(y(p) - x), \quad (3.13a)$$
$$y(p) = \arg \max_y U^B(y) + W^B(p) + S'(p)(y - x(p)), \quad (3.13b)$$

and the transversality conditions

$$\lim_{t \to \infty} e^{-rt}Q(p) = \lim_{t \to \infty} e^{-rt}S(p) = 0,$$

which guarantee that the resulting strategies $x(p), y(p)$ are admissible. In fact $Q$ and $S$ denote the value functions of the two parties:

$$Q(p_0) = \max_x \left[ \int_0^\infty e^{-rt} [U^A(x(t)) + W^A(p(t))] dt, \right.\left. p(t) = -x(t) + y(t), p(0) = p_0 \right], \quad (3.14a)$$

$$S(p_0) = \max_x \left[ \int_0^\infty e^{-rt} [U^B(y(t)) + W^B(p(t))] dt, \right.\left. p(t) = -x(t) + y(t), p(0) = p_0 \right], \quad (3.14b)$$

The first order condition of (3.13) gives

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Eqs. (3.15) simply state that in closed loop equilibrium the marginal cost of fund raising activity is equal to the marginal benefit. They are equivalent to

\[ c_1 + c_2 x - Q'(p) = 0, \]  
\[ d_1 + d_2 y + S'(p) = 0. \]

Eqs. (3.15) yield the optimal strategy in close loop form, provided that the value function \( Q \) and \( S \) are known.

A closed loop Nash equilibrium consists of a pair of strategies \( x^N \) and \( y^N \) that is determined by Eqs. (3.16). To specify the equilibrium strategies, we need more information about the value functions \( Q(p) \) and \( S(p) \). Thus, it is necessary for us to solve these Hamilton-Jacobi-Bellman Eqs. (3.12) and (3.13). Formally, one has to solve a pair of simultaneous ordinary differential equations. But, in general, there exists tremendous technical difficulties to solve it by quadrature. Due to the bliss functional forms are linear quadratic, we can take the usual procedure to reduce the analysis to linear quadratic games.

Now, we assume that the solutions for the value functions \( Q(p) \) and \( S(p) \) are linear quadratic:

\[ Q(p) = q_0 + q_1 p + \frac{1}{2} q_3 p^2, \]  
\[ S(p) = s_0 + s_1 p + \frac{1}{2} s_2 p^2. \]

Inserting the terms in (3.16) into the HJB Eqs. (3.12) yields
Using the functional forms for the value function in (3.17), we get the following six equations from the comparison of coefficient of $p$ in (3.18):

\[
\begin{align*}
\text{(3.18a)} \quad rQ(p) &= \frac{c_1^2}{2c_2} + \left(\frac{c_1}{c_2} - \frac{d_1}{d_2}\right)Q(p) - \frac{Q'(p)^2}{2c_2} - \frac{Q'(p)S'(p)}{d_2} + W^S(p), \\
\text{(3.18b)} \quad rQ(p) &= \frac{d_1^2}{2d_2} + \left(\frac{c_1}{c_2} - \frac{d_1}{d_2}\right)S'(p) - \frac{S'(p)^2}{2d_2} - \frac{Q'(p)S'(p)}{c_2} + W^S(p).
\end{align*}
\]

Using the functional forms for the value function in (3.17), we get the following six equations from the comparison of coefficient of $p$ in (3.18):

\[
\begin{align*}
\text{(3.19a)} \quad rq_0 &= a_0 - \frac{c_1^2}{2c_2} - \left(\frac{d_1}{d_2} - \frac{c_1}{c_2}\right)q_1 - \frac{s_1q_1}{d_2} - \frac{q_1^2}{2c_2}, \\
\text{(3.19b)} \quad rq_1 &= a_1 - \left(\frac{d_1}{d_2} - \frac{c_1}{c_2}\right)q_2 - \frac{q_1q_2}{c_2} - \frac{s_1q_2 + s_1q_2}{d_2}, \\
\text{(3.19c)} \quad \frac{1}{2} rq_2 &= \frac{1}{2} a_2 - \frac{q_2s_2}{d_2} - \frac{q_2^2}{2c_2}, \\
\text{(3.19d)} \quad rs_0 &= b_0 - \frac{d_1^2}{2d_2} - \left(\frac{d_1}{d_2} - \frac{c_1}{c_2}\right)s_1 - \frac{s_1q_1}{c_2} - \frac{s_1^2}{2d_2}, \\
\text{(3.19e)} \quad rs_1 &= b_1 - \left(\frac{d_1}{d_2} - \frac{c_1}{c_2}\right)s_2 - \frac{s_1s_2}{c_2} - \frac{s_2q_1 + s_1q_2}{c_2}, \\
\text{(3.19f)} \quad \frac{1}{2} rs_2 &= \frac{1}{2} b_2 - \frac{q_2s_2}{c_2} - \frac{s_2^2}{2d_2}.
\end{align*}
\]

Substituting variables among the six equations in (3.19) gives a consecutive elimination of the variables that produces a fourth order polynomial equation for $q_2$ or $s_2$. Since it is impossible to get analytical solution for a fourth order polynomial equation, we appeal to the symmetric case to illustrate the feedback strategy.

Now, we go straightforward to the quasi-symmetric case, in which $a_i = -b_i$, $a_2 = b_2$ and $c_i = d_i, i \in \{1,2\}$, and hence $q_i = -s_i$. The game leads to a three dimensional non-linear simultaneous system:
The value of \( q_0 \) might be calculated from (3.14), but we will omit these calculations, as this term does not appear in the equilibrium strategies of (3.16). Solving (3.20a) and (3.20b) gives

\[
\begin{align*}
\frac{r_q}{2} &= \frac{-c_1^2}{2c_2} + \frac{q_1^2}{2c_2} + a_0, \\
\frac{r_q}{2} &= \frac{q_1q_2}{2c_2} + a_1,
\end{align*}
\tag{3.20a, 3.20b}
\]

\[
\frac{1}{2}r_q = \frac{-3q_1^2}{2c_2} + \frac{1}{2}a_2
\tag{3.20c}
\]

Since the value functions \( Q \) and \( S \) are concave in \( p \), we take the negative solution of (3.20c). Inserting (3.21) into (3.16), we have the optimal closed loop fund raising strategies for both parties respectively:

\[
\begin{align*}
\frac{d_x}{d} &= \frac{c_1 + \frac{6a_1}{c_2[5r + (\sqrt{r^2 + \frac{12a_2}{c_2})]}} - \frac{1}{6}[r - (\sqrt{r^2 + \frac{12a_2}{c_2}})]p_N^e, \\
\frac{d_x}{d} &= \frac{-d_1 - \frac{6a_1}{d_2[5r + (\sqrt{r^2 + \frac{12a_2}{c_2})]}} + \frac{1}{6}[r - (\sqrt{r^2 + \frac{12a_2}{c_2}})]p_N^e.
\end{align*}
\tag{3.22a, 3.22b}
\]

Eqs. (3.22) state that in equilibrium, play A's fund-raising activity increases with respect to its own popularity, but player B's activity decreases with respect to its opponent's popularity.
Since \( p = y - x \), \( p^* = 0 \). For the subgame perfect equilibrium, the stationary fund raising strategies for both parties are

\[
x^*_m = -\frac{c_1}{c_2} + \frac{6a_1}{c_2[5r + (\sqrt{r^2 + \frac{12a_2}{c_2}})]} = y^*_m.
\]

Now, looking at (3.10), for the symmetrical case, we have open loop stationary solution

\[
x^{o}_m = -\frac{c_1}{c_2} + \frac{a_1}{c_2 r} = y^{o}_m.
\]

The difference between \( x^*_m \) and \( x^{o}_m \) is

\[
\mathcal{G} = x^*_m - x^{o}_m = -\frac{c_1}{c_2} \left( \frac{1}{r} - \frac{6}{5r + (\sqrt{r^2 + \frac{12a_2}{c_2}})} \right) > 0.
\]

Thus, we claim the following proposition:

**Proposition 2.** For the symmetrical case, there exists a unique and stable linear Markov subgame perfect equilibrium with the appealing properties that there are higher campaign fund collection compared with the open loop equilibrium.

This proposition is great contrast to the results in Wirl (1994). Wirl presents a dynamic model on lobbying in which the subgame perfect equilibrium lowers lobbying activity and expense, and then argues it provides a partial explanation of the puzzle that rent-seeking expense are often small compared with the prize sought. Since Wirl does not compare the interest groups' gains with their contributions, he really fails to supply an explanation to what he observes. Our model focuses on the phenomena that two competing parties chase soft money and ignore the general public's criticism about their corruption. Comparing with Wirl's work, it is far persuasive for us to apply a differential
game to explain this observation. We also extend the one agent model in Feichtinger and Wirl (1994) to a world in which each political party faces the challenges from its opponent. It provides a more complete feature to portray a real world.

3.5 Conclusion

This paper considers the phenomena of political fund raising race between two political parties. We assume that each individual rational political party derives bliss from political campaign money and popularity. If collecting political fund implies the party sells its services to the rent seekers, the party's popularity will fade as its fund raising activity is more vigorous than its opponent's is. Given these dynamic reactions, the two parties have to trade off between popularity and money collection. We model the interaction between the two competing political parties as a differential game. For this game, we consider first a time consistent and then a subgame perfect equilibrium of the linear Markovian type. In the symmetrical case, we find the parties trash popularity in time consistent and subgame perfect equilibrium. Moreover, the subgame perfect equilibrium considerably lifts political money collecting activities compared with time consistent. This supplies a partial description about the observation: The political parties in the United States ignore the general public's criticism on their soft money collecting activities and engage in an ever-escalating fund raising race. The reason for this outcome is that the dynamic closed loop strategies have built in the mechanism that each party collect like political money for like. Whenever collecting mass money, the political party knows that this gives a chance to its opponent to follow up and instead harms its own popularity. This common knowledge encourages aggressive strategies in the beginning and thus lifts the fund-raising activities of the political parties.
CHAPTER 4. IMMIGRATION, BORDER CONTROL AND THE RELATIVE BARGAINING POWER OF GOVERNMENT UNDER LOBBYING PROCESS

4.1 Introduction

The conventional theory of immigration proceeds on the assumption that the immigrants to rich countries from poorer countries have less education and skill than the rich-country workforce and therefore compete with the lower-income natives. But, the increase of unskilled immigrants has the effects upon workplace productivity and raises the income of skilled labors. As a result, the immigration of foreign labor may affect the income redistribution of the host country.

In fact, there have been a lot of theoretical works on the effects of labor migration, which is across country border. They have focused on the welfare impact of migration, the distributive effects of factors, and policy measures (Ramaswami 1968, Rivera-Batiz 1982, Ethier 1985 and 1986, Kuhn and Wooton 1987, and Davies and Wooton 1992). Among them, there exists a consensus that the distributive effects are sizeable. In United States, the native or incumbent low skilled will lose as a result of immigration — primarily through lower wage rather than increased unemployment; while the additional low-skilled labor that results from immigration does raise the earnings of high-skilled workers (Johnson 1980). Consequently, the immigration policy can be a political issue because of the interest conflict between the two production factors.

To ease the interest conflict among different factors caused by the entry of foreign labors, a government that maximizes its object function concerning immigration policy is to control the inflow of immigrants. It is a straight perception that immigration can not be controlled unless the host country secures its border against the illegal entry of foreign labor. However, it will be a tremendous cost for a host country to make its border...
seamless. So, the optimal border control may just keep illegal immigrants in a certain level rather than shut down the border. In fact, one often observes a host country permitting some legal entries of foreign labors and simultaneously deterring the illegal entries of foreign labors. This observation gives rise to two related puzzles.

**Puzzle 1.** An optimal policy for a small country concerning labor mobility is to let foreign labor in until the marginal labor product is driven down to the foreign wage. That implies that all foreign labor should be legally admitted for a national optimum. **Puzzle 2.** If for some reason illegal entry should be deterred, why allow for legal entry of foreign labor?

The existing models cannot account for the simultaneous existence of legal and illegal immigrants and hence cannot answer these two puzzles. They either do not distinguish two types of labor (most models) or assume that legal labor is non-existent or outside the model (for example, Ethier 1986, Bond and Chen 1987 and Djajic 1997).

In this chapter, first we develop a simple small country model in an attempt to study these puzzles. Here, there are three production factors and one sector in this economy. The owners of production factors may overcome the free rider problem to form interest groups and lobby the government for the favor of immigration policy. The interaction between the government and the lobby is portrayed as in the common agency model developed by Grossman and Helpman (1994). The basic frame we adopt is that of Ethier (1986), in which the effects of enforcement policies designed to reduce the level of illegal immigrant are analyzed.

Second, since our basic setup about the political economy is based on the framework of Grossman and Helpman (1994), inevitably, it inherits some of their extreme characteristics. One of them is that: The government has monopoly power in
relation to multi-lobby but has no power at all facing a unique lobby. To relieve it, we expand our one sector setting to be multi-sector and portray the lobby-government negotiation as a Nash bargaining process. The relative bargaining power of the government is no more extreme in the extension section.

Our main conclusion is that: If the production factors' interests are not all presented by the lobbies, the political equilibrium is to set up a quantitative restriction on the inflow of immigrants. In addition, if the marginal cost of deterring illegal immigrant can not be covered by the marginal benefit of allowing legal immigrant for entry, the host country's best policy is to ban any legal entry and set up an optimal border control. However, if a high enough lump-sum tax is imposed on the legal immigrant, the policy, deterring illegal entry to allow a certain level of foreign labors for legal entry, is the political equilibrium policy of host country. In the extensive part, we show that in the short run the relative bargaining power of the government only affects the distribution of the surplus, derived from the political process, between the government and the lobby; however, it affects the immigration policy in the long run.

The remaining of this chapter is organized as the following: Section 4.2 introduces a basic model in which all the owners of production factors are presented by lobbies. Section 4.3 analyzes the pseudo-political level of legal temporary immigrants. In Section 4.4, we study the condition, for which border control and legal entry of foreign unskilled labor can simultaneously coexist. In Section 4.5 we expand our one sector setting to multi-sector, and model the lobby-government negotiation as a Nash bargaining.
4.2 The Basic Model

Consider a small economy with three production factors, capital $K$, skilled labor $H$ and unskilled labor $L$. They are the inputs to produce a single goods, a numeraire. The production function is linearly homogeneous and takes the form

$$f(K, L, H) = K^\alpha L^\beta H^\gamma$$

where $\alpha$, $\beta$, $\gamma > 0$, and $\alpha + \beta + \gamma = 1$. (4.1)

Thus $\alpha$, $\beta$ and $\gamma$ respectively are the shares of output for capital, skilled labor and unskilled labor. We assume that the reward for a unit of unskilled labor $f_L()$ is greater than the foreign wage of unskilled labor $w^*$; that is $f_L() > w^*$. So, the foreign labors have incentives to move to the host country to earn a wage, which is higher than that in their own countries. The number of foreign labors, who want to move to host country, is a function of the discrepancy between host country's wage and foreign wage. The larger the difference, $f_L - w^*$, the more foreign labors, who want to immigrate to host country, there are.

Suppose that the immigrants' income does not belong to the host country. The government is to choose immigrant policies to maximize the country's income, and the social optimal of immigrants allowed satisfies

$$M^* = \arg \max_{M \in (0, \infty)} f(K, H, L + M) - Mf_L.$$ (4.2)

Because the marginal social welfare with respect to immigrants (unskilled foreign labors) is positive, the available social optimal level of immigrants should satisfy:

$$f_L() = w^*.$$

Now, we assume that the government pursues its goals but not just the aggregate income. It cares about the total level of political contributions and the national aggregate income. The government is a democratically elected political entity, which the incumbent

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politician during his first term collects campaign contributions to finance the expenditure of a later election. Here, we assume that, in this economy, every individual only owns one kind of factors, and the owners of different factors may organize to influence the government's immigrant policy.

As Olson (1965) mentions, the internal organization of lobby group is a matter that is difficult to deal with because it involves so many details. Of course, tackling this kind of difficulty will be an interesting challenge for us. However, this is not the issue that we want to focus on. Instead, we assume that, in this economy, some of the production factors can overcome the free rider problem and organize lobby groups to lobby the government for favorable immigration policies.

The lobby that represents an organized specific factor makes its political contribution contingent on the policy vector implemented by the government. We denote by $C^i(.)$ the contribution schedule presented by the interest group $i$, where $i \in \{ K, H, L \}$. The net joint welfare of factor $i$ can be expressed as

$$V^i = \delta_i - C^i(.)$$  \hspace{1cm} (4.3)

To simplify our analysis, we choose a linear form for the government's objective function, namely

$$G = \sum_{i \in \{ K, H, L \}} C^i(.) + \theta W$$  \hspace{1cm} (4.4)

where $W(.) = \sum_{i \in \{ K, H, L \}} \delta_i$ is the aggregate income of domestic production factors and $\theta$ is the weight that the government attributes to the aggregate income.

We are interested in the political equilibrium of a two stage, non-cooperative game in which the lobbies simultaneously choose their political contribution schedules in the first stage (prior to the election) to induce the government to set policies in the second
stage (after the election). The interaction between various lobbies and the government is portrayed as a game that has the structure of common agency problem. Moreover, the features of contribution schedules also add another characteristic, menu action problem, for the relationship between incumbent politician and interest groups. At a political equilibrium, the immigrant policy and political contributions are determined as a subgame perfect Nash equilibrium, which is initially developed by Bernheim and Whinston (1986). If the government's choice set is allowed to be continuous and the contribution schedules are differentiable, we can closely follow Grossman and Helpman (1994) and Dixit (1996) to derive the equilibrium in differentiable strategies.

Bernheim and Whinston suggest that there are multiple equilibria for this menu auctions game; however, they further argue truthful Nash equilibria are stable to the non-binding communication among players and hence may be focal among the set of Nash equilibrium. To simplify our exposition, we assume that the contribution schedules are globally truthful. Under the assumption of global truthful political contributions, which everywhere reflect interest groups' preferences, we can get a neat property. The equilibrium policy vector $P^0$ of any TNE satisfies:

$$\max \sum_{i \in \Omega} y_i(\cdot) + \theta W(\cdot)$$  \hspace{1cm} (4.5)$$

where $\Omega$ is the set of interest groups. This equation tells us that when the lobby groups are following truthful strategies, in equilibrium, the government is induced to behave as if it were maximizing a social welfare function, which the incumbent politician puts more weight on the lobby groups than on non-lobby groups.

Now, suppose that the owners of these three factors well organize respectively to be interest groups and the lobbies offer contribution schedules to the incumbent politician.
in exchange for favorable immigrant policies. The outcome will be the same as Grossman and Helpman (1994) mention: "When all voters are active in the process of buying influence, the rivalry among competing interests is most intense, and the government captures all of the surplus from the political relationship."

This can be more detailed if we look at equation (4.5). The government objective function is equivalent to \((1 + \theta)\hat{W}(.)\) Consequently, the lobbying activity will not affect the government's decision. If the lobbies use truthful contribution schedules, in equilibrium, the outcome under complete political influence (which all the production factors exert influences by offering contributions to incumbent politician) will be same as the outcome of social optimum. Thus, we have the following remark:

**Remark 1.** If all production factors have their interests represented by a lobby, then the political equilibrium is social optimal, leading to a completely political internalization of immigration policy.

The rivalry among competing interests induces the government behaves as if it were maximizing a social welfare function that weighs different members of society equally. Remark 1 highlights the interpretation of complete political economy.

### 4.3 Incomplete Political Economy

In contrast to the situation that all the production factors make political effort, we assume that the unskilled labors and skilled labors can overcome the free rider problem and respectively organize lobby groups to offer campaign contribution to the government in exchange for immigration policies, while the capital fails to do so. This assumption is crucial because it allows the government makes inefficient immigration policies to favor the interest groups at the expense of not-lobbying groups. The lobby represented the organized specific labor (skilled or unskilled) makes its political contribution contingent...
on the immigrant policy vector implemented by the government. We denote by $C^l$ the contribution schedule presented by the interest group $l$ where $l \in \{L, H\}$. The net joint welfare of skilled labors can be expressed as

$$V^H = Hf_H(\cdot) - C^H(\cdot)$$

and for the unskilled labor, the net joint welfare is

$$V^L = Lf_L(\cdot) - C^L(\cdot).$$

The government's objective function is

$$G = C^L(\cdot) + C^H(\cdot) + \theta W(\cdot).$$

(4.6)

The immigrant policy of TNE can be derived as the solution to the following problem:

$$\max_{\sum_{l \in \{L, H\}} M_l} \sum_{l \in \{L, H\}} \theta W(\cdot).$$

(4.7)

This equation tells us that when the lobby groups are following truthful strategies, in equilibrium, the government is induced to behave as if it were maximizing a social welfare function that the incumbent politician puts more weight on the welfare of skilled labor and unskilled labor.

Under this theme, we will lay out our analysis about the issues of immigration policy. First of all, we set up some pseudo-benchmarks in which the host country is immune to illegal immigrants, and the government under political influence is to choose the equilibrium level of legal immigrants allowed for entry.

If there is no political influence from any interest groups, the social optimum still calls for inflows of unskilled foreign labors until the marginal product of unskilled labor equals foreign wage. Let $M^o$ denote the number of immigrants that the unskilled labor's wage of host country is equal to foreign wage; that is, $M^o$ satisfies
\[ w^* = f_L(K, L + M, H). \]  

Now, \( K \) or the owners of capital are not politically organized; however, the well-organized interest groups, skilled labors \( H \) and unskilled labors \( L \), use contribution schedules to exert political influence. Because the temporary immigrants are not the residents of host country, the social welfare of host country does not include the earning of temporary immigrants. It is equal to \( f(\cdot) - Mf_L \).

The equilibrium immigrant policy of any TNE \( M^{PT} \) satisfies

\[ M^{PT} = \arg \max_{M \in (0, M^o]} Hf_H(\cdot) + Lf_L + \theta(f(\cdot) - Mf_L). \]  

The first-order condition for (4.9) is equal to

\[ Hf_{HL} + Lf_{LL} - \theta Mf_{LL} = 0. \]  

Rearranging (4.10) and using the property of Euler's equation for the homogenous degree zero function, we can get

\[ -Kf_{KL} - (1 + \theta)Mf_{LL} = 0. \]  

So, the equilibrium level of immigrants, which are allowed for entry under political influence, is equal to

\[ M^{PT} = \frac{-Kf_{KL}}{(1 + \theta)f_{LL}} > 0. \]

We name \( M^{PT} \) to be the pseudo-political level of legal temporary immigrants that under the lobbying process the political economy outcome is to allow this level of foreign labor in the host country legally. Under mild condition, this level should be smaller than \( M^o \). We plug \( M^o \) the social optimal level of legal immigrant into

\[ -Kf_{KL} - (1 + \theta)Mf_{LL} \]

and then we have

\[ 56 \]
Thus we can use (4.12) to check whether $M^PT$ is less than $M^o$. Obviously, if

$$
\alpha - (1 - \beta)(1 + \theta) \frac{M^o}{L + M^o} > 0,
$$

then $M^PT < M^o$.

To satisfy (4.13), the share of output for capital $\alpha$ should be large enough, and $\theta$ should be small enough. For fulfilling (4.13), the population of unskilled labors should be large enough relative to the population of legal immigrants. Because the capital is not politically organized, the interest group can induce the government to set up an immigrant policy, which is much more at the expense of capitalists for a larger $\alpha$. In fact, its presence introduces inefficiency in an otherwise socially efficient equilibrium. As Dixit (1996) interprets, the coefficient $\theta$ sizes up the government's deal between social welfare and contributions; smaller $\theta$ suggests a less benevolent government. Furthermore, if the population of unskilled labors is relatively large, it means that the interest group of unskilled labors has more strength to exert political influence on the government and the population of legal immigrant will not have much impact on the welfare of native unskilled works.

4.4 Immigration and Border Control

Suppose that the foreign labors are not allowed to freely enter into the host country. Then there are some public policies, which concern illegal immigrants. One of the measures against illegal entry is to control the inflow of illegal foreign workers by border control. To finance the expenditure of enforcing border control, the government
collects income tax from the whole economy at a tax rate $t$. To simplify our analysis, we assume that the firm of host country can completely discern illegal immigrant. Thus, illegal workers and legal temporary immigrants are not included in the host country’s tax base. The net joint welfare of skilled labors can be expressed as

$$V^H = (1-t)HF^H() - C^H(),$$

(4.14)

and the net joint welfare of the unskilled labors is

$$V^U = (1-t)LF^U() - C^U().$$

(4.15)

The government's objective function is

$$G = C^L() + C^H() + \theta W(),$$

(4.16)

where $W() = \sum_{i \in \{L,K,H\}} (1-t)f_i$. The immigrant policy vector of TNE can be derived as the solution to the following problem:

$$\max_{\mathbf{z}} (1-t)LF^U() + (1-t)HF^H() + \theta W()$$

(4.17)

where $Z$ is immigrant policy set and $z$ is a component of $Z$.

4.4.1 Border Control and No Legal Immigrant Allowed

First of all, we assume that the host country forbids any foreign labor to enter. Furthermore, we assume that the cost of cross border is constant and smaller than the discrepancy between host country's wage and foreign wage. Since the wage of native labor is much higher than the world average wage, there is an incentive for foreign workers to illegally enter the host country.

To control the level of immigrants, the government chooses expenditure $B$ to enforce border control. Corresponding to $B$, there is a positive probability $P(B)$ that an illegal worker will be apprehended while attempting to illegally cross the border. The probability is increasing in the resource that the government devotes to border control.
enforcement. Furthermore, following Ethier (1986) we establish a standard assumption that

\[ P(0) = 0, \; P'(\cdot) > 0, \; P''(\cdot) < 0, \; P(\cdot) < 1. \]  \hspace{1cm} (4.18)

Let \( R(f_L) \) be the foreign unskilled labors, who attempt to immigrate illegally to the host country; it is a function of the wage of host country. Moreover, we assume that \( R'(f_L) > 0 \); that is, the foreign unskilled labors that want to illegally enter the host country will increase as the unskilled labor wage of host country raises up. Because there is no legal immigrant, the foreign unskilled labor who successfully enter the host country is equal to

\[ I(B) = R(f_L)[1 - P(B)]. \]  \hspace{1cm} (4.19)

Then we have

\[ I'(B) = \frac{-R(f_L)P'(B)}{1 - R'(f_L)f_L[1 - P(B)]}. \]  \hspace{1cm} (4.20)

Eq. (4.20) states that the enhancing the border enforcement can deter more foreign unskilled workers from illegally immigrating into the host country.

After some illegal immigrants successfully enter the work force of unskilled labor in the host country, the production function of host country can be express as \( f(K, L + I(B), H) \). The government of host country finances the border control by levying income tax on these three domestic production factors, capitals, skilled labors and unskilled labors. Because the illegal workers are excluded from the tax base, the expenditure of border control can be expressed as

\[ B = n[f(\cdot) - I(B)f_L]. \]
Actually, \( t(B) = B[f(.) - I(B)f_L(.)] \), and hence the tax rate is a function of the expenditure of border control.

If there is an interior solution for truthful Nash equilibrium, \( B^\circ \), the equilibrium enforcement of any TNE, satisfies

\[
B^\circ = \arg \max_{B \in [0, \infty)} [1 - t(B)][H_{H'}(.) + Lf_L(.) + \theta[f(.) - I(B)f_L(.)]]. \tag{4.21}
\]

Then the first order condition of (4.21) with respect to border control is

\[
[1 - t(B)][H_{H'}I'(B) + Lf_LI'(B) + \theta I(B)f_LI'(B)]
- t'(B)[H_{H'}(.) + Lf_L(.) + \theta[f(.) - I(B)f_L(.)]]
= 0. \tag{4.22}
\]

Now, let \( B^* \) is the border enforcement that satisfies \( I(B) = M^{\text{fr}} \), and thus

\[
[1 - t(B^*)][H_{H'}I'(B^*) + Lf_LI'(B^*) + \theta I(B^*)f_LI'(B^*)]
- t'(B^*)[H_{H'}(.) + Lf_L(.) + \theta[f(.) - I(B^*)f_L(.)]]
\]
\[
= -t'(B^*)[H_{H'}(.) + Lf_L(.) + \theta[f(.) - I(B^*)f_L(.)] < 0. \tag{4.23}
\]

Looking (4.10) and (4.23), we can make a comparison of the equilibrium illegal immigrants with the pseudo-political level of legal temporary immigrant.

**Proposition 1.** If the lobbies from skilled labor and unskilled labor use contribution schedules that are differentiable around the equilibrium point, then the equilibrium illegal immigrants will be greater than the pseudo-political level of legal temporary immigrant.

This proposition simply states that: Since the border control is resource consuming, increasing border control always lifts tax rate and hence decreases the income of the host country. Considering the negative marginal effect of border control on the resource of host country, the government will set a looser immigrant constraint than the pseudo-political level of legal temporary immigrants.
4.4.2 Legal Temporary Immigration and Border Control

Suppose that the government of host country allows a certain level of legal immigrants for entry and chooses expenditures $B$ to control the inflow of illegal foreign workers. The production function of host country can be express as $f(K, L + M + I(B), H)$. The unskilled labors include three parts, native labors $L$, legal immigrants $M$, and illegal immigrants $I(B)$. Given the level of legal immigrants, $M$, the number of illegal immigrant is equal to

$$I(M, B) = R(f_L)[1 - P(B)].$$

So, the marginal illegal immigrant level with respect to border control and the marginal illegal immigrant level with respect to legal immigrant level respectively are

$$I_M(M, B) = \frac{R'(f_L)[1 - P(B)]}{1 - R'(f_L)[1 - P(B)]} < 0,$$

$$I_B(M, B) = \frac{-R'(f_L)(B)}{1 - R'(f_L)[1 - P(B)]} < 0.$$

Since increasing the immigrants allowed for entry decreases the wage of the host country, the marginal illegal immigrant level with respect to legal immigrant level is negative.

The Government finances the border control by levying tax on the domestic production factors. The expenditure of border control can be expressed as

$$B = t\{f(.) - [M + I(B)]f_L\}.$$  

Specifically, the tax rate can be written as $t(M, B) = B/\{f(.) - [I(B) + M]f_L\}$, and hence the marginal tax rate with respect to border control is

$$t_B = t(\frac{1}{B} + \frac{[M + I(B)]f_L I_B}{f(.) - [I(B) + M]f_L}) > 0.$$  \hfill (4.25a)

Moreover, the marginal tax rate with respect to the legal immigrant is
\[ t_M = t(\frac{M + I(B)(f_{uu}I_M + f_{uL})}{f(\cdot) - [I(B) + M]f_L}) < 0. \]  

(4.25b)

Since increasing the legal immigrants increases the income of host country, the marginal tax rate with respect to the legal immigrant is negative. Plugging (4.25a) into (4.25b) gives

\[ t_M = \frac{f_{uu}}{f_L} \left( \frac{M}{B} - \frac{t}{B} \right) + \frac{t}{[f(\cdot) - [I(B) + M]f_L]} . \]  

(4.25c)

Under the process of lobbying, the government decides the equilibrium level of legal immigrants and the equilibrium border enforcement. If there exists an interior solution in equilibrium, the equilibrium border enforcement of any TNE \( B^0 \) satisfies

\[ B^0 = \arg \max_{B \in (0, \infty)} (1 - t(B))\{Hf_{uu} + Lf_L + \theta(f(\cdot) - [I(B) + M]f_L)\} . \]  

(4.26)

The first order condition of (4.26) with respect to border control is equal to

\[ (1 - t(B,M))(Hf_{uu}I_M + Lf_{uL}I_L + \theta[I(B,M) + M]f_{uu}I_M) \]
\[ - t \left( Hf_{uu} + Lf_L + \theta[f(\cdot) - [I(B,M) + M]f_L} \right) \]
\[ = 0. \]  

(4.27)

Therefore, we can find an equilibrium border control \( B(M) \) which is a function of legal immigrant. Further, we try to find an equilibrium level of legal immigrant. Taking differential of the TNE objective function with respect to legal immigrant of unskilled labor, we have

\[ (1 - t(B,M))(Hf_{uu}I_M + Lf_{uL}I_M + \theta[I(B,M) + M]f_{uu}I_M) \]
\[ + (1 - t(B,M))(Hf_{uu} + Lf_{uL} + \theta[I(B,M) + M]f_{uu}) \]
\[ - t \left( Hf_{uu} + Lf_L + \theta[f - [I(B,M) + M]f_L} \right) \]  

(4.28)

Rearranging (4.28) and plugging (4.27) and (4.25c) into (4.28), we can simplify (4.28) to be
\[
\left(\frac{I_M}{I_B} + \frac{1}{I_B}\right) \frac{t}{B} \left\{Hf_H + Lf_L + \theta (f - [I(B,M) + M]f_L) \right\} < 0.
\]

Since
\[
\frac{I_M}{I_B} + \frac{1}{I_B} = \frac{1}{I_B} \left( \frac{R'()f_{ul}[1-P(B)]}{1-R'()f_{ul}[1-P(B)]} + 1 \right) < 0
\]
and
\[
\frac{t}{B} \left\{Hf_H + Lf_L + \theta (f - [I(B,M) + M]f_L) \right\} > 0,
\]
we have
\[
\left(\frac{I_M}{I_B} + \frac{1}{I_B}\right) \frac{t}{B} \left\{Hf_H + Lf_L + \theta (f - [I(B,M) + M]f_L) \right\} < 0.
\]

It means that the cost of deterring an additional illegal immigrant from entry cannot be covered by the benefit of allowing an additional legal immigrant for entry. So, if the government chooses a legal immigrant level at arbitrary \(M^p\) and sets the border open, this kind of policy is the same as open border for foreign unskilled workers, and then native unskilled labor will be no better than the situation that no political contribution is made.

**Proposition 2.** Because the cost of deterring an illegal immigrant cannot be covered by the benefit for allowing an additional legal immigrant for entry, the policy of allowing limited legal immigrants for entry cannot coexist with border control. Since no unskilled foreign applicants will be admitted to enter legally, for the host country the only remaining immigration policy-decision is to set up an optimal border control.
There is a corner solution, for which the incumbent politician is to set up the level of legal unskilled immigrants at zero. As a result, the possible choice for the government is to set up optimal border control to deter foreign unskilled labor from crossing border.

4.4.3 Border Control and Imposing Lump-Sum Tax on Legal Immigrants

From the analysis of above, we recognize that it is impossible to find the compatibility between border control and allowing a certain level of legal foreign labor for entry. However, under the case in which the host country's government imposes a lump-sum tax $T$ on legal unskilled immigrant, allowing a certain level of legal immigrant for entry and border control may be compatible. We assume that legal immigrants should pay the lump sum tax immediately after they get the admission to host country. If $T$ is less than the expected cost of illegally crossing the border, the foreign workers of being legally admitted to enter will be willing to pay.

Suppose that the tax revenue levied from legal immigrants' lump-sum tax is contributed to the border control. The tax base for border control can be expressed as

$$B = MT + t(f(.) - (M + I(B)f_L)).$$

Obviously, $t(B,M,T) = (B - TM)/(f(.) - (M + I(B)f_L))$, and hence the tax rate is a function of the expenditure of border control and the tax revenue collected from legal immigrant. Thus the marginal tax rate with respect to border control is

$$t_B = t\{-\frac{1}{B - TM} + \frac{M + I(B)f_L I_B}{f(.) - (M + I(B)f_L)}\} > 0,$$  \hspace{1cm} (4.29a)

and the marginal tax rate with respect to legal immigrant level is

$$t_M = t\{-\frac{T}{B - TM} + \frac{[M + I(B)f_L (I_M + 1)]}{f(.) - (M + I(B)f_L)}\} < 0.$$  \hspace{1cm} (4.29b)
Since increasing the legal immigrants can increase the host country's tax revenue and income, the marginal tax with respect to the legal immigrant is negative. Plugging (4.29a) into (4.29b) gives

\[
t_M = \frac{I_M}{I_B} t_B - \frac{t}{B - TM} \left( \frac{I_M}{I_B} + T \right) + \frac{t(M + I(B))f_L}{f(\cdot) - (M + I(B))f_L}.
\]

(4.29c)

If there exists an interior solution for truthful Nash equilibrium, the equilibrium immigrant policy vector of any TNE \((B^*, M^*)\) satisfies

\[
(B^*, M^*) = \arg \max_{B \in (0, \infty), M \in (0, \infty)} \left\{ 1 - t(B, M, T) \left( Hf_H(\cdot) + Lf_L(\cdot) + \theta \{ f(\cdot) - \{1(B) + M\} f_L(\cdot) \} \right) \right\}.
\]

(4.30)

The first order condition of (4.30) with respect to border control is

\[
(1 - t)\{ Hf_H(\cdot) + Lf_L(\cdot) + \theta \{ f(\cdot) - \{1(B) + M\} f_L(\cdot) \} \} I_B
- t_B \{ Hf_H(\cdot) + Lf_L(\cdot) + \theta \{ f(\cdot) - \{1(B) + M\} f_L(\cdot) \} \} = 0,
\]

(4.31)

and the first order condition of (4.30) with respect to the level of legal immigrants is

\[
(1 - t)\{ Hf_H(\cdot) + Lf_L(\cdot) + \theta \{ f(\cdot) - \{1(B) + M\} f_L(\cdot) \} \} I_B
- t_M \{ Hf_H(\cdot) + Lf_L(\cdot) + \theta \{ f(\cdot) - \{1(B) + M\} f_L(\cdot) \} \} = 0.
\]

(4.32)

Rearranging (4.32) and plugging (4.31) into (4.32), we have

\[
\frac{t}{B - TM} \left( \frac{1 + I_M}{I_B} + T \right) \{ Hf_H(\cdot) + Lf_L(\cdot) + \theta \{ f(\cdot) - \{1(B) + M\} f_L(\cdot) \} \} = 0.
\]

(4.33)

In fact, equation (4.33) is equal to

\[
-1 \frac{1}{(B - TM)I_B} = \frac{T}{B - TM} + \frac{I_M}{(B - TM)I_B}.
\]

(4.34)
Proposition 3. Suppose that the government of host country imposes a lump-sum tax on legal unskilled immigrants, which is less than the expected cost of illegally crossing border. If the tax is so high that in equilibrium the marginal cost of deterring an illegal immigrant from entry can be covered by the benefit of allowing a legal immigrant for entry, it can be observed that a host country permits legal entry of foreign labor and simultaneously deters the foreign labor from illegal entry.

The left-hand side of (4.34) is the cost of deterring an additional illegal immigrant from entry to allow an extra legal immigrant's entry. The right hand side of (4.34) is the benefit of allowing an extra legal immigrant for entry. The first part of right hand side in (4.34) is accrued by the reduction of foreign labors' incentive of illegal entry because allowing an extra legal immigrant's entry lowers the host country's unskilled labor's wage. The second part of (4.34) is the lump-sum tax collected from a legal immigrant.

4.5 Government's Bargaining Power and Immigration Policy

The model of common agency has an extremely unrealistic feature: When there are more than two lobbies in an economy, the government seizes all the surplus of lobbying process. On the contrary, when there is only one lobby in the economy, the interest group captures all the surplus. Since there are more than two interest groups in our previous analysis, the government has the monopoly power in the relation to the interest groups. To undo this uncommon restriction, we adopt the model of Harris and Todaro (1970), in which the economy comprises a developed urban sector and a less-developed rural sector. We also follow the work of Maggi and Rodriguez-clare (1997) and portray the deal between the government and the interest group as a Nash bargaining process. In this section, what the relative bargaining power of the government in the lobby-government negotiation affects the immigration policy is analyzed.
4.5.1 The Economic Structure

The economy is composed of two sectors, urban (U) and rural (R), and there are two production factors of production, labor and capital. The total population of native labor is fixed and equal to \( L \). The capitals are sector-specific and fixed; \( K_U \) and \( K_R \) denote the sector specific capitals for urban sector and for rural sector respectively. The economy is small in the sense that it can not influence world wage \( w^* \).

The native labors can move cross sectors in the long run but not in the short run. Let \( L \) denote the native labors located in rural sector. The production technologies in the both sectors are constant-return to scale. The production function of urban sector is \( Q^U = F(K_U, L - L) \) and the production function of the rural sector is \( Q^R = G(K_R, L) \).

We can write the marginal labor productivity in the urban sector as \( \phi(L) = F_L \) where \( F_i \) denotes the partial derivative of \( F \) with respect to factor \( i \). Diminishing returns to labor in the urban sector implies \( \phi'(L) > 0 \).

Here, we assume that the foreign immigrants can work for the rural sector but not for the urban sector. If there is no immigrant in this economy, in equilibrium, \( F_L(K_U, L - L^*) = G_L(K_R, L^*) \) where \( L^* \) is the equilibrium level of native labors that work in the rural sector (see Figure 4.1).

If there are \( l \) units of foreign labors in this economy, the total labors in the rural sector are \( l + L \). If there is no lobbying process within this economy, the objective function of host country's government is taken to be the aggregate income of all native agents in this economy. The aggregate income can be written as

\[
Q(L, l) = F(K_U, L - L) + G(K_R, L + l) - lG_L(K_R, L + l).
\]
Figure 4.1. Wage curves and equal-income curve.
The aggregate income of host country is equal to the output of the economy minus the payment to foreign labors.

4.5.2 The Political Structure

We assume that the native labors in the rural sector organize as a labor union; they are able to solve the free-rider problem and form a lobby. However, the native labors in the urban sector and capitalists in the both sectors fail to do so. The union of the rural sector lobbies the government to regulate the entry of foreign labor. Here, we assume that the government can totally control the amount of immigrants. To compensate the government for setting quantitative constraints on the entry of foreign labor, the lobby collects contribution from the native labors of rural sector according to the number of immigrants permitted. Suppose that the government does not allow for any immigrant entry, then the labor union will reward the government contribution $B(L)$, which is a function of the native labors located in the rural sector. If the government allows $l$ units of foreign labor for entry, the contribution to the government is given by $B(L) - c_l$, where $c$ is the contribution deduction per unit of immigrant entry allowed. The lobby seeks to maximize the welfare of labors in the rural sector. Its objective is to maximize the returns to labor in the rural sector net of contribution:

$$V = LG_L(K_R, L + l) - [B(L) - c_l]. \quad (4.33)$$

To keep the consistency with the previous setup we assume that the government's objective is a weighed average of total welfare and contribution from the lobby:

$$\Psi = Q(L, l) + \alpha [B(L) - c_l]. \quad (4.34)$$
If $a = 0$, the government will set its immigration policy so as to maximize the national income and choose open border. If $a > 0$, however, the government will regulate foreign labors' entry if the appropriate contribution are made available by the lobby.

Here, we assume that the native labors in the rural sector can come together and form an interest group but the native labors in the economy neither overcome the free rider problem nor form an economy-wide interest group. In our simple setting, it is equivalent to assume that there does not exist an economy-wide lobby. Thus, only the short-run lobby influences the government's immigration policy. We also assume that the native labors can make their decisions alone, and can not communicate with the others to coordinate the places they work. We can thus think of the lobby from rural sector as an independent player in the lobby-government negotiation; it takes as given the urban-rural migration decision of native labor.

The timing of the game can be described in two stages. At stage 1, the native labors choose where to work. Each native labor is small so he can not strategically influence the other native labors' decision on work place. The native labors $L$ allocated in the rural sector outline the choices of the native labors in the first stage.

At stage 2, the government and lobby are involved in bargaining the level of foreign labors allowed for entry and contribution. Since the bargaining between government and lobby is occurred in the second stage, the government does not have any commitment concerning the immigration policy in relation to the native labors. The level of immigrants allowed for entry and the contribution are determined via the Nash bargaining process between the government and the lobby. The disagreement point $(d_1, d_2)$ is assumed to be the existing state of affairs, in which the lobby contributes
nothing to the government and the government keeps border open. \( d_1 \) and \( d_2 \) are the values of the status quo for the government and interest group respectively. Let \( L^o \) be the total labors that work in the rural sector when the border is open. Then the disagreement point can be written as

\[
d_1 = F(K_U, L - L) + G(K_R, L^o) - (L^o - L)w^*
\]

and

\[
d_2 = Lw^*.
\]

The lobby and government bargain to solve

\[
\max_{\{\psi, \upsilon\}} \{\psi - d_1\}^\beta \{\upsilon - d_2\}^{1-\beta},
\]

in which \( \beta \in [0,1] \) is the relative bargaining power of the government. Eq. (4.35) is called the generalized Nash product, which is a product of the bargaining participants' gain net of the disagreement point payoff. Eq. (4.35) can decide an efficient outcome; however, the bargaining power \( \beta \) decides the distribution of the gain in the lobby-government negotiation.

Here, we assume that the government lasts long enough to care about the long run state of the economy. For a government that can maintain its power for a long period, the incumbent politicians are involved their chance of being elected. Thus, the government’s objective function must include the political contributions and the welfare of general publics; they are the significant factors deciding the chance of reelection in the modern democracy.

4.5.3 The Political Equilibrium and the Relative Bargaining Power of Government

First, let us determine the equilibrium urban-rural migration of native labors under open border, which will be an reference for comparing with political equilibrium. Since the return to the labor allocated to the urban sector is increasing in \( L \), the curve \( \phi(L) \) is
upward sloping and intersects the horizon line \( w^* \) at most once. Moreover, since the production technology is constant return to scale, \( G_L(K_r, L) \) is downward sloping and intersects the horizon line \( w^* \) at most once. We assume that \( \phi(0) < w^* < \phi(L) \) and \( G_L(K_r, L) < w^* < G_L(K_r, 0) \) to ensure that the both curves have an interior intersection with the horizon line \( w^* \). The level at the intersection for the curve \( \phi(L) \), denoted by \( L^* \), represents the equilibrium allocation of native labor under open border (see Figure 4.1).

If the government sets up the open border policy, the wage in the rural sector is equal to \( w^* \). Some of the native labors will move to the urban sector until the wages of both sectors are equal. When the labors in the rural sector can come together to form the lobby, they shall lobby the government to set up a quantitative constraint on the inflow of foreign labors. The presence of the rural sector's lobby leads to that the wage in the rural sector should be higher than \( w^* \); otherwise, the lobby will not contribute anything to the government. Under the lobbying process, the native labors settle down in the rural sector will be more than under open border. However if the government's gain in the lobby process is too small, the government may open border to increase the country's income.

As usual, to solve a two stage game we proceed by backward induction. In the second stage, given the native labors' rural-urban migration choices (summarized by \( L \)), the first order conditions of (4.35) are

\[
\begin{align*}
\beta(-L G_{LL} - ac)(V - d_2) + (1 - \beta)(LG_{Lx} + c)(\Psi - d_1) &= 0, \\
- \beta a_l(V - d_2) + (1 - \beta)(\Psi - d_1) &= 0.
\end{align*}
\]

Rearranging the first order conditions gives the outcome of the bargaining, \( \{\tilde{I}(L), \tilde{C}(L)\} \).

The equilibrium level of immigrants maximizes
\[ Q(l, L) + aG_L(K_R, L + l)L. \] (4.37)

The expression of (4.37) can be interpreted as a social welfare function that weighs different members of society differently: The native labors in the rural sector receive a weight of \( 1 + a \) and the others receive the smaller weight of one. This is consistent with the expression in Grossman and Helpman (1994). The first order condition of maximizing (4.37) gives

\[ \bar{T} = aL. \] (4.38)

The schedule \( \bar{T}(L) \) can be interpreted as the short-run equilibrium immigrants allowed for entry, given the allocation \( L \) of native labors. The level of immigrants allowed for entry is proportionally increasing in the size of native labor located in rural sector. This result is a direct consequence of the fact that \( \bar{T} \) maximizes the weighed objective function (4.37). If \( a = 0 \), the function in (4.37) is simply the aggregate income, which is maximized by opening border. If \( a > 0 \) the gross income component of rural sector’s native labors, \( G_L L \), is assigned an extra weight in (4.37). More labor located in rural sector implies a higher marginal labor income loss of allowing immigrant for entry. To compensate this loss, the aggregate income must increase via increasing the labor in the rural sector. As a result, the optimal \( \bar{T} \) increases with \( L \).

The contribution can be express as a weighed sum of the aggregate income loss from an quantitative constraint on the entry of foreign labor and the lobby's willingness to pay for the government's immigration policy:

\[ B(L) - \bar{T} = \beta[L[G_L(K_R, L + \bar{T}) - w^*]] + \frac{1}{a}[(1 - \beta)[Q(L, \bar{T}^*) - Q(L, \bar{T})]] \] (4.39)
where \( l^o = L^o - L \) (see Figure 4.1). Note that if \( \beta = 0 \), the contribution is just enough to compensate the government for the income loss caused by regulating the entry of foreign labors. On the other hand, if \( \beta = 1 \) the government obtains the all surplus derived by the native labor of rural sector via restricting the entry of foreign in a certain level; the surplus is equal to \( L(G_L - w^*) \). Dividing (4.37) by \( L \) gives

\[
\tilde{d}(L) = \frac{B(L) - \beta \tilde{I}}{L}
\]

\[
= \frac{1}{L} \{ \beta \{ L[G_L(K_R, L + \tilde{I}) - w^*] \} + \frac{1-\beta}{aL} [Q(L, l^o) - Q(L, \tilde{I})] \}
\]

\( \tilde{d} \) is the payment of each native labor, who works in the rural sector, to the contribution.

In the first stage, the native labors have their expectation about the immigration policy and contribution, which will be selected. The native labors migrate to the sector where they expect they can earn higher income. At a subgame-perfect equilibrium, the incomes of the native labors in the both sectors must be equal and hence the native labors have no incentive to migrate across sectors.

Under the lobbying process, the wage in the rural sector is equal to \( G_L(K_R, (1 + a)L) \) given the allocation of native labor \( L \). In the long run equilibrium the returns of native labor in the both sectors must be equal; this income of the native labor is the one that solve

\[
G_L(K_R, (1 + a)L) - \tilde{d}(L) = \phi(L).
\]

Plugging (4.40) into (4.41) yields the following expression for the equal-income curve:

\[
G_L^* = \phi(L) + \frac{1}{L} \{ \beta \{ L[G_L(K_R, L + \tilde{I}) - w^*] \} \} \\
+ \frac{1-\beta}{aL} [Q(L, l^o) - Q(L, \tilde{I})].
\]
If $G_L(K_R, (1 + a)L) > G_L^*$, the labors of the rural sector have higher income than those of the urban sector do; the opposite happens when $G_L(K_R, (1 + a)L) < G_L^*$.

For $\beta = 1$,

$$G_L - G_L^*(L) = w^* - \phi(L).$$

Since $\phi(0) < w^*$,

$$\lim_{L \to 0^+} (G_L - G_L^*(L)) > 0.$$

In Figure 4.1 we have drawn the short-run equilibrium wage curve and the equal-income curve, which the government has all the bargaining power. Since

$$G_{LL} < \frac{\partial G_L^*}{\partial L}$$

for all $L > 0^*$, these curves intersect once. Let $\hat{L}$ denote the level of $L$ at the intersection. Since $\hat{L}$ is the only allocation of native labor for which the short-run equilibrium labor income are equal across sectors, $\hat{L}$ constitutes the unique equilibrium allocation. The long-run equilibrium domestic labor income is then given by $G_L(K_R, (1 + a)\hat{L})$.

However for $\beta < 1$ the short-run equilibrium wage curve and the equal-return income curve may intersect not just once. Let $\hat{X}$ denote the set of these intersections. For $\hat{L} \in \hat{X}$ and $\hat{L} < L^*$, $\phi(\hat{L}) < w^*$, so in this case $\hat{L}$ is impossible to be an equilibrium urban-rural migration. If the rural sector's labor income is less than $w^*$, the labor union of the rural sector will not contribute anything to the government. So, the equilibrium allocation $\hat{L}$ of native labor must be greater than $L^*$.

Assumption 1. $G_{LL} \leq \frac{\partial G_L^*}{\partial L}$ for $L \geq L^*$ and $G_L \succeq G_L^*$ for $L = L^*$.
To make this assumption holds, the marginal product of labor in the urban sector decreases fast enough to dominate the possible change in the marginal individual payment of the rural sector to the contribution. This assumption guarantees the curve \( G_L(K_R, (1 + a)L) \) intersect once for \( L \geq L^* \). So \( \hat{L} \) is the only equilibrium allocation of the native labors, for which the short run equilibrium incomes of labor in the both sectors are equal; that is, \( \hat{L} \) makes up the unique equilibrium allocation of the native labors. The long run equilibrium income of the native labors is then given by \( \phi(\hat{L}) \).

Now, we give some intuitive interpretations for the fact that the equilibrium level of the native labor in the rural sector is greater under the lobbying process than under open border. Suppose for a moment that the allocation of labor is given by \( L = L^* \). At this allocation, the return to labor would be equal across sectors if the border is open. Since a small amount of quantitative restriction on the entry of foreign labor causes a second-order aggregate income loss but first-order gain for the lobby, the lobby can profitably buy the regulation of immigration from the government at a low expense. Therefore, the lobbying activity makes the income of the rural sector's native labors higher than the income of the urban sector's native labors; that is, \( G_L(K_R, (1 + a)L) > G_L^* \). As a result, the native labors in the urban sector have incentive to migrate to the rural sector. The migration from the urban sector to the rural sector drives the native labors \( L \) allocated in the rural sector above \( L^* \). When the rural sector is organized to lobby the government, the government immigration policy will make the level of native labors in rural sector larger than open border. Rearranging (4.41) gives
\[ G_L^* = \phi(L) + \frac{1}{aL} [Q(L, L^*) - Q(L, \bar{L})] \]
\[ + \frac{1}{L} \beta [L[G_L(K_R, L + \bar{L}) - w^*] + \frac{1}{a} [Q(L, L^*) - Q(L, \bar{L})]]. \]

So, if
\[ L[G_L(K_R, (1 + a)L) - w^*] > \frac{1}{a} [Q(L, L^*) - Q(L, \bar{L})] \] (4.43)

for \( L > L^* \), the unique political equilibrium level of immigrants is decreasing with \( \beta \). Eq. (4.43) simply states that the rural native labors' surplus under the lobby process is greater than the weighted loss from restricting the entry of foreign labor.

Consider the extreme case \( \beta = 1 \). In equilibrium, we have
\[ G_L^* = \phi(L) + [G_L(K_R, (1 + a)L) - w^*] = G_L(K_R, (1 + a)L) ; \] (4.44)

it implies \( \phi(\hat{L}) = w^* \). Thus starting at the open border, there is no migration between the two sectors; the level of native labors in the rural sector will remain at the open border size, \( L = L^* \). This happens because when the government has all the bargaining power, the native labors in the rural sector get the wage \( G_L(K_R, (1 + a)L^*) \) at the expense of their lobbing surplus. On the other hand, notice that the wage of native labor in the rural sector is equal to \( G_L(K_R, (1 + a)L^*) \), which is higher than \( w^* \). But the lobby pays \( G_L(K_R, (1 + a)L^*) - w^* \) for restricting the immigrant level at \( aL^* \). This suggests that when the government is strong in relation to the lobby, the rural sector still has quantitative constraint on the entry of foreign labors, but the allocation of native labor will be the same as under open border. The next proposition summarizes these findings.
Proposition 4. If \( L[G_L(K_R, L + \tilde{T}) - w^*] > \frac{1}{a}[Q(L, L^*) - Q(L, \tilde{T})] \) for \( L > L^* \), then (i) the equilibrium allocation is decreasing in \( \beta \), with \( \hat{L} > L^* \) for \( \beta \in [0, 1) \) (ii) the equilibrium wage \( G_L(K_R, (1 + \alpha)\hat{L}) \) is increase in \( \beta \), with \( G_L(K_R, (1 + \alpha)\hat{L}) > w^* \) for all \( \beta \) in \([0, 1)\). If \( \beta = 1 \) then \( \hat{L} = L^* \), the net income of individual native labor is equal to \( w^* \).

The bargaining power plays an important role for the determination of immigration policy. In the short run, the bargaining power has no impact on the equilibrium immigrant policy; it affects only the distribution of the surplus between government and the rural sectors' native labor. When the migration of the native labors between rural sector and urban sector is possible in the long run, the distribution of surplus and the determination of immigration policy are no longer separate. In the long run equilibrium the incomes of native labors should be equal. A change in the government's bargaining power affects the allocation of native labors, and hence the equilibrium level of immigrants is affected. In particular a higher bargaining power on the part of the government leads to less native labors, who work in the rural sector, and there are less foreign labor allowed for entry.

4.5.4 The Value of Commitment to Open Border

Suppose now that the government has a choice to commit to open border at early stage say time zero. Assume that the lobby from rural sector starts operating only after the native labors settle down in the both sectors, so that there is no lobby at time zero. To examine the value of commitment to open border, we derive the government's bliss in the political equilibrium and compare it with the bliss it would derive by committing to open border. The bliss of the government in the political pressure can be derived from Eqs. (4.34), (4.37) and (4.39):
\[ \Psi^p = Q(\hat{L}, \hat{L}^*) + \beta[(1 + a)\hat{L}(G_L(K_R, (1 + a)\hat{L}) - \omega^*) - K_R[r - G_k(K_R, (1 + a)\hat{L})]] \] (4.45)

where \( \hat{L}^* = L^* - \hat{L} \) and \( r = G_k(K_R, L^*) \). Conversely, if the government commits to open border, its bliss is given by \( \Psi^o = Q(L^*, l^*) \) where \( l^* = L^* - L^o \). To understand the relationship between \( \Psi^p \) and \( \Psi^o \), let us first consider two extreme cases.

First, for the case \( \beta = 0 \) from proposition 1 we know that in the political equilibrium the allocation of native labor is given by \( \hat{L} > L^* \). Since \( Q(\hat{L}, \hat{L}^*) \) has highest value at \( \hat{L} = L^* \), (because increasing immigrant always increases aggregate income), it follow that \( \Psi^o \) is higher than \( \Psi^p \).

Next, in this case \( \beta = 1 \) the native labor allocation is \( \hat{L} = L^* \). The equilibrium level of native labors allocated in the rural sector is the same as the level under open border. The only difference between \( \Psi^o \) and \( \Psi^p \) is created by the fact that under no commitment of open border the government receives contribution from the lobby, which obviously implies \( \Psi^p > \Psi^o \). Moreover

\[
a\hat{L}[G_L(K_R, (1 + a)\hat{L}) - \omega^*] - K_R[r - G_k(K_R, (1 + a)\hat{L})] \\
= a\hat{L}[G_L(K_R, (1 + a)\hat{L}) - \omega^*] - \frac{1}{a}[Q(\hat{L}, l^*) - Q(\hat{L}, \hat{L}^*)].
\]

If (4.43) holds \( \Psi^p \) is directly increasing with \( \beta \). Under the condition (4.43) \( \hat{L} \) is decreasing in \( \beta \) and \( \Psi^p \) is increasing in \( \beta \) given \( \hat{L} \). It is easily for us to argue that \( \Psi^p \) is continuous function of \( \beta \), thus we claim the following proposition:

**Proposition 5.** If \( L[G_L(K_R, L + \hat{L}) - \omega^*] > \frac{1}{a}[Q(L, l^*) - Q(L, \hat{L}^*)] \) for \( L > L^* \), there exits a level of the government bargaining power, \( \bar{\beta} \) such that \( \Psi^p > \Psi^o \) if and only if \( \beta > \bar{\beta} \).
In this economy, the lobby from rural sector faces no opposition from competing interest. For a common agency model as in Grossman and Helpman (1994), even though the lobby from rural sector captures the all surplus from its political relationship with the government, the government at least keeps a bliss level, which is the same as the one under open border. However, in our model, when the government's bargaining power is limited, the government's bliss may be less under political equilibrium than under open border.

To analyze this, first we consider the case $\beta = 0$. In the second period, given the native labor allocation, the government establishes a quantitative restriction on the entry of foreign labors to benefit the native labors of rural sector and receives bliss, which is the same as that under open border. In the first period, since the native labors rationally expect that the government will restrict the foreign labors' entry on a certain level in the forthcoming period, they migrate to the rural sector. If the economy is immune to the lobby activity, the government will open border and the allocation of the native labors is equal to $L^\star$. The allocation of native labor $\hat{L}$ under rational expectation is greater than $L^\star$. As a result, the government's current bliss under open border is different from the bliss in which there is no political pressure on the government. In fact, the government's bliss is less when the native labors anticipate that the lobbying process affects the immigration policy than when there is no lobby activity in the economy.

When the government has some bargaining power $\beta > 0$, the government can collect political money by restricting the entry of foreign labors at a certain level. The government must face a trade-off between the political money collection and the aggregate income loss. The immigration policy pushed by the lobbying process leads to
three effects: (a) A production loss in the rural sector is generated by forbidding foreign labor moving in; since the government's reservation bliss is given by the open border given $\hat{L}$, this is always compensated by the lobby's contribution. (b) The Government receives contribution in excess of the bliss that the border is open, according to its bargaining power; the extent of these rents represents the government's cost of opening border. (c) An allocation of native labors, which is greater than $L^*$, arises in the long run. This allocation makes the aggregate income less than the aggregate income when the allocation of native labor is $L^*$. This kind of loss presents even if the bargaining between lobby and government breaks down, so the government does not get compensated for it. Thus the loss represents the cost of not opening border. The effect b is opposite to the effect c. The government weighs the two effects, and a higher $\beta$ inclines to set a certain level of quantitative constraint on the entry of foreign labor.

Now, we consider how the bargaining power $\beta$ affects the equilibrium level of immigrants allowed for entry when the government can exercise the option of opening the border. If $\beta$ is low, the government allow the foreign labor for free entry. If $\beta$ is higher than a critical level, the government chooses not to commit to open border and the native labors receive higher wage than $w^*$. The foreign labors allowed for entry are decreasing with $\beta$. Also the net income of the native labor is decreasing with $\beta$.

4.6 Conclusion

In this chapter we have incorporated the political model of common agency, developed by Grossman and Helpman (1994), into the model Ethier (1986), which works on the matters of illegal immigration. This kind of setup is to explain the issue concerning the fact that a host country permits the legal entry of foreign labor and simultaneously
deters illegal entry of foreign labor. First, we argue that if all the factors have their interests represented by the lobbies, the political equilibrium is social optimum, in which the national income is maximized and there is no restriction on the entry of foreign labor. Second, it is shown that if not all the production factors' interests are presented by the lobbies, the political equilibrium is to set up a quantitative restriction on the inflow of foreign labors. Moreover, we confirm that if the marginal cost of deterring illegal immigrant can not be covered by the marginal benefit of allowing legal immigrant for entry, the immigration policy of host country is to ban legal entry of foreign labors and set up an optimal border control. However, if a high enough lump-sum tax is imposed on the legal immigrants, the policy, setting an optimal border control to deter the foreign labors from illegal entry and simultaneously allowing certain level of legal foreign labor for entry, is the political equilibrium.

To undo the unrealistic aspect of common agency model that the government has the monopoly power in relation to several interest groups, we further extend the one sector setting to multi-sector and adopt the Nash bargaining process to examine the immigration policy when the government have varied bargaining power. We show that in the short run the bargaining power only affects the distribution of the surplus between government and lobby in the political process. However, the relative bargaining power of the government affects the immigration policy in the long run. Moreover, we also argue that the government may commit the policy of opening border to increase the aggregate income and forgo the political contribution. The main reason is that committing to open border increase the aggregate income; this benefit may outweigh the rent that the government derives from the political process.
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Poll: "Most say Clinton acted illegally or unethically". Allpolitics, Oct. 8 1997. CNN.

APPENDIX. THE PROOF OF PROPOSITION 1 IN CHAPTER 3

To obtain a feedback characterization of the open loop equilibrium, we guess a linear feedback representation of the controls to be

\[ x^0 = f_0 + f_1 p, \]

\[ y^0 = g_0 + g_1 p. \]

The substitution of Eqs. (3.5) into the differentiation of Eqs. (A1) with respect to time, and the comparison of the coefficient in the differentiation of Eqs. (A1) yield the following system of equations:

\[ f_1 (g_0 - f_0) = rf_0 + \frac{rc_1}{c_2} - \frac{a_1}{c_2}, \]

\[ f_1 (g_1 - f_1) = rf_1 - \frac{a_2}{c_2}, \]

\[ g_1 (g_0 - f_0) = rg_0 + \frac{rd_1}{d_2} - \frac{b_1}{d_2}, \]

\[ g_1 (g_1 - f_1) = rg_1 - \frac{b_2}{d_2}. \]

The ratio between (A2.2) and (A2.4) yields

\[ f_1 = -M g_1, \]

where \( M = d_1 a_2 / b_2 c_2 \). Plugging (A3) into (A2.4) gives

\[ (1 + M) g_1^2 - rg_1 - \frac{b_2}{d_2} = 0. \]

Solving (A2) gives

\[ g_1 = \frac{r \pm \sqrt{r^2 + 4(1 + M) \frac{b_2}{d_2}}}{2(1 + M)}. \]
Only the smaller root in (A5) leads to a stable solution. Since the constant terms \( g_0 \) and \( f_0 \) are irrelevant to our proof of proposition 1 in Chapter 3, all we need here is coefficients \( f_1 \) and \( g_1 \). Because (3.7) ensures stability of dynamic system (3.5), we can rewrite the optimal fund raising strategies for the both parties as

\[
x^0 = x_0^0 + f_1 (p - p_e^0),
\]

\[
y^0 = y_0^0 + g_1 (p - p_e^0).
\]

Specifically,

\[
g_1 = \frac{r - \sqrt{r^2 + 4(1 + \frac{d_2 a_2}{b_2 c_2}) \frac{b_2}{d_2} \frac{b_2}{b_2 c_2} \frac{b_2}{d_2} < 0}}{2(1 + \frac{d_2 a_2}{b_2 c_2})} \tag{A6}
\]

and

\[
f_1 = - \frac{d_2 a_2}{b_2 c_2} \frac{r - \sqrt{r^2 + 4(1 + \frac{d_2 a_2}{b_2 c_2}) \frac{b_2}{d_2} \frac{b_2}{b_2 c_2} \frac{b_2}{d_2} > 0}}{2(1 + \frac{d_2 a_2}{b_2 c_2})} \tag{A7}
\]
Far-tsair Lai was born and grew up in Taiwan, a wonderful country. He is proud to be a Taiwanese.

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