1999

Spectrum of Solar Neutrinos Above 6.5 MeV.

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SPECTRUM OF SOLAR NEUTRINOS
ABOVE 6.5 MEV

A Dissertation
Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy
in
The Department of Physics and Astronomy

by
Robert Ellis Sanford, Jr.
B.S., Tulane University, 1988
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May 1999
In memory of my father
Robert Ellis Sanford
(1932–1998)
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ABSTRACT

The spectrum of recoil electrons from solar neutrino scattering above 6.5 MeV has been measured using the first 504 days of Super-Kamiokande detector data. The scattering rate is found to be $13.56 \pm 0.42\text{(stat.)} \pm 0.29\text{(syst.)}$ events/day/22.5kton, which is a factor of $0.474 \pm 0.015\text{(stat.)} \pm 0.010\text{(syst.)}$ of the expected rate. The measured spectrum and the expected spectra from $^8\text{B}$ and HeP Neutrino scattering are compared using a $\chi^2$ minimization process to find the best-fit match between the measured neutrino rates and a linear combination of the expected rates from $^8\text{B}$ and HeP neutrinos. Using only the $^8\text{B}$ expected spectrum the fitting procedure results in a best-fit scaling factor of 0.479; that is, the $^8\text{B}$ expected spectrum best matches the measured spectrum if it is scaled by 0.479. This best-fit has a $\chi^2$ value of 19.05 with 15 degrees of freedom, which corresponds to a confidence level of 21.2 %. The ratio of the measured spectrum to the scaled $^8\text{B}$ expected spectrum results in an upturn at higher energies which may be an artifact of statistics or an indication of neutrino oscillations or of some other phenomenon. Using both the $^8\text{B}$ and HeP expected spectra in the fitting procedure results in a $^8\text{B}$ best-fit scaling factor of 0.446 and a HeP best-fit scaling factor of 25.1. This best-fit has a $\chi^2$ value of 12.85 with 14 degrees of freedom, which corresponds to a confidence level of 53.8 %. The larger-than-standard HeP neutrino contribution flattens the upturn at higher energies and, thus, results in a lower $\chi^2$ value. This dissertation describes in detail the analysis that produced these results.
CHAPTER 1

INTRODUCTION

Neutrinos are one of the most interesting and perhaps the most difficult to study elementary particles known today. The “standard model” of particle physics, the well-tested description of the nature of fundamental particles and forces, describes neutrinos as weakly interacting, chargeless, and massless members of the lepton family. Because neutrinos weakly (and thus rarely) interact with normal matter, they have been dubbed “ghost” and “shadow” particles. Three types or “flavors” of neutrinos are thought to exist \(^1\) and are named after the massive lepton with which they are associated: electron \((\nu_e)\), muon \((\nu_\mu)\), and tau \((\nu_\tau)\) flavors. Neutrinos are created by nuclear decays, nuclear reactors, high energy particle accelerators, and the fusion reactions that power the Sun.

Photons are the traditional carriers of information used to study the Sun. However, photons created in the core of the Sun where the fusion processes proceed may take 100,000 years or longer to percolate to the solar surface and escape into space. During that time any interesting characteristics of core reactions, e.g. a time dependence in energy production, would be smeared out. Solar neutrinos on the other hand easily travel through the Sun due to their weak interactions with matter. Solar neutrinos travel essentially at

---

\(^1\)The relation between neutrino flavors and the massive leptons can be written as:

\[ \nu_e + \text{nucleon} \rightarrow e + \ldots \]

\[ \nu_\mu + \text{nucleon} \rightarrow \mu + \ldots \]

\[ \nu_\tau + \text{nucleon} \rightarrow \tau + \ldots \]

---
the speed of light and arrive at Earth within minutes, therefore providing a window into the interior mechanics of the Sun. Because neutrinos are created by the solar fusion reactions, the neutrino fluxes reflect the rates of nuclear fusion reactions, which cannot be measured otherwise.

Optical studies of the Sun provide almost all of what is known about the Sun including: mass ($M_\odot$), radius ($R_\odot$), surface chemical composition, and luminosity ($L_\odot$). The age of the Sun is determined from meteoritic studies. Table 1.1 lists the measured solar parameters. Models of the Sun using these measured parameters have been developed since the 1950's to understand the processes which power the Sun (and other stars). These solar models generally begin with a homogeneous composition of gases with hydrogen “burning” in the core and their equations of state are numerically time evolved with the following assumptions:

- The Sun is in hydrostatic equilibrium i.e. the thermal and radiated luminosity pressures exactly counter the gravitational collapse of the volume.

- Energy transport is provided by photons or convective motion.

- The primary source of energy is nuclear fusion.

- Chemical abundances change solely due to nuclear reactions.

Once time evolved to the current age of the Sun, the predicted parameters (radius, luminosity, etc.) should match observed values. These models combine the standard electroweak model of particle physics with a solar structure
Table 1.1: Measured solar parameters: Numbers given in parentheses indicate one standard deviation uncertainty in the last digits of the proceeding number [2, 3].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$1.989(2) \times 10^{30}$ kg</td>
</tr>
<tr>
<td>Radius</td>
<td>$6.9599(7) \times 10^8$ m</td>
</tr>
<tr>
<td>Luminosity</td>
<td>$3.826(8) \times 10^{26}$ W</td>
</tr>
<tr>
<td>Age</td>
<td>$4.57(2) \times 10^9$ yr</td>
</tr>
</tbody>
</table>

Modeling of stellar evolution by SSMs has been very successful in the field of observational astronomy. Models have provided the relation between stellar luminosity and mass and the basic understanding of the distribution of stars on a Hertzsprung–Russell (H-R) diagram, a stellar luminosity verses color or temperature distribution. The H-R diagram has been hailed as one of the most incisive tools in observational astronomy [2].

Models which successfully time evolve into a system with the current observed properties of the Sun can be used to predict other solar properties, such as solar neutrino fluxes.

---

**Solar models which incorporate entirely new physical effects that are not incorporated in the standard electroweak theory e.g. free quarks or use input parameters beyond the range of recognized uncertainties are called nonstandard solar models.**
**Figure 1.1: PP-Chain: the main nuclear reactions which occur in the Sun while converting protons into α-particles. Also shown are the predicted relative frequencies of each fusion initiating process and each α-particle producing chain endpoint from BP98 SSM.**

### 1.1 Predicted Solar Neutrino Fluxes

The Sun generates energy by nuclear fusion. Figure 1.1 shows the main nuclear reactions which occur in the Sun while converting protons into α-particles. Collectively these reactions are known as the proton-proton or “PP” chain. Labels for individual reactions are listed in parentheses. Figure 1.2 shows the carbon-nitrogen-oxygen (CNO) cycle, which is a circular α-particle producing chain using carbon, nitrogen, and oxygen as catalysts. Note that neutrinos are created in several different reactions in the PP-chain and the CNO cycle.

The relative rates of the PP-chain and CNO cycle reactions are determined by each SSM so as to model the observed solar properties. Thus,
the predicted solar neutrino rates depends upon the SSM used. However, the agreement among different SSMs in the predicted solar neutrino fluxes is within about 2% when the same input parameters are given [6]. In this analysis the Bahcall-Pinsonneault 1998 (BP98) SSM is taken as the representative SSM and will be primarily referenced [7]. The BP98 SSM incorporates helium and metal\textsuperscript{3} gravitational settling into the model, which leads to a larger concentration of helium in the core region than is caused by nuclear processes alone; see Ref. [7] for details.

Also shown in Figure 1.1 in parentheses are the predicted relative frequencies of each fusion initiating process and each $\alpha$-particle producing chain endpoint. Since all modern SSMs result in a CNO cycle energy contribution

\textsuperscript{3}In astronomy metals are any element heavier than helium.
Table 1.2: Predicted solar neutrino fluxes at Earth from BP98 SSM with 1σ uncertainties [7].

<table>
<thead>
<tr>
<th>Reaction Source</th>
<th>Flux ( \times 10^{10} \text{ cm}^{-2} \text{s}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>pp</td>
<td>( 5.94 \times (1.00^{+0.01}_{-0.01}) )</td>
</tr>
<tr>
<td>pep</td>
<td>( 1.39 \times 10^{-2} (1.00^{+0.01}_{-0.01}) )</td>
</tr>
<tr>
<td>hep</td>
<td>( 2.10 \times 10^{-7} )</td>
</tr>
<tr>
<td>(^{7}\text{Be})</td>
<td>( 4.80 \times 10^{-1} (1.00^{+0.09}_{-0.09}) )</td>
</tr>
<tr>
<td>(^{8}\text{B})</td>
<td>( 5.15 \times 10^{-4} (1.00^{+0.19}_{-0.14}) )</td>
</tr>
<tr>
<td>(^{13}\text{N})</td>
<td>( 6.05 \times 10^{-2} (1.00^{+0.19}_{-0.13}) )</td>
</tr>
<tr>
<td>(^{15}\text{O})</td>
<td>( 5.32 \times 10^{-2} (1.00^{+0.22}_{-0.15}) )</td>
</tr>
<tr>
<td>(^{17}\text{F})</td>
<td>( 6.33 \times 10^{-4} (1.00^{+0.12}_{-0.11}) )</td>
</tr>
</tbody>
</table>

of about 1 %, these reactions will not be discussed [8]. Almost 100 % of the fusion reactions begin with the “pp” reaction (upper left of Figure 1.1) with a small fraction initiating from the “pep” reaction (upper right). There are four termination reactions where α-particles are created: “pp-I” (86 %), “pp-II” (14 %), “pp-III” (0.02 %), and the rare “HeP” reaction (0.00002 %). Figure 1.3 shows the corresponding predicted neutrino fluxes for each neutrino-producing reaction in the PP-chain as a function of neutrino energy. (Note the log scales.) The neutrinos from the pp reaction dominate the spectrum. And most neutrinos have energies less than 2 MeV, although energies extend to about 15 MeV for the relatively rare \(^{8}\text{B}\) neutrinos and to almost 19 MeV for the even more rare HeP neutrinos. Table 1.2 lists the integrated predicted solar neutrino fluxes at Earth. This analysis will concentrate upon the higher energy \(^{8}\text{B}\) and HeP neutrinos.
As mentioned earlier the nuclear reaction cross sections, radiative opacity, chemical composition, diffusion coefficients, and observed solar constants are the most significant input parameters in a solar model, and so the uncertainties in the neutrino fluxes originate in the uncertainties of these input parameters. The radiative opacities are sensitive to temperature which are in turn sensitive to the assumed chemical composition. The correlation between the solar core temperature $T_c$ and some neutrino fluxes $\phi$ are \[10\]:

\[
\phi_{pp} \propto T_c^{-1.2} \\
\phi_{7Be} \propto T_c^8 \\
\phi_{8B} \propto T_c^{18}
\]

The pp neutrinos are relatively unaffected by temperature uncertainties, whereas the $^7\text{Be}$ and especially the $^8\text{B}$ neutrinos are highly sensitive.
The dominating uncertainty in the $^8$B flux originates from the uncertainty in the cross section of the $^7Be + p \to ^8B + \gamma$ reaction. Because the pp-III termination reaction only occurs about 0.02 % of the time, this reaction contributes insignificantly to the energy generation of the Sun. So changing the cross section of this reaction results in no observable affect in solar processes other than to modify the $^8$B neutrino flux. The cross sections of solar reactions are generally parameterized by [10]:

$$\sigma(E) = S(E) E^{-1} e^{2\pi \eta(E)},$$

where:

- $\sigma(E)$ = cross section of reaction
- $E$ = center-of-mass kinetic energy of the reaction partners
- $\eta = Z_1 Z_2 e^2 \nu^{-1}$
- $Z_{1,2}e$ = charges of reaction partners
- $S(E)$ = S-factor
- $\nu$ = relative velocity of reaction partners

Measurement of the $S_{17}$-factor (the S-factor of the $^7Be + p \to ^8B + \gamma$ reaction) has only been accomplished at energies higher than those at which the reaction occurs in the Sun ($\sim 20$ keV). Therefore, the $S_{17}$-factor must be extrapolated down to the astrophysical energy regime. Since the S-factor is directly proportional to the reaction cross section, the resulting uncertainty in the $S_{17}$ value from the extrapolation causes a large uncertainty in the $^8$B neutrino flux ($\sim 10$ %).
Once the predicted neutrino fluxes and their uncertainties have been calculated, they can be compared to measured values.

1.2 Solar Neutrino Experiments

Three different types of solar neutrino experiments have thus far been performed and each have different energy thresholds. The three experiment types are chlorine, gallium, and water types, named after their target materials. The top of Figure 1.3 shows the energy regime of each experiment type and thus the types of neutrinos to which each experiment type is sensitive.

Since neutrinos interact with matter only through the weak force, the cross sections for neutrino interactions are very small. Consequently, the solar neutrino detectors are large and massive so as to increase the interaction rates. Solar neutrino detectors are also always located underground to shield the detectors from cosmic and atmospheric particles which can cloak the measured solar neutrino signals.

1.2.1 Homestake Chlorine Detector

The first solar neutrino observations were pioneered by R. Davis et al. with the Homestake solar neutrino detector, so named for the Homestake Gold Mine in Lead, South Dakota, U.S.A. in which it is located [11]. The experiment began operation in 1968 and still operates today. The detector utilizes 615 tons of liquid C\textsubscript{2}Cl\textsubscript{4} to detect the solar neutrinos via the inverse \(\beta\)-decay reaction:

\[
\nu_\text{e} + ^{37}\text{Cl} \rightarrow e^- + ^{37}\text{Ar}
\]  

(1.3)

The isotopic abundance of \(^{37}\text{Cl}\) is 24.23\%. The neutrino energy threshold of this reaction is 0.814 MeV, making this experiment sensitive to \(^7\text{Be}\), \text{pep}, \(^8\text{B}\),
and HeP neutrinos (refer to Figure 1.3). The $^{37}$Ar produced by the neutrino capture are removed by pumping a He carrier gas through the liquid and are counted by a proportional counter which observes the decay of $^{37}$Ar via 2.82 keV Auger electrons (the half-life of $^{37}$Ar is 35.0 days). Exposure times between counting vary from one to three months.

The expected rate of neutrinos using the BP98 SSM for the Homestake experiment is [12]:

$$R_{BP98}^{Home} = 7.7^{+1.3}_{-1.0} \text{ Solar Neutrino Units (SNU)}, \quad (1.4)$$

where:

$$1 \text{ SNU} = 1 \text{ capture per } 10^{38} \text{ target atoms per second.} \quad (1.5)$$

(A SNU is pronounced “snew.”)

The measured rate from 1970 to 1995 is:

$$R_{obs}^{Home} = 2.54 \pm 0.14 \pm 0.14 \text{ SNU}, \quad (1.6)$$

which is about 33 % of the expected rate [13]. The measured rate incorporates data from several exposures and is equivalent to about 14 interactions per month. Some exposures resulted in neutrino interaction rates consistent with zero, and it is unclear whether this is due purely to Poisson statistics. The discrepancy between observed and expected solar neutrino rates is now known as the "Solar Neutrino Problem." About twenty years after the commencement of the Homestake experiment, the second solar neutrino detector began operation to look for a similar deficit.

1.2.2 Kamiokande Water Cherenkov Detector

In early 1987 the Kamiokande solar neutrino detector, the predecessor of the Super–Kamiokande detector, commenced taking data in the Kamioka
Mozumi Mine in Japan [14]. The experiment ran for most of Solar Cycle 22 and ended in 1996. The Kamiokande detector is an imaging Cherenkov light detector, which measures the solar neutrino rate by observing the Cherenkov radiation emitted by recoil electrons after neutrino–electron scattering (this is discussed in more detail in Section 2.1). The 3000 tons of water target are contained in a cylindrically shaped water tank whose walls are about 20% covered with 50 cm photomultiplier tubes (PMTs), which detect the Cherenkov light produced from neutrino interactions.

The recoil electron total energy threshold of the Kamiokande detector began at 9 MeV, but dropped to 7 MeV in mid-1988. Thus, this experiment is sensitive to only the high energy (and relatively rare) \(^{8}\)B and HeP neutrinos. The expected flux predicted by the BP98 SSM for the Kamiokande detector is [12]:

\[
R_{BP98}^{Kam} = 5.15^{+0.98}_{-0.72} \times 10^6 \text{ cm}^{-2}\text{s}^{-1},
\]

while the measured flux is [15]:

\[
R_{obs}^{Kam} = 2.80^{+0.19}_{-0.33} \times 10^6 \text{ cm}^{-2}\text{s}^{-1}.
\]

The measured rate is equivalent to about 14 neutrino–electron scatterings per month.

Kamiokande not only confirmed the solar neutrino problem, but it also showed by using the detector’s timing and direction information that the neutrino interactions it was measuring originated from the Sun.

In the early 1990’s two more solar neutrino detectors of a third type joined the search for the “missing” neutrinos.
1.2.3 SAGE and GALLEX Gallium Detectors

Two gallium-based experiments began in the early 1990's: the SAGE experiment located in Mount Andyrchi in the Caucasus Mountains, Russia (1990) [16] and the GALLEX experiment in the Gran Sasso tunnel near Rome, Italy (1991) [17]. Both experiments detect solar neutrinos via the inverse-beta decay reaction:

\[ \nu_e + ^{71}\text{Ga} \rightarrow e^- + ^{71}\text{Ge}. \] (1.9)

The SAGE detector began with 27 tons of metallic gallium, which was increased to 55 ton in July 1991. The GALLEX detector used 100 tons of GaCl$_3$-HCl as the target of which 30.3 tons is gallium. The extraction processes of these experiments, which are different for each experiment, are far more complicated procedures than those of the chlorine experiment; refer to Ref. [16] and Ref. [17] for details. However, the energy threshold of the gallium reaction is 0.233 MeV, thus making these experiments sensitive to pp neutrinos. Almost 100\% of fusion reactions begin with the pp reaction, and so the pp fluxes are thought to be the least sensitive to uncertainties in the solar models. The expected rate of neutrinos interactions for the the gallium experiments is [12]:

\[ R_{BP}^{Ga} = 129^{+8}_{-6}\text{(SNU)}, \] (1.10)

while the observed rates are [18, 19]:

\[ R_{obs}^{SAGE} = 67 \pm 8\text{(SNU)} \] (1.11)

\[ R_{obs}^{GALLEX} = 78 \pm 6\text{(SNU)}, \]

which are equivalent to about 19 interactions per month.
Again measurements of the solar neutrino fluxes fall far short of predicted values. Two possible solutions to this enigma are discussed in the next section.

1.3 Possible Solutions to the Solar Neutrino Anomaly

Two possible solutions have been proposed to explain the discrepancy between measured and predicted solar neutrino rates. The first proposes that the standard solar models are incorrect and must be modified. This would involve changing input parameters or modifying their dynamic prescriptions. The second proposes that the neutrino physics needs to be modified and not the solar models. This would introduce neutrino properties that are not currently included in the standard model, specifically the non-conservation of neutrino flavor (neutrino flavor oscillations).

1.3.1 Non-Standard Solar Models

One proposed solution to the discrepancy between measured and expected solar neutrino rates is the modification of the standard solar models. This would involve the changing of input parameters, such as the diffusion coefficients or the S-factors discussed in Section 1.1, or modifying the models' dynamic prescriptions. Recall that the expected $^8$B neutrino flux is highly sensitive to the modeled temperature gradient in the Sun (Eqn. 1.1) and the $S_{17}$-factor used in the reaction cross section (Eqn. 1.2).

Hata and Langacker [20] compared the results of the experiments described in Section 1.2 with several modified SSMs. The Kamiokande experiment is sensitive to essentially only $^8$B neutrinos (HeP neutrino contribution
is expected to be negligible since the expected flux at Earth is three orders of magnitude lower than that of $^8$B neutrinos). Since the $^8$B is created from $^7$Be (refer to Figure 1.1), the rates of $^8$B and $^7$Be solar neutrinos are linked. The gallium experiments are sensitive to pp, pep, $^7$Be, and $^8$B neutrinos, and they measured on average 72.5 SNU. This measurement is fully accounted for by the expected pp and pep rates of alone: 73 SNU [8]. (Recall that the basic pp and pep reactions are thought to be the least sensitive to SSM uncertainties.) But the $^8$B neutrinos measured by Kamiokande must also contribute to the gallium measurements. The Kamiokande measured $^8$B flux should make up about 7 SNU of the gallium measurement. This implies that the expected $^7$Be neutrino flux from the gallium experiments must be zero or negative.

For the chlorine experiment, the $^8$B neutrino flux measured by Kamiokande would contribute about 3.2 SNU to the total measured flux [8]. This contribution, however, is larger than the total measured flux of the Homestake experiment ($2.54 \pm 0.14 \pm 0.14$ SNU). These results are summarized in Figure 1.4 along with the predicted $^8$B and $^7$Be neutrino fluxes from several modified solar models [21]. All $^8$B and $^7$Be flux values in this figure are normalized by $\phi(Be)_{SSM} = 5.15 \times 10^9$ and $\phi(B)_{SSM} = 6.62 \times 10^6$ in units of cm$^{-2}$s$^{-1}$, respectively. The combined measured rates form the distribution on the left, while the modified solar models all lie far outside the 99 % confidence level contour. From this analysis it is not apparent that changing the solar models can account for the discrepancy between measured and expected neutrino rates.
Figure 1.4: The constraints of $^8$B and $^7$Be solar neutrino fluxes from the combined chlorine, gallium, and water experiments at the 90 (shaded), 95 (dot-dash), and 99 % (dot-dot-dash) confidence levels [21]. Also shown are the standard and nonstandard solar model predictions (see Reference [21] for references). The Sun core temperature $T_c$ dependence is shown by the dashed curve and the effect of a $S_{17}$-factor reduction is shown by the arrow. All $^8$B and $^7$Be flux values in this figure are normalized by $\phi(Be)_{SSM} = 5.15 \times 10^9$ and $\phi(B)_{SSM} = 6.62 \times 10^8$ in units of cm$^{-2}$s$^{-1}$, respectively.
1.3.2 Neutrino Oscillations

Standard electroweak theory defines the masses of neutrinos to be zero although there are no physical reason for this assumption. However, if all three neutrinos are massless then it is rather odd to have three states with distinct quantum numbers that are degenerate in energy. If neutrinos do have mass then it is possible for the non-conservation of neutrino flavor. Since the experiments described in Section 1.2 are mostly or exclusively sensitive to electron flavor neutrinos, if the neutrinos changed flavor during their travel to the Earth the experiments would detect fewer electron flavor neutrinos than SSMs would predict. This changing of flavor is known as neutrino flavor oscillations and requires that at least one neutrino mass be non-zero and that there be differences among the neutrino masses.

Another characteristic of neutrino oscillations is that the passage of neutrinos through matter (such as the Sun or Earth) may affect the rate of oscillations. The effect of matter on neutrino oscillations was first proposed by S. P. Mikheyev and A. Yu. Smirnov based on the original work of L. Wolfenstein and so is called the "MSW" effect [22, 23].

The MSW effect originates from the extra interaction potential the electron flavor neutrinos have while traveling through matter that is not present for $\nu_\mu$ and $\nu_\tau$. Electron neutrinos can interact with electrons via charged current (CC) and neutral current (NC) current interactions, while $\nu_\mu$ and $\nu_\tau$ can only interact via NC interactions. This difference may influence the rate of neutrino oscillations. Neutrino oscillations that are induced by matter are called MSW or matter oscillations, and oscillations that may occur without the presence of matter are named vacuum oscillations. The following sections
are brief descriptions of Vacuum and MSW neutrino oscillations; refer to Ref. [24] for instance for more details.

### 1.3.3 Vacuum Oscillations

If there are slight mass differences among the neutrinos, then the mass eigenstates might be a mixture of the flavors. For example:

\[
\nu_1 = a_1 \nu_e + a_2 \nu_\mu + a_3 \nu_\tau
\] (1.12)

where:

- \( \nu_1 \) = first neutrino mass eigenstate such that 
  
  \( m_1 < m_2 < m_3 \), where \( m_i \) is the mass of the \( i^{th} \)
  mass eigenstate

- \( a_i \) = coupling constants, where \( \sum a_i^2 = 1 \).

In a two component approximation, the mass eigenstates are mostly a mixture of two neutrino flavors, for example:

\[
\nu_1 = a_1 \nu_e + a_2 \nu_\mu; \text{ where } \sqrt{a_1^2 + a_2^2} = 1.
\] (1.13)

This may be written in matrix notation with respect to the flavor eigenstates as:

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix} \equiv \mathbf{R}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
\] (1.14)

where \( \theta \) is the mixing angle of the mass eigenstates. If \( \theta = 0 \) then there is no mixing of states, and there is maximal mixing if \( \theta = \frac{\pi}{4} \) radians.

The time dependence of the mass eigenstates \( \nu_1 \) and \( \nu_2 \) is obtained by the time-dependent Shrödinger equation\(^4\):

\(^4\)In this analysis \( \hbar = c = 1 \).
\[ i \frac{d}{dt} \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix} = \mathbf{H} \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix}, \] (1.15)

where $\mathbf{H}$ is the Hamiltonian operator, $t$ is time, and $E_i$ is the energy of $\nu_i$ ($i = 1, 2$). The solution to the above equation is:

\[ \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix} = \begin{pmatrix} e^{-iE_1t} & 0 \\ 0 & e^{-iE_2t} \end{pmatrix} \begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix}. \] (1.16)

The time-dependent flavor eigenstates can be then written as:

\[ \begin{pmatrix} \nu_\alpha(t) \\ \nu_\mu(t) \end{pmatrix} = \mathbf{R} \begin{pmatrix} e^{-iE_1t} & 0 \\ 0 & e^{-iE_2t} \end{pmatrix} \mathbf{R}^{-1} \begin{pmatrix} \nu_\alpha(0) \\ \nu_\mu(0) \end{pmatrix}. \] (1.17)

Assuming that the masses $m_i$ are small compared with the energy $E_i$ ($i = 1, 2$) and that the mass eigenstates have about the same momentum $p$, then the energy can be approximated as:

\[ E_i = \sqrt{p^2 + m_i^2} \approx p + \frac{m_i^2}{2p} \approx E + \frac{m_i^2}{2E}, (i = 1, 2). \] (1.18)

Using this result and Eqn. 1.17 the probabilities that a $\nu_e$ at time $t = 0$ remains a $\nu_e$ ($P(\nu_e \rightarrow \nu_e; L)$) or oscillates into a $\nu_\mu$ ($P(\nu_e \rightarrow \nu_\mu; L)$) can be calculated:

\[ P(\nu_e \rightarrow \nu_e; L) = 1 - \sin^2 \theta \sin^2 \frac{\delta m^2 L}{4E} \] (1.19)

\[ = 1 - \sin^2 \theta \sin^2 \left(1.27 \frac{\delta m^2 [eV^2]}{E[MeV]} L[m] \right) \]

and

\[ P(\nu_e \rightarrow \nu_\mu; L) = \sin^2 \theta \sin^2 \frac{\delta m^2 L}{4E} \] (1.20)

\[ = \sin^2 \theta \sin^2 \left(1.27 \frac{\delta m^2 [eV^2]}{E[MeV]} L[m] \right), \]

where $L$ is the distance traveled in time $t$ and $\delta m^2$ is the difference of the squares of the masses: $\delta m^2 = m_2^2 - m_1^2$. The oscillation probabilities are
functions of distance of travel $L$, neutrino energy $E$, the difference in the squares of the neutrino masses $\delta m^2$, and the mixing angle $\theta$. In the case of solar neutrinos the distance traveled $L$ is the Earth–Sun distance. The corresponding oscillation length $L_{\text{osc}}$ is:

$$L_{\text{osc}} = \frac{4\pi E}{\delta m^2} \simeq 2.48[m] \frac{E[\text{MeV}]}{\delta m^2[\text{eV}^2]} \quad (1.21)$$

These results, however, may be affected by matter induced oscillations.

### 1.3.4 Matter Oscillations

The presence of matter may affect neutrino oscillations because $\nu_e$ feels an extra interaction potential that is not present for $\nu_\mu$ and $\nu_\tau$. The time-dependent Schrödinger equation is given by:

$$i \frac{d}{dt} \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = \begin{bmatrix} R \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} R^{-1} + \begin{pmatrix} V_A & 0 \\ 0 & 0 \end{pmatrix} \end{bmatrix} \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix}, \quad (1.22)$$

where $V_A$ is the additional potential felt by the electron neutrinos:

$$V_A = \sqrt{2} G_F N_e \quad (1.23)$$

and $G_F$ is the Fermi coupling constant and $N_e$ is the electron number density.

A mixing matrix analogous to the vacuum oscillations can be formed under the assumption of a constant electron density:

$$\begin{pmatrix} \nu^m_1 \\ \nu^m_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_m & -\sin \theta_m \\ \sin \theta_m & \cos \theta_m \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \quad (1.24)$$

where $\nu^m_i$ ($i = 1, 2$) are the mass eigenstates in matter and $\theta_m$ is the mixing angle in matter. The matter mixing angle $\theta_m$ is defined to be:

$$\tan 2\theta_m = \frac{\sin 2\theta}{\cos 2\theta - L_{\text{osc}}/L_0}, \quad (1.25)$$

----

5In this analysis $\hbar = c = 1$. 

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where $L_0$ is the neutrino–electron interaction length:

$$L_0 = \frac{\sqrt{2\pi}}{G_F N_e} = \frac{2\pi}{V_A} \quad (1.26)$$

The mixing angle in matter $\theta_m$ becomes maximal when $\cos 2\theta = (L_{osc}/L_0)$, which is called the resonant condition. The resonant condition can be rewritten in terms of the electron density:

$$N_{e,res} = \frac{\delta m^2 \cos 2\theta}{2\sqrt{2} G_F E} \quad (1.27)$$

If the electron neutrino passes through a volume with this resonant electron density then maximal mixing can occur even if the vacuum mixing angle is very small. The corresponding transition probability is:

$$P(\nu_e \rightarrow \nu_\mu; L) = \sin^2 2\theta_m \sin^2 \frac{\pi L}{L_m}, \quad (1.28)$$

where:

$$L_m = L_{osc} \left( \frac{\sin 2\theta_m}{\sin 2\theta} \right). \quad (1.29)$$

This solution is valid if the electron density variation is relatively small such that the constant solar density assumption can be relaxed adiabatically. Otherwise, Eqn. 1.22 must be solved numerically.

1.4 Oscillation Limits

The results of the solar neutrino experiments discussed in Section 1.2 were combined by Hata et al. [21] to limit the neutrino oscillation ($\sin^2 2\theta$ verses $\delta m^2$) parameter space. Many combinations of the oscillation parameters $\sin^2 2\theta$ and $\delta m^2$ were tested using solar models and the resulting predicted electron flavor neutrino flux values were compared with measured values.
Figure 1.5 shows the combined allowed regions of the chlorine, gallium, and water experiments. There are three overlapping allowed regions (shaded) known as the “Large Angle Solutions” (LAS) centered near $(\sin 2\theta, \delta m^2) = (0.6, 10^{-5})$, the “Small Angle Solutions” (SAS) centered near $(\sin 2\theta, \delta m^2) = (0.006, 6 \times 10^{-6})$, and the “Vacuum Solutions” shown on the right-hand-side of Figure 1.5.

Each region predicts a unique characteristic of neutrino oscillations. The LAS predict a variation in the $\nu_e$ flux as the neutrinos pass through the Earth, resulting in a higher flux measurement when the Sun is below the horizon. This is called the “Day/Night” effect. In this region all neutrino energies are uniformly affected. In the SAS region there is an oscillation energy dependence. Thus, a comparison of the expected and measured neutrino energy spectrum shapes would show a characteristic distortion in the measured spectrum. The Vacuum Solutions are valid if the oscillation length $L_{\text{osc}}$ just happens to be about the Earth-Sun distance. These solutions are often called the “Just-So” oscillations for this reason. Just-So oscillations might be evident in seasonal neutrino flux differences as the Earth–Sun distance varies.

The Day/Night effect, spectrum shape distortions, and seasonal variations are all directly measurable criteria that are independent of solar models. The Super–Kamiokande detector was designed so as to investigate each of these allowed regions with higher statistics than the previous solar neutrino experiments. This analysis focuses mainly upon the spectrum shape measurement.
Figure 1.5: The result of the MSW parameter space allowed by the combined observations at the 95% confidence level assuming the BP95 SSM (shaded) [21]. The constraints from the chlorine, water, and gallium experiments are shown in the left figure by the dot-dash, solid, and dashed lines, respectively. Also shown in the left figure are the regions excluded by Kamiokande spectrum and day/night data (dotted lines). (The first year of data from Super-Kamiokande is included in the water experiments' allowed regions.) The figure on the right includes the Kamiokande spectrum data.
CHAPTER 2

DETECTOR DESCRIPTION

2.1 Detection Method

A charged particle moving within a dielectric medium will radiate via Cherenkov radiation if the particle velocity $u$ exceeds the speed of light in the medium, namely $c/n$, where $c$ is the speed of light in vacuum and $n$ is the index of refraction of the medium. The Cherenkov photons are emitted in a cone defined by the angle $\theta_{ch}$ with respect to the particle's direction of motion, where:

$$\theta_{ch} = \cos^{-1}\left(\frac{c}{n(\lambda)u}\right)$$  \hfill (2.1)

The energy loss per unit length of traveled path $L$ in terms of the number of emitted photons $N_{\text{photon}}$ is:

$$\frac{d^2 N_{\text{photon}}}{dL d\lambda} = \frac{2\pi \alpha}{\lambda^2} \sin^2 \theta_{ch}.$$  \hfill (2.2)

The Super-Kamiokande detector is sensitive to Cherenkov photons produced in the water contained within the detector. For a water index of refraction of 1.33 and relativistic velocities ($u \approx c$), the cone angle $\theta_{ch}$ is about 42°. The minimum total energy required for an electron to produce Cherenkov photons, the Cherenkov threshold energy, in water is 0.768 MeV. About 390 Cherenkov photons are emitted for every centimeter of particle travel.
travel. The Super-Kamiokande detector looks for Cherenkov photons produced by electrons that have been scattered by neutrinos to reconstruct the interaction vertices and determine the recoil electron directions and energies.

2.2 General Detector Description

The Super-Kamiokande detector is situated at a depth of 2700 meters water-equivalent in the Japanese Kamioka Mozumi Mine (36°25′N latitude, 137°18′E longitude). Locating the detector underground serves to reduce the number of cosmic rays entering the detector, which can overwhelm detector electronics and create long-lived β-decay nuclei within the detector. The rate of entering cosmic ray muons, which are still able to penetrate the rock over-burden, is reduced by a factor of about $10^{-5}$ when compared to surface rates. The muon rate through the detector is about 3 Hz. A cutaway view of the detector is shown in Figure 2.1.

The detector tank is a steel water tank of cylindrical geometry containing 50,000 tons of ultra-pure water (39.3 m in diameter and 41.4 m in height). Figure 2.2 shows a cross sectional diagram of the detector.

The water tank volume is divided into two concentric volumes by a light barrier of cylindrical shape, which also houses all photomultiplier tubes (PMTs), the sensitive elements of the detector. The central 32,000 tons (36.2 m in height and 33.8 m in diameter), called the inner detector (ID), is viewed by 11,146 inward-facing PMTs. The ID PMTs are of 50.8 cm (20") diameter and cover 40.4 % of the inner surface. The PMTs convert Cherenkov photons that strike them into electrical impulses that provide both timing and pulse height information. Figure 2.3 shows a muon which
Figure 2.1: A general cut-away view of the Super-Kamiokande detector. The inset shows the location of the detector within Mt. Ikenoyama.

entered the detector at the top and stopped within the interior of the ID. In this figure the barrel of the detector has been unrolled to form a rectangular plane and the top folded up and the bottom folded down. Each colored pixel represents one 50.8 cm ID PMT which produced a signal from the muon Cherenkov light. The entry point is indicated by the cluster of hit PMTs at the top and the Cherenkov cone is clearly visible along the barrel wall. Reflected and scattered light causes the other, random PMT signals. The color of each PMT represents the amount of light striking the PMT measured in photo-electrons; the color scale is found at the bottom of the figure.

The remaining ID surface is covered by black polyethylene sheets to reduce light reflections within the central volume and to provide a light barrier.
Figure 2.2: Cross section view of the Super-Kamiokande detector. (Figure by K. Martens)
Figure 2.3: A view of a cosmic ray muon entering the inner volume of the Super-Kamiokande detector. In this figure the barrel of the detector has been unrolled to form a rectangular plane and the top folded up and the bottom folded down. The entry point is indicated by the cluster of hit PMTs at the top and the Cherenkov cone is clearly visible along the barrel wall. Reflected and scattered light causes the other, random PMT signals. The color of each PMT represents the amount of light striking the PMT measured in photo-electrons; the color scale is found at the bottom of the figure.
Table 2.1: The dimensions and volumes of the Super-Kamiokande water tank divisions.

<table>
<thead>
<tr>
<th>Tank Section</th>
<th>Dimensions</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>41.4 m (height)</td>
<td>50,000 tons</td>
</tr>
<tr>
<td></td>
<td>39.3 m (diameter)</td>
<td></td>
</tr>
<tr>
<td>Inner Detector</td>
<td>36.2 m (height)</td>
<td>32,000 tons</td>
</tr>
<tr>
<td></td>
<td>33.8 m (diameter)</td>
<td></td>
</tr>
<tr>
<td>Analysis Fiducial</td>
<td>34.2 m (height)</td>
<td>22,500 tons</td>
</tr>
<tr>
<td>Volume</td>
<td>31.8 m (diameter)</td>
<td></td>
</tr>
</tbody>
</table>

between the inner and outer volumes. The outer volume surrounding the ID, called the outer detector (OD), is viewed by 1,885 outward-facing 20.3 cm (8") diameter PMTs and is used to identify in-coming particles such as cosmic rays and to passively reduce $\gamma$ and neutron backgrounds radiating from the rock surrounding the detector. The thickness of OD is 2.6 m on the top and bottom of the detector and 2.75 m around the barrel. This is equivalent to 7.2 radiation lengths (RL) and 4.3 nuclear interaction lengths (NL) on the top and bottom and 7.6 RL and 4.6 NL around the barrel. All OD surfaces are covered with white Tyvek$^{TM}$ sheets, which have a reflectivity of greater than 80%, to allow for the collection of reflected light from outward-moving particles exiting the detector.

The fiducial volume, the volume of the detector used in this analysis, starts 2 m inward from the ID walls and contains 22,500 tons of water. Table 2.1 summarizes the dimensions and volumes of the SK tank divisions.

Because the Super-Kamiokande detector reconstructs scattering vertices, directions, and energies from the emitted Cherenkov light of recoil electrons,
it is called an imaging water Cherenkov detector. The advantages of this type of neutrino detector over the chlorine and gallium radiochemical detectors are that the exact time of arrival of the incident neutrino is known; the scattering vertex and the direction of the recoil electron can be reconstructed from PMT information, which gives information about the incident neutrino direction (Eqn. 5.5); and the energy spectrum of recoil electrons can be measured, which reflects that of the incident neutrinos (Eqn. 5.3). These qualities enable the measurement of the neutrino flux, any time dependence in the measured flux, and the energy spectrum of the flux.

2.3 Photomultiplier Tubes (PMTs)

Photomultiplier tubes are the sensitive elements of the Super-Kamiokande detector. PMTs convert a single photon into an electric pulse which can be measured electronically. A photon strikes the glass face of the PMT, and (with a certain probability) the photon is absorbed by a thin photocathode material which is bonded to the inner surface of the glass. The photocathode will (also with a certain probability) eject an electron via the photoelectric effect. Electrons generated in this manner are called photo-electrons. The photo-electron accelerates towards the rear of the PMT, attracted by the electric fields generated by a series of dynodes with large electric potentials between them. When the photo-electron hits the first dynode it knocks out a few electrons which accelerate toward the next dynode where each knock out more electrons. By the last dynode the original single electron has multiplied by several orders of magnitude ($10^7$ for the ID and $10^8$ for the OD PMTs at 2000 V), creating a large electric signal which can be easily registered by
electronics (about 3 mV for the ID PMTs). The ratio of output signal to input signal (in this case several magnitudes of electrons to one photo-electron) is know as “gain.”

2.3.1 Inner Detector (ID) PMTs

The PMTs mounted in the ID are Hamamatsu R1449 PMTs, the improved versions of the 50.8 cm (20”)diameter PMTs used in the Kamiokande experiment; Figure 2.4 shows a schematic [25]. The photocathode is made of a bialkali (Sb-K-Cs), chosen for its high sensitivity to blue light and low thermal emission. Figure 2.5 shows the quantum efficiency (QE) of the photocathode and PMT glass window to convert a photon into a photo-electron as a function of wavelength. The average quantum efficiency at $\lambda = 390$ nm (a typical Cherenkov wavelength in the SK detector) is 22 %. 

Figure 2.4: Diagram of an inner detector 50.8 cm (20”)diameter photomultiplier tube.
Figure 2.5: The quantum efficiency (QE) of an inner detector PMT to convert a photon into a photo-electron as a function of incident photon wavelength.

Good timing response from each ID PMT is crucial to determining the scattering vertex and direction of a recoil electron. Time of travel differences ("jitter") of photo-electrons emitted from the photocathode are due to differences in initial velocities and directions and different electric fields between the cathode and first dynode and between subsequent dynodes. A typical relative transit time distribution for one PMT tested with 410 nm single photo-electron light is shown in Figure 2.6. Fitting the distribution to an asymmetric Gaussian results in a 1 σ width of 2.2 ns. The mean 1 σ width for all 11,146 ID PMTs is about 3 ns.

Most of the Cherenkov light received by each PMT will be at the single photo-electron level for most solar neutrino-electron scattering events. A single photo-electron peak in the PMT pulse height distribution, which is separated from the dark noise (thermal emissions of electrons from the photocathode or dynodes) peak, allows an efficient means of eliminating dark noise signals. The design of the PMTs is successful in this respect as is evidenced in Figure 2.7, which shows the pulse height distribution of 410 nm
single photo-electron light as a function of ADC\(^1\) counts. A clear single photo-electron peak is visible near 400 ADC counts; dark noise causes the large spike near 1 ADC counts.

The large diameter of the PMTs make them extremely sensitive to the geomagnetic field passing through the detector which has been measured to be 450 mG. The PMTs do not incorporate magnetic shield material, yet require that any external fields be less than 100 mG to retain their good timing characteristics. To this end Helmholtz coils are employed to counter the geomagnetic field, which results in a net magnetic field within the detector of less than 100 mG.

### 2.3.2 Outer Detector (OD) PMTs

The PMTs mounted in the OD are Hamamatsu R-1408 20.3 cm (8") PMTs and are the same PMTs which served in the IMB detector; the base

\(^1\) Analog-to-Digital Converter
design is shown in Figure 2.8. The PMT cables are Belden Cable shielded coaxial type YR39515 with polyethylene jacket, chosen for their low radon emission (less than 0.00005 Bq/m). Two outward facing OD PMTs are mounted for every 12 inward facing ID PMTs in the light barrier super structure. Refer to Figure 2.9.

Each OD PMT face sits in the center of a 60×60 cm wavelength-shifter (WS) plate, which has been drilled out to accommodate the PMT. The WS plates increase the net effective photo-collection area of each PMT by absorbing photons of wavelengths 300–400 nm and isotropically re-emitting the photons at longer wavelengths. Some of these photons are trapped within the WS plate due to total internal reflection and strike the PMT glass window. Light coupling between the PMT and the WS plate is achieved via simple physical contact. The outer edges of the WS plates are lined with aluminum coated mylar tape with a reflectivity of about 80 % to improve the chances of a re-emitted photon reaching the PMT. The effective photo-coverage of
Figure 2.8: Outer detector photomultiplier tube base design.
Figure 2.9: Schematic view of the frame sub-structures which support both the inner and outer detector PMTs.
the OD is almost doubled with the addition of the WS plates. The timing uncertainty of these PMTs with and without the WS plates is about 9 ns and 5 ns, respectively.

2.4 PMT Read-out Electronics

The inner and outer detectors not only have different types of PMTs, but also have different electronics to read their PMT signals. This is due to the different functions of the inner and outer detectors and to budget constraints. One of the main requirements of the ID is that there should be almost no situations in which the entire detector is "dead," meaning that the detector cannot take new data. The ID read-out electronics avoid dead time due to signal digitizing by processing only the PMTs that are active during a physics event and allowing the remaining PMTs to record any new events. It is also possible for the ID read-out electronics to process a PMT's information from a previous physics event, while recording the PMT's information from a new event.

The primary function of the OD is to detect muons and other particles which enter the detector from the surrounding rock. These particles may subsequently produce physics events, which may appear to be neutrino-electron scattering events. Hence, the OD read-out electronics are designed to measure and record PMT signals for a relatively long time period before and after an ID event.

The signal cables connecting the inner and outer PMTs to their respective electronics are all 70 m long. This keeps signal travel times along the cables the same no matter where in the detector a PMT may be located and to keep the relative timing between inner and outer signals as close as possible.
2.4.1 Inner Detector Electronics

Each ID PMT is connected to a channel of a Analog Timing Module (ATM), which records the PMT charge and timing information; that is it measures the PMT pulse height and the relative time of occurrence (relative to a global detector trigger time). Each ATM accepts 12 input channels and 20 ATMs are nominally included in each TKO\(^2\) crate. A total of 934 ATMs are distributed in 48 TKO crates with 12 crates in one of 4 electronics huts sitting on the top of the detector. Also included in one TKO crate is a Super Control Header (SCH) module, which is responsible for sending ATM data to a Super Memory Partner (SMP) in a VME crate for temporary data storage, and a GONG\(^3\) module, which distributes incoming global trigger signals and event identification numbers to each ATM. Refer to Figure 2.10.

Each ATM channel is divided into 2 identical subchannels A and B. The ATM toggles between subchannels A and B such that if one subchannel e.g. A is busy digitizing the PMT signal from a previous event, subchannel B can measure and digitize the current PMT signal. This enables one subchannel, for example, to record the charge and time information from a stopping muon (a cosmic ray muon which loses kinetic energy and stops in the detector) and the second subchannel to record the charge and time information from the subsequent decay electron. An ATM subchannel first splits the PMT signal. One signal is amplified with a gain of 100 and discriminated; if the value surpasses a threshold of about 0.32 PE a "HIT" signal is generated for 900 ns. The HIT signal initiates the charging of a Time-to-Analog Converter (TAC)

\(^2\)TRISTAN/KEK On-Line
\(^3\)GO/NoGo
Figure 2.10: Schematic of inner detector PMT data flow from the PMTs (on left) to the on-line Host CPU. Also shown is the detector triggering system.
and the integration of the second signal in a Charge-to-Analog Converter (QAC). The TAC charges a capacitor with a constant current source until a global detector trigger signal arrives at the ATM; the amount of charge on the capacitor is a measure of the relative time between the HIT signal generation and the global trigger signal arrival. The QAC integrates the PMT pulse height for 300 nsec; the amount of charge in the QAC is a measure of the number of PEs generated in the PMT. (Photomultiplier tube pulses are nominally about 3 mV in height.)

If a global trigger signal is not received by the ATM within 1.1 μsec of the HIT signal, the TAC and QAC are reset, which takes 200 ns. If a global trigger signal is received, then the ADC begins digitizing the voltages stored in the TAC (1024 ns range with 0.25 ns resolution) and the QAC (409.5 pCoulomb range with 0.1 pC [0.1 PE] resolution). Digitization requires 6 μs per subchannel.

An individual ATM channel will be unable to process a PMT signal under the following conditions:

- if one ATM sub-channel is digitizing a signal and a second PMT signal occurs during the 900 ns HIT signal generation of the previous signal (any PMT signal that is received while a HIT signal is being generated is ignored as to prevent recording PMT signals from cable reflections);
- or

- if both subchannels are busy processing previous signals; or
• if the current subchannel did not receive a global trigger signal within 1.1 \( \mu \text{sec} \) and the second subchannel is still digitizing its data from a previous hit. It takes 200 ns for a subchannel which did receive a global trigger signal to clear. Refer to Ref. [26] for more details.

The output of the ADC is written into a 1 kword FIFO\(^4\) memory with an event number and is sent via the SCH to the SMP. Data is read from FIFO memory independently of PMT signal processing to again reduce the probability of detector dead time. There is one SMP module for each of the 48 TKO crates. Eight on-line “Slave” CPUs read the data from the SMPs and transmit it via a FDDI\(^5\) optical fiber connection to another on-line “Host” CPU, which merges the inner and outer PMT information with the same event number along with the corresponding trigger information into a single event data structure. Figure 2.10 shows a diagram of the data flow from the ID PMTs on the left to the on-line Host CPU on the right.

The data is then sent out of the mine to the nearby computer center via another FDDI connection. There the raw digitized TAC and QAC counts are converted into time (\( \text{ns} \)) and charge (\( PE \)) units and the on-line data structure is changed to a ZEBRA format. The data is copied and archived to tape with one copy remaining at the computer center and one copy sent to the United States.

2.4.2 Outer Detector Electronics

Each OD PMT has only one cable attached, requiring PMT signals to travel along the same cable, which supplies the high voltage. The PMT

\(^{4}\)First In/First Out

\(^{5}\)Fiber Distributed Data Interface; capable of data transfer of 100 Mbits/sec
cable is connected to a custom "paddle" card, which separates the PMT signals from the supplied voltage through a high voltage capacitor. Each paddle card can accommodate 12 PMTs and fans out the supplied voltage to each attached PMT. An individual PMT can be disconnected from the high voltage supply by disconnecting a jumper located on the paddle card. There are 20 paddle cards per high voltage supply crate and 2 high voltage crates in each of the 4 electronics huts.

A PMT signal is passed via a ribbon cable to a custom Charge-to-Time (QTC) module where the signal is discriminated. If a PMT pulse height surpasses a threshold of 25 mV or about 0.5 PE, then the QTC integrates the charge of the PMT for a 200 ns window and produces an Emitter-Coupled Logic (ECL) pulse. The leading edge of the ECL pulse is the time of the leading edge of the PMT signal and the width is proportional to the natural logarithm of the charge. Each QTC module can accommodate 48 paddle card channels.

An ECL pulse from a QTC then travels via another ribbon cable to a LeCroy 1877 Time-to-Digital Converter (TDC) where the leading and falling pulse edges are recorded. Each TDC module has 96 input channels (1 per PMT), and there are 5 TDC modules per Fastbus crate and 1 Fastbus crate in each of the 4 electronics huts. Each TDC channel works as a circular buffer that can store up to 16 edges over a window up to 32 μs wide. Since two edges (leading and falling) are required to define each ECL pulse, 8 signals from one PMT can be stored in each TDC channel. However, if a ninth signal is received, then the first buffered signal is overwritten; if a tenth
signal is received the second buffered signal is overwritten; etc. An overflow deletes the pulse information from the earliest signals, which may affect the determination of the entry or exit time of an event. The window of OD data taking originally was centered about the global detector trigger time with data taken 16 $\mu$s before and 16 $\mu$s after the trigger, but this was changed (starting with run 2800) in September of 1996. Outer detector data taking window is now shortened to 10 $\mu$s before and 6 $\mu$s after the global detector trigger time. This not only reduced the data size of each physics event, but also reduced the probability of an overflow of the circular buffers.

All TDC channels are digitized and read out at the same time, a process which takes 2 to 8 $\mu$s depending on the number of edges in the event. The resolution of digitization is 0.5 ns. During digitization the OD electronics can no longer take data, but after read out a channel is free to take new data. If a global trigger signal is received during OD dead time, a flag bit is introduced into the data stream to record this condition.

The TDC signals are read by a Fastbus Smart Crate Controller (FSCC) which sends the data via a FDDI connection into storage in a large VME crate buffer (DDC2-DM115) that is read out by an on-line Slave CPU and sent on to the on-line Host CPU, where the data is merged with the ID event data. The OD data also incorporates Global Positioning System (GPS) timing information with the PMT data. Figure 2.11 diagrams the OD PMT signal path from the PMT on the left to an on-line Slave CPU on the right; the GPS system is also shown. Refer to Ref. [27] for further details.
Figure 2.11: Schematic of outer detector PMT data flow from the PMTs (on left) to an on-line Slave CPU (Sukant). Also shown is the Global Positioning System (time) system.

2.5 Detector Triggering

Detector triggering electronics determine when PMT signals are recorded. There are 4 self-generating trigger types as well as 3 external types. The self-generating trigger types are: Low Energy (LE), High Energy (HE), and Outer Detector (OD). The LE trigger selects solar neutrino event candidates, while the HE trigger selects higher energy events such as cosmic ray muons and atmospheric neutrino and proton decay candidates. An OD trigger indicates activity in the outer detector. Normally a physics event will cause several triggers: an entering cosmic ray muon, for example, will trigger the LE, HE, and OD triggers, while a muon decay electron event may only make a LE trigger. All relevant event trigger types are recorded with the event PMT data. Because it is preferable to base the trigger time upon ID timing for
event reconstruction, the OD trigger is delayed 100 ns as to allow any inner
detector trigger to supersede it; otherwise the OD will produce the global
detector trigger.

The LE, HE, and SLE triggers rely upon inner detector ATM HITSUM
signals for triggering. Each ATM channel which has a PMT signal above
threshold produces a HITSUM signal for 200 ns with an amplitude of 11 mV.
All HITSUM signals are added together and if the summed signal surpasses
one or more of the trigger type thresholds a global detector trigger signal is
produced. The summing electronics attempt to reduce the contribution by
the constant dark noise by AC coupling the HITSUM signals. For the entire
detector PMT dark noise causes about 2 HITSUM signals in any 50 ns time
window. Dark noise signals occur at random times in the 200 ns window
and will tend to appear as a DC level background. The threshold voltage
for the LE trigger is 320 mV, equivalent to about 29 HITSUM signals (over
background) and corresponding to a energy threshold of about 5.7 MeV. The
HE trigger threshold is 340 mV, requiring about 31 HITSUM signals. Outer
detector triggering occurs when there is a coincidence of 19 OD PMTs above
QTC threshold in a 200 ns window.

All triggers are sent to the Trigger (TRG) module in the central electron-
ics hut and the TRG issues a global detector trigger signal to all ID GONG
modules and OD TDC modules 30 ns after the first trigger type input. The
TRG also generates a 16 bit event number and the relative time of the event
is recorded with 20 ns resolution by an internal 50 MHz 48 bit clock along
with all relevant trigger types. This information is sent to the on-line Host
CPU where it is merged with the ID and OD PMT data.
The three external trigger types include a clock (CLK) trigger, which provides a prescaled, unbiased sample of detector PMT dark noise, and two calibration trigger types, which are used to trigger the detector during special calibration periods.

2.6 Detector and Data Quality Monitoring

The detector status and data quality are monitored by an on-line, real-time computer system named Kingfish. Kingfish provides an early detection of equipment failures, communication problems among the on-line CPUs, and interesting physics events. Before the on-line "Host" computer sends data events out of the mine for calibration and archival storage, the data is copied and sent to the Kingfish monitor via an FDDI connection. This ensures that, if a problem occurs with Kingfish, the data flow from the mine will be unaffected. Kingfish monitors the data using four software utilities named SKIMMER, SKIM, GETREAL, and SCAN; a data flow diagram is shown in Figure 2.12.

The SKIMMER utility monitors the FDDI port for incoming data, unpacks each event from the on-line data format into a readable format, performs a quick check of the data (detailed below), and stores the event data in a circular ring buffer called the Receiver Ring Buffer (RRB). After unpacking an event SKIMMER makes a quick check of the event data to determine whether the event is a possible atmospheric neutrino event or a possible supernova event. To determine the latter SKIMMER simply tracks the number of events that occur each second; if the event rate surpasses 500 Hz (default)
Figure 2.12: Data flow diagram for the Kingfish monitor. Data events are received by the SKIMMER utility via the FDDI connection from the on-line Host CPU. Events are placed in the Receiver Ring Buffer (RRB) and candidate super nova and atmospheric neutrino candidates are copied and placed in the Contained Ring Buffer (CRB). The GETREAL utility calibrates and saves to disk events placed in the CRB. The SKIM utility gathers detector and data information from each event placed in the RRB, provides histograms of the information, and places the event data into either the Big Ring Buffer (BRB) or the Small Ring Buffer (SRB). The SCAN utility produces graphical reconstruction of BRB and SRB events.
or a user set threshold, the event is flagged. Possible atmospheric neutrino events require that the events have only an LE trigger type (i.e. no HE, OD, or calibration trigger types), have more than 250 ID PMT signals, and no more than 300 OD PMT signals. Events which pass the above criteria are flagged. A further requirement for both supernova and atmospheric neutrino candidates is that no error or calibration flags from the inner or outer detector electronics can be present in the event data record. Copies of the flagged events are placed into a second ring buffer, called the Contained Ring Buffer (CRB), and all events (both flagged and unflagged) are placed in the RRB.

Ring buffers are required because the on-line Host CPU does not send a continuous stream of events to the Kingfish monitor (or out of the mine), but instead sends a 2 Mbyte “spill” of many data events every few seconds. Kingfish cannot process each event quickly enough to keep up with the high rate event spill, and so it stores the events in ring buffers so as not to lose data. The RRB is capable of storing 1000 maximum size events, which are events with PMT data for every ID and OD PMT, while the CRB has a capacity of 100 similarly sized events. The ring buffers are designed so that one process can insert an event into a buffer “slot” while another process is reading and removing an event from a different slot. If a ring buffer becomes full then any new data events are lost.

The GETREAL process monitors the CRB buffer for new events and reads them in one at a time from the buffer. GETREAL calibrates each event to convert the raw PMT charges and times into photoelectrons and

\[7\] During a supernova, the Super-Kamiokande detector may trigger at rates which exceed 500 Hz.
nanoseconds. Once calibrated GETREAL saves the event to disk; this provides a backup in case there is a problem with the tape archiving process.

The SKIM process monitors the RRB for new events and reads them in one at a time. SKIM then extracts information from each event and compiles it to provide statistical information about the detector and data quality. Information that is monitored includes:

- the number of times each PMT (both ID and OD) has been “hit” (PMTs which have signals for almost all events may be “flasher” PMTs; these are PMTs which are arching internally and perhaps creating sparks that trigger the entire detector);

- the number of times each PMT was the PMT with the largest charge pulse in an event (PMTs which continuously have the largest pulses may also be “flasher” PMTs);

- the event trigger types (checks the relative triggering rates and confirms that the proper triggers are enabled or disabled);

- the number of hit PMTs in each event (provides a measure of the the energies on which the detector is triggering).

The SKIM process normally gathers statistics on over 60 detector and data characteristics and provides real-time histograms of those statistics.

Once SKIM has processed an event, it writes the event to either the Small Event Ring Buffer (SRB) or the Big Event Ring Buffer (BRB), depending upon the size of the event (measured in kBytes). Events larger than 3 kBytes, which are generally cosmic ray muon events, are placed in the BRB, while
the remainder are placed in the SRB. Events in the SRB and BRB buffers are free to be read by one or more on-line analysis programs. Currently, only the SCAN utility reads from these buffers.

The SCAN utility reads from either SRB or LRB buffer (the user decides which) and provides a graphical reconstruction of the events. Figure 2.3 is a SCAN image of a cosmic ray muon event. Visual inspection of events, especially of cosmic ray muons events which produce large amounts of Cherenkov light, provides a fast and effective means to verify that all sections of the detector are operating and that the data is flowing from the on-line Host CPU. Figure 2.13 shows a SCAN display which indicates the location of all dead and low occupancy inner detector PMTs.

2.7 Water Purification System

Clean water is essential to the solar neutrino analysis, since Cherenkov light scatter and absorption by water impurities affect event reconstructions. Also important is the removal of dissolved radon gas. The decay of radon is a source of background of the solar neutrino analysis. Although β—decay from $^{222}\text{R}$ has an energy endpoint of 3.26 MeV, the energy resolution of the detector can smear the energy into the solar neutrino analysis energy range. The source of the water used to fill the water tank came from an underground aquifer, a readily available clean source. During filling (which took about 3 months) the water first passed through the purification system before entering the tank. The water is continuously being recycled through the purification system with a turnover time taking about one month. Figure 2.14 compares the measured electrical resistance of the water both exiting and
Figure 2.13: SCAN display which indicates the location of all dead and low occupancy inner detector PMTs (squares).
Figure 2.14: The measured resistance of the tank water both from the input and return of the water purification system from June 1996 to June 1997.

entering the purification system. The exiting water has a resistance of about 18.20 MΩcm, which is very close to the chemical limit of 18.24 MΩcm.

Water is removed from the tank at the top and reinjected at the bottom. Figure 2.15 shows a schematic of the water purification system. The water passes through the following components:

- 1 μm nominal filter: Removes relatively large particulate contaminates.

- Heat exchanger: Water pumps increase the temperature of the water; the heat exchanger decreases the temperature to about 14°C to control bacterial growth.

- Ion exchangers: Removes metal ion impurities in the water.

- UV sterilizers: Kills bacteria to counts less than \((10^3 \sim 10^4)/100\) ml.

- Vacuum de-gasifier: Removes gases dissolved in the water; about 96% of dissolved radon gas is removed at this stage.
- Reverse Osmosis Filters: Removes gases dissolved in the water.

- Cartridge polishers: Removes metal ion impurities in the water.

- Ultra Filters: Removes sub-μm contaminants.

2.8 Air Purification System

Since the Super-Kamiokande detector is situated in a mine, the detector is literally surrounded by sources of radon gas. The rock dome above the water tank as well as the walls and tunnels leading to the detector are sealed with Mineguard® polyurethane material to retard entry of radon gas. Double doors at all entrances also limit the amount of mine air that enters the detector area, and fresh air is pumped directly into the experiment area from outside of the mine.
An air purification system is employed to pump radon-free air into the top of the water tank, where there is an air gap about 60 cm between the surface of the water and the top of the steel tank. A positive pressure is always maintained in this volume with the radon-free air to hinder the migration of radon gas into the detector. The air purification system is comprised of heating elements and particulate and activated carbon filters; the system is diagramed in Figure 2.16. Figure 2.17 shows the concentration of radon in the mine air immediately outside the sealed experiment area and in the air entering the top of the water tank between 1 June 1996 and 30 June 1997. The average radon concentration is about $10^{-2}$ Bq/m$^3$. Also note the seasonal variation of the mine air concentration. In winter months the mine air temperature is warmer than the outside air and so the mine air flows out of the mountain, while in warmer months the air flow stops, allowing radon concentrations to increase.
Figure 2.17: Radon concentration in the mine air immediately outside the sealed experiment area and in the air entering the top of the water tank from the Radon-free air system between 1 June 1996 and 30 June 1997.
Chapter 3

Detector Calibration

Understanding the detector responses to electrons of different energies and directions in all locations of the detector are essential for the spectrum measurement of electrons scattered by solar neutrinos. To accomplish this analysis several steps are required for each event:

- the vertex of the event must be reconstructed
- the direction of the electron must be reconstructed
- the energy of the electron must be reconstructed

To perform these reconstructions the timing and charge (pulse height) responses of each PMT channel at various light levels must be understood. The timing and charge values are read by the data acquisition system in TAC and QAC charge units and must be converted into physically meaningful quantities, such as nanoseconds and photoelectrons. This is accomplished by comparing the responses of each PMT channel to known inputs from controlled sources. Translating the detector responses into physical quantities is known as detector calibration. Each PMT channel must be calibrated, since each may give slightly different responses to the same inputs. For instance, different PMTs may produce different pulse heights for the same input light level (the ratio of output signal to input light level is known as PMT “gain”). Thus, the relative responses of the PMTs to similar light levels must be measured.
The timing responses of each PMT may vary as well, and the time that a PMT channel registers a hit can vary as a function of the intensity of the light causing the signal. Time variation due to light intensity is known as time "slewing." Thus, PMT time slewing must be measured for each PMT at various light levels.

Once the PMT channel signals can be translated into nanoseconds and photoelectrons, they can be used in the vertex, direction, and energy reconstructions. How well the detector can reconstruct these items from the PMT data must be measured as a function of location, direction, and energy, so these can be taken into account during the determination of uncertainties in the signal measurement and during the calculation of expected neutrino signal rates. Further, the energy reconstruction is required to be accurate to within an uncertainty of less than ±1 % to be able to investigate neutrino oscillation hypotheses.

The water quality (transparency) affects the vertex, direction, and energy reconstructions, since Cherenkov photons can be scattered or absorbed in the water. Thus, measurements of the water transparency, which vary in time, are important to the neutrino signal analysis.

The vertex and direction reconstruction algorithms are detailed in Chapter 4. An event's energy is reconstructed by measuring the number of photoelectrons detected by the PMTs. The number of detected photoelectrons (called "hits") can be corrected for light attenuation, PMT acceptance angle, effective PMT density, the number of nonfunctioning PMTs, and the probability of a two photoelectron emission in one PMT to compute the number
of effective hits, $N_{\text{eff}}$. The $N_{\text{eff}}$ values, however, need to be converted into electron energies. This is accomplished by calibrating the detector responses to monoenergetic electrons of known energy. These electrons are injected into the detector by a linear electron accelerator (LINAC). The detector responses are measured for thousands of monoenergetic electrons, which provide a statistical distribution of measured photoelectrons. The corresponding $N_{\text{eff}}$ distribution is fit to a Gaussian curve and the mean provides the $N_{\text{eff}}$ relation to the known input electron energy (measured in MeV). The $1\sigma$ width of the distribution defines the energy resolution of the detector for that electron energy. The energy calibration is repeated using electrons of seven different energies and this results in a nearly linear relation between $N_{\text{eff}}$ and electron energy. Other calibration sources are also used to cross-check and monitor the LINAC energy calibration.

The LINAC is also used to measure the vertex and direction reconstruction resolutions, since the LINAC electrons are injected into the detector at a known position and direction. Vertex resolution is defined as the radius of the sphere around the LINAC electron injection location that contains 68% of the reconstructed vertices. Angular resolution is defined as the opening angle of the cone around the LINAC electron beam direction which contains 68% of the reconstructed directions. Vertex resolution is important to the neutrino signal analysis, since a fiducial volume cut is made to remove events within 2 m of the detector walls (refer to Chapter 6). The direction reconstruction resolution is important, since a direction cut is made during neutrino signal extraction (refer to Chapter 4).
The calibrations and resolution measurements briefly described above are
detailed in the following sections. The sections are divided into: PMT cali-
brations, water transparency measurements, energy calibrations, and detec-
tor resolutions. A final section is included which discusses the measurement
of the efficiency of detector to self-trigger on electrons of various energies
(trigger efficiency).

3.1 PMT Calibrations

3.1.1 PMT Relative Gain

To ensure that the inner detector PMTs provide uniform responses to low
intensity photons, the voltage of each PMT is adjusted to make PMT relative
gains about equal. Gain is the ratio of output signal to input signal. The
relative gain of the PMTs is measured using an optical system consisting of a
Xenon lamp, optical fiber, and a scintillator ball. The system is diagramed in
Figure 3.1. The Xe lamp can produce intense flashes of light over relatively
short time periods on the order of tens of nanoseconds. Light from the Xe
flash lamp is filtered by Ultraviolet (UV) and Neutral Density (ND) filters;
the ND filter reduces the intensity without introducing wavelength depen-
dency. The light is then split and sent to a monitor/trigger module and to
the interior of the detector via optical fibers. At the end of the detector fiber
is an acrylic ball filled with BBOT scintillator and MgO powder. The BBOT
absorbs the UV light entering the ball and emits light near the wavelength
of Cherenkov light in water. The MgO powder serves to diffuse the light.

The goal of this calibration is to measure the relative gains of the PMTs.
This is done by measuring the "corrected Q" of each PMT for each light
Figure 3.1: The Xenon calibration system used to measure PMT relative gains [28].

Flash. Corrected $Q$ means the pulse height of a PMT that is corrected for angle of acceptance\(^1\), effective PMT density, the number of nonfunctioning PMTs, and the probability of a two photoelectron emission. The Corrected $Q$ is then normalized by the Xe monitor pulse height and corrected for the uniformity of the scintillator ball. The system is flashed several times with the scintillator ball at different locations within the inner detector. Figure 3.2 shows the final Corrected $Q$ distribution for all 11,146 PMTs; the relative gain spread ($1\sigma$) is 7%. These differences in gain are compensated for in software during event reconstruction and detector simulations, so to provide uniform response from the PMTs.

\(^1\)the angle at which the light hits the PMT; this can affect the response of the PMT due to asymmetries in the dynodes and electric fields within the PMT. Refer to Section 2.3.1
3.1.2 PMT Absolute Gain

The intensity of Cherenkov light produced by solar neutrino interactions within the detector is at the single photo-electron level for each "hit" PMT. Thus it is critically important to know the absolute gain for each PMT at the single photo-electron level. The ATM modules (see Section 2.4.1) record PMT pulse heights in units of picoCoulombs and so the conversion to the number of photoelectrons (PE) is needed. To make this calibration, the same equipment used in the "nickel" energy calibration is utilized (see Section 3.3.2). Electrons in the detector water Compton scatter with photons generated by the nickel source and produce Cherenkov light at the single photo-electron level. Figure 2.7 shows the distribution of pulse heights measured in picoCoulombs for a typical PMT during a nickel calibration. The large peak near zero is caused by PMT dark noise (thermal emissions of electrons from the photocathode or dynodes), while the second peak shows the single photo-electron distribution. Fitting the second distribution to a partial Gaussian curve results in the mean single photo-electron level of about

Figure 3.2: The "Corrected Q" distribution of all inner detector PMTs. The relative gain spread is 7 %.
400 ADC counts for this PMT. Every PMT's single photo-electron level is similarly measured.

3.1.3 PMT Timing

The relative timing of "hit" PMTs in an event is important when reconstructing the event vertex and direction. However, the time that a PMT channel registers a hit can vary as a function of the intensity of the light causing the signal. Time variation due to light intensity is known as time "slewing." Time slewing is due to two phenomena called the "first photo-electron effect" and "discriminator walk." The first photo-electron effect is a statistical effect. Photons striking a PMT have a probability (quantum efficiency) of being converted into a photo-electron of about 20%. Photons from the same event may strike a PMT face at slightly different times due to different path lengths or scattering; but it is the first photo-electron that triggers the ATM channel discriminator. A more intense spray of photons will create a photo-electron earlier than a less intense spray on average, thus causing a slightly earlier hit time to be recorded. The discriminator walk causes a similar effect. Large PMT pulse heights will cross the ATM discriminator threshold earlier (with respect to the peak of the pulse) than smaller pulse heights, thus causing an earlier hit time to be recorded.

The slewing of each PMT is mapped using a laser and filter system diagramed in Figure 3.3. The laser is a Laser Science, Inc. LSI V337-ND-S nitrogen laser with a pulse width of 3–4 ns, an average pulse energy of 300 $\mu$J, and a wavelength of 337 nm. Light from the laser pumps an Exalite$^{TM}$ 384 dye to produce 384 nm light. The light then travels through a logarithmically
Figure 3.3: Schematic of laser calibration system used in PMT timing calibration [28].

graded filter, then is split into 2 optical fibers. One fiber leads to a detector triggering system and the other to the detector interior. The end of the detector fiber is tipped with TiO$_2$ epoxy/diffuser and is suspended within a UV plexiglass container filled with LUDOX$^TM$ diffuser. The TiO$_2$ and LUDOX$^TM$ combination provide moderately diffused light with a relatively small increase in the pulse time width.

The laser is fired using several different filter levels to produce light levels from the single photoelectron level to several hundred PE. The timing and corresponding pulse heights are mapped in Figure 3.4 for all PMTs. Closed circles are measured data and open circles are the mean values with 1 σ bars. To understand this figure one needs to recall that an ATM channel begins
charging its TAC when a PMT signal is received and stops when the global detector trigger signal arrives. Therefore, signals that travel faster through a PMT will have higher TAC charges than slower moving signals. This results in longer TAC time values for large charge pulses, while low charge pulses have smaller time values, as is evident in Figure 3.4. The mapping of time and charge (TQ Map) for each PMT is used in event reconstructions.

3.2 Water Transparency

Water transparency affects event vertex, direction, and energy reconstruction, since Cherenkov light can be scattered or absorbed in the water. The water transparency in the Super-Kamiokande detector is measured using two different methods. One method provides a direct measurement of the transparency, however the value is only valid for a time period immediately...
proceeding and following the measurement. Since normal data taking must be suspended for several hours to make this type of measurement, a second, non-intrusive method is employed to monitor relative changes in water transparency between direct measurements.

3.2.1 Direct Measurement

A nitrogen laser and a charged couple device (CCD) camera system, shown in Figure 3.5, is employed to make direct water transparency measurements. Light from the laser pumps a dye to produce monochromatic light. Dyes can be chosen such that light between 337 and 600 nm can be produced. The laser light is then split with one beam entering an integrating sphere, which is monitored by a 2 inch PMT. The other beam travels through an optical fiber to an acrylic diffuser ball in the interior of the detector. The CCD camera sits at the water surface with its lens submerged and directed towards the diffuser ball. The laser is fired several times with the diffuser ball at different distances from the CCD camera. The CCD camera records the number of photons received and this signal is normalized by the monitor PMT signal height. The effects of light scattering are removed by only using CCD pixels near the diffuser CCD image. Figure 3.6 shows a typical CCD image taken of a 400 nm laser pulse, and Figure 3.7 shows the normalized CCD signal as a function of the distance between the CCD camera and the diffuser ball. The water transparency during this calibration using 400 nm light was 72.1 m. This procedure is repeated several times using different laser dyes to provide measurements at 337, 400, 500, and 580 nm wavelengths.
Figure 3.5: Water transparency direct measurement system.

Figure 3.6: CCD camera image taken of a 400 nm laser pulse using the direct water transparency measurement system.
Figure 3.7: CCD signal normalized by the corresponding monitor pmt signal as a function of the distance between the CCD camera and the diffuser ball. The water transparency during this calibration for 400 nm light was $(72.1 \pm 3.2)$ m.
3.2.2 Relative Measurements

Cosmic ray muon decay events which occur naturally within the detector are used to measure relative changes in the water transparency. This is advantageous since these events are recorded during normal detector operation, reducing detector downtime. The criteria for selecting electron events which decay from muons are:

- a candidate decay electron event must follow a stopping muon event within 1.5 to 8 $\mu$s (Figure 2.3 shows a muon that entered the detector at the top and stopped within the interior of the detector.)

- a candidate event must have $N_{eff} > 70$. ($N_{eff}$ is defined in Section 3.3; it is the number of "hit" PMTs with corrections for light attenuation, PMT acceptance angle, effective PMT density, the number of nonfunctioning PMTs, and the probability of a two photoelectron emission in one PMT. This criterion ensures decay electrons of energies $> 10$ MeV.)

In each decay event only the PMTs with times that are within a 50 ns window ($N_{50}$) and that lie within a cone of opening angle 32° to 52° with respect to the reconstructed decay electron direction are used in the analysis. This removes effects from scattered and reflected light from the water transparency measurement. The relation between Corrected $Q$ (see Section 3.1.1) for each PMT and the distance between the PMT and the reconstructed vertex is shown in Figure 3.8. The relation is fit to a best-fit line and the water transparency is calculated by applying the following relation:

$$-\frac{1}{\lambda_{wt}} = \frac{\ln(Q_{corr})}{r}, \quad \text{where:}$$

$$Q_{corr}$$

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Figure 3.8: Corrected $Q$ ($Q_{corr}$) for each PMT as a function of the distance between the PMT and the reconstructed vertex. Also shown is the best-fit line with slope $= -0.117 \times 10^{-3}$.

$\lambda_{wt}$ = water transparency

$Q_{corr}$ = Corrected $Q$

$r$ = distance from reconstructed vertex to PMT

Cosmic ray muons enter the Super-Kamiokande detector at a rate of about 3 Hz of which about 4% stop within the detector. Muon decay events are collected in one week periods and then are used in the relative water transparency measurement. Figure 3.9 shows the relative water transparency values as a function of time. The transparency has increased, since detector turn-on time due to water recirculation through the water purification
Figure 3.9: Relative changes in water transparency values as function of time from June 1996 to June 1997 as measured by muon decay electrons.

system. Periods of steady increases in water transparency follow upgrades or maintenance of the water purification system.

3.3 Energy Calibrations

Precise energy scale calibration of the detector is essential for the energy spectrum measurement of recoil electrons. The total amount of Cherenkov light emitted during an event is used in the energy determination. Calibration events of known energies are observed in the detector and the detector Monte Carlo (MC) is tuned using these events. The tuned MC can then be used to relate the detector response and the absolute energy of any event observed within the detector. To investigate different neutrino oscillation hypotheses, the uncertainty in the absolute energy determination must be < 1 %.
An electron linear accelerator (LINAC) is utilized as the primary absolute energy scale calibrator. The absolute energy scale is monitored for stability by muon decay electrons and spallation products induced by cosmic ray muons. The energy scale is also cross-checked using $^{16}$N produced by stopping muon capture on oxygen and a Ni(n,γ)Ni source.

Electron energy is measured by calculating the effective number of “hit” PMTs, $N_{\text{eff}}$, which is the number of hit PMTs with corrections for light attenuation through the water, the angular dependence of PMT acceptance, the effective density of PMTs, the number of nonfunctioning PMTs, and the probability of a two photo-electron emission in one PMT. Further corrections are made for noise hits due to the PMT dark noise rate (~3.3 kHz, which contributes about 1.8 hits within 50 ns) and for the tail of the hit PMT time distribution (up to 100 ns), caused by the scattering of light in the water and by reflections on the PMT and light barrier surfaces. Equation 3.2 defines $N_{\text{eff}}$:

$$N_{\text{eff}} = \sum_{i=1}^{N_{50}} \left[ R_{\text{oper}} \cdot R_{\text{cover}}(\theta_i, \phi_i) \cdot R_{\text{type}} \cdot e^{L_i/\lambda} \cdot (X_i + \epsilon_{\text{tail}} - \epsilon_{\text{dark}}) \right], \quad (3.2)$$

where:

- $N_{50} = \text{number of “hit” PMTs in an event whose times lie within a 50 ns window}$
- $R_{\text{oper}} = \text{ratio of total PMTs (11,146) to functioning PMTs}$
- $R_{\text{cover}}(\theta_i, \phi_i) = \text{effective PMT coverage corrected for photon acceptance angle}$
\[ R_{\text{type}} = \text{PMT type gain correction} \]
\[ = \begin{cases} 0.833, \text{KEK}^2 \text{ PMTs (375 PMTs)} \\ 1.000, \text{Normal PMTs (10771 PMTs)} \end{cases} \]
\[ e^{L_i/\lambda} = \text{correction factor for light attenuation} \]
\[ L_i = \text{distance from reconstructed vertex to } i^{th} \text{ PMT} \]
\[ \lambda = \text{water transparency} \]
\[ \epsilon_{\text{tail}} = \text{correction for hits outside the 50 ns time window (reflected light)} \]
\[ = \frac{N_{100} - N_{50}}{N_{50}} \]
\[ N_{100} = \text{number of "hit" PMTs in an event whose times lie within a 100 ns window} \]
\[ N_{50} = \text{number of functioning PMTs} \]
\[ \epsilon_{\text{dark}} = \text{correction for dark noise hits} \]
\[ = \frac{n_{\text{pmt}} \cdot Rate_{\text{dark}}}{N_{50}} \cdot 100 \text{ ns} \]
\[ n_{\text{pmt}} = \text{number of functioning PMTs} \]
\[ Rate_{\text{dark}} = \text{dark noise rate} \]

The \( X_i \) term is the expected number of photoelectrons detected by the \( i^{th} \) PMT, which is determined by the number of hit PMTs in the \( 3 \times 3 \) patch of PMTs centered about the \( i^{th} \) PMT, and is defined to be:

\[ X_i = \begin{cases} \lambda_i = \ln [(1 - x_i)^{-1}], & x_i < 1. \\ 3.0, & x_i = 1, \end{cases} \]

where:

\[ x_i = \text{ratio of hit PMTs to functioning PMTs in the } 3 \times 3 \text{ patch centered about the } i^{th} \text{ PMT} \]
Figure 3.10: Average number of photoelectrons per PMT in a 3×3 patch (λ) as a function of the number of hit PMTs in the patch.

\[ \lambda_i = \text{average number of photoelectrons per PMT in} \]

the patch centered about the \( i^{th} \) PMT,

and \( \lambda_i \) satisfies the Poisson distribution for the probability of no hits:

\[ P_0 = \frac{\lambda_i^0 \cdot e^{-\lambda_i}}{0!} = 1 - x_i. \]  \hspace{1cm} (3.4)

Figure 3.10 shows the average number of photoelectrons per PMT in a 3×3 patch (λ) as a function of the number of hit PMTs in the patch. The value at 9 hit PMTs was calculated by extrapolation.

The \( N_{\text{eff}} \) corrections are designed to remove position and water transparency related effects so as to give uniform response over the fiducial volume. \( N_{\text{eff}} \), as above described, is closely related to electron visible energy. However, the energy used in this analysis includes the energy deposition below
the Cherenkov threshold and the rest mass of the electron and is, therefore, the total electron energy. Thus, any difference between the measured total energy obtained by $N_{\text{eff}}$ and the true electron total energy is due to detector energy resolution smearing and position dependent response. The LINAC is employed to relate $N_{\text{eff}}$ to absolute energy.

### 3.3.1 Linear Electron Accelerator (LINAC)

The LINAC, located near the Super-Kamiokande detector, injects downward-moving monoenergetic electrons with a tunable energy ranging from 4.89 to 16.09 MeV. Figures 3.11 and 3.12 show various views of the LINAC calibration system. The LINAC sits in an adjacent cavern, separated from the Super-Kamiokande detector by solid rock. The LINAC is a medical grade accelerator (Mitsubishi ML-15MIII) modified for this calibration. Accelerated electrons travel from the LINAC within evacuated beam pipes ($\leq 10^{-4}$ torr) to the detector interior, passing 3 bending magnets, 4 collimators, steering magnets, and 2 sets of focusing quadrupole magnets. The beam pipe end-cap, shown in Figure 3.13, is tapered as to reduce shadowing effects, and the cap window is made of 100 $\mu$m Ti. The end-cap can be inserted into one of several access portals along the detector’s x-axis and its depth adjusted by the number of vertical beam pipe segments used; refer to Figure 3.11.

Energy calibration utilizes LINAC data taken at 8 representative positions within the Super-Kamiokande 22.5 kton fiducial volume with 7 different energies ranging from 4.89 to 16.09 MeV. Details of the LINAC calibration can be found in Ref. [29].
Figure 3.11: Schematic of the LINAC and beam line in relation to the Super-Kamiokande detector. The dots represent the locations where LINAC calibrations have been performed. The detector coordinates of the calibration locations are listed in Table 3.1. The origin \((x,y,z) = (0,0,0)\) is located at the center of the inner detector. The negative \(y\)-direction is 42.5° West of the Magnetic North Direction and 49.4° West of the Geodesic North Direction.

Figure 3.12: Detail of the LINAC and first bending magnet (D1). The beam momentum is defined by the setting of the D1 magnet.
Figure 3.13: Detail of the LINAC beam pipe end-cap.

Table 3.1: Locations where LINAC calibrations has been taken. Also refer to Figure 3.11. The origin \((x,y,z) = (0,0,0)\) is located at the center of the inner detector.

<table>
<thead>
<tr>
<th>Beam Pipe End-Cap Position</th>
<th>X (m)</th>
<th>Y (m)</th>
<th>Z (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-3.88</td>
<td>-0.71</td>
<td>12.28</td>
</tr>
<tr>
<td>B</td>
<td>-3.88</td>
<td>-0.71</td>
<td>0.27</td>
</tr>
<tr>
<td>C</td>
<td>-8.13</td>
<td>-0.71</td>
<td>12.28</td>
</tr>
<tr>
<td>D</td>
<td>-8.13</td>
<td>-0.71</td>
<td>0.27</td>
</tr>
<tr>
<td>E</td>
<td>-12.37</td>
<td>-0.71</td>
<td>12.28</td>
</tr>
<tr>
<td>F</td>
<td>-12.37</td>
<td>-0.71</td>
<td>0.27</td>
</tr>
<tr>
<td>G</td>
<td>-3.88</td>
<td>-0.71</td>
<td>-11.73</td>
</tr>
<tr>
<td>H</td>
<td>-12.37</td>
<td>-0.71</td>
<td>-11.73</td>
</tr>
</tbody>
</table>
The absolute energy of the LINAC electron beam is measured by a germanium detector, which was in turn calibrated by gamma-ray sources and internal-conversion electrons from a $^{207}$Bi source at the Tanashi branch of KEK. The germanium detector is used to calibrate the LINAC beam during each calibration period. The uncertainty in the beam energy deposition in the Super-Kamiokande detector is 0.55% at 6 MeV and 0.3% at 10 MeV, resulting from the uncertainty in the beam energy (<20 keV) and the reflectivity of the beam pipe end-cap materials.

Beam spills can occur at a maximum rate of 60 Hz with an average intensity at the beam pipe end-cap of about 0.1 electrons per spill. The beam spread at the end-cap is about 1 to 2 cm. Electrons passing through the end-cap trigger the Super-Kamiokande detector via the “Trigger counter” scintillator/pmt combination in the end-cap; see Figure 3.13. “Veto counters” mounted along the interior perimeter of the beam pipe above the Trigger counter veto off-center electron events.

LINAC calibration data is passed through the same data reduction chain as the solar neutrino analysis (refer to Chapter 6) with one additional cut applied to remove LINAC events which have more than one electron in them. Figure 3.14 shows examples of timing distributions for one, two, and three electron events. About 5% of all LINAC triggers have multiple peaks and are rejected. Figure 3.15 shows a scatter plot of the reconstructed vertex positions of LINAC events taken at (x,z)=(-4 m,0 m) with a beam momentum of 16.31 MeV/c. Projections are shown to the right and bottom of the figure.
Figure 3.14: Examples of event timing distributions with one (top), two (middle), and three (bottom) electrons.

Figure 3.15: Scatter plot of the reconstructed vertex positions for LiNAC events taken at $(x,z)=(-4 \text{ m}, 0 \text{ m})$ and beam momentum 16.31 MeV/c. Projections are shown to the right and bottom of the figure.
The absolute energy scale, the relation between $N_{eff}$ and the total electron energy, is obtained from a MC simulation program for which various parameters are tuned to reproduce the LINAC data taken at the various positions and energies (also refer to Section 5.1). Figure 3.16 compares the LINAC-tuned MC and the measured electron energy distributions for each LINAC energy setting. Each measured and MC distribution is fit to a Gaussian curve and the respective mean value is taken to be the energy “scale” of the events. The $1\sigma$ widths of the fits are defined to be the energy resolutions. The fractional difference between MC and measured energy scales is shown as a function of energy in Figures 3.17 and 3.18. Figure 3.17 shows the differences at each LINAC position and Figure 3.18 shows the position averaged differences. The dotted and solid lines mark the $\pm0.5$ and $\pm1\%$ levels respectively. All position averaged differences are within the $\pm1\%$. Further, the MC reproduces the position dependence of the energy scale to within $0.5\%$ on average as indicated in Figure 3.19.

Figure 3.20 shows the fractional difference in measured and MC energy resolutions as a function of beam energy. Averaged over all positions and energies, the MC reproduces the measured energy resolution to better than $2\%$. Table 3.2 lists the position averaged energy resolutions for each LINAC beam energy.

The measured water transparency is used in calculating $N_{eff}$ for each event to correct for Cherenkov photon attenuation. The variation of the water transparency has caused $\sim3.8\%$ change in the energy scale over the data taking period considered in this paper. By adjusting $N_{eff}$ for the variations
Table 3.2: Super-Kamiokande detector resolutions as measured by LINAC calibration system.

<table>
<thead>
<tr>
<th>Total Energy (MeV)</th>
<th>Energy Resolution (%)</th>
<th>Angular Resolution (degree)</th>
<th>Vertex Resolution (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.89</td>
<td>20.9±0.6</td>
<td>36.7±0.2</td>
<td>182±21</td>
</tr>
<tr>
<td>5.84</td>
<td>19.2±0.5</td>
<td>34.6±0.2</td>
<td>133±8</td>
</tr>
<tr>
<td>6.79</td>
<td>18.0±0.3</td>
<td>32.0±0.1</td>
<td>108±5</td>
</tr>
<tr>
<td>8.67</td>
<td>16.2±0.2</td>
<td>28.4±0.2</td>
<td>85±2</td>
</tr>
<tr>
<td>10.78</td>
<td>14.7±0.3</td>
<td>25.3±0.2</td>
<td>73±2</td>
</tr>
<tr>
<td>13.44</td>
<td>13.5±0.3</td>
<td>22.5±0.1</td>
<td>65±2</td>
</tr>
<tr>
<td>16.09</td>
<td>12.6±0.3</td>
<td>20.6±0.1</td>
<td>50±2</td>
</tr>
</tbody>
</table>

in water attenuation length, the stability of the energy scale is better than 0.5 % over the time period described here and ~0.2 % in r.m.s.

3.3.2 Nickel Calibration

Before the LINAC system became operational in early 1997, an energy calibration system based on a neutron source and nickel wire combination suspended in the detector was used. The nickel calibration source is sketched in Figure 3.21. It consists of a cylindrical canister with the an ionization counter with $^{252}$Cf applied to its electrode at its center. About 3 % of the $^{252}$Cf decays result in fission events in which an average of 3.8 neutrons are released (the remaining 97 % are $\alpha$ decays). The ionization counter produces a signal for each fission; this signal is amplified and is used to trigger the Super-Kamiokande detector. The neutrons thermalize via scattering with protons in the water within the canister and are absorbed by the nickel wire producing gammas. These gammas escape the canister and Compton scatter with electrons in the Super-Kamiokande detector.
Figure 3.16: Measured and LINAC-tuned MC electron energy distributions for several beam energies. Data are crosses and MC are histograms.
Figure 3.17: Fractional difference between measured and MC absolute energy scales at each LINAC position (A–H) defined in Table 3.1 as a function of LINAC beam energy. The errors shown are statistical. The dotted and solid lines mark the ±0.5 and ±1 % levels respectively.
Figure 3.18: Fractional difference between measured and MC absolute energy scales averaged over LINAC positions as a function of LINAC beam energy. The inner error bars are the r.m.s. of the spread of over all positions and the outer bars are the systematic errors. The dotted and solid lines mark the ±0.5 and ±1 % levels respectively.
Figure 3.19: Fractional difference between measured and MC energy scales as a function of position.
Figure 3.20: Fractional difference between measured and MC energy resolutions as a function of LINAC beam energy.
Figure 3.21: Schematic of the nickel calibration source. The $^{252}$Cf neutron source is applied to the electrode of the ionization counter at the center.

There are many difficulties with this calibration procedure. The nickel wire is comprised of several isotopes and each isotope has many nuclear de-excitation branches, most with multi-gamma emissions. The transition modes of $^{58}$Ni(n,γ)$^{59}$Ni for example are shown in Figure 3.22. Many branches are not modeled because there are so many branches and because all excitation modes are not known.

Comparison of nickel calibrations taken at LINAC calibration positions show a systematic shift in the energy scales between the two calibrations of the order of 3%. Nickel calibrations also show an up/down asymmetry when only upward- or only downward-moving Compton scattered electrons are used in the analysis. To investigate these phenomena a special nickel calibration was performed by replacing the nickel wire with a boron and
Figure 3.22: Transition diagram for the $^{58}\text{Ni}(n,\gamma)^{59}\text{Ni}^*$ reaction [28].
water solution such that the boron solution had approximately the same total neutron cross section as the nickel wire. The energy distribution for events in this special calibration were subtracted from the normal nickel calibration to remove any unknown and unmodeled backgrounds. The energy scale shift between nickel and LINAC calibrations reduces to 1.4% with this background subtraction technique.

It is believed that the current nickel source is not understood well enough to be modeled properly e.g. it incorporates too many materials which complicate modeling e.g. metals within the ionization counter, steel hardware to suspend the canister, and electric cables containing chlorine in the insulation. A new nickel calibration source is now being prepared, which is spherically symmetric and contains a minimum of metal components and no chlorine components. Nickel calibrations, however, are still performed monthly using the current source to track any temporal or positional changes in the energy scale, since these calibrations can be performed in a shorter time period and at more locations within the detector than can be done with the LINAC. The nickel and LINAC energy scales are further discussed in Section 3.3.3.

3.3.3 Nickel Calibration

Stopping muons captured by oxygen in the water of the detector produce $^{16}$N a fews times a day. The decay of $^{16}$N has only four major decay modes (all easily calculable) and so it is much easier to model than the nickel calibration. Because $^{16}$N is produced naturally by stopping cosmic ray muons, $^{16}$N decay events are uniformly distributed throughout the detector. Figure 3.23 shows the measured energy distribution of $^{16}$N events with
Figure 3.23: Measured energy distribution of $^{16}$N events (dashed line) with the corresponding nickel-tuned MC distribution (solid line).

the corresponding nickel-tuned MC distribution, while Figure 3.23 shows the measured $^{16}$N energy distribution, but with the corresponding LINAC-tuned MC distribution. The difference in energy scales between that obtained by the measured $^{16}$N decay beta spectrum and the MC tuned by LINAC is $0.2 \pm 0.6 \%$, while agreement with the nickel-tuned MC is visibly very poor.

3.3.4 Calibrations using Spallation Products

The time variation and directional dependence of the energy scale was monitored using spallation events, which are beta- and gamma-rays from radioactive nuclei created by cosmic ray interactions within the detector. Because spallation events are distributed uniformly in time and throughout the detector volume, they can be used to monitor the time variation and the directional dependence of the energy scale on a more continuous basis and at more points in the volume than is possible with the LINAC or nickel
calibrations. The resulting time variation of the energy scale is less than 0.5 % over the entire 504 day time period.

The spallation events are subdivided into 10 data sets according to the reconstructed zenith angle and the relative difference of the energy distribution among the 10 data sets is compared. The obtained angular dependence of the energy scale is less than 0.5 %. This result allows the use of the LINAC absolute energy calibration, which thus far has been taken electrons moving only in the downward-going direction, for all directions.

3.3.5 Absolute Energy Scale and Uncertainty

Summing the known uncertainties in the absolute energy scale described above, the net uncertainty in the energy scale is estimated to be 0.8 % at 10 MeV, which comes from the uncertainty in the LINAC electron energy deposition (0.3 %), the position dependence of the energy scale (0.5 %),
Figure 3.25: The values of $N_{\text{eff}}$ as a function of energy. Each data point in this figure was generated by the LINAC-tuned MC and represents the mean values of thousands of simulated electrons at random vertices and random initial directions throughout the 22.5 kton fiducial volume. The line is a best-fit $4^{th}$ order polynomial.

the uncertainty of the water transparency determination ($0.2 \%$), and the directional dependence of the energy scale ($0.5 \%$).

The absolute energy scale, the relation between $N_{\text{eff}}$ and the total electron energy, is obtained from the LINAC-tuned MC. Figure 3.25 shows $N_{\text{eff}}$ values as a function of input electron energy. Each data point in this figure was generated by the LINAC-tuned MC and represents the mean values of thousands of simulated electrons at random vertices and random initial directions throughout the 22.5 kton fiducial volume. The line is a best-fit $4^{th}$ order polynomial.

3.4 Detector Resolutions

Since the LINAC electrons are injected into the detector at known positions with a known direction, the vertex and direction reconstruction resolutions can be measured using LINAC electrons. Vertex resolution is defined
as the radius of the sphere around the LINAC electron injection location that contains 68% of the reconstructed vertices. Figure 3.26 compares the measured vertex distributions and the corresponding LINAC-tuned MC simulations as a function of the distance from the beam pipe end-cap for various beam energies taken at \((x,z)=(-12 \text{ m}, +12 \text{ m})\). Figure 3.27 shows the fractional difference between the measured and MC vertex resolutions. Table 3.2 summarizes the position averaged resolutions as measured by the LINAC calibration system.

Angular resolution is defined as the opening angle of the cone around the LINAC electron beam direction which contains 68% of the reconstructed directions. Figure 3.28 shows the fractional difference between the measured angular resolutions and the corresponding MC simulations. The measured angular resolution is 2–3% smaller than the corresponding MC simulation. The difference could be due to an inaccurate amount of light scattering in the current MC simulation, but it is not yet fully understood. Table 3.2 summarizes the angular resolutions as measured by the LINAC calibration system.

3.5 Trigger Efficiency

An accurate measurement of the solar neutrino flux depends upon the ability of the Super-Kamiokande detector to self-trigger on electron scattering events with total energies <20 MeV. To measure the trigger efficiency of the detector the same equipment as used in nickel energy calibrations is utilized (see Section 3.3.2). While the nickel gamma source is suspended within the detector, data is taken with both normal and special calibration
Figure 3.26: Measured and LINAC–tuned MC vertex distributions as a function of the distance from the beam pipe end-cap and various beam energies taken at $(x,z)=(-12 \, \text{m}, +12 \, \text{m})$. Data are crosses and MC histogram.
Figure 3.27: Fractional difference between data and MC vertex resolutions averaged over all LINAC positions as a function of beam energy. The error bars are the r.m.s. of the spread of over all positions.

Figure 3.28: Fractional difference between measured and MC angular resolutions averaged over all LINAC positions. The error bars are the r.m.s. of the spread of over all positions.
triggers (also refer to Section 2.5). The special “CAL” calibration trigger operates just as the LE and HE triggers, except that it is set to a very low trigger threshold of 150 mV corresponding to about 14 PMT signals (above dark noise) in a 200 ns time window. The LE trigger threshold is 320 mV, requiring about 29 PMT signals. The efficiency of the LE trigger to trigger the detector on low energy events is calculated by measuring by the ratio:

\[
\text{eff}_{\text{trg}} = \frac{\text{Number of events with both LE and CAL triggers}}{\text{Number of events with CAL triggers}}
\]

The positional dependence of the LE trigger is measured by placing the nickel source at different locations within the detector. The efficiency of the LE trigger as a function of the number of corrected PMT “hits” \( N_{\text{eff}} \) is shown in Figure 3.29 with the nickel source at the detector center position \((x, y, z) = (0.35 \ m, -0.71 \ m, 0 \ m)\) (solid line) and at \((x, y, z) = (0.35 \ m, -0.71 \ m, +12 \ m)\) (dashed line). The trigger efficiency is almost 100 % for energies > 7 MeV \( N_{\text{eff}} > \sim 47 \) throughout the analysis fiducial volume. The MC detector simulations agree with the measured trigger efficiencies, except for a 1.2 % difference in the 6.5 to 7 MeV energy range (MC has the greater efficiency). This difference is considered during the neutrino flux calculation.
Figure 3.29: Efficiency of the LE trigger as a function of the number of corrected PMT “hits” \( N_{eff} \) as measured by the nickel gamma source. Solid line is data measured with nickel source located at \((x, y, z) = (0.35 \, m, -0.71 \, m, 0 \, m)\) and dashed line is data measured at \((x, y, z) = (0.35 \, m, -0.71 \, m, +12 \, m)\).
CHAPTER 4

EVENT RECONSTRUCTION

The vertex and direction of each solar neutrino candidate event is reconstructed using the PMT timing and hit pattern information. For electrons of total energy of about 10 MeV, the distance traversed through water is less than 10 cm. This is smaller than the possible vertex resolution of the detector, and so each neutrino candidate event can be treated as a point source of Cherenkov radiation during vertex reconstruction. The pattern of “hit” PMTs produced by the Cherenkov photons allows the reconstruction of the particle’s direction of travel.

Each cosmic ray muon which enters the detector has its entry location, exit location (if it exits), and track direction reconstructed. Muon track information is used in removing spallation events from the neutrino candidate data set.

4.1 Vertex Reconstruction

The vertex of a solar neutrino candidate event is reconstructed using hit PMT timing information. First, the PMT timing information in each event is evaluated to remove random noise and reflected light hits that will adversely affect the reconstruction. Figure 4.1 shows a typical timing distribution of hit PMTs in a candidate event. A peak of in-time PMTs can be seen and is preceded and followed by several PMT hits that may be random dark noise or reflected light hits or late (long path length) signal hits.
Figure 4.1: Typical timing distribution of hit PMTs in a neutrino scattering candidate event [30]. Times $t_1$, $t_2$, $t_3$, and $t_4$ are defined in the text.

The selection of which PMT timing hits to be used in vertex reconstruction is made using by the following algorithm:

1. Define the earliest PMT hit time to be $t_1$ and the latest $t_4$; refer to Figure 4.1.

2. Locate the 200 ns time window which contains the maximum number of hit PMTs. Label the beginning time of this window $t_2$ and the ending time $t_3$ ($= t_2 + 200$ ns); refer to Figure 4.1.

3. Estimate the number of dark noise hits $N_{\text{noise}}$ in this window by the relation:

$$N_{\text{noise}} = \frac{(t_3 - t_2)}{(t_2 - t_1) + (t_4 - t_3)} \cdot [N_{\text{hit}}(t_1 : t_2) + N_{\text{hit}}(t_3 : t_4)],$$

(4.1)

where $N_{\text{hit}}(t_i : t_j)$ is the number of PMT hits between time $t_i$ and time $t_j$. 

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4. Calculate the “significance” of the window, where significance is defined to be:

\[
\text{significance} = \frac{N_{\text{signal}}}{\sqrt{N_{\text{noise}}}} = \frac{N_{\text{hit}}(t_2 : t_3) - N_{\text{noise}}}{\sqrt{N_{\text{noise}}}} \tag{4.2}
\]

5. Locate the time sub-window of width \([200 \text{ ns} \cdot \frac{n}{11}, (n = 1, 2, 3, ...10)]\) within the 200 ns window which contains the maximum number of hit PMTs.

6. Calculate the \(N_{\text{noise}}\) and significance values of each time sub-window as done in steps 3 and 4.

7. Use only the PMT hits within the window/sub-window with the maximum significance value for the vertex reconstruction. Or if a wider time window exists with a significance greater than 80 % of the maximum significance, use the hits within that larger time window instead.

After the selection of PMT hits the vertex is found using a grid search method. Grid points are distributed throughout the detector as shown in Figures 4.2 and 4.3. The vertex of the event is assumed to be at each grid point in turn. At each grid point each selected PMT hit has its time-of-flight (TOF), the theoretical time for a photon to reach the PMT from the grid point, subtracted from the measured PMT time. This time difference is called the “residual time” \(t_{\text{res}}\). The “goodness” of the grid point is calculated using the following definition:

\[
\text{goodness} = \frac{1}{N_{\text{sel}}} \cdot \sum_i \exp \left( \frac{-t_{\text{res},i}^2}{2\sigma_i^2} \right), \tag{4.3}
\]
where the summation is over selected PMT signals, $N_{sel}$ is the number of selected PMT hits, and $\sigma_t$ is the PMT timing resolution (5 ns is used). If a grid point is very near to the vertex of the event the residual times for each PMT will be very small and the goodness value will approach its maximum value of 1. However, if a grid point is very far from the vertex the time residuals will be very large and the goodness value will approach the minimum value of 0. The best-fit grid point is chosen by maximizing the goodness value. Once a grid point is chosen, a finer grid is laid about that grid point and the process is re-iterated. This is repeated several times until a final grid spacing of 5 cm is used.
The vertex resolutions were measured using the LINAC calibration system at eight representative locations within the detector. Table 3.2 lists the combined measured vertex reconstruction resolutions for each LINAC beam energy. Vertex resolution is defined as the radius of the sphere around the LINAC electron injection location that contains 68% of the reconstructed vertices.

4.2 Direction Reconstruction

Once the vertex position of an event has been reconstructed, the electron's direction of travel can be reconstructed by examining the pattern of hit PMTs created by the Cherenkov ring. The direction of travel is found by varying the particle direction about the reconstructed vertex in 20° steps in azimuth and
zenith and choosing the direction with the largest direction likelihood value. The direction likelihood value is determined by the below defined function, which evaluates the likelihood that the observed pattern of hit PMTs resulted from a Cherenkov cone with the given vertex and direction. The direction likelihood function $L_{\text{dir}}$ is defined as:

$$L_{\text{dir}} = \sum_i \log [P(\cos\theta_{\text{dir},i})] \cdot \frac{\cos\theta_{\text{acc},i}}{A(\theta_{\text{acc},i})},$$

where the summation is over selected PMTs and:

- $\theta_{\text{dir},i}$ = angle between the assumed direction and the vector from the vertex to the $i^{th}$ PMT
- $P(\cos\theta_{\text{dir},i})$ = probability that a Cherenkov photon radiated at cone angle of $42^\circ$ w.r.t. particle direction will be detected at cone angle of $\cos\theta_{\text{dir},i}$
- $\theta_{\text{acc},i}$ = angle between the vector from the vertex to the $i^{th}$ PMT and the direction in which the PMT faces i.e. the acceptance angle
- $A(\theta_{\text{acc},i})$ = relative probability of photon detection of the $i^{th}$ PMT as a function of acceptance angle

The functions $P(\cos\theta_{\text{dir}})$ and $A(\theta_{\text{acc}})$ are derived from MC simulations of 10 MeV electrons and are shown in Figure 4.4. The particle direction is fine tuned by scanning about the best-fit direction with increasingly smaller angular steps with the final scan utilizing $1.6^\circ$ step sizes.
Figure 4.4: The functions $P(\cos\theta_{dir})$ (left) and $A(\theta_{acc})$ (right) as derived from MC simulations of 10 MeV electrons [28].

The direction resolutions as measured by the LINAC calibration system are listed in Table 3.2 as a function of beam energy. Angular resolution is defined as the opening angle of the cone around the LINAC electron beam direction which contains 68% of the reconstructed directions.

4.3 Muon Track Fitting

Cosmic ray muons entering the detector at a rate of about 3 Hz can breakup the nuclei of oxygen atoms in the water and create radioactive products; the decay of these products can resemble solar neutrino–electron scattering. Such decay events are known as “spallation” events. To remove spallation events from the neutrino scattering candidate sample, the tracks of muons traveling though the detector are fit and these tracks are used in the data reduction process described in Chapter 6. Details of the fitting procedure can be found in Reference [28]. About 6% of muons cannot be fit due to very short path lengths in the detector (called “corner clippers”) or
two or more muons entering the detector simultaneously. The resolution of the muon fitter was calculated by comparing the fit entry and exit positions with those fit by eye for about 1000 muon events. The $1 \sigma$ track difference between software and human fitting is 67 cm.
CHAPTER 5

MONTE CARLO (MC) SIMULATIONS

5.1 Detector Simulation MC

Electromagnetic interactions affecting the recoil electrons and Cherenkov photons within the detector are simulated by incorporating GEANT 3.21, a particle physics simulation package developed at CERN [31], into the detector MC. The GEANT package is capable of simulating electromagnetic interactions from about 10 keV to about 10 TeV. For electrons with total energies on the order of 10 MeV, the dominate processes modeled are multiple Coulomb scattering, Cherenkov light and δ-ray generation, and Bremsstrahlung. For the Cherenkov photons absorption and scattering are the dominate processes modeled. The number and direction (with respect to the simulated recoil electron) of generated Cherenkov photons are calculated using Eqns. 2.2 and 2.1, respectively, however, the wavelengths of the generated photons are limited to between 300 and 700 nm, since PMT sensitivity (quantum efficiency) is limited to this range.

Most parameters in the MC are modeled using measured and theoretical values. Photomultiplier tube quantum efficiencies and pulse height responses are modeled using the measured values described in Section 2.3.1. Reflections from the PMT glass surfaces and the black sheet light barrier covering the ID walls are modeled using theoretical values, which have been validated by measurement.
Other parameters in the detector MC are set by tuning the MC responses to match measured detector values. The three tuned parameters are PMT timing resolutions, PMT collection efficiencies, and the water transparency scattering-to-absorption ratio. Photomultiplier timing resolutions for single photo-electrons in the MC are tuned to reproduce the measured vertex resolutions. Figure 3.27 shows the fractional difference between measured and MC vertex resolutions as a function of LINAC beam energy. Errors are the r.m.s. spread over all beam pipe positions. The tuned PMT timing resolution is 2.4 ns, which is in good agreement with the measured value of 2.2 ns (see Section 2.3.1).

Like PMT timing resolution the water transparency scattering-to-absorption ratio is also tuned. Although water transparency is set by measured values, the ratio of scattering to absorption is not. Rayleigh scattering dominates for photon wavelengths shorter than about 400 nm, while for longer wavelengths absorption dominates. This ratio is set by tuning the MC positional dependence to the measured positional dependence of the LINAC calibrations. Figure 5.1 shows the theoretical, measured, and tuned attenuation coefficients (inverse of attenuation length) as a function of wavelength. The dashed lines are theoretical predictions and the data points are measured values. The solid line is the MC tuned values, which agrees very well with both the measured and theoretical values.

The final tuned parameter is the PMT collection efficiency, which was measured by ICCR\(^1\) to have a mean value of 70 % [25]. The collection

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\(^1\)Institute for Cosmic Ray Research, University of Tokyo
efficiency affects the overall energy scale of the MC and is the final parameter which is tuned to energy calibration data. The tuned MC collection efficiency value is 78 %. The fractional difference between tuned MC and measured energy scales using the LINAC calibration system is shown as a function of energy in Figure 3.18. All values are within the ±1 %.

5.2 Solar Neutrino Interaction Simulation

Simulations of solar neutrino scattering within the Super-Kamiokande detector are divided into two steps. The first step calculates the expected recoil kinetic energy spectrum of neutrino-scattered electrons within the Super-Kamiokande detector from best-estimate (standard) solar neutrino spectra and neutrino-electron scattering cross sections. The second step simulates
the Super-Kamiokande detector's response to these recoil electrons. The detector MC used in the second step is described in Section 5.1.

The Super-Kamiokande detector is sensitive to both $^8$B and HeP neutrinos, although HeP neutrinos are not expected to contribute significantly to the measured neutrino spectrum. The simulation of $^8$B solar neutrinos is discussed in detail in Sections 5.2.1 and 5.2.2. Simulation of HeP neutrinos is performed using the same procedures as described for $^8$B neutrinos. A brief description of the HeP neutrino simulation is provided in Section 5.2.3.

5.2.1 $^8$B Solar Neutrino Simulation

The best-estimate (standard) normalized $^8$B solar neutrino energy spectrum $\lambda(E_\nu) \, [\text{MeV}^{-1}]$ is shown in Figure 5.2 [32]. The shape of the spectrum is independent of the conditions in the solar interior\(^2\) and so is independent of solar models. Uncertainties in the shape of the spectrum due to the plasma environment in which the $^8$B neutrinos are being created are small; the 3\(\sigma\) uncertainties are shown as the $\lambda^\pm$ spectra in Figure 5.2. However, solar models furnish the normalization factor to the neutrino energy spectrum. This spectrum and the normalization factor $N^{BP}_{^8B}$ from Bahcall et al. [7] (BP98) of $5.15 \times 10^6/\text{cm}^2/\text{sec}$ provide the theoretical $^8$B solar neutrino flux incident at Earth as a function of neutrino energy, $E_\nu$. The expected $^8$B neutrino flux peaks near 6.7 MeV and falls to zero at about 15 MeV. Solar neutrinos are indirectly detected by the observation of recoil electrons that have been scattered by the neutrinos.

\(^2\)at least up to terms of order $kT/q_{\text{max}}$, where $k$ is Boltzmann's constant, $T$ is the Sun temperature in the region of neutrino production, and $q_{\text{max}}$ is the maximum neutrino energy [33]
Figure 5.2: The best-estimate (standard) $^8$B neutrino spectrum $\lambda$, together with the spectra $\pm \lambda$ allowed by the maximum ($\pm 3\sigma$) theoretical and experimental uncertainties [32].
Neutrino-electron elastic scattering proceed by the following reaction:

\[ \nu + e^- \rightarrow \nu + e^- \]  

(5.1)

The neutrino and electron interact via the weak force, and so the cross sections for neutrino-electron scattering are very small. Ref. [34] furnishes a neutrino-electron scattering differential cross section equation, that can be numerically integrated to calculate the cross sections. The differential cross section is given by:

\[
\frac{d\sigma(E_{\nu})}{dT} = \frac{2G_F^2m_e(hc)^2}{\pi} \left\{ g_L^2(T) \left[ 1 + \frac{\alpha}{\pi} f_-(z) \right] + g_L^2(T)(1-z)^2 \left[ 1 + \frac{\alpha}{\pi} f_+(z) \right] - g_R(T)g_L(T)\frac{m_e}{E_\nu} z \left[ 1 + \frac{\alpha}{\pi} f_-(z) \right] \right\},
\]

(5.2)

where:

- \( E_{\nu} \) = energy of the incident neutrino [MeV]
- \( T \) = kinetic energy of recoil electron [MeV]
- \( m_e \) = electron mass
- \( z = \frac{T}{E_{\nu}} \)
- \( G_F \) = Fermi coupling constant
  
  \[ G_F = (1.16639 \pm 0.00001) \times 10^{-11} \text{MeV}^{-2} \]

\[
g_L^{(\nu_{\tau}, e)}(T) = \rho_{NC}[\frac{1}{2} - \kappa(\nu_{\tau}, e)(T) \sin^2 \theta_W(m_Z)] - 1
\]

\[
g_L^{(\nu_{\mu}, e)}(T) = \rho_{NC}[\frac{1}{2} - \kappa(\nu_{\mu}, e)(T) \sin^2 \theta_W(m_Z)]
\]

\[
x = \mu \text{ or } \tau
\]

\[
g_R^{(\nu_{\tau}, e)}(T) = -\rho_{NC}\kappa(\nu_{\tau}, e)(T) \sin^2 \theta_W
\]
\[ g_R^{(\nu, e)}(T) = -\rho_{NC}^{(\nu, l)} \kappa^{(\nu, e)}(T) \sin^2 \theta_W \]
\[ \rho_{NC}^{(\nu, l)} = 1.0126 \pm 0.0016 \]
\[ \theta_W = \text{Weinberg angle} \]
\[ \sin^2 \theta_W(m_\tau) = 0.23124 \pm 0.00024 \]
\[ \kappa^{(\nu, e)}(T) = 0.9791 + 0.0097I(T) \pm 0.0025 \]
\[ \kappa^{(\nu, e)}(T) = 0.9970 + 0.00037I(T) \pm 0.0025 \]
\[ I(T) = \frac{1}{6} \left[ \frac{1}{3} + (3 - y^2) \left( \frac{1}{2} y \ln \left( \frac{y + 1}{y - 1} \right) - 1 \right) \right] \]
\[ y = \sqrt{1 + \frac{2m_e}{T}} \]

Eqn. 5.2 includes terms which take into account Quantum Chromodynamic (QCD) effects, Quantum Electrodynami (QED) effects, and radiative corrections. The dependence of the \( \kappa(T) \) expression upon the recoil electron kinetic energy \( T \) for example arises from radiative corrections shown as Feynman diagrams in Figure 5.3. The QCD corrections arise from diagrams such as that shown in Figure 5.4. The functions \( f_+(z), f_-(z), \) and \( f_{+ -}(z) \) describe QED effects and are long and complicated; they can be found in Ref. [34]. Also note that different definitions for \( g_L, g_R, \) and \( \kappa(T) \) are required for \( (\nu_e, e) \) and \( (\nu_x, e) \) scatterings \( (x = \mu \text{ or } \tau) \), since \( (\nu_x, e) \) scattering can only take place via Neutral Current (NC) interactions while \( (\nu_e, e) \) scattering occurs via both NC and Charged Current (CC) interactions. Only \( (\nu_e, e) \) scattering is considered in this simulation.
Figure 5.3: Feynman diagrams for electroweak radiative corrections.

Figure 5.4: Feynman diagram for QCD corrections.
Table 5.1: Comparison of published neutrino–electron scattering cross-sections [34] with those calculated in this analysis.

<table>
<thead>
<tr>
<th>( E_\nu ) [MeV]</th>
<th>( \sigma(E_\nu) ) [barn]</th>
<th>This Analysis ( \sigma(E_\nu) ) [barn]</th>
<th>( \Delta % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.38</td>
<td>( 1.92 \times 10^{-21} )</td>
<td>( 1.89 \times 10^{-21} )</td>
<td>-1.56</td>
</tr>
<tr>
<td>0.86</td>
<td>( 5.79 \times 10^{-21} )</td>
<td>( 5.78 \times 10^{-21} )</td>
<td>-0.173</td>
</tr>
<tr>
<td>1.00</td>
<td>( 6.98 \times 10^{-21} )</td>
<td>( 6.99 \times 10^{-21} )</td>
<td>0.143</td>
</tr>
<tr>
<td>1.44</td>
<td>( 1.09 \times 10^{-20} )</td>
<td>( 1.09 \times 10^{-20} )</td>
<td>0.0</td>
</tr>
<tr>
<td>2.00</td>
<td>( 1.59 \times 10^{-20} )</td>
<td>( 1.59 \times 10^{-20} )</td>
<td>0.0</td>
</tr>
<tr>
<td>3.00</td>
<td>( 2.51 \times 10^{-20} )</td>
<td>( 2.51 \times 10^{-20} )</td>
<td>0.0</td>
</tr>
<tr>
<td>4.00</td>
<td>( 3.42 \times 10^{-20} )</td>
<td>( 3.43 \times 10^{-20} )</td>
<td>0.292</td>
</tr>
<tr>
<td>5.00</td>
<td>( 4.35 \times 10^{-20} )</td>
<td>( 4.35 \times 10^{-20} )</td>
<td>0.0</td>
</tr>
<tr>
<td>7.00</td>
<td>( 6.19 \times 10^{-20} )</td>
<td>( 6.19 \times 10^{-20} )</td>
<td>0.0</td>
</tr>
<tr>
<td>10.0</td>
<td>( 8.96 \times 10^{-20} )</td>
<td>( 8.97 \times 10^{-20} )</td>
<td>0.112</td>
</tr>
<tr>
<td>12.0</td>
<td>( 1.08 \times 10^{-19} )</td>
<td>( 1.08 \times 10^{-19} )</td>
<td>0.0</td>
</tr>
<tr>
<td>14.0</td>
<td>( 1.27 \times 10^{-19} )</td>
<td>( 1.27 \times 10^{-19} )</td>
<td>0.0</td>
</tr>
<tr>
<td>16.0</td>
<td>( 1.45 \times 10^{-19} )</td>
<td>( 1.45 \times 10^{-19} )</td>
<td>0.0</td>
</tr>
<tr>
<td>18.0</td>
<td>( 1.64 \times 10^{-19} )</td>
<td>( 1.64 \times 10^{-19} )</td>
<td>0.0</td>
</tr>
<tr>
<td>20.0</td>
<td>( 1.82 \times 10^{-19} )</td>
<td>( 1.82 \times 10^{-19} )</td>
<td>0.0</td>
</tr>
</tbody>
</table>

A neutrino of given energy \( E_\nu \) can impart a maximum kinetic energy to the recoil electron of:

\[
T_{\text{max}} = \frac{2E_\nu^2}{(2E_\nu + m_e)}
\]  

(5.3)

due to conservation of momentum and energy and a minimum kinetic energy of zero. The differential cross-section can then be numerically integrated from zero to \( T_{\text{max}} \) to obtain the scattering cross-section for each neutrino energy, \( \sigma(E_\nu) \).

Table 5.1 compares the scattering cross-sections as calculated in this analysis with the published values [34]. All differences are less than 0.3 % except for the value at \( E_\nu = 0.38 \) MeV with a difference of 1.6 %. Because the
The energy threshold of the solar neutrino analysis is 6.5 MeV, cross-sections at such low neutrino energies do not contribute to the results of this analysis.

The product $\sigma(E_\nu) \cdot \lambda(E_\nu)$ provides the expected relative probability of $^8\text{B}$ neutrino–electron scattering as a function of neutrino energy; this distribution is shown in Figure 5.5. Notice that the distribution peaks near 8 MeV and vanishes near 14 MeV. The total number of expected $^8\text{B}$ solar neutrino interactions within the Super–Kamiokande inner detector is then:

$$N_{nB} = N_e \int_0^\infty \sigma(E_\nu) \lambda(E_\nu) dE_\nu = 287.4 \text{ recoil electrons per day}$$

where:

$$N_e = \text{Number of electrons within the inner detector volume}$$

$$= 1.08 \times 10^{34}$$

Note that this is the total expected rate of $^8\text{B}$ solar neutrino–electron scattering within the Super–Kamiokande detector and can result in recoil electron kinetic energies of 0 to about 16 MeV. Due to the energy resolution smearing of the detector and the energy threshold of about 5.7 MeV, the detection of a recoil electron depends upon its kinetic energy.

To determine the expected recoil electron spectrum resulting from neutrino–electron scattering, a two step process is followed. First, a $^8\text{B}$ neutrino energy $E_\nu$ is randomly chosen from the relative probability distribution shown in Figure 5.5. Once the neutrino energy is fixed, a random recoil electron kinetic energy $T$ is then chosen with the relative probability given by the normalized differential cross-section distribution $\frac{d\sigma(E_\nu)}{dT}$. Figure 5.6 shows a
Figure 5.5: The expected relative probability of $^8$B neutrino–electron scattering $\sigma(E_\nu) \cdot \lambda(E_\nu)$. The distribution is normalized to 1.

typical differential cross-section distribution as a function of recoil electron kinetic energy for a fixed incident neutrino energy of 10 MeV. A 1 MeV recoil electron, for example, is more likely to result than a 9 MeV electron for this incident neutrino. Repeating this process 700,000 times results in the composite expected recoil electron kinetic energy spectrum shown in Figure 5.7. For the expected rate in Eqn. 5.4, the 700,000 simulated $^8$B events represent about 2,436 days of data taking.

5.2.2 Detector Response Simulation

Once the kinetic energies of the scattered MC electrons are calculated, each electron is given a random scattering vertex within the inner detector and a random recoil direction in local detector coordinates. Figure 3.11 defines the local coordinate system. Electron recoil direction, relative to the
Figure 5.6: Partial cross-section $d\sigma(E_\nu)/dT$ of neutrino-electron scattering for a 10 MeV neutrino as a function of electron kinetic energy. The plot has been zero suppressed.

Figure 5.7: Kinetic energy spectrum of $^8\text{B}$ neutrino scattered electrons. This spectrum is input into the detector simulation MC.
Figure 5.8: Schematic of neutrino–electron scattering.

original neutrino direction, is a function of neutrino incident energy and recoil electron kinetic energy and is given by the equation:

\[
\cos^2 \theta_{\text{scatter}} = \frac{(1 + \frac{M_e}{E_v})^2}{(1 + \frac{2M_e}{T})},
\]

where \( \theta_{\text{scatter}} \) is the angle between the incident neutrino’s direction and the recoil electron’s direction; refer to Figure 5.8.

To simulate measured data as closely as possible, each MC neutrino is projected as coming from the Sun relative to the detector at times covering the detector live time of this solar neutrino analysis. Each recoil electron is then given a direction relative to the incident neutrino direction with the corresponding \( \theta_{\text{scatter}} \) and a random \( \phi_{\text{scatter}} \).

The direction information along with the kinetic energy of each recoil electron is then fed into the detector simulation MC described in Section 5.1. Each output event of the detector simulation MC has its vertex, direction, and energy reconstructed, just as the measured data, and is passed through the same data reduction chain as measured data (refer to Chapter 6). To simulate the spallation cut, a simple reduction of 20% is made on the remaining MC events which pass all other cuts (see Chapter 6 for details of
the spallation cut). The $\cos \theta_{\text{sol}}$ distribution of the surviving MC events from the original 700,000 is shown in Figure 5.9, where the angle $\theta_{\text{sol}}$ is the angle between the reconstructed event direction and the vector from the Sun to the reconstructed event vertex; refer to Figure 5.10. A value of $\cos \theta_{\text{sol}} = 1$ indicates that the event appears to originate from the Sun’s direction.

The event rates of the surviving neutrino–electron scattering events are shown in Figure 5.11 as a function of electron total energy. Event rates in this figure have been grouped into 16 energy intervals: 6.5–7 MeV, 7–7.5 MeV, ..., 13.5–14 MeV, and 14–20 MeV. The expected rate of detected $^8$B neutrino-scattered electrons in the 6.5–20 MeV energy interval is 28.6 events/day/22.5 kton. Also shown in Figure 5.11 are the simulated event rates of HeP neutrino–electron scatterings.

### 5.2.3 HeP Solar Neutrino Simulation

Figure 5.12 shows the normalized expected HeP neutrino energy spectrum [35]. This spectrum and the normalization factor $2.1 \times 10^3/\text{cm}^2/\text{s}$ (BP98) [7] provide the means to calculate the expected recoil electron energy spectrum resulting from neutrino–electron scattering within the Super–Kamiokande detector. This procedure is fully described in Section 5.2 for $^8$B neutrinos, and the same procedure is employed for HeP neutrinos. Figure 5.13 shows the resulting expected recoil electron kinetic energy spectrum for 700,000 simulated solar HeP neutrino–electron scattering events. Note that the electron recoil energies extend to higher energies than those scattered by $^8$B neutrinos (Figure 5.7).
Figure 5.9: The $\cos \theta_{sol}$ distribution for MC $^8$B neutrino scattered recoil electrons for the 6.5–20 MeV energy range.

Figure 5.10: Definition of the angle $\theta_{sol}$. 

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Figure 5.11: Detector simulated $^8$B and HeP neutrino–electron scattering rates as a function of recoil electron total energy.

Figure 5.12: The best-estimate (standard) HeP neutrino spectrum [32]. The distribution is normalized to 1.
Figure 5.13: The recoil electron kinetic energy spectrum resultant from 700,000 simulated HeP neutrino–electron scatterings.

For the best-estimate HeP flux given above, 700,000 simulated events represent about 3,870,000 days of data taking:

\[
N_{\text{hep}} = N_e N_{\text{hep}}^{BP98} \int_0^\infty \sigma(E_\nu) \lambda_{\text{hep}}(E_\nu) \, dE_\nu = 0.181 \text{ recoil electrons per day,}
\]

where:

\[N_{\text{hep}} = \text{expected rate of HeP neutrino–electron scattering events within the inner detector volume} \]

\[N_e = \text{Number of electrons in the inner detector} = 1.08 \times 10^{34} \]

\[N_{\text{hep}}^{BP98} = \text{expected HeP flux} = 2.1 \times 10^3 \text{ /cm}^2/\text{s} \]

\[E_\nu = \text{neutrino energy} \]

\[\sigma(E_\nu) = \text{neutrino–electron scattering cross section} \]

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\[ \lambda_{\text{hep}}(E_{\nu}) = \text{normalized expected HeP solar neutrino energy spectrum}, \]

and:

\[
700,000 \text{ events} \div 0.181 \text{ events/day} = 3,870,000 \text{ days} \quad (5.7)
\]

of data taking.

The recoil electron kinetic energy spectrum is then fed into the detector simulation Monte Carlo to calculate the expected detector response. The neutrinos are projected as coming from solar directions covering the analysis live time and the scattering vertices are randomly distributed throughout the detector volume.

The simulated solar neutrino-electron scattering events are then subjected to the same reduction cuts as are data events. The event rates of the surviving HeP neutrino-electron scattering events are shown in Figure 5.11 as a function of electron total energy. Event rates in this figure have been grouped into 16 energy intervals: 6.5–7 MeV, 7–7.5 MeV, ..., 13.5–14 MeV, and 14–20 MeV. The expected rate of detected HeP neutrino-scattered electrons in the 6.5–20 MeV energy interval is 0.033 events/day/22.5 kton. Also shown in Figure 5.11 are the simulated event rates of $^8\text{B}$ neutrino-electron scatterings.

The shapes of the $^8\text{B}$ and HeP neutrino-electron scattering spectra shown in Figure 5.11 are independent of solar models, but the overall rates are determined by the normalization factor of the solar model used. In Chapter 8 these expected spectra are compared to the measured energy spectrum. In the comparison the normalizations of the expected $^8\text{B}$ and HeP spectra are
allowed to vary so as to find the combination of expected spectra which best matches the measured spectrum.
CHAPTER 6
DATA REDUCTION

The data used in this analysis was recorded between 31 May of 1996 and 25 March of 1998 and represents 504 days of detector live time. About $7 \times 10^8$ events were recorded during this period, but the expected number of solar neutrino interactions is only about 14,400 events above 6.5 MeV (see Section 5.2.2). Before a neutrino signal can be extracted, the data must be reduced to remove events that are either obviously not solar neutrino events (e.g. cosmic ray muons) or events caused by known background sources (e.g. spallation events, background emissions from the surrounding rock, and "flasher" PMTs). However, the cuts must be carefully chosen so as not to introduce any energy dependence in the final sample. Most of the cuts described below are energy independent.

General cuts are applied to the data to ensure data quality. Data gathering is divided into "runs," which are typically 24 hours in duration. Dividing the data into runs allows for more manageable archiving and analysis. Each run is further subdivided into "subruns," which are about 10 minutes long in duration. Data quality is checked by making simple checks on the duration of each run and subrun. Runs and subruns are rejected from the analysis if:

- the run duration is less than 5 minutes (This normally indicates a problem with the data acquisition system during the start of the run.)
• the subrun duration is less than 30 seconds (This normally indicates that the subrun was the last in the run, which can have incomplete information, or that a hardware or software malfunction occurred during the subrun.)

Other data quality requirements are:

• all detector equipment must be operating normally during data taking (e.g. ATM modules are not in a self-calibrating mode)

• the data acquisition system must completely record all event information (e.g. no missing trigger or SMP information and no electronics error flags)

Another general cut applied to the data is to remove all events that were triggered with an OD trigger, which indicates activity in the outer detector. These events are normally due to cosmic ray muon events entering the detector.

The remaining data reduction cuts are grouped into three main categories: the “first and second reductions” target the removal of cosmic ray muon events, decay-muon electron events, flasher PMT events, and electronic noise events; the “spallation cut” targets radioactive decay products produced by cosmic ray muons; and the fiducial volume cuts target backgrounds radiating from the rock surrounding the detector and the PMT glass. Details of cuts can be found in Ref. [28], but they are briefly described here.

The “first reduction” is comprised of separate cuts with the following descriptions:
Figure 6.1: Typical event total charge distribution. Hatched area contains solar neutrino candidate events. The total charge reduction cut is indicated by the arrow.

- The total charge recorded by the inner detector PMTs must be less than or equal to 1000 PE ($\simeq 100$ MeV). Events above this threshold are generally cosmic ray muon events. Figure 6.1 shows the total charge distribution for a typical data sample. Two peaks are clearly visible: the lower energy peak which contains the low energy solar neutrino candidates and the higher energy peak which contains the cosmic ray muon events. The efficiency of this cut to keep all solar neutrino events is 100%.

- The time since the previous event must be greater than 20 $\mu$s. This cut primarily targets the removal of muon-decay electron events. Figure 6.2 shows the distribution of time-since-previous-event for a typical data sample. The 20 $\mu$s (indicated by the arrow in the figure) generates a
Figure 6.2: Distribution of time-since-previous-event for a typical data sample. Arrow indicates reduction time cut.

extensive detector dead time of less than 0.001 %.

- Noise events generated by the electronics (e.g. triggering on a flickering florescent light fixtures) are cut by comparing the number of PMT signals with charges less than 0.5 PE (including negative charges) to the total number of PMT signals in the event. Events in which over 40 % of the PMT signals are from PMT pulses less than 0.5 PE are rejected. Figure 6.3 shows the distribution of the ratio of signals with charges less than 0.5 PE to the event total charge ($R_{\text{noise}}$) for a data sample which is known to contain noise events. The cut is indicated by the arrow. In normal data runs this cut has a negligible effect.

- A second electronic noise cut rejects events in which over 95 % of channels in one ATM module have a PMT signal.
Figure 6.3: Distribution of the ratio of signals with charges less than 0.5 PE to the event total charge ($R_{\text{noise}}$) for a data sample which contains noise events.

- "Flashers" are PMTs that electrically arch internally and trigger neighboring PMTs. Flasher events can look like a neutrino scattering events and so must be removed from the data. Figure 6.4 shows scatter plots of the maximum charge PMT in each event and the number of PMTs with signals which neighbor this maximum charge PMT. The upper plot uses data which includes a flasher PMT. The lower plot uses normal data. To remove flasher events from the data, the cut shown in Figure 6.4 is applied. This cut removes about 0.8 % of MC $^8$B events.

The "second reduction" removes events based upon their vertex reconstruction:

- Events in which the vertex reconstruction time window with the maximum significance (see Section 4.1) contains less than 10 PMTs are rejected. This cut is a vertex reconstruction quality requirement and cuts events with very low energy (less than $\sim 4$ MeV).
Figure 6.4: Scatter plots of the maximum charge PMT in each event and the number of PMTs with signals which neighbor this maximum charge PMT. The upper plot uses data which includes a flasher PMT. The lower plot uses normal data. The cut to the data is shown in the upper plot.
Figure 6.5: Scatter plots of the goodness value and the number of hit PMTs $N_{\text{hit}}$ of each event. The upper plot uses data which includes a flasher PMT, while the lower plot uses normal data. The cut is shown in both plots.

- The vertex reconstruction "goodness" value (see Section 4.1) must be greater than or equal to 0.4. This is another reconstruction quality requirement and targets flasher PMT events, which tend to have low goodness values. Figure 6.5 shows scatter plots of the goodness value and the number of hit PMTs $N_{\text{hit}}$ of each event. The upper plot uses data which includes a flasher PMT, while the lower plot uses normal data. The cut is shown in both plots.

No energy dependence is apparent for this cut. Higher energy events are generally expected to have higher goodness values, since these events produce more Cherenkov light which makes vertex fitting easier.
(Recall that the number of hit PMTs is a rough measure of the energy of the event). This trend can be seen in Figure 6.5. However, the goodness value cut does not remove low energy (low $N_{hit}$) events preferentially, but removes flasher events which tend to have higher energies but poorer fit goodness values. This cut removes about 0.02 % of the MC $^8$B events.

- A second vertex reconstruction is fit using a different PMT selection algorithm. If the new fit vertex is more than 5 m away from the original fit vertex, the event is rejected. This cut targets events generated by gammas radiating from the PMT glass. Since these vertices are very close to the PMT wall, vertex reconstructions can be very volatile. This cut removes about 0.4 % of MC $^8$B events.

After the first and second reduction cuts the “spallation cut” is applied to the remaining data events. The spallation cut targets the removal of events produced by the breakup of oxygen nuclei by cosmic ray muons. Table 6.1 lists many of the $\beta$ and $\gamma$ producing spallation products which are important to this analysis (i.e. their energies are within the 6.5–20 MeV analysis energy range) [28, 36]. About 5 % of cosmic ray muons entering the Super-Kamiokande detector produce spallation products. The half-lives of the products vary from about 1 $\mu$s to several seconds. Although many of spallation products produce electrons and gammas well below the 6.5 MeV energy threshold of this analysis, the energy resolution of the Super-Kamiokande detector (described in Section 3.4) can smear the energy of these events into the analysis energy range.
<table>
<thead>
<tr>
<th>Isotope</th>
<th>$\tau_{1/2}$ (sec)</th>
<th>Decay Mode</th>
<th>Kinetic Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^8\text{He}$</td>
<td>0.119</td>
<td>$\beta^-\gamma$</td>
<td>$10.65 + 0.98(\gamma)$ (84%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-n$</td>
<td>$\sim 6.3(&lt; 16%)$</td>
</tr>
<tr>
<td>$^8\text{Li}$</td>
<td>0.838</td>
<td>$\beta^-$</td>
<td>16.0</td>
</tr>
<tr>
<td>$^8\text{B}$</td>
<td>0.770</td>
<td>$\beta^+$</td>
<td>18.0</td>
</tr>
<tr>
<td>$^9\text{Li}$</td>
<td>0.178</td>
<td>$\beta^-$</td>
<td>13.6 (50.5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-\gamma\nu$</td>
<td>$11.2 + 2.43(\gamma)$ (34%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-n$</td>
<td>$\sim 8 (15.5%)$</td>
</tr>
<tr>
<td>$^9\text{C}$</td>
<td>0.126</td>
<td>$\beta^+$</td>
<td>16.5 (60%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^+p$</td>
<td>$\sim 8.9$ (23%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^+\alpha$</td>
<td>14.1 (17%)</td>
</tr>
<tr>
<td>$^{11}\text{Li}$</td>
<td>0.0085</td>
<td>$\beta^-$</td>
<td>20.6</td>
</tr>
<tr>
<td>$^{14}\text{Be}$</td>
<td>13.8</td>
<td>$\beta^-$</td>
<td>11.5 (54.7%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-\nu\nu$</td>
<td>$9.38 + 2.12(\gamma)$ (31.4%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-\nu\nu\nu$</td>
<td>$4.71 + 6.79(\gamma)$ (6.5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-\nu\nu\nu\nu$</td>
<td>$3.53 + 5.85(\gamma)$ (4.0%)</td>
</tr>
<tr>
<td>$^{12}\text{Be}$</td>
<td>0.0236</td>
<td>$\beta^-$</td>
<td>11.7</td>
</tr>
<tr>
<td>$^{12}\text{B}$</td>
<td>0.0202</td>
<td>$\beta^-$</td>
<td>13.4 (97.22%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-\gamma$</td>
<td>$8.9 + 4.4(\gamma)$ (1.23%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-\alpha$</td>
<td>$5.71 + 3.8(\gamma)$ (1.50%)</td>
</tr>
<tr>
<td>$^{12}\text{N}$</td>
<td>0.011</td>
<td>$\beta^+$</td>
<td>17.3 (94.55%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^+\gamma$</td>
<td>$12.9 + 4.4(\gamma)$ (1.9%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^+\alpha$</td>
<td>$9.68 + 3.8(\gamma)$ (2.7%)</td>
</tr>
<tr>
<td>$^{13}\text{B}$</td>
<td>0.0174</td>
<td>$\beta^-$</td>
<td>13.4</td>
</tr>
<tr>
<td>$^{13}\text{O}$</td>
<td>0.00858</td>
<td>$\beta^+$</td>
<td>17.8 (89.2%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^+\gamma$</td>
<td>$14.3 + 3.5(\gamma)$ (9.8%)</td>
</tr>
<tr>
<td>$^{14}\text{B}$</td>
<td>0.0138</td>
<td>$\beta^-\gamma$</td>
<td>$14.5 + 6.09(\gamma)$ (82%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-\nu\nu\nu$</td>
<td>$13.9 + 6.73(\gamma)$ (7%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-\nu\nu\nu\nu$</td>
<td>$20.6 (5%)$</td>
</tr>
<tr>
<td>$^{15}\text{C}$</td>
<td>2.45</td>
<td>$\beta^-\gamma$</td>
<td>$4.47 + 5.30(\gamma)$ (63.2%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-\nu\nu\nu$</td>
<td>$9.77$ (36.8%)</td>
</tr>
<tr>
<td>$^{16}\text{C}$</td>
<td>0.747</td>
<td>$\beta^-$</td>
<td>$\sim 4.5$</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>7.13</td>
<td>$\beta^-\gamma$</td>
<td>$4.29 + 6.13(\gamma)$ (66.2%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-\nu\nu\nu\nu$</td>
<td>$3.30 + 7.11(\gamma)$ (4.8%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta^-\nu\nu\nu\nu\nu$</td>
<td>$10.4$ (28.0%)</td>
</tr>
</tbody>
</table>

Table 6.1: Possible $\beta$ and $\gamma$ spallation backgrounds with energies near the energy range of the solar neutrino analysis.
Identification of spallation events is done by a likelihood method. Spallation events are highly correlated with the “parent” muon in space and time. Candidate neutrino events, which occur very close to a muon track in both space and time, are very likely to be spallation events. Also, muon events which deposit more energy in the detector than the average expected 24.1 PE per cm traveled in the detector are more likely to have spallation events associated with them [28].

The spallation cut compares the location and timing of each candidate event with the tracks of the previous 100 muons to enter the detector. For each muon a likelihood value that the candidate event is a spallation event of that muon is calculated. For muons which stop within the detector or whose tracks could not be fit, only the timing and muon event total energy are used to determine the likelihood values. The largest likelihood value determines whether the event is cut or not. Figure 6.6 shows the maximum likelihood distributions for normal data and for data in which the events were given a random vertex location within the detector. (The random vertex events are not associated with any muon and serve to model solar neutrino events.) All muons represented in this figure have track fits. Figure 6.7 shows a similar distribution, however none of the muons represented in this figure have track fits. The likelihood cuts used in the spallation cut are shown in the respective figures. The spallation cut results a 20 % dead time upon the analysis as measured by the fraction of random-vertex events cut.

The final cuts made upon the data are fiducial volume and energy cuts. Figure 6.8 shows the reconstructed vertex radius and z-coordinate.
Figure 6.6: Spallation maximum likelihood distributions for normal data and for data in which the events were given a random vertex location within the detector. All muons represented in this figure have track fits. Arrow indicates cut value.

Figure 6.7: Spallation maximum likelihood distributions for normal data and for data in which the events were given a random vertex location within the detector. All muons represented in this figure do not have track fits, thus likelihood values are calculated using timing and excess charge information only. Arrow indicates cut value.
Figure 6.8: Reconstructed vertex radius-squared (top) and z-coordinate (bottom) distributions of a sample of the final candidate neutrino signal events. The 2 m fiducial volume cut is indicated by the arrows.

distributions of a sample of the candidate events which have passed all other cuts. The excess of events along the detector walls are thought to be background events radiating from the surrounding rock and the PMT glass windows. A fiducial volume cut is made which removes all events that have their fit vertices within 2 m of a detector wall; events with vertices located within $R < 1490$ cm and $|Z| < 1610$ cm are kept in the final sample. The 2 m fiducial volume cut is indicated in the figure by arrows.
A second fiducial cut, called the "gamma cut" targets background events from the PMT glass whose vertices are fit within the 2 m fiducial volume. The reconstructed direction of each candidate event is projected backward in the opposite direction, and the distance from the fit vertex to the detector wall along this path is measured. If this distance is less than 4.5 m, the event is cut. This removes events that are just within the 2 m fiducial volume and are moving inward from the wall of PMT faces. Figure 6.9 shows the distribution of distances from the wall as above described, and the arrow shows the cut. Figure 6.10 shows the vertex and direction distributions of the remaining events before (unhatched) and after (hatched) the gamma cut. Note that the direction distributions after the gamma cut are flat, indicating that there are no direction preferences in the final sample.

One of the final cuts made upon the data is the energy cut of the analysis. Events with energies less than 6.5 MeV or greater than 20 MeV are removed from the data.

The remaining events are then placed through a final flasher cut. Figure 6.11 shows the distributions of $Dir_{k_s}$ values (left) and vertex reconstruction goodness values (right) for a data set which contains flasher events (top) and for the flasher events alone (bottom). The $Dir_{k_s}$ values result from a Kolmogorov-Smirnov test, which compares the expected wall projection shape of the Cherenkov cone using the reconstructed vertex and direction with the measured shape. The $Dir_{k_s}$ value is a measure of the difference between the measured and expected shapes and can be used to calculate a probability of agreement [37]. Flasher events are removed from the data by
Figure 6.9: Distribution of distances from the wall as described in the text; the arrow shows the gamma cut value.
Figure 6.10: Vertex and direction distributions of the final candidate neutrino signal events before (unhatched) and after (hatched) the gamma cut.
removing events with $\text{Dir}_{KS}$ values greater than 0.25 and goodness values less than 0.6. This cut removes about 0.15 % of MC $^8$B events.

Table 6.2 enumerates the effects of each data cut upon the data, and Figure 6.12 shows the energy distribution of the candidate events after each major reduction cut.

Figure 6.13 shows the direction-to-Sun ($\cos \theta_{\text{sol}}$) distribution for the final sample of 87051 events, where the angle $\theta_{\text{sol}}$ is the angle between the reconstructed event direction and the vector from the Sun to the reconstructed event vertex; refer to Figure 5.10. A value of $\cos \theta_{\text{sol}} = 1$ indicates that the event appears to originate from the Sun’s direction, which is predicted
Table 6.2: Reduction cuts made upon the data sample and the number of events remaining after each cut. Begin with $\sim 7 \times 10^8$ events and end with 87051 candidate events. Monte Carlo event reduction is also shown. “D.T.” means “dead time” and “Eff” means “efficiency.”

<table>
<thead>
<tr>
<th>Reduction Step</th>
<th>Cut Description</th>
<th>Data Events</th>
<th>Monte Carlo Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Reduction</td>
<td>Total Charge $&lt; 1000$ PE</td>
<td>$\sim 7 \times 10^8$</td>
<td>$1.06 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>Time since previous event $&gt; 20 \mu s$</td>
<td>$5.03 \times 10^8$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Outer detector hits $&lt; 20$</td>
<td>$4.27 \times 10^8$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Electronic noise cut</td>
<td>$3.91 \times 10^8$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Flasher cut</td>
<td>$3.90 \times 10^8$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$3.88 \times 10^8$</td>
<td></td>
</tr>
<tr>
<td>Second Reduction</td>
<td>PMTs used in vertex fit $&gt; 10$</td>
<td>$3.86 \times 10^8$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Vertex “goodness” $&gt; 0.4$</td>
<td>$3.84 \times 10^8$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.5 m fiducial cut</td>
<td>$2.12 \times 10^7$</td>
<td>$1.05 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>Energy $&gt; 5$ MeV</td>
<td>$8.64 \times 10^6$</td>
<td>$1.00 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>Second vertex fit cut</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spallation Cut</td>
<td>2 m fiducial cut</td>
<td>$4.33 \times 10^6$</td>
<td>$20 %$ D.T.</td>
</tr>
<tr>
<td></td>
<td>Bad run/subrun cut</td>
<td>$3.91 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>Energy Cut</td>
<td>(6.5–20 MeV)</td>
<td>$126277$</td>
<td></td>
</tr>
<tr>
<td>Gamma Cut</td>
<td>DirKS $&gt; 0.25$; goodness $&lt; 0.6$</td>
<td>$90070$</td>
<td>$92.2 %$ Eff</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$87051$</td>
<td>$99.8 %$ Eff</td>
</tr>
</tbody>
</table>
Figure 6.12: The energy distribution of the candidate events after each major reduction cut. The dotted line shows the lower energy cut.

for solar neutrino-electron scattering events. A prominent solar neutrino-electron scatter peak is visible near $\cos \theta_{\text{sol}} = 1$ over a nearly isotropic background. The background events are thought to be due to radon decay events and remaining spallation events. Measurements of radon levels in the water of the detector range within $\pm 15\%$ of 1.4 mBq/m$^3$. Beta decays of $^{214}\text{Bi}$ which originate from $^{222}\text{Rn}$ have an energy endpoint of 3.26 MeV, but the finite energy resolution of the detector can smear the energy of these events to higher energies. Whether all of the remaining background can be explained by radon and residual spallation events is still under investigation.
Figure 6.13: The direction-to-Sun (cos $\theta_{sol}$) distribution for the final sample of 87051 events, where the angle $\theta_{sol}$ is the angle between the inverse direction of reconstructed event and the Sun direction at the time of the event; refer to Figure 5.10.
CHAPTER 7

NEUTRINO SIGNAL EXTRACTION

7.1 First Order Signal Extraction

The cosine of the angle between the observed electron direction and the Sun’s direction at that time (\(\cos \theta_{\text{sol}}\)) is shown in Figure 6.13 for each event in the final sample with energies between 6.5 and 20 MeV. The distribution clearly displays the excess solar neutrino-electron scattering peak (solar neutrino signal) in the solar direction (\(\cos \theta_{\text{sol}} = 1\)). The distribution also shows a non-negligible amount of nearly isotropically distributed background. To first order in this analysis, the background is assumed to be isotropically distributed (flat), and signal events candidates are defined to be those events with \(\cos \theta_{\text{sol}} \geq 0.5\). The assumption of a flat background and the restriction of signal candidates to \(\cos \theta_{\text{sol}} \geq 0.5\) are later modified to allow for non-flat backgrounds and signal events with \(\cos \theta_{\text{sol}} < 0.5\).

The following definitions are made by partitioning the \(\cos \theta_{\text{sol}}\) distribution into 40 evenly spaced bins (see Figure 7.1):

- Region \(R_1 \equiv\) Bins 1-30 (30 bins total), corresponding to \(\cos \theta_{\text{sol}} < 0.5\)
- Region \(R_2 \equiv\) Bins 31-40 (10 bins total), corresponding to \(\cos \theta_{\text{sol}} \geq 0.5\)
- \(N_{\text{SIG}} \equiv\) Number of signal events in Region \(R_2\) (To first order only Region \(R_2\) contains signal events. This will be corrected for later.)
- \(N_{\text{BG}} \equiv\) Number of background events in Region \(R_2\)
Figure 7.1: Regions and bin numbering of the $\cos \theta_{\text{sol}}$ distribution used in this analysis and described in Section 7.1.

- $N_1$ $\equiv$ Number of events in Region $R_1$ (To first order all events in this region are assumed to be background events. This will be corrected for later.)

- $N_2$ $\equiv$ Number of events in Region $R_2 = N_{SIG} + N_{BG}$ (Events in this region are comprised of both signal and background.)

Using these definitions, the background in Region $R_2$ is:

$$N_{BG} \equiv \frac{N_1}{3}, \quad (7.1)$$

since background is assumed to be flat and Region $R_1$ contains three times as many bins as Region $R_2$. The signal in Region $R_2$ is then:

$$N_{SIG} = (\text{Number of events Region } R_2) - (\text{Background in Region } R_2)$$
$$= N_2 - \frac{N_1}{3}. \quad (7.2)$$
Table 7.1: Angular cut efficiency $\epsilon$ (fraction of MC events with $\cos \theta_{\text{sol}} \geq 0.5$) by energy interval.

<table>
<thead>
<tr>
<th>Energy Interval [MeV]</th>
<th>Efficiency %</th>
<th>Energy Interval [MeV]</th>
<th>Efficiency %</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5–7.0</td>
<td>90.7</td>
<td>10.5–11.0</td>
<td>94.9</td>
</tr>
<tr>
<td>7.0–7.5</td>
<td>92.1</td>
<td>11.0–11.5</td>
<td>96.0</td>
</tr>
<tr>
<td>7.5–8.0</td>
<td>92.8</td>
<td>11.5–12.0</td>
<td>96.3</td>
</tr>
<tr>
<td>8.0–8.5</td>
<td>93.2</td>
<td>12.0–12.5</td>
<td>96.6</td>
</tr>
<tr>
<td>8.5–9.0</td>
<td>93.7</td>
<td>12.5–13.0</td>
<td>96.2</td>
</tr>
<tr>
<td>9.0–9.5</td>
<td>94.3</td>
<td>13.0–13.5</td>
<td>96.8</td>
</tr>
<tr>
<td>9.5–10.0</td>
<td>94.7</td>
<td>13.5–14.0</td>
<td>96.4</td>
</tr>
<tr>
<td>10.0–10.5</td>
<td>95.2</td>
<td>14.0–20.0</td>
<td>97.4</td>
</tr>
</tbody>
</table>

The determination of the number of signal events then can be reduced to a simple counting of events within regions $R_1$ and $R_2$. This definition of neutrino signal provides a straightforward and easily understood first approximation to the measured solar neutrino signal. This definition will be refined below to allow for signal events with $\cos \theta_{\text{sol}}$ values of less than 0.5 and for non-isotropically distributed background. To adjust for the former, Monte Carlo simulations of solar neutrino interactions within the Super-Kamiokande detector (described in Section 5.2.1) are utilized.

7.2 Angular Cut Efficiency Correction

Some of the MC recoil electron events have $\cos \theta_{\text{sol}}$ values of less than 0.5, as is evident from Figure 5.9, and so data signal events should also be expected to have values of less than 0.5. Signal event candidates, as defined in Section 7.1, are those events with $\cos \theta_{\text{sol}} \geq 0.5$. Increasing the angular size of this window to, say, $\cos \theta_{\text{sol}} \geq 0$, would serve to include most of these signal events and decrease the systematic error due to direction fitting, but it would
also greatly enhance the statistical error in the neutrino signal calculation, since many more background events would be considered signal candidates. The results of the MC detector simulation can be used to adjust the signal measurement for the angular cut at $\cos \theta_{sol} = 0.5$ without increasing the window size. The MC events are grouped by the electron observed total energies into several energy intervals: 6.5-7 MeV, 7-7.5 MeV, 7.5-8 MeV, ..., 13.5-14 MeV, and 14-20 MeV. Since the recoil electron direction (with respect to the original scattering neutrino direction) is dependent upon the kinetic energy of the recoil electron (Eqn. 5.5), the number of signal events with $\cos \theta_{sol}$ less than 0.5 is thus expected to be energy dependent. For each energy interval the fraction of MC events with $\cos \theta_{sol} \geq 0.5$ is calculated and is taken as the efficiency of the $\cos \theta_{sol} = 0.5$ angular cut in that energy interval. Table 7.1 lists the $\cos \theta_{sol}$ angular cut efficiency for each energy interval.

The total number of measured signal events must be adjusted for signal events with $\cos \theta_{sol} < 0.5$. The number of signal events is affected in two ways by these events. First, signal events with $\cos \theta_{sol} < 0.5$ are not even considered to be signal event candidates and so the number of possible signal events are undercounted. Second, these events are counted as background events, resulting in over-counting the number of background events. These miscounting effects are corrected for by rewriting Eqn. 7.2 as:

$$N_{SIG} = (N_2 + x) - \frac{(N_1 - x)}{3},$$

(7.3)

where:
\[ x = \text{number of signal events with } \cos \theta_{sol} < 0.5 \]
\[ = (1 - \epsilon)N_{SIG} \]
\[ \epsilon = \text{efficiency of the } \cos \theta_{sol} = 0.5 \text{ angular cut as} \]
measured by MC events; \( 0 \leq \epsilon \leq 1 \)

Solving this equation for \( N_{SIG} \) results in the adjusted signal equation:

\[ N_{SIG} = \frac{N_2 - N_1/3}{(4\epsilon - 1)/3} \quad (7.4) \]

Note that the numerator of the right-hand-side of the above equation is just the right-hand-side of the original signal equation (Eqn. 7.2). So Eqn. 7.4 may be written as:

\[ N'_{SIG} = \frac{N_{SIG}}{\epsilon'} \quad (7.5) \]

where:

\[ N'_{SIG} = \text{number of signal events adjusted for the} \]
angular cut at \( \cos \theta_{sol} = 0.5 \)
\[ N_{SIG} = \text{unadjusted number of signal events} \]
\[ \epsilon' = \frac{(4\epsilon - 1)}{3} \]

The measured data signal rates for each energy interval is then corrected by the corresponding MC efficiency \( \epsilon' \) to produce the an adjusted signal rate. Table 7.1 lists the \( \cos \theta_{sol} \) angular cut efficiency for each energy interval.
7.3 Non-Flat Background Correction

Another refinement to the data signal measurement is made to allow for non-isotropically distributed (non-flat) background noise events. Placement of a data event in the \( \cos \theta_{\text{sol}} \) distribution depends upon the reconstructed event direction and the Sun direction at the time. If it is assumed that background events occur independently of the time of day or season, then the location of a background event in the \( \cos \theta_{\text{sol}} \) distribution depends upon the chance location of the Sun at that time.

To demonstrate the concept of the non-flat background method, we first choose an event so far removed from the solar direction that it can be labeled as a background event with certainty e.g. an event with \( \cos \theta_{\text{sol}} = -0.9 \). Recalculating the \( \cos \theta_{\text{sol}} \) value of this event many times with the Sun position taken at different moments throughout the 504 day analysis live time results in the \( \cos \theta_{\text{sol}} \) distribution shown in Figure 7.2; the area under the figure has been normalized to 1. The distribution reflects the temporal asymmetries of the solar direction with respect to the event direction in local detector coordinates. Figure 7.2 is then the \( \cos \theta_{\text{sol}} \) distribution for this assumed typical background event as if it had occurred at many different times throughout the solar neutrino analysis and not just at one time. In essence, this process is “time-smearing” the background event over the entire period of the analysis.

Background events as defined in Section 7.1 include all events with \( \cos \theta_{\text{sol}} \) values less than 0.5 and compose a portion of the events with \( \cos \theta_{\text{sol}} \geq 0.5 \). Each event with \( \cos \theta_{\text{sol}} < 0.5 \) is time-smeared, as described above, and its
Figure 7.2: The $\cos \theta_{sol}$ time-smeared distribution for one background event with a measured $\cos \theta_{sol}$ value of $-0.9$. The area under the distribution has been normalized to 1.
distribution is normalized to 1. Each time-smeared distribution is then added together to form a composite distribution. Events with $\cos \theta_{sol} \geq 0.5$ are also time-smeared, but are normalized not to 1 but to the event’s probability of being a background event (since not every event in this region is a background event). An event’s probability of being background depends upon to which bin in the measured $\cos \theta_{sol}$ distribution it belongs. The probability is determined by:

$$Prob_{bg}(i) = \frac{(N_{BG}/10)}{N(i)}, \quad (7.6)$$

where:

$$\frac{(N_{BG}/10)}{N(i)} = \text{number of flat background events per bin}$$

(Recall $N_{BG}$ is the total number of background events in the 10 bins of Region R$_2$.)

$$N(i) = \text{total number of events in the } i^{th} \text{ bin}$$

of the original $\cos \theta_{sol}$ distribution

$$(31 \leq i \leq 40).$$

For example, if the number of events in bin j is 42 and the number of flat background events in each bin is calculated to be 21, then each event in bin j has a probability of being background of 21/42 or 50%. This method avoids attempting to distinguish between events near the solar direction that are true signal events and those which are background events, since all events in this region are utilized. Each of these time-smeared distributions is also added to the composite distribution.
Figure 7.3: First iteration $\cos \theta_{\text{sol}}$ time-smeared distribution for 6.5–20 MeV energy interval. Note that the background distribution is not isotropically distributed (flat).

The completed composite distribution then provides a new and better estimate of the background than the original flat-background assumption by smearing each event over the analysis period with its proper weighting. Figure 7.3 shows the composite time-smeared background $\cos \theta_{\text{sol}}$ distribution for the 6.5–20 MeV energy interval.

The time-smearing procedure is repeated for a second time using the time-smeared background distribution instead of the flat-background estimation to calculate $Prob_{bg}(i)$:

$$Prob_{bg}(i) = \frac{N_{BG}(i)}{N(i)},$$

(7.7)
where:

\[ N_{BG}(i) = \text{number of events in the} \]
\[ i^{th} \text{ bin of the time-smeared} \]
\[ \cos \theta_{sol} \text{ composite distribution} \]
\[ (31 \leq i \leq 40). \]

The entire time-smearing procedure is then iteratively repeated several times with each iteration using the time-smeared background distribution of the previous iteration until the time-smeared distributions converge. In this analysis convergence is defined to be a 0.1 % or less difference between two successive time-smeared background distributions. Typically, the time-smeread distributions for the different energy intervals converge within three to five iterations. It is important to note that this method makes no assumptions about the spatial, directional, or energy distributions of background or signal events to calculate background levels i.e. no Monte Carlo events are generated to determine background levels, only data events are used.

The final composite time-smeared background distribution is then compared to the original \( \cos \theta_{sol} \) distribution and the number of neutrino signal events is calculated by:

\[ N_{SIG} = N_2 - \sum_{i=31}^{40} N_{BG}(i), \]  

(7.8)

where:

\[ N_2 = \text{Number of events with } \cos \theta_{sol} \geq 0.5 \]

in the original distribution
\[ N_{BG}(i) = \text{number of events in the}\]
\[ i^{th} \text{ bin of the final time-smeared}\]
\[ \cos \theta_{sol} \text{ composite distribution}\]

The number of signal events \( N_{SIG} \) is then corrected for the \( \cos \theta_{sol} = 0.5 \) cut by using Eqn. 7.5.

The original measured \( \cos \theta_{sol} \) distribution and the final time-smeared distribution for the 6.5–20 MeV energy interval are over-laid in Figure 7.4. The non-flat background time-smearing procedure can also be performed in other energy intervals; Figures 7.5 and 7.6 show the over-laid measured and time-smeared \( \cos \theta_{sol} \) distributions for the energy intervals: 6.5–7 MeV, 7–7.5 MeV, 7.5–8 MeV,..., 13.5–14 MeV, and 14–20 MeV. Table 7.2 compares the extracted signal rates for each energy interval using the flat background assumption (Eqn. 7.2) and using the non-flat background time-smearing procedure; only small differences are observed.

7.4 Errors

Systematic errors in this analysis are classified into three categories: (1) energy-bin-correlated experimental errors (called "correlated" from now on), (2) energy-bin-correlated error in the expected energy spectrum calculation, and (3) energy-bin-uncorrelated ("uncorrelated") errors. The sources of correlated experimental errors are uncertainties in the absolute energy scale, energy resolution, the directional anisotropy of the background (non-flat background), and angular resolution (direction) errors. The correlated error in the expected spectrum calculation is obtained by using the 1\( \sigma \) error of \( ^8 \text{B} \) neutrino energy spectrum. The sources of uncorrelated errors are: the
Figure 7.4: Over-lay of the original $\cos \theta_{\text{sol}}$ distribution and that of the final iteration time-smeared distribution.
Figure 7.5: Over-lay of the original $\cos \theta_{\text{Sol}}$ distribution and that of the final iteration time-smearred distribution for several energy intervals.
Figure 7.6: Over-lay of the original $\cos \theta_{\text{sol}}$ distribution and that of the final iteration time-smeared distribution for several energy intervals.
Table 7.2: Comparison of extracted signal rates using the flat background assumption (Eqn. 7.2) and using the non-flat background time-smearing procedure. Negative signs indicate that the flat background approximation is the larger value.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5-7.0</td>
<td>2.5</td>
<td>10.5-11.0</td>
<td>0.91</td>
</tr>
<tr>
<td>7.0-7.5</td>
<td>-0.038</td>
<td>11.0-11.5</td>
<td>-0.51</td>
</tr>
<tr>
<td>7.5-8.0</td>
<td>1.7</td>
<td>11.5-12.0</td>
<td>0.026</td>
</tr>
<tr>
<td>8.0-8.5</td>
<td>-1.1</td>
<td>12.0-12.5</td>
<td>-1.4</td>
</tr>
<tr>
<td>8.5-9.0</td>
<td>1.9</td>
<td>12.5-13.0</td>
<td>-2.6</td>
</tr>
<tr>
<td>9.0-9.5</td>
<td>-3.8</td>
<td>13.0-13.5</td>
<td>0.92</td>
</tr>
<tr>
<td>9.5-10.0</td>
<td>2.7</td>
<td>13.5-14.0</td>
<td>-3.1</td>
</tr>
<tr>
<td>10.0-10.5</td>
<td>2.1</td>
<td>14.0-20.0</td>
<td>3.9</td>
</tr>
</tbody>
</table>

uncertainty in trigger efficiency, the uncertainty in the data reduction cut efficiencies, and the uncertainty in the live time calculation. Errors which may be energy-bin-correlated, but whose energy dependence is not well known, are categorized as uncorrelated systematic errors by assigning the largest possible deviation in the energy spectrum to each energy bin. The uncertainty in the cross section of $\nu$-e scattering is such an error. Determination of these errors as well as the statistical errors are detailed in the following sections.

7.4.1 Angular Resolution (Direction) Systematic Error

The fractional difference between LINAC calibration data and the corresponding MC detector simulation angular resolutions is shown in Figure 7.7 as a function of LINAC beam energy. The data points in this figure were fit to a straight line, however the values at 6 and 11 MeV were not used in the fit. By ignoring these data points more conservative values for the angular resolution differences at these energies are assumed, since the central values
at 6 and 11 MeV are much lower than their neighboring data points. The resulting best-fit line has the following parameters:

\[ \Delta_{\text{ang}}(E) = E \cdot b_1 + b_0 \]  

(7.9)

where:

\[ \Delta_{\text{ang}}(E) = \text{angular resolution fractional difference} \]

between MC and LINAC data

\[ E = \text{electron total energy [MeV]} \]

and \( b_0 \) and \( b_1 \) are 0.0912 and -0.00391, respectively.

The effect of the systematic angular resolution error on the signal calculation is evaluated by changing the \( \cos \theta_{\text{sol}} \) value for each MC event to:

\[ \cos \theta_{\text{sol}} \Rightarrow \cos \theta_{\text{sol}} \pm \Delta_{\text{ang}}(E) \cdot \cos \theta_{\text{sol}} \]

(7.10)

\[ \Rightarrow \cos \theta_{\text{sol}}(1 \pm \Delta_{\text{ang}}(E)) \]

The \( \cos \theta_{\text{sol}} \) value for each \( ^8 \text{B} \) MC event is first shifted towards the solar direction (\( \cos \theta_{\text{sol}} \Rightarrow \cos \theta_{\text{sol}}(1 + \Delta_{\text{ang}}(E)) \)) and a composite \( \cos \theta_{\text{sol}} \) distribution is formed. The angular cut efficiencies (the fraction of MC events with \( \cos \theta_{\text{sol}} \geq 0.5 \); refer to Section 7.2) are then calculated using the shifted \( \cos \theta_{\text{sol}} \) distributions in the energy intervals of 6.5–7 MeV, 7–7.5 MeV, 7.5–8 MeV, ..., 13.5–14 MeV, and 14–20 MeV. Shifting the \( \cos \theta_{\text{sol}} \) values towards the solar direction increases the MC angular resolution and the angular cut efficiencies will change due to the shifted composite distributions. This will affect the measured data signal values, since they are adjusted by these efficiencies.
Figure 7.7: Systematic error of angular resolution as a function of LINAC beam energy. Cosine values used are the position average at each energy. The line is the best fit linear curve ignoring data points at 6 and 11 MeV.

The process is repeated by shifting away from the solar direction (\( \cos \theta_{\text{sol}} \Rightarrow \cos \theta_{\text{sol}}(1 - \Delta_{\text{ang}}(E)) \)) to reduce the MC angular resolution. Table 7.3 tabulates the differences in measured signal rates between shifted and non-shifted \( \cos \theta_{\text{sol}} \) distributions. Increasing the MC angular resolution in the 6.5–20 MeV energy interval reduces the measured data neutrino signal rate by 0.59% relative to the signal rate adjusted using the non-shifted MC distribution. Reducing the MC angular resolution increases the measured rate by 0.66%. The angular resolution error for the 6.5–20 MeV signal rate is then taken to be 0.66%.

7.4.2 Non-Flat Background Systematic Error

The measured signal rates as extracted by the non-flat background method described in Section 7.3 are affected by the number of bins used to divide
Table 7.3: Differences in measured neutrino signal rates as adjusted using positively-shifted \((\cos \theta_{\text{sol}}(1 + \Delta_{\text{ang}}(E)))\) and negatively-shifted \((\cos \theta_{\text{sol}}(1 + \Delta_{\text{ang}}(E)))\) MC \(\cos \theta_{\text{sol}}\) distributions with respect to rates adjusted by non-shifted \(\cos \theta_{\text{sol}}\) distributions. Negative values indicate a rate loss due to shifting.

<table>
<thead>
<tr>
<th>Energy Interval [MeV]</th>
<th>+Shifted (\Delta%)</th>
<th>-Shifted (\Delta%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5-7.0</td>
<td>1.62</td>
<td>-1.41</td>
</tr>
<tr>
<td>7.0-7.5</td>
<td>1.48</td>
<td>-1.40</td>
</tr>
<tr>
<td>7.5-8.0</td>
<td>1.48</td>
<td>-1.14</td>
</tr>
<tr>
<td>8.0-8.5</td>
<td>1.31</td>
<td>-1.04</td>
</tr>
<tr>
<td>8.5-9.0</td>
<td>1.18</td>
<td>-0.86</td>
</tr>
<tr>
<td>9.0-9.5</td>
<td>0.839</td>
<td>-0.825</td>
</tr>
<tr>
<td>9.5-10.0</td>
<td>0.884</td>
<td>-0.800</td>
</tr>
<tr>
<td>10.0-10.5</td>
<td>0.847</td>
<td>-0.433</td>
</tr>
<tr>
<td>10.5-11.0</td>
<td>0.688</td>
<td>-0.631</td>
</tr>
<tr>
<td>11.0-11.5</td>
<td>0.615</td>
<td>-0.554</td>
</tr>
<tr>
<td>11.5-12.0</td>
<td>0.560</td>
<td>-0.494</td>
</tr>
<tr>
<td>12.0-12.5</td>
<td>0.502</td>
<td>-0.521</td>
</tr>
<tr>
<td>12.5-13.0</td>
<td>0.407</td>
<td>-0.618</td>
</tr>
<tr>
<td>13.0-13.5</td>
<td>0.347</td>
<td>-0.375</td>
</tr>
<tr>
<td>13.5-14.0</td>
<td>0.112</td>
<td>-0.187</td>
</tr>
<tr>
<td>14.0-20.0</td>
<td>0.232</td>
<td>-0.147</td>
</tr>
</tbody>
</table>
Table 7.4: Differences in measured neutrino signal rates using 40 and 20 bins to divide the \( \cos \theta_{\text{sol}} \) distribution in the non-flat background method. Negative values indicate that the measured rate using 20 bins is lower than that using 40 bins.

<table>
<thead>
<tr>
<th>Energy Interval [MeV]</th>
<th>( \Delta % )</th>
<th>Energy Interval [MeV]</th>
<th>( \Delta % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5-7.0</td>
<td>-0.175</td>
<td>10.5-11.0</td>
<td>-0.159</td>
</tr>
<tr>
<td>7.0-7.5</td>
<td>0.181</td>
<td>11.0-11.5</td>
<td>-0.245</td>
</tr>
<tr>
<td>7.5-8.0</td>
<td>-0.167</td>
<td>11.5-12.0</td>
<td>-0.083</td>
</tr>
<tr>
<td>8.0-8.5</td>
<td>0.130</td>
<td>12.0-12.5</td>
<td>0.005</td>
</tr>
<tr>
<td>8.5-9.0</td>
<td>-0.072</td>
<td>12.5-13.0</td>
<td>0.066</td>
</tr>
<tr>
<td>9.0-9.5</td>
<td>-0.172</td>
<td>13.0-13.5</td>
<td>-0.519</td>
</tr>
<tr>
<td>9.5-10.0</td>
<td>-0.184</td>
<td>13.5-14.0</td>
<td>-0.340</td>
</tr>
<tr>
<td>10.0-10.5</td>
<td>-0.034</td>
<td>14.0-20.0</td>
<td>-0.209</td>
</tr>
</tbody>
</table>

the \( \cos \theta_{\text{sol}} \) distribution. The systematic error in the non-flat background method was calculated by changing the number of bins from 40 to 20 (i.e. doubling the angular size of the original bins) and recalculating the non-flat background levels. Table 7.4 compares the measured signal rates using background levels calculated with 40 bins with those using 20 bins for each energy interval. The change in measured signal rate in the 6.5–20 MeV energy interval due to utilizing 20 bins is 0.24%.

7.4.3 Energy Scale Systematic Error

The effects of the systematic error in energy scale, caused by the slight differences in the energy scales between LINAC calibration data and the corresponding MC detector simulation, is measured by shifting the measured energy of each data event by the uncertainty in the energy scale and remeasuring the signal rates in each energy interval. Shifting an event’s energy may cause the event to spill into an energy bin different than the one the
non-shifted event would occupy. Many events will also have their energies cross the analysis lower energy limit of 6.5 MeV. The energy scale fractional difference between LINAC data and MC simulation is shown in Figure 3.18 from which the energy scale uncertainties are taken. The uncertainties used in this analysis are the ±1σ values of each data point. Uncertainty values are linearly extrapolated between data points and data events with measured energies greater than that of the highest energy data point (at 16.3 MeV) are given the uncertainty values at the highest energy data point. An event's measured energy is shifted by:

\[ E_{\text{shift}} = E \pm \sigma_{\text{scale}}(E) \cdot E \]

\[ = E(1 \pm \sigma_{\text{scale}}(E)) \]

where:

\[ E_{\text{shift}} = \text{shifted electron total energy [MeV]} \]
\[ \sigma_{\text{scale}}(E) = \text{energy-dependent energy scale uncertainty} \]

First, all event energies are shifted to higher values \( E_{\text{shift}} = E(1 + \sigma_{\text{scale}}(E)) \) and the signal rates are remeasured in each energy interval using the non-flat background method. Then, all event energies are shifted to lower values \( E_{\text{shift}} = E(1 - \sigma_{\text{scale}}(E)) \) and signal rates are again remeasured. Table 7.5 summarizes the differences between the signal rates of both the higher- and lower-shifted energy events with respect to the non-shifted energy events. The difference in the measured signal rates between higher- and non-shifted energies in the 6.5–20 MeV energy interval is 0.14% and that between lower- and non-shifted energies is −2.0%. The large difference caused by
Table 7.5: Differences in measured signal rates with data energies shifted to higher values (plus-shifted $E_{\text{shift}} = E(1 + \sigma_{\text{scale}}(E))$) and shifted to lower values (minus-shifted $E_{\text{shift}} = E(1 - \sigma_{\text{scale}}(E))$) with respect to non-shifted energies. Negative values indicate a rate loss due to shifting.

<table>
<thead>
<tr>
<th>Energy Interval [MeV]</th>
<th>minus-Shifted Δ%</th>
<th>plus-Shifted Δ%</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5-7.0</td>
<td>-7.38</td>
<td>-2.72</td>
</tr>
<tr>
<td>7.0-7.5</td>
<td>-0.439</td>
<td>4.32</td>
</tr>
<tr>
<td>7.5-8.0</td>
<td>3.77</td>
<td>-0.956</td>
</tr>
<tr>
<td>8.0-8.5</td>
<td>-2.46</td>
<td>1.19</td>
</tr>
<tr>
<td>8.5-9.0</td>
<td>-0.770</td>
<td>0.267</td>
</tr>
<tr>
<td>9.0-9.5</td>
<td>-3.32</td>
<td>-2.08</td>
</tr>
<tr>
<td>9.5-10.0</td>
<td>-3.77</td>
<td>-1.46</td>
</tr>
<tr>
<td>10.0-10.5</td>
<td>-2.20</td>
<td>-1.13</td>
</tr>
<tr>
<td>10.5-11.0</td>
<td>-1.75</td>
<td>1.51</td>
</tr>
<tr>
<td>11.0-11.5</td>
<td>-7.95</td>
<td>-5.35</td>
</tr>
<tr>
<td>11.5-12.0</td>
<td>-1.02</td>
<td>4.37</td>
</tr>
<tr>
<td>12.0-12.5</td>
<td>-0.862</td>
<td>-2.37</td>
</tr>
<tr>
<td>12.5-13.0</td>
<td>9.76</td>
<td>12.3</td>
</tr>
<tr>
<td>13.0-13.5</td>
<td>-7.38</td>
<td>-8.77</td>
</tr>
<tr>
<td>13.5-14.0</td>
<td>6.93</td>
<td>7.18</td>
</tr>
<tr>
<td>14.0-20.0</td>
<td>-8.19</td>
<td>13.9</td>
</tr>
</tbody>
</table>

Shifting to lower event energies is due to the steeply falling neutrino signal rate with increasing electron total energy. The energy scale error adopted for the 6.5–20 MeV energy interval is ±2.0 %.

7.4.4 Energy Resolution Systematic Error

The detector simulation MC used in this analysis has better energy resolution than that observed with LINAC calibration data by about 2%. This is shown schematically in Figure 7.8, which depicts the distribution of LINAC electron measured energies and the corresponding MC distribution about the same monoenergetic electron input energy $E_0$. The spread of measured

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energies of the LINAC events is slightly larger than that of the MC. The effect of this systematic energy resolution error upon the expected signal rates is evaluated by changing the energy resolution of the MC events to match that of the LINAC data. This is accomplished by shifting the measured energy of each MC event in the following prescribed fashion:

if \( E_{\text{meas}} < E_{\text{input}} \):

\[
E_{\text{meas}} = E_{\text{meas}} - E_{\text{meas}} \cdot \Delta_{\text{res}}(E_{\text{input}}) \quad (7.12)
\]

\[
= E_{\text{meas}}(1 - \Delta_{\text{res}}(E_{\text{input}}));
\]

if \( E_{\text{meas}} > E_{\text{input}} \):

\[
E_{\text{meas}} = E_{\text{meas}} + E_{\text{meas}} \cdot \Delta_{\text{res}}(E_{\text{input}}) \quad (7.13)
\]

\[
= E_{\text{meas}}(1 + \Delta_{\text{res}}(E_{\text{input}}));
\]

where:

\( E_{\text{meas}} \) = measured MC electron total energy [MeV]

\( E_{\text{input}} \) = input MC electron total energy [MeV]

\( \Delta_{\text{res}}(E_{\text{input}}) \) = Gaussian correction factor

\[
= (\sigma_{\text{data}}(E_{\text{input}}) \cdot 2\%) \cdot G_{\text{ran}} \quad (7.14)
\]

\( \sigma_{\text{data}}(E_{\text{input}}) \) = LINAC energy resolution

\( G_{\text{ran}} \) = Gaussian-distributed random number with \( \sigma = 1.0 \)

In other words, if a MC event's measured energy is higher than the MC input energy, the measured energy value is increased; and if the measured energy is lower than the MC input energy, the measured energy is reduced. This is done in such a way as to widen the spread of the MC distribution.
Figure 7.8: Schematic of the LINAC measured energy distribution and the corresponding MC distribution about a common electron input energy $E_0$. $\delta \sigma$ is the difference in the standard deviations between the two distributions and is exaggerated for clarity.
Figure 7.9: Energy resolution of LINAC data as a function of electron total energy. The central value of the different positions A–H at each energy is used in this analysis. The coordinates of positions A–H are listed in Table 3.1.

about a given energy to match that of the calibration data. Figure 7.9 shows the energy resolution of LINAC data as a function of electron total energy from which $\sigma_{data}(E)$ is taken. Table 7.6 lists the differences between expected $^8$B signal rates with the MC energy resolution broadening and without. This difference is 0.15% for the 6.5–20 MeV energy interval.

7.4.5 Energy Uncorrelated Errors

Energy uncorrelated errors which introduce uncertainties in the spectrum analysis are:

- the uncertainty in the $^8$B neutrino spectrum
Table 7.6: Comparison of MC signal rates with and without energy resolution broadened energies. Negative values indicate that the unbroadened resolutions result in larger expected rates than the broadened resolutions.

<table>
<thead>
<tr>
<th>Energy [MeV]</th>
<th>$^8\text{B MC}$ Δ%</th>
<th>HeP MC Δ%</th>
<th>Energy [MeV]</th>
<th>$^8\text{B MC}$ Δ%</th>
<th>HeP MC Δ%</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5–7.0</td>
<td>0.24</td>
<td>-0.025</td>
<td>10.5–11.0</td>
<td>0.72</td>
<td>-0.44</td>
</tr>
<tr>
<td>7.0–7.5</td>
<td>-0.42</td>
<td>-0.25</td>
<td>11.0–11.5</td>
<td>0.23</td>
<td>0.17</td>
</tr>
<tr>
<td>7.5–8.0</td>
<td>0.36</td>
<td>-0.30</td>
<td>11.5–12.0</td>
<td>0.47</td>
<td>-0.099</td>
</tr>
<tr>
<td>8.0–8.5</td>
<td>-0.61</td>
<td>0.072</td>
<td>12.0–12.5</td>
<td>0.49</td>
<td>-0.034</td>
</tr>
<tr>
<td>8.5–9.0</td>
<td>0.15</td>
<td>-0.31</td>
<td>12.5–13.0</td>
<td>4.0</td>
<td>0.38</td>
</tr>
<tr>
<td>9.0–9.5</td>
<td>-0.014</td>
<td>0.72</td>
<td>13.0–13.5</td>
<td>0.48</td>
<td>-0.22</td>
</tr>
<tr>
<td>9.5–10.0</td>
<td>0.86</td>
<td>-0.58</td>
<td>13.5–14.0</td>
<td>2.7</td>
<td>0.13</td>
</tr>
<tr>
<td>10.0–10.5</td>
<td>-0.16</td>
<td>-0.051</td>
<td>14.0–20.0</td>
<td>3.4</td>
<td>0.90</td>
</tr>
</tbody>
</table>

- the uncertainty in the trigger efficiency
- the uncertainty in the first reduction “flasher” cut
- the uncertainty in the “Dirks” flasher cut
- the uncertainty in the spallation dead time
- the uncertainty of the fiducial volume cut
- the uncertainty in the neutrino–electron cross sections
- the uncertainty in the live time determination

The error introduced in the bin-by-bin expected flux due to the uncertainty in the $^8\text{B}$ neutrino energy spectrum were calculated by using the ±1σ spectrum error (3σ/3) shown in Figure 5.2. Using the ±1σ spectra cause a ±1.2% error in the expected $^8\text{B}$ neutrino energy spectrum.
Measurement of the trigger efficiency is described in Section 3.5. The MC detector simulations agree with the measured trigger efficiencies, except for a 1.2 % difference in the 6.5 to 7 MeV energy interval (MC has the greater efficiency). The error introduced by the uncertainty in the trigger efficiency used in this analysis is ±1.2 % for the 6.5 to 7 MeV energy interval.

The efficiency of the first reduction “flasher” cut described in Chapter 6 is measured using both nickel calibration events (Section 3.3.2) and MC events. The difference between the reduction efficiencies ±0.2 % is used as the systematic error of this cut for each measured flux energy bin.

The efficiency of the Dirks reduction cut described in Chapter 6 is measured using a sample of spallation events and MC events. The difference between the reduction efficiencies ±0.7 % is used as the systematic error of this cut.

The dead time incurred by the spallation cut is determined using Monte Carlo events as described in Chapter 6. The spallation dead time (20 %) was initially measured by a subset of the muons in the 504 day analysis and this value was used to adjust MC flux values. The difference in the spallation dead time using the subset and using all muons in the 504 day data is 0.2 %. The spallation cut systematic error used is then ±0.2 %.

The uncertainty in the fiducial volume cut is due to the finite vertex reconstruction resolution. The error caused by the uncertainty in the fiducial volume cut is measured by shifting the reconstructed vertices of a sample of final candidate events by the 1σ vertex resolution toward and then away from the PMT wall and measuring the difference in the number of events removed.
Table 7.7: Energy non-correlated errors applied to each energy bin.

<table>
<thead>
<tr>
<th>Energy Non-Correlated Error</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^8$B Spectrum</td>
<td>± 1.2</td>
</tr>
<tr>
<td>$^8$B Cross-Section</td>
<td>± 0.5</td>
</tr>
<tr>
<td>Dirk, Cut</td>
<td>± 0.7</td>
</tr>
<tr>
<td>Flasher Cut</td>
<td>± 0.2</td>
</tr>
<tr>
<td>Spallation</td>
<td>± 0.2</td>
</tr>
<tr>
<td>Live time</td>
<td>± 0.1</td>
</tr>
<tr>
<td>Vertex</td>
<td>± 1.3</td>
</tr>
<tr>
<td>Trigger Efficiency (6.5–7 MeV only)</td>
<td>±1.2</td>
</tr>
</tbody>
</table>

by the 2 m fiducial volume cut. The uncertainty in the fiducial volume cut introduces a ±1.3 % error in the spectrum measurement.

Uncertainties in the neutrino–electron scattering cross section calculations introduce uncertainties in the expected spectra. This is measured by comparing the total cross sections using the $+1\sigma$ and $-1\sigma$ values of $\kappa(T)$ and $\rho_{NC}$ in Eqn. 5.2. The maximum difference in total cross section values is 0.5 %, and so ±0.5 % error is used as the systematic error.

The final error to be considered is the uncertainty in the live time calculation. Live time is calculated using two methods: one uses summary files from the on-line data acquisition system and the other uses the time and day information of each event in the analysis. The difference in the live time calculations are ±0.1 %, which is used as the systematic error in the calculation. Table 7.7 summarizes the magnitudes of the energy uncorrelated errors.
Table 7.8: Measured signal statistical error for each energy interval.

<table>
<thead>
<tr>
<th>Energy Interval [MeV]</th>
<th>%</th>
<th>Energy Interval [MeV]</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5-7.0</td>
<td>8.94</td>
<td>10.5-11.0</td>
<td>11.3</td>
</tr>
<tr>
<td>7.0-7.5</td>
<td>8.60</td>
<td>11.0-11.5</td>
<td>12.2</td>
</tr>
<tr>
<td>7.5-8.0</td>
<td>8.25</td>
<td>11.5-12.0</td>
<td>13.3</td>
</tr>
<tr>
<td>8.0-8.5</td>
<td>8.45</td>
<td>12.0-12.5</td>
<td>14.2</td>
</tr>
<tr>
<td>8.5-9.0</td>
<td>8.53</td>
<td>12.5-13.0</td>
<td>17.8</td>
</tr>
<tr>
<td>9.0-9.5</td>
<td>11.7</td>
<td>13.0-13.5</td>
<td>15.7</td>
</tr>
<tr>
<td>9.5-10.0</td>
<td>9.51</td>
<td>13.5-14.0</td>
<td>21.4</td>
</tr>
<tr>
<td>10.0-10.5</td>
<td>11.1</td>
<td>14.0-20.0</td>
<td>17.5</td>
</tr>
</tbody>
</table>

7.4.6 Statistical Errors

The statistical error in the flat background method (Eqn. 7.2) is used to calculate the statistical error in the signal measurement of this analysis. Using simple counting errors of $\sqrt{N_1}$ and $\sqrt{N_2}$ for $N_1$ and $N_2$, respectively, the resulting signal statistical error $\sigma_{SIG}$ is:

$$\sigma_{SIG} = \sqrt{\frac{1}{9}N_1 + N_2}$$  \hspace{1cm} (7.15)

Table 7.8 lists the statistical error percentage of each energy interval. The statistical error of this analysis in the 6.5–20 MeV interval is 3.1%.

Now that the neutrino spectrum has been measured and the expected $^8$B and HeP spectra and the associated spectra errors have been calculated, the measured and expected spectra are compared in the following chapter.
CHAPTER 8

RESULTS AND DISCUSSION

8.1 Measured Flux and Spectrum

The recoil electron rate from solar neutrino scattering (signal) with total electron energies ranging between 6.5 and 20 MeV is:

\[
\text{Data} = 13.56 \pm 0.42(\text{stat.}) \pm 0.29(\text{syst.}) \text{ events/day/22.5kton} \quad (8.1)
\]
during the first 504 days of the Super-Kamiokande detector operation. The ratio of this rate to the corresponding expected rate from $^8$B and Hep neutrinos using the BP98 SSM normalization factors is [7]:

\[
\frac{\text{Data}^{SK}}{SSM_{BP98}} = 0.474 \pm 0.015(\text{stat.}) \pm 0.010(\text{syst.}) \quad (8.2)
\]

The stability of this measurement over time is demonstrated by dividing the 504 days of data into four subsets of approximately 126 days each. The measured signal rate (using the flat-background approximation) of each data subset is shown with its statistical errors in Figure 8.1. The right-most point is the mean value of the four subsets. The dashed line indicates the signal rate measured using all data.

Figure 8.2 shows the measured signal rate and the combined standard $^8$B and Hep expected signal rate for each energy interval: 6.5–7 MeV, 7–7.5 MeV, 7.5–8 MeV,..., 13.5–14 MeV, and 14–20 MeV. The errors shown are the systematic and statistical errors added in quadrature. These spectra are compared in the following sections.
Each data subset represents approximately 126 days.

Figure 8.1: Measured signal rate (using the flat-background approximation) of each data subset. Each subset is approximately 126 days in duration. Error bars are statistical only. The right-most point is the mean value of the four subsets. The solid line indicates the signal rate measured using all data.
Figure 8.2: Measured and expected solar neutrino scattered electron rate as a function of recoil electron total energy.
8.2 Comparison of Measured and Expected Spectra

The expected $^8$B and HeP neutrino scattering rates once simulated can be compared with the measured rates. Recall that the shapes of the expected spectra are solar model independent as discussed in Section 5.2.1. However, the normalization factors of solar models determine the overall rates of the expected signal. In the following sections the normalizations of the expected $^8$B and HeP spectra are allowed to vary so as to find the combination of expected spectra which best matches the measured spectrum.

8.2.1 Possible HeP Neutrino Contribution

Comparison of the normalized expected HeP and $^8$B solar neutrino energy spectra, shown in Figures 5.12 and 5.2, respectively, reveal that the HeP neutrinos extend to higher energies than $^8$B neutrinos. From these normalized distributions HeP neutrinos appear to dominate the solar neutrino spectrum at the higher energies. However, the best-estimate normalization factors for these distributions provided by the BP98 SSM show that the expected flux of HeP neutrinos at Earth is about 3 orders of magnitude lower than that of $^8$B neutrinos: $2.1 \times 10^3/cm^2/sec$ and $5.15 \times 10^6/cm^2/sec$ for HeP and $^8$B neutrinos, respectively. With such a small flux HeP neutrinos are not expected to contribute significantly to the neutrino spectrum measured by Super-Kamiokande. However, the possible errors associated with the expected HeP flux calculation are not well known due to large uncertainties in the low-energy $^3$He–p cross sections.

The parent reaction which produces HeP neutrinos in the Sun occurs by weak interactions, and so the cross sections are too small to measure directly.
Table 8.1: Expected HeP fluxes calculated using various theoretical HeP production cross section models [38].

<table>
<thead>
<tr>
<th>Model Author(s)</th>
<th>Year</th>
<th>Expected HeP Flux $[cm^{-2}s^{-1}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salpeter</td>
<td>1952</td>
<td>$575 \times 10^3$</td>
</tr>
<tr>
<td>Werntz and Brennan</td>
<td>1967</td>
<td>$3.38 \times 10^3$</td>
</tr>
<tr>
<td>Werntz and Brennan</td>
<td>1973</td>
<td>$7.40 \times 10^3$</td>
</tr>
<tr>
<td>Tegnér and Bargholtz</td>
<td>1983</td>
<td>$(3.65-22.8) \times 10^3$</td>
</tr>
<tr>
<td>Wolfs et al.</td>
<td>1989</td>
<td>$14.0 \times 10^3$</td>
</tr>
<tr>
<td>Wervelman et al.</td>
<td>1991</td>
<td>$52.0 \times 10^3$</td>
</tr>
<tr>
<td>Carlson et al.</td>
<td>1991</td>
<td>$1.19 \times 10^3$</td>
</tr>
<tr>
<td>Schiavilla et al.</td>
<td>1992</td>
<td>$(1.28-2.83) \times 10^3$</td>
</tr>
</tbody>
</table>

Further, no limits can be placed upon the cross sections by standard solar models, since the rate of the HeP reaction is so small that it does not affect the modeled solar luminosity or sound speed. Theoretical calculations for low energy $^3$He–p cross sections have been attempted since the 1950's, however these vary by more than 2 orders of magnitude. Two major factors which make this calculation so difficult are that the HeP reaction is a forbidden transition and that the matrix elements connecting small components of the wave function and the mesonic exchange corrections are difficult to calculate. Table 8.1 lists the expected HeP fluxes using different published cross section values, demonstrating the large variance in the calculated cross section values. Consequently, the total uncertainty in HeP fluxes is difficult to quantify, although some authors have adopted a value of a factor of 6 [10]. Since the uncertainty in HeP fluxes is so large, the HeP normalization will be allowed to vary between factors of 0 and 50 during the comparison of the measured and expected spectra. The $^8$B normalization will be constrained...
Figure 8.3: Standard (BP98) expected signal rate from $^8$B and HeP neutrinos and the measured signal rate as a function of recoil electron total energy.

within factors of 0 and 1. Figure 8.3 shows the standard expected signal rates of $^8$B and HeP neutrinos and the measured signal rate as a function of recoil electron total energy. Tables 8.2, 8.3, and 8.4 list the measured and MC rates and the associated systematic and statistical errors.

8.2.2 Best-Fit Routine

A $\chi^2$ minimization routine was used to calculate the best-fit match between measured neutrino signal rates and a linear combination of the expected
Table 8.2: Measured solar neutrino scattering rates and the associated systematic and statistical errors for 504 live time days. Units are in \(\text{events/day/22.5kton}\).

<table>
<thead>
<tr>
<th>Energy Range [MeV]</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rate</td>
</tr>
<tr>
<td>6.5–7.0</td>
<td>2.27</td>
</tr>
<tr>
<td>7.0–7.5</td>
<td>1.97</td>
</tr>
<tr>
<td>7.5–8.0</td>
<td>1.91</td>
</tr>
<tr>
<td>8.0–8.5</td>
<td>1.61</td>
</tr>
<tr>
<td>8.5–9.0</td>
<td>1.45</td>
</tr>
<tr>
<td>9.0–9.5</td>
<td>0.843</td>
</tr>
<tr>
<td>9.5–10.0</td>
<td>0.952</td>
</tr>
<tr>
<td>10.0–10.5</td>
<td>0.655</td>
</tr>
<tr>
<td>10.5–11.0</td>
<td>0.532</td>
</tr>
<tr>
<td>11.0–11.5</td>
<td>0.400</td>
</tr>
<tr>
<td>11.5–12.0</td>
<td>0.310</td>
</tr>
<tr>
<td>12.0–12.5</td>
<td>0.232</td>
</tr>
<tr>
<td>12.5–13.0</td>
<td>0.148</td>
</tr>
<tr>
<td>13.0–13.5</td>
<td>0.155</td>
</tr>
<tr>
<td>13.5–14.0</td>
<td>9.10E-02</td>
</tr>
<tr>
<td>14.0–20.0</td>
<td>0.158</td>
</tr>
</tbody>
</table>
Table 8.3: Expected $^8$B solar neutrino scattering rates and the associated systematic and statistical errors for 504 live time days. Units are in events/day/22.5kton.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5-7.0</td>
<td>4.87</td>
<td>6.42E-02</td>
<td>3.84E-02</td>
</tr>
<tr>
<td>7.0-7.5</td>
<td>4.3</td>
<td>5.90E-02</td>
<td>3.63E-02</td>
</tr>
<tr>
<td>7.5-8.0</td>
<td>3.83</td>
<td>5.16E-02</td>
<td>3.43E-02</td>
</tr>
<tr>
<td>8.0-8.5</td>
<td>3.29</td>
<td>4.73E-02</td>
<td>3.19E-02</td>
</tr>
<tr>
<td>8.5-9.0</td>
<td>2.80</td>
<td>3.66E-02</td>
<td>2.95E-02</td>
</tr>
<tr>
<td>9.0-9.5</td>
<td>2.30</td>
<td>2.99E-02</td>
<td>2.68E-02</td>
</tr>
<tr>
<td>9.5-10.0</td>
<td>1.82</td>
<td>2.84E-02</td>
<td>2.38E-02</td>
</tr>
<tr>
<td>10.0-10.5</td>
<td>1.45</td>
<td>1.90E-02</td>
<td>2.13E-02</td>
</tr>
<tr>
<td>10.5-11.0</td>
<td>1.14</td>
<td>1.69E-02</td>
<td>1.89E-02</td>
</tr>
<tr>
<td>11.0-11.5</td>
<td>0.856</td>
<td>1.13E-02</td>
<td>1.64E-02</td>
</tr>
<tr>
<td>11.5-12.0</td>
<td>0.623</td>
<td>8.62E-03</td>
<td>1.40E-02</td>
</tr>
<tr>
<td>12.0-12.5</td>
<td>0.473</td>
<td>6.56E-03</td>
<td>1.22E-02</td>
</tr>
<tr>
<td>12.5-13.0</td>
<td>0.299</td>
<td>1.22E-02</td>
<td>9.74E-03</td>
</tr>
<tr>
<td>13.0-13.5</td>
<td>0.205</td>
<td>2.84E-03</td>
<td>8.02E-03</td>
</tr>
<tr>
<td>13.5-14.0</td>
<td>0.134</td>
<td>3.96E-03</td>
<td>6.47E-03</td>
</tr>
<tr>
<td>14.0-20.0</td>
<td>0.219</td>
<td>7.95E-03</td>
<td>8.35E-03</td>
</tr>
</tbody>
</table>

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Table 8.4: Expected HeP solar neutrino scattering rates and the associated systematic and statistical errors for 504 live time days. Units are in \textit{events/day}/22.5kton.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5–7.0</td>
<td>3.14E-03</td>
<td>7.76E-07</td>
<td>2.36E-05</td>
</tr>
<tr>
<td>7.0–7.5</td>
<td>2.98E-03</td>
<td>7.58E-06</td>
<td>2.32E-05</td>
</tr>
<tr>
<td>7.5–8.0</td>
<td>2.89E-03</td>
<td>8.55E-06</td>
<td>2.29E-05</td>
</tr>
<tr>
<td>8.0–8.5</td>
<td>2.71E-03</td>
<td>1.94E-06</td>
<td>2.22E-05</td>
</tr>
<tr>
<td>8.5–9.0</td>
<td>2.44E-03</td>
<td>7.58E-06</td>
<td>2.11E-05</td>
</tr>
<tr>
<td>9.0–9.5</td>
<td>2.25E-03</td>
<td>1.62E-05</td>
<td>2.04E-05</td>
</tr>
<tr>
<td>9.5–10.0</td>
<td>2.10E-03</td>
<td>1.23E-05</td>
<td>1.97E-05</td>
</tr>
<tr>
<td>10.0–10.5</td>
<td>1.91E-03</td>
<td>9.70E-07</td>
<td>1.88E-05</td>
</tr>
<tr>
<td>10.5–11.0</td>
<td>1.70E-03</td>
<td>7.58E-06</td>
<td>1.78E-05</td>
</tr>
<tr>
<td>11.0–11.5</td>
<td>1.52E-03</td>
<td>2.52E-06</td>
<td>1.69E-05</td>
</tr>
<tr>
<td>11.5–12.0</td>
<td>1.37E-03</td>
<td>1.36E-06</td>
<td>1.60E-05</td>
</tr>
<tr>
<td>12.0–12.5</td>
<td>1.14E-03</td>
<td>3.88E-07</td>
<td>1.46E-05</td>
</tr>
<tr>
<td>12.5–13.0</td>
<td>1.01E-03</td>
<td>3.87E-06</td>
<td>1.37E-05</td>
</tr>
<tr>
<td>13.0–13.5</td>
<td>8.60E-04</td>
<td>1.94E-06</td>
<td>1.27E-05</td>
</tr>
<tr>
<td>13.5–14.0</td>
<td>7.27E-04</td>
<td>9.70E-07</td>
<td>1.17E-05</td>
</tr>
<tr>
<td>14.0–20.0</td>
<td>2.54E-03</td>
<td>2.28E-05</td>
<td>2.20E-05</td>
</tr>
</tbody>
</table>
HeP and $^8\text{B}$ rates. The initial $\chi^2$ equation is defined as follows:

$$
\chi_0^2 = \sum_{i=1}^{16} \frac{[R_{i}^{\text{dat}} - \alpha_0 R_i^{^8\text{B}} - \beta_0 R_i^{\text{HeP}}]^2}{(\sigma_i^{\text{dat}})_{\text{stat}}^2 + \alpha_0^2 (\sigma_i^{^8\text{B}})_{\text{stat}}^2 + \beta_0^2 (\sigma_i^{\text{HeP}})_{\text{stat}}^2 } \cdots + (\sigma_i^{\text{dat}})_{\text{nc}}^2 + \alpha_0^2 (\sigma_i^{^8\text{B}})_{\text{nc}}^2 ,
$$

(8.3)

where:

- $R_{i}^{\text{dat}}$ = measured signal rate of the $i^{th}$ energy interval
- $R_i^{^8\text{B}}$ = expected $^8\text{B}$ rate of the $i^{th}$ energy interval
- $R_i^{\text{HeP}}$ = expected HeP rate of the $i^{th}$ energy interval
- $\alpha_0$ = free parameter indicating the relative flux of $^8\text{B}$ neutrinos
- $\beta_0$ = free parameter indicating the relative flux of HeP neutrinos
- $(\sigma_i^{\text{dat}})_{\text{stat}}$ = statistical uncertainty in the measured signal rate of the $i^{th}$ energy interval
- $(\sigma_i^{^8\text{B}})_{\text{stat}}$ = statistical uncertainty in the $^8\text{B}$ expected rate of the $i^{th}$ energy interval
- $(\sigma_i^{\text{HeP}})_{\text{stat}}$ = statistical uncertainty in the HeP expected rate of the $i^{th}$ energy interval
- $(\sigma_i^{\text{dat}})_{\text{nc}}$ = energy non-correlated errors in the measured signal rate of the $i^{th}$ energy interval
- $(\sigma_i^{^8\text{B}})_{\text{nc}}$ = energy non-correlated errors in the $^8\text{B}$ expected rate of the $i^{th}$ energy interval

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This $\chi^2$ equation only utilizes statistical errors and energy non-correlated errors; all other errors will be considered shortly. Table 7.8 lists the statistical error values in each energy interval and Table 7.7 enumerates the energy non-correlated error values.

The free parameters $\alpha_0$ and $\beta_0$, which indicate the relative flux of $^8$B and HeP neutrinos respectively, are scanned within the following limits:

$$0 \leq \alpha_0 \leq 1 \quad (8.4)$$
$$0 \leq \beta_0 \leq 50 \quad (8.5)$$

The combination of values which returns the smallest value of $\chi^2$ represents the best fit to the measured spectrum.

The $\chi^2$ minimization process is repeated several more times, each time taking into account a different energy correlated error:

$$\chi_1^2 = \sum_{i=1}^{16} \frac{[R_{i, \text{dat, scale+}} - \alpha_1 R_{i, B}^8 - \beta_1 R_{i, \text{hep}}^8]^2}{\text{same as Eqn. 8.3}}$$

$$\chi_2^2 = \sum_{i=1}^{16} \frac{[R_{i, \text{dat, scale-}} - \alpha_2 R_{i, B}^8 - \beta_2 R_{i, \text{hep}}^8]^2}{\text{same as Eqn. 8.3}}$$

$$\chi_3^2 = \sum_{i=1}^{16} \frac{[R_{i, \text{dat, resol+}} - \alpha_3 R_{i, B}^8, \text{resol} - \beta_3 R_{i, \text{hep, resol}}^8]^2}{\text{same as Eqn. 8.3}}$$

$$\chi_4^2 = \sum_{i=1}^{16} \frac{[R_{i, \text{dat, ang+}} - \alpha_4 R_{i, B}^8 - \beta_4 R_{i, \text{hep}}^8]^2}{\text{same as Eqn. 8.3}}$$

$$\chi_5^2 = \sum_{i=1}^{16} \frac{[R_{i, \text{dat, ang-}} - \alpha_5 R_{i, B}^8 - \beta_5 R_{i, \text{hep}}^8]^2}{\text{same as Eqn. 8.3}}$$

$$\chi_6^2 = \sum_{i=1}^{16} \frac{[R_{i, \text{dat, nby}} - \alpha_6 R_{i, B}^8 - \beta_6 R_{i, \text{hep}}^8]^2}{\text{same as Eqn. 8.3}},$$

where:
\( R_{i}^{dat, scale+} \) = measured signal rate of the \( i^{th} \) energy interval with the energy scale shifted positively by the \( 1 \sigma \) energy scale error.

\( R_{i}^{dat, scale-} \) = measured signal rate of the \( i^{th} \) energy interval with the energy scale shifted negatively by the \( 1 \sigma \) energy scale error.

\( R_{i}^{8B,resol} \) = expected \( ^8B \) rate of the \( i^{th} \) energy interval not corrected for the \( 1 \sigma \) energy resolution systematic error

\( R_{i}^{HeP,resol} \) = expected HeP rate of the \( i^{th} \) energy interval not corrected for the \( 1 \sigma \) energy resolution systematic error

\( R_{i}^{dat,ang+} \) = measured signal rate of the \( i^{th} \) energy interval with angular resolution increased by the \( 1 \sigma \) angular resolution error

\( R_{i}^{dat,ang-} \) = measured signal rate of the \( i^{th} \) energy interval with angular resolution decreased by the \( 1 \sigma \) angular resolution error

\( R_{i}^{dat,nfbg} \) = measured signal rate of the \( i^{th} \) energy interval with \( 1 \sigma \) non-flat background errors

\( \alpha_j \) = free parameter indicating the relative flux of \( ^8B \) neutrinos of the \( j^{th} \) \( \chi^2 \) iteration (\( 1 \leq j \leq 6 \))

\( \beta_j \) = free parameter indicating the relative flux of HeP neutrinos of the \( j^{th} \) \( \chi^2 \) iteration (\( 1 \leq j \leq 6 \))

Equations 8.6 and 8.7 replace the measured signal rates with those measured with the energy scale shifted positively and negatively \( 1 \sigma \) of energy scale error, respectively. Eqn. 8.8 replaces the expected HeP and \( ^8B \) neutrino
rates, which have been corrected for energy resolution systematic error, by
rates which have not been corrected. Equations 8.9 and 8.10 replace the mea-
sured signal rates with those adjusted for increased and decreased angular
resolutions, respectively. And Eqn. 8.11 replaces the measured signal rates
with those measured with 1 σ of non-flat background error.

The free parameters of each χ² equation are scanned for the combination
which returns the smallest χ² value. The relative flux of ⁸B neutrinos α and
the associated error are then defined to be:

\[
\alpha = \alpha_0 + (\alpha_0 - \alpha_1) + (\alpha_0 - \alpha_2) \\
+ (\alpha_0 - \alpha_3) + (\alpha_0 - \alpha_4) \\
+ (\alpha_0 - \alpha_5) + (\alpha_0 - \alpha_6) \\
\equiv \alpha_0 (1 + \sigma_1^\alpha + \sigma_2^\alpha + \sigma_3^\alpha + \sigma_4^\alpha + \sigma_5^\alpha + \sigma_6^\alpha) \quad (8.12)
\]

Similarly, the relative flux of Hep neutrinos β and the associated error are
defined to be:

\[
\beta = \beta_0 + (\beta_0 - \beta_1) + (\beta_0 - \beta_2) \\
+ (\beta_0 - \beta_3) + (\beta_0 - \beta_4) \\
+ (\beta_0 - \beta_5) + (\beta_0 - \beta_6) \\
\equiv \beta_0 (1 + \sigma_1^\beta + \sigma_2^\beta + \sigma_3^\beta + \sigma_4^\beta + \sigma_5^\beta + \sigma_6^\beta) \quad (8.13)
\]

The χ² minimization process was first implemented by requiring the rel-
ative contribution of Hep neutrinos to be zero, since standard solar models
expect that Hep neutrinos should not significantly contribute to the mea-
sured neutrino spectrum. This is accomplished in the minimalization process
by defining $\beta_j = 0.0 \ (0 \leq j \leq 6)$ in Eqn. 8.3 and Eqns. 8.6–8.11. By requiring no HeP contribution the $\chi^2$ minimization process results in a minimum $\chi_0^2$ value of 19.05 and a relative $^8$B neutrino contribution of:

$$\alpha_{B,only} = 0.479_{-0.0108}^{+0.00594} \quad (8.14)$$

With 15 degrees of freedom, this $\chi^2$ value represents a confidence level of 21.2% that the $\alpha$-weighted expected neutrino signal matches the measured spectrum. Figure 8.4 shows the ratio of the measured signal rates to the expected $^8$B rates weighted by $\alpha_{B,only}$.

Rerunning the $\chi^2$ minimization process without restricting the HeP neutrino contribution free parameter $\beta$ to zero results in a smaller $\chi_0^2$ value of 12.85 and $^8$B and HeP neutrino contributions of:

$$\alpha_{B,HeP} = 0.446_{-0.0122}^{+0.00630} \quad (8.15)$$

$$\beta_{B,HeP} = 25.1_{-2.14}^{+8.10},$$

respectively. This $\chi^2$ value with 14 degrees of freedom represents a confidence level of 53.8%, over 2.5 times the value evaluated with $^8$B alone. Figure 8.5 shows the measured to expected ratio with the expected $^8$B and HeP contributions weighted by $\alpha_{B,HeP}$ and $\beta_{B,HeP}$, respectively. The larger-than-standard HeP contribution flattens the upturn at the higher energies thus resulting in a lower $\chi^2$ value.

8.3 Discussion

The ratio of measured signal to expected (using the BP98 SSM $^8$B solar neutrino flux) shown in Eqn. 8.2 is consistent with the Kamiokande experiment result of:
Figure 8.4: Ratio of the measured neutrino signal to the weighted (by $\alpha_{Bonly}$) expected $^8\text{B}$ signal rates.
Figure 8.5: Ratio of the measured to expected signal rates with the expected $^8$B and HeP rates weighted by $\alpha_{B,HeP}$ and $\beta_{B,HeP}$, respectively.
The Super-Kamiokande result is stable over time as demonstrated in Figure 8.1.

The "absence" of $^8$B neutrinos indicates either errors in the solar models or neutrino oscillations (or perhaps some other currently unknown phenomenon). The MSW neutrino oscillation hypothesis predicts a distortion in the shape of the measured spectrum for the small angle solution parameters. Figure 8.4 shows the spectrum ratio of the measured and the best-fit weighted $^8$B expected neutrino signal. The upturn at the high energies of this figure may indicate such a distortion. Although statistics in the higher energy data points are poor (meaning that the upturn may flatten in time), the upturn has been present in the spectrum since the first 100 days of data taking. Using 504 days of data the confidence level that the expected spectrum shape using only $^8$B neutrinos matches the measured shape is 21.2%. A more recent analysis using 700 days of data calculates the confidence level to be about 6%. The $\chi^2$ value is being driven primarily by the ratio upturn at higher energies.

The Super-Kamiokande detector is sensitive to $^8$B neutrinos and also to HeP neutrinos, which are thought not to contribute significantly to the measured solar neutrino signal. However, the uncertainty in the best-estimate HeP flux calculation is very large. Since the HeP spectrum extends to higher energies than the $^8$B neutrinos the upturn may be explained by a larger-than-standard HeP contribution. A large HeP neutrino contribution flattens the upturn at higher energies as is shown in Figure 8.5. The corresponding confidence level is 53.8%.
Table 8.5 lists the standard $^8$B and HeP neutrino signal expectations and the measured rates for the integrated energy intervals: 14–20 MeV, 15–20 MeV, 16–20 MeV, ..., and 19–20 MeV. Energies above 14 MeV are chosen since HeP neutrinos extend to higher energies than $^8$B neutrinos and so any large HeP neutrino contribution would first be evident at higher energies. Integrated energy ranges are used to maintain statistical significance. The fourth column of the table totals the standard expected rates of $^8$B and HeP neutrinos and the fifth column weights the total standard expectation by the $^8$B neutrino-only best-fit result $\alpha_{B\text{only}}$. In the 18–20 MeV and 19–20 MeV energy ranges, where a large HeP neutrino contribution should be most evident, the measured rates and the weighted total expectations are very consistent without HeP contribution enhancement. (The measured rate of $-1.6$ events/504 days/22.5 kton in the 19–20 MeV energy range is consistent with 0.) It is in the lower energies of Table 8.5 (14–20 MeV to 17–20 MeV) that measured rates and the weighted total expectations agree the least.

Table 8.6 lists the standard $^8$B and HeP neutrino signal expectations weighted by the best-fit values $\alpha_{B,\text{HeP}}$ and $\beta_{B,\text{HeP}}$, respectively. The weighting of the HeP signal results in a larger-than-standard HeP contribution. The fourth column totals the two weighted expectations. Measured rates are also listed for comparison. In the lower energy intervals of Table 8.6 (14–20 MeV to 17–20 MeV), the total weighted expectations are apparently very consistent with measured values. In the 18–20 MeV and 19–20 MeV energy ranges, where a large HeP neutrino contribution should be most evident, agreement is ambiguous. However, statistics are very poor at these high energies.
Table 8.5: Standard HeP Contribution: The integrated energy rates from the Standard Solar Model BP98 (SSM) expectations of $^8$B and HeP neutrinos are listed in columns 2 and 3, respectively. Column 4 totals the $^8$B and HeP neutrino expectations. Column 5 weights the total $^8$B and HeP neutrino expectations by the best-fit factor $\alpha_{Bonly}$ of Eqn. 8.14. Column 6 lists the measured rates for comparison with column 5. All rates are in units of $\text{events/}504\text{ days/}22.5\text{kton/energy bin}$.

<table>
<thead>
<tr>
<th>Energy Range [MeV]</th>
<th>$^8$B SSM</th>
<th>HeP SSM</th>
<th>$^8$B SSM + HeP SSM</th>
<th>0.48 ($^8$B SSM + HeP SSM)</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 – 20</td>
<td>110.4</td>
<td>1.3</td>
<td>111.7</td>
<td>53.6</td>
<td>79.6</td>
</tr>
<tr>
<td>15 – 20</td>
<td>37.6</td>
<td>0.7</td>
<td>38.3</td>
<td>18.4</td>
<td>42.3</td>
</tr>
<tr>
<td>16 – 20</td>
<td>12.4</td>
<td>0.4</td>
<td>12.8</td>
<td>6.1</td>
<td>15.3</td>
</tr>
<tr>
<td>17 – 20</td>
<td>3.1</td>
<td>0.2</td>
<td>3.3</td>
<td>1.6</td>
<td>6.3</td>
</tr>
<tr>
<td>18 – 20</td>
<td>1.0</td>
<td>0.1</td>
<td>1.1</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>19 – 20</td>
<td>0.3</td>
<td>0.0</td>
<td>0.3</td>
<td>0.1</td>
<td>-1.6</td>
</tr>
</tbody>
</table>

Table 8.6: Enhanced HeP Contribution: The integrated energy rates from the Standard Solar Model BP98 (SSM) expectations of $^8$B and HeP neutrinos (listed in columns 2 and 3 of Table 8.5) are weighted by the best-fit factors $\alpha_{B,Hep}$ and $\beta_{B,Hep}$ of Eqn. 8.15 in columns 2 and 3, respectively. Column 4 totals the individually weighted $^8$B and HeP expectations. Column 5 lists the measured rates for comparison with column 4. All rates are in units of $\text{events/}504\text{ days/}22.5\text{kton/energy bin}$.

<table>
<thead>
<tr>
<th>Energy Range [MeV]</th>
<th>$0.446 \times (^8$B SSM)</th>
<th>$25.1 \times (\text{HeP SSM})$</th>
<th>$0.446 \times (^8$B SSM) + 25.1(HeP SSM)</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 – 20</td>
<td>49.2</td>
<td>34.4</td>
<td>83.6</td>
<td>79.6</td>
</tr>
<tr>
<td>15 – 20</td>
<td>16.8</td>
<td>20.0</td>
<td>36.8</td>
<td>42.3</td>
</tr>
<tr>
<td>16 – 20</td>
<td>5.5</td>
<td>10.6</td>
<td>16.1</td>
<td>15.3</td>
</tr>
<tr>
<td>17 – 20</td>
<td>1.4</td>
<td>5.0</td>
<td>6.4</td>
<td>6.3</td>
</tr>
<tr>
<td>18 – 20</td>
<td>0.4</td>
<td>2.0</td>
<td>2.4</td>
<td>0.4</td>
</tr>
<tr>
<td>19 – 20</td>
<td>0.1</td>
<td>0.6</td>
<td>0.7</td>
<td>-1.6</td>
</tr>
</tbody>
</table>

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Figure 8.6: The measured energy distribution of 10.78 MeV LINAC electrons is shown by the data points. The boxes are the summation of values from the corresponding MC simulations, where the vertical size of a box indicates the estimated systematic errors in energy scale and resolution added in quadrature with statistical error.

Another possible explanation of the upturn at the higher energies is that the detector simulation MC does not accurately model the detector energy resolution. Figure 8.6 shows the measured energy distribution of 10.78 MeV LINAC electrons (data points) and the corresponding MC (boxes). There is good agreement in the shape over two orders of magnitude. The energy resolution is defined to be the $1\sigma$ width of the Gaussian fit to the energy distribution. If a non-Gaussian tail exists beyond the $1\sigma$ width, an upturn in the spectrum ratio could result if it is not modeled by the MC.

To investigate this possibility, the energy resolution of the $^8\mathrm{B}$ and HeP MC events were broadened to match the data as described in Section 7.4.4,
however a non-Gaussian tail was added beyond the $1\sigma$ width. This is accomplished by shifting the measured energy of each MC event in the following prescribed fashion:

if $E_{\text{meas}} < E_{\text{input}}$:

$$E_{\text{meas}} = E_{\text{meas}} - E_{\text{meas}} \cdot \Delta_{\text{res}}(E_{\text{input}}) = E_{\text{meas}}(1 - \Delta_{\text{res}}(E_{\text{input}}));$$

if $E_{\text{meas}} > E_{\text{input}}$ and

if $E_{\text{meas}} < [E_{\text{input}} + 1\sigma_{\text{data}}(E_{\text{input}})]$:

$$E_{\text{meas}} = E_{\text{meas}}(1 + \Delta_{\text{res}}(E_{\text{input}}));$$

if $E_{\text{meas}} > E_{\text{input}}$ and

if $E_{\text{meas}} > [E_{\text{input}} + 1\sigma_{\text{data}}(E_{\text{input}})]$:

$$E_{\text{meas}} = E_{\text{meas}}(1 + \Delta_{\text{res}}(E_{\text{input}})) + \kappa \cdot (E_{\text{meas}} - E_{\text{input}})^\gamma;$$

where:

$$E_{\text{meas}} = \text{measured MC electron total energy [MeV]}$$

$$E_{\text{input}} = \text{input MC electron total energy [MeV]}$$

$$\Delta_{\text{res}}(E_{\text{input}}) = \text{Gaussian correction factor}$$

$$= (\sigma_{\text{data}}(E_{\text{input}}) \cdot 2\%) \cdot G_{\text{ran}} \quad (8.19)$$

$$\sigma_{\text{data}}(E_{\text{input}}) = \text{LINAC energy resolution}$$

$$G_{\text{ran}} = \text{Gaussian-distributed random number with } \sigma = 1.0$$

$$\kappa \text{ and } \gamma = \text{non-Gaussian tail parameters} \quad (8.20)$$
The non-Gaussian tail parameters $\kappa$ and $\gamma$ were numerically scanned, and the combination which provide the expected spectra with the smallest corresponding $\chi_0^2$ value\(^1\) (see Eqn. 8.3) are chosen. The parameters which provide the best-fit to the shape of the measured spectrum are:

\[ \kappa = 0.1 \quad (8.21) \]
\[ \gamma = 1.9 \]

Figure 8.7 shows the energy distribution of 10.78 MeV MC electrons with the best-fit non-Gaussian tail parameters (top) and the resulting spectrum ratio (bottom). These parameters result in a minimum $\chi_0^2$ of 12.8. With 14 degrees of freedom, this $\chi^2$ represents a confidence level of 53.9\%, which is the same confidence level as the large HeP contribution. However, the energy distributions required to produce this spectrum ratio are not observed with the LINAC calibration and do not appear likely to cause the spectrum distortion. A new, high statistics energy calibration using a $^{16}$N generator (refer to Section 3.3.3) is currently being developed to confirm the LINAC calibrations.

The statistics are poor at the high energies of the upturn, which means that much more data must be collected to differentiate a large HeP neutrino contribution from spectral deformations caused strictly by neutrino oscillations (assuming that the upturn is not a statistical structure and does not flatten with time). At current data collection rates of events with energies of 18 MeV or greater, several more years of data taking will be required to answer the question of a possible large-than-standard HeP contribution to the measured solar neutrino spectrum.

\(^1\)The $\chi_0^2$ parameters $\alpha_0$ and $\beta_0$ are set equal to 1 during this procedure.
Figure 8.7: Energy distribution of 10.78 MeV MC electrons with the best-fit non-Gaussian tail parameters (top) and the resulting spectrum ratio (bottom).
8.4 Summary

The spectrum of recoil electrons from solar neutrino scattering above 6.5 MeV has been measured using the first 504 days of Super-Kamiokande data. The scattering rate recorded is:

\[ 13.56 \pm 0.42 \text{ (stat.)} \pm 0.29 \text{ (syst.)} \text{ events/day/22.5kton}, \]

which is a factor of \( 0.474 \pm 0.015 \text{ (stat.)} \pm 0.010 \text{ (syst.)} \) of the expected rate. The measured spectrum is shown in Figure 8.3 along with the expected spectra from \(^8\text{B}\) and HeP neutrinos. These spectra were compared using a \( \chi^2 \) minimization process to find the best-fit match between the measured neutrino rates and a linear combination of the expected rates from \(^8\text{B}\) and HeP neutrinos. Using only the \(^8\text{B}\) expected spectrum resulted in a best-fit scaling factor of 0.479; that is, the \(^8\text{B}\) expected spectrum best matches the measured spectrum if it is scaled by 0.479. This best-fit has a \( \chi^2 \) value of 19.05 with 15 degrees of freedom, which corresponds to a confidence level of 21.2%. Figure 8.4 shows the ratio of the measured spectrum to the scaled \(^8\text{B}\) expected spectrum. Note the upturn at higher energies which may be an artifact of statistics, or an indication of neutrino oscillations or some other phenomenon. Using both the \(^8\text{B}\) and HeP expected spectra in the fitting procedure results in a \(^8\text{B}\) best-fit scaling factor of 0.446 and a HeP best-fit scaling factor of 25.1. This best-fit has a \( \chi^2 \) value of 12.85 with 14 degrees of freedom, which corresponds to a confidence level of 53.8%. Figure 8.5 shows the ratio of the measured spectrum to the scaled \(^8\text{B}\) and HeP expected spectra. The larger-than-standard HeP contribution flattens the upturn at higher energies, thus
results in a lower $\chi^2$ value. The upturn does not appear to be due to an inaccurate modeling of the detector energy resolution, since the resolution error required to produce such an upturn is much higher than the measured energy resolution error. If the upturn is due to a larger-than-standard HeP neutrino contribution, then the HeP contribution would be most evident at higher energies. At the current rate of data collection of events with energies of 18 MeV or greater, several more years of data taking will be required for the Super-Kamiokande detector to differentiate between a larger-than-standard HeP contribution and possible neutrino oscillation modes.
BIBLIOGRAPHY


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Robert Sanford, Jr. was born in New Orleans, Louisiana in 1966. After graduating *Summa Cum Laude* from Jesuit High School, he received his bachelor of science degree in Mechanical Engineering from Tulane University and was ranked first in his M.E. class. One year of his college years was spent studying at the University of Sheffield, England. Robert worked as an engineer for Shell Oil Company in New York and Florida for nearly four years before returning to Louisiana and attending Louisiana State University, where he earned a master of science degree and a doctor of philosophy degree in physics.
DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: Robert Ellis Sanford, Jr.

Major Field: Physics

Title of Dissertation: Spectrum of Solar Neutrinos above 6.5 MeV

Approved:

[Signatures]

Major Professor and Chairman
Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:
March 19, 1999