Nonaxisymmetric Equilibrium Models for Gaseous Galaxy Disks.

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NONAXISYMMETRIC EQUILIBRIUM MODELS
FOR GASEOUS GALAXY DISKS

A Dissertation
Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy
in
The Department of Physics and Astronomy

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May, 1999
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Although the office where I write these words is very chilly, I feel a definite warmth at the thought of all the people who have brought me to this place. This section of my thesis is supposed to be the most personal and should, therefore, be more my own creation than any other part. I now realize however, that this section is not mine at all; its content was dictated by the generous acts of many other people long before I began to write it. I can only try, however imperfectly, to record their generosity here and express my gratitude.

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ABSTRACT

Three-dimensional hydrodynamic simulations show that, in the absence of self-gravity, an axisymmetric, gaseous galaxy disk whose angular momentum vector is initially tipped at an angle $i_0$ to the symmetry axis of a fixed spheroidal dark matter halo potential does not settle to the equatorial plane of the halo. Instead, the disk settles to a plane that is tipped at an angle $a = \arctan(q^2 \tan i_0)$ to the equatorial plane of the halo, where $q$ is the axis ratio of the halo equipotential surfaces. The equilibrium configuration to which the disk settles appears to be flat but it exhibits distinct nonaxisymmetric features. Whereas a purely stellar dynamical system in the same configuration would be destroyed by differential precession of the stellar orbits, the gaseous disk appears to be secularly as well as dynamically stable. This result has important implications for models of galaxy evolution because, over time, any stellar population that is formed from such an inclined disk should naturally fill a larger volume of space (form a "thicker disk") that surrounds the gaseous disk.
CHAPTER 1

INTRODUCTION

1.1 Observations of H I Disk Structure

A basic picture of how stars are distributed in thin disks within spiral galaxies has been understood ever since Edwin Hubble established that these "island universes" lie outside our own galaxy. The classification scheme introduced by Hubble (1936) assigned labels to spirals based upon their optical appearance, ranging from S0 for flattened, lenticular shapes with no spiral arms, onward through the Sa class for disks with tightly wound spiral arms, and culminating in the Sc class of disks which are characterized by very open, loosely wound spiral arms.

A clear picture of the distribution of the gas between the stars in these galaxies, and how it varies with the morphological types defined above, emerged later. We now know that the gaseous interstellar medium (ISM) of spiral galaxies consists of five components (cf. Mihalas and Binney 1981):

1. Cold, dense molecular clouds at temperatures $T \sim 20$ K and number densities $n \sim 10^3$ cm$^{-3}$. These clouds contain molecular gases such as H$_2$ and CO, as well as grains of dust.

2. Cold, neutral gas in large clouds at $T \sim 100$ K with densities $n \sim 20$ cm$^{-3}$. This component is sometimes referred to as the cold neutral medium (CNM).
3. The warm neutral medium (WNM) of gas surrounding the cold neutral clouds. This gas is characterized by temperatures $T \sim 6 \times 10^3$ K and densities $n \sim 0.3 \text{ cm}^{-3}$.

4. Warm, ionized gas at $T \sim 8000$ K and $n \sim 0.5 \text{ cm}^{-3}$ which exists in both localized regions heated by hot stars (H II regions) and in a diffuse component throughout the disk (the warm ionized medium or WIM).

5. The hot interstellar medium (HIM) of ionized gas at $T \sim 10^6$ K with densities $n \sim 10^{-3} \text{ cm}^{-3}$ which has been heated by shock waves from supernovae.

The last four of these components exist in approximate pressure equilibrium with each other. The primary constituent of the ISM is atomic hydrogen, in both the neutral (H I) and ionized (H II) components, although molecular hydrogen (H$_2$) is found in the giant molecular clouds and is particularly concentrated in the inner regions of spiral galaxies (Kulkarni and Heiles 1988).

The neutral hydrogen of both the CNM and WNM is usually referred to collectively as simply the H I disk. This H I disk is often the only component included in discussions of the structure of gaseous disks of spiral galaxies. One reason for this is that the H I usually accounts for most of the mass that is present in gaseous form (Kulkarni and Heiles 1988). Another eminently practical reason for focusing on the H I is that it is more easily detected than the other components. Molecular hydrogen is detected at millimeter wavelengths, which are difficult to observe since the atmosphere of the Earth...
is relatively opaque at such wavelengths, while the HIM component is detectable only at the opposite end of the spectrum in X-rays, which can only be detected by spacecraft above the Earth's atmosphere. Neutral hydrogen, on the other hand, is easily detected by ground-based radio telescopes due to its well-known spectral line at a wavelength of 21 cm which was first predicted by van de Hulst (1945) and detected in the Milky Way a few years later by Ewen and Purcell (1951).

A clearer picture of the H I content of spiral galaxies emerged as the resolution of radio telescopes improved. Neutral hydrogen comprises about 1% of the total mass of many spiral galaxies, although the actual fraction varies with morphological type and may range from less than 0.01% in many S0 galaxies to more than 50% in some Sc and irregular galaxies (Giovanelli and Haynes 1988). The central regions of most spiral galaxies are deficient in neutral hydrogen, so generally the H I disk forms a thin, flattened torus (Giovanelli and Haynes 1988). While the stellar disks of spirals are thin, typically having scale heights of less than 1 kpc and radii of perhaps 10 kpc, the H I disks are even thinner; the average thickness is only about 200 pc but the H I is often detectable out to radii beyond 30 kpc (Mihalas and Binney 1981). Thus, while the Holmberg radius, $R_H$, marks the radius at which the optical brightness of a galaxy fades (reaching a surface brightness of 26.5 mag arcsec$^{-2}$), H I gas may remain detectable out to more than twice the Holmberg radius.

When such thin disks are viewed edge-on, any deviation from a planar structure becomes obvious. Figure 1.1 illustrates the appearance of such a
Figure 1.1: Schematic depiction of a warped disk as viewed at an inclination of 90°.

disk, based upon model data. The gas in the outer regions warps above the plane of the inner disk on one side and below the plane on the opposite side. Such "integral sign" warps are a common phenomenon. Sanchez-Saavedra et al. (1990) found such warps in 42 out of 86 galaxies viewed edge-on. As Binney (1992) has pointed out, this finding suggests that almost all spiral galaxies may be warped since the warp of a disk viewed edge-on will only be apparent if the line of nodes of the warp is closely aligned with our line of sight.

Luckily, warps may also be detected even in galaxies that are not so fortuitously aligned as to show the familiar "integral sign" shape. Gas orbiting in a perfectly flat disk viewed at an inclination of 0° (i.e. face-on) will have no component of velocity along our line of sight and will not exhibit any Doppler shift (neglecting random thermal motions perpendicular to the
disk). Gas moving in a warped disk, on the other hand, must display some velocity perpendicular to the plane of the inner disk. Viewed face-on, this velocity normal to the disk will lie along the line of sight and will produce a measureable Doppler shift in the 21 cm emission line of neutral hydrogen. In the case of a galaxy viewed at an intermediate inclination, $i$, both the rotational velocity in the plane of the galaxy and the warp-induced velocity perpendicular to that plane will have components along the line of sight. The inclination of a galaxy to the line of sight is not usually known with any certainty, so the true three-dimensional velocity cannot be determined from the single measured component unless we make some assumption about the symmetry of the velocity.

Rogstad, Lockhart, and Wright (1974) demonstrated that the observed patterns of Doppler shifts in 21 cm observations of spiral galaxies could be fit by kinematical “tilted ring” models which assumed the galaxies to be composed of concentric rings of gas in circular orbits as shown in Figure 1.2. The orientation of each ring may be specified by a “tilt” which measures the inclination of the ring to the inner disk plane, and by a “twist” which measures the direction of that tilt. Specifically, the twist angle measures the azimuth in the inner disk plane of the line of nodes at which the ring intersects that plane. Rings sharing a common line of nodes produce a disk which is warped, but not twisted. If each successive ring of a warped disk is twisted through a greater angle, the line of nodes will trace out a spiral as one moves radially outward from the innermost ring.
Observations of many galaxies at 21 cm (see, in particular, the exhaustive study by Bosma (1981)), confirmed the expectation that most spiral galaxies have warped H I disks. Briggs (1990) summarized the following set of rules obeyed by most H I disks:

1. Warps typically develop at radii where the surface brightness lies between $25.0 \text{ mag arcsec}^{-2}$ and $26.5 \text{ mag arcsec}^{-2}$. The latter figure defines the Holmberg radius, $R_H$.

2. The line of nodes is straight within $R_H$, but usually twists into a leading spiral beyond this radius.
1.2 Theory of H I Disk Structure

Given the prominence of warps in the observations of H I disks described above, it is not surprising that most of the effort in modeling H I disks has been devoted to explaining warps. The fact that warps are so common indicates that they must be a fairly long-lived phenomenon. Early dynamical models for warped disks focused on describing the warp as a normal mode of a self-gravitating system of particles in circular orbits, but Hunter and Toomre (1969) showed that an isolated disk was stable against such bending modes, provided the surface density of the disk decreased sufficiently slowly toward the edge.

An alternative mechanism for creating and maintaining warps soon appeared, however. Both optical and radio observations revealed that the orbital velocities of stars and gas in the disks of spiral galaxies are inconsistent with the galaxian masses inferred from the optical brightness of those galaxies. Rotation curves, which are simply plots of the orbital velocities of stars (or gas) in a galaxy as a function of radius, allow one to determine the gravitational potential of the galaxy, and therefore the mass responsible for that potential. For example, particles moving in circular orbits around a point mass, \( M \), would generate rotation curves in which velocity declines with distance according to the Keplerian prescription \( v^2 = \frac{GM}{R} \). The rotation curves of most galaxies on the other hand, tend to be quite flat, showing velocities that remain nearly constant with radius, indicating that the amount of mass in these galaxies continues to increase with distance from the centers of the galaxies, out to the most distant parts of the H I disks observable. In
virtually all spiral galaxies, the mass inferred from such dynamical analyses is much greater than that of the visible stars and gas.

The most direct solution to this "missing mass" problem has been the assumption that the visible disks of galaxies lie within unseen halos of dark matter. Although the nature of the dark matter remains an unresolved issue, it is quite reasonable to assume that it must possess some amount of angular momentum, and in the simplest case, this would result in a halo structure that is flattened into a spheroid to some degree. Toomre (1983) and Dekel and Shlosman (1983) independently suggested that a warp might be sustained if the disk were embedded within such a massive halo and inclined at an angle to the equatorial plane of the halo. In these models, the gravitational forces of the disk and the halo combine to sustain a warp as a normal mode of the system which persists much like a standing wave on the surface of a drum. Following up on these ideas, Sparke and Casertano (1988) studied the normal modes of a self-gravitating disk composed of material in circular orbits inclined with respect to the equatorial plane of an oblate halo and found that interaction between the disk and the halo could prevent the winding up of the warp for a limited range of halo flattening and inclinations. Their models showed that a steady-state warp can be supported but the warp should not be twisted as Briggs (1990) found, i.e. the line of nodes of the tilted disk orbits should be straight.

The works cited above focused on the steady-state structure of warped disks. More recently, as an extension of the above referenced work on steady-state warped disks, Hofner and Sparke (1994) examined the time-dependent
settling of disks from an unwarped initial disk inclined to the halo equatorial plane to the final warped state described above. The disk was assumed to be composed of material in circular orbits initially inclined to the halo equator. Like the earlier work of Sparke and Casertano, their simulations did not model fluid effects, but analyzed the response of a disk subject only to the force of gravity in the form of the self-gravity of the disk and its gravitational attraction to a fixed external halo. They employed a linearized treatment of the gravitational forces and discovered that a warped structure developed as the outermost parts of the disk retained their inclination while the inner regions settled toward the halo equator as a result of the outward transport of energy by dispersive bending waves. Dubinski and Kuijken (1995) considered the response of a halo that was not fixed, but allowed to respond to the gravitational force of the disk. Their models showed that the inner regions of the halo would quickly align with the disk, but the outer regions could remain misaligned and drive warped bending modes.

While gravitational interactions such as those modeled by Hofner and Sparke (1994) can drive the settling of a disk composed of discrete particles, one would also expect a fluid disk to be subject to viscous effects that could dissipate energy and lead to settling. The individual clouds which constitute a fluid disk may interact to produce a viscous effect in a manner analogous to the way molecules in a fluid interact to produce a bulk viscosity (Tohline 1982). That is, clouds moving past one another will experience gravitational attractions, even physical collisions at times, that alter their momentum and dissipate some of their kinetic energy into heat. When neighboring
clouds have significantly different velocities, the opportunity for dissipation in turbulent motion is greatest. Such a condition can arise when different regions of an extended H I disk precess at different rates.

Just as a spinning top will precess when tipped from a vertical alignment because it feels a torque, the orbit of a particle (or an individual fluid element) in a non-spherical potential will precess if it moves out of the equatorial plane of the potential. The flattened shape of an oblate potential means that the gravitational force outside the equatorial plane is not directed radially toward the center of the potential. Since the gravitational force is not aligned with the position vector, the particle will experience a torque and the orbit of the particle will precess about the symmetry axis of the potential. Furthermore, the shape of the potential dictates that the magnitude and direction of the torque will vary radius. The resulting rate of precession therefore varies with radius in a manner which will be described quantitatively in a later chapter.

This differential precession of fluid elements at different radii implies that some fluid elements will encounter others moving in significantly different directions. These collisions between clouds should create turbulent viscous effects like those described above (cf. Tohline, Simonson, and Caldwell 1982).

Steiman-Cameron and Durisen (1988) employed a “cloud-fluid” model of such collisions between clouds in order to describe viscous effects in gaseous galaxy disks. Their method, which neglected gas pressure within the disk, averaged the effects of viscosity along with gravitational and Coriolis effects over an orbital period and produced a set of equations which described the time-dependent settling of gas disks on precessional timescales.
These studies suggest a general picture of the evolution of gaseous galaxy disks that reside in nonspherical, dark matter halos. Gas falling into a spheroidal halo will generally have its angular momentum vector inclined at some angle to the symmetry axis of the halo. If the gas is cold, on a dynamical timescale it will form into a rotationally flattened disk that is in dynamical equilibrium but inclined at an angle to the equatorial plane of the halo. Subsequently, differential precession is expected to distort the disk into a warped and twisted structure on a precessional timescale at each radius. The precessional timescale may be many orbital periods, depending upon the distortion of the halo and the inclination of the disk (Tohline, Simonson, and Caldwell 1982). As differential precession leads to interactions between fluid elements, dissipative effects should cause the disk to settle to the equatorial plane; alternatively, settling may be driven by the dispersive bending waves described by Hofner and Sparke (1994). Since the precession rate is much slower at larger radii, the observed twists in the outer regions of warps probably represent transient features resulting from time-dependent settling of gas into a preferred plane (New et al. 1998).

None of the works cited above modeled H I disks as a fluid subject to all of the dynamical effects present in a nonequilibrium state. The present work began as an attempt to study this time-dependent structure using a full three-dimensional hydrodynamic simulation to examine what occurs in the simplest case of a massless disk embedded in a static, axisymmetric halo. Following the evolution of such a disk shows that it eventually settles into a previously unexpected equilibrium state in which the disk remains inclined
to the equatorial plane of the halo potential. This result may have important implications for the structure of spiral galaxies.
CHAPTER 2
HYDRODYNAMIC SIMULATION TECHNIQUE

2.1 The Equations of Hydrodynamics

To fully understand the behavior of a gas like the H I disks in spiral galaxies, we must include a description of fluid effects such as pressure by employing the equations of hydrodynamics. We know from everyday experience that the behavior of fluids can be quite different from that of solid objects, even though both obey the same physical laws. For example, a solid object will normally have a constant volume, so conservation of matter for such an object naturally implies that its density, $\rho$, remain constant also. One might then express the concept of conservation of matter for a solid object by simply stating $\frac{\partial \rho}{\partial t} = 0$. A fluid on the other hand, may conserve matter even as it experiences local changes in density as matter flows from one region to another. This distinction illustrates the two different ways in which the equations of fluid dynamics may be specified. The Lagrangian specification defines the concept of an individual fluid element which is small compared to the size scale of the problem. The trajectory of each fluid element of constant mass is then followed in a manner analogous to the trajectory of a solid object, although the fluid element may distort and change its volume as it moves. Conservation of matter is expressed in the Lagrangian formulation as

$$\frac{D\rho}{Dt} = 0,$$  \hspace{1cm} (2.1)
where the derivative operator is the total derivative which includes not only the explicit variation of $\rho$ with time but also the variation due to spatial gradients in the fluid density.

An alternative to the Lagrangian specification is the Eulerian specification which expresses how the quantities of interest vary at fixed reference points in space. If one considers a fixed small volume, $dV$, bounded by a surface, $dA$, the density within that volume must change continuously; matter cannot suddenly appear or disappear without a source or sink. The conservation of matter for a fluid may then be stated in the form of the Eulerian equation of continuity (cf. Landau and Lifshitz 1987) as

$$\frac{\partial}{\partial t} \int \rho dV + \int \rho \mathbf{v} \cdot dA = 0.$$  \hspace{1cm} (2.2)

Applying the divergence theorem to the above equation and requiring that it be satisfied at any arbitrary location enables one to write the equation of continuity in its familiar differential form as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0.$$  \hspace{1cm} (2.3)

Similarly, the familiar expression of Newton's Second Law for the conservation of momentum may be applied to a fluid by equating the change in momentum within a volume element $dV$ to the sum of the momentum flux across $dA$ due to the flow of the fluid at velocity $\mathbf{v}$ plus the forces acting on the fluid within the volume element. If the only forces present are assumed to be those arising from a gravitational potential, $\Phi$, and a gas pressure, $P$, the rate of change of the momentum in $dV$ may then be written
\[
\frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla P - \rho \nabla \Phi. \tag{2.4}
\]

One more conserved quantity remains to be accounted for: energy. Conservation of energy implies that the specific entropy of an adiabatic fluid must remain constant. A scalar equation for conservation of energy, combined with the equation of continuity and the three components of the equation of motion, give five scalar equations describing the time evolutionary behavior of the fluid. There are, however, six unknown quantities: \( \rho, v_1, v_2, v_3, P, \) and internal energy \( u \) (The potential is known everywhere since this model ignores self-gravity. A self-gravitating model would require that the potential be determined from the Poisson Equation). Clearly, one more relation is needed. The remaining information is specified through an equation of state for the fluid. Using the equation of state to express the pressure as a function of density closes the system of equations and allows the state of the fluid to be determined by solving simultaneously equations (2.3), (2.4) and the energy equation described below.

Many astrophysical fluids are well described by a polytropic equation of state

\[
P = K \rho^{(n+1)/n}. \tag{2.5}
\]

Comparison with the ideal gas law shows that an adiabatic monatomic ideal gas may be described as a polytrope of index \( n = 3/2 \). This simple ideal gas equation of state will be used in this work. The constant \( \gamma = (n + 1)/n \) may be specified instead of \( n \). This form of the equation of state gives a particularly simple form of the energy equation.
If $u$ represents the internal energy of a small amount of fluid, conservation of energy for an adiabatic fluid flow demands

$$\frac{D}{Dt} \left( \frac{u}{\rho} \right) = -\frac{P}{\rho} \nabla \cdot \mathbf{v}, \quad (2.6)$$

where the left-hand side is the total derivative of the internal energy and the right hand side represents the work done by the fluid. Using the polytropic equation of state and defining an entropy tracer

$$\tau = u^{1/\gamma}, \quad (2.7)$$

the above equation may be written

$$\frac{\partial \tau}{\partial t} + \nabla \cdot (\tau \mathbf{v}) = 0. \quad (2.8)$$

Note that all three of the equations for conserved quantities may be expressed in the consistent form

$$\frac{\partial Q}{\partial t} + \nabla \cdot (Q \mathbf{v}) = S_Q, \quad (2.9)$$

where $Q = \rho$ in the continuity equation, $Q = \rho \mathbf{v}$ in the equation of motion, and $Q = \tau$ in the energy equation. For the equation of motion, the source term, $S_Q$, represents the forces which appear on the right hand side of that equation. Since there are no corresponding sources in the equation of continuity or the energy equation, $S_Q = 0$ in those equations. This consistent form of the three hydrodynamic equations will be exploited in the numerical solution of these equations.
2.2 The Hydrodynamic Code

Once the initial conditions are specified, a solution of the three basic hydrodynamic equations given above will describe how the state of the fluid evolves with time. For the simulations presented in this dissertation, the hydrodynamic equations are solved using a numerical finite-difference hydrodynamic (FDH) code originally developed by Tohline (1978) and subsequently modified as described by Woodward (1992). The code was adapted to the parallel architecture of the MasPar computer by Woodward and a more complete description is given by Woodward, Tohline, and Hachisu (1994). The current version of the FDH code is second-order accurate in both space and time and is implemented on a uniformly zoned, cylindrical grid of resolution $128 \times 128 \times 64$ in the radial, vertical, and azimuthal directions, respectively.

Since all three equations to be solved have the same form, given by equation (2.9), it will suffice to describe in general terms how that equation is solved by the FDH code. The source term, $S_Q$, is non-zero only in the case of the equation of motion, (2.4). For the three components of equation (2.4), the source term consists of gradients of the pressure and the gravitational potential. For this work, the version of the code described in Woodward, Tohline, and Hachisu (1994) was modified to incorporate a logarithmic potential of the form described by Richstone (1980). Gradients in the potential and the pressure are estimated to second-order accuracy using a familiar Taylor series expansion. For example, the radial gradient of the potential at the interface between cells centered at radius $R_j$ and $R_{j-1}$ may be approximated as
In every case, care is taken to evaluate the gradient in the potential at the same location as the velocities in the divergence term.

The divergence term in equation (2.6) represents the effects of advection, or flow of material from one grid cell to another. It may be computed at the location of a given grid cell by summing the flux of the desired quantity across all faces of the grid cell. The simplest method of computing this flux is the donor cell method which assumes that the value of the quantity (density, for example) used to compute the flux at the face of a cell may simply be represented by the value found in the adjacent cell. Unfortunately, this method is only accurate to first order, so it is not used in this version of the FDH code. Woodward (1992) has described how the technique of Van Leer (1976) was adapted to the FDH code to provide a numerically stable, second order method of computing the fluxes.

The methods described above provide finite difference approximations to the spatial derivatives in equation (2.4). If the partial derivative with respect to time in equation (2.4) is replaced by a finite difference, the equation may be written

\[
\frac{Q(t_{n+1}) - Q(t_n)}{\Delta t} + \nabla \cdot (Qv) = S_Q.
\]  

Rewriting equation (2.11) as

\[
Q(t_{n+1}) = Q(t_n) + \Delta t [S_Q - \nabla \cdot (Qv)]_n,
\]

shows that the value of \( Q \) at time \( t_{n+1} \) may be explicitly determined from
the value at the previous time step, \( t_n \), if the values of \( Q \) and \( v \) specified in the divergence term on the right-hand side of the equation are known at time \( t_n \). Using the known values at \( t_n \) in the divergence however, constitutes a linear extrapolation from \( t_n \), which is only accurate to first order in \( \Delta t \).

To improve accuracy, the algorithm employed in the FDH code uses a more involved procedure to estimate the values of the desired quantities at one-half of a time step. The procedure may be summarized as follows:

1. The source terms of the momentum equations are computed based upon current values of the density and pressure.

2. The values of the source terms are then used to estimate the velocities one half time step later by advancing the solution to equation (2.12) forward by \( \Delta t/2 \) while ignoring the divergence term, i.e. solving

\[
Q(t_n + \frac{\Delta t}{2}) = Q(t_n) + \frac{\Delta t}{2} S_Q(t_n). 
\] (2.13)

3. The velocities derived from the newly determined momenta are next used in the advection term of equation (2.12) to advance the solution forward one half time step in order to obtain better estimates of the velocities at \( t = t_n + \Delta t/2 \).

4. The improved velocities are then used to compute the advection term for the full time step from \( t_n \) to \( t_{n+1} \).

5. Finally, the source terms are updated to further accelerate the flow from \( t = t_n + \Delta t/2 \) to \( t = t_{n+1} \).
To obtain a numerically stable solution, the time step, $\Delta t$, must satisfy the Courant condition first described by Courant, Friedrichs, and Levy (1928):

$$\Delta t \leq 0.7 \frac{\Delta l}{c_s + |v|},$$

(2.14)

where $\Delta l$ refers to the width of the grid cell in a given direction, $v$ is the velocity of the fluid in that direction, and $c_s$ is the sound speed of the fluid. The Courant condition merely states that one must choose a time step that is significantly less than the time required for information about the fluid properties to propagate across the grid cell.

Since the finite-difference approximation described above is accurate to second order, it must introduce errors of order $(\Delta R)^3$, where $\Delta R$ represents the typical size of a grid cell. These error terms mimic the effects of viscous forces in the equation of motion and give rise to what is known as "numerical viscosity" (cf Tohline 1982; Norman Wilson, and Barton 1978). Thus, although this version of the FDH code includes no explicit treatment of viscosity, some viscous dissipation will nonetheless be present due to the effects of numerical viscosity. However, the second-order accuracy of the code implies that numerical viscosity will be very small compared to the other forces present.

In order to adapt the FDH code to the problem of gaseous galaxy disks, some minor modifications were made to the version described by Woodward, Tohline, and Hachisu (1994). Since the current work ignores self-gravity, the potential specified in the equation of motion is not determined by a solution to Poisson's equation but may be simply specified a priori. Richstone (1980) has described an axisymmetric logarithmic potential of the form

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which accurately reproduces the flat rotation curves observed in spiral galaxies. The parameter \( q \) is the axis ratio of the equipotential surfaces, so \( q = 1 \) represents a spherically symmetric potential while \( 0 < q < 1 \) for an oblate spheroidal potential.

Since simpler models of galaxy disks in the past have often been described in terms of tilted ring models, an output routine was added to the FDH code to display the evolving structure of the disk in this manner. In this routine, the computational grid is divided into a large number of concentric spherical shells. For each shell, the current three-dimensional density distribution determined by the FDH code is analyzed to compute the inertia tensor, \( I \), of the mass within that shell, according to the prescription

\[
I_{i,j} = \sum_\alpha m_\alpha \left[ \delta_{ij} \sum_k x_{\alpha,k}^2 - x_{\alpha,i} x_{\alpha,j} \right],
\]

where \( \alpha \) enumerates the grid cells contained within a given shell, \( m_\alpha \) refers to the mass within each of those cells, and \( x_i \) refers to the \( i \)th coordinate of the cell center. When this tensor is diagonalized the resulting eigenvectors specify the orientation of the principal axes of the mass distribution. If we envision the mass distribution within each shell as very roughly planar (to a zeroth-order approximation), the largest of these vectors corresponds to the normal to that plane. Therefore, the orientation of this vector gives the orientation of the ring of material which lies within the current shell. The inertia tensor for each shell is computed several times during the evolution of the model and the values are written to disk. Afterward, another program

\[
\Phi = \frac{v_0^2}{2} \ln \left( x^2 + y^2 + \frac{z^2}{q^2} \right)
\]

(2.15)
is used to find the orientation of the eigenvectors as a function of time. This allows us to approximate the actual three-dimensional density distribution as a set of tilted rings. The time-dependent tilt and twist of each ring are then examined in order to see a simpler summary of how material is distributed throughout the disk.
CHAPTER 3

CREATION OF AXISYMMETRIC INITIAL MODELS

The FDH code described previously will solve the hydrodynamic equations which characterize the evolution of the system in time, but before that can be done, some initial state for the gas must be specified. As described in Chapter 1, it is to be expected that gas falling into the potential well of a galaxy halo will quickly form into a disk which may be inclined at an angle to the symmetry plane of the halo. The density and velocity distribution within that disk will then serve as the initial model to be input to the FDH code.

The structure chosen for the initial disk is that of a disk in hydrodynamic equilibrium in a spherically symmetric, logarithmic potential. Since a fluid disk can exist in equilibrium at any angle to the (arbitrary) equatorial plane of a spherically symmetric potential, such a potential seems a good choice for creating an initial structure that will be inclined to the equatorial plane of the true, non-spherical, potential. The spherically symmetric potential employed in the creation of the initial model has the same logarithmic form as the potential used in the FDH code, so both may be described by equation (2.15) with the obvious requirement that $q = 1$ for the spherically symmetric potential. The density and velocity distribution of the equilibrium disk in this potential represent a reasonable initial state for the gas. Since this initial configuration is not an equilibrium state in the oblate potential, the FDH code then models the time-dependent evolution of the system.
A disk that is in equilibrium in the potential described by equation (2.15) with \( q = 1 \) will be radially supported by both pressure and rotation. One may define a cylindrical coordinate system oriented so that the \( \z \)-axis is perpendicular to the disk. At times, it will be convenient to refer to the equivalent Cartesian system in which the disk lies in the \((x,y)\) plane. Since the orientation in a spherically symmetric potential is irrelevant, the Cartesian coordinates employed in this description may be taken to be the coordinates used in the definition of the potential in equation (2.15).

The gas cannot move on intersecting paths, since that would lead to collisions which would quickly dissipate energy and change the orbit of the gas. Instead, it follows closed circular orbits. Gas in the midplane of the disk moves in a circular orbit about the origin which is determined by the effects of rotation, pressure, and the gravitational potential. The forces associated with all of these effects are collinear on a fluid element that resides precisely in the midplane of the disk. Gas above (or below) the midplane however, does not orbit the origin. Instead, it follows a circular path about the \( z \)-axis at a constant height above (or below) the \((x,y)\) plane. The \( z \) component of the gravitational force is now balanced by the pressure gradient within the disk, so the circular motion of the gas may serve to support it against the component of gravitational force parallel to the \((x,y)\) plane.

A Fortran-77 program was written to determine the density and velocity distribution within the initial equilibrium disk necessary to maintain this equilibrium. The program was modeled after a program written by Andalib (1990) which was based upon the Hachisu Self-Consistent-Field technique.
(hereafter, HSCF). The HSCF algorithm has been described by Hachisu (1986). The HSCF technique allows one to model equilibrium structures of rotating, self-gravitating fluids, but self-gravity is not an important consideration for the present work. Omiting consideration of self-gravity greatly simplifies the task, eliminating the iterative solution method used in the HSCF technique and merely requiring a direct solution for the balance of forces at all points in the fluid.

Specifically, in equilibrium the equation of motion given earlier must reduce to the steady-state form

$$\nabla \cdot (\nu \rho \mathbf{v}) + \nabla P + \rho \nabla \Phi = 0. \quad (3.1)$$

For gas moving in circular orbits determined by a spherically symmetric potential, the first term reduces to the simple $v^2/R$ centripetal term for circular motion. Writing the velocity in terms of an angular velocity, $\Omega$, one may write the radial component of equation (2.4) in integral form as

$$- \int \Omega^2 R dR + \int \rho^{-1} dP + \Phi = C, \quad (3.2)$$

where $C$ is a constant. The polytropic equation of state

$$P = K \rho^{1+1/n} \quad (3.3)$$

allows one to write the second term as

$$\int \rho^{-1} dP = (1 + n) \frac{P}{\rho} \quad (3.4)$$

which is the enthalpy, $H$. The density therefore may be determined trivially from the enthalpy and the polytropic relation. Representing the second term of equation (3.2) by $H$, the equilibrium condition may be restated as
\[ H = C - \Phi + \int \Omega^2 R \, dR. \] (3.5)

Since the form of the potential is given, the equilibrium density distribution may be determined from equation (3.5) if the angular velocity \( \Omega \) is known. The assumption that the fluid moves in closed circular orbits about the \( z \)-axis implies that \( \Omega \) may depend only upon the distance \( R \) from the \( z \)-axis. The variation of \( \Omega \) with \( R \) indicates the degree to which the disk is supported by pressure, in addition to the support derived from rotational motion. A test particle in the potential described by equation (2.15), immune to the effects of pressure and travelling in a circular orbit about the origin, would move with a velocity \( v_0 \) at any radius \( R \). A fluid element located at the pressure maximum, \( R_c \), must move at this same velocity since the pressure gradient at this point is zero. Fluid elements at radii \( R < R_c \) feel a pressure gradient pushing toward the origin. This additional force requires the fluid to travel at an azimuthal velocity greater than that needed for a particle, free from the influence of pressure, to maintain circular orbit. Similarly, fluid elements at radii \( R > R_c \) feel a pressure gradient directed outward that counters some portion of the gravitational force and allows the fluid to orbit at a velocity somewhat less than that of a particle. The degree of pressure support in a given initial configuration may therefore be described by specifying a rotation law for the gas of the form

\[ \Omega = \Omega_0 \left[ \frac{R}{R_{\text{OUT}}} \right]^m, \] (3.6)

where \( \Omega_0 \) and \( m \) are constants and \( R_{\text{OUT}} \) indicates the outer edge of the disk. In the chosen potential, the case \( m = -1 \) corresponds to a complete lack of
pressure support, for which each disk particle would move at the constant linear velocity $v_0$ at every radius $R$ and the disk would have zero vertical thickness.

The value of $m$ determines the degree of pressure support and, hence, the thickness of the disk. As noted earlier, the H I disks of galaxies are quite thin, with $z/R_{\text{OUT}} \sim 0.01$. The value of $m$ chosen for the initial rotation law must reflect this. Most of the models in this work were constructed using $m = -1.02$, which gives typical values of $z/R_{\text{OUT}} \approx 0.05$. It is difficult to model disks much thinner than this due to the resolution of the computational grid.

Once the rotation law index, $m$, has been specified, equation (3.5) may be solved for the density distribution if the constants $C$ and $\Omega_0$ are known. These constants are determined by the boundary conditions. The density, and hence the enthalpy, must go to zero at the inner and outer edges of the disk. The inner and outer radii, $R_{\text{IN}}$ and $R_{\text{OUT}}$, are the remaining free parameters of the model. When they are specified, evaluating equation (3.5) at these two points yields the two desired constants and equation (3.5) may then be used to determine the equilibrium density at all locations from the specified potential.
RESULTS OF AXISYMMETRIC INITIAL MODELS

The FDH code was used to follow the evolution of models for a variety of initial conditions. Specific values of halo distortion, $q$, and initial inclination $i_0$, are shown in Table 4.1. The other quantities in the table will be defined later.

Axisymmetric initial models created by the HSCF code described in the previous chapter were placed onto the computational grid of the FDH code with their axis of rotation aligned with the $z$-axis of the FDH grid. In each case, the equatorial plane of the halo was therefore inclined at an angle, $i_0$, to the equatorial plane of the FDH grid. Each combination of $q$ and $i_0$ represents a different initial model. Since all of the models exhibited qualitatively similar behavior, the evolution of one model, which may be viewed as a typical case, will be described in detail. Similar results for the other models are summarized in Table 4.1.

Figure 4.1 shows the evolution of Model 3, for which $q = 0.8$ and $i_0 = 20^\circ$. A cross section of the three-dimensional density is shown in a plane perpendicular to the line of nodes defined by the intersection of the initial disk with the halo equator. The plane of the halo equator is marked by the solid line. The times indicated in the figure are in units of orbital periods at the outer edge of the disk.

The disk settles rapidly toward the equator, with the inner regions settling most quickly. Surprisingly, the disk stops settling before reaching the
Figure 4.1: Evolution of an initially axisymmetric model (Model 3). Isodensity contours in the plane normal to the line of nodes are shown at levels from 10% to 90% of the maximum density. Times are in units of rotation periods at the outer edge. The solid line indicates the halo equator while the dashed line indicates the plane inclined to the equatorial plane of the halo at an angle, $a$, predicted by equation (5.5).
equatorial plane and remains at an inclination of about 13°. At each radius, the disk material reaches this inclination in less than the time required for one orbit at that radius. The last frame of Figure 4.1 shows that the simulation continued for five orbits at the outer edge of the disk. Since the orbital velocity is nearly constant with radius in this potential, the orbital frequency goes as $1/R$ and this time corresponds to about fifty orbits of the inner region. It appears that the disk can remain in a quasi-equilibrium state in this orientation for a time that is not only long compared to an orbital period, but long compared to the expected precessional timescale as well.

Steiman-Cameron and Durisen (1988) described how the precessional timescale of a particle in an orbit inclined at an angle $i_0$ to the equatorial plane of a logarithmic potential may be estimated. One begins by expanding the expression for the potential given by equation (2.15) in spherical harmonics and dropping terms of higher order than quadrupole. Averaging this precession over a circular ring of radius $R$ gives an estimate of the ring's precession frequency at $R$ as

$$\Omega_p = -\frac{3}{4} \frac{v_0}{R} \eta \cos i_0,$$

(4.1)

where the constant $\eta$ is given by

$$\eta = \frac{2 - 2q^2}{1 + 2q^2}.$$

(4.2)

The ratio of the precession frequency to the orbital frequency, $\Omega = v_0 / R$, is listed in Table 4.1 for each model. A halo distortion of $q = 0.80$ and inclination of $i_0 = 20°$ implies a precessional period that is 4.5 times the orbital period. Since most of the disk remains at an inclination of 13° for several
precessional periods, it appears that differential precession is not acting to drive settling from this configuration.

To obtain a more quantitative picture of the settling process, the data were examined in a form similar to that of the tilted ring models described earlier. The orientation of a series of planes corresponding to the distribution of matter within a series of concentric spherical shells was determined by analyzing the moment of inertia as described in chapter 2. The resulting polar angles specifying the normal to each plane may be considered to specify a tilt and twist angle for a ring of matter lying in that plane.

Figure 4.2 shows a series of tilt and twist angles for twenty different radii. As the time labels in Figure 4.2 indicate, the first three frames show data measured at the same time as the corresponding density evolution shown in Figure 4.1 previously. The final frame reflects the orientation of the disk at \( t = 0.89 \) orbits, which is earlier than the time depicted in Figure 4.1d, but nevertheless shows that the disk has reached its final inclination of 13°. The earlier time was chosen for the final frame to facilitate comparison with Figure 4.3, which will be described below. The plot of tilt angle vs. radius clearly shows that the disk settles to a plane inclined at about 13° to the equator, while the line of nodes appears relatively straight. Henceforth, we will refer to this plane which contains the maximum density of the evolved disk as the preferred plane.

Since it appears that the plane of the halo equator is not the preferred plane for this disk, it is instructive to repeat the above procedure but now to measure the tilt and twist angles of each ring’s symmetry axis relative
Figure 4.2: Orientation of the disk for Model 3, relative to the halo equator as function of radius. The tilt angle is plotted in the left-hand column and the twist angle in the right-hand column. The times correspond to those of Fig. 4.3.
to the preferred plane instead of the halo equator. Figure 4.3 shows that this choice of reference plane makes the behavior of the system much clearer. The material in the disk settles toward the preferred plane and precesses as it settles. Material in the preferred plane has a tilt angle of zero, so the twist is then undefined. For this reason, the final frames of this figure and Figure 4.2 are plotted at an earlier time than the final frame of Figure 4.1.

The plots shown in Figure 4.2 and Figure 4.3, are reminiscent of the plots shown by Briggs (1990) in his analysis of observations of 12 galaxies. Fitting observed velocities to tilted ring models, his plots of the tilt and twist of the rings relative to the innermost region of the disk appear similar to the plots relative to the preferred plane, shown in Figure 4. Briggs also plots the tilt and twist of rings relative to another plane which he chooses in an attempt to make the line of nodes of both the inner and outer regions as straight as possible, although at a different angle for each of the two regions. Briggs comments that the physical significance of this reference plane is unclear, and later points out that the apparent straightness of the line of nodes relative to this plane may simply be a consequence of choosing to measure the twist from a reference point which makes some portions of the spiral line of nodes lie along two line segments (Briggs and Kamphuis 1991). Nevertheless, Briggs' work shows how a tilted ring model may appear when referred to a plane that is not the proper reference plane. His plots relative to this plane look quite similar to the plots relative to the halo equator shown in Figure (4.2).

The important result is that the halo equator is not the equilibrium plane for a settling gas disk.
Figure 4.3: Orientation of the disk relative to the preferred plane as a function of radius. Tilt and twist angles are again shown for Model 3, but the angles are now measured relative to the preferred plane represented by the dashed line in Fig. 4.1. Times correspond to those of Fig. 4.2.
Although the disk stays at the preferred inclination of 13°, that orientation is not a true steady-state in the sense that the density distribution remains unchanged with time. The temporal evolution of the disk is best seen when an isodensity surface of the disk is rendered at several times during the evolution and the images are combined into an animation. Such animations reveal that even after the disk settles to the preferred plane, the surface of the disk undulates in a wave-like motion as small ripples move across the disk. The ripples propagate radially outward, travelling from the inner edge of the disk to the outer edge in about the time required for an orbit at the outer edge. The ripples do not appear to be reflected at the edge of the disk, but the pattern continuously repeats with new ripples appearing at the inner edge. The rippling motion shows no sign of damping out on the timescales modeled in these simulations.

More insight may be gained into the behavior of the disk by examining slices of the density and velocity in the preferred plane. Figure 4.4 shows a contour plot of the density in the plane of the dashed line in Figure 4.1, i.e. the preferred plane of maximum density lying at an angle of 13° to the halo equator. Figure 4.4 illustrates the distribution of density after 2.7 orbits at the outer edge of the disk, so it reflects the condition of the disk after it has settled to its final orientation. The highest density regions show a faint spiral character. This spiral pattern is clearly evident in the disk as it settles but gradually diminishes over one or two orbital periods (measured at the outer radius of the disk) after the disk reaches the preferred plane. Figure 4.5 shows a series of plots of density (relative to the maximum density of the
initial model) as a function of azimuthal angle, $\phi$, measured around circles of successively larger radii in the preferred plane. The plots clearly show the azimuthal variation of the density and illustrate as well how the spiral structure causes the relative density maximum to shift its azimuthal location with radius. Note that in some regions, the maximum density reaches values greater than the maximum found in the initial model.

Figure 4.6 shows a similar plot of the radial velocity, $\dot{R}$, as a function of azimuth in the preferred plane. The $\sin 2\phi$ dependence of the velocity indicates that the gas is moving not in circular orbits (which should have $\dot{R} \sim 0$) but in elliptical orbits. Unlike the density variation shown in Figure 4.5, curves of $\dot{R}(\phi)$ appear to have little azimuthal phase shift, indicating that the elliptical orbits are aligned at all radii. The azimuthal velocity in all cases is very close to a value of 1.0, so the plots indicate that the radial velocity is
Figure 4.5: Density as a function of azimuth at several radii in the preferred plane. The density has been scaled relative to the maximum density of the initial model, $\rho_{HSCF}$. The vertical lines mark the location of the line of nodes at azimuths of $0$ and $\pi$. 

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Figure 4.6: Radial velocity as a function of azimuth at several radii in the preferred plane.
typically a few percent of the azimuthal velocity. Thus, the orbits of the gas are characterized by low eccentricities, but they exhibit a clear indication of elliptical shapes.

Although the disk remains in the preferred plane after the initial settling, the rippling motion described earlier indicates that the velocity at any instant may not lie entirely within that plane. Figure 4.7 confirms this. In this figure, the component of the velocity perpendicular to the preferred plane is plotted as a function of azimuth. Although the variation in the velocity is small, and complicated by the scatter arising from sampling points at slightly different heights above or below the plane, there is a clear indication of a systematic variation in the vertical component of velocity with azimuth. When the same quantities are plotted at a later time however, the vertical velocities in corresponding frames appear quite different. Figure 4.8 plots the velocities normal to the disk at the same locations as the previous figure, but at a time more than half an orbital period later (as measured at the outer edge of the disk). The small velocities normal to the settled plane of the disk therefore are not indicative of a steady-state velocity pattern, but exhibit continuous changes with time. These changing velocities are probably associated with the rippling motions seen in animations of the three-dimensional isodensity surfaces.
Figure 4.7: Vertical velocity at $t = 2.7$ as a function of azimuth at several radii in the preferred plane.
Figure 4.8: Vertical velocity at $t = 3.3$ as a function of azimuth at several radii in the preferred plane.
Table 4.1: Models evolved from initially axisymmetric disks.

<table>
<thead>
<tr>
<th>Name</th>
<th>$q$</th>
<th>$i_0$</th>
<th>$a_{\text{SIM}}$</th>
<th>$a$</th>
<th>$\Omega_p/\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0.75</td>
<td>20°</td>
<td>11.4° ± 0.4°</td>
<td>11.5°</td>
<td>0.291</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.80</td>
<td>10°</td>
<td>6.2° ± 0.4°</td>
<td>6.4°</td>
<td>0.233</td>
</tr>
<tr>
<td>Model 3</td>
<td>0.80</td>
<td>20°</td>
<td>13.2° ± 0.8°</td>
<td>13.1°</td>
<td>0.223</td>
</tr>
<tr>
<td>Model 4</td>
<td>0.80</td>
<td>30°</td>
<td>20° ± 1.7°</td>
<td>20.3°</td>
<td>0.205</td>
</tr>
<tr>
<td>Model 5</td>
<td>0.80</td>
<td>40°</td>
<td>28° ± 1.0°</td>
<td>28.2°</td>
<td>0.181</td>
</tr>
<tr>
<td>Model 6</td>
<td>0.90</td>
<td>10°</td>
<td>7.9° ± 0.4°</td>
<td>8.1°</td>
<td>0.107</td>
</tr>
<tr>
<td>Model 7</td>
<td>0.90</td>
<td>20°</td>
<td>16.5° ± 0.5°</td>
<td>16.4°</td>
<td>0.102</td>
</tr>
<tr>
<td>Model 8</td>
<td>0.90</td>
<td>30°</td>
<td>25.0° ± 0.6°</td>
<td>25.1°</td>
<td>0.0942</td>
</tr>
<tr>
<td>Model 9</td>
<td>0.95</td>
<td>20°</td>
<td>18.2° ± 0.6°</td>
<td>18.2°</td>
<td>0.0490</td>
</tr>
</tbody>
</table>
CHAPTER 5

ANALYSIS OF THE SIMULATIONS

The results described in the previous chapter are quite surprising and, at first glance, seem contrary to expected physical principles. In trying to understand in approximate analytical terms how the disk could remain in a plane other than the equatorial plane of the halo, it is instructive to consider the analogy of a rotationally flattened gas disk in a spherically symmetric potential. The solution to this problem was described in chapter 3 as the means for constructing initial models.

As described in chapter 3, the angular velocity of a gaseous disk in a spherically symmetric potential depends only upon the distance, $R$, from the symmetry axis. In terms of a cylindrical coordinate system which has the $z$-axis aligned with the rotation axis of the disk, every fluid element of the gas moves in plane parallel layers at constant $z$, supported vertically by pressure. The problem of a gaseous disk in a spherically symmetric potential is inherently two-dimensional (the density will be a function only of the cylindrical coordinates $R$ and $z$), but the symmetry of the gas flow couples the $R$ and $z$ dependence in such a way that only one component of the vector equation of motion need be solved explicitly. Once equation (3.5) has been used to determine the radial run of density at a given height, $z$, explicit treatment of the vertical pressure balance for other values of $z$ is not required; the $z$-dependence of the potential on the right-hand side of the equation determines the $z$-dependence of the enthalpy (and therefore
the density) on the left-hand side. Since the vertical pressure force exactly balances the $z$-component of the potential gradient everywhere, the net force on any fluid element has no $z$-component and each fluid element will continue to move in a plane of constant $z$.

A similar approach of considering separately the effects of motion in each dimension may be profitably employed in the case of a disk in an oblate spheroidal potential. As a first approximation to understanding the preferred orientation of the disk, it is convenient to assume that there are no ordered vertical motions and demand that the pressure gradient balance the component of the gravitational potential gradient parallel to the axis of rotation. In the plane perpendicular to the axis of rotation, pressure gradients will be neglected and it will be assumed that the disk is supported radially by motion in closed orbits in the prescribed external potential.

Specifically, then, defining a primed coordinate frame in which the $x'$-axis is aligned with the $x$-axis of the halo potential but in which the $z'$-axis is tipped at an angle $i_0$ to the $z$-axis of the halo, the axisymmetric potential described in the unprimed (halo) coordinate system by equation (2.15) assumes the form

$$\Phi(x', y', z') = \frac{v_0^2}{2} \ln \left[ x'^2 + (y' \cos i_0 - z' \sin i_0)^2 + \frac{1}{q^2} (y' \sin i_0 + z' \cos i_0)^2 \right]. \tag{5.1}$$

The assumption that the pressure gradient in the $z'$ direction balances the gradient in the gravitational potential implies

$$\frac{\partial \Phi}{\partial z'} = -\frac{1}{\rho} \frac{\partial P}{\partial z'} \tag{5.2}.$$
In particular, at each \((x', y')\) location in the disk, the pressure maximum (and the density maximum) must reside at the \(z'\) position where \(\partial \Phi / \partial z' = 0\).

If the halo is spherically symmetric (i.e. for \(q = 1\)), at any angle of inclination \(i_0\) the solution to this expression will demand that the density maximum reside in the \(z' = 0\) plane and that the pressure distribution be symmetric about that plane. (The initial disk structure shown in Figure 4.1a has this property.) However, for any \(0 < q < 1\), this constraint requires that the density maximum be displaced a distance \(z'_0\) above (or below) the \(z' = 0\) plane. Specifically, the location of the density maximum may be determined by setting the right-hand side of equation (5.2) to zero and evaluating the resulting expression, which shows that \(\partial \Phi / \partial z' = 0\) at

\[
z'_0 = \frac{q^2 - 1}{1 + q^2 \tan^2 i_0} y' \tan i_0. \tag{5.3}
\]

One can readily see that this expression defines a plane whose normal is tipped about the \(x'\)-axis at an angle \(b\) to the \(z'\) axis defined by

\[
\tan b = \frac{q^2 - 1}{1 + q^2 \tan^2 i_0} \tan i_0, \tag{5.4}
\]

or, referencing back to the unprimed coordinate system of the halo potential, tipped about the \(x\)-axis at an angle

\[
a = \arctan \left[q^2 \tan i_0 \right], \tag{5.5}
\]

to the \(z\)-axis. It is important to emphasize that this predicted value of the angle \(a\) follows directly from the assumption of equation (5.2) and is not explicitly implemented in the FDH code. For Model 3, this expression gives \(a = 13.1^\circ\), which is essentially identical to the angle \((a_{\text{SIM}} = 13.2^\circ \pm 0.8^\circ)\) to.
which the disk was observed to settle in the simulation. Figure 5.5 plots the predicted settling angle $\theta$ versus the angle $\alpha_{\text{SIM}}$ observed in several different simulations. The value of $\alpha_{\text{SIM}}$ for each simulation was determined by examining plots of the inclination, $i$, as a function of radius such as Figure 4.2a and averaging $i$ over radius. The average inclination was computed after the disk had ceased settling and remained at essentially the same orientation for several orbits. Error bars in the figure represent one standard deviation of angle with radius due to the rippling waves propagating throughout the disk. The close agreement between simulation results and this prediction suggests strongly that the initial disk settling is primarily a dynamical effect aimed at bringing the disk material into vertical pressure balance. Secondary effects such as the rippling motion of the disk may modify this simple treatment slightly, but they do not change the fact that the final inclination of the disk is predicted quite accurately by equation (5.5).

The excellent agreement of the results with the prediction of equation (5.5) suggests that gas pressure is primarily responsible for the support of the disk perpendicular to the $z' = 0$ plane. As mentioned above, however, orbital motion is primarily responsible for radial support of the disk parallel to the $z' = 0$ plane. In the plane lying orthogonal to the $z'$-axis, a steady-state flow demands that the fluid move along closed orbits in order to avoid collisions. Fluid confined to the plane $z' = 0$ will feel the effects of the three-dimensional, axisymmetric potential as a two-dimensional, non-axisymmetric potential in that plane. Setting $z' = 0$ in equation (5.1) shows that the form of the relevant two-dimensional potential is given by
Figure 5.1: Final inclination of disk predicted from equation (5.5) vs. the inclination observed in simulations.
\[ \Phi(x', y') = \frac{v_0^2}{2} \ln \left[ x'^2 + \frac{y'^2}{q'^2} \right], \quad (5.6) \]

where \( q'^2 = (\cos^2 i_0 + 1/q^2\sin^2 i_0)^{-1} \). If \( q' = q = 1 \), the equipotential lines are concentric circles and the desired closed orbits are circular. Hence, for a spherically symmetric halo potential a steady-state gas disk will be composed of fluid elements moving along nonintersecting, circular orbits. However, when \( 0 < q < 1 \), equation (5.6) has the form of a two-dimensional bar potential. There are no circular orbits in a two-dimensional bar potential (cf. Binney and Tremaine 1987). If \( q' \approx 1 \) however, one can derive an analytic expression for closed elliptical orbits in the plane. It seems reasonable to suggest, therefore, that in the plane perpendicular to its rotation axis, an axisymmetric gas disk that is placed in the three-dimensional halo potential defined by equation (2.15) will have to undergo dynamical readjustment, aligning its fluid orbits into radially nested elliptical orbits if it is to achieve a steady-state configuration. The analysis of the velocity flowfields of the settled disks described in chapter 4 suggest that indeed the gas is flowing in approximately elliptical orbits.
CHAPTER 6

NONAXISYMMETRIC INITIAL MODELS

The close agreement between the observed settling angles of the models and the angles predicted by the analysis of the previous chapter suggests that vertical pressure balance is the primary effect determining the equilibrium configuration of the disk. However, the analytic description outlined in the previous chapter is not a rigorous calculation of the motion. It correctly predicts the final inclination of the disk, but it is not completely self-consistent. Equation (5.2) expressed the assumption that the pressure gradient within the disk is the only force acting to counter the gravitational force in the $z'$ direction, but the result that the disk settles to a final orientation at an angle, $\alpha$, corresponding to a plane other than the $z' = 0$ plane, indicates that the orbital motion of the fluid has a small component in the $z'$ direction. Since the disk does not lie within the $z' = 0$ plane, there must be some acceleration normal to that plane.

To further test the hypothesis that the final orientation of the disk is primarily determined by vertical pressure balance, it was necessary to modify both the HSCF code and the FDH code to determine if a different class of initial model would exhibit the same behavior when computed using the modified codes. Since fluid disks in initially circular orbits inclined at an angle, $i_0$, to the halo equator were observed to settle to elliptical orbits inclined at an angle, $\alpha$, to the equator, it is reasonable to expect that initial models composed of fluid in elliptical orbits inclined at the angle, $\alpha$, to the
equator should remain in that orientation if the codes are giving a physically meaningful result.

6.1 Creation of Nonaxisymmetric Initial Models

The key to constructing such modified initial models is the assumption made in chapter 5 that the problem may be separated into two-dimensional motion within a plane inclined to the halo equator and the requirement that the pressure distribution within the disk balance the gravitational force normal to that plane. As in chapter 5, the problem may be described in terms of an unprimed coordinate system in which the z-axis is defined to be the symmetry axis of the halo and in terms of a primed coordinate system defined such that the z'-axis coincides with the angular momentum vector of the initial disk, which is inclined to the halo symmetry axis at an angle $i_0$. Since the disk will settle from this initial orientation to the preferred plane which lies at an angle, $\alpha$, to the equator, it will sometimes be useful to define a third, double-primed coordinate system in which the z''-axis is normal to the final preferred plane.

The computational grid of the FDH code corresponds to the primed coordinate system. Since it is much easier to compute the pressure gradient in this coordinate system than in the double-primed system, the improved initial models were based upon the requirement that they satisfy the condition for pressure balance in the $z'$ direction, i.e. they must satisfy equation (5.2). Similarly, equation (5.6) is most easily employed to describe the two-dimensional potential in the $(x', y')$ plane. Motion in the $(x'', y'')$ plane may then be described as a combination of motion in the $(x', y')$ plane plus a
periodic oscillation in the \( z' \) direction which result in a path confined to the \((x'',y'')\) preferred plane.

As discussed in chapter 5, it is reasonable in a first approximation to assume that all of the vertical support is due to pressure, while neglecting any radial pressure gradients within the disk. This means that the orbits of fluid elements in the \((x',y')\) plane may be approximated by particle orbits in the same potential. Determination of the particle orbits in the \((x',y')\) plane is the first step in creating nonaxisymmetric initial models which are closer to the true equilibrium state than the axisymmetric models previously employed. The procedure for creating such models is described in detail below, but it may be summarized as follows:

1. Closed elliptical orbits in the \( z' = 0 \) plane were computed for test particles.
2. The orbital velocities of the test particles at every point in the \( z' = 0 \) plane were assumed to represent the velocities of fluid at the same points in that plane.
3. The surface density of the disk, \( \sigma(x',y') \), was computed from the known velocities and the steady-state equation of continuity, \( \nabla \cdot (\sigma \mathbf{v}) = 0 \).
4. Equation (5.2) was used to determine the three-dimensional density distribution, \( \rho(x',y',z') \), from the surface density \( \sigma \).
5. A periodic oscillation in \( z' \) was added to the velocities in order to limit the motion of the fluid to the \( z'' = 0 \) plane.
Once these nonaxisymmetric initial models were created, the FDH code was modified to read in the three-dimensional distributions of density and velocity from the initial models and the models were evolved in the same manner as the axisymmetric initial models previously used.

Equation (5.6) describes the appearance of the potential in the plane \( z' = 0 \). Gas moving in this plane must follow closed, non-precessing orbits to avoid collisions between streams of gas. When \( q' \approx 1 \), it is possible to determine that the only closed orbits in this potential are nested ellipses (cf. Binney and Tremaine 1987) but there is no analytical solution to the problem of an orbit in this potential for arbitrary values of \( q' \). Therefore, it was necessary to determine the closed orbits numerically.

A useful tool for identifying closed orbits is the surface of section method first described by Poincaré (1892). The physical motion of a particle in the \((x', y')\) plane may be described as a trajectory in a four-dimensional phase space defined by the coordinates \( x', y' \), and the velocities \( \dot{x}' \) and \( \dot{y}' \). In general, the trajectory of an orbit will eventually fill a finite four-dimensional volume in phase space. Since the energy of a given orbit must be conserved, one may choose to eliminate one of the variables, e.g. \( x' \), and consider the motion in a three-dimensional phase space of \( y', \dot{x}' \), and \( \dot{y}' \), with the value of \( x' \) determined by the other three coordinates and the constant energy, \( E \). To further reduce the difficulty in visualizing the nature of the orbit, one may consider only the motion of the particle at the moment it crosses the \( x' \)-axis. At that time, the region of phase space available to the particle is a surface formed by the intersection of the three-dimensional phase space volume with
the surface \( y' = 0 \). The total energy of the orbit determines a curve that bounds the \((x', y')\) surface accessible to the given orbit. The particle may have any velocities within the bounded \((x', y')\) surface at different times when it crosses the \(x'\)-axis. If the motion of the particle is such that some other quantity is conserved in addition to the energy, the points generated by that motion will not fill the bounded \((x', y')\) surface, but must lie along a curve within that surface. Most realistic galactic potentials, including the potential of equation (2.15), do indeed conserve this quantity, known as the third integral of motion (cf. Binney and Tremaine 1987). Therefore, the surface of section for an orbit in this potential consists of a closed curve in the \((x', y')\) plane, formed by plotting the value of these two velocities each time the particle crosses the \(x'\)-axis. Since the particle crosses the \(x'\)-axis twice in each orbital period (once with \( y' > 0 \) and once with \( y' < 0 \)), two identical curves are created; one for positive values of \( y' \) and one for negative values. Figure 6.1 shows one of these curves (for \( y' > 0 \)) for a typical orbit.

In the special case of a closed orbit, the particle should cross the \(x'\)-axis (going in the positive \( y' \) direction) at the same location each time and should have the same velocity every time it returns to the same \((x', y')\) location, so the curve in the surface of section should reduce to a single point which specifies the unique values of \( x' \) and \( y' \) at the point \((x'_0, 0)\).

An automated procedure was developed to locate this point in phase space which identifies the velocities required for the existence of a closed orbit. The procedure began by assuming an initial velocity of \( x' = 0 \) and \( y' = v_0 \) in the \( y' > 0 \) direction for test particles crossing the \(x'\)-axis at each point.
corresponding to a cell on the $x'$-axis of the computational grid. A trial orbit was then computed for each of these particles using a fourth-order Runge-Kutta algorithm, and each particle was followed for several orbital periods. Each time the particle crossed the $x'$-axis, the location of the crossing, $x'$, and the velocity, $y'$ were noted. After several dozen crossings, an improved estimate for the initial velocity $y'_0$ at the point $(x'_0, 0)$ was then determined by setting $y'_0$ equal to the midrange of $y'$ values observed on previous crossings. Equivalently, one might envision plotting a surface of section in the $(x', y')$ plane and estimating the location of the point corresponding to a closed orbit by picking a point halfway between the greatest and smallest values of $y'$. The new estimate for the initial velocity was then used to compute another trial orbit which had less variation in position and velocity than the previous orbit. The procedure was repeated until the variation in position at which
the particle crossed the $x'$-axis was less than one tenth the width of the cells used in the computational grid of the FDH code. Although very nearly circular (to within a few percent), the closed orbits are slightly elongated in the $y'$ direction.

Once the closed orbits for a test particle in the two-dimensional potential of equation (5.6) were determined, the velocities associated with those orbits were assumed to apply to the fluid at every point in the disk. The assumption that the fluid moves in plane-parallel layers meant that the fluid flow could be treated as a two-dimensional problem, since velocity was assumed to be independent of $z'$. The two-dimensional structure of the disk was represented by a surface density, $\sigma(x', y')$ representing the integral of the three-dimensional density, $\rho(x', y', z')$, integrated in the $z'$ direction.

The value of $\sigma(x', y')$ could be estimated along the $x'$-axis, which is the line of nodes. Along this line, there is no component of the gravitational force normal to the disk, i.e. the gravitational force is purely radial as in the case of the axisymmetric models constructed with the HSCF code described in chapter 3. The HSCF code was therefore used to calculate the radial variation of density along the line of nodes and the density at each grid cell along the $x'$-axis was integrated in the $z'$ direction to establish a reasonable estimate for the the surface density $\sigma(x', 0)$ along the line of nodes.

To determine the value of $\sigma(x', y')$ everywhere in the two-dimensional disk, one must solve the steady-state continuity equation,

$$\nabla \cdot (\sigma \mathbf{v}) = 0.$$  \hspace{1cm} (6.1)

A finite difference approximation to this solution may be obtained by simply
requiring that the sum of the mass flux through all the faces of a given cell be zero. Unlike the calculation of the advection in the FDH code which was described in chapter 2, in which the right-hand side of the continuity equation represented the variation of density with time in a dynamical model, the equilibrium problem requires a time-independent solution for which the right-hand side of equation (6.1) is always zero. The simplest method is to use a first-order, donor cell method in which the flux into a cell is computed from the density and velocity in the adjacent cell (upstream in the sense of the direction of velocity flow in each coordinate) and the flux out of the cell is computed from the values of density and velocity associated with the given cell (the signs are reversed if the velocity is in the opposite direction). Since the current equilibrium code is only intended to create models that are a closer approximation to equilibrium than axisymmetric models, the donor cell method should suffice, given the other approximations already included in the equilibrium model. Therefore, the donor cell method was employed to solve equation (6.1) for $\sigma(x',y')$ on a two-dimensional cylindrical grid.

Figure 6.1 illustrates the geometry of the cylindrical grid in which the index $j$ labels the radial coordinate, $R$, and $l$ labels the azimuthal coordinate, $\phi$. It is clear from Figure 6.1 that equation (6.1) may be restated in finite difference form as

$$
\sigma_{j-1,l} \dot{R}_{j-1,l} R_{j,l} \Delta \phi - \sigma_{j,l} \dot{R}_{j,l} R_{j+1,l} \Delta \phi + \sigma_{j,l-1} R_{j,l-1} \dot{\phi}_{j,l-1} \Delta R - \sigma_{j,l} R_{j,l} \dot{\phi}_{j,l} \Delta R = 0,
$$

(6.2)

where the first two terms describe the net radial flux into the $(j,l)$ cell and the second two terms describe the net flux in the azimuthal direction.
To solve equation (6.2) simultaneously for all locations on the grid one may write the finite-difference equation in matrix form as

$$A\sigma = 0$$  \hspace{1cm} (6.3)

where $A$ represents the matrix of coefficients in equation (6.2). The solution of equation 6.3 is complicated by the fact that the shape of the elliptical disk does not match the circular grid used for the computations, so the inner (or outer) edge of the disk does not lie at a constant value of $R_j$ (see Figure (6.2)). In order to solve equation (6.3), an iterative method was adopted in which each coordinate was treated separately and the two one-dimensional solutions iterated until they converged. First, the finite-difference formula of equation (6.2) was applied to every cell in a circle of fixed radius, $R_j$, starting at the inner edge of the disk which was represented by $R_{jm}$. The known value of $\sigma$ on the line of nodes serves as a boundary value for the
problem, and the surface density at \( R_{j+1} \) is known to be zero since that cell lie inside the inner edge of the disk. Thus all densities in cells at a radius of \( R_{j-1} \) are known (they are zero for the inner edge) and may be moved to the right-hand side of equation (6.2), allowing it to be written as

\[
\left( -\dot{R}_{j,l} - R_{j,l} \dot{\phi}_{j,l} \right) \sigma_{j,l} + R_{j,l-1} \dot{\phi}_{j,l-1} \sigma_{j,l-1} = f_{j,l}
\]  

(6.4)

where the known (j-1) terms have been collected into the constant \( f_{j,l} \). Equation (6.4) may be written concisely in matrix form as

\[
A \sigma = f .
\]  

(6.5)

The only non-zero elements of the coefficient matrix, \( A \), occur on the diagonal (the terms associated with the \([j, l]\) cell) and in a single band adjacent to the diagonal (the terms associated with the \([j, l-1]\) cell). Such a matrix may be easily solved by back-substitution, using the known boundary value at \( l = 1 \) to compute a solution at \( l = 2 \) and proceeding around the circle, computing the density at each azimuth from the previous cell. Once complete, all densities at that radius are considered as known quantities and the procedure repeated for the circle of cells at \( R_{j+1} \) and so on. After an azimuthal sweep has been completed for every radius within the disk, a complementary procedure is performed to solve a matrix of radial coefficients at constant azimuth. The result is an array of surface densities everywhere in the disk. Since the radial and azimuthal flows were treated separately, these densities are only approximations. The procedure is repeated, computing new densities from those determined by the previous iteration, until the solution converges. When the sum over the entire disk of the absolute values of the
differences at each location between the two iterations becomes less than ten percent of the total surface density summed over the entire disk, the solution is considered to have converged.

Once the surface density has been determined, equation (5.2) is used to determine the three-dimensional density distribution \( \rho(x', y', z') \) from the surface density. Equation (5.3) gives the \( z' \) coordinate of the maximum density that lies along each line of constant \((x', y')\). When equation (5.2) is integrated in \( z' \) (holding \( x' \) and \( y' \) constant) from the density maximum, \( \rho_{\text{max}} \) (located at \( z'_0 \)), to a height \( z' \) and combined with the polytropic equation of state, one finds that the density profile is given by the relation

\[
\rho^{1/n}(x', y', z') = \rho_{\text{max}}^{1/n} + \frac{1}{k(n+1)} [\Phi(x', y', z'_0) - \Phi(x', y', z')] .
\]  

(6.6)

The unknown value of the maximum density, \( \rho_{\text{max}} \) is related to the thickness of the disk. Since the density must go to zero at the surface of the disk, one may set the left-hand side of equation (6.6) to zero and obtain a relation between \( \rho_{\text{max}} \) and the coordinates of the lower surface of the disk, \((x', y', z'_1)\),

\[
\rho_{\text{max}}^{1/n} = \frac{1}{k(n+1)} [\Phi(x', y', z'_1) - \Phi(x', y', z'_0)] .
\]  

(6.7)

A similar relation follows for the upper surface of the disk located at \( z'_2 \). An iterative procedure was devised to compute the value of \( \rho_{\text{max}} \) from equation (6.7) and its counterpart for the upper surface. After making an initial guess for the value of \( z'_1 \) (and \( z'_2 \)), \( \rho_{\text{max}} \) was computed from equation (6.7) and in turn used to compute a trial density distribution, \( \rho(x', y', z') \) from equation (6.6). The trial density distribution was integrated in \( z' \) and the
resulting surface density compared to the value of $\sigma(x', y')$ determined earlier. If the two estimates for surface density at any given $(x', y')$ location differed by more than five percent, the thickness of the disk was incremented by increasing the values of $z_1'$ and $z_2'$ and a new density distribution was computed. The process was repeated until the new surface density converged to an acceptably close value to that determined from the two-dimensional code.

In order to increase accuracy, the increments in $z_1'$ and $z_2'$, as well as the integration in $z'$, were performed using a resolution in $z'$ at least ten times that of the FDH code and the final results were interpolated onto the grid of the FDH code.

Once a three-dimensional density distribution had been created based upon the assumptions outlined above, the only task remaining was to ensure that the velocity of the fluid corresponded to motion in the plane lying at an angle $a$ to the halo equator (the $z'' = 0$ plane). As described above, the calculation of the closed orbits was actually performed in the $z' = 0$ plane which lay at a small angle, $b$, to the desired plane. Although the orbits determined in the neighboring plane do not represent an exact solution to the problem in the $z'' = 0$ plane, they certainly satisfy the required criterion of representing a closer approximation to the true equilibrium condition than the circular orbits previously employed. To put the disk in the $z'' = 0$ plane, it was necessary to add a small periodic oscillation perpendicular to the $z' = 0$ plane. The addition of the term

$$v_{z'} = \frac{2v_0}{\pi} \tan b \cos \phi'$$

(6.8)

to the velocity added that oscillation. This ordered motion, which is inde-
pendent of \( z' \), describes the movement of the disk in the \( z'' = 0 \) plane in terms of the single-primed coordinate system.

6.2 Results of Nonaxisymmetric Models

To test the hypothesis that the vertical pressure balance condition described by equation (5.2) is responsible for the maintenance of the disk in the preferred plane, the methods described in the previous section were used to create nonaxisymmetric density and velocity distributions based upon that condition. These initial models were then evolved using the FDH code in the same manner as the axisymmetric initial models described in chapter 4. The results of one such simulation are shown in Figure 6.3. The model shown corresponds to Model 3, described in chapter 4 for which \( q = 0.80 \) and \( i_0 = 20^\circ \). The nonaxisymmetric equilibrium code predicts that such a disk should settle to a permanent inclination of 13° to the halo equator. This inclination is marked by the dashed line in Figure 6.3, which identifies this plane of the nonaxisymmetric initial model. Figure 6.3 shows that the disk readjusts itself somewhat about this plane as the evolution proceeds, but essentially maintains its initial orientation. This strongly suggests that the improved initial disk model with elliptical orbits and a revised vertical pressure distribution is a fairly accurate representation of the true equilibrium conditions.

A nonaxisymmetric initial model was created for each of the simulations described in chapter 4 and evolved in the same manner as the model shown in Figure (6.3). In every case, the disk remained at the inclination predicted by the nonaxisymmetric equilibrium code.
Figure 6.3: Evolution of a nonaxisymmetric initial model created using the code described in chapter 6. This model, which lies within the same potential as the model shown in Fig. 4, remains in the initial plane.
CHAPTER 7

CONCLUSION

The result of this work is quite unexpected. Previous models for galaxy disks as systems of self-gravitating particles, suggested that a disk initially inclined to the equatorial plane of the halo should settle to that plane. Only by modelling the disk as a three-dimensional fluid is it possible to see the effects of pressure gradients within the gas. Models based upon systems of particles could not predict that a disk would settle from an initial inclination to an angle where the gravitational force normal to the initial plane vanishes. Nevertheless, two strong pieces of evidence support this numerical result. First, the excellent agreement of the observed final inclination of the disk with that predicted by equation (5.5) indicates that the hypothesis of vertical pressure balance expressed in equation (5.2) must play a primary role in determining the final orientation of the disk. Second, the fact that the nonaxisymmetric initial models described in chapter 6 do not move to a new, intermediate inclination when evolved in the FDH code, but maintain their original orientation in the preferred plane strongly suggests that the elliptical orbits and vertical pressure distribution of the improved initial models do indeed represent a close approximation to an equilibrium condition. Once in the plane in which the gravitational forces perpendicular to the disk are balanced by pressure forces, the gas quickly establishes an equilibrium characterized by elliptical orbits in the preferred plane. The elliptical orbits arise because gas confined to the preferred plane feels a two-dimensional bar potential within

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that plane. The final nonaxisymmetric disk structure appears to be secularly as well as dynamically stable since the disk maintains its orientation on a timescale that is long compared to the expected precessional timescale for a particle.

The spiral character seen in Figure 4.4 may have important implications for the structure of spiral galaxies. Since orbital periods are longer at greater radii, material arms in a spiral should wind up on a timescale that is short compared to the age of the universe. Lin and Shu (1964) first proposed the density wave hypothesis to explain the continued existence of spiral arms. In their model, the spiral arms arise as a standing density wave in a self-gravitating system. Matter streams through the high density regions of the spiral arms but the location of the arms themselves remains relatively constant. To obtain a long-lived spiral density wave from an initially axisymmetric density distribution requires some initial nonaxisymmetric perturbation. While the models used in the current work are not self-gravitating, and hence cannot support standing density waves, the spiral enhancement of density could serve as a “seed” for the development of spiral density waves.

Since the pressure and density distributions in the final disk configuration are nonaxisymmetric (see, for example, Figure 4.5), fluid elements throughout the disk experience periodic compressions during each orbit. Such compressions could promote a steady rate of star formation throughout the disk. More importantly, however, any stars which form in such a disk would not be subject to the hydrodynamical forces which confine gas to the preferred plane. Stellar orbits should then precess about the symmetry axis of the potential.
As differential precession carries the stars out of the preferred plane, the stars should form a thicker stellar disk around the H I gas disk. Stars which formed from a gas disk inclined at an angle, \( a \), to the equatorial plane of the halo would, after precessing through a full 360°, form a cone of half-angle \((90° - a)\) about the symmetry axis of the halo.

Stellar disks are, in fact, observed to be much thicker than H I disks. While gas disks typically have thicknesses \( \sim 200 \text{ pc} \), stellar disks are often nearly five times as thick (Mihalas and Binney 1981). The stellar disks of S0 galaxies are much thicker, giving those galaxies a lenticular shape. The relative thickness of stellar disks has traditionally been ascribed to the scattering of disk stars by dense molecular clouds (Spitzer and Schwarzschild 1951). The observed velocities of stars perpendicular to the disk (velocity in the \( z'' \) direction in the notation of chapter 6) tend to agree quite well with the predictions of the mechanism proposed by Spitzer and Schwarzschild. Quantitative modelling of the precessional mechanism proposed above would therefore be necessary to determine if it could be consistent with observed velocities in the case where both mechanisms are active. In particular, the collisions described by Spitzer and Schwarzschild predict a \( z'' \) velocity dispersion that is a function of stellar age, which is in agreement with observation (Mihalas and Binney 1981). Since the \( z'' \) velocity due to precession is characterized by a systematic dependence on radius and the stellar age (the \( z'' \) velocity will depend upon the amount of time during which the star has precessed), constraints on the velocity might impose severe constraints on the halo distortion, \( q \).
The degree to which the stellar disk might be increased in thickness by precession out of the preferred plane of the gas depends upon two parameters: the precession frequency \( \Omega_p \) at a given radius, and the time, \( t_s \), during which that precession acts on each star. The value of the former is determined by the flatness of the halo and the angle, \( \alpha \), between the preferred plane of the gas and the equatorial plane of the halo, while the latter depends upon the stellar age. Individual stars old enough to have precessed far from the plane of their birth are generally not bright enough to be resolved in other galaxies, but the integrated light of the older stellar population will reveal the region of space filled by those stars. If the stars are indeed precessing out of the preferred plane of the gas, they should fill a volume similar to that of the cone described above, but modified by the effects of scattering as described by Spitzer and Schwarzschild.

Within the Milky Way, most studies of stellar distribution and kinematics of disk stars are limited to stars of spectral type A and earlier, since later types are generally not visible at distances greater than about 2 kpc (cf. Mihalas and Binney 1981). The short (\( t_s \sim 10^8 \) yr) lifetimes of these stars indicate that they could not precess very far from the plane of their birth. Table 4.1 shows that the ratio of the precessional period, \( \Omega_p \), to the orbital period for Model 3 is almost a factor of five.\(^1\) Since the orbital period in the solar neighborhood is about \( 2 \times 10^8 \) years, the resulting precessional period of \( 10^9 \) years indicates that an A star would only precess through

\(^1\)The precessional period of stars formed from Model 3 should actually be computed from the preferred plane, at \( \alpha = 13^\circ \), not the initial plane at \( i_0 = 20^\circ \), but since we have no \textit{a priori} reason to specify values for \( q \) and \( i_0 \), the numbers in Table 4.1 will suffice to illustrate the principle.
about 1/10 of a full cycle in its lifetime. Serious comparisons of the amount of precession with observed galactic structure would require constraints on the value of the halo flattening, $q$, possibly from models of the $z''$ velocity resulting from precessional motion. It is interesting to note, however, the existence of Gould’s Belt in the Milky Way as an example of a population of nearby B stars which lie in a plane inclined at about 16° to the plane of the disk.
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Paul Fisher was born in 1957 in Houston, Texas. He graduated from Texas A & M University in 1979 with a bachelor of science degree in physics. In 1982, he earned a master of science degree in physics from the New Mexico Institute of Mining and Technology. After working for several years as a telescope operator at the National Radio Astronomy Observatory Very Large Array near Socorro, New Mexico, he enrolled as a graduate student at Louisiana State University. He will receive the degree of Doctor of Philosophy in physics from Louisiana State University in May, 1999.
DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: Paul Fisher

Major Field: Physics

Title of Dissertation: Nonaxisymmetric Equilibrium Models for Gaseous Galaxy Disks

Approved:

[Signatures]

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

6 April 1998