1998

# Multilocation Inventory Systems With Centralized Information.

Jianxin Hu
*Louisiana State University and Agricultural & Mechanical College*

# MULTILOCATION INVENTORY SYSTEMS WITH CENTRALIZED INFORMATION

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Interdepartmental Program in Business Administration

by
Jianxin Hu
B.S., Zhejiang University, 1985
M.S., Louisiana State University, 1994
December 1998

UMI Number: 9909824

## ACKNOWLEDGEMENTS

I would like to express sincere gratitude to Dr. Helmut Schneider, Major Professor and Chairman of my doctoral program committee, for his guidance, support, and insight toward the entire research process. A special appreciation goes to Dr. Edward Watson, Co-Chairman of the doctoral committee, for his invaluable assistance, continuous encouragement, and able direction throughout the graduate program. I am also grateful to Dr. Peter Kelle, Dr. Linguo Gong, and Dr. Hong Zhang, who served on the committee and provided me with perceptive comments and suggestions for improving the manuscript.

Finally, I wish to express a deep feeling of gratitude to my family. Thanks are given to my parents for their understanding and encouragement. I also thank my children, Ivan and Allan, for providing me with much fun, joy, and hope. Most of all, I dedicate this work to my wife, Jing Gu, whose love, support, forbearance, and sacrifice made this possible. This achievement would have little meaning without her to share it with.

TABLE OF CONTENTS

ACKNOWLEDGMENTS ..... ii

LIST OF TABLES ..... v

LIST OF FIGURES ..... vi

ABSTRACT ..... vii

CHAPTER
1. INTRODUCTION ..... 1
   1.1 OVERVIEW ..... 1
   1.2 RESEARCH FOCUS ..... 4

2. LITERATURE REVIEW ..... 6
   2.1 BRIEF HISTORY ..... 6
   2.2 PERIODIC REVIEW STOCHASTIC DEMAND MODELS ..... 8
      2.2.1 CLASSICAL MULTI-ECHELON INVENTORY MODELS ..... 12
      2.2.2 CENTRALIZED PLANNING MODELS ..... 16
      2.2.3 DECENTRALIZED PLANNING MODELS ..... 21
   2.3 CONTINUOUS REVIEW STOCHASTIC DEMAND MODELS ..... 23
      2.3.1 ONE-FOR-ONE REPLENISHMENTS ..... 25
      2.3.2 (Q, r) ORDERING POLICIES ..... 28
   2.4 INVENTORY MODELS WITH TRANSSHIPMENTS ..... 30

3. INDIVIDUAL VERSUS CENTRALIZED ORDERING
   IN MULTILOCATION INVENTORY SYSTEMS ..... 37
   3.1 MOTIVATION AND BACKGROUND ..... 37
   3.2 INVENTORY MODELS ..... 43
      3.2.1 INDIVIDUAL ORDERING ..... 44
      3.2.2 CENTRALIZED ORDERING ..... 46
   3.3 COMPUTATIONAL RESULTS AND DISCUSSION ..... 51
      3.3.1 COMPUTATIONS ..... 51
      3.3.2 IMPORTANT ASSUMPTIONS ..... 53
      3.3.3 EXTENSIONS ..... 57
   3.4 CONCLUSION ..... 58

4. MULTILOCATION INVENTORY SYSTEMS
   WITH TRANSSHIPMENTS ..... 59
   4.1 PROBLEM DESCRIPTION ..... 59
   4.2 MODEL FORMULATION - BASE STOCK POLICY ..... 64
      4.2.1 GENERAL FORMULATION ..... 64
      4.2.2 SIMPLIFYING ASSUMPTIONS ..... 66
      4.2.3 SIMPLIFIED FORMULATION ..... 67
   4.3 MODEL FORMULATION - (*s*, *S*) POLICY ..... 70
   4 4 COMPUTATIONAL STUDY ..... 74

4.4.1 OPTIMAL VERSUS SIMPLIFIED ORDERING POLICIES............74
4.4.2 BASE STOCK VERSUS (s, S) ORDERING POLICIES...................79
4.5 EXTENSIONS AND CONCLUSIONS.....................................................83
4.5.1 EXTENSIONS.......................................................................83
4.5.2 ALLOCATION ASSUMPTION.......................................................84
4.5.3 CONCLUSIONS.....................................................................85

5. SUMMARY AND CONCLUSIONS.............................................................87
5.1 INDIVIDUAL VERSUS CENTRALIZED ORDERING ..........................87
5.2 INVENTORY AND TRANSSHIPMENTS..............................................88
5.3 LIMITATIONS AND FUTURE RESEARCH DIRECTIONS...................89

BIBLIOGRAPHY.......................................................................................92

VITA.....................................................................................................100

**LIST OF TABLES**

1.1 Traditional and Supply Chain Management Approaches Compared ..................2

3.1 Computational Results for Centralized and Individual Ordering When $K_c=K$ ..................52

3.2 Computational Results for Centralized and Individual Ordering When $K_c=N^*K$ ..................54

4.1 Experimental Parameters ..................75

4.2a Simplified vs. Optimal Policies for Poisson Distributions ..................77

4.2b Simplified vs. Optimal Policies for Negative Binomial Distributions ..................77

4.3 Effects of $p$ and $h$ on the Magnitude of Cost Savings of Optimal Policies ..................79

4.4 Experimental Parameters ..................81

4.5a Base Stock vs. $(s, S)$ Policies for Poisson Distributions ..................82

4.5b Base Stock vs. $(s, S)$ Policies for negative Binomial Distributions ..................82

## LIST OF FIGURES

2.1 A Two-Echelon Inventory System ........ 16

3.1 Within-State Distribution Network ........ 37

3.2 $\Phi(x)$ vs. $(x-\mu)/\sigma$ ($\mu$=10,12,14,16,18,20) ........ 56

## ABSTRACT

The management of multi-echelon inventory systems has been both an important and challenging research area for many years. The rapid advance in information technology and the emphasis on integrated supply chain management have new implications for the successful operation of distribution systems. This research focuses on the study of some fundamental issues related to the operation of a multilocation inventory system with centralized information.

First, we do a comparative analysis to evaluate the overall performance of individual versus centralized ordering policies for a multi-store distribution system where centralized information is available. This study integrates the existing research and clarifies one of the fundamental questions facing inventory managers today: whether or not ordering decisions should be centralized.

Next, we consider a multi-store distribution system where emergency transshipments are permitted among these stores. Based on some simplifying assumptions, we develop an integrated model with a joint consideration of inventory and transshipment components. An approximately optimal (s, S) policy is obtained through a dynamic programming technique. This ordering policy is then compared with a simplified policy that assumes free and instantaneous transshipments. We also examine the relative performance of base stock policies for a centralized-ordering distribution system. Numerical studies are provided to give general guidelines for use of the policies.

# CHAPTER 1

# INTRODUCTION

## 1.1 OVERVIEW

Inventory management for the wholesale/retail distribution systems has been both an important and challenging research area for many years. In the current business environment characterized by intensified competition and diversified markets, efficient operation and management of distribution systems is increasingly important for retailers. The stocking and control of inventory is a key component of distribution systems. Good inventory management often determines the success of a business.

The major goal of inventory management is to minimize inventory investment while providing the best service possible to the customer. It is plain to see that different functional areas of an organization will have different, and often conflicting, perspectives on the issue of inventory management. Marketing's strategy of high customer service and maximum sales conflict with manufacturing and distribution goals. Manufacturing is mainly concerned with high throughput and low cost production with little consideration of its impact on inventory levels and distribution capabilities. Purchasing decisions are made with very little information beyond historical buying patterns. The result of these factors is the lack of coordination among the different channel members of a supply chain, a network of facilities/activities that transfers goods from the supplier to the ultimate user. The challenge of meeting the demanding needs of the customer, the pace of change in today's competitive market requires an integrated approach to the management of

different supply chain functions. Supply chain management is a strategy through which such integration can be achieved. In Table 1.1, Cooper and Ellram (1993) present a framework for differentiating between traditional systems and supply chain management systems. These characteristics can be used as guidelines for establishing and managing a supply chain.

Table 1.1 Traditional and Supply Chain Management Approaches Compared

| **Element** | **Traditional** | **Supply Chain** |
|---|---|---|
| Inventory Management Approach | Independent efforts | Joint reduction in channel inventories |
| Total Cost Approach | Minimize firm costs | Channel-wide cost efficiencies |
| Time Horizon | Short term | Long term |
| Amount of Information Sharing and Monitoring | Limited to needs of current transactions | As required for planning and monitoring processes |
| Amount of Coordination of Multiple Levels in the Channel | Single contact for the transaction between channel pairs | Multiple contacts between levels in firms and levels of channel |
| Joint Planning | Transaction-based | On-going |
| Compatibility of corporate Philosophies | Not relevant | Compatible at least for key relationships |
| Breadth of Supplier Base | Large to increase competition and spread risk | Small to increase coordination |
| Channel Leadership | Not needed | Needed for coordination focus |
| Amount of Sharing of Risks and Rewards | Each on its own | Risks and rewards shared over the long term |
| Speed of Operations, Information and Inventory Flows | "Warehouse" orientation (storage, safety stock) interrupted by barriers to flows: localized to channel pairs | "DC" orientation (inventory velocity) interconnecting flows: JIT, quick response across the channel |

One major reason for the emerging interests in supply chain management, which first appears in the logistics literature as an inventory management approach, is the effective control of channel inventories. So it is not surprising that multi-echelon inventory models are plentiful in the supply chain literature. The major concern of these inventory models is the development of inventory control policies, considering multi-locations and several levels or echelons together. Since most of these models focused on the distribution side, complexities due to the production component of the total supply chain are avoided. However, distributors usually have to consider the interrelationship between transportation and inventory in the determination of distribution strategies. As we know, inventory and transportation are primary components of a typical distribution system in terms of cost and service levels. One of the limitations of the current research on multi-echelon inventory models is the exclusion of transportation component since the research has largely been focused on the inventory system only.

The rapid advance in information technology and the emphasis on integrated supply chain management have significant implications for the successful operation of a distribution system. The widespread use of electronic data interchange (EDI) technology, point-of-sale scanners, and on-line electronic data transmission lead to new opportunities for the efficient management of inventory and distribution systems. These advances in computer technology help reduce lead-time and costs, improve data accuracy and customer service. They also make centralized decision making possible, and enable management's better planning and control of the whole system performance. As integrated distribution is pursued, the trend towards

centralized distribution is expected to grow in the future. Although considerable progress has been made in the research on distribution systems with centralized information, further studies are needed to clarify one of the fundamental questions of the operation of distribution systems: to what extent should information and control be centralized?

## 1.2 RESEARCH FOCUS

This research involves the study of a multilocation inventory system with centralized information. It is motivated by the analysis of a real distribution system that consists of several stores, each carrying replacement parts for a major industrial and agricultural equipment manufacturer. The research is focused on the investigation of aforementioned issues with the current trend of centralized distribution. It mainly consists of two parts as follows: (1) determination of replenishment policies for distribution systems with centralized information, and (2) development of operational control rules for defining joint order and redistribution policies for inventory systems with emergency transshipments.

First, a comparative analysis is carried out to evaluate the overall performance of a multi-store distribution system under individual and centralized ordering policies respectively. This study answers the basic question facing many inventory managers today: whether or not ordering decisions should be centralized based on the consideration of relevant inventory costs. It builds on, and integrates the existing research by establishing a general guideline on stock replenishment policies for multilocation inventory systems with centralized information.

Next, we consider a multi-store distribution system where transshipments among these stores are permitted as an emergency measure to stock-out situations. The objective of this research is to determine the optimal inventory policies that minimize joint inventory and transshipment costs. We focus on the investigation of (s, S) type policies for centralized-ordering inventory systems with emergency transshipments. With some simplifying assumptions, we obtain an approximately optimal inventory policy through a dynamic programming technique. This ordering policy is then compared with a simplified policy that assumes free and instantaneous transshipments. Numerical studies are provided to show the importance of integrating inventory and transshipment decisions. We also examine the relative performance of base stock policies, a special case of (s, S) policies that assume zero ordering set-up costs, for a centralized-ordering distribution system.

There is always a trade-off involved in the research of multi-echelon inventory theory. A gap exists between theory and practice because inventory models are either overly complicated or contain very restrictive assumptions. In this research, we attempt to simplify the complex problem of inventory management of distribution systems but keep the relevant important characteristics of the system studied. Hopefully, this study can provide some insights into the effective management of distribution systems with centralized information.

## CHAPTER 2

## LITERATURE REVIEW

### 2.1 BRIEF HISTORY

The use of analytical techniques in studying inventory problems seems to be started with the development of simple lot size formula, or economic order quantity (EOQ) formula. Ford Harris (1915) is generally credited with the first derivation of this formula. It is often referred to as the Wilson formula since it was also derived by Wilson (1934) as an integral part of the inventory system he marketed.

It was not until the 1950s when more rigorous mathematical analyses of various inventory problems were undertaken. The book by Whitin (1953) was an important development since it was the first English book dealt in any detail with stochastic inventory models. Starting from the early fifties, two distinct approaches have been developed in the analysis of inventory problems. One approach considers inventory problems as multi-stage decision processes. The objective is to find conditions on the cost functions that will ensure simple forms of optimal ordering policies. Generally, an iterative procedure is used to solve a sequence of functional equations whose solution yields the optimal policies. This technique, which Bellman (1957) has used to form the basis of dynamic programming, was first applied by Arrow, Harris, and Marshak (1951) for the analysis of inventory problems, and was then examined with more generality by Dvoretzky, Kiefer, and Wolfowitz (1952,1953). Bellman, Glicksberg, and Gross (1955) showed how the methods of dynamic programming could be used to obtain structural results for a stochastic inventory problem. In that paper ordering cost is assumed to be linear (no set-up

cost), and the holding and shortage cost functions are also linear (may be relaxed by the weaker assumption of convexity of single-period cost function). It was then demonstrated that the optimal policy assumed a simple form that was characterized by a single critical number. The second approach focused on the characterization of steady state properties of the stochastic process of inventory problems. Based on certain simple ordering policy (usually depending on one or two parameters), the stationary distribution of the inventory level can be obtained through the knowledge of a branch of probability theory known as renewal theory (see e.g. Karlin, 1958). This stationary distribution, if it exists, will be dependent on the demand distribution and the inventory policy used, but independent of any cost structure. The conditional expected average cost per period is then obtained by imposing the cost structure on the stationary distribution. Finally, this average cost can be minimized with respect to the one or two parameters that characterize the inventory policy being used. A collection of important papers by Arrow, Karlin, and Scarf (1958) provided an excellent summary of the early modeling efforts in both dynamic programming and stationary analysis of inventory problems, it was the basis for later developments of mathematical inventory models. In the early sixties, multi-echelon inventory systems were first studied by Clark and Scarf (1960,1962). Some sophisticated mathematical models considering emergency order and transshipments could be found in the book edited by Scarf, Gilford, and Shelly (1963). The text by Hadley and Whitin (1963a) had an excellent coverage on the mathematical basis for single location inventory models, including the heuristic approximation treatments and the exact formulation of inventory models for systems with unit Poisson

demand. Clark (1972) presented a very comprehensive survey of research in multi-echelon inventory theory covering published results through 1971. The research on multi-echelon inventory models had not been very active in the 1970s. The publication of the book *Multi-Level production/Inventory Control system: Theory and Practice* edited by Schwarz (1981) generated renewed interests in the subject. Since then, a number of researchers have studied multi-echelon inventory systems focusing on various aspects.

Since the focus of this thesis is on the study and analysis of inventory problems of retail and distribution systems, our primary interest will be on inventory models dealing with stochastic demands. While deterministic demand may be valid in certain production-inventory systems, e.g., assembly plants operating under regular schedules, it is generally not a good assumption for retail applications. For deterministic demand there are very effective models for production and distribution systems with multiple products and locations, allowing for rather general system structures, interdependencies between different items, and constrained work centers (Roundy 1985, Maxwell and Muckstadt 1985). Muckstadt and Roundy (1993) had a comprehensive review of planning models of multistage production systems with constant demands. Extensions to the integrated inventory-vehicle routing problems for multi-echelon distribution systems with deterministic demands were developed by Anily and Federgruen (1990, 1993).

## 2.2 PERIODIC REVIEW STOCHASTIC DEMAND MODELS

Stochastic inventory problems have traditionally been considered as multi-stage decision processes. They were successfully approached by the iterative

functional equation procedure of dynamic programming. In this method, periodic review is usually assumed because continuous review would be extremely difficult to handle by the same technique. The classical dynamic programming approach used in single location problems often includes some essential elements of inventory control problems, and it is the basis for later development of multi-echelon inventory models. Most of the research focused on finding the optimal policies that would minimize a cost function consisting of three parts: (a) costs of ordering, (b) expected one-period holding and shortage costs, and (c) expected future costs. Under suitable restrictions upon cost and demand processes, simple types of cost minimizing policies exist, which can be obtained by a recursive process starting at some future time periods and working backwards to the beginning period. If we assume linear holding and shortage costs, the form of ordering costs will determine the structure of the optimal policy. Generally, the ordering cost function is defined as $K\delta(u) + cu$ where

$\delta(u) = \begin{cases} 0 & \text{if } u = 0 \\ 1 & \text{if } u > 0 \end{cases}$ $u$ is the number of units ordered, $c$ is the unit ordering cost,

and $K$ is the order set-up costs. For the special case of $K$=0, Bellman, Glicksberg, and Gross (1955) demonstrated that the optimal policy assumed a simple form characterized by a single critical number, the base stock level S of the system. This policy is commonly known as the base stock policy: if the inventory position (inventory on hand plus inventory on order minus backorders) is below the base stock level S, it is increased to S. For the general cases of $K$>0, Scarf (1960) were able to show that the optimal policy is of (s, S)-type, i.e., the inventory position is raised to some order-up-to-level S if the starting position is at or below a reorder

point s, otherwise do not order. It should be noted that values of s and S could change from one period to the next for a finite horizon problem. That a stationary (s, S) policy is optimal for the infinite horizon problem was proven by Iglehart (1963).

With the structure of optimal policies determined, the next step would be the calculation of the values of the optimal (s, S) policies. Exact calculations of the optimal (s, S) policies were traditionally considered as prohibitively expensive. The difficulty of the problem is mainly due to the ill behavior of the policy cost function, which in general fails to be quasiconvex and may have several local optima. Veinott and Wagner (1965) developed a complete computational approach for finding optimal (s, S) policies based on renewal theory and stationary analysis. They established upper and lower bounds for the optimal values of $s^*$ and $S^*$, and applied essentially full enumeration of the two dimensional grid on the (s, $\Delta$) plane ($\Delta \equiv$S-s). Wagner, O'Hagan and Lundh (1965) applied successive approximation methods of dynamic programming to obtain the optimal (s, S) policies, and compared them with a number of numerical approximations under a wide variety of system parameter values. Considerable progresses have been made in the development of more efficient algorithms in the last decade. Federgruen and Zipkin (1984a) presented an efficient algorithm based on an adaptation of the general policy-iteration method for solving Markov decision problems. The knowledge on (s, S) inventory policies from the renewal theory is exploited to simplify the computations. Zheng and Federgruen (1991) developed a new algorithm that has greatly simplified the exact calculations of the optimal (s, S) policies. Their algorithms provided an efficient search on the (s,

S) plane itself based on a number of properties of the cost function, as well as new tight upper and lower bounds for $s^*$ and $S^*$, which are iteratively and easily updated.

Although these recent algorithms are very efficient for computing optimal (s, S) policies, one notable limitation is that they only consider the case of stationary demand distribution and inventory costs parameters. Stationary (s, S) policies are of only limited interest in practice since the distribution of demand is time varying in most real environments. The dynamic programming techniques are more flexible since they can deal with non-stationary data easily. However, they provide us with no information about the dependency of optimal policies on the many parameters involved in the model or about the sensitivity of costs as a function of the policies. With these considerations, it is expected that some approximations developed earlier are still of practical value.

Analytical approximations to optimal policies may be obtained more naturally from a stationary approach than by the analysis of functional equations. Therefore, most of the approximation methods were based on the stationary analysis of inventory problems. Roberts (1962) presented mathematical backgrounds of renewal theory applied to inventory systems with (s, S) policies. He derived simple approximations for the optimal value of s and S for the case of large K (ordering set-up costs) and p (unit shortage costs) through the study of asymptotic behavior of the renewal function. Roberts' results were first extended to consider service levels by Schneider (1978). He developed approximation methods for (s, S) policy when a certain service level is required. Numerical investigations demonstrated that the approximations are very accurate. Ehrhardt (1979) developed a power

approximation for computing (s, S) inventory policies. He generalized Roberts' approximation function to an appropriate form of power series, then constructed regression analysis to obtain values of parameters using a grid of 288 known policies. The resulting approximation policies are easy to compute and require only knowledge of the mean and variance of demand, they can also be generalized for systems with demand distributions that exhibit nonstationarity or correlation from period to period. A revision of the power approximation was made by Ehrhardt and Mosier (1984). The revision incorporated modifications that would ensure the proper limiting behavior of S-s for the case of very small variance of demand as well as homogeneous results for different demand units chosen. Schneider and Ringuest (1990) developed similar power approximations for computing (s, S) policies using service level. They did not assume the knowledge of shortage costs, which are difficult to measure in practice. Instead, they define a $\gamma$-service level, which measures the average backlog relative to the average demand. The resulting approximations were demonstrated to be simple and accurate.

### 2.2.1 CLASSICAL MULTI-ECHELON INVENTORY MODELS

One of the earliest multi-echelon inventory models involving uncertain demands was developed by Clark and Scarf (1960). They considered a system, in which N facilities are arranged in series. External demand occurs at the lowest echelon (N) only, and the stock is transmitted from echelon i ( $\geq 1$ ) to echelon i+1 incurring linear transfer cost. The first facility in the series places its orders to the outside supplier with a fixed cost as well as proportional cost. The Stocks are reviewed and decisions made periodically. Clark and Scarf showed that the optimal

policy for this system can be computed by decomposing the problem exactly into N separate single location problems which can be solved recursively for echelons N, N-1,..., 1. The decomposition procedure starts from the lowest echelon N. The problem for this echelon is a typical single location inventory problem with linear ordering cost. The optimal policy is characterized by the base stock of the echelon. A slight modification of the policy is necessary considering the constraint of multi-echelon structure. If the inventory position for the lowest echelon is below the base stock level, it is increased to the latter provided the next facility up (N-1) has sufficient stock; Otherwise ship as much as possible. Such policy is called modified base stock policy. For the intermediate echelon i ( $1<i<N$), define this echelon's inventory as inventory at echelon i plus inventory at the next lower echelon (i-1) plus shipments in transit between these two echelons. Clark and Scarf showed that the cost for echelon i includes the echelon's holding cost, linear ordering cost (shipments cost from echelon i-1), and a convex *induce penalty cost*. The induced penalty cost represents the increase in expected costs at echelons i+1, ..., N due to insufficient inventory at echelon i, it can be obtained based on the optimal policy and expected cost function for echelon i+1. Again, modified base stock policies are optimal for all intermediate echelons. The problem for the first facility can be approached in the same way. However, the optimal policy is of (s,S)-type because of the inclusion of some fixed ordering cost.

In a subsequent paper, Clark and Scarf (1962) considered the case where fixed ordering costs were applied at all facilities. It was found that the optimal solution could not be broken down into a sequence of single-state variable problems

because the induced penalty cost is dependent on both total echelon inventory and the stock level of the echelon itself for a sequence of (s, S) policies. Since no exact solution method is available for this model, Clark and Scarf replaced the induced penalty cost functions by certain lower and upper bound approximations that would depend on the echelon inventory only. They suggested that the ordering policies associated with the upper bound be used as approximate optimal policies.

The Clark and Scarf model generated considerable interests in the study of multi-echelon inventory systems. There have been various extensions of the Clark-Scarf series multi-activity model. Fukuda (1961) included an option of stock disposing in his studies. He considered the optimal ordering and disposal policies in the series echelon structure using induced shortage costs for disposal in analogy with the Clark-Scarf model. Hochstaedter (1970) considered the case of inventory system of parallel facilities with a common supplier. As in Clark and Scarf (1962), upper and lower bounds were established, with each set of bounds yielding (s, S)-type policies for each facility. He also discussed the difference between the upper and lower cost functions. Federgruen and Zipkin (1984b) considered the case of infinite planning horizons of the basic Clark and Scarf model. They showed that the decomposition technique extended to the infinite horizon case under both criteria of discounted costs and long-term average costs. The resulting formulation of the infinite-period problem leads to significant computational simplifications. Rosling (1989) extended the Clark and Scarf model to general assembly networks with linear ordering and assembly costs.

The assumption of simple serial system in Clark and Scarf model seriously limited the model's applicability in practice since few actual production-inventory or retail distribution systems have this type of structure. For distribution systems, a more appropriate assumption is the inverted tree structure, or an arborescent structure, where each facility can only be supplied by a single source at the next higher echelon, but it can support a number of facilities at the next lower echelon. Bessler and Veinott (1966) studied a multi-period multi-echelon inventory system with a rather general arborescent structure. Each facility may order stock from an exogenous source with proportional ordering cost. Delivery lag was assumed to be zero. They examined a special supply policy in which each facility except facility 1 (top supplier) immediately passes its shortages on to a given supplier with backlogging occurring only at the top supplier. Bounds on the base stock levels were obtained. The Bessler-Veinott approach is more general than the Clark-Scarf model in terms of included features of the problem. It is significant because multi-echelon inventory systems with arborescence structure are good approximations for typical distribution systems. Most of the multi-echelon inventory models developed later were confined to arborescence structure. Recently, Chen and Zheng (1994) summarized the existing results on the characterization of optimal policies for serial, assembly, and one-warehouse multi-store distribution systems. They also obtained lower bounds on the minimum costs of managing theses systems based on cost-allocation, physical decomposition framework.

### 2.2.2 CENTRALIZED PLANNING MODELS

Most of the published research on multi-echelon inventory systems deals with a special kind of arborescence structure. As illustrated in Figure 2.1, it is a two-echelon inventory system with one upper echelon facility supporting N lower echelon facilities, each facing independent and identically distributed (IID) random demand from customers.

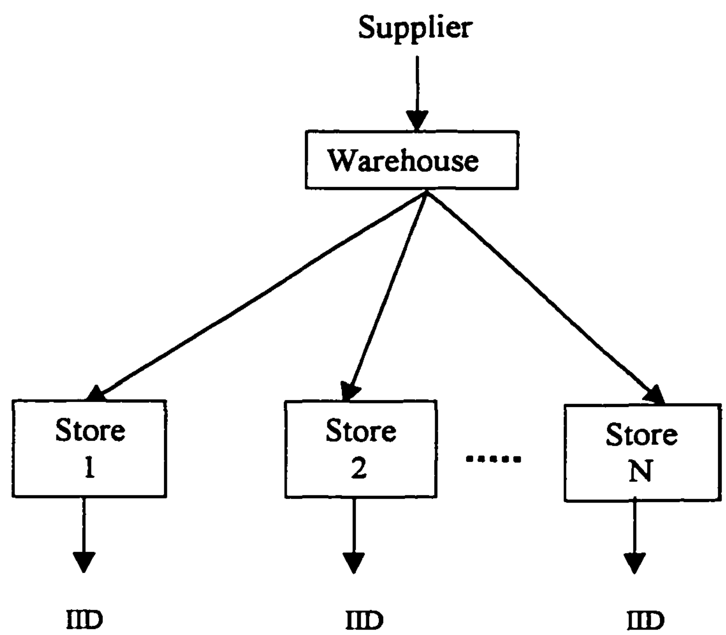



Figure 2.1 A Two-Echelon Inventory System

To avoid possible confusions on terms, we will use "warehouse" to represent the upper echelon and "stores" to represent lower echelon facilities throughout the thesis. External demands for different periods are generally assumed to be independently and identically distributed. The warehouse could be virtual instead of a physical location in the sense that it only serves centralized planning functions such as ordering and inventory allocation rather than holding stocks. In fact, most

published research on centralized planning models considered the inventory systems where the warehouse holds no inventory. Centralized planning systems are also called PUSH systems because a central decision maker possessing all the necessary information will order replenishment stocks (based on system wide inventory positions) and then allocates them to different stores.

#### 2.2.2.1 SYSTEMS WITHOUT CENTRAL INVENTORIES

For systems without central inventories, centralized planning would still be beneficial in that ordering costs could be reduced with economies of scale. Also, there is risk pooling over the supplier lead time. By postponing the allocation from the time of ordering to the time of receiving the shipments at the warehouse, we can observe the demands in the intervening periods, and thus make more balanced allocations.

Eppen and Shrage (1981) seems to be the first to study a two-echelon inventory system with centralized ordering where the warehouse does not carry any inventory. Exogenous demands occur at the stores following stationary normal distribution. They derived approximately optimal policies and costs of: (1) a base stock policy assuming no fixed order costs, and (2) an (m, y) policy assuming fixed order costs. An (m, y) policy is one in which every m periods the system inventory position is raised to a base stock of y. They presented analysis on the optimal allocation assumption that requires enough replenishments for inventory allocation to ensure the same probability of stock out at each store. Simulation study showed that the assumption was good for systems with low variability of demands.

Federgruen and Zipkin (1984c) studied the same system of Eppen and Schrage (1981) with some important extensions. They used the dynamic programming approach to study the inventory system. In the exact formulation of the dynamic program, the state space would be N+L (the number of stores + the number of review periods of the time lag of order) dimensional. Exact solutions would be impossible except for small values of N and L. They devised approximations by relaxation to solve the problem of "curse of dimensionality". A myopic allocation scheme is suggested that require minimizing the expected one-period inventory costs. By using this assumption, they showed that the multi-location problem could be simplified to a single location problem with time lag. The state space of the dynamic program reduced to 1 dimension after proper aggregation of lead time demands (see, Karlin and Scarf, 1958). The dynamic programming approach described above can easily deal with non-stationary data. Ordering policies were obtained from solutions of functional equations rather than restricted forms assumed as in Eppen and Schrage (1981).

Jönsson and Silver (1987) also studied the two-echelon inventory system but with emphasis on adjusting the imbalance of stores' inventories through stock redistribution. Demands at the stores are assumed following normal distributions. The warehouse uses base-stock replenishment policies with a predetermined order cycle of H time periods. They demonstrated that most of the expected shortages would occur in the final period of the order cycle. So they considered a complete redistribution of all stores' inventories one period before the end of the order cycle.

Numerical evaluations showed that substantial reduction in safety stock investment is possible by using redistribution.

Schwarz (1989) considered essentially the same model as Jönsson and Silver (1987), but focused on the evaluation of risk-pooling effect over outside supplier lead time. He compared the risk-pooling incentive with the cost of: (1) increased overall lead time to the stores; (2) pipeline inventory holding costs. The primary conclusion of the research is that the value of the warehouse depends most importantly on the pipeline inventory costs.

Erkip, Hausman, and Nahmias (1990) extended the study of Eppen and Schrage (1981) to the case of correlated demands. They allowed item demands to be correlated across stores and also correlated in time, but ignored the fixed ordering costs. An explicit expression for the optimal safety stock as a function of the level of correlation through time was developed, and numerical evaluations were included to illustrate the impact of the various magnitude of correlation.

Kumar et al. (1995) used a similar approach as that of Eppen and Schrage (1981) to study a different distribution system. A warehouse coordinates the replenishments of system inventories and the allocations of stocks among N stores located along a fixed delivery route. Except the difference on this delivery arrangement, exactly the same assumptions were used as in Eppen and Schrage. Kumar et al. studied the risk-pooling effect of dynamic allocation policy. Under the dynamic policy, allocation decision for each store is delayed until the delivery vehicle arrives at that store.

### 2.2.2.2 SYSTEMS WITH CENTRAL INVENTORIES

Several researchers extended the centralized-planning, multi-echelon inventory models to allow the warehouse to hold stock during the cycle. One purpose of holding this stock is to permit the warehouse to make dynamic allocations to the stores rather than one single allocation at the beginning of a cycle. Dynamic allocation will lead to a more efficient distribution of stock towards the end of cycle. This kind of risk pooling was often referred to as " the depot effect".

Federgruen and Zipkin (1984b) discussed the extension of Clark and Scarf model to multi-echelon distribution systems. The multiple stores at the lower-echelon could be modeled as a single location inventory problem if the probability of stock imbalance of the system is small.

Jackson (1988) studied the benefits of holding central stock at the warehouse for a two-echelon distribution system. An (m, y) ordering policy is assumed at the warehouse, and the allocation policy is predefined to be a base stock policy for each store. This means that the warehouse will make shipments to restore the inventory position of each store to some ship-up-to level in every period when the warehouse has enough stock. When the stock on hand at the warehouse is insufficient to raise the inventory position of every store to its ship-up-to level, certain runout allocation rule has to be developed to optimally allocate stock in this situation. No shipments are made after the runout period until the beginning of the next cycle. Jackson developed an approximate cost function model for the case of identical retailers. He demonstrated that holding stock at the warehouse would significantly improve the service performance.

An extension of Jackson's work was presented by Jackson and Muckstadt (1989). They investigated the risk pooling benefits of centralized stock with the case of two allocation periods per order cycle. Two aspects of the risk pooling effects were identified. The first aspect of risk pooling is that the distribution of stock will be more balanced as a result of holding stock in reserve. The second aspect is that risk pooling removes some of the uncertainty involved in planning stock levels.

McGarvin et al. (1993) had detailed discussion on optimal inventory allocation policies for one-warehouse N-identical-retailer distribution system. They developed an infinite-retailer model to determine two-interval allocation heuristics for N-retailer systems. Simulation tests suggest that the infinite-retailer heuristic policies are near optimal for as few as two retailers.

### 2.2.3 DECENTRALIZED PLANNING MODELS

Generally, it is very difficult to determine optimal order and supply policies for centralized-control, multi-echelon inventory systems. The complexity of managing such systems arises when the distribution of stock in the system becomes unbalanced. Under such circumstances, the ideal stock level for each store based on the total system stock only might not be attainable without other measures such as transshipments etc. Thus the optimization problem requires knowledge of more than just the total amount of system stock, and the problem of finding the optimal policies tends to be computationally intractable. In addition, centralized control over the entire system is not always possible in practical situations, particularly if different echelon facilities are owned by different owners. So some researchers

studied the multi-echelon inventory systems considering the approach of decentralized inventory control.

Ehrhardt et al. (1981) considered the problem of inventory control for a two-echelon distribution system. They assumed that demands at the warehouse are comprised of replenishment orders from the stores that follow (s, S)-type policies. It is showed the demand at the warehouse will be correlated over time. Because of this demand correlation, the optimal policies at the warehouse are not necessary of the (s, S) type, but they adopt that form because of its widespread use in practice. They suggest that the warehouse policy be computed by using the power approximation method of Ehrhardt (1979) which incorporates the period to period demand correlation.

Nahmias and Smith (1994) considered a two-echelon inventor system with partial lost sales. Both stores and the warehouse are restocked to some order-up-to level at fixed time intervals, where the replenishment frequency of the stores is an integer multiple of the frequency of the warehouse. All lead times are assumed to be zero. A newsboy type model is developed to determine the optimal base stock levels at the stores. Because of partial lost sales, the probability distribution of the total demand at the warehouse is complex. They concluded that the benefit of retaining stock at the warehouse is most significant for low demand, high value items.

Schneider et al. (1995) studied the same system as that of Ehrhardt et al. (1981) but focused on the issue of service levels at the warehouse and stores. They investigated the interrelationship between the safety stocks at the warehouse and the stores. Two types of service levels were defined: stockout occasions $\alpha$, and time

weighted backorders $\gamma$. They used $\alpha$ service level at the warehouse to adjust the effect of warehouse stockouts on the lead time to the stores. Near optimal policies for both stores and the warehouse were obtained by power approximations. Numerical studies were provided to evaluate the accuracy of approximations.

Graves (1996) used a unique approach to study the multi-echelon inventory systems with a very general distribution topology. He assumed perfect flows of information at all levels, so that as demands occur, stocks at upper echelons would be committed to satisfy these demands. He also assumed that each site orders at preset times according to base stock policies. His model focused on the characterization of inventory levels for each site at every time instant. The inventory level is related to the coverage time of shipments, which in turn is related to the depletion or runout time. The difference between runout and coverage time is the buffer time provided by the base stock. The iterative processes of characterization of inventory levels are continued up the supply chain until the source site is reached. Numerical studies for a two-echelon distribution system showed that most of the safety stock should be kept at the stores.

## 2.3 CONTINUOUS REVIEW STOCHASTIC DEMAND MODELS

In a continuous review inventory system, all demand transactions are monitored as they occur. Therefore, inventory-ordering decisions can be made more responsively than the periodic review system. Treating stationary continuous review systems as the limiting cases of periodic review models discussed above, we can relate the (s, S) policies in periodic review systems to the widely used lot size/ reorder point (Q, r) models for the continuous review systems by setting r = s and

Q=S-s. So we can think of (Q, r) policies as a special case of the larger class of (s, S) policies. Although (s, S) or (Q, r) policies are widely assumed in the study of continuous review inventory systems, in general, little appears to be known about the optimality of these policies.

For single location problems, Hadley and Whitin (1963a) discussed a heuristic method of formulating (Q, r) models though approximate analysis of average costs. The heuristic method can be applied for both cases of backorders and lost sales. They also developed an exact formulation of the (Q, r) model for the backorders case with unit Poisson demand and fixed lead times. Moinzadeh and Nahmias (1988) extended Hadley and Whitin's (1963a) approximate analysis of the basic (Q, r) model and developed a four-parameter ($Q_1$, $Q_2$, $R_1$, $R_2$) model for a single-location inventory system with two supply modes.

From the consideration of information costs related to inventory review and decision making, continuous review policies are of special interest for low demand items, such as spare parts. Indeed, most of the research on continuous review policies deals with inventory systems for repairing and supplying recoverable items. It has significant impacts on the successful operation of many military inventory control systems. The classical model of Multi-Echelon Technique for Recoverable Item Control (METRIC) developed by Sherbrooke (1968) provided much of the impetus for the extensive research on optimal inventory control of high-value, low-demand, and reparable items. These models are specifically suited for multi-echelon systems where failed components are replaced and then repaired at service depots. One-for-one, (S-1, S) ordering and random depot allocation procedures are

assumed. These analyses make use of a queuing model analog to derive the steady state distribution of outstanding orders. Nahmias (1981) presented a comprehensive survey of these models. Besides these important special models, considerable progress has also been made for multi-echelon inventory systems using general (Q, r) policies. Axsäter (1993a) provided a general survey on multi-echelon inventory models with continuous review policies.

### 2.3.1 ONE-FOR-ONE REPLENISHMENTS

The classic one-for-one replenishment model for a multi-level system is METRIC, developed by Sherbrooke (1968). The model assumed a two-echelon structure where demands (failures) at the bases follow stationary compound Poisson distributions. Routine repair is done at the bases, and more serious repair is done at the depot. Also, it is assumed repair times are independently and identically distributed, this is equivalent to the ample-server, no-queueing-for-repair assumption. The objective of the model is to obtain the policy that minimizes expected backorders subject to a constraint on system investment. The METRIC model approximates outstanding orders at the bases by Poisson random variables ( Feeney and Sherbrooke, 1966). It means that the depot backorder level is also Poisson, and the problem is greatly simplified. The METRIC model is significant in a number of respects. It includes many important features of determining suitable spare levels in a large-scale reparable item inventory system. As a result, it is one of the few multi-echelon inventory models to be successfully implemented. Muckstadt (1973) provided an important generalization of METRIC to include a hierarchical or indentured parts structure (MOD-METRIC).

Muckstadt and Thomas (1980) performed an empirical study to investigate the advantages of multi-echelon models comparing with using adaptations of single location models for high cost, low demand items. Data from a large industrial inventory system were used to provide comparative analysis on overall inventory required to meet a given service level. They concluded that substantially better performance could be obtained for multi-echelon models than single-location models. Hausman and Erkip (1994) did the same analysis using the same data. However, their conclusion was somewhat different from that of Muckstadt and Thomas (1980). Hausman and Erkip expanded the search range on optimal service levels at both the stores and warehouse, and found an improved policy for the single-echelon model. They demonstrated that the suboptimality penalty is only 3% to 5% when single-echelon systems are appropriately parameterized.

The METRIC approximation can be regarded as a single parameter approximation because it models the number of outstanding orders with Poisson process, which is characterized by the mean. Graves (1985) studied a similar multi-echelon inventory system for reparable items with one-for-one replenishments. He derived an exact model for the system under the assumption that demand at each site follows compound Poisson distribution and the shipment time from the repair depot to each base is deterministic. He then used a negative binomial distribution to approximate the distribution of outstanding orders at the base. As expected, this two-parameter approximation is more accurate than the METRIC approximation.

Albright (1989) considered the same problem as in METRIC model but with more general settings. He assumed that there are only a finite number of servers at

each site and the failure rates at the bases depend on the current number of items online rather than a constant. He developed an approximation to the stationary distribution of the queueing model. Extension of the model to include the multi-indenture structure was considered by Gupta and Albright (1992).

Axsäter (1990a) presented a new approach to model the two-echelon inventory system with one-for-one replenishments. Instead of using the traditional approach of steady state analysis, he focused directly on evaluating the average costs associated with an inventory policy. A recursive procedure is developed to determine the optimal stockage policy.

Svoronos and Zipkin (1991) also studied a multi-echelon inventory system with exogenous Poisson demand and one-for-one replenishments, but focused on modeling the stochastic transportation times. In contrast with the METRIC assumption of parallel processing of ample-server, they assumed that units were processed sequentially in a queue following FIFO rule. Approximations for the steady state behavior were described. They showed that, in sharp contrast to prior multi-echelon models, lead-time variances played an important role in system performance. The study was extended to the case of exogenous compound Poisson demand by Zipkin (1991).

Some researchers have extended the basic METRIC model to allow lateral transshipments. Lee (1987) assumed grouping of similar bases into disjoint clusters, with lateral made within each cluster only. Approximations were derived and tested for the expected level of backorders and the quantity of emergency lateral transshipments for the case of identical bases, and they were used to obtain the

optimal stocking levels. Axsäter (1990b) studied the same problem with different techniques. His approximation can be applied to dissimilar bases, and results in smaller errors than Lee based on simulation tests. Sherbrooke (1992) used regression analysis on simulation results to obtain approximations to estimate the expected backorders in a multi-echelon system with lateral transshipments.

**2.3.2 (Q, r) ORDERING POLICIES**

For many consumable low-value items, considerable fixed ordering costs more or less prohibit one-for-one replenishments. This means that the general lot size/reorder point (Q, r)-type models should be investigated. Unfortunately, the problems are much more complicated when we study the general (Q, r)-type batch ordering policies. Let us consider a typical two-echelon inventory system where a warehouse supports N stores that each faces a Poisson demand. If each store operates based on the general (Q, r)-type ordering policies, the demand process at the warehouse will be the superposition of N Erlang renewal processes instead of the simple Poisson process in the case of one-for-one ordering (Q=1). Except for some very special cases, exact analysis of (Q, r) policies for a multi-echelon inventory system is impossible. So it is usually practical to develop various approximations to facilitate system analysis.

Deuermeyer and Schwarz (1981) developed an approximate model to study the service performance of the above two-echelon inventory system operating under (Q, r) policies. The model is a generalization of the exact single-facility (Q, r) model of Hadley and Whitin (1963a). The difficulty to model such a multi-echelon inventory system includes (1) modeling the demand process at the warehouse (2)

modeling the effect of stock out at the warehouse on the stores lead time. The warehouse demand process will be nonstationary (fortunately, the nonstationality turns out to be small), and the effective lead time of each store is stochastic due to the random delay of warehouse shortages. Facing these difficulties, they first proposed an approximation that replaces the N independent Poisson demands at the stores by an aggregated Poisson demand at an artificial store. This approximation yields a stationary renewal process that is easy to work with. Then a deterministic approximation is applied to replace the random delay due to the warehouse stock out by its average value. Finally a complete model is formed by synthesis of the system as a whole based on above approximations. Simulation results showed that the model works well for a wide range of operating parameters. Their model was used by by Schwarz et al. (1985) to examine the problem of maximizing system fill rate subject to a constraint on system safety stock.

Badinelli and Schwarz (1988) studied the same system but focused on the problem of optimal allocation of safety stock among the warehouse and stores. In particular, the so-called "portfolio" motive for holding warehouse safety-stock inventory is investigated. A heuristic for minimizing expected backorders with respect to a constraint on average system on-hand inventory is introduced. They showed that optimal warehouse inventories should be small.

Svoronos and Zipkin(1988) proposed several refinements of the Deuermeyer and Schwarz (1981) model. The key innovation in their model is the use of second-moment information in the approximations. They derived the expressions for the mean and variance of the warehouse demand then chose an appropriate distribution

to model it. The same idea is used to model the stochastic time delay due to warehouse stock out. As one would expect, their model is more accurate than that of Deuermeyer and Schwarz.

Axsäter (1993b) extended his analysis on one-for-one replenishment policies to the general (Q, r) policies. He assumed identical stores with Poisson demand. Both exact and approximate models were developed to estimate inventory costs. His model is as accurate as that of Svoronos and Zipkin(1988). He also extended his analysis to a one-warehouse, N non-identical stores system under compound Poisson demand (Axsäter, 1995).

## 2.4 INVENTORY MODELS WITH TRANSSHIPMENTS

For multi-location inventory systems, a major concern is the imbalance of inventory levels at different locations facing stochastic demands. Transshipments and redistribution can be used as an anticipatory measure to balance inventories among the locations. In many practical operations, transshipments are also used as an emergency action, by which stockouts at certain locations are filled with units from neighboring facilities with available inventories. Transshipments/redistribution could reduce system-wide inventory costs significantly, and improve the service levels provided to customers. There are, of course, significant cost and response time tradeoffs associated with the use of transshipments. Transshipments also affect stock replenishment policies.

Allen (1958) was perhaps the first to consider a multi-location inventory system with transshipment/redistribution. He assumed random demands and known initial base stock levels at different locations. The objective was to determine stock

redistribution policies that would minimize transportation and shortage costs. Based on various properties of the solution to this problem, he developed a simple iterative procedure of computation to determine the optimal values of transshipments.

Gross (1963) extended Allen's (1958) analysis with the development of a single period model that allowed for both replenishment and redistribution at the stores. He provided detailed analysis for a two-location inventory system with linear ordering and transshipments costs. The problem of optimal transshipment is very complex because it depends not only on the stock level at each store but also on the inventory level of the whole system. For a two-location distribution system, a complete specification of the optimal policies can be obtained by an appropriate division of the plane of initial base stock levels and separate analysis over different regions. For systems with more than two locations, Gross suggested that a search technique, similar to Allen's (1958) iterative procedures, be used to obtain numerical solutions.

Hadley and Whitin (1963b) studied a centralized planning system consisting of N stores where redistribution is permitted. Replenishments are controlled centrally based on system-wide stock levels according to a continuous review (Q, r) policy. Demand at each store is assumed following stationary Poisson distribution with unsatisfied demands backlogged. Redistribution among the N stores can be made in two modes of transportation referred to as fast and slow, respectively. The objective of the study is to determine the optimal ordering, allocation, and redistribution policies that minimize the long-run average inventory and transportation costs. Using heuristic method of average costs, they derived a

formula of total costs with respect to the expected costs of redistribution under different scenarios. A number of simplifying assumptions was made for determining Q and r. The resulting (Q, r) policy includes an explicit consideration of the effects of redistribution costs. If redistribution is assumed free and instantaneous, the system can be treated as a single installation, thus the effects of redistribution costs on Q and r are neglected. It is demonstrated that total system costs could increase substantially by neglecting the effects of redistribution costs on optimal ordering policies. Finally, the optimal allocation and redistribution policies can be obtained from the solutions of dynamic programming models.

Krishnan and Rao (1965) considered a similar system as Gross (1963) but approached the problem from a different perspective. They assumed that transshipments were used as an emergency measure to stockout situations after the demands were realized, and they only considered the determination of optimal base stock levels that minimize the expected inventory and transportation costs. As in Gross (1963), analytical results were easily obtained for a two-location system. The analysis could be extended to the case where there are N stores with known and independent distributions of demand if the unit transportation cost between any two stores is a constant.

Das (1975) also studied a two-store stochastic inventory problem with periodic review. During each replenishment cycle, one transfer of stocks between the two stores is permitted at certain predetermined time within this cycle. Various properties of the cost function were established, and optimal stock and transfer rules were investigated.

Hoadley and Heyman (1977) developed a general one-period two-echelon inventory model that allows for stock purchasing or disposing at the warehouse, returning or shipping items between the echelons, and transshipments among the stores. For all these activities, they assumed constant linear costs independent of the locations and distances. The decision problem is to choose an initial stock level at the warehouse and initial allocation so as to minimize the initial stock movement costs and the one-period inventory costs. The general model took different forms, depending on the relative magnitudes of the various shipping costs. Each form was discussed and analyzed respectively.

Cohen, Kleindorfer, and Lee (1986) considered a complex multi-echelon inventory system for managing low-demand, high-cost items. They developed a comprehensive one-period stochastic inventory model that takes account of a unique set of characteristics such as pooling mechanisms, emergency and normal replenishments, part scrapping and/or recycling. Stocking locations at each echelon are grouped into several disjoint "location groupings" based on geographical and managerial considerations. Transshipments are considered only within the same location grouping. The objective of the study is to determine the optimal stock levels at different locations subject to service level constraints. Solutions to the constrained optimization problem are found using a branch and bound procedure.

Tagaras and Cohen (1992) presented an extensive analysis on the problem of pooling/transshipments in a two-location inventory system. They discussed the possible impacts of replenishment lead times on pooling policies. Their study extended some other two-location models with zero replenishment lead time, which

are characterized by complete pooling in that if there is an economic incentive, the maximum amount (minimum of supply and demand) will be sent. When replenishment lead time is non-negligible, some managers may hold back stock and practice partial pooling as a hedge against demand uncertainty over the supply lead time. There are a variety of pooling options based on different managerial considerations. Simulation study was used to compare the complete pooling policy with a class of partial pooling policies. The cost performance of these partial pooling policies is shown to be inferior to that of complete pooling. A simple heuristic method is introduced to compute near-optimal stock levels for the complete pooling case.

Generally, all of the above research focused on the development of operational control rules for defining joint order and transshipment policies. In contrast, some other researchers put more emphasis on the analysis of structural results for the problem by utilizing stochastic programming or dynamic programming models.

Karmarkar and Patel (1977) reexamined the two-location distribution system of Gross (1962) and developed a simpler approach to solve the problem. They decomposed the problem into two parts: (1) a linear transportation problem, and (2) nonlinear decoupled newsboy problems. This approach leads to a characterization of optimal policies in terms of the dual of the transportation problem. This method is not suitable for systems with more than two locations. For the numerical solutions of larger problems, the problem was formulated as a linear program with column generation. A qualitative analysis of the more general convex programming

problems was discussed by Karmarkar (1979). Various properties of the optimal policy and of the optimal value function are described. The stochastic multi-location inventory problem can be generalized to allow transshipment, disposal activities, capacity constraints, and multiple products.

Karmarkar (1981) extended his qualitative analysis of single-period multi-location inventory problems to the multi-period multi-location problems. He studied the structure of optimal policies in general multi-period, multi-location inventory problems with proportional costs. It is shown that the multi-period problem has the same generic form in each period as a single-period problem. Therefore, properties of the single-period problem are inherited in the multi-period formulation. Karmarkar (1987) then discussed the computational issues for the general multi-period inventory problems. He developed lower and upper bounds of the total cost function, and suggested approximations to the optimal solution.

Robinson (1990) considered a more specific multi-period, multi-location distribution system with transshipments. He examined the effects of transshipments on optimal ordering policies and the associated total costs of the system. He assumed linear ordering costs and zero replenishment and transshipment lead times as in Karmarkar (1981). It is shown that the optimal policy is a base stock policy, and the optimal order-up-to-level is stationary if the base stock level for the final period is not negative. The problem is formulated as a stochastic inventory problem with linear recourse of transshipments. Though analytical solution is available for the special case where all cost parameters are equal among retail outlets, approximations are necessary for the general stochastic linear problem. Using

Monte Carlo sampling to approximate the demand by a discrete distribution, Robinson transformed the stochastic problem into a deterministic linear problem, which is solved readily. The approximation method is evaluated for special cases where analytical solutions are available.

## CHAPTER 3

## INDIVIDUAL VERSUS CENTRALIZED ORDERING IN MULTILOCATION INVENTORY SYSTEMS

### 3.1 MOTIVATION AND BACKGROUND

This research is motivated by the analysis of a distribution system of an equipment distributor in the state of Louisiana. As shown in Figure 3.1, the distributor has seven retail stores located in the following cities: Alexandria, Baton Rouge, Lake Charles, Monroe, New Orleans, and Shreveport. Each store carries replacement parts for a major industrial and agricultural equipment manufacturer.

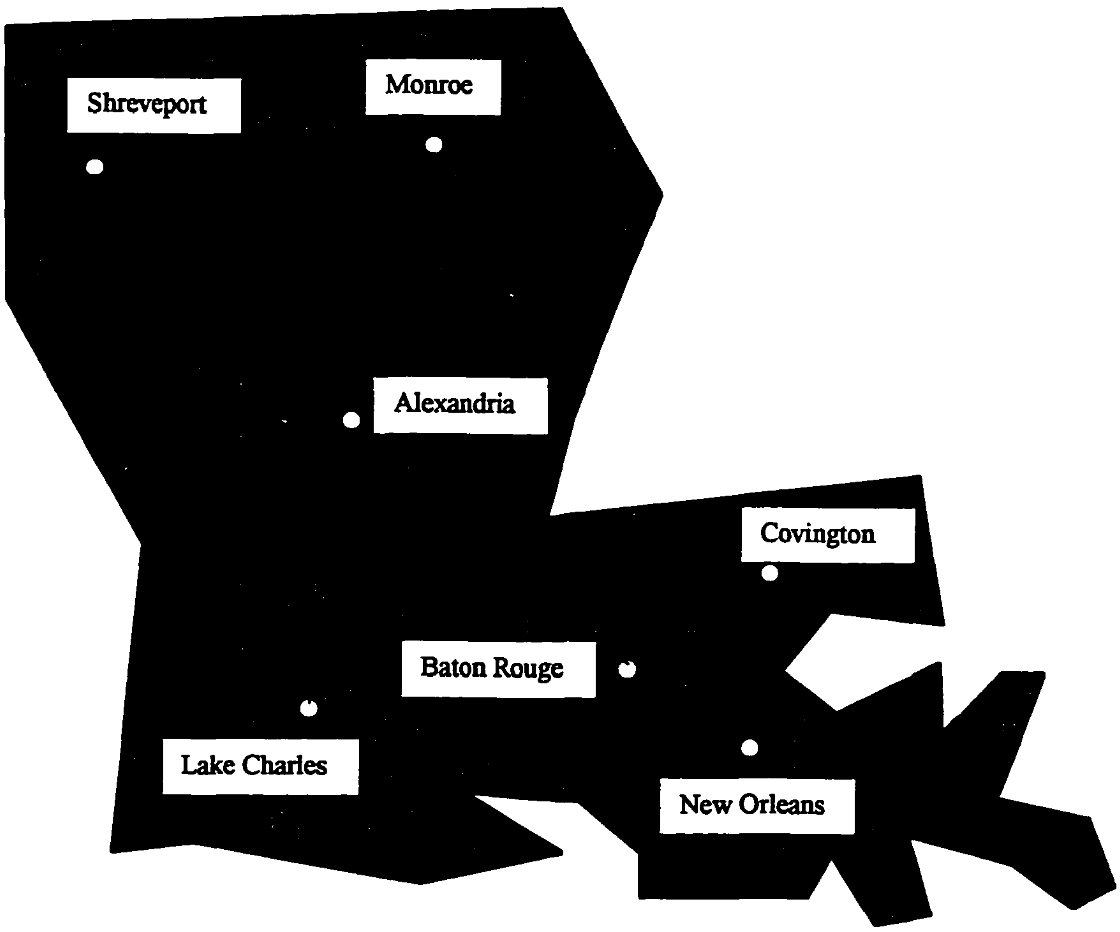



Figure 3.1. Within-State Distribution Network

Currently, each store operates independently. Stocked parts are subject to periodic (weekly) review policy. At the end of each week, an inventory manager reviews stock levels for various items and orders replenishment parts for delivery to each store from the manufacturer. The inventory policy for each stocked part is determined based on the order formula code (OFC). The OFC is assigned based on each part's sales and list price. The order quantity is calculated from simple economic order quantity (EOQ) formula, and the reorder point /safety stock is determined based on the last 12 months of sales activity. When a stock-out occurs, the company may experience a lost sale but, in general, the customer will wait 1 or 2 days for an expedited part.

Recently, the company implemented a new business information system to assist inventory control of this distribution network. This computerized inventory system removed previous communication barriers between different stores. Now, the inventory manager has instant access to the inventory records at each store. How to take advantage of the new information system to reduce overall system costs and maintain/improve high levels of customer service is a key issue facing the management today.

This study consists of a part of our analysis of the above distribution system. Previously, Schneider and Watson (1997) examine the opportunity of reducing inventory costs by implementing the centralized warehouse concept. They consider decentralized control (PULL systems) of the distribution system under a two-echelon structure, with the addition of a central warehouse or using the so-called virtual warehouse arrangement. They assumed that replenishment decisions are

made individually at different stores, centralized information is only used for coordination of central stock (at the warehouse) and local stock (at the stores). They investigated the effects of different inventory policies on the distribution system costs of this company and illustrated the effect of number of stores on central warehousing decisions.

The primary argument for adopting centralized warehouse is to take advantage of risk pooling effects. The idea of risk pooling is conceptually easy to understand. As long as the demands at various stores are not perfectly correlated, the aggregated demand at the central warehouse will have less variation hence requiring smaller safety stock comparing with the requirements of meeting each individual store's demand separately at the same service level. In a real application many issues should be considered: capital investment and operation costs on the central warehouse, severance costs for removing existing warehouses, effects on customer service level, and information and transportation networks. If we simply add a central warehouse to the existing distribution system without any careful analyses of the trade-off involved with centralized warehousing, operation cost of the distribution system may increase significantly.

When ordering decisions are made independently at each store the distribution system is a typical one-echelon inventory system, which has been studied extensively. For distribution systems with centralized information, a common alternative of inventory control practice is to use the so-called PUSH control system, where *a central decision maker* possessing all the necessary information orders replenishment stocks based on system-wide inventory level and

then allocate them to each store appropriately. Centralized ordering may result in cost savings from economies of scale such as reduced ordering set-up costs, and quantity discounts. Additional benefits may also be available from risk pooling over the supplier lead time because the central decision maker can postpone the allocation from the time of ordering to the time of receiving the shipments and make more balanced allocations by observing the demands in the intervening periods.

The centralized ordering system can be considered as a special two-echelon inventory system with one virtual warehouse at the upper echelon and multiple stores at the lower echelon. The virtual warehouse performs centralized ordering functions and serves as a transshipment point for stock allocations among different stores. A number of researchers have studied this type of two-echelon inventory systems. Some important references are summarized below.

Eppen and Schrage (1981) appears to be the first to study the two-echelon inventory system with centralized ordering where the warehouse holds no stock. Exogenous demands at the stores are assumed following stationary normal distribution. If there is no fixed ordering costs, base stock policy is considered. Otherwise, an $(m, y)$ policy is assumed, in which the system inventory position is raised to a level of $y$ every $m$ periods. They derive a newsboy-like formula for determining an approximately optimal quantity of system stock. The solution reveals the effect of risk pooling over the supplier lead time. They also present detailed analysis on the optimum allocation assumption that requires enough replenishment for inventory allocation to ensure the same probability of stock out at each store. Simulation studies show that the assumption is good for systems with a

low variability of demand. Erkip, Hausman, and Nahmias (1990) extend the model to allow demand to be correlated over time and between locations.

Federgruen and Zipkin (1984c) consider the same system of Eppen and Schrage (1981) with some important extensions. They use a dynamic programming approach in their analysis. An approximation method based on relaxation is developed to reduce the state space dimensions of the dynamic program. A myopic allocation scheme is suggested that requires minimizing the expected one-period inventory costs. By using this assumption, they show that the multi-location problem could be simplified to a single location problem. This dynamic programming approach is applicable for systems with non-stationary data. For some important special cases, it also allows for other classes of demand distributions, including the exponential and the gamma distributions. Ordering policies are obtained from solutions of functional equations rather than restricted forms assumed as in Eppen and Schrage (1981).

Kumar et al. (1995) use a similar approach as that of Eppen and Schrage (1981) to study a different inventory distribution system. Their system consists of a warehouse coordinating the replenishment and allocation of inventory among $N$ retailers located along a fixed delivery route. They study the risk pooling effect of using dynamic policy comparing with the usual static allocation policy. Under the dynamic policy, allocation decision for each store is delayed until the delivery vehicle arrives at that store.

Despite the progresses made in the research area of centralized ordering inventory systems, the following basic question remains to be answered: under what

circumstances should we use centralized ordering instead of individual ordering for multilocation inventory systems with centralized information? While the system-wide ordering set-up cost is expected to be lower for centralized ordering, we are not sure if centralized ordering will perform as well as individual ordering regarding other inventory costs including inventory holding and penalty costs. In centralized ordering, replenishment decisions are made based on the total system stock only. Therefore, all the information about each store's inventory level is not fully utilized. From the perspective of individual stores, the ordering decisions might not be optimal. Although the imbalance of inventories at different stores is adjusted during the subsequent allocation process, the overall efficiency of this strategy has to be evaluated against the individual ordering policies.

In this chapter, we focus on the comparative analysis of individual versus centralized ordering policies for the distribution system. This study builds on, and integrates the existing research by establishing general guidelines for making replenishment decisions. Problems related to transshipment/redistribution will be considered in the next chapter.

Since the company already has the information network that allows for centralized decision making, we will not discuss any information requirements in our analysis. Instead, we want to decide whether or not ordering decisions should be centralized based on the consideration of inventory costs, which include ordering set-up costs, inventory holding cost, and shortage cost for stockouts. Since it is not our major concern to do any detailed analysis on the risk pooling effect, we will assume instantaneous replenishments and allocations to simplify the analysis.

**3.2 INVENTORY MODELS**

We consider a periodic-review multilocation inventory system. Demand at each store is assumed to be independent and identically distributed (IID) random variables with either Poisson or negative binomial distribution functions. In the literature, it is common to assume demand follows normal distribution. However, this is not an appropriate model for items with low demand and high variation because a significant portion of the demand would take on negative values according to the normal distribution. The Poisson distribution is a common model for low demand items, but it requires that the variance be the same as the mean of the demand distribution. For items with significantly larger variances we will assume the negative binomial distribution because its variance is greater than its mean. We also assume that linear costs are incurred for holding inventory or for backorders. An ordering set-up cost, which includes the review and administrative cost, will be assessed. The following notations are used in the model formulation:

$K$: ordering set-up cost;
$N$: number of stores;
$i$: the index of the set of stores: $i$=1,…,$N$;
$t$: the index of the time periods; $t$=1,…,$T$;
$s$: reorder point;
$S$: order-up-to level;
$h$: unit holding cost per period;
$p$: unit stockout cost per period;
$\mu$: mean of demand per period at a store;
$\sigma$: standard deviation of demand per period at a store;
$x_i$: inventory level at location $i$ at the beginning of a period;
$x^t$: $(x_i)_{i=1}^N$ vector of stores inventory levels;
$z_i$: allocation to location $i$ at period $t$. $z^t$: vector of stores allocations;
$u_i$: demand at location $i$. $u^t$: vector of stores demand;
$\varphi(\xi)$: probability density function (pdf) of the demand at a store;
$\Phi(\xi)$: cumulative distribution function (cdf) of the demand at a store;
$c(x)$: total cost for ordering $x$ units, $c(x) = 0$ if $x = 0$, $K + cx$ if $x$>0;

### 3.2.1 INDIVIDUAL ORDERING

The individual ordering system is a typical one-echelon inventory system. Scarf (1960) shows that (*s*, *S*) type policy will be optimal for the system described above. In individual ordering, the inventory level at each store is reviewed and ordering decisions taken every week. Replenishment orders are filled from either a central depot or the manufacturer's warehouse.

We use a dynamic programming approach to compute the optimal (*s*, *S*) policies. Scarf (1963), among others, presents an excellent discussion about this technique in inventory theory. The procedure is briefly reviewed here. Considering a $T$-period inventory problem beginning with $x$ units of stock on hand, let $y$ represents the inventory level immediately after an order delivery. If we define $f_t(x)$ as the minimum expected cost from period $t$ to $T$, the total expected cost for the $T$-period problem is $f_1(x)$. Assume that unsatisfied demand is fully back ordered, and orders are delivered instantaneously, the following functional equation is obtained

$$f_t(x) = \min_{y \geq x}\{c(y-x) + L(y) + \int_0^{\infty} f_{t+1}(y-\xi)\varphi(\xi)d\xi\} \quad \text{and } f_{T+1}(\cdot) = 0 \tag{3.1}$$

where $L(y)$ is the expected one-period inventory holding and shortage costs

$$L(y) = \begin{cases} \int_0^{y} h(y-\xi)\varphi(\xi)d\xi + \int_y^{\infty} p(\xi - y)\varphi(\xi)d\xi & y \geq 0 \\ \int_0^{\infty} p(\xi - y)\varphi(\xi)d\xi & y < 0 \end{cases} \tag{3.2}$$

If we assume the delivery lag is of fixed length and is equal to $\lambda$ review periods, then the minimum expected costs not only depend on the starting inventory but also

on the outstanding orders $y_j$ ($j$=1,2, ... $\lambda$-1) due in the subsequent $\lambda$-1 periods. The state space of the dynamic programming will be $\lambda$ dimensional. The recursive calculations involved in the computation of optimal policies will be prohibitively long except for very small values of $\lambda$. Karlin and Scarf (1958) show that if excess demands are backlogged, the functional equation may be reduced to one involving only a single variable. Let $f_t(x, y_1, ..., y_{\lambda-1})$ represent the expected cost from period $t$ to $T$ if an optimal policy is followed, then

$$\begin{aligned} f_t(x, y_1, ..., y_{\lambda-1}) &= L(x) + \int_0^\infty L(x + y_1 - \xi)\varphi(\xi)d\xi + ... \\ &+ \int_0^\infty L(x + \sum_1^{\lambda-1} y_j - \xi)\varphi^{(\lambda-1)}(\xi)d\xi + g_t(x + \sum_1^{\lambda-1} y_j) \end{aligned} \tag{3.3}$$

where $\varphi^{(j)}$ represents density of demand over $j$ periods, and $g_t(u)$ satisfies the following functional equation:

$$g_t(u) = \min_{y \geq u}\{c(y-u) + \int_0^\infty L(y-\xi)\varphi^{(\lambda)}(\xi)d\xi + \int_0^\infty g_{t+1}(y-\xi)\varphi(\xi)d\xi\} \tag{3.4}$$

The optimal policy is determined from the on hand plus total stock on order. Other than replacing $L(y)$ by $\int_0^\infty L(y-\xi)\varphi^{(\lambda)}(\xi)d\xi$, the functional equation is identical with the zero lead time case. So precisely the same analysis can be applied. More specifically, we can view the lead time problem in the following way. Since the lead time is $\lambda$ periods, ordering decisions in period $t$ will affect the inventory costs only at period $t$+$\lambda$ and later. The inventory costs for the intervening $\lambda$ periods is not affected by the ordering decision, so they can be excluded from the consideration.

The only related cost will actually be incurred in period $t+\lambda$, but it is viewed as cost for period $t$. This transformation is a standard procedure for problems with time lag.

### 3.2.2 CENTRALIZED ORDERING

Generally, we follow the procedure developed by Federgruen and Zipkin (1984c) in the analysis for centralized ordering system. The expected total cost of the system will depend on the vector of initial inventory levels at the stores $x^t$. If $y$ represents the order quantity at the warehouse, the following conditions must be satisfied:

$$\begin{gathered} y \geq 0, \ \sum_{i=1}^{N} z_i = y \\ z_i \geq 0, \ i = 1, \ldots, N \end{gathered} \tag{3.5}$$

The functional equation for the dynamic program of the problem is

$$\begin{gathered} f_t(x^t) = \min_{y, z^t} \{ c(y) + \sum_{i=1}^{N} L_i(x_i + z_i) + Ef_{t+1}(x^t + z^t - u^t) \} \\ \text{and } f_{T+1}(\cdot) = 0 \\ \text{subject to:} \\ \sum_{i=1}^{N} z_i = y, y \geq 0, z_i \geq 0 \end{gathered} \tag{3.6}$$

The state space of this dynamic program is $N$ dimensional. Exact solution of the equation is impossible except for small values of $N$ and $T$.

Following Federgruen and Zipkin (1984c), we apply an approximation by relaxing the non-negativity constraints on the allocation quantity $z_i$. Under this relaxation, the myopic allocation policy that minimizes the current period inventory costs is also optimal up to the approximation. The problem defining the myopic allocation policy with relaxation of non-negativity constraint is as follows:

$$R(x^t) = \min_{z^t}[\sum_{i=1}^{N} L_i(z_i + x_i)]$$
$$\text{subject to } \sum_{i=1}^{N} z_i = y \tag{3.7}$$

Let $k_i \equiv x_i + z_i$, and applying Lagrange multiplier $\lambda$ to include the constraint

$$R(x^t) = \min_{z^t}\{\sum_{i=1}^{N}[h\int_0^{k_i}(k_i - \xi)\varphi_i(\xi)d\xi + p\int_{k_i}^{\infty}(\xi - k_i)\varphi_i(\xi)d\xi] + \lambda(y - \sum_{i=1}^{N} z_i)\} \tag{3.8}$$

Taking the partial derivative with respect to $z_i$ and set to zero, we get

$$\Phi_i(k_i) = \frac{p+\lambda}{p+h} \tag{3.9}$$

We see $\Phi_i(k_i)$ is a constant for different stores. So the optimal allocation policy will ensure the same probability of stockout at each store.

The determination of the value of $k_i$ depends on the demand distribution. Federgruen and Zipkin (1984) assumes that there exists a common distribution function $F(\cdot)$ for $\Phi_i(k_i)$ such that

$$\Phi_i(k_i) = F(\frac{k_i - \mu_i}{\sigma_i}) \tag{3.10}$$

Clearly, normal demand will meet this condition. In that case $F(\cdot)$ is the standardized normal distribution function. As they point out the above assumption is satisfied for several other classes of demand distributions under special circumstances. Federgruen and Zipkin (1984c) then proceed with this assumption to get the one-period inventory holding and shortage costs. It can be easily shown that the cost function will depend on the vector of inventory positions $x^t$ only via its sum $X_t$ (not its distribution among the stores). Also, $R(x^t)$ takes the form of a single

location one-period cost function, with mean of demand $\sum_{i=1}^{N}\mu_i$, and variance of demand $(\sum_{i=1}^{N}\sigma_i)^2$. Reviewing the whole procedure, we can easily see that the assumption of Equation (3.10) is critical for the whole analysis. However, this assumption is not valid for most demand distributions. For Poisson and negative binomial demand, we can not find such a distribution function of $F(\cdot)$. So we have to rely on a certain approximation again in order to solve the problem. Now, instead of using explicitly the assumption of equation (3.10) and a known common distribution function of $F(\frac{k_i-\mu_i}{\sigma_i})$, we will only assume that equation (3.9) will lead to the following equation:

$$\frac{k_i-\mu_i}{\sigma_i}=\frac{x_i+z_i-\mu_i}{\sigma_i}=c \tag{3.11}$$

Using the constraint on y, we have

$$\sum_{i=1}^{N}(x_i+z_i-\mu_i)=c\sum_{i=1}^{N}\sigma_i \rightarrow c=\frac{X_t+y-\sum_{i=1}^{N}\mu_i}{\sum_{i=1}^{N}\sigma_i} \tag{3.12}$$

and

$$k_i \equiv x_i+z_i=\mu_i+\frac{\sigma_i(X_t+y-\sum_{i=1}^{N}\mu_i)}{\sum_{i=1}^{N}\sigma_i} \tag{3.13}$$

We see that inventory levels at different stores only depend on the total stock of the system after the order delivery. And the expected one-period inventory cost of the system $R(x^t)$ will also depend only on the total stock of the system. It is easy to

show that the minimum expected inventory cost of the system depends only on the total stock of the system and the myopic allocations are optimal in all periods in the resulting problem up to the approximation by straightforward induction.

Although $R(x^t)$ is dependent on the vector of inventory positions only via its sum, generally it will not take the form of a single location cost function. However, if we follow Federgruen and Zipkin's (1984c) assumption, $R(x^t)$ will be in the form of a single location cost function. Consider the following example where the demand at each store follows normal distribution. Then the common function of $F(\frac{k_i - \mu_i}{\sigma_i})$ is the standardized normal distribution function. $R(X_t)$ is determined from the from the following equation

$$R(X_t) = \sum_{i=1}^{N}[h\int_0^{k_i}(k_i - \xi)\varphi_i(\xi)d\xi + p\int_{k_i}^{\infty}(\xi - k_i)\varphi_i(\xi)d\xi]$$

$$\text{where} \quad k_i = \mu_i + c\sigma_i = \mu_i + \frac{\sigma_i(X_t + y - \sum_{i=1}^{N}\mu_i)}{\sum_{i=1}^{N}\sigma_i} \tag{3.14}$$

The one-period inventory cost $L(k_i)$ for store $i$ is

$$L(k_i) = L(\mu_i + c\sigma_i) = h\int_0^{k_i}(k_i - \xi)\varphi_i(\xi)d\xi + p\int_{k_i}^{\infty}(\xi - k_i)\varphi_i(\xi)d\xi \tag{3.15}$$

Rearrangement yields

$$\begin{aligned} L(k_i) &= h(k_i - \mu_i) + (h + p)\int_{k_i}^{\infty}(\xi - k_i)\varphi_i(\xi)d\xi \\ &= hc\sigma_i + (h + p)\sigma_i E(c) \end{aligned} \tag{3.16}$$

Where $E(c)=\int_c^\infty (t-c)\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt$, the unit normal loss function.

Substituting (3.16) into (3.14)

$$R(X_t)=\sum_{i=1}^{N}L(k_i)=hc\sum_{i=1}^{N}\sigma_i+(h+p)E(c)\sum_{i=1}^{N}\sigma_i \tag{3.17}$$

However, Equation (3.17) is exactly the expected one-period inventory cost function for a single location where the demand follows normal distribution with mean of $\sum_{i=1}^{N}\mu_i$ and standard deviation $\sum_{i=1}^{N}\sigma_i$.

After the myopic allocation assumption and approximation, the recursive equation of the dynamic program is finally formulated as

$$f_t(X_t)=\min_{y\ge 0}\{c(y)+\sum_{i=1}^{N}[h\int_0^{k_i}(k_i-\xi)\varphi_i(\xi)d\xi+p\int_{k_i}^{\infty}(\xi-k_i)\varphi_i(\xi)d\xi]$$
$$+\int_0^\infty f_{t+1}(X_t+y-\xi)\varphi^{(N)}(\xi)d\xi\}$$
$$\text{and} \tag{3.18}$$
$$f_{T+1}(\cdot)=0$$
$$\text{where}\quad k_i=\mu_i+\frac{\sigma_i(X_t+y-\sum_{i=1}^{N}\mu_i)}{\sum_{i=1}^{N}\sigma_i}$$

We see the multilocation inventory system is finally modeled as a single location problem with 1-dimensional state space $X_t$ and one decision variable $y$. The problem is then solved easily through the dynamic programming technique.

It should be pointed out that the above formulation would result in a lower bound of the inventory cost because the non-negativity constraint of allocation quantities is relaxed in our approximation. Generally, the approximation is quite

good if the coefficients of variation of the demand distributions at various stores are close. Eppen and Schrage (1981) provide some detailed analysis for a closely related issue. If imbalance of inventories at the stores is very large, the ideal stock levels for various stores, which depend only on the total stock of the system, might not be attainable. In that case, negative allocation quantities are required for certain stores. It means some inventories must be withdrawn from these stores and reallocated to other stores. However, this is infeasible without using any other inventory adjustment measures such as transshipments or redistribution.

A complete specification of inventory policies will require that ordering policies be complemented with a feasible allocation policy. So after we obtain the approximate optimal ordering policies we need to revisit the problem of allocation policies. Federgruen (1993) suggests the myopic policy be used, i.e, every incoming order is allocated to minimize the expected cost of the very first period after the shipment arrives. However, the non-negativity constraint on allocation quantities must be re-imposed. This problem is a typical knapsack problem with separable convex objective.

## 3.3 COMPUTATIONAL RESULTS AND DISCUSSION

### 3.3.1 COMPUTATIONS

The dynamic program is solved by the successive approximation method. Calculation is stopped when the average cost per period is stabilized. The optimal inventory policy is reached and stabilized usually within a few periods. In our calculation, we assume identical stores for simplicity. The computation results are summarized in Table 3.1. For individual ordering with $N$ identical stores, the total

Table 3.1 Computational Results for Centralized and Individual Ordering when $K_c$=$K$ ($h$=1, $\sigma^2/\mu$=2 for Negative Binomial Distributions)

| Factors considered | | | | Individual ordering ($N$=1) | | | Centralized ordering ($N$=2) | | | Centralized ordering ($N$=10) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***K*** | ***P*** | **Dist.** | **Mean** | ***s*** | ***S*** | **cost** | ***s*** | ***S*** | **cost** | ***s*** | ***S*** | **cost** |
| 1 | 9 | Poisson | 10 | 12 | 14 | 6.9 | 25 | 28 | 12.7 | 139 | 140 | 59.7 |
| 10 | 9 | Poisson | 10 | 9 | 22 | 15.6 | 21 | 29 | 21.7 | 129 | 140 | 68.7 |
| 1 | 99 | Poisson | 10 | 16 | 18 | 10.3 | 33 | 36 | 19.7 | 179 | 180 | 94.4 |
| 10 | 99 | Poisson | 10 | 14 | 18 | 19.3 | 31 | 37 | 28.7 | 169 | 180 | 103.4 |
| 1 | 9 | NegBin | 10 | 13 | 16 | 9.8 | 29 | 33 | 18.7 | 159 | 160 | 89.5 |
| 10 | 9 | NegBin | 10 | 10 | 21 | 17.3 | 23 | 33 | 27.6 | 139 | 160 | 98.5 |
| 1 | 99 | NegBin | 10 | 20 | 23 | 16.0 | 41 | 47 | 31.0 | 219 | 230 | 150.8 |
| 10 | 99 | NegBin | 10 | 17 | 27 | 23.9 | 37 | 47 | 39.8 | 209 | 230 | 159.8 |
| 1 | 9 | Poisson | 20 | 23 | 26 | 9.2 | 49 | 52 | 17.4 | 259 | 260 | 82.9 |
| 10 | 9 | Poisson | 20 | 19 | 26 | 18.2 | 43 | 52 | 26.4 | 239 | 260 | 91.9 |
| 1 | 99 | Poisson | 20 | 29 | 31 | 13.9 | 59 | 62 | 26.7 | 309 | 310 | 129.7 |
| 10 | 99 | Poisson | 20 | 26 | 31 | 22.9 | 55 | 62 | 35.7 | 299 | 310 | 138.7 |
| 1 | 9 | NegBin | 20 | 25 | 28 | 13.2 | 53 | 56 | 25.3 | 279 | 280 | 122.5 |
| 10 | 9 | NegBin | 20 | 20 | 28 | 22.1 | 45 | 56 | 34.3 | 259 | 280 | 131.5 |
| 1 | 99 | NegBin | 20 | 34 | 37 | 20.9 | 71 | 74 | 40.8 | 369 | 370 | 200.0 |
| 10 | 99 | NegBin | 20 | 30 | 37 | 29.9 | 65 | 74 | 49.8 | 349 | 370 | 209.0 |

cost of the inventory system is $N$ times as large as the inventory cost for one store. From the table, we see the total inventory cost for centralized ordering system is always lower than that of individual ordering. This is not very surprising. Since we assume same ordering set-up costs in our calculation, centralized ordering has the advantage of placing fewer orders at the warehouse than individual ordering at each store.

From the consideration of inventory review and other administrative costs, it might be more reasonable to assume higher ordering set-up costs for centralized ordering. It is of interest to find when the total cost for centralized ordering will be the same as that of individual ordering. So we increase the ordering set-up costs for centralized ordering ($K_c$) until the total cost of the inventory system is the same for centralized and individual ordering. The computation results for $N$-identical-store ($N$=2,10 respectively) inventory systems are summarized in Table 3.2. It shows that when $K_c$ happens to be $N*K$ ($K$ is the ordering set-up costs at each store) centralized ordering will have the same total inventory cost as in individual ordering. If $K_c < N*K$, centralized ordering will lead to lower inventory costs, and vice versa. Judging from this result, we conclude that up to the allocation assumption and approximation the ordering costs alone determine the system's performance. Centralized ordering will achieve the same system-wide inventory holding and penalty costs as that of individual ordering.

### 3.3.2 IMPORTANT ASSUMPTIONS

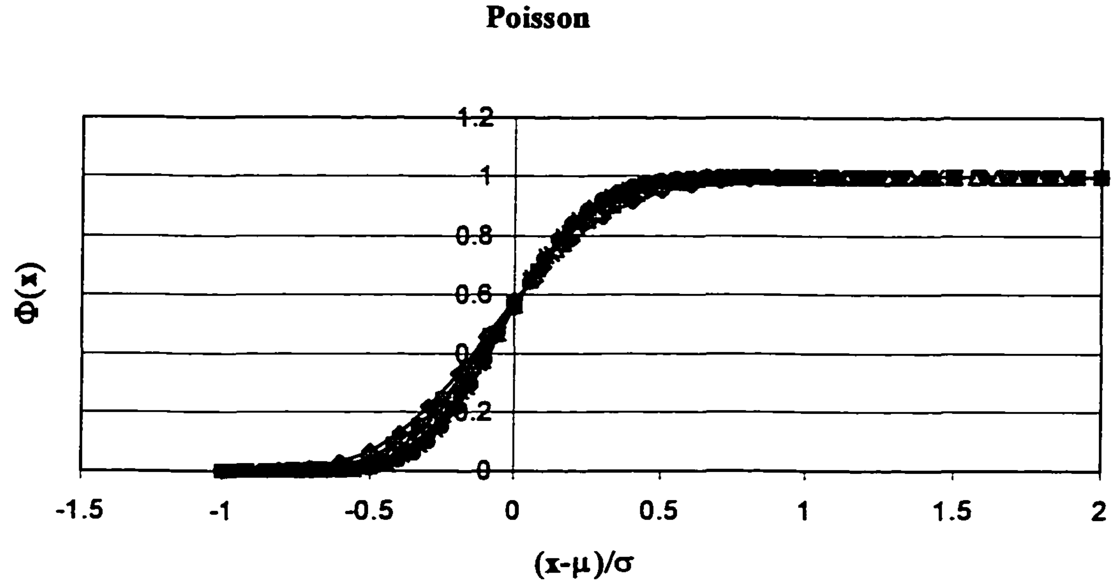
Federgruen and Zipkin (1984) required that for $\Phi_i(k_i) = c$ a common distribution function $F(\frac{k_i - \mu_i}{\sigma_i})$ exists. This assumption is relaxed in our analysis.

Table 3.2 Computational Results for Centralized and Individual Ordering When $K_c=N*K$ ($h=1$, $\sigma^2/\mu=2$ for Negative Binomial Distributions)

| Factors considered | | | | Individual ordering ($N$=1) | | | Centralized ordering ($N$=2) | | | Centralized ordering ($N$=10) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***K*** | ***P*** | **Dist.** | **Mean** | ***s*** | ***S*** | **cost** | ***s*** | ***S*** | **cost** | ***s*** | ***S*** | **cost** |
| 1 | 9 | Poisson | 10 | 12 | 14 | 6.9 | 25 | 28 | 13.7 | 129 | 140 | 68.7 |
| 10 | 9 | Poisson | 10 | 9 | 22 | 15.6 | 19 | 46 | 31.2 | 99 | 140 | 158.7 |
| 1 | 99 | Poisson | 10 | 16 | 18 | 10.3 | 33 | 36 | 20.7 | 169 | 180 | 103.4 |
| 10 | 99 | Poisson | 10 | 14 | 18 | 19.3 | 29 | 36 | 38.7 | 149 | 180 | 193.4 |
| 1 | 9 | NegBin | 10 | 13 | 16 | 9.8 | 27 | 32 | 19.7 | 139 | 160 | 98.5 |
| 10 | 9 | NegBin | 10 | 10 | 21 | 17.3 | 19 | 46 | 35.4 | 109 | 241 | 175.3 |
| 1 | 99 | NegBin | 10 | 20 | 23 | 16.0 | 41 | 46 | 32.0 | 209 | 230 | 159.8 |
| 10 | 99 | NegBin | 10 | 17 | 27 | 23.9 | 35 | 60 | 48.7 | 179 | 230 | 249.8 |
| 1 | 9 | Poisson | 20 | 23 | 26 | 9.2 | 47 | 52 | 18.4 | 239 | 260 | 91.9 |
| 10 | 9 | Poisson | 20 | 19 | 26 | 18.2 | 39 | 52 | 36.4 | 199 | 260 | 182.0 |
| 1 | 99 | Poisson | 20 | 29 | 31 | 13.9 | 59 | 62 | 27.7 | 299 | 310 | 138.7 |
| 10 | 99 | Poisson | 20 | 26 | 31 | 22.9 | 53 | 62 | 45.7 | 269 | 310 | 228.8 |
| 1 | 9 | NegBin | 20 | 25 | 28 | 13.2 | 51 | 56 | 26.3 | 259 | 280 | 131.5 |
| 10 | 9 | NegBin | 20 | 20 | 28 | 22.1 | 41 | 56 | 44.3 | 209 | 280 | 221.6 |
| 1 | 99 | NegBin | 20 | 34 | 37 | 20.9 | 69 | 74 | 41.8 | 349 | 370 | 209.0 |
| 10 | 99 | NegBin | 20 | 30 | 37 | 29.9 | 61 | 74 | 59.8 | 309 | 370 | 299.0 |

Instead of explicitly using function of $F(\frac{k_i - \mu_i}{\sigma_i})$, we only require that $\frac{k_i - \mu_i}{\sigma_i}$ be constant. In Figure 3.2, we show the relationship between the distribution function $\Phi(x)$ and the variable of $(x-\mu)/\sigma$ for six different Poisson and Negative binomial distribution functions respectively. The curves are well overlapped if the mean of the demand is not too small ($\mu \geq 10$). So if $\Phi(\cdot)$ is a constant we can reasonably assume that $(x-\mu)/\sigma$ is also a constant. Hence this is a good approximation for both Poisson and negative binomial distribution functions. However, this good approximation does not imply the validity of the existence of a common distribution function of $F(\frac{k_i - \mu_i}{\sigma_i})$. There is no such distribution function for both Poisson and negative binomial demands.

We note that although our assumption differs from that of Federgruen and Zipkin (1984c), both of us essentially assume that inventory policies do not dependent on the skewness of demand distributions. So it requires only knowledge of the mean and variance of demand distributions to determine inventory policies. This is basically a normal approximation that ignores the possible effect of higher moments of demand distributions. This approximation seems to work sufficiently well for our purposes as shown above. Schneider and Ringuest (1990) consider a similar problem by examining the relationship between reorder point and demand distributions through regression analysis. Their computational results show that normal approximation works well for Poisson distribution. For negative binomial distribution, they improve the normal approximation by adding an adjustment term



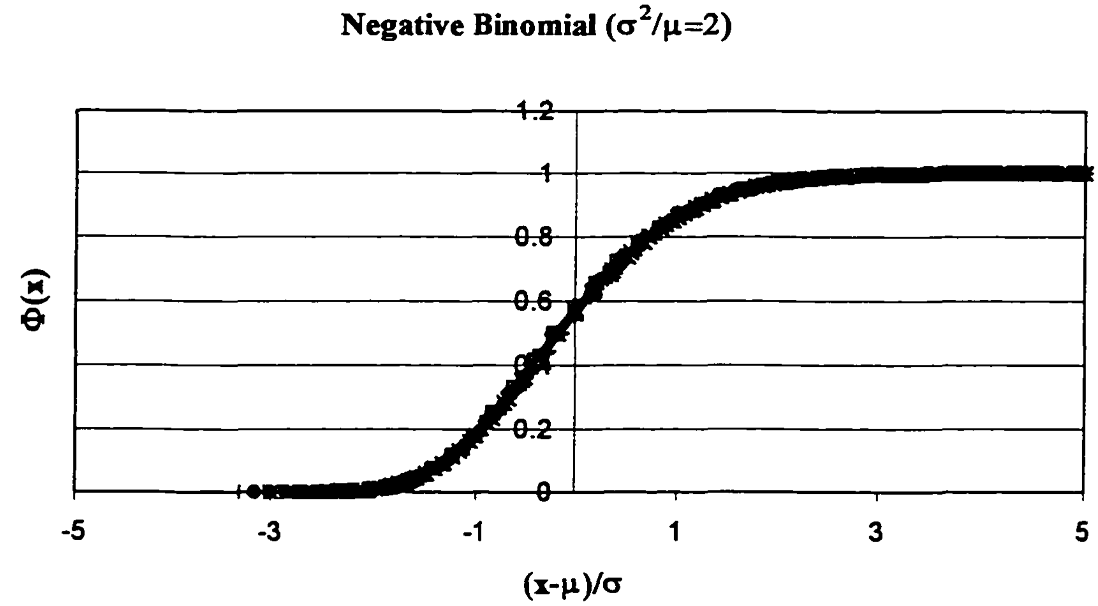




Figure 3.2 $\Phi(x)$ vs. $(x-\mu)/\sigma$ ($\mu$=10,12,14,16,18,20)

that accounts for the skewness of demand distributions measured by the variance to mean ratio. A similar subject is also discussed by Ehrhardt (1979).

### 3.3.3 EXTENSIONS

Extensions to positive allocation lead times are straightforward. The standard transformation technique discussed in decentralized ordering systems can be applied without any difficulty. However, it is not so simple to incorporate positive ordering lead times in our analysis. The reason for this problem is that the inventory system has to be characterized differently during the two different periods, i.e., the periods of ordering delay and the periods of allocation delay. In centralized ordering system, there is a risk pooling effect in the period of ordering delay. However, in the following allocation period, replenishment stocks are separated and shipped to each store (based on the store's inventory level and the allocation policy). Since the allocations are related to each individual store's operation no risk pooling effect is involved. Because of this difference care should be taken in the proper aggregation of demands in different periods. Since the mean/average of the aggregated demand can always be determined easily we only discuss variance of the aggregated demand. The variance of the aggregated demand of the $N$ stores over the ordering lead time is the summation of variance from each store. In contrast, the standard deviations (not variance) of the N stores are summed to get the standard deviation (not variance) of the aggregated demand over the allocation time lags. In our model formulations, the expected one-period inventory cost of the system is expressed as the summation of the costs of the $N$ stores. For each store, the standard transformation can be applied directly to model the

allocation lead time. It will not cause any problem because each store is treated separately during the period of allocation delays. However, if we apply the same method to each store individually during the period of ordering lead time, we essentially neglect the risk pooling effect of centralized ordering. The resulting cost is thus the upper bound of the cost of the inventory system. If the expected one-period inventory cost of the distribution system can be approximately represented by a single location cost function with a certain demand distribution then the same transformation may be applied to deal with the problem of ordering lead time.

## 3.4 CONCLUSION

In this chapter, an analysis is carried out to compare the overall performance of individual versus centralized ordering policies for a multilocation inventory system. It is found that ordering costs alone determine the system's performance. The total cost of the inventory system will be the same for both ordering policies when centralized ordering set-up costs equal to the summation of ordering set-up costs at each store in individual ordering. The above result is valid for systems without risk pooling effects (zero lead time). Allocation lead times do not affect this result. But ordering lead times will provide additional advantage for a centralized ordering inventory system. Issues related to the extension of the model are also discussed.

# CHAPTER 4

# MULTILOCATION INVENTORY SYSTEMS WITH TRANSSHIPMENTS

## 4.1 PROBLEM DESCRIPTION

In Chapter 3, we presented the background information of a distribution system that motivated this dissertation research. We also performed a comparative analysis of individual versus centralized ordering policies for this multi-store distribution system. Since the main objective of that study is to provide a general guideline on ordering policies for multilocation inventory systems with centralized information, we do not specifically address the issue of stock-out procedure in our analysis. We simply assume that if a store experiences a stock-out for a part, the part is expedited through an emergency order to the manufacturer's warehouse. In practice, placing an emergency order will be very costly to the distributor. In contrast with the regular replenishments for which the manufacturer pays the freight charge, the distributor has to pay the overnight freight charge and also an expedite premium for an emergency order. So in real operations, when a store experiences a stock-out for a part, other stores are checked first. If the part is available at another store, then it is pulled and transshipped. If the part is not available at any other location then the original store will place an emergency order to the manufacturer's warehouse. To deal with the requirement of transshipments, the company has negotiated a contract with a local trucking firm to handle all the daily shipping duties among these stores with monthly charges based on the total transshipment quantities. With this shipping arrangement, transshipments among the stores are essentially instantaneous since all deliveries are overnight.

Transshipments provide the opportunity of using stocked items at nearby stores without the need of placing emergency orders. Thus it may lead to savings on freight and premium charges (These costs are considered to be the penalty costs for stock-outs in our analysis) as well as inventory holding costs. However, these savings must be balanced against the transshipment costs. In addition, one should also consider the difficulties involved in the determination of inventory policies for distribution systems with transshipments.

In this chapter, we try to integrate both inventory and transshipment components in our analysis of the distribution system introduced in Chapter 3. The objective of this study is to determine the optimal inventory policies that minimize joint inventory and transshipment costs. We start with base stock policies by assuming zero ordering set-up costs. In that case, individual ordering is preferred because it does not involve any allocation constraints that might affect centralized ordering (although this is generally not a big issue as discussed in Chapter 3). We first formulate a general model that integrates inventory and transshipment problems for multi-store inventory systems using individual-ordering, base-stock policies. Since the general model is extremely difficult to work with analytically, we make some simplifying assumptions based on the real operation of the distribution system we studied. With these simplifications, optimal base stock policies can be obtained easily. Some qualitative results about the effect of transshipments on base stock policies are presented.

Next, we extend the analysis to $(s, S)$ type policies for the inventory system with non-negligible ordering set-up costs. As is shown in Chapter 3, centralized

ordering offers the advantage of economies of scale in ordering costs. So we study (*s*, *S*) type policies by assuming centralized inventory control of the distribution system. A dynamic programming technique is used to obtain approximately optimal (s, S) policies. Numerical studies are provided to give general guidelines for use of the policies.

In the literature, very few inventory models adequately address the multilocation, multiperiod stochastic inventory system with emergency transshipments. Most research focuses on a single period analysis of multilocation inventory systems where transshipments are used as an anticipatory measure to balance the inventory levels at the stores. Allen (1958) considers the problem of stock redistribution to minimize transshipments and shortage costs. Gross (1963) determines both optimal redistribution and replenishments policies for a two-store inventory system. Das (1975) studies a similar system where transshipment is permitted at a certain predetermined time in a period. Hoadley and Heyman (1977) develop a general single period model that allows for stock purchasing and disposing at the warehouse, returning or shipping items between the warehouse and various stores, and transshipments among the stores. Karmarkar and Patel (1977) reexamine the two-location problem of Gross (1963) and develop a simpler solution procedure. Karmarkar (1979) also provides a qualitative analysis of the more general convex programming problems. Structural properties and computational issues associated with the general multiperiod inventory problems are discussed by Karmarkar (1981,1987). Jönsson and Silver (1987) examine a centralized ordering inventory system with emphasis on adjusting the imbalance of inventory levels at the stores through stock

redistribution. They assume a base stock replenishment policy and a predetermined order cycle of certain time periods. They consider a complete redistribution of stocks one period before the end of an order cycle to balance the inventory levels at all stores. It is shown that a substantial reduction in safety stock is possible by using redistribution.

In many practical operations, transshipments are used as an emergency measure to stockout situations after demands are realized. Cohen, Kleindorfer, and Lee (1986) consider a complex multi-echelon inventory system for managing low-demand, high-value items. They develop a one-period inventory model that considers pooling mechanisms for shortages, emergency and normal replenishments, and stock repositioning. Transshipments are allowed only among the locations belonging to a "location grouping". They obtain near-optimal base stock policies for different locations subject to a service level constraint. Extensions to ($s$, $S$) type policies are considered by Cohen et al. (1992). Tagaras and Cohen (1992) discuss the possible effects of replenishment lead times on pooling policies. They perform simulation analysis on different pooling policies for a two-location inventory system. Some researchers extend the basic METRIC model of continuous review, one-for-one replenishment inventory systems for repairable items to allow lateral transshipments. They include Lee (1987), Axsäter (1990), and Sherbrooke (1992).

The most directly relevant studies to this research are provided by Krishnan and Rao (1965), and Robinson (1990). Krishnan and Rao (1965) consider the determination of optimal base stock policies that would minimize the one period inventory and transportation costs for inventory systems with emergency

transshipments. They assume equal transshipment rate between any two stores. Robinson (1990) extends their analysis to allow for multiple periods, and varying transshipment rates across the stores. He assumes individual ordering at each store according to base stock policies, and obtains near optimal ordering and transshipment policies using approximations of linear programming. In this research, we also assume equal transshipment rates across the stores as in Krishnan and Rao (1965), but our main interest is in studying centralized-planning, $(s, S)$-type ordering policies for a multiperiod, multilocation inventory system with emergency transshipments. The key issue of this study is to investigate the effects of transshipment costs on the optimal ordering policies that would minimize joint inventory and transshipment costs.

For computational tractability, the system is approximated by aggregating demand throughout the week and allowing for a single transshipment opportunity at the end of the week right after the weekly inventory review. The sequence of events in any period is delivery, demand, review, transshipment, and order. Demands at the stores are assumed to be independently and identically distributed random variables with either Poisson or negative binomial distribution functions. To simplify the analysis, we also assume that both replenishment and transshipment lead times are zero. Linear holding or shortage costs are incurred for each store based on its ending inventory level. Excess demand for the system is fully backlogged.

The following notations are used in our model formulations:

$h$: unit inventory holding cost per period;
$p$: unit inventory shortage costs per period;
$K$: ordering set-up cost;
$\tau$: unit transshipment cost;

| | |
|---|---|
| $i$: | the index of the set of stores: $i$=1, ..., $N$; |
| $t$: | the index of the time periods; |
| $s$: | reorder point; |
| $S$: | order-up-to point; |
| $\mu$: | mean of demand per period at a store; |
| $\sigma$: | standard deviation of demand per period at a store; |
| $x$: | starting inventory level at a store, $x^t \equiv (x_i)_{i=1}^N$ corresponding vector for $N$ stores; |
| $y$: | inventory level right after an order delivery at a store, $y^t \equiv (y_i)_{i=1}^N$ corresponding vector for $N$ stores; |
| $z$: | allocation quantity, $z^t \equiv (z_{it})_{i=1}^N$ vector of allocation quantities; |
| $w_{ij}$: | transshipment quantity from store i to store j, $w^t$: matrix of transshipment quantities; |
| $d$: | random demand, $d^t \equiv (d_i)_{i=1}^N$ vector of random demands of $N$ stores; |
| $u$: | inventory position after the demand is observed but before any recourse action is taken. |
| $\varphi(\xi)$: | probability density function (pdf) of the demand at a store; |
| $\varphi^{(N)}(\xi)$: | convolution of demand density functions of $N$ stores; |
| $\Phi(\xi)$: | cumulative distribution function (cdf) of the demand at a store; |
| $c(x)$: | total cost for ordering $x$ units, $c(x) = 0$ if $x = 0$, $K + cx$ if $x$>0. |

## 4.2 MODEL FORMULATION - BASE STOCK POLICY

### 4.2.1 GENERAL FORMULATION

The characterization of inventory systems with transshipments requires the determination of both inventory and transshipment policies. If we assume base stock policies for inventory replenishments, then we need to determine the optimal order-up-to point for each part. With regard to transshipment decisions, we need to specify the directions and quantities involved in transshipments among different stores.

Considering a general $T$-period problem with starting inventory vector of $x^t$ at $N$ stores, define $f_t(x^t)$ as the minimum expected total cost from period $t$ to the end of the horizon, which is denoted as period $T$, then the minimum expected total cost of the $T$-period problem is $f_1(x^1)$. The general formulation of inventory systems with transshipments can be stated as

$$f_t(x^t) = \min_{y^t \geq x^t, w^t \geq 0} \{ \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \tau_{ij} w_{ij} + \sum_{i=1}^{N} [h \int_0^{q_i} (q_i - \xi) \varphi_i(\xi) d\xi +$$

$$p \int_{q_i}^{\infty} (\xi - q_i) \varphi_i(\xi) d\xi] + E[f_{t+1}(q_i - \xi)] \}, \text{ and } f_{T+1}(\cdot) = 0 \qquad (4.1)$$

where

$$q_i = y_i + \sum_{\substack{j=1 \\ j \neq i}}^{N} (w_{ji} - w_{ij}).$$

Although it is theoretically possible to obtain the optimal ordering and transshipment policies simultaneously from the above optimization problem, the procedure is extremely difficult. It requires an integrated solution of the stochastic convex model with linear recourse constraints.

The problem can somewhat be simplified under certain conditions. Robinson (1990) demonstrated that the optimal base stock policies would be stationary as long as the order-up-to point for the last period is non-negative. This non-negativity condition is usually met for most of the retail distribution systems in practice. Based on the above observation, the general problem is essentially decomposed into two simpler problems. First, we only need to consider the last time period in the determination of optimal base stock policies. This requires the solution of a single-period model of the multilocation inventory system with transshipments. After the determination of optimal ordering policies, the general problem reduces to stochastic linear programming problems of transshipments.

Even with the above simplification, the stochastic convex model of the single-period, multilocation inventory system with transshipments remains difficult to solve analytically. Some numerical solution techniques based on certain simplifications and approximations are needed in order to obtain near-optimal

ordering and transshipment policies simultaneously from the above equation. Generally, these solution procedures require a lot of computations. If they were to be implemented in a typical inventory system with thousands of stock keeping units (SKUs), the computations involved would be prohibitively expensive.

### 4.2.2 SIMPLIFYING ASSUMPTIONS

From a managerial point of view, the major difficulty of managing inventory systems with emergency transshipments is to determine the optimal ordering policies that minimize inventory and transshipment costs. Since we consider transshipments as a recourse action to stockouts after demands are observed, transshipment decisions can easily be determined based on intuitive judgements under the assumption of zero replenishment lead time. Some complications may arise if the replenishment lead time is non-negligible. In that case, transshipment decisions may affect subsequent inventory costs.

In the following analysis, we try to simplify the general transshipment problem by focusing on the overall effect of transshipments on the inventory ordering policies without considering the details of transshipments. For that purpose, some further assumptions are introduced as follows:

(1). Cost parameters for all stores are the same.

(2). Transshipment costs are linear and proportional to the quantities involved.

(3). Transshipment rates are the same between every pair of stores.

(4). Transshipment costs are less than the savings from inventory holding and shortage costs so that it is always desirable to apply transshipments as long as opportunities exist.

With these assumptions, complexities related to transshipment decisions are avoided. We know the optimal transshipment policies are shipping from stores with excess stocks to those with shortages until either the system wide available stocks or shortages at the end of a review period deplete. Then, the system-wide transshipment costs will only depend on the total expected transshipment quantities involved.

Since the above assumptions are crucial for our analysis some explanations may be necessary. Assumptions (1) and (2) are very common in the literature, they are generally representative of many real situations. Assumption (4) is necessary. Otherwise, there is no reason to consider transshipments. Assumption (3) appears to be the most restrictive one. Although it is a good assumption for the contractual transshipments arranged by the company we studied, generally transshipment rates would be different between different pairs of stores because of the difference in distances. Since transshipments are typically used among retail outlets in close proximity the difference in distances, hence transshipment rates, are expected to be small. Then we may simply use the average rate as an approximation. This is a reasonable approximation because we are only interested in the overall effect of transshipment costs on the optimal ordering policies.

### 4.2.3 SIMPLIFIED FORMULATION

According to Robinson (1990), the optimal order-up-to point of the base stock policy can be obtained from a single period inventory model. The total cost of the inventory system consists of transshipment costs as well as inventory holding and shortage costs. Define $e$ to be an $N$-vector of 1's, $d^t$ as the vector of random

demands at $N$ stores. Let $u^t = y^t - d^t$, $(x)^+ = \max(0, x)$, and $(x)^- = \max(0, -x)$ (If $x$ is a vector, the maximization is taken component by component). Also, define $H(u^t)$ to be the total cost of the inventory system then

$$H(u^t) = \tau[\min\{e \cdot (u^t)^+, e \cdot (u^t)^-\}] + h(e \cdot u^t)^+ + p(e \cdot u^t)^- \tag{4. 2}$$

Since

$$e \cdot (u^t)^+, e \cdot (u^t)^-\} = e \cdot (u^t)^+ - (e \cdot u^t)^+ \tag{4. 3}$$

Equation (4.2) can be rewritten as

$$H(u^t) = (h - \tau)(e \cdot u^t)^+ + p(e \cdot u^t)^- + \tau e \cdot (u^t)^+ \tag{4. 4}$$

Then the minimum expected cost of the inventory system $f(x^t)$ is

$$\begin{aligned} f(x^t) = \min_{y^t \geq x^t} \{ [(h - \tau) \int_0^{Y_t} (Y_t - \xi) \varphi^{(N)}(\xi) d\xi + p \int_{Y_t}^{\infty} (\xi - Y_t) \varphi^{(N)}(\xi) d\xi ] + \\ \tau \sum_{i=1}^{N} \int_0^{y_i} (y_i - \xi) \varphi_i(\xi) d\xi, \text{ where } Y_t = \sum_{i=1}^{N} y_i \end{aligned} \tag{4. 5}$$

Taking the partial derivative of $f(x^t)$ with respect to $y_i$ and setting it to zero

$$\frac{\partial f(x_t)}{\partial y_i} = -p + (p + h - \tau)\Phi^{(N)}(Y_t) + \tau \Phi_i(y_i) = 0 \qquad i = 1, \ldots, N \tag{4. 6}$$

where $\Phi^{(N)}(\cdot)$ denotes the cumulative distribution function of the convolution of demands over the N outlets. Rearrange the above equation, we finally get the following result for the inventory system with transshipment using individual-ordering, base stock policies:

$$\begin{aligned} &\Phi_1(y_1) = \Phi_2(y_2) = \ldots = \Phi_N(y_N) \\ &and \\ &\Phi^{(N)}(Y_t) = \frac{p - \tau \Phi_i(y_i)}{p + h - \tau}. \end{aligned} \tag{4. 7}$$

If we know the demand distributions at the N stores we should be able to obtain the optimal order-up-to points for the $N$ stores simultaneously from equation (4.7). Even without any information about the demand distributions, we can still obtain some interesting results from the above equation. By definition

$$\Phi^{(N)}(Y_t) = \frac{p - \tau\Phi_i(y_i)}{p + h - \tau} \leq 1 \tag{4. 8}$$

So, $\tau \leq \frac{h}{1 - \Phi_i(y_i)}$. Because $\Phi^{(N)}(Y_t) = 1$ when $\tau = \tau_{max} = \frac{h}{1 - \Phi_i(y_i)}$, $\Phi^{(N)}(Y_t)$ must be a non-decreasing function of $\tau$. Then we have

$$\frac{d\Phi^{(N)}(Y_t)}{d\tau} = \frac{d}{d\tau}\{\frac{p - \tau\Phi_i(y_i)}{p + h - \tau}\} = \frac{p - (h + p)\Phi_i(y_i)}{(p + h - \tau)^2} \geq 0. \tag{4. 9}$$

This implies

$$\Phi_i(y_i) \leq p/(p + h). \tag{4. 10}$$

It is well known that $\Phi_i(y_i) = p/(p + h)$ for inventory systems where transshipments among the retail outlets are not allowed. So, the optimal order-up-to points of the stores for an inventory system with transshipments must be equal or less than that of the corresponding inventory system without transshipments.

It is also very interesting to compare the result with that of a centralized control inventory system. We know from Equation (4.7) of the following optimal replenishment policies for an individual ordering inventory system. Every store orders up to the same fractile points of the demand distributions, and the total inventory of the system is raised to a certain base stock level of a virtual location with aggregated demand over all of the N stores. For the centralized ordering system, replenishment decisions are made based on the total stock of the system and

the aggregated demands of the stores. The arriving stocks are then properly allocated to different stores. It is well known that the optimal allocation policy is to achieve the same fractile point of the demand distributions for all stores (see for instance, Eppen and Schrage, 1981). So, if we view the order-up-to points for various stores in the individual ordering system as the stores' inventory levels right after the allocations in the centralized planning system, then the results are the same for both systems where no fixed ordering costs are involved.

### 4.3 MODEL FORMULATION - (*s*, *S*) POLICY

Generally, it is very difficult to determine the optimal (*s*, *S*) policy that minimizes the joint inventory and transshipment costs for multilocation inventory systems with centralized ordering. Before we do any further analysis on that subject, let us first consider an easier problem. It has been assumed that the transshipment lead-time is zero. If transshipment costs were also assumed to be zero, the problem would be solved easily. When transshipments are free and instantaneous, the multilocation inventory system can be treated as a single location problem with aggregated demand. Define $Y_t$ as the order-up-to point for the inventory system and $X_t$ as the total system inventory at the beginning of period $t$. The dynamic programming formulation for the single location problem with aggregated demand is as follows

$$\begin{aligned} f_t(X_t) = \min_{Y_t \ge X_t} \{ & c(Y_t - X_t) + h\int_0^{Y_t} (Y_t - \xi)\varphi^{(N)}(\xi)d\xi + \\ & p\int_{Y_t}^{\infty} (\xi - Y_t)\varphi^{(N)}(\xi)d\xi + Ef_{t+1}(X_{t+1})\}, \text{ and } f_{T+1}(\cdot) = 0 \end{aligned} \qquad (4.11)$$

The last term of the above minimization objective function represents the minimum expected costs for the remaining periods. It depends on the starting inventory levels at the outlets.

It is well known that $(s, S)$ type policies are optimal for this inventory system when ordering set-up costs are non-negligible (Scarf, 1960; and Iglehart, 1963). While the computation of the optimal $(s, S)$ policies is traditionally considered to be time-consuming and expensive, considerable progress has been made in developing efficient computational algorithms during the past two decades. Some recent efforts are reported in Federgruen and Zipkin (1984a), and Zheng and Federgruen (1991). In addition to exact methods, numerous approximations have been proposed (see for instance, Ehrhardt, 1979, Schneider, 1978, Schneider and Ringuest, 1990).

If transshipment costs are non-negligible they must be included in the inventory model. Furthermore, the characterization of centralized-ordering inventory systems requires the specification of allocation policies in addition to the designation of ordering policies. The simplified transshipment model for base stock policies in Section 4.2.3 can be easily extended to $(s, S)$ type policies. Adding the ordering set-up costs of $c(Y_t - X_t)$ to Equation (4.5) and extending it to the multiperiod analysis, we get

$$f_t(x^t) = \min_{y^t \geq x^t, z^t} \{c(Y_t - X_t) + (h - \tau)\int_0^{Y_t} (Y_t - \xi)\varphi^{(N)}(\xi)d\xi +$$

$$p\int_{Y_t}^{\infty} (\xi - Y_t)\varphi^{(N)}(\xi)d\xi + \tau[\sum_{i=1}^{N} \int_0^{y_i} (y_i - \xi)\varphi_i(\xi)d\xi] + Ef_{t+1}(x^{t+1})\}, \tag{4. 12}$$

$$\text{and } f_{T+1}(\cdot) = 0$$

$$\text{to} : \sum_{i=1}^{N} (x_i + z_i) = Y_t .$$

where $z_i$ represents the replenishment quantity that is allocated to store $i$, $y_i$ is the inventory level at store $i$ right after the delivery of $z_i$, and $Y_t = \sum_{i=1}^{N} y_i$ the order-up-to point of the inventory system.

The transition of inventory levels at stores is obtained from the following equation

$$x_{it+1} = y_{it} - \xi_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{N} (w_t^{ji} - w_t^{ij}),\ i = 1, \dots, N \tag{4. 13}$$

We see that it depends on the demand distribution as well as the transshipments involved. Since the only knowledge we have about transshipments is the expected total quantity involved, we can not specify each store's inventory level at the beginning of the next time period. Besides that, we also have the problem of "curse of dimensionality". The state space of the functional equation is $N$ dimensional. Except for very small $N$, exact solutions of the dynamic program are not attainable. So, we need to develop some approximation procedures to solve this problem. Here we will use an approximation based on the myopic allocation policy suggested by Federgruen and Zipkin (1984c).

The myopic allocation policy is to allocate the arriving replenishment stocks to different stores so that inventory costs for the current period are minimized. Myopic allocation policies are determined from solutions to the following allocation problem

$$\begin{aligned} &\min_{z^t} \{\tau \sum_{i=1}^{N} \int_0^{x_i + z_t} (x_i + z_t - \xi) \varphi_i(\xi) d\xi\} \\ &\text{Subject to: } Y_t = \sum_{i=1}^{N} (x_i + z_t). \end{aligned} \tag{4. 14}$$

Applying a Lagrange multiplier λ to include the equality constraint, we get

$$\tau\sum_{i=1}^{N}\int_{0}^{x_i+z_i}(x_i+z_i-\xi)\varphi_i(\xi)d\xi+\lambda[Y_t-\sum_{i=1}^{N}(x_i+z_i)].$$

Taking the partial derivative with respect to $z_i$ and setting it to zero, we have

$$\Phi_i(x_i+z_i)=\lambda/\tau=\text{constant} \tag{4.15}$$

In general, the determination of the value of $y_i$ requires an iterative procedure based on the specific demand distribution. To determine the value of $y_i$ explicitly, we apply the same analysis as in Chapter 3 by assuming that equation (4.15) will lead to the following equation

$$\frac{x_i+z_i-\mu_i}{\sigma_i}=c\text{ (constant)}. \tag{4.16}$$

Then $y_i$ is easily obtained as follows

$$y_i=x_i+z_t=\mu_i+\frac{\sigma_i(Y_t-\sum_{i=1}^{N}\mu_i)}{\sum_{i=1}^{N}\sigma_i}. \tag{4.17}$$

The above equation tells us that the inventory level at each store will only depend on the total stock of the system following the myopic allocation rule. Substituting this result into Equation (4.12) and applying simple induction, we find that the minimum expected total cost of the inventory system depends on the vector of stores' inventory levels $x^t$ only via its sum $X_t$. Then the multilocation inventory system can finally be modeled as a simple dynamic program with single state variable of $X_t$, and single decision variable of $Y_t$ as follows:

$$
\begin{aligned}
f_t(X_t) = \min_{Y_t \geq X_t} \{ & c(Y_t - X_t) + p\int_{Y_t}^{\infty}(\xi - Y_t)\varphi^{(N)}(\xi)d\xi \\
& + (h-\tau)\int_0^{Y_t}(Y_t - \xi)\varphi^{(N)}(\xi)d\xi \\
& + \tau\sum_{i=1}^{N}\int_0^{\mu_i + \frac{\sigma_i}{\sum_{i=1}^{N}\sigma_i}(Y_t - \sum_{i=1}^{N}\mu_i)} [\mu_i + \frac{\sigma_i}{\sum_{i=1}^{N}\sigma_i}(Y_t - \sum_{i=1}^{N}\mu_i) - \xi]\varphi_i(\xi)d\xi \\
& + \int_0^{\infty} f_{t+1}(Y_t - \xi)\varphi^{(N)}(\xi)d\xi \}, \text{ and } f_{T+1}(\cdot) = 0.
\end{aligned}
\tag{4. 18}
$$

Here we assume full backordering of system-wide shortages at the end of a review period to obtain the transition of state variables.

## 4.4 COMPUTATIONAL STUDY

### 4.4.1 OPTIMAL VERSUS SIMPLIFIED ORDERING POLICIES

In Section 4.3, we first derived a simplified ordering policy based on the assumption of free and instantaneous transshipments. Then we developed an inventory model that incorporates both inventory and transshipment costs. We also established a solution procedure to obtain an approximately optimal (*s*, *S*) policy by applying myopic allocation assumption and approximation. It is of interest to compare the performance of these two inventory policies.

Experiments are designed to study the magnitude of cost savings with the optimal ordering policy obtained from equation (4.18) comparing with the simplified policy from equation (4.11), which assumes free transshipments. Both dynamic programs are solved by the successive approximation method. The calculation is stopped when the average cost per period stabilizes.

To investigate the effects of ordering policies on the total cost of the inventory system under different scenarios, we employ a total of 16 experiments for each ordering policy. In the experiments, we consider cases of identical stores for simplicity. Experimental parameters are listed in Table 4.1.

Table 4.1 Experimental Parameters (* Negative Binomial)

| Factor | Level |
|---|---|
| Demand distribution | Poisson, NB* ($\sigma^2/\mu$=2) |
| $P$ | 5,10 |
| $N$ | 2,10 |
| $\tau$ | 2, $h$+$p$ |
| $K$ | 10 |
| $\mu$ | 10 |
| $h$ | 1 |

The high level of transshipment rate $\tau$ is taken at the maximum value of $h$+$p$, at which the inventory system with transshipments will have the same performance as that of an inventory system without transshipments. If $\tau$ is greater than $h$+$p$ then we should not consider transshipments at all. To verify this observation, we can simply put $\tau = h$+$p$ into equation (4.18), then we get

$$f_t(X_t) = \min_{Y_t \ge X_t}\{c(Y_t - X_t) + p\int_{Y_t}^{\infty}(\xi - Y_t)\varphi^{(N)}(\xi)d\xi - p\int_{0}^{Y_t}(Y_t - \xi)\varphi^{(N)}(\xi)d\xi$$

$$+(h+p)\sum_{i=1}^{N}\int_{0}^{\mu_i + \frac{\sigma_i}{\sum_{i=1}^{N}\sigma_i}(Y_t - \sum_{i=1}^{N}\mu_i)}[\mu_i + \frac{\sigma_i}{\sum_{i=1}^{N}\sigma_i}(Y_t - \sum_{i=1}^{N}\mu_i) - \xi]\varphi_i(\xi)d\xi \qquad (4.\ 19)$$

$$+\int_{0}^{\infty} f_{t+1}(Y_t - \xi)\varphi^{(N)}(\xi)d\xi\}, \text{ and } f_{T+1}(\cdot) = 0.$$

If we use the same notations as in Chapter 3, then $y = Y_t - X_t$ (the order quantity), and $k_i = \mu_i + \frac{\sigma_i}{\sum_{i=1}^{N}\sigma_i}(Y_t - \sum_{i=1}^{N}\mu_i)$. Making these substitutions into the above equation and with some simple algebraic rearrangements, we get

$$f_t(X_t) = \min_{y\geq 0}\{c(y) + \sum_{i=1}^{N}[h\int_0^{k_i}(k_i - \xi)\varphi_i(\xi)d\xi + p\int_{k_i}^{\infty}(\xi - k_i)\varphi_i(\xi)d\xi] \\ + \int_0^{\infty} f_{t+1}(X_t + y - \xi)\varphi^{(N)}(\xi)d\xi\} \text{ and } f_{T+1}(\cdot) = 0 \tag{4. 20}$$

Equation (4.20) is exactly the same as equation (3.18), the model for the centralized-ordering, multilocation inventory system without transshipments. This concludes our proof of the previous observation regarding the maximum transshipment rate.

Computational results are summarized in Tables 4.2a and 4.2b for Poisson and negative binomial distributions respectively. In Tables 4.2a and 4.2b, we see very similar patterns between Poisson and negative binomial distributions. It is found that ordering policies incorporating the effects of transshipment costs have significant advantage over the simplified policies that assume free transshipments for inventory systems with many stores and significant transshipment costs. As expected, the simplified policies only work well for systems with few stores and very low transshipment rate.

A surprising finding from this computational study is that cost savings are extremely sensitive to the relative magnitude of penalty costs. Consider the case of 10 stores with Poisson demands, the maximum cost savings increase from 18% to 62.6% if $p$ increases from 5 to 10. From the table, we also see that ordering policies assuming free transshipments consistently underestimate both $s$ and $S$ values under

Table 4.2a Simplified vs. Optimal Policies for Poisson Distributions.

| Factor | | | Policy | $s$ | $S$ | Average Cost/Period | Savings % |
|---|---|---|---|---|---|---|---|
| $N$=2 | $p$=5 | $\tau$=2 | Simplified | 17 | 24 | 18.1 | 0 |
| | | | Optimal | 17 | 24 | 18.1 | |
| | | $\tau$=6 | Simplified | 17 | 24 | 20.4 | 2.5 |
| | | | Optimal | 17 | 26 | 19.9 | |
| | $p$=10 | $\tau$=2 | Simplified | 20 | 26 | 19.3 | 0 |
| | | | Optimal | 20 | 26 | 19.3 | |
| | | $\tau$=11 | Simplified | 20 | 26 | 23.1 | 4.5 |
| | | | Optimal | 21 | 28 | 22.1 | |
| $N$=10 | $p$ =5 | $\tau$=2 | Simplified | 99 | 110 | 40.2 | 0 |
| | | | Optimal | 100 | 110 | 40.2 | |
| | | $\tau$=6 | Simplified | 99 | 110 | 70.0 | 18.0 |
| | | | Optimal | 119 | 130 | 59.3 | |
| | $p$ =10 | $\tau$=2 | Simplified | 104 | 113 | 44.1 | 6.3 |
| | | | Optimal | 109 | 120 | 41.5 | |
| | | $\tau$=11 | Simplified | 104 | 113 | 114.8 | 62.6 |
| | | | Optimal | 129 | 140 | 70.6 | |

Table 4.2b Simplified vs. Optimal Policies for Negative Binomial Distributions.

| Factor | | | Policy | $s$ | $S$ | Average Cost/Period | Savings % |
|---|---|---|---|---|---|---|---|
| $N$=2 | $p$ =5 | $\tau$=2 | Simplified | 17 | 26 | 21.7 | 0 |
| | | | Optimal | 17 | 26 | 21.7 | |
| | | $\tau$=6 | Simplified | 17 | 26 | 25.1 | 2.0 |
| | | | Optimal | 19 | 28 | 24.6 | |
| | $p$ =10 | $\tau$=2 | Simplified | 21 | 29 | 24.1 | 1.7 |
| | | | Optimal | 21 | 30 | 23.7 | |
| | | $\tau$=11 | Simplified | 21 | 29 | 31.2 | 10.6 |
| | | | Optimal | 23 | 32 | 28.2 | |
| $N$=10 | $p$ =5 | $\tau$=2 | Simplified | 101 | 114 | 56.3 | 6.6 |
| | | | Optimal | 109 | 120 | 52.8 | |
| | | $\tau$=6 | Simplified | 101 | 114 | 105.0 | 26.2 |
| | | | Optimal | 119 | 140 | 83.2 | |
| | $p$ =10 | $\tau$=2 | Simplified | 108 | 119 | 62.2 | 11.5 |
| | | | Optimal | 110 | 120 | 55.8 | |
| | | $\tau$=11 | Simplified | 108 | 119 | 177.5 | 75.0 |
| | | | Optimal | 139 | 160 | 101.4 | |

the experimental settings. Therefore, the simplified policies will lead to higher penalty costs. Total inventory cost of the distribution system will also increase correspondingly with $p$.

Since this research is motivated by the practical problem of the equipment distribution system we studied. Accordingly, our modeling approach and computational study are developed with this application in mind. As is shown above the relative magnitude of penalty costs is a key factor that affects computation results. It will determine the relevancy of our analysis of inventory policies. While it is usually difficult to specify the penalty cost, for the particular system studied, a fairly good estimate of this parameter can be obtained by considering the opportunity costs of premium charges and freight costs of placing an emergency order. We believe the current experimental settings for the penalty costs match fairly well with the system studied. For many inventory systems requiring high service levels, the relative magnitude of the penalty costs are much higher. In that case, simplified policies that assume free transshipments will significantly increase the inventory costs of the distribution system. Even rough adjustment of ordering policies to include the effects of transshipment costs will greatly improve the operation.

For inventory systems with relatively low penalty costs, the results may be totally different. Since this is not our major concern we will not do any detailed analysis on that issue. Here, we only give a simple example to show the sensitivity of cost savings with respect to the relative magnitude of penalty costs. In the last experiment for Poisson demand ($N$=10, $\mu$=10, $h$=1, $p$=10, $\tau$=11), the cost saving is

62.6%. If we set $h$=5, $p$=6 and use the same values for other factors the cost saving is only 3.4%, see the details in Table 4.3. It is also interesting to note that the simplified policy now overestimates both $s$ and $S$ values.

Table 4.3 Effects of $p$ and $h$ on the Magnitude of Cost Savings of Optimal Policies

| Experimental Settings (Poisson demand, $N$=10, $\mu$=10, $\tau$=11) | | | s | S | Average Cost/Period | Savings % |
|---|---|---|---|---|---|---|
| $p$=10 | $h$=1 | Simplified | 104 | 113 | 114.8 | 62.6 |
| | | Optimal | 129 | 140 | 70.6 | |
| $p$=6 | $h$=5 | Simplified | 94 | 101 | 152.6 | 3.4 |
| | | Optimal | 89 | 100 | 147.6 | |

### 4.4.2 BASE STOCK VERSUS (s, $S$) ORDERING POLICIES

For multilocation inventory systems with centralized ordering, each order needs to cover the total demands at all of the stores. If the total demands are substantial and ordering set-up costs are not extremely high, the system will probably place an order in every period. In that case, we may expect base stock policies work well for the inventory system. Since base stock policies are easier to work with, it is desirable to investigate how well they will perform comparing with the more complex ($s$, $S$) type policies. If the penalty cost for using base stock policies is not significant, it might be more reasonable to implement base stock policies in real application.

The following computational study is designed to evaluate the relative performance of base stock policies with respect to ($s$, $S$) type policies. In this study, both policies are obtained from the solutions to the dynamic program of equation (4.18) although it might be easier to determine base stock policies directly from

equation (4.7). For base stock policies, we first assume $K$=0 to obtain the optimal base stock level from equation (4.18), then we substitute the real settings for $K$ and the above base stock level into the dynamic equation to calculate the average cost per period. Values of $s$ and $S$, together with the corresponding average cost for the ($s$, $S$) policy are obtained from the solutions to the optimization problem of the dynamic program for the set value of $K$. From the above procedure, we know that the total inventory cost of base stock policies cannot be less than that of ($s$, $S$) policies. And obviously, the relative performance of base stock policies will depend on the magnitude of ordering set-up cost $K$. If $K$ is small, the total cost of the inventory system under base stock policies may be close to, or even exactly the same as the value under ($s$, $S$) policies. However for inventory systems with large values of $K$, using base stock instead of ($s$, $S$) policies may incur significant penalty costs.

It is clear that the experimental settings for the value of $K$ will determine computation results. Unfortunately, it is very difficult to estimate ordering set-up costs for the distribution system studied. Generally, they include labor costs incurred in reviewing inventories, making decisions, and processing orders. They may also include costs of some accounting operations such as preparing shipping invoice, making transaction records etc., as well as parts of receiving and inspection costs which are independent of the order size. Because of the difficulties of measuring all these costs, it is impossible to determine accurately the ordering set-up costs. So in the following computational study, values of $K$ are chosen quite subjectively to show their effect on the relative performance of base stock policies.

It should be noted that set-up costs for centralized ordering will typically increase with the number of stores involved in the inventory system (not necessary in linear relationship). So we choose to represent the $K$ values based on $N$, the number of stores in the inventory system. The experimental settings for other parameters are the same as in previous computations. Table 4.4 summarizes the parameter settings for this computation study.

Table 4.4 Experimental Parameters (* Negative Binomial)

| Factor | Level |
|---|---|
| Demand Distribution | Poisson, NB* ($\sigma^2/\mu$=2) |
| $P$ | 5,10 |
| $N$ | 2,10 |
| $\tau$ | 2, $h+p$ |
| $K$ | 2*$N$, 20*$N$ |
| $\mu$ | 10 |
| $h$ | 1 |

Computational results are summarized in Tables 4.5a and 4.5b for Poisson and negative binomial distributions respectively. From these tables, we can see that the average costs of the inventory system under base stock policies are the same as that of ($s$, $S$) policies when the values of $K$ are at low level of 2*$N$. Under these circumstances, the order-up-to points $S$ for both policies take on the same value. The only difference between these two policies is reorder points. The reorder point $s$ for a base stock policy will always be $S$-1 because of the assumption of zero ordering set-up costs. Reorder points for the ($s$, $S$) type policies will depend on the value of $K$. In the experiments with low values of $K$, the difference between order-up-to and reorder points for the ($s$, $S$) policies, denoted as $\Delta$ ($\Delta$=$S$-$s$), is much less than the expected demand of the inventory system (the average value of $\Delta/\mu_T$ is about 0.2).

Table 4.5a Base Stock vs. (*s*, *S*) Policies for Poisson Distributions.

| Factor | | | | Policy | | | | Avg. Cost/Period | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *N* | *P* | *τ* | *K* | Base Stock | | (*s*, *S*) | | Base Stock | (*s*, *S*) | Diff. % |
| | | | | *s* | *S* | *s* | *S* | | | |
| 2 | 5 | 2 | 4 | 23 | 24 | 20 | 24 | 12.1 | 12.1 | 0 |
| | | | 40 | 23 | 24 | 13 | 43 | 48.1 | 37.4 | 28.6 |
| | | 6 | 4 | 25 | 26 | 21 | 26 | 13.9 | 13.9 | 0 |
| | | | 40 | 25 | 26 | 13 | 45 | 49.9 | 38.7 | 28.9 |
| | 10 | 2 | 4 | 25 | 26 | 22 | 26 | 13.3 | 13.3 | 0 |
| | | | 40 | 25 | 26 | 17 | 46 | 49.3 | 39.5 | 24.8 |
| | | 11 | 4 | 27 | 28 | 23 | 28 | 16.1 | 16.1 | 0 |
| | | | 40 | 27 | 28 | 17 | 48 | 52.1 | 41.7 | 24.9 |
| 10 | 5 | 2 | 20 | 109 | 110 | 99 | 110 | 50.2 | 50.2 | 0 |
| | | | 200 | 109 | 110 | 79 | 206 | 230.2 | 174.8 | 31.7 |
| | | 6 | 20 | 129 | 130 | 109 | 130 | 69.4 | 69.4 | 0 |
| | | | 200 | 129 | 130 | 79 | 206 | 249.4 | 193.5 | 28.9 |
| | 10 | 2 | 20 | 119 | 120 | 103 | 120 | 51.5 | 51.5 | 0 |
| | | | 200 | 119 | 120 | 84 | 206 | 231.5 | 182.6 | 26.8 |
| | | 11 | 20 | 139 | 140 | 119 | 140 | 80.6 | 80.6 | 0 |
| | | | 200 | 139 | 140 | 89 | 206 | 260.6 | 223.9 | 16.4 |

Table 4.5b Base Stock vs. (*s*, *S*) Policies for Negative Binomial Distributions.

| Factor | | | | Policy | | | | Cost | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *N* | *P* | *τ* | *K* | Base Stock | | (*s*, *S*) | | Base Stock | (*s*, *S*) | Diff.% |
| | | | | *s* | *S* | *s* | *S* | | | |
| 2 | 5 | 2 | 4 | 25 | 26 | 21 | 26 | 15.8 | 15.8 | 0 |
| | | | 40 | 25 | 26 | 13 | 47 | 51.8 | 39.5 | 31.1 |
| | | 6 | 4 | 27 | 28 | 21 | 28 | 18.7 | 18.7 | 0 |
| | | | 40 | 27 | 28 | 13 | 50 | 54.7 | 41.3 | 32.4 |
| | 10 | 2 | 4 | 29 | 30 | 24 | 30 | 17.8 | 17.8 | 0 |
| | | | 40 | 29 | 30 | 17 | 50 | 53.8 | 42.5 | 26.6 |
| | | 11 | 4 | 31 | 32 | 27 | 32 | 22.3 | 22.3 | 0 |
| | | | 40 | 31 | 32 | 19 | 54 | 58.3 | 45.8 | 27.3 |
| 10 | 5 | 2 | 20 | 119 | 120 | 99 | 120 | 62.8 | 62.8 | 0 |
| | | | 200 | 119 | 120 | 72 | 210 | 242.8 | 184.6 | 31.5 |
| | | 6 | 20 | 139 | 140 | 109 | 140 | 93.2 | 93.2 | 0 |
| | | | 200 | 139 | 140 | 69 | 221 | 273.2 | 207.9 | 31.4 |
| | 10 | 2 | 20 | 119 | 120 | 108 | 120 | 65.8 | 65.8 | 0 |
| | | | 200 | 119 | 120 | 87 | 220 | 245.8 | 190.6 | 29.0 |
| | | 11 | 20 | 159 | 160 | 139 | 160 | 111.4 | 111.4 | 0 |
| | | | 200 | 159 | 160 | 99 | 250 | 291.4 | 228.5 | 27.5 |

This result indicates that the reorder point will be reached and the inventory system has to place replenishment orders in almost every period. So the small difference in reorder points will not affect the placement of replenishment orders, and because of the same value of order-up-to points for both ordering policies, the total cost of the inventory system under base stock policies will be the same as that of $(s, S)$ policies. However, in the experiments with high level of $K$, the value of $\Delta$ is larger than the expected demand of the inventory system (the average value of $\Delta/\mu_T$ is about 1.4). So the reorder point will not be reached in certain periods, and the inventory system under $(s, S)$ policies will not order in every period. This will lead to savings on ordering costs comparing with the base stock policies. From the tables, we also notice that the order-up-to points of $(s, S)$ policies are generally different from those of base stock policies when $K$ is large. So inventory holding and shortage costs will also be different between these two policies.

Although it is difficult to draw a precise conclusion from this computational study, the following observations are at hand for the use of these policies. As a first approximation, base stock policies can be used for centralized ordering inventory systems with small or moderately. large ordering set-up costs. However for inventory systems with high ordering set-up costs, it may be worth the effort to develop optimal $(s, S)$ type policies.

## 4.5 EXTENSIONS AND CONCLUSIONS

### 4.5.1 EXTENSIONS

For general distribution systems with emergency transshipments, there are several directions for further research that are evident. First, our modeling approach

is not directly applicable if transshipment rates between different pairs of locations are significantly different. An efficient stochastic programming technique needs to be developed for the determination of optimal ordering and transshipment policies for general inventory systems with transshipments.

Next, extensions to positive replenishment lead times may be necessary for some distribution systems. It will be a complex problem to study the possible impacts of replenishment lead times on pooling/transshipment policies. As pointed out by Tagaras and Cohen (1992), some managers may hold back stock and practice partial pooling as a hedge against demand uncertainty over the supply lead-time. They perform simulation studies on several partial pooling policies and find that all of them are inferior to the complete pooling policy where the maximum amount (minimum of supply and demand) will be sent. Although their studies are not conclusive for different situations we expect complete pooling be a good policy. With complete pooling policy, extensions to positive lead times are straightforward if we assume system-wide excess demand is backlogged. A standard transformation technique for time lag proposed by Karlin and Scarf (1958) can be used. The same method is applicable for transshipment lead times. However, since the basic time unit is a review cycle we can generally assume instantaneous transshipment in practice.

### 4.5.2 ALLOCATION ASSUMPTION

In Section 3, we apply the myopic allocation assumption originally proposed by Federgruen and Zipkin (1984b) for inventory systems where transshipments are not allowed. The assumption is then approximated by relaxing a non-negativity

constraint on the allocation quantity to obtain the inventory level for each store. They show the approximation is good if the coefficients of variation of the demands at different stores are close. Otherwise, the ideal inventory level might not be attainable. For inventory systems with transshipments, the ideal inventory level for each store, which depends only on the total stock of the system, are always attainable because negative allocation quantities are permitted with transshipments. So the allocation assumption is better for this case comparing with inventory systems without transshipments. However, in the model formulation we have neglected the possible redistribution costs for adjusting the stock at each store to the ideal inventory level. Since the equal fractile allocation assumption is a good approximation for systems that do not exhibit large imbalance of inventories at different stores, the initial redistribution quantity is expected to be small. So, it is reasonable to neglect the transshipment costs involved for that purpose.

### 4.5.3 CONCLUSIONS

In this chapter, we consider the problem of determining optimal ordering policies for multilocation inventory systems with emergency transshipments. Considerations on simplifying assumptions for the general transshipment problem are discussed. Approximate inventory models are developed for both base stock and $(s, S)$ type policies. We use dynamic programming techniques to obtain the optimal ordering policies that minimize transshipment and inventory costs. The optimal ordering policy is then compared with the simplified policy that assumes free and instantaneous transshipments. We also perform a comparative analysis on base stock versus (s, S) type policies for a centralized-ordering inventory system.

Computation results show that the relative magnitude of penalty cost is the most important factor to consider in planning inventory replenishment policies. For inventory systems with high penalty costs or high service level requirements, even rough adjustment of ordering policies to include the effect of transshipments will greatly improve the operation. The simplified policies only work well for systems with few stores and very low transshipment rate. It is also found that base stock policies work well for a centralized-ordering inventory system with small or moderately large ordering set-up costs.

# CHAPTER 5

# SUMMARY AND CONCLUISION

The management of multi-echelon inventory systems is a challenging research area that offers tremendous potential for both rich theory in management science and large pay-offs in practical applications. While the manufacturing sector may place more emphasis on better planning and "just-in-time" methods to reduce investment in in-process inventories, retailers in the commercial world are making use of more sophisticated inventory models to manage their complex distribution networks. With the current advances in computer technology and information systems, inventory managers today can have easy access of stock information at various geographically dispersed locations. These developments, coupled with the emphasis on total supply chain management, have significant implications for operating retail distribution systems.

This research focuses on the study of some fundamental issues related to the integration and coordination of different distribution activities. Our research provides important insights into the effective operation of multilocation inventory systems with centralized information.

## 5.1 INDIVIDUAL VERSUS CENTRALIZED ORDERING

First, we consider different ordering policies for a multi-store distribution system. Prior to the development and advances of distribution information systems, each store operates independently by placing individual orders directly from the manufacturer. When centralized information is available, the ordering decisions may be planned centrally through appropriate coordination and allocation of

inventories at different stores. We have performed a comparative analysis to evaluate the overall performance of individual versus centralized ordering policies. This study builds on, and integrates the existing research by establishing a general guideline on ordering policies for multilocation inventory systems with centralized information.

From the computational results, we find that generally ordering costs alone will determine the systems' performance. Centralized ordering will result in cost savings as long as its ordering set-up cost is lower than the total of that of individual ordering. Additional benefits for centralized ordering may also include quantity discounts and a risk-pooling effect over the supplier lead-time.

## 5.2 INVENTORY AND TRANSSHIPMENTS

Due to the complexity of the problem in modeling inventory systems with transshipments, very few inventory models in the literatures can adequately describe the multi-location, multi-period stochastic inventory systems with emergency transshipments. Existing research in this field has been limited to the simple base stock policies that assume zero ordering set-up costs. For inventory systems with non-negligible ordering set-up costs, solution procedures for the general (s, S) type policies remain to be developed.

In this research, we have developed approximate inventory models for both base stock and (s, S) type policies for a centralized-ordering distribution system with emergency transshipments. Approximately optimal (s, S) policies, which minimize joint inventory and transshipment costs, are obtained through a dynamic programming technique. The optimal ordering policy is then compared with a

simplified policy that assumes free and instantaneous transshipments. The later policy can be obtained easily from a traditional single-location inventory model. We also perform a comparative analysis to evaluate the relative performance of base stock policies comparing with the (s, S) polices for inventory systems with non-negligible ordering set-up costs.

Computation results show that inventory shortage costs and transshipment rates are two most important factors to consider in planning inventory policies. For inventory systems with high shortage costs (or high service level requirement) and high transshipment rates, even rough adjustment of ordering policies to include the effect of transshipment will reduce the total cost of the distribution system significantly. The simplified policies can only be used for distribution systems with few stores and very low transshipment rates. It is also found that base stock policies generally work well for a centralized-ordering inventory system with small or moderately large ordering set-up costs.

## 5.3 LIMITATIONS AND FUTURE RESEARCH DIRECTIONS

In the development of inventory models for centralized-ordering, multi-store distribution systems, we follow a myopic allocation assumption (Federgruen and Zipkin, 1984b) with an approximation of relaxation of the non-negativity constraint on allocation quantities. Then it is shown that the ideal inventory level at each store will only depend on the total stock of the system. This result is critical for the application of dynamic programming to the multi-period analysis of the system studied. Although we can obtain the optimal inventory level at each store without relaxing the constraint, we choose not to do that because it will make the multi-

period analysis overly complicated. The exact optimal inventory level at each store will not only depend on the total stock of the inventory system but also on the initial inventory at each store. The state space of the dynamic program thus becomes $N$-dimensional. Federgruen and Zipkin (1984b) show that the above approximation of relaxation is good if the coefficients of variation of the demands at different stores are close. For systems exhibiting large difference of demand variations at different locations, it may be necessary to impose the non-negativity constraint on allocation quantities. And again the resulting dynamic program will be $N$-dimensional. Before the development of more efficient dynamic programming procedures and other good approximations of the inventory models, we may continue to use the current inventory policies in real application. As for the allocation, we can disregard the negative quantities involved and simply allocate the remaining available inventory following the equal fractile allocation assumption for the remaining stores. Some simulation analysis will be helpful to evaluate this practice.

Throughout this research, we assume all lead times are zero. Generally, it is a good approximation to assume instantaneous allocation and transshipment for the wholesale/retail distribution systems. The centralized warehouse is usually located near the stores, and transshipments are typically used among retail outlets in close proximity. Therefore, both allocation and transshipment lead times are negligible comparing with the basic time unit of a review cycle. For few special distribution systems with significant allocation delays, a standard transformation technique for time lag (Karlin and Scarf, 1958) is available if we assume excess demand is fully backlogged. Perhaps, it is of greater value to incorporate a positive replenishment

lead time in our inventory models. The same transformation can be used to include replenishment lead times in our analysis of centralized ordering policies if we can approximately represent the one-period inventory costs as a single location cost function with a certain demand distribution. Further research is necessary to find such a cost function that is a good approximation of the one-period inventory costs.

If the replenishment lead time is non-negligible, the analysis of inventory systems with transshipments will involve an additional complexity in that we need to study the possible impacts of lead times on the transshipment policies. As pointed out by Tagaras and Cohen (1992), some managers may hold back stock and practice partial pooling/transshipment as a hedge against demand uncertainty over the supply lead-time. Further research may include considering different pooling policies and developing new mathematical models to determine the optimal inventory and transshipment policies.

Finally, our inventory/transshipment model is not directly applicable if transshipment rates between different pairs of locations are significantly different. An efficient stochastic programming technique is desirable for the simultaneous determination of optimal ordering and transshipment policies for general inventory systems with transshipments.

## BIBLIOGRAPHY


Albright, S.C. (1989), "An Approximation to the Stationary Distribution of a Multiechelon Repairable-Item Inventory System with Finite Sources and Repair Channels", *Naval Research Logistics*, Vol.36, pp.179-195.

Allen, S.G. (1958), "Redistribution of Total Stock over Several User Locations", *Naval Research Logistics Quarterly*, Vol.5, pp.337-345.

Anily, S., and Federgruen, A. (1990) "One Warehouse Multiple Retailer Systems with Vehicle Routing Costs", *Management Science*, Vol.36, No.1, pp.92-149.

Anily, S. and Federgruen, A. (1993) "Two-Echelon Distribution Systems with Vehicle Routing Costs and Central Inventories", *Operations Research*, Vol.41, No.1, pp.37-47.

Arrow, K., Karlin, S., and Scarf, H. (eds.) (1958), Studies in the Mathematical Theory of Inventory and Production, Stanford University Press, Stanford, CA.

Arrow, K.J., Harris, T., and Marschak, J. (1951), "Optimal Inventory Policy", *Econometrica*, Vol.19, No.3, pp.250-272.

Axsäter, S. (1990a), "Simple solution Procedures for a Class of Two-Echelon Inventory Problems", *Operations Research*, Vol.38, No.1, pp.64-69.

Axsäter, S. (1990b), "Modelling Emergency Lateral Transshipments in Inventory Systems", *Management Science*, Vol.36, No.11, pp.1329-1338.

Axsäter, S. (1993a), "Continuous Review Policies for Multi-Level Inventory Systems with Stochastic Demand", in Logistics of Production and Inventory, Handbooks in Operations Research and Management Science, (Graves, S.C. et. al., eds.), Vol. 4, North-Holland, Amsterdam, pp.175-197.

Axsäter, S. (1993b), "Exact and Approximate Evaluation of Batch-Ordering Policies for Two-Level Inventory Systems", *Operations Research*, Vol.41, No.4, pp.777-785.

Axsäter, S. (1995), "Approximate Evaluation of Batch-Ordering Policies for a One-Warehouse, N Non-Identical-Retailer System Under Compound Poisson Demand", *Naval Research Logistics*, Vol.42, pp.807-819.

Badinelli, R.D., and Schwarz, L. B. (1988), "Backorders Optimization in a One-Warehouse N-Identical retailer Distribution System", *Naval Research Logistics*, Vol.35, pp.427-440.

Bellman, R. (1957), Dynamic Programming, Princeton University Press, Princeton, NJ.

Bellman, R., Glicksberg, I., and Gross, O. (1955), "On the Optimal Inventory Equation", *Management Science*, Vol.2, pp.83-104.

Bessler S. A., and Veinott, A. F. (1966), "Optimal Policy for a Dynamic Multiechelon Inventory Model", *Naval Research Logistics Quarterly*, Vol.13, pp.355-389.

Chen, F., and Zheng, Y. (1994), "Lower Bounds for Multi-Echelon Stochastic Inventory Systems", *Management Science*, Vol.40, No.11, pp.1426-1443.

Clark, A.J. (1972), "An Informal Survey of Multi-Echelon Inventory theory", *Naval Research Logistics Quarterly*, Vol.19, pp.621-650.

Clark, A.J., and Scarf, H. (1960), "Optimal Policies for a Multi-Echelon Inventory Problem", *Management Science*, Vol.6, pp.475-490.

Clark, A.J., and Scarf, H. (1962), "Approximate Solutions to a Simple Multi-Echelon Inventory Problem", in Studies in Applied probability and Management Science (Arrow, Karlin, and Scarf, eds.), Stanford University Press, Stanford, CA., pp.88-110.

Cohen, M. A., Kleindorfer, P. R., and Lee, H. L. (1986) "Optimal Stocking Policies for Low Usage Items in Multi-Echelon Inventory Systems", *Naval Research Logistics Quarterly*, Vol.33, pp.17-38.

Cooper, M.C., and Ellram, L.M. (1993), "Characteristics of Supply Chain Management and the Implications for Purchasing and Logistics Strategy", *The International Journal of Logistics Management*, Vol.4, No.2, pp.13-24.

Das, C. (1975), "Supply and Redistribution Rules for Two-Location Inventory Systems: One-Period Analysis", *Management Science*, Vol.21, No.7, pp.765-776.

Deuermeyer, B. L., and Schwarz, L. B. (1981), "A Model for the Analysis of System Service Level in Warehouse-Retailer Distribution Systems: The Identical Retailer Case", in Multi-Level Production/Inventory Control Systems: Theory and Practice, TIMS Studies in the Management Sciences, (Schwarz, L., B., ed.), Vol. 16, North-Holland, Amsterdam, pp.163-193.

Dvoretzky, a., Kiefer, J., and Wolfowitz, J. (1952a), "The Inventory Problem: I. Case of Known Distributions of Demand", *Econometrica*, Vol.20, No.2, pp.187-222.

Dvoretzky, a., Kiefer, J., and Wolfowitz, J. (1952b), "The Inventory Problem: II. Case of Unknown Distributions of Demand", *Econometrica*, Vol.20, No.3, pp.450-466.

Ehrhardt, R. A. (1979), " The Power Approximation for Computing (s, S) Inventory Policies", *Management Science*, Vol.25, No.8, pp.777-786.

Ehrhardt, R. A., Schultz, C.R., and Wagner, H.M. (1981), "(s, S) Policies for a Wholesale Inventory System", in Multi-Level Production/Inventory Control Systems: Theory and Practice, TIMS Studies in the Management Sciences, (Schwarz, L.B. ed.), Vol.16, North-Holland, Amsterdam, pp.145-161.

Ehrhardt, R.A., and C. Mosier (1984), "A Revision of the Power Approximations for Computing (s, S) Policies", *Management Science*, Vol.30, No.5, pp.618-622.

Eppen, G., and Schrage, L. (1981), "Centralized Ordering Policies in a Multi-Warehouse System with Lead Times and Random Demand", in Multi-Level Production/Inventory Control Systems: Theory and Practice, TIMS Studies in the Management Sciences, (Schwarz, L.B. ed.), Vol.16, North-Holland, Amsterdam, pp.51-67.

Erkip,N., Hausman,H., and Nahmias, S. (1990), "Optimal Centralized Ordering Policies in Multi-Echelon Inventory Systems with Correlated Demands", *Management Science*, Vol.36, No.3, pp.381-392.

Federgruen, A., and Zipkin, P. (1984a), "An efficient Algorithm for Computing Optimal (s, S) Policies", *Operations Research*, Vol.32, pp.1268-1285.

Federgruen, A., and Zipkin, P. (1984b), "Computational Issues in an Infinite-Horizon, Multiechelon Inventory Model", *Operations Research*, Vol.32, No.4, pp.818-836.

Federgruen, A., and Zipkin, P. (1984c), "Approximations of Dynamic, Multilocation Production and Inventory Problems", *Management Science*, Vol.30, No.1, pp.69-84.

Feeney, G. J., and Sherbrooke, C. C. (1966), "The (s-1, s) Inventory Policy Under Compound Poisson Demand", *Management Science*, Vol.12, No.5, pp.391-411.

Fukuda, Y. (1961), "Optimal Disposal Policies", *Naval Research Logistics Quarterly*, Vol.8, pp.221-227.

Graves, S.C. (1985), "A Multi-Echelon Inventory Model for a Repairable Item with One-For -One Replenishment", *Management Science*, Vol.31, No.10, pp.1247-1256.

Graves, S. C. (1996) "A Multiechelon Inventory Model with Fixed Replenishment Intervals", *Management Science*, Vol.42, No.1, pp1-18.

Gross, D. (1963), "Centralized Inventory Control in Multilocation Supply Systems", in Multistage Inventory Models and Techniques, (Scarf, Gilford, and Shelly, eds.), Stanford University Press, Stanford, CA., pp47-84.

Gupta, A. and Albright, S. C. (1992) "Steady-State Approximations for a Multi-Echelon Multi-Indentured Repairable-Item Inventory System", *European Journal of Operational Research*, Vol.62, pp.340-353.

Hadley, G., and Whitin, T.M. (1963a), Analysis of Inventory Systems, Prentice-Hall Inc., Englewood Cliffs, NJ.

Hadley, G., and Whitin, T.M. (1963b), "An Inventory-Transportation Model with N Locations", in Multistage Inventory Models and Techniques, (Scarf, Gilford, and Shelly, eds.), Stanford University Press, Stanford, CA., pp.116-142.

Harris, F. (1915), Operations and Cost, Factory Management Series, A.W. Shaw Co., Chicago, IL, pp48-52.

Hausman, W.H., and Erkip, N.K. (1994), "Multi-Echelon vs. Single-echelon Inventory Control Policies for Low-demand Items", *Management Science*, Vol.40, No.5, pp.597-602.

Hoadley, B., and Heyman, D.P. (1977), "A Two-Echelon Inventory Model with Purchases, Dispositions, Shipments, Returns and Transshipments", *Naval Research Logistics Quarterly*, Vol.24, pp.1-19.

Hochstaedter, D. (1970), "An Approximation of the Cost Function for Multi-Echelon Inventory Model", *Management Science*, Vol.16, pp.716-727.

Igelhart, D. (1963), "Optimality of (s, S) Policies in the Infinite Horizon Dynamic Inventory Problem", *Management Science*, Vol.9, pp.259-267.

Jackson, P.L. (1988), "Stock Allocation in a Two-Echelon Distribution System or 'What to Do Until Your Ship Comes in' ", *Management Science*, Vol.34, No.7, pp. 880-895.

Jackson, P.L., and Muckstadt, J.A. (1989), "Risk Pooling in a Two-Period, Two-Echelon Inventory Stocking and Allocation Problem", *Naval Research Logistics Quarterly*, Vol.36, pp.1-26.

Jönsson, H., and Silver, E. A. (1987), "Analysis of a Two-Echelon Inventory Control System with Complete Redistribution", *Management Science*, Vol.33, No.2, pp.215-227.

Karlin, S. (1958), "The Application of Renewal Theory to the Study of Inventory Policies", in Studies in the Mathematical Theory of Inventory and Production, (Arrow, Karlin, and Scarf eds.), Stanford University Press, Stanford, CA., pp.270-297.

Karlin, S., and Scarf, H. (1958), "Inventory Models of the Arrow-Harris-Marschak Type with Time Lag", in Studies in the Mathematical Theory of Inventory and Production, (Arrow, Karlin, and Scarf, eds.), Stanford University Press, Stanford, CA., pp.155-178.

Karmarkar, U.S., and Patel N.R. (1977), "The One-Period N-location Distribution Problem", *Naval Research Logistics Quarterly*, Vol.24, pp.559-575.

Karmarkar,U.S. (1979), "Convex/Stochastic Programming and Multilocation Inventory Problems", *Naval Research Logistics Quarterly*, Vol.26, pp.1-19.

Karmarkar,U.S. (1981), "The Multiperiod Multilocation Inventory Problem", *Operations Research*, Vol.29, pp.215-228.

Karmarkar,U.S. (1987), "The Multilocation Multiperiod Inventory Problem: Bounds and Approximations", *Management Science*, Vol.33, pp.86-94.

Krishnan, K.S., and Rao, V.R.K. (1965), "Inventory Control in N Warehouse", The *Journal of Industrial Engineering*, Vol.16, pp.212-215.

Kumar, A., Schwarz, L.B., and Ward, J.E. (1995), "Risk-pooling Along a Fixed Delivery Route Using a Dynamic Inventory-allocation Policy", *Management Science*, Vol.41, No.2, pp.344-362.

Lee, H.L. (1987), "A Multi-Echelon Inventory Model for Repairable Items with Emergency Lateral Transshipments", *Management Science*, Vol.33, pp.1302-1316.

Maxwell, W., and Muckstadt, J. (1985), "Establishing Consistent and Realistic Reorder Intervals in Production-Distribution systems", *Operations Research*, Vol.33, No.6, pp.1316-1341.

McGavin, E. J., Schwarz, L. B., and Ward, J. E. (1993), "Two Interval Inventory Allocation Policies in a One-Warehouse N-Identical-Retailer Distribution System", *Management Science*, Vol.39, No.9, pp.1092-1107.

Moinzadeh, K., and Nahmias, S. (1988), "A Continuous Review Model for an Inventory System with Two Supply Modes", *Management Science*, Vol.34, No.6, pp.761-773.

Muckstadt, J.A. (1973), "A Model for a Multi-Item, Multi-Echelon, Multi-Indenture Inventory System", *Management Science*, Vol.20, pp.472-481.
Muckstadt, J. A. and Thomas, L. J. (1980), "Are Multi-Echelon Inventory Methods Worth Implementing in Systems with Low-Demand-Rate Items? ", *Management Science*, Vol.26, No.5, pp.483-494.

Muckstadt, J., and Roundy, R. (1993) "Analysis of Multistage Production Systems", in Logistics of Production and Inventory, Handbooks in Operations Research and Management Science, (Graves et al., eds.), Vol. 4, North-Holland, Amsterdam, pp.59-131.

Nahmias, S. (1981), "Managing Repairable Item Inventory Systems: A Review", in Multi Level Production/Inventory Control Systems: Theory and Practice, TIMS Studies in the Management Sciences, (Schwarz, L. B., ed.), Vol. 16, North-Holland, Amsterdam, pp.253-277.

Nahmias, S., and Smith, S. A. (1994), "Optimizing Inventory Levels in a Two Echelon System with Partial Lost Sales", *Management Science*, Vol.40, No.5, pp.582-596.

Roberts, D. (1962), "Approximations to Optimal Policies in a Dynamic Inventory Model", in Studies in Applied probability and Management Science (Arrow, Karlin, and Scarf, eds.), Stanford University Press, Stanford, CA., pp.207-229.

Robinson, L. W. (1990) "Optimal and Approximate Policies in Multiperiod Multilocation Inventory Model with Transshipments", *Operations Research*, Vol.38, No.2, pp.278-295.

Rosling, K, (1989), "Optimal Inventory Policies for Assembly Systems Under Random Demands", *Operations Research*, Vol.37, pp.565-579.

Roundy, R. (1985), "98%-Effective Integer-Ratio Lot-Sizing for One-Warehouse Multi-Retailer Systems", *Management Science*, Vol.31, No.11, pp.1416-1430.

Scarf, H. (1960), "The Optimality of (s, S) Policies in the Dynamic Inventory Problem", in Mathematical Methods in the Social Sciences, (Arrow, Karlin, and Suppes, eds.), Stanford University Press, Stanford, CA., pp.196-202.

Scarf, H. (1963), "AnalSytical Techniques in Inventory Theory", in Multistage Inventory Models and Techniques, (carf, Gilford, and Shelly, eds.), Stanford University Press, Stanford, CA, pp.185-225.

Scarf, H., Gilford, D., and Shelly, M. (eds.) (1963), Multistage Inventory Models and Techniques, Stanford University Press, Stanford, CA.

Schneider, H. (1978), "Methods for Determining the Reorder Point of an (s, S) Ordering Policy when a Service Level is Specified", *Operational Research Quarterly*, Vol. 29, pp.1181-1193.

Schneider, H., and Ringuest, J. L. (1990), "Power Approximation for Computing (s, S) Policies using Service Level", *Management Science*, Vol.36, No.7, pp.822-834.

Schneider, H., Rinks, D. B., and Kelle, P. (1995), "Power Approximations for a Two-Echelon Inventory System Using Service Levels", *Production and Operations Management*, Vol. 4, No. 3, pp.1-20.

Schneider, H., and Watson, E. (1997), "An Analysis of Warehouse and Distribution Strategies, A Case Study", *OR Spektrum*, to appear.

Schwarz, L. B. (ed.) (1981), Multi-Level Production/Inventory Control Systems: Theory and Practice, TIMS Studies in the Management Sciences, Vol.16, North-Holland, Amsterdam.

Schwarz, L. B. (1989), "A Model for Assessing the Value of Warehouse Risk-Pooling: Risk-Pooling Over Outside Supplier Leadtimes", *Management Science*, Vol.35, No. 7, pp.828-842.

Schwarz, L.B., Deuermeyer, B.L., and Badinelli, R.D. (1985), "Fill-Rate Optimization in a One-Warehouse N-Identical-Retailer Distribution System", *Management Science*, Vol.31, No.4, pp.488-498.

Sherbrooke, C.C., (1968), " METRIC: A Multi-Echelon Technique for Recoverable Item Control", *Operations Research*, Vol.16, No.1, pp.122-141.

Sherbrooke, C. C., (1992), "Multiechelon Inventory Systems with Lateral Supply", *Naval Research Logistics*, Vol.39, pp.29-40.

Svoronos, A., and Zipkin, P. (1988), "Estimating the Performance of Multi-Level Inventory Systems," *Operations Research*, Vol.36, No.1, pp.57-72.

Svoronos, A., and Zipkin, P. (1991), "Evaluation of One-For-One Replenishment Polices for Multi-Echelon Inventory Systems", *Management Science*, Vol.37, No.1, pp.68-83.

Tagaras, G. and Cohen, M. (1992) "Pooling in Two-Location Inventory Systems withNon-Negligible Replenishment Lead Times", *Management Science*, Vol.38, No.8, pp.1067-1083.

Veinott, A. F., and Wagner, H. M. (1965), "Computing Optimal (s, S) Inventory Policies", *Management Science*, Vol.11, No.5, pp.525-552.

Wagner, H., O'Hagen, M., and Lundh, B. (1965), "An Empirical Study of Exactly and Approximately Optimal Inventory Policies", *Management Science*, Vol.11, pp.690-723.

Wilson, R.H. (1934), "A scientific Routine for Stock Control", *Harvard Business Review*, Vol.13, pp.116-128.

Whitin, T.M. (1953), The Theory of Inventory Management, Princeton University Press, Princeton, NJ.

Zheng, Y., and Federgruen, A. (1991), "Finding Optimal (s, S) Policies Is as Simple as Evaluating a Single Policy", *Operations Research*, Vol.39, No.4, pp654-665.

Zipkin, P. (1991), "Evaluation of Base Stock Policies in Multiechelon Inventory Systems with Compound-Poisson Demands", *Naval Research Logistics*, Vol.38, pp.397-412.

## VITA

Jianxin Hu was born in Yongkang, Zhejiang Province, China, on September 25, 1965. He is the first of three children born to Ruqing Hu and Yuting Wu. He enrolled at Zhejiang University, China, in September 1981, and graduated with a Bachelor of Science degree in Chemical Engineering in July, 1985. He was employed as an engineering consultant and project engineer at Beijing Research Institute of Chemical Industry from August, 1985 to July, 1991. He enrolled in the Graduate School at Louisiana State University in August, 1991 and earned a Master of Science degree in Chemical Engineering in 1994. He entered the doctoral program in business administration in the Department of Information Systems and Decision Sciences in 1995. He is currently a candidate for the degree of Doctor of Philosophy in Business Administration.

## DOCTORAL EXAMINATION AND DISSERTATION REPORT

**Candidate:** Jianxin Hu

**Major Field:** Business Administration
(Information Systems and Decision Sciences)

**Title of Dissertation:** Multilocation Inventory Systems with Centralized Information

**Approved:**

Major Professor and Chairman

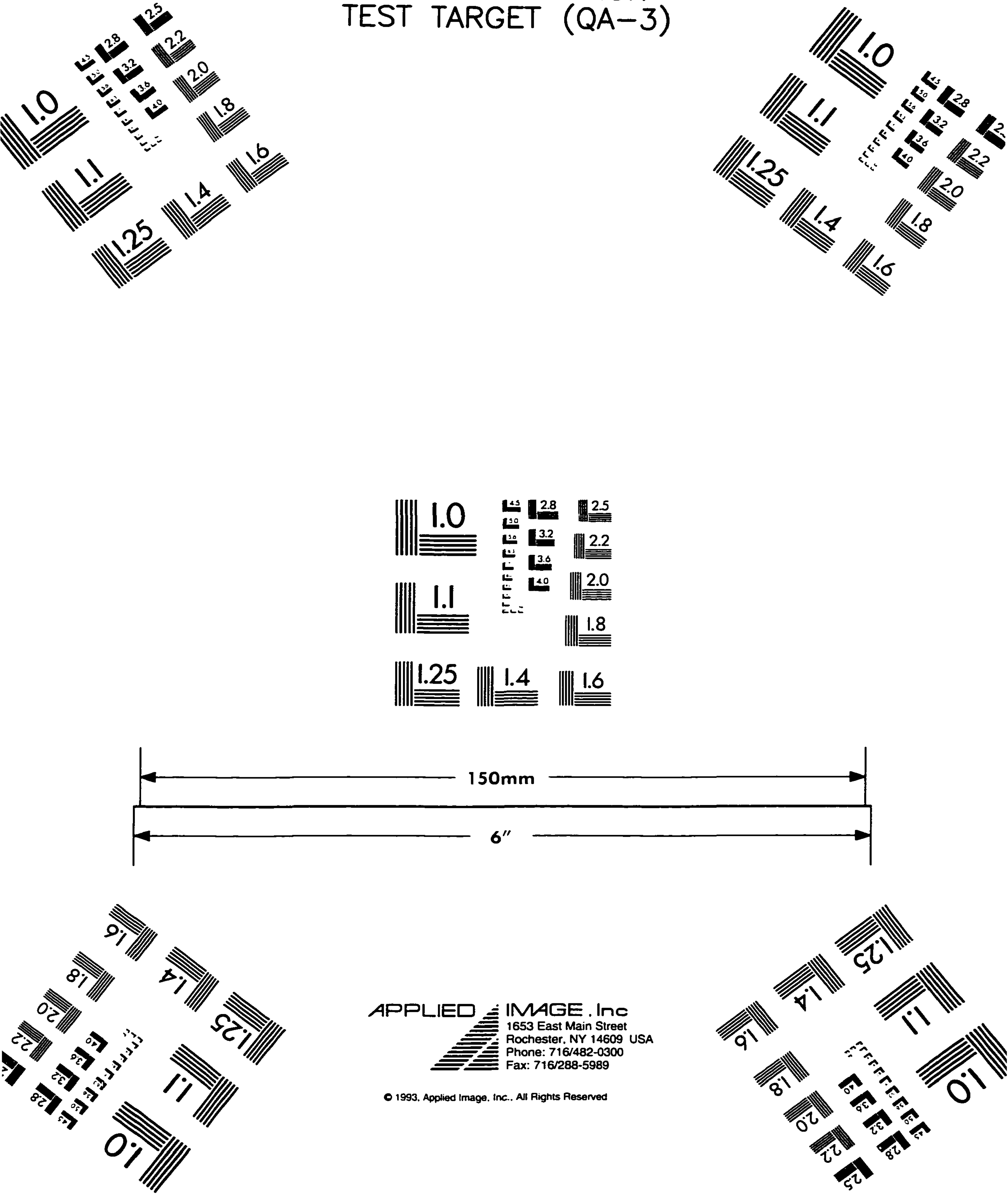
Dean of the Graduate School

**EXAMINING COMMITTEE:**

**Date of Examination:**

May 8, 1998

# IMAGE EVALUATION TEST TARGET (QA-3)



APPLIED IMAGE . Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989