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THE STUDY OF HIGH ENERGY
HEAVY NUCLEUS INTERACTIONS
IN NUCLEAR EMULSION CHAMBERS
USING DIGITAL IMAGE PROCESSING

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Physics and Astronomy

by

Philip Deines-Jones
B.S., University of Wisconsin, 1985
August 1996
To Courtney
and my mother and father,
Judy and Dumont
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Abstract

Nuclear emulsion chambers with lead targets have been exposed to a 158 GeV per nucleon beam of $^{208}\text{Pb}$ nuclei, by far the heaviest ion accelerated to such a high energy to date. These interactions frequently produce more than 1000 charged particles. In order to measure multiplicities and secondary particle trajectories in these extremely large events, an automatic CCD-based microscope system has been developed to analyze images, count secondary tracks, measure their trajectories, and estimate their charge.

Based on the analysis of 40 high-multiplicity Pb-Pb events measured using this system, we assess the degree to which these interactions can be described as a superposition of individual nucleon-nucleon interactions. The measured pseudorapidity distributions agree very well with the superposition-based FRITIOF parton model, although the actual multiplicities are somewhat lower than the calculated ones. The Pb-Pb pseudorapidity distributions are compared to those from emulsion exposures to other beams. At energies of 158-200 GeV per nucleon, the target and projectile tails of the pseudorapidity distributions are consistent with limiting fragmentation of the target and beam nucleons, supporting the wounded nucleon description of superposition. The variations in the central pseudorapidity regions can also be described as the sum of production from wounded target and projectile nucleons.
In agreement with previous studies, we find that multiplicities of heavy nucleus interactions increase more rapidly with energy than in nucleon-nucleon interactions. However, the produced multiplicity per wounded nucleon in central Pb-Pb interactions is no greater than in central interactions of protons, oxygen, or sulfur on silver-bromine targets. This suggests that the increase in multiplicity may not be the effect of re-interaction of particles in large nuclei, as previously conjectured.
Chapter 1
Introduction

1.1 Heavy Ion Interactions

Ultra-relativistic heavy ion collisions are the subject of a hybrid field that combines particle and nuclear physics. The program of particle physics is to describe interactions of leptons, quarks, hadrons, etc., in terms of fundamental gauge theories (quantum electro- or chromodynamics). Generally speaking, these interactions involve two or three particles. In contrast, nuclear physics treats the particles inside the nucleus, which may number two hundred or more, as bulk material. The many-body interactions between the nuclear constituents (nucleons) are essential. At collision energies near 1 GeV per nucleon (GeV/n), neither point of view is separately sufficient because the energy is too low to neglect collective effects, with energy scales of 10-100 MeV, and because electromagnetic forces compete with nuclear ones. If the energy is raised to 10 GeV/n, the nominal lower limit of ultra-relativistic interactions, the picture simplifies. These interactions are so violent and sudden that collective interactions within the nuclei become less important, and the main experimental features can be understood by thinking of the collision as a collection of individual nucleon-nucleon interactions. This point of view is the superposition picture or "incoherent superposition model."
At even higher energies, bulk effects may once again become important. Quarks and gluons are confined in hadrons. However, if a sufficiently dense state of hadronic matter can somehow be created, an effect akin to Debye screening may allow gluons to screen the strong interaction between quarks [Halzen 84], resulting in a reduced coupling between them. The quarks can then move freely within this dense matter, creating a quark conductor or “quark-gluon plasma” (QGP). This intuition appears to be supported by lattice gauge calculations [Engels 90], which predict a phase transition at (kinetic) energy density \( \epsilon \) on the order of 2 GeV/fm\(^3\) or temperature \( T \) around 200 MeV. This bulk material thermal energy corresponds to single interaction momentum transfers which are low enough to be in the energy range of soft interactions, where the interactions are between whole hadrons, not individual quarks. This is the regime of “ordinary” hadrons, i.e., p, n, \( \pi \), K, etc.

There is no guarantee that such a hot, dense state can be made by colliding nuclei together. In addition to sheer available energy, a system must exhibit several other features if a QGP is to be created [Schmidt 93]:

- The system must be larger than the strong force interaction scale (\( \sim 1 \) fm).

- It must contain a thermodynamically large number of particles.

- The system must approach thermal equilibrium. If a strict superposition picture is correct, nucleus-nucleus (AA) interactions do not meet this requirement, since by definition there is no interaction between nucleons other than the pairs which collide. Thus, for heavy ion collider experiments to produce a QGP, there must be an energy exchange mechanism between nucleons. The likely candidate mechanism is for particles produced in the nucleon-nucleon (NN) interactions to reinteract within the nuclear material. Whether this reinteraction actually occurs is critical to determining whether the conditions are right to form a QGP.
While the search for the QGP drives the field, there are other reasons to study heavy ion interactions, as well. The interactions usually leave behind a noninteracted piece of nuclear matter which is extremely far from equilibrium. There is a sub-field devoted to characterizing and understanding this "spectator" fragmentation. The data collected in these experiments is also useful for the design and calibration of high energy cosmic ray detectors. The problem here is in some sense the inverse of that in interaction studies: given some measured quantity from a cosmic ray interaction (e.g., the energy deposited in a calorimeter), the incident energy must be reconstructed. Finally, "normal" heavy ion interactions are not without intrinsic interest. Although there is a consensus that the superposition model is successful in describing the main features of AA interactions, the exact way in which NN interactions are added together to describe AA interactions is far from obvious, as will be seen in Chapter 4. It is worth asking whether we completely understand the connections between the phenomena of NN interactions and their AA counterparts.

This work concerns results from the exposure of nuclear emulsion to the heaviest and highest energy beam to date, the 158 GeV/n lead beam accelerated at the European Center for Nuclear Research (CERN) in December 1994. A new automatic microscopy system, the first of its kind, was used for these measurements. The primary purposes of the analysis are to assess the degree to which superposition alone describes these interactions, to determine the extent to which increasing beam mass can help create conditions conducive to QGP formation, and to clarify the precise relationship between NN and AA collisions. The emulsion measurements are compared to parton model calculations and to data from previous emulsion exposures by the Krakow-Louisiana-Minnesota-Moscow Collaboration (KLMM). We observe no additional deviation from superposition over these previous beams. In fact, the deviation is less than expected from an extrapolation.
of the previous results. These results are among the first to come from this beam, and so our conclusions are necessarily provisional. However, it would seem that our previous explanations for these deviations may be incorrect. This would suggest that we are further from creating the conditions for a QGP than had been thought.

1.2 Concepts

The range of the nuclear force, \( \sim 1 \text{ fm} \), is smaller than the diameters of all but the lightest nuclei. For example, C has a radius of 4.6 fm. At the same time, the interaction energy is high enough so that diffractive scattering is not very important \((\hbar c = 0.2 \text{ GeV fm})\), and the interaction occurs on a time scale faster than any relaxation times in the nucleus. It should therefore be quite accurate to view two colliding nuclei as classical geometrical objects. (Due to Lorentz contraction, a speeding nucleus appears flattened in the direction of travel. This classical picture breaks down if the Lorentz contracted extent is less than \( \sim 1 \text{ fm} \).) In general, the two nuclei do not collide head on but rather with some offset, or impact parameter \( b \), measured in fm. Then, the nuclei only partly overlap. (The \( b \approx 0 \) events are called "central", as opposed to "peripheral" interactions.) In the superposition picture, the region of overlap contains the participant nucleons which partake in the interaction. The crescent-shaped remainders, one each for the target and projectile, are called the spectators. These consist of noninteracted nuclear matter which is nonetheless very far from equilibrium due to its odd shape. The spectators break up into fragments (conventionally defined as pieces with charge \( Z > 2 \)), alphas, deuterons, protons, neutrons, etc. These pieces typically have kinetic energies on the scale of the Fermi energy inside nuclear matter. This is fortunate for the study of the participants, since it means that the fragments in general are not relativistic.
in the rest frame of the spectator. At fixed-target accelerators, the target spectators (i.e., the target spectator pieces), have low energy, are heavily ionizing, and can be discerned from produced particles on that basis. Multiply charged spectator fragments can be differentiated from singly charged particles, also on the basis of their higher ionization. The projectile spectators are relativistic in the lab frame, but are emitted at angles smaller than most of the produced particles, which are governed by higher energy processes.

Baryon conservation dictates that there be two nucleons in the final state, plus, possibly, a small number of $p\bar{p}$ and $n\bar{n}$ pairs. A nucleon may undergo an isospin flip from $p$ to $n$ or vice versa, but otherwise can be thought of as coming through the interaction more or less intact. The nucleon pairs participating in an interaction shed on the order of half of their energy in the form of produced particles, mostly pions. The degree of energy loss, or "stopping power," is greater in nuclei than in proton targets. However, the energy loss in high energy ($\sim 200 \text{ GeV/n or greater}$) collisions is not as great as one would expect based on the number of intra-nuclear NN collisions. This phenomenon is referred to as nuclear "transparency."

Secondary particle production is not instantaneous or spatially localized, but occurs over some (proper) formation time $\tau \sim 1 \text{ fm/c}$ as particles materialize out of the excited QCD field. The interaction path length in nuclear material is on the order of 1.8 fm, so nuclei are thick targets – a given nucleon may interact several times while passing through the target nucleus. If we could follow an individual projectile nucleon through a target Pb nucleus, for example, we might see it interact before it gets very far through the target and becomes excited. The interacted target nucleon is likewise excited, and is eventually ejected from the target. The projectile nucleon, which we should now perhaps call an excitation, is scattered ($p_t \approx 450 \text{ MeV/c}$) but continues on in almost the same direction. The $p_t$ data in heavy nuclei are consistent with multiple scattering, but the dynamics
and particularly the energy loss in these multiple interactions is still unclear. The excitation is, at any rate, extremely relativistic, and therefore time dilated, as are its final state products. The formation zone, where the final state particles materialize or “hadronize” is therefore a function of final state particle energies (Fig. 1.1). Whether or not reinteraction of the final state particles plays an important role in redistributing energy therefore depends on the underlying NN produced particle momentum distributions, the true value of the formation time, and the size of the target nucleus.

If thermalization is achieved, and if a phase transition is realized, then the experimental task will be to study the hot nuclear material based on the final state particles. These particles will be the “frozen out” remnants of the plasma. This is a daunting prospect, but not a hopeless one. The situation has been likened to the study of Big Bang cosmology, where probes like the three degree microwave background, which decoupled a few minutes after the Big Bang, can be used to study the universe at that epoch. From a nuclear emulsion experimentalist’s point of view, one interesting suggestion is that the final state momentum and angular distributions of the particles might reflect macroscopic fluctuations in temperature and density during the phase transition. Particle multiplicities would also be expected to increase, as they would reflect the temperature at the time of phase transition, much as photon flux indicates the temperature of a black body (the Stefan-Boltzmann law). There are many other, perhaps more decisive, suggestions for experimental QGP signals [Schmidt 93].

1.3 The KLMM Nuclear Emulsion Chamber Experiments

This work concerns the first results from exposures of nuclear emulsion chambers with Pb targets to a beam of 158 GeV/n $^{208}$Pb ions. It is part of a larger
Figure 1.1: Schematic illustration of the formation zone. A 200 GeV nucleon traveling in the z-direction with impact parameter $\rho = 0$ is assumed to interact in the center of a target nucleus. Assuming its products have a typical transverse momentum $p_t = 350$ MeV/c, the horizontal lines represent the edge of the formation zone where pions hadronize. Also shown for comparison are radii of several nuclei (O, Ag, and Pb). The volume available for reinteraction depends strongly on the size of the nucleus and the actual formation time.
program conducted by KLMM, which over the last ten years has analyzed nuclear emulsions exposed to beams ranging from O to Pb at energies from a few GeV to 200 GeV. This program is itself the successor to a series of proton studies at energies as high as 800 GeV. Nuclear emulsion is better suited to counting particles and to precisely measuring their trajectories than other techniques. The trajectory information gleaned from emulsion experiments, although not sufficient to reconstruct the interaction dynamics, constrains it by measuring the total production and two kinematic variables, the particle space angles. (App. A defines the "pseudorapidity" variable \( \eta \) for characterizing opening angles \( \theta \) in ultra-relativistic collisions, and explains its relationship to the more familiar momentum variables.)

If the superposition model is substantially correct, the multiplicity data, when compared to multiplicities from NN collisions at the same energy, provide a direct probe of the number of participating nucleons.

Lead-208 is by far the heaviest ion to be accelerated to such a high energy, which is important for several reasons. In order to make predictions for these interactions, models must extrapolate over a factor of 5 in the number of participating nucleons from the nearest previous data at this energy (Fig. 1.2). Thus, the Pb beam provides the most stringent test yet of superposition's predictions of how collisions scale with nuclear mass. The beam also provides the highest energy density at the time of collision so far available. The interactions of Pb ions on heavy targets produce so many particles that for the first time, it is possible to study spatial fluctuations in individual events [Wosiek 95]. Perhaps most significantly, though, the interaction volume is much larger than in previous interactions at such a high energy. The Pb beam presents the best possibility currently available of observing reinteraction or other thermalization processes.
Figure 1.2: Schematic illustration of relative energy density and interaction volume for AGS and SPS beams. The beams are organized by the number of participant nucleons $W$, which characterizes the volume of the interaction. It also suggests how appropriate taking a thermodynamic limit may be, since it represents the number of initial state particles, and is proportional to the number of final state particles. The ordinate is the peak pseudorapidity density per participant nucleon, which scales with the energy density. The data are selections of central events used in this work, but are representative of the regions of parameter space examined thus far.
Emulsion chambers, as opposed to emulsion stacks, are a relatively recent technical innovation for KLMM. (The distinction between chambers and stacks will be described in the next chapter.) The author supervised an exposure of emulsion stacks and chambers to the 10.6 GeV/n Au beam from the AGS (Alternating Gradient Synchrotron, at Brookhaven National Laboratory) in 1992. These Au beam chambers were of little scientific value, but were useful for developing and testing a new microscope system for the automated measurement scheme, the subject of Chapter 2. This automated CCD camera microscopy system was then applicable, with only minor modification, to the much larger events from the 158 GeV/n Pb beam which was accelerated at the SPS (Super Proton Synchrotron) at CERN in December, 1994. The multiplicities of these high energy Pb interactions frequently exceed 1000 charged particles, and the automated system made possible the first analysis of these interactions. These results are presented in Chapters 3 and 4.

Chapter 3 compares the Pb angular distributions to a "state-of-the-art" Monte Carlo superposition simulation and characterizes the multiplicity of head-on events. In general, the model agrees very well with the data, although it predicts multiplicities which are higher than observed. Although angular measurements alone are not sufficient to determine particle momentum and interaction dynamics, it is likely that any unexpected change in dynamics will nevertheless be reflected in the particle trajectories, and this is not observed.

After ten years of measurements from heavier and heavier beams, the experimental community has reached a plateau with the SPS Pb beam. There are no heavier or higher energy beams planned until the Relativistic Heavy Ion Collider (RHIC), scheduled for completion in 1999, begins taking data. It is an appropriate time to put results from individual beams into a broader perspective. The SPS beams, ranging from protons to Pb, are compared in Chapter 4, and a surprisingly simple description of the angular distributions emerges. The multiplicities in
beams of energies from 14.6 GeV to 800 GeV are compared as well. This analysis indicates that the underlying relationship between NN and AA interactions is still somewhat cloudy, although very simple empirical relationships can describe the data.
Chapter 2

Automated Track Recognition and Event Reconstruction

2.1 Introduction

Nuclear emulsion is an excellent charged particle detector. It combines sensitivity to minimum ionizing particles (MIPs) with spatial resolution superior to the best electronic techniques available. This combination accounts for emulsion's usefulness in high energy cosmic ray detectors like JACEE [Burnett 86] and neutrino oscillation searches [Winter 95], and makes it ideal for analyses of high multiplicity interactions like the present work. Unfortunately, it has proven difficult to analyze emulsion in a systematic and automatic way, although attempts to do so date back at least to the 1950's [Powell 59]. Instead, measurement has been a slow, manual task, requiring a high degree of training, a fact which has limited both the number of analyzed events and the study of systematic errors in individual datasets. Automatic charge measurement in emulsion has long been possible in certain circumstances [Fowler 77], and semi-automatic aides to measurement have been employed for some time [Iyono 90, Garpman 88, Olson 93]. But track counting and measurement in emulsion has remained a labor-intensive task despite success in automating the same task in passive etch detectors [Price 91]. Ironically, this difficulty is a consequence of emulsion's advantages — high spatial resolution
and sensitivity to MIPs—which make automatic track detection computationally challenging. Large quantities of imaging data must be acquired and processed, and the analysis routines must efficiently detect tracks yet reject the background from knock-on electrons, secondary particle production, etc. Until recently, this data acquisition and analysis was impractical.

To illustrate both the advantages and the difficulties associated with emulsion measurements, several typical microscope views of nucleus-nucleus interactions are shown in Figs. 2.1-2.4. In each of these images, the event axis is perpendicular to the plate and near the center of the field of view. The nominal emulsion thickness is 55 μm (before development), and the field of view is approximately 110 μm from top to bottom.

Figs. 2.1 and 2.2 show the same field of view, but at different focus depths. The interaction occurred about 50 μm upstream of the emulsion, and one can see that the tracks noticeably spread out between the two focus planes. But it is not easy to identify the secondary tracks from these two images. Not all the dark spots in these images are secondary particles; there are features in each frame, like delta rays (the wandering tracks) and isolated grains, that have no partner in the other. Likewise, there are real secondary tracks which appear in one frame but not the other, or perhaps in neither. This is because the mean distance between developed grains along a MIP track is 3.5 μm, so a microscope image with a 1-2 μm depth of focus will sometimes fall on a gap between grains. Increasing the depth of focus alleviates this problem, but captures more background grains in each frame.

Figures 2.3 and 2.4 show more extreme examples, where even with a small depth of focus, the background spots easily outnumber the real tracks. In order to reliably identify tracks, manual scanners continually adjust the microscope's focus slightly and look for tracks that persist from the top of the emulsion to the bottom. To imitate this behavior, an automatic system must acquire many frames, each at
Figure 2.1: Event 20-06, approximately 80 $\mu$m downstream of the interaction in the Pb foil. The field of view is 140 $\mu$m $\times$ 108 $\mu$m, and the depth of the field is about 1 $\mu$m. The focal plane is 4 $\mu$m into the emulsion.
Figure 2.2: Same as Fig. 2.1, but 10μm into the emulsion. If one compares a small region near the edge of Fig. 2.2 with the same area in Fig. 2.1, one notices that the correspondence between individual features is poor.
Figure 2.3: Event 20-06 in plate 16 (upstream), 3.3 cm from the vertex.
Figure 2.4: Au chamber event 43023, approximately 2 mm from the vertex. The image has much more background but is still analyzable. Also note the emulsion distortion, which causes the normally incident tracks to appear to have entered from the direction of the left side of the image.
a slightly different focus depth (together, the frames are called a "focus sequence"),
and the image analysis software must search for persistently dark paths through
the resulting three-dimensional image (depth of focus being the third dimension).

At LSU we have developed the first successful system known to us which auto-
matically measures and reconstructs nuclear interactions in emulsion "chambers",
in which thin emulsion plates are exposed perpendicular to the beam. The system
identifies the particles in its field of view which emanate from a vertex, efficiently
rejects background tracks, measures the track space angles, and provides a rough
charge assignment which distinguishes minimum ionizing tracks from heavier frag-
ments. The overall reconstruction accuracy is 95% or better. As part of the
Krakow-Louisiana-Minnesota-Moscow collaboration (KLMM, CERN experiment
EMU-13), we have used this system to analyze a set of 40 semi-central 158 GeV/c
Pb-Pb events with a mean multiplicity of 1097. Work is also under way to employ
this new technique to analyze cosmic ray interactions in balloon-borne JACEE
emulsion calorimeter chambers.

The next section describes the KLMM Pb-Pb emulsion chamber experiment.
The image acquisition and analysis is treated in Section 3, and Section 4 covers
track reconstruction.

2.2 Chamber Design and Exposure

There are two emulsion detector geometries in common use: stacks and chambers.
KLMM exposed 10 stacks and 32 chambers to the CERN $^{208}$Pb beam. This work is
concerned mainly with the chambers; preliminary stack results have been published
elsewhere [Wosiek 95]. Stacks are solid volumes of emulsion arranged in pellicles, in
this instance 600 $\mu$m thick. A stack then serves as both detector and target. This
affords $4\pi$ acceptance, which is useful in studies in which, for example, particle
production is correlated with target fragments. The stack geometry also makes charge assignment possible by track densitometry, grain counting, or delta ray counting along several millimeters of track length [Powell 59]. However, stacks also have several shortcomings. There are measurement biases in the z-direction. One potential source of bias is emulsion "shrinkage" by a factor of 2–3 in volume in the z-direction when developed due to the removal of AgBr grains. Although this bias can be corrected, a more serious bias is introduced by the non-isotropic resolution of the microscope. Resolution along the focus axis (∼ 2 μm or worse) is not as good as the resolution in the plane of focus (∼ 1 μm). Further, a track lying directly "under" another may be completely shadowed and therefore invisible. In addition to measurement bias, the emulsion induces secondary interactions, particularly electron pair production, which adds spuriously to multiplicities, and, even worse, introduces spurious spatial correlations in the data. These correlations are especially important in searches for anomalous angular fluctuations.

Chambers, on the other hand, are oriented perpendicular to the tracks. By using emulsion only to sample the path of the track, chambers present far less grammage both to the incident beam and to produced particles. Chambers can have a thin metal foil target instead of a composite emulsion target. The developed emulsion is viewed in the microscope from above, so there is no azimuthal measurement bias. The emulsion is affixed to the acrylic substrate with adhesive, so the developed emulsion shrinks perpendicular to the plate. Fig. 2.5 illustrates a typical KLMM emulsion chamber. Each emulsion plate consists of a 200 μm thick acrylic base coated with a 55 μm Fuji ET7B emulsion layer on each side. Each plate is an extremely "light" detector, consisting of only ∼ 0.06 g/cm² of material. (Most tracks are fully measured before they pass through 4 such plates.) Each of the 32 chambers has a 10 cm × 5 cm front area, and holds 3–4 100 μm thick lead target foils.
Figure 2.5: KLMM Pb chamber used at CERN. A chamber with three target modules is shown. Some of the chambers had four targets. The right-hand columns show details of the upstream chamber structure at 10 and 100 times the scale of the left column. The horizontal scale is arbitrary.
The exposure of the chambers to the 158 GeV/c $^{208}$Pb beam resulted in an average of ~350 primary $^{208}$Pb ions/cm$^2$ across the face of the chambers, concentrated in three $1.5 \times 2$ cm$^2$ beam spots. This density is small enough to ensure a low delta-ray background and to keep the data cuts due to interactions occurring too close to a non-interacting primary to an acceptably low level.

### 2.3 Data Acquisition

Event reconstruction through the analysis of microscope images is done in several stages. The processing chain is shown in Fig. 2.6. The data-taking phase consists of:

- scanning, which locates and selects events for study,
- image acquisition, which records microscope fields around the event in several plates, each spanning a different range in opening angle,
- image analysis, which finds the track candidates in individual fields of view, and
- track reconstruction, which combines the measurements of track candidates measured in individual plates, and assigns space angles and charges to the tracks.

#### 2.3.1 Scanning

To select a sample of relatively central interactions, the emulsion plates directly below each target were visually scanned at low power (200x) for high multiplicity events. Fig. 2.7 illustrates a typical low power field of view containing a large event. After the initial scanning selections were made, each event was examined in all the plates upstream of the interaction and rejected if the primary was noticeably less ionizing (approximately 5 charge units) than nearby Pb tracks or if
Figure 2.6: Overview of the reconstruction analysis chain.
Figure 2.7: Typical low-power field of view. The field is 1.8 mm x 2.3 mm. The large spots are Pb primaries. The event shown in Fig. 2.1 appears as a small spot near the center of the image (arrow). The other small spots are fragments from peripheral interactions. The horizontal streak is an emulsion surface imperfection.
the primary had suffered an additional observable interaction. The plates adjacent to the target allowed rejection of interactions occurring in emulsion rather than in the lead target. The event was also examined downstream and rejected if the remnants of the projectile contained fragments noticeably heavier than alphas. (Only two events were rejected on this basis.) Events with nearby (less than 60 $\mu$m) non-interacting primaries which might obscure secondary tracks were also rejected. These high-multiplicity events are as conspicuous in the emulsion as the Pb primaries themselves. Few if any of the very largest events are missed in scanning. Events with charge multiplicities above $\sim 1000$ are scanned efficiently, but those with lower multiplicities are sampled incompletely. The appraisal of multiplicity during scanning is very rough, and therefore we expect a gradual roll-off of scanning efficiency at low multiplicities. The smallest event found has a multiplicity of 590. We estimate that we have selected $(22.2 \pm 2.7)$% of all nuclear charge-changing interactions in the lead targets of the scanned chambers. Scanning efficiency is discussed further in Chapter 3.

2.3.2 Image Acquisition

To digitize the emulsion images for event reconstruction, we have constructed several microscopy systems equipped with PC-controlled stages and CCD cameras (Fig. 2.8). In the usual "high-power" mode of operation, a 100x microscope objective together with a 0.45x coupling lens yields a useful image which is 108 $\mu$m x 140 $\mu$m, and which has about a $\sim 2 \mu$m depth of field. (Fig. 2.1-2.4 showed several images taken at high power). In the typical "low-power" mode, a 6x objective gives a 2.3 mm x 1.8 mm field of view, with a depth of about 200 $\mu$m (Fig. 2.7). The digitized pictures are 512 pixels x 480 pixels x 8 bits. The microscope stage is equipped with stepping motors and linear optical encoders on all three axes. It
Figure 2.8: The LSU automated microscopy system. The monitor can display the contents of either buffer or the "live" digitized image, allowing it to be used as a blink comparator between live and stored images, or between two stored images.
can be stepped under software control in 1 μm steps in three directions, or it can be operated manually.

During acquisition of a focus sequence, the stage is controlled by the image acquisition program. This program monitors the CCD image and begins acquisition when it finds the upper surface of the emulsion. It then steps the focus vertically in 0.8 μm steps until it finds the lower surface, at which time it terminates acquisition and writes the focus sequence to file. Surfaces are detected by subtracting consecutive frames and finding the largest absolute residual in a selected window. If $I(x, y, z)$ is the image brightness at a pixel located at coordinates $(x, y, z)$, the focus signal $F$ is

$$F = \max |I(x, y, z + \Delta z) - I(x, y, z)|$$  

resulting in a focus signal like the example in Fig. 2.9(a). (This works because there are almost always at least a few grains in focus when the microscope is focused in the emulsion. Moving the focus 0.8 μm makes these in-focus grains significantly more blurry and brighter.) To avoid triggering on dust or oil bubbles outside the emulsion, this calculation is performed in four separate windows [Fig. 2.9(b)]; the second highest value is kept and the other three are thrown out. The focus signal is filtered and the result compared to a preset threshold to determine whether the microscope is focused inside or outside the emulsion. Depending upon the exact emulsion thickness, approximately 20 frames are acquired in each focus sequence.\(^1\)

The determination of the emulsion thickness is repeatable to ±1 μm.

### 2.3.3 Reference Systems

Ideally, track coordinates would be measured with respect to some point in the chamber whose position is well-determined. In practice, the procedure is more

\(^1\)Note that $20 \times 0.8 \, \mu m = 16 \, \mu m$ is a typical emulsion thickness after development, and is substantially less than the nominal 55 μm pre-development thickness quoted above.
Figure 2.9: Automatic focusing subsystem. (a) Focus signal as a function of focus depth. (b) The focus signal is measured in four separate windows.
subtle. Partly, this has to do with the accuracy of mechanical microscope stages. One micron absolute accuracy over 10 cm pushes the state of the art, yet MIPs within a high-power field of view can be measured to ±0.2 μm relative to one another. So instead of using a single global coordinate system, a separate system is established for each event, based on the positions of local non-interacting primary tracks (LPs). (There is a global coordinate system for locating events, but these coordinates are only accurate to ~ 300 μm. To distinguish between the global and the LP systems, we refer to the former measurements as "event coordinates" and the latter as "plate coordinates." ) Lead primaries in Fuji emulsion are ~ 30 μm in diameter, and we have found that the axis of a primary can be routinely determined to ±5 μm. To measure the position of the event axis, the event primary is measured relative to the LPs upstream of the vertex. Positions on subsequent plates are then measured relative to the same LPs.

This reference system is easily established using a blink comparator. In the plate upstream of the interaction, the event primary is centered in the field of view, a low-power image of the LPs is recorded, and then the blink comparator is used to line up the rest of the plates on the stage by comparing the microscope image on the camera monitor to the previously recorded upstream image. This procedure also quickly identifies heavy particles which have undergone a peripheral interaction, since these large fragments do not travel parallel to their non-interacting neighbors, and appear to jump back and forth as the comparator is blinked. These fragments are disregarded when establishing the reference system. After alignment using the LPs, the event axis is centered in the field of view to ±5 μm (Fig. 2.10), which is much better than the plate coordinates, but still far worse than the relative uncertainty in MIP positions. Therefore the residual plate shifts must be fitted out during track reconstruction, a process described in Section 2.5. Thus, the reconstruction software must still treat the position of each plate
Figure 2.10: Errors in plate alignment using local noninteracting reference pri­
maries. The determination of alignment errors is discussed in Section 2.5.
as a free parameter. What the LP alignment establishes is the position, in each image, of the event axis (to \( \pm 5\mu m \)).

The longitudinal plate coordinates are also uncertain at the level of a few hundred microns, due to small air gaps, spacer non-uniformity, and variation in emulsion thickness. The reconstruction software must also fit these parameters. A fiducial Rohacell foam spacer 1.500 cm thick located 3.3 cm downstream provides one well-known spacing, and the other plate spacings are inferred from this and the fits.

In the transverse plane, the fitted track location has a statistical uncertainty of \( \sim 0.2 \mu m \) and tracks typically leave the field of view at transverse distances \( \sim 40 \mu m \) from the event axis; the resulting 0.5% uncertainty in the transverse position corresponds to \( \delta \eta = 0.005 \). A systematic uncertainty in the transverse positions derives from the absolute determination of the event axis. Typically, the last measured plate is 3.3 cm downstream. This results in a typical systematic uncertainty in the azimuthal angle \( \theta \) of \( 5 \mu m/3.3 \text{ cm} = 0.15 \text{ mrad} \) in the absolute positioning of the event with respect to the reference system. The uncertainty in the longitudinal track positions has a statistical component which is greatest at large angles but does not exceed 1%, and an estimated 1% systematic component due to uncertainty in the fiducial spacer thickness. The overall uncertainty in the pseudorapidity ranges from \( \sim 0.01 \) at small \( \eta \) to 0.03 at \( \eta = 6 \). The value of the pseudorapidity loses significance beyond \( \eta = 9 \).

### 2.4 Image Analysis

Image analysis begins with a focus sequence of images and ends with a list of track candidates and their coordinates for that sequence's field of view. The analysis must efficiently discriminate secondary tracks (the signal) from the various back-
grounds. It must do it fairly quickly, and therefore simply; since 15-20 such fields of view are analyzed to reconstruct one typical event, speed is an issue if the system is to be practical. To develop the analysis, the ideas of emulsion "signal" and "background" need to be articulated precisely enough so that they can be translated into computer code. The software might be written to hunt for individual grains, and then assemble them into tracks; it might treat the tracks themselves as primitive objects; or it might recognize an interaction vertex as a "gestalt". We have settled on the last strategy, which provides excellent signal-background separation while at the same time being computationally practical.

Secondary tracks in emulsion have a straight, ray-like appearance,² appearing either as a series of distinct grains, randomly distributed along the track, or a more or less solid track of ionization, accompanied by occasional delta rays (knock-on electrons). A track which is viewed almost end-on is not easily resolved into distinct grains. (See App. B for a discussion of the impact of the microscope response function on track imaging.) In any case, a minimum ionizing particle produces on average one developed grain every 3.5 \( \mu m \) along its path, yielding 16 ± 4 grains in 55 \( \mu m \) of emulsion. The individual grains appear at high power as small regions (\( \sim 0.5 \, \mu m \)) which are 40-70% as bright as their surrounding neighborhood. (The variation is a reflection of the variation in the size of the grains themselves [Powell 59].) Small angle Coulomb scattering is negligible in 55 microns of emulsion for even the lowest energy produced particles. Secondary interactions are quite rare; the pion nuclear m.f.p. in emulsion is 35 cm. The geometry of

---

²There are two cases in which trajectories are not straight lines. The first is when there is "C-distortion" in the emulsion resulting from differential stresses freezing in the emulsion during drying, causing straight trajectories to appear curved. Due to the excellent development of the Pb plates, C-distortion is practically non-existent in this exposure. However, the image analysis routines have been successfully tested in the AGS Au chambers, where this effect was observed. The other instance in which trajectories are not straight is target fragments (the so-called "black tracks"). These are emitted with a typical energy of 20 MeV, and range out within 3 mm. They are isotropically emitted, so a chamber's black track acceptance is low. Therefore, while black tracks are considered signal in stacks, they are viewed as background in chambers.

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secondary tracks is therefore simple: we are looking for straight tracks that point back to a common vertex. The other feature which distinguishes secondary tracks from background is that they have a minimum specific ionization and therefore a minimum mean darkness. They may be darker than minimum, however, as is the case with projectile spectator fragments.

The physical backgrounds can be grouped into two categories. In the first group are "random tracks," which are straight but are not associated with the event under study. The only way to distinguish these real but unrelated tracks from those which are created by the interaction is by confirming whether or not they point back to the vertex. The other kind of background tracks are delta rays. These electrons are minimum ionizing, but scatter significantly in a single emulsion layer. They therefore deposit more ionization energy in emulsion than more massive MIPs. Heavy ion beam tracks produce copious numbers of long-range deltas, and some of these escape the emulsion plate in which they were produced, giving rise to a fairly uniform distribution of deltas on top of the local distribution surrounding each beam track.

Among the instrumental backgrounds are "chemical fog," consisting of developed grains which are not associated with any ionizing track, but are an artifact of the development process. Emulsion surface defects may also be prominent enough to cause problems, especially if the emulsion is thin.

The last kind of background, shadowing, is not strictly a background at all; rather, it is an instrumental effect. In ordinary transmitted light microscopes, the light passes through the entire two-sided emulsion plate before reaching the eye or CCD. Thus, the objects near the plane of focus are not uniformly illuminated, but are shadowed by out-of-focus objects below (and above) them. The magnitude of the darkening of the field due to shadowing is of the same order of magnitude as the darkness of the grains themselves.
The nature of the signal, and backgrounds, give us some clues about how a successful track recognition algorithm should work. Because the individual grains in a track are not always resolved, and also because many or most grains are not part of secondary tracks, it is reasonable to try to detect the entire track rather than the grains of which it is composed. We could therefore operationally define a track to be a straight path through the emulsion which has some minimum average darkness. This criterion excludes chemical fog, which is dark only in some small neighborhood (Fig. 2.11) and therefore contributes only a small amount of darkness to any path through the emulsion. It also discriminates against delta rays. Because of their scattering, delta rays develop more grains than other MIPs, and sometimes mimic real secondaries, especially if they are energetic enough to follow more or less straight paths for 20-30 μm. We therefore need a second criterion — that the dark path be small and compact in the transverse direction in order to ensure that the particle that produced the path did not scatter. We can accomplish this by demanding that the path be darker than similar paths in its local neighborhood. This criterion simultaneously solves the shadowing problem, since we measure the darkness of tracks not in terms of the intensity of light incident on the plate, but relative to the brightness in their immediate neighborhood.

The final criterion to be folded in is the requirement that all selected tracks point back to a common vertex. It is important to understand that the vertex point one sees in the emulsion does not correspond directly to the event coordinates of the interaction point due to emulsion shrinkage and distortion (Fig. 2.12), as well as to the uncertainty in the measurement reference system. We need to identify tracks which point back to this "apparent vertex" whose position is known a priori to ±5 μm in the transverse direction and to within 5-50% in the longitudinal direction, relative to the center of the microscope field of view. (The larger value applies to the plate closest to the target.) Because of the uncertainty in the
Figure 2.11: Patterns of ionization from tracks, delta rays, and chemical fog.
Figure 2.12: Relation between real and apparent vertex.
apparent vertex position we need to modify the vertex criterion slightly: we demand that all secondary tracks point to a common apparent vertex whose position will have to be found as we search for tracks.

This new vertex requirement brings us to the conclusion promised above: the software will be an apparent vertex finder, rather than searching for individual tracks. Once the vertex is found, the individual tracks of which it is composed can be identified and characterized.

Fig. 2.13 illustrates how the vertex finding is actually done. For each trial vertex, the intensity in individual frames is averaged along paths radiating from the trial vertex. This produces a processed image which can be thought of as what the emulsion would look like from the standpoint of the trial vertex. Tracks passing through the trial vertex appear as dark spots, while isolated grains, coincidental tracks, and delta rays appear washed out. The vertex finder evaluates trial vertices and searches for the one with the maximum number of small dark spots. Fig. 2.14 shows such a processed or “accumulated” image. To count the number of tracks in the accumulated image, the vertex finder first high-pass filters the image, which implicitly imposes the compactness criterion by removing large (diameters greater than 1 \( \mu m \) or so) objects, and also removes the shadowing bias. The pixel darknesses in the resulting image are then compared to a threshold (Fig. 2.15), producing yet another image in which the dark pixels are turned on and the bright pixels turned off. Each distinct cluster of dark pixels is counted as a candidate track, and the optimization routine in the vertex finder maximizes the number of clusters to determine the best apparent vertex and produce the final accumulated image, which is stored for further analysis. App. C describes the cluster finding, optimizing, and lookup routines in more detail.

The filtering routine in the vertex finder is optimized for speed, since it is not necessary to find every single track to accurately determine the apparent vertex.
Figure 2.13: Schematic illustration of the vertex finding process.
Figure 2.14: Typical accumulated image. This is a processed image of the same field of view shown in Figs. 2.1-2.2. A 20-frame focus sequence has been used to generate this image.
Figure 2.15: Histogram of filtered image values on every darkness peak. The threshold is individually determined for every field of view based on the position of the background peak.
position. The final accumulated image is therefore handed off to a second-stage image analysis routine which performs essentially the same analysis but in a more careful manner (App. C.1.4). Each resulting cluster is centroided to measure the track positions. In addition, each track's darkness is measured by comparing the mean brightness of pixels around the track centroid to the pixel brightness off-track. The resulting list of track positions and darknesses of each candidate track is saved for later submission to the plate fitting and reconstruction routines.

To give a qualitative idea of what has been accomplished so far, track candidates from the same event measured in two different emulsions are compared in Fig. 2.16. The measurements in the upstream emulsion have been scaled and shifted to overlap with the downstream layer. The correspondence is quite good, but, as expected, there are some candidates in one emulsion that do not appear in the other. Either a real track was missed in one layer, or background was incorrectly identified as a secondary track. One can see from this comparison that it is possible to clean the candidate track lists by comparing consecutive emulsions. The vertex finder analyzes each field of view independently, and this allows us to use coincidence techniques both to clean the track list and to systematically estimate backgrounds and efficiencies. To do this, we must first assemble all of the individual emulsion track lists into a single list for the entire event. This is the subject of the next section.

2.5 Reconstruction

The image analysis produces track lists from each individual measured emulsion. The reconstruction routine must then search all the emulsions for the individual measurements along each track and join them together to form a single track list. Reconstruction entails precisely determining the emulsion positions relative to one
Event 2006

- 5U
- + 4U

\[ \Delta x = -0.97 \]
\[ \Delta y = 0.41 \]
\[ \text{scale} = 1.2672 \]

Figure 2.16: Coincidence of measurements in two emulsions. The units are arbitrary.
another and to the vertex, and then comparing the individual measurements in all the plates to find the real tracks and reject the background.

Fig. 2.17 displays measurements from two emulsions side by side. It is not immediately clear which measurement pairs belong to the same tracks. In order to connect the measurement pairs, one must know the relative positions of the emulsions and the vertex. However, the uncertainties in these positions, which have been determined from local non-interacting primaries (±5 µm) and knowledge of the chamber structure (± ~300 µm), are far too large for positive assignment of individual measurements to particular tracks. Fig. 2.18 schematically illustrates the plate alignment problem. Almost all produced particles emitted from the interaction have virtually straight trajectories [Fig. 2.18(a)]. After disassembling the chamber for development and image acquisition, we have imprecise knowledge of the plate and vertex positions, which makes track reconstruction ambiguous [Fig. 2.18(b)].

In principle, the apparent vertices provide information about the plate positions, but using this information for plate alignment works poorly in practice, mainly for two reasons. A small amount of linear emulsion distortion can shift the apparent vertex horizontally many microns. In addition, precise knowledge of the emulsion shrinkage factor, a function of relative humidity and temperature, is required and entails careful manual measurement of every plate at the time of image acquisition. Instead, the plate alignment is done using pattern matching software [Fig. 2.18(c)]. As the figure shows, the pattern matching determines positions relative to the vertex up to a transverse shift (i.e., an uncertainty in the direction of the event axis) and a longitudinal scale. The transverse ambiguities are removed by assuming the event axis is parallel to the local non-interacting primaries (LPs), and the longitudinal scale is determined using the fiducial spacer.
Figure 2.17: Measurement maps before plate alignment.
Figure 2.18: The plate alignment problem. (a) Plate alignment prior to disassembling the chamber. The large circles represent a local noninteracting primary, smaller dots represent shower tracks. (b) Upon disassembly, plate registration is lost. (c) Pattern matching reconstructs plate positions up to an overall transverse shift and longitudinal scale. (d) Local noninteracting primaries determine the shift and the fiducial spacer determines the scale.
Once this information is incorporated, the original event geometry is reconstructed [Fig. 2.18(d)].

The pattern matching algorithm aligns a pair of emulsions by shifting the upstream emulsion measurements with respect to the downstream points by an offset \((\Delta x, \Delta y)\) and by scaling the upstream measurements by a factor \(s\) in order to maximize the overlap between the two emulsions (Fig. 2.19). To characterize the quality of the overlap, the figure of merit \(S\) that is maximized is

\[
S = \sum_{i=1}^{N_{DS}} e^{-\left(d_{(nn)i}/\rho_0\right)^2}, \tag{2.2}
\]

where \(N_{DS}\) is the number of tracks in the downstream side, \(d_{(nn)i}\) is the distance between downstream track \(i\) and its nearest neighbor in the upstream emulsion, and \(\rho_0\) is set to 1.0 \(\mu\)m. For close pairs \(d_{(nn)} \ll 1.0 \mu\)m, the individual exponential terms approach

\[
1 - \left(d_{(nn)i}/\rho_0\right)^2, \tag{2.3}
\]

and \(S\) is a measure of the sum of the squares of distances between nearest neighbors. The exponent discounts tracks whose nearest neighbors are more than 1.0 \(\mu\)m away, as these are likely to be spurious measurements. This fitting procedure is performed in a pairwise fashion, starting with the most downstream pair of emulsions and chaining up to the most upstream. For example, the downstream side of plate 5 is fitted to the upstream side of plate 5, which is fitted to the downstream side of plate 4, which is fitted to the upstream side of plate 4, etc. Every matched emulsion pair is plotted (e.g., Fig. 2.19) for visual inspection to confirm the fits. The fitted emulsion positions are also compared with the known chamber structure to check for gross fitting errors (Fig. 2.20).

Once the plate matching is complete, direction vectors which cluster together are assigned to each measurement, and the individual measurements can be grouped together into track candidates. The direction vector to a point from the
Event 2006

- 3U
- 2U
- \( \Delta x = 0.95 \)
- \( \Delta y = 2.37 \)
- scale = 2.9683

Figure 2.19: Coincidence of measurements in two emulsions. Compare these superimposed measurements with the same measurement in Fig. 2.17. In the upstream emulsion (crosses), which is about 80 \( \mu m \) from the vertex, the tracks in the center of the field of view are not resolved. The downstream emulsion (squares) is 2.97 times farther downstream, and the tracks are resolved almost to the event axis. On the other hand, the upstream emulsion shows wide-angle tracks which have passed out of the field of view of the downstream emulsion. The units are arbitrary.
Figure 2.20: An additional test of the plate fits is provided by the reconstructed longitudinal chamber structure. Physically, the chamber structure is nearly identical for every event, although not every plate is always measured. The fitted distance from the vertex to each measured plate is plotted for several events. The two incorrect events, 18-16 and 20-27, stand out clearly.
vertex is characterized by \((x_{\text{ref}}, y_{\text{ref}})\), the point at which the trajectory intersects an arbitrary reference plane parallel to the plates at distance \(z_{\text{ref}}\) from the vertex. The direction vector is related to the space angles through

\[
\tan \theta = \sqrt{\frac{x_{\text{ref}}^2 + y_{\text{ref}}^2}{z_{\text{ref}}}},
\]

\[
\tan \phi = \frac{y_{\text{ref}}}{x_{\text{ref}}}. \tag{2.4}
\]

Qualitatively, a track candidate is a cluster of individual measurements with similar values of \(x_{\text{ref}}\) and \(y_{\text{ref}}\) (within \(\sim 1.0 \mu\text{m}\)). A list of clusters is generated according to criteria which are unrestrictive enough to include almost all real tracks, and also some spurious candidates. The main requirements for a candidate to be considered a confirmed track are:

- A candidate must be measured in at least two emulsions. This coincidence requirement efficiently discriminates against residual background.

- The candidate cannot be missed in more than two consecutive emulsions. This requirement cuts accidental coincidences, and also tracks which do not point precisely back to the vertex.

In addition, there are further tests against low-energy tracks and tertiary electron-positron pairs, and against spurious close pairs (caused by one false measurement close to a real track). Appendix E describes these selections, as well as the details of the cluster finding algorithm.

### 2.6 Results

To test the quality of the reconstruction against traditional techniques, we measured two events, with multiplicities \(\sim 670\) and 1300, both manually and automatically. The manual measurements were done using the semi-automatic microscope
Table 2.1: Comparison of manual and automatic measurements of two 10.6 GeV/n Au-Au events. Column 1 shows the number of tracks that were found with both methods. Column 2 is the number of tracks which were mistakenly identified, and Col. 3 is the number missed by one method but not the other.

<table>
<thead>
<tr>
<th>Event 1:</th>
<th>Agree</th>
<th>False Tracks</th>
<th>Missed Tracks</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automatic</td>
<td>121</td>
<td>0</td>
<td>2</td>
<td>123</td>
</tr>
<tr>
<td>Manual</td>
<td>116</td>
<td>1</td>
<td>6</td>
<td>123</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Event 2:</th>
<th>Agree</th>
<th>False Tracks</th>
<th>Missed Tracks</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automatic</td>
<td>139</td>
<td>0</td>
<td>0</td>
<td>139</td>
</tr>
<tr>
<td>Manual</td>
<td>134</td>
<td>5</td>
<td>0</td>
<td>139</td>
</tr>
</tbody>
</table>

system at INP in Krakow. A CCD camera is used to display the microscope image on a monitor, allowing tracks to be measured by the operator using a mouse and cursor. These track measurements are stored in a computer file as they are taken. The events were each measured twice by two physicists at INP, and manually reconstructed. The discrepancies in the manual reconstructions were then resolved on a track-by-track basis, to provide as clean a track list as possible against which the automatic measurements could be compared. Using an earlier version of the software, a similar test (with one manual measurer) was performed using two AGS Au events with multiplicities of about 120 (Table 2.1). The Au comparison provides a more rigorous test of background rejection (indeed, background grain counts in the Au chambers are similar to those found in Antarctic JACEE flights), but the Pb events have larger multiplicities, the tracks more densely populate the plates, and there are many more plates applied in the reconstruction (~12 for Pb, compared to 3 or 4 for Au).

Initially the manual-automatic comparison of the Pb events indicated that the automatic reconstruction overestimated the multiplicities by about 10%. After detailed comparison of the automatic and manual reconstructions of Event 18-02, more stringent coincidence criteria were applied in the automatic reconstructions,
Table 2.2: Comparison of manual and automatic measurements of two 158 GeV/n Pb-Pb events. Column 2 shows the multiplicity, estimated by the technique described in Chapter 3. Columns 3-6 display the number of tracks detected by one method but not by the other. These may include both real and spurious tracks. Column 3 shows the number of discrepancies in the automatic measurement. Columns 4-6 show the manual discrepancies. The number of manual discrepancies which are within ~ 1μm of another discrepancy are shown in Col. 5. This category accounts for most of the differences between the two methods.

<table>
<thead>
<tr>
<th>Event</th>
<th>Est. Mult.</th>
<th>Extra Tracks</th>
</tr>
</thead>
<tbody>
<tr>
<td>18-02</td>
<td>672</td>
<td>5</td>
</tr>
<tr>
<td>18-22</td>
<td>1299</td>
<td>5</td>
</tr>
</tbody>
</table>

and the overall discrepancy in each event was reduced to the order of 5% (Table 2.2). The revised automatic track lists actually have significantly fewer tracks than the manual lists. Many of the extra tracks in the manual lists are close pairs (i.e., separated by ~ 1 μm or less in the farthest downstream plate in which they are measured). Most of the discrepancies are at wide angles (η < 4). Excluding the close pairs, the discrepancies amount to 4.6% of the counted tracks in Event 18-02, and 2.1% in Event 18-22.

The excess of close pairs in the manual measurements appears to be largely an artifact of the manual reconstruction. The most important known production mechanism of close pairs, π⁰ decay into e⁺e⁻ pairs, contributes only a few percent to the total charge multiplicity in these very light emulsion chambers, and cannot account for the entire excess (App. F). Fig. 2.21 shows the "nearest neighbor separation" distribution, i.e., the distribution of nearest neighbor distances ρₘₙ for pairs of tracks in the farthest downstream plate that is used for the reconstruction. The lack of pairs closer than about 1 μm in the automatic measurements is explained by the limited instrumental pair resolution [Fig. 2.22]. If the ~72 extra manually measured tracks are real, Fig. 2.21 indicates that they are spatially
Figure 2.21: Nearest neighbor distribution for Event 18-22. The transverse distance between nearest neighbor tracks is calculated for the most downstream measurable plate (inset shows the geometry), using the fitted track trajectories.
Figure 2.22: Fraction of resolved close pairs as a function of pair separation for Event 18-22.
correlated. But it is unlikely that a physical mechanism could produce pairs which all have nearest neighbor separations less than 1 μm, since the plate separations reflect a variety of angular separations. It is also unclear why these pairs would be produced at a substantially smaller rate in the smaller event. We conclude that the manual measurements overestimate the number of pairs with separations close to the microscope's optical resolution. Based on these comparisons, we have a value of 5% for the systematic counting uncertainty, which reflects manual/automatic discrepancies, as well as an estimate of undercounting of pairs with \( \rho < 1 \mu m \) (assuming these pairs are not correlated) in the automatic analysis.

These systematics illustrate not only the difficulty of counting the tracks in these large events but also the utility of an automatic measuring system. The nearest-neighbor and optical resolution studies performed on the automatic measurements are difficult to repeat manually. The systematic, repeatable nature of the automatic measurements has other advantages, as well. For instance, the efficiency with which tracks are detected can be estimated for every event, plate-by-plate. After the event is reconstructed, each track is examined for "missing" measurements, i.e., emulsions in which the track could have been detected (because it was well-separated from other tracks) but was not seen. A record of misses is kept for every event and every plate. Fig. 2.23(c) shows that the miss rates in our example event are on the order of 1% or less. Similarly, the "singles" background is estimated from the number of measurements which are not used in reconstructed tracks [Fig. 2.23(b)], and thus are presumed to be background. The measurement background and efficiency are used to estimate the number of false and undetected tracks. The errors for Event 20-06 are displayed in Table 2.3. (The propagation

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3A recent reanalysis of the semiautomatic measurement of Event 18-22 supports this hypothesis. This new analysis finds only 36 close pairs identified as singles in the automatic measurement. When these tracks were manually traced outward from the event axis, 21 were found to really be double tracks, and 15 to be singles.
Figure 2.23: Track count, singles rate, and measurement inefficiency by plate for Event 20-06. In plate 16, 12 tracks are visible. Two of these are missed in emulsion 16U, giving rise to a large miss rate in this emulsion.
Table 2.3: Reconstruction statistics for Event 20-06. (The multiplicity is estimated by the method described in Chapter 3.)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Est. Multiplicity (4π)</strong></td>
<td>1198</td>
</tr>
<tr>
<td><strong>Totals:</strong></td>
<td></td>
</tr>
<tr>
<td>Measurements</td>
<td>4329</td>
</tr>
<tr>
<td>Track Count</td>
<td>762</td>
</tr>
<tr>
<td>Singles</td>
<td>400</td>
</tr>
<tr>
<td>Misses</td>
<td>17</td>
</tr>
<tr>
<td><strong>Track Measurements:</strong></td>
<td></td>
</tr>
<tr>
<td>Tracks with 2/3/4 or More Measurements</td>
<td>171/95/396</td>
</tr>
<tr>
<td>Avg. Measurements. per Track</td>
<td>6.58</td>
</tr>
<tr>
<td>False Close Pairs</td>
<td>48</td>
</tr>
<tr>
<td>Large Dispersion Tracks</td>
<td>2</td>
</tr>
<tr>
<td>Good Tracks with 1 Unique Measurement</td>
<td>118</td>
</tr>
<tr>
<td><strong>Estimated Track Counting Errors Due To:</strong></td>
<td></td>
</tr>
<tr>
<td>Missed Measurements</td>
<td>-2.0</td>
</tr>
<tr>
<td>Unresolved Close Pairs</td>
<td>~ -4</td>
</tr>
<tr>
<td>Accidental Coincidences</td>
<td>+3.4</td>
</tr>
<tr>
<td>False Close Pairs</td>
<td>+7.4</td>
</tr>
<tr>
<td>Electrons from π⁰ decay</td>
<td>+19</td>
</tr>
<tr>
<td><strong>Estimated bias:</strong></td>
<td>+24 (+3.1%)</td>
</tr>
</tbody>
</table>

of individual measurement errors into track counting errors is described in App. E.) The track counting errors add up to about 3% of the track count, which is consistent with these comparisons if the extra manually measured close pairs are indeed spurious. It is likely that the automatic measurements have a bias toward overestimating the charge multiplicity by about this amount.

Having established that the reconstruction procedure produces “clean” track lists, we can examine some of the properties of these tracks. Fig. 2.24 shows the standard deviation of individual measurements about their fitted tracks. The mean standard deviation is 0.14 μm in both the x and y-directions, and the transverse measurement uncertainty is therefore 0.20 μm. Since the field of view is about 50 μm from center to perimeter, the track angles are determined to ~ 0.4%. 

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Figure 2.24: Standard deviations of individual measurements around their fitted track trajectories.

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Besides a track's space angle, its other main property is its darkness. Fig. 2.25 shows the darkness distribution near the interaction axis as a function of opening angle for 40 semi-central events. Most of the tracks within 5 mrad of the axis are minimum ionizing, but a more heavily ionizing component can also be observed, corresponding to spectator alphas and heavier fragments.

Using the automatic system, a single operator can measure and reconstruct events several times faster than previously possible. Because the analysis can be performed in parallel on several machines simultaneously, the measurement "bottleneck" is the image acquisition. With the current setup, a chamber with 20 events can be digitized in 3-5 days, and the analysis can be started while data from the next chamber is being acquired. A single Pb event is processed on a 66 MHz 486 PC in ~ 10 hours.

In summary, when the entire analysis chain from image analysis to track reconstruction is tested as a whole, the results agree well with careful manual measurements. Further, automatic measurement opens up new possibilities to rigorously understand counting systematics by providing consistent, detailed background and efficiency measures. This accomplishment augments one of emulsion's main strengths: the ability to characterize individual tracks. At the same time, automation ameliorates emulsion's chief weakness by making measurement much faster and simpler. It will be seen in the following chapters that the quality of the track counts and angular measurements makes possible not only a more detailed examination of kinematics, but may also provide new clues about interaction dynamics.
Figure 2.25: Darkness distribution of tracks near the event axis.
Chapter 3

High-Multiplicity Lead-Lead Interactions at 158 GeV/n*

3.1 Introduction

Current interest in studies of relativistic heavy nucleus collisions is based on the expectation that fundamentally important physical phenomena may occur as a result of the formation of high density, high temperature nuclear matter. Under such extreme conditions, matter may undergo a transition into a deconfined quark-gluon plasma phase [Schmidt 93]. The required conditions may have existed in the early universe, and they may be created in the interiors of neutron stars and in central collisions of energetic heavy ions. This last possibility provides an opportunity to study such extreme conditions in terrestrial laboratories. If high-multiplicity lead-lead central collisions are characterized by sufficiently high transverse momenta, $p_T$, and central pseudorapidity densities, $dN/d\eta$, the energy densities may reach the level at which a quark-gluon plasma (QGP) could be formed [Bjorken 83]. Although the produced particle multiplicities and their space angle distributions will surely be dominated by common features that reflect kinematical constraints and variations in the impact parameter, new phenomena (if they exist) may be observable above this anticipated background in forms such as very large multiplicities,

*This material has appeared in modified form in Physical Review C.
or non-statistical variations or fluctuations in the distributions of the secondary particles.

In December 1994, $^{208}$Pb ions were accelerated at CERN to a momentum of 158 GeV/c per nucleon, by far the highest energy ultra-heavy nucleus beam ever produced. The Krakow-Louisiana-Minnesota-Moscow collaboration (KLMM, CERN experiment EMU-13) exposed a series of nuclear emulsion chambers with Pb targets to this beam in order to study charged particle multiplicities and angular distributions from interactions in the symmetric lead-lead system. Emulsion's excellent spatial resolution allows accurate track counting and angular measurement, with relatively small systematic uncertainties. In this chapter, we present the first results from the measurement of a sample of 40 of the highest multiplicity Pb-Pb collisions. In this analysis we consider only the gross properties of the angular distributions and the multiplicities. However, individual event multiplicities are sufficiently high in these collisions that it is now possible to search individual events for deviations from the behavior expected from models based on incoherent superpositions of nucleon-nucleon collisions.

### 3.2 Experiment and Analysis Procedures

As described in Chapter 2, the emulsions were exposed perpendicular to the beam in chambers of 20 double-sided plates each, spaced out over a distance of approximately 17 cm from the first Pb target to the final emulsion plate. An emulsion chamber is an extremely "light" detector, as each plate consists of only $\sim 0.06$g/cm$^2$ of material.

To select a sample of relatively central interactions, the emulsion plates directly below each target were scanned for high multiplicity events. (Section 2.3.1 describes the event scanning and selection.) Few if any of the very largest events

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are missed in scanning. However, the appraisal of multiplicity during scanning is very rough, and therefore we expect a gradual roll-off of scanning efficiency with decreasing multiplicity. Events with charge multiplicities above \( \sim 1000 \) are scanned efficiently, but those with lower multiplicities are sampled incompletely. The smallest scanned event has a multiplicity of 590. Scanning efficiency is discussed further in Section 4 in connection with the multiplicity distribution.

As a result of the selection process, we have chosen events for analysis at a rate of \( (1.42 \pm 0.18) \times 10^{-3} \) event per incoming primary. By using the parameterization of the charge-changing cross section for ultra-heavy ion interactions found by Nilsen et al. [Nilsen 95] and Geer et al. [Geer 95], we expect a nucleon charge-changing cross section for 158 GeV/c per nucleon Pb-Pb interactions of 6.9 barns. Using this calculated cross section, we estimate that we have selected \( (22.2 \pm 2.7)\% \) of all nuclear charge-changing interactions in the lead targets of the scanned chambers.

A CCD camera-equipped microscope with stepper motors controlling all three microscope stage axes is used for this analysis. The acquisition is controlled by software which steps the focus vertically in 0.8 \( \mu \)m increments through the emulsion layer and automatically detects the surfaces of the emulsion to begin and end acquisition. Image analysis software searches the focus sequence for a persistent series of dark pixels radiating out from a common vertex, while rejecting isolated dark grains and tracks which do not point back to the vertex. The track "darkness," a measure of the ionization density, is also recorded in order to distinguish minimum ionizing tracks from those of alphas and heavier projectile fragments.

Projectile fragments are expected to be confined to the very forward direction. Fig. 3.1(a) relates the track darkness to the track emission angle \( \theta \), and shows a population of dark fragments mostly confined to a 2 mrad cone. Fig. 3.1(b) shows the darkness distribution for individual tracks inside the forward 2 mrad cone, corresponding to pseudorapidity \( \eta = -\ln(\tan(\theta/2)) = 6.9 \). Two peaks can be
seen corresponding to minimum ionizing particles and to heavier particles (mostly alphas). We have identified tracks within this 2 mrad cone with darkness less than 15 as minimum ionizing particles and tracks with darkness of 15 or more as fragments. The rms opening angle of the particles identified as fragments is \( \sim 0.7 \text{ mrad} (\eta = 8.0) \). Fields 108 \( \mu \text{m} \times 140 \mu \text{m} \) across are digitized in an average of nine plates along the axis of the event, and successive measurements from the individual plate sides are then fitted together to reconstruct the tracks in the event. By comparing the reconstructed tracks to their constituent measurements, we have determined the imaging system's pair resolution to be 1.0 \( \mu \text{m} \) and the rms scatter of individual measurements within an emulsion layer to be 0.2 \( \mu \text{m} \). To further discriminate secondary tracks from backgrounds, measurements in at least two emulsion layers are required within \( \sim 1.0 \mu \text{m} \) of each track [Deines-Jones 93]. This requirement results in the suppression of tracks below \( \eta = 2.6 \). All tracks in the data sample have been fully measured inside the \( \eta \geq 2.9 \) cone. In each event, the track detection efficiency and background rejection are estimated for each measured emulsion layer by counting the missing and rejected measurements in the successive plate sides, respectively. The image processing software detects tracks with an average 96% efficiency or better for \( \eta \geq 2.6 \).

In the transverse plane, the fitted track location has a statistical uncertainty of \( \sim 0.2 \mu \text{m} \) and tracks typically leave the field of view at transverse distances \( \sim 40 \mu \text{m} \) from the event axis; the resulting 0.5% uncertainty in the transverse position corresponds to \( \delta\eta = 0.005 \). A systematic uncertainty in the transverse positions derives from the absolute determination of the event axis. This is measured manually under the microscope by observing the positions of nearby non-interacting primary ions as reference tracks. The reference track positions are determined to 5 \( \mu \text{m} \); over a typical distance of 3.3 cm (corresponding to 15 emulsion plates), this results in a typical systematic uncertainty of 0.15 mrad in the
Figure 3.1: Track darkness in the forward cone. (a) Track darkness vs. opening angle $\theta$. (b) Darkness distribution of all tracks in the forward 2 mrad cone.
absolute positioning of the event with respect to the reference system. The un­
certainty in the longitudinal track positions has a statistical component which is
greatest at large angles but does not exceed 1%, and an estimated 1% system­
atic component due to uncertainties in the absolute mechanical spacing between
plates during the exposure. The overall uncertainty in the pseudorapidity ranges
from \(\sim 0.01\) at small \(\eta\) to 0.03 at \(\eta = 6\). The value of the pseudorapidity loses
significance beyond \(\eta = 9\).

### 3.3 Pseudorapidity Distributions

Fig. 3.2(a) shows the pseudorapidity distribution for the single event with the
highest multiplicity. In order to compare the data to expectations based on an
incoherent superposition model, we have simulated a sample of 1267 \(^{208}\text{Pb-Pb}\)
collisions using the FRITIOF 7.02 Monte Carlo code [Pi 92] with an unrestricted
range of impact parameters. In this preliminary study we have run FRITIOF
in its default configuration. The dotted curve shows the average pseudorapidity
distribution of the nine simulated events with restricted multiplicities \(N_{2.9-6}\) within
the region \(2.9 \leq \eta < 6\) which most closely match that of the measured event. (We
base our window on the region above \(\eta = 2.9\), where all tracks are measured
in at least two layers, and below \(\eta = 6.0\), above which spectators are expected
to appear in the measured data. Individual spectators are not included in the
FRITIOF calculations.) The two distributions are in good agreement \(\chi^2 = 0.83\)
over the entire measured range).

The mean pseudorapidity distribution \((dN/d\eta)\) for the entire data sample of
40 events is shown in Fig. 3.2(b) as the solid line. We have matched the measured
events with 40 events selected from the FRITIOF set with restricted multiplici­
ties most nearly equal to those of the real events, and have plotted their average
Figure 3.2: Pseudorapidity distributions of Pb-Pb interactions. (a) Pseudorapidity distribution of the highest multiplicity measured event (solid line), and the average of nine simulated FRITIOF events with similar multiplicities (dotted line). (b) The mean pseudorapidity distribution for the entire measured data sample (solid line) and that for a set of FRITIOF events selected with the same multiplicity distribution as the data (dotted line). Inset shows the region above $\eta = 6$ in detail.
distribution as the dotted line. In the region between 2.9 and 6, the difference between the distributions in Fig. 3.2(b) corresponds to a $\chi^2$ per degree of freedom of 1.33. Again, the shapes of the distributions agree well except for $\eta > 6.5$, where the data show the expected contribution by spectators. We note that in Fig. 3.2(a), the measured and calculated distributions agree well even up to the highest $\eta$ values. In this most central event, few if any spectators are observed.

To study the shape of the pseudorapidity distribution in the forward cone, we have separated the dataset into a "central" sample of 21 events containing two or fewer projectile fragments and a "semi-central" sample comprised of the remaining 19 events (Table 3.1). To compare the shapes of the distributions, we have normalized their areas between $\eta = 2.9$ and $\eta = 6$ to the area of the mean inclusive distribution, and plotted the normalized distributions for $\eta > 6$ in Fig. 3.3 along with the FRITIOF distribution from Fig. 3.2. The semi-central sample shows a component above the (spectator-free) FRITIOF prediction which is almost completely absent in the central sample, suggesting that FRITIOF predicts the shape of the forward produced distribution reasonably well, and that the "central" sample consists of events in which almost all of the projectile nucleons participate in the interaction. (According to FRITIOF, our central event sample should contain an average of 16 spectator protons distributed over the pseudorapidity range $\eta \geq 6$. We would therefore expect to see an excess of the number of measured tracks above the value for the FRITIOF pions equal to 16. In fact, we see an excess of $2.4 \pm 5.0$. 

Table 3.1: Central and semicentral datasets.

<table>
<thead>
<tr>
<th>Sample</th>
<th>No. Events</th>
<th>$\langle N_{\geq 2}\rangle$</th>
<th>$\langle N_{\text{prod}}\rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central</td>
<td>21</td>
<td>0.9 $\pm$ 0.8</td>
<td>1314 $\pm$ 210</td>
</tr>
<tr>
<td>Semicentral</td>
<td>19</td>
<td>5.3 $\pm$ 2.2</td>
<td>845 $\pm$ 160</td>
</tr>
</tbody>
</table>
Figure 3.3: Comparison of the shapes of the $\eta > 6$ region for central, semicentral, and spectator-free FRITIOF distributions. See text for details.
Table 3.2: Rate of increase of pseudorapidity densities with multiplicity.

<table>
<thead>
<tr>
<th>Interval</th>
<th>2.9 – 3.6</th>
<th>4 – 5</th>
<th>5 – 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.457 ± 0.004</td>
<td>0.340 ± 0.004</td>
<td>0.158 ± 0.003</td>
</tr>
<tr>
<td>FRITIOF</td>
<td>0.448 ± 0.001</td>
<td>0.337 ± 0.001</td>
<td>0.167 ± 0.001</td>
</tr>
</tbody>
</table>

These values differ by 2.7σ, suggesting that FRITIOF may be overestimating the pion production in the forward direction by perhaps 30%.

Deviations from the superposition model, if they occur, might be expected to be strongest in the largest events. We look for trends in pseudorapidity shape with changing event size in Fig. 3.4. In Fig. 3.4(a) the mean pseudorapidity density appears to be directly proportional to the restricted multiplicity both near the peak and in the forward direction. In particular, there is no indication of a flattening of the central peak even for the high multiplicity central events. This scaling implies that on average the shapes of the pseudorapidity distributions are independent of the event multiplicity. This linear behavior is reproduced by FRITIOF. Table 3.2 compares the one-parameter linear fits shown in Fig. 3.4 and the corresponding fits to the FRITIOF data. The shapes of the pseudorapidity distributions agree quantitatively with FRITIOF from η = 2.9 to η = 6.0 up to the highest measured multiplicities. (The uncertainties in Table 3.2 are statistical only. The 5% difference between the measured and the calculated slopes in the η = 5 – 6 interval is on the same order as our systematic counting error, and does not appear to be significant.)

Fig. 3.4(b) shows the total (unsigned) charge in five cones centered on the beam axis from η > 5.5 to η > 8.0. The fits are shown for all five cones. For clarity, the data from only three representative cones are shown. The forward cones include spectator protons and fragments as well as some produced particles. We have assumed that the fragments are all alphas, and calculated the total charge.
Figure 3.4: Relationships of forward charge and multiplicity density to multiplicity for several regions of the pseudorapidity distributions. (a) Produced multiplicities in three intervals. Fits are constrained to pass through the origin. (b) Total charge in several forward cones. The fits are all statistically weighted.
in the interval accordingly. As Figs. 3.2 and 3.3 illustrate, the region forward of \( \eta = 6.5 \) contains most of the spectator contribution. In this region, increasing the multiplicity (and the centrality) decreases the number of spectators and therefore the total forward charge. Widening the cone to include \( \eta > 5.5 \) (the top curve) includes enough produced particles so that the charge in this cone increases with increasing multiplicity.

The data in Fig. 3.4(b) are consistent with a linear relationship between event multiplicity and total charge. An additional test of linearity is possible for the \( \eta > 5.5 \) and \( \eta > 6.0 \) cones, which contain essentially all of the spectator charge. Very peripheral events must therefore have a charge of nearly 82 inside these cones. This is what the linear extrapolations predict. Larger cones have charge intercepts that are also consistent with 82.

Summarizing, the pseudorapidity distributions are consistent with superposition in general, and agree well with FRITIOF in particular. The shapes of the distributions are independent of multiplicity. When we compare the shapes of the measured Pb-Pb distributions to the shapes of simulated events with similar multiplicities, we see no significant differences except those in the forward region, which can be attributed to spectators.

### 3.4 Multiplicities

To estimate the produced charged particle multiplicities (i.e., the multiplicity excluding spectators) over all angles, we have scaled the restricted multiplicity \( N_{2.9-6} \) by a factor \( N_{\text{prod}}/N_{2.9-6} = 1.82 \pm 0.06 \) determined from the FRITIOF sample with \( N_{\text{prod}} > 600 \) (to mimic our scanning selections). Adding the uncertainty in the scaling factor in quadrature with the estimated systematic uncertainty based on our comparisons of manual and automated reconstructions, we estimate a typical
uncertainty in the produced multiplicity of 6%. The produced multiplicity for the largest event [Fig. 3.2(a)] is then $1729 \pm 100$.

The multiplicity distribution of our 40 measured events is presented in Fig. 3.5. We estimate in Chapter 2 that we have analyzed $(22.2 \pm 2.7)\%$ of all events in the chambers. To make a direct comparison with the data, we calculate the FRITIOF multiplicities using the same prescription $N_{\text{prod}} = 1.82N_{2.9-\sigma}$ as used to estimate the produced multiplicities of the measured events. (FRITIOF multiplicities computed using the entire $\eta$ range produce a distribution which is very similar to the one shown, but which falls off somewhat more steeply around 1850.) As expected from our event selection technique, we appear to undersample events with multiplicities less than 1000. At higher multiplicities, there is no evidence for an enhanced production probability. Indeed, we see fewer events above $N_{\text{prod}} = 1400$ than expected. This apparent deficit is statistically unconvincing, but intriguing. It cannot be fully explained by normalization uncertainties in $N_{\text{prod}}$ or our scanning rate.

To further investigate this possible deficit of large events, we examine the spectator region in greater detail. As the impact parameter $b$ decreases to 0, the number of spectators decreases. By using FRITIOF to estimate the number of produced particles in the forward region, we can calculate the multiplicity $N_0$ corresponding to events with no spectators, i.e., events in which the forward multiplicity is entirely due to produced particles. This multiplicity turns out to be rather insensitive to the FRITIOF model assumptions. Fig. 3.6(a) shows the total charge $Z_{\eta > 6}$ in the cone $\eta > 6.0$ vs. $N_{\text{prod}}$. The filled circles represent the central sample with two or fewer fragments, and the large open circles represent the semicentral sample with more than two fragments. Fig. 3.4(b) shows that this cone contains essentially all of the spectator charge. The total charge of the FRITIOF events inside the $\eta > 6.0$ cone has therefore been calculated by adding the produced
Figure 3.5: Probability distribution $dP/dN_{\text{prod}}$ of the estimated produced particle multiplicity $N_{\text{prod}} = 1.82N_{2,9-6}$. The distribution of the data (solid line) has been normalized to an area of 0.222 based on the calculated cross section and event selection efficiencies. The dotted line shows the results from an unbiased FRITIOF sample normalized to an area of unity. The shaded region shows the central events with two or fewer fragments. The right-hand axis shows the number of events in each multiplicity bin.
Figure 3.6: Measured charge in several forward cones (large filled circles are the central sample, large open circles are the semicentral sample), predicted produced charge (small crosses), and predicted total charge (points in upper band). Note that the vertical scales vary. The straight lines are statistically weighted fits, with the pion baseline fit constrained to pass through the origin.

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forward multiplicity (the “pion baseline”, shown as small crosses) to the spectator charge. (FRITIOF does not propagate individual spectators, but does report the total spectator charge.) The FRITIOF calculation of $Z_{n>6}$ is displayed as the small points in the top band. The FRITIOF distribution converges to charge 82 on the left, and merges into the pion baseline near $N_{\text{prod}} = 1850$, which a zero impact parameter ($b = 0$) run confirms as the mean multiplicity $N_0$ of head-on events predicted by FRITIOF.

The FRITIOF distribution lies significantly above the measured points. In addition, the $Z_{n>7}$ and $Z_{n>8}$ distributions in Fig. 3.6(b) and 3.6(c) merge into the pion baselines near $N_{\text{prod}} = 1500$, not near the expected $N_{\text{prod}} = 1850$. We cannot explain the difference as a systematic counting error, or in terms of a bias introduced by our event selection criteria. The intercept of the fit to the measured events at $Z_{n>6} = 82 \pm 4$ argues against a large systematic error in fragment charge assignments. In any case, such an error would not greatly affect the central sample, which has an average of only 0.9 fragments per event. We conclude that the discrepancy is real.

The difference indicates that FRITIOF cannot be correctly predicting both $N_0$ and the pion baseline in the forward direction. We first consider the possibility that FRITIOF predicts $N_0$ correctly, and that the difference is entirely due to an incorrect pion baseline. If $N_0 = 1850$ as FRITIOF predicts, one consequence is that our so-called “central” sample is not actually very central, despite the relative lack of alphas and heavier fragments. From Table 3.1, we estimate that these events would have on average $82 \times (1850 - 1314)/1850 = 24$ spectator protons, equal to the entire mean multiplicity forward of $\eta = 6.5 (25 \pm 1)$. Thus, the produced particle pseudorapidity distribution, which agrees with FRITIOF to within 5% up to $\eta = 6.0$, would have to abruptly cut off around $\eta = 6.5$, and the tracks forward of $\eta = 6.5$ in Fig. 3.3 would have to be essentially all spectators. Figs. 3.6(b) and

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Table 3.3: Spectator depletion analysis in five forward cones.

<table>
<thead>
<tr>
<th>Cone</th>
<th>$N_0$</th>
<th>$\sigma_{\text{stat}}$</th>
<th>$\sigma_{\text{syst}}$</th>
<th>$N'_0 - N_0$</th>
<th>$N'_0$</th>
<th>sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0</td>
<td>1370</td>
<td>60</td>
<td>70</td>
<td>330</td>
<td>1700 ± 340</td>
<td>1.49</td>
</tr>
<tr>
<td>6.5</td>
<td>1430</td>
<td>80</td>
<td>70</td>
<td>200</td>
<td>1620 ± 220</td>
<td>0.59</td>
</tr>
<tr>
<td>7.0</td>
<td>1480</td>
<td>50</td>
<td>70</td>
<td>120</td>
<td>1600 ± 150</td>
<td>0.31</td>
</tr>
<tr>
<td>7.5</td>
<td>1470</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>1550 ± 120</td>
<td>0.17</td>
</tr>
<tr>
<td>8.0</td>
<td>1570</td>
<td>110</td>
<td>80</td>
<td>60</td>
<td>1630 ± 150</td>
<td>0.13</td>
</tr>
</tbody>
</table>

3.6(c) confirm that for the data to be consistent with $N_0 = 1850$, an essentially complete absence of produced particles is required in these cones. The agreement in Fig. 3.3, the deficit in the multiplicity distribution, and the lack of fragments in the central sample all favor the interpretation that the difference between the data and FRITIOF in Fig. 3.6(a) is not entirely due to an incorrect model of the forward region, but is in significant measure caused by FRITIOF's overestimate of produced multiplicities.

We now consider the case in which FRITIOF correctly models the forward pseudorapidity distribution, but overestimates the produced multiplicities. In this case, it is possible to estimate the number of spectators in the measured events by subtracting the pion baseline. The mean multiplicity $N_0$ of head-on events, which have almost no spectators, is then estimated by the intersection in Fig. 3.6 of the fits to the measured events and to the pion baseline. (For simplicity, we neglect the small correction due to the fact that even head-on events probably have an estimated 4 charged spectator nucleons. This causes us to slightly overestimate $N_0$.) FRITIOF's total charge distribution in Fig. 3.6(a) crosses the produced particle line at 1840, which agrees well with the direct calculation of $N_0 = 1850$, demonstrating the reliability of the analysis technique. The analysis has been applied in five cones from $\eta > 6.0$ to $\eta > 8.0$, and the results are summarized in the first column of Table 3.3.
As discussed in Section 3, FRITIOF may actually overestimate the forward production by an amount on the order of 30%, in which case the pion baseline slopes in Fig. 3.6 are too steep, and the values of $N_0$ calculated in Table 3.3 are slightly too low. Table 3.3 also gives a corrected value $N_0'$ of the head-on multiplicity in the case where the slope $m$ of the pion baseline is decreased by 30%.

The systematic error $\sigma_{\text{syst}}$ in $N_0$ is dominated by the uncertainty in $N_{\text{prod}}$. There is also an uncertainty in the fragment charge assignment which propagates into $N_0$, but this contribution turns out to be negligible. Even assuming that the fragments are all carbon only changes the value of $N_0$ by 40: the intersection is mainly determined by the fragment-poor central points near the pion baseline. The systematic uncertainty due to the uncertainty in the forward production can be estimated from $N_0' - N_0$. Combining the statistical and systematic uncertainties, we find values of $N_0'$ from the five cones ranging from $1550 \pm 120$ to $1700 \pm 340$, all smaller than the FRITIOF value of 1850.

The sensitivity $\frac{\Delta N_0}{N_0} / \frac{\Delta m}{m}$ of $N_0$ to the pion baseline slope $m$ can be reduced as shown in Table 3.3 by choosing a narrow cone. However, the statistical uncertainty increases as the cone is restricted. We choose the $\eta > 7.5$ cone as the best compromise, giving a value of $N_0' = 1550 \pm 120$. The smallest that the pion baseline slope can possibly be is 0 ($\Delta m/m = -1$), corresponding to no pions at all in the $\eta > 7.5$ cone. In this case, the head-on multiplicity would rise to 1730, still below the FRITIOF value of 1850.

Finally, we combine our multiplicity and pseudorapidity results to find a relationship between multiplicity and peak pseudorapidity density. We use the data from the $\eta = 2.9 - 3.6$ interval in Fig. 3.4 along with the factor of 1.82 from FRITIOF to quantify the relationship between produced multiplicity and $(dN/d\eta)$ at the peak of the pseudorapidity distribution. The best linear fit (constrained to

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pass through the origin) gives $(dN_{\text{prod}}/d\eta)_{\text{peak}} = (0.25 \pm 0.01) \times N_{\text{prod}}$. Thus the mean pseudorapidity density for $b = 0$ events should be $390 \pm 30$. The highest pseudorapidity density we observe in a particular event is 425.

Summarizing, our study of event multiplicities shows a significant difference from FRITIOF when the forward charges are compared to the event multiplicities. Our estimate of the mean multiplicity of head-on events $N_0$ is $1550 \pm 120$, corresponding to a mean peak pseudorapidity density of $390 \pm 30$. No matter what the forward distribution, the best estimate of 1550 cannot increase to more than 1730, corresponding to $(dN/d\eta)_{\text{peak}} = 430$.

### 3.5 Discussion

Comparison of the data to FRITIOF shows good agreement in the pseudorapidity distributions at pseudorapidity densities as high as 425. There is no evidence in the data for flattening of the central pseudorapidity peak, even at pseudorapidity densities six times higher than in experiments at similar energies ($200 \text{ GeV/nucleon O and S on emulsion}$ [Dąbrowska 93]). Such flattening might be expected if a quark-gluon plasma had been formed [Bjorken 83]. It should be noted, however, that even for the largest event, with a central pseudorapidity density of 425, assuming $\langle p_T \rangle = 350 \text{ MeV/c}$ (FRITIOF’s value) and an interaction distance $2ct = 2 \text{ fm/c}$, the energy density evaluated with the standard expression [Schmidt 93] from Bjorken’s model

$$\epsilon = \frac{3}{4\pi A^{2/3}} \left( \langle p_T \rangle^2 + m^2_\perp \right)^{1/2} \frac{dN}{d\eta}$$  \hspace{1cm} (3.1)

(where $A = 208$ is the mass number) corresponds to only $1.1 \text{ GeV fm}^{-3}$.\(^1\) Although this energy density is significantly higher than in previous experiments at similar

\(^1\)There is a great deal of uncertainty in this number [Schmidt 93]. NA49 [Margitis 95] uses a prescription which gives an energy density about twice as high as cited here, mainly because the formation times differ by a factor of two.
energy, it may still be below the point at which a quark-gluon plasma should be formed.

Our determination of the mean multiplicity of head-on events, $N_0 = 1550 \pm 120$, is significantly lower than the value that FRITIOF predicts. It should be noted, however, that Adamovich et al. [Adamovich 92] report FRITIOF simulations with a mean production rate of 7.68 particles per nucleon-nucleon collision, implying $N_0 = 208 \times 7.68 = 1600$, in agreement with our measurements. The suggestion that these events are smaller than FRITIOF predicts has also been made by the EMU-01 collaboration based on the analysis of their first two events [Stenlund 95].

The first results of the NA49 experiment [Margetis 95] showed a peak negative particle multiplicity $dN^-/d\eta = 230$ for central events, indicating a charged particle multiplicity density of 460. This is higher than our value but perhaps consistent with it.

The pseudorapidity distributions of central events are dominated by mesons (mostly pions), which outnumber the approximately 164 scattered protons by a factor of $(1550 - 164)/164 = 8.5$. FRITIOF's excellent agreement with the pseudorapidity distributions is an indication that it correctly predicts the pion momentum distributions (c.f. App. A), but it says little about the proton momenta. (FRITIOF's agreement with the pseudorapidity distributions is not trivial. VENUS, in contrast, appears to produce a Pb-AgBr distribution which is slightly too narrow [Wosiek 95], suggesting that the simulated pions have too little longitudinal momentum. NA49 has directly compared VENUS and FRITIOF transverse energy distributions and finds that VENUS puts 7% more transverse energy in the central peak.) However, even though few of the charged particles are protons, nucleons are important in the energy balance of the interaction, accounting for $43 \pm 5\%$ of the final-state energy in central collisions. The energy balance of the
interaction,

\[ E_{\text{beam}} - E_{\text{spect}} = E_{\text{nucleon}} + E_{\text{meson}} + E_{\text{other}}, \]  \hspace{1cm} (3.2)

requires that the energy of the participants (the beam energy \( E_{\text{beam}} \) minus the spectator energy \( E_{\text{spect}} \)) manifests itself in final state nucleons, mesons, or other particles (gammas, electrons, nucleon-antinucleon pairs) that occur more rarely. It is therefore plausible that the unexpectedly small multiplicities are due to a somewhat larger than predicted fraction of the available energy going into scattered nucleons, i.e., a smaller than predicted nuclear stopping power.

The value of \( N'_0 \) marks the beginning of the tail of the multiplicity distribution. In the superposition model, the width of this tail is determined by the width of the p-p multiplicity distribution and the statistics of 208 independent nucleon-nucleon collisions. FRITIOF predicts the standard deviation of the \( b = 0 \) multiplicity distribution to be 60. If Pb-Pb interactions are indeed simply the result of independent nucleon-nucleon interactions, then with better statistics one would expect to see a rather rapidly diminishing tail beyond \( N'_0 \) with a width of approximately 60.

Although the method used to determine \( N'_0 \) requires a model of the pion baseline, it has some noteworthy compensatory advantages that distinguish it from other techniques. It does not rely on multiplicity cuts which could bias the result. (Fig. 3.5 distinguishes the central and semicentral samples used in Section 3, but the entire dataset is used in the multiplicity analysis.) It is insensitive to sampling biases, and does not require that the tail of the distribution be fully sampled. The result is almost independent of the absolute calibration of the forward charge measurement. And, it can be performed with a small set of carefully measured events in which the tracks have been individually counted.

In conclusion, charged particle multiplicities and pseudorapidity distributions have been measured by counting individual tracks in a sample of 40 high multiplicity
Pb-Pb collisions. The shapes of the pseudorapidity distributions are in good agreement with the results expected from calculations based on a superposition model of individual nucleon-nucleon collisions. Despite calculated energy densities twice those of previous experiments, we see no indication of QGP formation in the form of flattened pseudorapidity distributions or enhanced multiplicities. Indeed, our best estimate of the mean multiplicity of zero impact parameter events is $1550 \pm 120$, about 16% lower than predicted by FRITIOF.
Chapter 4

Pseudorapidity and Multiplicity Dependencies on Mass and Energy

4.1 Introduction

Our present understanding of nucleus-nucleus (AA) collisions is based on the notion that they are an incoherent superposition of individual nucleon-nucleon (NN) interactions. To apply this idea to the Pb-Pb system is to extrapolate over two orders of magnitude in mass from protons on protons and three orders of magnitude in the number of nucleon-nucleon collisions. Before attempting to search for exotic phenomena in this system, we need to ask if we actually understand its normal behavior. There are two motivations for this. In the first place, the main interest in the field today is the search for collective (that is, non-superposition) processes at high energy densities. If we are to recognize such new physics at the onset and to trust the finding, it will be necessary to have a quantitative understanding of the standard physics that governs these interactions. Aside from this important but technical reason for pursuing this study, we are interested in finding a simple framework for predicting multiplicities and angular distributions. As demonstrated in the previous chapter, the current generation of Monte Carlo programs give fairly accurate results, but it is hard to interpret the small but significant deviations
from the measurements or attach meaningful uncertainties to the calculations. In this chapter, multiplicities are calculated using the much simpler wounded nucleon model (WNM) [Bialas 76]. We find that the WNM makes multiplicity predictions which are as accurate as those from FRITIOF for energies from 14 GeV/n to 200 GeV/n and nuclei from protons to lead. A detailed analysis of the pseudorapidity distributions also provides evidence that the physical picture underlying the WNM must be essentially correct. We show that the distributions of a variety of projectile-target combinations at 200 GeV/n can be predicted with a simple extension to the WNM.

4.2 Particle Production Model

One can estimate the interaction path length of a nucleon passing through a nucleus from the pp cross section and the density of nuclear matter to be on the order of 1.5 fm. This is much smaller than the diameters of most nuclei, so one expects multiple nucleon-nucleon interactions as two nuclei pass through one another. Curiously, this expectation is not borne out by the experimental multiplicities. For example, for head-on Pb-Pb collisions at 158 GeV/n, we expect that almost all 208 of the nucleons in each nucleus will suffer at least one collision (i.e., will be "wounded"), and that on average each wounded nucleon undergoes $\bar{n} = 5.1$ interactions. (This calculation is explained below.) At this energy, the mean charge multiplicity of pp interactions is $7.2 \pm 0.7$. Naively, this implies an average head-on multiplicity of $208 \times 5.1 \times 7.2 = 7600$, a factor of 5 higher than the result discussed in the previous chapter.

Whereas this calculation gives too high a multiplicity for Pb-Pb interactions, it gives too small an answer for proton-nucleus (pA) collisions. A compilation of the 200 GeV pA multiplicities [Dąbrowska 93] is consistent with the expression
This kind of observation about pA multiplicities led to the essential assumption in the wounded nucleon model [Bialas 76]: the mean multiplicities in pA and AA collisions are proportional to the number of wounded nucleons, not the number of interactions. In a pA collision with $\nu$ NN interactions, there are $W_t = \nu$ wounded target nucleons, and there is $W_p = 1$ wounded projectile nucleon. In the wounded nucleon model, the average pA multiplicity is therefore

$$\langle n_{pA} \rangle = (7.56 \pm 0.30) [\nu + (1.10 \pm 0.10)]. \quad (4.1)$$

where $n_{pN}$ is the average proton-nucleon multiplicity. The factor $\frac{1}{2}$ accounts for the fact that a pp interaction wounds two nucleons. This formula also gives a much more reasonable value of $208 \times 7.2 = 1500$ for the Pb-Pb example.

The wounded nucleon model predicts particle multiplicities, but not their angular distributions. To do this, we make the additional assumption that a given pseudorapidity distribution is the sum of a target contribution $\rho_t(\eta)$ proportional to $W_t$, and a projectile contribution $\rho_p(\eta)$ proportional to $W_p$. The distributions $\rho_t(\eta)$ and $\rho_p(\eta)$ certainly depend on energy, but all the target and projectile dependence is assumed to be contained in the factors $W_t$ and $W_p$. Neglected in this model are the possibilities of reinteraction, which introduces an additional target/projectile thickness dependence, or a separate central production mechanism [Albrecht 91], which would require a third component. The motivation for this model comes partly from the observation of charge separation in pp and pp interactions [Breakstone 83], indicating that particle production in this system can be decomposed into a forward projectile component (in the lab frame) and a retarded target component, with some overlap in the central region.
We now briefly describe the wounded nucleon calculation. Except where noted, we are following the procedure described by Pruet [Pruet 90]. Let \( A_t \) (\( A_p \)) represent the number of target (projectile) nucleons, and \( \sigma_{N_t}, \sigma_{N_p}, \) and \( \sigma_{pt} \) stand for the total inelastic cross sections of nucleon-target, nucleon-projectile, and target-projectile interactions, respectively. The basic result for inclusive interactions is that the number of wounded nucleons \( W \) is

\[
W = A_t \frac{\sigma_{N_p}}{\sigma_{pt}} + A_p \frac{\sigma_{N_t}}{\sigma_{pt}}. \tag{4.3}
\]

The first term corresponds to the number of wounded target nucleons \( W_t \), and the second corresponds to \( W_p \). To evaluate cross sections, we assume that np, nn, and pp interactions have the same inelastic cross sections (although cross sections for individual channels may differ), and use the Particle Data Group empirical formula for pp interactions [Hikasa 92]. For pA and AA cross sections, we use the Letaw and Westfall semi-empirical formulas [Letaw 83, Westfall 79]. For the ultra-heavy Pb-Pb interactions, however, the estimate [Nilsen 95] of 6.4 b is used, which is 10% lower than the Westfall estimate. To characterize the significant systematic differences between measurements of pA and AA interactions [Wilson 91], we adopt a 10% systematic uncertainty which is propagated through the calculations.

The calculation has been extended to handle central collisions assumed to have impact parameters ranging from 0 to \( b_{\text{max}} \) [Sumiyoshi 83]. The result is

\[
W_t(b_{\text{max}}) = A_t \frac{\sigma_{N_p}(b_{\text{max}})}{\sigma_{pt}(b_{\text{max}})}; \quad W_p(b_{\text{max}}) = A_p \frac{\sigma_{N_t}(b_{\text{max}})}{\sigma_{pt}(b_{\text{max}})}. \tag{4.4}
\]

The partial cross sections for impact parameters between \( b \) and \( b + db \) are evaluated using a Glauber calculation, and integrated between \( b = 0 \) and \( b = b_{\text{max}} \). The physical inputs to this calculation are the nuclear density as a function of nuclear radius [Negele 70] and the NN inelastic cross section. The results are not very sensitive to the density, except for the pA case, where the number of wounded
nucleons varies \( \sim 10\% \) over a reasonable range of choices for the density function. However, the result has a roughly linear dependence on the NN cross section. We assume that projectile nucleons which strike one target nucleon have the same pp cross section for interacting with a second.

To experimentally isolate a set of central data, a cut, described below, is made to select central events. This is related to the maximum impact parameter through

\[
\frac{\sigma_{pt}^c}{\sigma_{pt}} = \frac{N_{pt}^c}{N_{pt}},
\]

where \( N_{pt}^c \) and \( N_{pt} \) are the number of events in the central and inclusive samples, and \( \sigma_{pt}^c \) and \( \sigma_{pt} \) are the partial and inclusive cross sections. The maximum impact parameter is related to the central partial cross section geometrically:

\[
\sigma_{pt}^c = \pi b_{\text{max}}^2.
\]

Fig. 4.1 shows the mean number of wounded target and projectile nucleons, and Fig. 4.2 the average number of interactions \( \nu_p \) and \( \nu_t \) a nucleon experiences while passing through the target or projectile, at 200 GeV/n, for the beam-target combinations used in this analysis. (Recall that the mean number of interactions is \( \nu = \frac{1}{2}[\nu_p + \nu_t]. \)) Results are almost identical for 15, 60, and 800 GeV/n.

### 4.3 Data and Analysis

To isolate the phenomenon of particle production, we must attempt to experimentally separate the produced particles from the tracks related to the nuclear breakup. In the projectile region, spectator fragment tracks \( (Z \geq 2) \) can be recognized by their higher ionization and removed. Projectile spectator protons, however, can only be distinguished from produced particles in a statistical sense. In the target region, the low energy target fragments \( (Z \geq 1) \) can be separated from produced particles because of their higher ionization. The tracks in the stack measurements
Figure 4.1: Number of wounded nucleons versus maximum impact parameter $b_{\text{max}}$. In (a-c), the curves represent, from top to bottom, the $W_t$ for the projectile interacting on Ag, $W_p$ for the projectile on Ag, $W_t$ for the projectile on Br, and $W_p$ for the projectile on Br.
Figure 4.2: Number of interactions $\nu_t$ ($\nu_i$) as a nucleon passes through the target (projectile) nucleus as a function of maximum impact parameter $b_{\text{max}}$. For simplicity, the Br curves are only shown for a S projectile. The Br curves for other projectiles are analogous to the S case, being from a few to 10% lower than their Ag counterparts.
are categorized either as shower particles or heavy tracks depending on whether their ionization $I$ is less than $1.4I_0$, where $I_0$ is the minimum ionization produced by a singly charged particle. The heavy group contains protons with energies below 400 MeV, and pions with energies below 70 MeV. Heavy tracks include the target spectator protons and fragments, promptly scattered protons (wounded target nucleons), and a few percent admixture of low energy pions. The separation of target spectator protons may not be entirely clean. For example, in the projectile region, a few percent of the spectator protons have pseudorapidities as low as $\eta = 6$ (e.g., Fig. 3.3), corresponding to $\theta = 5$ mrad, so their transverse momentum $p_t = 0.005p_{beam} = 1$ GeV/c. Based on their transverse momentum alone, these tracks would appear as minimum ionizing tracks in the projectile rest frame.

The datasets chosen for this analysis are taken from central interactions on targets at least as heavy as the projectile. In these events, there are few if any spectator protons in the projectile region, so almost all the particles in this region are either produced in the interaction or are projectile protons that have interacted in the target. Heavily ionizing heavy tracks, concentrated in the target region, are excluded from the analysis to remove target fragments and tracks associated with the target breakup. Note that cutting heavy tracks also cuts promptly scattered target protons. Scattered projectile protons, which are minimum ionizing particles, cannot be individually identified and cut from the data. Thus, the target and projectile regions are necessarily treated differently.

The datasets used in the analysis are shown in Table 4.1. The central Pb-Pb events are chosen by the criteria laid out in Chapter 3, namely, these events have no large fragments and have two or fewer spectator alphas. The other central datasets come from emulsion stacks, which are composite targets (Table 4.2). The centrality criterion is imposed by requiring that there be no multiply charged fragments ($n_f = 0$) in the forward region. (This centrality selection criterion is
Table 4.1: Proton-nucleus and nucleus-nucleus interactions used in the analysis.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Energy</th>
<th>Events</th>
<th>(&lt; n_+ &gt;)</th>
<th>(b_{max})</th>
<th>(W_p)</th>
<th>(W_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p (periph.)</td>
<td>200</td>
<td>607</td>
<td>9.2 ± 0.2</td>
<td>-</td>
<td>0.97 ± 0.03</td>
<td>1.2 ± 0.2</td>
</tr>
<tr>
<td>p-AgBr</td>
<td>200</td>
<td>481</td>
<td>22.1 ± 0.4</td>
<td>3.0 ± 0.3</td>
<td>0.97 ± 0.03</td>
<td>4.4 ± 0.3</td>
</tr>
<tr>
<td>p-AgBr</td>
<td>800</td>
<td>286</td>
<td>32.7 ± 0.8</td>
<td>2.8 ± 0.3</td>
<td>0.97 ± 0.03</td>
<td>4.5 ± 0.3</td>
</tr>
<tr>
<td>O-AgBr</td>
<td>14.6</td>
<td>215</td>
<td>51.2 ± 1.0</td>
<td>4.7 ± 0.4</td>
<td>12.8 ± 0.6</td>
<td>25 ± 2</td>
</tr>
<tr>
<td>O-AgBr</td>
<td>60</td>
<td>226</td>
<td>106 ± 2</td>
<td>4.7 ± 0.4</td>
<td>12.8 ± 0.6</td>
<td>25 ± 2</td>
</tr>
<tr>
<td>O-AgBr</td>
<td>200</td>
<td>151</td>
<td>172 ± 4</td>
<td>4.7 ± 0.4</td>
<td>13.0 ± 0.6</td>
<td>26 ± 2</td>
</tr>
<tr>
<td>Si-AgBr</td>
<td>14.6</td>
<td>154</td>
<td>28 ± 2</td>
<td>4.3 ± 0.3</td>
<td>22.5 ± 0.8</td>
<td>37 ± 2</td>
</tr>
<tr>
<td>S-AgBr</td>
<td>200</td>
<td>106</td>
<td>295 ± 7</td>
<td>4.2 ± 0.5</td>
<td>25.9 ± 1.5</td>
<td>41 ± 3</td>
</tr>
<tr>
<td>Pb-Pb (cent.)</td>
<td>158</td>
<td>21</td>
<td>1314 ± 60</td>
<td>4.9 ± 0.6</td>
<td>161 ± 10</td>
<td>161 ± 10</td>
</tr>
</tbody>
</table>

Table 4.2: Emulsion Composition.

<table>
<thead>
<tr>
<th>Component</th>
<th>A</th>
<th>BR-2 Density [atoms/cm³ x 10²²]</th>
<th>Fuji ET7B Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>1.008</td>
<td>3.148</td>
<td>3.165</td>
</tr>
<tr>
<td>C</td>
<td>12</td>
<td>1.412</td>
<td>1.295</td>
</tr>
<tr>
<td>N</td>
<td>14</td>
<td>0.396</td>
<td>0.280</td>
</tr>
<tr>
<td>O</td>
<td>16</td>
<td>0.956</td>
<td>0.975</td>
</tr>
<tr>
<td>S</td>
<td>32</td>
<td>0.004</td>
<td>0.017</td>
</tr>
<tr>
<td>Br</td>
<td>79.9</td>
<td>1.031</td>
<td>0.987</td>
</tr>
<tr>
<td>Ag</td>
<td>107.9</td>
<td>1.036</td>
<td>0.982</td>
</tr>
<tr>
<td>I</td>
<td>126.9</td>
<td>0.002</td>
<td>0.008</td>
</tr>
</tbody>
</table>
stricter than the one imposed on the Pb events. There are very few Pb events with no alphas or fragments because the beam and target are of equal size.) Interactions on the Ag or Br in emulsion can be selected by choosing those events with \( n_h > 15 \) heavy tracks (Fig. 4.3). The peripheral proton sample is selected by requiring that \( n_h = 0 \). It is not a pure hydrogen target sample: most of these events are on targets other than hydrogen (mostly C, N, or O).

KLMM [Barbier 88, Pruet 90] has validated the stack target selection criteria and centrality cuts by studying multiplicities in central and inclusive p, O, Si, and S samples and finds that, within errors, the multiplicities in the central AgBr samples compare in the predicted way with the multiplicities expected from the WNM calculation.

### 4.4 Comparisons to the Wounded Nucleon Model

We first focus attention on the angular distributions at wide angles near \( \eta = 0 \) and very narrow angles near the beam rapidity (\( \eta \approx 6 \) at SPS energies). In pp, p\( \bar{p} \) [Giacomelli 90], pA [Fredriksson 87], and \( \pi \)-Emulsion interactions [Cherry 94], these regions show striking regularities, illustrated in Fig. 4.4. The pseudorapidity distributions in the target region depend on the target mass, but are approximately independent of energy. The tail of the distribution in the projectile region is independent of energy (for a particular beam) in shape and amplitude, but is shifted by the kinematically determined beam rapidity. If the kinematic shift is removed, the right-hand tails superimpose well. This energy independence is called nucleon "limiting fragmentation." The range of rapidity or pseudorapidity over which limiting fragmentation holds is called the "fragmentation region," and encompasses roughly \( \eta < 1 \) on the target side and \( \eta > 5 \) on the projectile side. The observation can be summarized by saying that on average, a struck nucleon
Figure 4.3: Number of shower tracks $n_s$ in a sample of $n_f = 0$ events on emulsion targets analyzed by $n_A$ for (a) 200 GeV protons, (b) oxygen, and (c) sulfur. A cut $n_A > 15$ is applied to all data from emulsion targets to select a sample of AgBr events.
Figure 4.4: Demonstration of limiting fragmentation in pA beams. In the target region, the pseudorapidity densities are independent of energy, and only depend on the target. In the projectile region, the densities only depend on energy and are independent of the target. Further, the energy dependence is a simple shift $\Delta y_{\text{beam}}$, and the distributions would appear the same in the projectile rest frame.
produces pions in the target and projectile fragmentation regions in a way which is independent of how hard the nucleon is struck.

To see how fragmentation depends on how many times a nucleon is struck, the target and projectile regions for several beam-target combinations at SPS energies are plotted in Figs. 4.5(a) and 4.5(b). In Figs. 4.5(c) and 4.5(d), the target (projectile) distributions have been normalized to the calculated number of target (projectile) wounded nucleons $W_t$ or $W_p$ (Table 4.1). The tails of these scaled distributions are almost independent of target and projectile.

In the projectile region, the nucleus beams show a slight excess over the proton beam, which is to be expected since there are a few spectators even in the central AA samples. (It has already been noted that there may be on average $\sim 16$ spectator protons in the central Pb events.) In the target region, the proton beam shows a slight excess over the nucleus beams. This excess increases as the minimum $n_h$ of the proton sample is increased, perhaps because the distinction between target breakup and production phenomena based on track ionization is imperfect. In the $0 < \eta < 2$ region of the $n_h > 15$ sample, the heavy tracks outnumber the shower tracks, so a small fraction of relativistic tracks coming from the target breakup could account for the excess. Except for this possible discrepancy, however, the fragmentation regions scale with the number of wounded nucleons. In the projectile region, this scaling is valid over two orders of magnitude, from protons to lead.

Given this scaling behavior in the fragmentation regions, it is reasonable to ask if the distributions over the entire range of pseudorapidity are an admixture of two components, one component proportional to $W_t$, and the other to $W_p$, i.e.,

$$\rho_{Pt}(\eta) = W_t \rho_t(\eta) + W_p \rho_p(\eta).$$  \hspace{1cm} (4.7)

We construct $\rho_t(\eta)$ and $\rho_p(\eta)$ by minimizing the sum of the squares of the residuals
Figure 4.5: The tails of the pseudorapidity distributions. (a,b) Pseudorapidity densities in the target and projectile regions. (c,d) Pseudorapidity densities per wounded target (projectile) nucleon in the target (projectile) region.

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between the model and the data in each pseudorapidity bin $i$:

$$\sum \frac{1}{\sigma_i^2} \left( \rho_i^{\text{meas.}} - W_t \rho_{t,i} - W_p \rho_{p,i} \right)^2,$$

where $\rho_i^{\text{meas.}}$ is the measured pseudorapidity density in bin $i$, $\sigma_i$ is the statistical uncertainty of the measurement, and the sum is over the datasets (i.e., the beam-target combinations) used in the fit. The minimization leads to a set of two linear equations in $\rho_{t,i}$ and $\rho_{p,i}$:

$$\sum \frac{1}{\sigma_i^2} \left( \rho_i^{\text{meas.}} - W_t^2 \rho_{t,i} - W_p W_t \rho_{p,i} \right) = 0,$$

$$\sum \frac{1}{\sigma_i^2} \left( \rho_i^{\text{meas.}} - W_t W_p \rho_{t,i} - W_p^2 \rho_{p,i} \right) = 0.$$

These equations are solved to give the components $\rho_{t,i}$ and $\rho_{p,i}$ in pseudorapidity bin $i$. The analysis has been performed on the SPS data ($p$, $O$, and $S$ on AgBr, and $Pb$ on Pb). For the 158 GeV/n Pb-Pb data, the pseudorapidities have been shifted by the change in beam rapidity $(\ln(200) - \ln(158) = 0.235$ pseudorapidity units) to compare it to the other data at 200 GeV/n. In addition, the Pb-Pb densities have been scaled up by a factor of 1.061 to correct for the lower isospin-averaged NN multiplicity at 158 GeV/n (see below). The number of wounded nucleons are treated as fixed parameters of the model, and have not been optimized to improve the fits. The peripheral data are excluded from the fit, since the value of $W_t$ to adopt for this set (1.2) is little more than a reasonable guess. The derived target and projectile components, shown in Fig. 4.6, have similar areas ($4.1 \pm 0.5$ and $4.7 \pm 0.5$, respectively).

The slightly smaller area of the target component is consistent with the removal of the heavily ionizing scattered wounded target protons. The negative excursion of $\rho_p$ in the interval $\eta = 0 - 2$ indicates that in this region, the pseudorapidities increase somewhat more slowly with target mass $A_t$ than does $W_t$. This lower dependence can be accounted for if a few percent of the spectator charge is
Figure 4.6: Derived decomposition of average NN interaction at SPS energies.
converted to relativistic protons. This effect would be almost negligible in all but the $p$-AgBr sample, the lowest shower multiplicity sample with a large number of heavy tracks.

The pseudorapidity distributions calculated from $\rho_p$, $\rho_t$, and Eq. 4.7 are compared to the data in Fig. 4.7. The model reproduces the main features of the observed distributions. The peripheral data are also reproduced with a reasonable choice of $W_t$, suggesting that these events with the smallest number of interactions behave in much the same way as the more central collisions. We conclude that the two-component decomposition is a physically plausible model which reproduces the observed target, projectile, and central regions of the pseudorapidity distributions.

The SPS data are compared to beams of other energies in Fig. 4.8, which shows the relationship between produced multiplicity per wounded nucleon $W = W_t + W_p$ and the NN multiplicity at the same energy. To estimate the produced multiplicity (i.e., excluding final state protons) for NN interactions, we first calculate the total charge multiplicity $n_{\text{ch}}^{pp}$ of pp interactions from an empirical fit to the data [Thomé 77]. This value is corrected to give the mean produced multiplicity by subtracting the mean number of final state protons $< p >$ [Gazdzicki 94]. The results are almost identical to previous calculations which summed multiplicities of individual meson species. To estimate the produced multiplicity in the AA interactions, the shower multiplicity is corrected by subtracting the expected number of final state participant protons coming from the projectile and also the expected number of projectile spectator protons. To a first approximation, the number of participant projectile protons is equal to the charge of the projectile. There is a probability, however, that incident proton changes to a neutron or vice versa. We therefore calculate the fractions of pp, pn, np, and nn interactions expected from the number of target and projectile protons $(Z_t, Z_p)$ and neutrons $(A_t - Z_t, A_p - Z_p)$ and subtract an isospin weighted expected value for $< p >$. The values used for the expected
Figure 4.7: Reconstructed pseudorapidity densities based on the decomposition of Fig. 4.6.
Figure 4.8: Average produced multiplicity per wounded nucleon compared to the average produced NN multiplicity at the same energy. The estimated number of participant and spectator protons are subtracted from the measured shower multiplicity $n_s$ to obtain $n$. The isospin averaged proton multiplicity is subtracted from the measured pp multiplicity to obtain $n_{NN}$.
number of protons are the values measured at 200 GeV: \(< p >_{pp}= 1.34 \pm 0.15, \:< p >_{pn}= 1.00 \pm 0.08, \text{ and } < p >_{n}= 0.61 \pm 0.30. \) We assume these values are independent of energy. This appears to be consistent with the data at 60 GeV, but the proton multiplicity is known to be slightly higher at ISR energies. The estimated Pb-Pb multiplicities include projectile and target participant protons, and in this case, both contributions are subtracted. The correction for the number of spectators protons is \(\frac{N_p}{A_p} (A_p - W_p).\) The participant corrections range from 5\% for 200 GeV/n S-AgBr to 17\% for 14 GeV/n Si-AgBr. The spectator correction is smaller in all cases, being 0 for the proton beams, and 4\% for Si-AgBr.

The solid line in Fig. 4.8 shows the WNM prediction, and the dotted lines represent the systematic uncertainty in the pp measurements and the various corrections. The data agree well with the model at 10 GeV, but show a significant multiplicity excess at higher energies. The most striking feature, however, is that the multiplicity per wounded nucleon is independent of the beam and target.

In summary, the pseudorapidity distributions strongly support a physical picture in which \(W_p\) incoming projectile nucleons hadronize into final state particles emitted in the central and projectile regions, and likewise for the target nucleons. For the beam-target combinations examined here, the average number of final state charged particles depends only on \(W_t, W_p,\) and energy. The pseudorapidity densities in the projectile regions are particularly interesting because they allow us for the first time to compare the beam contribution to particle production from beams spanning most of the periodic table. The densities in this region scale with the number of wounded projectile nucleons, and are independent of whether the target is light (\(\bar{\nu} \approx 1.2), \text{ AgBr (} \bar{\nu} = 2.7 - 3.0), \text{ or Pb (} \bar{\nu} = 4.8).\)

At SPS energies, the total mean number of particles in both AA and pA interactions are about 15-20\% higher than in NN interactions. This excess is absent at lower AGS energies. Based on the limited data above 200 GeV/n, which
consists exclusively of \( pA \) interactions, the excess appears to grow linearly with the same energy NN multiplicity.

### 4.5 Interpretation and Conclusions

Experimentally, the multiplicities and pseudorapidity distributions can be parameterized in terms of energy, the number of wounded nucleons, and the number of interactions:

\[
\begin{align*}
    n &= n(E, W, \nu), \\
    \frac{dN}{d\eta} &\equiv \rho(\eta) = \rho(\eta; E, W_t, W_p, \nu_t, \nu_p).
\end{align*}
\]  

(4.11)  

(4.12)

As Fig. 4.2 shows, \( \text{Pb} \) has a much larger \( \bar{\nu} \) than \( \text{O} \) or \( \text{S} \) (i.e., \( \text{Pb} \) is thicker), and it is also significantly thicker than \( \text{Ag} \) or \( \text{Br} \). Yet the multiplicities and angular distributions of all beam-target combinations are well described by the number of wounded nucleons. They have no explicit dependence on the number of interactions. This suggests that the common practice of organizing data by \( \nu \) and ignoring \( W \) may be confusing. Most of these works consider only \( pA \) interactions or \( AA \) interactions over limited ranges of beam and target, but now that it is possible to compare a very broad span of beams, it is clear that the number of wounded nucleons is more physically significant than the number of interactions.

The pseudorapidity distributions can be described as the sum of two physically plausible density functions \( W_t\rho_t \) and \( W_p\rho_p \). It is tempting to interpret the density functions as the distributions of "target pions" and "projectile pions", but this is necessarily somewhat speculative. It is remarkable, though, that the distributions are well described by these two components, without recourse to a separate central component, as has sometimes been suggested. Even in the central region, the densities are consistent with a simple admixture of target and projectile components. This tends to argue against there being much energy mixing or thermalization in
the central region, which would be expected to introduce a ħ, mass, or energy
density dependence. In the superposition picture, this result is to be expected.
The decomposition analysis gives us a simple way of examining the central region
for deviations from linear behavior.

We find a 15-20% excess in all the SPS beams over the values expected from
NN multiplicities. This excess in p, O, and S measurements has already been re­
ported by KLMM [Barbier 88, Pruet 90], and has been reported in pA interactions
by other groups as well [Otterlund 79, Andersson 79]. In agreement with previous
emulsion work, we find that the pA and AA multiplicities rise more rapidly as a
function of energy than do NN multiplicities. This excess is independent of the
beam mass. Results from non-emulsion techniques tend to be inconclusive. For
example, NA35 [Bächler 94] finds \( n/W = 3.6 \pm 0.5 \) for 200 GeV/n central S-S inter­
actions in streamer chambers, agreeing with the value 3.2 \( \pm 0.1 \) for NN interactions,
but also agreeing with the higher KLMM measurements. Higher multiplicities are
also consistent with the increased slowing of protons measured in AA collisions.
The surprising result presented in this work is that Pb-Pb interactions have the
same magnitude excess as the other beams.

It is unlikely that this difference is due to a systematic bias. This work finds
no evidence for a significant target selection bias, and finds the same effect in
pure Pb targets. It is possible that there is a small contamination of the p-AgBr
data from relativistic target spectator protons, but the effect will not significantly
affect the AA data, where the multiplicities are much higher in relation to target
spectator charge. The NN multiplicities in this work have been estimated from
inclusive pp data, whereas the previous KLMM work has summed estimated meson
multiplicities, but the two techniques give essentially identical answers. This work
applies isospin averaging in calculating the number of participant protons in the
spectator region, but as this is essentially a correction of a correction, the net effect on the data is quite small.

In the projectile region, the peripheral proton beam sample agrees well with the p-AgBr and AA samples. Its mean shower multiplicity, $9.0 \pm 0.2$, is higher than the expected $7.1 \pm 0.7$. This can be attributed to enhanced multiplicity, or to a number of wounded target nucleons greater than one. The peripheral sample is certainly dominated by C, N, and O events, since it constitutes 13% of the minimum bias sample, and only 4.4% of the p-Em events are on H. In p-C interactions, there are $2.5 \pm 0.2$ wounded nucleons (1.5 wounded target nucleons and 1 wounded projectile nucleon), averaged over all impact parameters. Hence the peripheral multiplicities can be attributed either to a fairly rich mixture of CNO events or to enhanced multiplicities. The issue is undecidable without knowing the composition of the sample.

The multiplicity excess has been attributed to reinteraction of produced particles inside the nucleus. This possibility is not ruled out, but it appears to have difficulties. Reinteraction would be expected to be more important for larger nuclei, and such an effect is not observed. Further, particles from secondary interactions should have a larger $\langle p_t \rangle$. Enhanced $E_t$ tails have been observed and fairly convincingly associated with reinteraction [Baym 87, Margetis 95], but at a smaller level than the 20% multiplicity excess. Indeed, such a large reinteracted component would be expected to measurably increase $\langle p_t \rangle$ by 5–10% over the pp value, as well as appear in the $p_t$ distributions. This is not observed.

It has been suggested that the observed multiplicity may indeed depend on the number of interactions $\nu$, but this effect is mitigated because a nucleon passing through a target loses energy with each interaction, so each successive interaction produces fewer and fewer pions. There appear to be difficulties with this explanation as well. At 200 GeV/n, the first interaction produces 6.6 particles. If the
average NN inelasticity is 0.5, a second interaction would produce 4.3 particles, and a third 2.4 particles. The observed effect shows no such $\nu$ dependence and is much smaller than this calculation suggests.

The fact that the multiplicity per participant nucleon is constant over a wide range of nuclei appears to coincide well with NA49's observation that the nuclear stopping power (i.e., how much an incident nucleus is slowed in passing through a nucleus) is independent of the projectile mass, but the complete situation is far from clear. The rapidity loss of the protons $\langle y - y_{\text{incident}} \rangle$ in proton interactions on heavy targets at 200 GeV [Abe 88] is significantly larger than in S-S interactions, which is the nearest analog to the Pb-Pb system for which detailed stopping data exists ($\sim 2$ units for protons-heavy target data compared to 1.6 for S-S). It is possible that there is a stopping power dependence in pA collisions which may not exist in AA collisions. At the same time, the multiplicities indicate that NN collisions are not quite the same in isolation as they are in nuclei. The superposition picture in general does an excellent job of describing how the various physical quantities vary with nuclear mass and energy, but exactly how these quantities are tied to their analogs in NN collisions remains somewhat obscure.

In conclusion, the average multiplicities and angular distributions in AA interactions are determined by the number of participating nucleons. In thick nuclear targets and projectiles, the number of participants is quite distinct from the number of interactions, which plays at most a secondary role. The main features of the beam and target dependencies of the angular distributions can be explained by the WNM. The wounded nucleon model, which calculates AA multiplicities in terms of NN multiplicities at the same energy, produces accurate multiplicity predictions for AGS energies, but predicts values which are too low for SPS and higher energies. This discrepancy may have its source in nuclear effects, such as reinteraction, but this explanation is not entirely satisfying. The fact that the excess appears
in a beam as small as O perhaps suggests that we do not fully understand NN collisions in a nuclear environment.
References


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Appendix A

Pseudorapidity, Angles, and Momentum

In interaction studies (and almost nowhere else), angles are usually reported in terms of pseudorapidity. Rapidity distributions are items of particle physics esoterica; pseudorapidity distributions are even more abstruse. Even within the community, misperceptions persist about the relationship between pseudorapidity, rapidity and the more familiar kinematical variables. Before discussing the analysis of these distributions, it is therefore worthwhile to examine their physical meaning.

The rapidity of a particle with energy $E$ and longitudinal momentum $p_l$ is defined to be

$$y = \frac{1}{2} \ln \frac{E + p_l}{E - p_l} = \ln \frac{p_l}{E}.$$  \hspace{1cm} (A.1)

It is straightforward to show that the shape of an interaction's rapidity distribution $dN/dy$ is Lorentz invariant: the only effect of a Lorentz boost is to shift the distribution by an amount $\arctan \beta$ [Perkins 87, Barbier 87]. A particle with no longitudinal momentum in the center-of-mass (CM) frame has rapidity 0 in the CM frame. In the target frame this transforms to [Hikasa 92]

$$y_{CM} \approx \frac{1}{2} \ln \frac{\sqrt{s}}{m} \approx \ln 2\gamma,$$ \hspace{1cm} (A.2)

where $s$ is the usual Mandelstam variable and $\sqrt{s}$ is the center-of-mass energy. (The approximation is valid in the limit $e^{-2y_{CM}} \ll 1$, which covers essentially all
data in the present work.) There are kinematical lower and upper limits to the
rapidity given approximately by 0 and $\ln \sqrt{s}/m$, respectively, in the target frame.

Pseudorapidity bears a close relationship to rapidity. It is defined to be
\[ \eta = -\ln \tan \frac{\theta}{2}, \]  
(A.3)
and is a function only of scattering angle $\theta = \arctan(p_t/p_i)$. The definition of
rapidity can be expanded to obtain
\[
y = \frac{1}{2} \ln \frac{E + p_i}{E - p_i}
= \frac{1}{2} \ln \frac{1 + \frac{m^2}{p^2} + \cos \theta}{\sqrt{1 + \frac{m^2}{p^2} - \cos \theta}}
= \frac{1}{2} \ln \frac{\cos^2 \theta/2 + (m/2p)^2 + \ldots}{\sin^2 \theta/2 + (m/2p)^2 + \ldots}
\approx -\frac{1}{2} \ln \tan \frac{\theta}{2} \equiv \eta
\]  
(A.4)
(A.5)

Note that there are really two approximations involved: $m/(2p\cos^{\theta}/2)$ and $m/(2p\sin^{\theta}/2)$ must both be small. To examine the validity of these assumptions in a practical example, we cast these lowest-order corrections in a slightly different form:

\[
\text{Term 1} = \frac{1}{2} \left( \frac{m \sin \theta}{2p_t \cos \theta/2} \right)^2,
\]  
(A.7)
\[
\text{Term 2} = \frac{1}{2} \left( \frac{m \sin \theta}{2p_t \sin \theta/2} \right)^2.
\]  
(A.8)

At energies typical of fixed target accelerators, soft sector physics predominates.
By far the most commonly produced particle in nucleon-nucleon interactions at
these energies is the pion, whose transverse momentum distribution is rather well
described by a simple hard sphere scattering model [Perkins 87]. The $p_t$ distribu­
tion is approximately exponential with a $\langle p_t \rangle = 350$ MeV/c, and is more or less
independent of energy up to beam energies of several hundred GeV. Further, Feyn­
man scaling is approximately valid at these energies, and $p_t$ is weakly correlated
with $p_t$ and therefore $\theta$. Thus $\langle p_t \rangle = 350$ MeV/c is a good scale to apply in the correction terms. It is often stated that $\eta$ approximates $y$ if $\theta \gg 1/\gamma$ and if $p \gg m$. This is misleading. The pseudorapidity approximation is more or less valid over the entire range of angles, but is best for high $p_t$ particles. The shift $\Delta = \eta - y$ is a function only of the particle's mass $m$, its transverse momentum $p_t$, and its pseudorapidity $\eta$ (or, equivalently, zenith angle $\theta$). Keeping only the highest order terms, we can approximate the shift as

$$\Delta \approx \frac{m^2}{2p_t^2} \left( [\frac{\sin \theta}{2 \sin(\theta/2)}]^2 - [\frac{\sin \theta}{2 \cos(\theta/2)}]^2 \right) = \frac{m^2}{2p_t^2} \cos \theta. \quad (A.9)$$

The shift is approximately bounded by $\frac{m^2}{2p_t^2}$, and is positive for all $\eta > 0$. Fig. A.1 shows the $\eta$ dependence of this function.

We can now qualitatively understand the main features of a produced particle pseudorapidity distribution. For the majority of particles, which have $2p_t^2 > m^2$, the pseudorapidity approximation is a valid one. It is for this reason that pseudorapidity distributions share their main features in common (approximately) with rapidity distributions. In particular, they are approximately Lorentz invariant. However, the low $p_t$ component causes them to be shifted slightly to the right of their rapidity analog, to be somewhat asymmetric (i.e., skewed to the right), and in the target frame, to have some particles above the upper limit $y_{\text{max}}$. The main relationships are summarized schematically in Fig. A.2. The location of the distribution is controlled by the beam energy, and to a smaller extent by the pions' $\langle p_t \rangle$. Particles farther from the center of the pseudorapidity distribution have, on average, larger center-of-mass longitudinal momentum than those near the peak. The width of the distribution is therefore controlled mainly by the mean longitudinal momentum $\langle p_t \rangle$, or equivalently, the mean pion energy. The area under the distribution is equal to the total number of produced particles.
Figure A.1: Pseudorapidity shift $\Delta$ as a function of pseudorapidity.
Figure A.2: Qualitative meaning of the position and width of the pseudorapidity distribution.
Appendix B

Optical Resolution

It is well known that diffraction limits optical resolution to about the wavelength of light (although, given enough photons, this limit can be beaten statistically). However, we are using the microscope to image objects that have not only transverse but also longitudinal extent, which is a less familiar case. The resolution limit in the longitudinal direction is dominated by geometrical rather than diffraction effects.

Let $I_-$ be the light removed from the optical path by a point-like source (such as a developed grain), $\rho$ the transverse distance from the microscope axis, and $z$ the height of the objective focal plane. The “numerical aperture,” $\tan \alpha$, of the objective is the effective aperture divided by the distance between the focal plane and the objective’s principal plane. (The condenser’s numerical aperture is similarly defined. In practice, it is adjusted to be about 80% of the objective’s numerical aperture.) It is sufficient for present purposes to assume that the emulsion gelatin is a perfect transmitter of light, and the only removal of light from the optical path (scattering or absorption) is done by developed grains. An object located at $\vec{x} \equiv (x, y, z) = (0, 0, 0)$ will cause a shadow above and below it of radius $\rho = z \tan \alpha$. The geometrical point response function is then

$$R(\vec{x}) = \begin{cases} \frac{I_-}{\pi (z \tan \alpha)^2} & \text{if } \frac{\rho}{z} < \tan \alpha \\ 0 & \text{otherwise} \end{cases} \quad (B.1)$$
The response falls off rather slowly with $z$, and in fact has no true scale, unlike the transverse behavior, which falls off rapidly for distances much larger than a wavelength. This accounts for the relatively poor resolution in the $z$-direction.

Clearly, the longitudinal response affects the ability to resolve details like individual grains in nearly end-on tracks. It is less obvious that the point response function also affects the imaging of wide-angle tracks. For tracks with an angle of inclination $\sim \alpha$, resolution of optically close pairs in the radial direction is adversely affected. In fact, the characteristic hour-glass response function can be seen in accumulated images of tracks near the vertex (e.g., at the edges of Fig. 2.14). In addition, MIP darkness abruptly changes when inclination angles exceed $\tan \alpha$ as grains emerge from the shadows of their neighbors (Fig. B.1).

For emulsion with a shrinkage factor $s$, the critical track angle $\theta_C$ is

$$\theta_C = \arctan \left( \frac{\tan \alpha}{s} \right). \tag{B.2}$$

For a 100x objective with numerical aperture 1.2, the condenser aperture should be set at about 1.0. The Fuji emulsion has a shrinkage factor of about 2.7 at typical humidity and temperature, so $\theta_C = 20^\circ$, corresponding to a pseudorapidity of $\eta = 1.7$. For the present work, we have cut tracks with $\eta < 2.9$, which is well away from the critical angle. This consideration may be important, however, in future work, such as with JACEE.
Figure B.1: Shadowing of tracks. The track image changes abruptly when the angle of inclination goes from less than the numerical aperture (a) to greater than the numerical aperture (b).
Appendix C

Image Processing Algorithms

C.1 Vertex Finder

C.1.1 The Accumulator

Given the coordinates of a trial vertex \( \vec{v} \equiv (v_x, v_y, v_z) \), the job of the accumulator routine is to produce a trial accumulated image by adding the pixels of the individual frames from the focus sequence into an accumulator array. Where does each image pixel go in the accumulator array? Fig. C.1 shows the geometry. It is convenient to define the origin to be on the optical axis (the center of the field of view) at the most downstream frame in the focus sequence. All pixels on a particular ray emanating from the vertex are to be added to the same accumulator pixel. Let us assume a one-to-one mapping for the most downstream frame. Then, by similar triangles, a point \( \vec{x} = (x, y, z) \) in the raw image is added into the accumulator array at a point \( \vec{y}_{acm} \) in a way such that

\[
\frac{x_{acm} - v_x}{v_z} = \frac{x - v_x}{v_z - z} \quad \text{and} \quad \frac{y_{acm} - v_y}{v_z} = \frac{y - v_y}{v_z - z},
\]

which means that the proper place to add the pixel at location \( \vec{x} \) in the focus sequence is at

\[
x_{acm} = \frac{x - v_x}{v_z - z} v_z + v_x,
\]
\[
y_{acm} = \frac{y - v_y}{v_z - z} v_z + v_y.
\]

(C.2)
Figure C.1: Vertex geometry.
For points at fixed $z$, i.e., in a particular frame of the focus sequence, the formula is linear in $x$ and $y$. The transformation amounts to a shift and an enlargement.

When enlarging an image on a computer, one must ensure that every output pixel has some input pixel in it, even though there are more output pixels than input pixels. Since we are going to high-pass filter (essentially, differentiate) the result, gaps would be a disaster. There is a standard technique to handle this problem efficiently. Instead of raster scanning the input array and filling the output array, one reverses the procedure. We fix $z$ (the frame number), invert Eq. C.2, and raster scan the output array, looking up the corresponding input pixel $(x,y)$.

Because the accumulator routine scans a large array on every call, and may be called 100 times for every image, it is the most important computational bottleneck in the entire track recognition chain. It therefore needs to be coded efficiently. The most important point is to recognize that Eq. C.2 is separable. That allows us to economically build separate lookup tables for $x$ and $y$ for every frame in the focus sequence. The logic is as follows:

For every frame $f$:

- Build lookup tables $x(x_{acm})$, $y(y_{acm})$
- Map valid array bounds for frame $f$ onto acm array
- For every valid $x_{acm}$:
  \[ x_{acm} = x_{acm} + \bar{z}(x_{acm}) \]

As an added benefit, inverting the mapping allows array boundaries to be checked outside of the innermost loop. Properly coded and optimized, the inner loop consists of only about 15 op-codes on a 486 PC, and the bounds-checking and table building overhead is negligible. On a 66 MHz 486 PC with an L2 memory cache, this loop executes at very nearly one instruction per cycle. Experiments on slower PCs as well as DEC Alphas indicate that execution speed scales with CPU
clock speed. This scaling is not very surprising, since most of the work in the loop consists of sequential memory accesses and integer arithmetic.

To further speed execution, the vertex finder can initially be run in a mode which undersamples the focus sequence by a user-specified amount. For the Pb analysis, we sampled every third pixel in the x and y direction for a step size of approximately 1.0 μm, but used every frame in the focus sequence, which sped execution by a factor of 9, at the cost of lower resolution and less accurate track counts in the vertex fitting. After the vertex finder has run in undersampling mode, it is run again in full sampling mode, using the best vertex from the undersampling run as the initial input. The full sampling run then converges quickly. On a 66 MHz 486, the vertex finder usually requires 20–30 minutes per image. Because the bottleneck is the inner accumulation loop, execution time depends mainly on the number and size of the input images, not on the number of tracks in the field.

C.1.2 Filtering

After the accumulator compiles an accumulated image for the trial vertex, this image has to be high-pass filtered. This process should be fast compared to the accumulation. As described above, the filtering is therefore done in two stages, pass 1 and pass 2. To find the vertex, it is not crucial to correctly count every single track in the field, and this more lenient criterion allows the pass 1 filter to be simpler and faster than pass 2.

The vertex finder convolves the accumulated image with the laplacian1 kernel:

\[
\begin{bmatrix}
-1 & -1 & -1 \\
-1 &  8 & -1 \\
-1 & -1 & -1 \\
\end{bmatrix}
\]  

(C.3)

This high pass filter returns 0 on a flat field, and a positive value on an upward-going peak. It is the image-processing analog of a coupling capacitor. The problem

---

1 More generally, a laplacian filter is any band-pass filter implemented as a convolution. This particular one is the simplest, and is widely known as “the” laplacian.
with the kernel as it is written is that the pixels in the input images are 0.25 μm across, and it is necessary to detect 1 μm objects. This kernel is not large enough to pass such large structures through to the output image. It also unnecessarily reduces the signal-to-noise ratio by emphasizing the images’ highest frequencies. The vertex finder solves this problem in the simplest way, by sampling every fourth pixel in the accumulated array. (The actual sampling rate is a user-definable parameter. A 4:1 rate was used in the Pb analysis.) This technique invites aliasing problems, but experimentally, it works satisfactorily. The final pass 2 filter, discussed in Section 1.4, is more sophisticated.

C.1.3 Vertex Optimization

The function to be maximized, i.e., the number of tracks seen end-on \( N(\vec{v}) \), is a somewhat atypical function to optimize. It is not smooth: it is an integer, not a real number, and it is determined through lookups of noisy data. If the trial vertex is not very close to the apparent vertex, no tracks are seen. Optimization routines that rely on the smoothness of a function (and possibly its derivatives as well) work poorly on this problem.

The vertex finder processes hundreds or thousands of images, and it does this rather slowly. It is therefore desirable to make the vertex fitter very reliable, so it can be run in batch mode, without user intervention. Since the user does not have precise a priori knowledge of the apparent vertex positions, the fitter should be as globally stable as possible.

A grid search is as globally stable a search routine as can be found, and this is what has been used – as a front end to a slightly smarter fitter. To speed the grid search, the input images are undersampled (section B.1.1). The grid search routine finds tracks that point back to within about 20 mrad of the trial vertex, corresponding to tracks 0.5 μm in radius and 20 μm in length (in the developed
emulsion). This acceptance cone is used to determine an \((x,y)\) step size for the search which ensures that the grid search will detect at least some tracks. The step size in the \(z\)-direction is generally set to increase proportional to \(z\). A step increase of 40% usually gives good results. If the imaged plate is several \(\text{mm}\) from the vertex, only one \(z\)-plane is required. The emulsion closest to the target may require 3-4 planes.

The pass 2 fitter uses three golden section searches [Press 92], one each for \(v_x\), \(v_y\), and \(v_z\). (The golden section algorithm converges rather slowly but is robust in the sense that it does not make unwarranted assumptions about smoothness.) The fitter is run three times. The first time, it starts on the best trial vertex from the grid search. The second run re-starts on the vertex from the first iteration. Both of these runs undersample to increase their speed. The final run uses full sampling. The routine is quite stable, yet acceptably fast. Failures to converge occur most often in emulsion very close to the vertex. Convergence can generally be achieved in this situation by adjusting the darkness threshold.

### C.1.4 Final Image Processing

To achieve maximum discrimination between real tracks and background, we require a pass 2 filter which leaves small-scale \((\sim 1\mu\text{m})\) features alone, but filters out larger features, like delta rays. At the same time, we are interested in separating close pairs of real tracks. To satisfy both these criteria requires careful tuning of the filter.

Unlike its one-dimensional cousin, the field of digital filter design in two or more dimensions is not yet very fully developed. We have followed Jähne's approach [Jähne 91], which builds spatial domain filters from one-dimensional binomial kernels. These building blocks are used to construct filters that can be analyzed analytically and implemented very efficiently. In addition, the results are approxi-
mately isotropic and exhibit no ringing. These filters are constructed from kernels of the form
\[ B^{2R+1} = \frac{1}{2^{2R} (R-r)! (R+r)!}, \quad r = -R, \ldots, R. \] (C.4)

For example, we can write a row vector
\[ B^5_x = 1/16[1, 4, 6, 4, 1], \]
and we can similarly write a column vector for \( B^5_y \). The effective width of such a filter is
\[ \sigma = \sqrt{\frac{R-1}{2}}. \] (C.5)

Nearly isotropic filters \( B^n \) (\( n = 2R + 1 \)) can be constructed by sequentially convolving input images with binomial row and column vectors:
\[ F = B^n_x B^n_y I \equiv B^n I, \] (C.6)
where \( I \) is the input image and \( F \) is the filtered image. The definition
\[ B^n = B^n_x B^n_y \] (C.7)
is actually a prescription for computing the two-dimensional filtered image in \( \mathcal{O}(2n) \) operations, as opposed to the \( \mathcal{O}(n^2) \) operations it would take to do the filtering directly.

A class of well-behaved laplacian filters can be constructed from these binomial elements as follows:
\[ n^l \mathcal{L} = (I - B^n_x B^n_y)^l, \] (C.8)
where \( I \) is the identity kernel. (N.B.: the exponent \( l \) represents successive application of the operators.) The behavior of this class of filters is demonstrated in Fig. C.2. Experiments on a sample dataset of images indicate that the lowest error rate is achieved when \( n \approx 33 \) (\( \sigma = 4 \) pixels = 1\( \mu \)m), and that the pair resolution is
Figure C.2: Transfer functions of the filters $n^L$. The combination of $n$ and $l$ control the width and steepness of the response. The construction of the filters from the binomial smoothing filter $B^n$ guarantees that they are approximately isotropic. The response of the pass 1 filter is also shown. Note that it is quite similar in behavior to $^{33,1}L$, but with substantial aliasing.
improved slightly by going from $l = 1$ to $l = 2$. Thus the filter used is a laplacian of order 2 with width $1 \mu m$.

To measure the intrinsic darkness of tracks, we need to normalize the track's darkness to the intensity of light incident on the track. Therefore, the input pixel values are logarithmically transformed so that the high pass filter responds according to the ratio of the pixel intensity to the intensity in the pixel neighborhood. After the image is filtered, the next steps are to set a threshold and identify the pixels which lie on tracks. The obvious approach to setting a threshold is to construct the filtered image's darkness histogram, and look for two peaks: one representing pixels on dark tracks, the other containing all the other pixels. Unfortunately, the darkness histogram is never bimodal (c.f. App. D). We have devised another approach, namely, to histogram only those pixels which are darkness peaks (i.e., as dark or darker than all 8 of their nearest neighbors). A typical darkness histogram for this subset of pixels is shown in Fig. 2.15. Evidently, if one defines background to be any dark spot, then the background vastly outnumbers the signal. In fact, it is more reliable to set the threshold based on the background peak than on the track peak, since the track peak is rather broadly distributed in darkness, and the number of tracks per image is highly variable (from a few to several hundred). Further, the background peak turns out to have a characteristic shape, which is a fact that can be exploited. The prescription used for setting the threshold is

$$d_{\text{thresh}} = d_{90} + \Delta,$$

(C.9)

where $d_{\text{thresh}}$ is the threshold, $\Delta$ is a fixed offset determined by experience, and $d_{90}$ is the darkness corresponding to the $80^{th}$ percentile in darkness, corresponding to the background peak's right-hand shoulder. (Note that for a gaussian distribution, the one-sigma point corresponds to the $84^{th}$ percentile. For a wide variety of reasonable peak shapes, this prescription determines a point on the distribution
where it is falling rapidly.) If there are a large number of tracks in the image, the track peak may spuriously pull \( d_{60} \) slightly to the right. To reduce this bias, the prescription is iterated once: everything above the first \( d_{\text{thresh}} \) is thrown out, and the procedure is repeated on the restricted histogram to give a final darkness threshold. This technique has proven to be quite reliable.

Using the threshold \( d_{\text{thresh}} \), a final bitmap image is created in which all pixels darker than threshold are set, and all others are cleared. The result has clusters of pixels each representing a track candidate (or perhaps close pairs of candidates). We wish to count and classify each cluster (i.e., calculate its centroid and darkness). In one dimension, this task would be trivial, but in two dimensions, there are more cases to consider, and the job is more complex. We have employed the Hoshen-Koppelman [Gould 88, Hoshen 76] algorithm for this purpose. This algorithm assigns each cluster a unique number, and returns a bitmap in which each pixel in a cluster is set to the cluster number. Classifying each cluster is simply a matter of analyzing the pixels set to the corresponding cluster number.

If a cluster contains multiple darkness peaks, it may represent a close pair (or multiplet) of tracks. Multiply-peaked clusters are separated into clusters containing exactly one peak before continuing. Centroids and darknesses are then tabulated in a straightforward fashion. The results are recorded and submitted to the plate fitter.
Appendix D

Why the Intensity Histogram is Never Bimodal

No matter how much effort is lavished on filtering, the resulting intensity histograms apparently never separate into a “signal” peak and a “noise” peak. This reflects a rather general feature of signal processing in two or more dimensions.

Suppose we have measured a gaussian signal with a uniformly sampling detector (e.g., a CCD with square pixels) with bin size \( \Delta \). The signal need not have anything to do with darkness or be a spatial image, but assume for the sake of concreteness that it is of the form

\[
D = D_0 e^{-(r/r_0)^2}. \tag{D.1}
\]

The goal is to segment this image into pixels which are either on-peak or off-peak. We would like to choose a threshold such that, if the threshold is changed slightly, the change has little impact on the segmentation, i.e., we would like the threshold to be between two histogram peaks.

The number of measurements per darkness interval is

\[
\frac{dN}{dD} = \left| \frac{dr}{dD} \right| \frac{dN}{dr} = \frac{1}{2 \sqrt{\ln(D_0/D)}} \frac{D_0}{D} \frac{dN}{dr}. \tag{D.2}
\]
where the phase space factor \( \frac{dN}{dr} \) is

\[
\frac{dN}{dr} = \begin{cases} 
1/\Delta & \text{in 1D} \\
2\pi r/\Delta^2 & \text{in 2D} 
\end{cases}
\]

FIG. D.1 shows examples of the 1D and 2D cases using the same parameters. Whereas in one dimension there are two distinct peaks, only a signal "tail" is seen in two (or more) dimensions. Looking for small, nearly point-like objects is harder in two dimensions than one because there is much more space to search.
Figure D.1: Darkness distributions in one and two dimensions.
Appendix E

Reconstruction Algorithms

E.1 Plate Fitting

Essentially, the plate fitting program is a minimization routine which minimizes the sum of some measure of distance between nearest neighbors in two adjacent plates. The standard quantity to minimize is the sum of the squares of the distances, but in this case, the sum of the squares emphasizes distant pairs which are physically unrelated. Optimization routines based on summing the squares sometimes converge but are extremely unstable. Instead, we have chosen a function which acts like the distance squared for small distances, but contributes little to the sum at large distances (Eq. 2.2). The detailed behavior of this function appears to be irrelevant – we have obtained equally good results, for example, by substituting a lorentzian for the exponential. What is important is the behavior at small and large distances $d_{nn}$.

The figure of merit $S$ is a function of the relative transverse plate shifts $(\Delta x, \Delta y)$ and the ratio $s$ of the distances of the downstream plate's distance to the vertex to the upstream plate's vertex distance. Because of the large number of measurements in both plates, the function frequently has several local minima. To correctly align the plates, the routine must find the global minimum. Like the vertex finder, the plate alignment routine has a first stage grid search followed by a conventional
optimization routine (Powell’s Method [Press 92]). To do the grid search, each plate’s track positions are binned into a two-dimensional array representing the field of view. The downstream plate is binned on a 2 micron square grid. The upstream coordinates are first transformed according to

\[ x_T = s(x - \Delta x), \quad y_T = s(y - \Delta y) \]  

(E.1)

and then are also binned on a 2 micron grid. The quality of the overlap is found by calculating a match-to-miss ratio, where the number of “matches” is the number of array bins that contain tracks upstream and downstream, and the “misses” are the number of bins with an upstream track but no downstream track. (The mean nearest-neighbor track spacing is roughly 5 \( \mu \)m near the edges of the plates. Therefore, the 2 \( \mu \)m array elements are mostly 0 near the plate edges. The size of the array element must be small enough that there are nonzero elements, but large enough that the calculation is performed quickly.) The grid search is performed on a 2 \( \mu \)m transverse grid. The scale step size is chosen so that a single step changes the upstream positions \((x_T, y_T)\) by no more than 2 \( \mu \)m.

E.2 Reconstruction

Reconstruction starts with the most downstream emulsion, and matches for each measurement are sought in the next most upstream emulsion. An upstream measurement is considered a possible mate if it falls within 1 \( \mu \)m of the ray joining the vertex to the downstream measurement. If more than one measurement exists in the search radius, the nearest is selected. If no match is found in that emulsion, the next one is searched, and so on. This procedure allows an upstream measurement to be shared among two or more tracks, but ensures a branching structure, in which two tracks never rejoin downstream.
To be considered a confirmed track, each cluster must pass four tests, illustrated by the examples in Fig. E.1:

**Coincidence** A track must be measured at least twice. This discriminates against residual background from the image processing stage. All the cluster in Fig. E.1 except (b) meet this criterion.

**Dispersion** The RMS scatter around the fitted track trajectory must be less than 1.0 μm, corresponding to 5 standard deviations. This cuts tracks that do not point back accurately to the vertex, as well as low-energy tracks emitted by the struck target.

**Accidental background** This tests for random tracks which happen to almost point toward the vertex, as well as spurious tracks created by background coincidence. The candidate is vetoed if it is missed in two or more consecutive emulsions, i.e., if it would be in the CCD field of view and also well-separated from nearby tracks, but is not measured. Thus, (g) is accepted but (h) is rejected. The detection efficiency for well-separated tracks is 99% on average, and tracks are measured in no more than 25 separate emulsions, so the probability of two or more consecutive misses in a real track is typically less than 25 × (0.01)² ~ 0.25%.

**Close Pair** A background measurement in proximity to a real track can imitate a pair of tracks. To be accepted as a real track, a cluster must have at least two measurements which belong uniquely to that cluster. In Fig. E.1, clusters (c) and (d) pass this test, but (f) does not. An exception is made for tracks with a single unique measurement if the unique measurement occurs in the most downstream measurable emulsion. Such a track is likely to be one of a close pair which is resolved just before it leaves the field of view.
Figure E.1: Features of track candidate clusters. Solid lines connect measurements in accepted tracks. Dotted lines represent rejected clusters. (a) A cluster consists of measurements with similar space angles. (b) Isolated single measurements are rejected. (c,d) Measurements can belong to more than one track. These examples are typical of close track pairs. (e,f) A close pair must be confirmed in the next layer. Track (e) is confirmed; (f) is not. (g) Gaps of one emulsion are allowed. (h) Gaps of more than one emulsion are not allowed.
Systematic errors are estimated on a track by track basis using background and efficiency information derived from the reconstruction. Values are computed for four kinds of systematics:

**Missing Measurements** There is a probability $1 - \epsilon$ of about 1% that a track will be missed in a particular emulsion. If the track can only be measured in two emulsions and is missed in one, it is then measured only once and is incorrectly considered to be background. If the track can be measured in three emulsions, according to the consistency tests above, it is only rejected if it is missed in two consecutive emulsions. The probability that a track will be missed because of missing measurements drops rapidly with the number of possible measurements, and the expected number of missed tracks is very nearly twice the number of double measurements times the measurement inefficiency. For Event 20-06, the expected number of missed tracks is approximately $2 \times 171 \times 0.06 = 2.0$ tracks.

**Spurious Doubles** Spurious doubles are created when a background measurement in one emulsion coincides with a background measurement in another, or when a random track which nearly points back to the vertex passes through the field of view. Accidental coincidences are cut if the supposed track is missing in at least two consecutive emulsions in which it could be measured if it were real. This cut is only about 50% efficient, since it can only be used if there are four or more measurable emulsions. In Event 20-06, 8 spurious doubles were detected, suggesting another 8 remain undetected.

**False Close Pairs** The main source of false close pairs appears to be single measurements incorrectly identified in image processing as two measurements. False close pairs are cut if one of the tracks has a missing downstream measurement. About 6% (48) tracks were cut on this basis. False close pairs

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remain undetected in that sample of tracks which branch only in the most
downstream emulsion. Event 20-06 has 118 such tracks. We estimate that
\( \frac{1}{2} \times 118 \times 0.062 = 3.6 \) false close pairs remain undetected.

**Unresolved Close Pairs** These are tracks which are too close together to be
optically resolved. By extrapolating the nearest-neighbor distribution (Fig.
2.21) to zero microns, we estimate that we miss roughly 6 tracks in a typical
high-multiplicity event.
Appendix F

The Electron Pair Background

A heavy ion interaction produces neutral particles as well as charged ones. In particular, the $\pi^0$ meson is produced with the same frequency as either the $\pi^+$ or the $\pi^-$. Neither the $\pi^0$ nor the products of its main decay mode, $\pi^0 \rightarrow 2\gamma$ ($\delta=25$ nm), are directly observable in emulsion. However, a gamma ray can interact with matter in the chamber and produce an $e^+e^-$ pair, and these are observable. The probability $p$ of producing an electron pair is governed by the path length $\tau$ through the material and the radiation length (R.L) of the material:

$$p = 1 - e^{-\tau/(R.L)}.$$  \hfill (F.1)

In the Pb chambers, the important radiators are the Pb foils (R.L. = 0.56 cm) and the emulsion (2.9 cm). Thus, the 100 $\mu$m foils each convert 1.8% of the gammas to electrons, and each emulsion plate, with 110 $\mu$m of emulsion, converts 0.38%. The exact number of pairs produced in each event depends on the thickness of Pb and the number of plates traversed by the gammas. These lengths can be calculated from the reconstructed position of the interaction in the Pb foil and the path lengths of the measured charged particles. For instance, for Event 18-22 the Pb path length is 60 $\mu$m, and on average the tracks traversed 2.6 plates (5.2 emulsion layers) before exiting the field of view. There were 822 charged particles counted, implying the existence of approximately 411 $\pi^0$ in the same angular region.
of acceptance. Each $\pi^0$ decays into two gammas, each of which has a chance of producing two electrons. Thus, the expected number of electrons expected to materialize in the Pb is $60 \, \mu m/0.56 \, cm \times 822 \times 2 = 17.6$, and the number in emulsion is $2.6 \times 110 \, \mu m/2.9 \, cm \times 822 \times 2 = 16.8$, for a total of 34.4.

The electron tracks are rejected either if the pair materializes in a downstream plate, in which case they will be cut because there are two or more missing measurements upstream (if they are not too close to another track), or if the electrons diverge measurably from trajectories pointing back to the vertex. Qualitatively, the efficiency for detecting the former case is fairly high, since the tracks are almost fully resolved by the second plate. The divergence of the pair is more difficult to detect, since the angle of divergence is very small (see below) and several resolved measurements of the electrons are required. Therefore, to a rough approximation, the uncut electrons are those that materialize in the target or the first plate. A reasonable estimate of the average number of uncut electrons is then $(50 \, \mu m/0.56 \, cm + 1.0 \times 110 \, \mu m/2.9 \, cm) \times 2 = 2.5\%$ of the track count. For an event like 20-06, with a track count of 762, this amounts to an expected 19 tracks. This value compares favorably with the number of electrons produced in stacks, which is 9\% of the produced multiplicity [Wosiek 95].

We would also like to know whether the electron pairs diverge sufficiently to be resolved. The most probable angle for an electron to make with its parent $\gamma$-ray is [Borsellino 53, Powell 59]

$$\delta = 4 \frac{m_e c^2}{E_{\gamma}} F\left(\frac{E'}{E_{\gamma}}\right),$$

where $m_e$ is the electron rest mass, $E_{\gamma}$ the energy of the parent gamma ray, and $E'$ is the energy of the less energetic of the two electrons. The function $F$ is always at least one but may range as high as two or more. The opening angle distribution has its mean about $1.4\delta$, and has a significant tail out to $\sim 5\delta$. When a $\pi^0$ decays,
it shares its longitudinal momentum $p_l$ among the two produced gammas, so that on average

$$\langle p_{\pi^\pm} \rangle = \frac{1}{2} p_{\pi^\pm} = \frac{1}{2} p_{\pi^\pm}$$

(E.3)

for small pion opening angles $\theta$. The electron-positron opening angle is, on average, twice $\delta$. Combining this with Eqs. F.2 and F.3,

$$\frac{2\delta}{\theta} = 16 \frac{m_{e}c}{p_{\pi^\pm}}$$

(F.4)

The left hand side is directly related to the emulsion pair resolution $\delta_{\text{pair}} = 1.0 \mu\text{m}$, since for a plate at any distance $z$ from the interaction,

$$\frac{2\delta}{\theta} = \frac{\delta_{\text{pair}}/z}{\rho/z} = \frac{\delta_{\text{pair}}}{\rho},$$

(F.5)

where $\rho$ is the distance on the plate of the pair from the event axis. Most electron pairs descended from a pion with transverse momentum $p_{\pi^\pm}$ are then resolved if

$$p_{\pi^\pm} < 16 \frac{m_{e}c}{\delta_{\text{pair}}/\rho}.$$  

(F.6)

The critical value of $p_{\pi^\pm}$ is 440 MeV/$c$, assuming $F = 1$ and using $\rho = 50$ micron for the maximum observed distance from the event axis. If $\langle p_{\pi^\pm} \rangle = 350$ MeV/$c$, this calculation says that about 70% of the pairs should have plate separations of more than $1 \mu\text{m}$. For example, there should be about 10 unresolved electron tracks in Event 18-22. We would expect about 4 of these to be cut by the reconstruction software.

Although this calculation is somewhat sensitive to the details of the electron production physics and the reconstruction efficiencies, it is evident that differences in electron pair acceptance cannot account for the excess of manually measured close pairs. We conclude that the excess is probably a measurement or reconstruction artifact.
Appendix G

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Title: Signaling Lead-Lead Interactions at 155 GeV/c

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Vita

Philip Deines-Jones was born in 1962 in Milwaukee, Wisconsin, and raised in Oconomowoc, Wisconsin. He earned two degrees while at the University of Wisconsin, Madison, a bachelor of science in physics and a bachelor of science in applied mathematics and physics. Both degrees were awarded in 1985.

In his graduate study under Prof. Michael L. Cherry, Deines-Jones has majored in physics and minored in astronomy, with research centering on analysis of the first measurements from 158 GeV/n Pb ions. During the course of his research, Deines-Jones developed a CCD microscope system which allowed automatic measurement of nuclear emulsion, the first system of its kind to yield published data. Initially supported as a research assistant, Deines-Jones was selected as a Louisiana Space Consortium Fellow in 1994, an appointment which extended through completion of his graduate work. His doctor of philosophy degree in physics was awarded by Louisiana State University, Baton Rouge, Louisiana, in 1996.


Deines-Jones lives with his wife, Courtney, and their four cats, Boris, Ivan, Kisa, and Nadja.
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Major Field:  Physics

Title of Dissertation:
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