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Systematics of low energy collective states in neutron-rich Cd isotopes

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Systematics of Low Energy Collective States in neutron-rich Cd Isotopes

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Abstract. It has been shown that there are significant deviations from the expected U(5) dynamical symmetry for $^{110,112,114,116}\text{Cd}$. However, there is very significant mixing with intruder states in this region. In this paper, we investigated states in the heavier $^{120,124,126}\text{Cd}$ populated via beta decay. These nuclei exhibit similar patterns to the lighter Cd isotopes even though the intruder states are much higher in energy.

1. Introduction

Up until recently, the neutron-rich even-even Cd isotopes $^{110-114}\text{Cd}$ are often cited as textbook examples [1,2] of vibrational nuclei and the best examples of U(5) symmetry. Nuclei that are close to the limits of U(5) symmetry would be expected to exhibit harmonically spaced multiphonon states. In the case of the Cd isotopes. This picture is complicated by low-lying intruder states (caused by the elevation of 2-protons across the $Z = 50$ shell gap) which at the proton mid-shell are in the same energy region as the 2-phonon states. As such, the lowest lying levels in the neutron-rich even-even Cd isotopes have energies that resemble an anharmonic vibrator coupled to 2-proton intruder states.

The experimental signature of these N-phonon states is a strong preference for decay of a N-phonon state to the (N-1)-phonon states over other lower energy states. The contribution of the intruder states to the B(E2) values of the transitions from the N-phonon states can be accounted for by calculations in the IBM-2 framework [3,4].

Previous studies have reported candidates for the 3-phonon states in ^{110}Cd [5], ^{112}Cd [3], ^{116}Cd [6], ^{118}Cd [7] and ^{120}Cd [8]. However, this simple picture was shown to be at odds with the experimental data for the cases of $^{110,112,114}\text{Cd}$ [4,9] and ^{116}Cd [10] where it was demonstrated that while the 6^+ , 4^+ and 3^+ states decay as expected, the previously proposed 0^+ and 2^+ candidates for the three-phonon quintuplet are not in agreement with IBM-2 calculations. The disagreements with calculations can not be due to experiments simply missing the transitions, as the nuclei $^{110,112,114,116}\text{Cd}$ have all been studied by a variety of methods with very high statistics.

This discrepancy in the decay of the suggested three phonon 0^+ and 2^+ states with the U(5) description is a consistent feature in all the well-studied even-even neutron-rich Cd nuclei. Figure 1 shows the decays of the 0^+ and 2^+ members of the reported 3-phonon quintuplet for ^{112}Cd [11], ^{114}Cd [12], ^{116}Cd [10], and ^{118}Cd [8]. None of these nuclei decay as expected in a vibrational picture.

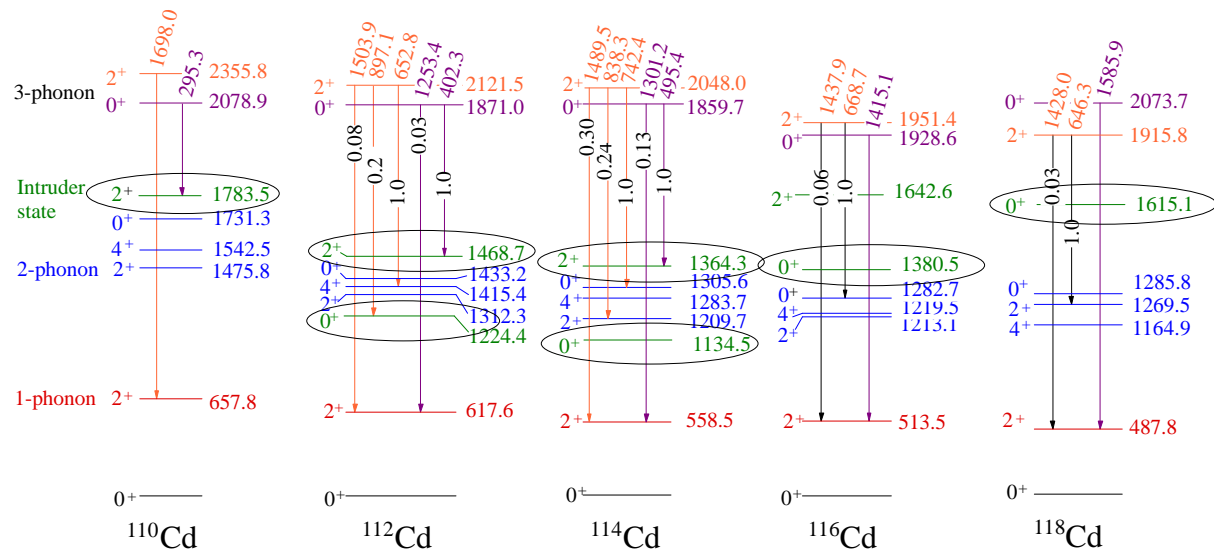


Figure 1. Comparison of the decays of the 0^+ and 2^+ members of the three phonon levels in $^{112,114,116,118}\text{Cd}$. Relative B(E2)s are shown. The intruder states are circled for clarity.

It is well established that as one moves away from the neutron mid-shell the energy of the intruder states would be expected to rise. If the explanation for the discrepancy between observed decays of the candidate 3-phonon states and those expected from IBM-2 calculations at least partially arises from mixing with intruder states, the picture should become more clear the further from neutron mid-shell. A systematic study of these nuclei is therefore needed to fully understand what is happening in the neutron rich Cd nuclei. In particular, how well does this relatively simple model describe the low-lying levels in the neutron-rich even-even Cd nuclei? With this as our motivation, we have remeasured the low energy states in $^{120,122,124,126}\text{Cd}$ via the beta decay of $^{120,122,124,126}\text{Ag}$.

2. Experimental Method

Silver-120,122,124,126 were produced via the proton-induced (40 MeV) fission of ^{238}U at the Holifield Radioactive Ion Beam Facility (HRIBF) in a plasma ion source and then transported to a moving tape collector. At the HRIBF, there are two locations available for these types of experiments. The first is the On-Line Test Facility (OLTF) (formally called UNISOR) which is connected to an online mass separator ($\Delta m/m \sim 2000$). The 40 MeV protons delivered to this facility are from the 25MV tandem and have an administrative beam current limit of 50 nA. The second involves 50 MeV protons of up to 20 μA from the ORIC cyclotron. The beam is then separated with a high resolution ($\Delta m/m \sim 10,000$) magnet and delivered to the LeRIBSS (Low-Energy Radioactive Ion Beam Spectroscopy Station) detector array. The experiment presented herein for the decay of ^{120}Ag was performed at the OLTF separator, and the experiments on $^{122,124,126}\text{Ag}$ are from the ORIC/LeRIBSS setup. Results are presented from the $^{120,124,126}\text{Ag}$ decays, while results from ^{122}Ag will be presented in a future paper [13].

The detector setup at the OLTF is the CARDS array (Clover Array for Radioactive Decay Spectroscopy), which consisted of three segmented-clover Ge detectors and a Si conversion-electron spectrometer. The clover detectors were mounted in a close geometry surrounding the tape forming

three sides of a square ~ 10 cm per side. The data acquisition system utilized in this work was a digital spectroscopy system based on DGF-4C modules (produced by X-ray Instrumentation Associates). These modules incorporate 40 MHz flash ADC's, and serve as a replacement for amplifiers, discriminators, conventional ADC's and TDC's. There were two sets of data taken. The first set of data was taken in "take-away" mode where the beam is collected in the center of the detector array, and the tape is moved every three seconds to remove longer-lived contaminants and daughter products. This setting was the bulk of the run with a total of ~ 75 hours of data taken. In the second set of data (of ~ 5 hours), the beam was collected ~ 30 cm in front of the Ge-detectors and then moved into the center of the array. The tape was moved every three seconds with a tape transport time of ~ 800 ms. This setting was used primarily to measure half-lives and resulted in a very reduced rate of ^{120m}Ag . More details on the experimental setup may be found in Refs. [14,15,16].

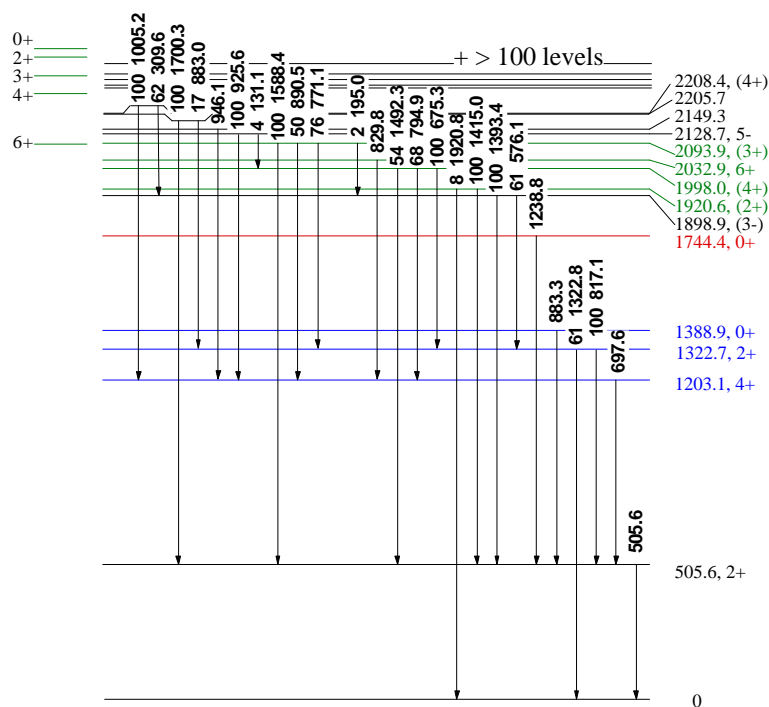


Figure 2. Low energy levels in ^{120}Cd populated in the beta decay of $^{120m,gs}\text{Ag}$.

At LeRIBSS the detector setup was similar, except there were four segmented clover Ge detectors, and two plastic scintillators in the CARDS setup. The electronics used were the PIXIE-16 modules [17] which are a newer, faster version of the DGFs. Due to the very short half-lives involved, the activity was collected in the center of the array. In order to measure half-lives, the beam was pulsed via electrostatic plates. The total tape cycle used for these measurements is 3 seconds, with the beam on for the first 2 seconds, and then turned off for one second to allow the half-life measurement. Overall, very high statistics (more than a factor of 50 larger than previous in all cases) were collected for these nuclei compared to previously published results. The number of events observed in the $2^+_1 \rightarrow 0^+$ transition were ^{120}Ag : $1e8$, ^{122}Ag : $1e6$, ^{124}Ag : $6e5$, and ^{126}Ag : $1e5$.

3. Experimental Results

3.1. ^{120}Ag

The beta decay of ^{120}Ag was first reported by Fogelberg *et al.*, [18]. In that work, 6 γ -rays from 5 levels in ^{120}Cd were measured from the decay of two isomers of ^{120}Ag . The most recently published

work by Wang *et al.*, [8] increased this to 62 γ 's depopulating 26 levels. Four of these levels 1899.0 (3^+), 1920.5 (2^+), 1997.9 (4^+), and 2032.8 (6^+), were assigned as candidates for three-phonon states in that work. Of these levels, the 6^+ and 4^+ decay as expected with a strong preference for decay to the 2-phonon states, but the 2^+ and 3^+ states do not. The assigned (3^+) state at 1899.0 keV decays only to the 2^+ 2-phonon state but not the expected 4^+ 2-phonon state. Previous experimental work based on $^{124}\text{Sn}(d,^6\text{Li})$ had assigned this state as 3^- [19]. The 2^+ identified as a 2-phonon state in that work decays only to the 2^+ 1-phonon state. No candidates for a 3-phonon 0^+ state were reported.

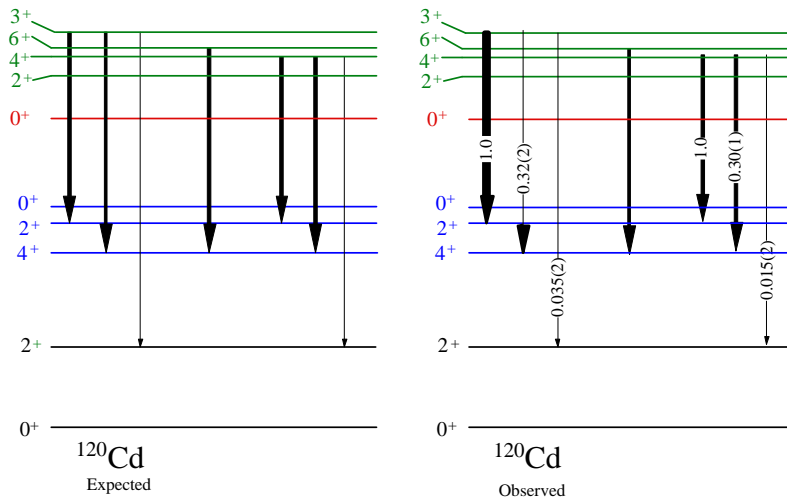


Figure 3. Comparison of expected versus observed decays of the 3^+ , 6^+ , and 4^+ "3-phonon states" in ^{120}Cd .

In the current work, a grouping of nine states (1898.9, 1920.6, 1998.0, 2032.9, 2093.9, 2128.7, 2149.2(new), 2205.7(new), and 2208.4 keV) between 1899 and 2208 keV were observed (see Figure 2). Of these, this work supports the J^π assignments from previous work for 1920.6 (2^+), 1998.0 (4^+), 2032.9 (6^+), 2128.7 (5^-), and 2208.4 (2^+). The state at 2093.9 was assigned 2^+ in Ref [20] based on a reported transition to the 0^+ ground state. No evidence of this transition is present in the current work either in singles or coincidence with the gammas feeding this state. This state feeds the levels at 505.6 (2^+), 1203.1 (4^+), and 1322.7 (2^+). We therefore assign a J^π of 3^+ to this state. We also adopt the 3^- value for the 1898.9 keV state from Ref [19]. The two new states at 2149.2 and 2205.7 keV likely have J^π of 2^+ or 4^+ , however, neither decays in a manner consistent with the systematics of the lighter Cd "3-phonon states".

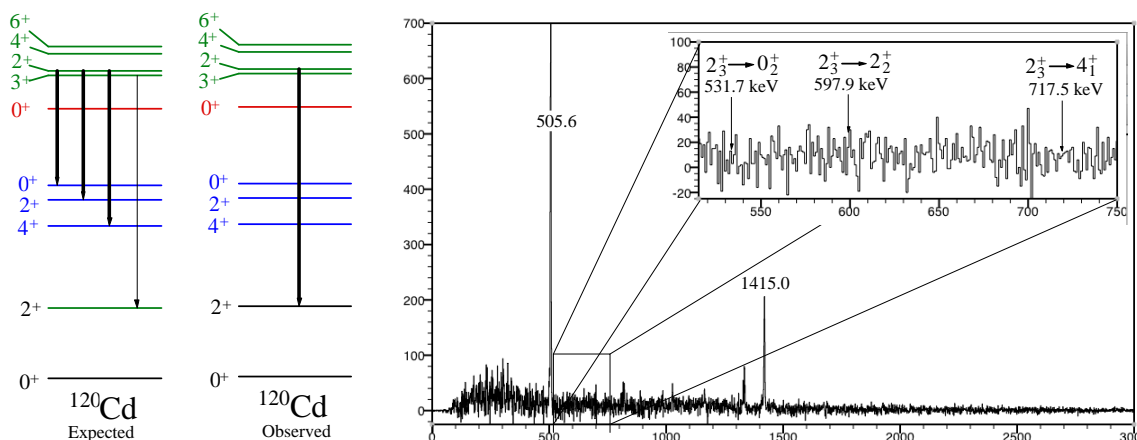


Figure 4. Coincidence with the 1580.0 keV transition that feeds the 1920.6 keV 2^+ two-phonon state. The expected $2^+_3 \rightarrow 0^+_2$, $2^+_3 \rightarrow 2^+_2$ and $2^+_3 \rightarrow 4^+_1$ transitions are clearly absent (see inset).

We assign the states at 1920.6 (2^+), 1998.0 (4^+), 2032.9 (6^+), and 2093.9 (3^+) as "3-phonon states" in ^{120}Cd . No candidates for the 0^+ member of the quintuplet were observed in the data. The 3^+ , 4^+ , and 6^+ states decay in a manner consistent with that of a 3-phonon state. Figure 3 shows a comparison of experimental relative B(E2)s versus the expected decay of 3^+ , 6^+ and 4^+ multiphonon states. All three of these states decay as expected.

The 2^+ state is only observed to decay to the 1-phonon 2^+ state. A 2^+ member of the three-phonon quintuplet would be expected to strongly decay to all three two-phonon states (4^+ , 2^+ , 0^+) and weakly to the 2^+ one-phonon state. Figure 4 show the spectrum coincident with a 1580.0 keV transition that feeds the state. In this spectrum, the expected $2^+_3 \rightarrow 0^+_2$, $2^+_3 \rightarrow 2^+_2$ and $2^+_3 \rightarrow 4^+_1$ transitions are clearly absent. In these data, no suitable candidates for a 0^+ state in this energy region were observed.

3.2. ^{124}Ag

Previous work (via the beta decay of ^{124}Ag [21], and a $^{238}\text{U}(\alpha, f\gamma\gamma\gamma)$ study [22] resulted in the discovery of a total of 16 states and 26 gammas (with spins up to 12^+). Of these, positive parity states at 613 (2^+_1), 1385 (4^+_1), 1428 (2^+_2), 1915 (3^+ , 4^+), 1924 (4^+ , 3^+), 1978 (2^+), and 2139 (6^+) are in the energy range that multi-phonon states would be expected. Notably absent are any states assigned as 0^+ (either intruder or phonon state).

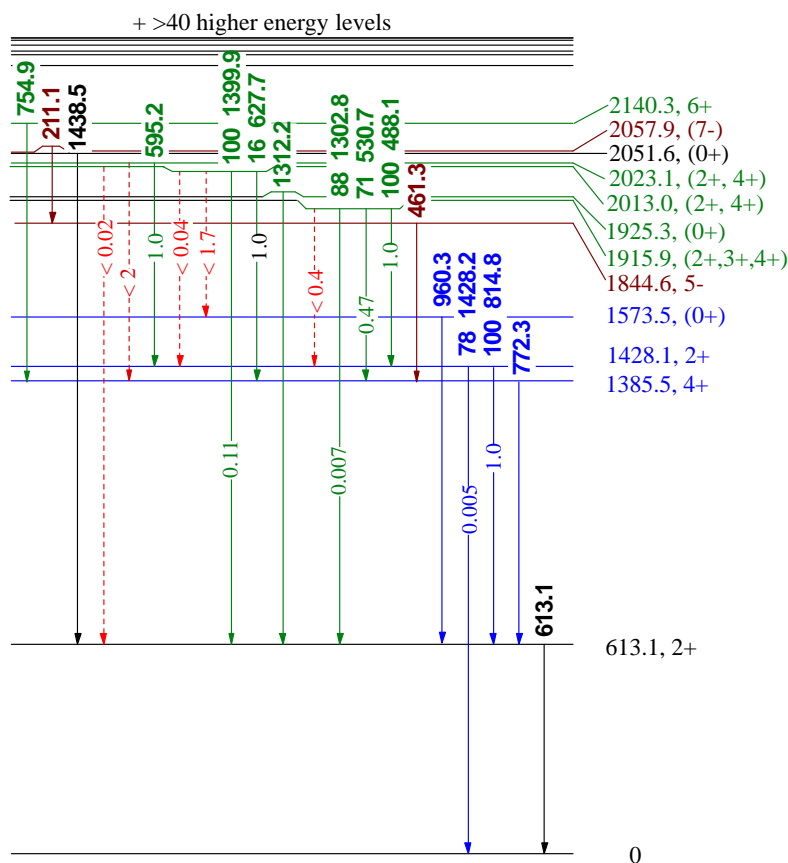


Figure 5. Low lying levels in ^{124}Cd showing a grouping of states between 1900 keV and 2150 keV that are possible three-phonon states. Relative B(E2)s are shown.

The 0^+_2 state would be expected to have a transition de-exciting to the 2^+_1 state with an energy ~ 700 - 900 keV. In addition, this state in all the lighter even mass Cd isotopes is only weakly fed (if at all) from above. Only one peak (960.3 keV) close to the expected energy is coincident with the 2^+_1 613.5 keV state and nothing else. We therefore assign this state at 1573.5 as (0^+_2). Many other new levels have been added to the decay scheme from this work, including 8 levels grouped in the energy region where a three phonon quintuplet would be expected (see Figure 5). The two levels at 1844.6 (5^-) and

2057.9 keV (7^-) were previously assigned as members of a negative parity band [22]. Our interpretation in the context of possible multiphonon states of the other levels are detailed below.

3.2.1. 1915.9 keV level

This level shows an enhanced decay to the 2^+_2 and 4^+_1 levels, and a weak decay to 2^+_1 level. No feeding into this state was observed from higher energy states. The decay of this state is similar to the decays of the 3^+_1 states in $^{116,118,120}\text{Cd}$. We therefore assign this state as 3^+_1 , however 2^+ and 4^+ are not ruled out.

3.2.2. 1925.3 keV level and 2051.6 keV level

The 1925.3 keV level is only observed to decay to the 2^+_1 level. No evidence of a 1925.5 keV gamma is present in the data. We have observed 2 levels feeding it from above. The two levels are at 2526.7 keV (which also decays to the 2^+_1 state), and 2763.0 keV which only decays to this state. The 2051.7 keV only decays to the 2^+_1 level. We assign both these levels as 0^+ . The energies of the 0^+ intruder states in $^{118,120,122}\text{Cd}$ are 1615, 1744, and 1992 keV respectively. Cadmium-124 is even further from mid-shell so the energy of its intruder state will increase. Therefore, we assign the 2051.6 keV state as the 2p-2h intruder state.

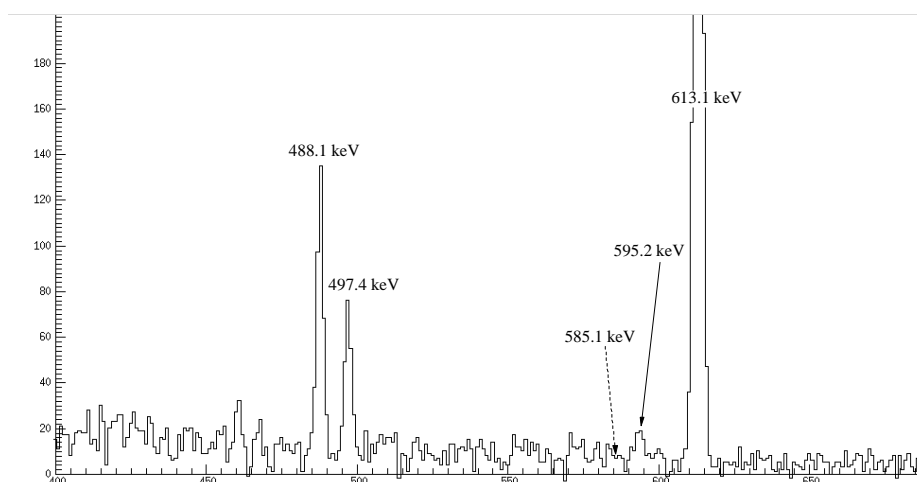


Figure 6. Gammas coincident with the 814.8 keV transition.

3.2.3. 2013.0 keV level

The 2013.0 keV level has an enhanced decay to the 4^+_1 level, and a weak decay to the 2^+_1 level, with no observed feeding from above. An expected 585.1 keV transition to the 2^+_2 level is not observed. Figure 6 shows a portion of the spectrum created by gating on the 814.8 keV $2^+_2 \rightarrow 2^+_1$ transition. The 585 keV gamma is clearly absent. The character of this state is not clear, but can be assigned as either 4^+ or 2^+ .

3.2.4. 2023.1 keV level

The only observed decay from this state is to the 2^+_2 level. This state is only weakly populated in the beta decay of ^{124}Ag , and no feeding from above was observed. The decay of this state is similar to the lighter Cd "3-phonon" 4^+_1 states. We therefore assign this state as (4^+).

3.2.5. 2140.3 keV level

This level is only observed to decay to the 4^+_1 level. It is fed from above by the known (7^-) 2383.7 keV and 2675.2 keV (8^+) states [22]. We therefore accept the previous assignment of 6^+ for this state.

Overall, we have found five states that could be considered as candidates for three-phonon states. The decay of these states is very similar to the lighter Cd isotopes. The 6^+ (2140.3 keV), 4^+ (2023.1 keV) and 3^+ (1916.1 keV) members decay in a manner consistent with multi-phonon states, while the 0^+ (1925.3 keV) and likely 2^+ member at 2013.0 do not.

3.3. ^{126}Ag

Previous studies on ^{126}Cd reported a total of 5 levels with 6 gammas via the beta decay of ^{126}Ag [21], and decay of a μs (12^+) isomer of ^{126}Cd [23]. The reported levels form what appears to be a ground state band and a negative parity band (with spins up to 9^-). These studies did not observe the 2^+_2 state. In this work, we report the observation of a state at 1579.4 keV that de-excites by a 927.3 gamma transition to the 2^+_1 and a 1579.4 keV gamma to the ground state. We assign this level as the 2^+_1 state based on the systematics of lighter Cd isotopes and predictions by the shell model code OXBASH [21]. The previous beta decay study did not observe this transition because it was masked by contamination from a 928.3 keV gamma-ray from the decay of ^{126}Sb [24].

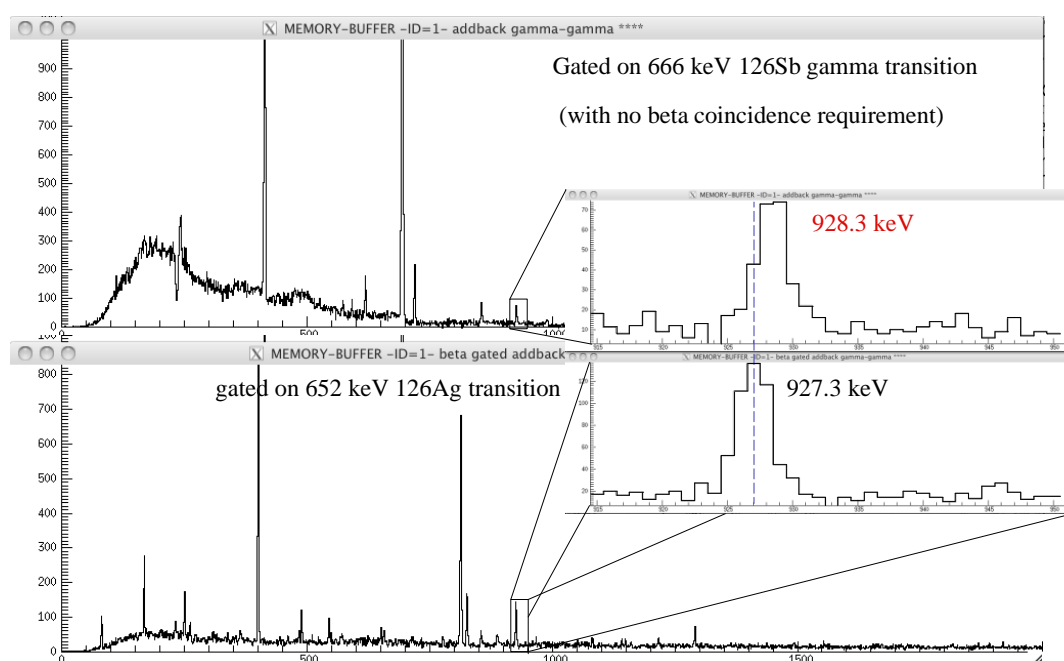


Figure 7. Gamma spectra created by gating on the (a) 666.4 keV $2^+_1 \rightarrow 0^+$ from the ^{126}Sb beta decay, and (b) 652.0 keV $2^+_1 \rightarrow 0^+$ gamma transition from the beta decay of ^{126}Ag .

The present study also contained a small amount of contamination from ^{126}Sb . Figure 7a (*top panel*) shows the gammas coincident with the 666.4 keV $2^+_1 \rightarrow 0^+$ transition in ^{126}Te populated via the beta decay of ^{126}Sb . In this spectrum, a 928.3 keV gamma is present. In Figure 7b (*bottom panel*) gammas coincident with the 652.0 keV $2^+_1 \rightarrow 0^+$ transition in ^{126}Cd are presented, which clearly shows a peak at 927.3 keV peak -1 keV different from the ^{126}Sb contaminant peak (see inset).

Figure 8 shows the decay scheme below 2450 keV. Although there are a few new levels observed in the energy region of interest, they are spread out in energy, and are not good candidates for 3-phonon levels. Unlike the lighter even-mass Ag isotopes, the beta decay of ^{126}Ag is dominated by a large amount of feeding to negative parity states. Analog states to the ones discussed above for $^{120,124}\text{Cd}$ may be present in this nucleus, but are not observed in this work due to the small feedings of these states via beta decay. In order to make a conclusion about whether or not these states exist, either a lot more statistics is needed, or the states must be populated by another method.

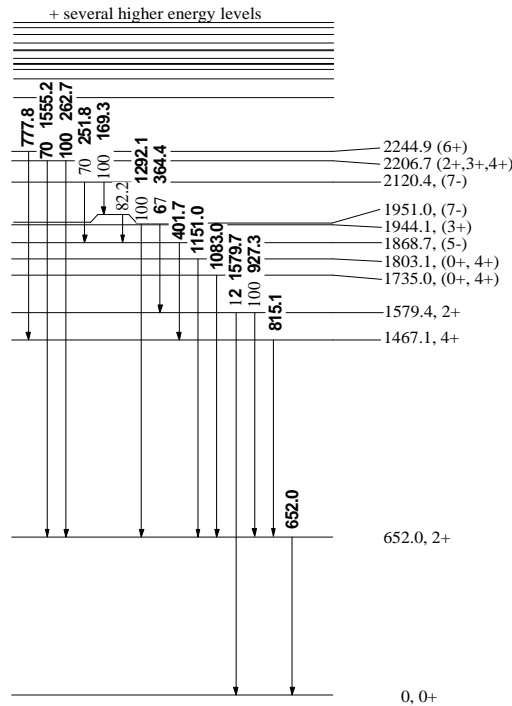


Figure 8. Partial level scheme of ^{126}Cd below 2450 keV populated by the beta decay of ^{126}Ag .

4. Conclusions

The decay pattern of $^{120,124}\text{Cd}$ are similar to lighter Cd isotopes despite the higher energy of the intruder states. Cadmium-120 and ^{124}Cd show a similar pattern as the lighter Cd isotopes, with no good candidates for 2^+ and 0^+ 3-phonon states. The current situation for ^{126}Cd is unclear as candidate 3-phonon states are absent in this work. Whether this is due to the states not being populated in beta decay or are simply absent cannot be determined.

Are the Cd nuclei really vibrational? Overall, the multi-phonon vibrational approach fails to explain the low-energy structure of these Cd isotopes. We propose an alternate way to describe the low-lying states in Cd by placing them in independent band structures rather than as N-phonon states. In all of these cases there is a ground state band, a quasi-gamma band, and an intruder band. This is illustrated in Figures 9-12 for ^{116}Cd [10], ^{118}Cd [8], ^{120}Cd , and ^{124}Cd . This interpretation is more consistent with the experimental data especially for the 0^+_4 and 2^+_4 states, than the three-phonon picture, as the decay pattern from these levels is what would be expected from the decay of independent bands.

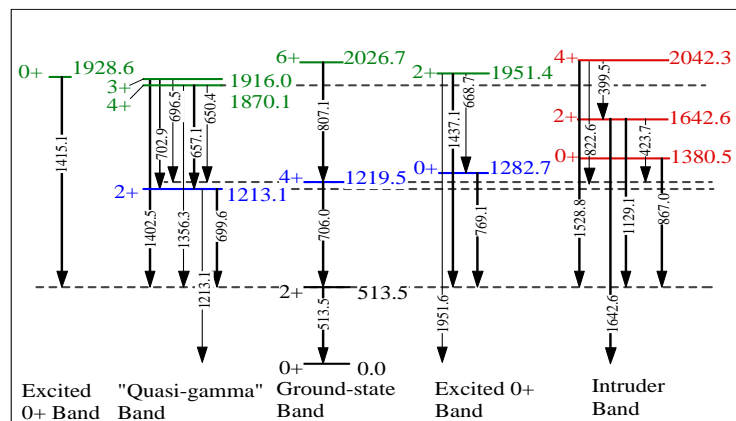


Figure 9. Low energy levels in ^{116}Cd drawn as band structures.

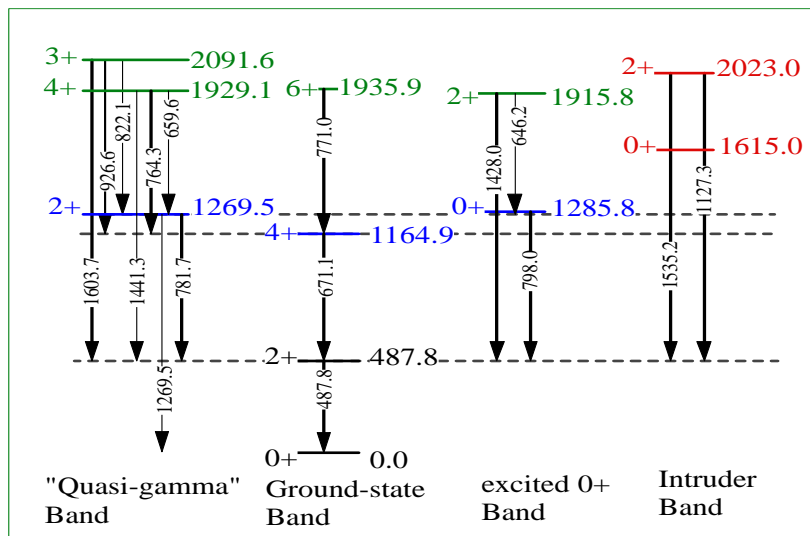


Figure 10. Low energy levels in ^{118}Cd drawn as band structures.

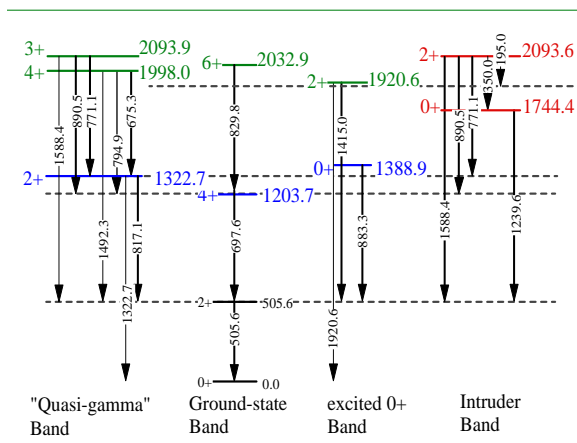


Figure 11. Low energy levels in ^{120}Cd drawn as band structures.

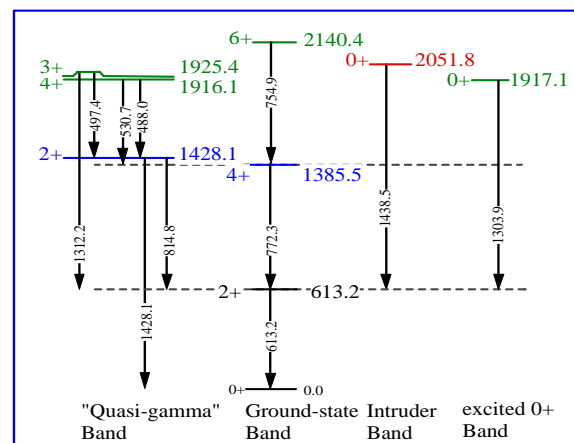


Figure 12. Low energy levels in ^{124}Cd drawn as band structures.

In this work, we have shown that the deviations from U(5) dynamical symmetry for the heavy even-mass Cd isotopes continue to higher masses even though the intruder states are further away in energy. While these states do not appear to be multiphonon states, their exact nature remains unclear. Future studies using other experimental methods (g-factors, etc.) will be needed to better understand the nature of these states.

Acknowledgments

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