Model for Planning Well Control Operations Involving an Induced Fracture.

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MODEL FOR PLANNING WELL CONTROL OPERATIONS INVOLVING AN INDUCED FRACTURE

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy

in

The Department of Petroleum Engineering

by

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B.S., Universidade de Sao Paulo, 1979
M.S., Universidade de Campinas, 1989
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ABSTRACT

This dissertation presents a numerical simulator for designing a dynamic kill of an underground blowout. The simulator consists of three sub-programs: a reservoir model, a wellbore model, and a fracture model.

Previously published procedures have modeled the fracture by assuming a constant pressure in the wellbore at the depth of fracture. This assumption has sometimes led to unrealistic results. In this work, a hydraulic fracture model is coupled with the reservoir and wellbore model using a system analysis approach.

The hydraulic fracture model is based on a pseudo-3D model with some modifications introduced to more accurately model the extension of the fracture and to account for the use of a two-phase fluid. To obtain a more accurate prediction in the fracture model, an experimental procedure was planned to measure the leak-off volume inside the fracture for drilling mud and gas, and a correlation based on experimental data was presented. The correlation uses three parameters, (spurt loss volume, pack buildup factor, and equilibrium Darcy flow velocity coefficient), to determine the leak-off volume.

Simulations of underground blowouts were run, and these simulations showed significant differences between the current and the proposed models for some cases. The results show that, with the proposed model, well control can be achieved with lower pumping rates than indicated with conventional models. These lower pumping rates can be important since the time and cost of gathering the required pumping equipment and, in some cases, drilling additional relief wells is greatly dependent on the pumping rates required to overcome the blowout.

Also, the proposed model can be used as a tool for verifying the applicability of the shut-in procedure in shallow wells when controlling a blowout.
CHAPTER I

INTRODUCTION

The drilling of a well in the oil industry is a very complex operation, and its success depends essentially on well planning. Good well planning must account for the possibility of encountering abnormal formation pressure, lost circulation zones, and other problems related to drilling.

The fluid pressure within the formations to be drilled establishes one of the most critical parameters in planning and drilling a well because it determines (among several parameters) the density of the drilling fluid, the depth of the casing shoes, and the casing specifications.

A formation fluid influx occurs in the well when the pressure of a drilled formation containing this fluid is greater than the hydrostatic pressure caused by the drilling fluid. The amount of the fluid influx is proportional to the permeability of the formation and to the pressure differential between the formation and the wellbore. The situation will become less or more complex depending on the kind of formation fluid, which can be water, oil, or gas. If the fluid is water or oil with a low gas-oil ratio, the procedure to regain control of the well will not be difficult. However, if the fluid is gas or oil with a high gas-oil ratio, the situation will be more complex due to the compressible nature and low density of the gas, and if the appropriate action is not taken at the right time, control of the well can be lost, and the influx can turn into a blowout with devastating consequences.

Due to the dangerous situation that an influx of formation fluid can cause, early detection of its presence is very important. An appropriate well control operation must be performed to circulate the invading fluid out of the well through an
adjustable choke at the surface. A schematic illustrating the hydraulic flow paths during the well-control operations is shown in Fig. 1.1.

![Fig. 1.1. Schematic of Well Control Operations](image)

An appropriate well control operation must keep the bottom hole pressure of the well slightly above the pore pressure of the formation to prevent additional formation fluid from flowing into the well and to prevent fracturing a weaker formation that can be exposed to the wellbore pressure. Unfortunately, control of well is lost in some cases because an improper well control procedure.

The blowout that may result from this loss of well control can be one of two types: either surface or underground. A blowout is called a surface blowout when an uncontrolled flow reaches the wellhead and produces formation fluid to the atmosphere or the seafloor. Surface blowouts are very dangerous because they cause an immediate risk to the rig crew and to the equipment. A blowout is called an underground blowout when a flow of high-pressure fluids occurs along an open hole section from the producing formation into lower-pressured intervals. This kind of
flow happens whenever an induced fracture or a lost circulation occurs in the lower-pressured formation, and this flow can cause a pressurization of the shallower interval with hydrocarbons originally contained in producing formation.

In an underground blowout, the problem is to determine whether the hydrocarbons are likely to remain contained within the shallower interval or will continue to move through the earth to the surface by means of natural rock fractures or the wedging open of fault planes. If formation fluids move upward, they may reach near-surface unconsolidated formations that can be liquefied by the high hydrocarbon pressures, resulting in crater formation that may cause equipment loss beneath the ground surface. Fig. 1.2 shows the principal mechanisms of sediment failure leading to crater formation that may take place following one underground blowout.

Fig. 1.2. Sediment Failure Mechanisms That May Follow an Underground Blowout
Although an underground blowout does not cause an immediate risk to the crew and rig, it can overpressurize a shallower formation or deplete the productive reservoir from which the hydrocarbons are flowing. An overpressured shallow formation can later increase the risks of a blowout when drilling subsequent wells (Fig. 1.3).

Another possible complication of an underground blowout is the loss of return of fluid at the rig flow line (lost circulation). Lost circulation can occur in any of the following four sub-surface conditions in a borehole: (a) flawless surface; (b) borehole with a closed fracture; (c) irregular borehole; and (d) highly fractured formations, vuggy zones or granular zones. A loss of circulation can be total or partial in any of the four cases; a total loss of circulation means that no drilling fluid returns to the surface, with all of the fluid going into the lost circulation zone. In the first three cases, the loss of circulation depends on the stress in the formation and in the wellbore. The lost circulation due to condition (d) occurs wherever the wellbore
pressure exceeds pore pressure, and it can be controlled only by using lost circulation products.

Therefore, an underground blowout results from a scenario where an influx of formation fluids and a total loss of circulation in a shallower formation occur concurrently. This study will be restricted to a situation in which the total lost circulation is caused by an induced fracture. Three possible cases of this situation can occur. The first case occurs when a late detection of an influx of formation fluids induces a lost circulation zone in the open hole section of the well. If not recognized in time, this lost circulation zone can lead to errors in the well control procedure being employed, causing further influx into the wellbore. This additional influx can cause an increase in the annulus pressure that can lead to an induced fracture with total loss of circulation, and further loss of control in the well.

The second case is when a total loss occurs due to excessive mud weight or to mud pressure surges caused by running in the drillstring or casing too fast. This total loss can cause the fluid level in the well to fall and result in an influx of formation fluid into the bottom of the well.

The third case in which a drilling operation is being performed with a partial lost circulation when a permeable highly pressurized formation is encountered, and an influx occurs. There are two possible results in this case. One is the influx can be circulated out of the well because partial circulation continues during the well control. Another possible result is that the lost circulation becomes complete because of an increase in wellbore pressure, which causes a high flow rate into the induced fracture. This total loss of circulation will cause an underground blowout.

As stated before, the causes of underground blowouts are numerous, but this study focuses on the control of an underground blowout caused by an induced fracture during a well control operation. It is assumed that the producing reservoir is a gas reservoir, and that the induced fracture has two wings extending in opposite
directions from the well and is oriented in the vertical plane. Although other fracture configurations such as horizontal fractures are known to exist, they constitute a low percentage of the situations experienced to date. Also horizontal fractures are usually limited to relatively shallow depths (less than 2,000 ft (610 m)); therefore this study will always assume the fracture to be vertical.

Although many ways (the use of barite plugs, cementing, packers, etc.) to control underground blowouts exist, this study will focus on the dynamic kill method as a means to regain control of the well. The dynamic kill method is a well control procedure that calls for pumping down the drill pipe, thus displacing the formation fluids out of the annulus. If the influx rate is sufficiently low, this action will stop the flow because the kill fluid being pumped will cause a bottom hole pressure that exceeds the formation pressure. On the other hand, if the influx rate is high, the kill fluid will be greatly diluted by formation fluid and may not increase the bottom hole pressure sufficiently to stop the flow. In such a case, a relief well may be necessary to achieve a high enough pump rate to regain the control of the well. In the case of an underground blowout, the mud is displaced with the annulus closed by the blowout preventer.

As part of planning a dynamic kill method, the current approaches for predicting the appropriate flow rate to regain the control of the well assume the pressure at the fractured formation to be constant, equal to or slightly greater than the value of the fracture initiation pressure. In our opinion, this assumption is unrealistic because this pressure will change with time as the fracture propagates and as the flow rate is increased. This change has been shown by two-dimensional models, such as those described by Perkins-Kern (1961) and Geerstma-de Klerk (1969) and by fracture treatment data. In addition, three-dimensional models have confirmed this change in pressure as the fracture propagates. All these observations regarding induced fracture are explained in Chapter II. Thus, assuming a constant fracture
injection pressure can lead to an inappropriate estimate of the mud flow rate needed to regain control of the well.

The main objective of this study is to evaluate when the assumption of a constant fracture injection pressure will lead to unacceptable errors in the design of a dynamic kill procedure. The evaluation is accomplished using a new computer model that couples a hydraulic fracture model with a conventional reservoir and wellbore model using a systems analysis approach. In addition, the model developed in this study can be used as a tool in designing a contingency plan for an underground blowout with emphasis on determining if the well must be shut or diverted. This is the most important phase in the control because, through this planning, all the economic, operational, and safety factors are defined.

The contingency plan for an underground blowout considers two strategies: (a) how to avoid the occurrence; and (b) the control of the underground blowout. The first part depends on many factors, but, among them, three of the most important are: (a) hydrostatic pressure in the well; (b) pore pressure within the formations to be drilled; and (c) minimum fracture pressure in the formations. Therefore, besides other factors, the occurrence of an underground blowout is directly dependent on the properties of the drilling fluid and the formation characteristics.

The choice of the drilling fluid density depends on the formation and the fracture pressure because the hydrostatic pressure created by the drilling fluid must be larger than the formation pressure but smaller than the fracture pressure. Normally, in well planning, the minimum fracture pressure is taken to be equal to the pressure determined in leak-off tests of adjacent wells or, in the case of wildcat wells, calculated through correlation. These values are valid only for the fracture initiation and are different during the fracture propagation, as will be shown later.

Formation pore pressure must also be estimated during well planning. The pressure of an abnormally pressured formation increases the complexity of the
planning process, so the engineer responsible for well planning must first determine whether abnormal pressures are present.

As stated by Bourgoyne et al. (1991), the origin of abnormal formation pressure is not understood completely, but several mechanisms that cause abnormal formation pressure have been identified, among them compaction, diagenetic, differential density, and fluid migration effects.

When fluid migration occurs between formations, unexpected abnormal pressures may develop in shallow depths. A shallower formation can be pressurized by the upward flow of fluids from a deep reservoir to the shallow one. For example, an improperly abandoned underground blowout, as shown in Fig. 1.3, can pressurize a shallow formation. Overpressurization of a shallow formation can also occur because of a leaky fault and leaky cement or casing as shown in Fig. 1.4. Because an unexpected influx in shallower formations can turn into a blowout easier than when abnormal pressure exists in deeper formations, drilling operations at shallower depths are more complex.

![Fig. 1.4. Shallow Formation Overpressurization Due to Leaky Fault or Leaky Casing](image-url)
In addition to the three most important factors that must be considered in the first part of the contingent plan and because underground blowouts are sometimes difficult to avoid, the plan must be timely. The contingency plan should be available at the rig site so that the staff understands how to control an underground blowout if it occurs. This plan should prepare the rig crew to recognize an underground flow, to calculate the combination of the drilling fluid density and flow rate required to control the underground flow, and to determine whether the procedure with the rig equipment is feasible or whether additional high-pressure pumps will be required. The rig personnel should also be prepared to calculate the volume of drilling fluid required for the procedure and to recognize whether or not the control method is proceeding successfully.

Once the well control contingency plan establishes the pump rate and density of the fluid, the pumping capacity of the rig equipment must be verified. When the rig equipment is known to be inadequate for the underground flow potential of a certain hole section, the contingency plan should specify the additional pump units that would be required at the well location and how these units would have to be rigged.

Information on the available kill fluid volume should be part of every well control contingency plan. The total kill fluid volume required must be determined so the control procedure can be executed without interruption. If the fluid is not enough to complete the whole control operation, the mud in the annulus will be produced into the fractured formation, and the well will continue flowing. Consequently, a well control contingency plan should also include the minimum volume of kill fluid that should be on the rig before the pumping operation starts.

In the planning phase of the contingency plan, the model developed in this study can be used to calculate the flow rate and density of the fluid necessary to control the underground blowout. Also with this model, the value of the fracture
initiation pressure can be updated as each casing string is set and actual leak-off data is obtained.

As an example of the previous discussion, Walters (1991) reports on the occurrence of an uncontained underground blowout with total lost circulation. Figure 1.5 shows the geological cross section indicating the trajectory of the drilling well. The 12 1/4-in (31.1-cm) hole was drilled to a target depth of 7,546 ft (2,300 m). The well was inadvertently fractured after logging the 4,872 to 7,546 ft open hole section (1,485 to 2,300 m). Drilling fluid losses occurred while running 9 5/8-in (24.3-cm) casing. The casing was retrieved, and a bit was run to ream the borehole and control fluid losses. During the reaming operation, the bit became stuck at 5,266 ft (1,605 m), and total loss of circulation occurred. Due to the decreasing hydrostatic pressure, formation fluid flowed through the well, establishing full communication along the open hole section from the overpressured gas in the deep reservoir sands (about 7,513 ft (2,290 m)) to the 13 3/8-in (34-cm) casing shoe. Cement was pumped through the drillstring to attempt to regain control of the well, but, after 11 days a mixture of oil, gas, water, and drilling fluid erupted at surface about 1,959 ft (600 m) from the rig, indicating that the underground blowout had reached the surface.

This case history is an example that contributes to the understanding of the events that may take place following an underground blowout. It appears that the contingency planning was not adequate to control the well. Although cement was pumped into the well, the use of cement as a plugging technique for gas formations only works when the cement has special components. Because the cement was inappropriate for the operation, the well continued flowing after the cementation. This well could be controlled with a fluid that had an equivalent circulating pressure of 4,232 psi (29.2 MPa) at 5,299 ft (1,615 m), but this value was approximately 359 psi (2.5 MPa) in excess of the minimum total stress in the formation.
In this case, the in-situ minimum stress at 5,299 ft (1615 m) where the loss probably occurred was 3,873 psi (26.7 MPa), the overburden stress (stress caused in a given point underground by the geostatic load of the sediments above this point) was 5,149 psi (35.5 MPa) and the original fluid formation pressure was 2,408 psi (16.6 MPa). These values were based on fracture closure pressures obtained from a microfracture test previously conducted in the field. In addition, the microfracture data from the field indicated the fracture propagation pressure exceeded the sand and shale sequence pressures by 300 psi (2.07 MPa).

These data gave a good indication that an unconfined fracture propagation with vertical orientation occurred at the time of the mud losses. This is an example of a case in which the pressure in front of the fracture does not remain at the fracture initiation pressure, but instead, changes as the fracture propagates.
In the analysis of this underground blowout, the approach was to assume that the prevailing stress directions were consistent with those at the time of faulting. Therefore, the fracture would propagate parallel to the fault strike, with the fracture intersecting the first fault above the point where the well was fractured (Fault A, Fig. 1.5). In this case there was a fault, and, after the intersection by the fracture, the pressure in front of the fracture remained constant because there was no more propagation. The gas pressure reduced the horizontal and total vertical effective stresses in the fault plane by the same amount and equal to the increase in the pore pressure. The increase of the pore pressure due to gas was estimated to be 1,850 psi (12.75 MPa). The effective stresses before and after the underground blowout were used to construct a Mohr's circle, and therefore, to analyze the consequences of the underground blowout in the faults.

The main conclusion was that the fault plane's normal effective stress declined to zero at the time of the internal blowout, causing dilatational shearing along parts of the fault plane, which in turn, allowed gas migration along and up the fault with eventual gas breakthrough at the surface. This mechanism also explained the observed charging of intervals which could have been caused by the migration of gas along or across the fault in sand streaks, on the downthrown side of the fault.
CHAPTER II

LITERATURE REVIEW REGARDING INDUCED FRACTIONS

This review focuses on the drilling aspects related to stabilization of the well, particularly of induced fracture. Although almost all of the previous works treated the subject of hydraulic fracturing technique as a formation stimulation method, the technique itself is mechanically related to three other phenomena existing in the petroleum industry, all of which appear to induce fractures by pressure applied in a wellbore. These phenomena are: (a) pressure parting in water injection wells; (b) lost circulation during drilling or well control operations; and (c) the breakdown of formations during squeeze-cementing operations.

Hydraulic fracturing remains poorly understood. After 40 years of fracturing experience, fracture pressure, in-situ fracture shapes, dimensions, symmetry about the wellbore, azimuths, and fracture conductivity often cannot be predicted accurately. In addition, in-situ rock properties and stress fields that significantly affect fracture pressure often cannot be determined using current field practices. Consequently, the results of the models are often limited. However, technology in fracturing has advanced significantly in the last years.

Therefore this chapter describes the most important progress that has being made in induced fracture in the last 40 years and is divided into two parts: the fracture initiation process, and the fracture propagation process.

2.1. Fracture Initiation Pressure

Before Hubbert and Willis (1957), one of the earliest and most important works in hydraulic fracture literature, the prevalent opinion about the mechanism of hydraulic fracture was that the fracture induction was caused by the pressure parting the formation along a bedding plane and lifting the overburden. This opinion had
already been contested by some authors who showed that the pressures required to induce a fracture were less than those required to lift the overburden, but the prevalent opinion began changing only after Hubbert and Willis' work (1957).

Fracture initiation depends mainly on the geologic processes that have occurred in the area of interest or in other words, it depends on the in-situ stress condition underground.

The in-situ stress, as it affects hydraulic fracturing, is the local stress state in a given rock mass at the depth of interest. The three principal stress components of the local stress state are influenced strongly by the weight of the overburden, pore pressure, temperature, rock properties, diagenesis, tectonics, and viscoelastic relaxation. In addition, drilling, fracturing, or production can alter some of these parameters, changing the local stress field.

In-situ stresses control the fracture azimuth and orientation (vertical or horizontal), vertical height growth, surface injection pressures, fracture cross-sectional width profiles, and other factors of fracture behavior. Therefore, it is important to know the stresses and their variations for better prediction in fracture. One example of stress influence on the geometry and orientation of a hydraulic fracture can be seen in Fig. 2.1.

There are two major schools of thought regarding the state of stress within the earth's crust: (1) the stress state is hydrostatic. That means the three principal stresses are equal. (2) The horizontal principal stresses are a function of the effective vertical stress and Poisson's ratio.

The first one is described as the standard state, and it states that stresses in rock tend to become equal because of the ability of the rocks to creep, such that any stress difference becomes alleviated.
The second one describes the stress state in an elastic, flat-lying stratum of semi-infinite extent that is constrained laterally, so the vertical and horizontal effective stress components are proportional.

The concept of effective stresses was first introduced by Terzaghi (1923), and it states that hydrostatic stress within a pore fluid has no influence on deformation, which is controlled by the effective stresses. Thus, this hydrostatic stress is a neutral stress, one that acts in all directions and in the same amount. This stress is regarded to exist in both the solid and the liquid, so the effective stresses arise exclusively from the solid structure, and it is equal to the compressive stress minus the pore
pressure. Major studies on rock formation have shown that fracturing is controlled by the effective stresses, provided the rocks have a sufficient permeability to allow movement of fluid, a connected pore system, and an inert pore fluid, so that the effects are purely mechanical. In addition, the values of those stresses are different in the three principal directions, so the second theory regarding fracture is used in this work.

Designating as principal stresses those stresses that are normal to planes where no shear stress occurs, the general stress condition underground, as pointed out by Hubbert and Willis (1957), is one in which the three perpendicular principal stresses are unequal. Since for unequal stresses the only planes on which the shear stresses are zero are those perpendicular to the principal stresses, it follows that one of the three trajectories of principal stress must terminate perpendicular to the surface of the ground, and the other two must be parallel to this surface.

Therefore, it means that in simple geologic structures and smooth surfaces the principal stresses are respectively nearly horizontal and vertical with the vertical stress equal to the pressure of the overlying strata. A schematic of the stress element is shown in Fig. 2.2. Denoting the horizontal matrix stresses as $\sigma_x$ and $\sigma_y$ and the vertical matrix as $\sigma_z$, the horizontal stresses in tectonically relaxed sediments are approximately equal but smaller than the vertical matrix stress.

In the induced fracture process when pressure greater than the least principal stress is applied in rocks within this stress condition, the fracture plane will most likely occur in the plane perpendicular to the direction of the least principal stress. The existent stress state in sedimentary basins containing oil and gas generally occurs in tectonically relaxed sediments where the cause of stress is primarily due to weight of the sediments over the layer in study, and so the probable direction of the fracture is vertical.
Under the assumption of the rocks showing an elastic behavior, the relation between horizontal and vertical stresses is given from elastic theory by:

$$\sigma_x = \sigma_y = \sigma_h = \frac{\nu}{1 - \nu} \sigma_z$$

(2.1)

The equation (2.1) can be applied only to a young deltaic deposition environment where normal faulting is common and the horizontal stresses are less than the vertical stresses. The reason for that is this equation, within the normal range of Poisson's coefficient for sedimentary rocks, gives values for the horizontal stress in the range 22-37 percent of the vertical stresses. In addition, in areas where thrust faults and folding are occurring, the horizontal stresses tend to be greater than the vertical stresses as pointed out by Hubbert and Willis (1957).

A hydraulic fracturing mechanism can be better understood by the introduction of fracturing fluid into a cavity located in the center of the rock element, as shown in Fig.2.3. For the fracturing fluid to enter the cavity, the pressure of the fracturing fluid must exceed the original fluid pressure in the element. This
Introduction increases the pressure inside the element which causes compression of the rock matrix. The compression of the rock will be greatest in the direction of the minimum matrix stress. The rock element will fracture when the pressure of the fracturing fluid exceeds the sum of the minimum principal matrix stress, the pore pressure, and the tensile strength of the rock element.

As stated previously, the preferential orientation of one fracture is perpendicular to the least principal stress, and so the induced fracture in tectonically relaxed areas characterized by normal faulting will be vertical. In contrast, in active thrust faulting regions where the least principal stress is vertical, the fracture will be horizontal.
The general stress condition underground is affected by the presence of a wellbore in the region. An approach for this case was used by Hubbert and Willis (1957) assuming that the rock is elastic, the borehole smooth and cylindrical, and the borehole axis vertical and parallel to one of the pre-existing regional principal stresses. The calculated stresses are the effective stresses or in other words, the stresses carried by the rock in addition to a hydrostatic pressure which exists within the wellbore as well as in the rock. The calculation was made from the solution in elastic theory for the stresses in an infinite plate containing a circular hole, with its axis perpendicular to the plate, similar as obtained by Timoshenko and Goodier (1951) in polar coordinates with the center of the hole as the origin.

The plane-stress components at a point \( \theta, r \) exterior to the hole in a plate are given by:

\[
\sigma_r = \left( \frac{\sigma_y + \sigma_z}{2} \right) \left( 1 - \frac{r_r^2}{r^2} \right) + \left( \frac{\sigma_y - \sigma_z}{2} \right) \left( 1 + \frac{3r_r^4}{r^4} - \frac{4r_r^2}{r^2} \right) \cos 2\theta \tag{2.2}
\]

\[
\sigma_\theta = \left( \frac{\sigma_y + \sigma_z}{2} \right) \left( 1 + \frac{r_r^2}{r^2} \right) - \left( \frac{\sigma_y - \sigma_z}{2} \right) \left( 1 + \frac{3r_r^4}{r^4} \right) \cos 2\theta \tag{2.3}
\]

\[
\tau_{r\theta} = \left( \frac{\sigma_y - \sigma_z}{2} \right) \left( 1 - \frac{3r_r^4}{r^4} + \frac{2r_r^2}{r^2} \right) \sin 2\theta \tag{2.4}
\]

Using these equations Hubbert and Willis (1957) calculated the horizontal stresses across the principal planes near the borehole for various relative values of \( \sigma_y / \sigma_z \) ratio as shown in Fig. 2.4. It can be seen that the stress concentrations are local, and the stresses rapidly approach the undisturbed regional stresses within a few hole diameters for any of the cases.

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The vertical component of the stress that is equal to the effective pressure of the overburden is also distorted around the borehole. However, the distortion in the vertical stress which is function of $\sigma_x$ and $\sigma_z$ is small in magnitude in comparison with the concentration of the horizontal stresses and disappears within a near distance from the wellbore. In addition to the stress distortion caused by the borehole, the effect of fluid pressure in excess of the original formation pressure applied in the borehole produces additional stresses and must be considered.

![Stress States Around a Borehole for Different Regional Stress Ratios $\sigma_y/\sigma_x$ (after Hubbert and Willis, 1957)](image)

Fig. 2.4. Stress States Around a Borehole for Different Regional Stress Ratios $\sigma_y/\sigma_x$ (after Hubbert and Willis, 1957)

If the fluid is a non-penetrating fluid, as drilling fluid is considered in almost all cases, the stresses may be derived from the Lamé solution for the stresses in a thick-walled elastic cylinder. One must also consider a very large outer radius, and the external pressure must be equal to zero. The solution for this case, if the excess of pressure is equal to $\Delta P$ is given by:
\[ \sigma_r = \Delta P \frac{r^2}{r^2} \] \hfill (2.5)

\[ \sigma_\theta = -\Delta P \frac{r^2}{r^2} \] \hfill (2.6)

\[ \sigma_z = 0 \] \hfill (2.7)

Fig. 2.5 shows the stresses caused by two cases where in the first one (a) the pressure applied in the borehole (\(\Delta P\)) acts alone and in the second one (b) the pre-existing regional stresses are superposed upon the internal pressure.

A fluid which cannot permeate a rock formation is defined as a non-penetrating fluid. If a fluid is able to permeate a rock formation it is termed penetrating. Penetrating fluids influence the distribution of pressure around the wellbore differently than non-penetrating fluids, as seen in Fig. 2.6.
Besides wellbore effects on formation stress conditions, the rupture pressure to initiate fractures depends upon rock properties, the kind of fluid being used, and the formation pore pressure.

![Diagram of pore fluid pressure around a borehole](image)

Fig. 2.6. Possible Distribution of Pore Fluid Pressure Around the Borehole (after Haimson and Fairhurst, 1967)

It can be seen from Fig. 2.4 the least compressive stress across a vertical plane at the walls of the well varies from $2\sigma_z$ to zero, depending on the ratio of $\sigma_y / \sigma_x$. The fracture pressure of formation in a vertical plane occurs when the effective tangential stress passes from compression to tension. Therefore, for the case of tectonically relaxed sediments, with $\sigma_x = \sigma_y = \sigma_h$, using elastic theory for the stresses in an infinite plate containing a circular hole, and considering the formation has a tensile strength of rock, $S_t$, the fracture initiation pressure will be given by:

$$P_f = 2\sigma_h - S_t + P_o$$

(2.8)

where tension in this equation is considered as negative and compression as positive.
It is important to note that the tensile strength of rock is used only for the calculation of the fracture initiation pressure. Once the fracture initiation has occurred, the formation loses the tensile strength, and this strength is not considered anymore during the phase of fracture propagation.

The overburden stress, $S_z$, is the stress caused by the weight of overlaying material on the point of interest, and can be calculated by:

$$S_z = \sigma_z + P_o$$

Substituting equations (2.8) and (2.9) in equation (2.1), the value of fracture initiation pressure in function of overburden stress is given by:

$$P_f = \frac{2v}{1-v}(S_z - P_o) - S_i + P_o$$

Normally, the best way to predict the overburden stresses and pore pressure in a formation is by means of using density or porosity data available from well logs. Correlation for calculations of overburden stresses or pore pressure can work well in a determined area, but it can lead to significant errors if used in another one.

The tensile strength of rock for flawless specimens can vary from zero for unconsolidated materials to several hundred pounds per square inch for the strongest rocks. However, as stated by Hubbert and Willis (1957), flawless specimens of linear dimensions greater than a few feet rarely occur. Normally they are intersected by one or more systems of joints containing divisions with only slight normal displacements. Across these joint surfaces the tensile strength is reduced essentially to zero.

As in any section of a wellbore with few tens of feet probably intersects such joints, the tensile strength of most rocks that are to be subjected to hydraulic fracture will be essentially zero. Using these facts in equation (2.10) gives:
\[ P_f = \frac{2v}{1-v}(S_z - P_o) + P_o \] .................................(2.11)

The theoretical treatment used by several authors (Haimson and Fairhurst (1967), Hagoort et al. (1980), Campos (1983)), in determining the fracture pressure is similar to that used in determining the equations (2.2) through (2.4). The additional stresses caused by the fluid being pumped into the well and by fluid movement into the formation are considered by the principle of superposition as developed by Hubbert and Willis (1957).

The solution of the fracture pressure under the assumptions that the formation is elastic, porous, isotropic, homogeneous, and where Poisson's ratio and Biot's constant are known is given by:

\[ P_f = \frac{3\sigma_z - \sigma_x - S_z}{2 - \alpha - \frac{2v}{1-v}} + P_o \] .................................(2.12)

where \( \alpha \) is the Biot's constant. Appendix A shows the derivation of equation (2.12)

In addition to these assumptions, the fracture fluid must be penetrating, the borehole must be smooth and cylindrical, and the borehole axis must be vertical and parallel to one of the pre-existing regional principal stresses.

Biot's elastic constant is a property of the rock, being a function of the rock matrix and the bulk rock compressibility under the assumptions that the rock is homogeneous, isotropic, and linearly elastic. Common values of the Biot's constant for sandstone reported in the literature are within the range of 0.70 and 0.90.

The solution for the case of non-penetrating fluid, also derived in the Appendix A is given by:
\[ P_f = 3\sigma_y - \sigma_x - S_i + P_o \] ..............................................................(2.13)

If \( \sigma_x = \sigma_y = \sigma_h \), the equation (2.13) becomes equation (2.11) which was first derived by Hubbert and Willis (1957).

Similar works were developed to determine the fracture pressure of one particular formation. Matthews and Kelly (1967) used the concept of the matrix stress coefficient, \( K_f \), that is a function of depth, and the consideration that \( S_i \) was equal to zero to develop the following expression for fracture initiation pressure:

\[ P_f = K_f \sigma_z + P_o \] ........................................................................................................(2.14)

Comparing equations (2.1), (2.11), and (2.14), it can be seen the matrix stress coefficient is a function of the ratio of horizontal effective stress to vertical effective stress.

Pennebaker (1968) presented a correlation similar to equation (2.14) where \( K_f \) was referred to as the effective stress ratio, and he correlated this ratio with depth. The most important feature in the Pennebaker's work was a correlation for determining vertical overburden stress gradient as a function of depth and geologic age for various depths. The interval transit time obtained from seismic data for those depths was 100 \( \mu sec/ft \).

Eaton (1969) developed a correlation assuming that equation (2.1) describes the relationship between horizontal and vertical matrix stress. In addition, he correlated Poisson's ratio as a function of depth using observed fracture data from Texas and Louisiana. Eaton's correlation is given by:

\[ P_f = \frac{V}{1 - V}(S_z - P_o) + P_o \] ........................................................................................................(2.15)
Christman (1973), examining bulk density logs of the Santa Barbara channel found that the formation bulk density correlates with stress ratio, given one estimation of the degree of compaction. The equation to predict formation fracture pressure is:

\[ P_f = F_a(S_z - P_o) + P_o \]  

(2.16)

In this equation, fracture is assumed to be at a depth of the highest stress ratio and lowest rock density. His work showed that different types of rocks have different fracture pressure initiation when in the same conditions of fracture, reinforcing the concept that fractures do not always happen at the casing shoe. The plot of stress ratio \( F_a \) versus the rock density observed in the Santa Barbara channel is shown in Fig. 2.7.

Daines (1982), using laboratory-derived physical properties of typical sedimentary rocks, and using Hubbert and Willis model, proposed a model for predicting fracture pressures in a wildcat well after the first fracture test in a compact formation. The prediction of fracture is given by:

\[ P_f = \sigma_i + \frac{V}{1 - V} (S_z - P_o) + P_o \]  

(2.17)

where the superposed horizontal tectonic stress \( \sigma_i \) varies between the limits:

\[ 0 \leq \sigma_i \leq 3\sigma_v - \sigma_i \left( \frac{V}{1 - V} \right) \]  

(2.18)
The result of the first fracture test in compact formation is then used to calculate the effective stress ratio of the superposed tectonic stress by means of the equation (2.17) where $\sigma_t$ is calculated and

$$\beta_s = \frac{\sigma_t}{\sigma_v}$$

Fig. 2.7. Stress Ratio Versus Rock Density (after Christman, 1973)

Since the stress ratio is considered constant with depth, and $\sigma_v$ is known at any point within the drilled hole, the superposed tectonic stress can be calculated in any point of the well.

Aadnoy and Larsen (1989) derived a new method to predict fracture pressure. In this method they adjusted certain parameters in the equations developed by Aadnoy and Chenevert (1987) to fit better the field measured fracture data.
Although their method considered borehole inclination, only the case of a vertical borehole is considered in this review. The equation to predict the fracture pressure gradient is given by:

\[
\frac{\partial P_f}{\partial z} = 2 \frac{\partial s}{\partial z} - 2K_i - a_i - (1 - b_i) \frac{\partial P_o}{\partial z} \tag{2.20}
\]

where the coefficients \( K_i, a_i \) and \( b_i \) are determined from field results.

The collection of pore pressure, depth, and the lithology for each fracture point or lost circulation datum is necessary for determining these coefficients. Referring to Fig. 2.8 and 2.9, Aadnoy and Larsen (1989), with overburden stress data from sonic logs, plotted the overburden stress gradient curve, the pore pressure gradient curve, and the fracture gradient points or lost circulation points vs. depth, as in Fig. 2.8. In addition, they calculated, for each fracture point, the value of horizontal in-situ stress gradient \( \sigma_{r_i} \) with:

\[
\sigma_{r_i} = \frac{1}{2} \left( \frac{\partial P_f}{\partial z} + \frac{\partial P_o}{\partial z} + A_i \right) \tag{2.21}
\]

where the correlation coefficient \( A_i \) given by:

\[
A_i = a_i - b_i \frac{\partial P_o}{\partial z} \tag{2.22}
\]

was set equal to zero. The coefficient \( K_i \) refers to the distance between the horizontal stress gradient curve and the overburden stress gradient curve. The curve parallel to the overburden stress gradient curve that passes through the minimum distance point was denominated corrected in situ stress, as shown in Fig. 2.8.
The values of the coefficient $a_i$ and $b_i$ are determined by plotting the correlation coefficient $A_i$ versus the pore pressure gradient (Fig. 2.8), where

$$ A_i = 2(\sigma_{Ticor} - \sigma_{T1}) $$

Morita et al. (1990), in an experimental work with cubic Berea sandstone samples 30x30x30 inches, studied the borehole breakdown when using drilling fluid as an injection fluid. They stated that the borehole breakdown does not occur until the well pressure exceeds the pressure which results in a tangential stress equal to the rock tensile strength even with a large surface flaw. The reason for this is the sealing effect of the mud in narrow natural fractures or in fractures created by high borehole pressure. Drilling fluids contain solids that form bridges in the fracture aperture (width) and therefore, they can plug minute cracks.
Although they found that borehole breakdown and extension pressures were abnormally high in their experiment when compared with the field data, they quantified two approximate fracture widths based on split rock samples used in the experiment: one that does not allow drilling fluid entry ($W_T$) and one that increases depending on the dehydration process ($W_M$), as shown in Fig. 2.10. Consequently,
the fracture tip is always several inches ahead of the drilling fluid front during the borehole breakdown and fracture extension process.

They also verified that the drilling fluid cannot penetrate into the fracture when the fracture initiation occurs, but only after the fracture aperture becomes wide enough to allow the drilling fluid entry. This occurs at the moment of the borehole breakdown and after that, the stable fracture propagates repeating the fracture inflation and propagation. The typical sequence of the observations of Morita et al. (1990) can be seen in Fig. 2.11.

![Diagram showing fracture sequence](image)

Fig. 2.11. Typical Sequence of Fracture Extension for Water Base Mud (after Morita et al., 1990)

The equations with respective boundary conditions that Morita et al. (1990) proposed as solution of the borehole breakdown pressure for those fractures starting from a smooth borehole and from a borehole with mud invaded crack can be seen in Table 2.1. Also the solution of fracture aperture (width) is shown in this table.
Table 2.1. Solution for Borehole Breakdown Pressure (after Morita et al., 1990)

<table>
<thead>
<tr>
<th>Borehole Breakdown Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ( A = r_0 \Rightarrow P_r = -3G_2 = G_0 + \frac{K_r}{1212.5(n(A/r_0))} )</td>
</tr>
<tr>
<td>(2) ( 10 &lt; A &lt; 100r_0 \Rightarrow )</td>
</tr>
<tr>
<td>[ P_r = -\frac{K_r}{1212.5(n(A/r_0))} \left( \beta \sqrt{\frac{r_0}{r}} - \sqrt{\frac{r_0}{r}} \right) ]</td>
</tr>
</tbody>
</table>

where \( \beta = \frac{1 - \nu_s}{A} \) and \( \beta = \frac{1 - \nu_s}{A} \)

(3) For a large \( A/r_0 \), use \( F_1 = 1/2, F_2 = 1 \) and \( F_3 = 1 \).

For a small \( A/r_0 \), use the following table for estimating \( F_1, F_2 \) and \( F_3 \).

Equation of Fracture Aperture

\[ W = \frac{W_t + W_s + W_f + W_o}{E} \]

1. For hydrostatic boundary stress: \( W_t = \frac{4(1 - \nu_s)A\sigma_{th}}{E} \)
2. For directional stress: \( W_t = \frac{4(1 - \nu_s)A\sigma_{th}}{E} \)
3. For borehole pressure: \( W_t = \frac{4(1 - \nu_s)A\sigma_{th}}{E} \)

The values of \( G_1, G_2, \) and \( G_3 \) are given by:

<table>
<thead>
<tr>
<th>( A/r_0 )</th>
<th>( G_1 )</th>
<th>( G_2 )</th>
<th>( G_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>1.011</td>
<td>1.013</td>
<td>1.026</td>
</tr>
<tr>
<td>0.11</td>
<td>1.013</td>
<td>1.015</td>
<td>1.028</td>
</tr>
<tr>
<td>0.12</td>
<td>1.015</td>
<td>1.018</td>
<td>1.030</td>
</tr>
<tr>
<td>0.13</td>
<td>1.017</td>
<td>1.021</td>
<td>1.033</td>
</tr>
<tr>
<td>0.14</td>
<td>1.019</td>
<td>1.024</td>
<td>1.035</td>
</tr>
<tr>
<td>0.15</td>
<td>1.021</td>
<td>1.027</td>
<td>1.038</td>
</tr>
<tr>
<td>0.16</td>
<td>1.023</td>
<td>1.031</td>
<td>1.042</td>
</tr>
<tr>
<td>0.17</td>
<td>1.026</td>
<td>1.035</td>
<td>1.045</td>
</tr>
<tr>
<td>0.18</td>
<td>1.029</td>
<td>1.039</td>
<td>1.050</td>
</tr>
<tr>
<td>0.19</td>
<td>1.031</td>
<td>1.043</td>
<td>1.054</td>
</tr>
<tr>
<td>0.20</td>
<td>1.033</td>
<td>1.046</td>
<td>1.058</td>
</tr>
</tbody>
</table>

(4) A larger value than 10 \( r_0 \), \( G_2 = 3.0 \) and \( G_3 = 0.39 \)

(5) Breakdown Pressure from an Open Crack

\[ P_r = K_r + 1212.5(3G_2 = G_0 + \frac{K_r}{1212.5(n(A/r_0))} + S \]

where \( S = \frac{1}{\sqrt{\beta}} \frac{\beta \sqrt{r}}{x} \frac{A\sigma_{th}}{E} \)

(3) For a large \( A/r_0 \geq 3.0 \), use \( F_1 = 1/2, F_2 = 1 \) and \( F_3 = 1 \).

For a small \( A/r_0 \), use the previous table for estimating \( F_1, F_2 \) and \( F_3 \).

Equation of Fracture Aperture

\[ W = \frac{W_t + W_s + W_f + W_o}{E} \]

1. For hydrostatic boundary stress: \( W_t = \frac{4(1 - \nu_s)A\sigma_{th}}{E} \)
2. For directional stress: \( W_t = \frac{4(1 - \nu_s)A\sigma_{th}}{E} \)
3. For borehole pressure: \( W_t = \frac{4(1 - \nu_s)A\sigma_{th}}{E} \)

The values of \( H_1, H_2, \) and \( H_3 \) are given by a function of \( A/B \) and \( B/r_0 \). For example, when \( B/r_0 = 5 \), \( A/B = 5 \), \( H_1 = H_2 = H_3 = 1 \); and for \( B/r_0 = 0.1 \), \( H_1, H_2, \) and \( H_3 \) are given in the following table:

<table>
<thead>
<tr>
<th>( A/B )</th>
<th>( H_1 )</th>
<th>( H_2 )</th>
<th>( H_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>1.5</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>2</td>
<td>1.6</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>2.5</td>
<td>1.9</td>
<td>1.9</td>
<td>1.9</td>
</tr>
<tr>
<td>3</td>
<td>2.2</td>
<td>2.2</td>
<td>2.2</td>
</tr>
<tr>
<td>3.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>2.8</td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td>4.5</td>
<td>3.1</td>
<td>3.1</td>
<td>3.1</td>
</tr>
<tr>
<td>5</td>
<td>3.4</td>
<td>3.4</td>
<td>3.4</td>
</tr>
</tbody>
</table>

(b) Breakdown Pressure from an Open Crack

The study of fracture propagation has been done through fracture models that have been developed in the last forty years. Those models identified, theoretically, some of the factors that affect fracture propagation, and they are: (1) variations of
in-situ stresses existing in different layers of rock, (2) relative bed thickness of formations near the fracture, (3) bonding between formations, (4) variations in mechanical rock properties (including elastic modulus, Poisson's ratio, toughness, or ductility), (5) fluid pressure gradient inside the fracture, (6) variations in pore pressure from one zone to the next.

Many authors pointed out that the fracture orientation and vertical fracture growth are dominated by the local stresses and variation in the stresses between adjacent formations. The horizontal fractures have been reported at shallow depths, and experience has shown that at depths below 1,000 to 2,000 ft (305 to 610 m) fractures are usually vertical. Vertical fracture growth can be stopped or reduced by higher lateral stresses in the formations above and below the fracture initiation zone.

![Fig. 2.12. Theoretical Fracture Propagation Model vs. Possible Actual In-situ Behavior (after Veatch, 1983)](image)

Stress changes between rock layers influence fracture configuration. The models to predict fractures presume a rather simple fracture configuration, as shown in Fig. 2.12.a, but probably a more complicated configuration as in Fig. 2.12.b occurs in the fracture.
This section gives a brief description of fracture propagation models and works related to them for a vertical fracture that has been developed up to date, as well as the influence of the factors that affect fracture propagation in these models. It is divided into two parts: the two-dimensional models, and the three-dimensional models.

2.2.1. Two-Dimensional Models

During propagation fracture in a two-dimensional (2D) fracture models, height is assumed to be constant and the figures that change are width and length (or radius).

Two basic types of approaches are used in 2D fracture propagation simulators: one by Perkins and Kern (1961) and other by Geerstma and de Klerk (1969). The difference between the two models is that the Perkins and Kern's model assumes the cross section of fracture in the vertical plane has an elliptical configuration. On the other hand, Geerstma and de Klerk's model assumes an approximately elliptical configuration.
configuration in the horizontal plane and a rectangular shape in the vertical plane. These configurations can be seen in Fig. 2.13.

The rate of fluid leak-off during hydraulic fracturing propagation is one of the most critical factors involved in determining the fracture geometry for a given condition. The rate of fluid leak-off to the formation is governed by the fracturing fluid leak-off coefficient $C$. The leak-off coefficient $C$ is defined by a combination of three types of linear flow mechanisms encountered during a fracture process.

Howard and Fast (1957) developed the fundamental concept of fluid leak-off coefficient which can be used in calculating the fracture area. They showed that this coefficient can give an estimation of the loss of fluid to the formation adjacent to the fracture. The loss of fluid governs the fracture extent and depends upon three flow mechanisms. These are controlled by the viscosity and the permeability of the formation to the fracturing fluid, its wall building properties, or the combined effects of viscosity and compressibility of the reservoir fluid.

Carter (1957) derived one general equation for estimating the extent of the fractured area given by:

$$A(t) = \frac{q_1 W}{4 \pi C^2} \left[ e^{-\left(\frac{2C}{W}\right)^2 t} \text{erfc}\left(\frac{2C}{W} \sqrt{\frac{t}{W}}\right) + \frac{4C \sqrt{t}}{W} - 1\right]$$ ...........................................................(2.24)

under the assumptions of uniform width within the fracture, linear flow between fracture and formation, flow perpendicular to the fracture face, and constant pressure inside the fracture being equal to the wellbore injection pressure. In addition, the velocity of flow into the formation at a point on the fracture face depends on the time this point has been exposed to flow.

The fracturing fluid leak-off coefficient reflects the fracturing fluid properties and defines the three types of linear flow mechanisms encountered in the assumptions.
of Carter (1957). The three types are effluent viscosity and relative permeability effects \((C_v)\), reservoir fluid viscosity/compressibility effects \((C_c)\), and wall building effects \((C_w)\). The fluid leak-off test for viscosity and relative permeability effect is given by:

\[
C_v = \sqrt{\frac{k_i \Delta P \phi}{2 \mu_a}}
\]  

and for reservoir fluid compressibility effects by:

\[
C_c = \Delta P \sqrt{\frac{k_i \phi c_i}{\pi \mu_f}}
\]  

In many analyses, \(C_v\) and \(C_c\) are combined in the form of \(C_{vc}\) where:

\[
C_{vc} = \frac{2C_v C_c}{C_v + \sqrt{C_v^2 + 4C_c^2}}
\]  

The fracturing fluid coefficient for the third case must be determined experimentally through a plotting of cumulative filtrate volume versus the square root of flow time, as indicated by Howard and Fast (1970), and shown in Fig. 2.14.

The plot is correlated mathematically by:

\[
V = m \sqrt{t} + V_{sp}
\]

where the spurt loss volume per unit of area \(V_{sp}\) is determined in the intercept of the straight line on the cumulative filtrate volume axis in the plot, and \(m\) is the slope of the plot.
The coefficient $C_w$ is calculated by taking the derivative of the leak-off velocity with respect to time

$$\frac{dv}{dt} = \frac{d}{dt}(m\sqrt{t}) = \frac{m}{2\sqrt{t}} = \frac{C_w}{\sqrt{t}}\text{.........................................................(2.29)}$$

or

$$C_w = \frac{m}{2}\text{.........................................................(2.30)}$$

Several methods have been proposed for combining the three separate leak-off coefficients. The simplest method used today is to assume that the wall building coefficient dominates the other coefficients, and its value is used in the prediction of fracture geometry. Another is given by calculation of $C_w$ in equation (2.27) and by
comparing the result with $C_w$, the lesser of the two values is used in the fluid loss calculation.

Another procedure was done by Smith (1965) who combined the effect of the three cases and defined a total leak-off test coefficient by

$$C = \frac{1}{C_c} + \frac{1}{C_v} + \frac{1}{C_w}$$  \hspace{1cm} (2.31)

Williams (1970) gave another approach of combining the three mechanisms calculated at the stated $\Delta P$ to arrive at a total leak-off coefficient. In his approach, he assumed a $P^{0.5}$ relationship for the influence of pressure differential on $C_w$, and the final expression for the total leak-off coefficient is

$$C = \frac{2C_c C_v}{C_v C_w + \sqrt{C_w^2 C_v^2 + 4C_v^2 (C_v^2 + C_w^2)}}$$  \hspace{1cm} (2.32)

Although the leak-off test coefficient can be calculated through equations, several authors determined its value through experimental tests. In these tests they used different apparatus and methods to handle shear effects and approach the real conditions of fracture.

Roodhart (1985) proposed a method based in his experiment where he considered the leak-off process as a result of three stages: spurt loss, buildup of filter cake, and erosion of filter cake. The equation that is most appropriate for his observations is:

$$V = V_{sp} + 2C_w \sqrt{t_A} + C_d t_B$$  \hspace{1cm} (2.33)
In his proposed equation $t_A$ is the time at which fluid loss is proportional to $t^{0.5}$, $t_B$ is the time at which leak-off is proportional to $t$. In his observations he also pointed out that, although equation (2.33) describes better the fluid loss volume, little accuracy is lost by describing the leak-off volume by equation (2.28).

Penny et al. (1985) analyzed the dynamic fluid loss data with a plot of log (volume) and log(time) and stated the equation describing the volume of fluid lost to the formation takes the form:

$$V = V_p + m_l t^j$$

where the exponent of time $j$ is the slope and the leak-off rate $m_l$ is the y-intercept determined in the log-log plot.

Ford and Penny (1988) used one apparatus where they approached the conditions of the fracture and presented a method of converting dynamic data to an effective leak-off coefficient to be used in fracture models. The equation to convert dynamic data to an effective leak-off coefficient is given by:

$$C_{	ext{eff}} = \frac{V - V_p}{2\sqrt{j}}$$

where the value of $V$ can be determined by the method or equation that fits better the conditions of the fracture process.

The influence of differential pressure and shear rate on dynamic leak-off test can be seen in Fig. 2.15 for the data collected by Ford and Penny (1988) in their apparatus.

In a related analysis, Clark and Barkat (1990) proposed that the leak-off profile of a dynamic test can be modeled by:
\[ V = V_s p \left(1 - e^{-bt}\right) + v_D t \tag{2.36} \]

where \( b \) is the pack buildup constant which is a function of gel type and additives. The magnitude of \( b \) indicates the rate at which the filter cake starts to control the fluid loss.

\[ \text{Fluid: } 40 \text{ lb/1000 gal} \text{ HP Guar complexed with delayed titinate } \theta 175^\circ \]
\[ F; \text{Permeability : } 0.1 \text{ md}; \text{Shear Rate: } 40 \text{ sec}^{-1} \]

\[ \text{Fluid: } 40 \text{ lb/1000 gal} \text{ HP Guar complexed with delayed titinate } \theta 175^\circ \]
\[ F; \text{Permeability : } 0.1 \text{ md}; \text{Shear Rate: } 102 \text{ sec}^{-1} \]

**Fig. 2.15. Effect of \( \Delta P \) on Leak-off Coefficient (after Ford and Penny, 1988)**

In using their experimental data, Clark and Barkat also observed that dynamically obtained fluid loss coefficients are proportional to an average \( P^{0.25} \) instead of \( P^{0.5} \).

Calculations of leak-off volume by using equations (2.28), (2.33), (2.34) and (2.36) vary by a factor of 2, and it is not clear which method is more adequate. It is first necessary to determine which method of calculation is most appropriate for the experimental data for a specific fluid used in the fracture process.

Perkins and Kern (1961) developed a model to predict the width of fracture where they showed that the crack width is essentially controlled by a fluid pressure drop in the fracture. In that work, they discussed the cases of vertical and horizontal fractures, but in this review the discussion will be restricted for vertical fractures using Newtonian and non-Newtonian flow in laminar and turbulent flow regime. In
the derivation of the width equations they did not consider the leak-off of the injected fluid, and they assumed the fracture is vertical and has constant height. They also assumed the cross-sectional shape of the crack at any point is elliptical, as shown in Fig. 2.13, with the maximum width proportional to the difference between pressure and stress at that point. The equations developed by Perkins and Kern (1961) are given by:

(a) Vertical Fracture Width Using Newtonian Fluid in Laminar Flow:

\[ W = 3.005 \left( \frac{q \mu L(1 - v^2)}{E} \right)^{0.25} \]  \hspace{1cm} (2.37)

(b) Vertical Fracture Width Using Newtonian Fluid in Turbulent Flow:

\[ W = 1.463 \left( \frac{f F_c \rho q^2 L(1 - v^2)}{E h_f} \right)^{0.25} \]  \hspace{1cm} (2.38)

(c) Vertical Fracture Width Using Power-Law Fluid in Laminar Flow:

\[ W = \left[ 13.5812(n + 1)^{\frac{2n+1}{n}} \left( \frac{2n+1}{n} \right)^n \left( 1 - v^2 \right) \right]^{\frac{1}{2n+2}} \left[ \frac{K q^n L h_f^{1+n}}{E} \right]^{\frac{1}{2n+2}} \]  \hspace{1cm} (2.39)

(d) Vertical Fracture Width Using Power-Law Fluid in Turbulent Flow:

\[ W = 1.463 \left( \frac{f F_c \rho q^2 L(1 - v^2)}{E h_f} \right)^{0.25} \]  \hspace{1cm} (2.40)

The prediction of the fracture geometry using Carter's equation for fracture area with Perkins and Kern's equations for fracture width became rapidly one of most
used approaches due to the good results the method presented for some fracturing conditions.

As Perkins and Kern (1961) did not consider fluid losses into the formation, Nordgren (1972) remedied this shortcoming by considering the flow of the fluid in the fracture modeled by the continuity equation:

$$\frac{\partial q}{\partial x} + \frac{\pi h_f}{4} \frac{\partial q}{\partial t} + q_t = 0$$

(2.41)

In this equation, the fluid loss rate is given by:

$$q_t = \frac{2h_f C}{\sqrt{1 - \tau(x)}}$$

(2.42)

and the flow rate being related to the pressure gradient by:

$$q = -\frac{\pi W^3 h_f}{64\mu} \frac{\partial (P - \sigma)}{\partial x}$$

(2.43)

Nordgren (1972) considered the problem subjected to initial condition:

$$W(x, 0) = 0$$

(2.44)

and to boundary conditions:

$$W(x, t) = 0 \text{ for } x > L$$

(2.45)

$$- \left( \frac{\partial W^4}{\partial x} \right)_{x=0} = \frac{256(1 - \nu) \mu q_t}{\pi G}$$

(2.46)
The approximate solutions found by Nordgreen (1972) for large fluid loss were:

\[ L(t) = \frac{q_it^{0.5}}{\pi Ch_f} \]  \hspace{1cm} (2.47)

and

\[ W(0,t) = 4 (\frac{2(1-v)\mu q_i}{\pi^2 GCh_f})^{0.2} t^{0.125} \]  \hspace{1cm} (2.48)

and for small time or no-fluid loss:

\[ L(t) = 0.68 \left( \frac{Gq_i^3}{(1-v)\mu h_i^4} \right)^{0.2} t^{0.8} \]  \hspace{1cm} (2.49)

and

\[ W(0,t) = 2.5 \left( \frac{(1-v)\mu q_i^2}{Gh_f} \right)^{0.2} t^{0.2} \]  \hspace{1cm} (2.50)

Geertsm a and de Klerk (1969) followed the design method proposed by Zheltov and Khristianovitch (1955) in which the width is assumed stable in the vertical direction. From this they derived equations to predict the width fracture for radial and linear flow considering the effect of low fluid loss. The derivation assumed a homogeneous and isotropic formation, the deformation of the formation could be calculated using a linear elastic stress-strain relation, the fracturing fluid behaved like
a purely viscous liquid, the fluid flow was laminar everywhere, and the fracture had a simple geometric extension pattern. Their solutions for width and length in the case of vertical fracture are then:

\[ W(0, t) = 2.27 \left( \frac{(1 - v) \mu q L^2}{G h_f} \right)^{0.25} \]  

...(2.51)

and

\[ L = \frac{q}{32 \pi h_f C^2} \left( \pi W(0, t) + 8 V_{sp} \right) \left( \frac{2 \alpha_L}{\sqrt{\pi}} - 1 + e^{\alpha_L^2 e^{t/\alpha_L^2}} \right) \]  

...(2.52)

where:

\[ \alpha_L = \frac{8C\sqrt{\pi t}}{\pi W(0, t) + 8V_{sp}} \]  

...(2.53)

The Lagrangian variational method of Biot et al. (1986) can be used to find an analytic solution of Geerstma and de Klerk's model incorporating large fluid losses. In this case the expression for width is given by:

\[ W(0, t) = 1.58 \left( \frac{(1 - v) \mu q^3 F_1}{G h_j^2 C} \right)^{0.2} t^{0.3} \]  

...(2.54)

and the relation between length and width by:

\[ W(0, t) = 3.13 \left( \frac{(1 - v) \mu C^2 F_2}{G} \right)^{0.2} L^{0.6} \]  

...(2.55)
As stated before, these two methods are the base of fracture prediction in the two dimensional models. The main difference between Perkins and Kern's method and Geertasma and de Klerk's method is related to the potential for a bedding plane slip. The first one assumes that there is sufficient bonding between the fractured formation and the adjacent layers that the fracture closes at the tips. Also, because high horizontal stresses at the top and bottom layers tend to close the fracture, the height growth of vertical fractures into the adjacent layers is limited. Another difference between the two methods is the propagation pressure calculated.

Nolte (1988) using approaches similar to Nordgren (1972), observed that the Newtonian flow equation that relates fracture pressure to injection rate and fluid viscosity yields:

\[
P \propto \frac{(E'\mu qL)^{0.25}}{h_f}
\]

for the Perkins and Kern's model and:

\[
P \propto \frac{(E'\mu q)^{0.25}}{h_f^{0.25}L^{0.5}}
\]

for Geertasma and de Klerk's model. Thus, Perkins and Kern's model predicts that wellbore fracturing pressure increases proportionally with fracture length raised to

\[
F_i = \int_0^1 \frac{1 - \frac{2 \arcsin \lambda}{\pi}}{(1 - \lambda^2)^{3/2}} d\lambda = 0.75
\]

(2.56)
one fourth power, and the Geerstma and de Klerk model indicates that pressure decreases proportionally to fracture length raised to one half power.

Daneshy (1973) using the same approach of Khristianovich and Zhelthov developed a numerical method to calculate the width of a fracture with a non-constant pressure distribution inside of the fracture. The assumptions in his work were: (a) homogeneous, isotropic, and linearly elastic reservoir rock; (b) constant fracture height; (c) constant fracture geometry along the height of the fracture; (d) incompressible fracturing fluid with power-law behavior; (e) laminar flow inside the fracture; (f) negligible fracture width variations due to the stresses induced by fluid leak-off; and (g) negligible pressure drop in the fracture due to the loss of fluid.

Geerstma and Haafkens (1979) compared the assumptions of vertical fracture propagation theories, namely, Perkins and Kern (1961), Geerstma and de Klerk (1969), Nordgren (1972) and Daneshy (1973). The comparison among theories was made theoretically by using expressions of width and length for each model and, in the case of Daneshy, using his data. The results for Newtonian fluids with different viscosities are shown in Fig. 2.16.

The calculations with non-Newtonian fluid in the Nordgren's and in the Geerstma and de Klerk's models were made by replacing the viscosity by its apparent value, once those theories consider only Newtonian fluids. This substitution is only permitted for the cases where the ratio $q/W$ is constant over the fracture length.

The conclusions of this comparison showed that there are two different kinds of mechanical formulations on the fracture propagation process, one that Perkins and Kern's model, and the other that follows Geerstma and de Klerk's model. In addition, they suggested that Perkins and Kern's model is more appropriate for length/height ratios bigger than one, while the other model is more appropriate for ratios less than one.
Fracture data: pump rate=10 bbl/min, height=100 ft, fluid loss coef=0.0015 ft/min^{0.5}, spurt loss=0.01 gal/ft^2, Poisson's ratio=0.15, shear modulus=2.6x10^6

Fig. 2.16. Fracture Dimensions Calculated with Different Theories for Newtonian Fluids (after Geerstma and Haafkens, 1979)
Fig. 2.17. Ratio of Model to Actual Width for Constant Pressure (after Nolte and Economides, 1991)

Fig. 2.17 shows their conclusions where the most appropriate model is selected for the ratio of widths (predicted to actual) equal to unity. This leads to selection of Perkins and Kern's model for relatively long fractures, and Geerstma and de Klerk's model for relatively short fractures.

Smith et al. (1982) presented measurements made with a downhole closed-circuit television camera during a fracture stimulation of a relatively shallow 4,500 ft (1,371.6 m) oil bearing sandstone formation to investigate the applicability of the two fracturing models. Using this technique, the fracture width measurements could be made only at the wellbore.

Their conclusions showed the fracture geometry assumption of Perkins and Kern (1961) is appropriate even for small volumes injected in fracture tests, so it can be used for length/height ratios less than one.

Warpinski (1985) made some measurements of width and pressure in a propagating hydraulic fracture in tests conducted in Nevada. This was accomplished
by creating a fracture at 1400 ft (425 m) in a tunnel where realistic in-situ conditions prevail such as natural fractures and material anisotropy.

He concluded, based on his data, that the pressure drop along the fracture length was much larger than predicted by the viscous theory used in the models. This fact results in shorter and wider fractures than predicted by the models, and it should be taken into account when choosing a model for the prediction of a fracture.

Warpinski (1985) also concluded the theory of Perkins and Kern (1961) agrees with the data better, particularly regarding pressure behavior. His experiments showed, in case of high stress regions above and below the fracture zone, that the stress contrasts on hydraulic fractures cause a nearly rectangular shaped fracture for low viscosity, low flow rate test.

Two dimensional design models require a fracture height so that width and length can be calculated with volume and flow considerations. This is one of the biggest problems in these models. Several factors have been identified that contribute to the containment of hydraulic fractures, among them in-situ stress differences, Young's modulus, fracture toughness, and interface slippage.

Perkins and Kern (1961) stated that stress differences between the pay zone and the bounding layers affect fracture containment or restriction. Their intuition has been supported by theoretical, laboratory, and field data, which show that the stress differences are the most important factor controlling fracture height. Stress not only controls or influences most aspects of fracture behavior, but also influences the values of both reservoir and mechanical properties of the rock.

Warpinski et al. (1982) conducted an experiment with twenty separate fracture tests in a tunnel showing the dominant effect of the in-situ stress contrasts as opposed to rock properties. In this experiment fractures propagated upward into hard, high-modulus materials, but they did not propagate downward through a thin, high stress layer.
Warpinski and Smith (1989) presented two equations that can be solved for fracture height assuming that material property variations are negligible and vertical pressure distribution in the hydraulic fracture is constant. The equations, referring to the geometry in Fig. 2.18, for a layered stress medium are given by:

\[
\frac{\sqrt{\pi(K_{\text{leap}} + K_{\text{lebottm}})}}{2\sqrt{a}} = (\sigma_2 - \sigma_1)\sin^{-1}\left(\frac{b_1}{a}\right) + (\sigma_3 - \sigma_1)\sin^{-1}\left(\frac{b_2}{a}\right) - \frac{\pi(\sigma_2 + \sigma_3 - 2P)}{2}
\]

\[
\frac{\sqrt{\pi(K_{\text{lebottm}} - K_{\text{leap}})}}{2} = (\sigma_2 - \sigma_1)\sqrt{a^2 - b_2^2} - (\sigma_3 - \sigma_1)\sqrt{a^2 - b_1^2}
\]

The solution for the fracture half height, \(a\), is found in a trial and error procedure by assuming an initial value for \(a\), calculating \(b_2\) through equation (2.60) and finally the pressure in equation (2.59). This value has to be within an acceptable range when compared with the fracture propagation pressure, and if it is not, another
value for $a$ must be assumed until the calculated value of pressure is within the acceptable range.

Table 2.2 gives some values for fracture toughness in several rock types, but a value representative of many sedimentary rocks is 1,000 psi-in$^{0.5}$ (1.1 MPa-m$^{0.5}$).

Table 2.2. Fracture Toughness of Rock (after Warpinski and Smith, 1989)

<table>
<thead>
<tr>
<th>Formation</th>
<th>$K_{ic}$ (psi-in$^{0.5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cozzette sandstone</td>
<td>1,430</td>
</tr>
<tr>
<td>Mesaverde fluvial sandstones</td>
<td>1,230</td>
</tr>
<tr>
<td>Mancos shale</td>
<td>1,300</td>
</tr>
<tr>
<td>Indiana limestone</td>
<td>845</td>
</tr>
<tr>
<td>Westerly granite</td>
<td>2,365</td>
</tr>
<tr>
<td>Devonian shale</td>
<td>750 to 1,200</td>
</tr>
<tr>
<td>Green River oil shale</td>
<td>730 to 1,000</td>
</tr>
<tr>
<td>Benson sand</td>
<td>1,440 to 1,580</td>
</tr>
<tr>
<td>Benson shale</td>
<td>530</td>
</tr>
</tbody>
</table>

Fig. 2.19 shows an example calculation for a nonsymmetrical case, a plot of fracture height as a function of the treatment pressure above the closure stress. With the large confining stress in the barrier zone, the fracture is restricted to a narrow height for pressures less than 600 psi (4.1 MPa). For greater pressures, the top of the fracture grows excessively and becomes unbounded at 800 psi (5.5 MPa) when the treating pressure is equal to the stress difference.

Nolte (1988), in a study about fracturing pressure analysis, illustrated the evolution of the geometry and pressure during the fracture process. In this work, besides PKN and KGD models, he referred to a radial model. The radial model has a circular shape and its propagation is in the radial direction. This model is applied where the penetration is less than the formation thickness, and as the Geerstma and de Klerk model, it predicts a decreasing pressure in the propagation fracture. In our case, it will just serve as an illustration of the fracture process because this model is more...
applicable where the fluid injection is restricted essentially to a point source as in the case of a limited perforated section in a cased wellbore.

![Diagram](image.png)

**Fig. 2.19. Example Height Calculation (after Warpinski and Smith, 1989)**

Fig. 2.20 illustrates the characteristic changes in pressure for idealized vertical fracture geometries, as pointed out by Nolte (1988).

Referring to Fig. 2.20, cases A-I (point source) and A-II (line source) show a decreasing pressure with fracture propagation, being according to the radial and KGD models. For both cases, the fracture is permitted to evolve into its preferred shape of a circle and is not influenced by any restriction, such as a higher stress barrier. For the line source case, as it occurs in an uncased wellbore, the early stages of the evolution would be consistent with the assumption for the KGD model, as indicated by case A-II.

After the radial model encounters barriers above and below the fractured formation the pressure propagation starts increasing in a manner similar to the Perkins and Kern's model. The reason is that the barriers force the fracture from its preferred circular shape and this results in increasing pressure as the fracture becomes...
long relative to its vertical height, as shown by part C in Fig. 2.20. The net pressure is defined as the total fluid pressure minus the closure pressure. As the net pressure continues to increase and exceeds 0.8 of the stress difference, significant height growth occurs with small pressure increases. It ultimately reaches a limiting value of 0.9 of the stress difference which is equivalent a total pressure slightly below the stress of the barrier being penetrated. This condition is shown with the profile D in Fig. 2.20. If the injection continues with significant vertical growth, the fracture can reach a formation of lower stress and in this case, due to the unrestricted vertical growth, the pressure will decrease in a manner similar to the profile indicated with letter A in Fig. 2.20.

Fig. 2.20. Evolution of Fracture Geometry and Pressure (after Nolte, 1988)

Thus, based on Nolte's observations, the fracture process contains three phases: the first one in which decreasing net pressures during the initial period indicates an unrestricted growth of a radially evolving fracture in the vertical plane. The second one in which increasing net pressures after the initial growth period is indicative of a vertical fracture extending in length with restricted height growth. Finally, in the third phase, pressures begin to approach a phase of a nearly constant value.
probably due to a significant height growth through a stress barrier. In some cases, where it is possible that the occurrence of a lower stress zone below and above the barriers, an unrestricted vertical growth into a lower stress zone can occur with subsequent decreasing pressure.

The first phase does not last very long and can be considered negligible in the fracture process where the fracture length is much larger than the fracture height.

Besides the growth through a stress barrier, Nolte and Smith (1981) pointed both the opening of secondary fractures and the formation of a companion horizontal fracture as other causes for a phase of constant pressure.

Secondary fracturing which has a higher permeability than the formation matrix occurs in natural fissures or cracks within the formation that are crossed by the primary fracture. Consequently, the fracture fluid penetrates into the cracks with a pressure nearly equal to the pressure in the fracture and causes an opening in the cracks that start to act as a propagating fracture when the fluid exceeds the rock stress acting across them. The process acts similarly to a pressure regulator for this excess, and can thieve a significant portion of the injected fracture fluid. Nolte and Smith (1981) defined the net pressure that opens the pressure regulating cracks as:

\[ P_n = P - P_e = P - \sigma_h = \frac{\Delta \sigma}{1 - 2v} \]

where \( \Delta \sigma \) is the difference in horizontal stresses for a vertical fracture. On the other hand, when the pressure for a vertical fracture exceeds overburden pressure a horizontal fracture is also achieved.

In all of these three conditions a fraction of the injected fluid becomes unavailable for the horizontal extension of the fracture. Nolte and Smith (1981), based on this fact, defined the formation pressure capacity as a limiting pressure value where a loss of integrity and undesirable fluid loss can occur if this value is
exceeded. Fig. 2.21 shows the rock stresses that define the pressure capacity and their requirements to confine a net pressure efficiently. In the case of a vertical fracture, the fluid is lost through the top or bottom when the capacity of the stress barrier or overburden is exceeded and through the side when the formation capacity to maintain closed fissures is exceeded.

![Diagram of fracture-pressure capacity from in-situ stresses](image)

Fig. 2.21. Definition of Fracture-Pressure Capacity from In-Situ Stresses (after Nolte and Smith, 1981)

Nolte (1988), with field data for massive treatments in tight gas fields using crosslinked polymer fluids, verified five types of interpretative slopes in a log-log plot of net pressure versus time. Fig. 2.22 shows the resultant plot and his interpretation.

The five types verified by Nolte (1988) can be described as five periods. The first one where the type I slope indicates extension with restricted height. It is followed by the type II slope, a nearly constant pressure (formation pressure capacity) at constant injection conditions that indicates an accelerated fluid loss to opening natural fissures or stable height growth through a stress barrier. The slope III-a is
interpreted to indicate a restriction to penetration caused by proppant bridging and occurred after proppant was introduced, and the slope III-b is indicative that half of the prior fracture area became restricted to flow. Slope III-b could also have resulted from one wing of the fracture becoming blocked to flow. Finally, the slope IV indicates unstable vertical growth through a lower stressed formation.

![Log-log Slope Interpretation](image)

<table>
<thead>
<tr>
<th>Approximate Log - Log Slope</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/8 - 1/4</td>
<td>Restricted Height and Unrestricted Extension</td>
</tr>
<tr>
<td>0</td>
<td>a) Stable Height Growth</td>
</tr>
<tr>
<td></td>
<td>b) Fissure Opening</td>
</tr>
<tr>
<td>1-1 (unit)</td>
<td>Restricted Extension - two active wings</td>
</tr>
<tr>
<td>2-1 (double)</td>
<td>Restricted Extension - one active wing</td>
</tr>
<tr>
<td>negative</td>
<td>IV Unstable Height Growth - run away</td>
</tr>
</tbody>
</table>

**Fig. 2.22. Log-log Slope Interpretation (after Nolte, 1988)**

Morita et al. (1990) verified in an experiment the bridging effect that occurred in the fracture process when using mud as the fracturing fluid. Their data shows that the pressure profile is similar to the profile described by Nolte (1988), but in this case, it alternates increases and decreases in the fracture pressure during the propagation. The increase is due to the fracture inflation and the decrease is due to the sudden movement of fluid front. This effect can be seen as the pressure peaks in

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Fig. 2.23 after 100 cc of mud injection, but the pressure trend still followed Nolte's observation.

![Fracture Initiation and Extension Pressures from a Borehole](image)

Palmer and Veatch (1990) presented a theory to describe the increase in wellbore pressure based on a Perkins-Kern geometry assuming the limit of large leak-off coefficient and time in a step rate test. They also chose the Perkins and Kern's model because net wellbore pressure rises in many of the step rate tests and the length/height ratio was greater than 4 in these tests.

They verified an observed/theoretical pressure discrepancy of 1.7 to 8.9 and they concluded the reason was high friction losses in constrictions within a fracture. These are approximately the same conclusions of Warpinski (1985) concerning friction losses within a fracture.

Valkó and Economides (1993) introduced continuum damage mechanics as an extension to current hydraulic fracture propagation as a way to resolve...
inconsistencies in the two dimensional models. The fluid-flow-constrained tip propagation boundary condition of the Perkins-Kern-Nordgren (PKN) model was replaced by a new one derived from continuum damage mechanics. In this model they introduced two new parameters: the dimensionless distance between the microcracks $l_d$, and the Kachanov parameter $C_k$ which is similar to a reciprocal viscosity in hydraulics. This new theory presented good results in their cases, but the two new parameters are somewhat difficult to determine in the formation being fractured.

2.2.2. Three-Dimensional Models

The 3-D mathematical models were developed to describe the fracturing process allowing the fracture height to vary with fluid injection including the vertical components of fluid flow. They were also developed to account for widely varying rock properties, reservoir properties, in-situ stresses, fracturing fluids and proppant loads.

Warpinski et al. (1994) divided the 3-D models into two categories: planar 3-D models and planar 3-D finite difference models.

The planar 3-D models, where the TerraFrac and the HyFrac3D models are included, assume planar fractures of arbitrary shape in a linearly elastic formation, 2-D flow in the fracture, power law fluids, and linear fracture mechanics for fracture propagation. The difference between the TerraFrac and the HyFrac3D models is in the numerical technique used in each model. The TerraFrac uses an integral equation representation, while the other uses the finite element method. Both models use finite elements for 2-D fluid flow within the fracture and a fracture tip advancement proportional to the stress intensity factor on the fracture tip contour. The stress intensity factor quantifies the intensity of the stress singularity at a crack tip. These models assume the fracture advances when the stress intensity factor reaches a critical value equal to the plane-strain fracture toughness of rock.
The planar 3-D finite difference model uses a finite difference method. The fracture opening is calculated by superposition using the surface displacement of a half space under normal load and it assumes the fracture propagates when the tensile stress normal to the fracturing plane exceeds the tensile strength of the formation at some distance outside the fracture.

One overall approach in a 3-D planar model is to subdivide the fracture into discrete elements and to solve the governing equations which are constituted by elasticity equations, fluid flow equations, and a fracture criterion.

The elasticity equations in these models assume the formation behaves as a linear elastic solid in response to changes in crack-face pressures introduced by hydraulic fracturing. Therefore, poroelasticity effects are neglected and isotropic elasticity is assumed. It is also assumed that the formation is infinite in extent, and that the fracture develops as a plane with the vertical fracture oriented perpendicular to the direction of the minimum in-situ compressive stress.

The elasticity problem solved is that of the change in stress and displacement fields caused by increasing the normal compressive stress $\sigma_z(x,y,0)$ on crack surface from its initial values $\sigma_z^0(x,y,0)$ to the current pressure $P(x,y,t)$ in the fracturing fluid. Shear traction on the crack surface is assumed to be zero. By using a surface integral formulation the change in normal stress on the crack plane is related to the crack opening by:

$$\Delta P(x,y) = P(x,y) - \sigma_z^0(x,y,0) = E_e \int_A \bar{\nabla} \cdot w \cdot \frac{1}{R} dA'$$

where $E_e$, the effective elastic modulus, is:

$$E_e = \frac{G}{4\pi(1-v)}$$
\[ \nabla', \text{the gradient operator, is:} \]

\[ \nabla' \equiv \frac{\partial}{\partial x'} \mathbf{i} + \frac{\partial}{\partial y'} \mathbf{j} \] \hspace{1cm} (2.64)

and the distance between point \((x', y')\) at which the integrand is evaluated and the point \((x, y)\) at which the pressure is evaluated is given by

\[ R = \left( (x - x')^2 - (y - y')^2 \right)^{\frac{1}{2}} \] \hspace{1cm} (2.65)

The fluid flow is idealized as laminar flow of an incompressible, power law fluid in 2-D flow. The fluid is assumed to flow between parallel porous walls. Leak-off through the walls occurs at a rate determined by the difference between the pressure in the fracturing fluid and the pore fluid pressure, and by the time the fracturing process started. The governing equations which are solved by a finite element method are constituted by the continuity equation:

\[ \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = -q_t - \frac{\partial w}{\partial t} + q_i \] \hspace{1cm} (2.66)

and by the pressure-gradient equations:

\[ \frac{\partial p}{\partial x} + \eta \left( \left[ \frac{q_x}{w^2} \right]^{n-1} \right) \frac{q_x}{w^3} = 0 \] \hspace{1cm} (2.67)

\[ \frac{\partial p}{\partial y} + \eta \left( \left[ \frac{q_y}{w^2} \right]^{n-1} \right) \frac{q_y}{w^3} = \rho F_x \] \hspace{1cm} (2.68)
where \( \rho F \) is the body force caused by the weight of the fluid per unit of volume.

The viscosity parameter \( \eta' \) is related to the power law coefficients by:

\[
\eta' = 2^{(\alpha+1)} K \left( 2 + \frac{1}{n} \right)^n
\]  

(2.69)

The fluid loss term, \( q_L \), is obtained from a time-dependent leak-off relation of the form:

\[
q_L(x, y, t) = \frac{2C_L(P - P_f)}{\sqrt{t - \tau(x, y)}}
\]  

(2.70)

where the coefficient \( C_L \) is the leak-off coefficient normalized with respect to the stress state at depth by dividing by the difference between the minimum in-situ compressive stress and the in-situ pore fluid pressure:

\[
C_L = \frac{C}{\sigma_z(0, 0) - P_f}
\]  

(2.71)

The fracture criterion of linear elastic fracture mechanics controls the advance of the crack by imposing the stress-intensity factor which is kept nearly equal to the critical stress-intensity factor during crack extension at each node. The condition for crack advance can be expressed as:

\[
w_a(s) < w_c
\]  

(2.72)

for no crack advance and:
\[ w_a(s) > w_c \] ...........................................(2.73)

for crack advance, where:

\[ w_c = \frac{2(1 - v)K_r}{G} \left( \frac{2a(s)}{\pi} \right)^{0.5} \] ...........................................(2.74)

where \( w_a \) is the crack opening at distance \( a \) from the crack front, \( w_c \) is the critical crack opening required for crack advance, and \( a(s) \) is the width of near-crack-tip zone as can be seen in Fig. 2.24.

\( K_r \) is a measure of the intensity of the elastic stress field near the crack tip that is required for crack extension and as indicated before, fracture toughness experiments on rocks indicate that \( K_r \) is about 1,000 psi-in\(^{0.5}\) for many competent rocks. Characteristic values for reservoirs may be different from the laboratory values. There is also evidence that the propagation of cracks over long distances can increase the value of \( K_r \), and this fact, besides other factors, can also explain the observed pressures at the wellbore are often larger than the predicted pressures in the fracture models.

In this 3-D model, the value of the normal velocity at each node along the crack front is assumed in such a way that the crack opening at a distance \( a \) from the crack front remains at the critical value \( w_c \).

This process can be achieved only iteratively because it is necessary to know the crack opening at time \( t_{n+1} \), and this is extremely time consuming.

Another approach that can be done is to estimate the velocity at each time step and to solve the equations (2.62) through (2.71) for the pressure and crack opening at \( t_{n+1} \) for the estimated change in the crack front location during the time step.
During this time step, the crack advances a distance for which the volume in an annular element near the crack front becomes equal to the volume corresponding to a crack opening profile with $w_o(s) = w_c$.

The normal velocity of a boundary segment is computed from a volume conservation equation for the annular element by

$$v = \frac{(\bar{q}_n - 2C_L(d\bar{P} - \bar{P}_o))\Delta s - \frac{dV}{dt}}{(\bar{w}_d + 2V_{ext})\Delta t} \quad \text{(2.75)}$$

This equation assumes that $d$ is the width of the annular element equal to the sum of the width of the near crack tip zone and the annulus. It also depends on $dV/dt$ being the required rate of change for volume in the annular element if the volume after crack advance is to be equal to the volume corresponding to a crack.
opening $w_a(s) = w_c$. Overbars indicate quantities averaged over the length $\Delta s$ of the annular element.

After computing the crack velocity, the advance of each node is computed by multiplying the velocity by the time increment.

The global solution in this 3-D model is obtained at time $t_{n+1} = t_n + \Delta t$ by solving equations (2.62) through (2.71) from the solution at time $t_n$, the injection flow rate and the crack advance.

2.2.3. Pseudo-Three-Dimensional Models

These models were developed from the PKN model by removing the requirement of constant fracture height, and applying a fracture propagation criterion to height. It presents better results than a 2D model and in appropriate cases yields the same results as a 3D model with the convenience of much less computer time to run this kind of program.

Pseudo-3D models can be divided into two categories: (1) models that divide the fracture along its length into a finite number of cells and use local cell geometry to relate fracture opening with fluid pressure and (2) models that use a parametric representation of the total fracture geometry.

The first category is the only one to be dealt with in this review. In this category, formation elasticity is approximated by assuming the fracture length is large compared to height that the effective elastic stiffness (the relationship between the pressure and the crack opening), at all cross sections, is independent of the crack length and the horizontal distance from the cross section to the crack front. The crack opening (fracture width) is obtained from the plane strain elasticity solution for the case of homogeneous, isotropic, linear elasticity by:

$$w(x, y) = \int_{-a}^{a} \Delta P(x, y_0 + y) \left(\frac{1-v}{\pi G}\right) \ln(R_c(y, y')) dy' \tag{2.76}$$
where:

\[
R_y(y, y') = \frac{(a + y')^{0.5}(a + y_0 - y)^{0.5} + (a - y')^{0.5}(a - y_0 + y)^{0.5}}{(a + y')^{0.5}(a + y_0 - y)^{0.5} - (a - y')^{0.5}(a - y_0 + y)^{0.5}}
\] (2.77)

and

\[
\Delta P(x, y) = P(x) - g_p y - \sigma(y) + g_o y
\] (2.78)

The elevation \( y \) is measured from the midheight of the pay zone; the elevation \( y' \) is measured from the midheight of the fracture, and \( y_0 \) is the location of the midheight of the fracture relative to the midheight of the pay zone.

Assuming the in-situ stress normal to the crack plane and the pressure in the fracture are constant, which is appropriate when the vertical extension of the crack is slow relative to the horizontal extension, it is possible to apply explicit integration of equation (2.76). Referring to Fig. 2.18, for the case of uniform in-situ stress within each layer, the integration results in:

\[
w_T(x, y) = w_I - w_{II} - w_{III} + w_{IV} - w_V
\] (2.79)

where:

\[
w_I = \frac{4(P - \sigma_1)}{E'} \sqrt{a^2 - y^2}
\] (2.80)

\[
w_{II} = \frac{4(\sigma_2 - \sigma_1)}{E' \pi} \left( -(b_2 - y) \cosh^{-1} \left( \frac{a^2 - b_2 y}{a |y - b_2|} \right) + \cos^{-1} \left( \frac{b_2}{a} \right) \sqrt{a^2 - y^2} \right)
\] (2.81)
\[ w_{tt} = \frac{4(\sigma_3 - \sigma_1)}{E\pi} \left( -(b_3 + y)\cosh^{-1}\left( \frac{a^2 + b_3 y}{a|y + b_3|} \right) + \cos^{-1}\left( \frac{b_3}{a} \right) \sqrt{a^2 - y^2} \right) \] ...........(2.82)

\[ w_{tr} = \frac{2}{E} g_{yr} \sqrt{a^2 - y^2} \] ...........(2.83)

\[ w_r = \frac{2}{E} g_{yr} \sqrt{a^2 - y^2} \] ...........(2.84)

and

\[ E' = \frac{E}{(1 - \nu^2)} \] ...........(2.85)

The height of the fracture is determined from the stress intensity factor at the top of the crack through the equation:

\[ K_{hop} = \frac{1}{\sqrt{\pi a}} \int_a^x \Delta P(x, y) \sqrt{a + y} dy \] ...........(2.86)

For the case of uniform in-situ stress within each layer the integration results in:

\[ K_{le} = K_{l1} - K_{l2} - K_{l3} + K_{l4} - K_{l5} \] ...........(2.87)

where:

\[ K_{l1} = (P - \sigma_1) \sqrt{\pi a} \] ...........(2.88)
\[ K_{12} = \frac{(\sigma_2 - \sigma_1)\sqrt{a}}{\sqrt{\pi}} \left( \cos^{-1}\left(\frac{b_3}{a}\right) + f_a \frac{\sqrt{a^2 - b_3^2}}{a} \right) \]  \hspace{1cm} (2.89)

\[ K_{13} = \frac{(\sigma_3 - \sigma_1)\sqrt{a}}{\sqrt{\pi}} \left( \cos^{-1}\left(\frac{b_3}{a}\right) - f_a \frac{\sqrt{a^2 - b_3^2}}{a} \right) \]  \hspace{1cm} (2.90)

\[ K_{14} = \frac{a}{2} f_a g_0 \sqrt{\pi a} \]  \hspace{1cm} (2.91)

\[ K_{15} = \frac{a}{2} f_a g_0 \sqrt{\pi a} \]  \hspace{1cm} (2.92)

where \( f_a = +1 \) for upper or \(-1\) for lower fracture tip of a vertical section.

Therefore the final solution of equation (2.87) presents two equations, one for the upper and other for the lower tip. These two equations together with an additional geometry constraint of:

\[ b_3 = h_f - b_2 \]  \hspace{1cm} (2.93)

give the solution for \( a \), \( b_2 \), and \( b_3 \).

In a pseudo-3D model the fracture height cannot be less than the fractured formation thickness. When this happens, the value of the fracture height is replaced by the formation thickness because such a situation is interpreted as the crack not propagating into the boundary layers.

The fluid flow in a pseudo-3D model is considered as being a one dimensional flow along the fracture length. The governing equations are basically the continuity equation:
\[-\frac{\partial Q(x,t)}{\partial x} = Q_L(x,t) + \frac{\partial A_{\varphi}(x,t)}{\partial t}\] ......................................................(2.94)

and the pressure gradient equation:

\[\frac{\partial P}{\partial x} = \frac{\eta'(Q(x,t))\eta}{\left(\int_{-\infty}^{\infty} \frac{2n+1}{n} \frac{1}{W} \right)^{\frac{\eta}{n}}}\] ......................................................(2.95)

with the boundary conditions:

\[Q(0,t) = \frac{q_i(t)}{2}\] ........................................................................................................(2.96)

\[\Delta P(L(t)/2, t) = P_L\] ........................................................................................................(2.97)

where:

\[A_{\varphi}(x,t) = \int_{-\infty}^{\infty} w(x,y,t) dy\] ....................................................................................................(2.98)

\[Q_L(x,t) = \frac{4Ca}{\sqrt{t-\tau(x)}}\] ....................................................................................................(2.99)

and \(P_L\) is the pressure differential required to open the fracture at the crack front.

Various methods can be used to solve the two flow equations, but one that presents
good results is to advance the fracture front a distance (\(\Delta L\)) during an assumed time
step (\(\Delta t\)) and integrating the two equations (2.94) and (2.95) by a Runge-Kutta
method. The cross section area in equation (2.94) is also a function of pressure, as
can be seen in the equations (2.79) and (2.98). After substituting those equations in
equation (2.94), the final result will be function of a term involving \( \frac{\partial P}{\partial t} \) which can be substituted by the difference relationship:

\[
\frac{\partial P(x,t)}{\partial t} = \frac{(P(x,t + \Delta t) - P(x,t))}{\Delta t} \tag{2.100}
\]

The value of \( Q(0,t) \) is compared with \( q_i(t)/2 \), and if they do not agree, another value for \( \Delta t \) is assumed until a desirable agreement is reached.

### Table 2.3. Rock and Reservoir Data for Three Layer Case

(after Warpinski et al., 1994)

<table>
<thead>
<tr>
<th>Interval</th>
<th>Depth (ft)</th>
<th>Thickness (ft)</th>
<th>In-situ Stress (psi)</th>
<th>Poisson's Ratio</th>
<th>Young's Modulus ( (10^6 \text{ psi}) )</th>
<th>Toughness (psi/in(^{0.5}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8,990-9,170</td>
<td>180</td>
<td>7,150</td>
<td>0.30</td>
<td>6.5</td>
<td>2,000</td>
</tr>
<tr>
<td>2</td>
<td>9,170-9,340</td>
<td>170</td>
<td>5,700</td>
<td>0.21</td>
<td>8.5</td>
<td>2,000</td>
</tr>
<tr>
<td>3</td>
<td>9,340-9,650</td>
<td>310</td>
<td>7,350</td>
<td>0.29</td>
<td>5.5</td>
<td>2,000</td>
</tr>
</tbody>
</table>

### Table 2.4. Treatment Data (after Warpinski et al., 1994)

| Bottomhole Temperature, °F | 246 |
| Reservoir pressure, psi    | 3,600 |
| Spurt Loss                 | 0.0  |
| Fluid-leak-off height      | Entire fracture height |
| Fluid leak-off coefficient, ft / \( \sqrt{\text{min}} \) | 0.00025 |
| Viscosity - Case A, cp     | 200  |

<table>
<thead>
<tr>
<th>Viscosity - Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
</tr>
<tr>
<td>( K )</td>
</tr>
<tr>
<td>Fluid volume, bbl</td>
</tr>
<tr>
<td>Injection rate, bbl/min</td>
</tr>
<tr>
<td>Proppant</td>
</tr>
</tbody>
</table>
Warpinski et al. (1994) presented results of some 3-D and pseudo-3D models for various rocks and reservoir data. Stress and rock property measurements were averaged over the appropriate depths for each interval to yield the physical data in Table 2.3. Young's modulus and Poisson's ratio were obtained from sonic measurements, thus accounting for the elevated values of Young's modulus.

Table 2.3 and 2.4 give the complete set of data input, but Warpinski et al. (1994) restricted the data set to limit as many discretionary inputs as possible to allow more direct comparison of model performance.

<table>
<thead>
<tr>
<th>Model</th>
<th>Length (ft)</th>
<th>Height (ft)</th>
<th>Pressure (psi)</th>
<th>Maximum Width (ft)</th>
<th>Well - Well (ft)</th>
<th>Overall W (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAH</td>
<td>3,408</td>
<td>318</td>
<td>1,009</td>
<td>0.65</td>
<td>0.35</td>
<td>0.30</td>
</tr>
<tr>
<td>NSI</td>
<td>3,750</td>
<td>903</td>
<td>283</td>
<td>0.56</td>
<td>0.32</td>
<td>0.25</td>
</tr>
<tr>
<td>RES</td>
<td>1,744</td>
<td>544</td>
<td>1,227</td>
<td>0.90</td>
<td>0.54</td>
<td>0.36</td>
</tr>
<tr>
<td>Marathon</td>
<td>1,360</td>
<td>442</td>
<td>1,387</td>
<td>1.04</td>
<td>0.68</td>
<td>0.64</td>
</tr>
<tr>
<td>Meyer-1</td>
<td>3,549</td>
<td>291</td>
<td>987</td>
<td>0.58</td>
<td>0.35</td>
<td>0.29</td>
</tr>
<tr>
<td>Meyer-2</td>
<td>2,692</td>
<td>360</td>
<td>1,109</td>
<td>0.72</td>
<td>0.41</td>
<td>0.34</td>
</tr>
<tr>
<td>Arco</td>
<td>3,598</td>
<td>306</td>
<td>992</td>
<td>0.57</td>
<td>0.31</td>
<td>0.25</td>
</tr>
<tr>
<td>Texaco</td>
<td>1,938</td>
<td>435</td>
<td>1,132</td>
<td>0.72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Advani</td>
<td>2,089</td>
<td>357</td>
<td>1,113</td>
<td>0.66</td>
<td>0.33</td>
<td>0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>Length (ft)</th>
<th>Height (ft)</th>
<th>Pressure (psi)</th>
<th>Maximum Width (ft)</th>
<th>Well - Well (ft)</th>
<th>Overall W (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAH</td>
<td>3,259</td>
<td>371</td>
<td>1,093</td>
<td>0.75</td>
<td>0.38</td>
<td>0.31</td>
</tr>
<tr>
<td>NSI</td>
<td>3,289</td>
<td>329</td>
<td>1,005</td>
<td>0.67</td>
<td>0.35</td>
<td>0.26</td>
</tr>
<tr>
<td>RES</td>
<td>902</td>
<td>596</td>
<td>1,428</td>
<td>1.10</td>
<td>0.74</td>
<td>0.49</td>
</tr>
<tr>
<td>Marathon</td>
<td>1,326</td>
<td>442</td>
<td>1,433</td>
<td>1.08</td>
<td>0.71</td>
<td>0.66</td>
</tr>
<tr>
<td>Meyer-1</td>
<td>2,915</td>
<td>337</td>
<td>1,094</td>
<td>0.69</td>
<td>0.40</td>
<td>0.32</td>
</tr>
<tr>
<td>Meyer-2</td>
<td>2,120</td>
<td>413</td>
<td>1,212</td>
<td>0.86</td>
<td>0.48</td>
<td>0.40</td>
</tr>
<tr>
<td>Arco</td>
<td>3,235</td>
<td>353</td>
<td>1,083</td>
<td>0.65</td>
<td>0.33</td>
<td>0.26</td>
</tr>
<tr>
<td>Advani</td>
<td>2,424</td>
<td>435</td>
<td>1,171</td>
<td>0.74</td>
<td>0.34</td>
<td>0.21</td>
</tr>
</tbody>
</table>

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Table 2.5 shows the complete set of results for the final fracture geometries from model runs for a three-layer case. The three-layer case shows the models calculate widely different fracture geometries for the same input parameters. Warpinski et al. (1994) did not point out a model that gives more realistic results, but they suggested the use of the model that is more appropriate for the data of the reservoir that is being fractured.

The problem is that this procedure requires knowledge about the formation being fractured and this is only possible if guided by experience with the reservoir. For cases where there is no experience with the formation a single model should be chosen. After the completion of the fracture the results should be analyzed, then an appropriate model must be selected for this area.

The selection of one model for a determined area does not mean that the model will predict better results for another area, and the procedure described before should be applied for determining the most applicable model for this other area.
CHAPTER III

EXPERIMENTAL WORK

3.1. Introduction

An experimental apparatus and operational procedure were designed to study the leak-off volume occurring inside the induced fracture during an underground blowout. The result of this study is used in a fracture model to predict pressures developed during an underground blowout.

The experimental apparatus, set up at the LSU Blowout School facilities, consisted of a fluid loss cell where a two-phase fluid passes over and through a porous core due to pressure differential. The volume that passes through the core is the leak-off volume used in the fracture model.

The lack of data for leak-off volume when using drilling fluid and gas was the main factor that led the author to design this experimental work. Without this experimental work, only data collected for foams or specific fracturing fluids could be used in the fracture model which would be unrealistic.

3.2. Description of the experimental apparatus

The test apparatus used for all fluid runs, which are described as dynamic leak-off rate tests, is shown schematically in Fig. 3.1. It can be divided in three major parts: the mixing system, the fluid loss cell, and the collector system.

The mixing system consists in two nitrogen bottles charged with a maximum pressure of 2,500 psi, a 10 gal fluid vessel, a heater, and two 20 ft rheology loops.

Base drilling fluid is prepared in a small tank, pumped into the fluid vessel, and pressurized at the required level to run the experiment. The two-phase fluid mixture is obtained by small adjustments in two needle-valves situated on two lines upstream of the venturi. Once the pressure and gas flow rate are set at the start of the
experiment, very little adjustment is needed to keep them at required levels. The gas and mud flow rates are measured in the collector system described later in this text.

Fig. 3.1. Leak-off Volume Experimental Apparatus

The main function of the two 20 ft rheology loops is to determine the rheological fluid properties through the measurement of pressure in both loop ends. The two-phase mixture travels through tubing before it is heated to provide the desired shear history for the fluid. The shear history could be varied by changing the flow rate inside the tubing arrangement. In this test, the procedure to determine the shear history of the fluid is done by measuring the pressure for three different flow rates.

The heater is used to maintain the fluid temperature at formation conditions, and it consists of a loop immersed in hot water. The two-phase fluid is heated to
formation temperature in the heater to approximate the formation conditions, then it passes through the second rheology loop just for verifying changes in the shear history of the heated fluid.

The fluid loss cell is an apparatus where a 0.94 x 1 inch core is set and the fluid passes through a 1x1.5x0.13 inch slot over the core. The slot dimensions are sized to provide the same shear rate as produced in other parts of the apparatus. Figure 3.2 shows the components of the cell.

The collector system is used to collect all the fluids that pass over and through the core. It consists of two parts: one that collects the leak-off and the other that collects the remainder.

The filtrate collector immediately below the test cell contains a 20 cc graduated cylinder to measure liquid leak-off through the core. The connecting tubing, between the grooved drainage pad supporting the core and the graduated cylinder, is a small-diameter short tube intended to minimize fluid holdup before measurement. The gas passing the core is bled through the top of the cylinder and measured with a gasometer. The total leak-off volume is the sum of the leak-off gas and leak-off liquid volumes recorded as function of time. The leak-off gas volume is measured with a gasometer, and the leak-off liquid volume is measured with a small graduated cylinder.

The flow rate and the gas void fraction are determined in the collector system by recording, as function of time, the remaining gas volume with the wet test meter and the remaining liquid volume with a large cylinder. The total flow rate is determined by the sum of these two volumes with the total leak-off volume, and division by the time of the recording. The gas void fraction is determined by dividing the total gas volume per the total fluid volume.

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Fig. 3.2. Fluid Leak-off Cell Components

<table>
<thead>
<tr>
<th>Number</th>
<th>Part Name</th>
<th>Required</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Cell Cap</td>
<td>1</td>
<td>SAE 1020</td>
</tr>
<tr>
<td>2</td>
<td>Cell Core Seat</td>
<td>1</td>
<td>SAE 1020</td>
</tr>
<tr>
<td>3</td>
<td>Core Platform (Screen)</td>
<td>1</td>
<td>SAE 1020</td>
</tr>
<tr>
<td>4</td>
<td>Core</td>
<td>1</td>
<td>Nitrile</td>
</tr>
<tr>
<td>5</td>
<td>Parker Polypak - #25001000</td>
<td>1</td>
<td>Nitrile</td>
</tr>
<tr>
<td>6</td>
<td>Parker O-Ring - #2-224</td>
<td>1</td>
<td>SAE 1020</td>
</tr>
<tr>
<td>7</td>
<td>Cell Sleeve</td>
<td>1</td>
<td>SAE 1020</td>
</tr>
</tbody>
</table>
3.3. Experimental Procedure

Before running the experiments, twelve Berea sandstone cores were dried in an oven for 12 hours at 250 °F, and the lateral surfaces of the cores were coated with a very thin layer of epoxy to avoid possible lateral flow during the experiment. The liquid permeability was then measured with a gas permeameter considering the Klinkenberg effect, and the porosity was measured by using mercury and an air pump. Table 3.1 shows the results of the measurement.

<table>
<thead>
<tr>
<th>Core</th>
<th>Length (in)</th>
<th>Permeability (mD)</th>
<th>Porosity (d'less)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.969</td>
<td>175.6</td>
<td>0.201</td>
</tr>
<tr>
<td>2</td>
<td>1.000</td>
<td>117.4</td>
<td>0.189</td>
</tr>
<tr>
<td>3</td>
<td>0.980</td>
<td>149.4</td>
<td>0.196</td>
</tr>
<tr>
<td>4</td>
<td>0.975</td>
<td>106.7</td>
<td>0.188</td>
</tr>
<tr>
<td>5</td>
<td>0.975</td>
<td>106.7</td>
<td>0.191</td>
</tr>
<tr>
<td>6</td>
<td>0.970</td>
<td>104.1</td>
<td>0.186</td>
</tr>
<tr>
<td>7</td>
<td>0.963</td>
<td>206.8</td>
<td>0.203</td>
</tr>
<tr>
<td>8</td>
<td>0.963</td>
<td>238.6</td>
<td>0.204</td>
</tr>
<tr>
<td>9</td>
<td>0.957</td>
<td>146.2</td>
<td>0.195</td>
</tr>
<tr>
<td>10</td>
<td>1.000</td>
<td>102.2</td>
<td>0.184</td>
</tr>
<tr>
<td>11</td>
<td>0.969</td>
<td>224.1</td>
<td>0.201</td>
</tr>
<tr>
<td>12</td>
<td>0.980</td>
<td>331.8</td>
<td>0.215</td>
</tr>
</tbody>
</table>

At the end of the measurement, each core was put in a vacuum pump for three hours. Then, 50,000 ppm brine was introduced to the evacuated container to saturate the core. All the cores were allowed to saturate for a minimum of 10 days.

The fluid leak-off tests were run with five different pressure differentials of 200, 400, 600, 800, and 1,000 psi and gas void fractions varying from 0% up to 90% for each pressure differential. The pressure differential and gas void fraction were selected based on values that can be reached during an underground blowout as well as to comply with the apparatus capacity. The leak-off from any fracture fluid is
driven by the pressure inside the fracture and restrained by the pressure of the fluid that it must displace.

The mud used was a bentonite type with a viscosity of 10 cp and a density of 8.7 ppg. Also, some runs were performed with a mud viscosity of 20 cp to verify its effect on leak-off volume. The main flow rate used in the test was a flow rate to achieve a shear rate greater than 100 sec\(^{-1}\), or 65 cc/min in the apparatus, which is one of the values recommended by Conway and Harris (1982) in fluid lab tests. The reason for using greater flow rates was the fact that an underground blowout reaches higher shear rates than the recommended by Conway and Harris (1982).

It was necessary to use a screen pad inside the fluid cell during the experiment because the cores presented some small variation in their lengths. The use of that screen allowed the top of the core to be completely aligned to the flow path of the cell avoiding any disturbance in the fluid flow. In addition, the measurement of the leak-off volume had to be normalized to the same length by using Darcy's equation.

The first core was used in 40 runs. These were divided in the following way: seven runs with 200 psi, eight with 400 psi, eleven with 600 psi, seven with 800 psi, and finally seven with 1,000 psi. All of the runs had a temperature of 175 °F after heating. Although the main objective of those runs was to measure the leak-off volume, they were only used to calibrate the system and improve the design of the apparatus. The reason for not using the data was the large variation in gas void fraction and flow rate during all runs, as well leaks in the apparatus. After these runs, a venturi was set in the apparatus to mix gas and mud in a constant proportion and the leaks were fixed. These modifications caused a large improvement in keeping the gas void fraction fairly constant during all the experiment, but the start. The results of the remaining runs were used in the analysis. Another observation in those runs was that the procedure to determine the shear history of the fluid was not feasible. The
reason was the pressure variation inside the two loops was negligible even when using large variations in flow rates, so the plot of shear stress as function of shear rate for this kind of fluid was not possible.

A total of 107 conclusive tests was run after the improvement of the apparatus and the result of the measurements can be seen in Appendix C.

3.4. Result Analysis

The approach to analyze the leak-off volume is the same as has been used by several authors when working with foam, or in other words, by analyzing the total leak-off volume, gas plus liquid, as function of time. The analysis of gas and liquid leak-off volumes separately is not possible because the core is completely saturated with liquid, and this causes the liquid leak-off volume to be greater than its real value. Therefore the author plotted the results of the total leak-off volume as function of time for all runs. The parallax error for the total leak-off volume measured in the experiment was estimated as 0.1 ml/sqcm. The author used a curve fit program that found a general equation for the collected data. The curve fitting equations for all runs are shown in Appendix C with their respective values of the Pearson product moment correlation coefficient $R$ and chi-square test $\chi^2$ which is defined by:

$$\chi^2 = \sum \frac{(V_{pre} - V)^2}{V}$$

where $V_{pre}$ is the predicted value by the equation that fits best the measured data.

It can be seen from the tables in Appendix C that the general equation is the same as found by Clark and Barkat (1990) in an experiment with one phase fluid. The equation is written as:

$$V = V_{sp} \left(1 - e^{-bt}\right) + v_Dt$$

(3.2)
where the parameters in this equation are spurt loss volume, the pack buildup factor, and the equilibrium Darcy flow velocity.

It has been shown in several studies that those parameters depend on pressure differential, flow rate, core permeability, mud filtration property, and rheological properties of the fluid. However, the induced fracture in an underground blowout is caused by a mixture of gas and mud, so another property to be studied is the gas void fraction ($H_g$).

Figure 3.3 shows the plot of the total leak-off volume data as function of time and gas void fraction for core number 8 (points in the graph), as well as the plot of equation (3.2) that fits the leak-off volume data (curves in the graph). The respective equations can be seen in Table c.1.6 in Appendix C, but it is clear the general equation fits very well the total leak-off volume data in all runs. The numbers in parenthesis in the legend of figure 3.3 indicate the gas void fraction of the experiment at 1 minute which is called initial gas void fraction in this study.

The use of equation (3.2) to predict the leak-off volume is possible when the spurt loss volume, the pack buildup factor, and the equilibrium Darcy flow velocity are known. The author analyzed those parameters as function of permeability, flow rate, pressure differential, viscosity, and gas void fraction.

The influence of gas void fraction on total leak-off volume is clear because the leak-off increases proportionally to gas void fraction, as can be seen in Fig. 3.3. The same conclusion is achieved about the pressure differential when observing the leak-off volume data. The influence of permeability, viscosity, and flow rate was not clear with the available data. The influence of the mud filtration property was not studied in this work because only one kind of mud was used in the experiment.
The cores used in the experiment were a Berea sandstone formation and they did not present a large variation of permeability (table 3.1). In addition, the effect of permeability is largely decreased by the mud cake formed during the leak-off process, so its influence could not be detected when studying the leak-off volume by regression analysis.

Runs with mud viscosity of 20 cp and pressure differential of 200 psi did not present a significant variation when compared with runs of 10 cp. Therefore, the viscosity effect was also considered negligible for the kind of mud used. This was expected because the mud filtrate had almost the same viscosity for the two different mud viscosities, and it is filtrate instead of mud that flows through the core.
Although the total flow rate had some influence on total leak-off volume, its effect was much less than the pressure differential and the gas void fraction. This can be seen by comparing the regression analysis with those variables, as well as by plotting the total leak-off volume as function of gas flow rate.

Therefore, this study will focus on the influence of gas void fraction and pressure differential on the three parameters of equation 3.2.

3.4.1. Spurt Loss Volume Correlation

Two out of five independent variables, the gas void fraction and pressure differential have larger influence than the others on the spurt loss volume. Also, by analyzing the available data, the author verified that the spurt loss volume was much more influenced by the initial gas void fraction (gas void fraction at 1 minute) than the average gas void fraction of the experiment. It is important to observe again that it was not possible to have the gas void fraction at the start equal to the rest of the experiment, and for this reason the experiment had some inconsistencies in the total leak-off volume measured data.

Figure 3.4 shows the total leak-off volume as a function of time for core 2 with a pressure differential of 1,000 psi. The figure shows the total leak-off volume is higher when the gas void fraction is higher, but in some cases this trend is not observed. This inconsistency can be observed for the case of 50% average gas void fraction (6% initial gas void fraction) where the total leak-off volume curve is closer to the curve of 10% average gas void fraction instead of to the curve of 60% average gas void fraction. The same effect was observed in other runs when the initial gas void fraction was not close to the average value. This fact allowed to conclude the total leak-off volume was more influenced by the initial gas void fraction than the average gas void fraction in the experiment.
Due to this fact, a plot of spurt loss volume as function of initial gas void fraction was done for five different pressure differentials, and a trend was determined for each pressure differential. Figure 3.5 shows the plot for three pressure differentials.

It can be observed that the value of the total leak-off volume keeps fairly constant when the gas void fraction is below 10%. It increases when the gas void fraction is larger than 10%. For values of gas void fraction greater than 70%, the data spread out of the normal trend. A possible cause for this is the larger effect of the gas expansion in the total leak-off volume for gas void fraction larger than 70%.
Figure 3.5. Spurt Loss Volume versus Initial Gas Void Fraction

Table 3.2 shows the best trend equations determined in Figure 3.5 for the five pressure differentials used in this experiment as function of gas void fraction.

Table 3.2. Spurt Loss Volume Equations

<table>
<thead>
<tr>
<th>$\Delta P$ (psi)</th>
<th>Equation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>$H_g &lt; 0.14 \Rightarrow V_{sp} = 1.123$</td>
<td>$H_g \geq 0.14 \Rightarrow V_{sp} = 4.29H_g + 0.502$</td>
</tr>
<tr>
<td>400</td>
<td>$H_g &lt; 0.14 \Rightarrow V_{sp} = 1.269$</td>
<td>$H_g \geq 0.14 \Rightarrow V_{sp} = 4.04H_g + 0.704$</td>
</tr>
<tr>
<td>600</td>
<td>$H_g &lt; 0.07 \Rightarrow V_{sp} = 1.309$</td>
<td>$H_g \geq 0.07 \Rightarrow V_{sp} = 4.47H_g + 1.002$</td>
</tr>
<tr>
<td>800</td>
<td>$H_g &lt; 0.09 \Rightarrow V_{sp} = 1.389$</td>
<td>$H_g \geq 0.09 \Rightarrow V_{sp} = 5.12H_g + 0.899$</td>
</tr>
<tr>
<td>1,000</td>
<td>$H_g &lt; 0.08 \Rightarrow V_{sp} = 1.607$</td>
<td>$H_g \geq 0.08 \Rightarrow V_{sp} = 5.07H_g + 1.215$</td>
</tr>
</tbody>
</table>

3.4.2. Pack Buildup Factor

The same approach was used to analyze this factor, but the plot of the total leak-off volume as function of the gas void fraction did not show a clear relation as in the plot of spurt loss volume. This observation can be seen in Fig. 3.6 for three

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different pressure differentials, up to a gas void fraction of 70%, where the trend appears to stay around a fairly constant value. The data is scattered after this point.

![Pack Buildup Factor](image)

Fig. 3.6. Pack Buildup Factor

The data for the pack buildup factor was analyzed with a curve fitting program, and a clear relation between the gas void fraction and the pack buildup factor could not be found.

The hypothesis of non dependence on the gas void fraction was analyzed by the procedure showed in Ostle and Malone (1988), with the cumulative $t$ distribution calculated by:

$$t = \frac{a_i}{s_{b_i}}$$

where $a_i$ is the slope of the estimated regression equation using the least squares method and $s_{b_i}^2$ is the estimated variance of $a_i$. Since $t$ has a $t$-distribution with $(n-2)$ degrees of freedom if the hypothesis is true, the hypothesis is rejected if
\[ \left| t \right| > t_{1-\alpha/2(n-3)} \] The results can be seen in table 3.3 for \( t \) values using a regression analysis for \( b \) in function of \( H_g \) and for \( \log(b) \) in function of \( \log(H_g) \).

### Table 3.3. Calculated \( t \)-Values for Pack Buildup Factor

<table>
<thead>
<tr>
<th>( \Delta P ) (psi)</th>
<th>( n )</th>
<th>( R^2 )</th>
<th>( t )</th>
<th>( n )</th>
<th>( R^2 )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>29</td>
<td>0.304</td>
<td>3.437</td>
<td>27</td>
<td>0.073</td>
<td>1.405</td>
</tr>
<tr>
<td>400</td>
<td>19</td>
<td>0.081</td>
<td>1.938</td>
<td>18</td>
<td>0.149</td>
<td>1.673</td>
</tr>
<tr>
<td>600</td>
<td>23</td>
<td>0.023</td>
<td>0.523</td>
<td>20</td>
<td>0.115</td>
<td>1.530</td>
</tr>
<tr>
<td>800</td>
<td>22</td>
<td>0.024</td>
<td>-0.703</td>
<td>20</td>
<td>0.067</td>
<td>-1.134</td>
</tr>
<tr>
<td>1,000</td>
<td>15</td>
<td>0.069</td>
<td>-0.985</td>
<td>13</td>
<td>0.111</td>
<td>1.175</td>
</tr>
</tbody>
</table>

By comparing the calculated \( t \) with the cumulative \( t \)-distribution values, it can be observed that the hypothesis cannot be rejected because the calculated values are smaller than the cumulative values. This means the pack buildup factor is independent on gas void fraction.

### Table 3.4. Average Pack Buildup Factor (min\(^{-1}\)) and Equilibrium Flow Velocity Coefficient (cm/min)

<table>
<thead>
<tr>
<th>( \Delta P ) (psi)</th>
<th>Buildup Factor (min(^{-1}))</th>
<th>Standard Deviation (min(^{-1}))</th>
<th>Flow Coefficient (cm/min)</th>
<th>Standard Deviation (cm/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>1.918</td>
<td>0.990</td>
<td>0.0328</td>
<td>0.0090</td>
</tr>
<tr>
<td>400</td>
<td>1.737</td>
<td>0.910</td>
<td>0.0429</td>
<td>0.0109</td>
</tr>
<tr>
<td>600</td>
<td>1.597</td>
<td>0.584</td>
<td>0.0465</td>
<td>0.0134</td>
</tr>
<tr>
<td>800</td>
<td>1.610</td>
<td>0.663</td>
<td>0.0462</td>
<td>0.0163</td>
</tr>
<tr>
<td>1,000</td>
<td>1.454</td>
<td>0.377</td>
<td>0.0324</td>
<td>0.0060</td>
</tr>
</tbody>
</table>

Due to these observations, it was decided to use the average value of \( b \) for each pressure differential in equation 3.2. The average pack buildup factor for a bentonite mud is shown in table 3.4 for five different pressure differentials.

### 3.4.3. Equilibrium Darcy Flow Velocity Coefficient

The plot of this coefficient as function of the gas void fraction can be seen in figure 3.7 for three different pressure differentials.
The author generally observed that it looks like the equilibrium velocity coefficient has a trend to a small increase in its value before it reaches 50% of gas void fraction, after that there is a slight trend to a decrease in its value. However this trend is not clear. The cumulative $t$-distribution was also used to verify the dependence of the equilibrium velocity coefficient on gas void fraction.

![Equilibrium Flow Velocity](image)

Fig. 3.7. Equilibrium Darcy Flow Velocity Coefficient

Table 3.5 shows the $t$ values calculated using a regression analysis for $v_D$ in function of $H_g$, as well as for $\log(v_D)$ in function of $\log(H_g)$ for those two trends observed in Fig. 3.7.

It can be seen that the calculated $t$ values are smaller than the cumulative $t$-distribution values. This means the equilibrium Darcy flow velocity coefficient is also independent on gas void fraction.
Therefore, it was also decided to use the average value of $v_d$ for each pressure differential in equation 3.2. The average equilibrium Darcy flow velocity coefficient for a bentonite mud is also shown in table 3.4 for five pressure differentials.

Table 3.5. Calculated $t$-Values for Equilibrium Darcy Flow Velocity Coefficient

<table>
<thead>
<tr>
<th>$\Delta P$ (psi)</th>
<th>$H_i &lt; 50%$</th>
<th>$H_i &gt; 50%$</th>
<th>$H_i &lt; 50%$</th>
<th>$H_i &gt; 50%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n$</td>
<td>$R^2$</td>
<td>$t$</td>
<td>$n$</td>
</tr>
<tr>
<td>200</td>
<td>22</td>
<td>0.102</td>
<td>1.510</td>
<td>13</td>
</tr>
<tr>
<td>400</td>
<td>14</td>
<td>0.001</td>
<td>-0.031</td>
<td>8</td>
</tr>
<tr>
<td>600</td>
<td>19</td>
<td>0.215</td>
<td>2.157</td>
<td>10</td>
</tr>
<tr>
<td>800</td>
<td>16</td>
<td>0.041</td>
<td>0.773</td>
<td>11</td>
</tr>
<tr>
<td>1,000</td>
<td>11</td>
<td>0.020</td>
<td>0.424</td>
<td>6</td>
</tr>
</tbody>
</table>

3.5. Comparison of Predicted Results with Measured Data

A statistical analysis of the measured data compared with the predicted values calculated with equation 3.2 was performed. The results of the standard error and $t$-test for all runs can be seen in Appendix C. Figure 3.8 presents a plot of the measured data and the predicted values for core 2 with a pressure differential of 1,000 psi.

It can be seen in Fig. 3.8 that equation (3.2) gives good prediction for the leak-off volume value in almost all runs. The experimental data showed some divergence from the normal trend observed in other runs for a gas void fraction of 50\% (6\%). By observing the data, it can be seen that a faster increase in the leak-off volume occurred in that run after 12 minutes with further decrease in the leak-off rate. One reason for that could be operational due to some gas getting trapped inside the lines until 12 minutes, being released after this time. This fact could cause a increase in the leak-off rate faster than the normal trend as observed in this particular run.
Table 3.6 presents the standard error achieved, as well as the \( t \)-test for a significance level of 5\% when comparing the predicted results with the measured leak-off volume for core 2 and pressure differential of 1,000 psi. For other runs see Appendix C.

The standard error for each run is computed as:

\[
|\text{Std Error}| = \frac{\sum_{i=1}^{n} |V_i - V_{\text{pred}}|}{n}
\]

where \( n \) is the number of measured points in the run.
Table 3.6. Standard Error and t-Test - Core 2 - $\Delta P = 1000$ psi

<table>
<thead>
<tr>
<th>Initial Gas Void Fraction (%)</th>
<th>0</th>
<th>10</th>
<th>7</th>
<th>6</th>
<th>62</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Gas Void Fraction (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.041</td>
<td>0.061</td>
<td>0.025</td>
<td>0.154</td>
<td>0.007</td>
<td>0.009</td>
</tr>
<tr>
<td>t</td>
<td>0.515</td>
<td>-1.847</td>
<td>0.579</td>
<td>0.575</td>
<td>-0.258</td>
<td>0.482</td>
</tr>
<tr>
<td>hypothesis</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
</tr>
</tbody>
</table>

The $t$ value is calculated by considering the measured data and the predicted data as paired observations where the differences ($D_i$) between the observations are considered a random sample from a normal population. For testing the null hypothesis or in other words, if the predicted data can be considered a good approximation, the test statistic is:

$$t = \frac{\bar{D}\sqrt{n}}{s_D}$$ ......................................................... (3.5)

$$\bar{D} = \frac{\sum_{i=1}^{n} D_i}{n}$$ ..................................................... (3.6)

$$D_i = V_i - V_{prei}$$ ................................................................. (3.7)

$$s_D^2 = \frac{\sum_{i=1}^{n} (D_i - \bar{D})^2}{(n-1)}$$ ................................ (3.8)

The hypothesis is rejected if $|t| \geq |t_{(1-\alpha/2)(n-1)}|$. In table 3.6 this is indicated with a R. It can be seen that the prediction for the case shown in table 3.6 worked very

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well, with only one case that presented a divergence in the results with a standard error of 15%.

Figure 3.9 indicates the error occurring in all runs estimating the leak-off volume by using equation (3.2). The values of its coefficients, spurt loss volume, pack buildup factor, and equilibrium Darcy flow velocity coefficient should be determined with the correlation presented in this chapter.

3.6. The Use of the Proposed Correlation in the Fracture Model

The fracture model needs the value of the leak-off coefficient $C$ in its governing equations to predict the fracturing process in each cell of the fracture. The method used by the author is similar to the method proposed by Ford and Penny (1988) to convert the leak-off volume to an effective leak-off coefficient, but it was

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modified to consider the unsteady state condition of the fracturing process in an underground blowout. The value of pressure differential and gas void fraction for each cell in each time step is used in the calculation of the effective leak-off coefficient for the cell considered. This calculation is done by equation (2.35) modified to consider such feature, and it is written as:

\[
C = \frac{\sum_{i=1}^{n} \Delta V_i - V_{sp}}{2\sqrt{t_{av}}} \tag{3.8}
\]

where \( n \) is the number of time steps to reach the time \( t \), \( t_{av} \) is the average time the cell is exposed in the fracturing process, and \( \Delta V_i \) is given by:

\[
\Delta V_i = V(t_i) - V(t_{i-1}) \tag{3.9}
\]

where the leak-off volume at time \( t \) is given by equation (3.2).

The calculated values for the leak-off volume with a pressure differential different from the values used in the experiment are obtained by interpolating the results between them.

3.7. Conclusions

The experimental work showed the gas void fraction and the pressure differential have a great influence in the leak-off volume, so a correlation considering this influence is presented here. The general equation to calculate the leak-off volume has the same form as the one used in one phase fluid, but its coefficients depend on gas void fraction and/or pressure differential. The experiment could not verify the dependence on gas void fraction and flow rate in two coefficients of equation (3.2), (the pack buildup factor and the equilibrium Darcy flow velocity coefficient), which is an indication that other factors, such as relative permeability and rheologic properties
due to the presence of gas in the fluid, could influence those coefficients. However, this cannot be studied with the current experimental apparatus and different designs should be used only to study these properties.

A further study by using different range of permeability is necessary to better determine the effect of core permeability in the leak-off volume, although the drilling fluid decreases that influence in a large proportion.

A future study to verify the effect of the mud filtration property in the total leak-off volume by using different kinds of drilling fluid must be done because this research used only one kind of drilling fluid in the experimental work.
CHAPTER IV

DEVELOPED UNDERGROUND BLOWOUT MODEL

Often, dynamic kill operations must be used in cases where an underground blowout occurred because it is not possible to control the exit pressure from a well by means of surface equipment. In such cases a high flow rate of mud or water is used to overcome the formation pressure and to regain the control of the well.

This study considers the case of a gas underground blowout caused by an induced fracture due to high pressures in the wellbore. The main objective is the development of a new model to predict the pressure and mud flow rate in the control of an underground blowout. A computer model was developed to assist the well control operations in such cases.

The program consists of a system analysis procedure where a gas reservoir model, a wellbore model, and a fracture model are interconnected. Previous computer models used for this purpose have simplified the behavior of the fracture by assuming a constant fracture injection pressure. The aim is to predict a pressure profile in the wellbore according to the mud flow rate used in the dynamic kill method. The calculation procedure of the program is explained in detail in the following sections as well as the models used in the program.

4.1. Gas Reservoir Model

The gas reservoir model is based on Al-Hussainy and Ramey's (1966) expressions modified to account for changing in flow rate and pressure. To calculate the wellbore pressure of an infinite gas reservoir produced at a constant flow rate, including skin and the non-Darcy effects, is used:

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\[ m(P_{BHP}) = m(P_{RES}) - \frac{0.366534QP_{se}T}{khT_{sc}} \left[ \log(2.2454t_d) + 0.87S + 0.87DQ \right] \] ...............(4.1)

where the real gas pseudo pressure is defined as:

\[ m(P) = 2 \int_{\mu_g}^{p} \frac{P}{dP} \] ..................................................................................................................(4.2)

and the dimensionless time by:

\[ t_0 = \frac{kt}{\phi(\mu_g c_t) r_w^2} \] ............................................................................................................(4.3)

where the total compressibility is approximated as:

\[ c_t = c_s(1 - S_{wi}) \] .........................................................................................................................(4.4)

The non-Darcy factor is calculated with:

\[ D = 0.159243 \frac{\beta_s M_{pl,k}}{R \mu h r_w T_{sc}} \] ............................................................................................................(4.5)

where the velocity coefficient for consolidated sandstone was determined by Geertsma (1974) as:

\[ \beta_s = \frac{0.005}{\phi^{0.5} \sqrt{k}} \] .........................................................................................................................(4.6)
The gas compressibility and the gas viscosity in these equations are evaluated at the reservoir pressure.

The bottom hole pressure and gas flow rate vary with time in a case of underground blowout. Due to this fact, the solution for the wellbore pressure can be found by applying the principle of the superposition for different flow rates in equation (4.1). The result after that is:

\[ m(P_{BHP}) = m(P_{RES}) - \frac{0.366534P_wT}{khT_{sc}} \left[ \sum_{j=1}^{n} \left( Q_j - Q_{j-1} \right) \log(2.2454(t_D - t_{D_{j-1}})) \right] + \]

\[ +0.87Q_n(S + DQ_n) \] .................................(4.7)

After algebraic manipulation the solution for the flow rate can be obtained from the following quadratic equation:

\[ Q_n^2 + \frac{0.87S + \log(2.2454(t_D - t_{D_{n-1}}))}{0.87D} Q_n + \]

\[ + \frac{B - A - Q_{n-1} \log(2.2454(t_D - t_{D_{n-1}}))}{0.87D} = 0 \] ..............................................(4.8)

where:

\[ A = \frac{[m(P_{res}) - m(P_{BHP})]khT_{sc}}{0.366534P_wT} \] .......................................(4.9)

and

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\[ B = \sum_{j=1}^{\text{act}} \left[ (Q_j - Q_{j-1}) \log \left( 2.2454(t_D - t_{Dj-1}) \right) \right] \]  

The model can also consider in an approximate way the case of a finite volumetric gas reservoir. The average pressure in the reservoir at each time step is calculated by the material balance equation as stated in Craft and Hawkins (1959):

\[ P_{\text{RES}i} = \frac{z_i T}{V_i} \left( \frac{P_{\text{RES}i} V_i}{z_i T} - \frac{P_{\infty} G_p}{T_{sc}} \right) \]

and where the subscripts \(i\) and \(f\) mean the initial reservoir conditions and the conditions at the time step respectively. The average reservoir pressure is used in place of the pressure at infinity for the approximate solution for a bounded reservoir.

The flow rate for each time step in the computer model is calculated through equation (4.8) using the value of the bottom hole pressure in that time step.

4.2. Wellbore Model

The wellbore model is an unsteady state numerical procedure based on Santos model (1989). This model accounts for unsteady state flow effects by preserving all terms of the equation of continuity and equation of momentum for a two-phase mixture.

It has basically two periods: the first one, in which the gas-liquid mixture travels through the annulus from the bottom hole to the two-phase leading edge and the single phase (mud) from the leading edge to the fractured formation; and the second period where only two-phase flow occurs in the annulus.

The model for two-phase flow in the annulus was based on Nickens' methodology (1985) with some modifications due to the new configuration of the problem.
The solution for unsteady state flow of two-phase mixture is based on a simultaneous solution of the continuity equations for gas and liquid phases, a momentum balance equation for two-phase mixture, an equation of state for gas and a semi-empirical relationship between the gas and liquid in-situ velocities.

The continuity equation for the liquid phase is given by:

\[ \frac{\partial H}{\partial t} + \frac{\partial (v_i H)}{\partial x} = 0 \] \hspace{1cm} (4.12)

and for the gas phase by:

\[ \frac{\partial [\rho_g (1 - H)]}{\partial t} + \frac{\partial [v_i \rho_g (1 - H)]}{\partial x} = 0 \] \hspace{1cm} (4.13)

The momentum balance equation for two phase mixture is written as

\[ \frac{\partial [(v_i F_{\rho_i} H) + (v_i F_{\rho_i} (1 - H))]}{\partial t} + \frac{\partial [(v_i^2 F_{\rho_i} H) + (v_i^2 F_{\rho_i} (1 - H))]}{\partial x} + \\
+ \left( \frac{\partial P}{\partial x} \right)_{elev} + \left( \frac{\partial P}{\partial x} \right)_{fri} = 0 \] \hspace{1cm} (4.14)

where \( \frac{\partial P}{\partial x} \) is the pressure gradient in the interval studied.

The elevation term for two-phase flow is calculated with:

\[ \left( \frac{\partial P}{\partial x} \right)_{elev} = g (F_{\rho_i} H + F_{\rho_i} (1 - H)) \] \hspace{1cm} (4.15)
The friction term is computed using the Beggs and Brill (1978) correlation. This was modified to accounting for the non-Newtonian fluid used in drilling operations, being calculated through the equation:

\[
\left( \frac{\partial p}{\partial x} \right)_{fric} = \frac{f_d F_c \rho_{ns} \nu_{mix}^2}{25.8d} \tag{4.16}
\]

where the two-phase mixture velocity is calculated by:

\[

\nu_{mix} = \nu_{i} H + \nu_{s} (1 - H) \tag{4.17}
\]

and the mixture no-slip density by:

\[

\rho_{ns} = \rho_{i} \lambda + \rho_{s} (1 - \lambda) \tag{4.18}
\]

The two-phase flow friction factor in this case can be calculated through:

\[

f_{q} = e^* f \tag{4.19}
\]

where \( f \) is the Fanning friction factor which is dependent on the pipe relative roughness and the two-phase Reynolds number is given by:

\[

(N_{Re})_f = \frac{F_c \rho_{ns} \nu_{mix} d}{\mu_{ns}} \tag{4.20}
\]

and the non-slip viscosity is defined as:

\[

\mu_{ns} = \mu_{i} \lambda + \mu_{s} (1 - \lambda) \tag{4.21}
\]
where the liquid viscosity is the drilling fluid plastic viscosity.

The exponent \( s \) in equation (4.19) is defined as:

\[
s = \frac{\ln y}{-0.0523 + \ln y \left( 3.182 + \ln y \left( -0.8725 + 0.01853 (\ln y)^2 \right) \right)} \tag{4.22}
\]

where

\[
y = \frac{\lambda}{H} \tag{4.23}
\]

If \( y \) is greater than 1.2 or less than 1.0, the exponent \( s \) is calculated as:

\[
s = \ln(2.2 y - 1.2) \tag{4.24}
\]

The fluid density of the liquid phase is considered constant and the density of the gaseous phase is related to pressure and temperature by the real gas equation:

\[
\rho_g = \frac{PM}{zRT} \tag{4.25}
\]

In this wellbore model the gas in situ velocity is related to the liquid in situ velocity through the equation:

\[
v_z = v_g - C_P \Delta \rho = v_g - C_P \left( v_l H + v_k (1 - H) \right) \tag{4.26}
\]

or in terms of gas velocity:
\[ v_s = \frac{C_p v_i H + v_s}{1 - C_p + C_p H} \] (4.27)

where the factors \( C_p \) and \( v_s \) depend on flow regime. The liquid hold up defines the flow regime boundary in this study. This definition is based on Caetano Filho (1986). He verified that the bubble flow occurs when the liquid hold up is between 1.0 and 0.85, slug flow between 0.75 and 0.45 and annular flow for liquid hold up less than 0.1. For the range values of liquid hold up not covered in the definition of the regime (\( H \) between 0.85 and 0.75, and \( H \) between 0.45 and 0.1), a transition regime is adopted with the same procedure as Santos (1989) where the in situ gas velocity is calculated through a linear interpolation between the regimes. This procedure avoids numerical inconsistencies in the solution when changing flow regimes. The equations for gas in situ velocity and the values for the factor \( C_p \) were the same used by Santos (1989). They are written as:

(a) Bubble Flow:

\[ v_s = 1.53 H^{0.5} \sqrt{\frac{g_s (\rho_l - \rho_g) g \sigma_{li}}{\rho_l^2}} \] (4.28)

with \( C_p \) equal to 1.1.

(b) Slug Flow

\[ v_s = 0.289 K \sqrt{\frac{(\rho_l - \rho_g) D g}{\rho_l}} \] (4.29)

where \( D \) is the outside diameter of the annulus,
\( K_t = 0.345 - 0.037R - 0.235R^2 - 0.134R^3 \) \( \ldots \) 

and \( R \) is the ratio of the inner to outer diameters in the annulus. The value of coefficient \( C_{fr} \) is adopted equal to 1.1.

(c) Annular Flow

In this regime there is almost no slippage between phases. Therefore, the slip velocity is assumed equal to zero and the gas and liquid velocities are the same.

![Finite Difference Scheme for Annulus Cell](image-url)

The solution of the differential equation is achieved numerically by using a Finite Difference Method. This method consists of discretization of the annulus into equal finite cells where finite difference approximations of flow equations are solved. The finite difference approximation used is centered in distance and backward in time. Figure 4.1 shows a cell for two different time steps. The current time step is determined by the length of the cell divided by the mixture leading velocity of the previous time step.
Point 1 represents the flow properties at the previous time step and at the lower boundary and point 2 at the upper boundary. Points 5 and 6 represent the same as points 1 and 2, respectively, at the present time. Points 3 and 4 represent arithmetic averaging of the properties at the center of the grid at previous and present times respectively. The flow properties are known at points 1, 2, and 5. The finite difference approximation estimates the flow properties at point 6.

The approximation for the space derivative in the continuity equation is calculated as:

$$\frac{\partial f}{\partial x} = \frac{f_6 - f_5}{\Delta x}$$ .................................................................(4.31)

where $f$ is some function of $x$ and $t$.

Substituting this approximation in the continuity equations for the liquid phase leads to:

$$\frac{(v_\rho_1 H)_6 - (v_\rho_2 H)_5 + (p_1 H)_6 + (p_2 H)_5 - (p_1 H)_2 - (p_2 H)_1}{\Delta x \cdot 2 \Delta t} = 0 ..............................................(4.32)$$

and for the gas phase to:

$$\frac{(v_{\rho g_1}(1-H))_6 - (v_{\rho g_2}(1-H))_5 + (\rho_1(1-H))_6 + (\rho_2(1-H))_5 - (\rho_1(1-H))_2 - (\rho_2(1-H))_1}{\Delta x \cdot 2 \Delta t} = 0 ..............................................(4.33)$$
The momentum balance equation is approximated by using a centered-in-distance and centered-in-time finite difference scheme. The approach for time derivative is the same, but the spatial derivative becomes:

\[
\frac{\partial f}{\partial x} = \frac{f_6 + f_2 - f_5 - f_1}{2\Delta x}
\]  \hspace{1cm} (4.34)

Substituting the momentum equation into Equation 4.34 gives:

\[
\frac{F}{2\Delta x} \left[ \left( v_i^2 \rho_s (1 - H) \right)_2 + \left( v_i^2 \rho_s (1 - H) \right)_6 - \left( v_i^2 \rho_s (1 - H) \right)_1 - \left( v_i^2 \rho_s (1 - H) \right)_5 \right] + \\
+ \left( v_i^2 \rho_i H \right)_2 + \left( v_i^2 \rho_i H \right)_6 - \left( v_i^2 \rho_i H \right)_1 - \left( v_i^2 \rho_i H \right)_5 + \frac{F}{2\Delta t} \left[ \left( v_s \rho_s (1 - H) \right)_5 + \left( v_s \rho_s (1 - H) \right)_6 - \\
- \left( v_s \rho_s (1 - H) \right)_1 - \left( v_s \rho_s (1 - H) \right)_2 + \left( v_s \rho_i H \right)_5 + \left( v_s \rho_i H \right)_6 - \left( v_s \rho_i H \right)_1 - \left( v_s \rho_i H \right)_2 \right] - \\
- \frac{P_s - P_6}{\Delta x} + 0.25 \left[ \left( \frac{\Delta p}{\Delta x} \right)_1 + \left( \frac{\Delta p}{\Delta x} \right)_2 + \left( \frac{\Delta p}{\Delta x} \right)_5 + \left( \frac{\Delta p}{\Delta x} \right)_6 \right]_{fri} - \\
-0.25 \left[ \left( \frac{\Delta p}{\Delta x} \right)_1 + \left( \frac{\Delta p}{\Delta x} \right)_2 + \left( \frac{\Delta p}{\Delta x} \right)_5 + \left( \frac{\Delta p}{\Delta x} \right)_6 \right]_{ev} \hspace{1cm} (4.35)
\]

This is practically the same formulation used by Nickens(1985), and Santos(1989).

The calculation of the flow properties at point 6 requires an iterative process which uses the known flow properties at points 1, 2 and 5. The process consists of the following steps:

a) Assume an initial in-situ liquid velocity at point 6

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b) Calculate the liquid hold up through Equation (4.32) and determine the flow regime.

c) Calculate the in situ gas velocity

d) Calculate the gas density at point 6 using Equation (4.33)

e) Calculate the pressure at point 6 using Equation (4.25). Use the z value calculated for pressure at point 5.

f) With the flow properties calculated at point 6 and the assumed in situ liquid velocity, calculate the pressure at point 6 using Equation (4.35).

g) Compare the pressures calculated in (e) and (f). If they are within an acceptable range of tolerance, the process is over and the properties at point 6 are determined. If not, assume another in situ liquid velocity and repeat the process.

If there is more than one grid, the process is repeated for the adjacent downstream grid with the properties at point 6 of the previous cell becoming the flow properties at point 5.

4.3. The induced fracture model

The fractured induction model used in the computer model is based on the assumption of an infinite plate with a circular hole in it. Poisson's ratio, Biot's constant, horizontal matrix stresses and vertical matrix stresses may be estimated from fracture field data or correlated from a particular field.

In this work, compression is represented as positive and tension as negative. Therefore, tensile strength of the formation \( S_t \) is then a negative number.

4.3.1. Vertical Fracture Initiation

Vertical fracture starts when the maximum effective tangential stress \( \sigma_\theta \) exceeds the tensile strength of the formation \( S_t \). Then fracture initiation occurs when \( \sigma_\theta = S_t \), so:

\[
S_t = \left( \frac{2v}{1-v} \right) \sigma_z + 2P_o - P_w + \alpha \left( \frac{1-2v}{1-v} \right) (P_r - P_s) - P_s \tag{4.36}
\]
The derivation of all equations concerning fracture initiation is detailed in Appendix A.

For a penetrating type of fluid the pore pressure ($P_p$) at the borehole wall is equal to the wellbore pressure, or $P_p = P_w$. Substituting this expression in the last equation and solving for $P_w$ leads to:

$$P_f = P_w + \left( \frac{2v}{1-v} \sigma_i - S_i \right) \left( 2 - \frac{(1-2v)}{1-v} \right)$$

(4.37)

For a non penetrating type of fluid the pore pressure ($P_p$) at the borehole wall is the original formation pore pressure, or $P_p = P_a$. From equation (4.36), after substituting $P_p$ for $P_a$:

$$P_f = P_w + \left( \frac{2v}{1-v} \sigma_i - S_i \right) P_a$$

(4.38)

4.3.2. Horizontal Fracture Initiation

Horizontal fractures start when the maximum effective vertical stress ($\sigma_z$) exceeds the tensile strength of the formation ($S_i$). In other words, the fracture initiation occurs when $\sigma_z = S_i$ or:

$$S_i = \sigma_z + P_a - P_p + \left( \frac{1-2v}{1-v} \right) (P_p - P_a)$$

(4.39)

For a penetrating fluid $P_p = P_w$ and after substituting:
Substituting for a non-penetrating fluid $P_p = P_s$ leads to:

$$S_i = \sigma_i$$

which is just the condition for horizontal fracture initiation. Theoretically, a truly non-penetrating fluid in a perfectly vertical borehole could not initiate a horizontal fracture because there could be no vertical component of stress generated by the borehole fluid pressure. However, it can be induced if end effects occur at the well bottom or if joints have separated sufficiently to allow the entrance of flow. In addition, the pressure at the wellbore should be at least that of the overburden, then:

$$P_f = P_w = S_i$$

The equations for $P_w$ give the initiation criterion for vertical and horizontal fractures. Whether the actual fracture is horizontal or vertical, it is determined in the computer model by analysis of the fracture-pressure values calculated with the equations for $P_w$ for the type of fluid used. The fracture associated with the smaller $P_w$ is the one initiated. The computer model only considers the expansion for vertical fracture because this occurs in almost all cases in depths greater than 500 meters. The cases of shallow formations will not be studied here. Therefore, the program does not analyze cases where a horizontal fracture occurs.
4.3.3. Fracture Expansion Model

The main objective of the model is to determine the wellbore pressure at the fractured formation in each time step. Therefore, the determination of the flow rate that is being injected into the fracture is essential to analyze the expansion of the fracture. This is done by assuming the fracture propagates in two wings, so the flow rate is equal to half of the flow rate calculated in the last cell of the wellbore model.

Although models such as Geertsm a and de Klerk (1969), Nordgren (1972), Khristianovic and Zheltov (1955) and Perkins and Kern (1961) gave good results to predict the geometry of fracture in some cases, they just consider that the fracture expands in two dimensions. This can lead to miscalculations in cases where the fracture advances in three dimensions as shown in the literature review.

Also, 3-D models need parameters that are very difficult to measure. The comparison of predicted results among the existing models showed large variation in the results. In addition, the computer time to run a 3-D model is much larger than when compared with 2-D models. This becomes another limitation for this kind of model because in the main program the 3-D model is coupled with a reservoir and a wellbore model.

On the other hand, a pseudo 3-D model considers the expansion of the fracture in three dimensions. This model needs only few parameters which can be obtained from field data. The predicted results are realistic when compared with some 3-D models. A limitation of this model occurs when the stress contrast between the fractured formation and the bounding layers is small, with a prediction of unstable and unrealistic vertical fracture migration.

The unstable vertical fracture migration can be prevented by a vertical pressure gradient in the equations that predicts the fracture height. Due to those facts the author decided to develop a pseudo-3D model to analyze the fracture.
The governing equations on which the pseudo-3D models are based had to be modified to consider the compressible nature of the fluid, because in an underground blowout the fluid is a mixture of a gas and a liquid phase such as mud or water. As stated by Clark and Perkins (1980) in the case of a blowout when the formation fluid flows to the surface, the pressure is reduced, additional gas comes out of the solution and the flowing stream accelerates. Thus the two-phase flowing mixture passes from bubble flow, through slug and transitional flow, and finally it goes into the mist flow regime. In the case of using a dynamic kill method to control an underground blowout, the most probable regime will be the bubble flow because of the high flow rate of mud or water used. Even so, the wellbore model calculates the flow pattern. The fracture model also assumes that the liquid hold up of the two-phase flow inside the fracture in each time step is the same as that calculated in the last cell of the wellbore model, therefore it assumes that there is no slip or fluid segregation inside the fracture.

The pseudo-3D model developed in this study assumes linear elasticity and uniform in-situ stress within each layer, so the width is calculated by the same equations given in the literature review:

\[ w_t(x,y) = w_i - w_{ii} - w_{iii} + w_{iv} - w_v \] .........................................................(4.43)

where:

\[ w_i = \frac{4(P(x,t) - \sigma_1)}{E'} \sqrt{a^2 - y^2} \] .........................................................(4.44)

\[ w_{ii} = \frac{4(\sigma_2 - \sigma_1)}{E' \pi} \left( -(b_2 - y) \cosh^{-1} \left( \frac{a^2 - b_2 y}{b_2} \right) + \cos^{-1} \left( \frac{b_2}{a} \right) \sqrt{a^2 - y^2} \right) \] .........................................................(4.45)
\[ w_{ll} = \frac{4(\sigma_3 - \sigma_1)}{E'\pi} \left( - (b_3 + y) \cosh^{-1} \left( \frac{a^2 + b_3 y}{a|y + b_3|} \right) + \cos^{-1} \left( \frac{b_3}{a} \sqrt{a^2 - y^2} \right) \right) \] .................................(4.46)

\[ w_{lv} = \frac{2}{E'} g_{pyy} \sqrt{a^2 - y^2} \] .................................................................(4.47)

\[ w_{lv} = \frac{2}{E'} g_{py} \sqrt{a^2 - y^2} \] .................................................................(4.48)

and

\[ E' = \frac{E}{(1 - v^2)} \] .................................................................(4.49)

The height of the fracture is determined in terms of critical stress intensity factor at the top of the crack through the equation:

\[ K_{\text{top}} = \frac{1}{\sqrt{\pi a}} \int_a^y \Delta P(x, y, t) \sqrt{a+y} \sqrt{a-y} dy \] .................................................................(4.50)

where:

\[ \Delta P(x, y, t) = P(x, t) - g_{py} - \sigma_3 + g_{py} + g_y \text{ for } -a \leq y < -b_3 \] .................................................................(4.51)

\[ \Delta P(x, y, t) = P(x, t) - g_{py} - \sigma_1 + g_{py} + g_y \text{ for } -b_3 \leq y < 0 \] .................................................................(4.52)
\[ \Delta P(x,y,t) = P(x,t) - g_p y - \sigma_1 + g_\sigma y - g_v y \text{ for } 0 \leq y < b_2 \] \hspace{1cm} (4.53)

\[ \Delta P(x,y,t) = P(x,t) - g_p y - \sigma_2 + g_\sigma y - g_v y \text{ for } b_2 \leq y < a \] \hspace{1cm} (4.54)

and at the bottom by:

\[ K_{\text{bottom}} = \frac{1}{\sqrt{\pi a}} \int_{-a}^{a} \Delta P(x,y,t) \sqrt{\frac{a+y}{a-y}} dy \] \hspace{1cm} (4.55)

where:

\[ \Delta P(x,y,t) = P(x,t) - g_p y - \sigma_2 + g_\sigma y + g_v y \text{ for } -a \leq y < -b_2 \] \hspace{1cm} (4.56)

\[ \Delta P(x,y,t) = P(x,t) - g_p y - \sigma_1 + g_\sigma y + g_v y \text{ for } -b_2 \leq y < 0 \] \hspace{1cm} (4.57)

\[ \Delta P(x,y,t) = P(x,t) - g_p y - \sigma_1 + g_\sigma y - g_v y \text{ for } 0 \leq y < b_3 \] \hspace{1cm} (4.58)

\[ \Delta P(x,y,t) = P(x,t) - g_p y - \sigma_3 + g_\sigma y - g_v y \text{ for } b_3 \leq y \leq a \] \hspace{1cm} (4.59)

The vertical pressure gradient \((g_v)\) is determined under the assumption that pressure in any cross section decreases in the direction of the tips in proportion to the pressure gradient for the lateral flow. This is the main point in a pseudo-3D model to
avoid unstable vertical fracture migration. The author assumed that this proportion depends on the ratio of height to length growth rate, or in equation form:

\[ g_v = \frac{a_t - a_{t-1}}{dx} \frac{P(x,t) - P_L}{L} \] ..........................(4.60)

where \( a_t \) and \( a_{t-1} \) are the half height of the cross section at current and previous time steps respectively and \( P_L \) is the pressure differential required to open the fracture at the crack front.

In the situation of uniform in-situ stress within each layer the direct integration of equations (4.50) and (4.55) results in:

\[ K_{lc} = K_{l1} - K_{l2} - K_{l3} + K_{l4} - K_{l5} - K_{l6} \] ..........................(4.61)

where:

\[ K_{l1} = (P(x,t) - \sigma_1)\sqrt{\pi a} \] ..........................(4.62)

\[ K_{l2} = \frac{(\sigma_2 - \sigma_1)\sqrt{\pi a}}{\sqrt{\pi}} \left( \cos^{-1}\left(\frac{b_2}{a}\right) + f_a \frac{\sqrt{a^2 - b_2^2}}{a} \right) \] ..........................(4.63)

\[ K_{l3} = \frac{(\sigma_3 - \sigma_1)\sqrt{\pi a}}{\sqrt{\pi}} \left( \cos^{-1}\left(\frac{b_3}{a}\right) - f_a \frac{\sqrt{a^2 - b_3^2}}{a} \right) \] ..........................(4.64)

\[ K_{l4} = \frac{a}{2} \frac{f_a g_p \sqrt{\pi a}}{\sqrt{\pi}} \] ..........................(4.65)

\[ K_{l5} = \frac{a}{2} \frac{f_a g_p \sqrt{\pi a}}{\sqrt{\pi}} \] ..........................(4.66)
The final solution (4.61) presents two equations, one for the upper and other for the lower tip. These two equations together with an additional geometry constraint of:

\[ b_3 = h_f - b_2 \] ...............................(4.68)

give the solution for \( a, b_2, \) and \( b_3. \)

The fluid flow in a pseudo-3D model is considered as being one-dimensional flow along the fracture length. The governing equations are basically the continuity equation for a compressible flow and the pressure gradient equation.

The continuity equation which is developed in Appendix B is given by:

\[
\frac{\partial}{\partial x} \left( \rho_{\text{mix}}(x,t)Q(x,t) \right) = \rho_{\text{mix}}(x,t)Q_L(x,t) + \frac{\partial \rho_{\text{mix}}(x,t)A_{cr}(x,t)}{\partial t} \] ...............................(4.69)

where

\[ A_{cr}(x,t) = \int_{-a}^{a} w(x,y,t) dy \] ...............................(4.70)

\[ Q_L(x,t) = \frac{4Ca}{\sqrt{t - \tau(x)}} \] ...............................(4.71)
The thickness of the fractured formation is used instead of the height of fracture in equation (4.71) for cases where the bounding layers are impermeable.

Integration of equation (4.70) and the use of equation (4.43) allow equation (4.69) to be written as:

$$-\frac{\partial (\rho_{\text{mix}}(x,t)Q(x,t))}{\partial x} = \rho_{\text{mix}}(x,t)Q_e(x,t) + \frac{2\pi \partial \rho_{\text{mix}}(x,t)a^2P(x,t)}{E'}$$

The pressure gradient equation is derived from equation (2.67) by applying the concept of apparent viscosity, which means $n'$ is equal to unity and $K$ equal to the apparent viscosity of the mixture. Such modifications in equation (2.67) yields:

$$\frac{\partial P}{\partial x} + \frac{12\mu_{\text{mix}}}{w^3} q_s = 0$$

and after integrating $q_s$ over the fracture height gives:

$$Q(x,t) = \int_{\alpha}^{\beta} \left[ w^3(x,y,t) \frac{\partial P(x,t)}{\partial x} \right] dy$$

This equation can be solved for the pressure gradient to obtain:

$$\frac{\partial P(x,t)}{\partial x} = \frac{12\mu_{\text{mix}}Q(x,t)}{\int_{\alpha}^{\beta} w^3(x,y,t)dy}$$

This equation can also be obtained by using the Fanning equation for two-phase fluid in each vertical cell of the fracture, with further integration over the fracture height.
The two-phase fluid viscosity or apparent viscosity is given as stated in Brill and Beggs (1978) by:

$$\mu_{\text{mix}} = H_t \mu_{\text{apl}} + (1 - H_t) \mu_g$$

(4.76)

where the liquid apparent viscosity is calculated through:

$$\mu_{\text{apl}} = \mu_l + 5.441 \times 10^{-5} \frac{\tau_A A_L^2}{\text{per} Q_l}$$

(4.77)

and $\text{per}$ is the perimeter of the fracture cross section.

The boundary conditions of equations (4.72) and (4.75) are given by:

$$Q(0, t) = \frac{q_i(t)}{2}$$

(4.78)

$$\Delta P(L(t)/2, t) = P_L$$

(4.79)

The value of $P_L$ is calculated by assuming that the fracture height at its front is equal to the height of the fractured formation. So from equation (4.61):

$$P_L = \frac{K_n}{\sqrt{\pi a}} + \sigma_1$$

(4.80)

The pseudo-3D equations were solved by advancing the fracture front at a distance $\Delta L$ during an assumed time step $\Delta t$ and by integrating the two flow equations by Runge-Kutta method after substituting the term involving $\frac{\partial p_{\text{mix}}(x, t)}{\partial t}$ by the difference relation.

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\[ \frac{\partial p_{\text{mix}}(x,t)a^2P(x,t)}{\partial t} = \frac{a^2 p_{\text{mix}}(x,t+\Delta t)P(x,t+\Delta t) - p_{\text{mix}}(x,t)P(x,t)}{\Delta t} \] \hspace{1cm} (4.81)

where \( \bar{a} \) corresponds to the average half height of the fracture between the instant \( t \) and \( t + \Delta t \) respectively.

The value of \( Q(0,t) \) obtained is compared with \( q(t)/2 \), and if they do not agree, another value for \( \Delta t \) is assumed. By an iterative process the calculation is repeated until the values come within an acceptable range.

This calculation procedure, differently from other pseudo-3D models, allows the calculation of the leak-off coefficient with equation (3.8) for each cell instead of using an average leak-off coefficient for all fracture extension.

<table>
<thead>
<tr>
<th>Table 4.1. Summary of Input Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Formation Properties</strong></td>
</tr>
<tr>
<td>Young's Modulus, psi</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
</tr>
<tr>
<td>Fracture Toughness, psi√in</td>
</tr>
<tr>
<td>Pay Zone Height, ft</td>
</tr>
<tr>
<td><strong>Fluid Properties</strong></td>
</tr>
<tr>
<td>Fluid Consistency Index, lbf sec^2 / ft^2</td>
</tr>
<tr>
<td>Flow Behavioral Index</td>
</tr>
<tr>
<td>Fluid Loss Coefficient, ft / √min</td>
</tr>
<tr>
<td>Spurt Loss, gal / ft^3</td>
</tr>
<tr>
<td><strong>Pumping Scheme</strong></td>
</tr>
<tr>
<td>Pumping Rate, bbl / min</td>
</tr>
<tr>
<td>Total Fluid Volume, bbl</td>
</tr>
<tr>
<td><strong>In-Situ Stress</strong></td>
</tr>
<tr>
<td>Upper Stress Contrast, psi</td>
</tr>
<tr>
<td>Lower Stress Contrast, psi</td>
</tr>
<tr>
<td>Stress Gradient, psi / ft</td>
</tr>
</tbody>
</table>

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The author compared the prediction of a pseudo-3D model when a vertical pressure gradient is used with a 3D model for a case where the stress of a boundary layer is close to that of the fractured formation. The reason for choosing this case was the fact that the pseudo-3D model does not work well when the stresses of boundary layers are close to the fractured formation stress. The summary of input data as well as the 3D model results were obtained from Morales (1989) and the input data can be seen in table 4.1.

Table 4.2 gives the prediction for a 3D model and for a pseudo-3D model in a fracture treatment with the data from Table 4.1.

<table>
<thead>
<tr>
<th></th>
<th>3D</th>
<th>Pseudo-3D</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half-Length, ft</td>
<td>280</td>
<td>280</td>
<td>0</td>
</tr>
<tr>
<td>Fracture Pressure, psi</td>
<td>177</td>
<td>175</td>
<td>1</td>
</tr>
<tr>
<td>Fluid Volume Pumped, bbl</td>
<td>358</td>
<td>375</td>
<td>4.7</td>
</tr>
<tr>
<td>Time of treatment, min</td>
<td>23.9</td>
<td>25.0</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Figure 4.2 shows the comparison of net pressure at wellbore between this pseudo-3D model and a 3D solution in function of time. Net pressure means the difference between the fracture pressure and the formation pressure. It can be seen that the pseudo-3D agrees very well with a 3D model after 10 minutes. This was expected because at early stages the 3D model does not consider the fracture height equal to the fractured formation thickness as the pseudo-3D model does, so smaller pressures will exist in a pseudo-3D model at early times.

One other point to be considered is that the pseudo-3D model with a vertical pressure gradient is the best alternative among the three kinds of models used in fracture prediction because it gives good prediction with less computer running time. This is very important because the wellbore model is also time consuming, and the simulation of an underground blowout would require a main frame if the simulation...
is long. The use of personal computers to predict pressure and flow rate in an underground blowout is better than the use of main frames due to the availability of those computers in any place.

![Graph showing comparison between 3D and Pseudo-3D Models](image)

**Fig. 4.2. Comparison of Net Pressure with a 3D solution**

4.4 - Global Calculation Procedure

This procedure achieves the coupling among three sub models: the reservoir, the wellbore, and the fracture sub models. The procedure assumes that the pressures and liquid holdup are common to contiguous models and cells and the mass flow rate is conserved.

The first step of the procedure is the calculation of the fracture initiation pressure with equations (4.36) or (4.37) for vertical fracture and equations (4.39) or (4.41) for horizontal fracture. If the pressure for horizontal fracture is less than the...
pressure for vertical pressure, the calculation procedure will stop because the model is valid only for the case of vertical fractures, as explained before. If this is not the case, the calculation procedure will continue with the assumption that the vertical fracture initiates at the same moment as the two-phase leading edge reaches the fractured formation.

Due to this assumption, it is necessary to calculate the variables required in the fracture model at the moment the fracture starts. For that, the procedure is to calculate the properties of each cell in the annulus within each time step until the two-phase fluid reaches the fractured formation. This is done by the wellbore model that simulates a circulation of mud and gas from the moment the influx started using the pressure at the fractured formation equal to the fracture initiation pressure. Once the two-phase fluid reaches the fractured formation, the fluid properties of the last cell in the wellbore are used in the fracture model. The propagation process then starts.

The algorithm for this calculation consists of the following steps:

a) Assume the liquid/mixture interface position and calculate the time increment by dividing the grid length by the mixture leading edge velocity for the previous time step.

b) Assume a bottom hole pressure and determine the other boundary conditions at bottom hole.

c) Determine the gas flow rate using the reservoir flow model

d) Determine the pressure drop throughout the annulus

e) Add the pressure drop to the fracture initiation pressure to determine the bottom hole pressure

f) Compare the assumed and calculated pressure values. If they are within an acceptable range, repeat the process for the next time step. If not, assume another bottom hole pressure and repeat the process.
After the two-phase leading edge reaches the fractured formation, the program starts simulating the fracture propagation and the underground blowout. This consists of the following steps:

a) With the total flow rate calculated in the last cell when the two-phase leading edge reaches the fractured formation, calculate the time step and pressure change for an assumed increment in the length of the fracture.

b) With the wellbore model using the same time step and pressure change calculated in the previous item; calculate the bottom hole pressure, the gas flow rate with the reservoir model and the pressure drop in the annulus.

c) Determine the total flow rate in the cell at the fracture formation and repeat the calculation from item (a).

d) Continue the process until the variation of pressure is negligible. The simulation for this time step is now over and the process is repeated for the next time step.

This procedure gives the variation of fracture pressure and gas flow rate produced as a function of time for the assumed mud flow rate in an underground blowout. The calculation is repeated for different mud flow rates until the appropriate rate to control the underground blowout is determined. As it can be seen in the comparison of the results, this new procedure gives different results than those calculated in the previous models. This can completely change the planning of an underground blowout.
CHAPTER V

COMPUTER PROGRAM AND NUMERICAL SIMULATIONS

This chapter shows the implementation of the calculation procedures discussed in chapter IV in a computer program. It also shows three different sets of input data. The results of the simulation and a comparison between the proposed model with an existing model are shown in a graphical analysis.

5.1. Computer Program

A FORTRAN program was written to implement the global procedure discussed in chapter IV.

The main feature of the program is the algorithm that uses an iterative process to determine the corrected bottom hole pressure after the assumed and calculated pressures are iterated to within an acceptable range. All flow variables for the time step under consideration are calculated in the wellbore, reservoir, and fracture models. This enables them to be known at any location of the system.

Once the calculation in the current time step is complete, the program updates the flow variables by replacing previous flow variables with the current ones. The program continues the calculation for the next time step, where the algorithm is repeated. The number of time steps is chosen by the user.

The program also allows the division of the wellbore into a finite number of cells, as desired by the user. In this way the error in the iterative process becomes smaller as the number of cells becomes larger. On the other hand, the running time of the program can be impractical if the number of cells is very large.

The choice of the incremental fracture growth in length should be as small as possible to avoid errors due to leak-off in each cell. A convenient choice is to use an incremental in length equal to one foot.
5.2. Input Data for a Deep Well

Table 5.1 shows the input data for a simulated case in a deep offshore well where the induced fracture occurred when circulating a kick at 30 spm with a triplex pump. The formation is considered permeable in all simulations.

### Table 5.1. Input Data for Well A-1

<table>
<thead>
<tr>
<th>Well A-1 Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Hole Depth, ft</td>
<td>16,000</td>
</tr>
<tr>
<td>Sea Water Depth, ft</td>
<td>500</td>
</tr>
<tr>
<td>Wellbore Diameter, in</td>
<td>8.5</td>
</tr>
<tr>
<td>Drill Pipe Outside Diameter, in</td>
<td>5</td>
</tr>
<tr>
<td>Drill Collar Outside Diameter, in</td>
<td>6 3/4</td>
</tr>
<tr>
<td>Drill Collar Length, ft</td>
<td>300</td>
</tr>
<tr>
<td>Bottom Hole Temperature, °F</td>
<td>240</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reservoir Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Pressure, psi</td>
<td>10,816</td>
</tr>
<tr>
<td>Porosity, dimensionless</td>
<td>0.20</td>
</tr>
<tr>
<td>Thickness, ft</td>
<td>55</td>
</tr>
<tr>
<td>Permeability, miliDarcy</td>
<td>150</td>
</tr>
<tr>
<td>Skin Factor, dimensionless</td>
<td>0</td>
</tr>
<tr>
<td>Gas Density, dimensionless</td>
<td>0.65</td>
</tr>
<tr>
<td>Initial Water Saturation, dimensionless</td>
<td>0.20</td>
</tr>
<tr>
<td>Temperature, °F</td>
<td>250</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fractured Formation Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth, ft</td>
<td>12,000</td>
</tr>
<tr>
<td>Thickness, ft</td>
<td>75</td>
</tr>
<tr>
<td>Stress in Upper Bounding Zone, psi</td>
<td>8,700</td>
</tr>
<tr>
<td>Fracture Initiation Pressure, psi</td>
<td>7,540</td>
</tr>
<tr>
<td>Stress in Lower Bounding Zone, psi</td>
<td>8,900</td>
</tr>
<tr>
<td>Stress Intensity Factor, psi ft(^{0.5})</td>
<td>144</td>
</tr>
<tr>
<td>Poisson's Ratio, dimensionless</td>
<td>0.20</td>
</tr>
<tr>
<td>Young's Modulus, psi</td>
<td>1,400,000</td>
</tr>
</tbody>
</table>
The fluid used in the simulation was a 9.0 ppg mud with the plastic viscosity of 10 cp and yield point of 25 lbf/100 sqft. The mud properties were chosen based on the mud used in the experiments and on the method used to control an underground blowout. Normally light weight mud is used in a dynamic kill method due to the high flow rates used in such method.

The simulation was performed for different flow rates until one reached a bottom hole pressure equal to or greater than the reservoir pressure. This particular flow rate was the adequate to control the simulated underground blowout caused by an induced fracture.

All runs for different flow rates used a maximum half fracture extension equal to 140 ft in the simulation. The choice for equal length in fractures for all cases made it possible to observe the effect of gas void fraction in fracture propagation velocity.

Figure 5.1 shows the profile of bottom hole pressure as a function of time in a simulation for a deep well with the proposed model. The bottom hole pressure is higher for larger mud flow rates as expected in a dynamic kill method, but the pressure profile as a function of time is different for lower mud flow rates (200 and 400 gpm). For example, in the case of 200 and 400 gpm mud flow rate the pressure profile first reaches a peak and decreases further with a trend for stabilization. On the other hand, the pressure profile always increases for flow rates of 800 and 1,200 gpm, with a trend for stabilization as the fracture extends further from the wellbore.

This can be explained by the fact the gas void fraction is inversely proportional to the mud flow rate. The bottom hole pressure depends on the gas void fraction in the annulus as well as the pressure in front the fracture. So the bottom hole pressure profile will decrease for those cases until stabilization is reached.
The same conclusion can be achieved in figure 5.2 where the gas flow rate is higher for lower values of mud flow rate, as expected in a dynamic kill method.

The gas flow rate profile shows that for lower values of mud flow rate (200 and 400 gpm) it first decreases to a minimum value, then increases and finally reaches a period of slight decrease in the gas rate. The gas flow rate profile for larger mud flow rates only decreases. This fact was expected due to the larger pressure losses that occur in the wellbore annulus when using larger mud flow rate to control the well.
Figure 5.2 shows the fracture pressure at wellbore. Several points should be noted. First, the pressure profile is independent of the mud flow rate until a minimum value is reached. The reason is that the total flow at early stages going through the fracture has a gas void fraction calculated for the flow rate used in the kick control, in this case 30 spm or 150 gpm. After the fluid with the dynamic kill flow rate begins arriving in the fracture, the pressure increases depending on the dynamic kill flow rate. As it was expected, the larger the flow rate the larger will be the pressure into the fracture, when compared at the same extension length (140 ft).

Another very important point to be observed in this simulation is the fact that the velocity of propagation is higher for higher gas void fraction. Figure 5.3 indicates the fracture reached the length of 140 ft much faster for a 200 gpm flow rate than for 1,200 gpm. Therefore, the mechanism of fracturing process in an underground
blowout is similar to a fracture treatment with foams, where the propagation depends on gas void fraction.

![Fracture Pressure at Wellbore](image)

**Fig. 5.3. Fracture Pressure at Wellbore for a Deep Well**

The simulation with the proposed model showed that this underground blowout could be controlled if using a mud flow rate little higher than 1,200 gpm. The bottom hole pressure reached a value of 10,815 psi, which is practically the reservoir pressure, when simulating the underground blowout with this flow rate.

Figure 5.4 shows the bottom hole pressure calculated using the current and the proposed models in a dynamic kill method for a mud flow rate of 1,200 gpm. It can be seen that, even in the case of a deep well, the flow rate necessary to control an underground blowout is less than the one calculated with the current models. The current models calculate a lower bottom hole pressure. The flow rate necessary to
control the simulated underground blowout will be 1,300 gpm instead of the 1,200 gpm flow rate calculated under the proposed model.

![Comparison Between The Proposed and Current Models](image)

Fig. 5.4. Comparison Between The Current and The Proposed Models

5.3. Input Data for Medium Depth

Table 5.2 shows the input data for the case of an offshore well at a depth of 9500 ft where the induced fracture occurred when circulating a kick at 30 spm. The main difference between the previous case and this case is the length between the fractured formation and the bottom hole depth.

Also in this case, the fluid used in the simulation was a 9.0 ppg mud with plastic viscosity of 10 cp and yield point of 25 lbf/100 sqft. This choice was based on the experimental work and on the method used in the control of the underground blowout.
Table 5.2. Input Data for Well A-2

<table>
<thead>
<tr>
<th>Well A-2 Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Hole Depth, ft</td>
<td>9,500</td>
</tr>
<tr>
<td>Sea Water Depth, ft</td>
<td>500</td>
</tr>
<tr>
<td>Wellbore Diameter, in</td>
<td>8.5</td>
</tr>
<tr>
<td>Drill Pipe Outside Diameter, in</td>
<td>5</td>
</tr>
<tr>
<td>Drill Collar Outside Diameter, in</td>
<td>6 3/4</td>
</tr>
<tr>
<td>Drill Collar Length, ft</td>
<td>300</td>
</tr>
<tr>
<td>Bottom Hole Temperature, °F</td>
<td>190</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reservoir Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Pressure, psi</td>
<td>6,080</td>
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<tr>
<td>Porosity, dimensionless</td>
<td>0.25</td>
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<tr>
<td>Thickness, ft</td>
<td>35</td>
</tr>
<tr>
<td>Permeability, miliDarcy</td>
<td>160</td>
</tr>
<tr>
<td>Skin Factor, dimensionless</td>
<td>0</td>
</tr>
<tr>
<td>Gas Density, dimensionless</td>
<td>0.65</td>
</tr>
<tr>
<td>Initial Water Saturation, dimensionless</td>
<td>0.20</td>
</tr>
<tr>
<td>Temperature, °F</td>
<td>200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fractured Formation Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth, ft</td>
<td>7,870</td>
</tr>
<tr>
<td>Thickness, ft</td>
<td>55</td>
</tr>
<tr>
<td>Stress in Upper Bounding Zone, psi</td>
<td>5,930</td>
</tr>
<tr>
<td>Fracture Initiation Pressure, psi</td>
<td>4,640</td>
</tr>
<tr>
<td>Stress in Lower Bounding Zone, psi</td>
<td>5,930</td>
</tr>
<tr>
<td>Stress Intensity Factor, psi. ft(^{0.5})</td>
<td>216</td>
</tr>
<tr>
<td>Poisson's Ratio, dimensionless</td>
<td>0.20</td>
</tr>
<tr>
<td>Young's Modulus, psi</td>
<td>2,100,000</td>
</tr>
</tbody>
</table>

Figure 5.5 shows the bottom hole pressure which occurs in this case for different mud flow rates. The same observations done in the previous case are pertinent for this one, although the well in this case is shallower than the other. It can be noted in this figure, with exception of the 400 gpm mud flow rate, that the pressure profile

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increases faster at early stages of the fracture process. It becomes slower as the fracture extends further from the wellbore. Also in this case, the velocity of propagation is higher for cases where the gas void fraction is higher.

Figure 5.5 shows the gas flow rate produced in the control of this underground blowout using the dynamic kill method with different mud flow rates. The same conclusions as before can be verified in this figure, with exception that the gas flow rate profile always decreased during the fracture expansion. This decrease was faster in the beginning of the process, then slowing down with the extension of the fracture. One of the reasons for that is the pressure in front of the fracture has more influence on the reservoir than the previous case due to the smaller length between the fractured formation and the reservoir for this case. The shorter length causes...
smaller pressure loss in the annulus, so the effect of the fracture pressure will be apparent sooner than the previous case. As the bottom hole pressure depends on losses in the annulus and on the pressure in front of the fracture, the effect of the pressure change in front of the fracture will be more predominant for smaller lengths between the fractured formation and the reservoir.

Fig. 5.6. Gas Flow Rate for a Medium Depth

Figure 5.7 shows the fracture pressure at the wellbore for this case, and the same points observed before can be repeated here. First, the pressure profile is the same independent of the mud flow rate until a minimum value is reached. This is because the fluid that initializes propagating the fracture has a gas void fraction calculated with the flow rate used in the kick control, 30 spm or 150 gpm. After
reaching this minimum value, the pressure in front of the fracture increases, with this increase depending on the mud flow rate used in the dynamic kill method.

The same observations related to the fracture propagation velocity can be verified for this case, where the velocity is higher for higher gas void fraction.

![Fracture Pressure at Wellbore](image)

Fig. 5.7. Fracture Pressure at Wellbore for a Medium Depth

The simulation with the proposed model showed that this underground blowout can be controlled if using a mud flow rate of 1,300 gpm, reaching the reservoir pressure with an extension of 141 ft.

Figure 5.8 shows the bottom hole pressure calculated in this simulated case using the current and the proposed models in a dynamic kill method for a mud flow rate of 1,300 gpm.
Fig. 5.8. Comparison Between The Current and Proposed Models

It can be seen that the flow rate necessary to control the underground blowout with the proposed model is smaller than the one calculated with the current models. The current models calculate a lower bottom hole pressure. The appropriate mud flow rate calculated by these models to control this underground blowout is 1,760 gpm instead of the 1,300 gpm flow rate calculated using the proposed model.

This case shows the importance of the use of the proposed model as the appropriate mud flow rate is very different between the two models. It can be noted that in cases where the fracture formation is closer to the reservoir formation, the difference between the two models is larger.
5.4. Input Data for Shallow Depth

Table 5.3 shows the input data for a case of an offshore well where the induced fracture occurred at a depth of 6700 ft.

<table>
<thead>
<tr>
<th>Table 5.3. Input Data for Well A-3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Well A-3 Characteristics</strong></td>
</tr>
<tr>
<td>Bottom Hole Depth, ( ft )</td>
</tr>
<tr>
<td>Sea Water Depth, ( ft )</td>
</tr>
<tr>
<td>Wellbore Diameter, ( in )</td>
</tr>
<tr>
<td>Drill Pipe Outside Diameter, ( in )</td>
</tr>
<tr>
<td>Drill Collar Outside Diameter, ( in )</td>
</tr>
<tr>
<td>Drill Collar Length, ( ft )</td>
</tr>
<tr>
<td>Bottom Hole Temperature, (^\circ F)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Reservoir Characteristics</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Pressure, ( psi )</td>
</tr>
<tr>
<td>Porosity, ( \text{dimensionless} )</td>
</tr>
<tr>
<td>Thickness, ( ft )</td>
</tr>
<tr>
<td>Permeability, ( \text{miliDarcy} )</td>
</tr>
<tr>
<td>Skin Factor, ( \text{dimensionless} )</td>
</tr>
<tr>
<td>Gas Density, ( \text{dimensionless} )</td>
</tr>
<tr>
<td>Initial Water Saturation, ( \text{dimensionless} )</td>
</tr>
<tr>
<td>Temperature, (^\circ F)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Fractured Formation Characteristics</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth, ( ft )</td>
</tr>
<tr>
<td>Thickness, ( ft )</td>
</tr>
<tr>
<td>Stress in Upper Bounding Zone, ( psi )</td>
</tr>
<tr>
<td>Fracture Initiation Pressure, ( psi )</td>
</tr>
<tr>
<td>Stress in Lower Bounding Zone, ( psi )</td>
</tr>
<tr>
<td>Stress Intensity Factor, ( psi \cdot ft^{0.5} )</td>
</tr>
<tr>
<td>Poisson's Ratio, ( \text{dimensionless} )</td>
</tr>
<tr>
<td>Young's Modulus, ( psi )</td>
</tr>
</tbody>
</table>
The induced fracture occurred when circulating a kick at 30 spm with a triplex pump. The differences between the previous cases and this one are the diameter of the well, which is bigger and the length between the fractured formation and the bottom hole depth which is shorter.

Also in this case, the fluid used in the simulation was a 9.0 ppg mud with plastic viscosity of 10 cp, and yield point of 25 lbf/100 sqft.

Figure 5.9 shows the bottom hole pressure profile that occurred in this situation for different mud flow rates. The dynamic kill method for this case requires higher mud flow rates than the previous ones due to the larger diameters of this well.

![Bottom Hole Pressure in Dynamic Kill Method](image)

**Fig. 5.9. Bottom Hole Pressure for a Shallow Depth**

It can be noted in figure 5.9 that the bottom hole pressure profile for each mud flow rate behaves similarly. The pressure profile increases at early stages of the
fracture process, then decreases and finally increases until it reaches a period of fair stabilization. This profile is very clear for a mud flow rate of 1,400 gpm, where it reaches stabilization when the fracture extension is little less than 140 ft. The same observations about the propagation velocity can be done in this case, where the velocity is higher for higher gas void fraction. The simulation of the fracture for a mud flow rate of 1,800 gpm stopped at half length of 132 ft instead of 140 ft because the bottom hole pressure became equal to the reservoir pressure at that point.

![Gas Flow Rate in Dynamic Kill Method](image)

**Fig. 5.10. Gas Flow Rate for a Shallow Depth**

Figure 5.10 shows the gas flow rate produced in controlling this underground blowout using the dynamic kill method with three different mud flow rates. As in the case of the bottom hole pressure profile, the profiles are also very similar among the
three different flow rates. This was expected because the gas flow rate depends on the bottom hole pressure.

This example showed more influence of the fracture pressure over the wellbore model because of two main reasons. The first one is the losses in the annulus are smaller due to the short length between the fractured formation and the reservoir formation. The second one was the diameter of well which decreased still more such losses, causing the fracture pressure to have this larger influence.

![Fracture Pressure at Wellbore](image)

**Fig. 5.11. Fracture Pressure at Wellbore for a Shallow Depth**

Figure 5.11 shows the fracture pressure profile at the wellbore for three different mud flow rates used in this case. It can be observed that the pressure profiles are almost the same until 170 seconds, independent of the mud flow rate. The reason for that is the fluid that started propagation of the fracture had a gas void fraction.
calculated with the flow rate used in the kick control. Another important observation is in the fracture pressure profile, which reached a minimum value before starting a gradual increase. The factor responsible for this is still the gas void fraction, as can be seen by comparing figure 5.10 with 5.11. These figures show that the minimum value of the fracture pressure profile occurred in an interval of time posterior to that when the gas flow rate profile reached its maximum value. This means the minimum value for each fracture pressure profile occurred at the maximum of the gas void fraction. After reaching its minimum value, the fracture pressure profile followed the trend verified in the other cases, depending on the mud flow rate used in the dynamic kill method.

The calculation using the proposed model showed that this simulated underground blowout could be controlled using a mud flow rate of 1,800 gpm. For this flow rate, the bottom hole pressure reached the value of the reservoir pressure when its half length had a value of 132 ft.

Figure 5.12 shows the bottom hole pressure calculated in the simulated case using the current and the proposed models for a mud flow rate of 1,800 gpm. The difference between the proposed and the current models is much larger in the case of shallower depth than the other ones. Consequently, the flow rate necessary to control this underground blowout was much less than calculated with the current models. The current models calculated a lower bottom hole pressure and it would give a mud flow rate of 3,200 gpm instead of the 1,800 gpm flow rate calculated under the proposed model.

This last case shows how important can be the influence of using a fracture model in planning a dynamic kill method. This importance increases as the pressure losses in the wellbore become smaller. The pressure losses are not large in the annulus when the extension of the wellbore between the fractured formation and the
reservoir is not long or when the well has a diameter equal to or larger than 12 1/4 inches.

This case shows that the dynamic kill method becomes less effective as the diameter of the well increases and high mud flow rates may be necessary to control the underground blowout. In such cases, the use of another method as a way to control the well may be necessary. This choice will depend on the equipment and on the particular conditions of the underground blowout.

![Comparison Between The Proposed and Current Models](image)

Fig. 5.12. Comparison Between The Current and The Proposed Models

These three examples have shown the cases where the proposed model presents different results than those from the current models. It can be stated that the pressure in front of the fractured formation changes with its propagation, and this change can have a large influence in the bottom hole pressure. This influence will be
larger for cases with smaller losses in the annulus between the reservoir and the fractured formation. This is typical for two cases: a large diameter in the well or a short length between the reservoir and the fractured formation, with the effect being larger as the diameter increases and the length between the formations decreases.

The comparison between the proposed model and field data was not possible because the lack of data for underground blowouts. The two underground blowouts with field data collected were presented by Walters (1991) as described in Chapter 1 and that presented by Flak and Gloger (1994). Both works mentioned the increase in the bottom hole pressure being caused by the injection of the fluid into a fractured formation. The simulation in such cases was not possible because of the lack of data concerning the formations being fractured and complex flow path in one of the cases, where an additional flow occurred between two casings before the fluid reaching the fractured formation.

Finally, the necessity of the use of the proposed model in planning the control of an underground blowout is obvious because of the difference in results when compared to models.
CHAPTER VI

SHUT-IN OR DIVERT A WELL

A well control operation increases in complexity when an influx occurs in shallow formations. The low fracture gradient in shallow formations associated with higher pressures caused by the influx can lead to underground blowouts or blowouts.

Reports about the two most used procedures, one in which the well is shut-in and another in which the well is diverted, have not shown a clear conclusion which situation each method should be used in.

The procedure of diverting a well can lead to dangerous situations with loss of surface equipment and even with personal injuries, because of the high rate of erosion caused by the influx flow. On the other hand, the procedure of shutting-in can cause craters.

Several studies have shown that many factors can affect the applicability of each procedure. Among them are the stress state in the fractured and bounding layers, rock properties, confining pressure, existence of faults in the shallow formations and fluid used in the well control.

The proposed model developed in this study can be used to verify the effect of the shut-in procedure in well control under the conditions of no-fault in the shallow formations and vertical induced fracture. So it is useful to determine if such methods can be applied to well control.

To show this feature of the proposed model six simulations with four different shallow formations; clay shale, green river shale, sandstone, and siltstone; were run to show how the characteristics of those formations can affect the decision to shut-in or divert the well. Table 6.1 shows the common input data used in all simulations.
Table 6.1. Principal Characteristics of the Simulated Well

<table>
<thead>
<tr>
<th>Well Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Hole Depth, ft</td>
<td>2,500</td>
</tr>
<tr>
<td>Sea Water Depth, ft</td>
<td>500</td>
</tr>
<tr>
<td>Wellbore Diameter, in</td>
<td>12.5</td>
</tr>
<tr>
<td>Drill Pipe Outside Diameter, in</td>
<td>5</td>
</tr>
<tr>
<td>Drill Collar Outside Diameter, in</td>
<td>8</td>
</tr>
<tr>
<td>Drill Collar Length, ft</td>
<td>300</td>
</tr>
<tr>
<td>Bottom Hole Temperature, °F</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reservoir Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Pressure, psi</td>
<td>1,300</td>
</tr>
<tr>
<td>Porosity, dimensionless</td>
<td>0.25</td>
</tr>
<tr>
<td>Thickness, ft</td>
<td>35</td>
</tr>
<tr>
<td>Permeability, miliDarcy</td>
<td>150</td>
</tr>
<tr>
<td>Skin Factor, dimensionless</td>
<td>0</td>
</tr>
<tr>
<td>Gas Density, dimensionless</td>
<td>0.65</td>
</tr>
<tr>
<td>Initial Water Saturation, dimensionless</td>
<td>0.20</td>
</tr>
<tr>
<td>Temperature, °F</td>
<td>110</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fractured Formation Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth, ft</td>
<td>1,465-1535</td>
</tr>
<tr>
<td>Thickness, ft</td>
<td>70</td>
</tr>
<tr>
<td>Stress in Upper Bounding Zone, psi</td>
<td>810</td>
</tr>
<tr>
<td>Fracture Initiation Pressure, psi</td>
<td>800</td>
</tr>
<tr>
<td>Stress in Lower Bounding Zone, psi</td>
<td>850</td>
</tr>
</tbody>
</table>

To study the applicability of the shut-in procedure in this case the simulation considers that only gas is flowing in the well and the fracture is induced at the moment the well is shut-in. In addition, the simulation considers that there is no mud being pumped into the well during the fracture propagation and that the in-situ stress gradients for the upper and lower bounding layers are equal to 0.7 psi/ft.
Figure 6.1 shows the results of the simulation for four different formations: clay shale, sandstone, siltstone and shale. The characteristic formation values necessary to run the simulation such as Young’s modulus and fracture toughness of rock, were obtained from Warpinski and Smith (1989) and are given in Table 6.2.

The simulation stops when the fracture reaches the half-length of 340 ft for all cases because this allows the comparison of the fracture propagation.

It can be seen from figure 6.1 that the kind of formation has a great influence in the vertical migration. The migration in this case is faster for sand and slower for clay shale. Also, it is clear that if this well were shut-in in the case of sandstone, the fracture would reach the sea floor very fast. This concludes the shut-in procedure could not be used here.
Table 6.2. Formation Characteristics

<table>
<thead>
<tr>
<th>Simulated Formation</th>
<th>Young's Modulus $psi$</th>
<th>Fracture Toughness $psi. ft^{0.5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay Shale</td>
<td>56,181</td>
<td>245</td>
</tr>
<tr>
<td>Green River shale</td>
<td>1,215,000</td>
<td>231</td>
</tr>
<tr>
<td>Berea sandstone</td>
<td>2,200,000</td>
<td>418</td>
</tr>
<tr>
<td>Repetto siltstone saturated</td>
<td>698,900</td>
<td>245</td>
</tr>
</tbody>
</table>

Figure 6.2 shows the vertical propagation of the fracture into the lower layers, where the stress is higher than the upper layers.

The influence of stress in the layers is evident, as the bottom of the fracture in this case does not propagate the same as the top of the fracture. The propagation in the clay shale, in this case, is still less than in sandstone.
This influence is much more evident when the stresses in the bounding layers are higher than the fracture formation. To show this fact, another simulation was run for clay shale and sandstone where the stresses in the upper and lower layers are equal to 850 psi.

Figure 6.3 shows that the propagation still occurs in the case of sands, but not if the formation is clay shale. The height of the fracture keeps constant during the fracture propagation in the case of clay shale. This allows to conclude that the shut-in procedure could be used without any restriction if the formation were clay shale.

These simulations have shown the influence of the type of formation and the stress in the layers during the fracturing process. The type of formation was expected to have a big influence because the width and pressure equations depend on the
Young's modulus causing different formations to have different profile in the fracturing process. The simulations indicated that as the value of Young's modulus decreases, the better the chances for a restriction in the vertical migration occurs. Also, the layer stresses are preponderant in restricting a fracture migration.

The choice between the two procedures still continues with the knowledge of the formation characteristics, because they can have large variation for the same kind of rock as shown by Warpinski and Smith (1989). Specific values can only be known on the site of the well through logs or cores, and an estimation of such characteristics can only be done in very well known areas.

Depending on the type of formation, if there are no bounding layers in the fractured formation and if the injection continues, the fracture will propagate in height reaching the surface sooner or later. So it is also important to have knowledge of the stresses in the layers because if there are no bounding layers caused by such stresses, the shut-in procedure is not recommended.

Once those characteristics are known, this proposed model can be used to decide if the shut-in procedure should be used or not in a shallow well, eliminating some questions about which procedure to apply.
7.1. Summary and Conclusions

The dynamic kill method has been used by the oil industry as a way to control an underground blowout when it is not possible to use a heavy weight mud.

The current models calculate the appropriate mud flow rate to control an underground blowout by choosing a mud flow rate that gives a bottom hole pressure greater than the reservoir pressure. The process consists of varying the mud flow rate, and calculating for each flow rate the hydrostatic pressure and the losses occurring in the annulus of the well. The bottom hole pressure is then calculated by adding the hydrostatic pressure and the pressure losses to the fracture initiation pressure which is considered constant.

This study introduces a fracture model to the current model that verifies whether or not the assumption of constant fracture pressure alters the calculation of the appropriate mud flow rate to control the underground blowout.

The fracture model introduced is a pseudo-3D model with modifications to eliminate the problem of vertical migration in cases where the stress in bounding layers is close to the stress in the fractured formation. The pseudo-3D model, after these modifications, presents results that fit the results of other 3D-models. The great advantage in using this pseudo 3-D model is the run time of a computer, which is much less than that for a standard 3-D model.

The fracture model also assumes the occurrence of the leak-off volume inside the fracture. To predict such leak-off volume, an experimental apparatus was designed to measure the leak-off when using mud and gas. The apparatus consisted of a fluid
loss cell where a formation core could be introduced and the leak-off volume could be measured by pressure differential.

A correlation to calculate the leak-off volume was developed based on experimental data. The correlation related the leak-off volume assuming three parameters: the spurt loss volume, the pack buildup factor, and the equilibrium Darcy flow velocity coefficient.

In contrast to other pseudo 3-D models, this model allows the calculation of the leak-off volume within each cell inside the fracture. It does not use an average value of the leak-off volume for the entire fracture. In addition, this fracture model can also be used with a two-phase fluid.

An unsteady state two-phase flow numerical procedure was developed and implemented in a FORTRAN program to simulate the pressures in the wellbore and the fracture. The program can predict fracture dimensions and fracture pressure in any cross section as well as pressure in any part of the wellbore. The program emphasizes the bottom hole pressure, pressure in front of the fracture and producing gas during the underground blowout.

To verify the effect of using a fracture model in the calculation procedure for planning an underground blowout, three different underground blowout simulations were run. The simulation for a deep well did not show a great difference between the proposed and the current models; the difference in the appropriate mud flow rate between the current and the proposed models was around 8%. The simulation for a medium depth showed a larger difference (35%), resulting in a mud flow rate of 1,300 gpm with the proposed model and 1,760 gpm with the current models. Finally, the difference between the proposed and the current models increased still more (60%) in the simulation of a shallow depth.

Based on such simulations, the proposed model should be useful in any configuration, although the usefulness of the proposed model decreases significantly.
in cases where the fluid losses in the annulus are large, in which the diameter of the well is less or equal to 8 1/2 inches or the distance between the reservoir and the fractured formation are large.

Simulations for blowouts at shallow depths were run to verify the use of the shut-in procedure in different formations. The simulation showed that the shut-in procedure has more chance of success when the Young’s modulus is lower and there are bounding layers in the shallow formations. Under other conditions, the well should be diverted.

7.2. Recommendations

The following recommendations are made for future experimental and theoretical works that study an underground blowout with an induced fracture.

1) Another experimental apparatus should be designed to study the effect of relative permeability and porosity in the leak-off volume. This design should aid in identifying other properties that have more influence in the spurt loss volume, the pack build-up factor, and the equilibrium Darcy flow velocity coefficient. For this, cores from different formations should also be used.

2) Mud with additives should be used in an experimental work to identify the effect of such additives on the leak-off volume.

3) A modification in the wellbore model to accelerate the calculation in the iterative process may be necessary. This could also be accomplished by experimental work to verify the applicability of the equations used to define the flow regime and calculate the gas slip velocity in the wellbore model.

4) Comparisons between field data and the proposed model should be performed. The data collected up to date could not be used because the available records did not include all of the required information to perform the simulation. Underground blowouts occurred under much more complex situations, such as leaking in casings and flow between casings, before the induction of the fracture. These additional

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problems completely impaired the use of the proposed model as a simulator because additional data about these complex situations were not available.
BIBLIOGRAPHY


Harris, P.C.: "Dynamic Fluid Loss Characteristics of CO2 Foam Fracturing Fluids," 59th Annual Fall Meeting of the SPE, Houston, TX, 1984, SPE 13180.


NOMENCLATURE

English

\( a \) = fracture half height, ft [m]
\( A \) = fracture area, ft\(^2\) [m\(^2\)]
\( a_i \) = empirical coefficient, psi/ft [KPa/m]
\( A_i \) = correlation coefficient, psi/ft [KPa/m]
\( A_w \) = cross sectional area of core through which filtrate tests are running, ft\(^2\) [m\(^2\)]
\( A_{cr} \) = cross sectional area of fracture, ft\(^2\) [m\(^2\)]
\( a(s) \) = width of near-crack-tip, ft [m]
\( b \) = pack buildup constant, s\(^{-1}\)
\( b_i \) = empirical coefficient, dimensionless
\( b_2, b_3 \) = geometry coefficients for layered stress fracture analysis, ft [m]
\( C \) = total leak-off test coefficient, ft/s\(^{0.5}\) [m/s\(^{0.5}\)]
\( C_t \) = factor depending on two-phase flow regime, dimensionless
\( c_g \) = gas compressibility, psi\(^{-1}\) [KPa\(^{-1}\)]
\( C_l \) = normalized leak-off coefficient, ft/(s\(^{0.5}\) .psi)[m/(s\(^{0.5}\).KPa)]
\( c_i \) = total formation compressibility, psi\(^{-1}\) [KPa\(^{-1}\)]
\( C_k \) = Kachanov parameter, (psi.s\(^{-1}\))\(^{-1}\) [(KPa.s\(^{-1}\))\(^{-1}\)]
\( C_r \) = reservoir fluid viscosity/compressibility coefficient, ft/s\(^{0.5}\) [m/s\(^{0.5}\)]
\( C_d \) = dynamic leak-off coefficient, ft/s\(^{0.5}\) [m/s\(^{0.5}\)]
\( c_w \) = effluent viscosity and relative permeability coefficient, ft/s\(^{0.5}\) [m/s\(^{0.5}\)]
\( C_{off} \) = combined leak-off coefficient, ft/s\(^{0.5}\) [m/s\(^{0.5}\)]
\( C_{w} \) = wall building coefficient, ft/s\(^{0.5}\) [m/s\(^{0.5}\)]
\( C_{off} \) = effective leak-off coefficient, ft/s\(^{0.5}\) [m/s\(^{0.5}\)]
\( d \) = width of the annular element, ft [m]
\( d_x \) = horizontal length of fracture cell, ft [m]
\( D \) = non-Darcy factor, s/ft\(^3\) [s/m\(^3\)]
\( D \) = outside diameter of the annulus, ft [m]
\( E \) = Young's modulus of rock, psi [kPa]
\( E \) = elastic modulus in plain strain, psi [kPa]
\( E_e \) = effective elastic modulus, psi [KPa]
\( f \) = Fanning friction factor, dimensionless
\( f_n \) = constant equal to 1 or -1
\( F_i \) = Geerstma and de Klerk's parameter, dimensionless
\( F_c \) = conversion factor, 2.1587\times10^{-4}\text{psi.ft.s}^3/\text{lbfm} [10^{-3}\text{ KPa.m.s}^3/\text{kgm}]
\( F_p \) = stress ratio, dimensionless
\( g \) = gravity acceleration, ft/s\(^2\) [m/s\(^2\)]
\( g_e \) = conversion constant, lbfm.ft/lbft.s\(^2\) [Kgm.m/N.s\(^2\)]
\( g_f \) = fluid density pressure gradient, psi/ft [KPa/m]
\( g_s \) = in-situ stress gradient, psi/ft [KPa/m]
\( g_v \) = vertical pressure gradient, psi/ft [KPa/m]
$G =$ bulk shear modulus of formation, psi [kPa]
$G_p =$ produced gas at standard pressure and temperature, ft$^3$ [m$^3$]
$h =$ reservoir thickness, ft [m]
$h_f =$ fractured formation thickness, ft [m]
$H =$ liquid hold up, dimensionless
$H_g =$ gas void fraction, dimensionless
$f =$ slope in log-log plot of leak-off volume versus time, dimensionless
$k =$ permeability, ft$^2$ [m$^2$]
$K =$ consistency index of the fluid, psi.s$^n$ [KPa.s$^n$]
$k_r =$ relative permeability of formation to leak-off effluent, ft$^2$ [m$^2$]
$k_r =$ formation permeability relative to mobile reservoir fluid, ft$^2$ [m$^2$]
$K_r =$ empirical coefficient, psi/ft [KPa/m]
$K_i =$ geometrical coefficient, dimensionless
$K_i =$ matrix stress coefficient, dimensionless
$K_d =$ fracture toughness or critical stress intensity factor, psi/ft [KPa/m]
$K_{bottom} =$ stress intensity factor at the bottom of crack, psi.ft$^{0.5}$ [KPa.m$^{0.5}$]
$K_{top} =$ stress intensity factor at the top of crack, psi.$^{0.5}$ [KPa.m$^{0.5}$]
$l_d =$ dimensionless distance between the microcracks, dimensionless
$L =$ fracture length, ft [m]
$m =$ slope of curve filtrate volume versus square root of time, ft/s$^{0.5}$ [m/s$^{0.5}$]
$m_i =$ y-intercept determined in log-log plot of leak-off volume versus time, ft [m]
$M =$ gas molecular weight, dimensionless
$m(P) =$ real gas pseudo pressure, psi/s [kPa/s]
$n =$ power law exponent, dimensionless
$P_{BHP} =$ bottom hole pressure, psi [kPa]
$P_{RES} =$ reservoir pressure, psi [kPa]
$P_{sc} =$ standard condition pressure, psi [kPa]
$P =$ pressure, psi [kPa]
$P_c =$ confining pressure, psi [kPa]
$P_f =$ fracture pressure, psi [kPa]
$P_{FC} =$ formation pressure capacity, psi [kPa]
$P_L =$ pressure differential required to open fracture at crack front, psi [kPa]
$P_n =$ net pressure, psi [kPa]
$P_o =$ formation pressure, psi [kPa]
$P_p =$ porous pressure, psi [kPa]
$P_w =$ wellbore pressure, psi [kPa]
$per =$ fracture cross section perimeter, ft [m]
$Q,q =$ flow rate, ft$^3$/s [m$^3$/s]
$q_i =$ injection flow rate into the fracture, ft$^3$/s [m$^3$/s]
$q_i =$ injection flow rate per unit fracture area, ft/s [m/s]
$q_f =$ fluid loss rate per unit fracture length, ft$^2$/s [m$^2$/s]
$q_L =$ fluid loss rate per unit fracture area, ft/s [m/s]
$Q_l = \text{fluid loss rate in pseudo-3D model, ft}^2/\text{s [m}^2/\text{s]}$

$q_n = \text{flow rate in direction normal to crack front per unit length of front, ft}^2/\text{s [m}^2/\text{s]}$

$q_r = \text{resultant flow rate in 3-D model, } \left(q_x^2 + q_y^2\right)^{0.5}, \text{ft}^2/\text{s [m}^2/\text{s]}$

$q_x = \text{volume flow rate in x direction per unit length in y direction, ft}^2/\text{s [m}^2/\text{s]}$

$q_y = \text{volume flow rate in y direction per unit length in x direction, ft}^2/\text{s [m}^2/\text{s]}$

$r = \text{radius, cylindrical coordinates, ft [m]}$

$R = \text{distance, ft [m]}$

$R_f = \text{fracture radius up to the pressure sealing point, ft [m]}$

$R_{m} = \text{length of dehydrated mud around the fracture tip, ft [m]}$

$R_t = \text{length of the fracture tip without mud invasion, ft [m]}$

$R_q = \text{height quotient in pseudo-3D model, dimensionless}$

$r_w = \text{wellbore radius, ft [m]}$

$\delta = \text{skin factor, dimensionless}$

$\delta_i = \text{total stress } (i = x, y, z), \text{psi [KPa]}$

$\sigma = \text{formation tensile strength, psi [KPa]}$

$S_w = \text{reservoir water saturation, dimensionless}$

$t = \text{time, s}$

$t_A = \text{time at which fluid loss is proportional to } \sqrt{t}, \text{s}$

$t_B = \text{time at which fluid loss is proportional to } t, \text{s}$

$t_D = \text{dimensionless time, dimensionless}$

$T = \text{absolute temperature, } ^\circ R [\text{^\circ K]}$

$T_s = \text{standard condition absolute temperature, } ^\circ R [\text{^\circ K]}$

$v = \text{velocity, ft/s [m/s]}$

$v_D = \text{equilibrium Darcy flow velocity, ft/s [m/s]}$

$v_g = \text{gas velocity, ft/s [m/s]}$

$v_l = \text{liquid velocity, ft/s [m/s]}$

$v_m = \text{two phase velocity, ft/s [m/s]}$

$v_s = \text{slip velocity, ft/s [m/s]}$

$V_i = \text{initial gas reservoir in place, ft}^3 [\text{m}^3]$

$V = \text{cumulative filtrate volume, ft}^3/\text{ft}^2 [\text{m}^3/\text{m}^2]$

$V_{frc} = \text{predicted cumulative filtrate volume, ft}^3/\text{ft}^2 [\text{m}^3/\text{m}^2]$

$V_{sp} = \text{spurt loss volume per area unit, ft}^3/\text{ft}^2 [\text{m}^3/\text{m}^2]$

$w = \text{crack opening or fracture width}$

$w_f, w_{uf}, w_{ufn} = \text{crack opening resulting from uniform pressure over fractured formation and bounding layers respectively, ft [m]}$

$W = \text{maximum fracture width, ft [m]}$

$W_f = \text{average fracture width}$

$W_{uf} = \text{fracture width at the sealing point with mud cake, ft [m]}$

$W_p = \text{fracture width at the point which does not allow mud invasion, ft [m]}$

$x, y, z = \text{coordinate axis}$

$z = \text{gas compressibility factor, dimensionless}$
Greek
\( \alpha = \) Biot's constant, dimensionless
\( \alpha_L = \) parameter in Geerstma and de Klerk's model, dimensionless
\( \beta = \) dipping angle, radians
\( \beta_L = \) Geerstma velocity coefficient, ft\(^{-1}\) [m\(^{-1}\)]
\( \beta_i = \) effective stress ratio of the superposed tectonic stress, dimensionless
\( \Delta P = \) differential pressure, psi [KPa]
\( \Delta s = \) length of the annular element, ft [m]
\( \Delta t = \) time interval, s
\( \Delta x = \) incremental length, ft [m]
\( \phi = \) porosity, dimensionless
\( \eta' = \) viscosity parameter, psi.s\(^{\ast}\), [KPa.s\(^{\ast}\)]
\( \lambda = \) non-slip liquid hold-up, dimensionless
\( \rho = \) density, lbm/ft\(^3\) [Kgm/m\(^3\)]
\( \rho_F = \) weight density of fluid, psi/ft [KPa/m]
\( \rho_s = \) gas density, lbm/ft\(^3\) [Kgm/m\(^3\)]
\( \rho_L = \) liquid density, lbm/ft\(^3\) [Kgm/m\(^3\)]
\( \rho_{mix} = \) two-phase fluid density, lbm/ft\(^3\) [Kgm/m\(^3\)]
\( \rho_{ni, s} = \) non-slip density, lbm/ft\(^3\) [Kgm/m\(^3\)]
\( \sigma_x = \) gas/liquid surface tension, lb/ft [N/m]
\( \sigma_f = \) in-situ fractured formation stress, psi [KPa]
\( \sigma_u = \) in-situ upper formation stress, psi [KPa]
\( \sigma_s = \) in-situ bottom formation stress, psi [KPa]
\( \sigma_r = \) horizontal in-situ stress, psi [KPa]
\( \sigma_T = \) in-situ stress \((i = x, y, z)\), psi [KPa]
\( \sigma_v = \) in-situ vertical stress, psi [KPa]
\( \sigma_r = \) in-situ radial stress, cylindrical coordinates, psi [KPa]
\( \sigma_h = \) superposed tectonic stress, psi [KPa]
\( \sigma_{T, cor} = \) corrected horizontal in-situ stress gradient, psi/ft [KPa/m]
\( \sigma_{T, tang} = \) in-situ tangential stress, cylindrical coordinates, psi [KPa]
\( \tau = \) time when position in fracture plane is first exposed to fracturing fluid, s
\( \tau_y = \) yield strength, psi.s [KPa.s]
\( \tau_{sh} = \) shear stress, cylindrical coordinates, psi [KPa]
\( \theta = \) angle, cylindrical coordinates, radians
\( \mu = \) fluid viscosity, psi.s [KPa.s]
\( \mu_{app} = \) apparent liquid fluid viscosity, psi.s [KPa.s]
\( \mu_f = \) viscosity of effluent from fracturing fluid at fracturing conditions, psi.s [KPa.s]
\( \mu_r = \) viscosity of mobile formation fluid at reservoir conditions, psi.s [KPa.s]
\( \mu_g = \) gas viscosity, psi.s [KPa.s]
\( \mu_l = \) liquid viscosity, psi.s [KPa.s]
\( \mu_{mix} = \) apparent viscosity of two-phase fluid, psi.s [KPa.s]
\( \mu_n = \text{non-slip viscosity, psi.s [KPa.s]} \)
\( \nu = \text{Poisson's ratio, dimensionless} \)
\( \nabla = \text{gradient operator} \)
APPENDIX A

FRACTURE-INDUCTION EQUATIONS

The expressions of stress distribution around a hole caused by two horizontal principal stresses \( \sigma_x \) and \( \sigma_y \) in plane are readily available (Timoshenko, S. and Goodier, N.J.). The stress distribution is given by:

\[
\sigma_r = \left( \frac{\sigma_z + \sigma_x}{2} \right) \left( 1 - \frac{r_w^2}{r^2} \right) + \left( \frac{\sigma_z - \sigma_x}{2} \right) \left( 1 + \frac{3r_w^4}{r^4} - 4\frac{r_w^2}{r^2} \right) \cos 2\theta \tag{a.1}
\]

\[
\sigma_\theta = \left( \frac{\sigma_z + \sigma_x}{2} \right) \left( 1 + \frac{r_w^2}{r^2} \right) - \left( \frac{\sigma_z - \sigma_x}{2} \right) \left( 1 + \frac{3r_w^4}{r^4} \right) \cos 2\theta \tag{a.2}
\]

\[
\tau_\theta = \left( \frac{\sigma_z - \sigma_x}{2} \right) \left( 1 - \frac{3r_w^4}{r^4} + 2\frac{r_w^2}{r^2} \right) \sin 2\theta \tag{a.3}
\]

\[
\sigma_z = \sigma_z - \sqrt{2(\sigma_z - \sigma_x)\frac{r_w^2}{r^2}} \cos 2\theta \tag{a.4}
\]

and

\[
\tau_\theta = \tau_\phi = 0 \tag{a.5}
\]

The previous equations assume the well hole is under the combined action of stresses. The resulting stress distribution in terms of total stress \( S_\phi \) at the well-bore radius \( r_w \) in polar coordinates is given by:

\[
S_\phi = P_\phi \tag{a.6}
\]
\[ S_e = \sigma_e + \sigma_i - 2(\sigma_e - \sigma_i)\cos 2\theta + P_o \] \hspace{1cm} (a.7)

\[ S_i = \sigma_i - 2v(\sigma_e - \sigma_i)\cos 2\theta + P_o \] \hspace{1cm} (a.8)

and

\[ S_n = S_e = S_i = 0 \] \hspace{1cm} (a.9)

Campos (1983) determined that the stress induced by the borehole fluid pressure \( P_m \) at the wellbore in the depth under consideration to be equal to:

\[ S = P_m - P_o \] \hspace{1cm} (a.10)

\[ S_n = P_o - P_m \] \hspace{1cm} (a.11)

\[ S_i = S_n = S_i = 0 \] \hspace{1cm} (a.12)

The induced stress due to the penetration of well fluid into the formation, considering the penetrating fluid has the same viscosity as the formation fluid and the formation is isotropic, is given by:

\[ S_e = \alpha\frac{1-2v}{1-v}(P_m - P_o) \] \hspace{1cm} (a.13)

\[ S_i = \alpha\frac{1-2v}{1-v}(P_m - P_o) \] \hspace{1cm} (a.14)
\[ S_r = S_\sigma = S_\tau = S_\epsilon = 0 \] \hspace{1cm} (a.15)

The total stress is determined by the principle of superposition (Timoshenko and Goodier, 1951) of the three induced stress cases:

\[ S_r = P_w \] \hspace{1cm} (a.16)

\[ S_\sigma = \sigma_0 + \sigma_r - 2(\sigma_0 - \sigma_r)\cos 2\theta + 2P_o - P_r + \alpha \left( \frac{1-2\nu}{1-\nu} \right)(P_r - P_o) \] \hspace{1cm} (a.17)

\[ S_\tau = \sigma_\tau - 2\nu(\sigma_0 - \sigma_r)\cos 2\theta + P_o + \alpha \left( \frac{1-2\nu}{1-\nu} \right)(P_r - P_o) \] \hspace{1cm} (a.18)

and

\[ S_\sigma = S_\tau = S_\epsilon = 0 \] \hspace{1cm} (a.19)

Due to the fact that all shear stresses are zero at the borehole wall, \( S_r, S_\sigma, \) and \( S_\theta \) are the principal stresses at the borehole wall. The maximum stresses occur at \( \theta = 0 \) and \( \theta = \pi \). Substituting these values in the equations for \( S_r, S_\sigma, \) and \( S_\theta \) leads to:

\[ S_r = P_w \] \hspace{1cm} (a.20)

\[ S_\sigma = 3\sigma_r - \sigma_0 + 2P_o - P_r + \alpha \left( \frac{1-2\nu}{1-\nu} \right)(P_r - P_o) \] \hspace{1cm} (a.21)

\[ S_\tau = \sigma_\tau - 2\nu(\sigma_0 - \sigma_r) + P_o + \alpha \left( \frac{1-2\nu}{1-\nu} \right)(P_r - P_o) \] \hspace{1cm} (a.22)
For a tectonically relaxed formation:

\[ \sigma_t = \sigma_r = \left( \frac{v}{1 - v} \right) \sigma_r \]  

(a.23)

and substituting in \( S_r \), \( S_z \), and \( S_\theta \):

\[ S_r = \sigma_r \]  

(a.24)

\[ S_z = \left( \frac{2v}{1 - v} \right) \sigma_z + 2P_o - P_w + \alpha \left( \frac{1 - 2v}{1 - v} \right) (P_s - P_o) \]  

(a.25)

\[ S_\theta = \sigma_\theta + P_o + \alpha \left( \frac{1 - 2v}{1 - v} \right) (P_s - P_o) \]  

(a.26)

A.1 - Vertical Fracture Initiation

Vertical fractures start when the maximum effective tangential stress \( \sigma_\theta \) exceeds the tensile strength of the formation \( S_r \). Substituting this in equation (a.25) gives:

\[ S_r = \frac{2v}{1 - v} \sigma_z + 2P_o - P_w + \alpha \frac{1 - 2v}{1 - v} (P_p - P_o) - P_p \]  

(a.27)

For a penetrating type fluid \( P_p = P_w \), and after substituting in equation (a.27) and solving for the wellbore pressure yields:

\[ P_f = P_w = \frac{\frac{2v}{1 - v} \sigma_z - S_r}{2 - \alpha \frac{1 - 2v}{1 - v} + P_s} \]  

(a.28)
For a non-penetrating type of fluid $P_p = P_o$, and after substituting in equation (a.27) gives:

$$P_f = P_w = \frac{2v}{1-v} \sigma_z - S_t + P_o$$

(a.29)

A.2 - Horizontal Fracture Equation

Horizontal fractures start when the maximum effective vertical stress ($\sigma_z$) exceeds the tensile strength of the formation ($S_t$). Substituting this in equation (a.25) leads to:

$$S_t = \sigma_z + P_o - P_p + \alpha \frac{1-2v}{1-v} (P_p - P_o)$$

(a.30)

For a penetrating fluid $P_p = P_w$, and after substituting in equation (a.30) gives:

$$P_f = P_w = \frac{\sigma_z - S_t}{1-\alpha} + P_o$$

(a.31)

and for a non-penetrating fluid $P_p = P_o$ which leads to:

$$S_t = \sigma_z$$

(a.32)
APPENDIX B

CONTINUITY EQUATION FOR TWO-PHASE FLOW

Considering the flow through the fracture and by applying the material balance in the vicinity of a point P situated inside the control volume as can be seen in figure b.1, yields:

\[ \text{a - mass influx/unit of time} = \left( \rho v(x,t) - \frac{\partial(\rho v(x,t))}{\partial x} \right) A_{c1} \] ..............................................(b.1)

\[ \text{b - mass efflux/unit of time} = \left( \rho v(x,t) + \frac{\partial(\rho v(x,t))}{\partial x} \right) A_{c2} + \rho Q_L(x,t) A \] ..............(b.2)

\[ \text{c- net change in mass inside the control volume/unit of time} = \frac{\partial(\rho V)}{\partial t} \] .........................(b.3)

Fig. b.1. Control Volume of Material Balance Equation Applied for Point P

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d- mass influx/unit of time - mass efflux/unit of time = \frac{\partial (\rho V)}{\partial t} \cdots (b.4)

From equation (b.4), after dividing by \Delta x, and remembering that:

\[ A_v(x,t) = Q(x,t) \cdots (b.5) \]

gives:

\[ \frac{\rho Q_x(x,t) - \rho Q_x(x,t)}{\Delta x} + \frac{\partial (\rho Q(x,t))}{\partial x} + \rho Q_L(x,t) + \frac{\partial (\rho A_v)}{\partial t} = 0 \cdots (b.6) \]

By decreasing the length of \Delta x to an infinitesimal value, the first term of equation (b.6) will be negligible and the equation becomes:

\[ \frac{\partial (\rho Q(x,t))}{\partial x} + \rho Q_L(x,t) + \frac{\partial (\rho A_v(x,t))}{\partial t} = 0 \cdots (b.7) \]

The term of the leak-off in the above equations considers the average density of the fluid inside the control volume.
APPENDIX C

EXPERIMENTAL DATA

Table c.1. Leak-off Volume (cc/cm²) for Core 2 - µ = 10 cp - ΔP = 200 psi

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>10</th>
<th>17</th>
<th>30</th>
<th>31</th>
<th>60</th>
<th>70</th>
<th>76</th>
<th>77</th>
<th>93</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Rate (cc/min)</td>
<td>88</td>
<td>148</td>
<td>162</td>
<td>237</td>
<td>100</td>
<td>306</td>
<td>216</td>
<td>252</td>
<td>230</td>
<td>239</td>
<td>306</td>
<td>434</td>
</tr>
<tr>
<td>Initial Gas Void Fraction (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Average Gas Void Fraction (%)</td>
<td>0</td>
<td>0.836</td>
<td>1.102</td>
<td>0.836</td>
<td>1.015</td>
<td>4.797</td>
<td>1.059</td>
<td>1.523</td>
<td>3.176</td>
<td>4.375</td>
<td>1.348</td>
<td>6.078</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>1.578</td>
<td>1.788</td>
<td>1.359</td>
<td>1.533</td>
<td>1.711</td>
<td>5.247</td>
<td>1.828</td>
<td>3.008</td>
<td>3.795</td>
<td>5.063</td>
<td>2.276</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>1.750</td>
<td>2.016</td>
<td>1.644</td>
<td>1.878</td>
<td>5.313</td>
<td>1.946</td>
<td>3.293</td>
<td>3.910</td>
<td>5.135</td>
<td>2.498</td>
<td>6.821</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1.895</td>
<td>2.141</td>
<td>1.770</td>
<td>1.818</td>
<td>1.988</td>
<td>5.359</td>
<td>2.113</td>
<td>3.567</td>
<td>4.024</td>
<td>5.208</td>
<td>2.774</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>2.039</td>
<td>2.305</td>
<td>1.895</td>
<td>2.041</td>
<td>2.125</td>
<td>5.431</td>
<td>2.292</td>
<td>3.703</td>
<td>4.191</td>
<td>5.546</td>
<td>3.059</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>2.100</td>
<td>2.415</td>
<td>2.009</td>
<td>2.284</td>
<td>2.262</td>
<td>5.477</td>
<td>2.418</td>
<td>3.840</td>
<td>4.305</td>
<td>5.672</td>
<td>3.238</td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>2.157</td>
<td>2.525</td>
<td>2.074</td>
<td>2.399</td>
<td>2.399</td>
<td>5.524</td>
<td>2.589</td>
<td>3.950</td>
<td>4.419</td>
<td>3.301</td>
<td>6.947</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>2.320</td>
<td>2.639</td>
<td>2.359</td>
<td>2.653</td>
<td>2.756</td>
<td>4.529</td>
<td>3.363</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table c.2. Curve-fit Coefficients for Leak-off Volume - Core 2 - ΔP = 200 psi

<table>
<thead>
<tr>
<th>Hg (%)</th>
<th>Hg (% )</th>
<th>Equation</th>
<th>$R^2$ (d'less)</th>
<th>$\chi^2$ (d'less)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>V = 1.207(1-exp(-0.854t)) + 0.0300t</td>
<td>0.985</td>
<td>0.081</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>V = 1.400(1-exp(-1.284t)) + 0.0341t</td>
<td>0.992</td>
<td>0.056</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>V = 1.006(1-exp(-1.545t)) + 0.0351t</td>
<td>0.995</td>
<td>0.027</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>V = 1.006(1-exp(-1.995t)) + 0.0434t</td>
<td>0.998</td>
<td>0.012</td>
</tr>
<tr>
<td>17</td>
<td>17</td>
<td>V = 1.307(1-exp(-1.354t)) + 0.0340t</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>30</td>
<td>86</td>
<td>V = 4.991(1-exp(-3.124t)) + 0.0178t</td>
<td>0.999</td>
<td>0.012</td>
</tr>
<tr>
<td>31</td>
<td>20</td>
<td>V = 1.372(1-exp(-1.231t)) + 0.0367t</td>
<td>0.997</td>
<td>0.022</td>
</tr>
<tr>
<td>60</td>
<td>40</td>
<td>V = 2.358(1-exp(-0.964t)) + 0.0537t</td>
<td>0.996</td>
<td>0.055</td>
</tr>
<tr>
<td>70</td>
<td>70</td>
<td>V = 3.452(1-exp(-2.404t)) + 0.0300t</td>
<td>0.999</td>
<td>0.005</td>
</tr>
<tr>
<td>76</td>
<td>91</td>
<td>V = 4.529(1-exp(-3.129t)) + 0.0400t</td>
<td>0.999</td>
<td>0.028</td>
</tr>
<tr>
<td>77</td>
<td>30</td>
<td>V = 1.830(1-exp(-1.125t)) + 0.0444t</td>
<td>0.990</td>
<td>0.118</td>
</tr>
<tr>
<td>93</td>
<td>98</td>
<td>V = 6.358(1-exp(-3.686t)) + 0.0217t</td>
<td>0.998</td>
<td>0.088</td>
</tr>
</tbody>
</table>

166
Table c.3. Leak-off Volume (cc/cm²) for Core 7 - $\mu = 10$ cp - $\Delta P = 200$ psi

<table>
<thead>
<tr>
<th>Initial Gas Void Fraction (%)</th>
<th>Average Flow Rate (cc/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>161</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Average Gas Void Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>1</td>
<td>1.098 0.805 2.528 1.904 0.549 1.392 1.061 1.098 3.183 3.957</td>
</tr>
<tr>
<td>2</td>
<td>1.238 0.996 2.791 2.197 0.843 1.655 1.354 1.392 3.245 4.097</td>
</tr>
<tr>
<td>4</td>
<td>1.430 1.136 3.055 2.490 1.685 1.663 1.373 1.513 3.297 4.289</td>
</tr>
<tr>
<td>8</td>
<td>1.500 1.308 3.333 2.733 1.979 1.941 1.648 1.685 3.300 4.377</td>
</tr>
<tr>
<td>16</td>
<td>1.738 1.551 3.672 3.077 2.374 2.235 1.941 2.089 3.443 4.638</td>
</tr>
<tr>
<td>20</td>
<td>1.906 1.723 3.674 3.312 2.532 2.502 2.216 2.250 3.547 4.759</td>
</tr>
<tr>
<td>24</td>
<td>1.960 1.833 3.778 3.365 2.638 2.524 2.229 2.318 3.654 4.826</td>
</tr>
<tr>
<td>28</td>
<td>2.013 1.998 3.830 3.572 2.744 2.782 2.487 2.524 3.957 4.892</td>
</tr>
<tr>
<td>36</td>
<td>2.272 3.882 3.626 2.821 2.937 2.488 2.577 4.251 4.951</td>
</tr>
</tbody>
</table>

Table c.4. Curve-fit Coefficients for Leak-off Volume - Core 7 - $\Delta P = 200$ psi

<table>
<thead>
<tr>
<th>Hg (%)</th>
<th>Hgi (%)</th>
<th>Equation</th>
<th>$R^2$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>$V=1.315(1-exp(-1.569t))+0.0267t$</td>
<td>0.997</td>
<td>0.014</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>$V=1.003(1-exp(-1.421t))+0.0353t$</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>20</td>
<td>60</td>
<td>$V=3.077(1-exp(-1.526t))+0.0286t$</td>
<td>0.989</td>
<td>0.138</td>
</tr>
<tr>
<td>36</td>
<td>48</td>
<td>$V=2.334(1-exp(-1.498t))+0.0433t$</td>
<td>0.996</td>
<td>0.049</td>
</tr>
<tr>
<td>40</td>
<td>40</td>
<td>$V=1.979(1-exp(-0.313t))+0.0269t$</td>
<td>0.995</td>
<td>0.049</td>
</tr>
<tr>
<td>54</td>
<td>22</td>
<td>$V=1.532(1-exp(-2.189t))+0.0439t$</td>
<td>0.995</td>
<td>0.032</td>
</tr>
<tr>
<td>58</td>
<td>16</td>
<td>$V=1.275(1-exp(-1.668t))+0.0408t$</td>
<td>0.991</td>
<td>0.047</td>
</tr>
<tr>
<td>66</td>
<td>21</td>
<td>$V=1.432(1-exp(-1.317t))+0.0381t$</td>
<td>0.995</td>
<td>0.027</td>
</tr>
<tr>
<td>80</td>
<td>65</td>
<td>$V=3.093(1-exp(-4.976t))+0.0295t$</td>
<td>0.990</td>
<td>0.012</td>
</tr>
<tr>
<td>88</td>
<td>95</td>
<td>$V=4.124(1-exp(-3.762t))+0.0283t$</td>
<td>0.998</td>
<td>0.036</td>
</tr>
</tbody>
</table>

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Table c.5. Leak-off Volume (cc/cm²) for Core 8 - \( \mu = 20 \) cp - \( \Delta P = 200 \) psi

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0</th>
<th>25</th>
<th>33</th>
<th>46</th>
<th>60</th>
<th>72</th>
<th>81</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1.098</td>
<td>1.317</td>
<td>0.880</td>
<td>1.136</td>
<td>2.746</td>
<td>4.251</td>
<td>5.191</td>
</tr>
<tr>
<td>2</td>
<td>1.263</td>
<td>1.629</td>
<td>0.899</td>
<td>1.430</td>
<td>3.077</td>
<td>4.526</td>
<td>5.936</td>
</tr>
<tr>
<td>4</td>
<td>1.373</td>
<td>1.761</td>
<td>1.174</td>
<td>1.551</td>
<td>3.183</td>
<td>4.595</td>
<td>6.024</td>
</tr>
<tr>
<td>8</td>
<td>1.494</td>
<td>1.904</td>
<td>1.437</td>
<td>1.723</td>
<td>3.301</td>
<td>4.673</td>
<td>6.087</td>
</tr>
<tr>
<td>24</td>
<td>2.089</td>
<td>2.445</td>
<td>1.994</td>
<td>2.369</td>
<td>3.818</td>
<td>4.987</td>
<td>6.238</td>
</tr>
<tr>
<td>28</td>
<td>2.142</td>
<td>2.551</td>
<td>2.001</td>
<td>2.422</td>
<td>3.939</td>
<td>5.041</td>
<td>6.275</td>
</tr>
<tr>
<td>32</td>
<td>2.169</td>
<td>2.709</td>
<td>2.166</td>
<td>2.475</td>
<td>4.049</td>
<td>5.094</td>
<td>6.304</td>
</tr>
<tr>
<td>36</td>
<td>2.197</td>
<td>2.272</td>
<td>2.528</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table c.6. Curve-fit Coefficients for Leak-off Volume - Core 8 - \( \Delta P = 200 \) psi

<table>
<thead>
<tr>
<th>Hg (%)</th>
<th>Hgi (%)</th>
<th>Equation</th>
<th>( R^2 ) (d'less)</th>
<th>( \chi^2 ) (d'less)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>( V=1.304(1-\exp(-1.639t))+0.0281t )</td>
<td>0.991</td>
<td>0.041</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>( V=1.634(1-\exp(-1.544t))+0.0333t )</td>
<td>0.999</td>
<td>0.001</td>
</tr>
<tr>
<td>33</td>
<td>11</td>
<td>( V=1.020(1-\exp(-1.455t))+0.0357t )</td>
<td>0.989</td>
<td>0.071</td>
</tr>
<tr>
<td>46</td>
<td>23</td>
<td>( V=1.551(1-\exp(-1.175t))+0.0318t )</td>
<td>0.990</td>
<td>0.061</td>
</tr>
<tr>
<td>60</td>
<td>61</td>
<td>( V=3.044(1-\exp(-2.229t))+0.0315t )</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>72</td>
<td>62</td>
<td>( V=4.548(1-\exp(-2.652t))+0.0182t )</td>
<td>0.999</td>
<td>0.010</td>
</tr>
<tr>
<td>81</td>
<td>76</td>
<td>( V=6.011(1-\exp(-1.987t))+0.0094t )</td>
<td>0.999</td>
<td>0.001</td>
</tr>
</tbody>
</table>

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Table c.7. Leak-off Volume (cc/cm²) for Core 2 - $\mu = 10$ cp - $\Delta P = 400$ psi

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Average Flow Rate (cc/min)</th>
<th>Initial Gas Void Fraction (%)</th>
<th>Average Gas Void Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.886</td>
<td>0.066</td>
<td>2.322</td>
</tr>
<tr>
<td>2</td>
<td>1.181</td>
<td>1.041</td>
<td>2.617</td>
</tr>
<tr>
<td>4</td>
<td>1.476</td>
<td>1.161</td>
<td>2.906</td>
</tr>
<tr>
<td>8</td>
<td>1.762</td>
<td>1.446</td>
<td>3.348</td>
</tr>
<tr>
<td>12</td>
<td>1.882</td>
<td>1.731</td>
<td>3.460</td>
</tr>
<tr>
<td>16</td>
<td>2.067</td>
<td>2.027</td>
<td>3.848</td>
</tr>
<tr>
<td>20</td>
<td>2.362</td>
<td>2.312</td>
<td>4.015</td>
</tr>
<tr>
<td>24</td>
<td>2.647</td>
<td>2.483</td>
<td>4.072</td>
</tr>
<tr>
<td>28</td>
<td>2.768</td>
<td>2.595</td>
<td>4.569</td>
</tr>
<tr>
<td>32</td>
<td>2.888</td>
<td>2.707</td>
<td>4.681</td>
</tr>
<tr>
<td>36</td>
<td>2.953</td>
<td>4.848</td>
<td>3.632</td>
</tr>
</tbody>
</table>

Table c.8. Curve-fit Coefficients for Leak-off Volume - Core 2 - $\Delta P = 400$ psi

<table>
<thead>
<tr>
<th>Hg (%)</th>
<th>Hg1 (%)</th>
<th>Equation</th>
<th>$R^2$ (d'less)</th>
<th>$\chi^2$ (d'less)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>$V=1.357(1\exp(-0.884t))+0.0479t$</td>
<td>0.994</td>
<td>0.054</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>$V=1.025(1\exp(-1.445t))+0.0572t$</td>
<td>0.990</td>
<td>0.070</td>
</tr>
<tr>
<td>16</td>
<td>44</td>
<td>$V=2.747(1\exp(-1.616t))+0.0608t$</td>
<td>0.994</td>
<td>0.010</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>$V=2.070(1\exp(-1.583t))+0.0433t$</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>39</td>
<td>1</td>
<td>$V=1.498(1\exp(-1.020t))+0.0310t$</td>
<td>0.988</td>
<td>0.073</td>
</tr>
<tr>
<td>40</td>
<td>41</td>
<td>$V=2.486(1\exp(-1.707t))+0.0432t$</td>
<td>0.999</td>
<td>0.004</td>
</tr>
<tr>
<td>66</td>
<td>75</td>
<td>$V=4.245(1\exp(-1.514t))+0.0450t$</td>
<td>0.995</td>
<td>0.133</td>
</tr>
<tr>
<td>73</td>
<td>21</td>
<td>$V=1.698(1\exp(-1.504t))+0.0586t$</td>
<td>0.981</td>
<td>0.133</td>
</tr>
<tr>
<td>80</td>
<td>80</td>
<td>$V=4.194(1\exp(-2.245t))+0.0381t$</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>92</td>
<td>95</td>
<td>$V=7.547(1\exp(-0.622t))+0.0175t$</td>
<td>0.933</td>
<td>3.670</td>
</tr>
</tbody>
</table>
Table c.9. Leak-off Volume (cc/cm²) for Core 6 - $\mu = 10$ cp - $\Delta P = 400$ psi

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Average Flow Rate (cc/min)</th>
<th>Average Gas Void Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Gas Void Fraction (%)</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1.146</td>
<td>0.840</td>
</tr>
<tr>
<td>2</td>
<td>1.272</td>
<td>1.126</td>
</tr>
<tr>
<td>4</td>
<td>1.440</td>
<td>1.243</td>
</tr>
<tr>
<td>8</td>
<td>1.553</td>
<td>1.412</td>
</tr>
<tr>
<td>16</td>
<td>1.726</td>
<td>1.806</td>
</tr>
<tr>
<td>20</td>
<td>1.839</td>
<td>1.976</td>
</tr>
<tr>
<td>24</td>
<td>2.001</td>
<td>2.145</td>
</tr>
<tr>
<td>28</td>
<td>2.108</td>
<td>2.262</td>
</tr>
<tr>
<td>32</td>
<td>2.218</td>
<td>2.432</td>
</tr>
<tr>
<td>36</td>
<td>2.274</td>
<td>2.786</td>
</tr>
</tbody>
</table>

Table c.10. Curve-fit Coefficients for Leak-off Volume - Core 6 - $\Delta P = 400$ psi

<table>
<thead>
<tr>
<th>Hg (%)</th>
<th>Hgi (%)</th>
<th>Equation</th>
<th>$R^2$ (d'less)</th>
<th>$\chi^2$ (d'less)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$V=1.309(1-exp(-1.833t))+0.0278t$</td>
<td>0.998</td>
<td>0.009</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
<td>$V=1.123(1-exp(-1.249t))+0.0416t$</td>
<td>0.998</td>
<td>0.010</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>$V=1.404(1-exp(-1.408t))+0.0465t$</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>21</td>
<td>20</td>
<td>$V=1.363(1-exp(-1.454t))+0.0408t$</td>
<td>0.994</td>
<td>0.038</td>
</tr>
<tr>
<td>26</td>
<td>26</td>
<td>$V=1.882(1-exp(-1.497t))+0.0453t$</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>40</td>
<td>3</td>
<td>$V=0.850(1-exp(-2.495t))+0.0500t$</td>
<td>0.993</td>
<td>0.045</td>
</tr>
<tr>
<td>51</td>
<td>13</td>
<td>$V=1.333(1-exp(-1.490t))+0.0441t$</td>
<td>0.985</td>
<td>0.079</td>
</tr>
<tr>
<td>79</td>
<td>80</td>
<td>$V=2.997(1-exp(-2.342t))+0.0513t$</td>
<td>0.996</td>
<td>0.007</td>
</tr>
<tr>
<td>93</td>
<td>91</td>
<td>$V=4.400(1-exp(-5.094t))+0.0257t$</td>
<td>0.998</td>
<td>0.035</td>
</tr>
</tbody>
</table>

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Table c.11. Leak-off Volume (cc/cm²) for Core 2 - \( \mu = 10 \text{ cp} \) - \( \Delta P = 600 \text{ psi} \)

<table>
<thead>
<tr>
<th>Average Flow Rate (cc/min)</th>
<th>Initial Gas Void Fraction (%)</th>
<th>Final Gas Void Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0%</td>
<td>29%</td>
</tr>
<tr>
<td>200</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>257</td>
<td>1.023</td>
<td>2.003</td>
</tr>
<tr>
<td>195</td>
<td>1.308</td>
<td>2.567</td>
</tr>
<tr>
<td>400</td>
<td>1.711</td>
<td>2.859</td>
</tr>
<tr>
<td>293</td>
<td>2.167</td>
<td>3.308</td>
</tr>
<tr>
<td>283</td>
<td>2.849</td>
<td>3.867</td>
</tr>
</tbody>
</table>

Table c.12. Curve-fit Coefficients for Leak-off Volume - Core 2 - \( \Delta P = 600 \text{ psi} \)

<table>
<thead>
<tr>
<th>Hg (%)</th>
<th>Hgi (%)</th>
<th>Equation</th>
<th>( R^2 ) (d'less)</th>
<th>( \chi^2 ) (d'less)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>V=1.281(1-exp(-1.397t))+0.0535t</td>
<td>0.999</td>
<td>0.003</td>
</tr>
<tr>
<td>0</td>
<td>29</td>
<td>V=2.538(1-exp(-1.516t))+0.0477t</td>
<td>0.999</td>
<td>0.015</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>V=1.284(1-exp(-0.710t))+0.0380t</td>
<td>0.996</td>
<td>0.028</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>V=1.432(1-exp(-1.421t))+0.0513t</td>
<td>0.999</td>
<td>0.004</td>
</tr>
<tr>
<td>22</td>
<td>23</td>
<td>V=1.433(1-exp(-1.615t))+0.0521t</td>
<td>0.999</td>
<td>0.005</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>V=2.095(1-exp(-1.547t))+0.0491t</td>
<td>0.999</td>
<td>0.003</td>
</tr>
<tr>
<td>36</td>
<td>8</td>
<td>V=1.267(1-exp(-1.850t))+0.0613t</td>
<td>0.995</td>
<td>0.041</td>
</tr>
<tr>
<td>41</td>
<td>3</td>
<td>V=1.345(1-exp(-0.618t))+0.0315t</td>
<td>0.984</td>
<td>0.099</td>
</tr>
<tr>
<td>44</td>
<td>14</td>
<td>V=1.611(1-exp(-1.918t))+0.0574t</td>
<td>0.998</td>
<td>0.025</td>
</tr>
<tr>
<td>53</td>
<td>48</td>
<td>V=3.127(1-exp(-2.127t))+0.0596t</td>
<td>0.990</td>
<td>0.225</td>
</tr>
<tr>
<td>65</td>
<td>65</td>
<td>V=3.914(1-exp(-1.868t))+0.0445t</td>
<td>0.999</td>
<td>0.003</td>
</tr>
<tr>
<td>73</td>
<td>52</td>
<td>V=4.341(1-exp(-1.418t))+0.0384t</td>
<td>0.990</td>
<td>0.266</td>
</tr>
</tbody>
</table>

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### Table c.13. Leak-off Volume (cc/cm²) for Core 4 - \( \mu = 10 \) cp - \( \Delta P = 600 \) psi

<table>
<thead>
<tr>
<th>Average Flow Rate (cc/min)</th>
<th>Initial Gas Void Fraction (%)</th>
<th>Final Gas Void Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>122</td>
<td>345</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>1.668</td>
<td>1.410</td>
</tr>
<tr>
<td>52</td>
<td>2.447</td>
<td>4.392</td>
</tr>
<tr>
<td>56</td>
<td>2.558</td>
<td>3.081</td>
</tr>
<tr>
<td>58</td>
<td>2.614</td>
<td>3.300</td>
</tr>
</tbody>
</table>

### Table c.14. Curve-fit Coefficients for Leak-off Volume - Core 4 - \( \Delta P = 600 \) psi

<table>
<thead>
<tr>
<th>Hg (%)</th>
<th>Hgi (%)</th>
<th>Equation</th>
<th>( R^2 ) (d'less)</th>
<th>( \chi^2 ) (d'less)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11</td>
<td>( V = 1.871(1 - \exp(-1.979t)) + 0.0255t )</td>
<td>0.994</td>
<td>0.039</td>
</tr>
<tr>
<td>16</td>
<td>52</td>
<td>( V = 3.418(1 - \exp(-0.712t)) + 0.0464t )</td>
<td>0.982</td>
<td>0.329</td>
</tr>
<tr>
<td>20</td>
<td>22</td>
<td>( V = 1.799(1 - \exp(-2.231t)) + 0.0458t )</td>
<td>0.997</td>
<td>0.032</td>
</tr>
<tr>
<td>21</td>
<td>16</td>
<td>( V = 1.526(1 - \exp(-2.368t)) + 0.0433t )</td>
<td>0.996</td>
<td>0.028</td>
</tr>
<tr>
<td>37</td>
<td>37</td>
<td>( V = 2.624(1 - \exp(-1.647t)) + 0.0484t )</td>
<td>0.999</td>
<td>0.005</td>
</tr>
<tr>
<td>43</td>
<td>43</td>
<td>( V = 2.948(1 - \exp(-1.698t)) + 0.0476t )</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>50</td>
<td>7</td>
<td>( V = 1.246(1 - \exp(-0.662t)) + 0.0522t )</td>
<td>0.989</td>
<td>0.114</td>
</tr>
<tr>
<td>52</td>
<td>75</td>
<td>( V = 4.423(1 - \exp(-3.118t)) + 0.0469t )</td>
<td>0.994</td>
<td>0.155</td>
</tr>
<tr>
<td>54</td>
<td>16</td>
<td>( V = 1.465(1 - \exp(-1.160t)) + 0.0641t )</td>
<td>0.997</td>
<td>0.074</td>
</tr>
<tr>
<td>62</td>
<td>62</td>
<td>( V = 3.774(1 - \exp(-1.806t)) + 0.0446t )</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>75</td>
<td>98</td>
<td>( V = 4.906(1 - \exp(-1.335t)) + 0.0207t )</td>
<td>0.997</td>
<td>0.074</td>
</tr>
</tbody>
</table>
### Table c.15. Leak-off Volume (cc/cm²) for Core 2 - μ = 10 cp - ΔP = 800 psi

<table>
<thead>
<tr>
<th>xxxx</th>
<th>70</th>
<th>284</th>
<th>291</th>
<th>250</th>
<th>214</th>
<th>231</th>
<th>316</th>
<th>332</th>
<th>253</th>
<th>320</th>
<th>421</th>
<th>419</th>
<th>394</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Initial Gas Void Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>xxxx</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average Flow Rate (cc/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0</th>
<th>2</th>
<th>6</th>
<th>20</th>
<th>23</th>
<th>28</th>
<th>28</th>
<th>38</th>
<th>50</th>
<th>50</th>
<th>55</th>
<th>57</th>
<th>83</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Final Gas Void Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>24</td>
</tr>
<tr>
<td>40</td>
</tr>
</tbody>
</table>

### Table c.16. Curve-fit Coefficients for Leak-off Volume - Core 2 - ΔP = 800 psi

<table>
<thead>
<tr>
<th>Hg (%)</th>
<th>Hgi (%)</th>
<th>Equation</th>
<th>$R^2$ (d'less)</th>
<th>$\chi^2$ (d'less)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$V=1.551(1-exp(-1.491t))+0.0509t$</td>
<td>0.999</td>
<td>0.003</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$V=1.261(1-exp(-1.845t))+0.0527t$</td>
<td>0.995</td>
<td>0.036</td>
</tr>
<tr>
<td>6</td>
<td>77</td>
<td>$V=4.746(1-exp(-3.138t))+0.0278t$</td>
<td>0.999</td>
<td>0.020</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>$V=2.117(1-exp(-1.544t))+0.0480t$</td>
<td>0.999</td>
<td>0.005</td>
</tr>
<tr>
<td>23</td>
<td>5</td>
<td>$V=1.251(1-exp(-0.881t))+0.0612t$</td>
<td>0.997</td>
<td>0.038</td>
</tr>
<tr>
<td>28</td>
<td>26</td>
<td>$V=2.022(1-exp(-0.702t))+0.0718t$</td>
<td>0.995</td>
<td>0.111</td>
</tr>
<tr>
<td>28</td>
<td>71</td>
<td>$V=4.988(1-exp(-1.358t))+0.0394t$</td>
<td>0.987</td>
<td>0.427</td>
</tr>
<tr>
<td>38</td>
<td>85</td>
<td>$V=5.808(1-exp(-1.264t))+0.0049t$</td>
<td>0.996</td>
<td>0.126</td>
</tr>
<tr>
<td>50</td>
<td>4</td>
<td>$V=1.301(1-exp(-0.899t))+0.0504t$</td>
<td>0.986</td>
<td>0.152</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>$V=3.489(1-exp(-1.626t))+0.0466t$</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>55</td>
<td>2</td>
<td>$V=1.397(1-exp(-1.391t))+0.0762t$</td>
<td>0.996</td>
<td>0.038</td>
</tr>
<tr>
<td>57</td>
<td>61</td>
<td>$V=4.169(1-exp(-3.023t))+0.0566t$</td>
<td>0.999</td>
<td>0.036</td>
</tr>
<tr>
<td>83</td>
<td>78</td>
<td>$V=5.444(1-exp(-0.240t))+0.0167t$</td>
<td>0.966</td>
<td>1.592</td>
</tr>
</tbody>
</table>

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### Table c.17. Leak-off Volume (cc/cm²) for Core 5 - \( \mu = 10 \) cp - \( \Delta P = 800 \) psi

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0</th>
<th>10</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
<th>210</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Rate (cc/min)</td>
<td>324</td>
<td>251</td>
<td>280</td>
<td>275</td>
<td>331</td>
<td>483</td>
<td>310</td>
<td>690</td>
<td>534</td>
</tr>
<tr>
<td>Initial Gas Void Fraction (%)</td>
<td>10</td>
<td>2</td>
<td>30</td>
<td>29</td>
<td>65</td>
<td>74</td>
<td>71</td>
<td>85</td>
<td>93</td>
</tr>
<tr>
<td>Final Gas Void Fraction (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table c.18. Curve-fit Coefficients for Leak-off Volume - Core 5 - \( \Delta P = 800 \) psi

<table>
<thead>
<tr>
<th>Hg (%)</th>
<th>Hgi (%)</th>
<th>Equation</th>
<th>( R^2 )</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>( V=1.661(1-\exp(-1.735t))+0.0327t )</td>
<td>0.995</td>
<td>0.027</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>( V=1.300(1-\exp(-1.744t))+0.0613t )</td>
<td>0.988</td>
<td>0.137</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>( V=2.594(1-\exp(-1.499t))+0.0473t )</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>33</td>
<td>29</td>
<td>( V=1.922(1-\exp(-1.825t))+0.0569t )</td>
<td>0.993</td>
<td>0.082</td>
</tr>
<tr>
<td>63</td>
<td>65</td>
<td>( V=3.465(1-\exp(-1.989t))+0.0566t )</td>
<td>0.994</td>
<td>0.140</td>
</tr>
<tr>
<td>64</td>
<td>74</td>
<td>( V=4.471(1-\exp(-2.553t))+0.0454t )</td>
<td>0.993</td>
<td>0.165</td>
</tr>
<tr>
<td>70</td>
<td>71</td>
<td>( V=4.443(1-\exp(-1.699t))+0.0431t )</td>
<td>0.999</td>
<td>0.004</td>
</tr>
<tr>
<td>85</td>
<td>85</td>
<td>( V=5.139(1-\exp(-1.758t))+0.0417t )</td>
<td>0.999</td>
<td>0.008</td>
</tr>
<tr>
<td>99</td>
<td>93</td>
<td>( V=5.275(1-\exp(-1.221t))+0.0285t )</td>
<td>0.999</td>
<td>0.049</td>
</tr>
</tbody>
</table>

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Table c.19. Leak-off Volume (cc/cm³) for Core 2 - $\mu = 10$ cp - $\Delta P = 1,000$ psi

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Average Flow Rate (cc/min)</th>
<th>Initial Gas Void Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>119</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>400</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>326</td>
<td>7</td>
</tr>
<tr>
<td>400</td>
<td>326</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Average Gas Void Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>1.149</td>
</tr>
<tr>
<td>18</td>
<td>1.432</td>
</tr>
<tr>
<td>40</td>
<td>1.604</td>
</tr>
<tr>
<td>60</td>
<td>1.719</td>
</tr>
<tr>
<td>70</td>
<td>2.004</td>
</tr>
</tbody>
</table>

Table c.20. Curve-fit Coefficients for Leak-off Volume - Core 2 - $\Delta P = 1,000$ psi

<table>
<thead>
<tr>
<th>Hg (%)</th>
<th>Hgi (%)</th>
<th>Equation</th>
<th>$R^2$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$V=1.509(1-exp(-1.284t))+0.0363t$</td>
<td>0.994</td>
<td>0.033</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>$V=1.606(1-exp(-1.298t))+0.0320t$</td>
<td>0.998</td>
<td>0.005</td>
</tr>
<tr>
<td>18</td>
<td>7</td>
<td>$V=1.570(1-exp(-2.129t))+0.0367t$</td>
<td>0.999</td>
<td>0.002</td>
</tr>
<tr>
<td>49</td>
<td>6</td>
<td>$V=1.615(1-exp(-0.592t))+0.0439t$</td>
<td>0.964</td>
<td>0.376</td>
</tr>
<tr>
<td>60</td>
<td>62</td>
<td>$V=4.341(1-exp(-1.547t))+0.0316t$</td>
<td>0.999</td>
<td>0.005</td>
</tr>
<tr>
<td>70</td>
<td>70</td>
<td>$V=4.831(1-exp(-1.533t))+0.0298t$</td>
<td>0.999</td>
<td>0.006</td>
</tr>
</tbody>
</table>

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Table c.21. Leak-off Volume (cc/cm²) for Core 3 - $\mu = 10$ cp - $\Delta P = 1,000$ psi

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Hg (%)</th>
<th>Initial Gas Void Fraction (%)</th>
<th>Average Flow Rate (cc/min)</th>
<th>Average Gas Void Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.000</td>
<td>310</td>
<td>0.000</td>
</tr>
<tr>
<td>16</td>
<td>16</td>
<td>1.401</td>
<td>376</td>
<td>1.677</td>
</tr>
<tr>
<td>26</td>
<td>53</td>
<td>1.681</td>
<td>265</td>
<td>3.179</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>1.850</td>
<td>250</td>
<td>3.921</td>
</tr>
<tr>
<td>40</td>
<td>61</td>
<td>1.962</td>
<td>200</td>
<td>4.377</td>
</tr>
<tr>
<td>41</td>
<td>62</td>
<td>2.018</td>
<td>397</td>
<td>4.755</td>
</tr>
<tr>
<td>58</td>
<td>70</td>
<td>2.129</td>
<td>297</td>
<td>5.207</td>
</tr>
<tr>
<td>67</td>
<td>83</td>
<td>2.240</td>
<td>297</td>
<td>5.677</td>
</tr>
<tr>
<td>83</td>
<td>83</td>
<td>2.527</td>
<td>310</td>
<td>4.212</td>
</tr>
</tbody>
</table>

Table c.22. Curve-fit Coefficients for Leak-off Volume - Core 3 - $\Delta P = 1,000$ psi

<table>
<thead>
<tr>
<th>Hg (%)</th>
<th>Hgi (%)</th>
<th>Equation</th>
<th>$R^2$ (d'less)</th>
<th>$\chi^2$ (d'less)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$V=1.735(1-\exp(-1.546t))+0.0256t$</td>
<td>0.998</td>
<td>0.069</td>
</tr>
<tr>
<td>16</td>
<td>13</td>
<td>$V=1.827(1-\exp(-2.125t))+0.0212t$</td>
<td>0.995</td>
<td>0.048</td>
</tr>
<tr>
<td>26</td>
<td>53</td>
<td>$V=4.115(1-\exp(-1.468t))+0.0319t$</td>
<td>0.999</td>
<td>0.014</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>$V=2.861(1-\exp(-1.476t))+0.0356t$</td>
<td>0.999</td>
<td>0.015</td>
</tr>
<tr>
<td>40</td>
<td>39</td>
<td>$V=3.359(1-\exp(-1.543t))+0.0340t$</td>
<td>0.999</td>
<td>0.003</td>
</tr>
<tr>
<td>41</td>
<td>61</td>
<td>$V=4.461(1-\exp(-1.349t))+0.0315t$</td>
<td>0.999</td>
<td>0.013</td>
</tr>
<tr>
<td>58</td>
<td>62</td>
<td>$V=3.728(1-\exp(-1.502t))+0.0241t$</td>
<td>0.999</td>
<td>0.001</td>
</tr>
<tr>
<td>67</td>
<td>70</td>
<td>$V=4.802(1-\exp(-0.844t))+0.0427t$</td>
<td>0.950</td>
<td>1.713</td>
</tr>
<tr>
<td>83</td>
<td>83</td>
<td>$V=5.461(1-\exp(-1.570t))+0.0286t$</td>
<td>0.999</td>
<td>0.003</td>
</tr>
</tbody>
</table>

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Table c.23. Standard Error and *t*-Test - Core 2 - $\Delta P = 200$ psi

<table>
<thead>
<tr>
<th></th>
<th>Initial Gas Void Fraction (%)</th>
<th></th>
<th>Average Gas Void Fraction (%)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Gas Void Fraction (%)</td>
<td>0</td>
<td>30</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Err</td>
<td>0.066</td>
<td>0.199</td>
<td>0.059</td>
<td>0.047</td>
</tr>
<tr>
<td>t</td>
<td>-0.143</td>
<td>-3.449</td>
<td>-1.023</td>
<td>0.384</td>
</tr>
<tr>
<td>Hyp</td>
<td>NR*</td>
<td>R</td>
<td>NR</td>
<td>NR</td>
</tr>
</tbody>
</table>

Table c.24. Standard Error and *t*-Test - Core 7 - $\Delta P = 200$ psi

<table>
<thead>
<tr>
<th></th>
<th>Initial Gas Void Fraction (%)</th>
<th></th>
<th>Average Gas Void Fraction (%)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Gas Void Fraction (%)</td>
<td>1</td>
<td>10</td>
<td>60</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10</td>
<td>20</td>
<td>36</td>
</tr>
<tr>
<td>Err</td>
<td>0.053</td>
<td>0.055</td>
<td>0.057</td>
<td>0.052</td>
</tr>
<tr>
<td>t</td>
<td>0.916</td>
<td>1.028</td>
<td>-0.747</td>
<td>-0.701</td>
</tr>
<tr>
<td>Hyp</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
</tr>
</tbody>
</table>

Table c.25. Standard Error and *t*-Test - Core 8 - $\Delta P = 200$ psi

<table>
<thead>
<tr>
<th></th>
<th>Initial Gas Void Fraction (%)</th>
<th></th>
<th>Average Gas Void Fraction (%)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Gas Void Fraction (%)</td>
<td>0</td>
<td>25</td>
<td>11</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>25</td>
<td>33</td>
<td>46</td>
</tr>
<tr>
<td>Err</td>
<td>0.062</td>
<td>0.028</td>
<td>0.064</td>
<td>0.045</td>
</tr>
<tr>
<td>t</td>
<td>1.064</td>
<td>0.786</td>
<td>0.847</td>
<td>-0.135</td>
</tr>
<tr>
<td>Hyp</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
</tr>
</tbody>
</table>

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### Table c.26. Standard Error and t-Test - Core 2 - \( \Delta P = 400 \text{ psi} \)

<table>
<thead>
<tr>
<th>Initial Gas Void Fraction (%)</th>
<th>4</th>
<th>16</th>
<th>44</th>
<th>30</th>
<th>1</th>
<th>41</th>
<th>75</th>
<th>21</th>
<th>80</th>
<th>95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Gas Void Fraction (%)</td>
<td>4</td>
<td>15</td>
<td>16</td>
<td>30</td>
<td>39</td>
<td>40</td>
<td>66</td>
<td>73</td>
<td>80</td>
<td>92</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Err</th>
<th>0.086</th>
<th>0.122</th>
<th>0.129</th>
<th>0.050</th>
<th>0.069</th>
<th>0.037</th>
<th>0.095</th>
<th>0.184</th>
<th>0.042</th>
<th>0.492</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>0.933</td>
<td>-0.833</td>
<td>3.224</td>
<td>2.051</td>
<td>-0.142</td>
<td>1.935</td>
<td>3.584</td>
<td>2.434</td>
<td>1.820</td>
<td>2.406</td>
</tr>
<tr>
<td>Hyp</td>
<td>NR</td>
<td>NR</td>
<td>R</td>
<td>NR</td>
<td>NR</td>
<td>R</td>
<td>R</td>
<td>NR</td>
<td>R</td>
<td>R</td>
</tr>
</tbody>
</table>

### Table c.27. Standard Error and t-Test - Core 6 - \( \Delta P = 400 \text{ psi} \)

<table>
<thead>
<tr>
<th>Initial Gas Void Fraction (%)</th>
<th>0</th>
<th>20</th>
<th>15</th>
<th>20</th>
<th>26</th>
<th>3</th>
<th>13</th>
<th>80</th>
<th>91</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Gas Void Fraction (%)</td>
<td>0</td>
<td>12</td>
<td>15</td>
<td>21</td>
<td>26</td>
<td>40</td>
<td>51</td>
<td>79</td>
<td>93</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Err</th>
<th>0.103</th>
<th>0.250</th>
<th>0.056</th>
<th>0.095</th>
<th>0.052</th>
<th>0.194</th>
<th>0.055</th>
<th>0.188</th>
<th>0.041</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>-1.522</td>
<td>-3.486</td>
<td>1.418</td>
<td>-2.102</td>
<td>1.669</td>
<td>-2.539</td>
<td>0.937</td>
<td>-4.749</td>
<td>-0.131</td>
</tr>
<tr>
<td>Hyp</td>
<td>NR</td>
<td>R</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>R</td>
<td>NR</td>
<td>R</td>
<td>NR</td>
</tr>
</tbody>
</table>

### Table c.28. Standard Error and t-Test - Core 2 - \( \Delta P = 600 \text{ psi} \)

<table>
<thead>
<tr>
<th>Initial Gas Void Fraction (%)</th>
<th>0%</th>
<th>29%</th>
<th>8%</th>
<th>10%</th>
<th>23%</th>
<th>25%</th>
<th>8%</th>
<th>3%</th>
<th>14%</th>
<th>48%</th>
<th>65%</th>
<th>52%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final Gas Void Fraction (%)</td>
<td>0%</td>
<td>0%</td>
<td>1%</td>
<td>10%</td>
<td>22%</td>
<td>25%</td>
<td>36%</td>
<td>41%</td>
<td>44%</td>
<td>53%</td>
<td>65%</td>
<td>73%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Err</th>
<th>0.046</th>
<th>0.069</th>
<th>0.141</th>
<th>0.029</th>
<th>0.265</th>
<th>0.010</th>
<th>0.078</th>
<th>0.195</th>
<th>0.057</th>
<th>0.063</th>
<th>0.012</th>
<th>0.159</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>0.787</td>
<td>2.634</td>
<td>-2.098</td>
<td>0.653</td>
<td>-2.992</td>
<td>0.265</td>
<td>1.358</td>
<td>-2.213</td>
<td>1.517</td>
<td>1.618</td>
<td>-0.044</td>
<td>4.840</td>
</tr>
<tr>
<td>Hyp</td>
<td>NR</td>
<td>R</td>
<td>NR</td>
<td>NR</td>
<td>R</td>
<td>NR</td>
<td>NR</td>
<td>R</td>
<td>NR</td>
<td>NR</td>
<td>R</td>
<td>R</td>
</tr>
</tbody>
</table>
Table c.29. Standard Error and $t$-Test - Core 4 - $\Delta P = 600$ psi

| Initial Gas Void Fraction (%) | 11 | 52 | 22 | 16 | 37 | 43 | 7 | 75 | 16 | 62 | 98 |
|-------------------------------|----|----|----|----|----|----|---|---|----|----|----|----|
| Final Gas Void Fraction (%)  | 0  | 16 | 20 | 21 | 37 | 43 | 50| 52 | 54 | 62 | 75 |
| Err                          | 0.100 | 0.124 | 0.066 | 0.088 | 0.064 | 0.013 | 0.129 | 0.041 | 0.097 | 0.011 | 0.168 |
| $t$                          | 0.219 | -0.635 | -1.768 | -2.031 | 0.053 | 1.007 | -0.275 | 0.833 | 0.151 | -0.169 | -4.770 |
| Hyp                          | NR | NR | NR | NR | NR | NR | NR | NR | NR | NR | R |

Table c.30. Standard Error and $t$-Test - Core 2 - $\Delta P = 800$ psi

<table>
<thead>
<tr>
<th>Initial Gas Void Fraction (%)</th>
<th>0</th>
<th>2</th>
<th>77</th>
<th>20</th>
<th>5</th>
<th>26</th>
<th>71</th>
<th>85</th>
<th>4</th>
<th>50</th>
<th>2</th>
<th>61</th>
<th>78</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final Gas Void Fraction (%)</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>20</td>
<td>23</td>
<td>28</td>
<td>28</td>
<td>38</td>
<td>50</td>
<td>50</td>
<td>55</td>
<td>57</td>
<td>83</td>
</tr>
<tr>
<td>Err</td>
<td>0.088</td>
<td>0.037</td>
<td>0.074</td>
<td>0.070</td>
<td>0.113</td>
<td>0.157</td>
<td>0.055</td>
<td>0.054</td>
<td>0.090</td>
<td>0.001</td>
<td>0.258</td>
<td>0.068</td>
<td>0.297</td>
</tr>
<tr>
<td>$t$</td>
<td>2.356</td>
<td>-0.224</td>
<td>-1.321</td>
<td>2.515</td>
<td>0.602</td>
<td>0.562</td>
<td>1.956</td>
<td>-0.692</td>
<td>-0.381</td>
<td>0.896</td>
<td>-0.928</td>
<td>2.290</td>
<td>-2.491</td>
</tr>
<tr>
<td>Hyp</td>
<td>R</td>
<td>NR</td>
<td>NR</td>
<td>R</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>R</td>
<td>R</td>
</tr>
</tbody>
</table>

Table c.31. Standard Error and $t$-Test - Core 5 - $\Delta P = 800$ psi

<table>
<thead>
<tr>
<th>Initial Gas Void Fraction (%)</th>
<th>10</th>
<th>2</th>
<th>30</th>
<th>29</th>
<th>65</th>
<th>74</th>
<th>71</th>
<th>85</th>
<th>93</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final Gas Void Fraction (%)</td>
<td>0</td>
<td>10</td>
<td>30</td>
<td>33</td>
<td>63</td>
<td>64</td>
<td>70</td>
<td>85</td>
<td>99</td>
</tr>
<tr>
<td>Err</td>
<td>0.069</td>
<td>0.130</td>
<td>0.045</td>
<td>0.109</td>
<td>0.121</td>
<td>0.043</td>
<td>0.023</td>
<td>0.026</td>
<td>0.122</td>
</tr>
<tr>
<td>$t$</td>
<td>0.389</td>
<td>-0.733</td>
<td>2.119</td>
<td>-2.163</td>
<td>-3.671</td>
<td>-0.846</td>
<td>-1.583</td>
<td>-1.504</td>
<td>-4.165</td>
</tr>
<tr>
<td>Hyp</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>R</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
<td>R</td>
</tr>
</tbody>
</table>

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Table c.32. Standard Error and $t$-Test - Core 2 - $\Delta P = 1,000$ psi

<table>
<thead>
<tr>
<th>xxxx</th>
<th>Initial Gas Void Fraction (%)</th>
<th>xxxx</th>
<th>Average Gas Void Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>xxxx</td>
<td>0</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>xxxx</td>
<td>0</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>Err</td>
<td>0.041</td>
<td>0.061</td>
<td>0.025</td>
</tr>
<tr>
<td>t</td>
<td>0.515</td>
<td>-1.847</td>
<td>0.579</td>
</tr>
<tr>
<td>Hyp</td>
<td>NR</td>
<td>NR</td>
<td>NR</td>
</tr>
</tbody>
</table>

Table c.33. Standard Error and $t$-Test - Core 3 - $\Delta P = 1,000$ psi

<table>
<thead>
<tr>
<th>xxxx</th>
<th>Initial Gas Void Fraction (%)</th>
<th>xxxx</th>
<th>Average Gas Void Fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>xxxx</td>
<td>0</td>
<td>13</td>
<td>53</td>
</tr>
<tr>
<td>xxxx</td>
<td>0</td>
<td>16</td>
<td>26</td>
</tr>
<tr>
<td>Err</td>
<td>0.033</td>
<td>0.096</td>
<td>0.042</td>
</tr>
<tr>
<td>t</td>
<td>0.354</td>
<td>1.875</td>
<td>2.282</td>
</tr>
<tr>
<td>Hyp</td>
<td>NR</td>
<td>NR</td>
<td>R</td>
</tr>
</tbody>
</table>
VITA

Alvaro Felippe Negrão, son of Geraldo Felippe Negrão and Maria Ignez Marchesini Negrão, was born in São Paulo, São Paulo, Brazil, on November 26, 1955.

He graduated from Escola Politécnica da Universidade de São Paulo in December 1978, receiving his Bachelor of Science degree in Civil Engineering.

He joined Petróleo Brasileiro S.A. in January 1979 where he attended the course in Petroleum Engineering sponsored by Petrobrás until January 1981.

During the years of 1981 and 1982 he worked as a Drilling Engineering Instructor of Petrobrás Training Center. Following this, he joined the Executive Group of Offshore Drilling in Rio de Janeiro, Brazil.

In this new position he worked in offshore operations, with emphasis on well control operations and design of offshore drilling rigs.

In August 1987, he entered Universidade de Campinas, from which he received a Master's Degree in Petroleum Engineering in March 1989.

He was relocated to Petrobrás head office in April 1989, where he worked in well design, especially in deep water wells until his selection for a doctoral program.

He entered the Graduate School of Louisiana State University in August 1991. Upon graduation, he will return to work in the head office of Petrobrás, in Rio de Janeiro, Brazil.
Candidate: Alvaro Felippe Negrao

Major Field: Petroleum Engineering

Title of Dissertation: Model for Planning Well Control Operations Involving an Induced Fracture

Approved:

[Signatures]

Major Professor and Chairman
Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:
December 6, 1994