ADVANCED COMMUNICATION AND SENSING PROTOCOLS USING TWISTED LIGHT AND ENGINEERED QUANTUM STATISTICS

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Dedicated to my ancestors, whose inspirational resilience and character in the face of extreme adversity, I endeavor to honor.
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# TABLE OF CONTENTS

ACKNOWLEDGMENTS ................................................................. iii

ABSTRACT ........................................................................ vi

CHAPTER 1. INTRODUCTION ...................................................... 1

CHAPTER 2. PRELIMINARIES ..................................................... 5
  2.1 Light Sources ................................................................. 5
  2.2 Twisted Light ................................................................. 11
  2.3 Optical Fiber Modes ......................................................... 15
  2.4 Artificial Intelligence ......................................................... 20
  2.5 Plasmonics ................................................................. 23

CHAPTER 3. HIGH-DIMENSIONAL OPTICAL ENCRYPTION IN COMMERCIAL MULTIMODE FIBERS ......................................................... 25
  3.1 Motivation ................................................................. 25
  3.2 Conceptual Overview - Secure Communication with Machine Learning ......................................................... 26
  3.3 Experiment & Results ......................................................... 29

CHAPTER 4. QUANTUM PLASMONIC SENSING ........................................... 38
  4.1 Motivation ................................................................. 38
  4.2 Theoretical Overview - Conditional Quantum Measurements ......................................................... 40
  4.3 Simulation & Results ......................................................... 41

CHAPTER 5. ENGINEERING QUANTUM STATISTICS USING TWISTED LIGHT IN COMMERCIAL MULTIMODE FIBERS ......................................................... 49
  5.1 Motivation ................................................................. 49
  5.2 Conceptual Overview - Engineered Quantum Randomness ......................................................... 49
  5.3 Experiment & Results ......................................................... 52

CHAPTER 6. CONCLUSION ........................................................... 55

WORKS CITED ................................................................... 56

VITA ............................................................................. 72
ABSTRACT

Advanced performance of modern technology at a fundamental physical level is driving new innovations in communication [1–4], sensing capability [5–8], and information processing [9–13]. Key to this improvement is the ability to harness the power of physical phenomena at the quantum mechanical level, where light and light-matter interactions produce technological advancement not realizable by classical means. Theoretical investigation into quantum computing [14, 15], sensing capability beyond classical limits [16], and quantum information [17–19] has prompted experimental work to bring state-of-the-art quantum systems to the forefront for commercial use. This dissertation contributes to the latter portion of the work. A set of preliminaries is included highlighting pertinent physical foundations for the experiments herein. Three experiments are then presented: (1) using twisted coherent light in multimode fibers for secure communication with machine learning [20], (2) conditional measurements for signal-to-noise ratio improvement of quantum plasmonic sensors [21], and (3) post-selected photon-number resolving measurements for quantum information processing using twisted pseudo-thermal light [22], with key results discussed for each investigation.
CHAPTER 1.
INTRODUCTION

Optics is a subject the study of which has been millennia in the making. We will confine our conversation of optics to the late nineteenth century when James Clerk Maxwell derived his famous equations of light, or electromagnetic radiation, in free space and media [23]. His treatise on light is fundamental to modern day communication methods. Discretized energy excitations of the electromagnetic field, commonly referred to as photons, transport both energy and momentum through time and space, and are the fastest physical phenomena discovered to date [24]. Quantum optics and quantum photonics describe the interactions of very few photons with other photons or matter and is governed by the laws of quantum mechanics, whereas their classical counterparts describe the interactions of a great many photons. Applied optics and photonics, classical or quantum, in combination with information theory and metrology, provide a framework for rich exploration of secure communication and sensing protocols [25–31] which is the focus of this work.

For secure communication using light, messages are encoded in photons, the carriers of electromagnetic radiation. Because the information is stored in photons, the transfer of the information occurs at the speed of light attenuated by the given medium [32–34]. Information can be stored in the polarization and/or spatial degree(s) of freedom of the carrier light wave [35–38]. Spin angular momentum, fundamental to polarization states, gives a two-level single bit rate of information transfer. The bit rate scales as $\log_2 d$ bits/photon [39]. Hence, a spin-based optical setup will transfer one bit of information for a two-dimensional system. On the other hand, orbital angular momentum, fundamental to spatial states, is infinite-dimensional in theory and thus can have a theoretically infinite bit transfer rate per quantum system limited only by the dimension of the universe [29]. Realistic constraints, such as physical access to higher order orbital angular momentum states, significantly reduce the bit transfer rate to several bits per photon – still advantageous to a two-dimensional polarization-based system [40, 41]. We consider OAM mode approaches to secure information protocols high-
dimensional as the states are described with dimension greater than two. These quantum states of light are referred to as qudits in lieu of their two-level counterparts – qubits. The intrinsic nature of light is governed by quantum mechanics and has both classical and quantum applications, the latter of which physically guarantees the security of information transfer [42–47].

Quantum plasmonic sensing aims to interweave the topical areas of plasmonic sensing and quantum sensing[48]. Plasmons are understood as charge density oscillations in a metal[48]. Coupled with light and interacting between a surface and a dielectric, one has the surface plasmon polariton. This pseudo-particle allows for the sensing of analytes with high sensitivity and subdiffraction limit measurement in areas such as biology[49–52] and chemistry [53–55]. Unfortunately, plasmonic structures suffer loss in the measured signal due to statistical fluctuations in the light source [56]. These fluctuations disguise weaker signals within the noise floor. Quantum sensors reduce this loss to levels below the shot-noise or standard quantum limit, allowing for increased measurement sensitivity [49, 57, 58]. The combination of these two sensing methods is a recent area of interest known as quantum plasmonic sensing, which combines high-sensitivity and precision measurement with low-loss sensing for material characterization at the nanoscale, improving the sensing performance of modern plasmonic nanostructures [21, 48].

Quantum information processing describes a collaboration of ideas from the fields of computer science, engineering, physics, mathematics, and materials science, to investigate the transfer of information by exploiting quantum phenomena [59, 60]. For quantum computers, this is done at the single atom level whereas quantum communication is carried out via single photons. The former is limited via miniaturization of computer components where quantum effects dominate and the latter, via transmission capacity and speed limitations also governed at the quantum level [61, 62]. One of the key features of quantum information processing is its applicability to characterizing complex multiphotonic phenomena which cannot be simulated via classical means [11, 63–66].
This thesis is organized in the following fashion:

- Chapter 2. The preliminaries will begin with a brief theoretical discussion of and motivation for the light sources used in the various experiments. Spatial light is introduced and discussed from a theoretical perspective. The focus will be on Laguerre-Gaussian modes with emphasis on the orbital angular momentum degree of freedom. A brief introduction to optical fibers is provided with a focus on optical modes. Chapter two is based on refs. [67–69].

- Chapter 3. The first experiment emphasizes spatial light for classically secure information transfer. The focus is on using individual orbital angular momentum states of light and those in superposition for message encoding, their coupling to the multimode fiber as an encryption protocol, and convolutional neural networks for image classification as a method of message recovery. A brief discussion of machine learning is included. The experiment highlighted in chapter three is taken from ref. [20].

- Chapter 4. A simulated plasmonic structure is described where conditional measurements are used to modify the statistical properties of the thermal light source. These measurements result in the subtraction of vacuum plasmonic modes, reducing the statistical noise fluctuation loss in the plasmonic structure. This method provides a quantum enhanced signal-to-noise ratio which is presented and discussed. The discussion of the work included in in chapter four is taken from ref. [21].

- Chapter 5. Photon-number-resolving detection in conjunction with an experimentally generated orbital angular momentum cross talk matrix is used to generate quantum randomness. The experimental results can be helpful in implementing various processing methods such as quantum random number generation or quantum random walks. The resulting modified photon number statistics are underscored as the source of quantum randomness. The preliminary results of chapter five are motivated by ref. [70] and are in preparation ([22]).
Chapter 6. We conclude the thesis with a summary of the included experiments and overall experimental contributions to the field.
CHAPTER 2.
PRELIMINARIES

We provide several topical studies on areas that are relevant to the main experiments presented in this work. Twisted light covers Laguerre-Gaussian modes which serve as the experimental beams used for experiments in chapters three and five. Coherent and thermal states of light are covered here. Coherent light is used as a source for the experiment in chapter three. Thermal light is used as a simulated source for the experiment in chapter four and a realized source for the experiment in chapter five. Multimode fibers are used for OAM mode coupling in chapters three and five. Plasmonic nanostructures are discussed for nanoscale sensing as a foundation for the experiment in chapter four. Photon-number-resolving detection is introduced for the discussion of Fock state post-selected measurements in chapter five.

2.1 Light Sources

Electromagnetic radiation, or light, has been studied throughout history. From the religious practices of the ancient world to fundamental studies that are key to advancing modern technologies, we have yet to still fully comprehend the nature of light. The theoretical framework for light that we know and understand today has its origins in the works of Kepler, Galileo, Descartes, Snell, Grimaldi and Newton amongst others [71]. Newton’s particle theory of light was the first extensive and empiracle treatise on the topic [72]. Light exhibited another nature however - that of a wave. Hooke, Huygens, Euler, and Young proposed a wave nature of light in which Young’s double slit experiment explained Newton’s rings as interference phenomena[73–75]. Fresnel solidified the wave-particle duality of light’s nature with his work on diffraction and polarization [76]. The culmination of work of these historical figures is represented in the monumental achievements of Ludvig Lorenz and James Clerk Maxwell - a theoretical framework for electromagnetic radiation [77, 78]. From Maxwell, we start with his four famous self-styled equations which describe the behavior of light:
\[ \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \]
\[ \nabla \cdot \vec{B} = 0 \]
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]
\[ \nabla \times \vec{B} = \mu_0 (\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}) \]  

(2.1)

Light sources vary in their statistical profile and this often yields important implications for both theoretical and experimental data analysis [79, 80]. This exploration starts with the quantization of light as derived from Maxwell’s equations and discussed in [67]. We briefly review the framework here. Beginning with a single-mode standing wave field in one-dimension, linearly polarized in the x-direction, and satisfying Maxwell’s equations, we have the electric and magnetic fields given by

\[ E_x(z, t) = \left( \frac{2\omega^2}{V\epsilon_0} \right)^{\frac{1}{2}} q(t) \sin(kz) \]  

(2.2)

\[ B_y(z, t) = \left( \frac{\mu_0\epsilon_0}{k} \right) \left( \frac{2\omega^2}{V\epsilon_0} \right)^{\frac{1}{2}} \dot{q}(t) \cos(kz) \]  

(2.3)

where \( \omega \) is the frequency, \( k \) is the wave number, \( V \) is the effective volume of the cavity, \( \epsilon_0 \) is the permittivity of free-space, and \( q(t) \) serves as the canonical position. The field energy is given by the Hamiltonian

\[ H = \frac{1}{2} \int dz \left( \epsilon_0 E_x^2(z, t) + \frac{1}{\mu_0} B_y^2(z, t) \right). \]  

(2.4)

We also assume perfectly conducting walls of distance \( L \) for our single-mode field. For proper boundary conditions of a vanishing field at the walls, the frequencies take on discrete values of \( \omega_m = c \left( \frac{m\pi}{L} \right) \) for \( m = 1, 2, ... \) Substituting Eqs. 2.2 and 2.3 with the given frequencies into 2.4 gives the Hamiltonian in canonical coordinates

\[ H = \frac{1}{2}(p^2 + \omega^2 q^2). \]  

(2.5)
By convention, we make use of the non-Hermitian creation ($\hat{a}^\dagger$) and annihilation ($\hat{a}$) operators in place of the canonical coordinates $\hat{q}$ and $\hat{p}$. They are given as

$$\hat{a}^\dagger = \frac{1}{2\hbar\omega}(\omega q - ip)$$ \hspace{1cm} (2.6)$$

$$\hat{a} = \frac{1}{2\hbar\omega}(\omega q + ip)$$ \hspace{1cm} (2.7)$$

with the commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$. The Hamiltonian in terms of these new operators is of the form

$$\hat{H} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right).$$ \hspace{1cm} (2.8)$$

$\hat{n} = \hat{a}^\dagger\hat{a}$ is defined as the number operator. We also introduce the quadrature operators in terms of the creation and annihilation operators

$$\hat{X} = \frac{\hat{a} + \hat{a}^\dagger}{2}$$ \hspace{1cm} (2.9)$$

$$\hat{P} = \frac{\hat{a} - \hat{a}^\dagger}{2i}$$ \hspace{1cm} (2.10)$$

where $\hat{X}$ and $\hat{P}$ are the dimensionless conjugate position and momentum operators. These operators are in phase quadrature which means they are associated with the time-dependent electric field amplitudes experiencing a $\frac{\pi}{2}$ phase difference. They obey the commutation relation $[\hat{X}, \hat{P}] = \frac{i}{2}$. Eqs. 2.8, 2.9, and 2.10 will be used as a starting point to discuss commonly used sources of light.

2.1.1 Fock States

Fock states are synonymous with number states and are the energy eigenstates of the optical field Hamiltonian. Substituting the number operator $\hat{n}$ into Eq. 2.8 gives

$$\hat{H} |n\rangle = \hbar\omega\left(\hat{n} + \frac{1}{2}\right)|n\rangle$$ \hspace{1cm} (2.11)$$

$$= E_n |n\rangle$$ \hspace{1cm} (2.12)$$
The Fock states, denoted $|n\rangle$, are definite number states with no statistical fluctuation in photon number. They constitute a complete orthonormal set defined as

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}} |0\rangle$$

(2.13)

The creation and annihilation operators act on this state to create a quantum of energy $\hbar \omega$ or destroy said quantum. Number states may be generated or removed by repeated application of these operators

$$\hat{n} |n\rangle = \hat{n} |n\rangle$$

(2.14)

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle$$

(2.15)

$$\hat{a} |n\rangle = \sqrt{n} |n-1\rangle.$$  

(2.16)

It is important to note that, although the Fock states have a well-defined photon number, they do not have a well-defined electric field. This is shown by the electric field variance

$$\Delta E_x = \sqrt{2\mathcal{E}_0} \sqrt{n + \frac{1}{2}} \sin(kz)$$

(2.17)

which for $n = 0$ has a non-zero fluctuations. This implies that even in vacuum, there is some physical phenomena present. This is the minimum uncertainty for Fock states. The quadrature variance however is quite large: $\langle (\Delta \hat{X})^2 \rangle \langle (\Delta \hat{P})^2 \rangle = \frac{1}{16} (2n + 1)^2$ and highlights the uncertainty relationship between the number states and the electric field. Fock states will be used to discuss photon-number-resolving detection in chapter five.

### 2.1.2 Vacuum States

As mentioned in section 2.1.1, the variance of the electric field for $n = 0$ photon number is non-zero [67]. This is realized via electric field strength fluctuations $\Delta E_x = \mathcal{E}_0 \sin(kz)$ for $|0\rangle$, the vacuum state [67]. This is a truly interesting phenomena. The vacuum states have zero photon number but possess tangible outcomes to quantum measurements due to
the fluctuations in electric field amplitude and quadrature variance [81, 82]. In chapter four, we look at conditional quantum measurements as a way to remove the contribution of vacuum fluctuations to a desired signal output [21]. This increases the signal-to-noise ratio, enhancing sensing capability beyond the shot-noise limit.

2.1.3 Coherent States

Coherent states are unique in that they are the closest "classical" quantum state due to exhibiting classical photon statistics. They are the eigenstates of the annihilation operator and are denoted $|\alpha\rangle$. The relation between $\hat{a}$ and $\alpha$ is shown by

$$\hat{a} |\alpha\rangle = \alpha |\alpha\rangle . \tag{2.18}$$

The electric field fluctuations are the same as that for the vacuum state. Since the Fock states are a complete orthonormal set, we can expand the coherent states in terms of Fock states to determine the coherent state relation. We start with

$$|\alpha\rangle = \sum_{n=0}^{\infty} C_n |n\rangle \tag{2.19}$$

and apply the annihilation operator to both sides of the sum. A recursion relation results for the coefficient $C_n$

$$C_n = \frac{\alpha^n}{\sqrt{n!}} C_0 \tag{2.20}$$

and we have

$$|\alpha\rangle = C_0 \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \tag{2.21}$$

where normalization yields

$$|\alpha\rangle = \exp \left( -|\alpha^2|/2 \right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle . \tag{2.22}$$
Coherent states are also referred to as displaced vacuum states because they can also be generated by displacing vacuum in phase space. Using the displacement operator $\hat{D}$, we have

\[
|\alpha\rangle = \hat{D}(\alpha) |0\rangle \tag{2.23}
\]

\[
\hat{D}(\alpha) = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a}). \tag{2.24}
\]

Inserting Eq. 2.24 into Eq. 2.23, we obtain the same expression in Eq. 2.22, showing equivalence with coherent states. So although coherent states exhibit some classical characteristics, they are indeed quantum states. Coherent states exhibit Poissonian statistics and are ubiquitous in Gaussian beam lasers which are used for the experiments in chapters three and five.

2.1.4 Thermal States

Thermal states are black body radiators and they exhibit statistically random field fluctuations (i.e. they are noisy). The most common example of thermal radiation is sunlight. A single-mode field with cavity walls at at temperature T in thermal equilibrium is discussed here. The field can be described by a probability density operator

\[
\hat{\rho}_T = \sum_{n=0}^{\infty} P(n) |n\rangle \langle n| \tag{2.25}
\]

and further characterized by a microcanonical partition function

\[
Z = \exp\left(-\frac{\hbar \omega}{2k_B T}\right) \sum_{n=0}^{\infty} \exp\left(-\frac{n\hbar \omega}{k_B T}\right), \tag{2.26}
\]

where $n$ is the energy quantum number: $E_n = \hbar \omega \left(n + \frac{1}{2}\right)$. The defining characteristic of thermal states is that they exhibit super-Poissonian statistics. The photon variance is higher than the mean photon number for a given ensemble. The probability distribution is the
familiar Bose-Einstein relation

\[ P_{Th}(n) = \frac{1}{\exp\left(\frac{\hbar \omega}{k_BT}\right) - 1}. \] (2.27)

Pseudo-thermal states can be generated via a coherent source by introducing a rotating ground glass element which randomizes the coherent source and produces thermal light with the above statistics. This method is used to produce the thermal light source for the experiments in chapters three and five.

2.2 Twisted Light

When light interacts with any material, there is an exchange of energy and momentum. The latter physical phenomena can be realized as linear momentum and/or angular momentum. The angular momentum component can have both spin and spatial contributions due to polarization and orbital angular momentum (OAM) respectively [83, 84]. One common way to study this light-matter interaction is with a laser. For the experiments herein, we take the laser to be a quantum coherent source of light [85]. Lasers are ubiquitous in experimental studies involving light. It has been shown that the beam of a Gaussian laser carries OAM of \( \ell \hbar \), where \( \ell \) is the integer OAM quantum number [86]. The laser is defined by a complete set of eigenmode solutions to the Paraxial Helmholtz equation. The equation in cartesian coordinates gives the complete set Hermite-Gaussian (HG) solutions which are the standard eigenmodes of a Gaussian laser beam. It is common practice to use the paraxial approximation of the Helmholtz equation in cylindrical coordinates to derive the complete set of Laguerre-Gaussian (LG) eigenmode equations: [69, 87]:

\[
\begin{align*}
    u_{\ell p}^{LG}(r, \phi, z) &= \frac{C_{\ell p}^{LG}}{w(z)} \left( \frac{p \sqrt{2}}{w(z)} \right)^{|\ell|} \exp \left[ -\frac{\rho^2}{w^2(z)} \right] L_p^{|\ell|} \left( \frac{2 \rho^2}{w^2(z)} \right) \exp[-i\ell \phi] \\
    &\times \exp \left[ -i k_0 \rho^2 \frac{z}{2(z^2 + z_R^2)} \right] \exp \left[ i(2p + |\ell| + 1) \tan^{-1} \left( \frac{z}{z_R} \right) \right].
\end{align*}
\] (2.28)
where \( l \) and \( p \) are the azimuthal and radial mode indices, \( C_{lp}^{LG} = \sqrt{\frac{2p!\pi}{(p+|l|)!}} \) is the normalization constant, \( I_p^{|l|} \left( \frac{2\rho^2}{w_0^2(z)} \right) \) is a generalized Laguerre polynomial, and the remaining factors are Gaussian beam characterization parameters. Using the wave equation:

\[
(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) \tilde{u}(\vec{r}, t) = 0,
\]

(2.29)

where \( u(\vec{r}, t) \) is a wave function assumed to be separable, we can derive the paraxial Helmholtz solution by separation of variables. The result is a framework for describing beams of light carrying OAM. The wave equation \( u(\vec{r}, t) \) is indeed separable into spatial and temporal components. We will denote spatial factor as \( \bar{u}(\vec{r}) \). The resulting Helmholtz wave equation is given by

\[
(\nabla^2 + k^2) \bar{u}(\vec{r}) = 0
\]

(2.30)

\[
\nabla^2 \bar{u} = -k^2 \bar{u}
\]

(2.31)

\[
\bar{U}(\vec{r}) = A(\vec{r})e^{ikz},
\]

(2.32)

where the complex amplitude is defined in Eq. 2.6. The transverse beam profile of a laser has a slowly varying amplitude with respect to propagation direction. This feature satisfies the small angle approximation for paraxial rays, i.e. divergence of the ray from the optical axis satisfies \( \sin \theta = \theta \). Describing beam propagation via the paraxial approximation has been long studied [88]. This approximation is beneficial in that it reduces the mathematical complexity of solving the full Helmholtz equation for laser beams. It allows for the neglect of higher order derivative terms in Eq. 2.5 along the propagation axis:

\[
\left| \frac{\partial^2 u}{\partial z^2} \right| \ll \left| k \frac{\partial u}{\partial z} \right|
\]

(2.33)
resulting in the Paraxial Helmholtz wave equation which is used to describe the behavior of laser light

\[ \nabla^2_{\perp} \vec{u} + 2ik \frac{\partial \vec{u}}{\partial z} = 0. \]  

(2.34)

Here, \( \nabla^2_{\perp} \) is the transverse Laplacian operator. Solving Eq. 2.34 in cylindrical coordinates yields the LG modes of Eq. 2.28. LG modes are characterized by the \( e^{-i\ell\phi} \) factor which imparts a helical twist to the wave front as can be seen in Fig. 2.1. This helical twist is the OAM and, for the nodal intensity component \( p = 0 \), the resulting beams are referred to as OAM modes. Only for \( p \neq 0 \) do we have true LG modes (other than that of zero order). This is an important distinction as \textit{LG modes are modulated in beam intensity as well as phase} whereas \textit{OAM modes refer to phase modulated beams only}. For the experiments in this work, OAM modes are used exclusively. Recently, research has been reviewed for LG modes with nodal intensity index \( p \neq 0 \) for a variety of technologies and applications [38]. Again, \( \ell \) is the topological charge and \( \phi \) is the azimuthal component of the transverse field. The sign of \( \ell \) gives the handedness of the twist direction. We note that although circularly polarized light has a similar rotation, it is distinct from spatial or twisted light. In Fig. 2.1 (a) and (b), the rotation of the Poynting vector reflects the circularly polarized field carrying spin angular momentum. In (c) and (d), the twisting of the entire wavefront around the propagation axis, not just a given radial point, is indicative of the azimuthal contribution of a beam carrying...
Figure 2.2. An OAM beam can be generated from a Gaussian beam phase modulated by (a) a spiral waveplate, (b) a spiral phase hologram, or (c) a fork hologram resulting in $\ell$ windings of the wavefront around the axis producing the familiar optical vortex or donut mode. These figures are taken from ref. [90].

OAM. The topological charge $\ell$ determines the number of intertwined helices the wavefront makes around the axis. For non-zero $\ell$, the $z$ axis acquires a screw dislocation or optical vortex of zero field intensity. Herein, the topological charge is further defined as the number of full twists that the phase wraps around the propagation axis in a single wavelength. The size of the vortex is proportional to $\ell$. Generation of OAM modes can be done in a few ways; a Gaussian beam is phase modulated by a spiral waveplate, or by a phase hologram with a spiral phase or fork phase pattern [90]. These phase modulating tools can be machined or in device form where software programming is used to specify the topological charge. The resulting intensity profile with a given optical vortex for $\ell \neq 0$ is the affectionately called a donut mode due to its toroidal structure as seen in Fig. 2.2. Several experimental donut mode intensity profiles are shown for various $\ell$ values in Fig. 2.3. It is within this phase singularity that the capacity for information storage is increased for increasing $\ell$, and this capacity increase within OAM modes has been shown [91]. Communication with OAM modes is presented in chapter three with a high-dimensional optical encryption protocol. In addition to this work, exciting research for high-capacity information transfer in various media is underway [92–94].
Figure 2.3. Experimental spatial intensity distributions of LG modes with azimuthal quantum numbers, $\ell = -10$, $\ell = -1$, $\ell = 0$, $\ell = 1$, $\ell = 10$. This figure is included here from ref. [20].

2.3 Optical Fiber Modes

Any discussion on fiber optics starts with propagation of light in media. Fundamentally, an optical fiber is a thin cylinder of silica glass surrounded by a cladding material with a lower refractive index than that of the fiber. Fibers that exhibit such a marked change in refractive index at the fiber core-to-cladding interface are called step-index fibers. Graded index fibers have a more gradually changing index within the fiber core. In this work, experiments where optical fibers are used, they are step-index fibers exclusively. A description of Further reading on the guided wave within a fiber via total internal reflection can be found in [95]. Here, we will focus on fiber modes. In light of the silica glass material, we present Maxwell’s equations without sources in a medium to begin the discussion of optical modes

\[
\nabla \cdot \vec{D} = \rho_f
\]
\[
\nabla \cdot \vec{B} = 0
\]
\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}
\]
\[
\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}.
\] (2.35)

The vectors $\vec{D}$ and $\vec{B}$ are the field flux densities and $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields. A similar derivation to section 2.2 yields the electric field wave

\[
\nabla^2 \vec{E} + n^2(\omega)k_0^2 \vec{E} = 0
\] (2.36)
Figure 2.4. Cross sectional area for the (L) step-index and (R) graded-index optical fiber. The step or jump between refractive indices $n_1$ and $n_2$ for the step-index fiber can clearly be seen to differ from the more gradual change of its graded index counterpart. These figures are included here from ref. [95].

where $k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda}$. The field oscillates at frequency $\omega$ with wavelength $\lambda$. We will describe and define the concept of a fiber mode following the technique found in [95]. Similar to other wave phenomena, fiber modes constitute a complete set of solutions to Eq. 2.36. It is a boundary value problem with the characteristic that the spatial profile of the optical field is constant with respect to propagation. We are motivated by the geometrical symmetry of the fiber to write Eq. 2.36 in cylindrical coordinates

$$\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + n_2^2 k_0^2 E_z = 0. \quad (2.37)$$

We consider a separable solution of the form

$$E_z(\rho, \phi, z) = R(\rho) \Phi(\phi) Z(z) \quad (2.38)$$
which, when substituted into Eq. 2.37, produces three independent differential equations

\[ \frac{d^2 Z}{dz^2} + \beta^2 Z = 0 \]  
(2.39)

\[ \frac{d^2 \Phi}{d\phi^2} + m^2 \Phi = 0 \]  
(2.40)

\[ \frac{d^2 R}{d\rho^2} + \frac{1}{\rho} \frac{dR}{d\rho} + \left( n^2 k_0^2 - \beta^2 - \frac{m^2}{\rho^2} \right) R = 0. \]  
(2.41)

The solutions to Eqs. 2.39 - 2.41 characterize the propagation, phase, and core/cladding interaction of the propagating waves \( E_z \) and \( B_z \)

\[ Z = e^{i\gamma z} \]  
(2.42)

\[ \Phi = e^{im\phi} \]  
(2.43)

\[ R(\rho) = \begin{cases} 
A_1 J_m(p\rho) + A'_1 Y_m(p\rho); & \rho \leq a, \\
A_3 K_m(q\rho) + A'_3 I_m(q\rho); & \rho > a, 
\end{cases} \]  
(2.44)

where \( \gamma \) is the propagation constant, and \( m \) is an integer value by the constraint that the field must be periodic in \( \phi \). Eq. 2.44 is satisfied by the Bessel functions [96] with coefficients \( A_1 \) and \( A_3 \) determined by boundary conditions of the fiber core/cladding interface, and \( J_m \), \( Y_m \), \( K_m \), and \( I_m \) are Bessel functions of different kinds. The parameters \( p \) and \( q \) given by

\[ p^2 = n^2 k_0^2 - \gamma^2 \]  
(2.45)

\[ q^2 = \gamma^2 - n^2 k_0^2 \]  
(2.46)

where the refractive index \( n \) is of the form

\[ n = \begin{cases} 
n_1; & \rho \leq a \\
n_2; & \rho > a 
\end{cases} \]  
(2.47)
with the propagation constant $\gamma$ yet to be determined. After the application of boundary conditions discussed in [95], we present the $z$-component of the resultant electric and magnetic fields

\[
E_z = \begin{cases} 
A_1 J_m(p\rho) \exp(i m \phi) \exp(i \gamma z); & \rho \leq a \\
A_3 K_m(q\rho) \exp(i m \phi) \exp(i \gamma z); & \rho > a
\end{cases}
\]  

(2.48)

\[
H_z = \begin{cases} 
A_2 J_m(p\rho) \exp(i m \phi) \exp(i \gamma z); & \rho \leq a \\
A_4 K_m(q\rho) \exp(i m \phi) \exp(i \gamma z); & \rho > a
\end{cases}
\]  

(2.49)

where expressions for $E_\rho$, $E_\phi$, $H_\rho$, and $H_\phi$ are produced in terms of the $z$-component fields. These six equations are the field expressions governing light propagation in the region of the fiber between the core and cladding. Applying continuity boundary conditions for the tangential components of $\vec{E}$ and $\vec{H}$, renders the coefficients $A_1$, $A_2$, $A_3$, and $A_4$. If the determinant of the coefficient matrix is zero, the field equations have a nontrivial solution. Appreciable algebraic manipulations give resulting eigenvalue equation [97–99]

\[
\left[ \frac{J'_m(pa)}{p J_m(pa)} + \frac{K'_m(qa)}{q K_m(qa)} \right] \left[ \frac{J'_m(pa)}{p J_m(pa)} + \frac{n_2^2 K'_m(qa)}{n_1^2 q K_m(qa)} \right] = \frac{m^2}{a^2} \left( \frac{1}{p^2} + \frac{1}{q^2} \right) \left( \frac{1}{p^2} + \frac{n_2^2}{n_1^2} \frac{1}{q^2} \right). 
\]  

(2.50)

where the primed coordinates are derivatives with respect to the argument. The propagation constant $\gamma$ can be calculated numerically for given parameters $k_0$, $a$, $n_1$, and $n_2$. We obtain a different value $\gamma_{mn}$ for each integer value $m$. These are the optical modes of the fiber. They are denoted $HE_{mn}$ or $EH_{mn}$ with the dominant field $H_z$ or $E_z$ determining the order. For the experiments discussed in this work, the notation $LP_{mn}$ is used to reference linearly polarized modes for a weakly guided fiber [101]. Additionally, we use multimode fibers for said experiments and the $V$ parameter, the normalized frequency, is used to further characterize
Figure 2.5. Propagation constant $b$ normalized as a function of the normalized frequency $V$ for several low-index fiber modes. The right scale shows the mode index $\bar{n}$, This figure is taken from ref. [100].

This type of fiber as distinct from a single-mode fiber. $V$ is defined as

$$V = k_0 a \sqrt{n_1^2 - n_2^2}$$

(2.51)

determines the cutoff condition for the number of modes that are successfully guided through the fiber. The normalized propagation constant $b$ can also be defined

$$b = \frac{\bar{n} - n_2}{n_1 - n_2}$$

(2.52)

where $\bar{n} = \frac{\Lambda}{2\pi}$. For our experiments, we use the $V$ parameter to understand how many OAM modes can be coupled to the multimode fiber. Several optical fiber modes are shown in 2.5 with their normalized frequencies.
2.4 Artificial Intelligence

One of the forerunners of computer science, Alan Turing, posited that it would one day be possible for advanced computational machines to be able to reason or have a conscience [102]. John McCarthy, a pioneer in artificial intelligence (AI), notes that AI is “the science and engineering of making intelligent machines, especially intelligent computer programs. It is related to the similar task of using computers to understand human intelligence, but AI does not have to confine itself to methods that are biologically observable” [103]. This path has led researchers to the task of having to define knowledge in the human context and study how it can be ported to machines [104–112]. Indeed there have been numerous applications of artificial intelligence across a variety of fields, from new approaches to educate students at all levels, to testing AI performance against human gamers [113–121]. But how does this field improve? How does one teach a computer to “think” and “reason”? This is the basis for machine learning - often equated to AI, but is more accurately thought of as a subset of the field [122]. Machine learning has played a role in the disruptive technologies of speech recognition [123–126], image classification [3, 20, 127, 128], natural language processing [129–132], and the collaboration of augmented reality and robotics [133–138]. Machine learning refers to a system’s ability to acquire, and integrate knowledge through large-scale observations, and to improve, and extend itself by learning new knowledge rather than by being programmed with that knowledge [139]. It is modeled with linear algebra via matrix and convolutional operations [140]. Training machines to learn within this framework is the purview of artificial neural networks (ANN). The following discussion is motivated by [141]. Inspired by biological neural networks, ANNs are largea parallel computing systems made up of tiny interconnected processors. ANN models attempt to use some “organizational” principles believed to be used in the human brain. A neuron is a cell in the brain that processes information. It receives information from and transmits information to other neurons via synapses. These synapses release neurotransmitters which hinder or allow electrical pulses from neurons. Given certain signals received, the synapses learn from this activity and this
is realized as memory. Numerous signals are sent and received in massive parallel processing capability by the brain. This netowrk of interconnects inspired research of ANNs across several disciplines such as neuroscience, computer science, physics, and artificial intelligence to name a few. ANN pioneers Warren McCulloch and Walter Pitts introduced a binary threshold unit for an artificial neuron wherein this synaptic behavior could be studied further [142]. Their computational model of a weighted sum of inputs \( \{n\} \in \mathbb{N} \), is given by

\[
y = \theta \left( \sum_{i=1}^{n} w_{i} x_{i} - u \right),
\]

(2.53)

featuring a threshold or step activation function. The function will output one if the sum is larger than the given threshold \( u \). It will output zero otherwise. These outcomes signal an excited synapse or one that is inhibited [142]. With their binary threshold neuron, McCulloch and Pitts proved that, with properly chosen weights, a sequential arrangement of such neurons can perform universal computations [141]. The McCulloch-Pitts neuron, with its threshold activation function, is shown in Fig. 2.6. The step function used by McCulloch and Pitts has been generalized to several functions. The logistic sigmoid function given by

\[
g(x) = \frac{1}{1 + \exp(-\beta x)},
\]

(2.54)

with slope parameter \( \beta \), is a popular choice for ANNs today [141]. The learning architecture of the ANN is supported by neurons as nodes and synapses as weights, referred to as synaptic weights. This framework is written as an algorithms and, as feed-forward neural networks are used for the experiment in chapter three, we discuss them here. These networks are connected in unidirectional layers. They are fed with an initial set of inputs (features) connected to a weighted hidden layer of neurons. An activation function is used to update the synaptic weights per certain features in the hidden layer. This hidden layer is connected to an output layer which produces the output neurons (labels or classes). The algorithm runs for several iterations or epochs until the a cost function is optimized. The
cost function determines the accuracy or performance of a training algorithm. It is also used when testing a new set of inputs given a previously trained algorithm. Given a test data set not previously encountered, the algorithm uses the training data, its “memory” to correctly identify the proper outputs for the new inputs. The activation and cost functions can vary given the ANN task [119]. For the experiment to be discussed in chapter three, we used supervised learning. This is a framework in which the training algorithm is given the correct “answers” (outputs) for given inputs. We also use image classification via a convolutional neural network (CNN). CNNs are a type of ANN that model human sight as adapted by machines [143]. CNNs are a specialized kind of neural network for processing data that has a known, grid-like topology, most successfully that of a two-dimensional image topology [140]. A deep learning application with multiple possible layers in its hidden layer, the CNN is ubiquitous in image classification and has been used to identify different light sources [144], reconstruct aberrated spatial profiles due to turbulence in free-space propagation [3], and decode encrypted messages in spatial light modes [20]. A model of a CNN, used to identify and resolve several closely propagating light sources [144], is shown in Fig. 2.7. Details on our experiment-specific CNN are given in chapter three.
2.5 Plasmonics

Plasmonics involves the study of electronic oscillations in metallica nanostructures and nanoparticles or surface plasmon resonance at the nanoscale[145]. It is a multidisciplinary field with sensing applications in several areas including medicine [146–148], biology [149–151], chemistry [152–154], physics [155–157], and environmental science [158–160]. The field of plasmonics had its beginnings with several studies of light-metal interactions. These studies include the observation of light interacting with a metal grating by Wood [161], the color of glass with metal particles by Maxwell-Garnett [162], and the scattering and absorption of light by a spherical particle of arbitrary size Mie [163]. These early investigations of the interplay between light and metallic material motivated a large research thrust into the field of surface plasmons (SPs). This included the theoretically proposed and experimentally confirmed dispersion relations of surface plasmon polaritons (SPPs) in metallic films [164, 165]. Additionally, exciting SPPs using a prism, or the attenuated total reflection (ATR) technique...
[166, 167] has proven a valuable tool for modern biosensing [168]. There are two different types of surface plasmons. Those that propagate along a dielectric–metal interface are referred to as SPPs whereas those that are localized at metallic nanoparticles are referred to as localized surface plasmons (LSPs) [169]. As a sensor, the sensitivity inherent in LSP and SPP resonance is governed by characteristic adsorption of analytes based on surface area and surface energy due to a given nanostructure geometry, and the volume and geometry of the small metallic particulates that define the wavelength and symmetry of the plasmon [170]. This sensitivity is due to the ability to confine electronic fields at the nanoscale [171]. It is this sub-diffraction limit confinement combined with a quantum treatment of plasmons as quasiparticles that makes way for the study of quantum plasmonic sensing, where it has been shown that light can be controlled at the quantum level for quantum enhanced sensing [21, 48, 172, 173]. The latter point will be discussed in chapter four with the work of [21].
3.1 Motivation

Among the multiple families of structured optical beams, Laguerre-Gaussian (LG) modes have received particular attention for their ability to carry orbital angular momentum (OAM) or twisted light [68, 86, 174, 175]. The OAM in this kind of beam is induced by a helical phase front given by an azimuthal phase dependence of the form $e^{i\ell \phi}$, where $\ell$ represents the OAM number and $\phi$ the azimuthal angle [68, 86, 89, 174, 175]. Over the past decade, there has been an enormous interest in using photons carrying OAM for quantum communication [38, 90, 94, 176–179]. These structured beams of light allow for the encoding of multiple bits of information in a single photon [89, 174, 175]. Additionally, it has been shown that high-dimensional Hilbert spaces defined in the OAM basis can increase the robustness of secure protocols for quantum communication [28, 180, 181]. However, despite the enormous potential of structured photons, their vulnerabilities to phase distortions impose important limitations on the realistic implementation of quantum technologies [3, 27, 28, 89, 90, 175, 182–184]. Indeed, LG beams are not eigenmodes of commercial optical fibers and consequently their spatial profile is not preserved upon propagation. For this reason, quantum communication with structured photons has been limited to free-space platforms [37, 185–189], with limited recent work in optical fibers [176, 190–193].

Recently, there has been an enormous interest in employing artificial neural networks to boost the functionality and robustness of quantum technologies [194–199]. In the field of photonics, there has been extensive research devoted to developing artificial neural networks for the implementation of novel optical instruments [144, 200, 201]. Indeed, convolutional neural networks (CNNs) have enabled the demonstration of new imaging schemes working at the single-photon level [3, 202, 203]. These protocols have been employed to characterize structured photons in the Laguerre-Gaussian (LG), Hermite-Gaussian (HG), and Bessel-Gaussian (BG) bases [89, 90, 175, 202–207]. Here, we introduce a machine learning protocol
that exploits spatial modes of light propagating in multimode fibers for high-dimensional encryption. This is achieved by training artificial neural networks from experimental spatial profiles in combination with a theoretical model that describes the propagation of spatial modes in multimode fibers. The trained neural network enables us to decrypt information encoded in spatial modes of light. We demonstrate robust and efficient bit-by-bit and byte-by-byte encryption in commercial multimode fibers.

3.2 Conceptual Overview - Secure Communication with Machine Learning

The conceptual illustration of our smart encryption protocol is presented in Fig. 3.1a. Here, Alice prepares a message encoded in high-dimensional OAM modes that is then sent to Bob through a multimode fiber. The protocol entails the use of the 8-bit ASCII (American Standard Code for Information Interchange) code, allowing Alice to encode a message in two different ways using the alphabet shown in Fig. 3.1b. In the first approach, each character in the message is represented by a byte (eight bits). Then, the OAM modes from $\ell = +1$ to $\ell = +8$ are one-to-one correlated with the position of each bit. The mode $\ell = +1$ is the most significant bit (MSB) and $\ell = +8$ the least significant bit (LSB). Consequently, for each character in the message, Alice has an eight-bit binary string where each bit position is mapped to an LG mode and sequentially sent to Bob in a bit-by-bit fashion. In the second approach, Alice prepares one state (byte) composed of a superposition of eight bits representing a particular character. This enables Alice to send a message to Bob through a sequence of characters, or byte-by-byte, leading to a more computationally efficient process. This scheme enables the mitigation of some errors that may be introduced during the transmission and reception of information, such as the loss of bits. We decrypt messages by training artificial neural networks with experimental spatial profiles in combination with a theoretical model that describes our protocol. We now introduce our model to describe propagation of spatial modes in multimode fibers. For this purpose, we consider the coupling of an encoded message from free space to the transmission channel, namely the optical fiber. In this case, one has to decompose the initially injected field into the modes that
Figure 3.1. a Conceptual schematic of our encryption protocol. In this case, Alice sends the message ‘9’ to Bob in a bit-by-bit fashion or through a superposition of spatial modes (byte-by-byte). The resulting computationally efficient feature vector is used to train a neural network with high accuracy. The pre-processing details for encrypted modes can be found in the Methods section. b The OAM mode-to-bit-position relation is shown along with superposition states that correspond to the ASCII digits from zero to nine. This ‘alphabet’ is used to encode information in spatial modes carrying OAM. Our experimental setup is depicted in c. Here, Alice encodes a message using OAM modes generated through a spatial light modulator (SLM). The spatial modes are coupled into a multimode fiber that is used to transmit information to Bob. In this case, we emulate multiple transmission conditions by introducing stress to the fiber via mechanical manipulation. The resulting perturbations are used to encrypt the message. We train our artificial neural network by collecting multiple spatial profiles of the distorted beams produced by the multimode fiber. Each distorted spatial profile of the optical beam corresponds to a particular condition of stress exerted on the fiber. Remarkably, our neural network is capable of recovering the initial superposition modes, converting them to the standard alphabet characters for Bob at read out. These figures are included here from ref. [20].
are sustained by the specific features of the fiber. For the weakly guiding step-index fiber used in this experiment, the modes are described by the linearly polarized (LP) solution set. The field distribution, in polar coordinates, $\Psi (r, \phi)$, is thus described by the solution of the scalar Helmholtz equation, which for a cylindrical fiber with a core radius $a$ is given by [69, 208, 209]

$$\text{LP}_{\ell p} = N_{\ell p} \begin{cases} \text{J}_\ell (\kappa_{T\ell p} r) \exp (-i\ell \phi) & \text{if } r < a, \\ \text{K}_\ell (\gamma_{\ell p} r) \exp (-i\ell \phi) & \text{if } r \geq a, \end{cases} \quad (3.1)$$

where $N_{\ell p}$ is a normalization constant, $J_\ell (x)$ is the Bessel function of the first kind and order $\ell$, and $K_\ell (x)$ is the modified Bessel function of the second kind and order $\ell$. Note that the parameters $\kappa_{T\ell p}$ and $\gamma_{\ell p}$ determine the oscillation rate of the field in the core and the cladding, respectively. These are defined by

$$\kappa_{T\ell p}^2 = n_{\text{core}}^2 k_0^2 - \beta_{\ell p}^2, \quad (3.2)$$

$$\gamma_{\ell p}^2 = \beta_{\ell p}^2 - n_{\text{cladding}}^2 k_0^2, \quad (3.3)$$

where $k_0 = 2\pi/\lambda_0$, with $\lambda_0$ being the vacuum wavelength of the light inside the fiber, $\beta_{\ell p}$ is the propagation constant of the $p$th guided mode for each azimuthal index $\ell$, and $n_{\text{core}}$ and $n_{\text{cladding}}$ are the refractive indices of the core and the cladding, respectively. For the description of the LP modes, the additional fiber parameter $V$ is required, which is defined as

$$V^2 = \kappa_{T\ell p}^2 + \gamma_{\ell p}^2 = \left(2\pi \frac{a}{\lambda_0}\right)^2 \left(n_{\text{core}}^2 - n_{\text{cladding}}^2\right). \quad (3.4)$$

This fiber parameter determines the amount of modes and their propagation constants. In our experiments, we make use of a 1-meter long, 10 $\mu$m-diameter fiber, with N.A. = $\sqrt{n_{\text{core}}^2 - n_{\text{cladding}}^2} = 0.1$. In these conditions, an arbitrary field propagating along the fiber may be decomposed in six LP modes with indices:

$$(\ell, p) \in \{(-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (0, 2)\}$$. This implies that, regardless of the initial
condition, the output mode of the fiber can always be written as

\[ \Psi_{\text{out}}(r, \phi) = \sum_{\ell, p} c_{\ell, p} L_{\ell, p}, \] (3.5)

where the coefficients \( c_{\ell, p} \) are defined by the injected field and the properties of the optical fiber throughout the propagation length.

In a realistic scenario, the local random variations of the fiber properties produce significant distortions of spatial modes, thus making almost impossible to predict the spatial distribution of photons at the end of the fiber, i.e., the coefficients \( c_{\ell, p} \) in Eq. (3.5). This is the main motivation behind our machine-learning protocol for encryption in optical fibers. In our experiments, variations are produced by a mechanical strain, and in the case of superposition modes, by both strain and mixing of the modes during the propagation. Once optical spatial modes leave the fiber, the goal is to recover the sequence of transmitted modes either bit-by-bit (individual modes) or byte-by-byte (superposition modes) as the case may be and then effectively decode the optical profiles (images) to compose the message. In this respect, Bob exploits the self-learning features of artificial neural networks to decrypt the information encoded in the distorted spatial modes efficiently. To train the neural network, the data-set comprises a collection of down-sampled images, rearranged as column vectors that correspond to the aberrated optical profiles, as shown in Fig. 3.1a. After the training, Bob utilizes the high efficiency of the neural network to retrieve the message by identifying individual modes if the communication was bit-by-bit, or recognising superposition modes when the communication was byte-by-byte. Again, he and Alice have the option to communicate via both protocols. He then recovers the message by ascertaining the proper ASCII character that corresponds to the encoding scheme.

3.3 Experiment & Results

The schematic diagram of our experimental setup is shown in Fig. 3.1c. Alice use a He-Ne laser with a spatial light modulator (SLM) to prepare the message to be sent using OAM
states of light. The light beam is then sent to Bob through multimode fiber, and the output is measured by a camera. More details of the experiment can be found in the Methods section. Examples of different OAM intensity distributions collected experimentally are shown in Fig. 3.2. Spatial profiles of individual LG modes with different azimuthal quantum numbers \((\ell = -10, \ell = -1, \ell = 0, \ell = +1, \ell = +10)\) and LG superpositions, before the multimode fiber, are displayed in Figs. 3.2a.1 and b.1, respectively. Each superposition has unique intensity distribution given by combining two and up to eight OAM single modes, depending on the character to be represented (see the alphabet shown in Fig. 3.1b). For demonstration purposes, Fig. 3.2b.1 presents superpositions of LG modes for the numeric characters: 0, 3, 4, 5 and 9.

As mentioned above, once a spatial mode is transmitted through the fiber, there is significant distortion from the applied tension, the local variations of the fiber properties, and even the noise of the camera sensor, resulting in an encrypted mode. Fig. 3.2a.2 and b.2 show the encrypted modes corresponding to the spatial beams in Fig. 3.2a.1 and b.1, respectively, for a displacement of 50 mm, which represents the maximum strain that may be applied in the fiber in our experiment. Note that intensity distributions of the encrypted modes change drastically with respect to the distributions of the initial modes. Moreover, the shape of the intensity patterns can change significantly as a function of the strain experienced by the fiber. Fig. 3.2c displays spatial profiles for LG superposition of the character 1 with different applied tension represented by the displacements: 0, 12, 25, 37, and 50 mm. Importantly, these distortions are induced randomly, which leads to an unbounded set of encrypted modes. Nevertheless, our NN can decode these encrypted modes with high accuracy for both individual modes and mode superpositions. This effectively generalizes an unbounded set from a limited collection of labeled examples.

To study the LG mode cross-talk when the beam propagates through the fiber, we measured the cross-correlation matrix for modes with azimuthal quantum numbers from \(\ell = -10\) to \(\ell = +10\). In the cross-talk matrix, the diagonal elements represent the conditional proba-
Figure 3.2. Spatial intensity distributions of initial and encrypted LG modes obtained experimentally for the maximum strain in the fiber (50 mm). Intensity profiles of individual modes with azimuthal quantum numbers, $\ell = -10, \ell = -1, \ell = 0, \ell = +1, \ell = +10$, before (a.1) and after (a.2) the propagation through the multimode fiber. **b.1** Superposition of LG modes representing the numeric characters 0, 3, 4, 5, and 9. Each character has been encoded using the alphabet displayed in Fig. 3.1. The bottom row (b.2) shows the encrypted modes corresponding to each of the superpositions. **c** Spatial profiles for the numeric character 1 obtained after propagation for different displacements of the fiber: 5, 12, 25, 37, and 50mm. Note that the fiber experiences strain due to the displacement, resulting in a dynamic intensity output. These figures are taken from ref. [20].
Figure 3.3. a Cross-talk matrix for LG modes with azimuthal quantum numbers from $\ell = -10$ to $\ell = +10$. Note that the LG modes are distorted severely, so the identification is practically impossible. b Diagonal cross-talk matrix obtained after applying our neural network. Our approach provides a powerful tool to recognize OAM modes after the fiber with an efficiency of 98%. c The cross-talk matrix obtained for the LG superpositions that represent the numeric characters zero to nine. The diagonal elements indicate that the transmitted characters are correctly identified. This neural network exhibits a performance of 99%. d Smart communication protocol with Alice sending the message “This is my first message!” (upper message) and the image of a Mexican pyramid (bottom message) to Bob through the multimode fiber. The transmission of the text and image is bit-by-bit and byte-by-byte, respectively. Bob deciphers both messages by using the trained NN with near-unity accuracy. These figures are taken from ref. [20].
ilities among the transmitted and detected modes that were correctly recognized. Remarkably, the diagonal elements in our cross-talk matrix are erased completely as indicated by Fig. 3.3a. In fact, it is practically impossible to recognize any mode. However, as shown in Figs. 3.3b and 3.3c, we exploit the functionality of machine learning algorithms to design a NN sufficiently sensitive to discern LG modes after the multimode fiber, enabling us to reconstruct a diagonal cross-talk matrix. To show the ability of our machine-learning algorithms to recognize encrypted OAM modes, we first design, train, and test a multi-layer neural network with the capacity to identify LG beams with different positive and negative topological charges, which go from $\ell = -10$ to $\ell = +10$. It is known that two pure LG modes with identical radial numbers but with opposite topological charges are indistinguishable using intensity measurements solely because they present exactly the same distributions. However, we experimentally demonstrate that our approach enables the discrimination of oppositely charged LG modes from their intensity patterns. We exploit the fact that OAM propagation through the multimode fiber induces phase distortions. The fiber is interpreted to be a "disordered" medium due to the inherent noise and the local variations of its properties. This leads to distinct modal cross-talk for the LG modes and their conjugates, resulting in changes in the intensity profiles. Consequently, this allows the NN algorithms to distinguish opposite LG modes unequivocally. Thus, our approach overcomes the limitations of existing strategies based on projection measurements and phase-measurement interferometry techniques. As seen in Fig. 3.3b, we obtain a classification accuracy of 98%.

Now we describe the implementation of the smart communication protocol using the trained NN. For bit-by-bit communication, we select the LG modes from $\ell = +1$ to $\ell = +8$ to form 8-bit binary words that allow us to encode characters from the ASCII code. It is worth mentioning that by using these eight modes, our neural network reaches an overall accuracy of 99%. Again, Alice encodes a message using the alphabet shown in Fig. 3.1b. This process is presented in Fig. 3.3d. Alice prepares the plain text "This is my first message!" that is transmitted to Bob through the multimode fiber. Note that we show
the detailed encoding and decoding processes for a particular character. In the figure, we highlight the exclamation mark, however the same stages are applied for all the characters of the message. The communication channel acts as the encryption process, so Bob receives a sequence of indistinguishable intensity profiles. The goal is to recover the sequence of transmitted bits by Alice from the intensity distributions. Prior to the decryption process, Bob carries out image pre-processing that includes the transformation of an image from RGB to grayscale. This is followed by the down-sampling process and the rearrangement of the pixels from resulting matrices into column vectors. In the decryption process, Bob uses the neural network to decipher the message by identifying each received LG mode and translating it via the standard alphabet.

To describe the implementation of our proof-of-principle smart communication protocol for byte-by-byte communication, LG superposition modes are prepared using the alphabet in Fig. 3.1b. We begin by using the dataset of encrypted superposition modes to train, validate and test a neural network that maps the distorted mixtures to one of the transmitted digits. After training, the performance of our neural network is 99%. This demonstrates the ability of our neural network to discern, with near-unity accuracy, experimental superpositions of LG modes. This is highlighted via the cross-talk matrix in Fig. 3.3c. Furthermore, to unveil the utility and functionality of our smart communication protocol, Fig. 3.3d presents a scheme where Alice sends the image of a Mexican pyramid to Bob through the multimode fiber. As in the previous case for the plain text, we emphasize the involved processes in the communication protocol for three particular pixels from the image. Here, each pixel of the image is represented by an eight-bit word whose decimal value is “1” for white pixels and “0” for black pixels. Alice can employ the superposition modes that represent the digits “1” and “0” to map the image and transmit it byte-by-byte (or equivalently pixel-by-pixel) through the communication channel. Thus, Bob receives one by one the encrypted pixels that comprise the image and pre-processes them. Then, Bob uses the neural network to identify the digits encoded in the superposition modes, after which he can retrieve the
Mexican pyramid. At this point it is worth mentioning that, after the propagation, the image information cannot be inferred from the distorted beams. This decryption process requires the trained neural network to recover the plain image.

We quantify the integrity of the received information by calculating the mean squared error (MSE), defined by $MSE = \frac{1}{n} \langle e | e \rangle$ where $e = (\hat{y} - y)$. Here, $\hat{y}$ and $y$ are vectors that contain the received and transmitted bytes, respectively. The measured MSE for both the message and image is zero. This validates the robustness and high efficiency of our protocol to decode OAM modes transmitted through the multimode fiber.

In summary, we have demonstrated a machine learning protocol that employs spatial modes of light in commercial multimode fibers for high-dimensional encryption. This protocol was implemented on a communication platform that utilizes LG modes for high-dimensional bit-by-bit and byte-by-byte encoding. The method relies on a theoretical model that exploits the training of artificial neural networks for identification of spatial optical modes distorted by multimode fibers. This process allows for the recovery of encrypted messages and images with almost perfect accuracy. Our smart protocol for high-dimensional optical encryption in optical fibers has key implications for quantum technologies that rely on structured fields of light, especially those technologies where free-space propagation poses significant challenges.

### 3.3.1 Tabletop Details

The schematic diagram of our experimental setup is shown in Fig. 3.1c. We use a He-Ne laser at 633 nm that is spatially filtered by a single-mode fiber (SMF). The output beam with a Gaussian profile illuminates a spatial light modulator (SLM) displaying a computer-generated hologram. The SLM together with a 4f-optical system allows us to prepare any arbitrary spatial mode of light carrying OAM. We then use a telescope to demagnify the structured beam before coupling into a multimode fiber with diameter of 10 µm. The preparation of the modes used to store the message to be sent is performed by Alice. At the output of the fiber, Bob uses a camera to measure the collimated spatial profile of the
communicated modes. Mechanical stress is induced in the fiber channel to generate the neural network training palette. The fiber is configured in a loop with the base secured to the optical table. The top of the loop is secured to a 3D translation stage with displacement occurring along the y-axis (orthogonal to and away from the plane of the table). Displacing the top of the loop attached to the translation stage 50 mm produces strain in the fiber. As the fiber is being pulled taut, successive images show the dynamic change of the mode, so the output at detection is now an LG mode distorted both via the multimode-fiber beam transformation as well as the applied tension. A camera is used to detect and display the output image. Two sets of data are taken: 1) The SLM is programmed to produce holograms for each OAM mode from -10 to 10, 21 modes total. For each mode, one image is captured at 0.10 mm translation intervals for a total displacement of 50 mm producing 500 images. 2) The SLM is programmed to produce holograms of OAM superposition modes for the 8-bit ASCII characters zero to nine. Each character is represented by a superposition of two, three, four, or five OAM single modes. One image is captured per 0.25 mm displacement interval over 50 mm for a total of 200 images per each superposition mode.

3.3.2 Neural Network Training

In what follows, we describe technical aspects of the neural networks developed in this work. The acquired sets of images in combination with machine learning algorithms enable the identification of distorted LG modes. This renders the originally encoded modes (message). The machine learning algorithms are characterized by solve tasks where conventional algorithms offer low performances or limited efficiencies. Typically, these solve tasks exploit a given collection of labeled examples or past experiences to predict the outcome for new data [140, 196]. We implement feed-forward neural networks with sigmoid neurons in the single hidden layer and softmax neurons in the output layer, to identify spatial modes transmitted through a multimode fiber. In this architecture, each neuron in a specific layer is connected to each neuron of the next layer through a synaptic weight. These synaptic weights are optimized by using the scaled conjugate gradient back-propagation algorithm
in a direction that minimizes the cross-entropy [211, 212]. Because sigmoid neurons are ranged in the interval [0,1], the cross-entropy is used as the loss function, as it has been shown to be ideal for classification tasks [211].

As is standard in artificial neural networks, these algorithms undergo two stages, training and test. To train, validate and test all of our neural networks, we dedicated 70% of the dataset to training, 15% to validation, and 15% to testing, whereas the number of epochs was always limited to 1000. In all cases, the networks were trained and tested with balanced data to avoid bias in identification, and the testing data was always excluded from the training stage. More specifically, we train our neural networks [213] from distorted modes collected by the CCD camera after the propagation through the fiber. The collection of RGB high-resolution images (1200×1024 pixels) are converted into grayscale images by eliminating the hue and saturation information but retaining the luminance. To reduce the data dimension, a down-sampling process is performed on the resulting monochromatic images by averaging small clusters of 140×140 pixels to form images of 9×7 pixels. In this way, the feature vector is obtained by reorganizing the pixels of the resulting images as a column vector. At this point, it is important to stress that the proper choice of the feature vector can have a dramatic effect on the performance results. As shown in the main manuscript, our extreme reduction in the image resolution allows us to train neural networks in a short time with low computational resources while maintaining a high recognition rate. Once the NN has been trained, Bob can utilize its high efficiency to retrieve the message sent by Alice, even if the channel is under strain. In order to assess the performance of the neural networks, we compute the ratio of the sum of false negatives and false positives to the total number of input observations, the so-called accuracy. We have run all of our algorithms in a computer with an Intel Core i7–4710MQ CPU (@2.50GHz) and 32GB of RAM with MATLAB 2019a.
CHAPTER 4.
QUANTUM PLASMONIC SENSING

4.1 Motivation

The possibility of controlling the confinement of plasmonic near-fields at the subwavelength scale has motivated the development of a variety of extremely sensitive nanosensors [214–217]. Remarkably, this class of sensors offers unique resolution and sensitivity properties that cannot be achieved through conventional photonic platforms in free space [217–220]. In recent decades, the fabrication of metallic nanostructures has enabled the engineering of surface plasmon resonances to implement ultrasensitive optical transducers for detection of various substances ranging from gases to biochemical species [214, 215, 217]. Additionally, the identification of the quantum mechanical properties of plasmonic near-fields has prompted research devoted to exploring mechanisms that boost the sensitivity of plasmonic sensors [48, 221–224].

The scattering paths provided by plasmonic near-fields have enabled robust control of quantum dynamics [224–227]. Indeed, the additional degree of freedom provided by plasmonic fields has been used to harness the quantum correlations and quantum coherence of photonic systems [224, 226, 228, 229]. Similarly, this exquisite degree of control made possible the preparation of plasmonic systems in entangled and squeezed states [230–233]. Among the large variety of quantum states that can be engineered in plasmonic platforms [48, 223], entangled systems in the form of N00N states or in diverse forms of squeezed states have been used to develop quantum sensors [57, 217, 234–236, 236]. In principle, the sensitivity of these sensors is not constrained by the quantum fluctuations of the electromagnetic field that establish the shot-noise limit [49, 220]. However, due to inherent losses of plasmonic platforms, it is challenging to achieve sensitivities beyond the shot-noise limit under realistic conditions [218]. Despite this, recent work has shown the potential of exploiting nonclassical properties of plasmons to develop quantum plasmonic sensors for detection of antibody complexes, single molecules, and to perform spectroscopy of biochemical substances [50, 237–239].
Figure 4.1. (a) Schematic diagram of the interactions in a plasmonic nanoslit. The plasmonic nanostructure has three input and three output ports. The photonic mode at the input is described by the operator \( \hat{b} \), whereas the two plasmonic modes are represented by \( \hat{a} \) and \( \hat{c} \). These modes are coupled to the plasmonic modes \( \hat{d} \) and \( \hat{f} \), and to the photonic mode \( \hat{e} \) at the output of the nanostructure. As described by the transformation matrix, the parameters \( \kappa, \tau, r, \eta, \) and \( t \) represent the coupling coefficients among the ports of the nanostructure. For sake of clarity, the diagram only illustrates the coupling paths for the input modes \( \hat{b} \) and \( \hat{c} \). The diagram in (b) shows the design of our simulated plasmonic sensor, comprising a slit of width \( w \) in a 200 nm gold thin film. Here, the plasmonic structure is illuminated by two thermal multiphoton sources that excite two plasmonic fields with super-poisonian statistics (the input grating couplers are not shown in the figure). The two counter-propagating surface plasmon (SP) modes interfere at the interface between the gold layer and the SiO\(_2\) substrate. The interference conditions are defined by the phase shift \( \varphi \) induced in one of the plasmonic modes by the substance that we aim to sense. These figures are taken from ref. [21].

Here, we explore a new scheme for quantum sensing based on plasmon-subtracted thermal states [240–242]. Our work offers an alternative to quantum sensing protocols relying on entangled or squeezed plasmonic systems [57, 217, 230–236, 236]. We use a sensing architecture based on a nanoslit plasmonic interferometer [243]. It provides a direct relationship between the light exiting the interferometer and the phase shift induced in one of its arms by the substance to be sensed (analyte). We introduce a conditional quantum measurement on the interfering plasmonic fields via the subtraction of plasmons. We show that this process enables the reduction of quantum fluctuations of the sensing field and increases the mean occupation number of the plasmonic sensing platform[241, 242]. Furthermore, plasmon subtraction provides a method for manipulating the signal- to-noise ratio (SNR) associated with the measurement of phase shifts. We demonstrate that the reduced fluctuations of plasmonic
fields leads to an enhancement in the estimation of a phase shift. The performance of our protocol is quantified through the uncertainty associated to phase measurements. We point out that the reduced uncertainties in the measurement of phases leads to better sensitivities of our sensing protocol. This study is conducted through a quantum mechanical model that considers the realistic losses that characterize a plasmonic nanoslit sensor. We report the probabilities of successfully implementing our protocol given the occupation number of the plasmonic sensing fields and the losses of the nanostructure. Our analysis suggests that our protocol offers practical benefits for lossy plasmonic sensors relying on weak near-field signals [244]. Consequently, our platform can have important implications for plasmonic sensing of delicate samples such as molecules, chemical substances or, in general, photosensitive materials [50, 237–239].

4.2 Theoretical Overview - Conditional Quantum Measurements

We first discuss the theoretical model that we use to describe conditional quantum measurements applied to a thermal plasmonic system. Fig. 4.1a describes the interactions supported by the plasmonic nanoslit under consideration [245]. This nanostructure acts as a plasmonic tritter by coupling the photonic mode $\hat{b}$ and the two plasmonic modes, described by the operators $\hat{a}$ and $\hat{c}$, to three output modes [245]. The photonic mode at the output of the nanoslit is described by $\hat{e}$, whereas the two plasmonic output modes are represented by the operators $\hat{f}$ and $\hat{d}$. As indicated in Fig. 4.1b, and throughout this paper, we study the conditional detection of the output modes $\hat{d}$ and $\hat{e}$ for a situation in which only the input plasmonic modes of $\hat{a}$ and $\hat{c}$ are excited in the nanostructure. Thus, the photonic mode $\hat{b}$ is assumed to be in a vacuum state. In this case, the plasmonic tritter can be simplified to a two-port device described by the following $2 \times 2$ matrix

$$\begin{pmatrix}
\hat{d} \\
\hat{e}
\end{pmatrix} = \begin{pmatrix}
\kappa & r \\
\tau & \tau
\end{pmatrix}
\begin{pmatrix}
\hat{a} \\
\hat{c}
\end{pmatrix}$$

(4.1)

The photonic mode $\hat{e}$ is transmitted through the slit and its transmission probability is
described by $2|\tau|^2 = T_{ph}$. Here, $T_{ph}$ represents the normalized intensity of the transmitted photons. Moreover, the plasmon-to-plasmon coupling at the output of the nanostructure is given by $|\kappa|^2 + |r|^2 = T_{pl}$. Here, the renormalized transmission (after interference and considering loss) for the plasmonic fields is described by $T_{pl}$. From Fig. 4.1b, we note that the interference supported by the plasmonic nanoslit shares similarities with those induced by a conventional Mach-Zehnder interferometer (MZI). The two plasmonic modes, $\hat{a}$ and $\hat{c}$, interfere at the location of the nanoslit, which in turn scatters the field to generate the output.[243]. The interference conditions are defined by the phase shift induced by the analyte. Plasmonic sensors with nanoslits have been extensively investigated in the classical domain, showing the possibility of ultrasensitive detection using minute amounts of analyte [214–217, 235, 243].

4.3 Simulation & Results

We now consider a situation in which a single-mode thermal light source is coupled to the nanostructure in Fig. 4.1b exciting two counter-propagating surface plasmon modes. This can be achieved by using a pair of grating couplers (not shown in the figure) [243]. The statistical properties of this thermal field can be described by the Bose-Einstein statistics as $\rho_{th} = \sum_{n=0}^{\infty} p_{pl}(n)|n\rangle\langle n|$, where $p_{pl}(n) = \tilde{n}^n/(1 + \tilde{n})^{1+n}$, and $\tilde{n}$ represents the mean occupation number of the field. Interestingly, the super-Poissonian statistics of thermal light can be modified through conditional measurements [240–242]. As discussed below, it is also possible to modify the quantum statistics of plasmonic fields. The control of plasmonic statistics can be implemented by subtracting/adding bosons from/to thermal plasmonic systems [246, 247]. In this work, we subtract plasmons from the transmitted field formed by the superposition of the surface plasmon modes propagating through the reference and sensing arms of the interferometer. This is the transmitted mode $\hat{e}$ is conditioned on the output of the field $\hat{d}$. The successful subtraction of plasmons boosts the signal of the sensing platforms. This feature is particularly important for sensing schemes relying on dissipative plasmonic platforms.
Figure 4.2. Normalized far-field intensity distribution scattered by the plasmonic nanoslit. The blue dashed line indicates the interference pattern produced by the field transmitted through the 320-nm-wide slit, this corresponds to mode $\hat{e}$. The panels from (a) to (d) are obtained for $\varphi = 0$, whereas those from (e) to (h) and (i) to (l) are calculated for $\varphi = \pi/2$ and $\varphi = \pi$ respectively. The dashed line in all plots represents the intensity distribution of the fields transmitted through the slit indicative of dipolar and quadrupolar near-field symmetry for $\varphi = 0$ and $\varphi = \pi$. The red shaded regions correspond to the standard deviation for $\bar{n} = 3.75$. Panels (a),(e) and (i) depict the unconditional detection of the signal with its associated noise. As displayed in panels (b) to (d), (f) to (h) and (i) to (l), the signal-to-noise ratio of the plasmonic sensor improves as the fluctuations of the field are reduced through the conditional detection of plasmons. The vertical lines on panels (a), (e) and (i) represent the angular range used for the calculation of the intensity variation with phase (i.e. sensitivity depicted in Fig. 4.3b). This figure is taken from ref. [21].

The conditional subtraction of $L$ plasmons from the mode $\hat{d}$ leads to the modification of the quantum statistics of the plasmonic system, this can be described by

$$p_{pl}(n) = \frac{(n + L)!\bar{n}_{pl}^{n}}{n!L!(1 + \bar{n}_{pl})^{L+1+n}},$$

where $\bar{n}_{pl}$ represents the mean occupation number of the scattered field in mode $\hat{e}$. We quantify the modification of the quantum statistics through the degree of second-order correlation.
function $g^{(2)}(0)$ for the mode $\hat{e}$ as

$$g^{(2)}_L(0) = \frac{L + 2}{L + 1}. \quad (4.3)$$

We note that the conditional subtraction of plasmons induces anti-thermalization effects that attenuate the fluctuations of the plasmonic thermal system used for sensing. Indeed, the $g^{(2)}_L(0)$ approaches one with the increased number of subtracted plasmons, namely large values of $L$. This effect produces bosonic distributions resembling those of coherent states [248]. Recently, similar anti-thermalization effects have been explored in photonic lattices [249].

The aforementioned plasmon subtraction can be implemented in the plasmonic nanoslit interferometer shown in Fig. 4.1b. It consists of a 200 nm thick gold film deposited on a glass substrate [243]. This thickness is large enough to enable decoupled plasmonic modes on the top and bottom surfaces of the film, as required. The gold film features a 320 nm slit, defining the reference arm of the interferometer to its left and the sensing arm (holding the analyte) to its right. The analyte then induces a phase difference $\varphi$ relative to the reference arm, thereby creating the output $(\hat{a}, \hat{e}$ and $\hat{f})$ that depends on this parameter. To verify the feasibility of our conditional measurement approach, we perform a finite-difference time-domain (FDTD) simulation of the plasmonic nanoslit using a wavelength of $\lambda = 810$ nm for the two counter-propagating surface plasmon modes ($\hat{a}$ and $\hat{c}$). The nanoslit is designed to support two localized surface plasmon (LSP) modes, one with dipolar symmetry and other with quadrupolar symmetry. Depending on the phase difference $\varphi$, these two LSP modes can be excited with different strengths by the fields interfering at the nanoslit. being the dipolar (quadrupolar) mode optimally excited with $\varphi = 0 \ (\varphi = \pi)$. This is due to the fact that the near-field symmetries of the interfering field are well-matched to the dipolar and quadrupolar fields for those values of $\varphi$[243]. The dashed lines in Fig. 4.2 indicate the far-field angular distributions of the transmitted intensity associated with the dipolar LSP mode (panels a to
d) and the quadrupolar LSP mode (panels i to l). Only a small angular range of the far-field distribution (range within vertical lines in Fig. 4.2) is used as the sensing signal. Thus, the sensing signal varies monotonically from a maximum value at $\varphi = 0$ to a minimum value at $\varphi = \pi$ [243].

4.3.1 FDTD Simulation

The transmission parameters of our sensor are estimated from FDTD simulations. Specifically, the transmission values for the photonic and plasmonic modes are $T_{\text{ph}} \approx 0.076$ and $T_{\text{pl}} \approx 0.0176$ for $\varphi = \pi$. However, our subtraction scheme is general and valid for any phase angle $\varphi$ in the range of $0 \leq \varphi \leq 2\pi$. Moreover, the total amount of power coupled to modes $\hat{e}$ and $\hat{d}$ normalized to the input power of the plasmonic structure is defined as $\gamma = T_{\text{ph}} + T_{\text{pl}} \approx 0.0941$. For the results shown in Fig. 4.2, we assume a mean occupation number of $\bar{n} = 3.75$ for the input beam. As shown in panels (a), (e) and (i) of Fig. 4.2, the output signals, calculated from Eq. (4.2) and represented by the red shaded region across all panels, exhibit strong quantum fluctuations. Surprisingly, after performing plasmon subtraction, the quantum fluctuations decrease, as indicated in the panels (b)-(d), (f)-(h) and (j)-(l) of Fig. 4.2. Evidently, this confirms that our conditional measurement protocol can indeed boost the output signal and consequently improve the sensing performance of a plasmonic device. However, due to the probabilistic nature of our protocol and the presence of losses, it is important to estimate the probability rates of successfully performing plasmon subtraction. In Table 4.1 we list the degree of second-order correlation $g_L^{(2)}(0)$, and the probability of successfully subtracting one, two, and three plasmons for different occupation numbers of the plasmonic fields used for sensing. It is worth mentioning that conditional measurements in photonic systems have been experimentally demonstrated with similar efficiencies [241].

The quantities reported in Table 4.1 were estimated for a phase shift given by $\varphi = \pi$. This table considers realistic parameters for the losses associated to the propagation of the plasmonic sensing field, and the limited efficiency $\eta_{\text{ph}}$ and $\eta_{\text{pl}}$ of the single-photon detectors used to collect photonic and plasmonic mode respectively. In this case, we assume $\eta_{\text{ph}} =$
Table 4.1. The estimated probability of plasmon subtraction and the corresponding degree of second-order coherence $g^{(2)}_L(0)$. The losses of the plasmonic nanostructure reduce the probability of subtracting multiple plasmons $L$ from the scattered field with an occupation number of $\bar{n}$. In this case, we assume $\varphi = \pi$. This table is included here from ref. [21].

<table>
<thead>
<tr>
<th>$\bar{n}$</th>
<th>$L = 1$</th>
<th>$L = 2$</th>
<th>$L = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$1.0 \times 10^{-2}$</td>
<td>$1.0 \times 10^{-4}$</td>
<td>$1.1 \times 10^{-6}$</td>
</tr>
<tr>
<td>1</td>
<td>$5.2 \times 10^{-3}$</td>
<td>$2.7 \times 10^{-5}$</td>
<td>$1.4 \times 10^{-7}$</td>
</tr>
<tr>
<td>0.5</td>
<td>$2.6 \times 10^{-3}$</td>
<td>$7.0 \times 10^{-6}$</td>
<td>$1.8 \times 10^{-8}$</td>
</tr>
<tr>
<td>0.3</td>
<td>$1.5 \times 10^{-3}$</td>
<td>$2.5 \times 10^{-6}$</td>
<td>$4.0 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

$g^{(2)}_L(0) = 1.5 \times 10^{-2}$, $1.33 \times 10^{-4}$, $1.25 \times 10^{-6}$.

0.3 and $\eta_{pl} = 0.3$. The latter value is obtained from our simulation, whereas the former corresponds to the efficiency of commercial single-photon detectors [250]. In general, the value for $\varphi$ determines how strongly the dipolar and quadrupolar LSP modes are excited, and consequently their far-field angular distributions. However, the process is applicable for other phases $\varphi$. Our predictions suggest that plasmonic subtraction can be achieved at reasonable rates using a properly designed nanostructure.

### 4.3.2 Quantum Sensor Performance Improvement

We now quantify the performance of our conditional scheme for plasmonic sensing through the SNR associated to the estimation of a phase shift. The SNR is estimated as the ratio of the mean occupation number to its standard deviation. This is defined as

$$\text{SNR} = \sqrt{(1 + L)\bar{n} \gamma \eta_{ph} \xi \cos^2 \frac{\varphi}{2}} \cdot \sqrt{1 + \bar{n} \gamma (\xi \eta_{ph} + (1 - \xi) \eta_{pl}) \cos^2 \frac{\varphi}{2}}.$$  \hspace{1cm} (4.4)

Here, the parameter $\xi = T_{ph}/(T_{ph} + T_{pl}) = 0.80$ represents the normalized transmission of the photonic mode. In Fig. 4.3a, we report the increasing SNR of our plasmonic sensor through the process of plasmon subtraction by plotting the SNR for the subtraction of one, two, and three plasmons for different phase shifts $\varphi$. In addition, for sake of completeness, we evaluate the improvement in sensitivity using error propagation [251]. More specifically,
Figure 4.3. The panel in (a) reports the signal-to-noise ratio (SNR) as a function of $\varphi$ for the conditional detection of the plasmonic modes transmitted by a 320-nm nanoslit. The red dots represent the unconditional SNR. Furthermore, the blue, green, and purple dots indicate the SNR for the subtraction of one, two, and three plasmons, respectively. This plot shows the possibility of improving the SNR of our plasmonic sensor through the subtraction of plasmons. The panel in (b) indicates that an increasing SNR leads to lower uncertainties in the estimation of phase shifts induced by analytes. The lower uncertainties described by $\Delta \varphi$ imply higher sensitivities of our plasmonic sensor. These figures are taken from ref. [21].
we calculate the uncertainty of a phase measurement $\Delta \varphi$. This parameter is estimated as

$$
\Delta \varphi = \sqrt{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2} / \left| \frac{d\langle \hat{n} \rangle}{d\varphi} \right|
$$

Here, the observable $\hat{n}$ corresponds to the conditional intensity measurement within an angular range of the far-field distribution (specified in Fig 4.2 with the vertical lines). In the field of quantum metrology, the reduced uncertainty of a phase measurement $\Delta \varphi$ is associated to an improvement in the sensitivity of a quantum sensor [251, 252]. In this regard, the conditional detection of plasmons increases the sensitivity of our plasmonic sensor. This enhancement is reported in Fig. 4.3b. Here, we demonstrate that the attenuation of the fluctuations of a weak plasmonic field, through the subtraction of up to three plasmons, leads to lower uncertainties in the sensing of photosensitive analytes.

In conclusion, we have investigated a new method for quantum plasmonic sensing based on the conditional subtraction of plasmons. We have quantified the performance of this scheme, under realistic conditions of loss, by considering the design of a real plasmonic nanoslit sensor. We showed that conditional measurements offer an important path for controlling the statistical fluctuations of plasmonic fields for sensing. In our work, we considered the case for which the sensing field contains a mean plasmonic number lower than two. In this regime, we showed that the attenuation of the quantum fluctuations of plasmonic fields increases the mean occupation number of the sensing field. Interestingly, this effect leads to larger signal-to-noise ratios of our sensing protocol. Furthermore, this feature of our technique enables performing sensitive plasmonic sensing with weak signals [214–217]. We believe that our work offers an alternative approach to boost signals in quantum plasmonic platforms operating in the presence of loss at the few particle regime [48, 223].

4.3.3 Methods

The design of the plasmonic structure given in Fig. 4.1b is simulated with a 2-D FDTD simulations by a 130 $\mu$m domain in $x$ direction and 8 $\mu$m along the $y$ direction. The boundary
condition is satisfied via the perfect matching layers to efficiently absorb the light scattered by the structure. Besides, the simulations time was long enough so that all energy in the simulation domain was completely decayed. The upper clad is made of CYTOP, a polymer with refractive index that closely matches the refractive index of 1.33. The mesh size was as small as 0.03 nm along \( x \) direction and where we have highly confined field propagation. To create the propagating plasmonic modes, we use a pair of mode sources in both sides of the central slit. The generated SP modes propagate toward the central slit where they interfere. The near-fields along a linear line underneath the nanostructure were extracted and used for the far-field analysis. The coupled light to the mode \( \hat{e} \), i.e. \( T_{ph} \), was calculated by the power flow through to the same linear line beneath the slit normalized to the input power. To have a realistic estimation of the subtracted light, the mode \( \hat{d} \) was first propagated for a distance of \( 10\lambda \) (8.1 \( \mu \)m) along the gold-glass interface and then a grating coupling efficiency of 36\% was considered to out couple the plasmonic mode to the free space [243]. The out-coupling was done far from the slit to avoid interactions of slit near-fields with fields of the assumed grating.
CHAPTER 5.
ENGINEERING QUANTUM STATISTICS USING TWISTED LIGHT IN COMMERCIAL MULTIMODE FIBERS

5.1 Motivation

Random processes, classical or quantum, have shown promise for gaming [253], quantum computing [254], quantum cryptography [255–257], quantum networks [258, 259], and even quantum gravity [260]. But randomness is challenging to define. The painstaking care in defining a truly random process stems from the fact that the methods cannot be tested by merely looking at their output blindly. In this case there is no distinction between what is truly random and a simple lack of knowledge of a determined generation scheme. The ability to generate quantum randomness on demand for technologies that rely on indeterminate situational outcomes has motivated a variety of protocols to date [28, 176, 261–267]. Quantum random walks and quantum random number generators are key to secure communication in quantum computing and quantum communication though the inherent challenge of true quantum randomness is still being studied [177, 254, 257]. We aim to implement a quantum random walk in a commercial multimode fiber with OAM modes exhibiting pseudo-thermal statistics. Our work is motivated by a continuous and multiphoton interference implementation of the theoretical work presented in ref. [70]. We present preliminary results of a multiphotonic quantum random walk in a commercial multimode fiber using incoherent light. The experiment exploits statistical correlations of thermal fields carrying orbital angular momentum with that of photon-number-resolved detection.

5.2 Conceptual Overview - Engineered Quantum Randomness

Our quantum randomness generation protocol is shown in Fig. 5.1. We modify the coherent case of chapter three with thermal light. We use a continuous wave Helium-Neon laser at a 633-nm wavelength as a coherent source. It is then incident on a mechanized rotating ground glass that imparts statistical fluctuations having super-Poissonian photon statistics given by the density operator
Figure 5.1. a Tabletop setup. Coherent light is filtered through a rotating ground glass, imparting super-Poissonian statistics. This thermal light exhibits multiphoton interference which is directed onto an SLM, coupled into a multimode fiber, then incident on a second SLM for optimal interference effects. The output is sent to a SNSPD where photon-number-resolving measurements are done via number state post-selection. This figure is taken from ref. [22].

\[
\hat{\rho}_{Th} = \frac{1}{1 + n_{Th}} \sum_{n=0}^{\infty} \left( \frac{n_{Th}}{1 + n_{Th}} \right)^n |n\rangle \langle n|
\] (5.1)

in the Fock or number basis. These statistics are realized in the production of multiphoton interference effects in thermal light, which contributes to our randomness scheme. We use second-order coherence theory to verify our thermal statistics for a single-mode field

\[
g^2(\tau) = 1 + \frac{\langle (\Delta n)^2 \rangle - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2}.
\] (5.2)

For thermal statistics, \(g^2(\tau) = 2\) [67], and we verified this to be true for our experimental photon statistics. This now diffuse light source is coupled into and filtered by a single-mode optical fiber (SMF) to output a Gaussian beam profile. The beam is expanded and directed onto a spatial light light modulator(SLM). This programmable diffractive device produces
computer generated fork pattern holograms (discussed in chapter two) that impart orbital angular momentum (OAM) to the incident light. The Gaussian Beam is transformed into a Laguerre-Gaussian beam with radial index \( p \) set to zero. As discussed in chapter two, this choice of \( p = 0 \) implies that we are using OAM modes as there is no amplitude modulation in \( p \) for the full LG framework. The eigenstates take the form \( |OAM_m\rangle = \sum_m c_m |M_l\rangle \) where \( |c_m^2| \) is the probability that a measured state will return \( |M\rangle \). Note that although the OAM index is denoted \( ^m \) by convention, OAM modes are often referred to as \( \ell \) modes. The light now takes on the characteristic twist nature and, after several 4f-systems, is coupled into a multimode fiber governed by the following

\[
\text{LP}_{\ell p} = N_{\ell p} \begin{cases} 
J_\ell (\kappa_{T\ell p} r) \exp (-i\ell \phi) & \text{if } r < a, \\
K_\ell (\gamma_{\ell p} r) \exp (-i\ell \phi) & \text{if } r \geq a,
\end{cases}
\]

(5.3)

where \( N_{\ell p} \) is a normalization constant, \( J_\ell (x) \) is the Bessel function of the first kind and order \( \ell \), and \( K_\ell (x) \) is the modified Bessel function of the second kind and order \( \ell \). Note that the parameters \( \kappa_{T\ell p} \) and \( \gamma_{\ell p} \) determine the oscillation rate of the field in the core and the cladding. The optical fiber framework is discussed in chapter two and experimentally presented in chapter three. The \( V \) parameter for multimode fibers is given by

\[
V^2 = \kappa_{T\ell p}^2 + \gamma_{\ell p}^2 = \left(2\pi \frac{a}{\lambda_0}\right)^2 \left(n_{\text{core}}^2 - n_{\text{cladding}}^2\right).
\]

(5.4)

For our fiber, an arbitrary field propagating along the fiber may be decomposed in six LP modes with indices: \( \{(\ell, 1) \mid \ell \in \{-2, -1, 0, 1, 2\}\} \) giving output states as

\[
\Psi_{\text{out}} (r, \phi) = \sum_{\ell, p} c_{\ell, p} \text{LP}_{\ell, p},
\]

(5.5)

where the coefficients \( c_{\ell, p} \) are defined by the injected field and the properties of the optical fiber throughout the propagation length. These modes are excited in our fiber as well as a few minor modes. On coupling, the thermal light is transformed by the multimode fiber.
Figure 5.2. a Crosstalk matrix and b a select probability distribution for the given SLM settings. The crosstalk matrix is produced via OAM projective measurements. For example, SLM 1 set to project the hologram for \( l = 0 \) and SLM 2 projects \( l = 2 \) produces the matrix(0,2) pixel and the shown probability distribution. Each pixel in the crosstalk matrix produces such a distribution and the combined crosstalk matrix represents multiphoton interference. This matrix is then post-selected for \( |n\rangle, n0, 1, 2, ... \) which gives the multiphoton interference result. These figures are included here from ref. [22], further enhancing the multiphoton effects. Here, the beam encounters additional 4f-systems which propagate the randomized output onto a second SLM. The output is sent through a final 4f-system and coupled to a superconducting nanowire single photon counter (SNSPD).

5.3 Experiment & Results

To obtain the cross-correlation or crosstalk matrix seen in Fig. 5.2(a), projective measurements are carried out on SLM 2 for modes \( l \in [-5, 5] \) on SLM 1. For example, SLM 1 is programmed to display a phase hologram for \( \ell = -5 \). SLM 2 is used to take projective measurements from \( \ell = 5, \ell = -4, ..., \ell = 4, \ell = 5 \) to obtain the top row of the matrix. SLM 1 is then programmed for \( \ell = -4 \) and the projective measurements are repeated to obtain the second row of the matrix and so on until the 11x11 matrix is constructed. Each pixel corresponds to a dual SLM spatial light projection which is denoted by \( (SLM1, SLM2) = (l_1, l_2) \) where SLM1 is the OAM imparting device and SLM2 is the projective measurement counterpart. The rows represent SLM1 and the columns SLM2. The crosstalk matrix shows
Figure 5.3. Post-selected photon-number-resolved data for $|n\rangle, n = 0, 1, ..., 7, 8$ in the OAM basis for $l \in [-5, 5]$. Each number state produces a different matrix which has engineered super-Poissonian photon statistics based on post-selected counts of a given OAM cross-talk matrix for that number state. The distribution of different OAM matrices for different number states shows multiphoton interference within the multimode fiber - a quantum random walk. Additionally, preparing the OAM states in superposition provides an increase in the Hilbert space [29] inducing increased multiphoton interference, or more walker paths. This figure is taken from ref. [22].
the conditional probabilities between the input modes of SLM1 and the modes detected from projective measurements on SLM2. There is low correlation as expected due to the OAM-to-fiber mode transformation before the second SLM. Experimental misalignment and the physical parameters of the fiber also contribute to the matrix in addition to diffraction. We use post-selected measurements of single photon counts to carry out photon-number resolving detection. A time-tagged time-resolved event counter in combination with an SNSPD is used to collect photon counts with photon statistics being measured in the far field. Each pixel in the crosstalk matrix has a corresponding probability distribution from the count measurements that describes the mean photon number counts for that dual-SLM setting. One such distribution is included showing the likelihood of detecting a photon for $(SLM1, SLM2) = (l_1 = 0, l_2 = 2)$ and can be seen in Fig. 5.3(b). We then post-select for $|n\rangle$ from each distribution creating a new crosstalk matrix, herein referred to as cross-correlation for counting. These unique matrices per post-selected state $|n\rangle$ are a result of multiphoton interference within the multimode fiber. This is a simple simulation of a quantum random walk where the OAM modes are the walker states. The matrices are shown in Fig. 5.3 $|n\rangle \in [0,8]$, although for each successive $|n\rangle$, new matrices result. These matrices shown in Fig. 5.3 represent a randomized distribution from which information can be extracted. Future work will include a protocol to strengthen our random walk argument via NIST and DIEHARD randomness tests similar to those found in [268]. These tests will be used to characterize a min-entropy bound for our method. Currently, this protocol based on the theoretical proposal of ref. [70], presents a simple method to produce multiphoton interference within an optical fiber which can be used as a communication channel for quantum key distribution and/or quantum random number generation.
CHAPTER 6.
CONCLUSION

The works discussed in this thesis span a variety of proposed technological advancements to science. Secure communication affects humanity at large, quantum sensing will allow for the exploration of materials at a most fundamental level, and quantum information processing may one day ensure fairness in gaming, equity in voting, and physically secure information transfer.

We began this discussion with a review of the necessary preliminaries for understanding the experiments therein. Light sources are fundamental to science and their statistical properties must be taken into account particularly at the few photon level. Twisted light is the foundation for two experiments discussed [20, 22] and the exploitation of its degree of freedom for high capacity storage and transfer of information has become of high interest in recent decades [29]. Lastly, optical fibers provide a way to communicate free of atmospheric turbulence which has proven a challenge for free-space communication in the classical and quantum regime [3, 176, 269, 270].

Next, we highlighted three experiments that push forward the knowledge of the field: optical encryption [20], quantum sensing [21], and multiphoton interference in commercial multimode fibers for quantum information processing [22]. Optical encryption in multimode fibers using twisted light is proving to be a growing research area as we move to mitigate the effects of environmental factors distorting communication channels [30, 176, 191, 193]. In combination with machine learning, the implementation of neural network algorithms to reconstruct the transformed modes is also of interest [20, 192]. Using conditional measurements to modify the quantum statistics of scattered plasmons in a nanostructure lowers the inherent noise fluctuations allowing for sub-shot-noise measurement [21]. When trying to sense with weak fields at a level commensurate with the wavelength of probing light, this method offers an increased measurement sensitivity. The results of this protocol has applications from biosensing to chemical characterization of materials. Finally, we looked into
the multiphoton interference of OAM modes in an optical fiber. The result is a simulated quantum random walk [22] which has applications in quantum key distribution and quantum random number generation.
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57


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VITA

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