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Reservoir Performance History Matching Using Type-Curves.

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RESERVOIR PERFORMANCE HISTORY MATCHING
USING
TYPE-CURVES

A Dissertation
Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment for the
requirements for the degree of
Doctor of Philosophy

in

The Department of Petroleum Engineering

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ABSTRACT

Recently, decline-curve analysis has expanded to permit engineers to analyze a petroleum reservoir directly in regard to its fluid-flow characteristics and its volumetric extent using rate-time type-curves of the constant terminal pressure solution of the diffusivity equation. This analysis is of enormous value to reservoir managers whose goal is to maximize oil and gas production from a petroleum reservoir. Reservoir extent, continuity, and flow capacity are paramount characteristics that are considered when developing models that predict reservoir performance while using alternative depletion strategies, such as during fluid-injection projects or enhanced recovery.

Reservoir producing conditions to which this technique can be readily applied are those whose actual bottom-hole flowing pressure (BHFP) closely approximates a constant value. Most wells, however, produce with variable BHFP. The work presented here focuses on an alternative rate-cumulative type-curve format whereby variable BHFP is incorporated into dimensionless variables containing both the production rate and the cumulative production providing a unified approach that can be applied to any reasonable variability in the producing rate or flowing pressure history.

The proposed method, with application to single phase and multiphase flow, provides the practicing engineer a better method for decline curve analysis and therefore propagates better reservoir characterization from production data.
CHAPTER 1

INTRODUCTION

While working in the oil and gas industry as a reservoir engineer during the 1980s, much of my time was spent forecasting oil and gas production rates for producing properties. These forecasts were converted to cash flow projections in order to help determine either future exploration budgets for oil companies, fair market values for acquisitions, or loan values for companies wanting to mortgage their producing properties in order to leverage their investments in the oil and gas industry. The balance of my professional time was spent examining these same producing properties in order to increase revenue through reservoir management.

Reservoir management includes reservoir characterization and performance prediction. With the advent of advanced decline-curve analysis, these two separate tasks became one. Type-curve matching of rate-time data with analytic solutions provides an extrapolation of future production rates. Type-curve matching also aids in characterizing the volumetric extent of the reservoir and in evaluating the most critical fluid-flow parameter, i.e., permeability. Because this new science helped consolidate my efforts, it has become of particular interest to me throughout my career and is the subject of my dissertation research.

One advancement in decline-curve analysis presented here includes pressure normalization of cumulative production. Like pressure normalization of
production rate, variations in bottom-hole flowing pressure (BHFP) are accounted for by dividing cumulative production by the pressure difference between initial and bottom-hole flowing pressures. The technique of combining pressure-normalized production rate (PNR) and pressure-normalized cumulative production (PNC) is an improvement over rate normalization alone in the analysis of reservoirs based on production data.

To apply this technique, determination of BHFP from surface-measured flowing-tubing pressure (FTP) is required along with determination of the original static reservoir pressure. Data can then be presented by plotting PNR versus PNC. This technique is then extended for use with gas reservoirs and solution-gas-drive reservoirs by further incorporating changes in viscosity, compressibility, and relative permeability during reservoir depletion.

This technique relies heavily on either measured BHFP or FTP. However, unlike with superposition techniques, it does not require the entire flowing pressure history for a well, thus allowing for greater application to situations found in the industry. The incorporation of PNR and PNC into decline-curve analysis provides a single-performance curve which is applicable to wells producing at constant BHFP, to wells producing at constant rate, and to wells with both varying rate and varying flowing pressure.

The benefit of a single-performance type-curve is its usefulness as a diagnostic tool. Identification of flow regimes, geological heterogeneities or boundaries, and interference from offset production or injection make it the ideal
plot for advanced decline-curve analysis. Although radial flow in unbounded and bounded reservoirs are presented here, the same diagnostic type-curve can be used with type-curves generated for other common wellbore and reservoir conditions, such as hydraulically fractured wells, naturally fractured reservoirs, dual-porosity systems, water-drive reservoirs, and other systems with pressure support at the outer boundary.

The following chapters present the application of this rate-cumulative performance plot on single-phase liquid reservoirs, single-phase gas reservoirs, and multiphase solution-gas-drive reservoirs. Methods for calculation of BHFP from FTP are provided in Appendix A.

An advantage of using either rate-time or rate-cumulative decline-curve analysis is that reservoir size, formation capacity, and wellbore effectiveness can be determined without either closing in the well or running costly instruments down the wellbore. This capability is greatly extended by the use of rate-cumulative analysis because pressure normalization of cumulative production allows for variable BHFP in the producing well.
CHAPTER 2

SINGLE-PHASE LIQUID SOLUTION

2.1 Constant Pressure Rate-Time Type-Curves

The constant pressure solution presented in 1933 by Moore et al.\textsuperscript{1} for the production rate from a well for bounded and unbounded reservoirs is shown in Fig. 2.1. The branches of Fig. 2.1 represent the rate decline for bounded, circular reservoirs with various ratios of external radius to wellbore radius. The nearly

\[ tD \]

\[ 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 10^{0} \]

\[ 10^{-4} \quad 10^{-5} \quad 10^{-6} \]

\[ reD = 500 \quad 1000 \quad 2000 \quad 5000 \quad 10,000 \]

\[ tD \]

Fig. 2.1 - Constant Pressure Type-Curve (after Moore et al.\textsuperscript{1})
horizontal curve of Fig. 2.1 represents the rate decline for an unbounded or infinite reservoir. Assumptions inherent in this solution are constant flowing pressure at the wellbore which fully penetrates a reservoir containing a slightly compressible fluid of single phase and constant viscosity, flow is horizontal in a homogenous and isotropic porous medium of uniform thickness, with constant permeability and porosity. Even though many of these assumptions are violated in practice, solutions based on this theory are widely used in hydrology and petroleum engineering. The solution shown in Fig. 2.1 will be referred to as the single-phase liquid solution.

Dimensionless variables are used in Fig. 2.1 as they provide a general solution to any number of specific problems. Actual rate and time can be calculated from dimensionless rate and for any specific set of reservoir parameters contained in the dimensionless variables. Dimensionless production rate, $q_D$, versus dimensionless time, $t_D$, are shown if Fig. 2.1 for various dimensionless external radius, $r_e$. The single-phase dimensionless rate, $q_D$, is defined (in field units) as:

$$q_D = \frac{141.2qB\mu}{kh(P_i-P_{wf})}$$

(2.1)

Where $q$ is the production rate (STB/d), $B$ is the formation volume factor (rb/STB), $\mu$ is the fluid viscosity (cp), $k$ is the permeability (md), $h$ is the formation height (ft), $P_i$ and $P_{wf}$ are the initial reservoir pressure and the wellbore flowing pressure (psia) respectively.
Dimensionless time, $t_D$, is defined as:

$$t_D = \frac{0.006328kt}{\phi \mu c_i r_{wa}^2}$$  \hspace{1cm} (2.2)

The additional terms used in this expression are $t$ for time (days), $\phi$ for porosity (fractional), $c_i$ is the total system compressibility (psi^{-1}), and $r_{wa}$ is the apparent wellbore radius (ft). The dimensionless external radius, $r_{eD}$ is defined as:

$$r_{eD} = \frac{r_e}{r_{wa}}$$  \hspace{1cm} (2.3)

Where the external radius is $r_e$ (ft) and the apparent wellbore radius is $r_{wa}$ (ft). Apparent wellbore radius is a measure of effectiveness and is related to the actual wellbore radius, $r_w$ (ft) by:

$$r_{wa} = r_w \exp (-s)$$  \hspace{1cm} (2.4)

Use of the apparent wellbore radius and the van Everdingen\textsuperscript{2} skin factor, $s$, in constant pressure type-curve variables was investigated by Uraite and Raghavan\textsuperscript{3} to allow for near wellbore damage (+s) or improvement (-s).

Dimensionless flow rate, $q_D$, and dimensionless cumulative production, $Q_D$, are related using:

$$Q_D = \int_0^{t_D} q_D \, dt_D$$  \hspace{1cm} (2.5)
Where dimensionless cumulative production, $Q_D$, is defined by:

$$Q_D = \frac{0.8936QB}{\phi h c r_w^2 (P_1 - P_w)} \quad \quad \quad \quad \quad \quad (2.6)$$

And $Q$ is the cumulative production (STB).

### 2.2 Unbounded Reservoirs: Rate-Time Type-Curves

In 1952 Jacob and Lohman presented the dimensionless rate-time type-curve shown in Fig 2.2 (bolded line) for unbounded systems. In 1981 Ehlig-Economides and Ramey represented the type-curve with the addition of
reciprocal dimensionless pressure (middle curve). Dimensionless pressure is defined as:

\[ P_D = \frac{kh(P_i - P_w)}{141.2qB\mu} \]  

(2.7)

In comparing the definitions of dimensionless pressure and dimensionless rate two differences need to be noted. First, the right hand sides of eqs. 2.1 and 2.7 are the reciprocal of each other. Secondly, dimensionless pressure represents the decline in BHFP for a well produced at constant rate, while dimensionless rate represents the decline in rate for a well produced at constant BHFP.

Also shown in Fig. 2.2 is the logarithmic approximation (upper curve) good for calculating dimensionless rate or pressure at late times:

\[ q_D(t_D) = \frac{2}{\ln(t_D) + .80907} \]  

(2.8)

This expression is within 2% for \( t_D > 5 \times 10^3 \).

\[ P_D(t_D) = \frac{1}{2} (\ln(t_D) + .80907) \]  

(2.9)

This expression is within 2% for \( t_D > 5 \).

Use of eqs. 2.8 or 2.9 allows semilog techniques to determine permeability and skin for the applicable time region. When data prior to the logarithmic approximation are to be analyzed, Fig. 2.2 can be used to determine permeability and skin from type curve-matching techniques. Semilog analysis and type-curve
matching techniques for unbounded reservoirs is covered thoroughly by Earlougher.

2.3 Pressure-Normalization

In practice, wells do not produce either at constant pressure or at constant rate. PNR is the technique of modifying the production rate by dividing it by the pressure drop for use with type-curves or semilog techniques. The resulting field term is the productivity index and incorporates variations in flowing pressure as well as variations in rate. In 1965 Winestock and Colpitts introduced this concept for use with gas well drawdowns. One dilemma of the PNR method for early time data is which type-curve in Fig. 2.2 do you use. After both solutions converge the problem becomes less ambiguous.

2.4 Unbounded Reservoirs: Rate-Cumulative Type-Curves

Fig 2.3 presents an alternative approach to rate-time type-curves. Dimensionless rate is plotted against dimensionless cumulative production for constant pressure production. Reciprocal dimensionless pressure is also plotted against dimensionless cumulative calculated by Holditch et al. as:

\[ Q_D(t_Dp_D) = t_D \]

The curve for dimensionless rate comes from tabular data of Ehlig-Economides. The curve for dimensionless pressure comes from tabular data of van Everdingen and Hurst.
The rate-cumulative solution shown in Fig. 2.3 has two distinct advantages over the rate-time solution shown in Fig. 2.2. First, the convergence of the two curves occurs sooner; second, there exists greater curvature (convex to the origin), both of which add to the uniqueness of a match. Field data are plotted as PNR vs PNC. The rate-cumulative solution therefore increases the ability to match data with variable rate and variable pressure.

In infinite-acting reservoirs, permeability can be determined from the vertical match of PNR with the type-curve where:
The van Everdingen skin factor for wellbore damage or improvement can then be calculated from the apparent wellbore radius:

\[ s = \ln \left( \frac{r_w}{r_{wa}} \right) \]  

(2.13)
bounded reservoirs the solutions diverge for the rate-time type-curve but are identical for the rate-cumulative type-curve.

2.5 Bounded Reservoirs: Rate-Time Type-Curves

Tsarevich and Kuranov (1966) are credited with being the first to observe that the boundary-dominated data (branches from the stem for specific dimensionless external radius in Fig. 2.1) are exponential in the rate decline, giving credence to the semi-log decline-curve plot used by industry for decades. This discovery allowed a much simpler analytic expression for flow rate during the boundary-dominated flow period. The exponential decline equation using dimensionless variables normalized by area and geometry is:

\[ q_{\text{dD}} = \exp\left(-t_{\text{dD}}\right) \]  
\[ (2.14) \]

These variables have an additional lower case "d" for decline-curve and are more convenient for type-curve presentation during boundary-dominated flow. Decline-curve dimensionless time, rate, and cumulative become:

\[ t_{\text{dD}} = \frac{t_D}{(\alpha B)} \]  
\[ (2.15) \]

\[ Q_{\text{dD}} = \frac{Q_D}{\alpha} \]  
\[ (2.17) \]

Where the area and geometry normalizing factors for circular reservoirs are defined by:
\[ \alpha = \frac{r_{eD}^2}{2} \tag{2.18} \]

\[ \beta = \ln(r_{eD}) - \frac{3}{4} \tag{2.19} \]

when \( r_{eD} > 30 \)

For non-circular reservoirs the Dietz Shape factor, \( C_s \), is included.

Definitions in the general case and for circular reservoirs with \( r_{eD} < 30 \) are given by Chen and Poston in Table 2.1

**Table 2.1 Area and Geometry Normalizing Factors for Type-Curves**

<table>
<thead>
<tr>
<th>Normalizing Factors</th>
<th>Circular</th>
<th>( r_{eD} &gt; 30 )</th>
<th>General</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>( \frac{(r_{eD}^2 - 1)/2}{r_{eD}^2} )</td>
<td>( A/(2 \pi r_{wa}^2) )</td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>( \frac{4r_{eD}^4\ln(r_{eD}) - 3r_{eD}^4 + 4r_{eD}^2 - 1}{4(r_{eD}^2 - 1)^2} )</td>
<td>( \frac{\ln 2.2458A}{C_A r_{wa}^2} )</td>
<td></td>
</tr>
</tbody>
</table>

Eqs. 2.18 and 2.19 can be obtained from the General column by substitution of appropriate definitions of area and value for Dietz Shape factor for circular reservoirs.

Rate-time type-curves based on decline-curve dimensionless variables are shown in Fig. 2.4. Fetkovich and Ehlig-Economides and Ramey have also presented similar figures. Note that the branches in Fig. 2.1 now form the stem in Fig. 2.4 and the stem of Fig. 2.1 now forms the branches in Fig. 2.4. In Fig 2.4 the unbounded curves converge and at that inflection, boundary-dominated data becomes concave to the origin. Uraite and Raghavan provide expressions to
calculate the transition from infinite-acting to boundary-dominated flow periods as a function of dimensionless external radius and also state that for all dimensionless external radius the transition can be approximated by a dimensionless time based on drainage area of 0.1. Were this dimensionless time is defined as:

\[ t_{DA} = t_D \frac{r_w^2}{A} \]  \hspace{1cm} (2.20)

Fig. 2.4 - Rate-Time Decline Type-Curve (RTDTC) (after Fetkovich and Ehlig-Economides and Ramey)
2.6 Bounded Reservoirs: Rate-Cumulative Type-Curves

The alternative constant pressure type-curve for flow rate data is the rate-cumulative type-curve shown in Fig. 2.5. Rate-cumulative type-curves will be shown to offer a enormous advantage over rate-time type-curves because they are equally applicable for constant pressure performance as well as variable pressure performance.

For wells that are produced at constant back-pressure, rate versus cumulative data can be plotted and matched just as they would be using the rate versus time data. Wells that have variable flowing pressure histories, including
shut-in periods, can be plotted just as was done in section 2.4 using PNR and PNC. This data plotting technique greatly extends the use of type-curves for most of the conditions encountered in the field.

To examine the ability to predict flow rates as a function of dimensionless cumulative production, the exponential decline equation, eq. 2.14, is combined with the cumulative-time relationship:

\[ Q_{ad}(t_{ad}) = 1 - \exp(-t_{ad}) \] ................................. (2.21)

which yields the boundary-dominated rate-cumulative relationship:

\[ q_{ad}(Q_{ad}) = 1 - Q_{ad} \] ................................. (2.22)

Eq. 2.22 infers that the dimensionless rate during the boundary-dominated flow period is a function of dimensionless cumulative and is not dependent on the pressure and rate history. To illustrate this point with a variable BHFP case, the constant rate solution is presented on both the constant pressure rate-time decline type-curve (RTDTC) and the constant pressure rate-cumulative decline type-curve (RCDTC). In order to make this comparison, one must define a decline-curve dimensionless pressure as:

\[ p_{ad} = \frac{p_D}{B} \] ....................................................... (2.23)

Dimensionless tabular data from Earlougher et al.\textsuperscript{14} for a well in the center of a closed square with an equivalent dimensionless external radius of 1128 is shown in Figs. 2.6 & 2.7.
Figs. 2.6 & 2.7 reveal two very important properties. First, infinite-acting data lying on the dimensionless external radius of 1000 branch fits either type-curve equally well. This is due in part to the logarithmic approximation (eqs. 2.8 & 2.9) being valid for dimensionless rate or reciprocal dimensionless pressure over the dimensionless time period displayed.

Secondly, while dimensionless rate and dimensionless reciprocal pressure diverge at the end of the infinite-acting period (inflection from convex to concave) on the RTDTC, they continue to track during the boundary-dominated portion on the RCDTC.
This second observation, which has not been made previously, can be derived by starting with the well known pseudosteady-state expression for dimensionless pressure presented by Ramey and Cobb:\(^15\):

\[
P_D(t_{DA}) = 2 \pi t_{DA} + \frac{1}{2} \ln\left(\frac{2.2458A}{r_{wa}^2 C_A}\right)
\]

with the objective of obtaining the reciprocal of the rate-cumulative expression of eq. 2.22. The first step is to substitute normalizing factors from Table 2.1 under the General column into eq. 2.24 and obtain dimensionless pressure as a function
of dimensionless time:

$$p_d(t_d) = \frac{t_d}{\alpha} + B.$$

(2.25)

Dividing through by beta, and substituting definitions for decline-curve dimensionless pressure and time yields:

$$p_{dD} = t_{dD} + 1.$$

(2.26)

Solving for decline-curve dimensionless time and then dividing by decline-curve dimensionless pressure to obtain decline-curve dimensionless cumulative as a function of decline-curve dimensionless pressure yields:

$$Q_{dD}(p_{dD}) = \frac{1}{p_{dD}}.$$

(2.27)

Rearranging to solve for reciprocal decline-curve dimensionless pressure yields:

$$\frac{1}{p_{dD}} = 1 - Q_{dD}.$$

(2.28)

For which the right hand side is equivalent to the right hand side of 2.22 and therefore, dimensionless decline-curve rate as a function of dimensionless decline-curve cumulative is equivalent to reciprocal decline-curve dimensionless pressure as a function of decline-curve dimensionless cumulative:

$$q_{dD}(Q_{dD}) = \frac{1}{p_{dD}(Q_{dD})}.$$

(2.29)
2.7 Type-Curve Matching Techniques

Reservoir parameters such as permeability, apparent wellbore radius, and drainage area are determined conventionally, using rate-time type-curves and the graphical technique of plotting rate-time field data on tracing paper with a log-log scale equivalent to the scale used for the type-curve. The field data are aligned keeping the grids parallel to the type-curve and a match point is selected. The match point can be any point common to both graphs and contains an ordinate and abscissa for both curves. This method is outlined by Earlougher. For RCDTC matching field data are plotted as PNR vs PNC. The match point from the pressure normalized field data and the RCDTC are selected as above.

Solving for the drainage area or external radius, fixed by the shift in horizontal axes (using eqs. 2.6, 2.17, & 2.18):

\[ A = \frac{5.615B}{\phi h c_i} \frac{(Q/ \Delta P)_M}{(Q_{dD})_M}, ft^2 \]  \hspace{1cm} (2.30)

This can be rearranged to solve for the pore volume, \( V_p \):

\[ V_p = \frac{B}{c_i} \frac{(Q/ \Delta P)_M}{(Q_{dD})_M}, Bbl \]  \hspace{1cm} (2.31)

Eq. 2.30 can also be used to determine the external drainage radius:

\[ r_e = \sqrt{\frac{A}{\pi}}, ft \]  \hspace{1cm} (2.32)
To calculate permeability and skin, enough early time data must be available to determine a dimensionless external radius. Selecting a dimensionless external radius combined with the effective external radius calculated from the area (eq. 2.30) provides the apparent wellbore radius. Rearrangement of eq. 2.3:

$$r_{wa} = \frac{r_e}{r_{eD}}$$

(2.33)

Allows skin to be calculated using eq. 2.13.

An assumption of reservoir geometry is not required to solve for reservoir size or skin effect because the reservoir shape factor is not involved. To determine permeability, an assumed geometry (usually radial) is used to calculate $\beta$ (eq. 2.19 or Table 2.1 - General). No significant difference occurs between selecting among other symmetrical drainage patterns such as a well in the center of a square.

The vertical axes alignment along with a calculated or approximated value of $\beta$ is used to determine permeability:

$$k = \frac{141.2Bu\beta}{h} \frac{(q/ \Delta P)_M}{(q_{eD})_M}, \text{md}$$

(2.34)

Another technique, promoted here, is to obtain performance history matches in a computer spread-sheet. Incorporating the elements of Fig 2.5 with the field data and a parameter block, containing all reservoir parameters used in the dimensionless variables, can be utilized to non-dimensionalize the field data.
and compare it to the dimensionless liquid solution. **Fig. 2.8** shows the spreadsheet schematically.

External radius, permeability and skin can be adjusted until a suitable match of the data and the type-curve are made. One specific advantage of this technique is the match between the field data and the analytic solution can be displayed on one graph. Dimensionless rate and cumulative production data during the infinite-acting period used in Fig. 2.5 obtained from Ehlig-Economides \(^9\) can alternatively be obtained by combining van Everdingen and Hurst \(^10\) and Sengul \(^16\). With infinite-acting dimensionless rate and cumulative tabular data,
branches for any dimensionless external radius can be generated using eqs. 2.16 through 2.19. The exponential solution, Eq. 2.14, can be used to generate boundary-dominated data after a $t_{DA} > 0.1$. 
3.1 Gas Pseudopressure

Two major assumptions, constant fluid compressibility and constant fluid viscosity, inherent to the development of the liquid solution require additional handling for the prediction of flow rates and pressures for gas reservoirs. In 1967 Al-Hussainy et al. defined gas pseudopressure as:

\[ P_p = 2 \int_0^p \frac{P}{\mu z} dp \]  \hspace{1cm} (3.1)

Where the compressibility factor, \( z \), and the viscosity, \( \mu \) (cp), are pressure dependent functions.

Gas pseudopressure represents the potential difference or driving force of fluid flow in the reservoir. Substitution of pseudopressure in dimensionless rate results in the following definition for gas reservoirs:

\[ q_d = \frac{1422 q_g T}{k_g h (P_{wi} - P_{pw})} \]  \hspace{1cm} (3.2)

Where \( q_g \) is the gas production rate (MCF/d), \( T \) is temperature (°R) and \( k_g \) is the
permeability to gas (md). Decline-curve dimensionless rate can be obtained as in Chapter Two, eq. 2.16, by multiplying by the normalizing factor beta, $\beta$.

By replacing pressure with pseudopressure, drawdowns of gas reservoirs during the infinite-acting time period can be analyzed using semilog and type-curve matching techniques discussed in section 2.2.

### 3.2 Normalized Time

During boundary-dominated flow, gas wells producing at constant pressure do not follow the exponential decline predicted by the liquid solution. This was demonstrated in 1985 by Carter, who presented a family of type curves correlated by a parameter describing the severity of the drawdown; the greater the drawdown, the larger the deviation from the liquid solution for gas reservoirs producing under the condition of constant BHFP.

To account for the changes in viscosity and compressibility in dimensionless time, Fraim and Wattenbarger in 1987 introduced a normalized time function that drew together the family of curves presented by Carter into a single curve, the liquid solution.

Viscosity-Compressibility normalized time is defined as:

$$t_{n(\mu c)} = \int_0^t \frac{\mu c_i}{\mu c} \, dt$$  \hspace{1cm} (3.3)

In eq. 3.3, viscosity and compressibility are evaluated at average reservoir
pressure. Dimensionless normalized decline-curve dimensionless time becomes:

\[
\tau_{dD} = \frac{0.006328t_n(\mu, \phi)}{\phi(\mu_c_i) \frac{r_w^2}{r} \alpha B} 
\]

(3.4)

Fig. 3.1 presents simulator generated production versus both dimensionless time and versus dimensionless normalized time for "Case 1 - Circular reservoir" from Fraim and Wattenbarger. This technique involves successive
approximations of gas in place (GIP) using the gas material balance, to interrelate average pressure through cumulative production to time. The method of computation for normalized time requires a summation of time steps that is sensitive to step size.

3.3 Normalized Cumulative

The results of Chapter Two suggest that it would be desirable to handle pressure dependent viscosity and compressibility in the dimensionless cumulative term. Using this technique, gas wells with variable rate and variable flowing pressure could be plotted as pseudopressure normalized production rate (PPNR) and pseudopressure normalized cumulative production (PPNC) on the RCDTC. This was investigated and found to be effective. Viscosity-compressibility normalization of cumulative production can be defined as:

$$Q_{n(\mu-c)} = \int_{0}^{Q} \frac{(\mu c)_{i}}{\mu c_{i}} dQ$$

(3.5)

A derivation for normalized cumulative paralleling that of normalized time by Fraim and Wattenbarger is included as Appendix B and results in the definition of viscosity-compressibility normalized decline-curve cumulative:

$$Q_{dD} = \frac{9.00Q_{n(\mu-c)} T}{\phi h(\mu c)_{i} r_{w}^{2}(P_{pi} - P_{pw}) \alpha}$$

(3.6)
The additional subscript (\(n-c\)) in the variables defined in eqs. 3.3 and 3.5 indicate viscosity-compressibility normalization.

Handling viscosity and compressibility in the cumulative term also provides a simpler computation method for normalization since fractional recovery, \(Q/GIP\) and \(P/z\) are linearly related by the material balance equation:

\[
\left( \frac{P}{z} \right)_{avg} = \left( \frac{P}{z} \right)_n \left( 1 - \frac{Q}{GIP} \right)
\]  

(3.7)

The integration in Eq. 3.5 can then be evaluated at intervals of \(P/z\) as shown in Fig. 3.2. Also shown in Fig 3.2 is the ratio of normalized cumulative production to actual cumulative production, or the viscosity-compressibility normalizing factor \(F_{n(\mu-c)}\):

\[
F_{n(\mu-c)} = \frac{Q_{n(\mu-c)}}{Q}
\]  

(3.8)

The normalizing factor (upper curve) and the viscosity-compressibility product ratio (lower curve) are shown versus fractional recovery for the fluid properties associated with "Case 1 - Circular reservoir". Also shown as solid triangles along the lower curve are viscosity-compressibility product ratio data from Fraim and Wattenbarger. Techniques for calculating viscosity and compressibility are developed in Appendix A. Normalized cumulative production of field data can then obtained by rearrangement of eq. 3.8:
\[ Q_{\mu(\mu \rightarrow \infty)} = F_{\mu(\mu \rightarrow \infty)} \cdot Q \]  

(3.9)

\[ \begin{align*}
\text{Gas gravity} &= 0.601 \\
\text{Reservoir Temperature} &= 200 \text{ degrees F}
\end{align*} \]

Fig 3.2 - Viscosity-Compressibility Product Ratio and \( F_n \) Versus Recovery

Therefore, cumulative production combined with a choice of GIP yields fraction recovery. And fractional recovery yields the viscosity-compressibility normalization factor by numerical integration of gas fluid properties.

Rate data from Fig 3.1 was used with cumulative production obtained by re-simulating Fraim and Wattenbarger\(^\text{19}\) "Case 1 - Circular reservoir" using a personal computer (PC) version of Boast II\(^\text{20}\) and is presented on the RCDTC show in Fig. 3.3.
Fig. 3.3 - RCDTC: Gas Well With Constant BHFP

Two distinct advantages of using the RCDTC have now been demonstrated. Most importantly, constant pressure and constant rate solutions are identical, providing the basis for variable pressure variable rate analysis using PNR and PNC for single phase liquid flow and PPNR and PPNC for single phase gas flow. Secondly, for gas reservoirs, accounting for viscosity-compressibility normalization in the dimensionless cumulative term gives unique results without regard to step-size of the field data and normalizes single phase gas flow to the liquid solution. Both of these advantages will be demonstrated in the following application.
3.4 Example Application: Gas Well

Data for this example comes from Garb et al.\(^ 21 \), and also Rodgers et al.\(^ 22 \).

This example was selected because of the limited amount of flowing pressure data available and because the drawdown is variable in pressure and variable in rate.

Table 3.1 presents reservoir and production data.

**Table 3.1  Reservoir and Production Data for Garb "Case 1"**

<table>
<thead>
<tr>
<th>Permeability to gas</th>
<th>GIP</th>
<th>Height</th>
<th>Temperature</th>
<th>Porosity</th>
<th>Gas gravity</th>
<th>Gas Saturation</th>
<th>Initial Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3 md</td>
<td>4.85 BCF</td>
<td>80 ft</td>
<td>636 °R</td>
<td>10%</td>
<td>0.7</td>
<td>75%</td>
<td>2500 psia</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>Rate</th>
<th>Cumulative</th>
<th>BHFP</th>
<th>( P_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mcf/d</td>
<td>M M cf</td>
<td>psia</td>
<td>psi (^{-2/3})</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2500</td>
<td>.4767+E9</td>
</tr>
<tr>
<td>1</td>
<td>1000</td>
<td>365</td>
<td>1604</td>
<td>.2108+E9</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td>730</td>
<td>1361</td>
<td>.1538+E9</td>
</tr>
<tr>
<td>3</td>
<td>800</td>
<td>1022</td>
<td>1352</td>
<td>.1519+E9</td>
</tr>
<tr>
<td>4</td>
<td>800</td>
<td>1314</td>
<td>1153</td>
<td>.1116+E9</td>
</tr>
<tr>
<td>5</td>
<td>600</td>
<td>1533</td>
<td>1216</td>
<td>.1238+E9</td>
</tr>
<tr>
<td>6</td>
<td>600</td>
<td>1752</td>
<td>1071</td>
<td>.9762+E8</td>
</tr>
<tr>
<td>7</td>
<td>400</td>
<td>1898</td>
<td>1197</td>
<td>.1200+E9</td>
</tr>
<tr>
<td>8</td>
<td>400</td>
<td>2044</td>
<td>1107</td>
<td>.1032+E9</td>
</tr>
</tbody>
</table>

The numerically simulated data was generated for a well in the center of a square. The data plot for this is presented in Fig. 3.4 showing PPNR versus PPNC. The immediate observation is that all data is concave to the origin indicating boundary-dominated data and therefore the RCDTC can be used.
The cumulative normalization factor was determined as a function of gas fluid properties similar to Fig. 3.2 and a polynomial curve fit of the factor as a function of fractional recovery was generated:

\[
F_{n(\mu, \omega)} = a + b \left( \frac{Q}{GIP} \right) + c \left( \frac{Q}{GIP} \right)^2 + d \left( \frac{Q}{GIP} \right)^3 \quad \cdots \quad (3.10)
\]

With

\begin{align*}
  a &= 0.990 \\
  b &= -0.579 \\
  c &= 0.358 \\
  d &= -0.238
\end{align*}

Fig. 3.4 - Data Plot for Garb et al. Case 1.
Known permeability, GIP, and apparent wellbore radius were input into the parameter block within the spreadsheet resulting in the match shown in Fig. 3.5.

![Data from Garb's Case 1](image)

Fig. 3.5 - RCDTC: Gas Well with Variable BHFP

The data show excellent agreement with the liquid solution constant pressure RCDTC demonstrating the ability to handle the variable BHFP case for gas reservoirs.

Table 3.2 provides additional parameters required to non-dimensionalize field data to the RCDTC and calculate dimensionless decline-curve rate and cumulative shown in Fig. 3.5.
Table 3.2  Dimensionless Rate and Cumulative for Garb et al.\textsuperscript{21} Case 1

<table>
<thead>
<tr>
<th>Year</th>
<th>PPNR</th>
<th>PPNC</th>
<th>Q/GIP</th>
<th>F_{n(-e)}</th>
<th>Q_{n(u-e)}</th>
<th>q_{dd}</th>
<th>Q_{dd}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.713E-06</td>
<td>0.0014</td>
<td>0.076</td>
<td>0.945</td>
<td>0.0013</td>
<td>0.748</td>
<td>0.231</td>
</tr>
<tr>
<td>2</td>
<td>3.072E-06</td>
<td>0.0022</td>
<td>0.153</td>
<td>0.906</td>
<td>0.0020</td>
<td>0.619</td>
<td>0.366</td>
</tr>
<tr>
<td>3</td>
<td>2.443E-06</td>
<td>0.0031</td>
<td>0.214</td>
<td>0.879</td>
<td>0.0027</td>
<td>0.492</td>
<td>0.494</td>
</tr>
<tr>
<td>5</td>
<td>2.179E-06</td>
<td>0.0036</td>
<td>0.275</td>
<td>0.853</td>
<td>0.0031</td>
<td>0.439</td>
<td>0.550</td>
</tr>
<tr>
<td>5</td>
<td>1.689E-06</td>
<td>0.0043</td>
<td>0.321</td>
<td>0.835</td>
<td>0.0036</td>
<td>0.340</td>
<td>0.649</td>
</tr>
<tr>
<td>6</td>
<td>1.571E-06</td>
<td>0.0046</td>
<td>0.367</td>
<td>0.816</td>
<td>0.0037</td>
<td>0.316</td>
<td>0.674</td>
</tr>
<tr>
<td>7</td>
<td>1.115E-06</td>
<td>0.0053</td>
<td>0.397</td>
<td>0.804</td>
<td>0.0043</td>
<td>0.224</td>
<td>0.765</td>
</tr>
<tr>
<td>8</td>
<td>1.065E-06</td>
<td>0.0054</td>
<td>0.428</td>
<td>0.791</td>
<td>0.0043</td>
<td>0.214</td>
<td>0.775</td>
</tr>
</tbody>
</table>

3.5 Type-Curve Matching Techniques: Gas Wells

Two preparation steps are required to analyze field decline-curves for gas wells. First, calculation of BHFP from FTP must be performed for all data. This can be done most efficiently in a programming language and the results imported to a spread-sheet that contain the rate and cumulative data as described in section 2.7.

The second step is to, again, use a program to calculate compressibility factors, compressibility, and viscosity for the gas gravity and temperature of the reservoir. Integrations can be performed in the program to obtain gas pseudopressure and viscosity compressibility normalizing factor. Polynomial fits,
such as the one presented in the example application for the normalizing factor, can also be made for gas pseudopressures as a function of BHFP. The coefficients for these two fits can then be incorporated into the spreadsheet.

![Flow Chart for Gas Well Analysis](image)

**Fig. 3.6 - Flow Chart for Gas Well Analysis**

A data plot of PPNR versus PPNC is then made and flow periods present are determined. Infinite-acting data, convex to the origin, can be analyzed without viscosity-compressibility normalized cumulative using the rate-cumulative type-curve for unbounded reservoirs (Fig. 2.3) or semilog techniques. Boundary-dominated data, concave to the origin, can be analyzed with the RCDTC (Fig. 2.5) using viscosity-compressibility normalized cumulative. Permeability and skin
can be determined from a match of the infinite-acting data on either type-curve and Area (or GIP) can be determined from boundary-dominated data. A flow chart for this procedure is presented in Fig. 3.6.
4.1 Oil Pseudopressure

Like gas pseudopressure, oil pseudopressure represents the driving force for fluid flow in the reservoir and is defined:

\[ P_p = \int_0^p \frac{k_{ro}(S_o)}{B_o \mu_o} dp \]  

(4.1)

Where \( k_{ro} \) is the permeability to oil relative to absolute permeability, \( k \), and is a function of oil saturation, \( S_o \). \( B_o \) and \( \mu_o \) are the formation volume factor and viscosity of the oil phase respectively and are functions of pressure.

Evinger and Muskat \(^{23}\) used the integral of eq. 4.1 in 1942 for steady-state flow. In 1973 Fetkovich \(^{24}\) incorporated oil pseudopressure in the pseudosteady-state flow equation:

\[ \frac{q_o}{J_o} = (P_{pavg} - P_{pwf}) \]  

(4.2)

Where \( P_{pavg} \) and \( P_{pwf} \) are the pseudopresses evaluated at average and bottom hole flowing pressures and \( J_o \) is the solution-gas-drive productivity index:

\[ J_o = \frac{kh}{141.2B} \]  

(4.3)
Fetkovich based his work on field experiments. Eq. 4.3 was later derived by Chen and Poston and further examined by Camacho-V and Raghavan. Use of oil pseudopressure results in the following definition for dimensionless rate:

\[ q_D = \frac{141.2 q_o}{kh(P_{pi} - P_{pwf})} \]  \hspace{1cm} (4.4)

Decline-curve dimensionless rate from eq. 2.16 becomes:

\[ q_{dD} = \frac{141.2 q_o \theta}{kh(P_{pi} - P_{pwf})} \]  \hspace{1cm} (4.5)

The difficulty in evaluating oil pseudopressure is the determination of the interrelation between oil saturation and pressure. Fetkovich provides an approximation for the oil pseudopressure difference used in eq. 4.2:

\[ (P_{pavg} - P_{pwf}) \approx \left( \frac{k_{ro}}{B_o \mu_o} \right)_{avg} \frac{\left( p_{avg}^2 - P_{wfi}^2 \right)}{2P_{avg}} \]  \hspace{1cm} (4.6)

This can be extended to the pseudopressure difference used in eq. 4.4:

\[ (P_{pi} - P_{pwf}) \approx \left( \frac{k_{ro}}{B_o \mu_o} \right)_{i} \frac{\left( P_i^2 - P_{wfi}^2 \right)}{2P_i} \]  \hspace{1cm} (4.7)
4.2 Normalized Time

Chen and Poston\textsuperscript{12} developed a multiphase version of the normalized time used by Fraim and Wattenbarger\textsuperscript{19} for solution-gas-drive reservoirs with the same objective of linearizing the rate-time performance to the liquid solution. The definition of mobility-compressibility normalized time is:

\[
t_{n(m-c)} = \int_{0}^{t} \frac{\lambda_{tr}/\zeta_{t}}{(\lambda_{tr}/\zeta_{t})_{i}} \, dt \tag{4.8}
\]

Where \(\lambda_{tr}\) is the total mobility in terms of relative permeability:

\[
\lambda_{tr} = k_{ro}/\mu_{o} + k_{rg}/\mu_{g} + k_{rw}/\mu_{w} \tag{4.9}
\]

and \(\zeta_{t}\) is the total system compressibility:

\[
\zeta_{t} = S_{o}\zeta_{o} + S_{g}\zeta_{g} + S_{w}\zeta_{w} + \zeta_{t} \tag{4.10}
\]

The subscripts for relative permeability, viscosity, saturation, and compressibility used in eqs. 4.9 and 4.10 are for oil, gas, water, and formation. Mobility and compressibility are calculated at average reservoir pressure and saturations.

Like the normalized time for gas reservoirs, the initial mobility is incorporated so that normalized time has the same units as real time. Also like the viscosity-compressibility normalization for gas wells, normalized time is sensitive to step size.
Decline-curve dimensionless time for oil wells becomes:

\[ t_{ud} = \frac{0.006328k_{oi}f_{n(m,c)}}{\varphi(\mu c)}_i r_w^2 \alpha B \]  

\[ (4.11) \]

where \( k_{oi} \) is the initial oil permeability at irreducible water saturation.

---

Fig. 4.1 presents the linearization of oil well production utilizing the oil pseudopressure of eq. 4.7 and the normalized time of Chen and Poston\(^{12}\).
Numerical integration of eq. 4.8 was performed by trapezoidal rule using 106 time steps over the 10,000 days of simulation provided by Chen.

Reservoir data used in Fig. 4.1 and Figs. 4.2 through 4.4 is contained in Table 4.1.

<table>
<thead>
<tr>
<th>Table 4.1 Reservoir Data for Figs. 4.1 - 4.4 (after Chen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Pressure</td>
</tr>
<tr>
<td>BHFP</td>
</tr>
<tr>
<td>Water Saturation</td>
</tr>
<tr>
<td>Porosity</td>
</tr>
<tr>
<td>S_water</td>
</tr>
<tr>
<td>Permeability</td>
</tr>
<tr>
<td>r_wa</td>
</tr>
<tr>
<td>a</td>
</tr>
<tr>
<td>P_i - P_pwf</td>
</tr>
<tr>
<td>Oil Gravity</td>
</tr>
<tr>
<td>J_o</td>
</tr>
</tbody>
</table>

\[ K_{ro} = ((S_o - S_{or})/(1 - S_{or}))^{1.5} \]

\[ K_{rg} = 0.7 (S_g/(1-S_{or}))^{1.5} \]

4.3 Normalized Cumulative

Following the logic used in Chapter Three for gas reservoirs, a normalized cumulative is defined for linearization of multiphase flow for rate-cumulative analysis to allow for the variable BHFP condition. Normalized cumulative production is defined as:

\[ Q_{n(m-o)} = \int_{0}^{Q} \frac{\lambda_r / \Omega}{(\lambda_r / \Omega)_i} dQ \]

\[ (4.12) \]
Dimensionless decline-curve cumulative becomes:

\[ Q_{dD} = \frac{0.8936 Q_{n(m-o)} k_{roi}}{\phi h (\mu c)_i r_{wa}^2 (P_{wi} - P_{pw}) \alpha} \]  \hspace{1cm} (4.13)

where \( k_{roi} \) is the initial relative permeability to oil at irreducible water saturation.

**Fig. 4.2** displays the rate-cumulative data of Chen using the decline-curve dimensionless variables defined by eq. 4.5 for production rate and by eq. 4.13 for cumulative production.
4.4 Variable BHFP for Solution-Gas-Drive Reservoirs

Solution-gas-drive reservoirs with production wells that flow to the surface rarely perform under the condition of constant BHFP even when FTP at the surface is held constant for pressure separators. This is because of the nature of solution-gas-drive reservoirs, as pressure is reduced in the reservoir, solution-gas evolves from the oil. Once enough gas has evolved to create a continuous phase, gas flows simultaneous with the oil to the wellbore and up the flow string. The flow of gas in the wellbore lightens the column weight of the fluid and there is less pressure drop in the wellbore. The decrease in pressure drop in the wellbore results in a decrease in BHFP. This effect is pronounced in deeper wells.

Another significant cause of backpressure change for a flowing well is conversion to artificial lift by pump. The consequence of this operation is a sharp change in backpressure because of a negligible column weight above working fluid level of the downhole pump. A similar change in backpressure will occur for other types of artificial lift such as gas lift.

Oil wells with high GOR may also be restricted in their production due to field rules or market demand for the gas. This type of curtailment may cause an increase in BHFP due to restricted production or even cause the well to be shut-in for periods after quotas are met. For these reasons it is important to be able to analyze production rates under the variable BHFP condition.
To test the compressibility-mobility normalization for the variable BHFP condition a Boast II simulation run was made by producing a well at constant rate. For comparison, Figs. 4.3 and 4.4 present the constant production rate case on both the RTDTC and the RCDTC. In Fig. 4.3, the deviation from the liquid solution is shown at the onset of boundary-dominated flow similar to the liquid case (Fig 2.6). Prior to the deviation, during the infinite-acting period, rate type-curves and semi-log techniques can be applied. The limitation of using the constant BHFP RTDTC becomes very apparent during the boundary-dominated period.
Fig. 4.4 shows that the mobility-compressibility normalized cumulative successfully linearizes the multiphase production, even with variable BHFP, to the constant BHFP liquid solution in both the infinite-acting and boundary-dominated flow periods. This provides a single technique to analyze rate-pressure data for all flow periods.

4.5 Determination of the Mobility-Compressibility Normalized Cumulative

Unlike the viscosity-compressibility normalizing factor for gas wells, which can be calculated for fluid properties alone, the mobility-compressibility normalizing factor must be generated from a numerical simulation to interrelate
saturation and pressure. This can be done with Muskat's\(^2\)\footnote{Footnote text} differential material balance:

\[
\frac{dS_o}{dP} = \frac{S_o}{B_o} \frac{dB_o}{dP} + \frac{\lambda_o}{\lambda_1} \quad .................................................. \quad (4.14)
\]

where:

\[
\lambda_1 = \lambda_o + \lambda_g + \lambda_w = \frac{k_o}{\mu_o} + \frac{k_g}{\mu_g} + \frac{k_w}{\mu_w} \quad .................................................. \quad (4.15)
\]

To insure that the saturation pressure relation would not vary due to method of depletion, two Boast II\(^2\)\footnote{Footnote text} simulations were performed, one for the constant rate depletion and the other for depletion in the constant BHFP mode. Shown in Fig. 4.5 is the mobility-compressibility normalization factor for both simulations. Definition of the mobility-compressibility normalization is the ratio of normalized cumulative to actual cumulative:

\[
F_{n(m \rightarrow \infty)} = \frac{Q_{n(m \rightarrow \infty)}}{Q} \quad .................................................. \quad (4.16)
\]

Fig 4.5 indicates that there is little effect in the selection of simulation option. Either a constant rate or constant BHFP simulation adequately defines the mobility-compressibility normalizing factor as a function of fractional recovery.
To utilize Fig. 4.5, cumulative production is combined with an estimate of oil in place (OIP) yielding a fractional recovery, the mobility-compressibility normalization factor can then be computed from a curve fit of Fig. 4.5. Decline-curve dimensionless variables can be plotted using eqs. 4.5, 4.7 and 4.13. Successive approximations of OIP, permeability, and skin are made until a best fit on the liquid solution RCDTC is obtained.
4.6 Undersaturated Reservoirs

Solution-gas-drive reservoirs may exist initially in a saturated or undersaturated condition. Flow in undersaturated reservoirs behave as a single phase liquid while flowing pressure is still above the saturation or bubble point pressure. For the case of an undersaturated reservoir with a BHFP less than the saturation pressure and an average reservoir pressure above the saturation pressure, Fetkovich has shown that the pseudosteady-state flow equation is:

\[ q_o = J (P_{avg} - P_b) + J_o (P_{pb} - P_{pwp}) \]  

(4.17)

The single phase productivity index, \( J \), used in the first term of eq. 4.17 is defined:

\[ J = \frac{k_{oi}h}{141.2B_o \mu_o \bar{B}} \]  

(4.18)

The pressure range that applies to the single phase productivity index in eq. 4.17 is the saturation pressure, \( P_b \), to the average reservoir pressure. The viscosity and formation volume factor in eq. 4.18 are evaluated at the average of that pressure range. The second term of eq. 4.17 represents saturated flow, and the pseudopressure range is from BHFP to the saturation pressure. The decline-curve dimensionless production rate becomes:
\[ q_{\text{AD}} = \frac{141.2 q_o \beta}{k h \left[ \frac{k_{ro}}{B_o \mu_o} (P_i - P_b) + (P_{pb} - P_{pwl}) \right]} \] ............... (4.19)

And the decline-curve dimensionless cumulative production is:

\[ Q_{\text{AD}} = \frac{0.8936 Q_{n(m-co)} k_{ro}}{\phi h (\mu_c) \left[ \frac{k_{ro}}{B_o \mu_o} (P_i - P_b) + (P_{pb} - P_{pwl}) \right] a} \] ............... (4.20)

where viscosity and formation volume factor in the pressure difference terms of eqs. 4.19 and 4.20 are evaluated at the average pressure of that pressure interval.

To check the validity of these equations, a Boast II simulation run was made using the reservoir data of Table 4.1 with an adjustment of initial pressure from 4500 psia to 5000 psia while the BHFP remained at 4000 psia. The saturation pressure also remains at 4500 psia and consequently the drawdown represents fluid flow in a reservoir containing both a saturated and undersaturated region. Fig. 4.6 presents verification for use of the composite pressure and pseudopressure differences used in eqs. 4.19 and 4.20 and extends the use of the RCDTC to undersaturated oil reservoirs.
Fig. 4.6 - RCDTC: Undersaturated Oil Well with Constant BHFP

Table 4.2 presents the augmented reservoir data for the undersaturated reservoir case.

Table 4.2 Augmented Reservoir Data for Fig. 4.6

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Pressure</td>
<td>5000 psia</td>
</tr>
<tr>
<td>Saturation Pressure</td>
<td>4500 psia</td>
</tr>
<tr>
<td>BHFP</td>
<td>4000 psia</td>
</tr>
<tr>
<td>OIP</td>
<td>5.533 MMSTB</td>
</tr>
<tr>
<td>$B_{oi}$</td>
<td>1.93 rb/STB</td>
</tr>
<tr>
<td>$B_{ob}$</td>
<td>1.94 rb/STB</td>
</tr>
<tr>
<td>$\mu_{oi}$</td>
<td>0.1865 cp</td>
</tr>
<tr>
<td>$\mu_{ob}$</td>
<td>0.1789 cp</td>
</tr>
</tbody>
</table>
4.7 Pseudopressure Approximation for Severe Drawdowns

The pseudopressure approximation (eq. 4.7) used in Figs. 4.1 through 4.6 is effective only for limited drawdowns. The BHFP used in these figures is slightly more than a 10% reduction from the initial or saturation pressure. Camacho-V and Raghavan point out that the true pseudopressure is a composite integral that incorporates the average reservoir pressure:

\[
(P_{pi} - P_{pw}) = \int_{P_{w}}^{P_{avg}(t)} \frac{k_{ro}(S_o)}{B_o \mu_o}(p,t)dp + \int_{P_{avg}(t)}^{P_{i}} \frac{k_{ro}(S_o)}{B_o \mu_o}(p,\bar{r})dp \]  \hspace{1cm} (4.21)

where \((p,t)\) and \((p,\bar{r})\) implies that \(p\) is the variable of integration and \(t\) or \(\bar{r}\) is fixed. The first integral in eq. 4.21 requires a determination of the saturation profile from the wellbore, \(r_w\), to a radius were reservoir pressure is equal to average pressure, \(\bar{r}\), in order to resolve \(k_{ro}\) as a function of saturation. Saturation in the second integral can be determined from material balance at average pressures:

\[
S_o = B_o \left[ \frac{S_o}{B_o} - \frac{5.615}{\phi h A} Q \right] \]  \hspace{1cm} (4.22)

A valuable approximation of eq. 4.22 has been developed by determining three estimates of saturation at any point of depletion: 1) The initial saturation,
2) The saturation at average pressure, and 3) The saturation at the wellbore using the constant GOR assumption of Levine and Prats:

\[
\frac{k_g}{k_o} = (GOR - R_s) \frac{B_g \mu_g}{B_o \mu_o}
\] ................................. (4.23)

where \(R_s\) is the solution gas oil ratio and is determined with the other fluid properties at BHFP. The permeability ratio determined in eq. 4.23 is then interrelated through the relative permeability curves to determine saturation at the wellbore.

The approximation developed utilizing these three saturations is:

\[
(P_{pi} - P_{p_w}) \approx (P_{pi} - P_{p_{avg}}) + (P_{p_{avg}} - P_{p_w})
\] ................................. (4.24)

where the two pseudopressure differences on the right hand side of eq. 4.24 are:

\[
(P_{pi} - P_{p_{avg}}) \approx (P_i - P_{avg}) \frac{\bar{\alpha}_i - \bar{\alpha}_{avg}}{\ln(\bar{\alpha}_i/\bar{\alpha}_{avg})}
\] ................................. (4.25)

and

\[
(P_{p_{avg}} - P_{p_w}) \approx (P_{avg} - P_{w}) \frac{\bar{\alpha}_{avg} - \bar{\alpha}_{rw}}{\ln(\bar{\alpha}_{avg}/\bar{\alpha}_{rw})}
\] ................................. (4.26)
where \( \bar{\alpha} \) is the integrand of eqs. 4.1 and 4.21:

\[
\bar{\alpha} = \frac{k_r(S_o)}{B_o \mu_o}
\]

(4.27)

Use of eqs. 4.24 - 4.27 are demonstrated in Fig. 4.7 for the reservoir data of Table 4.1 with the exception that the BHFP is reduced from 4000 to 2000 psia. This represents an increase in the drawdown from only about 10% to over 55% of the initial pressure.

Fig. 4.7 - Pseudopressure Approximations: Oil Well with Severe Drawdown
Open squares shown in Fig. 4.7 represent the pseudopressure approximation of eqs. 4.24 - 4.27. The solid line was generated from a Boast II simulation back calculating the pseudopressure difference during the infinite-acting period using eq. 2.8 and 4.4 and during the boundary-dominated period using eqs. 2.21, 4.5, and 4.13. Also shown in Fig. 4.7 as a horizontal line, is the approximation of Fetkovich$^{24}$ (eq. 4.7).

Use of eqs. 4.24 - 4.27 thus allows calculation of the pseudopressure difference used in decline-curve dimensionless rate (eq. 4.5) and decline-curve dimensionless cumulative production (eq. 4.13).
5.1 Reservoir History: Sun Ranch Field

Sun Ranch Field was discovered in March, 1987. The discovery well was Sun Ranch Federal #1 (SRF #1). Drilled to a total depth of 10,427 feet, and encountering a porous Grieve Sandstone from a depth interval from 10,224 to 10,292 ft. An initial set of perforations (10,280-286) tested 1117 STB/d of oil at a FTP of 550 psig. Additional perforations were added (10,225-234 and 10,245-254) and the well continued to flow test until the initial bottom hole pressure buildup was run on April 9, 1987. The pressure buildup test indicated an initial pressure of 4330 psig at bomb depth of 10,100 ft and a permeability to oil of 27 md for the perforated interval of 24 ft.

Drilling development continued with Sun Oil drilling five wells and Broken Hills Properties (BHP) drilling six wells. The final well to be drilled and completed was the Sun Ranch Federal A#1 (SRF A#1) February, 1989.

A field wide shut-in occurred in November 1988 pending unitization with production and partial pressure maintenance by gas injection commencing October, 1989. Two wells were used for gas injection. Initially the BHP #12-22 with the highest structural position was used. Injection was transferred to the BHP #7-22 in June, 1990.
All wells were drilled directionally due to the surface terrain. Bottom hole well locations with well names are shown in Fig. 5.1.

Fig. 5.1 - Unit Outline and Well Locations: Sun Ranch Field

Peak production prior to unitization occurred in August, 1988 at a rate of 40,501 STB per month from six (6) wells. During partial pressure maintenance peak production occurred January, 1990 at a rate of 43,534 STB per month from five (5) wells. By that time two (2) downdip wells had already watered out due to encroachment from a weak aquifer to the northeast.

Cumulative production from Sun Ranch through January, 1993 is 1.163 MMSTB representing a recovery of 23% of the OIP.
5.2 Sun Ranch Reservoir Data

Table 5.1 Reservoir Data for the Sun Ranch Field

<table>
<thead>
<tr>
<th>Basin:</th>
<th>Wind River</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geologic age:</td>
<td>Cretaceous</td>
</tr>
<tr>
<td>Formation:</td>
<td>Muddy</td>
</tr>
<tr>
<td>Deposition:</td>
<td>Tidal back-fill of an incised valley (Valley Fill)</td>
</tr>
<tr>
<td>Location:</td>
<td>Sections 15,22,23T33N R86W</td>
</tr>
<tr>
<td>Initial Pressure</td>
<td>4300 psig (PVT analysis Sun Ranch #1)</td>
</tr>
<tr>
<td>Reservoir Temperature</td>
<td>158°F</td>
</tr>
<tr>
<td>Number of wells</td>
<td>11</td>
</tr>
<tr>
<td>Reservoir area</td>
<td>400 ac (approx.)</td>
</tr>
<tr>
<td>Spacing</td>
<td>40 ac</td>
</tr>
<tr>
<td>Dip</td>
<td>14-18° NE</td>
</tr>
</tbody>
</table>

| $B_{oi}$             | 1.596 rb/STB                   |
| Water Saturation     | 15 %                           |
| OIP                  | 5.0 MMSTB                      |
| $S_{or}$             | 20 %                           |
| Permeability         | 25 md                          |
| $r_{wa}$             | .3717 ft                       |
| Oil Gravity          | 37.8 °API                      |
| $c_{oi}$             | $4.78 \times 10^4$ psi$^{-1}$  |

\[
K_{ro} = K_{roi} \left( (S_o - S_{or}) / (1 - S_{or} - S_{w}) \right)^4
\]

\[
K_{rg} = K_{roi} \left( 1 - S_{og} \right)^2 \left( 1 - S_{og} \right)^2
\]

\[
S_{og} = \left( \frac{1 - S_{g} \left( krg = 1 \right)} {1 - S_{i} \left( krg = 1 \right)} \right)
\]

\[
S_{i} \left( krg = 1 \right) = .15
\]

Table 5.1 presents reservoir data for the Sun Ranch field. Data from Table 5.1 was used to simulate single well performance for testing the application of the RCDTC and pseudopressure approximations from Chapter Four. The simulation was also used to generate the mobility-compressibility normalization.
factor as a function of recovery. The BHFP used in the simulation was 1500 psia. The reservoir area used was 640 ac or one square mile. Fig. 5.2 presents the RCDTC for the simulation.

Fig 5.2 - RCDTC: Sun Ranch Field Simulation

Fig. 5.2 demonstrates the applicability of the pseudopressure approximation used in section 4.7 (eqs. 4.24 - 4.27) for the relative permeability and fluid properties of the Sun Ranch Field.
Fig. 5.3 - Mobility-Compressibility Normalizing Factor for Sun Ranch Field

Fig 5.3 presents the mobility-compressibility normalization factor as a function of recovery. The data points are shown as open squares and the line is the polynomial fit that can be used to normalize individual well cumulatives as a function of well recovery.
5.3 Pressure Buildup Analysis

Bottom hole pressure buildup tests were run routinely in all eleven (11) wells upon initial completion of the well. Permeability, skin and initial pressure were determined from these tests and are shown in Table 5.2.

Table 5.2 Pressure Buildup Results: Sun Ranch Field

<table>
<thead>
<tr>
<th>Well</th>
<th>Height (ft)</th>
<th>Permeability (md)</th>
<th>Skin</th>
<th>Extrapolate Pressure (psig)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRF #1</td>
<td>24</td>
<td>26.90</td>
<td>10.70</td>
<td>4330</td>
</tr>
<tr>
<td>SRF #2</td>
<td>24</td>
<td>40.58</td>
<td>1.27</td>
<td>4146</td>
</tr>
<tr>
<td>SRF A#1</td>
<td>43</td>
<td>7.27</td>
<td>1.17</td>
<td>3137</td>
</tr>
<tr>
<td>SRF A#2</td>
<td>25</td>
<td>23.24</td>
<td>0.16</td>
<td>3688</td>
</tr>
<tr>
<td>SRF A#3</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BHP #3-22</td>
<td>36</td>
<td>53.0</td>
<td>1.62</td>
<td>4059</td>
</tr>
<tr>
<td>BHP #4-22</td>
<td>24</td>
<td>3.13</td>
<td>-2.50</td>
<td>2668</td>
</tr>
<tr>
<td>BHP #5-22</td>
<td>60</td>
<td>11.12</td>
<td>6.60</td>
<td>3443</td>
</tr>
<tr>
<td>BHP #6-22</td>
<td>24</td>
<td>2.11</td>
<td>-2.23</td>
<td>3209</td>
</tr>
<tr>
<td>BHP #7-22</td>
<td>18</td>
<td>41.40</td>
<td>1.80</td>
<td>3590</td>
</tr>
<tr>
<td>BHP #12-22</td>
<td>28</td>
<td>25.28</td>
<td>37.46</td>
<td>3371</td>
</tr>
</tbody>
</table>

The pressure buildup test in the SRF A#3 was unusable due to pressure leaks. Three pressure buildups exhibited a second zone of lower permeability and higher pressure. They were the BHP #4-22, BHP #5-22, and the BHP #6-22. These wells are grouped together in the western portion of the field.
The extrapolated pressures in Table 5.2 were corrected to a pressure datum of 3750 subsea and shown in Fig. 5.4 as open squares. Cumulative production was converted to recovery using an OIP of 3.5 MMSTB. Additional pore volume and OIP is contained in the zone of lower permeability. Total OIP from material balance studies is indicated to be 4.5 to 5.0 MMSTB for the entire reservoir. The solid line was generated from the simulation presented in Fig. 5.2.

![Graph](image)

**Fig 5.4 - Initial Well Pressures Versus Recovery: Sun Ranch Field**

The pressure trend indicates pressure communication among the individual wells in the Sun Ranch Field.
5.4 Individual Well Performances

Results from the pressure buildups were used to non-dimensionalize the production data for the individual wells. With permeability, skin, and initial pressure provided, the only remaining unknown was reservoir area. Area was adjusted until a best fit of the production data during primary depletion was obtained. Fig. 5.5 presents the type-curve match with the RCDTC for the SRF #1 well.

![Fig. 5.5 - RCDTC: SRF #1](image)

This well was an early strong producer for the field and its production at the time of field shut-in was 29% of that for all wells. The area obtained from
the match was 200 ac. A good estimate of permeability could be made for this well from the RCDTC match if buildup permeability was not available. This would prove useful if pressure buildup data were not available, as is the case in many fields.

In Fig. 5.6 the RCDTC match for SRF A#3 is presented. This well did not use pre-determined permeability and skin.

![Graph](Image)

**Fig. 5.6 - RCDTC: SRF A#3**

Results obtained from this performance match are an area of 40 ac and a permeability to oil of 13.5 md. A skin value of zero was used.
Fig. 5.7 presents the RCDTC performance match for the BHP #12-22 well. This well was converted to injection and produced less than one month. The value of three (3) ac obtained from this history match may be considered a minimum value of area. Without a greater length of drawdown, it is difficult to determine drainage area.

![Graph showing RCDTC performance match for BHP #12-22 well.](image-url)

The wells shown in Figs. 5.5 through 5.7 display only production during the primary depletion of the field. The BHP #12-22 was converted to injection. Attempts to produce the SRF #1 were unsuccessful due to water encroachment, and the SRF A#3 was never placed on production after field unitization.
Fig. 5.8 presents the RCDTC performance match for the SRF A#2 well which is the most prolific producer in the field. In Fig. 5.8, data shown as open squares represent rates and cumulatives during primary depletion. Data shown as solid triangles represent rates and cumulatives during gas injection. Fig. 5.8 demonstrates the ability of the RCDTC to demonstrate interference from offset gas injection. The results of the match yielded an area of 160 ac. This well had produced 49% of the oil produced by the entire field by January, 1993. Good pressure communication between the injection wells and this producer (also indicated by initial pressures in Fig. 5.4) can be confirmed by the RCDTC.
The BHP #3-22 was the second largest producer during pressure maintenance. The RCDTC performance match for this well is shown in Fig. 5.9.

Like Fig. 5.8, interference during pressure maintenance can be clearly observed by the deviation of the solid triangles from the open squares. The area obtained from the primary performance history match is 30 ac. The production from the BHP #3-22 was restricted during pressure maintenance due to high gas production because of the proximity to the BHP #7-22 gas injection well.
Conversion of the field gas injection point from the BHP #12-22 to the BHP #7-22 was done over concerns that the distance from the BHP #12-22 was too great from the principal producers in the field. Fig. 5.10 displays the RCDTC for the BHP #7-22 indicating weak support, if any, from the initial injection well.

![RCDTC: BHP #7-22](image)

Fig. 5.10 - RCDTC: BHP #7-22

Other problems with this injection well could be caused by damage indicated from its skin value of 37. Also, the BHP #12-22 was in close proximity to the three wells that exhibited a strong influence of a second zone of higher pressure. Drainage area obtained for the performance is 14 ac.
Fig. 5.11 displays the RCDTC for the SRF #2 well. This well had the highest production rates during primary depletion of the field. Maximum production rate for this well was 14,383 STB for the month of February, 1988. Area obtained from its performance is 50 ac.

![RCDTC: SRF #2](image)

The SRF #3-22 well also shows interference from offset injection. Note that the solid triangles plot at a very low decline-curve dimensionless rate compared to Fig. 5.8. This is due to its low structural position and proximity to the water aquifer. Rates declined in this well due to water encroachment.
The SRF A#1 well also produced near the field oil-water contact and suffered high water-oil ratios (6 BW/BO) eliminating the well's ability to flow regardless of its high producing GOR (80,000 SCF/STB). Fig. 5.12 presents the RCDTC for this well.

![RCDTC: SRF A#1](image)

**Fig. 5.12 - RCDTC: SRF A#1**

Solid triangles exhibit the influence of the nearby gas injection well, BHP #7-22. Area determined by the RCDTC history performance is 4 ac.
Fig. 5.13 displays the RCDTC for the BHP #5-22. Interference and declining rates due to increasing GOR can also be observed.

Area determined for the type-curve match during primary depletion is 3 ac. This low value is due in part to the analysis procedure of utilizing the perforated interval as the net height. If a net height could be determined for the high permeability zone only, a much greater drainage area would be computed. Future work in determining permeability from well logs in order to resolve which intervals correlate to the two layers exhibited in the pressure buildup analysis is warranted.
The BHP #4-22 is another well whose pressure buildup analysis clearly exhibited a layered reservoir. Area obtained from the RCDTC match shown in Fig. 5.14 is 15 ac. The minimal data used in the match is due to the fact that the well was completed during the field wide shut-in and only one month of production was allowed for the well prior to unitization and gas injection. No unique match could be made without using the permeability and skin determined from the pressure buildup analysis. The solid triangles in Fig. 5.14 display a high degree of scatter and may be a combined effect of offset injection and contribution from a second zone of greater pressure.

Fig. 5.14 - RCDTC: BHP #4-22
Most of the comments regarding the BHP #5-22 and the BHP #4-22 also apply to the BHP #6-22. In addition, this well is adjacent to the current injection well, BHP #7-22. BHP #6-22 has been a continuous producer during pressure maintenance. The RCDTC for this well is shown in Fig. 5.15.

![Fig. 5.15 - RCDTC: BHP #6-22](image)

Area obtained from the performance match is 4 ac. Again, a better estimation of net pay for the more permeable zone in this well would lead to an improved estimation of drainage area during primary depletion. None the less, drainage pore volume (vhA) should be accurate.
5.5 Summary

A major goal in this study was to determine communication between wells in the Sun Ranch Field. Initial pressures obtained from pressure buildup analysis (Fig. 5.4) and interference displayed in the individual well RCDTCs confer that communication does exist field wide.

Reservoir characterization, including determination of permeability and drainage area, have been demonstrated for the RCDTC. This favorably ties together the use of the RCDTC as a transient pressure/transient rate analysis technique with time honored pressure buildup analysis. The RCDTC can be thought of as an extended drawdown test which accounts for variation in rate, and variation in pressure. In the pursuit of reservoir characterization, the RCDTC provides the engineer with an additional analysis technique utilizing readily available production data.
CHAPTER 6

CONCLUSIONS

6.1 Single-Phase Liquid Flow

Pressure normalized rate (PNR) is effective for analyzing drawdown data in the infinite-acting flow period. For small changes in either bottom-hole flowing pressure (BHFP) or production rate, PNR can be used with rate-time type-curves, rate-cumulative type-curves, and semilog techniques. The basis for the use of PNR is that the solutions for wells producing at constant BHFP and wells producing at constant pressure converge (Fig. 2.2).

The constant rate and constant BHFP solutions are identical for the boundary dominated flow period when taken as a function of cumulative production (Fig. 2.7). This provides the basis for the use of pressure normalized cumulative (PNC) in conjunction with PNR to analyze boundary-dominated flow which is variable in rate and pressure.

When production encompasses both infinite-acting and boundary-dominated flow periods, determination of permeability, skin, and area can be made by type-curve matching with the RCDTC.

Appendix A provides techniques for calculating BHFP from flowing tubing pressure (FTP) for producing wells and bottom hole pressure for injection wells.
6.2 Single-Phase Gas Flow

Use of the liquid solution constant pressure rate-cumulative decline type-curve (RCDTC) can be extended to single-phase flow of compressible gases via the use of the viscosity-compressibility normalization factor and gas pseudopressure. Like gas pseudopressure, the viscosity-compressibility normalization factor can be determined from fluid properties alone (Fig 3.2).

Because of the independence in step size of time intervals in the determination of the viscosity-compressibility normalization factor, use of the RCDTC is superior to use of the rate-time decline type-curve (RTDTC) even for wells producing at constant BHFP.

Appendix A provides techniques for determining BHFP from FTP for single-phase gas flow and for calculating fluid properties required to determine gas pseudopressure and viscosity-compressibility normalization factors.

6.3 Multiphase Flow

Use of the RCDTC has also been extended to the multiphase flow of gas and oil for solution-gas-drive reservoirs via the use of the mobility-compressibility normalization factor and oil pseudopressure. The interrelation between oil saturation and average reservoir pressure for the relative permeability and fluid properties of a particular reservoir has to be made. Two techniques have been offered for this determination. One is Muskat’s differential material balance (eq. 4.14) the other is a reservoir simulator.
Two approximations are provided for calculating oil pseudopressure. One which was previously developed but only applicable to reservoirs with non-severe drawdowns (eq. 4.7) and a new approximation applicable to both non-severe and severe drawdowns (eqs. 4.24 - 4.27). The approximation developed in this work was applied successfully to two different sets of relative permeability and fluid properties (Fig. 4.7 and Fig. 5.2).

Decline-curve dimensionless variables have been developed for either initially saturated or undersaturated solution-gas-drive reservoirs.

Appendix A provides techniques for calculating BHFP from FTP for solution-gas-drive oil wells utilizing fluid property correlations and numerical integration techniques.

Chapter Five presents a field case history showing the utility of the RCDTC as an important tool in reservoir characterization. This case history substantiates the RCDTC as variable rate extended drawdown analysis by showing excellent comparisons with permeability determined from pressure buildup analysis. Specific to this case history, the RCDTC was also shown to be an excellent diagnostic plot to interpret interference from offset injection of gas during a partial pressure maintenance project.

6.4 Future Work

The radial flow model used in this dissertation is probably the most common model used in transient pressure analysis and decline-curve analysis. Other models encountered by petroleum reservoir engineers are: hydraulically
fractured wellbores, naturally fractured reservoirs, dual-porosity systems, water-drive reservoirs, and other systems with pressure support at the outer boundary. Rate-cumulative decline type-curves need to be generated for these models to allow the engineer to select the most appropriate solution for their reservoir.

Finally, the effects of non-Darcy flow and pressure dependent permeability need to be investigated for inclusion in rate-cumulative decline type-curve analysis.
REFERENCES


NOMENCLATURE

A  area (sq ft)
°API  liquid gravity, eq. A.23
BHFP  bottom-hole flowing pressure (psi) same as Pwf
B  formation volume factor (rb/STB)
Bo  oil formation volume factor (rb/STB)
Bt  two phase formation volume factor (rb/STB) Table A.2
Bbl  barrel (5.615 ft³)
CA  Dietz shape factor
c  system total compressibility (psi⁻¹)
cg  gas compressibility (psi⁻¹)
D  vertical depth (ft)
d  pipe inside diameter (in)
f  Moody friction factor, eq. A.7
fo  objective function, eq. A.25
fo'  first derivative of the objective function, eq. A.26
F  friction term, eq. A.6
FTP  flowing tubing pressure (psia)
F_n(μ-c)  viscosity-compressibility normalizing factor, eq. 3.8
F_n(m-c)  mobility-compressibility normalizing factor, eq. 4.13
GIP  gas in place (Mcf)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOR</td>
<td>gas oil ratio (SCF/STB)</td>
</tr>
<tr>
<td>h</td>
<td>formation thickness (ft)</td>
</tr>
<tr>
<td>J</td>
<td>single phase liquid productivity index (STB/d-psi) eq. 4.18</td>
</tr>
<tr>
<td>$J_g$</td>
<td>gas productivity index (MCF/d-psi $^2$-cp) eq. B.3</td>
</tr>
<tr>
<td>$J_o$</td>
<td>oil productivity index (STB/d-psi-cp) eq. 4.3</td>
</tr>
<tr>
<td>k</td>
<td>permeability (md)</td>
</tr>
<tr>
<td>$k_g$</td>
<td>permeability to gas (md)</td>
</tr>
<tr>
<td>$k_{oi}$</td>
<td>permeability to oil at irreducible water saturation (md)</td>
</tr>
<tr>
<td>$k_{roi}$</td>
<td>relative permeability to oil at irreducible water saturation (fraction)</td>
</tr>
<tr>
<td>L</td>
<td>length of flow string (ft)</td>
</tr>
<tr>
<td>$N_{re}$</td>
<td>Reynolds number, eq. A.8</td>
</tr>
<tr>
<td>m</td>
<td>weight of flow per STB (lbs/STB) eq. A.19</td>
</tr>
<tr>
<td>OIP</td>
<td>oil in place (STB)</td>
</tr>
<tr>
<td>P</td>
<td>pressure (psia)</td>
</tr>
<tr>
<td>PNR</td>
<td>pressure normalized production rate (STB/d/psi)</td>
</tr>
<tr>
<td>PNC</td>
<td>pressure normalized cumulative production (STB/psi)</td>
</tr>
<tr>
<td>PPNR</td>
<td>pseudopressure normalized rate</td>
</tr>
<tr>
<td>PPNC</td>
<td>pseudopressure normalized cumulative</td>
</tr>
<tr>
<td>$P_{avg}$</td>
<td>average pressure (psia)</td>
</tr>
<tr>
<td>$P_{pavg}$</td>
<td>average pseudopressure ($psi^2/cp$)</td>
</tr>
<tr>
<td>$P_b$</td>
<td>saturation pressure (psia)</td>
</tr>
<tr>
<td>$p_c$</td>
<td>pseudocritical pressure (psia) eq. A.9</td>
</tr>
</tbody>
</table>
\( P_d \) dimensionless pressure, eq. 2.7
\( P_{dd} \) decline-curve dimensionless pressure, eq. 2.23
\( P_p \) gas pseudopressure \((\text{psi}^2/\text{cp})\) eq. 3.1
\( P_{pi} \) oil pseudopressure \((\text{psi/} \text{cp})\) eq. 4.1
\( P_i \) initial pressure \((\text{psia})\)
\( P_{pi} \) initial pseudopressure \((\text{psi}^2/\text{cp})\)
\( P_{inj} \) injection pressure \((\text{psia})\)
\( P_r \) pseudoreduced pressure \((\text{dimensionless})\) Table A.2
\( P_s \) surface injection pressure \((\text{psia})\)
\( P_{wf} \) flowing bottom-hole pressure \((\text{psia})\) same as BHFP
\( P_{pwf} \) flowing bottom-hole pseudopressure \((\text{psi}^2/\text{cp})\)
\( \Delta P_h \) pressure drop due to hydrostatic fluid column \((\text{psi})\)
\( \Delta P_f \) pressure drop due to friction \((\text{psi})\)
\( q \) flow rate \((\text{STB/d})\)
\( q_o \) oil flow rate \((\text{STB/d})\)
\( q_g \) gas flow rate \((\text{MCF/d})\)
\( q_D \) dimensionless flow rate, eqs. 2.1, 3.2, & 4.4
\( q_{dd} \) decline-curve dimensionless flow rate, eqs. 2.16, 4.5, & 4.19
\( Q \) cumulative production \((\text{STB for oil, MCF for gas})\)
\( Q_{n(p-c)} \) viscosity-compressibility normalized cumulative production \((\text{MCF})\) eq. 3.5
\( Q_{n(m-c)} \) mobility-compressibility normalized cumulative production \((\text{STB})\) eq. 4.12
Q_{D} \quad \text{dimensionless cumulative production, eq. 2.6}

Q_{dD} \quad \text{decline-curve dimensionless cumulative production, eqs. 2.17, 3.6, 4.13, and 4.20}

RCDTC \quad \text{rate-cumulative decline type-curve}

RTDTC \quad \text{rate-time decline type-curve}

\bar{r} \quad \text{radius at which pressure is equal to average reservoir pressure (ft)}

r_{w} \quad \text{wellbore radius (ft)}

r_{wa} \quad \text{apparent wellbore radius (ft) eq. 2.4}

r_{e} \quad \text{external radius (ft)}

r_{eD} \quad \text{dimensionless external radius, eq. 2.3}

R_{s} \quad \text{Solution gas oil ratio (SCF/STB)}

s \quad \text{dimensionless skin, eq. 2.13}

S \quad \text{saturation (fraction)}

S_{g} \quad \text{gas saturation (fraction)}

S_{o} \quad \text{oil saturation (fraction)}

S_{w} \quad \text{water saturation (fraction)}

S_{or} \quad \text{residual oil saturation (fraction)}

STB \quad \text{stock tank barrel (5.615 ft^3)}

T \quad \text{reservoir temperature (°R)}

T_f \quad \text{reservoir temperature (°F)}

T_{LM} \quad \text{temperature log mean (°R)}

t \quad \text{time, days}

t_{c} \quad \text{pseudocritical temperature (°R) eq. A.10}
\( t_{\text{n(u-c)}} \) viscosity-compressibility normalized time (days) eq. 3.3

\( t_{\text{n(m-c)}} \) mobility-compressibility normalized time (days) eq. 4.8

\( t_D \) dimensionless time eq. 2.2

\( t_{DA} \) dimensionless time based on drainage area, eq. 2.20

\( t_{\text{dD}} \) decline-curve dimensionless time, eqs. 2.15, 3.4, and 4.11

\( T_r \) pseudoreduced temperature (dimensionless) Table A.2

\( V_m \) volume per STB (lbs/STB)

\( \overline{V_m} \) integrated average volume (lbs/STB) eq. A.20

\( V_p \) pore volume (bbl) eq. 2.31

\( W_f \) Energy loss term (ft) eq. A.15 and A.16

\( z \) gas compressibility factor (dimensionless)

Greek

\( \bar{\alpha} \) integrand of oil pseudopressure, eq. 4.27

\( \alpha \) decline-curve normalizing factor, eq. 2.18

\( \beta \) decline-curve normalizing factor, eq. 2.19

\( \varepsilon \) absolute pipe roughness (in)

\( \gamma \) specific gravity of fluid; referenced to water for liquids, to air for gases

\( \lambda_{\text{fr}} \) total mobility in terms of relative permeability (cp^{-1}) eq. 4.9

\( \lambda_t \) total mobility (md/cp) eq. 4.15

\( \phi \) porosity (fraction)

\( \mu \) fluid viscosity (cp)
Subscripts

f  formation

g  gas

M  match point in type-curve matching

o  oil

w  water
APPENDIX A

CALCULATION OF BOTTOM-HOLE FLOWING PRESSURE (BHFP)

A.1 Single Phase Liquid

Calculation of BHFP from flowing tubing pressure (FTP) is primarily a concern for hydrologist dealing with aquifers. In the petroleum industry flow of single phase water is usually in terms of injection rather than production. Water is injected into oil reservoirs to increase recovery (waterflooding) and frequently salt-water, that was produced with oil and gas, is injected into disposal wells. For these situations bottom-hole injection pressure (\( P_{\text{inj}} \)) rather than BHFP is required. Also, the pressure difference, \( (P_i - P_w) \) used in eqs. 2.1 & 2.6 are replaced with \( (P_{\text{inj}} - P_i) \) when determining reservoir parameters; size, skin, and permeability.

Calculation of BHFP or \( P_{\text{inj}} \) only differ by the sign of the friction pressure drop term, \( \Delta P_f \):

\[
P_{\text{wf}} = FTP + \Delta P_h + \Delta P_f \text{, psia} \quad \text{................................. (A.1)}
\]

and

\[
P_{\text{inj}} = P_s + \Delta P_h - \Delta P_f \text{, psia} \quad \text{................................. (A.2)}
\]

Where \( \Delta P_h \) is the hydrostatic pressure:

\[
\Delta P_h = 0.433 \; D \; \gamma \text{, psi} \quad \text{................................. (A.3)}
\]
For turbulent flow, an absolute pipe roughness, \( e \), of 0.00065 in. and a logarithmic approximation for the friction factor presented by Blasius\(^{28,29}\), the friction pressure drop term can be approximated by:

\[
\Delta P_f = \frac{\gamma^{0.75} q^{1.75} \mu^{0.25}}{854,000 d^{0.75}} D, \text{ psi } \quad \text{(A.4)}
\]

Where \( \gamma \) is the specific gravity of the fluid, \( q \) is the flow rate (STB/d), \( \mu \) is the viscosity (cp), \( d \) is the pipe inside diameter (in), and \( D \) is the vertical depth to the production or injection zone (ft).

### A.2 Single Phase Gas

Sukkar and Cornell\(^{31}\) presented in 1955 a method of calculating BHFP for gas wells derived from basic energy relations in terms of reduced pressure:

\[
\int_{Pr_1}^{Pr_2} \frac{(z/Pr) dPr}{1 + F(z/Pr)^2} = \frac{0.01877}{T_a} \gamma_g L, \quad \text{(A.5)}
\]

Where the friction term, \( F \), found in the denominator is:

\[
F = \frac{f q_g^2 T_{LM}^2}{1500 d^5 p_c^2}, \quad \text{(A.6)}
\]

and the remaining terms are:
\[ \gamma_g, \text{ gas gravity (air = 1)} \]

\[ L, \text{ length of flow string (ft)} \]

\[ T_{LM}, \text{ log mean average temperature (°R)} \]

\[ T_{LM} = \frac{(T_f - T_p)}{\ln(T_f/T_p)} \]

\[ f, \text{ Moody friction factor (Fanning friction factor x 4)} \]

\[ q_g, \text{ gas flow rate (MCF/d)} \]

\[ d, \text{ inside tubing diameter (in)} \]

\[ p_c, \text{ pseudocritical pressure (psia)} \]

\[ Pr_1, \text{ pseudoreduced pressure at the formation = BHFP/p_c} \]

\[ Pr_2, \text{ pseudoreduced pressure at the surface = FTP/p_c} \]

\[ z, \text{ gas compressibility factor} \]

The Moody friction factor can be approximated by a curve fit provided by Jain\(^{32}\):

\[
\frac{1}{\sqrt{f}} = 1.14 - 2 \log \left(\frac{\epsilon}{d} + \frac{21.25}{N_{re}^{0.9}}\right) \tag{A.7}
\]

Where \( \epsilon \) is the absolute roughness factor usually taken to be 0.00065 in. for clean steel pipe and the Reynolds number, \( N_{re} \) is:

\[
N_{re} = \frac{20 q_g \gamma_g}{d \mu} \tag{A.8}
\]
Where \( q_g \) is the gas flow rate (MCF/d) and \( \mu \) is the gas viscosity (cp) taken at average wellbore pressure \((FTP + BHFP)/2\). Eq. A.5 can also be used for static pressures and is useful for calculating initial reservoir pressure when bottom-hole measurements are not made.

To determine BHFP using eqs. A.5-A.8 numerically, approximations of BHFP are made, then integration of eq. A.5 is made using either Simpson's rule or Guass-Legendre. Succeeding approximations can be made using the Secant Method. These techniques for numerical integration and root finding are covered by Chapra and Canale.

This computation scheme requires gas fluid properties to be known as functions of pressure and temperature or pseudoreduced pressure and temperature. Pseudocritical pressures, \( p_c \), and temperatures, \( t_c \), for hydrocarbon gases can be obtained from Sutton:

\[
p_c = 756.8 - 131.0 \gamma_g - 3.6 \gamma_g^2
\]

and

\[
t_c = 169.2 + 349.5 \gamma_g - 74.0 \gamma_g^2
\]

Gas compressibility and gas compressibility factor can be determined from the pseudoreduced pressure and temperature by the Benedict-Webb-Rubin (BWR) equation of state presented by Dranchuk et al. Gas viscosity can be calculated from gas density by Lee et al. The strategy is to program subroutines...
for gas pressure-volume-temperature (PVT) properties and gas viscosity that can be called when calculating BHFP or when calculating gas pseudopressures and viscosity compressibility normalizing factor. The numerical integration technique of Guass-Legendre has been successfully employed for gas pseudopressures (eq. 3.1), viscosity compressibility normalization factor (eq. 3.8 with eq. 3.5), and BHFP (eq. A.5) using the six (6) point formula:

\[ \int_{a}^{b} f(x) \, dx = \sum_{i=1}^{6} c_i f(x_i) a_i \] ........................................ (A.11)

where \( c_i \) are the weighting factors and \( x_i \) is the argument:

\[ x_i = a_0 + a_1 x_{d_i} \] ......................................................... (A.12)

Table A.1 presents the weight factors and normalized arguments \( x_{d_i} \), used in eq. A.11. The argument, \( x_i \) is determined from the normalized argument (eq. A.12) by the interval of integration using the average value of the interval, \( a_o \):

\[ a_o = \frac{b + a}{2} \] ............................................................... (A.13)

and one half the width, \( a_1 \):

\[ a_1 = \frac{b - a}{2} \] ............................................................... (A.14)
Table A.1  Weighting factors and normalized arguments used in Guass-Legendre formulas (after Chapra and Canale\textsuperscript{23})

<table>
<thead>
<tr>
<th>i</th>
<th>Weighting factor, ( c_i )</th>
<th>Normalized argument, ( x_{d_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.171324492</td>
<td>-.932469514</td>
</tr>
<tr>
<td>2</td>
<td>.360761573</td>
<td>-.661209386</td>
</tr>
<tr>
<td>3</td>
<td>.467913935</td>
<td>-.238619186</td>
</tr>
<tr>
<td>4</td>
<td>.467913935</td>
<td>.238619186</td>
</tr>
<tr>
<td>5</td>
<td>.360761573</td>
<td>.661209386</td>
</tr>
<tr>
<td>6</td>
<td>.171324492</td>
<td>.932469514</td>
</tr>
</tbody>
</table>

A.3  Multiphase Gas and Oil

Friction factor's for multiphase flow comes from a correlation of field data with the numerator of the Reynolds number developed by Poettmann and Carpenter\textsuperscript{37}. Energy losses then calculated using the Fanning friction equation:

\[
W_f = \frac{f q_o^2 V_m^2 D}{2.8510^6 d^5} \quad \text{................................................. (A.15)}
\]

must equal energy losses resulting from total energy balance:

\[
W_f = \frac{1}{m} \int_{p_2}^{p_1} V_m dp - D \quad \text{................................................. (A.16)}
\]

Where \( P_1 \) is the BHFP and \( P_2 \) is the FTP. The friction factor correlation as well as well as eqs. A.15 and A.16 are presented by Craft \textit{et al.}\textsuperscript{23}. A polynomial fit of
the friction factor correlation of the form:

\[ \log f = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (A.17) \]

with \( x \) being the logarithm of the Reynolds number numerator and the coefficients are:

\[ a_0 = 1.6983 \]
\[ a_1 = -3.7017 \]
\[ a_2 = 0.96245 \]
\[ a_3 = -0.11502 \]

In field units the Reynolds number numerator, \( d v p \) is:

\[ d v p = 1.7684 \times 10^4 \frac{q_o m}{d} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (A.18) \]

where \( q_o \) is the production rate (STB/d), \( d \) is the inside tubing diameter (in), and \( m \) is the weight flowing per stock tank barrel (lbs/STB):

\[ m = 350.17 \gamma_o + GOR \rho_a \gamma_g \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (A.19) \]

where \( \gamma_o \) is the specific gravity of the oil with respect to water, \( \gamma_g \) is the specific gravity of the gas with respect to air, \( \rho_a \) is the density of air (=0.0764 lb/ft\(^3\)), and GOR is the producing gas oil ratio (SCF/STB). Other terms used in eqs. A.15 and A.16 are the volume per STB, \( V_m \) (cf/STB) which is a function of location in
the wellbore, the integrated average volume per STB, \( \bar{V}_m \) (cf/STB) over the pressure interval from FTP to BHFP:

\[
\bar{V}_m = \frac{\int_{P_2}^{P_1} V_m \, dp}{(P_1 - P_2)}
\]

(A.20)

BHFP using these equations requires successive approximations until energy losses calculated by eq. A.15 and eq. A.16 are the same. The fluid volume as a function of pressure over a continuous pressure interval can be calculated by using Standing\(^{39}\) fluid property correlations and gas compressibility factors for the determination of the two-phase volume factor, \( B_t \) (rb/STB). The two-phase volume factor is related to the volume per STB by a constant:

\[
\int_{P_2}^{P_1} V_m \, dp = 808.56 \int_{P_2}^{P_1} B_t \, dp
\]

(A.21)

Table A.2 presents pseudo-code for calculating the two-phase volume factor as a function of pressure. All necessary equations are contained in this appendix with the exception of gas compressibility factors. Reference 35 provides this calculation procedure.
Table A.2 Pseudo-code for determining Two-phase volume factors

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure, ( P ) (psia)</td>
<td>Gas compressibility factor, ( z ) (ref. 35)</td>
</tr>
<tr>
<td>GOR (SCF/STB)</td>
<td>gas formation volume factor, ( B_g )</td>
</tr>
<tr>
<td>Oil Gravity (°API)</td>
<td>( B_g = 0.03197P/(zT) )</td>
</tr>
<tr>
<td>Gas Gravity (air=1)</td>
<td>solution gas oil ratio, ( R_s ) (eq. A.22)</td>
</tr>
<tr>
<td>Reservoir Temperature, ( T ) (°R)</td>
<td>single phase formation volume factor, ( B_o ) (eq. A.24)</td>
</tr>
<tr>
<td>pseudocritical temperature and pressure (eqs. A.9 and A.10)</td>
<td>two phase volume factor, ( B_i )</td>
</tr>
<tr>
<td>pseudoreduced temperature and pressure ( T_r, P_r )</td>
<td>( B_i = B_o + B_g(GOR-R_s)/5.615 )</td>
</tr>
</tbody>
</table>

Using a rearrangement of Standing's\textsuperscript{39} correlation for saturation pressure to solve for the solution gas oil ratio:

\[
R_s = \gamma_g \left( \left( \frac{P}{18.2} + 1.4 \right)^{10^{0.0125API-0.009T}^{1205}} \right) \]

(A.22)

where \( T_f \) here is for reservoir temperature (°F). Oil specific gravity and °API are related by:

\[
\gamma_o = \frac{141.5}{131.5 + \text{°API}} \]

(A.23)
and the Standing\textsuperscript{39} correlation for single phase formation volume factor is:

\[
B_0 = 0.9759 + 12 \times 10^{-5} \left( R_s \left( \frac{\gamma_r}{\gamma_o} \right)^5 + 1.25 f \right)^{12}
\]  \hspace{1cm} (A.24)

Once again, Gauss-Legendre can be used for the numerical integration in eq. A.16, and the secant method can be used to provide a successive approximations until convergence is obtained. Two initial approximations are required to use the secant method and then a new approximation can be made from the last two and their objective function. The objective function here is the difference between the energy losses calculated in eq. A.15 with the energy losses calculated for eq. A.16:

\[
f_0 = W_f (eq. \ A.15) - W_f (eq. \ A.16)
\]  \hspace{1cm} (A.25)

The first derivative of the objective function is approximated by the finite divided difference:

\[
f_0' = \frac{f_0(BHFP_{i-1}) - f_0(BHFP_i)}{BHFP_{i-1} - BHFP_i}
\]  \hspace{1cm} (A.26)

And the successive approximation for BHFP (i.e. \(x_{i+1}\)) can be made by using the secant formula:
Additional approximations of BHFP are made until a convergence criteria is met. The initial two guesses can be made by using a liquid gradient (.3 psi/ft) and a gas gradient (.1 psi/ft) times the depth if the well (ft).
We can examine the linear relationship for rate and cumulative by looking at the curves of Fig. 2.5 in cartesian coordinates as shown in Fig. B.1.

The straight line is the boundary-dominated relation of eq. 2.21 extrapolated to an ordinate intercept of unity, the curve is the infinite acting portion for a dimensionless external radius of 100.
The actual flow rate associated with the intercept, $q_i$, can be used to non-dimensionalize rate in an alternative matter:

$$q_{AD} = \frac{q}{q_i} \quad \ldots \quad (B.1)$$

To derive this linear relationship during boundary dominated flow for gas wells using viscosity compressibility normalized cumulative, begin with the pseudosteady-state flow equation (Al-Hussainy et al.):

$$q_g = J_g (P_{avg} - P_{pwt}) \quad \ldots \quad (B.2)$$

Where the gas productivity index, $J_g$, is:

$$J_g = \frac{k_i h}{1422TB} \quad \ldots \quad (B.3)$$

The intercept flow rate used in eq. B.1 can be obtained from the eq. B.2:

$$q_{gi} = J_g (P_{pi} - P_{pwr}) \quad \ldots \quad (B.4)$$

Differentiating eq. B.2 to express change of rate with depletion:

$$dq = J_g dP_{avg} \quad \ldots \quad (B.5)$$
An alternative expression for real gas pseudopressure (eq. 3.1) presented by Fraim and Wattenbarger \(^{19}\):

\[
P_{\text{pavg}} = 2 \int_0^{P_{\text{avg}}} \frac{1}{\mu c_g} d(p/z)_{\text{avg}} \quad \cdots \quad \text{(B.6)}
\]

Taking the derivative of this expression yields:

\[
dP_{\text{pavg}} = 2 \frac{1}{\mu c_g} d(p/z)_{\text{avg}} \quad \cdots \quad \text{(B.7)}
\]

Substituting this expression into eq. B.5 yields:

\[
dq = \frac{2J_g}{\mu c_g} d(p/z)_{\text{avg}} \quad \cdots \quad \text{(B.8)}
\]

Now using the gas material balance:

\[
\left( \frac{p}{z} \right)_{\text{avg}} = \left( \frac{p}{z}_i \right) \left( 1 - \frac{Q}{GIP} \right) \quad \cdots \quad \text{(B.9)}
\]

and the derivative with respect to cumulative gas produced:

\[
d(p/z)_{\text{avg}} = - \frac{(p/z)_i}{GIP} dQ \quad \cdots \quad \text{(B.10)}
\]
Substituting this into eq. B.7 yields

\[ dq = \frac{-2 J_g (p/z)_i}{\mu c_g GIP} dQ \] ........................................ (B.11)

Integration of this expression using the extrapolated initial rate, \( q_{gi} \), to any rate later in time:

\[ \int_{q_{gi}}^{q} dq = \frac{-2 J_g (p/z)_i}{(\mu c_g)_i GIP} \int_{0}^{Q} (\mu c_g) dQ \] ........................................ (B.12)

Performing the integration and substituting the definition of viscosity compressibility normalized cumulative:

\[ q_{gi} - q_g = \frac{-2 J_g (p/z)_i}{(\mu c_g)_i GIP} Q_{n(\mu c)} \] ........................................ (B.13)

This results in a linear relationship between rate and viscosity compressibility normalized cumulative for gas wells flowing at constant BHFP.

Dividing both sides by the extrapolated initial rate and using eq. B.4:

\[ q_{AD} = 1 - \frac{2 (p/z)_i Q_{n(\mu c)}}{(\mu c_g)_i GIP(P_{pi} - P_{pwf})} \] ........................................ (B.14)
The expression for GIP is:

\[
GIP = \frac{S_g \phi h r_g^2 (p/z)_i}{9.00T}, \text{ Mcf} \qquad \text{(B.15)}
\]

Substituting:

\[
q_{4D} = 1 - \frac{9.00 Q_{n(\mu c)} T}{\phi h (\mu c)_i (P_{pi} - P_{pw}) \alpha} \qquad \text{(B.16)}
\]

This becomes the linear rate cumulative relation of eq. 2.21 with dimensionless decline-curve cumulative for gas wells becoming:

\[
Q_{4D} = \frac{9.00 Q_{n(\mu c)} T}{\phi h (\mu c)_i r_{ws}^2 (P_{pi} - P_{pw}) \alpha} \qquad \text{(B.17)}
\]

which is eq. 3.6 of the main text with viscosity compressibility normalized cumulative, \(Q_n\) in Mcf.

Rock and water compressibility have been ignored and total system compressibility has been approximated by the gas saturation gas compressibility product:

\[
\zeta_1 = S_g \zeta_g \qquad \text{(B.18)}
\]
VITA

Jeffrey Guy Callard was born in Lansing, Michigan on December 11, 1952. He is a child of Robert D. and Joanne C. Callard of Telluride and Durango, Colorado. Upon graduation from East Lansing High School in June of 1971, he enrolled at the University of Oklahoma where he graduated with a Bachelor of Science Degree in Petroleum Engineering with Distinction. While at the University of Oklahoma he was a member of the Society of Petroleum Engineers and the Pi Epsilon Tau and Tau Beta Pi Honor Societies. In the fall of 1976, Jeffrey Callard entered Leeland Stanford Jr. University where he graduated in June 1977 with a Master of Science degree in Petroleum Engineering. Upon graduation he worked in the petroleum industry for a major oil company, a bank specializing in oil and gas loans, an independent oil company, as a petroleum engineer for the United States Bureau of Land Management, and as visiting associate professor prior to enrolling at Louisiana State University in the fall of 1991.
DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: Jeffrey G. Callard

Major Field: Petroleum Engineering

Title of Dissertation: Reservoir Performance History Matching Using Type-Curves

Approved:

[Signature]

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

November 1, 1994