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Optimal Preventive Maintenance Strategies for a Production System Subject to Random Shocks.

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OPTIMAL PREVENTIVE MAINTENANCE STRATEGIES FOR A PRODUCTION SYSTEM SUBJECT TO RANDOM SHOCKS

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ABSTRACT

The failure rate of production equipment is usually increasing as wear accumulates with usage. An improved preventive maintenance approach is accomplished through predictive maintenance, where an indicator of wear, like vibration or heat, is measured and used to determine the optimal time of an adjustment (or maintenance) such as realignment, oil change or replacing seals. The state of the unit is restored into the same original "as new" state both after adjustment or failure (followed by a repair).

This research will develop mathematical models for a single and two unit production system. In the development of the one unit model, a cost criterion and indicator variable will be used for deciding when adjustment should take place. The cost to be minimized is the long-run average cost of adjustments and failures. An optimal solution to this problem will be obtained via dynamic programming and compared to an approximate steady state solution based on renewal theory. This approximation (like other earlier works) disregards the fact that after failure (that has a small probability) the unit is restored to its original state. Both models provide an upper control limit (UCL) on the indicator variable which triggers an adjustment when exceeded. It will be shown that disregarding the restoration after failure in the cost approximation causes the UCL to be underestimated. The resulting cost penalty is considerable in most cases.

For the two unit system, an optimizing mathematical model will be developed by monitoring an external variable for each unit and using this information collectively as a predictor for failure. The cost to be minimized is the long-run average cost of adjustments, system overhaul, and failures.

In the general case, the system with n units, it is shown why the current approach becomes more intractable as n increases. An alternate methodology is suggested.

Finally, using a simplified decision policy, a simulation model is offered as a safeguard that the mathematical model is realistic. Sensitivity and factor analysis results are also provided for both the single and two unit systems.

CHAPTER 1. INTRODUCTION

1.1 Foreword

Many manufacturing processes exhibit an increasing wear of equipment during the course of production. As an example, consider a chemical plant which has several production units, which will be called systems. Each system has many pieces of rotating equipment such as motors, pumps, gear boxes and valves, hereafter denoted as units. These units exhibit performance deterioration of various types such as bearing wear, component misalignment, or complete failure caused by excessive temperature or vibration. If one of the units breaks, it may be entirely lost; but at the very least, some major repair of the unit will be required. For that reason, most chemical plants use preventive and predictive maintenance to anticipate failures and provide for service before a malfunction occurs.

1.2 Maintenance Definitions

Mann [1983] defines maintenance as "the activities required to keep a facility in as-built condition, continuing to have its original productive capacity". He uses time to further categorize maintenance on the basis of when the work must be done:

1. Emergency maintenance must be done immediately.
2. Routine maintenance must be done in the foreseeable future.

3. Preventive maintenance is performed according to a planned schedule.

Today, preventive maintenance is being augmented by predictive maintenance. Predictive maintenance anticipates component failure rates using nonintrusive diagnostic methods that identify signs of deterioration while the machinery is operating. Service can then be provided before a malfunction occurs.

1.3 Predictive Maintenance Methods

Predictive maintenance many employ one or more of the following methods to monitor units:

(1) Vibration analysis can be used with all rotating equipment and is a quick and relatively inexpensive way to discover minor mechanical problems before they escalate into costly unscheduled plant shutdowns. Mann [1983] describes vibration as "mechanical motion or oscillation about a reference point of equilibrium". During normal operation, a piece of properly functioning rotating equipment produces a specific vibration signal, or "signature." If the signature changes, something is wrong. The adverse effects of excessive vibration include destruction of small and large structures near the source, loss of balance, blurred vision, fatigue, or

permanent hearing loss to those workers exposed to the vibration [Mann, 1983].

(2) Infrared thermography detects and records heat loss and heat-radiation of on-line equipment. This method is extremely accurate and additionally allows an assessment of problem severity directly from the increase in temperature [Mann, 1983].

(3) Ultrasonic pulse-echo techniques allow fast and accurate wall-thickness determinations that are not affected by coke, scale, or liquid in the system. It requires only one exposed surface (insulation must be removed wherever measurements are made) and is also useful for flaw and weld inspection [Mann, 1983]. If a unit shows an increase of the monitored values, a decision may be made to repair it before failure occurs.

(4) Tribology is used to detect contaminants in oil. Specifically, spectroscopy determines the contaminants in an oil and its physical condition, whereas ferrography locates the large ferrous particle content in oil [Petersen, 1990].

(5) Motor current analysis is a relatively new technique used to determine the electro-mechanical condition of a motor. Clamp-on current sensors are used to collect motor current spectrums which

are subsequently scanned and analyzed by computer [Petersen, 1990].

1.3 Predictive Maintenance Advantages

This predictive maintenance strategy has several advantages as reported by users [Petersen, 1990]:

(1) Maintenance expenditures are cut by predicting the need for repairs thereby reducing catastrophic failure. In addition, the cost of spare parts inventory is cut by permitting lower inventory levels.

(2) Product quality increases because preventive maintenance focuses on reducing the vibration of a machine, which is the culprit of distortion, contamination, and out-of-tolerance product quality. The ultimate result is higher customer satisfaction.

(3) Operator safety increases because catastrophic failure is forestalled and total routine maintenance is reduced, thereby lessening worker exposure to potential danger.

(4) Production uptime is increased by eliminating unforeseen downtime due to machine failure as well as reducing the need for scheduled downtime for routine preventive servicing.

1.5 Need for Maintenance Models

As a system gets older, more and more units may exhibit an increase of the monitored variables, and thus more and more repairs may have to be done.

An alternate strategy to repairing individual units is to overhaul the entire system. This, however, results in a loss of production for some time. The decision problem here is then when to repair and when to overhaul the system.

In practice, this is done several ways. Sometimes management decides on overhaul intervals which are convenient (time of low production); sometimes the systems are monitored by a control chart showing the number or percentage of units above a warning limit. When this percentage shows values outside the three sigma control limit, a signal is given for overhauling the system. This is called a control limit policy: one where a replacement of the machine or component is made whenever the cumulative damage is greater than or equal to a predetermined (usually upper) limit, or when a failure occurs, whichever comes first. Otherwise, no corrective action is taken.

Clearly, the need exists for maintenance models of this type since Petersen [1990] relates that after implementing predictive maintenance systems, companies report the ratio of total savings to expenditures for equipment, training, and manpower ranges from 6:1 to 10:1. The level of savings is dependent on how aggressively the plant pursues its predictive maintenance program. Moreover, he reports that only 35% of American industry is performing predictive maintenance at a level that contributes to net profit.

1.6 Objectives of the Study

The objectives for this dissertation are:

(1) Develop a mathematical model for a single unit subject to random shock by monitoring a single external variable and using it as a predictor for failure.

(2) Progress to a model for a system of two units.

1.7 Justification of the Research

Predictive maintenance, used as an enhancement of preventive maintenance, can contribute to net profit at a minimum rate of six dollars of savings to one dollar of expenditure, but is currently being used by less than half of American industries. Therefore, the economic incentive for research in this area is substantial. Moreover, of all the studies reviewed in the next chapter, none of them monitors a random variable representing an external variable and uses it as a predictor for failure as our model will. Additionally, the great majority of models in the literature are for a single unit in a system; and even when the focus of study is for a system, in most cases, the methodology is to view the system as a single unit. This work's expected contribution to the body of knowledge in preventive maintenance is the model of a true system consisting of multiple units that are subject to random shocks originating from an external variable.

1.8 Organization of the Dissertation

This dissertation is organized into ten chapters. Chapters 1 and 2 contain the introduction and review of relevant research, respectively. Chapter 3 provides motivation for the research. The dynamic programming and steady state approximation model for a single unit are discussed in Chapter 4, while Chapter 5 displays and examines the numerical analysis of the single unit model. Chapter 6 explains the two unit system and its mathematical solution. Chapter 7 has the two unit numerical analysis. Chapter 8 shows a simulation cost solution to the two unit system, while Chapter 9 introduces the multiple unit model and outlines its limitations. Finally, Chapter 10 cites conclusions and offers model extensions.

CHAPTER 2. LITERATURE REVIEW

2.1 Introduction

The search for optimal maintenance policies originated with the work of Barlow and Proschan [1960] and has been the topic of an enormous amount of research for at least three decades. A perusal of the surveys written on this subject [McCall, 1965; Pierskalla and Voelker 1976; Sherif and Smith 1981; Valdez-Flores and Feldman 1989] clearly discloses the variety and quantity of models and solution techniques. Most of the works discussed here are those published since 1975.

2.2 General Structure of Maintenance Models

The general structure of maintenance problems ensues [Mc Call, 1965]. The equipment under consideration is assumed to occupy one of several states while in operation: the "new" state is at one extreme of the sequence of states occupied and the "failed" state at the other. The intermediate states represent different degrees of deterioration. A probability rule governs the movement from state to state. This probability law may be known, partially known, or completely unknown by the decision maker. When equipment is unattended, it moves stochastically from state to state until it reaches the absorbing state of failure. However, by choosing a particular action at each decision point, the performance of the equipment can be regulated. Some of the possible choices are:

do nothing, inspect (if the equipment is not continuously observed), repair, replace or completely overhaul (which has the effect of "renewing" the equipment). If the equipment consists of more than one part, a specific action must be chosen for each part, and the action chosen for one part may be dependent on the state of one or more of the remaining parts. A maintenance policy is defined by the sequence of actions chosen by the decision-maker. The policy's influence is measured by the difference between the equipment's performance under the maintenance policy and its unregulated action. The policy's performance can be measured economically by assigning a cost to the occupation of each state and one to each intervening action. A policy is optimal when it yields the lowest expected cost per unit time.

2.3 Classification of Maintenance Models

Maintenance models can be classified by various characteristics, and each of the surveys previously mentioned focuses on different aspects of the models to create mutually exclusive groupings. The categories used here will follow those of Valdez-Flores and Feldman [1989] who employ the following set to distinguish among the research topics: inspection models, minimal repair models, shock models, and miscellaneous replacement models. Each of these are discussed below.

2.3.1 Inspection Models

An inspection maintenance system is characterized by the following assumptions:

(1) The state of the system is completely unknown unless an inspection occurs. This inspection is presumed perfect in that it is assumed to reveal the true state of the system without error.

(2) Without repair or replacement action, the system evolves as a non-decreasing stochastic process.

(3) The decision space of the maintenance inspection problem is two dimensional because decisions about maintenance action and inspection timing must be made at every decision event. The maintenance decision is whether the system should be replaced or repaired to a particular state or left as it is, and the inspection decision is when the next inspection should occur.

These models are used when it is impossible to continuously observe the physical condition of a system, but its true status can be obtained from an inspection before beginning corrective action. The challenge in finding an optimal inspection schedule for aging or deteriorating systems of this type is to balance the losses resulting from down time with the number of inspections whose cost only increases the system operating cost.

2.3.2 Minimal Repair Models

A minimal repair is said to occur when the repair or replacement of a failed component restores the system to operation but the failure rate of the system remains unchanged. This concept is used when modeling complex systems consisting of several components that are regarded as single units for maintenance purposes. For example, changing a burst water hose on a car leaves the overall failure rate of the car basically unchanged.

Minimal repair maintenance models generally include the following assumptions:

(1) The system's failure rate function is increasing. The term "failure rate function" is better described as the "rate of occurrence of failure": the probability that a failure, not necessarily the first, occurs in any very small interval [Ascher and Feingold, 1984].

(2) Minimal repairs do not affect the failure rate of the system.

(3) The cost of a minimal repair is less than the cost of replacing the entire system.

(4) System failures are immediately detected.

2.3.3 Shock Models

Shock models are used with systems that suffer a random amount of damage from randomly occurring shocks. The assumptions used to portray the general problem setting are:

(1) The damage accumulates additively until a replacement is made or failure occurs. Replacements take a negligible amount of time.

(2) The time between shocks and the resulting damage are random variables whose distribution functions may depend on the accumulated damage at time t.

(3) When failure occurs, the system is replaced with an identical new one at a cost which is a function of the state of the system at the time of failure. However, replacing the system before failure occurs is less costly and is a function of the damage level at the time of failure. That is, the replacement cost function is a non-decreasing function of the accumulated damage.

The optimal policy in most shock models takes the form of a control limit policy.

2.3.4 Miscellaneous Replacement Models

Valdez-Flores and Feldman [1989] group all other replacement models that do not fit into the previous sections into this category. Most of these models view a repair equivalent to a replacement, except that a repair to a failed system is more expensive than a replacement to one that is operating. Most of them also assume that the deterioration of the system can be perfectly observed.

The model developed in this work uses a control limit and belongs to the shock model category. The remainder of this chapter will review the literature

available on traditional shock models of the control limit type, and will further categorize them into four classes: those which conform to the general problem setting previously described, cases where system deterioration occurs discretely and continuously, those where cumulative damage is allowed to decrease between shocks, models where several decisions are possible at every damage level, and decreasing deterioration studies.

2.4 Review of Literature

2.4.1 Models Conforming to the General Problem Setting

Taylor's paper [1975] is the first of the shock model studies and has been the subject of several subsequent generalizations. The primary problem in this simplification of the general setting is to find an optimal control strategy to minimize the total long run average cost per unit time of a single machine or production system that is subject to random failure. The failures are assumed to be a probabilistic increasing function of cumulative shock damage sustained by the system. They occur according to a Poisson process, and the magnitudes of shocks are modeled as independent, identically distributed positive random variables with known distribution functions. This means that the magnitude of failures can be represented as a compound Poisson process.

There are two costs involved in the analysis, each independent of the damage level at the time of replacement: one that corresponds to each replacement and a higher one that is incurred when the failure happens during

operation of the system. The purpose of this approach is to afford an incentive for replacing the machine before failure occurs. The optimal strategy, then, will balance the cost of replacement with the cost of failure.

The solution procedure represents the cumulative damage attributed to shocks occurring from time 0 to time t as a terminating Markov process in order to find the optimal time to repair the machine. This optimal Markov time is shown to be determined by a single critical control level of accumulated damage. The optimal policy that Taylor derives is to either replace when failure occurs or when the accumulated damage first exceeds the critical control level.

Other extensions explored are system failure when cumulative damage first exceeds a threshold and a generalization of the basic model to include income lost during repair time.

Nakagawa [1976] studies a variation of Taylor's first model by allowing the time between shocks and the magnitude of the shock to be two independent and identically distributed random variables. He finds the optimal control limit policy for the long-run expected cost per unit time.

Feldman [1977] generalizes Nakagawa's and Taylor's models by allowing the times between shocks to be arbitrarily distributed and dependent on the accumulated damage. He finds the optimal replacement rule for a system where the cumulative damage is a nondecreasing semi-Markov process. The optimal

policy is found among the set of control limit policies that replace at shock times and is based on the long run expected cost per unit time.

Feldman's 1975 paper studies the same problem, except that the cost criteria is to minimize the discounted cost of replacement instead of the expected long run cost per unit time, and the solution is not restricted to control limit policies. Feldman shows here that the optimal policy among those that replace at shock times, is a control limit policy under a specific set of conditions.

His 1976 paper explores the same semi-Markov replacement problem as the first one described, with the exception that the replacement cost due to failure is not a function of the cumulative damage at replacement time, but rather an additional fixed cost is added to the cost of replacement when it occurs before failure.

Replacements and failures are allowed only at shock times in Feldman's models, but Aven and Gaarder [1987] allow the system to fail at any time, conditional upon the probabilistic process history. They show that when the conditional failure rate of the system is nondecreasing, the minimal long run cost per unit time is given by a control limit policy. No algorithm is offered.

Siedersleben [1981] also generalizes Feldman's 1977 paper. He seeks an optimal replacement rule for a system that continuously deteriorates according to a Markov renewal process but is inspected only at random times, so that the state is observed only at random times. This formulation can be regarded as a shock

model where the cumulative deterioration between two successive time intervals t-1 and t, is a quantity X_{t-1}, and the magnitude of the shock at time t is the accumulated damage at time t minus X_{t-1}. This deterioration process is assumed to form a Markov renewal process, and the cost of inspection is regarded as negligible. He examines two cost criteria: to minimize the total costs of the first N replacements of the system where $1 \leq N < \infty$, and to minimize the total discounted costs, assuming an infinite horizon for the process.

Zuckerman [1977] generalizes Taylor's income model by eliminating the restriction that the amount of damage caused by each shock is an exponential random variable. Additionally, the replacement cost before failure is allowed to be a nondecreasing function of the accumulated damage with the replacement cost of a failed system as an upper bound. Conditions are derived under which the optimal policy is a control limit rule for both the maximum long-run expected net income per unit time and the maximum total expected discounted net income. In his 1978 work, Zuckerman extends Feldman [1976] by allowing a replacement at any time before failure instead of allowing one only at shock times.

Taylor's cost model is also generalized by Abdel-Hameed and Shimi [1978] who allow the replacement cost before failure to be a nondecreasing convex function of the cumulative damage. Damage caused by shocks are independent, identically distributed random variables. The optimal policy is shown to be a control limit rule when replacements are allowed only at shock

times. This work is then analyzed by Zuckerman [1980] who proves that the optimal policy given by Abdel-Hameed and Shimi and previously by Taylor [1975], does replace at shock times so that the restriction to replace only at shock times can be dropped from the two earlier models. Zuckerman's proof is limited to the case where the time between shocks is exponentially distributed.

In a later study, Abdel-Hameed [1984] investigates a system subject to shocks where the system is assumed to have failed once the cumulative damage exceeds a given threshold. The replacement cost before failure is a nondecreasing function of the accumulated damage and is bounded by the replacement cost at failure. It is shown that a control-limit policy is optimal for the long-run expected cost per unit time, if some cost function conditions are satisfied. In addition, the failure distribution of the system is shown to have an increasing failure rate.

Bergman [1978] offers a general optimal replacement model when the policy is based on measurement of an increasing state variable, such as the cumulative damage caused by shocks. The only assumption made about the damage process is that it is nondecreasing. Before failure, the replacement cost is fixed and lower than the replacement cost at failure. Replacements may be made any time before failure. The policy that minimizes the long-run expected cost per unit time is shown to be a control limit policy under certain conditions.

Bergman's same general model is studied by Nummelin [1980] with the exception that replacement costs before failure and at failure are modeled as random variables dependent on the history of the system up to that particular time. He also shows that the optimal rule is a control-limit policy. Aven [1987] presents a general setup for replacement models using a counting process approach.

Gottlieb [1982] investigates a system subject to shocks that occur according to a semi-Markov process where the time between shocks are random variables dependent on the level of deterioration. The system can be replaced any time before failure at a constant cost, and after failure at a higher constant cost. He deviates from earlier models by assuming the failure rate need not be increasing. Weaker conditions are shown to be sufficient for the optimal replacement policy to be of the control limit type based on the long-run expected cost per unit time. His optimal policy is a state-age-dependent rule that replaces as soon as the time since the last shock reaches a level that is a function of the accumulated damage. Valdez-Flores and Feldman [1989] define a state-age-dependent policy as " a function ρ where replacement is made whenever the sojourn time in a state x reaches $\rho(x)$". This research spawned three more studies which follow.

Feldman and Joo [1985] examine Gottlieb's problem where the time between shocks are independent and identically distributed random variables with

an increasing failure-rate distribution function. The random amount of damage caused by each shock is assumed to be dependent on the cumulative damage. They also determine the optimal state-age-dependent policy to minimize the long-run expected cost per unit time.

Mizuno [1986] transforms Gottlieb's problem into a generalized mathematical programming problem that can be reduced to a linear program if the state and action spaces are finite. His main contribution is to prove the optimality of the control limit policy under weaker sufficient conditions.

Posner and Zuckerman [1986] take Gottlieb's problem and present the same results under weaker sufficient conditions for both the long-run expected cost per unit time and the expected discounted cost. They also prove that, when specific conditions are satisfied, for the cases in which the system can be replaced at shock times only and also when it can be replaced at any time before failure, the optimal policy is to replace at shock times.

2.4.2 Concurrent Discrete and Continuous Deterioration

Some shock models consider the case where system deterioration occurs continuously as well as at discrete points of time when shocks occur. Feldman [1977] generalizes his previous works by assuming that the system is subject to intervals of continuous deterioration and additionally may fail at any time within the set of deterioration periods. A semi-Markov process is used to model cumulative damage for every deterioration period. Here, Feldman proves that,

from the policies that replace only within the sets of deterioration times, the one that minimizes the long-run expected cost per unit time is a control limit policy.

Zuckerman [1978] portrays a continuous wear process which ends in failure by allowing a system to suffer an infinite number of shocks in a finite period of time. Replacement can occur at any stopping time before failure at a fixed cost, or when failure occurs at a higher fixed cost. Again a control limit policy is the one that minimizes the long-run expected cost per unit time.

A model of this type was most recently produced by Hordijk and Van der Duyn Schouten [1983] who allowed the system to suffer continuous deterioration between shocks. They assume the system can be replaced at any time before failure and show the optimal policy to be a control limit type.

2.4.3 Cumulative Damage Decreases Between Shocks

All models discussed here-to-fore assumed that damage to the system was nondecreasing. However, Gottlieb and Levikson [1984] investigate the case where damage is partially repaired between shocks and decreases according to a Markov process. The cost of replacement before and after failure increases and is fixed. They assume that the failure rate is not necessarily increasing with cumulative damage. Their results show that (1) a control limit policy minimizes long run expected cost per unit time under certain conditions; (2) the shock rate increases with cumulative damage and decreases with the time since the last shock; (3) moreover, if the shock rate increases with both the cumulative damage

and the time since the last shock, the optimal policy is to replace as soon as the time since the last jump equals or exceeds some level. This makes the optimal policy a decreasing function of the cumulative damage.

2.4.4 Several Decisions at Every Damage Level

Only two possible maintenance actions, replace or not replace, have been allowed in models cited up to now. More general models that allow one of several maintenance decisions at every damage level are reviewed in this section. Chikte and Deshmukh [1981] and Anderson [1981] examine a shock system that can be controlled by continuous preventive maintenance expenditures. Higher maintenance expenditures are assumed to more effectively alleviate deterioration thereby decreasing the frequency and magnitude of shocks. They search for an optimal policy that defines replacement and maintenance expenditure schedules to maximize the expected discounted net profit. Results are that the maintenance expenditure rate should be reduced as the deterioration level approaches the control limit.

Zuckerman [1986] uses a diffusion process to model the damage process in an analogous problem. He allows M possible maintenance actions at every deterioration level. Deterioration is caused by Poisson shocks of independent and identically distributed magnitudes. He shows that the optimal maintenance expenditure rate should be increased as the cumulative damage increases.

2.4.5 Research Using Decreasing Deterioration

When a damaged system is repaired to a lesser damage level, the cumulative deterioration may decrease from one period to the next. Valdez-Flores [1987] considers such a system and allows the cost of repair to be dependent on the deterioration of the system and the extent of the repair. He uses a Markov renewal process to find sufficient conditions to minimize the long run expected cost per unit time.

CHAPTER 3. MOTIVATION FOR THE RESEARCH

3.1 General Description of a Chemical System

Stephanopoulos [1984] describes a chemical plant as "an arrangement of processing units (reactors, heat exchangers, pumps, distillation columns, absorbers, evaporators, tanks, etc.), integrated with one another in a systematic and rational manner. The plant's overall objective is to convert certain raw materials (input feedstock) into desired products using available sources of energy, in the most economical way."

The processing units in a chemical plant are characterized by Cook and Cullen [1979] as follows: a reactor is any vessel used to carry out the chemical reactions of the process, whereas a heat exchanger is equipment used to transfer heat between liquids or between liquids and gasses. Pumps are the most efficient and widely used means of transferring liquids throughout the system including to tank farms where they are stored in bulk. The authors cite the proper selection, use, and maintenance of pumps as perhaps the most important factor in the safe and efficient operation of any chemical plant.

Distillation columns are used to separate a mixture of two or more liquids, while absorbers remove one or more components of a mixture of gases by contact with a liquid. Evaporators are used to separate one liquid from another,

or to separate a liquid from a solution or suspension of solids by changing the liquid to the vapor state.

Tanks, of course, are used for storing the thousands of gallons of liquid used in the chemical processes.

3.2 Periodic Inspection Model

According to Mann [1983], the inspection of plant equipment is an important phase of a comprehensive preventive maintenance program. He defines inspection as the examination of equipment to:

1. Ensure that it is performing as designed.
2. Evaluate the mechanical, pneumatic, hydraulic, and electrical mechanisms in terms of potential problems.
3. Estimate when a breakdown could occur.
4. Identify the component or function that may precipitate a breakdown.
5. Schedule repairs at a convenient time to prevent a breakdown at an undesirable time.

Mann goes on to state that inspecting every piece of equipment in the plant is as unrealistic as not inspecting any equipment, and although these two extremes are practiced in reality, he recommends a more moderate approach.

Once a decision has been made about which equipment to monitor, a logical subsequent consideration is whether to use continuous or periodic

inspection. It has been assumed that a periodic inspection of equipment takes place because this is the predominant practice in the local chemical industry. That is, a measurement of the indicator variable, like vibration, is recorded for the single unit or individually for the collection of units in the system, as the case may be, every T time units. The measurement is taken at the beginning of the period to simulate what occurs in reality and also to avoid the introduction of extraneous factors into the model. Time between inspections could be set at single or multiple days, or single or multiple weeks, but once established, it is constant.

Failures may occur at any time in a period, but for simplification purposes, it is assumed that they happen at the end of a period. A repair is presumed to occur instantaneously. This is because the cost of repair is constant. That is, there are no costs connected to the length of the repair time; and consequently, there is no change in the value of model parameters during the repair time. In an economic model, there is no difference between assuming the repair occurs instantaneously and on the other hand, allowing a time for repair, but assigning no cost to the time involved.

3.3 Predictive Maintenance Variables

Among the many predictive maintenance methods, five specific techniques were described in chapter one: vibration analysis, infrared thermography, ultrasonic pulse-echo techniques, tribology, and motor current analysis. When

all techniques are applicable, the ideal situation would be that readings of all five variables (taken at the beginning of period t) could be used collectively to predict the state of an individual unit in that period as $X_t = f(Z_{1t}, Z_{2t}, Z_{3t}, Z_{4t}, Z_{5t})$ where X_t represents the state of the unit at time t, and Z_{it} (for $i=1,2,3,4,5$) represents the 5 different diagnostic readings. However, the problem with this idealistic scenario is the difficulty of modelling the effect of the individual factors and the possible interaction of factors, in combination, on the unit. Consequently, at the outset, a single (external) predictor variable is used to indicate the state of the unit at time t.

The approach will be to find the optimal economic policy for a single unit using dynamic programming, and then do the same for a system composed of two individual units.

3.4 The Practical Problem

The genesis for this research is a heuristic maintenance strategy currently being used by Monsanto. At their Baton Rouge location, there are upwards of 300 pumps used in the process plant. The prevailing practice is to take individual weekly measurements to determine, in inches per second, when a single pump is out-of-control using a variable control chart. This out-of-control state is deemed to occur when the vibration in inches per second is more than three standard deviations above the average vibration of all pumps. This average is determined from sample or historic data. In addition to tracking the individual

pumps with variable control charts, a p-chart is used to follow, over time, the fraction of all rotating equipment that is above the upper control limit just described. When this fraction exceeds .3, the system is disassembled and overhauled, meaning that all pumps are reworked to be "as good as new".

So it seems that having thirty percent of all rotating equipment more than three standard deviations above the mean vibration represents a "magic" rule of thumb for this industry giant. Consequently, it is worthy of investigation to determine whether this rule of thumb can be validated mathematically or not.

3.4.1 General Simplifications and Assumptions

It is necessary to make simplifications and assumptions in order to model the situation described above. Next, explanations and validation are offered for the assumptions inherent to the original conditions:

1. McCall [1965] describes the general structure of maintenance models to include the assumption that the equipment occupies one of several states while in operation. Here, it is presumed that the state of a unit can be described by the quantity of the predictor variable present at the beginning of any time period t. Consequently, the state space is continuous since the measurement of the external variable is on a continuous scale.

2. Measurements (X_t) of the external variable are taken at the beginning of each period to avoid the introduction of extraneous factors into the model and also because this is done in practice.

3. Time between failures is exponentially distributed, and the failure rate (or rate of occurrences of failure) is a function of the state of the unit: failure rate = $\lambda(X_t)$. Note that this presumption does not imply a constant failure rate because the parameter lambda is a function of the state of the unit which, in turn, is determined by the quantity of the predictor variable present at the beginning of each period t. Therefore, the failure rate would remain unchanged only as long as the quantity of the predictor variable remains constant. For example, if the failure rate is a linear function of the state of the unit, then the failure rate is actually increasing over time since below, it is assumed that the deterioration experienced by the unit is non-decreasing.

4. In each period a random amount of deterioration (Y_t) occurs which can only be non-negative. The unit cannot improve spontaneously. This assumption duplicates reality because, in truth, no equipment can fix itself.

5. The random variable, Y_t, representing the deterioration that occurs in every period is independent and identically distributed, a characteristic necessitated by the mathematical formulation.

6. In order to provide an incentive to repair before the unit fails, the adjustment cost, K, associated with an unfailed unit, is fixed and strictly less than the fixed cost, r, of a failed unit: $K < r$. There is no cost attributed to a period requiring no unit repair or adjustment.

7. After an adjustment or repair, the state of the unit will have an initial value X_0 which is fixed and not a decision parameter, an assumption used in practice.

CHAPTER 4. A SINGLE UNIT MODEL

4.1 Introduction

One of the objectives for this dissertation is to develop a mathematical model for a single unit by monitoring a single external variable and using it as a predictor for failure. Here an economically based model will be used to develop an optimal solution to the single unit problem of finding the best time for a unit overhaul.

For the remainder of this exposition, a unit is defined as an item of rotating equipment such as a motor, pump, gear box, or valve whose mechanical and operating condition and/or performance is subject to gradual deterioration due to the adverse effects of an external variable such as vibration. Further, a system is specified as a collection of two or more units which is designed to perform one or more functions.

4.2 Problem Setting

Consider a unit that is monitored in discrete, fixed time intervals where the characteristic, X_t, represents measurements of an external variable and describes the state of the unit. It is assumed that the measurement X_t is taken at the beginning of period t and the state space is continuous. Based on this value of the external variable (the state of the unit), a failure time distribution will describe the probability that this unit will fail within the next time interval. The

failure time distribution is assumed to be exponential, where the failure rate is a function of the state of the unit, that is, the external variable. During each inspection period a deterioration described by a random variable Y_t occurs. The deterioration can only be non-negative, that is the unit cannot get better by itself. It is assumed in this single unit model that the Y_t are independent and identically distributed (i.i.d.) random variables. Thus the measurement at the end of the period t which is equal to the measurement at the beginning of period t+1, is given by

$$X_{t+1}=X_t+Y_t \ . \tag{1}$$

So the state of the system at the beginning of period t+1 is the sum of the amount of deterioration present at the beginning of period t and the amount of decline that occurred during period t.

4.3 Assumptions

Below, the assumptions are described and some simplifications made to have a mathematically tractable model.

1. At equidistant time intervals, the unit is inspected, and it is presumed that the state of a unit can be described by a measured quantity, X_t, of the predictor variable at the beginning of any time

period t. The state space is continuous since the measurement of the external variable is taken on a continuous scale.

2. Time to failures follows a nonhomogeneous Poisson process, and the failure rate (or rate of occurrences of failure) is an increasing function, $\lambda(X_t)$, of the state variable X_t. In general, the functional form may be linear, quadratic, or of higher order; however, later, a linear relationship is assumed. For simplification, failures are presumed to occur at the end of a period.

3. The non-negative random variable, Y_t, representing the deterioration occurring in every period is independent and identically distributed: a characteristic necessitated by the mathematical formulation. In our numerical examples, the Poisson distribution was used to describe the deterioration. The unit cannot improve spontaneously.

4. Measurements, X_t, of an external variable are taken at the beginning of each period to decide which is the most economic state of the system to make an adjustment. It was found that an upper control limit (UCL) policy is optimal. If and only if X_t is greater than or equal to the UCL, the unit is adjusted (preventive maintenance).

5. In order to provide an incentive for preventive maintenance (adjustment), the adjustment cost, K, is constant and strictly less than the fixed cost, r, of a failed unit requiring repair: $K < r$. No cost is attributed to a period without failure or adjustment.

6. After an adjustment or failure (followed by a repair), the state of the unit is restored to the initial value X_0 which is fixed and not a decision parameter: an assumption used in practice.

7. When no failure occurs during period t, the measurement of the external variable at the end of the period, which is equal to the measurement at the beginning of period t+1, is given by

$$X_{t+1}=X_t+Y_t \ .$$

4.4 Notation

The notation used in the remainder of this chapter is defined below:

$H(X_t)$ conditional expected failure cost given X_t, the state of the unit.

C(U) long run average cost per inspection period of the system operating under an upper control limit policy parameter, U.

C(D) long run average cost per inspection period of the system operating under an upper control limit policy parameter, U, but written as a function of D.

D	difference between the upper control limit, U, and the initial value X_0.
D*	optimum value of D.
$\delta(X_t)$	control variable equal to 0 if the decision at the beginning of the period is NOT to adjust, and 1 if the decision is to adjust.
g(y)	probability density function of the deterioration Y_t.
K	cost of an adjustment (preventive maintenance) when the unit is restored to its initial state.
λ	parameter of the exponential failure distribution which is a function of the state of the unit: $\lambda(X_t)$.
M(.)	renewal function of deterioration Y_t.
m(.)	renewal density function.
$M_D(.)$	steady state distribution of X_t.
μ	average number of deteriorations per unit time.
r	cost of repair when the unit fails.
T	the length of the inspection period.
U	upper control limit of a control limit policy.
U*	optimum upper control limit.
X_0	initial state of the unit after a repair.

X_t measurement of the external variable taken at the beginning of each period t that describes the state of the unit.

X_t^* value of X_t for the unit at the beginning of period t depending on whether or not it was adjusted.

Y_t random amount of deterioration that occurs in each inspection period.

4.5 Dynamic Programming Model

The reliability of a single unit, defined as the probability that the unit will NOT fail by the end of time period t or as the probability that the time between failures is greater than T is given by

$$e^{-\lambda(X_t+Y_t)T} \quad . \tag{2}$$

The cost of failure per inspection period, given that the amount of deterioration at the beginning of period t was X_t, is the product of the cost of repair and the probability of failure during period t:

$$r[1-e^{-\lambda(X_t+Y_t)T}] \ . \tag{3}$$

The conditional expected cost of failure, $H(X_t)$, given X_t, is found by integrating the conditional cost expression (3) with respect to the deterioration density function, g(y):

$$H(X_t)=\int_0^\infty r[1-e^{-\lambda(X_t+y)T}]g(y)dy \ . \tag{4}$$

At the beginning of each period t, the indicator variable X_t is measured. Depending on X_t, the system is adjusted: at adjustment, a cost K is charged, the system is reset to X_0, and the expected cost of failure in period t is $H(X_0)$; if no adjustment is done in period t, the expected cost of failure is $H(X_t)$, and no adjustment cost is charged. Introducing the decision variable:

$$\delta(X_t) = \begin{cases} 0 \text{ if the unit is not adjusted} \\ 1 \text{ if the unit is adjusted} \end{cases}$$

and denoting

$f_t(X_t)$ = cumulative minimal cost in period t for that period and all future periods

and

$$X_t^* = X_t - [X_t - X_0]\,\delta(X_t)\,,$$

the dynamic programming formulation to find the most economic state of the system (X_t) to adjust the unit (perform preventive maintenance) is:

$$f_t(X_t) = \underset{\delta(X_t)}{Min}\{K\delta(X_t) + H(X_t^*)$$

$$+ \int_0^\infty f_{t-1}(X_t^* + y)\, e^{-\lambda(X_t^* + y)T} g(y)\,dy$$

$$+ f_{t-1}(X_0)[1 - \int_0^\infty e^{-\lambda(X_t^* + y)T} g(y)\,dy]\}\,. \tag{5}$$

In order, from left to right, the terms to be minimized represent the adjustment cost in period t, expected failure cost in period t, future optimum of expected total cost if no failure occurs in period t, and the future optimum of expected total cost if failure occurs in period t, when the system is repaired and reset to the initial state X_0 in period t. This last term, the case when the initial state is reset due to failure and not by adjustment, is neglected in the majority of models published. The probability in the last term is initially small, but can be considerable as X_t increases. The future cost of no failure (failure) for each period t is the cumulative minimum cost for period t and all future periods times the probability of no failure (failure) that depends on the decision variable $\delta(X_t)$.

To solve the dynamic program numerically, the state space must be discretized. In practice, the smallest significant decimal would be used. For instance, for vibration measurements, the vibration is often recorded up to two significant places. First, it is assumed that there is a smallest measurement amount of deterioration Δ, and then the Poisson distribution is used to model multiples of this deterioration. Thus $\Delta = 0.01$ would be used as the smallest amount of measurable deterioration, and $X_t = X_0 + N\Delta$ where N is an integer. It is assumed that N follows a discrete distribution, for instance Poisson or negative binomial. In the numerical solutions, the Poisson distribution with parameter μ was used. By assumption, $T = 1$ and the initial value of the external variable is $X_0 = 1$.

A special case is considered where the failure rate is a linear function of the external variable, that is $\lambda(X_t+Y_t)T = \lambda TX_t + \lambda TY_t$. The dynamic programming formula (equation 5) is computed using backward iteration for t = 1,2,... until the optimal policy stabilizes. A limit of t = 400 was set, but most solutions stabilized in less than 134 periods, one in only 26 periods. Only one solution took 340 periods to stabilize. It was observed that the optimal policy always tends to an upper control limit (UCL) policy. This result is in accord with the theoretical result of Taylor (1975). Though Taylor doesn't provide any solution procedure for the UCL value in his continuous model, here the optimal solution is obtained by discretizing and using a dynamic programming procedure. Since the dynamic programming solution is numerically intensive, an approximate solution is considered in the next section.

4.6 Steady State Approximation

In this section the simplification of the dynamic programming model is considered with the idea that, provided the cost of failure is very high, then the probability of failure will tend to zero. Therefore the last term of equation 5 in section 4.5 becomes negligible. Ignoring the fact that after failure the unit is restored to its original state, X_0, transforms the original process into a generalized stochastic clearing system as described by Stidham (1986): "A stochastic clearing system is characterized by a stochastic input process and an output mechanism that removes all the quantity present whenever the input

exceeds a critical level q. In a generalized stochastic clearing system, the system contents are restored to a level m, which can be different from zero."

This portion of our model is similar to Nakagawa (1976) because the possibility of restoring the initial state due to failure, not adjustment, is disregarded. The approximation error and its effect on the solution and cost increase will be discussed in section 5.4.

Allowing the deterioration to accumulate without failure until an adjustment resets the initial value to X_0 qualifies the model as a clearing system. Stidham (1986) shows that for a clearing system with convex costs, a control limit policy is optimal. The stochastic clearing system uses a renewal approach solution. Consequently, the steady state distribution of the state variable X_t, defined by

$$M_D(x) = \lim_{t \to \infty} P(X_t \geq x) \tag{6}$$

is used in order to investigate the long run average cost per inspection period. Since $D = U - X_0$,the steady state distribution can be represented by equation (7) (Ross, 1983)

$$M_D(x) = \frac{1 + M(x - X_0)}{1 + M(U - X_0)}, \quad X_0 \leq x \leq U \tag{7}$$

where M(.) is the renewal function of the deterioration distribution, Y_t.

It is, however, more convenient to write the cost as a function of

$D = U - X_0$

$$C(D) = \frac{K + \int_0^D [\int_0^\infty \{r[1 - e^{-\lambda(X_0+x+y)T}]\} g(y)dy] m(x)dx}{1 + M(D)}. \tag{8}$$

The optimal control limit has to satisfy the first order equation

Dc/dD = 0; that is equivalent to

$$\frac{\int_0^\infty \{r[1 - e^{-\lambda(X_0+D+y)T}]\}\ g(y)\ dy\ m(D)}{1 + M(D)}$$

$$-\frac{K + \int_0^D [\int_0^\infty \{r[1 - e^{-\lambda(X_0+x+y)T}]\} g(y)dy] m(x)dx}{[1 + M(D)]^2} m(D) = 0. \tag{9}$$

Multiplying both sides of the equation (9) by

$$\frac{1 + M(D)}{m(D)}$$

yields the following equation

$$C(D) = \int_0^\infty r[1 - e^{-\lambda T(X_0+D+y)}]\ g(y)\ dy \tag{10}$$

where the left-hand side is expressed in (8). Equation (10) provides the

necessary condition for the optimum D, call it D^*. The optimum U^* is then obtained from the relationship: $U^* = D^* + X_0$.

Stidham (1986), in Corollary 1, gives conditions under which the cost function (8) is convex, and thus equation (10) is sufficient for an optimum, namely, if the cost rate in function (4) is convex and the quantity 1 + M(D) is log-concave for $D \geq 0$. Since M(D) depends only on the distribution of the deterioration process, g(y), the log-concavity is actually a condition for the distribution of the deterioration process. It holds for decreasing failure rate (DFR) functions (see Sahin (1990)). Gamma and Weibull distributions with shape parameter, $\alpha < 1$, are DFR functions, and they can provide a good approximation in some practical cases.

Since discretization was necessary in the approximation and a Poisson distribution was used for the deterioration process, convexity couldn't be proved for the cost function. However, our numerical results show a convex shaped cost function, albeit discrete.

Equation (10) is rewritten by substituting equation (8) for C(D), eliminating denominators and using the moment generating function:

$$G_F(-\lambda T) = \int_0^\infty e^{-\lambda T y} g(y)\, dy \qquad \textbf{(11)}$$

and get (12)

$$M(D)=\frac{1-\frac{K}{r}+e^{-\lambda TX_0}\{\int_0^D e^{-\lambda Tx}m(x)dx\}\,G_F(-\lambda T)}{e^{-\lambda(X_0+D)T}G_F(-\lambda T)}-1\,. \tag{12}$$

Note that the optimal D depends only on the cost ratio, K/r, and not on the individual cost of repair, r.

The variable is discretized as in section 4.5. If μ is the mean of the Poisson distribution, the renewal density can be computed using this recursive formula from Wagner et al (1965)

$$f(i) = \frac{\mu^i}{i!}e^{-\mu}$$

$$m(0) = \frac{f(0)}{1-f(0)}$$

$$m(j)=\frac{f(j)+\sum_{i=1}^{j} m(j-i)f(i)}{1-f(0)}\quad . \tag{13}$$

There is no closed form solution for equation (12); however, a simple numerical solution for the D which minimizes (8) can be found by computing C(D) over a range of D values which contains the optimum D.

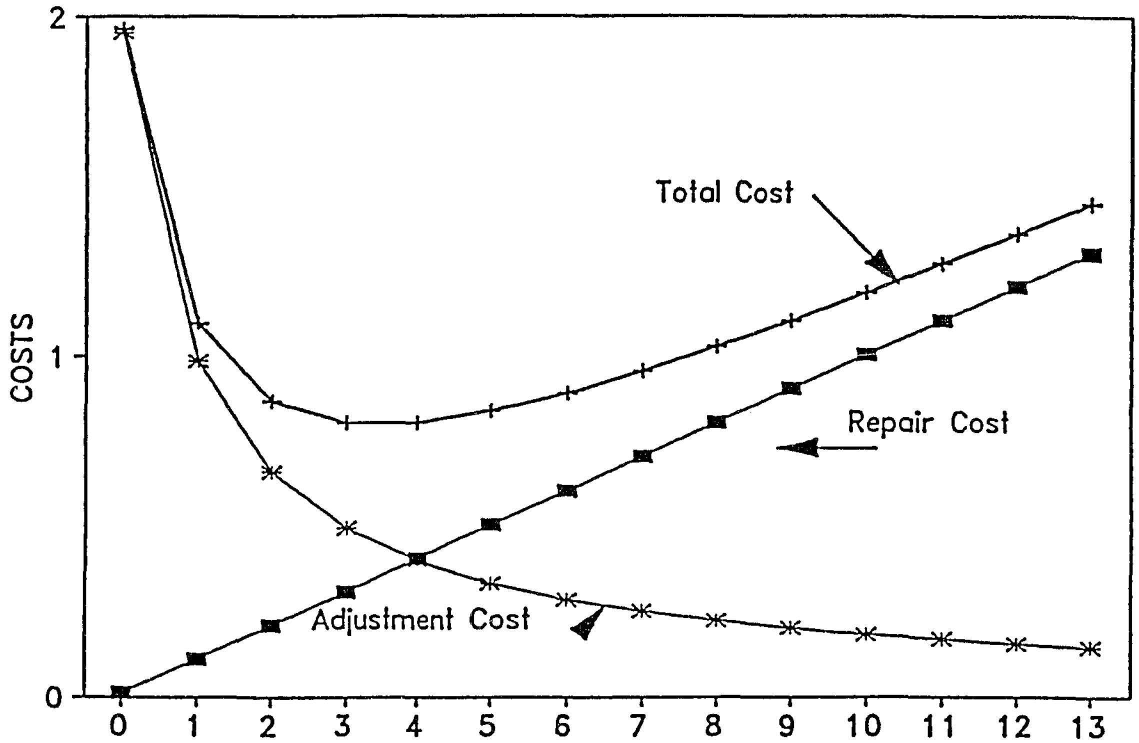
As an illustration using the parameter values: μ = .05, λ = .001, K/r = 1/5, and T = 1, the relationships of the repair cost, adjustment cost, and

resultant total cost are illustrated in Figure 1. The minimum total cost occurs at $D = U\text{-}X_0 = 3$ or 4, and the long run average cost for that D is both $C(3) = C(4) = 0.805$.

In summary, the approximation serves a dual purpose:

(1) it provides a basis for a simpler UCL search method to be used by the practitioner.

(2) it allows a comparison of the dynamic programming approach (incorporating the probability of failure during the period) to some of the earlier works (which ignore it.)



$D = U - X_0$

$\mu = .05$, $\lambda = .001$, $K/r = 1/5$, $T = 1$

Figure 1

Cost Functions

CHAPTER 5. NUMERICAL ANALYSIS OF THE SINGLE UNIT MODEL

5.1 Introduction

A two-level factorial analysis is used to evaluate the effect of changing one or more parameters. This methodology simultaneously investigates the effects of all combinations of the levels of two or more decision variables. It shows major trends and indicates a direction for further inquiry. Here, eight runs are analyzed, using a high and low level of three different parameters: mean of the deterioration distribution g(y) (μ = .10, .05), mean of the failure rate distribution (λ = .002, .001); and cost ratio (K/r = 80/200=2/5, 80/200 = 2/5); the high setting is always twice the low. Time between inspections in all cases is T = 1. Table 5.1 shows cost and optimum D values from the approximate and exact solutions and is the basis for much of our subsequent analysis. The optimal UCL is $U^* = D^* + X_0$ (where $X_0 = 1$ was used in our numerical examples).

The first three columns of Table 5.1 and succeeding tables in this chapter show whether the low (-1) or high (1) value of the parameter was used in the run, while the first two rows contain the parameters. Columns 4, 5, 6, and 7 contain the total expected cost for using D^*_{exact} in the exact solution, value of D^*_{exact} in the exact solution, cost of D^*_{approx} in the approximate solution, and value of D^*_{approx} in the approximate solution, respectively. Columns 8 and 9

contain the relative percent cost penalty (RCP) of the approximate solution and the relative percent error (RPE), respectively , defined by:

$$RCP = \frac{Exact\ Cost(D*_{approx}) - Exact\ Cost(D*_{exact}) * 100\%}{Exact\ Cost\ (D*_{exact})}$$

and

$$RPE = \frac{D*_{approx} - D*_{exact} * 100\%}{D*_{exact}}.$$

Table 5.1
Comparison of Exact and Approximate D* and their Costs

2/5	.002	.10	High					
1/5	.001	.05	Low					
K/r	λ	μ						
1	2	3	COST D^*_{ex}	D^*_{ex}	COST D^*_{approx}	D^*_{approx}	RCP	RPE
-1	-1	-1	0.909	5	1.04	3	14%	-40%
1	-1	-1	1.136	9	1.28	5	13%	-44%
-1	1	-1	1.316	4	1.57	2	19%	-50%
1	1	-1	1.666	6	2.00	3	20%	-50%
-1	-1	1	1.247	7	1.34	5	7%	-29%
1	-1	1	1.564	13	1.68	8	7%	-38%
-1	1	1	1.834	5	2.07	3	13%	-40%
1	1	1	2.271	9	2.53	5	11%	-44%

Factor analysis is used to show the effect of parameter changes on D* in section 5.2, the error of the approximate cost function (ECF) in 5.3, the error of the approximate solution as a relative percent error (RPE) in 5.4, and in 5.5, the effect of parameter changes on the relative percent cost penalty (RCP) from using D^*_{approx} instead of D^*_{exact}. Here, "cost" is the **real** expected total cost of adjustment and repair considering the restoration after failure during the period T (i.e. the exact cost).

5.2 Sensitivity Analysis

Table 5.2 shows the factorial analysis of D* obtained from using the exact solution in Equation 5. Column 4 has the optimum values of D at the indicated parameter settings, column 5 shows the effects of each factor: E1, E2, and E3 are the effects of changing K/r, λ, and μ respectively. E12, E13, E23, and E123 are interaction effects between E1 and E2, E1 and E3, E2 and E3, and E1, E2, and E3, respectively. The estimates of the effects produced by the factorial analysis are in column 6. The term "$\mu_{results}$" in the fifth column is the mean or average of all results in column 4.

As one would expect, the increase of repair cost to failure cost (K/r) effects an increase in the optimal D, resulting in a higher upper control limit U and an increase in the average time between adjustments. On the other hand, an increased failure rate, λ, is expected to decrease U resulting in more frequent

adjustments. Note that the cost effect "doubling K/r" is larger than the failure rate effect "doubling λ".

It is less evident that the increase in the average number of deteriorations per unit time (μ) results in a higher upper control limit. Doubling this parameter results in the same absolute change in D^*_{exact} as doubling λ, just the direction is opposite. The complexity of equation (5) defies a simple explanation for this result. It is interesting to note that the effects of combinations of the levels of two or more decision variables are much less than the simple effects indicating mainly pure effects of single parameters.

Table 5.2
Factorial Analysis of D* in Exact Solution

2/5	.002	.1	HIGH		
1/5	.001	.05	LOW		
K/r	λ	μ			
1	2	3	D^*_{exact}	FACTOR	EST.
-1	-1	-1	5	$\mu_{results}$	7.25
1	-1	-1	9	E1	4
-1	1	-1	4	E2	-2.5
1	1	-1	6	E12	-1
-1	-1	1	7	E3	2.5
1	-1	1	13	E13	1
-1	1	1	5	E23	-0.5
1	1	1	9	E123	0

5.3 Error of the Approximate Cost Function

Table 5.3 contains factorial analysis results on the error of the approximate cost function given by:

$$ECF = \frac{Approximate\ Cost\ (D*_{approx}) - Exact\ Cost\ (D*_{approx})}{Exact\ Cost\ (D*_{approx})} *100\%$$

Column 4 shows a high percent error, in most cases, using the approximate cost function (8) instead of the exact cost function (5). Columns 5 and 6 show the effects and estimates, respectively.

Table 5.3
Factorial Analysis of Error of
Approximate Cost Function (ECF)

2/5	0.002	0.10	HIGH		
1/5	0.001	0.05	LOW		
K/r	λ	μ			
1	2	3	ECF	FACTOR	EST.
-1	-1	-1	-22%	$\mu_{results}$	-15.25%
1	-1	-1	-9%	E1	13%
-1	1	-1	-31%	E2	-9.5%
1	1	-1	-20%	E12	0%
-1	-1	1	-12%	E3	10.5%
1	-1	1	1%	E13	1%
-1	1	1	-22%	E23	.5%
1	1	1	-7%	E123	1%

The error of the approximate cost function (ECF) rises considerably with the doubling of K/r and μ (13% and 10.5%) but as λ doubles the ECF falls by 9.5%. It should be noted that the approximate cost is almost always strictly less than the exact cost, which results from the fact that the approximation formulation ignores the cost that occurs in case of failure and restoration in a period, (the last term of Equation 5).

5.4 Error of Approximate Solution

Now the performance of the approximate solution developed in section 4.6 is compared to the exact dynamic programming solution of section 4.5 (see columns 5,7, and 9 of Table 5.1). Column five lists the optimum D obtained from the dynamic programming solution in Equation 5, column seven gives the optimum D obtained using the approximate solution from Equation 8, and column nine shows the relative percent error defined earlier as RPE.

The most obvious result is that the D* from the approximation always underestimates D* from the exact solution. The best approximations occur when μ is high and λ is low and the worst when μ is low and λ is high but the error is quite large even in these cases. Table 5.4 shows the factorial analysis of the RPE in column 9 of Table 5.1. Doubling μ and λ has the highest effect on the error of approximate D, and it is interesting to note that these results are equal in size but opposite in direction. The error of the approximate D seems to be unacceptable in most cases.

Table 5.4
Factorial Analysis of Relative Percent Error of Approximate D* (RPE)

2/5	.002	.1	HIGH		
1/5	.01	.05	LOW		
K/r	λ	μ			
1	2	3	RPE	FACTOR	EST.
-1	-1	-1	-40%	$\mu_{results}$	-41.9%
1	-1	-1	-44%	E1	-4.3%
-1	1	-1	-50%	E2	-8.3%
1	1	-1	-50%	E12	2.3%
-1	-1	1	-29%	E3	8.3%
1	-1	1	-38%	E13	-2.3%
-1	1	1	-40%	E23	-0.3%
1	1	1	-44%	E123	0.3%

Next the cost effect of using an approximate solution instead of the optimal D is examined. This measure gives the actual effect of decisions based on approximation instead of optimum.

5.5 Cost Penalty of Approximate Solution

The most important measure of the approximate solution is the relative cost penalty (RCP) defined earlier. It is immediately evident from column 8 of table 5.1 that the approximate solution provides higher than optimal costs in all cases: the largest difference being a 19% or 20% increase when λ is high and μ is low. In this situation, the approximate solution seems to be unacceptable.

However, when λ is low and μ is high, the cost penalty (around 7%) may be acceptable under certain practical considerations. The value of K/r is irrelevant. The mean percent difference for the eight values is 13% with a standard deviation of 4.5%.

It should be noted that evaluating the worth of the model only on the RCP is, in part, misleading. A deviation of 2 units below D^*_{exact} translates into a large percent difference when the optimal D is small (if $D^*_{approx} = 2$, and $D^*_{exact} = 4$ the percent difference is -50%, whereas if $D^*_{approx} = 5$ and $D^*_{exact} = 7$, the percent difference is only -29%).

The next table shows the factor analysis of the absolute percent difference of the exact cost of the approximate D* compared to the exact cost of D* from the dynamic programming solution.

These results indicate that cost differentials are primarily and almost equally influenced by λ and μ: K/r, and the two and three factor interactions are relatively insignificant. Specifically, the cost differential rises 5.5% when λ goes from its low to high level, and falls 7% when μ goes from its low to high level. The interesting aspect of this analysis is that λ and μ have close (in size) but opposite (in direction) effects on the cost deviation. This means that even though the approximation of D may not be as good as desired, the exact cost of using the approximate D instead of the exact D has a maximum increase in cost

differences of 5.5% and can even reduce the difference between the two costs as much as 7%.

Table 5.5
Factorial Analysis of Relative Cost Penalty (RCP) of Approximate Solution

2/5	.002	.10	HIGH		
1/5	.001	.05	LOW		
K/r	λ	μ			
1	2	3	RCP	FACTOR	EST.
-1	-1	-1	14%	$\mu_{results}$	13%
1	-1	-1	13%	E1	-0.5%
-1	1	-1	19%	E2	5.5%
1	1	-1	20%	E12	0%
-1	-1	1	7%	E3	-7%
1	-1	1	7%	E13	-0.5%
-1	1	1	13%	E23	-0.5%
1	1	1	11%	E123	-1%

CHAPTER 6. TWO UNIT MODEL

6.1 Introduction

As a system ages, growing numbers of units may exhibit an increase of the monitored variables, prompting escalating unit repairs. Thus, preventive maintenance will require an increasing amount of downtime and subsequent loss of production time for these individual repairs. Here, the decision is whether to overhaul the whole system or continue to endure downtime and repair costs from individual units.

Especially in the petro-chemical industry, one common pragmatic solution to this problem is to monitor the system. Using a p-chart to track the fraction of single units above three standard deviations of a heuristic mean, management orders an overhaul of the system when the fraction equals or exceeds the upper control limit. Alternatively, some strategies resort to overhauling the system at convenient intervals, generally times of low production.

The objective of this chapter is to develop a mathematical model for a two unit system by monitoring an external variable for each unit and use this information collectively as a predictor for failure. Recall the definition of a unit from Chapter 4 as an item of rotating equipment (such as a motor, pump, gear box, or valve) whose mechanical and operating condition and/or performance is subject to gradual deterioration due to the adverse effects of an external variable.

A system, then, is a collection of two or more units which is designed to perform one or more functions.

6.2 Assumptions

The assumptions of the two unit model are:

1. The system consists of two units arranged in a series.
2. Individual measurements of the external variable (X_{tn}) are taken at the beginning of each period t for each unit n, (n = 1,2), and used to determine whether or not the unit should be adjusted.
3. The state space (representing the quantity of the external variable) of each unit is continuous.
4. In each period t, a random amount of deterioration occurs in each unit n, (n = 1,2), which is independent, identically distributed, and strictly non-negative. No unit can improve spontaneously.
5. Failures are assumed to occur at the end of the period; the time between failures is exponentially distributed, with failure rate an increasing function of the state variable, X_{tn}. The failure rate for each unit is $\lambda(X_{tn})$, (n=1,2).
6. The probability of failure of each unit is independent and identically distributed. The cost of failure for each unit is r.
7. The decision to overhaul the system or adjust either unit individually is based on the level of the external variable present at the beginning of

period t. An overhaul has cost K. The cost of adjusting either unit individually is K_n, (n = 1,2), and $K_1 = K_2$. In addition, it is more economical to overhaul the system (adjust both units simultaneously) than to adjust each unit individually. So, $K_n < K < 2K_n$.

8. The cost of failure is much higher than the cost of overhauling the system and consequently much higher than adjusting a single unit individually:

$$r >> K > K_n.$$

9. After an adjustment or repair, the state of unit n, (n = 1,2), has an initial value, $X_{0n} = 1$, which is fixed and not a decision parameter. After an overhaul, both units are in state X_{0n}, (n = 1,2).

6.3 Notation

The notation pursuant to this chapter is:

δ_n decision variable for WHETHER or not unit n, (n=1,2), is adjusted in period t: δ_n has value 0 if unit n is NOT adjusted and 1 if unit n is adjusted.

δ indicator variable for overhaul of the system in period t: δ has value 0 when none or only 1 of the two units is adjusted. That is δ has value 0 if:

$\delta_1 = 0$ and $\delta_2 = 0$ OR

$\delta_1 = 0$ and $\delta_2 = 1$ OR

$\delta_1 = 1$ and $\delta_2 = 0$.

When the system is overhauled, δ has value 1. That is

δ has value 1 if:

δ_1 and $\delta_2 = 1$.

$g_n(y)$	probability distribution function of deterioration for unit n, (n = 1,2).
$g(z)$	distribution of the sum of deterioration in units 1 and 2 together.
$f_t(X_{t1},X_{t2})$	cumulative minimum cost in period t, for that period and all future periods.
$H(\cdot)$	conditional expected cost of failure.
λ	parameter of the exponential failure distribution which is a function of the state of the unit.
K	cost of adjusting units 1 and 2 simultaneously, synonymous to the cost of a system overhaul.
K_1	cost of an adjusting unit 1 and not unit 2.
K_2	cost of an adjusting unit 2 and not unit 1.
μ	average number of deteriorations per unit time.
r	cost of repair when unit n, (n = 1,2), fails.
R_n	reliability of unit n, (n = 1,2), the probability that unit n does not fail before time T.
T	length of the inspection period.

y_{tn} random amount of deterioration occurring in period t in unit n, (n = 1,2).

X_{0n} initial state of unit n, (n=1,2), after a repair, adjustment, or overhaul.

X_{tn} measurement of the external variable taken at the beginning of each period t that describes the state of unit n.

X_{tn}^{*} value of X_{tn} for machine n at the beginning of period t, depending on whether or not it was adjusted.

Z sum of deterioration in unit 1 and unit 2.

6.4 System Problem Setting and Model

Because the state space grows rapidly as the number of units increase, this analysis of a system solution begins with two units arranged in a series, and monitored in discrete, fixed time intervals of length T. Let the characteristic X_{tn} represent the measurements of an external variable taken at the beginning of each period t which describe the state of unit i, (i=1,2). This state space is continuous since the measurements are continuous.

Subsequently the vector $\vec{X}_t$ describes the system of 2 units by:

$$\vec{X}_t = \begin{Vmatrix} x_{t1} \\ x_{t2} \end{Vmatrix} .$$

Using these measurements of the external variable, X_{ti}, for each unit i, (i = 1,2), in period t, the probability that unit i will fail within the next time interval is assumed to be exponentially distributed, where the number of failures per unit time is a function of the state of the unit, that is, the rate of occurrence of failures is $\lambda(X_{ti})$, (i=1,2). In addition, as in the single unit model, non-negative random deterioration occurs for each unit i during each period t and is called y_{ti}. The values of y_{ti} are assumed to be independent and identically distributed for i=1,2. Therefore, the vector $\vec{y}_t$ consists of the amounts of deterioration occurring in units 1 and 2 during period t and is represented by

$$\vec{y}_t = \begin{Vmatrix} y_{t1} \\ y_{t2} \end{Vmatrix} .$$

Using the preceding information, the updating equation is written as

$$\vec{X}_{t+1} = \vec{X}_t + \vec{y}_t \quad . \tag{14}$$

In a series system, the reliability of unit i is usually represented by R_i, signifying the probability that the unit will NOT fail by time t or, alternatively, the probability that the time between failures is greater than t. Since the time

between failures is exponential, with the failure rate expressed as a function of the state of the unit, the reliability of unit i is expressed by:

$$e^{-\lambda_i(X_{ti}+y_{ti})T} \quad . \tag{15}$$

Accordingly, the reliability for a system of 2 independent units is represented by:

$$e^{-\sum_{i=1}^{2}\lambda_i(X_{ti}+y_{ti})T} \quad . \tag{16}$$

The cost of failure per inspection period in the two unit system, given that the amount of deterioration from the external variable at the beginning of that period for each unit i was X_{ti}, is the product of the cost of repair when a failure occurs and the probability of system failure. The mathematical expression is

$$r[1-e^{-\sum_{i=1}^{2}\lambda_i(X_{ti}+y_{ti})T}] \ . \tag{17}$$

The conditional expected cost of failure, given that the amount of deterioration due to the external variable at the beginning of that period for each unit i was the vector $\vec{X}_t$, is found by integrating the previous equation with respect to the vector of distributions of deterioration, $g(\vec{y})$:

$$H(\vec{X}_t)=\int_0^\infty r[1-e^{-\sum_{i=1}^{2}\lambda_i(X_{ti}+\vec{y})T}]\ g(\vec{y})d\vec{y} \ . \tag{18}$$

The remainder of this chapter explains the two unit model, dynamic programming equation and solution, and sensitivity analysis of the solution.

6.5 Two Unit System

The reliability of two independent units with equal failure rates, then, is simply

$$e^{-\lambda \sum_{i=1}^{2} (X_{ti}+y_{ti})T} . \tag{19}$$

The cost of failure per inspection period, given that the amount of deterioration in units 1 and 2 was X_{t1} and X_{t2}, respectively, is the product of the cost of repair and the probability of system failure during period t:

$$r[1-e^{-\lambda \sum_{i=1}^{2} (X_{ti}+y_{ti})T}] . \tag{20}$$

System failure means that the first, second, or both units fail.

For two units, the conditional expected cost of failure, $H(X_{t1},X_{t2})$, given X_{t1} and X_{t2}, is found by integrating the conditional cost in the previous equation with respect to the deterioration density function, g(z):

$$H(X_{t1},X_{t2}) = \int_0^\infty r[1-e^{-\lambda(X_{t1}+X_{t2}+z)T}]\; g(z)dz . \tag{21}$$

where $z = y_{t1} + y_{t2}$ and g(z) = distribution of the sum of deterioration in unit 1 and unit 2.

At the beginning of each period t, the external variables X_{t1} and X_{t2} are measured. Depending on these measurements, one of four actions occurs: unit 1 is adjusted and unit 2 is not, unit 1 is not adjusted and unit 2 is, both units are

adjusted, or neither unit is adjusted (that is, the system is allowed to continue as it was at the end of the previous period).

Table 6.1 below shows the possible decisions for each unit, related costs, and resultant levels of the external variable at the beginning of period t.

Table 6.1
Decision Alternatives at Beginning of Period t

Action to Unit 1	Action to Unit 2	Adjust Cost	X_{t1} value	X_{t2} value
Adjust	None	K_1	X_{01}	X_{t2}
None	Adjust	K_2	X_{t1}	X_{02}
Adjust	Adjust	K	X_{01}	X_{02}
None	None	0	X_{t1}	X_{t2}

Note that K is the cost of an overhaul (adjusting both units at once), whereas K_1 and K_2 represent the cost of adjusting each unit separately. When neither unit is adjusted, no adjustment cost is charged. The assumed relationship among the three costs for the two unit model is

$K_1 = K_2$ and

$K_1 < K < 2K_1$.

The decision variable δ_i is used to track whether or not each of the two units is adjusted at the beginning of a period:

$$\delta_i = \begin{cases} 0 \text{ if unit i NOT is adjusted} \\ 1 \text{ if unit i is adjusted,} (i = 1,2). \end{cases}$$

In addition, δ is used to indicate the NUMBER of units adjusted at the beginning of any single period.

$$\delta = \begin{cases} 0 \text{ when neither unit or only 1 unit is adjusted} \\ 1 \text{ when BOTH units are adjusted (system overhauled).} \end{cases}$$

By assumption, either of the units can fail at the end of a period, whether or not it was adjusted at the beginning of the period. For each unit, the cost of failure (repair cost) is r, and after a repair, the external variable assumes its initial level, X_{0i}, for repaired unit i,(i=1,2).

Using the notation:

$f_t(X_{t1},X_{t2})$ = cumulative minimal cost in period t for that period and all future periods,

the dynamic programming formulation to find the most economic state of the system (X_{t1},X_{t2}) to adjust either or both units is:

$$f_t(X_{t1},X_{t2}) = \underset{\delta_1,\delta_2,\delta}{Min}\{K_1\delta_1 + K_2\delta_2 + (K-K_1-K_2)\delta + H(X_{t1}^*,X_{t2}^*)$$

$$+\int_0^\infty\int_0^\infty f_{t-1}[X_{t1}^*+y, X_{t2}^*+w]e^{-\lambda(X_{t1}^*+X_{t2}^*+y+w)T}g_1(y)g_2(w)\,dy\,dw$$

$$+(1-R_1)\int_0^\infty f_{t-1}[X_{01},X_{t2}^*+w]e^{-\lambda(X_{t2}^*+w)T}g_2(w)\,dw$$

$$+(1-R_2)\int_0^\infty f_{t-1}[X_{t1}^*+y,X_{02}]e^{-\lambda(X_{t1}^*+y)T}g_1(y)\,dy$$

$$+f_{t-1}[X_{01},X_{02}](1-R_1)(1-R_2) \tag{22}$$

where

$$R_i = \int_0^\infty e^{-\lambda(X_{ti}^*+y)T}g_i(y)dy \tag{23}$$

and

$$X_{t1}^* = X_{t1} - [X_{t1} - X_{01}]\,\delta_1 \tag{24}$$

$$X_{t2}^* = X_{t2} - [X_{t2} - X_{02}]\,\delta_2. \tag{25}$$

In order, from top to bottom, the terms of the dynamic programming formulation, $f_t(X_{t1},X_{t2})$, represent:

·the adjustment cost plus the conditional expected cost in the current period,

·future minimum cost when neither unit fails,

·future minimum cost when unit 1 fails,

·future minimum cost when unit 2 fails,

·future minimum cost when both units fail.

In addition,

R_i is the reliability of unit i, (i = 1,2), the probability that unit i does not fail before time T.

X_{ti}^* is the value of X_{ti} for unit i, (i=1,2), at the beginning of period t, the amount of deterioration with which unit i begins period t, depending on whether or not it was adjusted.

For example:

X_{ti}^* is X_{t1} for unit 1 if no adjustment took place at the beginning of period t; but

X_{ti}^* is X_{01} for unit 1 if an adjustment did occur at the beginning of period t.

6.6 Numerical Solution

To solve the two unit dynamic program numerically, the state space is discretized and the Poisson distribution with parameter μ is used to model the deterioration process for each unit. By assumption, the length of the inspection period, T, is 1, and the initial value of the external variable for each unit after an adjustment, repair, or overhaul, is 1, ($X_{0i} = 1$ for $i = 1,2$). Again, the special case of linear failure rate is considered, as in the one unit model.

The optimal solution to the dynamic programming equation was computed with a fortran program for $t=1,2,3,\ldots,40$ until the optimal policy stabilized. As implied, a limit of $t = 40$ periods was set with each unit experiencing 200 increments in deterioration every period; all solutions stabilized by the end of 21 periods. Output was a matrix for the optimal decision policy and the average cost per period.

The optimal policy always tends to an upper control limit policy that describes when to: adjust unit 1 only, adjust unit 2 only, overhaul the system, or do nothing. The optimal decision matrix was consistent for all parameter settings satisfying initial assumptions. The general pattern of each decision matrix follows.

Policy Decision Matrix

X_{t2}	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0
X_{t1}																				
1	D	D	D	D	D	D	B	B	B	B	B	B	B	B	B	B	B	B	B	B
2	D	D	D	D	D	D	D	B	B	B	B	B	B	B	B	B	B	B	B	B
3	D	D	D	D	D	D	D	D	B	B	B	B	B	B	B	B	B	B	B	B
4	D	D	D	D	D	D	D	C	C	C	C	C	C	C	C	C	C	C	C	C
5	D	D	D	D	D	D	C	C	C	C	C	C	C	C	C	C	C	C	C	C
6	D	D	D	D	D	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
7	A	D	D	D	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
8	A	A	D	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
9	A	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
0	A	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
1	A	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
2	A	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
3	A	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
4	A	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
5	A	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
6	A	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
7	A	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
8	A	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
9	A	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
0	A	A	A	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C

The vertical axis represents unit 1 's increments in deterioration (from top to bottom) and the horizontal axis represents unit 2's increments of deterioration (left to right). The possible decisions are indicated at each combination of X_{t1} and X_{t2} (each state of the system) using:

A - adjust unit 1 only;

B - adjust unit 2 only;

C - overhaul system, adjust both units simultaneously;

D - do nothing.

For this example, the lowest average cost per period is achieved if:

unit 1 is adjusted (Decision A) when

$X_{t2} \leq 3$ and $X_{t1} - X_{t2} \geq 6$; and

unit 2 is adjusted (Decision B) when

$X_{t1} \leq 3$ and $X_{t2} - X_{t1} \geq 6$; and

the system is overhauled (Decision C) when

$X_{t1} \geq 4$ and $X_{t2} \geq 4$ and $X_{t1} + X_{t2} \geq 12$; and

otherwise, do nothing (Decision D).

Tables 6.2 - 6.5 contain the data generated by the 16 runs of the fortran program. Each table is separated into four horizontal sections:

- Parameter settings containing each run number and corresponding values of the decision variable.
- Simplified decision boundaries for:

  Adjust A = adjust unit 1 = A in policy matrix;

  Adjust B = adjust unit 2 = B in policy matrix;

  Adjust Both = overhaul = C in policy matrix.
- Average cost per period which is the long run average cost per period collected after the average cost per period stabilizes.
- CPU time (in seconds) required for the policy to stabilize.

All data for each run is in the column headed by that run number.

Table 6.2
Fortran Data: Runs 1-4

PARAMETER SETTINGS				
RUN NUMBER	1	2	3	4
μ =	.1	.1	.1	.1
λ =	.001	.001	.001	.001
K =	16	16	19	19
r=	200	200	200	200
K1=K2=	10	15	10	15
DECISION BOUNDARIES				
ADJUST A IF $X_{t1} \geq$	5	6	5	6
and $X_{t2} \leq$	2	1	3	2
ADJUST B IF $X_{t1} \leq$	2	1	3	2
and $X_{t2} \geq$	5	6	5	6
ADJ BOTH IF $X_{t1} + X_{t2}$ =	8	7	8	8
and $X_{t1} \geq$	3	2	4	3
and $X_{t2} \geq$	3	2	4	3
AVERAGE COST/PERIOD	1.37	1.38	1.40	1.47
CPU TIME (in seconds)	351	138	508	229

Table 6.3
Fortran Data: Runs 5-8

PARAMETER SETTINGS				
RUN NUMBER	5	6	7	8
μ =	.1	.1	.1	.1
λ =	.004	.004	.004	.004
K =	16	16	19	19
r=	200	200	200	200
K1=K2=	10	15	10	15
DECISION BOUNDARIES				
ADJUST A IF $X_{t1} \geq$	3	4	3	4
and $X_{t2} \leq$	1	1	2	1
ADJUST B IF $X_{t1} \leq$	1	1	2	1
and $X_{t2} \geq$	3	4	3	4
ADJ BOTH IF $X_{t1} + X_{t2} =$	5	5	6	5
and $X_{t1} \geq$	2	2	3	2
and $X_{t2} \geq$	2	2	3	2
AVERAGE COST/PERIOD	3.36	3.52	3.46	3.63
CPU TIME (in seconds)	197	172	147	91

Table 6.4
Fortran Data: Runs 9-12

PARAMETER SETTINGS				
RUN NUMBER	9	10	11	12
μ =	4	4	4	4
λ =	.001	.001	.001	.001
K =	16	16	19	19
r=	200	200	200	200
K1=K2=	10	15	10	15
DECISION BOUNDARIES				
ADJUST A IF $X_{t1} \geq$	27	34	22	32
and $X_{t2} \leq$	8	2	13	5
ADJUST B IF $X_{t1} \leq$	8	2	13	5
and $X_{t2} \geq$	27	34	22	32
ADJ BOTH IF $X_{t1} + X_{t2}$ =	36	36	36	38
and $X_{t1} \geq$	9	3	14	6
and $X_{t2} \geq$	9	3	14	6
AVERAGE COST/PERIOD	7.94	7.92	8.06	8.47
CPU TIME	2520	909	1560	1462

Table 6.5
Fortran Data: Runs 13-16

PARAMETER SETTINGS				
RUN NUMBER	13	14	15	16
μ =	4	4	4	4
λ =	.004	.004	.004	.004
K =	16	16	19	19
r=	200	200	200	200
K1=K2=	10	15	10	15
DECISION BOUNDARIES				
ADJUST A IF $X_{t1} \geq$	13	15	12	16
and $X_{t2} \leq$	4	1	7	3
ADJUST B IF $X_{t1} \leq$	5	1	7	3
and $X_{t2} \geq$	12	15	11	16
ADJ BOTH IF $X_{t1} + X_{t2} =$	18	17	20	19
and $X_{t1} \geq$	5	2	8	4
and $X_{t2} \geq$	6	2	8	4
AVERAGE COST/PERIOD	17.92	17.99	18.77	19.07
CPU TIME	952	324	1335	566

In the analysis of Chapter 7 and in the simulation chapter (Chapter 8), a simplified policy is adopted which is shown here with the simplified policy area in bold:

Simplified Policy Decision Matrix

X_{t2}	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0
X_{t1}																				
1	D	D	D	D	D	D	B	B	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**
2	D	D	D	D	D	D	D	B	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**
3	D	D	D	D	D	D	D	D	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**	**B**
4	D	D	D	D	D	D	D	C	C	C	C	C	C	C	C	C	C	C	C	C
5	D	D	D	D	D	D	C	C	C	C	C	C	C	C	C	C	C	C	C	C
6	D	D	D	D	D	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
7	A	D	D	D	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
8	A	A	D	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
9	**A**	**A**	**A**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
0	**A**	**A**	**A**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
1	**A**	**A**	**A**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
2	**A**	**A**	**A**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
3	**A**	**A**	**A**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
4	**A**	**A**	**A**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
5	**A**	**A**	**A**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
6	**A**	**A**	**A**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
7	**A**	**A**	**A**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
8	**A**	**A**	**A**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
9	**A**	**A**	**A**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
0	**A**	**A**	**A**	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C

The simplified policy description is:

unit 1 is adjusted when

$X_{t1} \geq 9$ and $X_{t2} \leq 3$; and

unit 2 is adjusted when

$X_{t1} \leq 3$ and $X_{t2} \geq 9$; and

both are adjusted when

$X_{t1} \geq 4$ and $X_{t2} \geq 4$ and $X_{t1} + X_{t2} \geq 12$; and

otherwise, do nothing.

In other words, in the simplified policy, the A and B regions are reduced to rectangular shapes (since this is where the policy stabilizes) rather than the actual trapezoid shapes that occur in the optimal solution. Most of the time, the two unit decision matrix exhibited the symmetry shown, resulting presumably from the assumption that both units experience the same average level of deterioration per unit time.

CHAPTER 7. NUMERICAL ANALYSIS OF THE TWO UNIT MODEL

7.1 Introduction

A two-level factorial analysis was used to evaluate the effects of four parameter changes on

a) cost and

b) control limits for:

doing nothing (decision D),

adjusting unit 1 (decision A),

adjusting unit 2 (decision B), or

system overhaul (decision C).

The advantages of this methodology is discussed in section 5.1.

All combinations of high and low levels of the following four decision variables yield 16 runs which are used as the basis for the 2^4 factorial analysis:

cost ratio ($K_1/r = K_2/r = 15/200, 10/200$),

cost ratio ($K/r = 19/200, 16/200$),

mean of the failure rate ($\lambda = .004, .001$), and

mean of the deterioration distribution ($\mu = 4, .1$).

Time between inspections in all cases is $T = 1$. The adjustment cost for unit 1 and unit 2 are the same ($K_1 = K_2$) and so is the level of deterioration. This has been done to allow the policy to be studied independent of these parameter

fluctuations. If the adjustment costs and deterioration levels had been allowed to change, the policy results would have been more difficult to interpret.

All tabular presentations in this chapter are organized so that:

columns 1-4 show whether the high or low value of the parameter (decision variable) was used in each run;

column 5 contains the cost or control limit being analyzed at the indicated parameter setting;

column 6 lists the effects of each factor where E1, E2, E3, and E4 are the effects of changing $K_1/r = K_2/r$, K/r, λ, and μ, respectively; the rest are the interaction effects between and among these factors as explained in section 5.2;

column 7 has the numeric estimates of the effects generated by the factorial analysis.

The "mean" (word is in column 6 and value in column 7) refers to the arithmetic average of all values in column 5, and refers only to the value to its immediate right.

7.2 Sensitivity Analysis

7.2.1 Cost Analysis

The cost analysis in Table 7.1 has average cost per period in column 5, with mean 7.9, and clear cut cost effects. The average cost per period is most affected by μ, the number of deteriorations per unit of time: as μ goes from low

Table 7.1
Cost Analysis

15/200	19/200	.004	4.0	≤HIGH		
10/200	16/200	.001	0.1	≤LOW		
Decision Variables						
K_1/r	K/r	λ	μ			
1	2	3	4	COST		Est.
-1	-1	-1	-1	1.37	MEAN	7.9
1	-1	-1	-1	1.38	E1	0.1
-1	1	-1	-1	1.40	E2	0.4
1	1	-1	-1	1.47	E12	0.1
-1	-1	1	-1	3.36	E3	6.2
1	-1	1	-1	3.52	E13	0.0
-1	1	1	-1	3.46	E23	0.2
1	1	1	-1	3.63	E123	0.0
-1	-1	-1	1	7.94	E4	10.8
1	-1	-1	1	7.92	E14	0.0
-1	1	-1	1	8.06	E24	0.3
1	1	-1	1	8.47	E124	0.1
-1	-1	1	1	17.92	E34	4.1
1	-1	1	1	17.99	E134	0.0
-1	1	1	1	18.77	E234	0.1
1	1	1	1	19.07	E1234	0.0

to high levels, the cost increases by 10.8. This seems reasonable since μ impacts all costs involved in the model. The influence of μ occurs through adjustments to unit 1, unit 2, or both units (costs K1, K2, and K, respectively) when the level of deterioration exceeds the UCL (upper control limit) and also through repairs, when either of the units fails. Recall that the failure rate is a function of accumulated deterioration, so that as a unit "wears down" (that is, vibration is increasing), the probability of failure increases. In other words, when μ is higher, average time between failures is smaller, raising costs through more frequent failures and repairs.

Factor 3, λ, has the second highest influence on cost. As λ goes from .001 to .004, the average cost per period rises 6.2. Failure cost is much higher than the adjustment costs. This would logically make adjustment or overhaul more desirable than repair, since it is more frugal. The fact that μ (which affects all costs in the model) has more impact on the cost structure than does λ, bears out this rational.

The interaction of λ and μ in E34 can't be ignored; it means that the effect of μ depends on the level chosen for λ and vice versa. This is not unexpected, because the failure rate is a function of the level of deterioration (by assumption and design) and as μ increases, the failure rate increases.

7.2.2 Boundary Analysis

Tables 7.2, 7.3, and 7.4 will show the effects of parameter changes on the decision boundaries of the optimal decision matrix.

Table 7.2 shows the analysis of the X_{t1} value (simplified policy) at which unit 1 is adjusted.

This response variable tells how much deterioration must exist in unit 1 before it should be adjusted. As μ goes from low to high, the X_{t1} value in the decision matrix increases 16.9. Logically, this seems reasonable because if the UCL is low and deterioration is high, then adjustments are triggered more often which raises costs. Therefore, in an economic model, it is logical for the UCL to rise when deterioration increases to forestall cost increases.

The failure rate in table 7.2, λ, causes X_{t1} to drop 8.4 units as levels switch. An explanation for this is that as the probability of failure increases and the cost of failure is very much higher than the cost of an adjustment, the control limit will fall to allow more frequent adjustment and circumvent higher costs due to failure.

The interaction of μ and λ lowers the UCL by 6.4 presumably because the failure rate is a function of deterioration, and since failure is such a high cost, the limit drops to adjust more often and avert higher costs from increased failures.

Table 7.2
Analysis of X_{t1} UCL for Unit 1

15/200	19/200	.004	4.0	$\leq$HIGH		
10/200	16/200	.001	0.1	$\leq$LOW		
Decision Variables						
K_1/r	K/r	λ	μ			
1	2	3	4	X_{t1}		Est.
-1	-1	-1	-1	5	MEAN	12.9
1	-1	-1	-1	6	E1	3.4
-1	1	-1	-1	5	E2	-0.9
1	1	-1	-1	6	E12	0.6
-1	-1	1	-1	3	E3	-8.4
1	-1	1	-1	4	E13	-1.4
-1	1	1	-1	3	E23	0.9
1	1	1	-1	4	E123	-0.1
-1	-1	-1	1	27	E4	16.9
1	-1	-1	1	34	E14	2.4
-1	1	-1	1	22	E24	-0.9
1	1	-1	1	32	E124	0.6
-1	-1	1	1	13	E34	-6.4
1	-1	1	1	15	E134	-1.4
-1	1	1	1	12	E234	0.9
1	1	1	1	16	E1234	-0.1

A small rise of 3.4 in X_{t1} is attributed to the parameter changes in K_1/r which again is explained by the action of reducing the number of adjustments and keeping costs low by adjusting less often.

Table 7.3 shows the X_{t2} values (simplified policy) at which unit 1 is adjusted. In essence, they indicate the degree of deterioration necessary in unit 2 to trigger an adjustment in unit 1.

In comparison to the X_{t1} values, these X_{t2} values are only slightly influential on the UCL for unit 1. The factors with the highest impact are K_1/r and μ with estimated effects of -3.0 and 3.8, respectively. Keeping in mind that K_1 and K_2 are equal, the 3.0 decrease in the X_{t2} UCL for unit one can be interpreted as implying that as the cost of adjusting 1 unit (either one) switches levels, the X_{t2} UCL for unit one decreases. That is, unit 1 now is adjusted at a lower level of unit 2 deterioration than before. This seems to imply that it becomes more cost effective to overhaul sooner, since the decision pattern in the simplified policy is A's then C's in the decision matrix.

The 3.8 rise in X_{t2} UCL for unit 1 that occurs when μ increases is deferring the adjustment of unit 1 and delaying a system overhaul. It is always cheaper to adjust a single unit than to overhaul, and the increase in deterioration would generate more adjustments and higher costs were the limits not raised.

Table 7.3
Analysis of X_{t2} UCL for Unit 1

15/200	19/200	.004	4.0	≤HIGH		
10/200	16/200	.001	0.1	≤LOW		
Decision Variables						
K_1/r	K/r	λ	μ			
1	2	3	4	X_{t2}		Est.
-1	-1	-1	-1	2	MEAN	3.5
1	-1	-1	-1	1	E1	-3.0
-1	1	-1	-1	3	E2	2.0
1	1	-1	-1	2	E12	-0.5
-1	-1	1	-1	1	E3	-2.0
1	-1	1	-1	1	E13	1.0
-1	1	1	-1	2	E23	-0.5
1	1	1	-1	1	E123	0.0
-1	-1	-1	1	8	E4	3.8
1	-1	-1	1	2	E14	-2.3
-1	1	-1	1	13	E24	1.3
1	1	-1	1	5	E124	-0.3
-1	-1	1	1	4	E34	-1.3
1	-1	1	1	1	E134	0.8
-1	1	1	1	7	E234	-0.3
1	1	1	1	3	E1234	0.3

The next set of analyses in this progression would logically be the X_{t1} and X_{t2} values at which unit 2 is adjusted, and then the X_{t1} and X_{t2} values at which both units are adjusted. However, having observed the symmetry present in the policy decision matrix, it is not surprising that the following results of factor analysis are identical (aside from the mean values):

X_{t1} value for adjusting unit 1 = X_{t2} value for adjusting unit 2

X_{t2} value for adjusting unit 1 = X_{t1} value for adjusting unit 2.

Additionally, since the A and B regions of the decision matrix are bordered by region C, these results (aside from the means) are also identical:

X_{t1} value for adjusting both = X_{t2} value for adjusting unit 1

X_{t2} value for adjusting both = X_{t1} value for adjusting unit 2.

Accordingly, these duplications will be omitted to avoid redundancy.

The last boundary to investigate in the decision matrix is the sum of X_{t1} and X_{t2} defining the diagonal for overhauling the system. It separates the decision area between do nothing (D) and an overhaul (C). Table 7.4 displays the parameter induced changes in the sum of X_{t1} and X_{t2} defining the diagonal area C (overhaul of the system). This sum indicates the joint aggregate amount of deterioration required in both units before they are adjusted simultaneously.

Table 7.4
Analysis of Sum of X_{t1} and X_{t2} Required for Overhaul

15/200	19/200	.004	4.0	≤HIGH		
10/200	16/200	.001	0.1	≤LOW		
Decision Variables						
K_1/r	K/r	λ	μ			
1	2	3	4	$X_{t1} + X_{t2}$		Est.
-1	-1	-1	-1	8	MEAN	17.0
1	-1	-1	-1	7	E1	-0.3
-1	1	-1	-1	8	E2	1.0
1	1	-1	-1	8	E12	0.3
-1	-1	1	-1	5	E3	-10.3
1	-1	1	-1	5	E13	- 0.5
-1	1	1	-1	6	E23	0.3
1	1	1	-1	5	E123	- 0.5
-1	-1	-1	1	36	E4	21.0
1	-1	-1	1	36	E14	0.3
-1	1	-1	1	36	E24	0.5
1	1	-1	1	38	E124	0.3
-1	-1	1	1	18	E34	- 7.8
1	-1	1	1	17	E134	- 0.5
-1	1	1	1	20	E234	0.3
1	1	1	1	19	E1234	0.0

The major influences on the sum of deterioration are well-defined: parameter increases in μ raise the sum by 21, the switch in λ lowers the sum by 10.3, and their interaction lowers the sum by 7.8.

The explanations for these effects parallel those for the individual unit UCLs. When μ increases, the boundary represented by the sum of deterioration should also increase to avoid cost spirals from more frequent overhauls. But, jumps in λ from low to high levels cause the boundary to retreat, inducing more frequent adjustment and deterring the much higher cost of failure. Again, the interaction of λ and μ is prominent and lowers the diagonal UCL for C. Recalling that the failure rate is a function of deterioration, one realizes that increases in μ imply higher failure rates and consequently, lower time between failures. It is more economic, then, to overhaul the system sooner at cost K, than to repair at much higher cost r.

The next chapter discusses the two unit simulation model, its results, and the error of the simulation solution compared to the dynamic programming solution.

CHAPTER 8. TWO UNIT SIMULATION

8.1 Introduction

Using the simplified policy, a simulation model for a two unit system was created to ensure that the average costs obtained from the optimal model coincide with reality. The complexity of boundary descriptions in the optimal model necessitates the relaxation.

Pegden et al (1990) describe a non-terminating system as one in which no event occurs to return the system to a fixed initial condition; the two unit system fits this category. They cite two major difficulties that are encountered in non-terminating systems:

1. Bias is introduced in the initial transient phase of the simulation because there is no natural basis for choosing the starting conditions or length of the run. In addition, in non-terminating systems, the focus is on the steady state behavior of the system, which is its behavior over a very long time. The initial phase varies with starting conditions, and there is no definite point where the system's behavior changes from transient to steady-state. The simulation is said to be in steady-state once the initial transient phase has declined to a point where the impact of initial conditions is negligible (even though this point is unpredictable). One solution

Pegden et al (1990) suggest is to use a warm-up period that is discarded along with its related bias. So, in this simulation, after observing a plot of the simulation response over time, a visual determination of the point where the system "settled down" (went into steady-state) was taken as 500 periods.

2. Estimation of the variance for determining confidence intervals is more difficult because there is no obvious point defining the end of a replication, and each new replication contains initial bias. If a single long run is used, the result is highly correlated observations within the run. This greatly complicates the analysis because then the observations are no longer independent. One solution offered to this dilemma is to estimate the variance of the mean by generating independent replications of the model. In this simulation, 5 replications of 10,500 periods each were used to estimate each mean and variance.

To reiterate: with the preceding information in mind, each of the 16 scenarios from the factorial design was simulated using 5 replications of 10,500 periods each. Every replication contained a warm-up of 500 periods.

8.2 Simulation Results

Table 8.1 displays the simulation results for the 16 runs organized so:

columns 1-4 show whether the high or low value of the particular parameter (factor) was used in each run;

column 5 contains the estimate of $\mu_{cost/period}$, the average cost per period for each of the 16 runs;

column 6 lists the 95% confidence interval estimates of the average cost per period constructed for each of the 16 runs. Each interval is built with the corresponding $\mu_{cost/period}$ from column 5 and corresponding standard error of the estimate (from column 6). The number of replications (sample size) used to obtain each μ was 5.

column 7 has the standard deviation of the average cost per period obtained from a sample of 5 replications, each 10,500 periods long, with a 500 period warm-up.

8.3 Error of the Simulation Solution

Next, the performance of the simulation cost solution described in this chapter is compared to the dynamic programming cost solution obtained in Chapter 6. Table 8.2 makes those comparisons.

Again, columns 1-4 contain the 16 different parameter setting of the 2^4 factorial experimental design. Column 5 has the average simulation cost per

Table 8.1
Simulation Results

Decision Variables						
k_1/r	k/r	λ	μ			
1	2	3	4	$\mu_{cost/per}$	95% c. i.	$\sigma_{\bar{x}}$
-1	-1	-1	-1	1.38	$1.30 \leq \mu \leq 1.45$	.0619
1	-1	-1	-1	1.43	$1.31 \leq \mu \leq 1.56$	.1010
-1	1	-1	-1	1.40	$1.34 \leq \mu \leq 1.47$	.0547
1	1	-1	-1	1.51	$1.41 \leq \mu \leq 1.61$	.0139
-1	-1	1	-1	3.35	$3.15 \leq \mu \leq 3.55$	.164
1	-1	1	-1	3.48	$3.30 \leq \mu \leq 3.66$	.144
-1	1	1	-1	3.42	$3.19 \leq \mu \leq 3.65$	.186
1	1	1	-1	3.62	$3.44 \leq \mu \leq 3.80$	.144
1	-1	-1	1	8.10	$7.71 \leq \mu \leq 8.49$	.314
1	-1	-1	1	8.07	$7.68 \leq \mu \leq 8.46$	.312
-1	1	-1	1	8.62	$8.44 \leq \mu \leq 8.79$	.143
1	1	-1	1	8.59	$8.25 \leq \mu \leq 8.93$	.274
-1	-1	1	1	18.00	$17.6 \leq \mu \leq 18.5$	.347
1	-1	1	1	18.20	$17.7 \leq \mu \leq 18.6$	.358
-1	1	1	1	19.20	$18.7 \leq \mu \leq 19.6$	.336
1	1	1	1	19.40	$19.1 \leq \mu \leq 19.8$	.312

Table 8.2
Relative Percent Error of the Simulated Average Cost/Period

Decision Variables				Simulation	DP	
k_1/r	k/r	λ	μ			
1	2	3	4	$\mu_{cost/per}$	$\mu_{cost/per}$	RPE
-1	-1	-1	-1	1.38	1.37	0.7%
1	-1	-1	-1	1.43	1.38	3.6
-1	1	-1	-1	1.40	1.40	0.0%
1	1	-1	-1	1.51	1.47	2.7%
-1	-1	1	-1	3.35	3.36	-0.3%
1	-1	1	-1	3.48	3.52	-1.1%
-1	1	1	-1	3.42	3.46	-1.2%
1	1	1	-1	3.62	3.63	-0.3%
1	-1	-1	1	8.1	7.94	2.0%
1	-1	-1	1	8.07	7.92	1.9%
-1	1	-1	1	8.62	8.06	6.9%
1	1	-1	1	8.59	8.47	1.4%
-1	-1	1	1	18.0	17.9	0.6%
1	-1	1	1	18.2	18.0	1.1%
-1	1	1	1	19.2	18.8	2.1%
1	1	1	1	19.4	19.1	1.6%

period for each of the 16 runs, and column 6 has the optimal average dynamic programming (DP) cost per period for each of the 16 runs. Column 7 has the Relative Percent Error (RPE) between the simulation average cost and optimal DP average cost defined by:

$$RPE = \frac{Simulation\ \mu_{cost/per} - DP\ \mu_{cost/per} * 100\%}{DP\ \mu_{cost/per}}.$$

As evidenced below, the simulation provides good estimates of the optimal DP average costs. Even though most of the time (11/16) the simulation average overestimates the DP results, it is always within 6.9% of the optimal average cost.

Next, Table 8.3 shows a cumulative frequency distribution of the RPE's. The classes are phrased as "within 1%", "within 2%",..., "within 7%". In the first row, this means that in 5 out of the 16 simulation runs, the average costs per period that were generated were no more than 1% larger or 1% smaller than the optimal average cost per period found in the dynamic programming solution. Therefore, in the final analysis, all of the averages produced by the simulation runs were no more than 7% bigger or smaller than their DP counterpart (equations 21-25), and 93.75% were no more than 4% bigger or smaller than the optimal mathematical solution produced using equations (21-25) in Chapter 6.

Table 8.3
Cumulative Frequency Distribution of the RPE's

Classes	Cumulative Frequency	Cumulative %
within 1%	5	31.25%
within 2%	12	75.00%
within 3%	14	87.50%
within 4%	15	93.75%
within 5%	15	93.75%
within 6%	15	93.75%
within 7%	16	100.00%

Most important from a statistical standpoint, only one of the sixteen DP averages does not fall into the 95% confidence interval estimate produced by the simulation.

An appraisal of the simulation model based on these calculations leads to the conclusion that it does provide an adequate estimate of the optimal mathematical average cost per period. Conversely, if the simulation model is used to determine whether the equations of Chapter 6 furnish a realistic model of the two unit system, again the conclusion is positive. In essence, the two models support each other.

CHAPTER 9. MULTIPLE UNIT MODEL

9.1 Introduction

A major benefit of the foregoing research, especially the two unit model, has been to demonstrate the existing policy structure. The objective of this chapter is to describe the system problem setting and model for n units using the same basic assumptions of the single and two unit systems. Constraints and limitations of the DP approach will be profiled.

9.2 Assumptions

The assumptions for the multiple unit model are:

1. The system consists of n units arranged in a series.
2. Individual measurements of the external variable (X_{ti}) are taken at the beginning of each period t for each unit i, (i = 1,2,...n), and used to determine whether or not the unit should be adjusted.
3. The state space (representing the quantity of the external variable) of each unit is continuous.
4. In each period t, a random amount of deterioration occurs in each unit i, (i = 1,2,...n), which is independent, identically distributed, and strictly non-negative. No unit can improve spontaneously.
5. Failures are assumed to occur at the end of the period; the time between failures is exponentially distributed, with failure rate an

increasing function of the state variable, X_{ti}. The failure rate for each unit is $\lambda(X_{ti})$, $(i=1,2....n)$.

6. The probability of failure for each unit is independent and identically distributed. The cost of failure for each unit is r.

7. The decision to overhaul the system or adjust any unit individually is based on the level of the external variable present at the beginning of period t. An overhaul has cost K. The cost of adjusting any unit individually is K_i, $(i = 1,2,\ldots,n)$, and $K_1=K_2=\ldots=K_n$. In addition, it is more economical to overhaul the system (adjust all units simultaneously) than to adjust any combination of units individually;

$$K_i < K < \sum_{i=1}^{n} K_i \; for \; i = 1,2,...,n.$$

8. The cost of failure is much higher than the cost of overhauling the system and consequently much higher than adjusting a single unit individually: $r >> K > K_i$.

9. After an adjustment or repair, the state of unit i, $(i=1,2,\ldots,n)$, has an initial value, $X_{0i} = 1$, which is fixed and not a decision parameter. After an overhaul, all units are in state $X_{0i}=1$, $(i = 1,2,\ldots,n)$.

9.3 Notation

The notation for the multiple unit model is defined below:

$g_i(y)$	probability distribution function of deterioration for unit i, (i = 1,2,...,n).
$g(z)$	distribution of the sum of deterioration in all units together.
$f_t(X_{t1},X_{t2})$	two unit cumulative minimum cost in period t, for that period and all future periods.
$H(\cdot)$	conditional expected cost of failure.
λ	parameter of the exponential failure distribution which is a function of the state of the unit.
μ	average number of deteriorations per unit time.
r	cost of repair when unit i, (i = 1,2,...,n), fails.
R_i	reliability of unit i, (i = 1,2,...,n), the probability that unit i does not fail before time T.
T	length of the inspection period.
y_{ti}	random amount of deterioration occurring in period t in unit i, (i = 1,2,...,n).
X_{0i}	initial state of unit i, (i=1,2,...,n), after a repair, adjustment, or overhaul.

X_{ti} measurement of the external variable taken at the beginning of each period t that describes the state of unit i.

9.4 System Problem Setting and Model

Our system consists of n units arranged in a series, and monitored in discrete, fixed time intervals of length T. Let the characteristic X_{ti} represent the measurements of an external variable which describe the state of unit i,(i=1,2,...,n) at the beginning of each period t. This state space is continuous.

The vector $\vec{X}_t$ describes the entire system of n units by

$$\vec{X}_t = \begin{Vmatrix} x_{t1} \\ x_{t2} \\ \cdot \\ \cdot \\ \cdot \\ x_{tn} \end{Vmatrix} .$$

Using these measurements of the external variable for each unit i, (i=1,2,...,n), in period t, X_{ti}, the probability that unit i, (i=1,2,...,n), will fail within the next time interval is assumed to be exponentially distributed, where the time between failures for that unit is a function of the state of the unit, that is, the rate of occurrence of failures is $\lambda(X_{ti})$, (i=1,2,...,n). In addition, as described in the single unit model, a non-negative deterioration occurs for each unit i,

(i=1,2,...,n), during each inspection period. This random amount of deterioration for unit i, (i=1,2,...,n), in period t is y_{ti}. The values of y_{ti} are assumed to be independent and identically distributed for i = 1,...,n. Therefore,

the vector $\vec{y}_t$ consists of the amounts of deterioration occurring in units i = 1,2,...,n during period t and is represented by

$$\vec{y}_t = \begin{Vmatrix} y_{t1} \\ y_{t2} \\ \cdot \\ \cdot \\ \cdot \\ y_{tn} \end{Vmatrix} .$$

Using the preceding information, we write the updating equation as

$$\vec{X}_{t+1} = \vec{X}_t + \vec{y}_t \quad . \tag{26}$$

In a series system, the reliability of unit i is usually represented by R_i, signifying the probability that the unit will NOT fail by time t or, alternately, the probability that the time between failures is greater than t. Since the time between failures is exponential, with the average time between failures expressed as a function of the state of the unit, this reliability for any one unit can be expressed with the following equation:

$$e^{-\lambda_i (X_{ti} + y_{ti})T} \quad . \tag{27}$$

Accordingly, the reliability for a system of n units is represented by:

$$e^{-\sum_{i=1}^{n}\lambda_i(X_{ti}+y_{ti})T} \quad . \tag{28}$$

The cost of failure per inspection period in the system, given that the amount of deterioration for each unit i was X_{ti}, can be found by taking the product of the cost of repair when a failure occurs for one unit and the probability of system failure. The mathematical expression is

$$r[1-e^{-\sum_{i=1}^{n}\lambda_i(X_{ti}+y_{ti})T}] \quad . \tag{29}$$

The conditional expected cost of failure, given that the amount of deterioration due to the external variable at the beginning of that period for each unit i was the vector $\vec{X}_t$, is found by integrating equation (29) with respect to the vector of distributions of deterioration, $g(\vec{y})$. The formulation is

$$H(\vec{X}_t)=\int_0^{\infty} r[1-e^{-\sum_{i=1}^{n}\lambda_i(X_{ti}+\vec{y})T}]g(\vec{y})d\vec{y} \ . \tag{30}$$

For n units, the conditional expected cost of failure, $H(X_{t1},X_{t2},\ldots X_{tn})$, given $X_{t1},X_{t2},\ldots X_{tn}$ is given by:

$$H(X_{t1},X_{t2},\ldots,X_n)=\int_0^{\infty} r[1-e^{-\lambda(X_{t1}+X_{t2}+,\ldots,tX_n+z)T}]g(z)dz \ . \tag{31}$$

where $z = y_{t1} + y_{t2} +,\ldots, + y_n$ and $g(z)$ = distribution of the sum of deterioration in unit 1, unit 2,..., unit n.

At this point in the formulation, the number of decision alternatives (concerning adjustments) at the beginning of period t goes from 4, using a two unit system, to 8, using a three unit system, to 2^n, with n units. Similarly, the number of failure alternatives at the end of the period goes from 4 with two units to 8 using three units to 2^n with n units. Therefore the number of total possible outcomes for each period escalates from 16 with two units, to 64 with three units, to 256 with 4 units, to 2^{2n} for n units.

This escalation overflows into the minimization equation. Recall equation 22 in Chapter 6, where the terms of $f_t(X_{t1},X_{t2})$, represent:

·the adjustment cost plus the conditional expected cost in the current period,

·future minimum cost when neither unit fails,

·future minimum cost when unit 1 fails,

·future minimum cost when unit 2 fails,

·future minimum cost when both units fail.

The last four terms of this equation compute the future minimum costs for the 4 different possible failure combinations in the two unit case settings referenced above. When the number of different outcomes jumps to 64, 256, and 2^{2n} with three, four, and n units, respectively, so will the number of terms necessary to

handle all possible failure combinations expand to 9, 17, and $2^n + 1$ in equation 22. Obviously, with this number of terms, the formulation becomes unwieldy even with three units. Furthermore, even if one had the stamina to pursue this solution approach, computer storage for vectors to handle deterioration (100 for 1 unit, 100^2 for 2 units, 100^3 for 3 vectors) and CPU time become insurmountable constraints. Average CPU time for 1 run of the two unit model was 716 seconds or about 12 minutes but the maximum was 42 minutes.

Accordingly, in future research the multiple unit model will probably be approached using the simplified policy and response surface simulation.

Using the two unit policy findings of Chapter 6, the simplified policy for a system of i units ($i = 1,2,\ldots,n$) is expected to parallel that of the two unit model:

Adjust unit i if

the deterioration for unit i $\geq UCL_{\text{unit } i}$ and

the deterioration of each OTHER unit j $\leq UCL_{\text{unit } j}$;

Overhaul the system when

sum of deterioration in all units $\geq UCL_{\text{sum}}$ and

the deterioration of each unit in the system $> UCL_{\text{overhaul}}$.

Otherwise do nothing.

CHAPTER 10. SUMMARY AND CONCLUSION

The question addressed by this research is "When is it cheaper to repair a deteriorating production system rather than fix its individual parts?" The question is answered for a single and a two unit system that suffer "shocks" of increasing wear.

In the single unit system, two models have been considered for deciding when to perform preventive maintenance on a single unit. Both models use a cost criteria, and the second model (steady state model) is an approximation of the first (dynamic programming model). The approximation is accomplished by disregarding the fact that a failure (which has a low probability) followed by an immediate repair restores the state back to state X_0 and viewing the resultant simplification as a clearing system.

A numerical study to compare the performance of each model singularly and comparatively reveals, first, that the best approximation for cost and D is achieved when μ is high and λ is low; the level of K/r is irrelevant. Moreover, even when D is poor, the cost differential for increasing the parameter levels is minimal: a rise of 5.5% when λ goes from its low to high level, and drop of 7% when μ goes from its low to high level. Second, the numerical analysis discloses that the approximation results are mostly a poor solution of when to perform preventive maintenance. Hence the conclusion is that restoring the system after

failure should NOT be ignored since this action results in underestimations as large as 50% from the exact values of D and overestimates in cost as high as 20%. It should be noted that earlier works in the literature have ignored the probability of failure during the period, which prompts the deduction that the control limit is larger in reality than these works purport them to be.

The contribution of the exact solution, which includes the cost of failure (omitted by most models), is significant as a basis of practical comparison for other models (which also omit the probability of failure) and as a benchmark for measuring the worth of more complicated solutions.

In the two unit system, the decision is whether to overhaul the entire system or continue to endure increasing downtime and repair costs from individual units. Two models were used to confront this issue: a dynamic programming formulation and a simulation design.

The dynamic programming model monitors an external variable for each unit and uses the two quantities collectively as a predictor for failure. This mathematical formulation provides the optimal average cost per period for a system of two units using a numeric process. It also generates a decision policy with which to achieve the optimum. A factorial experimental design was then used for sensitivity analysis of the model. The number of deteriorations per unit time, μ, and λ, the failure rate, and their interaction are major influences on the average cost per period. The boundaries, on the other hand, are also most

affected by μ, λ, and their interaction, but they also exhibit a small impact from K_1 and K_2, the adjustment cost for individual units.

Making use of a simplified version of the decision policy provided by the mathematical model, the simulation process also generated the optimal average cost per period. The 16 scenarios from the experimental design were run using 5 replications of 10,500 periods each. Every replication had a 500 period warm-up.

Comparisons of the simulation results to the mathematical results were favorable. Approximately ninety-four percent of all average costs generated by the simulation were no more than 4% larger or smaller than the optimal average cost per period found using dynamic programming.

The problem setting and model for n units was described along with a discussion of why the dynamic programming approach would be onerous. Response surface simulation was suggested as an alternative.

Further research and extensions would involve:

- adding more units to the system,
- dropping the independence assumption for the existing two unit system,
- searching for a system heuristic based on the single unit model,
- changing the failure distribution, and
- incorporating non-linear cost functions.

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APPENDIX A. ONE UNIT FORTRAN PROGRAM

```
FILE: ONEUNIT   FORTRAN   A1                 VM/ESA Conversational Monitor System

C********************* MAIN PROGRAM :********************
C
C                      ===>DESCRIPTIONS<==
C
C      PROGRAM FINDS THE OPTIMAL SOLUTION FOR A DYNAMIC PROGRAMMING
C      PROBLEM FORMULATED TO FIND THE MOST ECONOMIC TIME
C      TO REPAIR A SINGLE UNIT SUBJECT TO RANDOM SHOCKS
C
C                      ====>VARIABLES<====
C
C      DEL =    DELTA OF X SUB T = INDICATOR FOR WHETHER
C                AN ADJUSTMENT HAS OCCURRED: DELTA = 1
C                MEANS AN ADJUST HAS OCCURRED
C      F(Y)=     DISTRIBUTION OF DETERIORATION: ASSUMED POISSON
C      LAM =    NUMBER OF FAILURES PER UNIT TIME
C      LT  =    TIME = COUNTER FOR PERIOD NUMBER = BEGINNING OF PERIOD
C      MU  =    AVERAGE NUMBER OF DETERIORATIONS PER UNIT TIME
C                NOTE: LT STANDS FOR LOWER CASE T
C       R   =    COST OF FAILURE
C       T   =    LENGTH OF THE INSPECTION PERIOD
C       XT  =    CUMULATIVE VIBRATION MEASUREMENT TAKEN AT BEGINNING OF
C                 PERIOD I THAT DESCRIBES THE STATE OF THE UNIT = X SUB T
C       Y   =    RANDOM AMOUNT OF VIBRATION INCREASE OCCURRING IN EACH
C                 PERIOD I
C
C                       ====>COSTS<===
C
C       COST       = TOTAL MINIMUM COST FOR PERIOD T AND ALL PREVIOUS
C                     PERIODS FOR A VIBRATION AMOUNT OF X SUB T
C                    VIBRATION AMOUNT OF X SUB T
C       CXT        = C(X SUB T) = CONDITIONAL E(COST OF FAILURE)
C       FCNF       = EXPECTED FUTURE COST OF NO FAILURE
C       FCFAIL     = EXPECTED FUTURE COST OF FAILURE
C       FT(I,J)    = MATRIX /F SUB T OF X SUB T
C                  = COST AT PERIOD LT FOR STATE X SUB T
C                  = EQ 4/PAPER (I = PERIODS, J= VALUE OF X SUB T)
C       INUMPER    = NUMBER OF PERIODS PROGRAM WILL RUN
C       K          = COST OF AN ADJUSTMENT
C
C
C
C      ======> SUBROUTINES
C      SUBROUTINE CECOST===>FINDS CONDITIONAL EXPECTED COST OF FAILURE
C                           RETURNS CXT
C      SUBROUTINE FCNF   ===>FINDS E(FAILURE COST OF NO FAILURE)
C                           RETURNS FCNFSUM
C      SUBROUTINE FCFAIL===>FINDS E(FUTURE COST OF FAILURE)
C                           RETURNS FCFSUM
C
C
C
C                  ====>INITIAL VALUE OF VARIABLES<====
C
C      K = 16.0 = COST OF AN ADJUSTMENT AFTER X SUB T EXCEEDS UCL
C
```

```
C      LAM = .001 = LAMBDA = 2 FAILURES IN 1000 TIME UNITS
C      LAM = .001D0 FORCES ALL ZEROS AFTER THE 2 TO BE ZEROS ALSO
C      MU =4 = AVERAGE # OF UNITS OF DETERIORATION PER UNIT TIME
C      R = 200 = COST OF A REPAIR AFTER FAILURE: K < R BY ASSUMPTION
C      T = 1 = LENGTH OF INSPECTION PERIOD
C      XTO = 1 = INITIAL AMOUNT OF VIBRATION PRESENT IN UNIT = X SUBO
C      Y = 0 = Y SUB T = INCREASE IN VIBRATION PER PERIOD
C***********************************************************************
C
C******************** MAIN PROGRAM CODE *****************
C
C
C
C
       REAL K, LAM, R, FT(401,200), MU
       INTEGER LT, T, DELTA, Y, XT
       COMMON FT,LT,K,LAM,MU,R,T,XTO
C
C
C
C------------------------------------INITIALIZE VARIABLES
C
C
       INUMPER = 400
      K = 16.0
      LAM = .001D0
      R = 200
      MU = 4.0D0
      T = 1
      LT = 0
      XTO = 1.0
      CXT = 0.0
      FCNFSUM = 0.0
      FCFSUM = 0.0
      XT = 0
      Y = 0
             PRINT *
             PRINT *
             PRINT *
             PRINT *, '       *******RESULTS FROM PROGRAM*******'
      PRINT *,     'WHEN'
      PRINT *,     'K      = COST OF ADJUSTING             = ', K
      PRINT *,   'LAMBDA = # FALIURES PER UNIT TIME        = ', LAM
      PRINT *,     'R      = COST OF FAILURE               = ', R
      PRINT *,     'MU = AVG. # DETERIORATIONS PER UNIT TIME = ', MU
      PRINT *,     'T = LENGTH OF INSPECTION PERIOD        = ', T
      PRINT *,     'XTO    = INITIAL AMOUNT OF VIBRATION   = ', XTO
              PRINT *
              PRINT *
              PRINT *
              PRINT *
              PRINT *
              PRINT *
              PRINT *
              PRINT *
```

```
           PRINT *
           PRINT *
C
C
C
C     INITIALIZE ALL VALUES OF FT OF XT TO ZERO
C
      DO 10 I = 0, 400
         DO 50 J = 1, 200
         FT(I,J) = 0.0
50       CONTINUE
10    CONTINUE
C
C
C
C
C-------------------------------DO FOR NUMBER OF PERIODS = LT
C-------------------------------START NEW PERIOD
C
           DO 40 LT = 1, INUMPER
              ICOUNT = 0
              GRANSUM = 0.0
C-------------------------------FINDING F SUB T OF X SUB T
C-------------------------------FOR 200 VALUES OF X SUB T
C
C
              DO 20 XT = 1, 200
               ICOUNT = ICOUNT + 1
                 CALL CECOST(XT, CXT)
                 CALL FCNF(XT, FCNFSUM)
                 CALL FCFAIL(XT, FCFSUM)
                 FT(LT,XT) = CXT + FCNFSUM + FCFSUM
C
C
C-------------------------------GRANSUM WILL COLLECT COSTS
C-------------------------------IN AN INDIVIDUAL PERIOD
C-------------------------------TO FIND THE PERIOD AVERAGE
C
                 GRANSUM = GRANSUM + FT(LT,XT)
C
C
                 IF (XT .EQ. 1)
     X            PRINT *, 'THE COST FOR PERIOD',LT,'WHEN XT IS',
     X                   XT, 'WILL BE', FT(LT,XT)
C
C
C
C
           IF ((FT(LT,XTO) + K) .LT. FT(LT,XT))THEN
               PRINT *, 'THE COST FOR PERIOD',LT,'WHEN XT IS',
     X                 XT, 'WILL BE', FT(LT,XT)
           PRINT *, '===>CHEAPER TO ADJUST WHEN XT HAS REACHED',XT,
     X                 'IN PERIOD', LT, '<=========='
C
C--------------------------------------------------------------
C                     CALCULATE AND PRINT % DIFF FROM XTO TO UCL
```

```
C---------------------------------------------------------------------
C
      DIFF = ((FT(LT,XT) - FT(LT,XTO))/ FT(LT,XTO)) * 100
      PRINT *,
     X 'THE PERCENT DIFFERENCE IN COST FROM XTO TO UCL IS', DIFF,'%'
C
C---------------------------------------------------------------------
C                     CALCULATE AND PRINT AVERAGE FOR THIS PERIOD
C---------------------------------------------------------------------
C
      AVG = GRANSUM/ICOUNT
      PRINT *, 'AVERAGE AND ICOUNT FOR PERIOD',LT, 'ARE', AVG,ICOUNT
C     PRINT *
C
C ----------------------------SET REST OF COSTS IN F-SUB-T-OF-X SUB-T
C---------------------------- IN THIS PERIOD
C---------------------------- TO THIS MIN VALUE-[F(LT,XTO)+ K]
C---------------------------- THEN GO TO NEXT PERIOD
C
C
              DO 70 IREMAIN = ICOUNT, 200
                       FT(LT, IREMAIN) = FT(LT,XTO)+ K
70            CONTINUE
              GO TO 111
            ENDIF
C------------------------------------------------GET NEW-VALUE-OF-XSUBT
20          CONTINUE
C---------------------------- COMPUTE LONG-RUN-AVERAGE COST
      PRINT*,'ALL VALUES OF XT USED AND NO UCL'
      GO TO 40
C
111     ALONGAVG = AVG/LT
      PRINT *, 'THE LONGRUN AVG. COST FOR PERIOD',LT,' IS',
     XALONGAVG
           PRINT *, '***********************************************'
        PRINT *
C------------------------------ GO TO NEXT PERIOD
40      CONTINUE
      STOP
      END
C
C
C********************SUBROUTINE CECOST*******************************
C
C
C     THIS SUBROUTINE FINDS
C     THE CONDITIONAL EXPECTED COST OF FAILURE BASED ON
C                     LAM, MU, R, XT IN MAIN IS XTS HERE, T
C                     "COST" REPRESENTS ONE ITERATION
C                      IN THE NUMERICAL INTEGRATION PROCESS OF FINDING
C                      THE VALUE OF C OF X SUB T (EQUATION 4 OF PAPER)
C234567
      SUBROUTINE CECOST(XTS,CXT)
      REAL K, LAM, R, FT(401,200), COST, CXT, MU
      INTEGER LT, Y, XTS
```

FILE: ONEUNIT FORTRAN A1 VM/ESA Conversational Monitor System

```
      COST = 0.0
      CXT = 0.0
      Y = 0
      K = 16.0
      LAM = .001D0
      MU = 4.0D0
      T = 1
      R = 200
 100  COST = R*(1 - EXP(-LAM*(XTS + Y)*T))*(EXP(-MU)*MU**Y)/
     X (FAC(Y))
      IF (COST .GT. 1.0E-07 ) THEN
         CXT = CXT + COST
         Y = Y + 1
         GO TO 100
      ELSE
      GO TO 150
      ENDIF
 150  RETURN
      END
C
C
C*******************SUBROUTINE FCNF*************************************
C
C
      SUBROUTINE FCNF(XTS ,FCNFSUM)
      REAL K, LAM, R, FT(401,200), FCNFONE, FCNFSUM, MU
      INTEGER LT,T, DELTA, Y, XTS
      COMMON FT,LT,K,LAM,MU,R,T,XTO
C
C
C
C*****************************************************************
C     THIS SUBROUTINE FINDS
C          THE EXPECTED FUTURE COST OF NO FAILURE
C          IN EQUATION 4 PAPER
C          USING NUMERICAL INTEGRATION
C          FCNFONE REPRESENTS ONE ITERATION IN THE NUM. INT. PROCESS
C          XTS IN SUBROUTINE IS XT IN MAIN
C*****************************************************************
C
C
C
      FCNFSUM = 0.0
      FCNFONE = 0.0
            Y = 0
          XTO = 1.0
            K = 16.0
          LAM = .001D0
           MU = 4.0D0
            T = 1
            R = 200
C
C
 200  FCNFONE = (FT((LT-1), (XTS + Y))*EXP(-LAM*(XTS + Y)*T))
     X *(EXP(-MU))*MU**Y/FAC(Y)
```

FILE: ONEUNIT FORTRAN A1 VM/ESA Conversational Monitor System

```
C
C
C
CC        PRINT *, 'FCNFONE = ', FCNFONE, 'WHEN Y = ', Y
CC        PRINT *, 'FT((LT - 1), XTS + Y)) IS ',FT((LT - 1),(XTS + Y))
C
C
        IF (FCNFONE .GT. 1.0E-07)  THEN
         FCNFSUM = FCNFSUM + FCNFONE
C
C
CC        PRINT *, 'FCNFONE = ', FCNFONE, 'WHEN Y = ', Y
C
C
          Y = Y + 1
          GO TO 200
        ELSE
C
C
CC         PRINT *, 'FCNFONE .LE. 1.0E-7 WHEN Y = ', Y
CC         PRINT *, 'FCNFSUM =', FCNFSUM
CC         PRINT *
CC         PRINT *
C
C
           GO TO 250
        ENDIF
 250    RETURN
        END
C
C**********************SUBROUTINE FCFAIL****************************
C
C     THIS SUBROUTINE FINDS
C         THE EXPECTED FUTURE COST OF FAILURES
C         FOR EQUATION 4 USING NUMERICAL INTEGRATION
C         VALINT IS ONE ITERATION IN THE NUM. INT. PROCESS
C
C
      SUBROUTINE FCFAIL(XTS, FCFSUM)
C
C
C
      REAL K, LAM, R, FT(401,200), FCFSUM, MU, XTO
      INTEGER T, Y, XTS
      COMMON FT,LT,K,LAM,MU,R,T,XTO
      FCFSUM = 0.0
      Y = 0
      K = 16.0
      XTO = 1.0
      LAM = .001D0
      MU = 4.0D0
      T = 1
      R = 200
      SUMINT = 0.0
CC    PRINT *, 'Y = ',Y,'XTS =' ,XTS, 'LAM =',LAM, 'MU =', MU
```

```
 300  VALINT=EXP(-LAM*(XTS + Y)*T)*
     X(EXP(-MU))*MU**Y/FAC(Y)
      IF (VALINT .GT. 1.0E-07 )  THEN
         SUMINT = SUMINT + VALINT
CC       PRINT *, 'VALUE OF INTEGRAL IS ', VALINT, 'WHEN Y = ', Y
C
          Y = Y + 1
          GO TO 300
      ELSE
        FCFSUM=FT((LT - 1), XTO)*(1-SUMINT)
CC        PRINT *, 'VALUE OF INTEGRAL .LE. 1.0E-07 WHEN Y = ', Y
CC        PRINT *
CC        PRINT *
CC        PRINT *, 'FCFSUM =', FCFSUM
          GO TO 350
      ENDIF
 350  RETURN
      END
```

APPENDIX B. TWO UNIT FORTRAN PROGRAM

FILE: NUMATRIX FORTRAN A0 VM/ESA Conversational Monitor System

```
C
C
C                       <<<< TWOUNIT>>>>
C
C
C
C                       ===>DESCRIPTIONS<==
C
C        PROGRAM FINDS OPTIMAL SOLUTION FOR THE DYNAMIC PROGRAMMING
C        PROBLEM OF FINDING THE MOST ECONOMIC TIME
C        TO REPAIR A SYSTEM OF TWO UNITS SUBJECT TO RANDOM SHOCKS
C
C                       ====>VARIABLES<====
C
C        DEL1    = DELTA ONE = INDICATOR FOR WHETHER
C                  AN ADJUSTMENT HAS OCCURRED IN UNIT-1:
C                  DELTA 1 = 1 MEANS AN ADJUST HAS OCCURRED IN UNIT 1
C
C        DEL2    = DELTA TWO = INDICATOR FOR WHETHER
C                  AN ADJUSTMENT HAS OCCURRED IN UNIT-2:
C                  DELTA 2 = 1 MEANS AN ADJUST HAS OCCURREDIN UNIT-2
C
C        F(Y)    = DISTRIBUTION OF DETERIORATION FOR UNIT-1;
C                  ASSUMED POISSON
C
C
C        F(W)    = DISTRIBUTION OF DETERIORATION FOR UNIT-2;
C                  ASSUMED POISSON
C
C
C        INUMPER= NUMBER OF PERIODS PROGRAM WILL RUN
C
C        INUMXT1= NUMBER OF VALUES OF XT1 PROGRAM WILL COMPUTE EACH PERIOD
C
C        INUMXT2= NUMBER OF VALUES OF XT2 PROGRAM WILL COMPUTE EACH PERIOD
C
C        LAM     = NUMBER OF FAILURES PER UNIT TIME
C
C        LT      = TIME = COUNTER FOR PERIOD NUMBER = BEGINNING OF PERIOD
C                  NOTE: LT STANDS FOR LOWER CASE T
C
C        MU      = AVERAGE NUMBER OF DETERIORATIONS PER UNIT TIME
C
C        R       = COST OF FAILURE/ SAME FOR UNIT 1 AND UNIT 2
C
C        T       = LENGTH OF THE INSPECTION PERIOD
C
C        XT01    = INITIAL AMOUNT OF VIBRATION PRESENT AFTER ADJUSTMENT
C                  OR AFTER FAILURE IN UNIT 1
C
C        XT02    = INITIAL AMOUNT OF VIBRATION PRESENT AFTER ADJUSTMENT
C                  OR AFTER FAILURE IN UNIT 2
C
C        XT1     = CUMULATIVE VIBRATION MEASUREMENT TAKEN AT BEGINNING OF
C                  PERIOD T THAT DESCRIBES THE STATE OF UNIT 1
```

```
C                  = X SUB T1
C
C                    ===>NOTATION USED IN MAIN PROGRAM<===
C
C         XT2      = CUMULATIVE VIBRATION MEASUREMENT TAKEN AT BEGINNING OF
C                    PERIOD T THAT DESCRIBES THE STATE OF UNIT 2
C                  = X SUB T2
C                    NOTATION USED IN MAIN PROGRAM
C
C         XST1     = X* SUB T1 = SXUBT1 -(XSUBT1-XSUBT01)DELTA1
C                    NOTATION USED IN SUBROUTINES
C
C         XST2     = X* SUB T2 = SXUBT2 -(XSUBT2-XSUBT02)DELTA2
C                    NOTATION USED IN SUBROUTINES
C
C         Y        = RANDOM AMOUNT OF VIBRATION INCREASE OCCURRING IN
C                    UNIT 1 IN EACH PERIOD T
C
C         W        = RANDOM AMOUNT OF VIBRATION INCREASE OCCURRING IN
C                    UNIT 2 IN EACH PERIOD T
C
C
C                      ====>COSTS<===
C
C                    THE COST FUNCTION HAS BEEN COMPUTED USING
C
C                    FT(LT,XT1,XT2)= MATRIX F SUB (T-1) OF (XSUBT1, XSUBT2)
C                    FOUR DIFFERENT TIMES AS COST A, B, C, OR D
C                    DEPENDEING ON WHICH UNIT HAS REACHED THE UCL
C                    (IE RECEIVED AN ADJUSTMENT AT THE BEGINNING OF THE PERIOD
C
C
C         A    =     TOTAL MINIMUM COST FOR PERIOD T AND ALL PREVIOUS
C                    PERIODS WHEN ONLY UNIT 1 IS ADJUSTED (RESET)
C                    DELTA 1 = 1 AND   DELTA 2 = 0
C                    COST CALCULATED AT END OF PERIOD T
C
C
C         B    =     TOTAL MINIMUM COST FOR PERIOD T AND ALL PREVIOUS
C                    PERIODS WHEN ONLY UNIT 2 IS ADJUSTED (RESET)
C                    DELTA 2 = 1 AND DELTA 1 = 0
C                    COST CALCULATED AT END OF PERIOD T
C
C
C
C         C    =     TOTAL MINIMUM COST FOR PERIOD T AND ALL PREVIOUS
C                    PERIODS FOR ADJUSTING (RESETTING) BOTH UNIT 1 AND UNIT 1
C                    HERE DELTA = 1 AND DELTA 1 = 1 AND DELTA 2 = 1
C                    COST CALCULATED AT END OF PERIOD T
C
C
C         D    =     TOTAL MINIMUM COST FOR PERIOD T AND ALL PREVIOUS
C                    PERIODS FOR DOING NOTHING: NEITHER UNIT IS RESET
C                    COST CALCULATED AT END OF PERIOD T
C                        DELTA 1 = 0 AND DELTA 2 = 0
```

```
C
C     FT(LT,XT1,XT2)= MATRIX F SUB (T-1) OF (X SUB T1, X SUB T2)
C                   = MINIMAL COST FOR PERIOD LT AND ALL PAST PERIODS
C                     FOR STATE X SUB T1 IN UNIT 1
C                     AND STATE X SUB T2 IN UNIT 2
C                   WHERE
C                     LT = PERIODS
C                     XT1 = VALUE OF X SUB T1 = STATE OF UNIT 1
C                     XT2 = VALUE OF X SUB T2= STATE OF UNIT 2
C
C              THIS COST HAS BEEN COMPUTED FOR 4 DIFFERENT
C              SCENARIOS FOR EACH PERIOD AS COST-A, B, C, D
C
C
C
C     PCBOTH    = FINDS E(PAST MINIMIMAL COST) WHEN BOTH UNITS FAIL
C                 DURING THE PERIOD
C
C
C
C     INUMPER   = NUMBER OF PERIODS PROGRAM WILL RUN
C
C     K         = COST OF ADJUSTING UNIT 1 AND UNIT 2 SIMULTANEOUSLY
C     K1        = COST OF ADJUSTING UNIT 1 AND NOT UNIT 2
C     K2        = COST OF ADJUSTING UNIT 2 AND NOT UNIT 1
C
C               WHERE
C
C                 (K1 = K2) < K < (K1 + K2)
C                 (COST TO RESET UNIT 1 ALONE OR UNIT 2 ALONE
C                 IS LESS THAN COST OF RESETTING BOTH AT SAME TIME
C                 IS LESS THAN COST OF RESETTING BOTH INDIVIDUALLY
C                 BUT SEPARATELY)
C
C
C
C
C
C   ===================> SUBROUTINES <============================
C
C
C     HXT       = FINDS H(X* SUB T1, X* SUB T2)
C               = CONDITIONAL E(COST OF FAILURE THIS PERIOD)
C                 RETURNS SUMHXT
C
C     ONEHXT    = ONE ITERATION OF THE NUM. INT. PROCESS
C
C
C
C     PCNF      = FINDS E(PAST MINIMAL COST) WHEN NEITHER UNIT FAILS
C                 DURING THE PERIOD / RETURNS SUMNF
C
C     ONENF     = ONE ITERATION OF THE NUM. INTEGRATION PROCES
C
```

```
C
C
C     PC1F     = FINDS E(PAST MINIMIMAL COST) WHEN UNIT 1 FAILS
C                DURING THE PERIOD / RETURNS SUM1F
C
C     ONE1F    = ONE ITERATION OF THE NUMERICAL INTEGRATION PROCESS
C
C
C
C     PC2F     = FINDS E(PAST MINIMAL COST) WHEN UNIT 2 FAILS
C                DURING THE PERIOD / RETURNS SUM2F
C
C     ONE2F    = ONE ITERATION OF THE NUMERICAL ITEGRATION PROCESS
C
C     R1       = FINDS P(RELIABILITY UNIT-1) =-P(UNIT-1-DOES-NOT-FAIL)
C                RETURNS SUMR1
C
C     1-R1     = PROBABILITY UNIT 1 FAILS
C
C
C     R2       = FINDS P(RELIABILITY UNIT-2)-=-P(UNIT-2-DOES-NOT-FAIL)
C                RETURNS SUMR2
C
C     1-R2     = PROBABILITY UNIT 2 FAILS
C
C     --->> ALL "SUM" VALUES ARE COMPUTED-AND-RETURNED BY SUBROUTINES<<
C
C              ====>INITIAL VALUE OF VARIABLES<====
C
C     K   = 60.0 = COST OF ADJUSTING UNIT-1-AND-UNIT-2-AT-SAME-TIME
C     K1  = 40.0 = COST OF AN ADJUSTING ONLY UNIT-1 AT-BEGINNING OF PER.
C     K2  = 40.0 = COST OF AN ADJUSTING ONLY UNIT 2-AT-BEGINNING OF PER.
C
C     LAM = .001 = LAMBDA = 1 FAILURES IN 1000 TIME UNITS
C     LAM = .001D0 FORCES ALL ZEROS AFTER-THE-2-TO BE-ZEROS-ALSO
C     LAM IS AVERAGE !NUMBER! OF FAILURES PER UNIT TIME= POISSON MU
C     1/LAM IS AVERAGE !TIME-BETWEEN FAILURES!
C
C     MU =.1 = AVERAGE # OF UNITS OF DETERIORATION PER UNIT-TIME
C              AVERAGE FOR POISSON DISTRIBUTION OF DETERIORATION
C
C     R = 200 = COST OF A REPAIR AFTER FAILURE: K < R BY ASSUMPTION
C
C     T = 1 = LENGTH OF INSPECTION PERIOD
C
C     XT01= 1 = INITIAL AMOUNT OF VIBRATION IN UNIT-1 = X SUB 01
C     XT02= 1 = INITIAL AMOUNT OF VIBRATION IN UNIT 2 = X SUB 02
C
C     Y = 0 = INCREASE IN VIBRATION PER PERIOD FOR UNIT-1
C     W = 0 = INCREASE IN VIBRATION PER PERIOD FOR UNIT 2
C
C
C*********** TOWUNIT MAIN PROGRAM CODE ******************
      REAL K, K1, K2, LAM, R,  MU, SUMR1, SUMR2, LASTAVGPP
C
```

FILE: NUMATRIX FORTRAN A0 VM/ESA Conversational Monitor System

```
      INTEGER LT, T, Y, W, XT1, XT2, XT01, XT02,INUMXT1, INUMXT2
      COMMON/PARAMETERS/LAM, MU, R, T, XT1,XT2,LT,INUMXT1, INUMXT2
      COMMON/MATRIX/FT(41,200,200)
      CHARACTER *1 NAME(41,200,200)
C
      INUMPER = 40
      INUMXT1 = 200
      INUMXT2 = 200
C--------------------->RUN #1
      MU = 4.0D0
      LAM = .004D0
      K = 19
      R = 200.0
      K1 = 15
      K2 = 15
      T = 1
      XT01 = 1
      XT02 = 1
      Y = 0
      W = 0
      SUMHXT = 0.0
      SUMNF  = 0.0
      SUM1F  = 0.0
      SUM2F  = 0.0
      SUMBOTH = 0.0
      SUMR1  = 0.0
      SUMR2  = 0.0
      A = 0.0
      B = 0.0
      C = 0.0
      D = 0.0
      E = 0.0
      LASTAVGPP = 0.0
C
C
C
            PRINT *, ' *******                                   ******* '
            PRINT *
            PRINT *
      PRINT *,  '      RUN ------------> #-1- '
      PRINT *,     'MU = AVG. # DETERIORATIONS PER UNIT TIME   = ', MU
      PRINT *,     'LAMBDA = # FAILURES PER UNIT TIME          = ', LAM
      PRINT *,     'K      = COST OF ADJUSTING BOTH AT ONCE    = ', K
      PRINT *,     'R      = COST OF FAILURE                   = ', R
      PRINT *,     'K1     = COST OF ADJUSTING UNIT 1 ONLY     = ', K1
      PRINT *,     'K2     = COST OF ADJUSTING UNIT 2 ONLY     = ', K2
      PRINT *,     'T = LENGTH OF INSPECTION PERIOD            = ', T
      PRINT *,     'XT01   = INITIAL VIBRATION IN UNIT 1       = ', XT01
      PRINT *,     'XT02   = INITIAL VIBRATION IN UNIT 2       = ', XT02
      PRINT *
      PRINT *,'NUMBER OF PERIODS IS                  ', INUMPER
            PRINT *
      PRINT *,'NUMBER OF VALUES OF XT1,XT2 (RESP) ARE', INUMXT1,INUMXT2
            PRINT *, '******************************************'
            PRINT *
```

```
C
C
C----------------------------------------------------------
C     INITIALIZE ALL VALUES OF FT(LT,XT1,XT2) TO ZERO
C----------------------------------------------------------
      DO 10 I = 0, INUMPER
         DO 50 J = 1, INUMXT1
            DO 55 M = 1, INUMXT2
               FT(I,J,M) = 0.0D0
55          CONTINUE
50       CONTINUE
10    CONTINUE
C
C     INITIALIZE ALL VALUES OF NAME(LT,XT1,XT2) TO BLANKS
C
      DO 110 I = 0, INUMPER
         DO 510 J = 1, INUMXT1
            DO 515 M = 1, INUMXT2
               NAME(I,J,M) = ' '
515         CONTINUE
510      CONTINUE
110    CONTINUE
C-----------------------------------------------------------------------
C     **  DO FOR NUMBER OF PERIODS = LT= INUMPER
C-----------------------------------------------------START A NEW PERIOD
            DO 40 LT = 1, INUMPER
C-----------------------------------------------------------------------
C              FINDING FT SUB T (X SUB T1, X SUB T2)
C              FOR 200 VALUES OF XT1 AND 200 VALUES OF XT2
C--------------------------------------------------------START A NEW ROW
               DO 20 XT1 = 1, INUMXT1
C-----------------------------------------START A NEW COLUMN IN SAME ROW
                  DO 25 XT2 = 1, INUMXT2
C
C-----------------------------------------------------------------------
C    FIND COSTS A,B,C,D FOR THIS STATE IN THIS PERIOD
C        A =  COST OF RESETTING UNIT 1 ONLY
C        B =  COST OF RESETTING UNIT 2 ONLY
C        C =  COST OF RESETTING BOTH UNITS SIMULTANEOUSLY
C        D =  COST OF RESETTING NEITHER UNIT: DO NOTHING: LET IT RUN
C----------------------------------------------------------------------
C
C----------------------------------------------------------------------
C    FIND COST A
C----------------------------------------------------------------------
         CALL R1(XT01, SUMR1)
C
         CALL R2(XT2, SUMR2)
C
         CALL HXT(XT01,XT2,SUMHXT)
C
         CALL PCNF(XT01,XT2,SUMNF)
C
         CALL PC1F(XT01,XT2, SUMR1, SUM1F)
C
```

```
      CALL PC2F(XT01,XT02, SUMR2, SUM2F)
C
C
C-----------------------------------------------------------------
C            SUMBOTH FINDS LAST TERM IN COST EQUATION.
C            NOT NECESSARY TO USE SUBROUTINE
C-----------------------------------------------------------------
C
      SUMBOTH = (FT((LT-1), XT01,XT02))* (1 - SUMR1) * (1-SUMR2)
C
C
C
      A = K1 + SUMHXT + SUMNF + SUM1F + SUM2F + SUMBOTH
      PRINT *, 'COST A FOR PERIOD',LT,'IN-STATE',XT1,XT2,'IS',A
C
C
C-----------------------------------------------------------------
C     FIND COST B
C-----------------------------------------------------------------
      CALL R1(XT1, SUMR1)
C
      CALL R2(XT02, SUMR2)
C
      CALL HXT(XT1, XT02, SUMHXT)
C
      CALL PCNF(XT1, XT02, SUMNF)
C
      CALL PC1F(XT01, XT02, SUMR1, SUM1F)
C
      CALL PC2F(XT1, XT02, SUMR2, SUM2F)
C
      B = K2 + SUMHXT + SUMNF + SUM1F + SUM2F + SUMBOTH
C     PRINT*
C     PRINT *, 'COST B FOR PERIOD',LT,'IN STATE',XT1,XT2,'IS',B
C-----------------------------------------------------------------
C     FIND COST C
C-----------------------------------------------------------------
      CALL R1(XT01, SUMR1)
C
      CALL R2(XT02, SUMR2)
C
      CALL HXT(XT01,XT02,SUMHXT)
C
      CALL PCNF(XT01,XT02,SUMNF)
C
      CALL PC1F(XT01,XT02,SUMR1,SUM1F)
C
      CALL PC2F(XT01,XT02,SUMR2,SUM2F)
C
      C = K + SUMHXT + SUMNF + SUM1F + SUM2F + SUMBOTH
C     PRINT*
C     PRINT *, 'COST C FOR PERIOD',LT,'IN STATE',XT1,XT2,'IS',C
C-----------------------------------------------------------------
C     FIND COST D
C-----------------------------------------------------------------
      CALL R1(XT1, SUMR1)
```

```
C
          CALL R2(XT2, SUMR2)
C
          CALL HXT(XT1,XT2,SUMHXT)
C
          CALL PCNF(XT1,XT2,SUMNF)
C
          CALL PC1F(XT01,XT2,SUMR1,SUM1F)
C
          CALL PC2F(XT1,XT02,SUMR2,SUM2F)
C
          D = 0.0 + SUMHXT + SUMNF + SUM1F + SUM2F + SUMBOTH
C         PRINT *, 'COST D FOR PERIOD',LT,'IN STATE',XT1,XT2,'IS',D
C         PRINT*
C
C     FIND MINIMAL COST OF FOUR POSSIBLE CASES = E
C
      E = AMIN1(A,B,C,D)
       FT(LT, XT1, XT2) = E
C      PRINT*, 'MIN COST E FOR PERIOD',LT,'IN STATE',XT1,XT2,'IS',E
      IF (E .EQ. A) THEN
C         PRINT *, 'RESET A FOR', LT,XT1,XT2, 'AT COST', E
          NAME(LT,XT1,XT2) = 'A'
          ENDIF
C
C
C
      IF (E .EQ. B) THEN
C         PRINT *, 'RESET B FOR', LT, XT1, XT2, 'AT COST', E
          NAME(LT,XT1,XT2) = 'B'
C-----------------------------------IN THIS ROW
C-----------------------------------SET REST OF NAME COLMNS = 'B'
C-----------------------------------SET REST OF FT COLMNS = 'B'
          DO 60 IFINISH = XT2 + 1, INUMXT2
                NAME(LT,XT1,IFINISH) = 'B'
                FT(LT,XT1,IFINISH) = B
60        CONTINUE
C-------------------------------------BEGIN NEXT ROW AT "20 CONTINUE"
          GO TO 20
      ENDIF
C
C
C
C
C
      IF (E .EQ. C) THEN
         NAME(LT,XT1,XT2) = 'C'
C-----------------------------------IN THIS ROW
C-----------------------------------SET REST OF NAME COLMNS = 'C'
C-----------------------------------SET REST OF FT COLMNS = 'C'
                DO 61 IFINISH = XT2 + 1, INUMXT2
                      NAME(LT,XT1,IFINISH) = 'C'
                      FT(LT,XT1,IFINISH) = C
61              CONTINUE
C-------------------------------------BEGIN NEXT ROW AT "20 CONTINUE"
```

```
                GO TO 20
      ENDIF
C
C
C     IF E = D KEEP RUNNING: NO ACTION TAKEN ON UNITS
C
C
      IF (E .EQ. 0) THEN
         NAME(LT,XT1,XT2) = 'D'
         END IF
C               PUT MINIMUM VALUE,E, INTO MATRIX FOR THIS PERIOD/STATE
C
C
   25 CONTINUE
   20 CONTINUE
      PRINT *
      PRINT *, 'RESULTS AFTER======================>END OF PERIOD',LT
C---------------------------------------------------------------------
C     ACCUMULATE PERIOD TOTAL/CALCULATE,PRINT AVERAGE
C---------------------------------------------------------------------
      ONEPERAVG = 0.0
      PERIODSUM = 0.0
      AVGPERPD = 0.0
      ICOUNT = 0
      DO 30 IROW = 1, INUMXT1
         DO 31  ICOL = 1, INUMXT2
      IF (NAME(LT,IROW,ICOL) .EQ. 'D') THEN
        PERIODSUM = PERIODSUM + FT(LT,IROW,ICOL)
        ICOUNT = ICOUNT + 1
        GO TO 31
      ENDIF
      IF (NAME(LT,IROW,ICOL) .EQ. 'B') THEN
        PERIODSUM = PERIODSUM + FT(LT,IROW,ICOL)
        ICOUNT = ICOUNT + 1
        GO TO 30
      ENDIF
C-------------------IF FIND C W/C ABOVE IT GET OUT-AND-AVG
C-------------------OTHERWISE ADD/COUNT/GO TO NEW ROW
      IF (NAME(LT,IROW,ICOL) .EQ. 'C') THEN
         IF (NAME(LT,IROW - 1,ICOL) .EQ. 'C') THEN
C----------------------------------------------------QUIT AND-AVERAGE
            GO TO 36
         ELSE
            PERIODSUM = PERIODSUM + FT(LT,IROW,ICOL)
            ICOUNT = ICOUNT + 1
            GO TO 30
         ENDIF
      ENDIF
C------------------------IF THIS ENTRY IS NOT A 'D','B',OR 'C',
C------------------------THEN MUST BE AN 'A'
C------------------------DOES IT HAVE AN 'D'-ABOVE-IT?
C------------------------YES/ADD/COUNT/NEW COLM/ OTHERWISE NEW COLM
      IF (NAME(LT,IROW,ICOL) .EQ. 'A') THEN
         IF (NAME(LT,IROW - 1,ICOL) .EQ. 'D') THEN
            PERIODSUM = PERIODSUM + FT(LT,IROW,ICOL)
```

```
                ICOUNT = ICOUNT + 1
                GO TO 31
          ELSE
                GO TO 31
          ENDIF
      ENDIF
C***--------------------START NEW COLUMN/SAME ROW
31    CONTINUE
C***--------------------START NEW ROW
30    CONTINUE
36    ONEPERAVG = PERIODSUM/ICOUNT
C------------------------DON'T HAVE TO SUM ONEPERAVG BEC
C------------------------DPP VALUE IS THE CUM SUM ALREADY
      IF(LT .GT.1) THEN
      AVGPERPD = ONEPERAVG/LT
      PRINT *, 'PERIODSUM, ONEPERIODAVG ARE ----->', PERIODSUM,ONEPERAVG
      PRINT *, 'AVG/PER, ICOUNT ARE>>>>>>>>>>' , AVGPERPD,ICOUNT
         PCENTCHG = ((AVGPERPD - LASTAVGPP)/LASTAVGPP)*100
       PRINT *, '% CHANGE IN AVG/PERIOD IS-%-%-%-%-%-%-%', PCENTCHG, '%'
C-------------------------------------IF-% CHG VERY SMALL / QUIT
         IF(PCENTCHG .GT. 0 .AND. PCENTCHG .LT. .1) GO TO 444
      ENDIF
C-----------------------------------------------------------------------
C          PRINTING MATRIX OF COSTS FOR FIRST-10 STATES
C-----------------------------------------------------------------------
CMAT           PRINT *, '------ MATRIX OF COSTS FOR PERIOD------',LT
CMAT             WRITE(6,292) (M,M=1,10)
CMAT292          FORMAT(1X,6X,10(I3,8X))
CMAT             DO 120 JROW = 1,10
CMAT                WRITE (6,291)(FT(LT,JROW,JCOLM),JCOLM = 1,10),JROW
CMAT291                FORMAT (1X,10(F11.1),1X,I3)
CMAT120          CONTINUE
C-----------------------------------------BEGIN NEXT PERIOD
      LASTAVGPP = AVGPERPD
40    CONTINUE
C-----------------------------------------------------------------------
C                PRINTING MATRIX OF DECISIONS---% CHANGE-SMALL ENOUGH
C-----------------------------------------------------------------------
444            PRINT *, 'MATRIX OF DECISIONS: PERIOD',LT
               WRITE(6,290) (M,M=2,98,2)
290            FORMAT(1X,100(I2))
C-----------------------------------------------------------------------
C          J SHOULD GO TO NO. OF XT1 VALUES, K GOES TO NO. OF XT2 VALUES
C          206 FORMAT MUST BE NUMBER OF XT1 TIME A1
C-----------------------------------------------------------------------
                DO 333 J = 1,100
                   WRITE (6,206) (NAME(LT,J,KMAT),KMAT = 1,100),J
206                FORMAT(1X, 100(A1),1X,I3)
333             CONTINUE
      STOP
      END
C
C
C*******************SUBROUTINE HXT*******************************
C
```

```
C
C      THIS SUBROUTINE FINDS
C      THE CONDITIONAL EXPECTED COST OF FAILURE THIS PERIOD
C      AND RETURNS SUMHXT
C
C      XST1 = X*T1 IN DP FORMULATION
C      XST2 = X*T2 IN DP FORMULATION
C
C      PLAN IS TO USE XST1 AND XST2 IN EACH SUBROUTINE, BUT SEND
C             THE PARTICULAR VALUE (XT01, XT02, XT1, XT2) WHICH
C             IS DICTATED BY THE VALUE DELTA 1, DELTA 2, OR DELTA
C             IN EACH OF THE TERMS OF THE 4 DIFFERENT POSSIBLE CASES
C             AND IN R1 AND R2
C
C234567
      SUBROUTINE HXT(XST1, XST2, SUMHXT)
      REAL  LAM, R,  MU, SUMHXT, ONEHXT
      INTEGER LT, T, XST1, XST2, XT01, XT02, Z,XT1,XT2,INUMXT1,INUMXT2
      COMMON /PARAMETERS/ LAM, MU, R, T, XT1, XT2, LT,INUMXT1,INUMXT2
      SUMHXT = 0.0
      Z = 0
      ONEHXT = 0.0
      SUMINT = 0.0
 100  SUMINT = (1 - EXP(-LAM*(XST1 + XST2 + Z)*T))
     X*(EXP(-(2*MU))*(2*MU)**Z)/
     X(FAC(Z))
CC          PRINT*
CC          PRINT*, 'THE VALUE OF 1 ITERATION OF THE INTEGRAL IN HXT'
CC          PRINT*, 'IS==>',SUMINT, 'IN STATE===>',LT,XT1,XT2,'Z =',Z
C
      IF (SUMINT .GT. 1.0E-07 ) THEN
          ONEHXT =R * SUMINT
          SUMHXT = SUMHXT + ONEHXT
          Z = Z + 1
          GO TO 100
      ELSE
CC        PRINT *, 'IN HXT THE VALUE OF THE INTEGRAL IS',SUMHXT
CC        PRINT*,'                FOR STATE',LT,XT1,XT2
          GO TO 150
      ENDIF
 150  RETURN
      END
C
C
C*******************SUBROUTINE PCNF**********************************
C
C
C      THIS SUBROUTINE FINDS
C          THE EXPECTED PAST MINIMAL COST WHEN NEITHER UNIT FAILS.
C          RETURNS SUMNF
C
C          ONENF REPRESENTS ONE ITERATION IN THE NUM. INT. PROCESS
C
C          XST1 = X*T1
C          XST2 = X*T2 IN DP FORMULATION
```

```
C
C              THIS SUBROUTINE USES
C              X*T1 + Y FOR THE FIRST UNIT
C              X*T2 + W FOR THE FIRST UNIT
C
C
C
      SUBROUTINE PCNF(XST1 ,XST2, SUMNF)
      REAL  LAM, R,  ONENF, SUMNF, MU
      INTEGER LT,T,Y,W,XT1,XT2,XT01,XT02,XST1,XST2,INUMXT1,INUMXT2
      COMMON /PARAMETERS/ LAM, MU, R, T,  XT1, XT2,LT,INUMXT1,INUMXT2
      COMMON/MATRIX/FT(41,200,200)
C
C
C
      SUMNF = 0.0
      ONENF = 0.0
          Y = 0
          W = 0
C-----------------------------------------------------------------------
C     Y GOES TO INUMXT1 - XST1 BECAUSE THE COST MATRIX HAS NO
C            MORE THAN INUMXT1 VALUES FOR PERIOD (LT-1)
C-----------------------------------------------------------------------
        DO 250 Y = 0, 30
        DO 300 W = 0, 30
        ONENF = FT((LT-1), (XST1 + Y), (XST2 + W))
     X*EXP(-LAM*(XST1 + Y+ XST2 + W)*T)
     X*(EXP(-MU))*MU**Y/FAC(Y)
     X*(EXP(-MU))*MU**W/FAC(W)
          IF (ONENF .GT. 1.0E-07) THEN
             SUMNF = SUMNF + ONENF
          ELSE
             GO TO 250
          ENDIF
300          CONTINUE
250       CONTINUE
C
C
      RETURN
      END
C
C
C***********************SUBROUTINE PC1F***************************
C
C     THIS SUBROUTINE FINDS
C          THE EXPECTED PAST MINIMUM COST OF UNIT 1 FAILING
C          AT THE END OF THE PREVIOUS PERIOD.  THIS RESETS
C          X SUB T1 TO ITS INITIAL VALUE AT THE BEGINNING OF
C          THIS PERIOD.
C          THIS SUBROUTINE USES
C             XT01 FOR THE FIRST UNIT
C             X*T2+ W FOR THE SECOND UNIT
C
C          RETURN = SUM1F
C
```

```
C           ONE1F   = VALUE OF 1 ITERATION IN THE NUMERICAL
C                     INTEGRATION PROCESS.
C
C-----------------------------------------------------------------------
      SUBROUTINE PC1F(XT01, XST2, SUMR1, SUM1F)
C
C
C
      REAL  LAM, R,  MU, SUM1F, ONE1F
      INTEGER LT,T,XST1,XST2,XT01,XT02,W,XT1,XT2,INUMXT1,INUMXT2
      COMMON /PARAMETERS/ LAM, MU, R, T, XT1, XT2,LT,INUMXT1,INUMXT2
      COMMON/MATRIX/FT(41,200,200)
C
C
      SUM1F = 0.0
      ONE1F = 0.0
      W = 0
      SUMINT = 0.0
C
C
CC    PRINT *, 'W = ',W,'XST1 =',XST1,'LAM =',LAM, 'MU =', MU
C
 300  ONE1F = (FT((LT-1), XT01, (XST2 + W))*EXP(-LAM*(XST2 + W)*T))
     X *(EXP(-MU))*MU**W/FAC(W)
C
C
      IF (ONE1F .GT. 1.0E-07 ) THEN
        SUMINT = SUMINT + ONE1F
CC      PRINT *, 'VALUE OF INTEGRAL IS ', ONE1F, 'WHEN W = ', W
C
        W = W + 1
        GO TO 300
      ELSE
C
C              ONCE ESTIMATE OF INTEGRAL IS FOUND
C              MULTIPLY THAT VALUE BY (1 - R1)
        SUM1F = (1 - SUMR1) * SUMINT
CC      PRINT *, 'VALUE OF INTEGRAL .LE. 1.0E-07 WHEN W = ', W
CC      PRINT *
CC      PRINT *
CC      PRINT *, 'SUM1F =', SUM1F
        GO TO 350
      ENDIF
 350  RETURN
      END
C
C
C***********************SUBROUTINE PC2F*************************
C
C
C-----------------------------------------------------------------------
C     THIS SUBROUTINE FINDS
C       THE EXPECTED PAST MINIMUM COST OF UNIT 2 FAILING
C       AT THE END OF THE PREVIOUS PERIOD.  THIS RESETS
C       X SUB T2 TO ITS INITIAL VALUE AT THE BEGINNING OF
```

```
C         THIS PERIOD.  THIS SUBROUTINE USES XST2 AND XT02.
C         RETURN = SUM2F.
C         ONE2F  = VALUE OF 1 ITERATION IN THE NUMERICAL
C         INTEGRATION PROCESS.
C
C               THIS SUBROUTINE USES
C               X*T1 + Y FOR FIRST UNIT BEGINNING OF PERIOD DETERIORATION
C               XT02 FOR SECOND UNIT BEGINNING DETERIORATION
C-----------------------------------------------------------------------
      SUBROUTINE PC2F(XST1, XT02, SUMR2, SUM2F)
C
C
C
      REAL  LAM, R,  MU, SUM2F, ONE2F
      INTEGER LT,T,XST1,XST2,XT01,XT02,Y,XT1,XT2,INUMXT1,INUMXT2
      COMMON /PARAMETERS/ LAM, MU, R, T, XT1, XT2,LT,INUMXT1,INUMXT2
      COMMON/MATRIX/FT(41,200,200)
C
C
      SUM2F = 0.0
      ONE2F = 0.0
      Y = 0
      SUMINT = 0.0
C
C
CC    PRINT *, 'Y = ',Y,'XST1 =' ,XST1, 'LAM =',LAM, 'MU =', MU
C
 300  ONE2F = FT((LT-1), XST1 + Y, (XT02))*EXP(-LAM*(XST1 + Y)*T)
     X *(EXP(-MU))*MU**Y/FAC(Y)
C
C
      IF (ONE2F .GT. 1.0E-07 )  THEN
         SUMINT = SUMINT + ONE2F
C        PRINT *, 'VALUE OF INTEGRAL IS ', ONE2F, 'WHEN Y = ', Y
C
         Y = Y + 1
         GO TO 300
      ELSE
C
C                    FIND ESTIMATE OF INTEGRAL THEN MULTIPLY BY (1 - R2)
C
       SUM2F = (1 - SUMR2) * SUMINT
CC         PRINT *, 'VALUE OF INTEGRAL .LE. 1.0E-07 WHEN Y = ', Y
CC         PRINT *
CC         PRINT *
CC         PRINT *, 'SUM2F =', SUM2F
           GO TO 350
      ENDIF
 350  RETURN
      END
C
C
C
C
C
```

```
C
C*********************** SUBROUTINE R1 ***************************
C
C                THIS SUBROUTINE FINDS
C                   THE RELIABILITY OF UNIT 1 = P(UNIT 1 DOES NOT FAIL)
C                      THIS SUBROUTINE INVOLVES ONLY UNIT 1
C                      IT USES XST1 + Y FOR DETERIORATION IN UNIT 1
C-----------------------------------------------------------------
C
C
C
C
      SUBROUTINE R1(XST1, SUMR1)
C
C
      REAL  LAM, R,  MU, SUM2F, ONE2F
      INTEGER LT,T,XST1,XST2,XT01,XT02,Y,XT1,XT2,INUMXT1,INUMXT2
      COMMON /PARAMETERS/ LAM, MU, R, T, XT1, XT2,LT,INUMXT1,INUMXT2
      COMMON/MATRIX/FT(41,200,200)
C
C
      SUMR1 = 0.0
      ONER1 = 0.0
      Y = 0
C
C
CC    PRINT *, 'Y = ',Y,'XTS =' ,XTS, 'LAM =',LAM, 'MU =',MU
C
C
 300  ONER1=EXP(-LAM*(XST1 + Y)*T)*
     X(EXP(-MU))*MU**Y/FAC(Y)
      IF (ONER1 .GT. 1.0E-07 )  THEN
        SUMR1 = SUMR1 + ONER1
CC      PRINT *, 'IN SUBR1,ONER1 IS ', ONER1, 'WHEN Y = ', Y
C
         Y = Y + 1
         GO TO 300
C
      ELSE
C
CC       PRINT *, 'IN SUBR1, INTEGRAL .LE. 1.0E-07 WHEN Y = ', Y
CC       PRINT *
CC       PRINT *
CC       PRINT *, 'SUMR1 =', SUMR1
         GO TO 350
      ENDIF
350   RETURN
      END
C
C
C
C
C
C*********************** SUBROUTINE R2 ***************************
C
```

FILE: NUMATRIX FORTRAN A0 VM/ESA Conversational Monitor System

```
C     THIS SUBROUTINE FINDS
C        THE RELIABILITY OF UNIT 2 = P(UNIT 2 DOES NOT FAIL)
C                 THIS SUBROUTINE INVOLVES ONLY UNIT 2
C                 IT USES XST2 + W FOR DETERIORATION IN UNIT 2
C-----------------------------------------------------------------
C
C
C
C
      SUBROUTINE R2(XST2, SUMR2)
C
C
      REAL  LAM, R,  MU, SUMR2, ONER2
      INTEGER LT,T,XT1,XT2,XST1,XST2,XT01,XT02,W,INUMXT1,INUMXT2
      COMMON /PARAMETERS/ LAM, MU, R, T, XT1, XT2,LT,INUMXT1,INUMXT2
      COMMON/MATRIX/FT(41,200,200)
C
      SUMR2 = 0.0
      ONER2 = 0.0
      W = 0
C
C
CC    PRINT *, 'W = ',W,'XTS =' ,XTS, 'LAM=',LAM, 'MU=', MU
C
C
300   ONER2 = EXP(-LAM*(XST2 + W)*T)*
     X(EXP(-MU))*MU**W/FAC(W)
      IF (ONER2 .GT. 1.0E-07 ) THEN
       SUMR2 = SUMR2 + ONER2
CC     PRINT *, 'IN SUBR1, ONER2 IS ', ONER2, 'WHEN W = ', W
C
       W = W + 1
       GO TO 300
C
      ELSE
C
CC     PRINT *, 'IN SUBR2, INTEGRAL .LE. 1.0E-07 WHEN W = ', W
CC     PRINT *
CC     PRINT *
CC     PRINT *, 'SUMR2 =', SUMR2
       GO TO 350
      ENDIF
350   RETURN
      END
```

APPENDIX C. SIMAN SIMULATION PROGRAM

```
BEGIN;
PROJECT,            TwoUnit16,Fran Barbera,7/9/94;
ATTRIBUTES:         accum_a:
                    accum_b;
VARIABLES:          ucla,22:                      !ucl unit a
                    uclb,22:                      !ucl unit b
                    uclab,37:                     !ucl unit a + unit b
                    lcla,16:                       !unit a diagonal limit
                    lclb,16:                       !unit b diagonal limit
                    repcost,200:                  !repair cost
                    adj1cost,8:                 !adjust cost for 1 unit
                    adj2cost,14:                   !adjust cost for 2 unit:
                    Vibr(2),0:
                    Limits(2),16,22:
                    mean_e,.002:                  !mean for exp dist
                    mean_p,4:                    !mean for poisson dist
                    fail_a:
                    fail_b:
                    vib_a:
                    vib_b;
TALLIES:            1,A_Rep_Cost:
                    2,B_Rep_Cost:
                    3,A_Adj_Cost:
                    4,B_Adj_Cost:
                    5,AB_Adj_Cost;
;                    6,Avg_Cost_Per,"cost_per.dat";
REPLICATE,          5,,5500,,,500;
;DSTATS:             Limits(1),LDCL,"LDCL.dat":
;                    Limits(2),UCL,"UCL.dat":
;                    Vibr(1),Vibr_Level_a,
;                    "accum_a.dat":
;                    Vibr(2),Vibr_Level_b,
;                    "accum_b.dat";
OUTPUTS:            1,(TNUM(1)*TAVG(1)+TNUM(2)*TAVG(2)+
                    TNUM(3)*TAVG(3)+TNUM(4)*TAVG(4)+
                    TNUM(5)*TAVG(5))/(TFIN-500),"cost_per.dat",
                    Avg_Cost_Period;
END;
```

```
BEGIN;
          CREATE:     ,1;
          ASSIGN:     accum_a = 1:                  !vib total unit a
                      accum_b = 1;                  vib total unit b
loop1     DELAY:      0.5;
;          TALLY:      6,(TNUM(1)*TAVG(1)+
;                      TNUM(2)*TAVG(2)+TNUM(3)*TAVG(3)+
;                      TNUM(4)*TAVG(4)+TNUM(5)*TAVG(5))/
;                      (TNOW-500);
          ASSIGN:     vib_a = pois(mean_p,1):       !vibration deterioration
                      vib_b = pois(mean_p,2):       !vibration deterioration
                      accum_a = accum_a + vib_a:    !add to cumulative
                      accum_b = accum_b + vib_b:    !add to cumulative
                      Vibr(1) = accum_a:
                      Vibr(2) = accum_b:
                      fail_a = expo((1/accum_a)/(
                      mean_e),3):                   !time to failure a
                      fail_b = expo((1/accum_b)/(
                      mean_e),4);                   time to fail b
          DELAY:      0.5;
          BRANCH,     1:                            !check if unit a failed
                      if,fail_a.LE.1,repair_a:
                      else,check_b;
repair_a  TALLY:                                    !repair unit a
                      1,repcost;
          ASSIGN:                                   !reset vib accum unit a
                      accum_a = 1:
                      Vibr(1) = 1;
check_b   BRANCH,     1:                            !check if unit b failed
                      if,fail_b.LE.1,repair_b:
                      else,goon1;
repair_b  TALLY:                                    !repair unit b
                      2,repcost;
          ASSIGN:                                   !reset vib accum unit b
                      accum_b = 1:
                      Vibr(2) = 1;
goon1     BRANCH,     1:                            !make repair decision
                      if,(accum_a.ge.ucla).and.(
                      accum_b.le.lclb),adj_a:
                      if,(accum_b.ge.uclb).and.(
                      accum_a.le.lcla),adj_b:
                      if,(accum_a.gt.lcla).and.(
                      accum_b.gt.lclb).and.(
                      accum_a+accum_b.ge.uclab),
                      adj_ab:
                      else,loop1;
adj_a     TALLY:                                    !adjust unit a
                      3,adj1cost;
          ASSIGN:     accum_a = 1:
                      Vibr(1) = 1:
                      NEXT(loop1);
adj_b     TALLY:                                    !adjust unit b
                      4,adj1cost;
          ASSIGN:     accum_b = 1:
                      Vibr(2) = 1:
                      NEXT(loop1);
adj_ab    TALLY:                                    !adjust unit a and b
                      5,adj2cost;
          ASSIGN:     accum_a = 1:
                      accum_b = 1:
                      Vibr(1) = 1:
                      Vibr(2) = 1:
                      NEXT(loop1);
END;
```

VITA

Fran Barbera received a B.S. in Mathematics Education from Louisiana State University in 1963 and was a high-school mathematics teacher from 1963-1966. In 1978 Mrs. Barbera earned an M.S. in Applied Mathematics from Nicholls State University, and subsequently taught there as an instructor in Business Administration and Mathematics until 1989.

From 1989-1992 she studied at Louisiana State University in the Quantitative Business Analysis Ph.D. program. From 1992-1994, she was a Visiting Assistant Professor of Management at Southeastern Louisiana University in Hammond, Louisiana where she is currently an Assistant Professor.

DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: Frances Fertitta Barbera

Major Field: Business Administration (Quantitative Business Analysis)

Title of Dissertation: Optimal Preventive Maintenance Strategies for a Production System Subject to Random Shocks

Approved:

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

Date of Examination:

August 25, 1994