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Quantitative Models for the Transition to Just-In-Time Purchasing.

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QUANTITATIVE MODELS FOR THE TRANSITION TO JUST-IN-TIME PURCHASING

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ABSTRACT

Purchasing is a critical, external element of the Just-In-time (JIT) production system. JIT purchasing calls for small delivery quantities which arrive on time, with the quality and quantity required. Economic order quantity models which are adjusted for multiple deliveries or consider the cooperation between the purchaser and the supplier have been published in the literature. The JIT philosophy emphasizes both of these elements. We provide a model for single sourcing which incorporates multiple deliveries and cooperation for the case where there is a single reliable supplier. We call this perfect coordination. The transition to JIT, currently occuring in many companies in the United States, often involves problems with unreliable vendors. In in this case we have imperfect coordination. Also provided are extensions of this model for random lead times, random yield, and random demand which can occur in the transition to JIT. If a single, reliable supplier is not available, multiple sources can be used until a reliable source emerges. The question arises as to how to allocate the order among the suppliers. We propose dual sourcing models that allocate the quantity in order to minimize the stockout risk for deterministic and random demand. Sensitivity analysis, factorial analysis, and simulation studies are included.

CHAPTER 1

INTRODUCTION

The success of the Just-In-Time (JIT) production system in Japan has encouraged companies in other parts of the world to consider implementing it in manufacturing and service. The focus of our study, JIT purchasing, is probably the most critical element of a JIT system and may be more difficult to implement because the purchaser must deal with an external element, the supplier.

JIT purchasing calls for small delivery quantities which arrive on time, with the quality and quantity required. Generally, the supplier and the purchaser enter a long-term contract for a specified order quantity, quality level, and delivery schedule (Manoocheri 1984). In Japan, there are ideal purchasing conditions which enable the JIT system to succeed, such as suppliers which are located near their customers, and buyers that have power over their customers. In the United States, however, these conditions do not occur in most cases. The suppliers are frequently located hundreds of miles from the purchaser and even overseas.

Consider a Japanese radio manufacturer who runs a JIT production process at their plants in Japan and receive parts from suppliers located near the plants. The company also operates a plant in the United States, using a similar JIT production process. Many of the parts for this plant are supplied locally, but a few parts are shipped from Japan. The parts have a long lead time, and deliveries are often delayed due to strikes, breakdowns, and problems in Customs. Because of these delays the company must hold a large amount of safety stock for these parts. The plants in Japan do not have this

problem. The managers at the U.S. plant are considering making the items or ordering them from Mexico. This would provide a long term solution for the supply, but they must cope with the problem for now. The demand forecast horizon for buyers is short, but the lead time for parts may be long. If there are quality problems it may take a long time to receive replacement parts. Therefore, the radio manufacturers must order these parts in large quantities and hold a large amount of safety stock. This increases the holding cost and decreased flexibility can occur. This is a throwback to traditional inventory management compared to JIT purchasing.

Lawrence and Lewis (1993) report that many of the multinational companies that use JIT systems and have built plants in Mexico, may encounter problems implementing their systems at these plants. Most companies purchase their parts from outside of Mexico because the Mexican suppliers cannot satisfy their requirements. One manufacturer revealed that they purchase 95% of their parts outside Mexico and this can cause delivery problems. Although the goal of JIT purchasing is to find a single, reliable supplier, many companies that order from Mexican suppliers use multiple sources to prevent shortages. The problems which occur in these examples are similar to problems that companies encounter when they are converting to a JIT system.

The transition from a traditional type purchasing system to a JIT purchasing system can be a slow process or even unattainable. Problems such as the ones mentioned in the examples above can occur. During this transition the purchaser tries to cooperate with the vendor, with a goal of receiving smaller, more frequent deliveries from the vendor, on time, with the quality and quantity required. As in the examples above, problems

with timing or quality of deliveries can occur. Many small firms also encounter these obstacles (Golhar and Stamm 1993). Purchasing models are needed to aid the purchaser in the transition to a JIT system where purchasers and vendors may enter long term contracts for large quantities to be shipped in small lots. We propose a system which uses Economic Order Quantity (EOQ) type models to calculate the contract quantity and shipment frequencies.

In the ideal situation there is a *single reliable vendor* which will deliver quality items on time. We call this perfect coordination. There are simple EOQ extension models available in the literature which have been adjusted for the case of multiple deliveries. Since cooperation between the purchaser and the supplier is emphasized in the JIT philosophy, we develop joint inventory models which minimize the total cost of the purchaser and supplier. These models are extensions of joint purchaser/supplier models which are available for traditional inventory systems.

Another situation occurs when the *vendor is not reliable*, or is not able to provide small, frequent deliveries on time with the established quantity or quality levels. We call this situation imperfect coordination. There are delivery delays or quality problems which can cause shortages. In the spirit of JIT, we find the minimum safety stock, combined with the best shipment frequency, needed in order to protect against shortages which may be caused by delivery delays. The safety stock, required to provide the necessary service level, depends on the shipment frequency, among other things. Thus, the consideration of safety stock changes the optimal shipment policy. This safety stock is incorporated into the joint inventory models developed for perfect coordination. The

basic model considers random delays in shipments. It is extended for the cases of random yield and random demand.

The last situation we consider is where there is *not a single reliable supplier* that is willing or able to deliver small lot sizes, frequently, and on schedule with high quality levels. If available, multiple suppliers may be used until a reliable supplier is singled out. Multiple sourcing gives more protection against shortage and the competition can improve the price and quality of the product. Even Honda, a major pioneer in JIT purchasing, uses multiple sources in purchasing a considerable proportion of its parts. They use two suppliers for 44% of their parts, three suppliers for 16%, and four or five for 4%. (McMillan 1990)

In multiple sourcing, the order is split between the suppliers and a larger share will be allocated to the suppliers with better delivery characteristics, giving them incentive to improve (Ramasesh, Ord and Hayya 1991). Much work has been done to show the advantages of multiple sourcing. The question is how to allocate the order quantity among the suppliers. We give an allocation method for two suppliers which minimizes the stockout risk. The stockout risk is an appropriate service measure for important items, where the penalty for a stockout occasion is high and the cost of shortage is proportional to the number of shortage occasions rather than to the amount on short. We consider only a difference in lead time mean and variance and assume that all other characteristics of the suppliers are the same.

1.1 Research Objectives

The objectives of this study are to: 1) provide purchasing models to aid the purchaser in the transition to JIT which are adjusted for multiple deliveries and emphasize cooperation between the purchaser and the supplier, and 2) provide general tendencies of the models based on sensitivity and factorial analysis for practitioners.

1.2 Contribution of the Research

Economic order quantity models which are adjusted for multiple deliveries (Pan and Liao (1989), Ramasesh (1990)) or consider the cooperation between the purchaser and the supplier (Banerjee (1986), Goyal(1988)) have been published in the literature. The JIT philosophy emphasizes both of these elements. We provide a model for single sourcing which incorporates multiple deliveries and cooperation. Also provided are extensions of this model for random lead times, yield, and demand which can occur in the transition to JIT.

A reliable supplier may not be available and multiple suppliers may be used. Several studies have examined the advantages of multiple sourcing, but do not consider the allocation of the order between the suppliers. We provide an allocation model for dual sourcing which gives the optimal order split between suppliers in order to minimize the stockout risk. Insight into the models, for practitioners, is provided using sensitivity and factorial analysis.

1.3 Organization of the Research

This study is organized in six chapters. Chapter 2 provides a review of the relevant literature in single sourcing. An overview of previous research in multiple sourcing is given in Chapter 3. Purchasing models for the transition to JIT with single sourcing are derived in Chapter 4. Chapter 5 introduces a model for allocation of an order in dual sourcing. Finally, Chapter 6 summarizes the findings of the study and discuss the implications of the models for practitioners. Also, included are suggestions for future research directions.

CHAPTER 2

LITERATURE REVIEW - SINGLE SOURCING

Single sourcing is an integral part of the JIT philosophy. Companies want a single, reliable vendor that is willing to deliver small, frequent shipments, on time, with the quality and quantity required. JIT production systems require a level master production schedule (MPS), frozen for a certain time horizon. This requirement motivates the assumption of known demand with constant rate. Deterministic demand combined with deterministic lead time is surveyed in Section 1. In many cases the supplier is not able to provide reliable deliveries. Delays and/or quality problems may occur with deliveries. This situation is surveyed in Section 2. Other related literature is given in Section 3.

2.1 Deterministic Demand and Lead Times

Pan and Liao (1989) develop a simple Economic Order Quantity type model for a JIT delivery system. In a traditional EOQ model, it is assumed that the demand rate is known and constant, the unit cost is independent of the order size, there is a constant rate of items from inventory, and orders are received instantaneously. In order to determine the optimal order quantity, the sum of the ordering and holding costs are minimized. This model is extended, by Pan and Liao, to situation where the order is split into two equal parts and then they extend their result to n parts. It is assumed that a long-term purchasing agreement has been entered into with a supplier, and that the ordering cost is not affected by the number of deliveries.

The total cost includes the inventory holding cost and the ordering cost. The holding cost is affected by the number of shipments. In the case of n deliveries, the reduction in total cost due to the arrival of the order in small shipments is

$$1 - \frac{1}{\sqrt{n}} \tag{2.3}$$

This model yields the optimal order quantity for a specific number of deliveries, but how many deliveries should we use? The authors give three suggestions for determining the number of deliveries.

1) If a maximum inventory on-hand, M, is set then

$$n = \left(\frac{Q_1}{M}\right)^2 \tag{2.4}$$

where Q_1 denotes the traditional EOQ for the single shipment (n = 1) case.

2) If the average inventory on hand, H, is set then

$$n = \left(\frac{Q_1}{2H}\right)^2 \tag{2.5}$$

3) If a percentage rate, x%, of reduction in total cost is designated then

$$n = \frac{1}{(1-x)^2} \tag{2.6}$$

This model is simple and easy to implement. Since it is an extension of the basic EOQ model, most purchasers will find it easy to adjust to. A drawback to this model is that it ignores the extra shipping costs that the split deliveries will cause.

Ramasesh (1990) provides a similar model which adds the shipping cost and suggests that it "will enable us to achieve savings in cost and motivate our move toward the ultimate form of JIT purchasing."

The ordering cost and holding costs are assumed to be the same as in the Pan and Liao model. The new term is the shipping cost. Ramasesh finds the optimal number of shipments which will minimize the total cost and proposes several ways to specify the order quantity, Q.

1) Use the demand for the item over the order period if the production schedules are somewhat stable.
2) Specify a dollar amount for a vendor based on his past performance.
3) Order the requirements for an MRP type planning system if it is used.

4) If a kanban system is used, order an integer multiple of the number of items in a container.

5) Use the economic run length of the supplier.

The last option is also suggested by Newman (1988a). If this suggestion is chosen, then the economic manufacturing quantity of a JIT supplier can be found as in Golhar and Sarker (1992). They assume that a fixed interval delivery system is used with a single stage production system. The total cost is the sum of the holding costs for raw materials and finished goods, the setup cost for the production run, and the ordering costs.

To find the optimal production quantity, a specific procedure is required because of the integer variables involved. An iterative algorithm is provided in (Golhar and Sarker 1992) for the case of imperfect matching, where the production uptime and cycle time are not exact integer multiples of the shipment interval. The procedure results in an optimal or near optimal solution.

A formula for the manufacturing quantity is found for four special cases. They are as follows:

Perfect Matching (where the production uptime and cycle time are exact integer multiples of the shipment interval)

Fixed Interval Delivery System (with $P > D$)

Instantaneous Replenishment (with P> >D)

Instantaneous Replenishment without Processing
(with P> >D, where the demand rate is the production rate needed for JIT deliveries)

This model benefits the supplier, but may not give the best Q for the purchaser. Banerjee (1986) derives the joint economic lot size for the supplier and the purchaser. This is not adjusted for JIT purchasing though, since multiple delivery of an order in small shipments is not considered.

The assumptions for the model are that we have deterministic demand and lead times and that there is a sole supplier and no other purchasers. The vendor produces on a lot-for-lot basis.

The joint economic lot size JELS is derived from the minimization of a total cost function that is the sum of the total cost functions of the supplier and the purchaser. The minimum cost order quantity, Q_J^*, and the joint total relevant cost for that quantity are derived.

Banerjee also gives the cost increase for the vendor if the purchaser's Q is used, and the cost increase for the purchaser if the vendor's Q is used, and the cost penalties for each using Q_J*. He shows that in the typical case, the joint optimal total cost is considerably smaller than the sum of the individual costs for the supplier and buyer. This gives an initiative for cooperation. He feels this model is "at least an intermediate step toward the shift to JIT techniques," since the idea of cooperation between the supplier and the purchaser is used.

Goyal (1988) feels the lot-for-lot basis is too restrictive, and his model allows that the producer could produce in lot sizes of nQ where n is an integer. He adjusts the total cost given by Banerjee(1990) by computing the average inventory of the purchaser as the time-weighted inventory divided by the cycle length and derives a double inequality which can be used to determine n^*.

An example is given to show the reduction in cost by using this model. This model is more difficult to implement than the Banerjee model, but it may yield cost savings. Both models emphasize the cooperation of the vendor and the purchaser, but they are not adjusted for split deliveries, which are typical for JIT. This will be addressed in Section 4.1 of our study.

Hong and Hayya (1992) derive the conditions where multiple deliveries are worthwhile considering the aggregate ordering costs. They examine the cases of convex exponential, logarithmic, and linear step ordering cost functions.

The models in this section consider the ideal case where there is deterministic demand and lead times. This does not occur in most real world situations. The next section discusses the literature for the random lead time situation.

2.2 Random Lead Times

Many times there are delivery or quality problems which cause delays. Models are needed for use in the random lead time situation with multiple deliveries of a contract quantity according to JIT. Few results are given for the random demand and random lead time case with multiple deliveries. This is because of the difficulty in developing

mathematically tractable models. Kelle (1984) provides models to find the minimum safety stock for a prescribed service level where the multiple delivery times are random and the demand is deterministic. A fixed order period, T, is considered. The order quantity, that covers the demand of time T, is assumed to be a known quantity Q. It is also assumed that the demand rate is constant, thus, the delivery size is Q/n, and the deliveries follow a uniform pattern, so deliveries are scheduled to arrive at regular intervals. The vendor tries to deliver on schedule, but because of production, capacity, or transportation problems there is a random disturbance in the delivery times.

In order to protect against possible delays, the purchaser will hold a small amount of safety stock, M. There is no shortage in (0,t) if the cumulative amount of inventory, that is the safety stock + cumulative amount delivered in the period (0,t) is larger than or equal to the cumulative amount demanded up to time t. This must hold in the period $[0 \leq t \leq T]$ to insure a continuous supply. Since the delivery times are randomly disturbed, this can only be required to hold with some probability, $1 - \epsilon$. This probability is the service level provided by safety stock M. Using a transformation it can be assumed that T=1 and Q=1.

The cumulative demand function, F(t), is assumed to be uniform. Since the vendor is trying to meet the delivery schedule, but may be a little off the mark, the cumulative delivery function, $F_n(t)$, is an increasing step function with step sizes Q/n, and uniformly distributed random delivery. $F_n(t)$ will approach F(t), according to the coordination between the vendor and the supplier. Based on this coordination, $F_n(t)$ is

considered, in Kelle (1984), as the empirical distribution function of the uniform distribution.

The minimum amount of safety stock needed should provide a prescribed service level, $1 - \epsilon$. For a given, initial stock, M, the probability of no shortage can be expressed in the form of a distribution. This probability distribution is found in Birnbaum and Tingey (1951). An estimate of M, providing the required service level $1 - \epsilon$, is derived using the asymptotic distribution (as $n \rightarrow \infty$) given by Smirnov (1939). It is a simple formula, and is appropriate when there is a large number of deliveries, as in the case of JIT deliveries.

In an extension of the model, random yield or quality problems are also considered in Kelle (1984). There is the possibility that a part of the shipment is scrap, or inferior quality that cannot be used. A proportion of the order is guaranteed to be received in each delivery and the rest will be divided randomly among the deliveries. The probability of no shortage, for this case, is given in Kelle (1984). The exact solution is provided and also an approximation for large n.

Generalization is provided to allow any type of distribution for delivery quantities, not just the uniform. The quantity delivered is assumed to be a random variable. Numerical methods can be used to find the best value of the safety stock.

Kelle (1984) also considers the case where the demand rate, α, is random, with distribution G(x). In this case, the optimal safety stock value can be found using numerical methods. If it is assumed that α is normally distributed then an

approximation for the M, providing the required service level $1 - \epsilon$, is derived. We will give extensions to these models in Chapter 4.

Kelle and Schneider (1992) extend the above to model the multi-stage production process. They approximate the minimum work-in-process inventory target level needed to provide a prescribed service level.

The approximations in this section are based on asymptotic distributions (as $n \to \infty$) and therefore would be appropriate in the situation where there is a large number of deliveries. This is a characteristic of JIT purchasing. These models will be discussed in detail in Chapter 4.

2.3 Other Related Papers

Many papers discuss the benefits, problems, and implementation of JIT sourcing. (Schonberger and Gilbert (1983), Bartholomew (1984), Manoocheri (1984), and Ansari and Modaress (1986,1987) Golhar and Stamm (1993) examine three firms in order to study the effect of JIT purchasing implementation. Billesbach, Harrison, and Croom-Morgan (1991) compare JIT purchasing activities in the U.S. and the U.K., whereas, Giunipero and Keiser (1987) compare JIT purchasing in manufacturing and non-manufacturing environments with the use of a case study.

Hong, Hayya, and Kim (1992) compare the costs for a traditional economic manufacturing quantity (EMQ) system, which is the EOQ with a finite production rate, with a JIT order splitting model where there is investment in setup reduction. They use an integrated inventory model and find that the JIT system could produce better results.

In examining the buyer-supplier relationship, Rubin and Carter (1990) develop a general model to show that cooperation between them can be financially beneficial to both. Trevelyn and Schweikhart (1988) give a risk/benefit assessment model to compare single and multiple sourcing strategies. It is for use in specific situations.

The models in this chapter only consider the situation where the purchaser orders from a single supplier. In many instances, this is not possible because of government regulations or perhaps a reliable supplier is not available who is willing to deliver frequently. In this case multiple sourcing is an option. The purchaser may order from more than one vendor. Multiple vendor purchasing is surveyed in the next chapter.

CHAPTER 3

LITERATURE REVIEW - MULTIPLE SOURCING

One of the characteristics of JIT production is single sourcing. A single reliable vendor with small-lot deliveries is the goal. This ideal scenario many times does not occur. Since a reliable source is not available, multiple sources can be used to decrease supply uncertainty and provide frequent, small-lot deliveries.

Thus, in the transition to JIT purchasing, multiple sourcing can be a good strategy until an appropriate vendor is singled out. It promotes competition, which may improve quality and vendor reliability. The quantitative advantages of multiple sourcing have been examined in several studies. Since a reliable supplier is not available, we are considering the case where there are random lead times. In Section 1, we review papers where the authors assume identical lead time distributions for the suppliers. This is extended to the situation where the lead time distributions are different. It is reviewed in Section 2. The question of how to allocate the order quantity and how to choose the vendors arises. Papers dealing with these problems are surveyed in Sections 3 and 4. Section 5 reviews other related papers.

3.1 Identical Lead Time Distributions

Hayya, Christy, and Pan (1987) examine a reorder-point system with two vendors whose lead time distributions are identical. The order quantity is split evenly between the vendors. The demand distribution is normal and the lead time distribution is gamma. They also consider the possibility of order-crossing. This occurs when part of

an order from one period order can arrive before another order that was placed in a previous period. Because of the analytic intractability of this case, they use simulation. The purpose of the study is to compare the one vendor and the two vendor systems examining the lead time demand, on-hand inventory, inventory position, and the number of backorders.

The simulation results yield almost the same on-hand inventory, a smaller number of backorders for the two vendor system, and nearly the same inventory position. It also showed a smaller mean lead time demand for the two vendor situation and they explain this statistically using order statistics. It is pointed out that since the lead time demand is reduced, the stockout risk is lower.

Ramasesh, Ord, Hayya, and Pan (1991) also study a two vendor, (s,Q) re-order point system. The order is evenly split between two vendors with identical lead time distributions and constant demand. The exponential and uniform distributions are considered.

They assume that there is no order crossing. To insure this for the uniform case, they stipulate that the order quantity must be less than or equal to the maximum lead time demand. In the exponential case, to avoid the problems of order crossing, they assume that each order is a special one and cannot be exchanged.

The expected cost per unit time, ETCUT, is derived and a numerical search is used to find the optimal s and Q. The findings suggest that dual sourcing provides lower inventory holding and backorder costs which must be compared to the increased ordering costs. Dual sourcing gives larger savings for the case where the standard

deviation of the lead time demand is larger and for the exponential lead time when compared to the uniform case.

These studies only examine the two vendor case and very specific lead time distributions. Kelle and Silver (1990a) examine the situation where there are more than two vendors, further, Weibull lead times are considered which provide a good approximation for most of the practical forms of distributions. Here it is assumed that there is no order crossing. They give analytic expressions for the reduction in expected demand before the first delivery, and the interval of reduction for the variance of demand in this period.

When the demand rate is known, the reorder point, for a fixed service level, is derived and the reduction in order point is shown to be the same as for the expected demand before the first delivery. Lower bounds are provided for the quantity to order which yields a negligible risk of a stockout before a later delivery, and the quantity to give a negligible order crossing risk. Some numerical results are shown for the random demand case.

Kelle and Silver (1990b) re-examine the multiple vendor situation using expected shortage per cycle as a service measure instead of the probability of a stockout in a cycle. They find a reduction in expected shortage and the safety stock for order splitting. Numerical findings show that order splitting is advantageous when we have moderate lead time variability and when the order quantity is large relative to the expected lead time demand.

These studies exploit the benefits of order splitting. Since they use order statistics, they must assume identical lead times for the suppliers. It is more likely that the vendors will have different lead time characteristics.

3.2 Different Lead Time Distributions

Sculli and Wu (1981) consider a continuous review re-order point model with two vendors. The vendors have different normally distributed lead times with deterministic demand. The mean and standard deviation of the effective lead time demand (distribution of demand until the first delivery) and the inter-arrival time are numerically computed for different parameter combinations. The results imply that for a given p_1 service level, that is, the probability of no shortage, the reorder level is smaller for dual sourcing.

Sculli and Shum (1990) extend the above results to the case where there are more than two vendors. They show that the mean and variance of the lead time demand until the arrival of the first delivery is smaller than that of the individual suppliers.

Ramasesh, Ord, and Hayya (1991) extend their earlier results relaxing the stipulation that the order quantity is split evenly. Exponential lead times and deterministic demand are assumed. They compare the optimal total costs of the traditional one vendor model and two vendor model over different parameter values and find that dual sourcing provides savings in the following cases:

- when the lead time variability of the second supplier
  is within an upper bound.

- the shortage to ordering cost ratio is large.

- when the ordering cost increases, even by 50%.

The advantages of multiple sourcing have been examined in previous studies, but the question is, how can we allocate the order among the suppliers in order to exploit these advantages to the fullest.

3.3 Allocation of the Order Quantity Between Suppliers

In a JIT purchasing situation, multiple sources can be used until a single, reliable source is singled out. A larger percentage of the order can be allocated to the vendor with the best characteristics, i.e. quality, lead time, or price. This will give the vendors an incentive to improve, thereby, promoting competition. How much do we reward the vendor with the best characteristics?

Only a few studies have considered the problem of how to allocate the order quantity between the suppliers. Pan (1989) develops a linear program to find the optimal number of suppliers to use, and the allocation of the order between them. To measure supplier performance he uses price, quality, lead time, and service. The measure of most importance to the purchaser is used as the objective function, and the rest are constraints. The performance measures are calculated as the weighted average of the individual supplier performances. If quality is the most important to the purchaser, the linear program is as follows:

$$Max \quad \sum_{1}^{n} (q_i x_i)$$
$$st$$
$$\sum_{1}^{n} (p_i x_i) \geq Q$$
$$\sum_{1}^{n} (l_i x_i) \leq L$$
$$\sum_{1}^{n} (s_i x_i) \geq S$$
$$\sum_{1}^{n} (x_i) = 1$$
$$x_i \geq 0 \quad i=1...n$$

where x_i is the proportion of the order quantity allocated to vendor i, and p_i, q_i, l_i, and s_i are the price, quality, lead time, and service of vendor i, respectively. This is a simple, easy to use model, but the linear assumption is restrictive since there is no randomness in lead time.

Hong and Hayya (1992) develop a mathematical programming model for the selection of suppliers and the allocation of the order among those suppliers. The total relevant cost is minimized subject to constraints on the delivered cost and the product quality.

Chaundry, Forst, and Zydiak (1991) model the allocation as a multicriteria problem. The criteria are again price, quality, lead time, and service. They assume that the order quantity has already been determined. Integer goal programming is used to allocate the order quantity among the suppliers. The goals are as above, and vendor constraints are added.

Anupindi and Akella (1993) study the allocation problem for two vendors with continuous, known, stochastic demand. The ordering policy uses two reorder points. If the inventory is between the two points, only one vendor is used. If the level falls below the lower point, both vendors are used.

The optimal ordering policy is derived for three models. In Model I, each supplier either delivers the whole order with probability β, or they do not deliver the order at all. A random proportion of the shipment is delivered, by each supplier, in the current order period in Model II. The rest of the shipment is not delivered. In Model III, the random proportion is delivered from each supplier, but the rest of the order is delivered in the next period. The optimal policy is found for the single and multi-period problems.

Lau and Zhao (1993) find the optimal order quantity, reorder point, and proportion of split of an order between two suppliers for any stochastic form of demand and lead times using approximation and numerical search techniques. They give a procedure to approximate the lead time demand distribution. This is needed in order to consider the general demand case. They found that order splitting reduces the inventory carrying cost and that the optimal proportion of split for vendors varies with the difference in the mean lead times of the vendors.

3.4 Vendor Selection

Vendor selection is also an issue in multiple sourcing. Zhao and Lau (1992) study the dual sourcing situation with random lead times and demand. They suggest that the

average inventory can be reduced by choosing a second supplier with a larger average lead time than the first supplier. Narasimhan (1983) applies the analytic hierarchic process (AHP) to supplier selection and Akinc (1993) suggests a decision support approach to selecting suppliers in a JIT manufacturing setting.

Approved supplier lists are also used to select vendors. This is discussed in Plank and Kijewski (1991). Lockhart and Ettkin (1993) give seven steps to use in certifying vendors, and Newman (1988) gives guidelines for choosing a supplier. Giunpero (1990) did a study on the types of supplier performance measures in use, and Willis, Huston, and Pohlkamp (1993) suggest evaluating supplier performance with dimensional analysis.

3.5 Other Related Papers

Many papers discuss the advantages of multiple sourcing, such as enhanced competition and lower risk. These advantages are discussed in Ammer (1980), Kraljic (1983), and Morgan (1987). Kratz and Cox (1982) examine the competition factor. Multiple sourcing is widely used in government procurement. Drinnon and Hiller (1977), Sellers (1979), Kratz, Drinnon, and Hiller (1984), and Greer and Liao (1986) study this situation.

There are many other papers that deal with related topics such as multiple shipments from the same vendor, expedited shipments, and multiple vendors with different product prices. Moinzadeh and Lee (1989) give near optimal solutions for the

operating characteristics for a system where the shipments may arrive from a source in two shipments. The size of the first shipment is assumed to be random.

Horowitz (1986) examines the situation where buyers purchase the same product from different suppliers for different prices. He suggests that they do this because they wish to reduce the uncertainty in product delivery. The second source may be used as an emergency supplier. Moinzadeh and Nahmias (1988) give a heuristic procedure for this type of system, to determine the two reorder points and order quantities, one for each vendor. It is an extension of the (Q,R) system. The expected average cost per unit time is also derived. Simulation was used to verify the model and the results were very close to the analytic results. It was also shown, using a cost comparison, that it is economical to use an emergency supplier when the stockout cost is large relative to the other costs.

Estimating the lead time parameters for the multiple sourcing case is also of interest. Pan, Ramasesh, Hayya, and Ord (1991) estimate these parameters. They also give the lead time reduction for dual sourcing with uniform, exponential, and normal lead time distributions. Fong and Ord (1993) use a Bayesian approach with predictive distributions, to estimate the effective lead time mean and variance for multiple sourcing.

CHAPTER 4

SINGLE SOURCING PURCHASING MODELS FOR THE TRANSITION TO JIT

Single sourcing is a major goal in JIT purchasing. Purchasers want a reliable vendor that will provide small, frequent deliveries of quality product on time. Vendors and purchasers can work closely in order to achieve this perfect coordination. This cooperation is important for the success of JIT purchasing.

There are few quantitative JIT purchasing models available in the literature. Ramasesh (1990) and Pan and Liao (1989) have provided simple EOQ - type models, but they only consider the purchaser's costs and not cooperation which should be emphasized in JIT purchasing. Banerjee (1986) has developed a model to find the joint order quantity for the purchaser and the vendor in order to facilitate this cooperation. The Banerjee (1986) model considers the classic situation where the order quantity is delivered in one shipment, but JIT production emphasizes multiple shipments. We have adjusted the Banerjee (1986) model to include shipment cost and multiple deliveries of an order as in Ramasesh (1990). We also provide a simple solution. Section 4.1 details our inventory model for this situation.

In some cases, perfect coordination may not be possible. The transition to JIT is an example. During this time period there may be delivery delays, quality problems, or unstable demand, causing imperfect coordination. Section 4.2 contains models for this situation. In Section 4.2.1, a model is developed for the case where there are delivery delays. Another complication is quality problems. A model which handles

delays and random yield is given in Section 4.2.2. Section 4.2.3 contains a model for the random demand, random delay case. In Section 4.3, sensitivity and numerical analysis are given for the models.

4.1 Perfect Coordination - Model I

Buyer-supplier cooperation is emphasized in JIT. This will benefit both the purchaser and the supplier. They become partners in the process, rather than each trying to enhance their own position only. This should result in a much smoother operation that can include cost savings for both parties. Rubin and Carter (1990) detail an example, calculating costs, where the purchaser and supplier costs should be examined together. In order to incorporate this into our model the joint total relevant cost can be considered as in Banerjee (1986).

In the Japanese auto industry, the purchasers are typically strong and can force the vendors to produce in small lot sizes which benefits the purchaser. In the United States, the vendors consider frequent, in time deliveries as taking on the burden of holding inventories instead of the purchasers and are reluctant to implement JIT deliveries unless some compensation is provided by the purchaser. The vendors are strong in many cases and can force the purchasers into ordering large quantities which will benefit the vendor. In these cases, the vendor and purchaser have an adversarial relationship rather than a cooperative one, which is contradictive to the JIT philosophy. The joint ordering policy is considered as a compromise. One party may benefit more than the other, but they can use this model as a quantitative negotiation tool. One party

may compensate the other with some type of concession. The vendor may offer a discount or the purchaser may give the vendor a long-term contract. This allows the vendor and purchaser to both benefit from the cooperation.

The Banerjee model will be altered to handle the multiple delivery situation as in Ramasesh (1990). The purchaser's total cost will be the same as in Ramasesh and the supplier's cost will be adjusted for multiple deliveries and a finite production rate for the supplier is also examined.

Assumptions

1) The contract or order period is [0,T].

2) The demand is deterministic with known rate D.

3) The order quantity is Q = DT.

4) Cooperation exists between the buyer and the supplier, and total coordination is maintainable, that means that there will be n deliveries of size Q/n, delivered on schedule.

A known component or material demand with constant rate is generally valid for JIT systems, considering the level Master Production Schedule, MPS, requirement of JIT production. However, this assumption will be extended in Model IV. Random delays in deliveries are considered in Models II - IV, and random yield is considered in Model III.

<u>Notation</u>

D = Demand rate

Q = Order quantity

n = Number of shipments in a contract period

A = Purchaser's ordering cost per order

r = annual inventory carrying cost (% dollar value)

C_P = per unit purchasing cost

C_V = per unit production cost

S = vendor's setup cost per setup

Z = purchaser's shipment cost per shipment

The buyer orders a quantity to satisfy the demand for a period of time. The order quantity is split into n deliveries of size Q/n. The supplier ships the order, which is received on time, with the quality and quantity desired, when the buyer needs it. The entire order is received during the order period.

4.1.1 The Basic Model

The annual total relevant cost per unit time of the purchaser, (TRC_p), for a JIT purchasing system is given by Ramasesh (1990) as follows:

$$TRC_p(Q,n) = \underbrace{\frac{AD}{Q}}_{Ordering} + \underbrace{\frac{nZD}{Q}}_{Shipment} + \underbrace{\frac{QC_p r}{2n}}_{Holding} \qquad (4.1)$$

The total relevant cost for the vendor can be expressed as

$$TRC_v(Q,n) = \underset{Setup}{\frac{DS}{Q}} + \underset{Holding}{\frac{(n-1)}{n}\frac{Q}{2}rC_v} \tag{4.2}$$

The joint total relevant cost of the purchaser and vendor (JTRC), can therefore be expressed as

$$\begin{aligned} JTRC(Q,n) &= \left[\frac{AD}{Q}+\frac{Q}{2n}rC_p+\frac{nZD}{Q}\right] + \left[\frac{DS}{Q}+\frac{(n-1)}{n}\frac{Q}{2}rC_V\right] \\ &= \frac{D}{Q}(S+A+nZ)+\frac{Qr}{2}\left(\frac{(n-1)}{n}C_V+\frac{C_P}{n}\right) \end{aligned} \tag{4.3}$$

where the first term is the purchaser's cost and the second term is the supplier's cost. This is an extension of Banerjee's model (1986), considering multiple deliveries and cooperation in JIT purchasing. This equation is in the form of the total relevant cost, TRC, for the basic economic order quantity, EOQ, as shown below.

$$TRC(Q) = \frac{A_J D}{Q} + h_J\frac{Q}{2} \tag{4.4}$$

where the ordering cost A_J and the holding cost h_J are as follows

$$A_J = A + nZ + S \quad , \quad h_J = r\left(\frac{(n-1)}{n} C_V + \frac{C_P}{n}\right) \tag{4.5}$$

Hence, the joint economic order quantity that minimizes (4.3) has the form

$$\begin{aligned} Q_{J^*}(n) &= \sqrt{\frac{2A_J D}{h_J}} \\ &= \sqrt{\frac{2(A+nZ+S)D}{r\left(\frac{(n-1)}{n} C_V + \frac{C_P}{n}\right)}} \end{aligned} \tag{4.6}$$

which depends on the number of shipments, n. For a continuous n and fixed Q, (4.3) is a convex function of n. In that case, the optimal n can be found by taking the derivative of the JTRC with respect to n, setting it equal to 0 and solving, yielding

$$n^*(Q) = Q\sqrt{\frac{Q}{DZ}\left(\frac{Qr}{2} C_V + C_P\right)} \tag{4.7}$$

However, n must be integer. Thus, the two closest integers to n^* must be considered, substituted into (4.3) and the one which yields the smallest result provides the optimal number of deliveries with the lowest total cost for a fixed Q.

If both parameters are decision variables, we can proceed with the optimal solution as follows. For any integer n, $Q_J^*(n)$ as expressed in (4.6) provides the best Q. It must

be substituted into the JTRC of (4.3) and then the resulting equation must be optimized for n. Since we have a function in the form of the total relevant cost (4.3) for an EOQ-type model, the optimal total relevant cost equals $\sqrt{2A_j h_j D}$, which is in our case

$$JTRC(n) = \sqrt{2\,(A+nZ+S)\;r\,(\frac{(n-1)}{n}C_v+\frac{C_P}{n})\,D}$$

Finding the n which minimizes JTRC(n), is equivalent to finding the n which minimizes $(JTRC(N))^2$. Ignoring the terms which are independent of n, one can reduce the minimization problem to that of minimizing

$$F(n) = (A+nZ+S)\left(\frac{(n-1)}{n}C_v+\frac{C_P}{n}\right)$$

The optimal n cannot be separated and expressed explicitly, thus numerical methods must be used.

Since the JTRC is a convex function of n, the optimal number of shipments can be found by calculating the total cost for different values of n, in increasing order, and observing the point where the total cost ceases to decrease and begins to increase. The value of n at the minimum F(n) is optimal. This can be easily implemented with a spreadsheet.

4.1.2 Finite Replenishment Rate Model

It is more realistic to assume a finite production rate for the supplier. For a production rate P, (P > D), the total cost for the supplier is

$$TRC_v(Q,n) = \frac{DS}{Q} + Q_{AVG}rC_v \tag{4.8}$$

where Q_{AVG} is the average inventory per cycle. Q_{AVG} for a JIT supplier is provided in Golhar and Sarker (1992) as

$$\frac{Q}{2}\left(1-\frac{D}{P}-\frac{1}{n}\right) \tag{4.9}$$

Incorporating this expression into the JTRC yields

$$\begin{aligned} JTRC(Q,n) &= \left[\frac{AD}{Q}+\frac{QrC_P}{2n}+\frac{nZD}{Q}\right]+\left[\frac{DS}{Q}+\frac{QrC_v}{2}\left(1-\frac{D}{P}-\frac{1}{n}\right)\right] \\ &= \frac{D}{Q}(S+A+nZ)+\frac{Qr}{2}\left(\frac{1}{n}(C_p-C_v)+C_v\left(1-\frac{D}{P}\right)\right) \end{aligned} \tag{4.10}$$

We can proceed with the solution as in the previous section. The minimum cost order quantity, for a given n, is now

$$Q(n) = \sqrt{\frac{2D(A+nZ+S)}{r\left(\frac{1}{n}(C_p-C_v)+C_v\left(1-\frac{D}{P}\right)\right)}} \tag{4.11}$$

This is an overall optimum since the JTRC is a convex function of Q for fixed n. The optimal number of deliveries for a fixed Q can be found similarly as in the previous case except that

$$n(Q) = Q\sqrt{\frac{r(C_p-C_v)}{2ZD}} \tag{4.12}$$

must be used instead of (4.7) .

If both parameters are decision variables, then the same problems occur at the numerical solution as with the previous model. In the next section we provide a new formulation and a simple solution procedure that can handle both finite and infinite production rates.

4.1.3 Echelon Stock Model

We apply the idea of echelon stock to this situation, yielding simpler formulas and better control. The vendor-purchaser case with coordination can be considered as a multi-echelon system where the supplier is the primary stage and the buyer is the finishing stage. Echelon stock, at a stage, is the number of units at or beyond that stage, but still in the system. The value of units at any stage is only the value added at

that stage. Hence, the value of primary stage (vendor) inventory is C_v and the value of finishing (purchaser) stage inventory is $C_p' = C_p - C_v$, where C_p is the value of the inventory at the finishing (purchaser) stage so C_p' is only the value added at that stage. The expression for the JTRC simplifies to the form:

$$JTRC(Q,n) = \frac{D}{Q}(A+nZ+S) + \frac{Qr}{2}\left(\frac{C_P'}{n} + C_v(1-\frac{D}{P})\right) \qquad (4.13)$$

If we derive the optimal order quantity for a given n as previously, the following results.

$$Q(n) = \sqrt{\frac{2(A+S+nZ)D}{r\left(C_V(1-\frac{D}{P}) + \frac{C_P'}{n}\right)}} \qquad (4.14)$$

The expression can be substituted back into the JTRC. Finding the n which minimizes JTRC(n), is equivalent to finding the n which minimizes $(JTRC(N))^2$. Ignoring the terms which are independent of n, one can reduce the minimization problem to that of minimizing the following expression for n.

$$F(n) = [A+nZ+S]\left[C_V(1-\frac{D}{P}) + \frac{C_P'}{n}\right]$$

The resulting optimal number of shipments is

$$n^* = \sqrt{\frac{C_P'(A+S)}{ZC_V(1-\frac{D}{P})}} \qquad (4.15)$$

where n may not necessarily be integer, but one of the neighboring integers must provide the overall optimum because of the convexity property.

We now apply an algorithm the multiple-delivery situation and a finite production rate for the supplier. This procedure is used to find Q and n jointly. Our algorithm is as follows:

1) Compute

$$n^* = \sqrt{\frac{C_P'(A+S)}{ZC_V(1-\frac{D}{P})}}$$

If n^* is integer, go to 4 with $n = n^*$
If $n^* < 1$, go to 4 with $n = 1$
Otherwise, go to 2.

2) Find n_1 and n_2, the integer values that surround n^*

3) Find $F(n_1)$ and $F(n_2)$ where

$$F(n_1) = [A+n_1Z+S]\left[C_V(1-\frac{D}{P}) + \frac{C_P'}{n_1}\right]$$

and

$$F(n_2) = [A+n_2Z+S]\left[C_V(1-\frac{D}{P}) + \frac{C_P'}{n_2}\right]$$

If $F(n_1) \leq F(n_2)$ use $n = n_1$

If $F(n_1) > F(n_2)$ use $n = n_2$

4) Find Q such that

$$Q = \sqrt{\frac{2(A+S+nZ)D}{r\left(C_V(1-\frac{D}{P}) + \frac{C_P'}{n}\right)}}$$

5) The order quantity is Q for the purchaser.

This algorithm is similar to that given in Silver and Peterson (1985) for the multi-echelon situation. It was noted that if the n^* computed in step 1 is rounded to the nearest integer (.5 or larger rounded up) then the optimal n seems to be given. Values with decimal parts between .4 and .5 need to be checked.

Sensitivity and numerical analysis of this model is discussed in Section 4.3 and practical implications are given in Chapter 6.

4.2 Imperfect Coordination

In the perfect coordination case, we assume that the supplier is able to provide small, frequent deliveries of quality product on time. In many situations, this will not occur because of production limitations or transportation problems. A company which is in transition to JIT may have these troubles. A small amount of safety stock may be needed to guard against the shortages that these problems may cause. In the next sections, we develop Models II, III, and IV which will incorporate into the joint total relevant cost the cost of the minimum safety stock required to provide a prescribed level of service and check the effect of imperfect coordination on the optimal Q, n, safety stock, and the costs involved.

4.2.1 Random Lead Times - Model II

A supplier may be unable to deliver frequent shipments on time because of transportation problems, production constraints, or some other problem that cannot be solved at that time. This will result in delays which can cause shortages and a small amount of safety stock may be needed to protect against these potential problems. We suggest extending Model I by using safety stock approximations as given in Kelle (1984) and incorporating them into our joint total cost expression derived in the previous section. The optimal safety stock depends on the decision about the order

quantity and number of shipments. Thus, in calculating the optimal Q and n, the safety stock must be considered also.

The safety stock level will be calculated in order to provide a required service level. Several service measures are available. Some common measures of service are the probability of no stockout occasions in an order cycle, the fraction of demand to be met from the shelf, and the specified ready rate. (Silver and Peterson 1985) The probability of no stockouts in an order cycle gives the percentage of order periods in which a stockout does not occur. The second measure yields the required percentage of demand that is met directly, that is without backorders or lost sales. The ready rate is the percentage of time that there is stock on the shelf.

If one is only concerned that a stockout may occur, then the first measure is appropriate, however, if the amount of shortage is of interest then the second measure applies. The third measure is used when the length of a stockout is important. We consider the first measure, the probability of no stockouts in an order cycle, in our model. This is a popular service measure. The other two measures are more difficult to handle analytically especially for dual sourcing.

Assumptions

Assumptions 1 - 3 are the same as for Model I.

4) The required service level is the probability of no stockouts in an order period, P_α.

5) There will be n deliveries of size Q/n, but random delays may occur. There is cooperation between the buyer and the supplier.

The supplier tries to deliver on time, but there are production or transportation problems which cause random delays in the shipments. Hence, the actual delivery times will closely follow the pattern of the demand schedule with random disturbances.

In our basic mathematical model, we consider the case where the purchaser enters a contract with the supplier, for a contract period [0,T], to purchase a quantity, Q, which is the demand for this period. The demand rate, $D = Q/T$, is constant. This is generally a good approximation for JIT systems, considering the level production schedule (MPS) requirement. An extension will be given later.

We consider the contract (order) period, denoted by [0,T]. For any $0 \le t \le T$, the cumulative demand up to time t, $F^*(t)$, which can be normalized, for ease in computation, in the following form

$$\begin{aligned} F^*(t) &= Dt \qquad \text{in } (0,T] \qquad (4.16) \\ &= Qt/T \\ &= Q\,F(t/T) \\ &= Q\,F(u) \qquad \text{with } u = t/T \end{aligned}$$

where $F(u) = u$ in [0,1], $F(u) = 0$ for $u \le 0$, and $F(u) = 1$ for $u \ge 1$. These are the properties of the cumulative distribution function of the uniform distribution on [0,1], so known and new results can be used based on these properties.

We consider the case where there are problems for the supplier, with transportation or production, so the delivery times are disturbed by unpredictable random events. The

supplier is trying to deliver on schedule, but cannot meet the delivery time requested by the purchaser. There is a slight delay. We assume that each delivery has the same chance of delay and those delays are independent from each other. Under those assumptions, the actual delivery times, $t_1^* < t_2^* < \ldots < t_n^* \leq T$, can be considered as random, independent uniformly distributed times in [0,T] arranged in increasing order. This is an approximation since in reality, there is dependence among the delays, but usually not enough data is available to characterize these dependencies. Further, dependent random delivery times make the models analytically intractable.

We denote the cumulative amount delivered in the interval [0,t] as $F_n(t)$. Since at each delivery point, the delivery size is Q/n, $F^*_n(t)$ is given by the following expression:

$$F^*_n(t) = \begin{cases} 0 & \text{if } 0 \leq t \leq t_1^*, \\ \dfrac{kQ}{n} & \text{if } t_k^* < t \leq t_{k+1}^*, k=1,\ldots,n-1, \\ Q & \text{if } t_n^* < t \leq T \end{cases} \tag{4.17}$$

which can be normalized as in the case of $F^*(t)$ in (4.16) in the following form

$$\begin{aligned} F^*_n(t) &= QF_n\left(\frac{t}{T}\right) \\ &= QF_n(u) \qquad u = \frac{t}{T} \end{aligned} \tag{4.18}$$

where

$$F_n(u) = \begin{cases} 0 & if \ u \le \frac{t_1^*}{T} = u_1^*, \\ \frac{k}{n} & if \ \frac{t_k^*}{T} = u_k^* < u \le u_{k+1}^* = \frac{t_{k+1}^*}{T}, \ k=1,..,n-1, \\ 1 & if \ \frac{t_n^*}{T} = u_n^* < u \le 1 \end{cases} \tag{4.19}$$

and $u_1^* < u_2^* < \ldots < u_n^* < 1$ can be considered as a random, ordered sample of a uniform distribution on [0,1]. Thus, $F_n(u)$ has the same properties as an empirical distribution function of U, where U is a uniform random variable on [0,1]. An empirical distribution is a cumulative frequency distribution of a random sample of observed values and is a discrete approximation for the actual distribution. Empirical distribution functions have been widely used in statistics. Some of these results can be directly applied to our analysis providing approximate solutions. Other results are extensions to the previous results available in statistical literature. The cumulative delivery distribution is constructed in the same manner as an empirical distribution so we assume that as long as cooperation exists between the purchaser and supplier, it has the same pattern as the demand and hence the same underlying distribution.

Let M_n denote the safety or target stock that is to be planned at the beginning of the contract period to provide the required service level. The service measure considered is the probability of no shortage in an order cycle. As mentioned earlier, this measure is appropriate for the situation where a stockout carries a penalty and the length of the stockout is unimportant. $M_n + F_n^*(t)$ is the cumulative amount of stock available during the time interval [0,t]. There is no shortage in a period [0,t], if $M_n + F_n^*(t) \geq F^*(t)$ for

all times t $\in$ [0,t]. Therefore, the probability of no shortage in the period [0,T] is

$$P_\alpha(M_n) = Prob(M_n + F^*_n(t) \geq F^*(t), \text{ for all } 0 \leq t \leq T) \quad (4.20)$$

If we substitute $M_n = Qm_n$, then we have

$$P_\alpha(m_n) = Prob\left(Qm_n + QF_n\left(\frac{t}{T}\right) \geq QF\left(\frac{t}{T}\right), \quad 0 \leq t \leq T\right) \quad (4.21)$$

Using the notation u = t/T,

$$P_\alpha(m_n) = Prob(m_n + F_n(u) \geq F(u) \quad 0 \leq u \leq 1) \quad (4.22)$$

In this form of equation (4.20), we can see that we have a cumulative distribution function, F(u), and an empirical distribution function, $F_n(u)$, of the same uniform random variable U on [0,1]. $P_\alpha(m_n)$ is a statistic since it is a function of random variables. Since this statistic is based on order statistics $t_1^* < t_2^* < \ldots < t_n^*$, it is valid for any distribution considered (Wilks 1962). Therefore, any distribution defined on [0,1] is applicable. We are not restricted to the uniform distribution. This idea is discussed in Kelle (1984). That means that no specific information of the pattern of the variable demand is required. If the pattern of delivery follows the pattern of demand,

that is, they can be assumed to be the cumulative distribution function and empirical distribution function of the same random variable U. This is the main advantage of the model because, in practice, often we do not have the demand pattern or data, but as long as the coordination exists between the purchaser and supplier, the assumptions are realistic. This is the case for JIT purchasing.

For a prescribed service level, p_α (the probability of no shortage in [0,T]), we must solve the equation

$$P_\alpha(M_n) = p_\alpha \tag{4.23}$$

for m_n. The exact distribution, $P_\alpha(m_n)$, can be found in Birnbaum-Tingey (1951) as below.

$$P_\alpha(m_n) = 1 - (1 - m_n)^n - m_n \sum_{k=1}^{n} \binom{n}{k} \left(m_n + \frac{k}{n}\right)^{k-1} \left(1 - \left(m_n + \frac{k}{n}\right)\right)^{n-k} \tag{4.24}$$

Equation (4.23) can be solved numerically for m_n using (4.24) but it is a tedious procedure in practical application. Therefore, we want to a provide a simple approximate expression for $P_\alpha(m_n)$. The asymptotic distribution given by Smirnov (1935) is

$$\lim_{n\to\infty} P_\alpha(m_n) = 1-\exp(-2nm_n^2) \tag{4.25}$$

For finite n, 1 - $\exp(-2nm_n^2)$ can be considered as an approximation for $P_\alpha(m_n)$. Using this approximation, we can find an approximate solution for (4.23), thus, considering the linear transformation $M_n = Qm_n$, the necessary safety stock, M_n, can be expressed as

$$M_n \approx Q\sqrt{\frac{\ln\left(\frac{1}{1-p_\alpha}\right)}{2n}} \tag{4.26}$$

The safety stock increases as the service level increases, as one might expect. The error of approximation is discussed in Kelle (1984). It is acceptable if n > 10, which is usually valid for JIT deliveries. This is a simple formula which can be used easily in practice. As we see, the safety stock is decreasing as the number of shipments increases. This effect must be considered in determining the optimal shipment policy.

Holding this safety stock will add to our inventory cost, further, the optimal safety stock depends on the decision variables n and Q so we must incorporate this component into the total cost of our basic model (Model I), to provide the optimal n and Q. The result is the cost function of Model II.

$$JTRC(Q,n) = A\frac{D}{Q} + rC_p\left(\frac{Q}{2n} + Q\sqrt{\frac{\ln\left(\frac{1}{1-p_\alpha}\right)}{2n}}\right) + \frac{nZD}{Q} + \frac{DS}{Q} \quad (4.27)$$
$$+\frac{Q}{2}rC_v\left(1-\frac{D}{P}-\frac{1}{n}\right)$$

Solving as in Model I for a given n, yields the following optimal value for Q.

$$Q^*(n) = \sqrt{\frac{2D(A+S+nZ)}{r\left(\frac{C_p}{n} + C_p\frac{\sqrt{2\ln\left(\frac{1}{1-p_\alpha}\right)}}{\sqrt{n}} + C_v\left(1-\frac{D}{P}-\frac{1}{n}\right)\right)}} \quad (4.28)$$

Substituting $Q^*(n)$ in (4.27) provides the total cost as a function of n. Numerical methods must be used to find the optimal value for n as in Model I. The spreadsheet method mentioned earlier was used for the numerical calculations in this study. An advantage of this method is that the results are applicable for non-uniform demand, because of the properties of the distribution and empirical distribution function connection used in the model. The formulas provided for the safety stock can be used as an approximation as long as the coordination exists between the supplier and the purchaser.

The approximation is based on asymptotic results ($n \rightarrow \infty$), but generally the number of shipments in a just-in-time system is large so it usually provides an

acceptable accuracy. Numerical results of the approximation are discussed in Section 4.3.

4.2.2 Random Yield - Model III

We have considered the case where there may be delays in delivery. Quality problems can also arise. The effective delivery quantity may be reduced because of defective items that cannot be used. This is the case of random yield. We now consider this as an extension of Model II.

Assumptions

Assumptions 1 - 4 are the same as for Model II.

5) There will be n deliveries of size $\beta_i Q/n$, where β_i is a random variable with known distribution, $G_i(x)$.

In Model III, there are random delays in the deliveries and there is also a random part of the delivery which must be returned or scrapped due to quality problems. Therefore, the delivery size that can be used to satisfy the demand, $\beta_i Q/n$, is random.

The demand process for Model III is the same as for Model II, given by (4.7). The cumulative delivery quantity in the period [0,t] is

$$F_n^\beta(t) = QF_n^\beta(t/T)$$

where

$$F_n^{\beta}(u) = \begin{cases} 0 & if 0 \le u \le u_1^* \\ \frac{1}{n}\sum_{i=1}^{k} \beta_i & if u_k^* < u \le u_{k+1}^*, k=1,\ldots,n \end{cases} \tag{4.29}$$

and u = t/T.

The probability of no stockout, $P_\alpha(m_n)$ for this case, given by Kelle (1984), is

$$P_\alpha(m_n) = 1-(1-m_n)^n - m_n\sum_{k=1}^{n}\binom{n}{k}\int_0^{n(1-m_n)} A^{k-1}(1-A)^{n-k}dG_K(x) \tag{4.30}$$

where $A = m_n + x/n$. Setting $P_\alpha(m_n)$ equal to the prescribed service level, p_α, and solving for m_n gives the necessary safety stock level. The solution must be found using numerical methods.

If the assumption is made that $G_k(x)$ is Beta distributed $\beta_k(\mu,\sigma)$, and that a fixed fraction of the order, $\delta = 1 - \sigma/\mu$, is guaranteed to be delivered and a random fraction $1-\delta$ is also delivered, the asymptotic ($n \rightarrow \infty$) approximation for $P_\alpha(m_n)$ yields the approximate safety stock,

$$M_{n,\delta} \approx Q\sqrt{\frac{1+(1-\delta)}{2n}\ln\left(\frac{1}{1-p_\alpha}\right)} \tag{4.31}$$

for a prescribed service level p_α (see Kelle (1984)). This formula can be used if $\sigma < \mu$, and provides a good approximation for large n.

Thus, the total cost of Model III is

$$JTRC(Q,n) = A\frac{D}{Q} + rC_p\left(\frac{Q}{2n} + Q\sqrt{\frac{1+(1-\delta)}{2n}\ln\left(\frac{1}{1-p_\alpha}\right)}\right) + \frac{nZD}{Q} + \frac{DS}{Q} + \frac{Qr}{2}C_v\left(1-\frac{D}{P}-\frac{1}{n}\right) \quad (4.32)$$

For a fixed n, the optimal Q is

$$Q^*(n) = \sqrt{\frac{2D(A+S+nZ)}{r\left(\frac{C_p'}{n} + \frac{C_p'\sqrt{2\,[2-\delta]\ln\left(\frac{1}{1-p_\alpha}\right)}}{\sqrt{n}} + C_v\left(1-\frac{D}{P}-\frac{1}{n}\right)\right)}} \quad (4.33)$$

Substituting $Q^*(n)$ in (4.32) provides the total cost as a function of n that can be solved for n as in Model II.

4.2.3 Random Lead Times and Random Demand - Model IV

For a stable Master Production Schedule (MPS), which is typical in JIT production, a constant production rate that generates a deterministic demand for purchased parts and equipment is a good approximation. Typically the MPS is frozen because the cycle is short. However, in the transition to JIT, the contract with the supplier is relatively

long. The production level is fixed, but this level is influenced by the outside (consumer) demand which is not under the producers control and may be unpredictable. Therefore, since we are trying to predict for a longer period, there is uncertainty. We assume that the MPS is stable and hence a level production occurs, but the production level has an uncertainty at the time of the contract (order). This uncertainty is reflected in the demand rate, D.

The previous models have assumed that the demand rate is deterministic. We will now consider an extension of Model II where this assumption is relaxed to assume that the demand rate is random at the time of the contract. At the time when the contract quantity, Q, is to be specified, the total demand and thus, the production rate, is not exactly known.

Assumptions

Assumptions 1,4, and 5 are the same as for Model II.

2) The demand rate is random. The demand process is $\zeta(t) = \alpha t$ where α is a random variable with distribution G(x).

3) The order quantity is DT, where D is the expected demand per unit time, $E[\alpha]$.

Finding the safety stock that exactly provides the service level p_α is too complicated and can only be done under specific assumptions. However, if it is assumed that the demand rate distribution is Normal with expected value D, and standard deviation Σ,

then the following expression (based on asymptotic distribution results) is provided in Kelle (1984).

$$P(m_{n,s}) \approx 1-\exp[-2nm_{n,s}(m_{n,s}+1-(D+nm_{n,s}s^2))] \qquad (4.34)$$

where s = Σ/D. If we solve the equation

$$P_\alpha(m_{n,s}) = p_\alpha \qquad (4.35)$$

using approximation (4.25), the safety stock which will approximately provide the prescribed service level, p_α, is given by the following expression.

$$M_{n,s} = Q\sqrt{\frac{1}{2n(1-ns^2)}\ln\left(\frac{1}{1-p_\alpha}\right)} \qquad (4.36)$$

for s< 1/√n.

Using this approximation for the safety stock, the optimal order quantity is given by

$$Q^* = \sqrt{\frac{2D(A+S+nZ)}{r\left(\frac{C_p}{n}+C_p\frac{\sqrt{2\ln\left(\frac{1}{1-p_\alpha}\right)}}{\sqrt{n(1-ns^2)}}+C_v\left(1-\frac{D}{P}-\frac{1}{n}\right)\right)}} \qquad (4.37)$$

The inventory holding cost of the safety stock can be included in the total cost function and the optimal n and Q can be found by a similar numerical method as used in Model II.

4.3 Sensitivity and Cost Analysis

This section contains numerical results for sensitivity and cost analysis, and comparisons of the models in this chapter. Factor analysis is used to study the effects of the parameters on the models and we compare the costs and optimal decision variables of the four models.

4.3.1 Sensitivity Analysis

Production environments are not static. Many changes occur and it is important to consider how sensitive the parameters of a model are to these differences. We are interested in the effect of changes in the parameters on the decision variables, the optimal order quantity and number of deliveries. The parameters of major interest are Z, the shipment cost, C_v/C_p, the ratio of the per unit costs of the vendor and the purchaser, S/A, the ratio of the ordering and setup costs, and P/D, the ratio of the production rate and the demand rate.

For EOQ type models with continuous decision variables, sensitivity analysis is a very simple analytic procedure. It is well known from standard textbooks (Silver(1985), Naddor(1966), Hadley and Whitin(1963)), that the basic EOQ model is insensitive to parameter changes. That is one of the main reasons why the EOQ is still so popular

among practitioners and academicians alike for approximate solutions. The same properties are true for our models, if only the decision variable Q is considered. However, in our models, we have a discrete variable which makes sensitivity analysis more complex requiring numerical analysis, because the dependence and interactions between n and Q must be considered.

Numerical results reveal that as we increase the shipment cost, the optimal number of shipments decreases as we would expect. The larger the shipment cost, the smaller the optimal number of shipments. However, small changes in the shipment cost do not influence the optimal n. The joint optimal order quantity oscillates, but it is also insensitive to changes in the shipment cost. An illustration is shown in Table 4.1.

The effect of changes in parameter values on the total cost is also of interest. We examine the parameter effects on the percentage improvement in the joint total relevant cost achieved by ordering the joint optimal order quantity rather than the purchaser's optimal order quantity or the vendor's optimal order quantity. The percentage improvement in the joint total cost achieved by using the joint optimal order quantity rather than the purchaser's optimal order quantity is denoted by Q_p%. The respective percentage improvement by ordering the joint order quantity rather than the vendor's optimal order quantity is Q_v%. Both quantities are positive, indicating real improvement that can be considerably large going up to 30%.

As the shipment cost is increased, the value of Q_v% and Q_p% increase to a point and then decrease in most cases. So as the shipment cost increases a larger

improvement is achieved to a point by ordering Q_j^* rather than Q_v, but that improvement decreases for higher shipment costs.

The ratio C_v/C_p has an inverse effect on the optimal number of shipments as illustrated in Table 4.2. As C_v/C_p increases, the optimal number of shipments decreases. This is counter-intuitive since as C_v increases relative to C_p, and therefore C_v/C_p increases, the holding cost for the vendor increases. An increase in the optimal number of shipments would be expected. Also, the joint optimal order quantity decreases as C_v/C_p increases. Here a higher sensitivity occurs that emphasizes the influence of the cost ratio on the optimal n and Q. As the ratio P/D increases, the optimal number of shipments decreases as does the optimal joint order quantity. The effect on Q is larger than the effect on n. The above result is illustrated in Table 4.3. There are problems in the vendor model when the production rate is not much larger than the demand rate and the optimal number of shipments is small. Therefore, the cost savings for the vendor cannot be calculated, hence the omissions in the table. The practical implications of these results will be summarized in Chapter 6.

A 2^4 experimental design was developed to analyze and compare the effects and interaction of parameter changes, and an analysis run. The effects were Z, C_v/C_p, P/D, and S/A each observed at two levels, high and low. The parameter with the largest estimated effect for the optimal number of deliveries is C_v/C_p. The estimated effect is negative so there is an inverse relationship. As the parameter increases, the optimal number of deliveries decreases. C_v/C_p has a larger effect on the optimal number of deliveries for low levels of Z than for high levels. The shipment cost and the ratio P/D

followed with the next largest effects and also affect the number of deliveries inversely. This corroborates the earlier observations mentioned above. The ratio S/A has a positive estimated effect, therefore there is a direct relationship. The estimated effects for the interactions are small.

The factor with the largest effect on the joint optimal order quantity, Q_j^*, is P/D. There is an inverse relationship. The ratios, C_v/C_p and S/A, also have large estimated effects, but there is an inverse relationship between Q_j^* and C_v/C_p and a direct relationship between Q_j^* and S/A. The interaction between C_v/C_p and P/D and also the ratio S/A have a smaller estimated effect on Q_j^*. C_v/C_p has a larger effect on the optimal number of deliveries for low levels of P/D than for high levels. Tables 4.4-5 give results from the factorial analysis. Similar results were found in further runs.

The sensitivity of Models II-IV is also examined. In Model II, as P_α increases, there is a slight increase in n^*. As P_α increases from small, unrealistic values to largervalues, n^* may increase by one. In some cases, there is another increase in n^* by one when P_α reaches a value close to one. An illustration of this is given in Table 4.6. The joint order quantity oscillates.

In Model III, the parameter of interest is δ. As δ increases, there is a slight decrease in n^*. The value of n^* is lower, by one, for higher values of δ, generally. Table 4.7 gives an example of this trend. The joint order quantity oscillates.

The value of s, the standard deviation, in Model IV does not seem to have an effect on the optimal number of shipments or the order quantity. Table 4.8 is an illustration

TABLE 4.1

The Effect of Increasing the Shipment Cost on the Decision Variables and Cost Savings in Model I

Z	10	30	50	75
n^*	4	3	2	2
Q_j^*	600	619	608	632
Q_p%	2.75	3.26	8.15	8.72
Q_v%	.71	1.86	12.59	10.93
D=1000 A=100 S=400 C_p=25 C_v=20 P=3200 r=.2				
n^*	4	2	2	1
Q_j^*	540	530	553	501
Q_p%	9.09	18.13	12.2	28.74
Q_v%	1.65	18.37	16.37	---
D=1000 A=35 S=350 C_p=25 C_v=20 P=3200 r=.2				
n^*	4	2	2	1
Q_j^*	536	525	551	494
Q_p%	7.62	16.51	11.25	---
Q_v%	1.98	27.55	25.18	---
D=1000 A=40 S=300 C_p=25 C_v=20 P=2500 r=.2				
n^*	7	4	3	3
Q_j^*	707	705	696	748
Q_p%	3.43	5.73	7.18	4.13
Q_v%	.98	6.29	18.22	14.92
D=1000 A=35 S=300 C_p=25 C_v=15 P=1800 r=.2				

TABLE 4.2

The Effect of Increasing C_v/C_p on the Decision Variables in Model I

C_v/C_p	.04	.2	.4	.6	.8	1
n^*	19	8	5	3	2	1
Q_j^*	2726	1231	871	690	608	566
Q_p%	.06	.04	.08	2.61	8.15	27.07
Q_v%	.34	.05	.07	2.56	12.59	---
D=1000 A=100 S=400 Z=50 P=3200 r=.2						
n^*	19	8	5	3	2	1
Q_j^*	3244	1474	1051	820	731	710
Q_p%	.0001	.16	1.29	6.93	16.96	43.65
Q_v%	.003	.95	6	99.99	---	---
D=1000 A=50 S=300 Z=70 P=1500 r=.2						
n^*	18	7	5	3	2	1
Q_j^*	2595	1131	855	669	590	555
Q_p%	0	.36	.77	5.85	14.97	40.16
Q_v%	0	.92	1.21	11.03	---	---
D=1000 A=35 S=300 Z=50 P=2000 r=.2						
n^*	16	7	4	3	2	1
Q_j^*	2842	1317	903	762	665	612
Q_p%	.001	.07	1.74	3.88	16.66	35.56
Q_v%	.007	.11	1.83	3.57	16.13	---
D=1000 A=50 S=500 Z=75 P=3000 r=.2						

TABLE 4.3

The Effect of Increasing P/D on the Decision Variables in Model I

P/D	1.05	1.11	1.25	1.67	2.5
n^*	5	4	3	2	2
Q_j^*	1429	1039	703	421	292
A=30	S=200	C_p=25	C_v=20	Z=40	r=.2
n^*	5	4	3	2	2
Q_j^*	1843	1321	889	529	368
A=50	S=300	C_p=25	C_v=20	Z=70	r=.2
n^*	7	5	4	3	2
Q_j^*	1941	1342	924	568	364
A=50	S=350	C_p=25	C_v=20	Z=40	r=.2
n^*	12	8	6	4	3
Q_j^*	2298	1535	1047	629	411
A=50	S=350	C_p=25	C_v=15	Z=40	r=.2
n^*	10	7	5	4	3
Q_j^*	2657	1818	1217	775	501
A=50	S=500	C_p=25	C_v=15	Z=75	r=.2

TABLE 4.4

The Estimated Effects on n^* in the 2^4 Factorial Experiments

Factor	Estimated Effect	Direction
n^*		
1	4.75	-
2	7.25	-
1,2	2.75	+
3	4.00	-
1,3	1.50	+
2,3	2.50	+
1,2,3	1.00	-
4	2.50	+
1,4	1.00	-
2,4	2.00	-
1,2,4	0.50	+
3,4	0.75	-
1,3,4	0.25	+
2,3,4	0.75	+
1,2,3,4	0.25	-
$1 \rightarrow Z$ $2 \rightarrow C_v/C_p$ $3 \rightarrow P/D$ $4 \rightarrow S/A$		

TABLE 4.5

The Estimated Effects on Q_j^* in the 2^4 Factorial Experiments

Factor	Estimated Effect	Direction
	Q_j^*	
1	2.88	-
2	609.13	-
1,2	2.38	+
3	1103.13	-
1,3	0.88	+
2,3	366.63	+
1,2,3	2.38	-
4	376.63	+
1,4	4.38	-
2,4	177.63	-
1,2,4	30.63	-
3,4	222.63	-
1,3,4	2.88	+
2,3,4	113.63	+
1,2,3,4	20.13	+
$1 \rightarrow Z$ $\quad 2 \rightarrow C_v/C_p$	$3 \rightarrow P/D$	$4 \rightarrow S/A$

of this. Only small values of s are appropriate for this model. The model is an approximation and the approximation is not a good one for larger values of s.

4.3.2 Cost Analysis

In examining the costs involved for the vendor and purchaser in this model, each party has the lowest cost for their optimal order quantity. Each party also has the highest cost if the other party's optimal order quantity is ordered. If the joint optimal order quantity is agreed upon, this is a compromise. Each party does not benefit as much as they would if their optimal order quantity is ordered, however, their total cost is lower than if the other party's optimal order quantity is ordered. This fact is the basis of negotiations and motivation for JIT cooperation.

If the purchaser is strong, as in many industries in Japan, then they will urge the vendor to accept the purchaser's economic lot size, ELS. In this situation, the vendor may make some sort of concession to the purchaser in order to encourage the purchaser to agree to the joint order quantity. A price discount may be offered to the purchaser to equalize the increase in cost that will occur for the purchaser if the joint order size is ordered.

If the vendor is strong, as occurs in many situations in the United States, the vendor will urge the purchaser to agree to order the vendor's ELS. The purchaser may offer to pay the vendor a premium to accept the joint order quantity and number of shipments The purchaser may offer a unit price increase on the product, or may also seek out other suppliers.

TABLE 4.6

The Effect of Increasing P_α on the Decision Variables in Model II

P_α	.4	.6	.8	.9	.95	.99
n^*	3	4	4	4	4	5
Q_j^*	432	438	400	377	359	362
Q_p%	7.7	4.56	5.06	5.36	5.57	3.88
Q_v%	24.57	14.33	18.4	21.4	23.67	18.82
D=1000 A=40 S=300 C_p=25 C_v=20 Z=50 P=2000 r=.2						
n^*	4	4	5	5	5	5
Q_j^*	593	546	541	510	487	450
Q_p%	5.35	5.97	4.1	4.47	4.72	5.12
Q_v%	12.25	15.87	12.26	14.85	16.94	20.68
D=1000 A=60 S=500 C_p=25 C_v=20 Z=70 P=2000 r=.2						
n^*	5	5	5	5	6	6
Q_j^*	544	504	464	438	449	417
Q_p%	3.16	3.9	4.63	5.07	3.65	4.12
Q_v%	5.17	7.67	10.84	13.26	10.74	13.92
D=1000 A=40 S=400 C_p=25 C_v=15 Z=50 P=3000 r=.2						
n^*	3	3	3	3	4	4
Q_j^*	494	456	418	393	419	388
Q_p%	5.17	5.7	6.18	6.47	3.78	4.14
Q_v%	13.9	17.57	21.75	24.72	15.19	18.77
D=1000 A=50 S=400 C_p=25 C_v=20 Z=80 P=2500 r=.2						

TABLE 4.7

The Effect of Increasing δ on the Decision Variables in Model III

δ	.6	.7	.8	.9	.95	.99
n^*	5	5	4	4	4	4
Q_j^*	369	374	347	353	356	358
Q_p%	3.81	3.76	5.7	5.64	5.6	5.58
Q_v%	18	17.35	25.3	24.53	24.1	23.76
D=1000 A=40 S=300 C_p=25 C_v=20 Z=50 P=2000 r=.2 P_α=.95						
n^*	3	3	3	3	3	3
Q_j^*	448	454	461	468	472	475
Q_p%	4.41	4.36	4.3	4.23	4.2	4.17
Q_v%	21.26	20.6	19.92	19.18	18.79	18.47
D=1000 A=100 S=600 C_p=25 C_v=20 Z=150 P=3200 r=.2 P_α=.95						
n^*	4	4	4	4	4	4
Q_j^*	389	395	400	407	410	413
Q_p%	4.47	4.41	4.34	4.26	4.22	4.18
Q_v%	18.58	17.94	17.27	16.55	16.17	15.86
D=1000 A=50 S=400 C_p=25 C_v=20 Z=75 P=2500 r=.2 P_α=.95						
n^*	4	4	4	4	4	4
Q_j^*	413	419	425	432	435	438
Q_p%	4.6	4.53	4.46	4.39	4.35	4.32
Q_v%	19.83	19.19	18.52	17.8	17.42	17.1
D=1000 A=50 S=400 C_p=25 C_v=15 Z=75 P=2800 r=.2 P_α=.9						

TABLE 4.8

The Effect of Increasing s on the Decision Variables in Model IV

s	.001	.005	.01	.05	.1
n^*	4	4	4	4	4
Q_j^*	359	359	359	359	357
Q_p%	5.57	5.57	5.57	5.58	5.6
Q_v%	23.67	23.67	23.67	23.67	24.04
D=1000 A=40 S=300 C_p=25 C_v=20 Z=50 P=2000 r=.2 P_α=.95					
n^*	3	3	3	3	3
Q_j^*	476	476	476	475	473
Q_p%	4.16	4.16	4.16	4.17	4.18
Q_v%	18.39	18.39	18.39	18.45	18.64
D=1000 A=100 S=600 C_p=25 C_v=20 Z=150 P=3200 r=.2 P_α=.95					
n^*	6	6	6	6	6
Q_j^*	451	451	452	450	417
Q_p%	3.77	3.78	3.78	3.79	5.57
Q_v%	11.53	11.53	11.53	11.64	16.63
D=1000 A=40 S=400 C_p=25 C_v=15 Z=50 P=2800 r=.2 P_α=.95					
n^*	5	5	5	5	5
Q_j^*	435	435	435	434	431
Q_p%	5.38	5.38	5.38	5.98	5.44
Q_v%	13.57	13.58	13.58	13.67	13.95
D=1000 A=30 S=400 C_p=25 C_v=15 Z=50 P=3000 r=.2 P_α=.90					

If the vendor and the purchaser are equally strong, then the joint order quantity can be used as a compromise. In any case, the joint order size can be used as a negotiation tool between the buyer and the supplier and savings in total cost can be split between the parties. It promotes cooperation between the two parties, and may force each party to eliminate waste from their system in order to enhance their cost position.

We now refer to a specific example and illustrate the use of numerical and cost results provided by our models. The parameters are as follows:

$D = 1000$	$C_p = 30$
$A = 40$	$C_v = 20$
$S = 400$	$P = 3500$
$Z = 50$	$r = .2$

The optimal number of shipments is 3 and the joint optimal order quantity is 579. The costs for the vendor, purchaser, and the joint total costs and order quantities are given below. The costs are based on three shipments.

$$Q_j = 579$$
$$Q_p = 436$$
$$Q_v = 725$$

$TRC_p(Q_p) = 871.78$	$TRC_v(Q_v) = 1104.10$
$TRC_p(Q_j) = 907.01$	$TRC_v(Q_j) = 1132.13$
$TRC_p(Q_v) = 986.79$	$TRC_p(Q_p) = 1249.77$

$$JTRC(Q_j) = 2039.14$$

$$JTRC(Q_p) = 2121.55$$

$$JTRC(Q_v) = 2090.9$$

A graph displaying the cost functions is given in Figure 4.1. Line A is $TRC_p(Q_p)$, line B is $TRC_v(Q_v)$, and line C is $JTRC(Q_j)$.

We note that the properties mentioned earlier hold here. The purchaser and vendor achieve their lowest cost when their individual optimal order quantity is ordered. However, the lowest joint total cost occurs with the joint optimal order quantity.

If the vendor is strong, then the purchaser would benefit by \$79.78 if they can entice the vendor to agree to the joint order quantity. The vendor's cost would be higher by \$28.03. The purchaser could offer a premium or price increase to make up the \$28.03 and still be better off by \$79.78 - \$28.03 = \$51.75.

If the purchaser is strong, the vendor would benefit by \$117.64 if they can convince the purchaser to agree to the joint order quantity. The purchaser's cost would increase by \$35.23. The vendor could offer the purchaser a price discount which would make up the \$35.23 and still be ahead by \$117.64 - \$35.23 = \$82.41. In each case, the opportunity loss that the stronger party would incur if the joint order quantity is agreed upon can be used as a negotiation tool.

4.3.3 Model Comparisons

We have presented four models. One basic, deterministic model and three extensions which include a safety stock component to protect against shortages caused by a random component that was added to the model. The question arises as to how the safety stock component will change the decision variables and how the safety stock

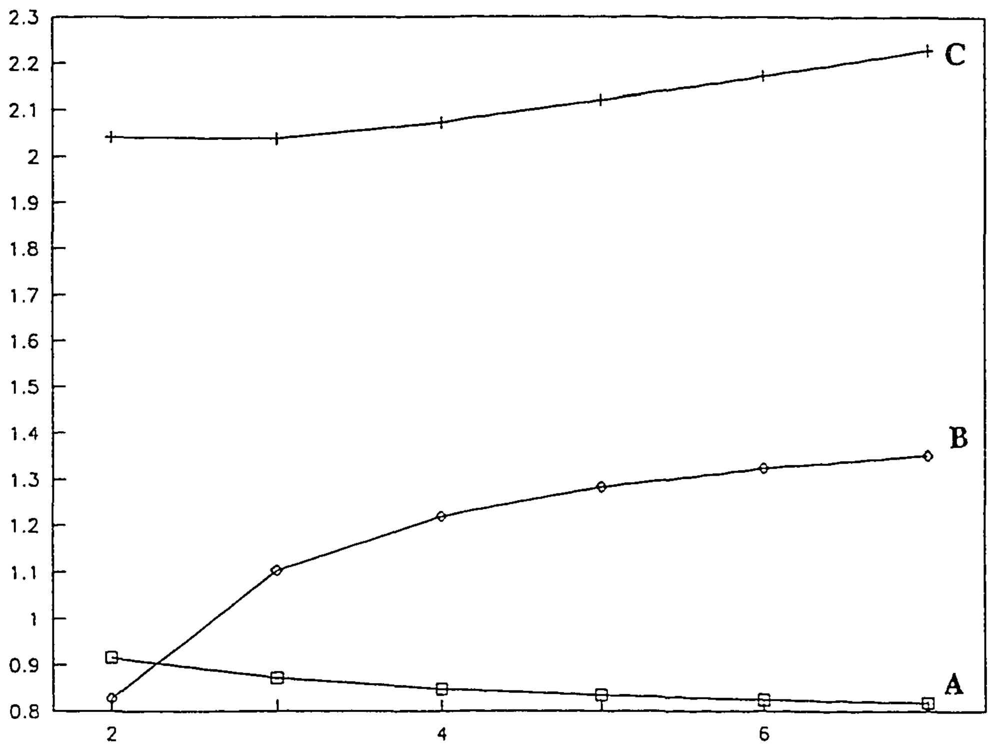

Figure 4.1
The Purchaser, Vendor, and Joint Total Cost Curves

models compare to the basic model. Next we summarize the comparison results of the models we achieved by analyzing different sets of parameter values.

In comparing Models II, III, and IV with Model I, the value of the optimal number of shipments is larger for Models II-IV than for Model I. However, there doesn't seem to be a difference in n^* for Models II-IV. The order quantity is smaller for Models II-IV than for Model I. Models II and III yield the same results, but the results from Model III differ slightly from that of Models II and IV in that the order quantity is slightly smaller. Model II, which is the simplest safety stock model, can likely be used to approximate the other two safety stock models. These results are illustrated in Table 4.9.

We can conclude that the complexity of Models III and IV doesn't result in significant changes in the optimal n and Q values. Thus, it seems to be sufficient for practical purposes to only deal with Model II in the transition state to JIT. It doesn't seem however, that in calculating the necessary safety stock, the effects of random yield and demand should be disregarded. Models III and IV give valuable information and help in the achievement of appropriate customer service in the transition state toward JIT.

TABLE 4.9

Comparison of the Decision Variables for
Models I - IV

Model	I	II	III	IV
n	1	2	2	2
Q	569	348	345	348
% decrease		38.8	39.4	38.8
D=1000 A=100 S=300 C_p=25 C_v=20 Z=150 P=2500 r=.2				
n	2	4	4	4
Q	593	359	356	359
% decrease		39.5	40	39.5
D=1000 A=40 S=300 C_p=25 C_v=20 Z=50 P=2000 r=.2				
n	2	5	5	5
Q	796	453	449	453
% decrease		43.1	43.6	43.1
D=1000 A=50 S=350 C_p=25 C_v=20 Z=60 P=1400 r=.2				
n	3	7	7	7
Q	883	528	523	527
% decrease		40.2	40.8	40.3
D=1000 A=100 S=400 C_p=25 C_v=20 Z=50 P=1500 r=.2				
n	5	9	9	9
Q	924	565	560	565
% decrease		38.9	39.4	38.9
D=1000 A=40 S=500 C_p=25 C_v=15 Z=40 P=1800 r=.2				
P_α = .95 δ = .95 s = .01				

CHAPTER 5

MULTIPLE SOURCING PURCHASING MODEL FOR MINIMIZING THE STOCKOUT RISK

Multiple sourcing can be used in the transition to JIT purchasing. If a reliable vendor is not available, then the order can be split among the vendors until a reliable source emerges. In order to facilitate this emergence, a larger part of the order quantity can be allocated to the supplier with the best characteristics. We suggest a model for dual sourcing in Section 5.1 that allocates the quantity in order to minimize the stockout risk. The supplier with better lead time characteristics receives a larger part of the order. Section 5.2 contains numerical analysis for the deterministic demand case. The random demand case is examined in Section 5.3, and simulation results are given in Section 5.4.

5.1 Minimizing the Stockout Risk for Random Lead Times and Dual Sourcing - Model V

We want the optimal split of order quantities for dual sourcing for the case where the objective is to minimize the stockout risk. The order quantity is assumed to be predetermined, and is divided between the two suppliers. We measure the stockout risk with the probability of a stockout. For strategically important items where there is a high cost penalty for stockout, independent of the magnitude and time of the stockout, the appropriate expression for the service level is the probability of no shortage in an order cycle. In most practical cases, the shortage before the first delivery is the critical

one, causing the most stockouts. Increasing the number of vendors, decreases this probability. The suppliers may have different, arbitrary lead time distributions. A continuous review (r,Q) inventory system is used.

Assumptions

1) The total order quantity is Q, which is pre-determined.

2) The split rate is k where the quantity for supplier A, $Q_A = kQ$, and the quantity for supplier B, $Q_B = (1-k)Q$ where $0 \leq k \leq 1$.

3) The orders are placed simultaneously for the two suppliers.

4) There are random lead times, L_A and L_B, for vendors A and B. The cumulative distribution function (cdf) and the probability density function (pdf) of the lead-time distributions for the two vendors, are F_A, f_A, and F_B, f_B, respectively.

5) A constant demand rate, D, is considered. This is extended for random demand later.

Model

In dual sourcing, if the order quantity is much larger than the expected lead time demand, the probability of a shortage just before the second delivery can be neglected. For smaller order quantities, however, we must consider also the probability of shortage just before the second delivery since this probability can also be considerable.

To provide the exact probability of having no stockouts, the interdependence of the stockouts must also be considered. The joint probability of no stockout is expressed for two and also for 3 vendors. The general random demand case is also discussed.

5.1.1 Probability of No Stockout Before the First Delivery

There is a stockout before the first delivery arrives if both $DL_A > r$ and $DL_B > r$ hold. For two different vendors, we can assume that the lead times are independent. So, the above event has the probability $[1-F_A(s)][1-F_B(s)]$, using the notation $s=r/D$. Thus, for dual sourcing, the probability of no stockout just before the first delivery is

$$P_1^{(2)} = 1 - [1-F_A(s)][1-F_B(s)] = F_A(s) + F_B(s) - F_A(s)F_B(s). \tag{5.1}$$

Splitting the order among n vendors, similarly, we can get the probability of no stockout before the first delivery

$$P_1^{(n)} = 1 - [1-F_1(s)][1-F_2(s)] \dots [1-F_n(s)], \tag{5.2}$$

where F_i denotes the cdf of vendor i $(i=1,2,\dots,n)$.

The relative decrease in stockout probability before the first delivery, i.e. the improvement in customer service before the first delivery can be expressed in the simple form

$$\begin{aligned} R_P(n) &= \{[1-P_1^{(1)}]-[1-P_1^{(n)}]\}/[1-P_1^{(1)}] \\ &= 1 - [1-F_2(s)][1-F_3(s)] \dots [1-F_n(s)] \end{aligned} \tag{5.3}$$

where $P_1^{(1)}$ is the probability of no shortage for a single supplier having the same r and Q. The above stockout probability decreases with the number of suppliers, independently of the order quantity and independently of the lead time distributions as long as $F_i(s)$, the probability of $L_i < r/D$ is positive. This is an extension of the result of Kelle and Silver (1990a), for arbitrary lead time distributions. It shows the advantage of increasing the number of vendors if the probability of stockout occurring before the second and subsequent delivery points can be neglected.

5.1.2 Probability of No Stockout Before the Second Delivery

For the case of dual sourcing, and random lead times, both L_A and L_B can be the second delivery. No stockout occurs before the second delivery if

$$L_B < L_A < s+q_B \quad \text{or} \quad L_A < L_B < s+q_A$$

holds, where $q_A=Q_A/D$, $q_B=Q_B/D$, and $s=r/Q$.

The probability that the first set of inequalities holds is

$$\int_0^{s+q_B} f_A(x_A) \int_0^{x_A} f_B(x_B)\, dx_B\, dx_A = \int_0^{s+q_B} f_A(x)F_B(x)\, dx.$$

The probability that the second set of inequalities holds is the same as the above expression, just the vendor subscripts, A and B, are switched.

For $Q_A=Q_B$, i.e. for even split, using integration by parts we can see that the probability of no shortage before the second delivery is

$$P_2^{(2)}= F_A(s+q/2)\, F_B(s+q/2) \tag{5.4}$$

with $q=Q/D$, where Q denotes the total order quantity for the two vendors. For $q_A \geq q_B$, the above probability is

$$P_2^{(2)}= F_A(s+q_B)\, F_B(s+q_A) + \int_{s+q_B}^{s+q_A} F_A(x)f_B(x)\, dx. \tag{5.5}$$

For an even split, it is easy to see that for large order quantity, Q, the probability of no shortage just before the second delivery is close to 1. For uneven split, the expression of $P_2^{(2)}$ also tends to 1 if both q_A and q_B tend to infinity. Thus, for large order quantity, the probability of shortage just before the second delivery can be neglected. In the above cases, dual sourcing clearly gives an advantage by decreasing the stockout risk relative to single sourcing.

5.1.3 Joint Probability of No Stockout for Two Vendors

If the stockout probability before the second delivery cannot be neglected, the probability of no stockout before the first and second delivery must be considered together since they may depend on each other. The joint probability of no stockouts, the so-called P_α service level for dual sourcing is expressed as follows.

THEOREM 1. For dual sourcing, with arbitrary lead time distributions F_A and F_B, given (r,Q) ordering policy, constant demand rate D, and split rate k/(1-k), $(0<k<1)$, the probability of no stockout (before first *and* second delivery)

$$P_\alpha^{(2)} = F_A(s)F_B(s+kq) + F_B(s)F_A(s+(1-k)q) - F_A(s)F_B(s) \tag{5.6}$$

with the notations $s=r/D$ and $q=Q/D$.

Proof: Because of the random lead times, the first delivery can arrive from vendor A or from vendor B. If A delivers first, there is no shortage if

$$\text{(A1): } L_A \leq s \text{ and } s < L_B \leq s+kq, \quad \text{or} \quad \text{(A2): } L_A \leq L_B \leq s$$

holds, where L_A and L_B are the random lead times. The appropriate probabilities (see Figure 5.1) are

$$P(A1) = F_A(s)\,[F_B(s+kq) - F_B(s)],$$

$$P(A2) = \int_0^s f_B(x_B) \int_0^{x_B} f_A(x_A)\, dx_A\, dx_B = \int_0^s f_B(x)F_A(x)\, dx.$$

If B delivers first, for the conditions of no shortage we have similar inequalities, (B1) and (B2), as above just the vendor subscripts, A and B, are switched and (1-k)q stands

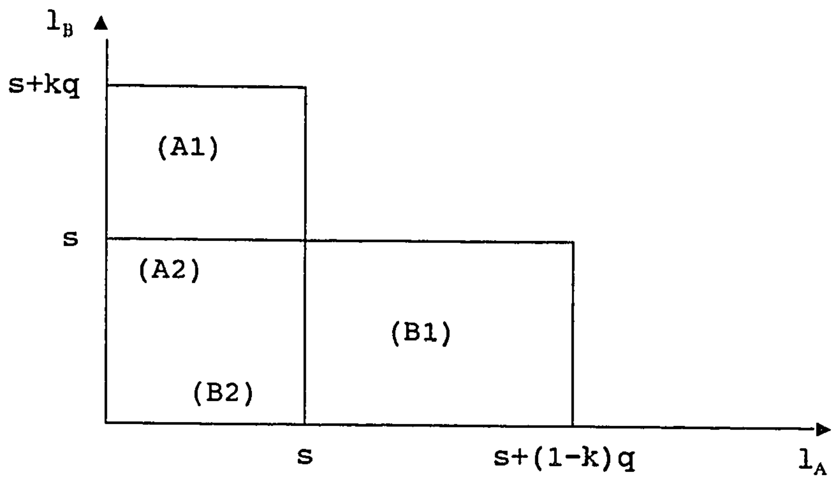

Figure 5.1
Probability of No Shortage for Dual Sourcing

$$P(A2) = F_A(s)F_B(s) - P(B2).$$

Thus, the sum of P(.)'s result in (5.6).

A simple, visual proof of the above result is as follows. The probability of the joint set $(A1) \cup (A2) \cup (B2)$ is the first term in (5.6). Similarly, the second term of (5.6) is the probability of $(A2) \cup (B1) \cup (B2)$. Since (A2) and (B2) has been considered twice, the probability of these sets has to be taken away; this is the third term of (5.6).instead of kq. Integrating P(A2) by parts, we get

COROLLARY 1. For identical lead time distributions, $F_A = F_B = F$

$$P_\alpha^{(2)} = F(s)\,[F(s+kq) + F(s+(1-k)q) - F(s)] \tag{5.7}$$

with $s=r/D$ and $q=Q/D$.

This formula is a generalization of the result published in Kim, Ord, and Hayya (1991) for even split.

5.1.4 Probability of No Stockout for Three Vendors

The probability of no stockout (the P_α service level) can be expressed for more than two vendors in a similar way as in Theorem 1.

THEOREM 2. For a given (r,Q) policy and constant demand rate D, 3 vendors with lead-time distributions F_i and split orders k_iQ ($i=1,2,3$; $k_1+k_2+k_3=1$), the probability of no stockout

$$
\begin{aligned}
P_\alpha^{(3)} = & \sum_{i,j,m=1,\, i\neq j\neq m}^{3} F_i(s)\,F_j(s+q_i)\,F_m(s+q_i+q_j) \\
& - \sum_{i,j,m=1,\, i\neq j,\, j<m}^{3} F_i(s)\,F_j(s+q_i)\,F_m(s+q_i) \\
& - \sum_{i,j,m=1,\, i\neq j,\, j<m}^{3} F_i(s)\,F_j(s)\,F_m(s+q_i+q_j) \\
& + F_1(s)\,F_2(s)\,F_3(s)
\end{aligned}
\tag{5.8}
$$

with the notations $q_i=k_iQ/D$ and $s=r/D$.

A simple visual proof is described here, the rigorous proof is provided in Appendix A. Let $L(i_1)$ denote the time until the first delivery and $q(i_1)=Q(i_1)/D$, where $Q(i_1)$ is the amount delivered the first time, i_2 for the second time, etc. The first set of terms expresses the probability of the events that the following three inequalities hold simultaneously:

$$L(i_1)\leq s,\ L(i_2)\leq s+q(i_1)\ \text{ and }\ L(i_3)\leq s+q(i_1)+q(i_2).$$

All 6 permutations i_1,i_2,i_3 of the three vendors 1,2,3 are considered.

The next two sets of terms express the probabilities of the events that the following inequalities hold:

$L(i_1) \leq s$, $L(i_2) \leq s+q(i_1)$ and $L(i_3) \leq s+q(i_1)$ (for set 2) and

$L(i_1) \leq s$, $L(i_2) \leq s$ and $L(i_3) \leq s+q(i_1)+q(i_2)$ (for set 3).

The probability of all 2 times 3 permutations of these events are taken away, because they are considered twice in the first set of terms. Finally, the event, expressed by

$L_1 \leq s$, $L_2 \leq s$ and $L_3 \leq s$

has been taken away twice in sets 2 and 3. Thus, to achieve the proper probability $P_\alpha^{(3)}$, the last term in (5.8) must be added.

COROLLARY 2. For identical lead time distributions, $F_1=F_2=F_3=F$

$$P_\alpha^{(3)} = \sum_{i,j=1,i\neq j}^{3} F(s)\,F(s+q_i)\,F(s+q_i+q_j) \tag{5.9}$$
$$-\sum_{i,j=1,i\neq j}^{3} F^2(s)\,F(s+q_i+q_j) - \sum_{i=1}^{3} F(s)\,F^2(s+q_i) + F^3(s)\,.$$

5.1.5 Probability of No Stockout for Random Demand Case

For random demand case, D_A and D_B, the demands during the lead times L_A and L_B are dependent random variables, thus the joint distribution function

$$P(D_A \leq x,\ D_B \leq y) = G(x,y) \tag{5.10}$$

has to be considered instead of the product of the lead-time distribution functions as we did for known demand rate case.

If A delivers first, $D_A \leq D_B$, and there is no shortage before the first and second delivery if

$$(A1):\ D_A \leq r \text{ and } r < D_B \leq r+kQ \qquad \text{or} \qquad (A2):\ D_A \leq D_B \leq r.$$

If B delivers first, for the conditions of no shortage we have similar inequalities, (B1) and (B2), as above just the vendor subscripts, A and B, are switched, and kQ is replaced by (1-k)Q. The appropriate probabilities

$$P(A1) = G(r,\ r+kQ) - G(r,\ r),$$

$$P(B1) = G(r+(1-k)Q,\ r) - G(r,\ r),$$

$$P[(A2) \cup (B2)] = P[D_A \leq D_B \leq r \text{ or } D_B \leq D_A \leq r)$$

$$= P(D_A \leq r \text{ and } D_B \leq r) = G(r,r).$$

Thus, in dual sourcing, with arbitrary joint lead time demand distribution (10), for given (r,Q) ordering policy, and split rate k/(1-k), $(0<k<1)$, the probability of no shortage before the first and second delivery is

$$P_\alpha^{(2)} = G(r,\ r+kQ) + G(r+(1\text{-}k)Q,\ r) - G(r,\ r) \tag{5.11}$$

Similarly, for 3 vendors, formula (5.8) can be extended for random demand case by replacing the product of the lead-time distribution by the joint distribution of the lead time demands

$$P(D_1 \leq x_1,\ D_2 \leq x_2,\ D_3 \leq x_3) = H(x_1,x_2,x_3).$$

Further, s is replaced by r, q_i is replaced by k_iQ for $i=1,2,3$.

5.1.6 The Optimal Split Rate for Two Vendors

The optimal split rate is considered that minimizes the stockout risk. First, general results are derived for the two and three-supplier case with deterministic and random demand. Next, we consider specific lead time distributions commonly used in literature and in practice.

THEOREM 3. For dual sourcing, with arbitrary lead time distributions F_A and F_B, given (r,Q) ordering policy, constant demand rate D, the optimal split rate k, minimizing the stockout risk, is the solution of the equation

$$\frac{f_A(s+(1-k)q)}{f_B(s+kq)}=\frac{F_A(s)}{F_B(s)} \tag{5.12}$$

if the solution $0< k <1$ (with the previous notations $s=r/D$, $q=Q/D$) and

$$F_A(s)f'_B(s+kq)+F_B(s)f'_A(s+(1-k)q)<0 \tag{5.13}$$

otherwise, k=0 or k=1 (double sourcing cannot be optimal).

Proof: The necessary condition $dP_\alpha^{(2)}(k)/dk=h(k)=0$ provides equation (5.12). The sufficient condition for max $P_\alpha^{(2)}(k)$ is $dh(k)/dk<0$, that means (5.13) must be fulfilled. If those conditions aren't fulfilled, the optimum must be on the border (k=0 or 1), i.e. single sourcing is optimal. (If the left hand side of (5.13) is zero, further derivatives of h(k) must be checked, but because of its very small chance, we disregard this case.)

Distributions, used for the approximation of lead time, are unimodal based on practical experiences. The s value must be larger than the modus of the lead-time

distribution at least for one of the suppliers. Otherwise, the probability of the shortage before the first delivery, $P(L_1 > s)$, is too high for the practically required risk level (.01 to .1). Thus, s must be at the end of the right tail of at least one of the density functions f_A and f_B. So, at least one of the inequalities $f'_B(s+kq)<0$ and $f'_A(s+(1-k)q)<0$ holds. If both inequalities hold or the other derivative is zero, (5.13) is satisfied. This is the most common case if the expected lead times of the two vendors are not very different.

Positive $f'_B(s+kq)$ means that the modus of F_B is larger than s+kq. Equation (5.13), in this case, is equivalent with

$$f'_B(s+kq) < - f'_A(s+(1-k)q)\, F_B(s) \,/\, F_A(s). \tag{5.14}$$

If s+kq is close to the modus of F_B, $f'_B(s+kq)$ can be close to zero and (5.14) may hold. Otherwise, the optimum is single sourcing. The same consequences hold for positive $f'_A(s+(1-k)q)$.

For identical lead time distributions ($F_A=F_B=F$), considering k=1/2, Theorem 3 provides the proof for the optimality of an even split.

In dual sourcing, for different lead time distributions, an even split is optimal if

$$\frac{f_A(s+q/2)}{f_B(s+q/2)}=\frac{F_A(s)}{F_B(s)}. \tag{5.15}$$

Equation (5.15) usually doesn't hold for general lead time distributions.

COROLLARY 3. Summarizing the above analysis:

- for identical lead-time distributions, an *even split* minimizes the stockout risk;
- for different lead time distributions, generally, an *uneven split* provides the minimal stockout risk;
- if in the different lead time case there is not a large difference in expected value, *split* orders provide a lower stockout risk;
- if the difference in the expected lead time is very large, *single sourcing* can provide a lower stockout risk.

The above results coincide with intuitive expectation. Additionally, for any specific situation, Theorem 3 provides the exact quantitative solution, the optimal split rate in dual sourcing for two arbitrary lead time distributions, or provides the answer that dual sourcing cannot ensure a lower risk than a single vendor.

5.1.7 General Results about the Optimal Split Rate

For the random demand case, the optimal split rate k must satisfy the necessary condition of optimality

$$\frac{\partial G(x,y)}{\partial y}\Big|_{\substack{x=r\\ y=r+kQ}} = \frac{\partial G(x,y)}{\partial x}\Big|_{\substack{x=r+(1-k)Q\\ y=r}}. \qquad (5.16)$$

Numerical methods can be applied for the solution of the above equation. The solution generally results that the even split, $k=1/2$, cannot be optimal except the case when the lead time distributions of the two suppliers are identical. In latter case, $G(x,y)=G(y,x)$, so $k=1/2$ is the optimal solution.

For three suppliers, the necessary condition of the optimal split rate k_1, k_2, $1-(k_1+k_2)$ is more complex than for dual sourcing. Numerical methods can be applied for the solution of the system of two equations, based on the partial derivatives of the expression (8) of $P_\alpha^{(3)}$ by k_1 and k_2. However, from the symmetric property of those equations, it follows that for the cases when all three suppliers or two out of the three suppliers have the same lead time distribution, the even split among identical lead time suppliers is the optimal one. Further, for nonidentical lead time distributions, an uneven split is generally optimal, similar to the case of dual sourcing.

5.1.8 The Optimal Split Rate for Specific Distributions

This part of the study considers specific lead time distributions commonly used in literature. We provide simple solutions and more specified quantitative analysis of the above questions.

The solution of the nonlinear equation (5.12) for a single variable is a simple numerical problem if the cdf and pdf of the lead-time distributions are known. First, we provide simple explicit formulas for the optimal k value in case of some common distributions frequently used for lead time approximations. Next, the more general Weibull distribution is considered.

Exponential lead times are characterized by the parameters λ_A and λ_B, the reciprocal of the expected lead time for the two vendors. Simple algebra provides the solution of equation (5.12):

$$k = [\lambda_A + s(\lambda_A-\lambda_B)/q - \ln R/q] / (\lambda_A+\lambda_B) \tag{5.17}$$

where

$$R = R(\lambda_A,\lambda_B,s) = \lambda_A[1-\exp(-\lambda_B s)] / \lambda_B[1-\exp(-\lambda_A s)], \tag{5.18}$$

$s=r/D$, $q=Q/D$. Inequality (5.13) is always satisfied.

Normal distributed lead times with the expected values m_A, m_B, and standard deviations σ_A, σ_B for the two vendors yield a quadratic equation for k that has the solution:

$$k = -b+\sqrt{(b^2-4ac)} / 2a\ , \tag{5.19}$$

where

$$a = q^2\ (\sigma_A{}^2-\sigma_B{}^2)$$

$$b = 2q(\sigma_A{}^2 s-\sigma_A{}^2 m_B+\sigma_B{}^2 s+\sigma_B{}^2 Q-\sigma_B{}^2 m_A)$$

$$c = -\sigma_A{}^2(s-m_B)^2+\sigma_B{}^2(m_A-s-q)^2$$
$$+2\sigma_B{}^2\sigma_A{}^2\ln\{\sigma_A\Phi_A(s)/[\sigma_B\Phi_B(s)]\},$$

with Φ denoting the standard normal cdf. If $\sigma_A=\sigma_B$, the solution of the appropriate linear equation is k=c/b. Inequality (5.13) is fulfilled if $s>m_A$ and $s>m_B$.

Uniform distributed lead times with lower bounds, l_A and l_B, and upper bounds, U_A and U_B, give the surprising result that the split rate doesn't influence the stockout risk,

$$P_\alpha^{(2)}=s\,(s+q)\,/\,(U_A-l_A)\,(U_B-l_B) \tag{5.20}$$

if

$$l_B\leq s,\ s+q_A\leq U_B \text{ and } l_A\leq s,\ s+q_B\leq U_A \tag{5.21}$$

Here, q_A=kQ/D, q_B=(1-k)Q/D, q=Q/D, and s=r/D, as earlier. The above result is intuitive, if we consider Figure 1 and know that the probabilities are directly proportional with the areas for uniform distribution, and we have the same total area in range of (5.21). Outside of the range (5.21), the different cases are easy to evaluate separately.

The assumption of *Weibull* distributed lead time is common in the literature. The shape parameters m_A , m_B and the scale parameters λ_A, λ_B allow a large variety of different cases to approximate (see e.g. Tadikamalla, 1978). The optimal split rate is the solution of the nonlinear equation

$$\frac{\lambda_A m_A(\lambda_A(s+q_B))^{m_A-1}\exp[-(\lambda_A(s+q_B))^{m_A}]}{\lambda_B m_B(\lambda_B(s+q_A))^{m_B-1}\exp[-(\lambda_B(s+q_A))^{m_B}]} = \frac{1-\exp[-(\lambda_A s)^{m_A}]}{1-\exp[-(\lambda_B s)^{m_B}]} \tag{5.22}$$

A numerical procedure can be applied for the solution of equation (5.22). From (5.13), the sufficient condition for the optimality is

$$\frac{\sqrt[m]{\frac{m-1}{m}}}{\lambda}<x \qquad (5.23)$$

with $x=s+kq$ for $m=m_A$ and $x=s+(1-k)q$ for $m=m_B$.

The *gamma* distribution is used also frequently for the approximation of lead time distribution. Referring to Tadikamalla (1978), the Weibull distribution is also a good approximation in these cases.

5.2 Numerical Analysis for Deterministic Demand

In this section, we summarize the results we derived from the numerical comparisons for different lead-time distributions and for several different parameter values. Tables are given for each of the lead time distributions. We examine the effect of a difference in lead time distributions for the vendors on the optimal proportion of split and the effect of random demand. In Section 1, the expected lead time for the second vendor is increased and the optimal order split is calculated. We also discuss the relative percentage improvement in the stockout risk for dual sourcing over single sourcing and an uneven split over an even split. In Section 2, the effect of the parameters on the optimal order split is examined. An approximation for the case of

random demand is given in Section 3 and the results are compared to the results for deterministic demand. Section 4 discusses verification of the models using simulation.

5.2.1 The Effect of the Expected Lead Time on the Optimal Order Split

In the basic situation, we assume, we have a single supplier A, with a given lead time distribution. For the sake of simplicity, we assume $E(L_A)=1$, and the order quantity, Q, is set (e.g. the EOQ). The reorder level, r, is determined by the required service level $P_\alpha^{(1)}$ in the simple form $r = DF_A^{-1}(P_\alpha^{(1)})$, using the inverse, F_A^{-1}, of the lead time distribution, F_A. Several different Q and $P_\alpha^{(1)}$ parameter values were considered in the numerical analysis.

The optimal split rate for the normal and exponential lead time cases can be found by using the simple formulas given previously. A simple search routine must be used for the Weibull lead time case. In order to calculate the shape parameter, m, for the Weibull distribution with a given coefficient of variation, $cv = \sigma/\mu$, we used the following equation which was found using regression by Kelle, Silver and Murphy (1992).

$$m = 1.387884/cv - .607253/\sqrt{cv} + .196309$$

This simple formula provides an approximation with a relative percentage error of less than 0.65% for $0.1 \le cv \le .08$, and less than 1.33% for $0.8 < cv < .95$. The value of the scale parameter, λ, was then determined using the following relationship

$$\lambda = \Gamma(1 + 1/m)/\mu$$

where μ is the mean.

Having the basic situation, a second vendor will be available. The first question is: if we split Q among the two suppliers what is the best split rate in order to achieve the highest service level?

The numerical results based on several different parameter combinations confirm the intuitive anticipation that by increasing the expected lead time of the second supplier, B, the optimal proportion of split, k, for vendor A increases. When $E(L_B)$ becomes large, a split is no longer optimal (k=1). Tables 5.1-5.3 illustrate the dependence of the optimal split rate on the expected lead time for a given P_α and for different time between orders, Q/D. Similar tendencies were observed for different parameters.

Notice that increasing Q/D, the time between orders, for a fixed $E(L_B)$, yields a decrease in the optimal split rate, k. Thus, with the decrease of Q/D, the split tends to be more and more uneven.

If we split the order, we may want to know the advantage of using an uneven split instead of an even split. The numerical results show, that an even split results in a higher stockout risk except in the case of identical lead time distributions. The improvement in service level depends on the parameter values. The Relative Percentage Improvement (RPI) is

$$RPI=[SR(.5)-SR(k)]/SR(.5) * 100\%,$$

TABLE 5.1

Optimal Split Rates Depending on Expected Lead Time, $E(L_B)$, with Exponential Lead Times ($E(L_A)=1$, $P_\alpha=.95$).

Q/D = .2							
$E(L_B)$	1	1.01	1.03	1.05	1.07	1.09	1.094
k	.5	.556	.666	.773	.878	.981	1
Q/D = .4							
$E(L_B)$	1	1.03	1.06	1.09	1.12	1.15	1.19
k	.5	.587	.67	.751	.829	.905	1
Q/D = .7							
$E(L_B)$	1	1.1	1.15	1.2	1.25	1.3	1.34
k	.5	.669	.746	.819	.889	.955	1
Q/D = 1							
$E(L_B)$	1	1.05	1.1	1.2	1.3	1.4	1.49
k	.5	.564	.625	.737	.838	.929	1
Q/D = 5							
$E(L_B)$	1	1.5	2	2.5	3	3.5	3.6
k	.5	.68	.8	.88	.95	.99	1
Q/D = 10							
$E(L_B)$	1	2	3	4	5	6	6.3
k	.5	.73	.85	.92	.96	.99	1

TABLE 5.2

Optimal Split Rates Depending on Expected Lead Time, $E(L_B)$
with Normal Lead Times
($E(L_A)=1$, $P_\alpha=.95$, $\nu_A=\nu_B=.1$)

Q/D = .2							
$E(L_B)$	1	1.03	1.06	1.09	1.12	1.15	1.17
k	.5	.58	.662	.749	.842	.945	1
Q/D = .4							
$E(L_B)$	1	1.05	1.1	1.15	1.2	1.25	1.3
k	.5	.566	.635	.711	.8	.895	1
Q/D = .7							
$E(L_B)$	1	1.05	1.1	1.2	1.3	1.4	1.47
k	.5	.537	.575	.66	.76	.89	1
Q/D = 1							
$E(L_B)$	1	1.1	1.2	1.3	1.4	1.5	1.63
k	.5	.552	.609	.673	.750	.844	1
Q/D = 5							
$E(L_B)$	1	1.5	2	3	4	5	5.5
k	.5	.55	.61	.71	.82	.93	1
Q/D = 10							
$E(L_B)$	1	2	4	6	8	10	10.5
k	.5	.55	.65	.75	.86	.93	1

TABLE 5.3

Optimal Split Rates Depending on the Standard Deviation of Lead Time, σ_B, with Normal Lead Times ($E(L_A)=E(L_B)=1$, $P_\alpha=.95$, and $\sigma_A=.1$)

Q/D = .2						
σ_B	.1	.2	.25	.3	.35	.4
k	.5	.846	.917	.955	.972	.978
Q/D = .4						
σ_B	.1	.2	.3	.4	.5	.6
k	.5	.77	.88	.92	.94	.943
Q/D = .7						
σ_B	.1	.2	.4	.6	.8	1
k	.5	.73	.835	.931	.94	.941
Q/D = 1						
σ_B	.1	.3	.5	.7	.9	1.2
k	.5	.814	.901	.932	.943	.9465
Q/D = 5						
σ_B	.1	1	2	3	4	5.7
k	.5	.929	.966	.975	.977	.9776
Q/D = 10						
σ_B	.1	2	4	6	8	10.1
k	.5	.963	.981	.985	.986	.9865

where SR(.5) is the stockout risk for an even split and SR(k) is the stockout risk for the optimal split. The general tendency is that with increasing the lead time of the second supplier, relative to the first supplier, the relative improvement (RPI) achieved by the optimal split, k, over an even split, k=.5, increases. Tables 5.4-5.7 give an illustration of the numerical results.

We may have the choice between single and dual sourcing. In this case, we ask, under which circumstances should we split the order? For several different parameter values, we compared the stockout risk, SR(1), provided by a single supplier, with the stockout risk, SR(2), provided by dual sourcing. We considered the same order quantity, Q, and reorder level, r, for both cases and the optimal split of Q for dual sourcing. When $E(L_B)$ becomes large, single sourcing provides a higher P_α. We denote the value of $E(L_B)$ where a split is no longer advantageous by $E(L_B)^*$.

Numerical calculations show that by increasing the expected lead time for the second supplier, the improvement due to dual sourcing decreases. The rate of decrease depends upon the parameter values. As Q/D, increases, the point where a split is no longer advantageous, $E(L_B)^*$, increases. That means if the time between orders increases, then dual sourcing is more attractive even for a second supplier with a much longer lead time. Tables 5.7-5.11 illustrate the above numerical results.

For small Q/D, SR(1) < SR(2) occurs for small values of $E(L_B)$, which implies that single sourcing is better. For cases where Q/D is low and the standard deviation of the lead time is high, dual sourcing may never be attractive. Incomparing the point where dual sourcing is no longer optimal, $E(L_B)^*$, there is a decrease for a given P_α and

TABLE 5.4

Service Improvement by Uneven Split Instead of Even Split with Exponential Lead Times
(P1=0.95, 1-P1=.05, $E(L_A)=1$)

$E(L_B)$	1	1.05	1.1	1.2	1.3	1.4	1.5
Q/D = .4							
k	.5						
SR(k)	.08						
SR(.5)	.08						
RPI	0%						
Q/D = 1							
k	.5						
SR(k)	.06						
SR(.5)	.06						
RPI	0%						
Q/D = 3							
k	.5	.53	.558	.609	.656	.699	.738
SR(k)	.024	.0264	.0293	.035	.042	.049	.056
SR(.5)	.024	.0265	.0297	.037	.045	.054	.064
RPI	0%	.37%	1.35%	5.41%	6.67%	9.26%	12.5%
$E(L_B)$	1	1.1	1.2	1.4	1.6	1.8	2.0
Q/D = 5							
k	.5	.544	.589	.653	.71	.76	.8
SR(k)	.01	.0133	.017	.025	.033	.043	.054
SR(.5)	.01	.0135	.018	.028	.042	.058	.075
RPI	0%	1.48%	5.56%	10.7%	21.4%	25.9%	28%

TABLE 5.5

Service Improvement by Uneven Split Instead of Even Split with Normal Lead Times and $\sigma_A = \sigma_B = .1$
(P1=0.95, 1-P1=.05, $E(L_A)=1$)

$E(L_B)$	1	1.1	1.2	1.3	1.4	1.5
Q/D = .4			$E(L_B)^* = 1.26$			
k	.5	.635	.796	1		
SR(k)	.003	.014	.037	.054		
SR(.5)	.003	.017	.079	.29		
RPI	0%	17.6%	53.2%	81.4%		
Q/D = .7			$E(L_B)^* = 1.45$			
k	.5	.575	.66	.761	.891	1
SR(k)	.002	.013	.032	.046	.05	.051
SR(.5)	.002	.013	.033	.061	.169	.47
RPI	0%	0%	3.03%	24.6%	70.4%	89.1%
Q/D = 1			$E(L_B)^* = 1.5$			
k	.5	.552	.609	.673	.75	
SR(k)	.002	.013	.032	.0456	.05	
SR(.5)	.002	.013	.032	.0457	.053	
RPI	0%	0%	0%	.22%	5.66%	
Q/D = 5			$E(L_B)^* = 2.0$			
k	.5	.51	.52	.531	.542	
SR(k)	.002	.013	.032	.046	.05	
SR(.5)	.002	.013	.032	.046	.05	
RPI	0%	0%	0%	0%	0%	

TABLE 5.6

Service Improvement by Uneven Split Instead of Even Split with Normal Lead Times and $\sigma_A = \sigma_B = .5$
(P1=0.95, 1-P1=.05, $E(L_A)=1$)

$E(L_B)$	1	1.1	1.2	1.3	1.4	1.5	1.6
Q/D = .4			$E(L_B)^* = 1.08$				
k	.5	.633					
SR(k)	.041	.052					
SR(.5)	.041	.054					
RPI	0%	3.7%					
Q/D = .7			$E(L_B)^* = 1.35$				
k	.5	.576	.653	.733	.817		
SR(k)	.021	.027	.035	.044	.055		
SR(.5)	.021	.028	.038	.054	.076		
RPI	0%	3.57%	7.89%	18.5%	27.6%		
Q/D = 1			$E(L_B)^* = 1.61$				
k	.5	.553	.606	.662	.719	.779	.842
SR(k)	.01	.014	.019	.025	.032	.04	.049
SR(.5)	.01	.014	.021	.03	.044	.063	.09
RPI	0%	0%	9.52%	16.7%	27.3%	36.5%	45.6%
$E(L_B)$	1	1.1	1.2	1.4	1.6	1.8	2.0
Q/D = 5			$E(L_B)^* = 2.0$				
k	.5	.52	.541	.585	.633	.687	.75
SR(k)	.002	.005	.01	.024	.039	.047	.05
SR(.5)	.002	.005	.01	.024	.039	.0473	.053
RPI	0%	0%	0%	0%	0%	.63%	5.66%

TABLE 5.7

Service Improvement by Uneven Split Instead of Even Split with Weibull Lead Times and $\sigma_A = \sigma_B = .1$
(Q/D=.4, P1=0.95, 1-P1=.05, $E(L_A)=1$)

$E(L_B)$	1	1.05	1.1	1.2	1.3	1.4	1.5
Q/D = .2			$E(L_B)^* = 1.12$				
k	.5	.628	.77	1			
SR(k)	.003	.012	.033	.10			
SR(.5)	.003	.018	.081	.393			
RPI	0%	33.3%	59%	74.6%			
Q/D = .4			$E(L_B)^* = 1.25$				
k	.5	.56	.623	.765	.94		
SR(k)	.002	.009	.019	.04	.06		
SR(.5)	.002	.009	.019	.11	.402		
RPI	0%	0%	0%	63.6%	85%		
Q/D = .7			$E(L_B)^* = 1.40$				
k	.5	.532	.565	.635	.712	.796	
SR(k)	.002	.009	.019	.038	.046	.05	
SR(.5)	.002	.009	.019	.038	.073	.165	
RPI	0%	0%	0%	0%	37%	70%	
Q/D = 1			$E(L_B)^* = 1.60$				
k	.5	.52	.542	.587	.636	.688	.743
SR(k)	.002	.009	.019	.038	.046	.049	.05
SR(.5)	.002	.009	.019	.038	.046	.057	.165
RPI	0%	0%	0%	0%	0%	14%	70%

TABLE 5.8

Service Improvement by Dual Sourcing Relative to Single Sourcing with Exponential Lead Times
($P1=0.95$, $1-P1=.05$, $E(L_A)=1$)

$E(L_B)$	1	1.05	1.1	1.2	1.3	1.4	1.5
Q/D = .2							
k	.5						
SR(2)	.08						
SR(1)	.05						
RPI	-60%						
Q/D = 1							
k	.5						
SR(2)	.06						
SR(1)	.05						
RPI	-20%						
Q/D = 3			$E(L_B)^* = 1.41$				
k	.5	.53	.558	.609	.656	.699	.738
SR(2)	.024	.0264	.0293	.035	.042	.049	.056
SR(1)	.05	.05	.05	.05	.05	.05	.05
RPI	52%	47%	41.4%	30%	16%	2%	-12%
$E(L_B)$	1	1.1	1.2	1.4	1.6	1.8	2.0
Q/D = 5			$E(L_B)^* = 2.92$				
k	.5	.544	.589	.653	.71	.76	.8
SR(k)	.01	.0133	.017	.025	.033	.043	.054
SR(.5)	.05	.05	.05	.05	.05	.05	.05
RPI	80%	73.4%	66%	50%	34%	14%	-8%

TABLE 5.9

Service Improvement by Dual Sourcing Relative to Single Sourcing with Normal Lead Times and $\sigma_A = \sigma_B = .1$
($P1=0.95$, $1-P1=.05$, $E(L_A)=1$)

$E(L_B)$	1	1.05	1.1	1.2	1.3	1.4
Q/D = .4			$E(L_B)^* = 1.26$			
k	.5	.635	.796	1		
SR(2)	.003	.014	.037	.054		
SR(1)	.05	.05	.05	.05		
RPI	94%	72%	26%	-8%		
Q/D = .7			$E(L_B)^* = 1.45$			
k	.5	.575	.66	.761	.891	1
SR(2)	.002	.013	.032	.046	.05	.051
SR(1)	.05	.05	.05	.05	.05	.05
RPI	96%	74%	36%	8%	0%	-2%
Q/D = 1			$E(L_B)^* = 1.7$			
k	.5	.552	.609	.673	.75	
SR(2)	.002	.013	.032	.0456	.05	
SR(1)	.05	.05	.05	.05	.05	
RPI	96%	74%	36%	8.8%	0%	
Q/D = 5			$E(L_B)^* = 2.0$			
k	.5	.51	.52			
SR(2)	.002	.013	.032			
SR(1)	.05	.05	.05			
RPI	96%	36%	0%			

TABLE 5.10

Service Improvement by Dual Sourcing Relative to Single Sourcing with Normal Lead Times and $\sigma_A = \sigma_B = .5$
($P1=0.95$, $1-P1=.05$, $E(L_A)=1$)

$E(L_B)$	1	1.1	1.2	1.3	1.4	1.5	1.6
Q/D = .4			$E(L_B)^* = 1.08$				
k	.5	.633					
SR(2)	.041	.052					
SR(1)	.05	.05					
RPI	18%	-4%					
Q/D = .7			$E(L_B)^* = 1.35$				
k	.5	.576	.653	.733	.817		
SR(2)	.021	.027	.035	.044	.055		
SR(1)	.05	.05	.05	.05	.05		
RPI	58%	46%	30%	12%	-10%		
Q/D = 1			$E(L_B)^* = 1.61$				
k	.5	.553	.606	.662	.719	.779	.842
SR(2)	.01	.014	.019	.025	.032	.04	.049
SR(1)	.05	.05	.05	.05	.05	.05	.05
RPI	80%	72%	62%	50%	36%	20%	2%
$E(L_B)$	1	1.1	1.2	1.4	1.6	1.8	2.0
Q/D = 5			$E(L_B)^* = 2.0$				
k	.5	.52	.541	.585	.633	.687	.75
SR(2)	.002	.005	.01	.024	.039	.047	.05
SR(1)	.05	.05	.05	.05	.05	.05	.05
RPI	96%	90%	80%	52%	22%	12%	0%

TABLE 5.11

Service Improvement by Dual Sourcing Relative to Single Sourcing with Weibull Lead Times and $\sigma_A = \sigma_B = .1$
(Q/D=.4, P1=0.95, 1-P1=.05, $E(L_A)$=1)

$E(L_B)$	1	1.05	1.1	1.2	1.3	1.4	1.5
Q/D = .2				$E(L_B)^*$ = 1.12			
k	.5	.628	.77	1			
SR(2)	.003	.012	.033	.10			
SR(1)	.05	.05	.05	.05			
RPI	94%	76%	34%	-100%			
Q/D = .4				$E(L_B)^*$ = 1.25			
k	.5	.56	.623	.765	.94		
SR(2)	.002	.009	.019	.04	.06		
SR(1)	.05	.05	.05	.05	.05		
RPI	96%	82%	62%	20%	-20%		
Q/D = .7				$E(L_B)^*$ = 1.40			
k	.5	.532	.565	.635	.712	.796	
SR(2)	.002	.009	.019	.038	.046	.05	
SR(1)	.05	.05	.05	.05	.05	.05	
RPI	96%	82%	62%	24%	8%	0%	
Q/D = 1				$E(L_B)^*$ = 1.60			
k	.5	.52	.542	.587	.636	.688	.743
SR(1)	.002	.009	.019	.038	.046	.049	.05
SR(2)	.05	.05	.05	.05	.05	.05	.05
RPI	96%	82%	62%	24%	8%	5%	0%

Q/D, as the standard deviation of the lead time increases. Therefore, for situations where the standard deviation of the lead time is larger, dual sourcing is less attractive.

5.2.2 The Effect of P_α and the Standard Deviation of the Lead Time on the Optimal Order Split

We now examine the effects of the parameters on the value of $E(L_B)$ where dual sourcing is no longer advantageous, $E(L_B)^*$, are examined. The numerical results are given in Table 5.12. In some cases, as mentioned previously, there is never an advantage in dual sourcing over single sourcing. In the table, this occurs where the value of $E(L_B)^*$ is absent. As the standard deviation of the lead time, σ, increases, the value of $E(L_B)^*$ decreases. (It should be noted that the model does not handle larger values of the standard deviation.) This implies that dual sourcing is more enticing for vendors with a smaller lead time variance. There is not much difference between the results for the different levels of P_α.

5.2.3 The Effect of the Shape of the Distribution on the Optimal Order Split Rate

We have seen earlier, that besides the parameter values, the optimal split rate also depends on the shape of the distribution. We observed, however, that we have the same tendencies in the change of the optimal split rate for different shapes, just the rate of the change is different. In the Weibull case, we change the shape of the distribution of vendor B. Moving from m=4 to m=1, the shape changes from a

TABLE 5.12

The Value of $E(L_B)^*$ Depending on P_α, Q/D, and the Standard Deviation of the Lead Time with Weibull Lead Times ($\sigma_A = \sigma_B$)

Q/D	.2	.4	.7	1
$P_\alpha = .9$				
$\sigma = .1$	1.11	1.24	1.39	1.59
$\sigma = .3$	---	1.12	1.29	1.43
$\sigma = .5$	---	---	1.12	1.26
$P_\alpha = .95$				
$\sigma = .1$	1.12	1.25	1.4	1.6
$\sigma = .3$	1.01	1.14	1.3	1.43
$\sigma = .5$	---	1.0	1.13	1.26
$P_\alpha = .99$				
$\sigma = .1$	1.13	1.26	1.42	1.56
$\sigma = .3$	1.04	1.15	1.28	1.4
$\sigma = .5$	---	1.03	1.12	1.22

TABLE 5.13

Optimal Split Rates Depending on the Shape of the Lead Time Distribution for Weibull Lead Times
($Q/D=.4$, $\mu_A=1$, $P_\alpha=.95$ $1-P_\alpha=.05$)

$E(L_B)$	1	1.05	1.1	1.15
m = 4				
k	.5	.599	.695	.789
SR(2)	.015	.024	.036	.05
SR(1)	.05	.05	.05	.05
RPI	70%	52%	28%	0%
m = 3				
k	.5	.606	.707	
SR(2)	.03	.04	.052	
SR(1)	.05	.05	.05	
RPI	40%	20%	-40%	
m = 2				
k	.5			
SR(2)	.052			
SR(1)	.05			
RPI	-40%			
m = 1				
k	.5			
SR(2)	.08			
SR(1)	.05			
RPI	-60%			

positively skewed density, through a closely normal shape and negatively skewed densities, to an exponential density. Table 5.13 implies that as m decreases, dual sourcing becomes less attractive. This can be explained by the fact that Q/D is small (.4). As we have noted earlier, in the case of exponential lead times (m=1), a longer time between orders, Q/D, yields better results in dual sourcing. As the shape parameter, m, increases, dual sourcing is more attractive for smaller Q/D.

5.3 Random Demand

As detailed earlier, changes in production level can occur causing fluctuation in demand. This situation is considered in this section. An approximate analytic procedure and simulation will be used.

5.3.1 Analytic Procedure

When random demand and random lead times are assumed in an inventory model, the distribution of demand during lead time must be considered. This distribution is difficult to derive, except for special cases, since it is a compound distribution. Bagchi, Hayya, and Ord (1984) present an approach to modeling lead time demand using its components rather than modeling it directly. Other approaches are given in Carlson (1982), Nahmias and Demmy (1982), Ord and Bagchi (1983), and Bagchi, Hayya and Ord (1983).

The components of lead time demand are the lead time (LT), the order intensity (OI), and the order size (OS). Intermediate components are formed. Period demand

(DPUT) is formed from combining OS and OI, whereas, lead time order intensity (LTOI) is formed from combining OI with LT. Lead time demand is then found from combining DPUT with LT, or LTOI with OS. Since the intermediate components are compound distributions, this adds to the difficulty.

For slow moving items, the daily demand can be approximated with the Poisson distribution. Using the Gamma distribution to model lead time, the lead time demand will be negative binomially distributed. (Taylor 1961) For fast moving items, the normal distribution can be used to model daily demand. For gamma distributed lead times, the distribution of the lead time demand is given by the following density function. (Burgin 1972)

$$f_W(w) = \frac{2\alpha^k}{\Gamma(k)\,(2\Pi\sigma^2)^{\frac{1}{2}}} \left(\frac{w}{\theta}\right)^{k-\frac{1}{2}} K_{k-\frac{1}{2}}\left(\frac{\theta w}{\sigma^2}\right) \exp\left(\frac{\mu w}{\sigma^2}\right) \quad (5.24)$$

where $\theta = \mu^2 + 2\alpha\sigma^2$ and K is the Bessel function of imaginary argument.

For these cases, we can calculate the $P_\alpha^{(2)}$ values from formula (5.6). The optimal k, for the deterministic demand case, can be found using numerical methods. This k value can be used as a starting point for a numerical search procedure used to find the optimal k for the random demand case.

Another approach is to approximate the lead time demand distribution for each vendor, $W_A(x)$ and $W_B(x)$, with a three parameter Weibull distribution. Using the first

three moments of both lead time and demand distributions, we express the first three moments of the lead time demand distribution and then fit the appropriate Weibull distribution. The procedure is as follows:

1) Compute the first three moments of the demand and lead time distributions where

$\mu_i(l)$ = the ith moment of the lead time distribution
i = 1,2,3

$\mu_i(d)$ = the ith moment of the demand distribution
i = 1,2,3

2) Compute the first three moments, μ_1, μ_2, μ_3, of the lead time demand distribution using the moments from step 1. The appropriate expressions are detailed in Carlson(1964), Kottas(1979), and Wan and Lau(1981).

$$\mu_1 = \mu_1(l)\mu_1(d)$$

$$\mu_2 = \mu_1(l)\mu_2(d) + \mu_2(l)\mu^2{}_1(d)$$

$$\mu_3 = \mu_1(l)\mu_3(d) + 3\mu_2(l)\mu_2(d)\mu_1(d) + \mu_3(l)\mu^3{}_1(d)$$

3) Fit the moments from step 2 to a three parameter (λ,m,c) Weibull distribution. The cumulative distribution function is below.

$$W(x) = 1-\exp\{-[\lambda(x-c)]^m\} \qquad (5.25)$$

This procedure is similar to the one given in Lau and Zhao (1993). They considered the service measure to be the expected number of shortages. The probability of no shortage used as a service measure in our work.

We will use an approach similar to the second one for approximating demand, based on the two parameter Weibull distribution rather than using the compound distributions previously discussed. This gives a unified treatment of the deterministic and random cases. It is an approximation in both cases. The approximation is validated through simulation experiments described in Section 5.4.

The values of m and λ are found using the regression formula and relationship between λ and m that were used in the deterministic case. The difference, however, is that the mean and variance used will be for the lead time demand rather than for the lead time only. The mean and variance for the lead time demand (LTD) is calculated with the following formulas

$$\mu_{LTD}=\mu_L\cdot\mu_D \qquad (5.26)$$
$$\sigma_{LTD}=\sqrt{\mu_L\cdot\sigma_D^2+\mu_D^2\cdot\sigma_L^2}$$

where μ_{LTD} is the mean lead time demand, μ_L is the mean lead time, μ_D is the mean demand, σ_{LTD} is the standard deviation of the lead time demand, σ_L is the standard deviation of the lead time, and σ_D is the standard deviation of the demand.

5.3.2 Numerical Results for Random Demand

In comparing the deterministic and random demand results, we examined three levels of the standard deviation for the demand, σ_D, and compared the results for each case with the deterministic demand case ($\sigma_D = 0$). The value of $E(L_B)^*$ decreases as σ_D

TABLE 5.14

Comparison of the Value of $E(L_B)^*$ for the Deterministic and Random Demand Cases with Weibull Lead Times and $P_\alpha = .9$
($\sigma = \sigma_A = \sigma_B$)

σ_D	0	.1	.3	.5
Q/D = .2				
$\sigma = .1$	1.11	1.09	---	---
$\sigma = .3$	---	---	---	---
$\sigma = .5$	---	---	---	---
Q/D = .4				
$\sigma = .1$	1.24	1.22	1.11	---
$\sigma = .3$	1.12	1.11	1.03	---
$\sigma = .5$	---	---	---	---
Q/D = .7				
$\sigma = .1$	1.39	1.4	1.28	1.11
$\sigma = .3$	1.29	1.29	1.19	1.05
$\sigma = .5$	1.12	1.11	1.05	---
Q/D = 1				
$\sigma = .1$	1.59	1.53	1.43	1.25
$\sigma = .3$	1.43	1.42	1.33	1.18
$\sigma = .5$	1.26	1.25	1.18	1.07

TABLE 5.15

Comparison of the Value of $E(L_B)^*$ for the Deterministic and Random Demand Cases with Weibull Lead Times and $P_\alpha = .95$
$(\sigma = \sigma_A = \sigma_B)$

σ_D	0	.1	.3	.5
Q/D = .2				
$\sigma = .1$	1.12	1.17	1	---
$\sigma = .3$	1.01	1.14	---	---
$\sigma = .5$	---	---	---	---
Q/D = .4				
$\sigma = .1$	1.25	1.24	1.12	---
$\sigma = .3$	1.14	1.13	1.05	---
$\sigma = .5$	1	---	---	---
Q/D = .7				
$\sigma = .1$	1.4	1.4	1.3	1.13
$\sigma = .3$	1.3	1.3	1.2	1.07
$\sigma = .5$	1.13	1.13	1.07	---
Q/D = 1				
$\sigma = .1$	1.6	1.55	1.44	1.26
$\sigma = .3$	1.43	1.42	1.34	1.19
$\sigma = .5$	1.26	1.25	1.18	1.08

TABLE 5.16

Comparison of the Value of $E(L_B)^*$ for the Deterministic and Random Demand Cases with Weibull Lead Times and $P_\alpha = .99$
$(\sigma = \sigma_A = \sigma_B)$

σ_D	0	.1	.3	.5
Q/D = .2				
$\sigma = .1$	1.13	1.12	1.03	---
$\sigma = .3$	1.04	1.03	---	---
$\sigma = .5$	---	---	---	---
Q/D = .4				
$\sigma = .1$	1.26	1.25	1.14	1.02
$\sigma = .3$	1.15	1.14	1.07	---
$\sigma = .5$	1.03	1.02	---	---
Q/D = .7				
$\sigma = .1$	1.42	1.42	1.3	1.13
$\sigma = .3$	1.28	1.27	1.2	1.08
$\sigma = .5$	1.13	1.12	1.08	1.01
Q/D = 1				
$\sigma = .1$	1.56	1.56	1.44	1.24
$\sigma = .3$	1.4	1.39	1.32	1.17
$\sigma = .5$	1.22	1.21	1.17	1.08

increases. This implies that dual sourcing is less attractive for a second supplier with a longer lead time when the variability of the demand is larger.

There is not that much difference in the results for $\sigma_D = .1$ and the deterministic results. The deterministic optimal split rates could be used to approximate the rates for the random demand case where the standard deviation of the demand is small. This is an advantage since our procedure used to estimate m may not be valid for a standard deviation smaller than .1. It should be noted that dual sourcing is not attractive for the case where there is a low Q/D and a high standard deviation of the lead time.

If we examine the effect of increasing the standard deviation of the demand, holding all else constant, we find that an even split remains optimal (k = .5), since the lead time characteristics are not changing but the stockout risk for dual sourcing increases. Therefore, as mentioned previously, dual sourcing is less attractive for larger demand variability. This is evidenced in Table 5.17.

5.4 Simulation

A simulation study was conducted to validate the analytic models for deterministic and random demand. The inventory system was simulated in SLAM, a simulation language. The demand was deleted from the inventory daily, and the reorder level checked. If the inventory falls below the reorder level, then an order is released to two vendors and is added to the inventory after the duration of the lead time. The order quantity was split between the vendors using the optimal split rates that were calculated from our analytic model. The stockout risk was calculated by counting the number of

TABLE 5.17

Comparison of the Stockout Risk for Dual Sourcing for Increasing Variability of Demand with Weibull Lead Times and $P_\alpha = .95$ ($\sigma = \sigma_A = \sigma_B$)

σ_D	.1	.2	.3	.4	.5	.6	.7
Q/D = .2							
σ=.1	.008	.03	.047	.06			
σ=.3	.047	.055					
σ=.5	.079						
Q/D = .4							
σ=.1	.003	.007	.021	.037	.051		
σ=.3	.021	.029	.039	.049	.058		
σ=.5	.05	.054					
Q/D = .7							
σ=.1	.002	.003	.006	.017	.03	.042	.052
σ=.3	.006	.011	.019	.029	.039	.048	.056
σ=.5	.03	.034	.039	.045	.052		
Q/D = 1							
σ=.1	.002	.003	.003	.008	.018	.029	.039
σ=.3	.003	.005	.009	.016	.026	.036	.044
σ=.5	.018	.021	.026	.032	.039	.046	.052

order periods in which a stockout occurs and dividing by the total number of orders. Random lead times were modeled using the Weibull distribution.

The simulation results for deterministic demand are compared to the analytic results in Table 5.18. The relative percentage improvement (RPI) in the stockout risk for dual sourcing over single sourcing is calculated for each case. The RPI values for the simulation is close to or larger than the analytically calculated RPI. In the simulation, the optimal split rate performs better for a second vendor with a larger expected lead time than the analytic model estimates.

The simulation was also run using the optimal split rates as calculated with the approximation procedure given for random demand. The demand in the random case was modeled with the Weibull distribution. The results are illustrated in Table 5.19. The comparison is similar to the deterministic demand results.

In order to check the performance as measured by other service measures the expected shortage per unit time was also calculated. Because of the high service levels examined, this measure is so close to zero that comparisons are not informative. Instead the relative percentage improvement in total amount short (RPI_{TS}) for dual sourcing over single sourcing is given in Table 5.20. In comparison, the relative percentage improvement in the stockout risk (RPI_{SR}) for dual sourcing over single sourcing is slightly larger in many cases, but there is still a large improvement in the total shortage for dual sourcing over single sourcing.

The simulation results show that our model provides a reasonable approximation for the optimal order split for dual sourcing which minimizes the stockout risk. The

relative percentage improvement is larger in most cases so it gives a conservative solution.

TABLE 5.18

Comparison of the Simulation and Analytic Results for Service Improvement by Dual Sourcing Relative to Single Sourcing with Weibull Lead Times, $\sigma_A = \sigma_B = .1$, and Deterministic Demand
($P1=0.95$, $1-P1=.05$, $E(L_A)=1$)

$E(L_B)$	1	1.05	1.1	1.2	1.3	1.4
Q/D = .4						
k	.5	.56	.623	.765		
SR(2)	.002	.009	.019	.04		
SR(1)	.05	.05	.05	.05		
RPI	96%	82%	62%	20%		
Simulation						
k	.5	.56	.623	.765		
SR(2)	.0035	.0015	.0055	.016		
SR(1)	.05	.05	.05	.05		
RPI	93%	97%	89%	68%		
Q/D = .8						
k	.5	.52	.542	.587	.636	.688
SR(2)	.002	.009	.019	.038	.046	.049
SR(1)	.05	.05	.05	.05	.05	.05
RPI	96%	82%	62%	24%	8%	5%
Simulation						
k	.5	.52	.542	.587	.636	.688
SR(2)	.0025	.0025	0	0	0	0
SR(1)	.05	.05	.05	.05	.05	.05
RPI	96%	96%	100%	100%	100%	100%

TABLE 5.19

Comparison of the Simulation and Analytic Results for Service Improvement by Dual Sourcing Relative to Single Sourcing with Weibull Lead Times, $\sigma_A = \sigma_B = .1$, and Weibull Demand
(P1=0.95, 1-P1=.05, $E(L_A)=1$)

$E(L_B)$	1	1.05	1.1	1.2	1.3	1.4
Q/D = .4						
k	.5	.579	.658	.822		
SR(2)	.003	.007	.013	.036		
SR(1)	.05	.05	.05	.05		
RPI	94%	86%	74%	28%		
Simulation						
k	.5	.56	.623	.765		
SR(2)	.0073	.0073	.0063	.0089		
SR(1)	.05	.05	.05	.05		
RPI	85%	85%	87%	81%		
Q/D = .8						
k	.5	.52	.542	.587	.636	.688
SR(2)	.002	.006	.012	.026	.038	.046
SR(1)	.05	.05	.05	.05	.05	.05
RPI	90%	88%	76%	48%	24%	8%
Simulation						
k	.5	.52	.542	.587	.636	.688
SR(2)	.0095	.0095	.0084	.005	.004	.003
SR(1)	.05	.05	.05	.05	.05	.05
RPI	82%	82%	84%	90%	92%	94%

TABLE 5.20

Comparison of the Improvement by Dual Sourcing Relative to Single Sourcing as Measured by the Stockout Risk and the Total Shortage with Weibull Lead Times, $\sigma_A = \sigma_B = .1$, and Deterministic Demand (P1=0.95, 1-P1=.05, $E(L_A)=1$)

$E(L_B)$	1	1.1	1.2	1.3	1.4	1.5
Q/D = .4						
k	.5	.623	.765			
RPI_{TS}	89%	89%	96%			
RPI_{SR}	93%	97%	89%			

$E(L_B)$	1	1.05	1.1	1.2	1.3	1.4
Q/D = .8						
k	.5	.527	.555	.615	.679	.951
RPI_{TS}	90%	90%	97%	97%	97%	0%
RPI_{SR}	90%	90%	97%	97%	97%	10.3%

CHAPTER 6

SUMMARY AND CONCLUSIONS

JIT purchasing is a critical part of a JIT production system because the purchaser must deal with an external element, the supplier. During the transition period to JIT production, smaller lots with high quality and on time deliveries are required. The purchaser tries to cooperate with the vendor in order to reach this goal, but problems with timing or quality can occur.

The ultimate goal of JIT is a single reliable supplier, who is willing and able to deliver small lots, frequently, and on time. We call this perfect coordination. If the supplier is not able to deliver in a timely manner with the required quality, we have imperfect coordination. In this case, safety stock must be held to protect against delays caused by these problems. The P_α service measure, the probability of no shortage, which is the complement of the stockout risk, $SR = 1 - P_\alpha$, is used in our models to calculate the safety stock necessary for a prescribed service level. For both cases, we supplied new quantitative models to help in decision making and cooperation. The optimal order quantity and number of shipments is given which yields the minimum joint total cost for the supplier and buyer. Also given, for imperfect coordination, is the optimal level of safety stock which provides the required service level. The models can serve as tool for negotiation between the vendor and the purchaser.

During the transition to JIT there may not be a single, reliable supplier willing to deliver small lot sizes, frequently and on schedule with high quality levels. Multiple sources must be used until a reliable supplier emerges. This is a frequent practice, and

is even used by typical Japanese JIT manufacturers such as Honda. Suppliers may be rewarded for better delivery performance by allocating a larger part of the order to them. This will enhance competition, yielding better supplier performance. There has been a lot of research on the benefits of multiple sourcing, but not on how the order should be allocated between the suppliers. Our quantitative model gives the answer to two main questions: when is it worth it to have another supplier, and what is the optimal split rate in order to minimize the stockout risk.

Quantitative models are provided to help in the transition state toward the ultimate goal of JIT purchasing. First, deterministic demand is considered in order to study the effects of random deliveries more clearly. The models are then extended to the situation where random demand occurs.

In Section 1, the implications of the single sourcing models are discussed from a practical point of view. This is continued in Section 2 for the dual sourcing case. Finally, future research suggestions are given in Section 3.

6.1 Implications of the Single Sourcing Models

Buyer-supplier cooperation is emphasized in JIT. Our model minimizes the joint total cost for the purchaser and the vendor in order to capitalize on this cooperation. Either party will have a lower total cost if their optimal order quantity is used rather than the joint optimal order quantity. However, if we add the cost of the buyer and the purchaser, the joint total cost will be lower for the joint optimal order quantity. This provides economic motivation for cooperation and price negotiations. Cooperation is

a major element of JIT purchasing and the companies may benefit monetarily and also in other ways, from compromising and agreeing to use the joint order quantity. We suggest that this model can be used as a bargaining tool. The stronger party may agree to the joint order quantity if some type of concession is given in the form of a long contract, a price discount, or maybe a premium and the savings can be shared. This promotes cooperation between the parties which provides a better working relationship.

Our model calculates the breakeven points for negotiation for both parties. In terms of cost, our results imply that a party, the vendor or purchaser, always benefits from using the joint ordering policy rather than the optimal policy for the other party. This supports the use of the joint ordering policy as a negotiation tool. As C_v/C_p increases, this savings becomes larger. As the shipment cost increases, there is also an increase in the savings to a point, but the improvement decreases for higher shipment costs.

Certain factors in the models influence the order quantity and shipment frequency and must be considered closely. The ratio of the per unit purchase cost and per unit production cost, C_v/C_p, and the shipment cost affect the delivery frequency. C_v/C_p has the largest effect on the optimal number of deliveries. As this ratio decreases the optimal number of shipments increases and the holding cost for the purchaser relative to the vendor's holding cost also increases. The purchaser would want more deliveries, hence, less inventory carried. The shipment cost has the next largest effect on the number of deliveries. As one would expect, for a higher shipment cost, the optimal number of shipments is lower.

The optimal order quantity is affected most by the ratio of the production rate to the demand rate, P/D. As P/D increases, the optimal order quantity decreases. The factors with the next largest effect are C_v/C_p and the ratio of the setup cost to the ordering cost, S/A. C_v/C_p has an inverse relationship with the optimal order quantity, and S/A has a direct relationship. On the other hand, sensitivity analysis results show that small changes do not affect the optimal number of shipments, and cause only very minor changes in the joint optimal order quantity. These results demonstrate the robustness of our models to errors in parameter estimations.

In the situation where safety stock is necessary, the parameter, P_α, the probability of no shortage in an order cycle, has a small effect on the number of deliveries. An extra delivery may be required to provide a higher level of service. The optimal order quantity decreases for a fixed number of shipments.

In the case where quality problems arise, the proportion of the order that is guaranteed to be delivered, δ, has an inverse effect on the optimal number of deliveries. One fewer delivery may be necessary for a larger percentage of the shipment to be delivered and usable. The optimal order quantity increases for a fixed number of shipments.

In the situation where the demand is random, the standard deviation of the demand does not affect the optimal number of deliveries, but there is a decrease in the order quantity as the standard deviation of the demand increases. A larger order may be necessary to combat the effects of the randomness in the demand.

We compared the basic model and its extensions. There is a larger difference between the basic model and the extensions, however, there is not much difference between the extensions. Thus, the simplest model, Model II, could be used to calculate the order quantity and number of deliveries for all three of the extensions. It requires less data and easier to calculate and implement. The optimal number of deliveries is larger for the extension models and the delivery size is smaller. An increase in the number of deliveries, along with the safety stock held, provides protection against shortages. There is, however, a considerable difference in the safety stock requirement among the three extensions. Models III and IV can provide the necessary safety stock to protect against shortages caused by random yield and random demand, thus, they must be considered in deciding the safety stock level, but not necessarily in the decision about the number of shipments and order quantity, as was discussed previously.

6.2 Implications of the Dual Sourcing Models

We provide a model to calculate the optimal proportion of split of an order between two suppliers in order to minimize the stockout risk. The model allocates the order between the suppliers based on the delivery characteristics, the mean and standard deviation of the lead time. The shape of the lead time distribution is also considered. It allocates a larger proportion of the order to the supplier with the better performance. At some point, the delivery characteristics of the second supplier reach the point where dual sourcing is no longer optimal and a single supplier should be used. This point was determined by numerical calculations.

For a higher order frequency, the point where dual sourcing is no longer optimal is reached more quickly. Therefore, dual sourcing is not as attractive for purchasers that order frequently. In JIT purchasing, a long term contract is usually entered into and therefore the order frequency may be smaller.

As the standard deviation of the lead time increases, the point where dual sourcing is no longer optimal decreases. Therefore, dual sourcing may be more enticing for vendors with a smaller lead time variance.

As mentioned earlier, there may be randomness in the demand. We give an approximate analytic procedure for this case based on the Weibull distribution that provides a good approximation for a wide range of common distributions common in practice. The analytic results for the deterministic and random cases were compared at several levels of the standard deviation of the demand. The point where dual sourcing is not longer attractive decreases as the standard deviation of the demand increases. This implies that dual sourcing is less attractive for a second supplier with a longer expected lead time when the variability of demand is larger. It was also noted that dual sourcing is not attractive for the case where there is a high order frequency together with a high standard deviation of the lead time. In other cases, dual sourcing provides a considerable decrease in stockout risk.

Simulation results supported the idea that dual sourcing yields a lower stockout risk than single sourcing when the optimal allocation of the order quantities is used. The simulation results demonstrated an even higher improvement for dual sourcing over single sourcing than the analytic results indicated. Simulation results showed that

whenever the stockout risk improves, there is also an improvement in the expected shortage per unit time, which is another frequently used measure of service.

6.3 Future Research Directions

In the single sourcing models, it was assumed that the ordering cost was not affected by the number of shipments. This effect may be investigated and also, other forms of shipment cost can be considered. Random demand was considered only in the specific case where the demand rate is random and normally distributed. Other, more general, cases for randomness could be investigated. Price discounts, constraints on the shipment sizes or number, were not considered either. In the determination of safety stock, other measures could also be considered.

In dual sourcing, our research did not consider vendors with different prices. Also, although the expected shortage for dual sourcing was investigated with simulation, models which consider other service measures should be investigated. In the dual sourcing model, the order quantity and reorder level were predetermined. Further research may include considering the total cost of dual sourcing and the joint calculation of the order quantity and reorder level with the order split ratio. The importance and difficulties of JIT purchasing warrants the extension of current results which provided some new quantitative models, managerial issues, and research directions in that area.

BIBLIOGRAPHY

Akinc, Umit, "Selecting a Set of Vendors in a Manufacturing Environment," Journal of Operations Management, 11, (1993), 107-122.

Ammer, D., Materials Management and Purchasing, Richard D. Irwin, Inc., Home Wood, IL, (1980).

Ansari, A., and J. Heckel, "JIT Purchasing: Impact of Freight and Inventory Costs," Journal of Purchasing and Materials Management, Summer, (1987), 24-28.

Ansari, A., and B. Modarress, "Just-In-Time Purchasing: Problems and Solutions," Journal of Production and Materials Management, Summer (1986), 11-15.

Ansari, A., and B. Modarress, "The Potential Benefits of Just-In-Time Purchasing for U.S. Manufacturers," Production and Inventory Management, 28, (1987), 30-35.

Anupindi, R. and R. Akella, "Diversification Under Supply Uncertainty," Management Science, 39, (1993), 944-963.

Bagchi, U., J. Hayya, and J.K. Ord, "The Hermite Distribution as a Model of Demand During Lead Time for Slow-Moving Items," Decision Sciences, 14, (1983), 447-466.

Bagchi, U., J. Hayya, and J.K. Ord, "Modeling Demand During Lead Time," Decision Science, 15, (1984), 157-176.

Baldwin, R., and R. Gagnon, "The Transfer of Japanese Manufacturing Principles and Their Impact on Purchaser-Vendor Policies," Production and Inventory Management, (1993), 26-30.

Banerjee, A., "A Joint Economic-Lot-Size Model for Purchaser and Vendor," Decision Science, 17, (1986), 292-311.

Bartholomew, D., "The Vendor-Customer Relationship Today," 25, (1984), 106-121.

Billesbach, T.J., Harrison, A., and S. Croom-Morgan, "Supplier Performance Measures and Practices in JIT Companies in the U.S. and the U.K.," International Journal of Purchasing and Materials Management, Fall, (1991), 24-28.

Birnbaum, Z., and H. Tingey, "One-sided Confidence Contours for Probability Distribution Function," An. Math Statist, 22, (1951), 592-596.

Burgin, T., "Inventory Control with Normal Demand and Gamma Lead Times," Operational Research Quarterly, 23, (1972), 73-80.

Burton, T., "JIT/Repetitive Sourcing Strategies: "Tying The Knot" with Your Suppliers," Production and Inventory Management, 29, (1988), 38-41.

Carlson, P., "An Alternative Model for Lead-Time Demand: Continuous-review Inventory System," Decision Sciences, 13, (1982), 120-128.

Chaundhry, S.S., Forst, F.G., and J.L. Zydiak, "A Multicriteria Approach to Allocating Order Quantity Among Vendors," Production and Inventory Management, (1991),82-85.

Coleman, B.J., and M.A. McKnew, "A Technique for Order Placement and Sizing," Journal of Production and Materials Management, Spring, (1990), 32-38.

Ding, F., and M. Yuen, "A Modified MRP for a Production System with the Coexistence of MRP and Kanbans," Journal of Operations Management, 10, (1991), 267-277.

Drinnon, J., and J. Hiller, Predicting the Costs and Benefits of Competitive Production Sources, The Analytic Sciences Corporation, TR-1511.

Fawcett, S.E. and L.M. Birou, "Just-In-Time Sourcing Techniques: Current State of Adoption and Performance Benefits," Production and Inventory Management, (1993), 19-24.

Fong, D. and J.K. Ord, "Estimating Moments of the Effective Lead Time for a Stock Control Model with Independent Normal Lead Times," Journal of the Operational Research Society, 44, (1993), 247-252.

Giunipero, L.C. and E.F. Keiser, "JIT Purchasing in a Non-Manufacturing Environment: A Case Study," Journal of Production and Materials Management, Winter, (1987), 19-25.

Giunipero, L.C., "Motivating and Monitoring JIT Supplier Performance," Journal of Production and Materials Management, Summer, (1990), 19-24.

Golhar, D.Y., and B.R. Sarker, "Economic Manufacturing Quantity in a Just-In-Time Delivery System, International Journal of Production Research, 30, (1992), 961-972.

Golhar, D.Y., and C.L. Stamm, "The Just-In-Time Philosophy: A Literature Review," International Journal of Production Research, 29, (1991), 657-676.

Golhar, D.Y., and C.L. Stamm, "JIT Purchasing Practices in Manufacturing Firms,"Production and Inventory Management, (1993), 75-79.

Goyal, S.K., ""A Joint Economic-Lot -Size Model for Purchaser and Vendor": A Comment", Decision Science, 19, (1988), 236-241.

Greer, W. and S. Liao, "An Analysis of Risk and Return in the Defense Market - Its Impact on Weapon System Competition," Management Science, 32, (1986) 1259-1273.

Hadley, G. and T. Whitin, Analysis of Inventory Systems, Prentice-Hall, Englewood Cliffs, New Jersey, (1963).

Hammel, T.R. and L.R. Kopczak, "Tightening th Supply Chain," Production and Inventory Management, 2nd Quarter, (1993), 63-69.

Hayya, J.C., D.P. Christy, and A. Pan, "Reducing Inventory Uncertainty: A Reorder Point System with Two Vendors," Production and Inventory Management, 2nd Quarter, 28 (1987), 43-48.

Hill, A.V., and T.E. Vollman, "Reducing Vendor Delivery Uncertainties in a JIT Environment," Journal of Operations Management, 6, (1986), 381-392.

Hong, J. and J.C. Hayya, "Just-In-Time Purchasing: Single or Multiple Sourcing?" International Journal of Production Economics, 27, (1992), 175-181.

Hong, Hayya, and Kim, "JIT Purchasing and Setup Reduction in an Integrated Inventory Model," International Journal of Production Research, 30, (1992), 255-266.

Horowitz, I. "On Two-Source Factor Purchasing," Decision Sciences, 17, (1986), 274-279.

Johansson, H., "Preparing for Accounting System Changes: Accountants Must Move from the Office to Factory Floor," Management Accounting, July, (1990), 37-41.

Jordon, H., "Inventory Management in the JIT Age," Production and Inventory Management, 29, (1988), 57-59.

Jordon, P. and W Schwendig, "Toward a More Flexible Ordering Policy," Journal of Purchasing and Materials Management, Fall, (1985), 25-28.

Joshi, K., "Storage Space Costs and the EOQ Model," Journal of Purchasing and Materials Management, Summer, (1990), 37-41.

Joshi, K. and J.F. Campbell, "Managing Inventories in a JIT Environment," International Journal of Purchasing and Materials Management, Spring, (1991), 32-36.

Kelle, P., "An Inventory Control Program Based on Reliability Type Models," Proceedings of the Second International Symposium on Inventories, Budapest, Hungary, (1982), 491-497.

Kelle, P., "On the Safety Stock Problem for Random Delivery Processes," European Journal of Operational Research, 17, (1984), 191-200.

Kelle, P. and H. Schneider, "A New Model for Service-Level Estimation in Just-In-Time Manufacturing," Proceedings of IFAC/IFORS Symposium, Budapest, July, (1991), 1572-1577.

Kelle, P., and H. Schneider, "Extension of a Reliability-Type Inventory Model to Just-In-Time Systems," International Journal of Production Economies, 26, (1992), 319-326.

Kelle, P., E.A. Silver, and G.F. Murphy, "Cost Analysis and Extension of a Simple Maintenance-Scheduling Heuristic," Working Paper, (1992).

Kelle, P. and E.A. Silver, "Safety Stock Reduction by Order Splitting," Naval Research Logistics, 37, (1990a), 725-743.

Kelle, P. and E.A. Silver, "Decreasing Expected Shortages through Order Splitting," Engineering Costs and Production Economics, 19, (1990b), 351-357.

Kim, J., J.K. Ord, and J.C. Hayya, "Reduction of Shortages in a Stochastic Order Splitting Model," Working Paper, Penn State University, (1991).

Kraljic, P., "Purchasing Must Become Materials Management," Harvard Business Review, 61, (1983), 109-117.

Kratz, L. and L. Cox, "Competition of Defense Procurements: Evidence, Theory, and Applications," The 1982 Federal Acquisition Research Symposium, Washington, D.C.

Kratz, L., J. Drinnon, and J. Hiller, Establishing Competitive Production Services: A Handbook for Program Managers, Defense Systems Management College, Fairfax, VA, (1984).

Lau, H. and L. Zhao, "Optimal Ordering Policies with Two Suppliers When Lead Times and Demands are all Stochastic," European Journal of Operational Research, 68, (1993), 120-133.

Lawrence, J., and H. Lewis, "JIT Manufacturing in Mexico: Obstacles to Implementation," Production and Inventory Management, (1993), 31-35.

Lockhart, M. and L. Ettkin, "Vendor Certification: Seven Steps to a Better Product," Production and Inventory Management, (1993), 65-69.

Manoochehri, G.H., "Suppliers and the Just-In-Time Concept," Journal of Purchasing and Materials Management, Winter, (1984), 16-21.

Mayer, R. "Optimum Safety Stocks When Demands and Lead Times Vary," Journal of Purchasing and Materials Management, Spring, (1984), 27-32.

McMillan, J., "Managing Suppliers: Incentive Systems in Japanese and U.S. Industry," California Management Review, Summer, (1990), 38-55.

Moinzadeh, K., and H. Lee, "Approximate Order Quantity and Reorder Points for Inventory Systems Where an Order Arrives in Two Shipments," Operations Research, 2, (1989), 277-287.

Moinzadeh, K., and Nahmias, "A Continous Review Model for an Inventory System with Two Supply Modes," Management Science, 34, (1988), 761-773.

Naddor, E., Inventory Systems, John Wiley and Sons, New York, (1966).

Nahmias, S., and W. Demmy, "The Logarithmic Poisson Gamma Distribution: A Model for Lead Time Demand," Naval Research Logistics Quarterly, (1982), 28, 667-677.

Narasimhan, R., "An Analytical Approach to Supplier Selection," Journal of Purchasing and Materials Management, Winter, (1983), 27-32.

Newman, R.G., "Single Source Qualification," Journal of Purchasing and Materials Management, Summer, (1988a), 10-17.

Newman, R.G., "The Buyer-Supplier Relationship Under Just-In-Time," Production and Inventory Management, 29, (1988b), 45-49.

Ord, J.K., and U. Bagchi, "The Truncated Normal-Gamma Mixture as a Distribution of Lead Time Demand," Naval Research Logistics Quarterly, 30, (1983), 359-365.

Pan, A.C., and C. Liao, "An Inventory Model Under Just-In-Time Purchasing Agreements," Production and Inventory Management, 30, (1989), 49-52.

Pan, A., "An Investigation of Order Splitting in an (s,O) Inventory System where Unit Demand is Constant and Lead Time Variable," Ph.D. Dissertation, Penn State University, 1987.

Pan, A.C., "Allocation of Order Quantity Among Suppliers," Journal of Purchasing and Materials Management, Fall, (1989), 36-39.

Pan, A.C., R. Ramasesh, J. Hayya, and J.K. Ord, "Multiple Sourcing: The Determination of Lead Times," Operations Research Letters, 10, (1991), 1-7.

Perry, J. "Lead Time Management: Private and Public Sector Practices," Journal of Purchasing and Materials Management, Fall, (1990), 2-7.

Plank, R.E. and V. Kijewski, "The Use of Approved Supplier Lists," International Journal of Purchasing and Materials Management, Spring, (1991), 37-41.

Presutti, W.D., "The Single Source Issue: U.S. and Japanese Sourcing Strategies," Journal of Purchasing and Materials Management, Winter, (1992), 2-9.

Ramasesh, R.V., "Two-Source Split Ordering in Stochastic Lead-Time Inventory Systems," Working Paper, Texas Christian University, (1989).

Ramasesh, R.V., "Recasting the Traditional Inventory Model to Implement Just-In-Time Purchasing," Production and Inventory Management, (1990), 71-75.

Ramasesh, R.V., J.K. Ord, and J.C. Hayya, "Two Source Split Ordering in Stochastic Lead Time Inventory Systems," Working Paper, Texas Christian University, (1991).

Ramasesh, R.V., J.K. Ord, and J.C. Hayya, "Note: Dual Sourcing with Nonidentical Suppliers," Naval Research Logistics, 40, (1993), 279-288.

Ramasesh, R.V., J.K. Ord, J.C. Hayya, and A. Pan, "Sole versus Dual Sourcing in Stochastic Lead-Time (s,Q) Inventory Models," Management Science, 37, (1991), 428-443.

Ramsay, J., "The Myth of the Cooperative Single Source," Journal of Purchasing and Materials Management, Winter, (1990), 2-5.

Rubin, P. and J. Carter, "Joint Optimality in Buyer-Supplier Negotiations," Spring, (1990), 20-26.

Schonberger, R. and J. Gilbert, "Just-In-Time Purchasing: A Challenge for U.S. Industry," California Management Review, 26, (1983), 54-68.

Sculli, D. and Y.W. Shum, "Analysis of Continuous Review Stock Control Model with Multiple Suppliers," Journal of the Operational Research Society, 41, (1990), 873-877.

Sculli, D. and S.Y. Wu, "Stock Control with Two Suppliers and Normal Lead Times," Journal of the Operational Research Society, 32, (1981), 1003-1009.

Sellers, B., "Competition in the Acquisition of Major Weapons", Naval Postgraduate Systems School, Monterey, CA, (1979).

Silver, E., and R. Peterson, Decision Systems for Inventory Management and Production Planning, John Wiley and Sons, (1985).

Smirnov, N., "On the Estimation of the Discrepancy Between Empirical Curves of Distribution for Two Independent Samples. Bull. Math. Univ. Moscow, 2, (1939), 3-16.

Tadikamalla, P., "Application of the Weibull Distribution in Inventory Control," Journal of the Operational Research Society, 29, (1987), 77-84.

Taylor, C., "The Application of the Negative Binomial Distribution to Stock Control Problems," Operational Research Quarterly, 12, (1961), 81-88.

Treleven, M., "Single Sourcing: A Management Tool for the Quality Supplier," Journal of Purchasing and Materials Management, Spring, (1987), 19-24.

Treleven, M. and S. Schweikhart, "A Risk/Benefit Analysis of Sourcing Strategies: Single vs. Multiple Sourcing," Journal of Operations Management, 7, (1988), 93-114.

Watts, C., K. Kim, and C. Hahn, "Linking Purchasing to Corporate Competitive Strategy," Journal of Purchasing and Materials Management, Fall, (1992), 2-8.

Wilks, S., Mathematical Statistics, John Wiley, (1962).

Willis, T.H., C.R. Huston, and F. Pohlkamp, "Evaluation Measures of Just-In-Time Supplier Performance," Production and Inventory Management, (1993), 1-5.

Zhao, L., and H. Lau, "Reducing Inventory Costs and Choosing Suppliers with Order Splitting," Journal of the Operational Research Society, 43, (1992), 1003-1008.

APPENDIX

Notation

D = demand rate

Q = order quantity

n = number of shipments in a contract period

A = purchaser's ordering cost per order

r = annual inventory carrying cost (% dollar value)

C_p = per unit purchasing cost

C_v = per unit production cost

S = vendor's setup cost per setup

Z = purchaser's shipment cost per shipment

TRC_p = total relevant cost for the purchaser

TRC_v = total relevant cost for the vendor

$JTRC$ = joint total relevant cost for the purchaser and vendor

Q_{AVG} = the average inventory per cycle

$F^*(t)$ = cumulative demand up to time t where $0 \leq t \leq T$

$F(u)$ = cumulative demand up to time t where $0 \leq t \leq 1$ and $D=1$

$F_n^*(t)$ = cumulative amount delivered up to time t where $0 \leq t \leq T$

$F_n(u)$ = cumulative amount delivered up to time t where $0 \leq t \leq 1$ and $Q=1$

P_α = the probability of no shortage in the period $[0,T]$

p_α = the prescribed service level

M_n = the amount of safety stock

δ = a fixed fraction of the order that is guaranteed to be delivered

Q_p = the optimal order quantity for the purchaser

Q_v = the optimal order quantity for the vendor

Q_j = the joint optimal order quantity for the vendor and the purchaser

$Q_p\%$ = the relative improvement that a purchaser achieves when the joint order quantity is ordered rather than the vendor's optimal order quantity

$Q_v\%$ = the relative improvement that a vendor achieves when the joint order quantity is ordered rather than the purchaser's optimal order quantity

k = the optimal proportion of the order for vendor A

r = the reorder level

L_A = the lead time for vendor A

L_B = the lead time for vendor B

F_A = the cumulative lead time distribution function for vendor A

F_B = the cumulative lead time distribution function for vendor B

f_A = the lead time probability density function for vendor A

f_B = the lead time probability density function for vendor B

$s = r/D$

$q = Q/D$

$P_1^{(1)}$ = the probability of no stockouts for a single vendor

$P_1^{(2)}$ = the probability of no stockouts before the first delivery for two vendors

$P_1^{(n)}$ = the probability of no stockouts before the first delivery for n vendors

$R_p(n)$ = the relative decrease in stockout probability before the first delivery

$P_2^{(2)}$ = the probability of no stockouts before the second delivery for two vendors

$P_\alpha^{(2)}$ = the probability of no stockouts for dual sourcing

D_A = demand during lead time L_A

D_B = demand during lead time L_B

RPI = relative percentage improvement

SR = stockout risk

$E(L_B)^*$ = the value of the mean lead time for vendor B where dual sourcing is no longer attractive

LT = lead time

OS = order size

OI = order intensity

DPUT = period demand

LTOI = lead time order intensity

μ_L = mean lead time

σ_L = standard deviation of the lead time

σ_D = standard deviation of the demand

σ_A = standard deviation of the lead time for vandor A

σ_B = standard deviation of the lead time for vandor B

μ_{LTD} = mean lead time demand

σ_{LTD} = standard deviation of the lead time demand

RPI_{SR} = relative percentage improvement in stockout risk

RPI_{TS} = relative percentage improvement in total shortage

VITA

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