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# Angle-dependent magnetothermal conductivity in $d$ -wave superconductors

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We analyse the behavior of the thermal conductivity,  $\kappa(H)$ , in the vortex state of a quasi-two-dimensional  $d$ -wave superconductor when both the heat current and the applied magnetic field are in the basal plane. At low temperature the effect of the field is accounted for in a semiclassical approximation, via a Doppler shift in the spectrum of the nodal quasiparticles. In that regime  $\kappa(H)$  exhibits twofold oscillations as a function of the angle between the direction of the field in the plane and the direction of the heat current, in agreement with experiment.

Experiments which show that the superconducting order parameter in the high- $T_c$  cuprates is  $d$ -wave are often sensitive to the surface effects, and there is still interest in the bulk probes of the symmetry of the gap. One piece of evidence for the linear nodes in the bulk comes from the verification of the universal low temperature limit of the in-plane thermal conductivity, [1] another is based on the observed non-linear dependence of the electronic specific heat on the applied magnetic field,  $H$ , in the mixed state.[2]

The latter result is based on the observation that the properties of a  $d$ -wave superconductor in the dilute vortex regime ( $H \ll H_{c2}$ ) are determined by the near-nodal quasiparticles in the bulk.[3] In a semiclassical treatment, the energy of quasiparticle with the momentum  $\mathbf{k}_n$ , where  $n$  labels a node, at point  $\mathbf{r}$  is shifted by  $\delta\omega = \mathbf{v}_s(\mathbf{r}) \cdot \mathbf{k}_n$ , where  $\mathbf{v}_s$  is the velocity field associated with the supercurrents. In the regions of the Fermi surface near the nodes where this shift exceeds the local gap, there exist unpaired quasiparticles. Since the typical supermomentum is  $\hbar/R$  where  $R^2 \approx \Phi_0/\pi H$ , and since the number of vortices  $n_v \propto H$ , the spatially averaged density of states in a pure  $d$ -wave material varies as  $\sqrt{H}$ . If the positions of the impurities and the vortices

are uncorrelated, the argument can be generalized to include the impurity scattering, which depends on the local density of states.[4] This simple approach has worked remarkably well in describing the low-temperature thermal and transport properties of the vortex state with the field perpendicular to the  $\text{CuO}_2$  layers. In particular, the increase of the  $T = 0$  limit of the in-plane thermal conductivity with  $H$  is well described by the semiclassical theory.[5]

In relatively three-dimensional cuprates, such as  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ , the same approach applies when the field is in the basal plane. In that case the magnitude of the Doppler shift depends on the relative orientation of the field with respect to the nodes, and the density of states in the pure limit exhibits fourfold oscillations as a function of the angle between the direction of the field and the crystalline axes.[6] Experimental verification of this prediction has not been possible so far and is hindered by the extrinsic contributions to the specific heat[7] and the orthorhombicity of the material.[8] On the other hand, the angular dependence of the thermal conductivity in the vortex state has already been observed, [9,10] and here we analyse it in the semiclassical framework.

As in Refs.[5,6] we assume a cylindrical Fermi surface and a  $d$ -wave gap, and approximate the superflow by the velocity field around a single vortex. The field and the thermal gradi-

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ent are applied at angle  $\alpha$  and  $\varepsilon$  to the  $\hat{a}$  axis respectively. We consider the regime  $T, \gamma \ll E_H \ll \Delta_0$ , where  $E_H \approx (v_f/2)(\pi\Phi_0\lambda_{ab}/H\lambda_c)^{1/2}$  is the average Doppler shift, and  $\gamma$  is the low-energy scattering rate. Following Refs.[5,6] we obtain that the local change in  $\kappa$  is  $\delta\kappa(\rho)/\kappa_{00} = E(\alpha)\sin^2\beta/\rho^2$ , where  $\kappa_{00}/T = \pi N_0 v_f^2/6\Delta_0$  is the universal thermal conductivity,  $N_0$  is the normal state density of states,  $\Delta_0$  is the gap amplitude,  $v_f$  is the Fermi velocity,  $\beta$  is the winding angle of the vortex,  $\rho$  is the distance from the center of the vortex normalized to  $R$ ,  $E(\alpha) = (\pi E_H^2/8\Gamma\Delta_0)\max(\sin^2\alpha, \cos^2\alpha)$ , and  $\Gamma$  is the bare scattering rate. The local  $\kappa(\mathbf{r})$  has to be spatially averaged to obtain the field dependence. When the heat gradient,  $\nabla T$ , is parallel to the field  $\kappa_{\parallel}(H) = \langle \kappa(\mathbf{r}) \rangle$ , where the brackets denote the average over a unit cell of the vortex lattice. For other relative orientations of  $\nabla T$  and  $\mathbf{H}$  the averaging procedure is not clear; it was argued[5] that  $\kappa_{\perp}(H) = [\langle (1/\kappa(\mathbf{r})) \rangle]^{-1}$  is appropriate for  $\nabla T \perp \mathbf{H}$ . We therefore take here a simple approach of averaging independently the components of the heat current along and normal to the vortex, and expect that this procedure gives at least qualitatively correct results. Then the longitudinal and the Hall thermal conductivity are given by  $\kappa_{\parallel} = \kappa_{\parallel} \cos^2(\alpha - \varepsilon) + \kappa_{\perp} \sin^2(\alpha - \varepsilon)$  and  $\kappa_{\perp} = (1/2)|(\kappa_{\perp} - \kappa_{\parallel}) \sin 2(\alpha - \varepsilon)|$  respectively, with  $\kappa_{\parallel} = \kappa_{00}[1 + E(\alpha) \ln(\Delta_0/E_H)]$  and

$$\frac{\kappa_{\perp}}{\kappa_{00}} = \left[ \sqrt{1 + E(\alpha)} - E(\alpha) \sinh^{-1} \frac{1}{\sqrt{E(\alpha)}} \right]^{-1}. \quad (1)$$

The result for  $\kappa_{\perp}$  is in agreement with the twofold pattern of Ref.[10] at  $T = 0.8\text{K}$  with  $\varepsilon = \pi/2$ , see Fig.1. The minima of  $\kappa_{\perp}(\alpha)$  correspond to increased scattering by the vortices when the heat current is normal to the field.

In the regime where  $T \geq E_H$  Yu *et al.* [9] have measured  $\kappa_{\pm} = (\kappa_{\parallel} \pm \kappa_{\perp})/\sqrt{2}$  with  $\varepsilon = 0$ , found a twofold pattern, and explained it as a consequence of Andreev reflection of quasiparticles in the presence of supercurrents; we note that the angular dependence of  $\kappa_{\pm}$  is very similar to what we obtain at  $E_H \gg T$ . In the same regime Aubin *et al.* [10] observed a *fourfold* symmetry of  $\kappa_{\perp}(\alpha)$ , consistent with the picture in which the scattering of quasiparticle by the vortices becomes more

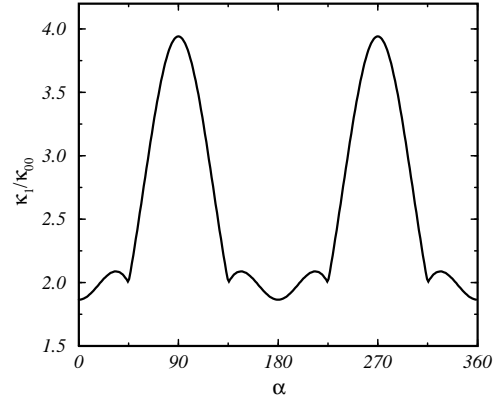


Figure 1. Angular dependence of  $\kappa_{\perp}$  for  $E_H = 0.05\Delta_0$  and  $\Gamma = 0.001\Delta_0$ .

important at high  $T$ [11] and this scattering has the same symmetry as the gap.[9,11] The work to explain in detail the high  $T$  dependence is in progress, and will be reported elsewhere.

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