Louisiana State University

LSU Digital Commons

Faculty Publications

Department of Physics & Astronomy

11-14-2008

Pauli-limited upper critical field in dirty d -wave superconductors

A. B. Vorontsov
Louisiana State University

I. Vekhter
Louisiana State University

M. J. Graf
Los Alamos National Laboratory Theoretical Division

Follow this and additional works at: https://digitalcommons.lsu.edu/physics_astronomy_pubs

Recommended Citation

Vorontsov, A., Vekhter, I., & Graf, M. (2008). Pauli-limited upper critical field in dirty d -wave superconductors. *Physical Review B - Condensed Matter and Materials Physics, 78* (18) https://doi.org/10.1103/PhysRevB.78.180505

This Article is brought to you for free and open access by the Department of Physics & Astronomy at LSU Digital Commons. It has been accepted for inclusion in Faculty Publications by an authorized administrator of LSU Digital Commons. For more information, please contact ir@lsu.edu.

Pauli-limited upper critical field in dirty d-wave superconductors

A. B. Vorontsov,[1,*] I. Vekhter,[1] and M. J. Graf[2]

[1] *Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana, 70803, USA*
[2] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA.*

(Dated: August 16, 2021)

We calculate the Pauli-limited upper critical field and the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) instability for *dirty* d-wave superconductors within the quasiclassical theory using the self-consistent $\hat{t}$-matrix approximation for impurities. We find that the phase diagram depends sensitively on the scattering rate and phase shift of nonmagnetic impurities. The transition into the superconducting state is always second order for weak (Born) scattering, while in the unitarity (strong) scattering limit a first-order transition into both uniform and spatially modulated superconducting states is stabilized. Contrary to general belief, we find that the FFLO phase is robust against disorder and survives impurity scattering equivalent to a T_c suppression of roughly 40%. Our results bear on the search of FFLO states in heavy-fermion and layered organic superconductors.

PACS numbers: 74.25.Ha, 74.25.Dw, 74.81.-g, 74.25.Op

arXiv:0806.2445v1 [cond-mat.supr-con] 15 Jun 2008

Introduction. In type-II singlet superconductors a magnetic field suppresses superconductivity for two reasons: (1) the phase of the Cooper pair wave function couples to the vector potential resulting in the appearance of vortices; (2) Zeeman coupling of the magnetic field to the electron spins polarizes and splits the conduction band, which destroys superconductivity when the loss in magnetic energy equals the energy gain from pair condensation[1, 2, 3, 4]. This latter mechanism is referred to as Pauli limiting and leads to a first or second-order transition from the normal (N) to superconducting (SC) state depending on the value of the magnetic field. It has been predicted that a clean system at high fields can remain superconducting beyond the Pauli limit by forming the nonuniform Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state with a spatially modulated order parameter [5]. This state, however, is suppressed by disorder [6].

In contrast to conventional (isotropic s-wave) superconductors, unconventional (d-wave) superconductors are affected by nonmagnetic impurities even at zero field; scattering averages the gap over the Fermi surface and suppresses T_c. The different rates of suppression of the uniform and FFLO states determine the phase diagram in the field-temperature (B-T) plane. Agterberg and Yang [7] found that in two-dimensional (2D) d-wave superconductors with purely Zeeman coupling, the N-SC transition is of second order at all T, with the Larkin-Ovchinnikov (LO) modulation, $\Delta_{LO} \sim \cos \mathbf{q}\cdot\mathbf{R}$, and the uniform (USC) state, $\Delta_{USC} = const$, favored at low and high temperatures, respectively, and a narrow intermediate T region, where the nodeless Fulde-Ferrell (FF) state, $\Delta_{FF} \sim e^{i\mathbf{q}\cdot\mathbf{R}}$, is stabilized. Ref. [8] reported that under combined orbital and Zeeman coupling in impure d-wave superconductors the first-order transition into the vortex state appears at intermediate temperatures. Very recently, Houzet and Mineev [9] studied orbital and impurity effects in s-wave and d-wave Pauli-limited superconductors and concluded that orbital effects are necessary for a first-order transition to occur in 2D d-wave superconductors. In contrast, for s-wave in 3D the transition to the FFLO state is first-order[10, 11].

Remarkably, our understanding of impurity effects in nonuniform states is still incomplete. In Refs. [7, 8, 9] the discussion was limited to weak (Born) impurity scattering and focused only on the Ginzburg-Landau (GL) regime close to the onset of the FFLO instability, using an expansion in the modulation wave vector $\mathbf{q}$. However, $q = |\mathbf{q}|$ increases rapidly to values comparable to the inverse superconducting coherence length, $q\xi_0 \sim 1$, so this expansion quickly becomes invalid away from the critical point.

In this Letter, we present a microscopic treatment of impurity effects on the superconducting states in purely Pauli-limited quasi-2D d-wave superconductors. Impurities are treated in the self-consistent $\hat{t}$-matrix approximation (SCTA) covering the weak (Born) and strong (unitarity) scattering limits [12]. The latter limit, never considered previously, is especially important because of a search for FFLO-like states in heavy-fermion and layered organic superconductors [13], where impurity scattering is strong [14]. Our approach is not limited to an expansion in q, and hence is valid for any temperature and impurity concentration along the second-order upper critical field B_{c2}. We show that the phase diagram of a Pauli-limited *dirty* d-wave superconductor is very different for nonmagnetic impurities in the Born and unitarity limits. The differences originate from the dependence on scattering strength of quartic and higher order coefficients in the GL functional. The first order N-SC transition, absent for Born scattering, is stabilized by strong impurities, and is therefore expected in heavy fermion systems.

Quasiclassical equations. We follow Refs. [12, 15] and solve the quasiclassical equations for the 4×4-matrix Green's functions in particle-hole and spin space, which satisfy the normalization condition, $\widehat{g}^2 = -\pi^2\widehat{1}$, and the transport equation,

$$[i\varepsilon_m\widehat{\tau}_3 - \mu\mathbf{B}\cdot\hat{\mathbf{S}} - \widehat{\Delta}(\mathbf{R},\hat{\mathbf{p}}) - \widehat{\sigma}^{imp}(\mathbf{R};\varepsilon_m)\,, \qquad (1)$$
$$\widehat{g}(\mathbf{R},\hat{\mathbf{p}};\varepsilon_m)] + i\hbar\mathbf{v}_f(\hat{\mathbf{p}})\cdot\boldsymbol{\nabla}_{\mathbf{R}}\,\widehat{g}(\mathbf{R},\hat{\mathbf{p}};\varepsilon_m) \;=\; 0\,.$$

Here μ is the magnetic moment, $\varepsilon_m = \pi k_B T(2n+1)$ are the Matsubara frequencies, $\widehat{\Delta}$ is the mean-field superconducting order parameter depending on the coordinate, $\mathbf{R}$, and momentum direction, $\hat{\mathbf{p}}$, at the Fermi surface with velocity $\mathbf{v}_f$. The electron spin operator is $\hat{\mathbf{S}} = \boldsymbol{\sigma}\frac{1}{2}(1+\widehat{\tau}_3)+\boldsymbol{\sigma}^*\frac{1}{2}(1-\widehat{\tau}_3)$. The Pauli matrices $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ operate in spin and particle-hole space, respectively. Eq. (1) is complemented by self-consistency equations for $\widehat{\Delta}$ and the impurity self-energy $\widehat{\sigma}^{imp}$. We use $\hbar = k_B = 1$.

In the SCTA $\widehat{\sigma}^{imp} = n_{imp}\widehat{t}$, with impurity concentration n_{imp}. For isotropic scattering the t-matrix satisfies $\widehat{t}(\mathbf{R};\varepsilon_m) = u_0\widehat{1} + u_0\mathcal{N}_f\langle\widehat{g}(\mathbf{R},\hat{\mathbf{p}};\varepsilon)\rangle_{\hat{\mathbf{p}}}\;\widehat{t}(\mathbf{R};\varepsilon_m)$, where angular brackets $\langle\dots\rangle$ denote a normalized Fermi surface average. The strength of the nonmagnetic impurity potential, u_0, is expressed via the isotropic scattering phase shift, $\delta_0 = \arctan(\pi u_0\mathcal{N}_f)$; $\mathcal{N}_f$ is the density of states per spin at the Fermi surface. For Born (unitarity) scattering $\delta_0 = 0$ ($\delta_0 = \pi/2$) and the normal-state scattering rate $\Gamma \equiv 1/2\tau_N = \Gamma_u\sin^2\delta_0$, with $\Gamma_u = n_{imp}/\pi\mathcal{N}_f$.

If we choose the direction of the spin quantization along $\mathbf{B} = B\hat{\mathbf{z}}$ (which is allowed if the hamiltonian has spin-rotation symmetry in the absence of the field), both $\widehat{g}$ and $\widehat{\sigma}^{imp}$ have block-diagonal structure corresponding to the two spin projections. Hence, the quasiclassical equations for the spin-up and spin-down sectors decouple [16], and we solve separately for the diagonal, g_s, and off-diagonal, f_s, f'_s, components of $\widehat{g}$, with $s = \pm1\{\uparrow,\downarrow\}$, with the constraint $g_{s}^2 - f_{s}f'_{s} = -\pi^2$. However, both spin projections enter the self-consistency equation for $\widehat{\Delta}$. We assume a separable pairing interaction $\mathcal{Y}(\hat{\mathbf{p}})\mathcal{Y}(\hat{\mathbf{p}}')$, where $\mathcal{Y}(\hat{\mathbf{p}})$ gives the angular dependence of the gap function with the normalization $\langle\mathcal{Y}^2(\hat{\mathbf{p}})\rangle = 1$. For $\Delta(\mathbf{R},\hat{\mathbf{p}}) = \Delta(\mathbf{R})\mathcal{Y}(\hat{\mathbf{p}})$, we find

$$\Delta(\mathbf{R})\ln\frac{T}{T_{c0}} = T\sum_{\varepsilon_m}\left(\langle\mathcal{F}(\mathbf{R},\hat{\mathbf{p}};\varepsilon_m)\rangle_{\hat{\mathbf{p}}} - \frac{\pi\Delta(\mathbf{R})}{|\varepsilon_m|}\right)\,, \quad (2)$$

$$\widehat{\sigma}^{imp}_{s} = \mathcal{S}_s\begin{pmatrix} \cot\delta_0 + \langle g_{s}\rangle/\pi & \langle f_{s}\rangle/\pi \\ \langle f'_{s}\rangle/\pi & \cot\delta_0 - \langle g_{s}\rangle/\pi\end{pmatrix}\,. \quad (3)$$

Here $\mathcal{F}(\mathbf{R},\hat{\mathbf{p}};\varepsilon_m) = \frac{1}{2}\mathcal{Y}(\hat{\mathbf{p}})[f_\uparrow(\mathbf{R},\hat{\mathbf{p}};\varepsilon_m) + f_\downarrow(\mathbf{R},\hat{\mathbf{p}};\varepsilon_m)]$ and $\mathcal{S}_s = \Gamma/[1 - \pi^{-2}\sin^2\delta_0(\langle g_{s}\rangle^2 - \langle f_{s}\rangle\langle f'_{s}\rangle + \pi^2)]$. To calculate the B-T phase diagram, we derive the Ginzburg-Landau functional (expansion in Δ for arbitrary q) by taking $\Delta(\mathbf{R}) = \sum_{\mathbf{q}}\Delta_{\mathbf{q}}\exp(i\mathbf{q}{\cdot}\mathbf{R})$ and solving Eqs. (1)-(3) together with the normalization condition for $\widehat{g}$ to third order in Δ. We substitute the n-th order solutions $f_{s}^{(1)}, f_{s}^{(3)}$ into Eq. (2) to obtain the GL free energy difference between the SC and N states,

$$\Delta\Omega^{GL} = \sum_{\mathbf{q}}\alpha(T,B;\mathbf{q})|\Delta_{\mathbf{q}}|^2 + \sum_{\mathbf{q}_1\mathbf{q}_2\mathbf{q}_3\mathbf{q}_4}\frac{1}{2}\beta(T,B;\mathbf{q}_1,\mathbf{q}_2;\mathbf{q}_3,\mathbf{q}_4)\;\Delta_{\mathbf{q}_1}\Delta_{\mathbf{q}_2}\Delta^*_{\mathbf{q}_3}\Delta^*_{\mathbf{q}_4}\delta_{\mathbf{q}_1+\mathbf{q}_2,\mathbf{q}_3+\mathbf{q}_4}\,, \quad (4a)$$

$$\alpha(T,B;\mathbf{q}) = \ln\frac{T}{T_{c0}} - 2\pi T\sum_{\varepsilon_m>0}\mathrm{Re}\left(\left\langle\mathcal{Y}\tilde{\mathcal{Y}}_{\mathbf{q}}D^{-1}_{\mathbf{q}}\right\rangle - \varepsilon_m^{-1}\right)\,, \quad (4b)$$

$$\beta(T,B;\mathbf{q}_1,\mathbf{q}_2;\mathbf{q}_3,\mathbf{q}_4) = \pi T\sum_{\varepsilon_m>0}\mathrm{Re}\left\{\left\langle\tilde{\mathcal{Y}}_{\mathbf{q}_1}\tilde{\mathcal{Y}}_{\mathbf{q}_2}\tilde{\mathcal{Y}}_{\mathbf{q}_3}\tilde{\mathcal{Y}}_{\mathbf{q}_4}\frac{D_{(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3+\mathbf{q}_4)/4}}{D_{\mathbf{q}_1}D_{\mathbf{q}_2}D_{\mathbf{q}_3}D_{\mathbf{q}_4}}\right\rangle - \Gamma\Upsilon_{\mathbf{q}_1\mathbf{q}_2\mathbf{q}_3\mathbf{q}_4}\right\}\,, \quad (4c)$$

$$\Upsilon_{\mathbf{q}_1\mathbf{q}_2\mathbf{q}_3\mathbf{q}_4} = \left(\frac{1}{2} - \sin^2\delta_0\right)\left(\Theta^{(2)}_{\mathbf{q}_1\mathbf{q}_3}\Theta^{(2)}_{\mathbf{q}_2\mathbf{q}_4} + \Theta^{(2)}_{\mathbf{q}_1\mathbf{q}_4}\Theta^{(2)}_{\mathbf{q}_2\mathbf{q}_3}\right) - \sin^2\delta_0\left(\Theta^{(1)}_{\mathbf{q}_1}\Theta^{(1)}_{\mathbf{q}_3}\Theta^{(2)}_{\mathbf{q}_2\mathbf{q}_4}\right.$$
$$\left. + \Theta^{(1)}_{\mathbf{q}_1}\Theta^{(1)}_{\mathbf{q}_4}\Theta^{(2)}_{\mathbf{q}_2\mathbf{q}_3} + \Theta^{(1)}_{\mathbf{q}_2}\Theta^{(1)}_{\mathbf{q}_3}\Theta^{(2)}_{\mathbf{q}_1\mathbf{q}_4} + \Theta^{(1)}_{\mathbf{q}_2}\Theta^{(1)}_{\mathbf{q}_4}\Theta^{(2)}_{\mathbf{q}_1\mathbf{q}_3} - 2\Theta^{(1)}_{\mathbf{q}_1}\Theta^{(1)}_{\mathbf{q}_2}\Theta^{(1)}_{\mathbf{q}_3}\Theta^{(1)}_{\mathbf{q}_4}\right)\,, \quad (4d)$$

where we introduced the angular averages $\Theta^{(1)}_{\mathbf{q}_i} = \left\langle\tilde{\mathcal{Y}}_{\mathbf{q}_i}D^{-1}_{\mathbf{q}_i}\right\rangle$, $\Theta^{(2)}_{\mathbf{q}_i\mathbf{q}_j} = \left\langle\tilde{\mathcal{Y}}_{\mathbf{q}_i}\tilde{\mathcal{Y}}_{\mathbf{q}_j}D^{-1}_{\mathbf{q}_i}D^{-1}_{\mathbf{q}_j}\right\rangle$, and defined $\eta_{\mathbf{q}} = \frac{1}{2}\mathbf{v}_f\cdot\mathbf{q}$, and $D_{\mathbf{q}} = \varepsilon_m + \Gamma + i(\mu B + \eta_{\mathbf{q}})$. We introduced $\tilde{\mathcal{Y}}_{\mathbf{q}} = \mathcal{Y} + \mathcal{Y}_{i,\mathbf{q}}$ with $\mathcal{Y}_{i,\mathbf{q}} = \Gamma\Theta^{(1)}_{\mathbf{q}}$.

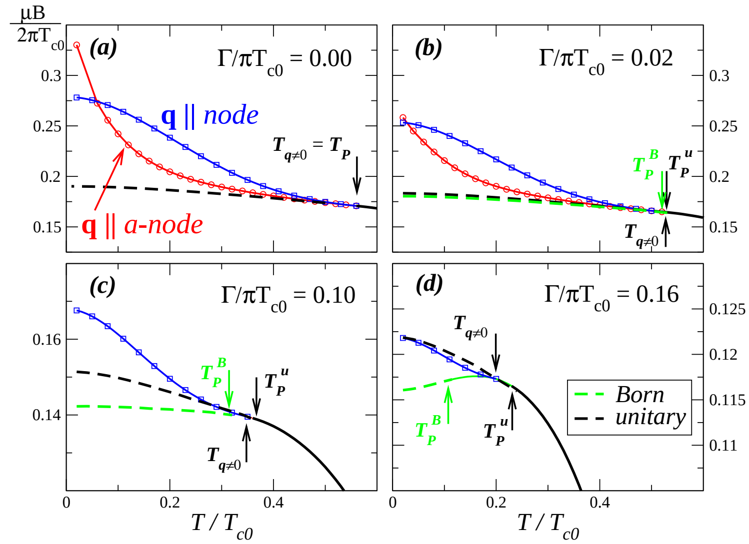
Results. The second-order N-SC transition is determined from the GL coefficient $\alpha(T,B;\mathbf{q}) = 0$ and depends only on Γ, but not on δ_0. Thus the transition line is independent of the phase shift. An instability into the modulated FFLO state becomes possible below $T_{\mathbf{q}\neq0}$, where the maximal B_{c2} is found for $q \neq 0$. This occurs when the GL coefficient κ in the q-expansion of $\alpha(T,B;\mathbf{q}) \approx \alpha_0 + \kappa q^2$ becomes negative,

$$\kappa = 2\pi T\;\mathrm{Re}\sum_{\varepsilon_m>0}\frac{1}{D^3_{\mathbf{0}}}\left(\langle\mathcal{Y}^2\eta^2_{\mathbf{q}}\rangle + \frac{\Gamma\langle\mathcal{Y}\eta_{\mathbf{q}}\rangle^2}{\varepsilon_m + i\mu B}\right)\,. \quad (5)$$

For d-wave SC the last term vanishes, since $\langle\mathcal{Y}\eta_{\mathbf{q}}\rangle = 0$.

In contrast, the quartic term in the GL functional ex-

FIG. 1: (Color online) The critical fields of Pauli-limited *dirty* d-wave superconductors for transitions to USC (first-order, dashed) and FFLO states (second-order, solid) for $\mathbf{q}$ along nodes (squares) and antinodes (circles). Superconductivity sets in at the highest B_{c2}. (a) Pure case: Below $T_P = T_{\mathbf{q}\neq 0}$ the N-USC transition is below the second-order N-FFLO transition. (b-d) Born and unitarity impurities split T_P and $T_{\mathbf{q}\neq 0}$, thus $T_P^B < T_{\mathbf{q}\neq 0} < T_P^u$. A modulation along antinodes quickly suppresses B_{c2} relative to $\mathbf{q}\|node$, panel (b). For Born impurities the second-order FFLO transition is above the N-USC line at all temperatures, as for the clean case. In the unitarity limit the first-order N-USC transition preempts a N-LO transition, see panel (d).



plicitly depends on the scattering phase shift, Eqs. (4c)-(4d). For example, it controls the location of the first-order transition to the USC state, T_P, which competes with the FFLO instability. In unconventional superconductors $\langle \mathcal{Y} \rangle = \mathcal{Y}_{i,\mathbf{0}} = 0$ and the critical point T_P is determined by a sign change of the GL coefficient β at $\mathbf{q} = 0$,

$$\beta_0 = \pi T \,\mathrm{Re} \sum_{\varepsilon_m > 0} \left(\frac{\langle \mathcal{Y}^4 \rangle}{D_{\mathbf{0}}^3} - \frac{\Gamma(1 - 2\sin^2 \delta_0)}{D_{\mathbf{0}}^4} \right) . \qquad (6)$$

For $\Gamma = 0$, both κ and β_0 become negative at exactly the same temperature, $T_{\mathbf{q}\neq 0} = T_P \simeq 0.5615 T_{c0}$. Since the transition into the FFLO state has a higher critical field at any temperature $T < T_P$, the first-order transition is superseded by the onset of the FFLO state [10].

A comparison of κ and β_0 shows that in *dirty* unconventional superconductors $T_{\mathbf{q}\neq 0} = T_P$ only for $\delta_0 = \pi/4$. For Born (B) and unitarity (u) scattering β_0 depends on δ_0, such that T_P^B and T_P^u shift in opposite directions relative to $T_{\mathbf{q}\neq 0}$, hence $T_P^B < T_{\mathbf{q}\neq 0} < T_P^u$ as shown in Fig.1. The latter inequality is especially important since it shows that for strong scatterers Pauli limiting leads to a first-order transition into the USC state at high fields/low temperatures in the B-T phase diagram. As the system becomes dirtier, i.e., the lifetime τ_N decreases, these characteristic temperatures are suppressed to zero in the following order, $T_P^B \to 0$ at $\Gamma/\pi T_{c0} \gtrsim 0.18$, $T_{\mathbf{q}\neq 0} \to 0$ at $\Gamma/\pi T_{c0} \gtrsim 0.20$, and $T_P^u \to 0$ for $\Gamma/\pi T_{c0} \gtrsim 0.22$. Note that for larger Γ the N-USC transition line is of second order at all T.

Fig. 1 gives the upper critical field lines for different states. Second-order transition lines are found by the largest spatial modulation vector $q \equiv Q$ that maximize B_{c2}. In clean d-wave SC [15, 17, 18, 19] the modulation is along a gap maximum (antinode) at low $T/T_{c0} < 0.06$, and along a gap node for $0.06 < T/T_{c0} < 0.56$, see Fig. 1(a). However, already for small impurity scattering, $\Gamma/\pi T_{c0} \gtrsim 0.02$, the critical field for $\mathbf{q}\|antinode$ is lowered below $B_{c2}^{\mathbf{q}\|node}$, and the stable configuration is with $\mathbf{q}\|node$ over the entire range of existence of the FFLO state, see Fig. 1(b).

Determining the first-order transition lines of B_{c2} requires a self-consistent calculation of the full free energy functional, the details of which will be given elsewhere [20]. We find that in the Born limit the first-order transition is always below B_{c2}^{FFLO}, in agreement with [7, 9]. In contrast, in the unitarity limit $T_{\mathbf{q}\neq 0} < T_P^u$ and B_{c2}^{FFLO} is below the first-order transition to the USC state, see Figs. 1(b-d).

For intermediate impurity scattering, the phase diagram is given in Fig. 2. To determine the structure of the SC state near B_{c2}, we analyze the GL free energy, Eq.(4), for four possible phases: USC [$\Delta(\mathbf{R}) = \Delta_{USC}$], FF with a single Fourier component $\mathbf{Q}_1 = (Q, 0)$ [$\Delta(\mathbf{R}) = \Delta_{FF} \exp(iQx)$], LO with $\{\mathbf{Q}_1, \mathbf{Q}_3\} = \{(\pm Q, 0)\}$ [$\Delta(\mathbf{R}) = \Delta_{LO} 2\cos Qx$], and square lattice (SQ) with $\{\mathbf{Q}_1, \mathbf{Q}_3, \mathbf{Q}_2, \mathbf{Q}_4\} = \{(\pm Q, 0), (0, \pm Q)\}$ [$\Delta(\mathbf{R}) = \Delta_{SQ} 2(\cos Qx + \cos Qy)$]. The $x, y-$axes are along the gap nodes. For each phase, we calculate $\Delta\Omega_i^{GL} = -\alpha^2/\beta_i$, with $\beta_{FF} = 2\beta_{1111}$, $\beta_{LO} = \beta_{1111} + 2\beta_{1313}$ and $\beta_{SQ} = 0.5(\beta_{1111} + 2\beta_{1212} + 2\beta_{1313} + 2\beta_{1414} + 2\beta_{1324})$, where $\beta_{ijkl} = \beta(T, B; \mathbf{Q}_i, \mathbf{Q}_j; \mathbf{Q}_k, \mathbf{Q}_l)$. Along the second-order transition line the phase with the lowest positive value of β has the lowest energy.

For Born impurities (Fig. 2 right), $\beta_i > 0$ for all nonuniform states, and the LO state is favored in most of the phase diagram except a small region below $T_{\mathbf{q}\neq 0}$, where the FF phase is stabilized for the impure case [7, 9]. Analysis of $\Delta\Omega^{GL}$ indicates that this phase is separated by a second-order transition from the USC and by first-order from the LO state.

The situation is very different for strong impurities (Fig. 2 left). Following the B_{c2} line from $T_c(B = 0)$ to

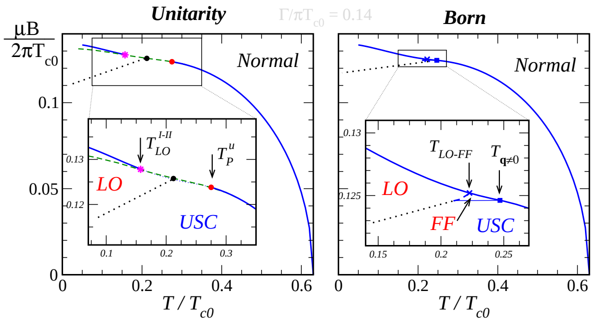


FIG. 2: (Color online) The phase diagram for $\Gamma/\pi T_{c0} = 0.14$. Left panel: transition into the LO state at low T becomes first order above T_{LO}^{I-II} for unitarity impurities. There is also a region of a first-order transition into the USC state below T_P. Right panel: Born impurities result in second-order transitions. The LO state is favored in large parts of the phase diagram over the FF state, except near $T_{\mathbf{q}\neq 0}$ [7, 9]. The transition line shown between the LO and USC states (dotted line) is qualitative.

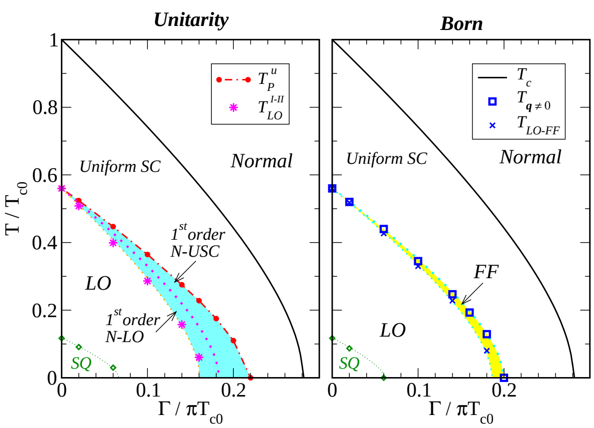


FIG. 3: (Color online) Phase transitions along the B_{c2} line in the Γ-T_c plane. The (black) solid line is $T_c(\Gamma)$ at $B = 0$. For Born impurities the LO state exists for $T < T_{\mathbf{q}\neq 0}$, with a small region occupied by the FF state. For unitarity scattering a first-order transition appears into the USC and LO states. At low T a square lattice FFLO state (SQ) [18] is rapidly suppressed with increasing Γ [22].

lower T, we reach the critical point T_P, below which the N-USC transition is of first order. At $T \to 0$ the transition is second order into the LO state, but becomes first order above T_{LO}^{I-II}. We estimate where the first-order N-USC and N-LO lines meet. However, determining the location of this point and the LO-USC transition line requires a fully self-consistent treatment of the nonuniform problem [15, 21], which is beyond the scope of this work.

Conclusions We summarize our results in Fig. 3, where we show all states that arise along the upper critical field line for fixed Γ. For nonuniform states, we only consider modulations along gap nodes, since states with $\mathbf{q}||antinode$ are destabilized even faster by impurities. We find for *dirty* d-wave superconductors that the FFLO state is quite robust and survives impurity scattering equivalent to $\sim 40\%$ of T_c suppression or a mean-free path ℓ of $\xi_0/\ell \lesssim 0.16$. This result is important for the search of an FFLO state in doped Ce-115 [23], and other heavy-fermion and layered organic superconductors.

Notably, the differences between weak and strong impurity scattering are significant. In the Born limit T_P is suppressed below the onset of the nonuniform state, $T_P^B < T_{\mathbf{q}\neq 0}$, and the transition is always of second order. Impurities stabilize a narrow region of the Fulde-Ferrell state just below $T_{\mathbf{q}\neq 0}$. In contrast in the unitarity limit (relevant to recent experiments) $T_{\mathbf{q}\neq 0} < T_P^u$ and the first-order transition into the uniform state preempts a modulated state. Below $T \sim T_{\mathbf{q}\neq 0}$ the transition into the Larkin-Ovchinnikov state begins as a first-order line and becomes second-order at lower T. Importantly, in this limit the interplay of Zeeman splitting and disorder, even without orbital effects, drives the transition between the normal and superconducting state first order.

We acknowledge support from the Louisiana Board of Regents (A.B.V. and I.V.) and the US Dept. of Energy at LANL, contract no. DE-AC52-06NA25396, (M.J.G.).

* Present address: Dept. of Physics, University of Wisconsin, Madison, WI; email:anton@physics.wisc.edu

[1] B. Chandrasekhar, Appl. Phys. Lett. **1**, 7 (1962).

[2] A. Clogston, Phys. Rev. Lett. **9**, 266 (1962).

[3] G. Sarma, J. Phys. Chem. Solids **24**, 1029 (1963).

[4] K. Maki and T. Tsuneto, Prog. Theor. Phys. **31**, 945 (1964).

[5] P. Fulde and R. Ferrell, Phys. Rev. **135**, A550 (1964); A. I. Larkin and Y. N. Ovchinnikov, Zh. Eskp. Teor. Fiz. **47**, 1136 (1964), [Sov. Phys. JETP **20**, 762 (1965)].

[6] L. G. Aslamazov, Sov. Phys. JETP **28**, 773 (1969).

[7] D. Agterberg and K. Yang, J. Phys.: Cond. Matt. **13**, 9259 (2001).

[8] H. Adachi and R. Ikeda, Phys. Rev. B **68**, 184510 (2003).

[9] M. Houzet and V. P. Mineev, Phys. Rev. B **74**, 144522 (2006).

[10] A. Buzdin and H. Kachkachi, Phys. Lett. A **225**, 341 (1997).

[11] S. Matsuo, et.al., J. Phys. Soc. Japan **67**, 280 (1998); C. Mora and R. Combescot, Phys. Rev. B **71**, 214504 (2005).

[12] L. J. Buchholtz and G. Zwicknagl, Phys. Rev. B **23**, 5788 (1981); S. Schmitt-Rink, K. Miyake, and C. Varma, Phys. Rev. Lett. **57**, 2575 (1986); P. J. Hirschfeld, D. Vollhardt, and P. Wölfle, Sol. State Comm. **59**, 111 (1986).

[13] A. Bianchi et al., Phys. Rev. Lett. **91**, 187004 (2003); C. Capan et al., Phys. Rev. B **70**, 134513 (2004); J. Singleton et al., J. Phys.: Condens. Matter **12**, L641 (2000); Y. Matsuda and H. Shimahara, J. Phys. Soc. Japan **76**, 051005 (2007); A. G. Lebed, *Physics of Organic Superconductors and Conductors*, (Springer, Berlin, 2007).

[14] C. J. Pethick and D. Pines, Phys. Rev. Lett. **57**, 118 (1986).

[15] A. B. Vorontsov, J. A. Sauls, and M. J. Graf, Phys. Rev.

B **72**, 184501 (2005); A. B. Vorontsov and M. J. Graf, ibid. **74**, 172504 (2006); J. Alexander, T. Orlando, D. Rainer, and P. Tedrow, ibid. **31**, 5811 (1985).
[16] U. Klein, Phys. Rev. B **69**, 134518 (2004).
[17] K. Maki and H. Won, Czech. J. Phys. **46**, 1035 (1996).
[18] H. Shimahara and D. Rainer, J. Phys. Soc. Japan **66**, 3591 (1997); H. Shimahara, ibid. **67**, 736 (1998).
[19] K. Yang and S. Sondhi, Phys. Rev. B **57**, 8566 (1998).
[20] A. B. Vorontsov, et al., (unpublished).
[21] H. Burkhardt and D. Rainer, Ann. Phys. **3**, 181 (1994).
[22] Q. Wang, C.-R. Hu, and C.-S. Ting, Phys. Rev. B **75**, 184515 (2007).
[23] L.D. Pham, et.al., Phys. Rev. Lett. **97**, 056404 (2006); E.D. Bauer, et.al., Phys. Rev. B **73**, 245109 (2006); Y. Tokiwa, et.al., arxiv:0804:2454 (unpublished).