The Interference of Non-Meaningful Learning on Subsequent Meaningful Learning.

Dolores Pesek Simoneaux

Louisiana State University and Agricultural & Mechanical College

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The interference of non-meaningful learning on subsequent meaningful learning

Simoneaux, Dolores Pesek, Ph.D.
The Louisiana State University and Agricultural and Mechanical Col., 1992
THE INTERFERENCE OF NON-MEANINGFUL LEARNING ON SUBSEQUENT MEANINGFUL LEARNING

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy

in

The Department of Curriculum and Instruction

by

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B.S., Our Lady of the Lake University, 1966
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December 1992
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ABSTRACT

The purpose of this study is to develop a framework for the notions of meaningful and non-meaningful and to test for learning interference from a non-meaningful then meaningful instructional sequence.

A review of the literature reveals that terminologies differ widely, but similarities in notions emerge. The terms meaningful and non-meaningful are employed here to indicate, respectively, richness in relationships, or a relative absence of relationships both within the knowledge structure, and in relation to previous knowledge.

Based on a Piagetian framework of learning involving assimilation/accommodation and consequently disequilibrium, I hypothesize that non-meaningful learning tends to establish constructs that interfere with subsequent meaningful learning. Two processes are possible: the non-meaningful knowledge structure may need to be discarded and a new structure formed; or the meaningful concepts may be rejected due to noncompliance with prior non-meaningful learning.
meaningful structures. Thus non-meaningful learning may hinder, or even preclude subsequent meaningful learning.

To test this hypothesis a two-treatment research design was framed: Treatment 1 has non-meaningful then meaningful instruction; Treatment 2, meaningful-only instruction. Posttests and a retention test provide evidence of learning. Two studies were conducted according to this design: a generic with eighth graders and a mathematics-specific with fifth graders. An analysis of quantitative and qualitative data was conducted.

In both studies students receiving only meaningful instruction scored significantly better than those receiving meaningful preceded by non-meaningful instruction. Interviews revealed Treatment 1 students were hindered in transferability and creativity in problem solving, and made errors by over-generalizing their learning. The results of this study suggest that behavioral and constructivist methodologies are inherently incompatible, which has implications for the
relations between administrative and professional branches of education.
CHAPTER 1

INTRODUCTION

The rationale for this study and an overview of this dissertation are included in this chapter.

Rationale

Mathematics proficiency in America is not at the level necessary to compete effectively with other industrialized nations, nor sufficient to provide for the personal needs required for quality living. These beliefs are reflected in publications titled *A Nation at Risk: The Imperative for Educational Reform* (National Commission on Excellence in Education, 1983), *The Mathematics Report Card: Are We Measuring Up?* (Dossey, Mullis, Lindquist, & Chambers, 1988), and *Everybody Counts: A Report on the Future of Mathematics Education* (National Research Council, 1989). This present situation forces us to look at the mathematics classroom for some causes as well as solutions.

One response to the present dilemma in mathematics education is to teach by rote. This
approach stresses memorization of disjointed bits of information through drill and practice and seems to offer a quick solution. Skemp (1987) states that on the surface at least, instrumental instruction seems to have some advantages. He discusses that it sometimes is easier to understand. It is easier, for example, to memorize the rule, "to divide by a fraction flip the second fraction and multiply" than it is to understand a meaningful basis for the procedure. Because sometimes memorization is easier than understanding, the rewards of a correct response can be more immediate and more apparent with this non-meaningful approach.

Even though memorizing a mathematical fact or procedure seems expedient, research suggests that when the objective is retention, transfer, and problem solving, meaningful instruction is far more advantageous than rote instruction (Brownell, 1947; Bruner, 1960; Ausubel, 1963; Skemp, 1987).

Meaningful learning in mathematics has been advocated by the National Council of Teachers of Mathematics (NCTM) for many years. Using manipulatives as one means for developing an
understanding of mathematical concepts has long been recognized. Over forty-five years ago NCTM supported this philosophy in their yearbook, Multi-Sensory Aids in the Teaching of Mathematics (NCTM, 1945). Since that publication many other yearbooks, articles, research studies, and the recent Curriculum and Evaluation Standards for Mathematics Education (NCTM, 1989) have advocated the same.

Yet when conducting a survey of kindergarten through fifth grade teachers in a large urban school district Scott (1983) found that few teachers reported using any manipulative materials more than five times a year. Even in first grade, fewer than 60% of the teachers claimed any use of manipulatives in their classrooms. Suydam (1986) reported that although most teachers, when asked, indicated a belief in the value of manipulatives, few put them to use during instruction.

The most typical mathematics instructional unit has the following structure: homework is checked; the teacher demonstrates the steps to some process and works some examples; then the students practice the process individually with pencil and paper. The
student’s role frequently is to memorize formulas, definitions, or algorithms through practicing many problems.

Even though Dewey’s (1910) philosophy of education and the related Gestalt psychology (Latner, 1973) gave a strong endorsement to meaningful instruction in many areas of education during the twentieth century, mathematics instruction has remained organized with a drill orientation of behaviorism (Suydam, 1987). Behaviorism has more completely dominated American psychology than that of other countries. Many theorists (e.g., Thorndike, 1922; Gagné, 1985) even singled out the discipline of mathematics for their treatment of stimulus-response. The behaviorist results-oriented pedagogy has remained the leading instructional mode in the American classroom, sacrificing long term benefit for short-term expediency.

American mathematics teachers as well as students’ parents have been, and still are, entrenched in a behaviorist philosophy in mathematics. Teachers’ elementary, secondary, and
gallery of college mathematics instruction, their pre-service training, and most mathematics textbooks provided to them in their classrooms emphasize memorization of discrete ideas and procedures. Parents, administrators, and school systems as well as other governmental agencies evaluate classroom effectiveness on behaviorist based assessments. Whether the judgement is on teacher performance, e.g., the Louisiana Teaching Internship and Teacher Evaluation Projects (Chauvin, Ellet, Loup, & Slan, 1989), or student achievement as indicated on standardized tests, the assessment is of rigidly defined behavioral objectives.

Schooling is a public, political institution. As long as education is measured by rigid behaviorist instruments, it will be difficult to truly change the emphasis of instruction. "Tests...are one way of communicating what is important for students to know" (NCTM, 1989, p. 189). Kulm (1990) addresses this issue.

It might be argued that the behavioral objectives craze and the focus on learning hierarchies were largely responsible for widening the gap between school mathematics and "real" mathematics. Anything that could not be
stated and measured behaviorally gradually disappeared from the curriculum. (p. 1)

There is general agreement that meaningful learning and critical thinking should be the primary outcomes of education. But the administrative branch of education still sees test scores as the major indicator of learning, and it is prepared to let test scores be the principal influence on the structure of the education system. Associations of educational administrators (American Association of School Administrators, National Association of Elementary School Principals, National Association of Secondary School Principals) and school boards (National Association of State Boards of Education and National School Boards Association) support the call for meaningful teaching as stated and described in the Curriculum and Evaluation Standards for Mathematics Education (NCTM, 1989), but at the same time they call for higher test scores.

This study seeks to influence the debate over educational practices by demonstrating the non-viability of a curriculum serving two masters. If a clear empirical demonstration can be offered that rote methods of instruction interfere with
subsequent meaningful learning, then the case for a reassessment of current administrative policies can begin to be formulated. At the same time, this study intends to contribute to our theoretical understanding of learning and its development.

Organization of the Dissertation

The review of the literature, Chapter 2, investigates the notions of meaningful and non-meaningful and explores research findings on the development of interference resulting from non-meaningful learning on subsequent meaningful learning.

A quasi-experimental design to test for the interference of non-meaningful learning on subsequent meaningful learning is described in Chapter 3. This design utilizes two treatments: Treatment 1 has non-meaningful instruction followed by meaningful instruction; Treatment 2 only has meaningful instruction. Two studies, a generic and a mathematics-specific, investigate this phenomenon. Quantitative as well as qualitative data were collected for analysis.
Reports of data gathered in this study are given in Chapter 4. The quantitative analysis comparing the means of the two treatment groups on post- and retention tests are reported. The analysis of qualitative data gathered through 12 student interviews describing treatment effects in greater depth also are reported in this chapter.

Conclusions from the quantitative and qualitative analysis are given in Chapter 5. Limitations are discussed and implications for pedagogy are made. Some questions have been answered in this study, but many others arose. Some of these questions are addressed in Chapter 5.
CHAPTER 2
LITERATURE REVIEW

This chapter reviews the literature in two areas: the notions of meaningful and non-meaningful; and the development of an interference resulting from non-meaningful learning prior to meaningful instruction.

Notions of Meaningful and Non-Meaningful

Much has been made in the literature of the notions of meaningful and non-meaningful; however, there is a lack of consensus as to what these notions refer. Terms such as understanding and skills (McLellan & Dewey, 1895; Bruner, 1960), meaningful and skills (Thorndike, 1922; Brownell, 1935; Gagné, 1985), meaningful and rote (Ausubel, 1963), relational and instrumental (Skemp, 1987), semantic and syntactic (Resnick, 1982), meaningful and mechanical (Baroody and Ginsburg, 1986), teleological and schematic (VanLehn, 1986), conceptual and procedural (Hiebert, 1986; Greeno, Riley & Gelman, 1984), and principles and skills
(Gelman & Meck, 1986) are some of the variations that have been used in mathematics education. Some of these distinctions (understanding and skills, and meaningful and skills) refer to pedagogical approaches to mathematics. Other pairs (meaningful and rote; relational and instrumental; semantic and syntactic; meaningful and mechanical; and teleological and schematic) make reference to types of mathematical understanding. Still other distinctions (principles and skills, and conceptual and procedural) relate to mathematical content.

**Pedagogical Approaches**

Pedagogy is the focus of most earlier writings on meaningful and non-meaningful notions. McLellan and Dewey (1895), Thorndike (1922), Brownell (1935), Bruner (1960), and Gagné (1985) present arguments defending the emphasis they feel should receive greater attention during instruction. Dewey, Brownell, and Bruner make forceful arguments for increased emphasis on understanding; explaining that knowledge of skills, including procedures and recall of facts, should be an outgrowth of an understanding of the concepts underlying the skills. Taking an
opposing view, Thorndike and Gagné advocate memorization of procedures and facts as being the heart of mathematics instruction. They hold that mathematics understanding is not required for many mathematics activities, and that through repeated practice of skills, necessary understanding evolves. Both groups agree that a complete mathematics program includes some meaningful as well as some non-meaningful understanding. One group believes skills knowledge is an outgrowth of meaningful understanding; whereas, the other group holds that meaning is an outgrowth of repeated practice of skills. Regardless of the focus of instruction recommended, skill knowledge for these authors refers to disconnected actions performed in mathematics, and understanding implies some knowledge of relationships.

Mathematical Understanding

More recently, cognitive psychologists and mathematics educators have shifted the emphasis from prescriptive instruction to descriptive understanding. Their focus is on describing the
inter-relatedness between meaningful and non-meaningful learning as well as their differences.

Ausubel (1963) uses meaningful and rote learning in reference primarily to a kind of learning process and secondarily to a learning outcome. He describes meaningful learning as a knowledge structure that relates substantively to existing cognitive structures and rote learning as a knowledge structure internalized "verbatim, as a discrete and isolated end in itself" (p. 22). Recognizing that learning cannot occur in a cognitive vacuum, he explains that rote learning relates to cognitive structures only in an arbitrary, non-substantive fashion. This arbitrary connection does not allow "the incorporation of derivative, elaborative, correlative, supportive qualifying or representational relationships" (p. 22) found in meaningful learning.

Skemp (1987) uses the terms relational and instrumental to express the polarity. With relational understanding the learner knows both what to do and why. Instrumental understanding (a
concept which Skemp hesitates to call understanding) he described as "rules without reasons" (p. 153).

Baroody and Ginsburg (1986) separate mathematics understanding using the terms semantic and mechanical. Semantic knowledge includes the concepts and principles of mathematics, while, mechanical refers to knowledge of facts and procedures.

Both Resnick (1982) and VanLehn (1986) use meaningful and non-meaningful notions when addressing the understanding of procedural action in mathematics. Resnick employs the terms semantic knowledge and syntactic knowledge to describe children's errors with the subtraction algorithm. She found that one can learn to correctly apply the subtraction algorithm without connecting the steps to their semantic bases; however, when procedural knowledge is not linked to the semantics of the procedure, systematic errors occur.

In discussing students' use of computer programming procedures, VanLehn (1986) uses the terms schematic and teleological to describe students' learning. Schematic refers to an
understanding of the program itself; teleological describes an understanding of the program as well as an understanding of the relationship of a program and its parts to its intended purpose. With teleological knowledge one has an understanding of more relationships than with schematic knowledge, thus enabling the subject the added ability to debug, extend, adapt, and optimize the program.

As stated above, Ausubel (1963), Skemp (1987), Baroody and Ginsburg (1986), Resnick (1982), and VanLehn (1986) use differing terminology in their descriptions of mathematics understanding. Though their terms and interpretations differ, the notions of meaningful and non-meaningful carry a common strand of relatedness and lack of relatedness, respectively.

**Mathematical Content**

Meaningful and non-meaningful notions are used by still other researchers to describe the content of mathematics instruction. Hiebert and Lefevre (1986), Greeno, Riley, and Gelman (1984), and Gelman and Meck (1986) all relate the notions of meaningful and non-meaningful to mathematics content.
Different terms are utilized and different meanings are applied, but again some common theme exists. Hiebert (1986) edited ten papers into a monograph entitled _Conceptual and Procedural Knowledge: The Case of Mathematics_. In the book Hiebert and Lefevre define conceptual knowledge as "knowledge that is rich in relationships" (p. 3), and state further, that, by definition, "a unit of conceptual knowledge cannot be an isolated piece of information" (p. 4). They define procedural knowledge to include formal language as well as algorithms in mathematics. They use the terms meaningful and rote to describe methods of pedagogy. Conceptual knowledge, they explain, cannot be taught non-meaningfully because it, by definition, involves relationships. Procedural knowledge, by contrast, can be taught and learned either meaningfully or by rote (non-meaningfully).

**Summary**

Whether the notions of meaningful and non-meaningful in mathematics education refer to an emphasis in pedagogy (Dewey, Thorndike, Brownell, Bruner, Gagné), a description of mathematics
understanding (Ausubel, Skemp, Baroody, Ginsburg, Resnick, VanLehn), or a distinction of mathematics content (Hiebert, Lefevre, Greeno, Riley, Gelman, Meck), a common definition surfaces. Despite a variety of terminology employed, there exists across studies a common theme that learning can be rich in relationships or relatively unconnected.

**Definition of Terms**

Consonant with many of the writers cited above, I define meaningful and non-meaningful as opposite extremes in relationship density. Meaningful learning results in knowledge structures that are rich in relationships; non-meaningful learning results in structures with few relationships. Meaningful instruction intends to assist students in relating a mathematical task to many other notions: previous learning, other operations, life situations, models, other mathematical concepts, etc. Non-meaningful instruction emphasizes memorization of terms, rules, procedures, and formulas as isolated pieces of knowledge separate from any existing relationships. Meaningful instruction does not guarantee meaningful
understanding just as non-meaningful instruction does not necessarily result in only non-meaningful learning.

Meaningful learning in this study includes both internal and external referential relationships. Internal relationships include vocabulary and symbol meanings, the reasons behind the steps of a procedure, and the relationship of the task at hand to other mathematical constructs. External relationships include references to other disciplines, real life situations, and models. The more numerous the relationships developed (both internal and external) the more meaningful is the understanding.

In contrast, non-meaningful learning at its extreme involves no referential relationships. A concept is learned or considered as an isolated fact. Theoretically, if no relationships are developed, recall would be impossible, therefore, learning would not occur (Ausubel, 1963). In practice, therefore, non-meaningful learning must include a lesser degree of relationships or else relationships to idiosyncratic non-extendable
features of the domain. Some instances of non-meaningful knowledge promoted in mathematics classrooms are captured in phrases like "with like signs you add, unlike signs you subtract," and "flip the second fraction and multiply." Another example of non-meaningful instruction occurs when the teacher has students memorize "D, M, S, BD" to assist in remembering the order of actions in the procedure for long division. Non-meaningful instruction stresses the "how to" in a procedure instead of the "why."

Pedagogical Issues

The advantages of one form of learning over the other form have been discussed by many writers.

Advantages to Meaningful Learning

Brownell (1947), Ausubel (1963), and Skemp (1987) have contributed much to the literature on the role of meaningful learning. Besides noting that a meaningful approach to instruction makes teaching more interesting for the teacher, Brownell (1947) lists ten values of meaningful learning for the student.

2. Equips him with the means to rehabilitate quickly skills that are temporarily weak.
3. Increases the likelihood that arithmetical ideas and skills will be used.
4. Contributes to ease of learning by providing a sound foundation and transferable understandings.
5. Reduces the amount of repetitive practice necessary to complete learning.
6. Safeguards him from answers that are mathematically absurd.
8. Provides him with a versatility of attack which enables him to substitute equally effective procedures for procedures normally used but not available at the time.
9. Makes him relatively independent so that he faces new quantitative situations with confidence.
10. Presents the subject in a way which makes it worthy of respect. (pp. 263-64)

Information gathered from teachers is the basis of Brownell’s list of advantages to meaningful understanding.

Ausubel (1963) presents evidence that learning and retention are enhanced when instantiated meaningfully. Ease in learning, speed in learning, transferability, and retention are advantages he lists. He states that research is available to show that materials presented meaningfully are much easier to learn, they are learned more rapidly, and
that the materials are more transferable than are those learned by rote.

Relational (meaningful) learning requires fewer principles of more general application than does instrumental (non-meaningful) learning explains Skemp (1987). Instrumental learning requires a multiplicity of rules. Skemp lists four advantages of relational learning: a) it is more adaptable to new tasks, b) it is easier to remember - there's more to learn but once learned, it is easier to recall because it is a part of a whole, c) it can be effective as a goal because it has built in motivation, and d) it is organic in quality in that it is "an agent of (its) own growth" (p. 5).

Advantages to Non-Meaningful Learning

There are advantages to non-meaningful learning as well. Non-meaningful learning, according to the definition used in the present study, includes constructs being considered apart from their referential relationships. Formal language is one such construct, and its emphasis without attention to referential meanings can provide some advantages. Hiebert and Lefevre (1986) propose that formal
language provides a powerful tool for dealing with complex ideas; cognitive effort is reduced by focusing on the symbols. "Viewed as cognitive aids, symbols help to organize and operate on conceptual [meaningful] knowledge. But that is not all. The symbol system can also produce conceptual knowledge" (p. 15).

Byers and Erlwanger (1984) divide mathematics into two aspects: content (meaningful) and form (non-meaningful). They draw attention to the fact that much content of mathematics would not have been developed had it not been for formal advances. Struik (1967) reminds us that the introduction of the Hindu-Arabic numerals and Leibniz's integral notation (both formal notations that can be considered apart from their meaningful contexts) often bring advances in related concepts in mathematics.

**Interference**

As well as an interest in clarifying the vital notions of meaningful and non-meaningful learning, this study also addresses the sequencing of the
instructional mode in educational practice. The above references suggest that both meaningful and non-meaningful understanding are vital to a complete mathematics program. The issue addressed here is how the sequence of learning affects the understanding achieved.

Theoretical Framework

Piaget's theory of learning has as its essence the active interplay of the subject with its environment. This interaction involves a constant assimilation of the environment with simultaneous and necessary accommodation of the subject to that environment. Assimilation and accommodation occur also at a deeper level. The information assimilated directly from the environment (exogenous knowledge) is further assimilated into larger internal frameworks (endogenous knowledge).

Piaget (1975/1977) describes exogenous and endogenous knowledge and their interdependence: "Exogenous knowledge originates in the observables, i.e., based on experience with external objects, or with material aspects and results of the actions of
the subject" (p. 804). Endogenous knowledge, on the other hand,
is derived from the internal coordination of the actions or operations on the subject....The distinctive character of the endogenous knowledge is thus its necessity, as opposed to the simple matters of fact that exogenous knowledge records....All exogenous knowledge presupposes an endogenous framework, since it implies an "assimilation"....Now assimilation requires assimilative instruments....These instruments apply endogenous frameworks or "forms," even if their "contents" are exogenous. (p. 804)

Piaget (1967/1977) explains further that assimilations by the subject of his environment, as well as the assimilating of endogenous knowledge into larger structures, is determined not only by the environment but also by the subject's endogenous knowledge. Endogenous knowledge or logico-mathematical knowledge, a term frequently used by Piaget in his later works, is itself influenced by heredity, and past experiences, as well as by all anticipated experiences by the subject, as these anticipations affect one's receptivity.

With every assimilation there is present some accommodation. On the physiological level, the eye has to accommodate its shape to the distance from the object assimilated. On the intellectual level,
experiences as well as mental reflections are assimilated. Whether they are assimilated into existing logico-mathematical structures, or form new structures, accommodations are occurring. Existing structures accommodate to the assimilated knowledge by self-modification.

In the process of assimilation/accommodation a disequilibrium and re-equilibration occurs. Some equilibration is sought with every interaction of the organism to its environment. Even though equilibration/disequilibration, form a continuous sequence, there is present a stabilizing of the organism through the structuring of constructs.

In discussing the equilibrium sought by every organism through assimilation and accommodation, Piaget (1967/1977) states:

Logico-mathematical structures involve a sui generis equilibrium situation with regard to the relationship between assimilation and accommodation. On one hand, they appear to be a continuous construction of new assimilation schemata: assimilation of a previous structure into a new one, which integrates it, and assimilation of the experimental datum into structures thus set up. But, on the other hand, logico-mathematical structures give evidence of a permanent accommodation, insofar as they are modified neither by the newly constructed structures (except, of course, by being improved thereby) nor by the experimental
data whose assimilation they make possible. (p. 849)

With each developmental stage the tendency toward equilibrium creates a disequilibration and a re-equilibration; however, with each assimilation and accommodation some disequilibrium occurs. This dynamic of learning is presented by Piaget (1967/1977) as follows:

We suggest that the equilibrium between assimilation and accommodation which is brought about by logico-mathematical structures constitutes a state - mobile and dynamic and, at the same time, stable - aspired to unsuccessfully by the succession of forms, at least where behavior forms are concerned, throughout the course of the evolution of organized creatures. Whereas this evolution is characterized by an uninterrupted succession of disequilibria and of re-equilibrations, logico-mathematical structures do, in fact, attain permanent equilibrium despite the constantly renewed constructions which characterize their own evolution. (p. 849)

Based on this Piagetian framework, every experience in the learning environment affects schemata in a subject. When the learning is non-meaningful, frameworks still are formed. If few or no relationships are learned, new, separate schemata are formed with few linkages to possibly related schemata. The learning is not as rich.
When material is given to a student to memorize or when a "trick" remembering instrument is learned as in much non-meaningful instruction, relationships are assimilated to non-substantive schemata. Upon subsequent meaningful learning, the subject must accommodate previous structures to fit the present task's referential relationships. The subject must unlearn and relearn, thus creating unnecessary disequilibria or interference to the meaningful learning.

Sequence and Interference

Psychology Literature

In the area of psychology, several studies (Mathews, Russ, Stanley, Blanchard-Fields, Cho, & Druhan, 1989; Reber, Kassin, Lewis, & Cantor, 1980) have been conducted exploring implicit and explicit learning, and the affects of varying the sequence of these two modes of learning. Upon initial examination, the notions and the resulting findings seem related to the objectives of this study; however, some major differences exist.

Non-meaningful learning as defined in this study is generally a result of conscious non-
meaningful instruction or it is a conscious consideration of a concept isolated from its' relations. Implicit learning according to Reber (1989) "is an unconscious process" (p. 219).

The concepts of implicit and explicit seem to relate more to inductive and deductive approaches to learning than to the notions of non-meaningful and meaningful as defined in this study. Implicit learning as discussed by Reber seems to be more closely related to a meaningful approach in education than to non-meaningful learning. For instance, implicit mathematics learning seems to offer some explanation for Ginsburg's (1977) description of a preschooler's very powerful informal mathematical knowledge. An explicit mode of learning as discussed in psychology (Reber, 1989) seems to relate to a deductive meaningful approach in mathematics education. For instance, a meaningful deductive approach is utilized if, after having developed some definition of triangle, students are directed in identifying the triangles in a set of geometric shapes.
Area of Mathematics

No studies were found in the area of mathematics education that were designed for the purpose of detecting interference created by a sequence of non-meaningful and meaningful learning. Several authors, however, have explored sequence effects and as a result of their findings have suggested that an interference had been created.

In discussing conceptual (meaningful) and procedural (non-meaningful) learning, Davis (1986) addresses the importance of conceptual knowledge preceding procedural knowledge. He states that schools seem to deny that "one needs to have a reality to describe before one can try to describe it....Children need a reality to write about, before they can do much writing. This is true whether they are writing (in English) about 'What I did last summer' or (in some appropriate symbolism) about some actions in combining two collections of Dienes' MAB blocks" (p. 266). For Davis, effective schooling in all areas requires that meaningful experience precedes formal (non-meaningful) actions.

Whitman (1976) researched the relationship between formal (non-meaningful) and informal
(meaningful) techniques to equation solving with seventh-grade students. Each of three groups was instructed in one of the following techniques:

a) intuitive techniques only (In solving \( x + 3 = 15 \) the student was guided to ask, "What plus 3 equals 15?");
b) formal techniques only (In solving \( x + 3 = 15 \) the student was instructed to subtract 3 from each side.); or
c) intuitive techniques followed by formal techniques. Whitman found that the best performers were from technique 1, intuitive only, the next best was technique 3, intuitive then formal, and the weakest were those getting only formal instruction. The intuitive approach was considered meaningful, as defined in this study, because it related to students' prior understanding of equality. The formal technique simply instructed students on a procedure that had no meaning to the students. But the Whitman study did not investigate the effects of non-meaningful instruction preceding meaningful.

Kieran (1984) also explored equation solving. She conducted an experiment with seventh-grade
students who had not yet begun to study algebra. The students were separated into two groups: those who had been taught transposing as a way to solve arithmetic sentences (e.g., when asked the value for the variable in $5 + a = 12$, they responded $12 - 5 = 7$.) and those having no prior skills in transposing. Both groups were administered an instructional unit on giving meaning to variable and equality. They were encouraged to use trial and error as a means to balance the equation. By the end of the experiment only the group with no previous transposing skills was able to apply the procedure of performing the same operation on both sides of an equation. The transposing group resisted accepting a new understanding for equation solving and made errors by over-extending their transpositions. Kieran suggested an interference had been created by the prior learning of transposition (non-meaningful learning), a skill technique on an equation without an understanding of equality. Thus Kieran’s study is parallel in intent to the present one.
Hiebert and Wearne (1988) suggested that cognitive interference explains the study that they conducted. In testing their theory of developing competence with the written symbols of decimal fractions, Hiebert and Wearne observed that students having received syntactic (non-meaningful) instruction prior to a semantic (meaningful) presentation on working with decimals scored significantly lower than students with no prior instruction. Students choosing semantic approaches to problem solving scored significantly better than students using non-semantic means. Furthermore, a greater number of students having had no previous syntactic instruction used semantic analysis for direct measure solutions than did students with previous syntactic instruction. Hiebert and Wearne suggest, "Prior instruction that encouraged the routinization of syntactic rules seemed to interfere with, and prevented the adoption of, semantic analyses of the affected tasks" (p. 380).

Mack (1990) drew similar conclusions on interference from her research. Nancy Mack conducted six weeks of individual instruction to
eighth-grade students on fraction concepts and symbolism. The initial instruction was to build on the students' informal knowledge of fractions so no connection to fraction symbols and procedures was made. Mack found that students' knowledge of rote procedures frequently interfered with the students' attempts to build on their informal knowledge. The students who had previously acquired rote procedural knowledge tended to focus on symbolic manipulations and didn't seem to recognize correct or incorrect responses. The influence of the rote procedural knowledge could be overcome, she found, but it required a great deal of time and effort.

Summary

All four of the above studies compared sequence of instructional approaches in mathematics. Whitman (1976) compared meaningful, non-meaningful, and meaningful followed by non-meaningful learning. He did not include the sequence of non-meaningful with subsequent meaningful learning.

The studies of Kieran (1984), Hiebert and Wearne (1988), and Mack (1990) included a sequence of non-meaningful then meaningful learning and an
interference was detected. None of these studies were designed to investigate interference, but the presence of an interference was suggested.

The studies of Kieran (1984), Hiebert and Wearne (1988), and Mack (1990) took a pre-existing situation of non-meaningful learning and exploited it to look for interference effects on subsequent meaningful learning. So there is a lack of experimental control of one of the conditions – the non-meaningful instruction. Thus the reliable production of interference effects have not been demonstrated in a fully controlled study. This study seeks to extend previous work by testing the reliability of the interference effect. Furthermore, choosing generic content can demonstrate the ubiquity of the phenomenon more conclusively.
CHAPTER 3

METHODOLOGY

It is a truism of educational theory that the time spent on task positively correlates with the learning that is accomplished (Suydam, 1987). This study questions the validity of that principle by considering the possible interference of meaningful learning with prior non-meaningful learning.

Non-meaningful learning is defined in this study as acquisition of knowledge structures that are isolated from related structures (see Chapter 2). As a method of instruction, memorization of procedures or definitions frequently results in non-meaningful learning (Skemp, 1987).

In contrast, meaningful instruction is defined as one that is rich in relationships. The emphasis here is not on isolated facts but rather on how the particular concept or procedure connects with many other related constructs. Instruction that emphasizes understanding rather than memorization assists students in integrating a particular concept
into the larger picture of related knowledge (Skemp, 1987).

This study explores the counterintuitive hypothesis that in education more may actually yield less; that non-meaningful instruction may interfere with subsequent meaningful learning. Particularly, it is proposed that preceding meaningful instruction with non-meaningful instruction will result in the achievement of less learning than meaningful instruction alone.

Design

The basic strategy of this study is straightforward. In a two-treatment design, one treatment group was first assisted in a non-meaningful memorization task and later received meaningful instruction related to the same memorization content. The other group received only the meaningful unit of instruction. Support for the hypothesis would be indicated if students who received both non-meaningful and meaningful instruction (N-M group) scored significantly lower
on post- or retention tests than students receiving meaningful-only instruction (M-0 group).

Along with treatment effects, gender, and mathematics achievement level also were used as independent variables. Mathematics achievement was measured by the student’s most recent California Achievement Test score in mathematics. Students who scored below 50% were assigned to Level 1; above, to Level 2. This 2 X 2 X 2 full factorial design provided for the analysis of main effects and interactions of treatment, gender, and achievement.

Two experimental studies were conducted using this basic design. The first study was generic in content; the second involved specifically mathematical content. To obtain full treatment effects it was important to find an instructional topic to which students had no prior non-meaningful exposure, thus a study with a generic content was designed. Also, a generic study makes claim to a more general learning phenomenon. The second study, featuring specific mathematical content, illustrates the applicability of the general claim to a specific school subject.
Table 1

Sequence of the Design Components for Generic and Mathematics-Specific Studies

<table>
<thead>
<tr>
<th>Sequence</th>
<th>N-M</th>
<th>M-O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-meaningful instruction</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Meaningful instruction</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Posttest</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Retention test</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Note. N-M = non-meaningful then meaningful instruction; M-O = meaningful-only instruction.

Table 1 lists the general sequence used for these two studies. The N-M group received non-meaningful instruction followed by meaningful instruction, whereas the M-O group received only meaningful instruction. The researcher administered the treatments to all groups. Both groups took post- and retention tests. Further discussion of the design is included in the description of each study.
Generic Study

The generic study utilized an artificial schema developed by Richard Skemp (1962) which includes 16 characters for which there is a slight, but not obvious, relation between visual structure and referential meaning. For example, \( \rightarrow \) translates to "knowledge" and \( \rightarrow\) symbolizes "moves" (see Appendix A). When the individual characters are grouped into a cluster, they take on a meaning that is suggested by the meanings of the component symbols. Thus \( \rightarrow\) represents "message" (knowledge that moves). In this study, the physical arrangement of symbols in a cluster does not influence the meaning of the cluster. Thus \( \rightarrow\) and \( \rightarrow\) are the same cluster.

Subjects

Four intact eighth grade mathematics classes at St. Amant Middle School, located in a small-town/rural community in Ascension Parish, Louisiana, were used for this study because of convenience to the researcher. The grouping used at the school was a homogeneous grouping of the top mathematics students and a deliberate heterogeneous grouping of the rest of the
students. All four classes used for this study were those of heterogeneous groupings. Thus, the highest achieving mathematics students were not a part of this study.

In assigning classes to treatment groups two factors were considered. Two of the classes met before lunch and two after; classes were selected so as to control for a possible time-of-day effect. California Achievement Test score means also were calculated for each class, and assignment to treatment groups attempted to minimize group mean differences. A profile of the groups is provided in Table 2.

As the gender mean (1.5) indicates (Table 2), both groups were about 50% male and 50% female. The CAT and the level means indicate a slightly higher mathematics achievement for the M-O group of students than the N-M group.
Table 2
Descriptive Statistics on Generic Study Groups

<table>
<thead>
<tr>
<th>Descriptors</th>
<th>N-M</th>
<th>M-O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender mean</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>CAT mean (math)</td>
<td>51.95</td>
<td>55.04</td>
</tr>
<tr>
<td>Level mean</td>
<td>1.5</td>
<td>1.6</td>
</tr>
<tr>
<td>Number of subjects</td>
<td>22</td>
<td>28</td>
</tr>
</tbody>
</table>

Note. N-M = non-meaningful then meaningful instruction; M-O = meaningful-only instruction.

*Male = 1 Female = 2

bLevel 1 < 50% Level 2 > 50% on the CAT

Design of the Generic Study

As described earlier in this chapter, this generic study had two treatment groups: N-M and M-O. The generic study followed the sequence presented in Table 1.
On the first day, only the N-M group received instruction. These students were given six clusters of symbols with definitions to memorize (see Appendix B).

The purpose of this instruction was to have the students learn the content of this task non-meaningfully. Therefore, the role of the instructor was to assist the students in memorizing the six clusters of symbols and their interpretations with minimum relationships made to the assigned meanings of the individual symbols.

At the start of a 50-minute instructional period students of this group were given a handout that listed the symbol clusters to be memorized (see Appendix B). Materials were made available for flash cards. The directives given by the teacher were for the students to work alone (the assumption was that small group interactions would assist in revealing the grammar), to do anything that they judged would help them memorize the content effectively (write the symbols many times, make flash cards, say the words aloud, etc.), and study the content at home. For the purpose of motivation the students were promised a reward if they learned the
material by the following day. The next day all of these students were given a token reward.

On the second day both groups (in their intact classes) were given the meaningful treatment for 20 minutes, and then a posttest was administered (see Appendix C). This meaningful instruction used only a lecture format supported by overhead projection. (See Appendix D for all content written on the overhead.)

The meaningfulness of the task came from the sequence utilized during the presentation of the grammar - development of relationships was the focus. One symbol with its definition was presented on the overhead projector; then a second symbol with its definition. As soon as two symbols were learned, a cluster were formed; and with each new symbol presented additional clusters were created by combining symbols in various relationships to form new definitions. At the termination of this instructional unit all 16 symbols of the grammar with their definitions were listed on the overhead along with cluster symbols. The material was offered slowly with some time for reflection between item presentations.
Students were not allowed to take notes. (The purpose of this directive was to eliminate the possibility of having notes available for sharing with other students or using in later evaluations.) The students were simply to listen, watch, and reflect.

The students received no feedback on the posttest results. Two weeks later they were given the same instrument to test for retention.

In summary, the N-M group had a 50 minute instructional period and time at home for memorization followed by 20 minutes of meaningful instruction on the study’s content the following day. The M-O group only had the 20 minutes of meaningful instruction. Statistical methods described below were used to compare various aspects of post- and retention test performance for the two groups.

**Description of the Instruments**

The post- and retention tests were identical (see Appendix C). This instrument consisted of 25 free response items subdivided into three sets: memorization, instruction, and transfer. Six items (# 14, 16, 18, 20, 21, and 22) had been given for memorization as non-meaningful tasks to the N-M group
Six items (#4, 6, 9, 13, 19, and 20) were familiar because they had been presented during the meaningful instruction phase to all students (the instruction subtest). (Number 20 was part of both the memorization and instruction subtests.) The other 14 items (#1, 2, 3, 5, 7, 8, 10, 11, 12, 15, 17, 23, 24, and 25) had never been seen before by the students, and required a transfer of understanding (the transfer subtest).

Of the 25 items on the test, the first 19 items had the symbols provided and the students were to give an interpretation. The last six items had a definition given and the students were to provide the appropriate symbols. Each subtest included some items of each of these forms: the memorization subtest had three of each; the instruction subtest had five with symbols given and two with definitions given; the transfer subtest had 11 with symbols and 2 with definitions.

Scoring

For scoring the instrument, the maximum value for items differed. The first 19 items were each worth 2, 5 or 10 points depending on whether the response was a simple definition of an individual symbol, some correct
interpretation of the cluster meaning or a completely correct response, respectively. For example, for the item \( \mathcal{P} \), a response of "makes knowledge" was scored at 2 points because it was a literal translation of the cluster. A response of "teacher" was worth 5 points since that response required some transference of meaning but was not completely correct because the symbol \( \bar{I} \) which represents "person" was not included in the cluster. A response of "teach" earned 10 points.

The last six items, in which students were required to produce symbols, were scored by a different set of criteria. Two points were given for each correct symbol and negative two for each incorrect symbol employed by the student. Zero was the minimum score that could be earned on these items; however, the maximum score varied according to the number of symbols that comprised the cluster. These last six items had an average maximum value of 10 points. (See Appendix E for the answer key and more examples of partial value responses.)

To protect against researcher bias, the scores assigned for items were discussed and agreed upon by
the researcher and two outsiders. All tests were scored by the researcher and at least two additional people.

Reliability

A coefficient for Cronbach's Alpha was determined for each of the three subtests of the generic post- and retention tests. The reliability coefficient on the memorization items was 0.677, the instruction items 0.663, and the transfer items 0.819.

Analysis

Students' performance was analyzed using a 2 X 2 X 2 (treatment-by-level-by-gender) analysis of variance on the means of the post- and retention tests. A report of this analysis is given in Chapter 4.

Content-Specific Study

The mathematical content chosen for this study was area and perimeter of squares, rectangles, triangles, and parallelograms. For this mathematics-specific study, it was necessary to identify a topic that was accessible to the students, but for which they had not previously received non-meaningful instruction. Finding area and perimeter measures with the use of
formulas (the non-meaningful content for this study) is initiated into the curriculum at the very end of the fifth-grade year according to the mathematics curriculum guide of Ascension parish. The *Curriculum and Evaluation Standards for Mathematics Education* (NCTM, 1989) suggest that students are capable of understanding the concept of area and perimeter and some informal method for obtaining these measures long before the fifth-grade level. Thus, finding the area and perimeter of geometric shapes with fifth-grade students seemed appropriate content for this study.

**Subjects**

Six intact regularly scheduled fifth grade mathematics classes at St. Amant Middle School in Ascension Parish were used for this study. All classes are grouped heterogeneously for mathematics instruction. Three of the classes selected met before lunch and three after lunch. There were two regular fifth grade mathematics teachers in the school. To control for the teacher variable, three classes from each teacher were selected for this study.

To control for the class variable, each class was separated into two treatment groups using random
stratification by gender and level as the criteria. A student's level was determined by the individual's California Achievement Test score in mathematics: those below 50% were assigned to Level 1; those above, to Level 2. A profile of each treatment group is provided in Table 3.

Table 3
Descriptive Statistics on Mathematics Study Groups

<table>
<thead>
<tr>
<th>Descriptors</th>
<th>N-M</th>
<th>M-O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender mean</td>
<td>1.55</td>
<td>1.46</td>
</tr>
<tr>
<td>CAT mean (math)</td>
<td>51.0</td>
<td>56.9</td>
</tr>
<tr>
<td>Level mean</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Number subjects</td>
<td>59</td>
<td>57</td>
</tr>
</tbody>
</table>

Note. N-M = non-meaningful then meaningful instruction; M-O = meaningful-only instruction.

*aMale = 1     Female = 2
*bLevel 1 < 50%   Level 2 = 50% and above
The gender means in Table 3 indicate there were a few more females in the N-M group than in the M-O group. The table also shows that the number of students per level in the two treatments was the same (though, when individual standardized scores were averaged, the M-O group had a slightly higher mean).

Twelve students, six from each of two classes, were selected for interviews. From each of the two classes three males and three females, three Level 1 and three Level 2 students were randomly selected.

Design of the Content-Specific Study

A somewhat more elaborate design was used in this study than in the generic content study. As in the generic design, two treatments were used. In addition to the generic content, this design included a pretest, a posttest immediately following the non-meaningful instruction (posttest1), and three interview sessions. The pretest was used as a covariate to control for previous exposure to the concepts of area and perimeter. Posttest1 was administered to establish the effectiveness of the non-meaningful instruction. Interviews were conducted to gain further insight into possible causes and characteristics of any interference
created. Table 4 shows the sequence followed for this study.

Table 4

Sequence of Components of the Mathematics Study

<table>
<thead>
<tr>
<th>Treatment groups</th>
<th>N-M</th>
<th>M-O</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sequence</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Interview I</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Non-meaningful instruction</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Posttest₁</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Interview II</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Meaningful instruction</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Posttest₂</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Interview III</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Retention test</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

**Note.** N-M = non-meaningful then meaningful instruction; M-O = meaningful-only instruction.
All instruction and testing was conducted during the students' regular mathematics class. The regular mathematics teacher was responsible for the M-O students during the time when the non-meaningful treatment was administered to the N-M students in the class. Remedial and enrichment work totally unrelated to the content of this study were utilized by the regular teacher during this time.

All instruction periods and interviews were audi-taped. To detect bias and to add validity, a middle school mathematics teacher from another school observed one instructional unit each day, as well as some of the interviews (see Appendix F for observer notes). The final interviews were videotaped, transcribed, and analyzed.

Day-By-Day Procedures

Day 1

The pretest was administered to all students (see Appendix G).

Day 2

Twelve students, six from each treatment, were interviewed. (See Appendix H for basic questions used for this interview.) The purpose of this interview was
to gather data on the student's attitude toward school, learning, and mathematics. These interviews were audio-taped.

**Days 3-7**

The N-M group received five days of non-meaningful instruction on using formulas. The purpose of this treatment was to have students memorize and use formulas non-meaningfully (i.e., without connections to previous understandings of area and perimeter or to the appropriateness of a particular formula for obtaining a specific measure). The formulas employed were for finding perimeter and area of squares \( P = 4s, A = s^2 \), rectangles \( P = 2[l + w], A = lw \), triangles \( P = a + b + c, A = \frac{1}{2}bh \), and parallelograms \( P = 2[b + w], A = bh \). The role of the instructor was to help the students memorize and apply the formulas. The students were not assisted in seeing relationships between the mathematical concepts being considered or between the concepts and real-life situations. The lesson plans were designed with this purpose in mind (see Appendix I).

At the beginning of the first day of instruction, all the formulas were written by the students on their
note pads with no explanations of meanings offered. Dissimilar formulas were presented on the same day. Perimeter and area formulas for the same shape were not presented together. The reasons for using a particular formula for a particular shape were not discussed. No connections to real-life applications were included.

Much effort was put into the design of the lesson plans to assist memorization. Pads were provided and students were encouraged to take notes. They worked in cooperative groups. Formulas were written many times and reviewed each day.

A typical day's lesson started with a review of formulas previously learned. Then, a shape was drawn, the dimensions were labeled with appropriate variables, and the formula for finding the desired measure was written on the overhead. The students were asked to write the new formula ten times. The instructor then explained the indicated operations in the formulas and connected the variable(s) in the formula to the variable(s) of the shape. Values for the variables were assigned and the teacher modeled the application of the formulas for finding the desired measure. The students worked three problems with the instructor and
then five additional problems in cooperative groups. The period terminated with a quick review of the formulas presented that day (see Appendix I for complete lesson plans).

Day 8

Posttest, (see Appendix J) was administered to the N-M group to assess the effectiveness of this treatment. The problems on this test replicated the simple straightforward format used during instruction; a shape was drawn, only necessary measures were given, and a space for the student's response was provided. (The use of a formula was not a requirement for credit.)

Day 9

A second interview with the original 12 students was conducted and audio-taped (see Appendix K for interview II questions). The purpose of this set of interviews was to gain insight into the student's understanding of the concepts of area and perimeter.

Days 10-12

The two treatment groups within each class were combined for these three days of meaningful instruction. The purpose of this treatment was to
assist students in constructing concepts of area and perimeter rich in relationships. The role of the teacher was to assist students in interrelating visual images, language, physical manipulations, and ideas of area and perimeter of squares, rectangles, triangles, and parallelograms. The lesson plans were designed with this role in mind (see Appendix L).

The construction of many different relationships were encouraged during this task. To assist students in forming connections, the area and perimeter for each shape were always presented together. Questioning techniques also were used for this purpose. For example, students were asked to construct a specific square on their geoboards and then a diagonal. They were asked questions such as, "What was our original shape? What two shapes do we now have? What is the area of the square? Does anyone know the area of each triangle? How many times larger is the square than the triangle?" Students were asked to discuss and share relationships observed.

The sequence throughout these three days of instruction was from concrete to abstract. For example for the initial activity, the students, grouped
cooperatively, were directed to find the area and perimeter of their table top using their hands as units of measure. One of the last activities involved students in drawing grids in the interior of shapes. The sequence of materials used was student hands, square tiles, geoboards, grid paper, and finally unlined paper. Actions were taken to encourage students to abstract from or extend beyond the manipulations experienced, but no pencil or paper ever was used for calculations during this meaningful task. For example, when using the geoboard, students were asked to make the largest possible square (4 X 4) and find its area and perimeter. Students shared their answers and the process they used to derive their solutions. If everyone used counting squares as a method for finding their solutions, the instructor asked, "Is there any way we can find the area without counting each square?" And to extend, the instructor asked, "What if the geoboard had one more row and column, what would the area then be?"

During this meaningful instruction shapes were related to each other. For example, a square was discussed as being a special kind of rectangle,
triangles were formed by halving rectangles, and the transformation of parallelograms into rectangles (by slicing off and then re-adding a right triangle) was experienced.

Day 13

All students were given posttest$_2$ (see Appendix M). The students were asked not to erase any markings required to obtain their test responses.

Day 14

The 12 students previously interviewed were interviewed for a third time (see Appendix N for Interview III questions). The purpose of this session was to probe more deeply into students' understanding, to discover any patterns in concept development, to detect the presence of possible interference and to further develop a holistic image of the mathematics student.

Day 15

Two weeks after the posttest$_2$, all students were tested for retention (See Appendix M. Posttest$_2$ and the retention test were the same instrument.)

In summary, this design was structured so that one group of students, the N-M group, received non-
meaningful instruction followed by meaningful instruction in mathematics, while a second group of students, the M-O group, received only meaningful instruction. Statistical methods described below were used to compare various aspects of post- and retention test performance for the two groups.

Description of the Instruments

Quantitative Instruments

The pretest, posttest, and the retention test were basically identical (Appendices G and M). This instrument had 37 open-ended items; however, three items (32, 35, and 37) on the pretest were deleted for analysis because they were poorly constructed (Items 32 and 35, Appendix G, had assigned measures that could not discriminate concepts of area and perimeter. Item 37 appeared only on some student's instrument and not on others.) This test was designed to evaluate a student's ability to calculate the area and perimeter of squares, rectangles, triangles, parallelograms, and irregular shapes in many different formats. (See Appendix 0 for a description of the instrument design.)

Posttest, was designed to evaluate the students' ability to calculate the area and perimeter of squares,
rectangles, triangles, and parallelograms (see Appendix J). This test had only eight items, one for each objective. The items were presented in the same format used during the non-meaningful instruction; a drawing of the shape was provided with only necessary measures given. The students were not required to use formulas to get full credit for an item.

For scoring the instruments in this study each correct response had a value of one point. The maximum score was 37. Because the items were open-ended, some decision on criteria had to be established. Only numerical values were considered; the unit of measure, when given by the student, was not evaluated. The determining factor for some of the responses was whether the student displayed an understanding of area or perimeter, as the case required. For example for item 17, students were asked to draw a triangle having a perimeter of 10 feet. If a student drew a rectangle with a 10 foot perimeter it was accepted because an understanding of perimeter seemed evident; the incorrectness of the shape, being considered a slip.
To protect against researcher bias, all tests were scored by the researcher and at least two additional people in education.

Cronbach's Alpha was used to calculate a reliability coefficient on the pretest, posttest$_1$, posttest$_2$, and the retention test. These were 0.699, 0.754, 0.873, and 0.840, respectively.

A treatment-by-level-by-gender-by-class analysis of variance was conducted on the quantitative results of this study with the pretest used as a covariate. The three dependent variables (treatment, level, and gender) were analyzed for main effects and interactions; and the one nuisance variable (class) was tested only for main effect.

It was hypothesized that if non-meaningful learning interfered with meaningful learning, than students who learned most from the non-meaningful instruction would have a greater interference effect than would students who learned fewer non-meaningful skills. Thus an interaction effect was expected.

An alternative hypothesis is that, aside from learning the meaningful treatment better, also the higher achievers will be more able to overcome the
interference effect caused by the non-meaningful treatment. In this case this group of students would tend to score even more highly above the scores of the weaker students than would be anticipated by the general effect of their academic facility. Thus two contrary interaction effects are postulated for treatment by achievement level.

To test this hypothesis a correlation analysis using the pretest, California Achievement Test (CAT), and posttest₁ scores as independent variables and the posttest₂ and retention test scores as dependent variables against level (levels 1 and 2 based on CAT scores as assigned for other analysis) was conducted. Only students in the N-M group were included in this analysis as they were the only students receiving non-meaningful instruction. A complete report of all analyses is given in Chapter 4.

Qualitative Instruments

Interviews were designed to gain insight into student's understanding of the content of this study. Further understanding of possible interference created by initial non-meaningful instruction also was sought. The first set of interview questions conducted prior to
any treatments was designed to obtain data on factors influencing students' attitudes towards schooling, learning, and mathematics (see Appendix H). The second set followed the non-meaningful task, and was designed to check for student's understanding of area and perimeter (see Appendix K). The third set, conducted after the meaningful task, attempted to probe student's concepts of area and perimeter and to detect and gain understanding of any confusion with the concepts (see Appendix N).

The third set of interviews was quite extensive. Students were asked which instructional method they preferred, about definitions of terms, and about application of the concepts. This application included drawing of figures and calculating their areas and perimeters, finding relationships between formulas and their purposes, and obtaining explanations for posttest responses. This set of interviews was audio-taped, video-taped, transcribed, and analyzed.

A coding system was designed for analysis on the transcriptions of the third set of interviews. Each child's interview was separated and bound with the name, treatment, gender, and achievement level.
concealed. The interview questions were divided into eight areas of inquiry, and each area assigned a color code. Student responses were then color matched to specific questions.

Each area of inquiry was analyzed separately. Responses by each student on a specific topic were read several times and summarized into a list of data disclosed during the dialogue. After all eight areas were analyzed in this fashion, students were separated by treatment and the summaries were studied for emerging patterns.
The purpose of this study was to investigate whether non-meaningful learning interferes with subsequent meaningful learning. To test for this hypothesis two treatment groups were formed: N-M and M-O. The N-M group received non-meaningful instruction followed by meaningful instruction, whereas the M-O group received meaningful-only instruction. For analysis this study used a 2 x 2 x 2 treatment-by-achievement level-by-gender design; there were two levels (N-M and M-O) of the treatment factor, two levels (1 and 2) of the achievement factor, and two levels (M and F) of the gender factor. The study was conducted on a generic as well as a mathematics-specific content.

Treatment effect was the main variable of interest in the study but effects of and interactions with achievement level and gender also were investigated. The N-M treatment included non-meaningful instruction followed by meaningful instruction, whereas the M-O treatment had
meaningful-only instruction. Achievement level was determined by each student's most recent standardized mathematics achievement score as measured by the California Achievement Test (below 50% assigned to Level 1; those above, Level 2). Retention effects also were analyzed.

Questions addressed in this research were the following:

1) Does non-meaningful learning followed by meaningful learning result in poorer performance than meaningful learning alone?

If there is an interference of non-meaningful learning in subsequent meaningful learning,

2) Is the interference retained?

3) Are students of different mathematics achievement levels affected differently?

4) Do males and females experience the same effect?

5) Are students who learn the most non-meaningful skills interfered with the most on subsequent meaningful learning?
Each of these questions was addressed in the generic-content study and the mathematics-specific study.

Generic Study

The two-treatment design for this study used a generic grammar for its instructional content. The first treatment (N-M group) was given 6 clusters of symbols to memorize (non-meaningful task) and then a 20 minute explanation of the generic grammar (meaningful task). The second treatment (M-O group) only received the meaningful instruction. A three-factor (treatment X achievement X gender) analysis of variance was conducted on both the post- and retention tests. An F-test was run on the data collected from a posttest immediately following the meaningful instruction (posttest) and a retention test administered two weeks later.

The post- and retention tests for this study, which were identical, consisted of three parts: memory; instruction; and transfer - each having a different purpose. The memory item subtest included the six clusters given for memorization to the N-M
group during their non-meaningful task (see items 14, 16, 18, 20, 21, and 22 in Appendix C). The instruction item subtest included those items on the test presented during the meaningful task to both groups. (See items 4, 6, 9, 13, 19, and 20 in Appendix C. Item 20 was a part of both the memorization and instruction subtest.) The transfer item subtest was of major interest because it was designed to measure students' ability to transfer recently learned concepts to new problem situations (see remaining items in Appendix C). No students had exposure to these transfer items prior to the administration of the test.

Posttest Results

Memory Item Subtest

![Figure 1. Memory Item Subtest Results](image-url)
The Figure 1 above displays the results of an analysis of treatment, achievement level, and gender means and their interactions on the posttest memory item subtest. Two different patterns of effects resulted on the memory item test. The high achieving males and low achieving females scored similarly under the two treatments. However, students receiving the memorization task in the low achieving male and high achieving female groups learned the memory items better than those students not participating in the memorization task.

The results of an analysis of variance on the posttest memory item subtest are given in Table 5. As indicated in Table 5 and illustrated in Graph 1, there was a significant effect of treatment; however, there was also a significant three-way interaction. The items in this subtest were the items given to the N-M group for memorization during non-meaningful instruction. The males of the lower achievement level and the females of the higher achievement level from the N-M group learned the items significantly better than did other students.
Table 5

ANOVA on Posttest Memory Item Subtest in Generic Study

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>F Value</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment (T)</td>
<td>1</td>
<td>1066.43</td>
<td>5.63</td>
<td>.0224</td>
</tr>
<tr>
<td>Level (L)</td>
<td>1</td>
<td>495.96</td>
<td>2.62</td>
<td>.1132</td>
</tr>
<tr>
<td>T X L</td>
<td>1</td>
<td>34.54</td>
<td>0.18</td>
<td>.6715</td>
</tr>
<tr>
<td>Gender (G)</td>
<td>1</td>
<td>106.84</td>
<td>0.56</td>
<td>.4568</td>
</tr>
<tr>
<td>T X G</td>
<td>1</td>
<td>0.54</td>
<td>0.00</td>
<td>.9577</td>
</tr>
<tr>
<td>L X G</td>
<td>1</td>
<td>165.28</td>
<td>0.87</td>
<td>.3555</td>
</tr>
<tr>
<td>T X L X G</td>
<td>1</td>
<td>858.56</td>
<td>4.54</td>
<td>.0392</td>
</tr>
<tr>
<td>Within</td>
<td>41</td>
<td>7761.71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>48</td>
<td>10340.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Critical value: p < .05.

Instruction Item Subtest

In order to assess whether items presented during instruction were actually learned, analysis was conducted on the instruction subtest. Table 6 illustrates group means; however, differences were not statistically significant (Table 7).
Table 6

Descriptive Statistics for Treatment on Posttest Instruction Item Subtest in Generic Study

<table>
<thead>
<tr>
<th>Factor</th>
<th>Level</th>
<th>% Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>N-M (n=21)</td>
<td>59.36</td>
<td>3.66</td>
</tr>
<tr>
<td></td>
<td>M-O (n=28)</td>
<td>65.04</td>
<td>3.21</td>
</tr>
<tr>
<td>Level</td>
<td>1 (n=20)</td>
<td>57.93</td>
<td>3.74</td>
</tr>
<tr>
<td></td>
<td>2 (n=29)</td>
<td>66.47</td>
<td>3.12</td>
</tr>
<tr>
<td>Gender</td>
<td>Male (n=26)</td>
<td>63.43</td>
<td>3.56</td>
</tr>
<tr>
<td></td>
<td>Female (n=26)</td>
<td>60.99</td>
<td>3.32</td>
</tr>
</tbody>
</table>
Table 7
ANOVA on Posttest Instruction Item Subtest in Generic Study

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>F Value</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment (T)</td>
<td>1</td>
<td>183.28</td>
<td>0.67</td>
<td>0.4182</td>
</tr>
<tr>
<td>Level (L)</td>
<td>1</td>
<td>413.44</td>
<td>1.51</td>
<td>0.2264</td>
</tr>
<tr>
<td>T X L</td>
<td>1</td>
<td>109.84</td>
<td>0.40</td>
<td>0.5302</td>
</tr>
<tr>
<td>Gender</td>
<td>1</td>
<td>33.63</td>
<td>0.12</td>
<td>0.7279</td>
</tr>
<tr>
<td>T X G</td>
<td>1</td>
<td>2.59</td>
<td>0.01</td>
<td>0.9230</td>
</tr>
<tr>
<td>L X G</td>
<td>1</td>
<td>16.43</td>
<td>0.06</td>
<td>0.8078</td>
</tr>
<tr>
<td>T X L X G</td>
<td>1</td>
<td>4.53</td>
<td>0.02</td>
<td>0.8983</td>
</tr>
<tr>
<td>Within</td>
<td>41</td>
<td>11235.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>48</td>
<td>12171.84</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Critical value: p < .05.

Transfer Item Subtest

The main focus of this study was to compare the effects of non-meaningful then meaningful instruction (N-M group) with meaningful-only instruction (M-O group) on solving new problems. To assess these effects, a three-way ANOVA was
conducted on the transfer subtest with treatment, achievement, and gender as independent variables. As indicated in Tables 8 and 9 students receiving only the meaningful instruction performed significantly better than students receiving both instructions. No other main effects or interactions were significant; though the achievement level effect, and the level X gender interaction were nearly significant.
Table 8

Descriptive Statistics for Treatment on Posttest Transfer Item Subtest in Generic Study

<table>
<thead>
<tr>
<th>Factor</th>
<th>Level</th>
<th>% Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N-M (n=21)</td>
<td></td>
<td>27.82</td>
<td>5.62</td>
</tr>
<tr>
<td>M-O (n=28)</td>
<td></td>
<td>45.24</td>
<td>4.93</td>
</tr>
<tr>
<td>Level</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (n=20)</td>
<td></td>
<td>31.07</td>
<td>5.74</td>
</tr>
<tr>
<td>2 (n=29)</td>
<td></td>
<td>41.97</td>
<td>4.79</td>
</tr>
<tr>
<td>Gender</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Males (n=23)</td>
<td></td>
<td>37.73</td>
<td>5.47</td>
</tr>
<tr>
<td>Females (n=26)</td>
<td></td>
<td>35.32</td>
<td>5.10</td>
</tr>
</tbody>
</table>
Table 9

ANOVA on Posttest Transfer Item Subtest in Generic Study

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>F Value</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment (T)</td>
<td>1</td>
<td>6479.08</td>
<td>10.03</td>
<td>.0029</td>
</tr>
<tr>
<td>Level (L)</td>
<td>1</td>
<td>2536.95</td>
<td>3.93</td>
<td>.0542</td>
</tr>
<tr>
<td>T X L</td>
<td>1</td>
<td>561.58</td>
<td>0.87</td>
<td>.3565</td>
</tr>
<tr>
<td>Gender</td>
<td>1</td>
<td>123.81</td>
<td>0.19</td>
<td>.6638</td>
</tr>
<tr>
<td>T X G</td>
<td>1</td>
<td>474.10</td>
<td>0.73</td>
<td>.3966</td>
</tr>
<tr>
<td>L X G</td>
<td>1</td>
<td>2070.10</td>
<td>3.21</td>
<td>.0808</td>
</tr>
<tr>
<td>T X L X G</td>
<td>1</td>
<td>721.92</td>
<td>1.12</td>
<td>.2966</td>
</tr>
<tr>
<td>Within</td>
<td>41</td>
<td>26479.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>48</td>
<td>41195.92</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Critical value: p < .05.

Generic Retention Test Results

The immediate treatment effects found on posttest were supported by the results on the retention test. Descriptive data are given in Table 10.
Table 10

Descriptive Statistics for Treatment on Retention Test for Memory, Instruction, and Transfer Item Subtests in Generic Study

<table>
<thead>
<tr>
<th>Subtest</th>
<th>Treatment</th>
<th>% Means</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memory</td>
<td>N-M (n=21)</td>
<td>38.89</td>
<td>2.61</td>
</tr>
<tr>
<td></td>
<td>M-O (n=28)</td>
<td>21.70</td>
<td>2.29</td>
</tr>
<tr>
<td>Instruction</td>
<td>N-M (n=21)</td>
<td>34.11</td>
<td>4.13</td>
</tr>
<tr>
<td></td>
<td>M-O (n=28)</td>
<td>42.70</td>
<td>3.63</td>
</tr>
<tr>
<td>Transfer</td>
<td>N-M (n=21)</td>
<td>15.43</td>
<td>5.07</td>
</tr>
<tr>
<td></td>
<td>M-O (n=28)</td>
<td>26.38</td>
<td>4.45</td>
</tr>
</tbody>
</table>

On the memory item subtest there was a significant treatment effect, $F(1,41) = 10.02$, $p < .01$, with the N-M group scoring higher than the M-O group. On the transfer item subtest there was a significant treatment effect, $F(1,41) = 4.87$, $p < .05$, with the M-O group scoring higher than the N-M group. The N-M group memorized and retained the
non-meaningful task; whereas, the M-0 group retained
the ability to transfer concepts better than did the
N-M group. There were no level or gender
interaction effects on the retention test for the
memory or transfer item subtests. As with the
posttest, no effects were present on the retention
test for the instruction subtest.

Mathematics-Specific Study

Quantitative Analysis

The purpose of this study was to investigate
whether the findings with generic content, thought
to require the kind of thinking involved in a
mathematics classroom, also are present in a
mathematics-specific context. Calculations of
perimeter and area of various geometric figures were
used as the mathematics content.

For analysis, class was included as a nuisance
factor. Thus, this study had a 2 X 2 X 2 X 6 design
with two levels (N-M and M-0) of treatment, two
levels (1 and 2) of achievement, two levels (M and
F) of gender, and six levels of class. Only the
three dependent variables (treatment, level, and
gender) were analyzed for main effects and interactions, and the nuisance variable (class) was tested only for main effect.

Four tests were used for this study: pretest; posttest₁; posttest₂; and a retention test. The pretest was used as a covariate to control for previous knowledge of perimeter and area.

The focus of the non-meaningful task was to memorize and apply formulas to obtain desired perimeter and area measures. The meaningful task utilized no formulas. Instead, the students were guided through a sequence of manipulative, pictorial, and abstract representations to develop meaningful constructs of area and perimeter and to create their own methods of deriving the desired measures.

The generic and mathematics-specific studies differed as to the nature of the non-meaningful tasks; consequently, the subtests differed. For the generic study, the N-M students were asked simply to memorize and recall 6 clusters of symbols; the memory item subtest evaluated this task. In the mathematics-specific study the N-M students were
assisted in memorizing and applying formulas to simple problems. The memory item subtest for this mathematics study did not test for recall of formulas. Instead, it evaluated whether students could solve problems presented in the same format utilized during the non-meaningful instruction (see posttest₁ in Appendix J).

Posttest₁, designed to evaluate the effectiveness of the non-meaningful instruction, consisted of 8 items, one for each formula taught during the task. Eight similar items, serving as the memory item subtest, were included as part of the pretest, posttest₂, and the retention test. This memory item subtest was used for further analysis as described in the following section.

Additional analysis was also conducted on the N-M group scores to investigate the hypothesis that students who acquire more non-meaningful learning are interfered with more than students who learn fewer non-meaningful skills. Student’s posttest₁ results (non-meaningful learning indicators) were correlated against posttest₂ and retention test results (interference indicators). Because students
assigned to level 2 (higher CAT scores) had higher mean results on posttests than level 1 students, the level groups were analyzed separately to test for interaction effects.

Mathematics Posttest Results

N-M memory item subtest versus posttest$_i$.

In order to assess the effectiveness of the non-meaningful instruction unit, the memory item subtest from the pretest was compared to posttest$_i$.

Table 11

Descriptive Statistics for Means on Memory Item Subtest from Pretest and Posttest$_i$ for N-M Group in Mathematics Study

<table>
<thead>
<tr>
<th>Test</th>
<th>% Means</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>46.00</td>
<td>0.16</td>
</tr>
<tr>
<td>Posttest$_i$</td>
<td>62.40</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 11 illustrates that students did improve on the items that they had memorized; and this improvement is statistically significant (Table 12). This indicates that the non-meaningful treatment was a bona fide learning experience for the students.
No gender or level effects were present for this subtest nor were there any interactions.

Table 12

ANOVA on Memory Item Subtest from Pretest Versus Posttest, for N-M Group in Mathematics Study

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>F Value</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1</td>
<td>21.82</td>
<td>9.99</td>
<td>0.0032</td>
</tr>
<tr>
<td>Class</td>
<td>1</td>
<td>12.87</td>
<td>1.18</td>
<td>0.3394</td>
</tr>
<tr>
<td>Level (L)</td>
<td>1</td>
<td>3.39</td>
<td>1.55</td>
<td>0.2212</td>
</tr>
<tr>
<td>Gender (G)</td>
<td>1</td>
<td>0.05</td>
<td>0.02</td>
<td>0.8834</td>
</tr>
<tr>
<td>L X G</td>
<td>1</td>
<td>0.36</td>
<td>0.17</td>
<td>0.6865</td>
</tr>
<tr>
<td>Within</td>
<td>35</td>
<td>76.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>44</td>
<td>114.94</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Critical value: p < .05.

Total posttest 2 mathematics test.

The central issue in the study is the comparison of the effect of non-meaningful followed by meaningful instruction (N-M treatment) to meaningful-only instruction (M-O treatment). To assess these effects, a three-way ANCOVA was
performed on posttest, with treatment, level, and gender as independent variables and pretest score as the covariate. As indicated in Tables 13 and 14, the group receiving meaningful-only treatment scored significantly better than those receiving both treatments. There was a significant level effect. Gender and all interaction effects were not significant.
Table 13

Descriptive Statistics for Treatment and Level on Total Posttest, in Mathematics Study

<table>
<thead>
<tr>
<th>Factor</th>
<th>Level</th>
<th>% Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Treatment</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N-M (n=44)</td>
<td>35.78</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>M-O (n=55)</td>
<td>42.30</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td><strong>Level</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (n=33)</td>
<td>32.14</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>2 (n=66)</td>
<td>45.86</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td><strong>Gender</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Males (n=49)</td>
<td>38.46</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td>Females (n=50)</td>
<td>39.54</td>
<td>0.75</td>
<td></td>
</tr>
</tbody>
</table>
Table 14

**ANCOVA on Total Posttest\textsubscript{2} in Mathematics Study**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>F Value</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>1</td>
<td>481.80</td>
<td>19.89</td>
<td>.0001</td>
</tr>
<tr>
<td>Class</td>
<td>1</td>
<td>62.73</td>
<td>0.52</td>
<td>.7621</td>
</tr>
<tr>
<td>Treatment (T)</td>
<td>1</td>
<td>115.92</td>
<td>4.78</td>
<td>.0315</td>
</tr>
<tr>
<td>Level (L)</td>
<td>1</td>
<td>471.27</td>
<td>19.45</td>
<td>.0001</td>
</tr>
<tr>
<td>T X L</td>
<td>1</td>
<td>35.83</td>
<td>1.48</td>
<td>.2273</td>
</tr>
<tr>
<td>Gender (G)</td>
<td>1</td>
<td>3.18</td>
<td>0.13</td>
<td>.7181</td>
</tr>
<tr>
<td>T X G</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>.9968</td>
</tr>
<tr>
<td>L X G</td>
<td>1</td>
<td>0.67</td>
<td>0.03</td>
<td>.8682</td>
</tr>
<tr>
<td>T X L X G</td>
<td>1</td>
<td>1.32</td>
<td>0.05</td>
<td>.8160</td>
</tr>
<tr>
<td>Within</td>
<td>85</td>
<td>2059.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>98</td>
<td>3464.44</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* Critical value: $p < .05$.

\textit{N-M and M-O posttest\textsubscript{2} memory item subtest.}

Whereas students did make immediate gains on the memory subtest as a result of the non-meaningful treatment (recall Tables 11 and 12), their overall
performance on posttest₂ was inferior to that of students receiving the meaningful-only treatment. But how does their performance on the memory subtest alone compare to the M-O group? To answer this question, the previous analysis was rerun on the memory subtest alone, using the memory item subtest from the pretest as a covariate.

As is indicated in Tables 16 and 17, students receiving only three days of meaningful instruction scored significantly higher than students receiving five days of non-meaningful followed by the three days of meaningful instruction, even on items that matched the non-meaningful instruction mode. Students with higher mathematics achievement scored significantly better than students on the lower achievement level. Gender and interaction effects were not significant.
Table 15

Descriptive Statistics for Treatment and Level on N-M and M-O Memory Item Subtest for Posttest, in Mathematics Study

<table>
<thead>
<tr>
<th>Factor</th>
<th>Level</th>
<th>Means</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>N-M (n=44)</td>
<td>46.00</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>M-O (n=55)</td>
<td>58.00</td>
<td>0.24</td>
</tr>
<tr>
<td>Level</td>
<td>1 (n=33)</td>
<td>42.00</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>2 (n=66)</td>
<td>62.00</td>
<td>0.21</td>
</tr>
<tr>
<td>Gender</td>
<td>Males (n=49)</td>
<td>52.13</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>Females (n=50)</td>
<td>51.75</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Table 16

**ANCOVA Results on Posttest Memory Item Subtest for Both N-M and M-O Groups in Mathematics Study**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>F Value</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>1</td>
<td>2.94</td>
<td>1.05</td>
<td>0.3091</td>
</tr>
<tr>
<td>Class</td>
<td>5</td>
<td>5.91</td>
<td>0.42</td>
<td>0.8333</td>
</tr>
<tr>
<td>Treatment (T)</td>
<td>1</td>
<td>18.60</td>
<td>6.62</td>
<td>0.0118</td>
</tr>
<tr>
<td>Level (L)</td>
<td>1</td>
<td>48.10</td>
<td>17.12</td>
<td>0.0001</td>
</tr>
<tr>
<td>T X L</td>
<td>1</td>
<td>2.72</td>
<td>0.97</td>
<td>0.3279</td>
</tr>
<tr>
<td>Gender</td>
<td>1</td>
<td>0.02</td>
<td>0.01</td>
<td>0.9293</td>
</tr>
<tr>
<td>T X G</td>
<td>1</td>
<td>0.43</td>
<td>0.15</td>
<td>0.6967</td>
</tr>
<tr>
<td>L X G</td>
<td>1</td>
<td>0.17</td>
<td>0.06</td>
<td>0.8068</td>
</tr>
<tr>
<td>T X L X G</td>
<td>1</td>
<td>0.30</td>
<td>0.11</td>
<td>0.7429</td>
</tr>
<tr>
<td>Within</td>
<td>85</td>
<td>238.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>98</td>
<td>316.69</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note.** Critical value: p < .05.

**Interference Interactions**

Recall from Chapter 3 (p. 66) that two contrary expectations for interaction effects were discussed. The first hypothesis was that greater non-meaningful
learning would cause greater interference, and the second hypothesis that higher achievers overcome interference more readily than do lower achievers. In either situation an interaction would occur. Therefore, the fact that no interaction effect could be discerned (see Table 18) could have two interpretations. Both hypothesis could be true and thus make the interactions nondiscernible; or neither hypothesis is true so no interaction effect is present.

In order to determine which of these is the case, additional analysis were conducted. Posttest\textsubscript{1} scores were correlated with posttest\textsubscript{2} against level. The statistics for the correlation for levels 1 and 2 had values of \(-0.44654\) and \(0.50363\), respectively. There was a significant interaction effect, \(F(1,43) = -2.936, p = 0.0055\) for the posttest.

**Mathematics Retention Results**

The retention test results were analyzed to see if the effects observed immediately after instruction would persist over time. Many of the
immediate effects found on the posttests were retained as indicated in the following analysis.

**Total mathematics retention test.**

A three-way ANCOVA was conducted on the retention test results with treatment, level, and gender as main effects and the pretest as the covariate. Class was analyzed as a nuisance variable. As indicated in Tables 17 and 18 there were significant treatment, level, and gender effects on the total mathematics retention test. After a lapse of two weeks, the group receiving only meaningful instruction still scored higher than did students receiving non-meaningful and meaningful instruction. The students having higher achievement in mathematics remained higher on the retention test than did students with lower achievement in mathematics. Females scored significantly higher on the retention test than did males. This gender effect was not present on posttest₂. There were no interactions on the retention test.
Table 17

Descriptive Statistics for Treatment, Level, and Gender on Total Mathematics Retention Test in Mathematics Study

<table>
<thead>
<tr>
<th>Factor</th>
<th>Level</th>
<th>% Means</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>N-M (n = 44)</td>
<td>33.97</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>M-O (n = 55)</td>
<td>39.81</td>
<td>0.65</td>
</tr>
<tr>
<td>Level</td>
<td>1 (n = 33)</td>
<td>31.00</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>2 (n = 66)</td>
<td>42.78</td>
<td>0.58</td>
</tr>
<tr>
<td>Gender</td>
<td>Male (n = 49)</td>
<td>33.95</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>Female (n = 50)</td>
<td>39.81</td>
<td>0.69</td>
</tr>
</tbody>
</table>
Table 18

**ANCOVA on Total Retention Test in Mathematics Study**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>F Value</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>1</td>
<td>214.63</td>
<td>10.22</td>
<td>.0019</td>
</tr>
<tr>
<td>Class</td>
<td>1</td>
<td>108.77</td>
<td>1.04</td>
<td>.4018</td>
</tr>
<tr>
<td>Treatment (T)</td>
<td>1</td>
<td>91.70</td>
<td>4.37</td>
<td>.0396</td>
</tr>
<tr>
<td>Level (L)</td>
<td>1</td>
<td>349.43</td>
<td>16.64</td>
<td>.0001</td>
</tr>
<tr>
<td>T X L</td>
<td>1</td>
<td>29.12</td>
<td>1.39</td>
<td>.2422</td>
</tr>
<tr>
<td>Gender</td>
<td>1</td>
<td>95.03</td>
<td>4.53</td>
<td>.0363</td>
</tr>
<tr>
<td>T X G</td>
<td>1</td>
<td>21.04</td>
<td>1.00</td>
<td>.3196</td>
</tr>
<tr>
<td>L X G</td>
<td>1</td>
<td>1.22</td>
<td>0.06</td>
<td>.8104</td>
</tr>
<tr>
<td>T X L G</td>
<td>1</td>
<td>0.55</td>
<td>0.03</td>
<td>.8721</td>
</tr>
<tr>
<td>Within</td>
<td>85</td>
<td>1784.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>98</td>
<td>2836.73</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note.** Critical value: p < .05.

Further analysis was conducted on the memory items subtest of the retention test. This subtest includes those items replicating the format utilized.
during the non-meaningful instruction. There were no significant effects or interactions on the mathematics retention subtest; however, there was a marginal treatment effect, with $F(1,98) = 3.90$, $p = .0515$. The M-O group remained above the N-M group with treatment percent means of 56.50 and 47.88, respectively.

Interference interaction with retention.

As was found with posttest$_2$, there also were interaction effects on the retention test when a correlation was conducted comparing posttest$_1$, learning of non-meaningful skills, with retention scores. The statistics for the correlation for levels 1 and 2 had values of -.25040 and 0.59027, respectively. There was a significant interaction effect, $F(1,43) = -3.115$, $p = 0.0034$.

Qualitative Analysis

School Environment

Data for these analysis were gathered through observations, interviews, and a Southern Association of Colleges and Schools: 10 Year Report (St. Amant Middle School Faculty, 1991-92) issued during the time of this study.
St. Amant Middle School serves a middle income, predominantly white (86% white, 13% black, 1% other), rural industrial community in south Louisiana. About half of the parents are semiskilled laborers working in construction or industry in the area. A summary of the parents' level of educational attainment are as follows: 20% did not complete high school; 56% completed high school; 13% received some training beyond high school; and 11% are college graduates. The families are relatively stable (25% of the students' parents were divorced) and the community was very stable. (Over 90% of the students have attended this or feeder schools for three or more years.) The community is rural but located within 50 miles of two major cities. The families have the cultural opportunity of the city and the outdoor sports of the country available; they seem to prefer and take advantage of the country living.

St. Amant Middle School (a fifth through eighth grade structure) has about 800 students. Students' mean score is near the 50 percentile in all subjects
on standardized tests. Besides academics, the school offers a strong band and sports program.

The mathematics instructional program in the school at the fifth grade level (the level utilized for the mathematics study) is very traditional. Lecture, drill, and practice are the dominant approaches evidenced. The use of manipulatives is not utilized for mathematics instruction.

**Student Profile**

Qualitative data were gathered through interviews from a sampling of students participating in the mathematics study. The purpose of the interviews was to gain further insight into student’s attitudes and understanding of mathematics, but more particularly, to investigate causes and effects of interference more fully.

Interviews were conducted with 12 students randomly selected from two classes. The selection was stratified so as to include an equal number of students from each treatment, level, and gender.

Three interviews were conducted: at the beginning of the study prior to any instruction; after the non-meaningful instruction; and after the
meaningful instruction. The goals for each set of interviews differed.

**Interview 1**

The purpose of the initial session was to help the students feel comfortable with being interviewed and recorded and to get a picture of their background. General questions about school, study patterns, parental support, and mathematics were asked (see Appendix H for Interview I questions).

When asked about school only one student said he'd "like to rest at home and then come to school." All other students expressed more positive feelings using expressions like: "It's fun," "I like the work," "I've always liked school," "It's even better than last year," and "I like being with my friends and teachers." There was an overall feeling of interest and enthusiasm expressed.

Half of the students claimed mathematics as their favorite subject and gave "because it's easy" as their reason. Others said they liked mathematics because "it's interesting" and "because you don't have to do a lot of writing." Spelling was a favorite for many. Several students selected
mathematics as their least favored subject with "it's too hard" as the reason for the choice. Social studies was the most disliked subject. Generally, students liked subjects they did well in and disliked those they found difficult.

In discussing study patterns, a variety of responses were given. "I listen hard in class," "read my notes," "practice on my computer," "discuss it with my parents," "read it alone," "use flash cards," and "review over and over" were but a few of the methods shared. A few students acknowledged they never study for math tests because they "already knew it." These students named mathematics as their favorite subject. The pattern that emerged the most on this topic was that for studying spelling their parents "called out the words," and that for studying mathematics students "worked a few problems."

All students said they study alone, usually in their own rooms, and got parental assistance when requested. No one suggested excessive academic pressure from their parents; however, most of these fifth graders said their parents expected them to
attend college and they wished to comply. Academic support was evidenced even though very few of the parents themselves were college graduates.

In discussing mathematics, a couple of students said there was nothing about the subject they liked; and word problems and division were the areas of greatest dislike. "Division is hard because there are so many steps to learn," said one student. Most students, however, liked mathematics and named one of the basic operations with whole numbers as their favorite areas.

A few students were not aware of their doing any mathematics outside of school work; however, most students were aware of other people's mathematics applications. Some mentioned they use math "while cooking," "when measuring my average speed in running," "in playing games," and "when shopping." One student said, "When my dad says he caught one third of a sack of crawfish, I know how much he caught." This same student knew one third is less than one half. Most students were able to connect mathematics to real-life experiences.
In summary almost all the students expressed positive feelings about school, parental support, and mathematics, even though mathematics was not everyone's favorite subject. Most students did not have difficulty relating mathematics to their life experiences. They seemed to like what they understood and dislike any areas of confusion.

Interview 2

The purpose of the second session was to determine student's understanding of area and perimeter (see Appendix K for Interview II questions). The students who had just completed non-meaningful instruction were able to give definitions for area and perimeter as "space inside" and "distance around something," but admitted to having much confusion with the formulas. The comment frequently made was that "it was hard to keep all the formulas straight." As anticipated, the M-0 group, students who had not received instruction on these geometric concepts since the previous year's mathematics instruction, displayed almost no knowledge of area and perimeter.
Interview 3

The third session was considered most critical because it sought to understand the causes and effects of any possible interference (see Appendices N and P for a list of Interview III questions and the transcription of this session). Data gathered during this third interview were audiotaped, videotaped, transcribed, coded, and analyzed. The resulting summaries contribute to a description of student perceptions and treatment effects. The analysis that follows is a summary of the 12 student responses, and a more in-depth reporting of one student from each treatment whose contributions to this study were especially informative. This information was gathered during the third session of interviews.

Summary of Third Session Interviews - All Students

This summary follows the general order of topics addressed during the interview: comparison of the instructional tasks (only the N-M group had this topic since only they received both instructions); feelings about the meaningful instruction; the role of manipulatives; definitions of concepts;
calculation methods; application of concepts; and formula-problem relationships.

*Comparison of the instructional tasks.*

In comparing the two methods of instruction, the N-M students were split on several issues. About half of them stated that the formula instruction was preferred, easier, and more enjoyable; the other half preferred the meaningful instruction. One student said, "They, (the formulas), got me confused. It's complicated to remember all the stuff about which formula goes with which problem." Almost everyone in this group felt they learned more during the non-meaningful instruction than during the meaningful instruction, though they admitted to infrequent use of formulas as a means to obtain area and perimeter on their test. There was a conflict between what they "felt" they learned and what they actually learned and were able to apply. Being able to recite memorized material perhaps gives the students a sense of "having learned."
Feelings about the meaningful instruction.

The two treatment groups had very similar feelings about the meaningful instruction. All the students said they enjoyed the unit, and that their favorite manipulative was the geoboard. They liked "playing" with the "boards and bands." They all agreed that their regular mathematics instruction was very different from the treatment of this study and the most frequent reason given was that during regular mathematics classes "we don't use all that stuff" (manipulatives). There was no evidence from the students that manipulatives were ever utilized for instruction. Some students said their regular mathematics class compared to the formula instruction (non-meaningful) because their teacher also explains many problems on the overhead.

Role of manipulatives.

Many interesting and revealing comments were made on explaining how the manipulatives helped them learn. One student from the M-O group seemed to summarize her treatment group's sentiments by saying the manipulatives "measured the inside - you could see the squares." Students from the first treatment
commented, "You can find out how to figure it out," and "you can understand it good if you use all that because you can use a certain amount of stuff...like the squares." One child seemed to reveal some of the feelings of the N-M group when she said, "Some people said they couldn't do it and they got to learn that they could do it" (find the measure). The use of manipulatives gave the students a sense of empowerment as well as enjoyment.

Definitions of concepts.

In defining and describing area and perimeter and their applications, the two treatment groups frequently displayed some subtle, and some not so subtle, differences. In defining area and perimeter the N-M students used terms like "inside" and "outside," respectively. The M-O students consistently used "whole" ("the whole room", "the whole thing," "the inside and the middle") and "all the way around" as their definitions for the same terms. This subtle difference may contribute to an understanding of the N-M students' consistent error in applying area and perimeter to the measures of a dog house (See the paragraph on this discussion
later in this section). All the N-M students claimed walls could only "have perimeter" not area since "they (walls) went around." No M-O students had this same misconception.

**Calculation methods.**

In explaining how one can find area and perimeter, the two treatment groups again differed. The N-M group almost always named operations in their explanations ("Sometimes you multiply, sometimes you add"), whereas the M-O group's explanation was frequently grounded in the concrete ("Use hands," "use books," "use tiles"). The N-M group gave more confused explanations for finding area and perimeter and included more errors than those of the M-O group. The N-M group focused more on the process (frequently applying an incorrect process) than on the object of measure; the M-O did not.

**Application of concepts.**

In discussing who needs to understand area and perimeter and why they need to understand it, the M-O group gave more concrete applications (for carpet, painting, wallpaper) than did the N-M group (for
tests, later study, college). The M-O group made no errors in their applications of area to life situations, whereas, the N-M students made many. We need area to measure "the lengths of boards," "how much liquid," "the thickness of concrete," and the "height of a pole" were some of the errors made by the N-M students. The concepts of area and perimeter appeared to be less clear for the N-M students than they were for the M-O students.

For specific applications of the concepts in this study, students were asked questions applying area and perimeter to the building of a dog house and wallpapering or painting a room. Students were asked, "To know how much wood to buy for the floor of a dog house does one need the area or perimeter measures?" The same question on the amount of wood needed for the walls of a dog house, wallpaper for a room, and paint for the walls was asked. A pattern in the responses emerged. Almost every N-M student said one needs to know the perimeter of the dog house or room to determine the amount of paint, wallpaper or wood required for the walls (a consistent error). A typical response of the N-M
student was that walls don't have area "because they go around." The M-O students realized one needs, and can obtain, area measurements for those determinations.

**Formula-problem relationships.**

The students were shown and given an explanation for the formulas for perimeter of a rectangle \( P=2(l+w) \) and area of a triangle \( A=1/2bh \). They were asked to explain why the particular formulas would produce the desired measure. Most of the N-M students could not make a connection; however, of those who did, some error patterns emerged. No one in this group could correctly explain the role of the "2" in the perimeter formula. Some erroneous explanations given were, "the length is 2 and the width is 2," "it has 2 numbers," "because it has 2 different sides," and it makes it "easier to remember." For the triangle, a few of these students seemed to suggest some correct connection between the formula and its meaning, others expressed confusion, and one was totally incorrect in explaining that the formula
has 3 parts and a triangle has 3 sides; therefore, it works.

For the same formulas, the M-O group responded quite differently. Some students said they simply could not see any connections. Over half of the students from this group gave partial or complete explanations, and no explanation given was incorrect. To add the length and width and then double it was understandable to several M-O students. In various ways they explained the sides were repeated twice. The students who were able to connect the area of a triangle to its formula explained that a triangle is half of a rectangle so the formula was connected.

An element of interest here is that the N-M group had been exposed to and used the formulas during the 5-day non-meaningful instruction. The M-O group had no previous exposure to these formulas. One of the N-M students was quick to use the formulas correctly but gave incorrect explanations for why the particular formulas worked for the particular measures calculated.
Summary of Two Students

To give an even deeper understanding of the findings of this study, a more holistic picture of two individual students, one from each treatment, is described. The names have been changed to preserve the students' anonymity.

These particular students were selected for various reasons. Most importantly, Tom and Ann seemed to most clearly exemplify the extremes of treatment effects. Tom, the N-M subject, had a much higher achievement level, 81 percentile versus Ann's 54 percentile, yet displayed far more confusion on concept development than Ann did. Other reasons these individual students were selected were that they represented both genders, their measured achievement levels differed significantly, and their verbal expressions were revealing as well as delightful.

Tom.

Tom is a high achieving (81 percentile in mathematics on a California Achievement Test) male student who participated in the first treatment (N-M group) of this study. His behavior seemed
representative of many students with this same treatment, and his verbalizations contributed to an understanding of the students’ thinking (see Student F in Appendix P for a complete script of this student’s interview).

Tom’s favorite subject is mathematics because, as he put it, "You don’t have to do a lot of writing." He likes most whole number operations and finds fractions rather complicated. To study, he reads the material, verbalizes repeatedly, and works a few problems. He has a very positive attitude about learning and plans to attend college; a decision apparently strongly supported by his parents.

Tom learned the content of the non-meaningful material well (7 of 8 correct on posttest,) and was the student to most quickly utilize formulas for computations during the interview sessions. Even to find the area of the table at which he was sitting (with 1X2 as the given dimensions), he asked for paper and pencil, wrote down the formula, and calculated the correct solution.
During discussions on content application he usually identified the shape, selected the appropriate formula, and obtained a correct solution quite readily. However, if he did not employ a formula, many errors resulted. When finding the area of a rectangle, for example, he initially added the measures of the sides instead of multiplying them. He repeatedly found perimeter instead of area until he decided to use a formula. Once that decision was made, he calculated correctly.

Tom also misapplied concepts. He said one reason we learn area is "to find (the) amount of liquid in something." In discussing the construction of a sidewalk at school, he said the workers needed to know area to "know how deep to put it" and for the cover they needed area to know "how long to get the boards for it and how long to put the top over." Much confusion was apparent. Even though Tom applied formulas and computed them correctly, he displayed no understanding of the connections between the formulas and their purposes. For example, after correctly utilizing the formula for the perimeter of a rectangle and being asked why
one multiplies by 2, he explained, "Because there is one length and one width and that equals 2."

Tom ranked in the top 25 percent on his post-and retention tests, of all subjects in this study, scoring 19 and 18 respectively. There were no indications on his tests that he used formulas for computations and evidence did exist that some solutions were found using other means, e.g., grids were drawn in the interior of a shape to find area. He was able to correctly solve some items on the tests that required strategies other than formulas, e.g., item 16 (see Appendix M) that required the students to sketch a rectangle having a specified perimeter.

In summary, Tom was a high achieving mathematics student (supported by his teacher’s evaluation and California Achievement Test score) who displayed many confusions and misconceptions typical of N-M students in this study. He memorized the formulas well (posttest, score) and learned from the meaningful instruction, but exhibited much confusion.
Ann.

Ann was a very delightful fifth grade girl who received only the meaningful treatment and was interviewed only once, at the termination of the treatment. Though Ann's California Achievement score was average (54 percentile), her teacher reported her class performance very poor in mathematics. Despite her weakness in mathematics; she was chosen for this in-depth discussion because her behavior displayed many characteristics typical of students in the M-O group. (see Student K in Appendix P for a complete script of this student's interview).

Ann's scores on the pre-, post-, and retention tests were very poor, 2, 8, and 9, respectfully. A recurring error on the pretest was to add all the numbers given or to find the perimeter, yet even that was not consistent. There was more pattern evident on the posttest. All eight points scored were from perimeter problems, yet there were some seemingly simple perimeter problems she missed (e.g., numbers 15 and 30). On the retention test she missed some items she had correct on the
posttest, solved additional perimeter problems, and had two area problems correct, even a rather complex one (# 26). Not much pattern was detected in her errors, and she showed almost no markings to determine her thinking.

Initially during her interview she demonstrated lack of academic confidence and responded to mathematical situations in a seemingly random fashion. She couldn’t remember how to pronounce perimeter; she accented the first syllable. When given the dimensions of the room in which we sat and asked how she would find the area, she responded, "I’d probably have to do some multiplying and dividing." When further questioned on what she’d multiply and what she’d divide she said, "Well, I’d multiply 8 times 8 and that would be the same length and I’d get my answer and then I’d add 12 and 12. (pause) I don’t know." There seemed to be no pattern to her responses.

Finally, the interviewer stated, "What if I promised you $100 for finding the area of this room, what would you do about that?" Ann responded very creatively and accurately, and gave some very
profound explanations during the rest of the interview session.

In response to the above stated challenge to find the area of the room she immediately said, "I'd start taking down the books (off the shelves) and start doing stuff." She proceeded to explain, "I'd take those books and start putting them out and I'd count how many books I put out. I would get my area, but they have different sizes just like your hands (Earlier she stated that measurement with hands is an estimate because different people have different sized hands). It wouldn't come out exact." When the interviewer responded to her explanation by saying one could then go to the store and ask for 80 books of carpet, she laughed. She then connected to standard measure by explaining that one "could see how much a yard is, like put all the books together and see how much a yard is." You could then "measure all around the room." Her explanations were very concrete and accurate.

Ann performed some very quick mental calculations. When asked to draw a shape, assign
dimensions, and calculate the perimeter, she drew an 8X3 rectangle and instantly gave its perimeter.

When asked to define area and perimeter, she said area is "the whole room" and perimeter "is just to see how far around it goes." She drew grids and counted each space individually to obtain the area of a rectangle.

When discussing why one needs to learn area, she said, "If I want to put in a shelf (in this room), I need to know how much area it’ll take up and how much I won’t have after I put (in the shelf)." Again, a very concrete and accurate concept.

Students in the N-M group thought a wall could not have area because it "went around" the room; Ann said that the wall has area. When asked how she would calculate the area, she explained, "It (would be) just like taking this room and pushing it over that way (down), that, (the wall), would be the floor and that, (the floor), would be the wall."

Initially, when shown the formula for the perimeter of a rectangle and asked to make connections to its function, Ann was not able to do
so. After a little prodding she realized appropriate relationships. She was even able to relate to a small degree the formula for area of a triangle and its function, a relationship considered more difficult than perimeter of a rectangle.

Ann operated on a very concrete level and did not abstract readily, i.e., when finding the area of an 8X5 rectangle she drew grids and counted the squares. She repeated the counting three times to check her solution - never attempting to multiply.

Even though Ann seemed very weak in mathematics by some standards, she displayed some profound understanding. Her explanations and solution methods were buried in the concrete, but they were accurate. Once she was challenged to think, she indicated no misconceptions or confusions. Understanding the concept was not the problem; dealing with mathematics abstractly was. However, the abstractions she constructed from the meaningful instruction were accurate.

Summary

The purpose of this study was to investigate the presence of an interference as a result of
learning non-meaningfully prior to learning meaningfully. In order to accept or reject the null hypothesis that there was no interference created, evidence was necessary to show that non-meaningful learning occurred and that greater meaningful learning occurred for students receiving only meaningful instruction than for students receiving non-meaningful instruction prior to meaningful instruction.

Evidence was found that non-meaningful learning occurred in both studies. In the generic study the N-M students knew significantly more on the Memory Item Subtest than did the M-0 students. In the mathematics study the N-M students improved significantly from the pretest to a posttest immediately following the non-meaningful task.

After the completion of both non-meaningful and meaningful instruction, statistical evidence indicated that students receiving only meaningful instruction learned more than did students receiving non-meaningful followed by meaningful instruction. Even stronger evidence was found in an analysis on the Memory Item Subtest in the mathematics study
that replicated non-meaningful instruction items. Even though the N-M group improved significantly as a result of non-meaningful instruction, when items presented during the non-meaningful instruction were analyzed at the termination of the study, the M-O group scored significantly higher even on those memory items.

Qualitative data supported the quantitative analysis and contributed to a deeper understanding of the interference created. The presence of confusions and misconceptions caused by non-meaningful learning were made evident during the interviews.
CHAPTER 5

CONCLUSIONS AND DISCUSSIONS

The purpose of this study was two-fold. The first goal was to develop a framework for the notions of meaningful and non-meaningful in mathematics education. The second was to investigate the possible development of an interference when non-meaningful learning precedes meaningful learning.

Notions of Meaningful and Non-Meaningful

The notions of meaningful and non-meaningful have been addressed by many authors over the past century (Chapter 2). Though the terminology and concepts differed slightly among authors, a common theme emerged.

Meaningful learning produces knowledge which is rich in relationships (Hiebert & Lefevre, 1986). These relationships may involve other concepts within the mathematical field, or areas outside the discipline of mathematics.
By contrast, the notion of non-meaningful (a term unique to this study) is knowledge with fewer relationships. Non-meaningful learning is not knowledge devoid of relationships, thus not opposite of meaningful learning. Rather, it is knowledge towards the opposite extreme of meaningful learning along a continuum of relationship density.

Interference

The second goal of this study was to investigate the possible development of an interference when non-meaningful learning precedes meaningful learning. In reviewing the literature no study was located in the area of mathematics education that was designed specifically for the purpose of investigating interference as a result of sequence in pedagogy. Several studies, however, detected negative effects on meaningful learning resulting from student’s prior non-meaningful instruction (Whitman, 1976; Kieran, 1984; Hiebert and Wearne, 1988; Mack, 1990). But these studies did not control the nature of the non-meaningful instruction within the experimental design. Thus
this study provides a more secure basis upon which to analyze such phenomena.

Interference resulting from initial non-meaningful learning can be understood in terms of Piaget's notions of disequilibrium. Non-meaningful learning sets up superficial associations related to solution procedures (e.g., defining a mean as adding and dividing). These may conflict with subsequent meaningful instruction (e.g., defining a mean as a mental act of equalizing). In such cases, either prior structures remain, thus making new (meaningful) relationships impossible; or, structures have to be unlearned and new relationships have to be constructed. This unlearning and relearning creates unnecessary obstacles.

This analysis suggests that when initial mathematics instruction of a concept focusses on memorizing procedures, facts, and definitions, subsequent meaningful learning of the concept may be impaired. To test this hypothesis a research design was established for comparing two groups of students. The first group would receive a period of
non-meaningful instruction (a memorization task) followed by a shorter period of meaningful instruction. The second group would receive only the shorter period of meaningful instruction. Posttest means would then be compared and analyzed. Greater success for the meaningful-only group would be taken as support for the hypothesis that non-meaningful learning may interfere with subsequent meaningful learning.

Two studies, a generic and a mathematics-specific, were implemented according to this basic design. The generic study investigated interference as a general cognitive effect, whereas the mathematics-specific study would indicate a direct relevance to schooling, in a particular domain. The generic study was conducted with 49 eighth-grade students; the mathematics-specific, with 99 fifth-graders.

Generic Study Conclusions

Students in the generic study receiving only meaningful instruction performed significantly better than did students receiving meaningful preceded by non-meaningful instruction. Analysis of
the items given for memorization during the non-
meaningful task indicated students did indeed learn
the memorization material. The non-meaningful
learning was accomplished. Therefore, it can be
concluded that this non-meaningful learning did
interfere with subsequent meaningful learning.
Students receiving less instruction, 20 minutes of
meaningful explanations, performed better than did
students receiving a day for memorization plus 20
minutes of meaningful instruction.

A retention test administered two weeks after
the treatments showed that the treatment effect was
retained. Students receiving both treatments
remained significantly higher on remembering the
memorization items, but remained significantly below
the second group on ability to transfer the grammar
to new situations. These findings support the
hypothesis that non-meaningful learning interferes
with subsequent meaningful learning.

There were no gender or mathematics achievement
level effects on posttests evaluating students’
ability to transfer information learned to new
situations. Neither the gender of the student nor
the amount of prior mathematics learning, as measured by a standardized test, had any significant effect in this study. It was proposed that the generic study resembled the kind of thinking required in the mathematics classroom. Its content, however, was not mathematics.

Mathematics Study Conclusions

Treatment, Achievement Level, and Gender Effects

The results of the mathematics study added further support to the hypothesis that non-meaningful learning interferes with subsequent meaningful learning. Students receiving only three days of meaningful instruction scored significantly better than students receiving three days of meaningful instruction preceded by five days of non-meaningful instruction. Retention test results indicated the effect of treatment was retained.

A separate analysis of items taught during the non-meaningful instruction, deviated somewhat from the generic study result. Immediately following non-meaningful treatment it was evident that the students had made significant gains from the pretest. But, even on these eight items (which were
modified only by changing numerals), the meaningful-only instruction group scored significantly better than students receiving both instructional tasks.

In the mathematics study, gender and achievement level were analyzed for effects. There was no gender effect on the immediate mathematics posttest; however, the girls scored significantly higher than the boys on the mathematics retention test. These data are in compliance with previous observations that "at ages 9 - 13 differences (in gender are) minimal and sometimes in favor of females" (Fennema, 1977, p. 83).

There was a level effect in this mathematics study on the post-, retention, and memory item subtest. Students above average in mathematics achievement performed significantly better in this mathematics experiment than did students below average in achievement. Top achievers in mathematics learned more from these treatments than lower mathematics achievers.

Though top achievers learned more from the non-meaningful task as well as the meaningful task regardless of the treatment, the question remains
whether the hypothesis that greater non-meaningful learning results in greater interference can be accepted or rejected. The correlation analysis indicated a differing effect of interference on the two achievement level groups. The top mathematics achievers had a significant positive correlation while the lower mathematics achievers had a significantly negative correlation.

Both hypothesis that high achievers overcome interference and that students who learn more non-meaningful skills experience more interference were supported by the correlation analysis. The high mathematics achievers, students who learned most during the non-meaningful instruction, overcame interference and learned most during the meaningful instruction (positive correlation). An opposite effect was observed with lower mathematics achievers. Within the level of lower achievers greater learning of non-meaningful skills resulted in a greater interference (negative correlation).
Interference Characteristics

Analysis of quantitative data evidenced the presence of interference. Data gathered through interviews contributed to identifying characteristics of the interference.

All subjects expressed enjoyment of and appeared to be motivated by the meaningful instruction; however, several students who received both instructions felt they learned more during their non-meaningful task. Somehow the experience of memorizing material and being able to regurgitate it equated to learning for these students. Perhaps just this expectation of what learning should be like in schools, contributed to the interference effect.

The inability to transfer learning was a characteristic of students experiencing interference. This study provides evidence that initial non-meaningful learning hinders students from adapting to new situations. For example, on applying area and perimeter concepts learned in the context of horizontal surfaces (drawings on their paper) to contexts of vertical or oblique surfaces
(dog houses or walls), there were consistent errors. The students receiving non-meaningful instruction repeatedly confused area and perimeter in the new context - an effect almost absent for students experiencing only meaningful instruction. Skemp (1987) lists the ability to adapt to new tasks a consequence of meaningful learning.

Similarly, students receiving non-meaningful instruction tended to overgeneralize. They applied area and perimeter to liquid measure (volume) and length (height of a pole), and otherwise seemed to be more concerned about what mathematical operations they were using, than why they were using them. This tendency was not observed in students receiving only meaningful instruction. Generally, the exposure to formulas seemed to inhibit the free, open-ended, creative explanations of ideas and materials in area and perimeter problems.

Limitations

There were some limitations to this study that if overcome in future replicated studies may contribute further to the findings of this study.
1. **Researcher bias.** Researcher bias is a concern in this study. The researcher was the instructor for all treatments. Several measures were taken to control for possible bias. To protect against favoritism to one treatment group, each class during the mathematics experiment was divided into two treatments. One half of each class was given non-meaningful instruction; then they were combined with the second half of the class for meaningful instruction.

Further protection against observer bias was attempted by having all mathematics-specific sessions audio-taped, and by having an unbiased observer present during at least one instructional period each day of treatment (see Appendix F for observer notes).

Practical considerations warranted against training other instructors to conduct the treatments and interviews. This training could have created other less controllable design problems.

2. **Two-study replication.** Closer replication of the generic study to the mathematics study could
have contributed to a deeper understanding of level and gender effects.

The highest mathematics achievers in the eighth grade were homogeneously grouped at the site of this research; therefore, it was not convenient for them to be included in the generic study. The mathematics study, however, included top mathematics achievers. The level effects differed in the two studies. (The generic had no level effect; whereas, the level effect in the mathematics study was significant.) These differences may be a result of the parameters of this study.

Gender effects also differed between the two studies. (The generic had no gender effect; in the mathematics study it was significant.) These differing effects may be a result of differing grade levels utilized, yet several factors dictated the incorporation of these specific levels in this study. Seventh and eighth graders were included in piloting the generic study. The seventh graders exhibited much frustration in attempting the memorization task; therefore, the generic study was administered to only eighth graders.
It was difficult to find an area of mathematics in which students had not had prior contamination of non-meaningful learning. The topic and grade level selected for the mathematics study satisfied the required parameters for the content-specific experiment. Thus area and perimeter to fifth graders was selected.

3. Treatment time. The time allotted for both instructions during both studies and the time lapsed between the treatments and the retention tests are limited. For the generic study one day of non-meaningful instruction and a twenty minute meaningful instruction was employed. The mathematics study used a five day non-meaningful and a three day meaningful instructional time. For both studies two weeks lapsed between the post- and retention tests. The other studies mentioned in the review of the literature (see Chapter 2) suggest that the interference is quite general. A later retention testing and longer instructional periods, however, would reflect more realistic treatments.
Implications for Pedagogical Practices

This study supports a general theory of pedagogical sequence: teach meaningfully first. Further research on helping specific students construct specific concepts meaningfully is left for other studies.

There is a danger of being prescriptive and recommending a simplistic solution to a very complex problem. However, the quantitative and qualitative findings of this study raise some fundamental questions about established practices in mathematics education.

**Pedagogy**

No longer is it possible to rationalize that the means that we use can nevertheless lead to good ends. With pressure to produce grades and improve scores, there’s a general practice in mathematics to show students "how" (the procedure) to find solutions through operations, equation solving or problem solving procedures. Some theorists believe that through drill and practice of many problems, understanding the underlying principles of the procedure will emerge (Thorndike, 1922; Gagnè,
1985). This hypothesis finds no support in this study. Rather the study finds that the teaching of a procedure without the meaning reduces the possible development of later understanding. Based on Piaget's (1975/1977) exogenous/endogenous theory of learning, the prior knowledge can either block new learning or necessitate "unlearning" of constructs. Ausubel (1978) states that this unlearning may prove to be the most determinative single factor in the acquisition and retention of knowledge.

**Formal Mathematics**

Formal mathematics, the basis of the power of the discipline, is an abstraction. This study implies that abstractions should evolve from meaningful learning. Formal mathematics should be an end product of an educational process rather than the initiation of the process (Ausubel, 1963; Bruner, 1960; Skemp, 1987). In the classroom context this means children should have established some concept of "number" before they are asked to write a numeral to represent a set of objects; students should explore finding area and perimeter before being given any formulas to do the same; and
students should experience variables before given equations with variables to solve.

**Initial Instruction**

This study implies that the initial development of mathematical concepts and procedures be done meaningfully. Whether the objective is number concepts in the primary grades, procedures for fraction computation in middle grades or factoring in algebra, the concept needs to be presented in the context of its many relationships.

**Evaluation**

This research calls for a focus on process instead of product. Teachers need to attend to guiding children’s thinking not to showing them how to simply arrive at the correct answer. This principle calls for a reassessment of the roll of formal evaluations. As long as standardized tests are administered for the purpose of evaluating an individual teacher or building site, there is pressure to focus on the product. If a student is tested on "2 + 3," and the results of the test are used to evaluate the instructor’s performance, teachers are pressured to have students practice
"2 + 3" instead of exploring the uniting of two objects with three objects.

The Curriculum and Evaluation Standards (NCTM, 1989) states that if we really want to change mathematics education, we must change the current method used to evaluate it - standardized tests. Kamii and Lewis (1991) compared primary student's scores on a standardized test against data on understanding obtained through interviews. They found much evidence that on standardized tests as they are currently designed students receiving traditional instruction do as well or better than students going through a constructivist program. However, when students were evaluated on higher order thinking skills through interviews, those from the constructivist program scored significantly better than did students from a traditional program. It was concluded that present standardized evaluations test for ability to apply symbols and rules rather than meaningful learning. Attention needs to be given to evaluation instruments, timing of evaluations, and use of evaluation results.
Textbooks

The research results of this study support a change in textbook structure. For example, the typical middle school mathematics text is a collection of 15 to 20 chapters on various, seemingly separate, areas of mathematics. Each chapter includes 8 to 12 lessons, often hierarchically arranged, with the intent of a lesson per day on some new skill. The lessons are often viewed by students to be a collection of unrelated skill instructions.

Perhaps, instead of the present textbook structure, the curriculum for a set of grades, i.e., middle grades, should identify a few strands of mathematics to be explored at that instructional level. Units of study for a particular topic, i.e., measurement, would be developed including resource materials and well developed exploration activities from which to select. These activities would include the use of several models (concrete as well as pictorial), incorporate many related skills, and connect the learning to real-life situations interesting to the students involved.
Implications for Policy

This study is relevant to major issues in mathematics education in America today. It speaks to a behaviorist versus a constructivist debate in education.

There is a general agreement that the purpose of mathematics education is to have students be able to perform and understand mathematics. Understanding enhances retention and transfer - the purposes of education. To understand is to be able to connect a concept to its many related fields - to know it meaningfully (Brophy, 1983).

With the demand for increased mathematics competency on the individual and national levels, increased political pressure has been placed on school systems (National Research Council, 1989; Brockett, 1992). Even though mathematics professionals and supporting associations have produced a document, Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), that supports new and more meaningful approaches to the teaching of mathematics, political systems and administrative agencies have maintained a narrow
focus on test scores. Single-minded attention to test scores supports the behaviorist philosophy in education that has dominated mathematics education during much of the twentieth century (Kulm, 1990).

The behaviorist, stimulus-response, philosophy offers a pedagogical technology to influence measurable outcomes, namely, test scores. This technology stresses repetitive practice as the primary learning vehicle. It eschews constructivist methods based on discussion and explanation because the cognitive structures which such constructivist methods seek to develop and enhance are beyond the purview of a psychology rooted completely in observable behavior. Furthermore, the behaviorist pedagogical technology is not ineffective in training students to perform rigidly circumscribed tasks in rigidly controlled settings, but it is less useful for promoting the robust knowledge structures needed to adapt learning to diverse settings of application (Skemp, 1987).

What has emerged in American education is an uneasy co-existence between administrative and professional branches. Administrators endorse the
broad professional objectives of education; but insist on regimes of rigid test-centered accountability and support the most expedient methods (behaviorist methods) to enhance the indicators. Professional educators attempt (for the most part) to work within the strictures of administrative design; hoping to invest education with relevance and meaning to students through a parallel track of enriched activities and practices.

This study challenges the premise of this accommodation by questioning the viability of the two-track approach. The data here suggest that approaches of meaningful instruction cannot be tacked onto a behaviorist framework without significant degradation of the resulting meaningful learning. Thus this study pushes for a basic realignment of administrative and professional influences on pedagogical practices.

Questions and Problems

Many questions arise as a result of this research. Some of them are addressed here.
1. Is there a better term or definition for the notion of non-meaningful? Whereas "meaningful" is a relative term, admitting for example of modifiers "more," "less," etc., "non-meaningful" has absolutist connotations that contradict the basic concept intended: the density of relationships. "Less-meaningful" did not seem to suggest the extremes on a continuum as desired. Perhaps a more descriptive terminology for the notions in this study can be found.

2. What is the role of non-meaningful learning? The notion of "non-meaningful" as indicated in the literature is ambiguous. It refers both to an initial learning without an understanding of existing relationships, as well as a resulting knowledge (an abstraction) understood without referents. Abstract learning, whether it is of a procedure, symbolic language, or other mathematical concepts, is the ultimate goal of mathematics instruction (Hiebert and Lefevre, 1986; Byers & Erlwanger, 1984). It is certainly desirable for students to separate two-ness, equality, and part-whole relationships from physical models. It is
from these abstractions that higher level mathematics can evolve.

This study speaks very strongly to the pedagogy required to develop this formal knowledge. The literature review for this study suggests abstractions should emerge from meaningful learning. Yet, teaching abstractly (non-meaningfully) hinders meaningful concept development.

A related question is whether all non-meaningful instruction interferes with subsequent meaningful learning. Despite the findings of this study, care must to be taken not to over-generalize. Is children's learning of number names through song or rote experiences detrimental to their conceptualization of number? Is their writing of symbols prior to number conservation appropriate? Should procedural learning of fraction computation (especially that of division) be delayed until a student constructs a personal procedure, or an understanding of a given procedure is accomplished? Is there a place for any initial non-meaningful learning? Further development of the relationship
between meaningful and non-meaningful, and their roles, is desired.

3. What instructional methods contribute to non-meaningful and meaningful learning? This research focused on memorization of definitions and procedures as a method for non-meaningful instruction. The meaningful instruction followed a sequence from concrete to less-concrete for the mathematics study, and a very sequential construction of a grammar structure for the generic study. These are but a few aspects of pedagogy.

Manipulatives provide means for aiding in the construction of meaningful concepts. Manipulatives are not an end in themselves, and if not used appropriately, could actually deter in the learning of mathematics (Bright, 1986). However, "Research findings from a number of studies indicate that lessons in which materials were used are more likely to provide achievement than lessons in which materials could have been used, but were not" (Suydam, 1987, p. 4). The meaningful instruction in the mathematics portion of this study moved gradually from the child's world of manipulatives
(student’s hands) "to the adult world of abstraction" (Baratta-Lorton, 1976, p. xiv). It incorporated the use of several manipulatives to assist the students in inductively arriving at concepts of area and perimeter measures. It was not the intent of this study to recommend a particular instructional task nor to suggest the one chosen was the most effective method available.

Perhaps the effectiveness of an approach is highly dependent on the concept to be developed. A renewed attention has been given to pedagogy in NCTM’s Curriculum and Evaluation Standards for Mathematics Education (1989).

4. Can cognitive interference be overcome? Both of these studies looked at relatively brief periods of meaningful instruction in comparison to the preceding non-meaningful instruction. Thus we have no way to assess the durability of the interference effect. We might anticipate that students exposed to non-meaningful instruction who show an initial resistance to learning from meaningful instruction would become even more resolute in their attachment to rote/routinized
methods. Alternatively, they might soon abandon their less productive learning modes in a sustained meaningful-learning environment. As more teachers begin to adopt the meaningful instructional practices recommended by the professional organizations, it will be useful for theoretical and empirical research to begin to address the longer-term consequences.

5. Are the notions "meaningful" and "non-meaningful" applicable to other disciplines? Language has its grammar; science its principles; government its philosophies; and music its theory. Perhaps these grammars, principles, philosophies, and theories also can be taught meaningfully or non-meaningfully. They are all abstractions of their disciplines, and the attainment of those abstractions is desired. The question is, how do we as educators enhance abstract constructions? Do the findings on interference in this research speak to other disciplines? The results of the generic study suggest they do. These are questions for educational theorists to further explore.
REFERENCES


APPENDIX A

GRAMMAR FOR GENERIC STUDY

container \(\equiv\) apparatus

cloth \(\uparrow\) measures

\(\rightarrow\) moves

\(\downarrow\) powers

\(\circ\) heats

\(\cdots\) cools

\(\mathbb{R}\) water

\(\uparrow\downarrow\) dries

\(\Delta\) controls

\(\mathbb{R}\) makes

\(\mathbb{R}\) electricity

\(\leftarrow\) writes

\(\leftarrow\) knowledge

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APPENDIX B

MEMORIZATION TASK - GENERIC STUDY

\[ \equiv \rightarrow \begin{pmatrix} \text{FAX machine} \\ \text{bus driver} \end{pmatrix} \]

\[ I \Delta \begin{pmatrix} \text{traffic} \\ \text{thermometer} \end{pmatrix} \]

\[ \equiv \uparrow \begin{pmatrix} \text{steam engine} \\ \text{crew member} \end{pmatrix} \]
APPENDIX C

OVERHEAD FOR MEANINGFUL GENERIC INSTRUCTION

(Words in parenthesis not written on overhead)

- Container
- Cloth
- Moves
- Powers
- Heats
- .....
- Cools
- Water
- Dries
- Controls
- Apparatus
- Measures
- Person
- Writes
- Knowledge
- Electricity
- Makes
- Telephone
- Call
- School
- Teacher
- Letter
- Message
- Fax Machine
- Building
- Thermometer
- Car Driver
- Crew Member
- Message
- Letter
- Fax Machine
- School
- Teacher
- Container
- Vehicle
- Automobile
- Cool Water
- Ship
- Towel
- Air Conditioner
- Building
- 

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16 \equiv \uparrow \ldots

15 \Delta \left( \frac{\partial}{\partial t} \right) \Delta \left( \frac{\partial}{\partial n} \right)

16 \Delta \left( \frac{O}{\omega_s} \right) \rightarrow (I)

17 \begin{array}{c}
\infty \\
\infty
\end{array}

18 \left( \frac{O}{\omega_s} \right)

19 \begin{array}{c}
\lambda \\
\lambda
\end{array}

20 F A H \text{ machine}

21 \text{crew member}

22 \text{steam engine}

23 \text{refrigerator}

24 \text{ship}

25 \text{school bus}
APPENDIX E

GENERIC POSTTEST ANSWER KEY

WITH EXAMPLES OF PARTIAL CREDIT

2 pts        5 pts        10 pts

1. \( \Delta \) makes electrician switch electricity

2. \( \odot \) heats air conditioner stove

3. \( \equiv \) power electricity motor

4. \( \Rightarrow \) knowledge writes letter moves

5. \( \bigcirc \) makes teacher teach knowledge

6. \( \rightarrow (\rightarrow) \) person moves postcards mail person

7. \( \rightarrow \) moves telephone fax-letter electricity

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8. \(\frac{C}{\omega} \rightarrow \{L\}\) moves people driver bus

9. \(\frac{G}{\text{A}}\) container teacher school of people

10. \(\equiv G \square\) make machine sewing cloth machine

11. \(\frac{O}{\omega} \rightarrow \{\Lambda\}\) moves post-office mail knowledge truck

12. \(\{\Omega\} \approx \{\Lambda\}\) person principal driving controls instructor

13. \(\{\omega\} \approx \{\nu\}\) hot water steam steam engine boat

14. \(\equiv \uparrow \ldots\) measures temperature thermometer heat

15. \(\Lambda \{L\}; \Lambda \{\alpha\}\) people rowboat captain
16. $I \Delta \left( \begin{array}{c} \circ \\ \rightarrow \\ \text{person} \end{array} \right)$

air-balloon

bus
driver

17. $\circ \quad \text{hot cloth}

paper towel
clothes
dryer

18. $\left( \begin{array}{c} \circ \\ \rightarrow \end{array} \right)$

container
car
traffic

19. $I \in \mathcal{A}$

knowledge
school
scientist

(Correct Response)

20. FAX machine

$\Rightarrow \left( \begin{array}{c} \circ \\ \rightarrow \end{array} \right)$

21. Crew member

$\Rightarrow \left( \begin{array}{c} \circ \\ \rightarrow \end{array} \right)$

22. Steam engine

$\left( \begin{array}{c} \circ \\ \rightarrow \end{array} \right)$

23. Refrigerator

$\circ \quad \circ \quad \cdots$

24. Ship

$\Rightarrow \left( \begin{array}{c} \circ \\ \rightarrow \end{array} \right)$

25. School bus

$\Rightarrow \left( \begin{array}{c} \circ \\ \rightarrow \end{array} \right)$

(Correct Response)
I, Theresa Martinez, am a certified elementary mathematics teacher in Ascension Parish with eighteen years of experience.

I observed Dolores Simoneaux during an experiment executed during the spring of 1992 at St. Amant Middle School with fifth grade students.

Dolores introduced lessons on perimeter and area by using two different methods. One group of students was taught perimeter and area through memorization and repetition. Each skill was introduced by presentation of the formula, repetition of problems by the students with the teacher, and follow-up practice in small groups. Ample time and attention was given to each skill. The process was fully explained before the next skill was introduced. Dolores interacted with the students and fully answered pertinent questions. She administered clear concise directions and periodic explanations. They were given reinforcement but no logical reasons or meanings for the formulas.
The second method was taught by hands-on activities and a logical approach to solving perimeter and area. These students were shown the why and the how behind the methods employed. The same techniques of using cooperative group interactions and of mastering one concept prior to introduction of another were used for this instructional method.

These two groups were taught the same skills and no preference was shown by the teacher as she interacted with the students. The students were unaware of the goal of the instruction. She gave each instruction as to what would occur and followed daily lesson plans.

Dolores met with twelve children in personal interviews to better familiarize herself with the students. I observed several of these interviews. Upon completion of the experiment a test was given and Dolores scheduled a follow up conference with the students previously interviewed. I observed several of her post conferences and the students interacted well with Mrs. Simoneaux.
I detected no bias in Dolores Simoneaux's treatment of the students during any part of the experiment.

Submitted April, 1992

Theresa Martinez
APPENDIX G
MATHEMATICS PRETEST

Name __________________________

1. Draw a rectangle. Label the length 8 feet and the width 5 feet. Find the area. _______

2. Area = ______ cm²

3. This room is 12 feet by 10 feet. How many square feet of carpet are needed for the room?

4. Area = ______ cm²

5. Perimeter = ______ cm

6. Area = ______ in²
Draw a square. If the perimeter is 36 inches, what is the measure of each side? ___________

Draw a square. If the area of this square is 36 cm², what is the measure of each side? ___________

This garden measures 8 feet on each side. If Bill wants to put a small fencing around the garden, how much fencing is needed? ___________
This small flower garden measures 6 feet on each side. How many square feet of garden are there?

Draw a rectangle with a perimeter of 12 miles. Label each side.

Perimeter = ______ mi

Draw a triangle that has a perimeter of 10 feet. Label each side.

Area of shaded part = ______ cm²

Perimeter = ______ m
Draw a square. Label each side 3 feet. Find the area.

Area = ___ in\(^2\)

\[\text{Area} = \text{___ ft}^2\]
Draw a rectangle. Label the width 2 miles and the length 3 miles. Find the perimeter.

\[ \text{Area} = \text{yd}^2 \]

\[ \text{Area} = \text{cm}^2 \]

\[ \text{Perimeter} = \text{mi} \]

\[ \text{Perimeter} = \text{cm} \]

Draw a triangle that has an area of 4 square feet. Label the sides.
Draw a rectangle with an area of 20 ft². Label the sides.

Area = ___ cm²

Draw a square. Label each side 2 miles. What is the perimeter?

Perimeter = ___ in.

This room is 15 feet by 10 feet. Wanda wants to put carpet stripping all around the room. How many feet of stripping does she need?
APPENDIX H

INTERVIEW 1 QUESTIONS

1. Tell me about school for you.
2. What is your favorite subject? Why?
3. What is your least favorite subject? Why?
4. How do you learn best?
5. What do your parents think about school?
6. What do they do at home to help you with your studies?
7. What is math?
8. How do you feel about math?
9. How do you study math?
10. What does your teacher do that most helps you learn math?
11. What do your parents do to help you learn math?
12. What do you like about math?
13. What do you not like about math?
14. What area of math do you like most? Least? Why?
15. For what subject do you have to think the hardest?
16. Do you do any kind of math at home besides homework? Tell me about it.
APPENDIX I

LESSON PLANS FOR MATHEMATICS

NON-MEANINGFUL INSTRUCTION

Objective: The students will be able to find the perimeters and areas of squares, rectangles, triangles and parallelograms using formulas.

Day 1:
Objective: The student will be able to find the perimeter of squares and parallelograms using formulas.

Materials: Prepared transparencies, overhead pens, blank transparencies, note pads for each student

I. Introduction

   A. Discuss definitions for area (measure of the space inside a plane shape) and perimeter (measure of the distance around a shape).

   B. Display "Perimeter and Area Formulas" transparency (perimeter and area formulas for square, rectangles, triangles and parallelograms).

   C. Have students copy formulas onto their note pads.
II. Squares

A. Write formula, \( P = 4s \), on overhead. Discuss meaning of "4s" as 4 times \( s \); "s" representing the length of a side.

B. Have students write the formula 10 times.

C. Draw a square; label measure of side 5.

D. Display "Square - Class Activity" transparency. Work problems as whole class activity, substituting assigned values for the variables in the formula.

E. Display "Square - Individual Practice" transparency. Have students work problems individually. Check work.

III. Perimeter of parallelograms

A. Write formula, \( P = 2(b + s) \), on overhead. Discuss operations implied by formula and that "b" represents measure of base and "s" represents measure of vertically diagonal side.

B. Have students write formula 10 times.

C. Draw a parallelogram; label side 6 and base 10. Demonstrate working problem using formula.
D. Display "Parallelogram - Class Activity" transparency. Work problems as whole class activity.

E. Display "Parallelogram - Individual Practice" transparency. Have students work individually. Check work.

IV. Practice finding perimeters of squares and parallelograms. Working in small groups have students work "Perimeter of Squares and parallelogram" worksheet; check work.

Day 2:

Objective: The students will find the perimeter of rectangles and triangles using formulas.

Materials: Prepared transparencies, overhead pens, blank transparencies, note pads

I. Introduction

A. Distribute note pads.

B. Use flash cards and have students practice the formulas for perimeter of squares and parallelograms, identifying purpose of each formula.
C. Form small groups; have students work the "Perimeter of Squares and Parallelograms" page.

II. Rectangles

A. Write formula, \( P = 2(l + w) \), on overhead. Discuss operations implied by formula and that "l" represents the measure of the length and "w" the width of the rectangle.

B. Have students write the formula 10 times.

C. Draw a rectangle, label the length 4 and the width 3. Model working problems using the formula.

D. Display "Rectangle - Class Activity" transparency. Work problems finding perimeters as whole-class activity.

E. Display "Rectangle - Individual Practice" transparency. Have students work problems individually finding perimeters. Check work.

III. Perimeters of triangles

A. Write formula, \( P = a + b + c \), on overhead. Discuss operation implied by the formula
and that "a, b, c" represent the lengths of the sides of the triangle.

B. Have students write the formula 10 times.

C. Draw a triangle, label the sides 1, 2, and 3. Model working the problem using the formula.

D. Display "Triangle - Class Activity" transparency. Work problems as whole-class activity.

E. Display "Triangle - Individual Practice" transparency. Have students work individually finding perimeters. Check work.

IV. Practice finding the perimeters of squares, parallelograms, rectangles, and triangles. Working in small groups have students work "Perimeter of Squares, Parallelograms, Rectangles, and Triangles" worksheet. Check work.

**Day 3:**

Objective: The students will review perimeter formulas and find the area of squares using a formula.
Materials: Prepared transparencies, overhead pens, blank transparencies, note pads

I. Review
   A. Have students complete "Perimeter - Squares, Parallelograms, Rectangles and Triangles" worksheet.
   B. Use flash cards to practice identifying and stating formulas.

II. Area of squares
   A. Review definition for area: space inside a figure.
   B. Give formula for area of square: \( A = s^2 \).
      Explain operations implied. Work one example. Have students write the formula 3 times.
   C. Display "Square - Class Activity" and work 3 problems as a whole-class activity.
      Have students work the rest of the problems individually, checking and assisting the students in their cooperative groups. Check work.
Day 4:
Objective: The student will find the areas of rectangles and triangles.
Materials: Prepared transparencies, overhead pens, blank transparencies, note pads

I. Introduction
    Review formula for area of a square. Practice one problem.

II. Area of rectangles
    A. Give formula for area of rectangle: \( A = lw \). Explain operation implied. Work one example.
    Have students write the formula three times.
    B. Display "Rectangle - Class Activity" and work three problems as a class group.
    Have students work the rest of the problems individually, checking and assisting the students in their cooperative groups. Check work.

III. Area of triangles
    A. Give formula for area of rectangle: \( A = \frac{1}{2}bh \). Explain operations implied. Work one example. Have students write the formula three times.
B. Display "Triangles - Class Activity" and work three problems as a class group. Have students work the rest of the problems individually, checking and assisting the students in their cooperative groups. Check work.

IV. Review
If time permits, have cooperative groups begin work on "Area of Squares, Rectangles and Triangles" worksheet.

Day 5:
Objective: The students will find the area of parallelograms and review all perimeter and area formulas.
Materials: Prepared transparencies, overhead pens, blank transparencies, note pads

I. Introduction
Review area of squares, triangles, and rectangles using flash cards.

II. Area of parallelograms
A. Give formula for area of parallelogram, A = bh. Explain operations implied. Work
one example. Have students write the formula three times.

B. Display "Parallelograms - Class Activity" and work three problems as a class group. Have students work the rest of the problems individually, checking and assisting the students in their cooperative groups.

III. Review

A. Have students work individually on review worksheet for 10 minutes.

B. Students are to work in cooperative groups for 10 minutes to complete worksheet.

C. Students are to use their class notes to check their work and make corrections.

D. Compare answers and discuss.
APPENDIX J

MATHEMATICS POSTTEST

Name ____________________
Hour ____________________

Post-test:
Date ____________________

Area = __________ cm²

Area = __________ cm²

Area = __________ cm²

Area = __________ cm²
APPENDIX K

INTERVIEW 2 QUESTIONS

(N-M group only)
1. How did you like the lessons we did?
2. In what ways was it different from your regular math class?
3. In what ways was it the same?
4. We did a lot of memorizing in the classes, how did you help yourself remember?
5. If I had a formula that looked like this, "C = ab," and I told you a value for "a" and a value for "b", could you find the value for C? What does "ab" mean for you to do?

(N-M and M-O groups)
1. What is perimeter?
2. How do we get the perimeter of something?
3. Do you know any other way to get perimeter?
4. Why do we need to learn perimeter?
5. What is area?
6. How do we calculate area?
7. Why do we need to learn area?
8. For what kind of jobs do we need to know area?
9. How would you find the area of this room?
10. How would you find the perimeter of this room?
11. If I gave you the measure of each side of this figure, could you find the area? Perimeter? How?
APPENDIX L

LESSON PLANS FOR MATHEMATICS

MEANINGFUL INSTRUCTION

Objective: The students will be able to find the perimeters and areas of squares, rectangles, triangles and parallelograms.

Day 1:
Objective: The students will be able to find the perimeters and areas of squares and rectangles.
Materials: Decimeter square tiles, inch tiles, blank transparencies and pens

I. Discuss definitions for area (measure of the space inside a plane shape) and perimeter (measure of the distance around a shape).

II. Working in cooperative groups, have students use their hands to discover the number of hands necessary to cover their desk tops. Compare and discuss answers in large group. Relate their findings to concept of area.

III. Working in cooperative groups, have students find the perimeter of their desk tops in hand units. Compare and discuss answers in large group. Relate their findings to concept of perimeter.
IV. Discuss need for standard measures.

V. Using five decimeter square tiles for reference, have each group of students estimate and record the area and perimeter of their desk tops in decimeter squares. Discuss findings.

VI. Direct student, using inch tiles, to:

Make 1X4 rectangle; find perimeter; find area.
Double the length; find perimeter; find area.
Double the width of new rectangle; find perimeter; find area.
Rearrange tiles to make a square; find perimeter; find area.

Day 2:

Objective: The students will be able to find the perimeters and areas of squares and rectangles.

Materials: Centimeter grid paper; overhead grid paper; geoboards, overhead geoboard, geo-bands, scissors, glue, blank transparencies and pens, note pads for each student

I. Direct students, using geoboards, to:

Make shortest segment possible; length = 1 unit.
Make smallest square possible; area = 1 square unit; find perimeter.
Make 2X3 rectangle; find perimeter; find area.
Make 3X3 square; find perimeter; find area.
Make largest square possible; find area; find perimeter.
Consider what the area of a 5X5 square would be? Its perimeter?

II. Using grid paper:
On overhead draw 3X4 rectangle; find perimeter; find area.
Have students draw, cut, paste into note pads and record the areas and perimeters of:
   3X5 rectangle;
   6X6 rectangle;
   5X10 rectangle.

III. Use blank transparency:
Model and have students draw a rectangle labeling dimensions 2 and 4. Draw in square units. Find area. Find perimeter.
Model and have students draw a square labeling dimension of 3. Draw in the square units of
area. Determine the area. Determine the perimeter.
Draw a rectangle labeling dimensions 7 and 8; find the area; find the perimeter.
Draw a square labeling dimensions 10; find the area; find perimeter.

Day 3:
Objective: The students will find the areas and perimeters of triangles and parallelograms.
Materials: Geoboards, geo-bands, overhead geoboard, blank transparencies and pens, grid paper, grid transparency, scissors
I. Review:
Have students draw a 3X4 rectangle; draw in grids; find area; find perimeter.
Teacher draws a 4X8 rectangle. Do not draw in grids. Have students try to find area and perimeter.
II. Using geoboard:
Make 1X1 region; find area. Make diagonal; discuss area of each triangle.
Make 2X2 region; find the area. Make diagonal; find area of each triangle.
Make 1X2 region; find area. Make diagonal; area of each triangle.
Make 1X4 right triangle; find area; verify.
Make 1X2 right triangle; find area; verify.
Make 2X3 right triangle; find area; verify.

III. Using blank overhead transparency:
Draw right triangle labeling vase and height 4 and 4, respectively. Find area. Verify by enclosing in a square and halving that area.
Draw in grids if necessary.
Draw and label right triangle 10 and 4; find area; verify.
Draw right triangle labeling dimensions 5, 4 and 3; find area; find perimeter.

IV. Parallelograms:
On overhead geoboard make 2X2 rectangle; find area. Move band over to make parallelogram that is not a rectangle; discuss area.
Have students make 2X3 rectangle on geoboards; find area. Move band over to make parallelogram that is not a rectangle; discuss area.
Draw diagonal from bottom corner to first vertical grid on centimeter grid paper. Cut and move cut corner to opposite end of rectangle to make a parallelogram; find area.
APPENDIX M
MATHEMATICS POSTTEST 2

Name ____________________  
Hour ________________

1. Draw a rectangle below. Label the length 8 feet and the width 5 feet. Find the area. ________ ft²

2. This room is 12 feet by 10 feet. How many square feet of carpet are needed for the room?

3. __________

4. __________

5. __________

6. __________  

Post test 2

Perimeter = _______ cm

Area = _______ cm²

Perimeter = _______ cm

Area = _______ in²
1. Draw a square below. If the perimeter is 36 inches, what is the measure of each side? __________ inches

2. Draw a square below. If the area of this square is 36 cm², what is the measure of each side? __________ cm

3. This garden measures 8 feet on each side. If Bill wants to put a small fencing around the garden, how much fencing is needed? __________ ft

4. 5 cm

5 cm

10 cm 10 cm

6 cm Perimeter = __________ cm
This small flower garden measures 6 feet on each side. How many square feet of garden are there?

A rectangle has a perimeter of 12 miles. Sketch this rectangle below and label each side.

Draw a triangle below that has a perimeter of 10 feet. Label the sides.

The area of the shaded part is __ cm².
Draw a square below. Label each side 3 feet. Find the area.

**Area = ___ ft²**

**Perimeter = ___ in.**
Name ______________________

Hour ______________________

25) Draw a rectangle below. Label the width 2 miles and the length 3 miles. Find the perimeter.

Perimeter = ______ miles

26) 8 yd.

Area = ______ yd²

27) 8 cm

Area = ______ cm²

28) 5 cm

4 cm

Perimeter = ______ cm

29) 3 cm

2 cm

Area = ______ cm²

30) 5 mi

Perimeter = ______ mi

A rectangle has an area of 20 ft². Sketch this rectangle below and label each side.
32. Draw a square below. Label each side 2 miles. What is the perimeter? _______ miles

33. This room is 15 feet by 10 feet. Wanda wants to put baseboards all around the room. How many feet of baseboards does she need? _______ ft.

34. Draw a square below. Label each side 5 cm. What is the perimeter? _______ cm.

35. 4 ft. 3 ft.

Perimeter = _______ ft.
APPENDIX N

INTERVIEW 3 QUESTIONS

1. a) How did you feel about this week’s classes?
   (1.b through 1.e was only for N-M group)
   b) Did you like the first part with formulas or this part better?
   c) Which one was easier? Why?
   d) Which one did you enjoy more? Why?
   e) In which one did you feel you learned more? Why do you think that’s so?

2. a) What did you like most in this week’s lessons? Why?
    b) Is there anything you didn’t like? Why?
    c) Were these classes different from your regular math classes? In what way?
    d) How did using the manipulatives help you learn?

3. a) What is area?
    b) How do you find the area of a region?
    c) Why do we learn area?
    d) What kinds of people need to know how to find area?

4. a) What is perimeter?
b) How do we find the perimeter of a region?
c) Why do we learn perimeter?
d) For what kinds of jobs do we need to know perimeter?
e) How would you find the area of this room?
f) How would you find the perimeter?

5. a) To know how much tile to buy for this room, do you need to know the area? Do you need to know the perimeter?
b) To find how much base board to buy for this room, do you need to know the area? Do you need to know the perimeter?

6. a) Draw some shape and explain to me how to get the area of that shape.
b) What is the perimeter of the shape you drew?

7. Look at the student's posttest and have him or her explain how some of the answers were obtained if the method is not obvious.
APPENDIX O
DESCRIPTION OF PRE- AND POSTTEST CONSTRUCTION

Items were designed for the mathematics pre- and posttests to include the following categories:

1. Area and perimeter of squares, rectangles, triangles, parallelograms and irregular shapes

2. Area and perimeter problems that could have been solved using formulas
   a. Problems with only necessary measures and including a drawing (items 2, 5, 9, 27, 30, 32, 35, 37)
   b. Word problems with only necessary measures and including a drawing (items 3, 11, 13, 36)
   c. Word problems with only necessary measures with no drawing (items 1, 18, 22, 25, 34)

3. Area and perimeter problems that could not have been solved using formulas
   a. Irregular shape with area divided into regular shapes (item 20)
   b. Irregular shape without area divided into regular shapes (item 15)
c. Problems with extra information included
   (4, 6, 8, 12, 14, 19, 21, 24, 26, 29, 33)

d. Problems requiring student to derive some necessary dimension (items 7, 10, 16, 17, 23, 28, 31)
APPENDIX P

INTERVIEW 3 TRANSCRIPTIONS

Student A (treatment 1, level 1, female, class 1)

T: Okey, (Student A), how do you feel about this past week's classes?
S: I liked it.
T: You liked it. Did you like this part better or the part where we studied formulas better?
S: Well, both the same.
T: Which one was easier?
S: Both about the same.
T: Okey, but which one of the instructions was easier. When I was helping you memorize formulas or when we were working with all the geoboards and grid paper?
S: Geoboards.
T: That was easier? Which one did you enjoy more? The formulas or working with geoboards and grid paper?
A: Geoboards and grid paper.
T: And with which one did you feel like you learned more?
S: With the formulas.
T: Why do you think that you learned more with that?
S: Cause you explained it to us. Like on the geoboard you had fun and everything doing it.
T: But you didn't feel like you understood as much.
S: Yes, Ma'am.
T: All right. What did you like most about this week's lessons? The one with the whole class together?
S: We got to work in groups and we got to understand what each other was thinking about the problem.
T: So you liked working in groups a lot.
S: Yes.
T: Is there anything you didn't like about this week's instruction?
S: No, Ma'am.
T: Maybe that you didn't feel like you understood as much?
S: No. I pretty much understood.
T: Okey. Were the classes we had this week different from your regular math classes that you have with your regular teacher?
S: Yes, Ma'am.
T: How were they different?
S: 'Cause with her we don't get to work on geoboards and stuff and we had a different teacher come in and she taught you different stuff. We're not working in our textbooks.
T: So it's different than your other classes. How did using the manipulatives, by manipulatives I mean the geoboards, the tiles, the hands and all of that, how did that help you learn? Or did it? Do you feel like that helped you learn?
S: Yes, Ma'am.
T: How did that help you learn?
S: Because maybe we got to learn how we can put them together. Like some people said they couldn't do it and they got to learn that they could do it. And we had to count all the measures with them and stuff.
T: So you found that they were beneficial.
S: Yes.
T: What is area?
S: You ask me that question again. The area is seeing how much length and all that is inside of a square or a rectangle or something.
T: Okey. So it's inside? How do you find the area of a region or shape? How do you get it?
S: Well there's two ways. You taught us one where you can count the stuff. But you said something that all of them aren't always going to have the squares, and you taught us another way. If they tell you how much it is, sometimes you times it and stuff and get the inside of it, the numbers.
T: Why do we have to learn about area? I asked you that question before also, right?
S: Yes, Ma'am.
T: Do you have any idea now? Why we study area? Why is that in your math book?
S: Because you have to learn the area of something; like if somebody asks you. Well they couldn't figure it out, and they ask you like what the area of this is.
T: Why would they want to know? Why would anyone want to know the area of something?
S: I don’t know.
T: Can you think of any kind of job or any kinds of people that need to know area?
S: Same thing I told you last time.
T: Which was...
S: Carpenter.
T: Why does a carpenter need area?
S: If the lady says, "I want this area of this room. The such and such length." He needs to know how much squares. Like you’re doing the kitchen or something you have these squares, you want to know how much squares you need for the carpet, I mean for this bathroom. And he has to make sure he knows all the area so he can get the right amount.
T: For what in that bathroom does he need the area?
S: The inside of it, the floor.
T: Is there anything else in that room that you might need to know the area for?
S: The ceiling.
T: Anything else?
S: No, Ma’am.
T: The ceiling or the floor. What is perimeter?
S: Perimeter is the outside. The outside, the outside of a house. Like if you’re building a house. You need to know how much wood you need to put up, so you can make the house just right. Like the person said. Like if you took a square, the inside of it is the area but the outside is the perimeter.
T: How do we find the perimeter of a region?
S: Counting by the things. Like I told you for the area. Sometimes you could count the width of it and the length.
T: If you see the grids you could count them. Okey.
S: And you could times the letters again.
T: Okey. You’re pretty sure about that? So if I told you that this desk is 2 feet by 3, and if I asked you to get the perimeter, what would you do with those numbers?
S: Times them.
T: Times them. Okey. Why do we learn perimeter?
S: I don't know.
T: Can you think of any people or any kind of job that may need to know how to get perimeter?
S: A school teacher.
T: That's right. Okey. Outside of a school teacher?
S: She has to teach the children how to...
T: I said besides a teacher. Is there any other profession, any other job? Does your mother ever need to know perimeter of anything?
S: Maybe.
T: For what?
S: Like she has to work in the rooms. And people, when I went to one. When I was in the hospital I had pneumonia and they said this room is such and such so I need such and such tall thing. And then you have to know; and then she has to figure out. Well, if this is the size of the room I have to figure out the same height of it and she has to know how tall.
T: So to know the height of something, she has to know perimeter?
S: Yes, Ma'am.
T: How do those two relate? Is the height of something its perimeter?
S: Yes, Ma'am.
T: Okey. How would you find the area of this room? What would you need and how would you go about it?
S: Well, in which way?
T: I don't know what way. What if I ask you to get the area of this wall, could you do that?
S: Maybe.
T: Is that possible?
S: Maybe.
T: How would you do it?
S: Well you tell me how wide it is and how long it is. And you put those 2 numbers together.
T: So if I tell you that this wall is 8 feet high and about 12 feet long. What do yo do with those 2 numbers?
S: Times them.
T: Okey. Could you get the perimeter of that wall?
S: I guess.
T: How would you get the perimeter?
S: Same thing like... No. I don’t know.
T: You’re not sure?
S: (Sakes head negatively.)
T: What if I’m trying to figure out how much of
that black little rubber baseboard I need for
the room. Would that be the area or the
perimeter if I need to figure out how much I
need to purchase?
S: Black what?
T: Okey, see the black little baseboard around the
room?
S: (Nods head.)
T: Okey, to know how much I need; if I tell you
how much I need, am I telling you the perimeter
of this room or am I telling you the area of
this room?
S: You’re telling me the area.
T: Can you explain that at all or do you just feel
like it’s just the area?
S: Because it’s the inside of the room. Oh, okey,
I know now. You cannot get the area by doing
the wall. That’s your perimeter of the wall.
T: You cannot get the area of the wall. That’s
part of the perimeter?
S: That’s part of the perimeter. And that’s the
area down there.
T: So I could get the area of the floor?
S: Yes, Ma’am.
T: But not of the wall?
S: Not of the wall.
T: Okey. I’ll give you some paper. I want you to
draw any shape. And then I want you to figure
out the area and perimeter of that. Give it
some measurements and tell me how you get the
area and how you get the perimeter.
S: I think the measurements?
T: Yes.
S: Okey, let’s see.
T: So one side’s 2 and one side’s 6. Get the area
of that space for me, and get the perimeter of
it for me, please.
S: Can I draw a little grid?
T: Whatever you need to do to get the area and
perimeter.
S: Oh, this side has to be bigger. I’m sorry.
T: Why?
S: Because that would look like a funny looking room.
T: Okey.
S: (Draws grid; counts.) Eighteen.
T: Eighteen is the what?
S: Perimeter.
T: Why don't you put $P = 18$ on that paper for me. All right. What is the area?
S: (Counts.) Eighteen.
T: Okey, so the area is also 18?
S: It shouldn't be.
T: Could it be?
S: It could. I guess. (Recounts.) I guess it could.
T: All right. In the very first week when I instructed you we had a formula for the perimeter of a rectangle. And we said that $P$ was equal to $2(1 + w)$. Can you tell me why mathematicians would give this formula to find perimeter of a rectangle? Why would those things together give us the perimeter?
S: The length and the width.
T: Okey, what about them?
S: Well, the length is 2 and the width is 2. But the 1 means length and w means width.
T: So is what we do with the 1 and the w here make any sense to how we got the perimeter here? Do you see why they came up with that formula? Why they didn't come up with another one?
S: No.
T: No. Let me ask you another formula. I gave you the formula for area of a triangle. I told you that area of a triangle is equal to $1/2bh$. Can you explain to me why anybody made up this formula to get the area of a triangle.
S: If you take a triangle, let me draw a triangle. Okey, I'm going to try this. When you take the numbers and you add them together, you take one half away from it. Like this is 4 and this is 2. I'm just giving a number. Four times 2 is 8 and half of 8 is 4. So it will be 4.
T: So the area is 4.
S: (Nods head.)
T: So the formula makes some sense.
S: Yes, Ma'am. That one does.
T: All right. Thank you very much. I want to ask you maybe one question on your test. On this side, number 3. It said this room is 12 feet by 10 feet. How many square feet of carpet are needed for the room? Can you explain to me how you got 10? I'm not telling that that's right or wrong. I just want you to explain it to me. Because you didn't show me your work here. You kind of scratched it out.


T: Okey, let's look at one more. Let's see if you remember. This is 2 days ago so you may not. All right. This garden. This garden measures 6 feet on each side. If Bill wants to put a small fence around the garden. How much fence does he need?

S: I times 6 and 6. Wait. That's 8? I thought that was an 8.

T: You thought that was an 8; so you multiplied 8 times 8 and got 68?

S: (Nods head.)

T: Okey, all right. So it looked like an 8 to you. Let me write that here. And maybe one more. What about this perimeter? How did you get this perimeter?

S: I added these up together, 10, 20, 25.

T: You have 35. What do you think it should have been?

S: Thirty.

T: You think it should have been 30. So you just added the distances around. All right. Let's see if there was one more. Yes, this one. I gave you this triangle and asked you to find the area. And you told me 3.

S: Which one, this one?

T: Yes. The area of the shaded part.

S: Half of 6 is 3.

T: Okey. I thought so. I thought that's how you got it but I wasn't sure. Okey. Thank you.
Student B (treatment 1, level 2, female, class 1)

T: Hi! I appreciate your coming out of P.E. This is more fun than that, isn’t it? Is it more fun?
S: Sometime.
T: Sometimes. Okay. How did you feel about this past week’s lessons, (Student B)?
S: It was fun.
T: Okay. You were at both lessons. The one we did the formula for and the other one. All right. Did you like the first one we did with the formulas, or did you like the one this past week with all of the objects? Which one did you like better?
S: The second one.
T: The second one with all of the objects. Why do you think you liked it better?
S: Because it’s just easier to learn.
T: It was easier to learn. Okay. My next question was which one was easier. It was easier learning with the objects? Okay. Which one did you enjoy more?
S: The second one.
T: Did you feel like you learned the formulas pretty well?
S: Yes, Ma’am.
T: Did you use the formulas on your test Friday?
S: Yes, Ma’am.
T: Which one do you think you used more; the formulas to answer the questions or drawing squares inside the figures?
S: Drawing.
T: You did more drawing than you did formula, but you did use formulas? Okay. With which one of those kinds of lessons do you feel you learned more; more with the formulas or did you learn more with the objects, the manipulatives?
S: Formulas.
T: You learned more with the formulas. Why do you think that you learned more with that? Can you explain? You don’t know, but you felt like you learned more?
S: Yes.
T: Is there anything you didn’t like about this past week’s lessons? We worked with the hands, getting perimeter and area, and then we worked
with tiles, and we worked with grid paper that we cut out, and we worked with geoboards. Can't remember it. Any of those activities we did that you really didn’t like?

S: No.

T: You liked it all. Were these classes this past week very different from your usual classes or were they kind of the same?

S: They were different.

T: They were different. Are your normal classes that you have in math more like the first week’s lessons or more like this past week’s lessons?

S: The first.

T: More like the first week. Why would you say that?

S: Because she uses an overhead a lot.

T: Like I did.

S: Yes. Sometimes we use special stuff to do the math, but not a lot.

T: But more overhead like I did the first week? Okay. How did using those manipulatives help you learn? Of course you said you learned more with the formulas, but let’s just go back to last week’s lessons when we used a lot of manipulatives, a lot of things with our hands. How did you feel those helped you learn?

S: It was easier to find out the area.

T: Which one was more confusing, the formulas or the manipulatives?

S: I guess the formulas.

T: You think the formulas were more confusing. Why do you think that was so?

S: Well, it wasn’t that confusing, but...

T: Just a little bit more?

S: Yes.

T: I got you. What is area?

S: It’s the part that’s inside of an object of a perimeter.

T: Of a figure, okay. How do you find the area of a region?

S: You get the two numbers beside it and multiply them.

T: Of almost any figure? Okay. Why do we learn area?
S: In case you have like a job that needs to find it.
T: What kind of job do you think needs to use area?
S: Carpenter.
T: Why does a carpenter need area? For what in particular does he need to know area?
S: Like find out; like if he has to put carpet in.
T: So if he wants to put carpet in a room, he needs to know the area of that room, right? That's what you're saying. Okay. What about for the wall? What if I wanted to do something with this wall, would area in any way help that carpenter?
S: Not really. You need to find the perimeter.
T: If it's the wall, he needs a perimeter? Okay. You would say to paint the wall, he would need the perimeter and not the area is what you're saying?
S: Yes.
T: All right. So carpenter. Can you think of any job or any other part of a carpenter's job that would need to know about area?
S: Well, like if he was putting furniture in, he would need to know where to put it so it wouldn't be all smashed together.
T: So area would help him with that also? Okay. Try to think of any other job. Do you think your mother or father ever use area at home for anything?
S: Yes.
T: For what?
S: For gardening.
T: All right. Tell me some more about that. What about gardening would they have to know the area for?
S: Like you have to roto-till a garden, you know, so much room. You know, you don't want it all tilled so you can't walk around.
T: So knowing the area would help that? Okay. Would perimeter help that also, the gardening? What would the perimeter help with?
S: Like if you were planting them in there, you would have to find out how much space. So you know how many plants you would plant in it.
T: And perimeter would help you to know that. Can you tell me some more about that? Can you explain that a little bit? We'll come back to that. What do I mean by perimeter?
S: The outer part of the area.
T: Why do we have to learn perimeter, besides for gardening?
S: Like if you’re putting baseboard on it.
T: Okay. Baseboards around the room. The black baseboards there under the shelves that go around; if I need to know how much baseboard I need for this room, would it help me to know the perimeter or help me to know the area of the room, to know how much baseboard to buy?
S: The perimeter.
T: The perimeter. All right. What if I want to know if I’m going to do a trim around this wall?
S: If you done that.
T: I still need the perimeter? Okay. And to paint the wall, I need to know how much paint I need.
S: Perimeter.
T: Perimeter for that also. Okay. But the carpet, you think that’s area, right? Okay. I wonder if you know anyone who sews? Do you think a seamstress ever needs to know area and perimeter?
S: Yes.
T: Tell me about that.
S: Like if you have to fit somebody, you got to know how big to make it or how small to make it.
T: So the measure, if I have to take off two inches, is that more perimeter or is that more area?
S: Area.
T: That’s area. Okay. Can you think of anything else that you would need perimeter for? No. Okay. So we decided we needed perimeter and area for gardening, a carpenter needs it, maybe for sewing; can you think of any other job? Okay. How do you find the area of this room? If I want you to get the area of this room?
S: You would add the sides together.
T: What do you mean be sides. What if this room measures eight feet by twelve feet and it's eight feet high?
S: You would have to like find like $8 + 8 = 16 + 12$ and plus another 12 for the other side.
T: Okay. That would give you the area? All right. Can you tell me how to get the perimeter in this room?
S: You multiply.
T: If I want to know how much tile to buy for the floor in this room, would I need to know the area or would I need to know the perimeter for that?
S: The area.
T: The area for that. We talked about the baseboards. I want you to draw on this paper any shape you want and explain how to get the area of that shape. Give it some dimensions, some measures, tell me.
S: You would times three (3 x 4) because it's four sides and you tell...
T: And that would give you the area? Okay. So you're saying that all the sides measured three, because it's in a square. All right. How would I get the perimeter of it? Why don't you put $a = 12$ just so I don't forget. How would I get the perimeter of that figure?
S: $3 + 3 + 3 + 3$.
T: I'm a little confused. Tell me what you're thinking. You'd get 12. So in this case, the area and perimeter can be the same number. That's possible. So you'd have $3 + 3 + 3 + 3$, right? Okay. When you were in that first week of lessons with us, you got the perimeter of a rectangle, now this was a square. For perimeter of a rectangle we'd take $2(l + w)$. Can you figure out why a mathematician, somebody, would decide that that's what you do to get perimeter? Why does that formula work? Explain that.
S: Easier to remember. It's just easier to do.
T: Why this formula? Why doesn't he make another formula for perimeter?
S: Because then it's like the length times the width. That's what they stand for.
T: So this means the length times the width. What about that 2?
S: Going to multiply it by 2.
T: When we wanted an area of a triangle, the formula for the area of the triangle was \( \frac{1}{2}bh \). What does all that mean? Why do we have this formula for getting the answer?
S: Because you’ve got to half it and then you got to times the two numbers right by each other and then all you do is multiply it.
T: So you multiply these two numbers, and then you take half of it. Why does this formula work for a triangle? Why does it work? Would it work for a rectangle? Why not?
S: Because there’s three sides on a triangle and four sides on a rectangle.
T: Okay. There are three sides; therefore, this formula works. What’s the connection between this formula and three sides? Tell me some more. I’m trying to figure out what’s happening in your head, okay? Just tell me all that you can.
S: I don’t know.
T: Do you think somebody just kind of made it up or do you think it has to mean something? Do you think you could have made that up if nobody would have ever given it to you?
S: I guess.
T: You guess. Okay. Tell me some more. Why do you think that formula fits with area of the triangle?
S: Because you have to multiply the perimeter here and add it.
T: I understand a little bit, but I’m going to ask you a few more questions. For a parallelogram I wouldn’t do this, right? Or would I?
S: You would do it.
T: I wouldn’t do this one, right? Why not?
S: It has a different formula.
T: It just has a different formula, right? Do you know why the other one has to be given a parallelogram or why this is a triangle? It just works, or is there another reason?
S: Because of the numbers because there’s three numbers.
T: Okay. So there are three numbers here, and the triangle has three sides. Does that make sense to you?
S: (Nods head.)
T: All right. I want to look at your test. There were just a couple of items that I wasn’t sure about how you arrived at your answer. I’m not telling you whether your answer is correct or not correct, but I want you to explain to me how you arrived at it. How did you get 20 from this? Think about it.
S: (No response.)
T: You were looking for area there. This was last week, Thursday, so you may not remember, but just try to remember how you got 20.
S: Because 2 x 6 = 12 + 12 because it’s the same size is 24 and then...
T: Where did you get 24?
S: Because 2 x 6 and this was the same as these two sides and these two sides are the same.
T: Okay. So you doubled the 12, then you got 24.
S: And then put this over.
T: That’s more than 20. Do you remember how you got 20? Maybe you made a mistake and maybe not.
S: Yes.
T: If you were to get the area right now, how would you do it?
S: I would take 2 x 6 = 12 and this would be the same so it’s 24 and 2 x 4 = 8 + 2; it would be 20.
T: 8 and 2 + your 24; you’re adding all of that together and got 33. So it (33) would be a better answer than what you’ve got? All right. Let’s look at another one. For this one, I gave you a square, number 18, shaded in half of that square, and told you that one side of that square measured six centimeters. You found the area. Do you remember how you got that area?
S: It would be, the whole thing all together would be 36 because 4 x 6 = 24 and half the 24 is 12 so I think it should be 12.
T: So you think it should have been 12 and not 18? Okay. We’ll do one more. You’re looking for a perimeter of that irregular shape, how did you get 24 there?
S: I added $6 + 6 = 12 + 4 = 16 + 3 = 19 + 5 = 24$. It would be 24.
T: And you didn't add this four, why not?
S: To find the perimeter it's just the outer edges.
T: Okay. All right. Good. You've got 8 here. I know it's kind of an interesting shape. Do you have any recall as to how you got 8 for number 28?
S: Yes. Make a long line so it's, you know, two rectangles and $1 + 1 = 2 + 2 + 2 + 5 = 11$. And that would be $1 + 12 + 3 = 15 + 3$ is 18. So maybe that should be 18 instead of 8.
T: Okay. I thank you very much. I appreciate you working with me for this experiment. Thank you much.
Student C (treatment 1, level 2, female, class 6)
T: You were with me for all of the sessions we had, right?
S: Yes, ma'am.
T: The first and the second part. How do you feel about this past week's classes; that we had last week? Did you like them?
S: Uh-huh.
T: Did you like them better than the part when we had formulas or did you like the formulas better?
S: Formulas.
T: You like the formulas better. Why?
S: It's just so easy to remember.
T: Which one did you enjoy more?
S: Formulas.
T: Formulas. Do you know why? Just because it's easier or is there another reason?
S: I don't know. It's just easier, and it's more fun.
T: You enjoyed the formulas better. All right. In which one did you feel like you learned more?
S: It was quiet.
T: I'm saying did you learn more when you learned the formulas, or did you learn more when you were using the manipulatives, all the tiles and geoboards.
S: Formulas.
T: You learned more with the formulas. All right. What did you like most about this past week's classes where we used all of the materials, all of the tiles and geoboards and all? What did you like most about that?
S: It was fun.
T: What was one of your favorite things we did?
S: Geoboards.
T: You liked the geoboards. All right. Was there anything this past week that you did not like?
S: I can't find anything.
T: Why do you think you like the geoboard so much?
S: It's fun to work with.
T: It's just fun. Were your classes this week different than your regular math classes when we worked with the geoboards and tiles and all that? How was that different?
S: It was different because we worked with everything and you didn’t just do stuff on the overhead.

T: So your regular math class was more like when you were learning formulas or more like what you went through this past week?

S: Past week.

T: Your regular class was more like this past week; is that what you’re saying?

S: (Nods head.)

T: Okay. Fine. How did those manipulatives help you learn or did they, using the geoboards and the tiles and your hands and so on? Can you tell me how that helped you learn?

S: Well you can understand it good if you use all that because you can use a certain amount of stuff like if you’re using a geoboard you can understand it because they have like the squares and stuff.

T: Okay. But you really still thought the formulas were better though.

S: (Nods head.)

T: That was even better. What is area?

S: The measure of something inside.

T: How do you find the area of a region?

S: Measure the inside.

T: How do you find that area to measure on the inside?

S: Well if this side was three and the other side was eight, three on the top and the bottom and eight on the sides, you make eight little lines on the side and three on the top and then you make them into squares and then you count the squares.

T: But you didn’t use the formula. Why didn’t you use the formula to find it?

S: It’s just, sometimes with the formula it’s easier, and sometimes I can’t figure it out.

T: So when you took your test last week, you used more of the formulas or more counting or figuring out?

S: Counting.

T: You did more counting. Okay. That’s interesting. Even though you liked the formulas better.

S: (Nods head.)
T: All right. Fine. Why do you think we learn about area? Why do we need to learn that?
S: So we know how much room we have inside.
T: How much what?
S: Room we have inside.
T: For example.
S: Like this room, you have to measure the inside and see how much room and stuff.
T: If I want to know how much room we have in here, then I would have to know the area, you're saying, right? What kind of people need to know, besides you and I, if I want to know how much space in here, are there any jobs that would require knowing area?
S: Construction worker.
T: Give me an example.
S: Like if he wanted to build these shelves, he'd have to know the measure of the wall and how big the shelves were going to be, how big from the ground it is.
T: So if I tell you that these walls are about eight feet high, does that tell me something about the area of this room?
S: No, not really.
T: You don't think so. Okay. What if I tell you that the floor measures eight feet that way and twelve feet this way; how would I figure out the area of this room?
S: Add 8 and 8, and 12 and 12.
T: And that would give me the area? Do you have to think about it some more? Think about it hard. I want to know the area of this room. How would I figure that out? Tell me what you're thinking so I can tell what's in your brain.
S: I'm kind of stuck on the area, cause on a test you'd always add up 8 + 8 and 12 + 12 when you had the measuring stick.
T: So you're thinking that's how you get the area. How would I get the perimeter of this room?
S: Measuring the walls with the measuring stick.
T: Doing what with the walls?
S: You said the walls are 8 x 8 feet, the walls you have to like, if the wall over there is 5 and this is 5. You'd add up 5 + 5 and 8 + 8.
T: Now I said it was eight feet high. That would help me to know the perimeter of this room?
S: (Nods head.)
T: What is perimeter?
S: The measure of something outside.
T: How do I find the perimeter of a region, a shape?
S: Measure the outside.
T: Just measure the outside?
S: (Nods head.)
T: Why do you think we learn perimeter? What kind of jobs require that? What kind of jobs would need perimeter?
S: I think of a construction worker. That's what my dad does; all kind of stuff.
T: Let's take building something. What if your dad was building a dog house and the floor measured three feet wide and four feet long; what is the area of that dog house?
S: Twelve.
T: Twelve square feet. What is the perimeter of that dog house?
S: Fourteen.
T: How did you get fourteen.
S: 4 and 4 is 8 and 3 + 3 is 6.
T: To know how much wood you'd need for the floor of that dog house, would you need to know area or perimeter?
S: Area.
T: To know how much wood you'd need for the walls of the dog house would you need the area or perimeter?
S: Perimeter.
T: Perimeter. Why don't you draw a shape of any kind on there, and show me how to get the area of that shape, please.
S: Area?
T: Yes. How do I get the area of that shape?
S: Measure.
T: All right. Give it some measures. Okay. So it's 4 by 2. So what is the area of that shape?
S: 4 x 2 = 8. Or do you just draw the squares?
T: Whatever it takes for you to learn, for you to give me the area. So you think the area is 8?
S: Uh-huh.
T: All right. So right down $a = 8$. And what is the perimeter of that shape?
S: Twelve.
T: How did you get 12?
S: Added up the sides.
T: All the sides. Put that down, please. When we had our class on formulas, I told you that the perimeter of a rectangle equals $2(l + w)$. Can you tell me the connection between this formula and getting the perimeter? Why do we have that formula to get the perimeter of a rectangle?
S: You put the length plus width and you just put what kind of sides it has.
T: So you’re putting the longer side, the shorter side. And why times 2?
S: Because it has two different sides.
T: Tell me some more. Because the length and the width are two different sizes; is that why you times it times 2?
S: Uh-huh.
T: For a triangle, we had for the area of the triangle; we gave the formula $\frac{1}{2}bh$. Do you remember what that all stood for?
S: Not really.
T: Okay. The base and the height. Why do you think this formula would give me the area of a triangle? One-half times the base times the height?
S: Because the height is like when you add up, like a square it has the same sides; but when you’re making a triangle it might not have the equal sides. And the height is -- because it’s not always going to be the same.
T: Tell me some more. I’m not sure I understand. What is the one half to do? Why do you think the one-half is in there? Any idea?
S: Because it’s one side of a square. Because you taught us to draw.
T: Square, a rectangle. So why do we have one-half of the square.
S: Because you should put one-half and one-half because it’s a whole. This is just part.
T: So part of that rectangle. It’s half of that rectangle? Okay. That’s fine. Let me look at your test and ask a couple of questions on how you got the answers. On number 6, you got 8
for the area of this shape. Do you remember
how you got 8? I'm not saying that's right or
wrong, I just want an explanation.
S: I added this up. I drew the dots.
T: And then what do you think you did with that?
S: And I added 2 and 4 and 2.
T: You added 4 and what?
S: Four and two twos.
T: That would not give you eight. That would give
you 12. Two fours and two twos.
S: There's one four and two twos.
T: Okay. That would give you the eight. That
would give you the area of this shape. Let's
look at another one. You're looking for area
of this shape, number 8. You've got 22.
S: I think I added up the 10, the 8, the 6 and the
8.
T: Try it. That's 32 and not 22.
S: Oh, I added up this 8 and this 8.
T: That's 16.
S: And the 10.
T: Twenty-six.
S: I know I added up the 10 and the 8, and I think
I added up the 6.
T: If I asked you to get the area of that shape
today, would you do it in a different way?
S: (Nods head.)
T: How would you do it? Do you know how to get
it?
S: Add this 8 feet and this 6 feet and this 10
feet.
T: That would give you 24. That would give you
the area? All right. Let see if there's
another one I could ask. How did you get the
area of this to be 10.
S: That's 5 and that's 5 and that's 5 and that's
5.
T: Okay. So how do you get 10?
S: I added 5 + 5 + 5 + 5. I don't know. I messed
up.
T: So it should have been 20 you think
instead of 10?
S: (Nods head.)
T: Don't rewrite it. That's okay. Okay,
(Student C). I think we're finished.
Thank you very much.
Student D (treatment 1, level 2, female, class 6)

T: This is the third interview, right? I've seen you a few times. Tell me how did you like this past week's lessons when we were working with geoboards and tiles and hands and graph paper?

S: It was okay, but some of it was hard.

T: What was easier, this past week when we worked on all these materials or the formulas? Which did you find easier?

S: The formulas with 4 sides of the square. That was easy, but the other part was kind of difficult.

T: What was the other part? What do you mean by other part?

S: You know the perimeter and the area inside, that's what got me confused with the formula.

T: Okay. Let me ask you some more. I'm going to talk to you about two different treatments. In that first week when you had all of the formulas...

S: That's the one that was easier.

T: That was easier. The second week when we worked with materials was harder for you?

S: Yes, ma'am.

T: In which one did you feel like you learned more?

S: The first one.

T: The formulas. Which one did you enjoy more?

S: The first one.

T: You enjoyed it.

S: I enjoyed working with the geoboards, but the test was kind of difficult to me because some it was hard, and I know I missed a lot.

T: I didn't grade them yet, so I don't know. What did you like the most in this past week's lessons?

S: Playing with the geoboards and that you can learn a lot from using that. It shows you all kind of shapes and squares and rectangles and all that.

T: And you felt like the geoboards helped you learn. Why do you think it helped you learn?

S: Because if I didn't know, like four squares and the inside was --- The geoboards helped me because the squares, you know, when you get the rubberband and put it on them. And it's easier
to tell how many squares they have inside of it, the area inside.

T: Because you would count them.
S: Yes.
T: You could actually see them. Good. All right. Which set of classes was more like your regular math class; the one where you had the formulas or this last week when you had all of the materials? Which was more like your regular math class?
S: The formulas.
T: The first class. Why do you say that?
S: Because it was easier because you had wrote on the board.
T: But why was it like your regular math class?
S: Because it was quiet and [the teacher] explains a lot like you did, explains and helps us on the test and all, but you couldn’t help us on the test. But we did what we know.
T: So you felt like that was alike. What is area, (Student D)?
S: The area is the inside of a square or something.
T: Okay. Some kind of shape, right? How do I find an area?
S: By, like if you have 4/5, wait, that’s where I’m confused. How do you find the area?
T: How do you find the area? You’re a little confused on that right now. I’ll get back to that in a little while. I’ll catch you on that one. Okay. Why do you think that we learn about area? Why is this in a math book?
S: To help us learn because when we get in the future or something, we might need to go back and learn it. If we go to college or something they might have it on the test or something for us, and it’s easier for us to learn and up in the CAT test and the LEAP test they might have that too.
T: All right. Outside of tests, what if you never had another test in your life, do you think you should learn area?
S: (Nods head.)
T: Why?
S: Because if you were doing a house or something, you would have to know the inside -- you would
have to know the bases and all of that to build a house to say...

T: Okay. So you think you would need to know area for a job. Can you think of any other job that you would need to know area for besides building a house?

S: Math. Like drawing and stuff.

T: Drawing? Okay.

S: Drawing map figures.

T: All right. So for drawing figures you need area. Can you think of anything else? Do you think your mother ever uses area?

S: (Shakes head negatively.)

T: No?

S: Huh-uh.

T: What about your dad?

S: I don't think so.

T: You don't think so. Well, that's all right. What is perimeter?

S: The perimeter is like when you walk and you count how many like — It's like when you have a rectangle or something and if you have four on one side and three on one side, you walk and you walk around it and you count how many distance, I think, or something. And then sometimes they will multiply it and get the answer or they will add it and then they will add the two and then they will multiply the other ones with it.

T: So you're telling me how to get perimeter; and you're telling me it is the distance around them. So you said to get the perimeter sometimes we multiply and sometimes we add. Okay. Let's deal with that in a little while. Why do we learn perimeter besides for other math classes? Do you think a construction worker also needs to know perimeter?

S: Yeah.

T: You're pretty sure about that? If you're building a house?

S: Yeah. Because they would have to know how much stuff they need. They would have to know how much wood or something they need, so they have like a tape measure or something and that's how they get it I guess.
T: They kind of measure around it. All right. If you wanted to get the area of this room, what would you need to know?
S: How many squares you think you have up in it or -- You said area, right?
T: Uh-huh. I said area.
S: I can't get that one.
T: Okay. What if I told you that this room measures 8 feet that way and 12 feet in length and it's about 8 feet high; could you get the area of this room and if so, how would you do it?
S: By multiplying.
T: What?
S: 8 x 12 or you would multiply the two corner sides and...
T: What are the two corner sides? Tell me.
S: Eight and 12. And then whatever it is, they would go back to the other 8. You said 8; 8 and 12?
T: Wait. I told you that the height -- Let's take the height as 7 so you'll have different numbers. So the height is 7, the length of that wall is 8, and the length of this wall is 12. If I want the area of the room, what would I do?
S: First you would add all three of them up and then afterwards you would times the number you got to the perimeter, I think, and...
T: Stop. How do I get the area of this room?
S: You would add all three of the numbers up and then you would take the perimeter...
T: That number that you get?
S: Yeah. And you would times that times like if it had 4 x 12 you would times that and then that would give you the area square something.
T: How would I get the perimeter of this room?
S: I think you would have to add them or when you add them all up and then -- That's kind of a tricky one.
T: It's a little confusing.
S: I thought that -- That's the part. That's the one that I got mixed up with. I thought that if it was 8, one was 8 and one was like 10, I thought that you times 10 x 8 and that would give...
To get what?
The perimeter.
Okay. So when you got three numbers, you got a little confused.
Yes.
So what if I want the area of this floor; what would I do to get the area of this floor?
You would like say if it was 4, you'd say 4. You would say 8 + 12 and then it's kind of confusing.
Okay. You got it confused. All right. Are you trying to remember the formulas or are you trying to remember the geoboards; do you know?
The formulas.
You're trying to remember the formula and you're having a little hard time. Okay. If I wanted to know how much carpet to buy for this room, would I need to know the area or the perimeter?
The perimeter.
Okay. And if I wanted to -- Do you see that black plastic around? It's called a baseboard.
The area.
If I wanted to go all the way around it, that would be the area. All right. What if I wanted to know how much wallpaper to put on this wall?
I think it would be the area 'cause it's inside. Yeah, I think it would be the area.
If I want to know how much paper to put up. Okay. If I'm building a little dog house, if I want to know how much wood to buy for the floor, would I need area or perimeter?
Perimeter? No. The floor you say?
The floor.
You would need the perimeter because the perimeter is like you say walking a path or something and you would need to know by walking it or measuring it.
And what about to know how much lumber to buy for the walls of the doghouse?
It would have to be the area because -- Can you repeat that question?
Okay. If I want to know how much wood to buy for the walls of the doghouse, would I need to know area or perimeter?
S: The area.
T: Why?
S: Because they want to know how much goes in the inside of it.
T: Okay. So the walls are the inside.
S: Yes.
T: Okay. But the floor...
S: Is the perimeter.
T: That's the outside. Why don't you draw a shape on here, any shape you want and then I want you to find the area of it. Just give each side some kind of measure and then figure out the area.
S: I can do the 3, the ABC one?
T: Sure.
S: But I guess that's not the one I wanted.
T: Just draw another one.
S: You want me to erase that?
T: No you don't have to erase it.
T: Okay. So it's 3, 5, and 6 for the triangle sides. Now how would you get the area of that?
S: I'm trying to remember. Oh, I see what I did wrong.
T: What kind or size you want to make; do you know?
S: I don't get it.
T: Can you give me the perimeter of it?
S: No. It can't be that much.
T: How much did you get?
S: I thought it was 90, but it ain't no 90.
T: It's not 90. Okay.
S: Sixteen.
T: How did you get 16?
T: Okay. So you're saying the perimeter or the area is 14?
S: Yeah.
T: Which one; the area or the perimeter?
S: The area. Not the area. The perimeter out, the outside.
T: So put p = 14. And how did you get 14?
S: I added 5 + 3 = 8 and I added, wait. I added it this way. I had 6 + 3 = 9 and 9 + 5 = 14.
T: Okay. Fine. And you don't know how to get the area?
S: I thought you times the insides of them, but I couldn’t get that right for nothing.

T: Couldn’t figure out how to do that? Okay. So I’m going to put a=? since you weren’t sure about that. Remember when we learned formulas, we learned that the perimeter of a rectangle is equalled to 2(l + w), and we said this was the length plus width times 2. Why do you think we have that formula to get the perimeter of a rectangle? What is the connection? Why do we have that? Why does the formula work?

S: Because l + w would be the only two numbers that will fit, would fit it, and the only number that I think that — The only length and width that make sense with it.

T: All right. So why do you multiply it times 2 then?

S: Because each side -- No that ain’t right. I was going to say each side has 2, but it don’t mean -- confusing. I’m going to tell you what I think.

T: Sure.

S: 2 and l + w? I think you have to multiply it by 2 is like it has two numbers or something -- they have two numbers and you multiply that 2 times whatever that number is going to be.

T: Because it’s two numbers, you have a length and width. All right. What about do you remember the area formula we put triangle A=½bh. Now what do you think? Why do we have that formula for area of a triangle; the base times the height?

S: Yeah. Because a part of it has, I think if it’s not right it would have one half of like, one half of a part, something like that. If you have one part of a -- one half of something, I remember. And you would times that b x h.

T: The base times the height?

S: Yeah. I don’t get it.

T: You don’t know why we’ve got this formula for the area of a triangle? What if they had said that would be the area of a square; do you think it would work?

S: Because it might have one area inside or one perimeter something. I don’t get it.
T: It sounds a little confusing. Okay. Let's look at your test for just a minute. Here it says draw a rectangle below, label the length 8 feet and the width 5 feet and find the area. Do you know how you got 15?

S: I added.

T: Okay. You added 8 + 2 + 5 and got 15? Is that what you're saying? Okay. All right. I just couldn't figure out what you did. This last one says a rectangle has an area of 20 feet. Sketch the rectangle below and label each side.

S: What I did was thought that they was saying that get -- How you got 20 feet square? That's why I thought it was like 4 and 5 and you times 4 x 5 and it would give you 20 in each square. That's what I thought they was talking about.

T: It could be. I'm not saying you're right or not. I just want to know how you got it. I couldn't tell exactly how you got that.

S: That's the one I didn't understand.

T: This one was hard. There's a room that's 15 feet by 10 feet. Dawn wants to put baseboards all around the room. How many feet of baseboards does she need?

S: This was like the same one I was just saying like -- I don't remember how I got that. I know that -- I think I added 15 + 20. I'm not too sure.

T: I wonder where you got 20 from; do you know?

S: I don't remember.

T: Do you know how you got area here? You said the area was equalled to 1?

S: 'Cause you couldn't count it, the inside. So I thought that's what I got it.

T: Because it was so small? Okay. Thank you very much, dear. I appreciate working with you. That was nice.
Student E (treatment 1, level 2, male, class 1)

T: You've been with me through 2 1/2 weeks, right?
S: Yes, ma'am.
T: Long time. Both weeks. How do you feel about this past week's lessons; the ones where we had lots of materials we were dealing with on the table? How did you like that?
S: I thought it was fun.
T: It was fun.
S: And I like doing stuff that you can work in groups.
T: Okay. You like group work.
S: Yes, Ma'am.
T: Did you like the first part when we had the formulas or this part where we worked with materials? Which lessons did you like best?
S: The second part.
T: The second part. Okay. Which one was easier for you?
S: The second part.
T: Why do you think it was easier than the previous one?
S: Because I didn't know as much formulas and stuff as they did when we first started going.
T: Why didn't you like the formulas?
S: They get me confused and it's complicated to remember all the stuff about which formula goes with which problem.
T: Okay. And you felt like you understood more with the second week than the first week?
S: Yes, Ma'am.
T: What did you like most about this past week's lessons?
S: What did I like?
T: Uh-huh. This past week when we were working we worked with hands trying to get area and perimeter, we worked with tiles.
S: I like with the table. With like, we would see how much area and perimeter they had in a table. The table with our hands and stuff.
T: You liked that? Okay. Was there anything we did this past week that you didn't like?
S: No.
T: Do you feel like getting the area and perimeter of the table with your hand helped you learn?
S: (Nods head.)
T: Okay. The lessons we did this past week, were they very different than your regular math classes?
S: Yes, Ma'am.
T: Why do you say that? In what ways were they different?
S: Well, we don't usually use, what's it called - algebra?
T: Uh-huh.
S: We don't usually use algebra as much as we did this past week than we do in our class.
T: What do you mean by algebra?
S: Like we don't do that stuff in our class.
T: What stuff do you not do in your regular class? Tell me some more.
S: With the formulas.
T: Okay. So you're saying the formulas were not like your regular class, because you don't use formulas. Okay. This past week we didn't use formulas, we just used tiles and grid paper and geoboards and so on. Were those classes like your regular classes? Is that how you normally have the classes? Do you use materials like that?
S: Yes, Ma'am.
S: We have geoboards in our class. And we didn't use them yet.
T: Okay, but one day you will?
S: Yes, Ma'am.
T: So this past week, how was what we were doing in the class like your regular math classes? What about them were the same?
S: We do problems on the paper.
T: And we did a few of those, not too many, right?
S: Yes.
T: Let me ask this question a little bit differently. Think about the first week we had instruction. We worked with formulas and so on, and the second week we worked with lots of manipulatives, lots of materials. Which set of lessons were more like your regular math class? Not what you learned, but how the classes were conducted.
S: The one with the formulas.
T: Why do you think they were more like your regular math class?
S: Because adding and multiplying problems, you know. We do a lot of that.
T: How do you feel about using those manipulatives? Did using those materials help you learn? What way did they help you learn? Can you tell me? How do you think they were more helpful to learn? You don't know. You just felt like you understood them better? Okay. What do I mean by area, (Student E)? What is area?
S: How many square feet or square inches and stuff inside that object that you are using.
T: Why did we learn area?
S: So if you want to build something, you know, like a house.
T: What if I were building just a dog house or something little, when in building that dog house do I need to know area? Why would I need to know area?
S: You need to know for inside the dog house.
T: Would I need area for the sides of the dog house also? What part of the dog house do I need to know the area for?
S: The inside floor.
T: The floor on the inside. Okay. But if I need to know how many boards and much wood I need for the outside, what do I need to know?
S: Perimeter.
T: Perimeter for that. How do I get the area if I want to know the area of that dog house?
S: If it's a rectangle you count, you see how many feet or inches one side is and go up to the other side that has the same length as it and see how many it is and add it together and then to the other side. The other side is longer. And you add them two sides together with the others.
T: And that would give me the...
S: Area. No, perimeter.
T: That would give me the perimeter. Now how can I get area of the dog house?
S: Measure how many feet are in it, around it.
T: So what if all around the dog house it measures twelve feet, what is the area of that dog
house? What if the dog house is four feet long and three feet wide, what is the area of that dog house?

S: Fifteen.

T: How did you get fifteen?

S: $4 + 4 = 8$. $8 + 3 = 12 + 3 = 15$.

T: Okay. So it's fifteen feet. All right. What kind of people besides people who build dog houses or real houses, people houses, can you think of any other people who might need to know how to figure out the area of a region? Do you think your mother or daddy ever need to know area?

S: Yes, Ma'am.

T: For what?

S: If they would want to put new carpet in the house or in their bedroom.

T: Then they would need area or perimeter?

S: Area.

T: What if they want to change the baseboards around the room?

S: The perimeter.

T: That would be the perimeter. What if they want to paint or wallpaper the walls. What would they need for that?

S: Area.

T: To paint the walls they would need to know the area?

S: Paint it or to wallpaper it?

T: Does it make a difference? To paint the room, they would need what?

S: Paint.

T: Paint. But how can they decide how much paint to buy? What would help them know that? Would perimeter or area help them? You don't know or you don't think so?

S: I don't think so.

T: What about if they want to wallpaper the wall, would they want the perimeter or the area of the room for that?

S: Perimeter.

T: Perimeter would help them decide how much paper to buy? All right. Tell me, what is perimeter? I have an idea from what you said, but tell me some more.

S: It's the square feet around the object.
T: Okay. The square feet around the object. How do I get the perimeter of a room? If I want to know the perimeter of this room, how can I get it?
S: Add all four sides of it and add all of them up and you can find the perimeter.
T: And that would be perimeter. All right. Can you think of any job for which you would need to know perimeter?
S: Construction workers.
T: When they do what?
S: Build houses.
T: So what in that house do they need perimeter for? You told me a little bit, but tell me one more time.
S: Boards.
T: For what?
S: Boards.
T: The sides of the house they need to know the perimeter for. How much wood to buy for the sides of the house? Okay. Just like that dog house, right?
S: (Nods head.)
T: All right. I want you to draw a shape on here, any shape you want, and I want you to find the area for that shape. It kind of looks like a trapezoid to me. Is it to you, or did you mean it to be a rectangle?
S: A rectangle.
T: Okay. It's just a little bit crooked. All right. Give it some kind of measures. That side is 5, that's 5, 10 and 10. What is the area of that rectangle? The area is 30 feet square. Okay. Is that area or perimeter?
That's area. So a = 30. Go ahead and write that down so I can make sure that I keep it straight. Okay. What is the perimeter? How did you get that 30?
S: 5 + 5 = 10 + 10 = 20 and 10 is 30.
T: Okay. Good. Now find the perimeter for me, please.
S: The area is 50 and the perimeter is 30.
T: Okay. So you changed your mind on that.
S: Yes, Ma'am.
T: All right. Let me show you a formula that I taught you the first week. I told that to get
the perimeter of a rectangle you take $2(l+w)$. I want you to tell me why does a mathematician have this formula for the perimeter of a rectangle. Why? Is there any connection between this formula and what you did?

S: "L" is the length and "w" is the width. I don’t know why the 2 is there.

T: You don’t know why the 2. You really don’t know why there is a 2 there? All right. The area for a triangle is equal to $\frac{1}{2} bh$. Okay. You learned that well. All right. Why this formula for area of a triangle? What does that (the formula) have to do with the area of the triangle?

S: Because a triangle is half a square.

T: Okay. And half of the square...

S: Base times the height. Half of a rectangle.

T: Okay. Half of a rectangle. All right. So a triangle is half of a rectangle and so what is the base on that triangle there? Where would the base of that triangle be?

S: Down at the bottom.

T: All right. Put $b$ there. Where would the $h$ be?

S: Right here.

T: Down the center, huh? All right, that’s the height. Actually you’re saying that. What do you do with the "$b$" and the "$h$" in this formula?

S: Add them together.

T: You add them together and then what do you do?

S: Get the answer.

T: What do you do with that answer, anything, or is that the answer for the area?

S: Answer for the area.

T: Okay. But you’re saying the half is here because it’s half of a rectangle. Okay. Let’s look at your test, and I’ll ask you just a couple of questions so you can explain to me how you got your answers. You got 20 for your answer for number 6. Can you explain to me how you got the answer for that irregular shape to be 20? Do you remember?

S: $6 + 2 = 8 + 4 = 12 + 2 = 14 + 2 = 16 + 4 = 20$.

T: Okay. That’s how you got your 20.

S: Perimeter.
T: You think it's really perimeter and not area? All right. If I told you that you're right. I told you that that's perimeter. How would you get the area? Any idea? No? Okay. Let's look at one or two more. For the area of this one, you got 46.
S: And that would give me 36.
T: Thirty-two probably, huh?
S: Thirty-two, I mean, + 6 = 38 + 10 = 46.
T: You got 18 for this one; this irregular shape for area. Do you remember at all how you got that?
S: 5 + 1 = 6 x 2 = 8 + 2 = 10. No wait. 5 + 1 = 6 + 1 = 7 + 2 = 9 + 2 = 11 + 3 = 14 + 3 = 17 + 1 = 18.
T: Okay. Thank you very much, (Student E). I appreciate all of your hard work.
You were with me just this past week. You took a pre-test and an interview before, right? How did you feel about this past week’s class?

All right.

You liked it?

(Nods head.)

Do you enjoy learning?

Uh-huh.

What was one of the things that we did that you kind of liked, maybe one of your favorite things?

The geoboards.

You liked the geoboards. Why did you like the geoboards?

They were easier to make up shapes.

Do you feel like it helped you understand area and perimeter?

(Nods head.)

Okay. Was there anything we did this past week that you didn’t like a whole lot?

(Shakes head negatively.)

No. You were okay with all of it. Were the classes that I had with you very different than your regular math classes?

(Nods head.)

They were different. How were they different?

We didn’t work with those squares.

So in the regular classes you don’t work with that. So what we worked with was different, was there anything else that made it different?

(Shakes head negatively.)

Otherwise it was pretty much the same as your regular math classes?

(Nods head.)

All right. All those things that we used liked the geoboards, we had grid paper that we cut out, the hands, the tiles, do you remember you covered the table with your hands?

(Nods head.)

And we used some square tiles. All of those are called manipulatives. How do you think using those manipulatives helped you learn about area and perimeter? Do you think they were a big factor? How do you think they helped?
S: Well, you can find out how to figure out what the answer was.

T: You could just count to see it and that helped you some. What is area, (Student F)?

S: The space inside something.

T: How do I find the area of a space? How do I figure it out? If I want the area of this desk top, how could I go about doing that?

S: Measure this side.

T: What if I tell you that this side is about 2 feet?

S: 2 x 2.

T: 2 x 2 would give me 4. So the area of this desk top is 4. Why did you multiply it times 2?

S: Because there’s four sides and...

T: What if I told you that this was 1 foot wide and this was 2 feet long; how would I figure out the area of that desk?

S: You could add this one and this one, and this one, and this one.

T: All together and that would give me the area? All right. How would I get the perimeter of that desk?

S: Oh, that was perimeter.

T: You’re a little confused. Stop and think about it. It was last week that we did it. How would I get the perimeter?

S: How long is this desk and this?

T: This is 2 feet and this is 1. We’re pretending because this is really more than 1 foot. Let’s pretend that this is 2 feet and this is 1 foot.

S: All right.

T: So what’s the perimeter? How would I get it?

S: You would add this one and that one and you would take -- can I write it out.

T: Sure. So you said that the perimeter is 6. You used a formula. Where did you learn that formula?

S: In that class.

T: You weren’t in my class with formulas, were you?

S: (Nods head.)

T: You were. Okay. So you learned that in that class and so you know that you can get the
perimeter with the formula. Can you say what that formula means? Why does that formula fit?
S: Because this is like a rectangle and this is a rectangle because this was the length and this was the width.
T: And so we added the length and the width and then we...
S: Multiply it times two.
T: Why do we times it times two?
S: Because there is one length and one width and that equals two and then you times that to get how long the width is, the width and the length.
T: But why times two? I still don’t understand. Tell me some more.
S: (Shakes head negatively.)
T: You don’t know why you double it, but you know that it works. All right. Now give me the area of this desk top, please.
S: The area?
T: Uh-huh.
S: Four.
T: The area is four. How did you get four?
S: Wait up. No, it’s two.
T: How do you get two?
S: Because take this and you would divide it in half and it would be just the same.
T: A two on each side.
S: Yes. You divide it times two on each side.
T: Maybe there’s two together. So the area is 2 or the area is 4?
S: Two.
T: Okay. Good. Why do you think that we learn about area in mathematics class?
S: To find the floor when you want to put a carpet in or something in your house or something.
T: And how does that help? What if I want to carpet this room, and I tell you that this room measures eight feet on that side and twelve feet on this side? What do I do to know how much carpet to buy?
S: Times 8 and 12.
T: Okay. 8 x 12 would tell me the area and that would tell me how much carpet to buy? All right. Can you think of anyone else or any other reason why we learn area?
S: To find amount of liquid in something?
T: Okay. So you think a chemist might need to know if he needed to know how much liquid was in something? That would help. Can you think of another job? When they were making the sidewalk outside in front of those new buildings, do you think those people who were working out there needed to know anything about area?
S: Yes.
T: Why would knowing area help?
S: They would have to know how deep to put it and how long to put it, the width and the length.
T: And that would help them know. What about when they were building the covers on top; did they need the area for that also?
S: Yea, they needed to know how long to get the boards for it and how long to put the top over.
T: Back to this room. What if I want to paint this wall; would the area help me know how much paint to buy? Would I use area or perimeter to know how much paint to put on this wall?
S: Perimeter.
T: Why?
S: Because you would know how long the sides are.
T: How about if I was putting wallpaper on the wall?
S: You need to know area.
T: Area, why?
S: You need to know how thick you need to get it.
T: How what?
S: Thick.
T: How thick, okay, or how wide. So I need to know area if I want to know how much paper to put on the wall, but I would have to know perimeter to know how much paint to put on the wall. All right. What is perimeter?
S: Perimeter is the amount around something.
T: What kind of jobs need perimeter?
S: Windowing.
T: Windowing. Making window frames. Can you think of any other job, any other thing that a person would do that would need perimeter?
S: (Shakes head negatively.)
T: No other job that you can think of. Okay. How do you find the perimeter? Say it one more time.

S: You would take how long it is on the sides across and then take the length up and down.

T: Take the length up and down and what do you do with that number?

S: Well it depends on what kind of formula you got.

T: What if it's this wall? How do you get the perimeter of that wall?

S: You would take the length and the width and you would add them together and times it times two.

T: Because that's the formula, right? Okay. What if that wall was square shaped?

S: You would take the length and you would times it times four.

T: You learned the formulas very well, didn't you?

S: (Nods head.)

T: So if I want to know how much carpet in this room, I need area or perimeter, which one?

S: Area.

T: If I want to know how much baseboard to put around, do you see that black thing, that's called a baseboard under the shelf; if I want to go all the way around this room, do I want to know area or perimeter?

S: Perimeter.

T: How about drawing a shape for me, any kind of shape you want and then I want you to get the area of that shape. You drew a rectangle. How would I get the area of that rectangle? Just make up measures. So you have a three by two. How could I figure out the area of that rectangle?

S: 3 x 2.

T: You would multiply 3 x 2?

S: (Nods head.)

T: So why don't you put down area equals, just put the answer. It's 6. And what's the perimeter? The perimeter is 10. Okay. Let me ask you one more question and then I'll ask you a little about your test. If you were building a dog house and you wanted to know how much floor you need for that dog house, would you need to know the area or perimeter?
S: Area.
T: How about to build the walls in that dog house to know how much lumber to buy for the walls, what would you need for that?
S: Perimeter.
T: Do you need to know area at all?
S: Yes.
T: Explain it to me. To know how much lumber to buy for the floor we need an area, right? But to know how much lumber we need for the walls?
S: Oh, perimeter.
T: Perimeter. Does that make sense to you?
S: (Nods head.)
T: All right. Let me ask you a couple of questions here on your test. You got 18 for number 6. I'm not saying that's right or wrong. I just want you to explain to me how you got 18.
S: Well, I took the 4 and divided it into 2, so I have sets of 2. Since this is 2, 4, 6, 8, 10 and 2 + 6 + 2 = 9.
T: 2, 6 and 2 is 10.
S: 10.
T: And 2 + 2 + 2 = 6 there. So 10 + 6 = 16. Do you remember how you got 18?
S: Oh, I think I added this to it to make this go down.
T: All right. Two more on that side to get the area of that shape. Do you think so? That would make it 18, right? Let's look at a couple of other ones. Number 14, for the perimeter you said 32. Do you remember how you got 32? What did you do to get 32? Perimeter.
S: That's 10 and 21 and 24.
T: So you think it should have been 24 instead of 32?
S: (Nods head.)
T: You don't know how you got 32? All right. Let's look at one more. For the area of this triangle you got 12. Do you remember how you got 12?
S: I added these.
T: All of those put together. I forgot one other thing on here I wanted to talk about. You explained to me about this formula pretty well. What about the formula for the area of a
triangle; we learned that it's equal to one-half the base times the height. Can you explain in any way why we have this formula for the area of a triangle? Does that make any sense?

S: The half would probably be -- I would think this would be half of this.

T: Okay. You think this is half of this, so therefore we use the half. All right. Base is the length of the bottom, right?

S: Yes.

T: And the height is 6. And that's why we have that formula. Okay. Thank you very much, (Student F). I appreciate it. Lot's of hard thinking.
Student G (treatment 2, level 1, female, class 1)
T: You were with me just this past week just for those three days and then we tested, right?
S: I had taken the test before.
T: Okay. And you were interviewed before, right? Okay. But you weren’t with me during those first five days when I had the other group, right?
S: No, Ma’am.
T: How did you feel about this past week’s lessons?
S: I felt that I was learning something.
T: You did. Good. It was easy to understand. What did you like most about those lessons? Think about all the different things we did. What was most pleasing to you?
S: Playing with the little boards.
T: Let’s see the little boards.
S: With the rubber bands.
T: Okay. The geoboards. You like that a lot. Do you feel like it helped you learn?
S: Yes, Ma’am.
T: What did you like the least? Was there anything that you did not like very much in those lessons?
S: Huh-uh.
T: You liked it all. Okay. Were the classes that you had with me pretty much like your regular math classes?
S: Huh-uh.
T: Why not?
S: Because that boy named (student’s name), I’m not used to sitting by him.
T: Okay. So sitting next to (student’s name) made it different. Is there anything else that was different?
S: And sitting by (another student).
S: He was kind of bothering you a little bit? All right. Was there anything else that was different about the class? Otherwise, it was pretty much like your regular math class? All right. How did using those manipulatives, you know, the geoboards and the tiles and the grid paper and your hands to measure area, how did that help you learn? Did it really help you
learn? How do you think that helped you? Explain.
S: How many hands it did. I don’t know.
T: It just kind of made some sense. What is area?
S: It is the whole thing. Like whenever you measure the whole thing.
T: The whole thing. Tell me a little bit more about that.
S: In area you don’t just measure the outside of it, you measure the inside and the middle.
T: Okay. That’s the area. How do you get the area of a region? If I want you to get the area of this desk top, how would you go about doing that?
S: You put one of your hands here and then one on the other side and you count how many hands go around it.
T: Around the desk? Over the entire desk, that would help you get the area and then see how many hands, right? Good. So if the desk was a triangle could you do hands again, kind of?
S: (Nods head.)
T: All right. Why do we learn about area?
S: Because you might want to be a construction worker.
T: So a construction worker needs to know area. What do they do that makes it necessary that they know area?
S: Because they have to measure stuff.
T: For example, construction, let’s take the sidewalk that they just built here at the school by the eighth grade building. Did those construction workers need to know area to make a sidewalk?
S: Yes.
T: They did. All right. How?
S: How big to build it. How much concrete to put in.
T: Now when they put a cover over the sidewalk did they need to know area for the cover?
S: I don’t know what you mean by cover.
T: Well, they have that cover that goes over the sidewalk. It keeps the rain from you.
S: (Nods head.)
T: They need to know the area for that too. All right. What if you were building a dog house?
That's a construction worker, right? Do you need to know area to build a dog house?
S: Yes. To see how big you want it.
T: Do you need to know area to make the walls of the dog house?
S: Uh-huh.
T: What do you need to know the area for?
S: You don't.
T: You really don't need to know area. So what does a construction worker need area for?
S: They don't.
T: They don’t. Can you think of any job that does need area?
S: The people that -- Is it still a construction worker whenever you're building a house?
T: Uh-huh. It's kind of a construction. Sometimes we call them carpenters, but he is constructing something, so that's a construction worker. What would a carpenter need to know area for? What part of the house would he need to know area for?
S: How tall to make the house. If they're adding on or something, they might need to know how tall it is.
T: So the height of it would tell area. The area would tell how high it is?
S: No. It's perimeter.
T: The perimeter would tell me how high it is. Let’s look at this room. Let’s say we’re adding this room onto a house. I want to know what area is related to and what perimeter is related to in this room. If I want to know how much paint to use for this wall, would area help me?
S: No.
T: Would perimeter help me?
S: (Nods head.)
T: How would the perimeter help me?
S: The perimeter, like you would measure that side and then that side and then that side.
T: And how would that help me know how much paint to buy or would it?
S: It wouldn’t.
T: It wouldn’t help me. So I don’t need to know perimeter do I?
S: No.
T: Or do I.
S: I don't know.
T: You're not real sure. You're a little confused now. What about the floor. If I want to put carpet on the floor, would area or perimeter help me more to know how much carpet to buy?
S: Area.
T: Area. All right.
S: Because...
T: Because what?
S: Because it might be different sizes like where the door is it goes bigger a little and it's longer.
T: All right. So area would help me know how much carpet to buy. Would perimeter help me at all? All right. Do you see that baseboard, the black baseboard that goes under that shelf? What if I want to put baseboards all the way around this room, would the area help me to know how much baseboard to buy? No. Would perimeter help me?
S: (Nods head.)
T: Okay. I want to get back to that wall. What if I want to wallpaper that wall? Does that wall have an area? You told me it has a perimeter and measure all around. Does it have an area?
S: (Nods head.)
T: Okay. What part of it? Where's the area of that wall?
S: In the middle.
T: Oh, okay. So if I want to know how much paint, maybe the area would help? You don't know. What about wallpaper? If I want to wallpaper that wall, would knowing the area of the walls help me know how much paper to buy? Okay. All right. Is there any other job that you can think of besides carpentry where people might need to know area and perimeter?
S: Not that I can think of.
T: Let me ask you one more time. What is perimeter? I didn't ask you that question, but I think you have an idea.
S: The length.
T: The length of something. So if this room is twelve feet long, the perimeter of this room is twelve feet?
S: (Nods head.)
T: Okay. So what if the width of this room is eight feet and the length is twelve feet, do I need to know about that eight feet to get the perimeter or not because that's just extra?
S: You do.
T: Think about it.
S: If you need to know what, the perimeter?
T: If I want the perimeter of this room, and I told you that this side measures twelve feet and that side measures eight feet, how could I figure out the perimeter of this room? Is it twelve feet, the length of the room? No. How would I get the perimeter of this room?
S: You add 8 + 12.
T: All right. And that's 20. So the perimeter of this room is 20?
S: (Nods head.)
T: Okay. And how do I get the area?
S: You have to add the other two too.
T: Oh, I have to add another 8 and another 12, right? So that's another 20. So the perimeter is 40. How would I get the area of this room?
S: See how many hands go over it.
T: Good. Can I use anything else other than hands?
S: You can use a measuring tape.
T: Okay. So I measure it and I find that that's 12 and that's 8, so what's the area? How do I get the area?
S: You divide. I don't understand.
T: Okay. You look confused. I guess my question to you, (Student G), is could I get the area of this room without laying my hands on it? Could I get how many square feet instead of how many hands?
S: Uh-huh.
T: And how would I do that?
S: You would measure it with a tape.
T: Okay. But I'm telling you I used a tape, and I measured it and it's twelve feet. And I measured that wall, and it's eight feet.
S: Measure it like from the door to here.
T: Okay. And it's about nine feet. All right. So what do I do with all of those numbers to get the area and perimeter? Any idea.
S: You divide 8 into 20. I mean 40.
T: 8 into 40 and that would give me the area.
S: 5.
T: Give me 5, so the area of this room is 5?
S: Uh-huh.
T: That doesn't sound right to you. Okay. I'm going to ask you to draw me a picture. All right. Draw me any shape you want to on this paper, then I want you to try the area of that shape. It can be any length or any width or any height. So you're drawing a rectangle. Just give it some kind of length and width. The length is 14 and the width is 7. That side is 14 and that side is 7. Is there any way that you could find the area of that rectangle?
S: You put 7.
T: You're making 7 rows.
S: And you put 14 this way [made grids].
T: Okay. When you finish that, what would you need to do?
S: And you count how many squares there are.
T: All right. Okay. Well let's pretend. Would you maybe make one more line there? It really should be squares, right? Okay. So now you just count the squares. Is there any kind of way -- Is that correct now? So you can count all of the squares and that would tell you the area. What if it was like 140 x 50, would I have to count all of the squares or is there another way I could get the area?
S: (Nods head.)
T: What do you think?
S: I'm thinking. I had it a while ago. 14 ten times.
T: Do you think that would give you the area?
S: 7 x 14.
T: Why does that make sense, (Student G), that 7 x 14 would give you the answer?
S: Because you've got seven this way and fourteen that way and instead of drawing squares you can multiply it.
T: Okay. Then multiply it. All right. So the area in this case would be how much?
S: 98.
T: So put here a = 98. How would I get the perimeter of that rectangle?
S: I can't tell you.
T: How do you get the perimeter?
S: Add 7 and 7 is 14; 14 and 14 = 28; and 28 and 14 = 42.
T: Let me show you something. There is a formula that some mathematicians have developed and they said that to get the perimeter of a rectangle, you take 2(l + w). In other words this stands for the length, this stands for the width. You add them and multiply it by 2. Can you think of any reason why this formula would help you find the perimeter of this rectangle? Does that make any sense why they put it like that?
S: Not to me.
T: Not to you. Okay. They also said to get the area of a triangle, you take one-half the base times the height. You multiply the base times the height and you take ½ of that. Why do you think they came up with that formula? Does that make any sense to you?
S: No.
T: No.
S: I'm confused.
T: I know. And this is new to you. But this says that I multiply the base times the height of this triangle and then I take half of it and that will tell me the area of that triangle.
S: You can draw a square. You can draw half of that and then count it.
T: Okay. So I can make a rectangle out of it.
S: Then you add this and like if this is 4 and this is 2, you can like 4 + 4 = 8 and 2 + 2 = and that's 12 and then half of that is going to be 6.
T: Now just a couple of questions on your test. On this one you said that the area was 22. Do you remember how you got that area?
S: 2 + 2 is, wait. I covered this side up and then I take two, two, and then two and then the two that was here, 2, 4, 6, 8 and then I would cover up this side and this looks like 6 + 6 =
T: So it really should be 18 instead of 22. Let me ask you just a couple more. Now for some of these you have correct answers, but you didn’t show me how you got your answer and I just want you to explain it to me.

S: Okay.

T: Number 14. How did you get 32? How would you get the perimeter of this irregular shape?

S: 6 and 6 is 12 and then 16 and 20 and 20 and 5 equals 25.

T: So do you think that should be 28?

S: No, I think it should be 32.

T: How did you get 32?

S: Take 28 and add 4 more from this side and it’s 32.

T: Okay. All right.

S: Am I right?

T: Yes. Sure.

S: Great.

T: Do you remember how you got 20 for the area of this irregular shape for number 28?

S: 1, 2, and 3 and then that’s 6, that’s 9 all of that together and then 13 and this, no 14, and that would be 18. I didn’t get 18.

T: You didn’t get 18. Do you think it should have been 18 instead of 20?

S: Wait. Let me do this over.

T: All right.

S: Yeah, 18.

T: All right. That’s fine. I didn’t know how you got your answers, and I was wondering if you could explain it to me. You just add all the way around. All right, (Student G). I thank you very much. I need to give you a note.

S: I’m in my same class.
Student H (treatment 2, level 1, female, class 6)

T: All right, (Student H). I've interviewed you before, right? Okay. Just a few more questions today. How did you feel about the lessons we had. You just joined us for that one week, right, last week? How did you like those lessons?

S: It was fun.

T: You enjoyed it? Okay. Was it like your usual math classes?

S: No.

T: How was it different?

S: You'll just use the geoboards and those block things.

T: All right. So you don't usually do that in math class?

S: We do it sometimes, but not as much.

T: Not as much. What did you like most about last week's lessons? What was one of your favorite thing?

S: Playing with the geoboards.

T: What was one of your least? Was there anything you didn't like?

S: (Shakes head negatively.)

T: You liked it all? Okay. We worked with our hands to get the area and perimeter and we worked with the tiles and geoboards and grid paper; all that is called manipulatives. Do you feel like manipulatives helped you understand and if so how?

S: It helped me because it was kind of hard if you don't have like the grid paper or things that like measure the inside of something.

T: Okay. It helped you see the inside. What is area?

S: It's around something inside the shape.

T: Inside the shape. How do you find area?

S: If it's a square, you use like -- If you had those little tile things, you can put them in squares to get how many fit in.

T: Okay. So you could get area. What kinds of people need to know area? Why do we learn area?

S: Because like if we didn't know -- Like if we were a carpenter when we grow up and we don't
know how to use perimeter and area we couldn't measure the size of the area inside.

T: Can you think of any other jobs in which you use area besides carpentry?
S: No.
T: Any other job that might -- Okay. What is perimeter?
S: It's the amount of the length and the width of the sides of an object.
T: Very good. What kind of jobs need perimeter?
S: Carpenter. That's all I know.
T: Do you think your mom and dad ever use perimeter or area?
S: Yeah. My mom does. Sometimes she sews stuff and she makes a lot of clothes. She doesn't any more.
T: Okay. But when she did -- Okay. Good idea. When in that sewing does she use perimeter or area? Explain that a little bit more.
S: To measure the length of something or to figure out how much it needs to be or how long it needs to be.
T: So is there a perimeter of say a dress that she could make you? What would you call the perimeter of that; do you know?
S: (Shakes head negatively.)
T: When she measures length and width, it's a little bit different than perimeter, huh? Can you think of anything she'd do that she'd actually have to use perimeter?
S: No.
T: No. Okay. Or area?
S: Huh-uh.
T: No. Not directly. How would you find the area of this room?
S: You could take like square blocks and put it in each one of those (shelves), but that would be a lot.
T: But it could be done. Okay. So could we get the area of the floor?
S: Uh-huh.
T: You do it by laying squares on the floor? All right. Could you get area of the wall?
S: (Nods head.)
T: That's possible also? Okay. Can you think of any other way to get the area of the floor
besides literally laying blocks on it? Laying blocks on it is a good way, but is there any other way you can think of that we could do that? What if I just told you, (Student H), that this room is 8 feet long and 12 feet deep or 8 feet wide and 12 feet long; is there any way that you could get the area of that?

S: You could multiply.

T: You multiply 8 x 12? Why would that work?

S: Because you know that they have eight at the top for the width and twelve for the length. And you times it and it might equal the same; the squares in there, they might equal the same.

T: Okay. It might be the same number as the squares. All right. How about the perimeter? How could you get the perimeter of the floor if its 8 x 12?

S: You know that its 8 at the top and you get the same thing at the bottom then you add 8 + 8 and they have 12 on this side and the other side equal the same just add 12 + 12 and you add them together.

T: Good. All right. Could you do the same thing to the wall?

S: Uh-huh.

T: If I want to lay carpet on this floor, would I need to know area or perimeter to do that?

S: Perimeter.

T: Why? If I wanted to buy carpet, maybe a better word is buy. Is it still perimeter? If I wanted to know how much carpet to buy? Why?

S: Because it comes in like square pieces and all and you have to find the perimeter. And then if it's like 8 by 12 you can do the same thing I said like add 8 and 8, 12 and 12, and add them together. And you tell the people you measured the floor, I guess.

T: If I told you that the perimeter of my room is 40 feet...

S: They would get 40 squares of carpet to put in the room.

T: Now what if I want to buy -- do you see that black baseboard, the board that goes all the way around it. If I wanted to put it all the way around the room, to know how much I need
would I need to know the area or the perimeter of a room?

S: Perimeter.

T: Perimeter. Okay. I want to go back to that carpet. I want to ask you a little bit more on that. When it’s 80 feet all the way around the room, does that also mean it takes 80 squares to cover the floor?

S: Uh-huh. It can be sometimes, but not all the time.

T: So a while ago you told me I need to know the perimeter, then I’ll know how much carpet to buy. But now you’re telling me that wouldn’t always work.

S: Sometimes you may need the area.

T: Okay. I just wanted to make sure that you were clear on that. So sometimes it ends up being the same number you’re saying, and sometimes it doesn’t.

S: (Nods head.)

T: Now what if I tell you that the area of the room is 100; should I buy 80 square feet to lay down or should I buy 100 square feet to lay down on the floor?

S: 100 square feet.

T: You find it’s 100 square feet. All right. I want you to draw any shape you want to on there and then I want you to find the area and perimeter of that shape. You can just make up the measures. Are you going to draw a rectangle? That side is 8 and it’s square. Okay. Now, what is the perimeter of that square?

S: Thirty-two.

T: Okay. And say p = 32 so I know what that stands for. So it’s 32 units around. What’s the area?

S: Twenty-four.

T: How did you get 24.

S: See how many squares fit in the area: 1, 2, 3, 4. And you have 6 rows across. 6 x 4 = 24.

T: Let me show you. You weren’t with the class when we did formulas, but there’s a formula that says you can get the perimeter of a rectangle by taking 2 times the length plus the width, if I add length and width and then
multiply it times 2. Can you explain any connection between this formula and how you got perimeter of the rectangle?

S: Because you know that you have an 8 and you have 4 sides and they all equal 8.

T: This is for perimeter on a rectangle.

S: Oh.

T: So what if you had a rectangle and that’s the formula; how does this formula connect with how you get perimeter of that shape?

S: Because you can — If it’s like 1 here, you could multiply 1 x 2 ’cause you have 2 sides. It equals 2 and if that was -- No, you get -- No, if this is 8 and this is 6, you can add that and then you can multiply by 2 ’cause there’s 2 more sides.

T: A triangle. There’s a formula for the area of a triangle that says if I take the base of the triangle, that’s the measure here, times the height of the triangle, which is the measure here, and take that times \( \frac{1}{2} \) or half of that, I would get the area of the triangle. Can you make a connection between the formula and how that’s done?

S: This could be 4 and this could be 7. Okay, add that and it would be 11 and then...

T: Let me explain this also because you didn’t have formulas. When I have "b" next to "h" it means you multiply these two. So I multiply these two and take \( \frac{1}{2} \) of it. Why would that fit?

S: Because you multiply 7 x 4 = 28 and then you take \( \frac{1}{2} \) of 28 and multiply it times -- Multiply it times half.

T: You could multiply \( \frac{1}{2} \) x 28 or when you take \( \frac{1}{2} \) of something what else could you do?

S: Divide.

T: All right.

S: One-half of 28 equals...

T: One-half of 28 is 14.

S: Fourteen.

T: So does that make sense that the area of this triangle is 14?

S: Uh-huh.

T: Why?
S: Because if you multiply this and you get 28 and you take a \( \frac{1}{2} \) of 28 and it would be 14 so I guess that’s how you do it.

T: I wonder why you have a \( \frac{1}{2} \) there for a triangle, but you don’t for a rectangle.

S: Because a rectangle has four sides and the opposites are the same but -- A rectangle has two different kind of sides and a triangle has three different kinds.

T: So, tell me some more.

S: Like a rectangle has two sides that are the same and two other sides are the same, but a rectangle has...

T: And you think that explains why I should take \( \frac{1}{2} \) of it? When I’m doing a triangle doesn’t -- Triangles are not a half of it when I’m doing rectangles? Think so?

S: (Nods head.)

T: Okay. That’s not bad. I’m not sure I’m understanding what you’re saying. All right. I want to cover just a few items on your test, not because they’re right or wrong but just because I don’t know how you got your answer and I’d like you to explain to me. Number 6, you got 18. Can you explain to me how you got 18?

S: I drew my squares.

T: I see, you do have squares there but you erased it.

S: I have 3 and I have 2 and then 3 \( \times \) 2 = 6 and then 2 \( \times \) 7 = 14 + 6 = 18. No it’s not. 14 + 6 = 20.

T: Don’t erase it. Let’s just leave it. I just wanted you to explain it because I wanted to know how you got the area. So that’s how you got your 18 and you just made a mistake there or something. But you counted the squares inside and drew them. All right. For this problem you got 32. You said this garden measures 8 feet on each side. If Bill wants to put a small fence around the garden, how much fencing is needed? How did you arrive at 32?

S: All the sides are 8 feet and I multiplied it times 4.

T: Okay. And that’s 32. This triangle here. I asked you to find the area of the shaded part
which would have just been the triangle part or half that square. How did you get 7; do you know?

S: I drew squares.

T: You drew squares. Okay. I can see that. You counted those squares and you had 7 squares. Okay. Thank you very much (Student H).
Student I (treatment 2, level 1, male, class 1)

T: I asked you a lot of questions in two interviews already, right? And I'll ask you some more today? How did you feel about this past week's lessons?

S: Good.

T: You liked them? Okay. What did you like most about those lessons? Let's first remember what all we did. What all did we do?

S: Blocks. Taking down notes.

T: All right. You worked with geoboards. Do you remember those tiles? You worked with your hands to get area and perimeter of the table, right?

S: Uh-huh.

T: And you also cut out some of that grid paper. Do you remember that, the rectangle?

S: Uh-huh.

T: Which part of that did you feel like, and we worked in small groups, did you like the most?

S: The geoboards.

T: You liked the geoboards? Okay. Was there any part that you did not like very much?

S: I liked it.

T: You liked it all? Okay. Were these classes that I had with you very different than your regular math class?

S: (Nods head.)

T: How were they different, (Student I)?

S: You didn't teach us the same thing.

T: Okay. So I taught something different? All right. Did the way I taught it, was that different than your usual classes?

S: No, Ma'am.

T: That was kind of the same. What is area, (Student I)?

S: Something that you find the area of something.

T: And when you find the area of something, what are you finding? What does that word area mean? If you wanted to get the area of this desk, what does that mean? What would you be looking for?

S: The whole.

T: The whole desk?

S: No. The middle.
T: The middle of the desk? Okay. Can you say more about that?

S: No, Ma'am.

T: No. Okay. How do you find the area of a desk? If I wanted to find the area of this desk, how would I do that?

S: By putting your hand on it.

T: Show me and tell me. Explain a little bit more. Okay. And would just putting one hand on it tell me the area?

S: Huh-uh.

T: No. What else do I have to do?

S: Put 2 hands on there and go around the whole desk.

T: All the way around it. All right. So if I go all the way around and I tell you there are fifteen hands, is that the area?

S: Yes, Ma'am.

T: Okay. Why do you think we have to learn the area? Why do you think that's in your book? It's in your book. You're getting to it at the end of this year. Any idea? No idea. Okay. Do you know for what people need to know area? What kind of jobs do people do for which maybe they need to know about area? Do you think a policeman needs to know area?

S: Uh-huh.

T: For what? Try to think of a lot of different kinds of jobs. What kind of people need area?

S: A fireman.

T: For what? Why does a fireman need to know area? Any idea? No. Okay. What is perimeter? We studied area and perimeter. What do I mean by perimeter?

S: The middle.

T: The middle of something. So the perimeter of this desk is what? Or how would I find the perimeter of this desk?

S: By putting your hand on it.

T: All right. Show me again. That's what you did for area. How would the perimeter be different than that?

S: You got to do the whole desk.

T: By whole desk, what do you mean?

S: With your hands.

T: Okay. The whole thing.
S: And the side.
T: And a side. Okay. And that will give you the perimeter. In other words, if you want to know how many hands cover all of the desks, that’s perimeter? Okay. Why do we learn perimeter? Hard question, huh? Think real hard. Why do you think we have that in the math book for fifth graders?
S: So we can learn it and when we get in the higher grade.
T: Okay. But why do think they have it in higher grades? Why does the person need to know the perimeter? Any idea? Can you think of any kind of job that needs to know perimeter? Okay. What if, in this room, (Student I), if I was trying to put carpet in this room, do you think I need to know perimeter and area to lay the carpet?
S: Uh-huh.
T: Would I need to know perimeter or area? What do you think?
S: Perimeter.
T: So if I told you that this room is eight feet wide and twelve feet long, how would I find out how much carpet I need?
S: Find the area.
T: I’d have to find the area. And how would I do that? If this is eight feet wide and twelve feet long, how do I figure out what is the area of this room or what would I have to do to find the area?
S: Get a tape measure.
T: And then what do I do with my tape measure?
S: Measure to see how long it is and how wide.
T: But what if I measure it, (Student I), and I find out that its twelve feet long and eight feet wide, what do I do then? Any idea? No. Okay. Do you think I need to know the perimeter or the area of the room to know how much paint to use on this wall? Do you think if I knew the area that it would help a little bit?
S: Yes.
T: Would the perimeter help me to know how much paint to put on the wall? Perimeter or area, which one?
S: Area.
T: You think the area will tell me how much paint to use? Okay. What if I wanted to put baseboards around this room. Do you see the black plastic that goes around under the shelves? If I wanted to put it all the way around the room, would I need to know the area of this room or the perimeter to know how much baseboard to buy?
S: Area.
T: The area would help me with that also? All right. I want you to draw me any shape you want and then find the area. Tell me how to get the area of that shape.
S: You need to see how many corners.
T: See how many corners. There are four corners. So is four the area?
S: Yes.
T: So the area of that shape is four? Is that what you’re telling me? You can pretend this is any measure you want; just tell me what measure you want that if you need it. What if this measures five? Five units. The area is four because it’s four corners, right? What’s the perimeter? Why don’t you write down area equals four. What is the perimeter? Do you know the perimeter? No idea? Okay. Let me try one other thing. With the first group, when I worked with them, we said that the perimeter of a rectangle, I gave them a formula that some mathematicians had to get the perimeter. You can take $2(l+w)$. This is the length of the rectangle. This is the width of the rectangle. And you add that together and multiply it by two. That’s for this formula. Do you know why? Does this tell you anything about how to get perimeter? Does that help you at all? Okay. In what way? What does that tell you about a rectangle.
S: How wide it is.
T: It tells you how wide. All right. So if the width of a rectangle was three feet and the length was four feet.
S: You add that up.
T: You add this up and then what happens?
S: You multiply it.
T: By two. All right. You're absolutely right.
Now why would they give you this formula for
the perimeter of a rectangle? Why do you think
that works to add these two sides and multiply
by two?
S: So you can find the perimeter.
T: That gives me the perimeter, you're right. But
would they use this same formula for a
triangle? What about a triangle? Could I use
that same formula to get the triangle
perimeter?
S: Yes.
T: I can also just add the length and the width
and multiply it by two to get the perimeter?
S: (Nods head.)
T: All right. There's another formula for the
area of a triangle. It says you take a half of
the base times the height. Now can you tell me
why that formula would tell me something about
that area of that triangle? Does it make any
sense to you? No. Okay. I want to ask you
just one or two questions about your test, and
I just want you to explain to me how you got
your answer if you remember. For number 6, the
area of this irregular shape. It looks like an
L. You told me it was 16. Do you remember how
you got 16?
S: I added.
T: You added all those numbers? Okay. To get the
area? This asks for the area of this shape.
Do you remember how you got 19 for number 28?
S: No.
T: Do you remember? You don't. Okay. Thank you
very much (Student I). I think that's all the
questions I need to ask you.
Student J (treatment 2, level 2, female, class 1)

T: You were with me just this past week and you were with me for interviews. How did you like this week’s classes?

S: It was fun.

T: It was fun. Okay. What was one of your favorite things about it?

S: We got to play with the geoboards.

T: You like geoboards. Was there anything we did that you really didn’t care for a lot? (-) No. You liked all of it. Were the classes we had pretty much like your usual math classes or were they different?

S: A little different.

T: In what way were they different?

S: In our usual math class, we don’t use all kinds of stuff.

T: Okay. All kinds of materials; you don’t usually do that? Do you feel like you learned a lot?

S: Uh-huh.

T: Good. Those things that we used, when you talk about the geoboards and the tiles or our hands or the grid paper, those are all called manipulatives.

S: (Nods head.)

T: Do you feel like those manipulatives helped you learn? Can you explain that a little bit? How do you think they helped you learn? Can you explain that?

S: (Shakes head negatively.)

T: No. That would be too hard. What is area, (Student J)?

S: The measure of something.

T: Okay. Tell me a little bit more, because you’re about how tall, about 4½ feet? Is that the area of you?

S: (Shakes head negatively.)

T: What do I mean by area? Tell me a little bit more.

S: Like the measurement of the whole thing, the whole part, like the inside.

T: The inside. Okay. Why do you think we learn area? Let me ask first how do we get the area of something? How would I get the area of this desk?
S: You add it. Sometimes multiply.
T: Sometimes multiply, sometimes we add. How do we find the area of any kind of space? You said sometimes we add and sometimes we multiply. Can you tell me any more about that?

S: (Shakes head negatively.)
T: Why do we learn area? Why do you think that's in our math books?
S: So if you need to measure how big it is on the inside.
T: Can you think of any kinds of jobs that maybe need to use area? Any particular men or women that need to know area?
S: (Shakes head negatively.)
T: You can't think of anyone who needs to know area?
S: (Shakes head negatively.)
T: No. What is perimeter?
S: The area around something.
T: The area around something. How do you find the perimeter of a region, of this room? How would you get the perimeter of this room?
S: You add all the sides of it.
T: Okay, can I get the perimeter of a wall? Could I get the perimeter of that wall? All right. Could I get an area of that wall?
S: Huh-uh.
T: Why not?
S: Well, yea.
T: I could? What would the area of that wall mean?
S: How large it is all on the side of it.
T: So I could have area of the wall and I could have area of the floor, either one. If I wanted to know how much carpet to buy, would I need to know area or perimeter?
S: Both.
T: All right. Tell me. Explain that a little bit.
S: You need to know how wide you want it.
T: Okay. I need to know how wide, maybe how long also. So if this room measures eight feet across the width and it's twelve feet long, could you explain to me how to get the area of this room? Try.
S: Try and multiply like eight rows and the twelve.
T: Eight rows times twelve feet. That would be the area? All right. What about the perimeter of the room?
S: Add up all the sides of it.
T: What numbers would I add in this case?
S: Eight and 12 and then 8 and 12.
T: If I want to know how much baseboard to buy, that's that black plastic, if I wanted to put it all the way around the room, would the area or the perimeter help me to know how much baseboard to buy.
S: Perimeter.
T: Okay. The area wouldn't give me too much information.
S: (Shakes head negatively.)
T: I want you to draw a shape on here, any shape you want, and then I want you to get the area of that shape. So choose some shape. You're going to do a rectangle. All right. Give it some kind of measures. So the width of that rectangle is...
S: Four.
T: And the length is...
S: Three.
T: Why don't you write that on there. It's three by four. That's four and that's three. All right. What is the area of that rectangle?
S: Twelve.
T: All right. On the side, why don't you put \( a = 12 \). All right. What is the perimeter?
S: Fourteen.
T: Okay, put \( p = 14 \). You just went all the way around and you added that up, right? What if I tell you that some mathematicians made up a formula and that if I want to know the perimeter of a rectangle, I have to take \( 2(l + w) \). Now what this means is the length plus the width and then I have to double it. Does that make any sense to you why we would have this formula to get the perimeter of this rectangle? Can you show me how this formula somehow connects with that?
S: Add 4 and 4 is 8 and 3 and 3 is 6. And 8 + 6 = 14.
T: I know that, but do you think this formula says the same thing that you just said in some kind of way? Tell me how those two compare.

S: Because when you add it up you double that number and double that number.

T: Okay. That gives you perimeter. Good. There's also a formula for area of a triangle. There's a triangle, there's a formula that says, I didn't teach you this formula, but I'm just telling you, \( \frac{1}{2}bh \). "B" is the length of the base of the triangle, "h" is the length of the height. Why would we have a formula that says to get the area of a triangle, you take one half times the base times the height? Does that make any sense to you?

S: Yes.

T: All right. Explain it to me.

S: Because when you have a square, you just draw it and half.

T: So you're saying the area of the triangle is...

S: Half of a square.

T: Could I get the area of the square by taking the base times the height?

S: (Nods head.)

T: Okay. All right. There are a few problems on your test that I want you to just kind of explain a little better to me, please. Number 6, you said the area of this shape is 64. Can you remember how you did that and explain it to me a little better?

S: I counted the squares.

T: Do you think they have 64 squares there?

S: No.

T: No. You don't have 64, so I was wondering where you got your 64. Do you remember?

S: Huh-uh.

T: Tell me what you think the area should be now that you look at it?

S: Seventeen.

T: You think it should be 17. How did you get 17? Explain it a little bit more.

S: It's 16.

T: Sixteen. So you're just counting all the squares and there's 16. So if you were to take this test again, you would change it to 16. You have no idea where you got 64 from?
S: (Shakes head negatively.)
T: No. Okay. You had a problem here where you drew a square for number 10, and I said that the area is 36, what is the measure of each side? And you said 6. How did you get that?
S: $6 + 6 + 6 + 6 = 36$.
T: $6 + 6 + 6 + 6$ I think is 24.
S: (Nods head.)
T: All right. Now it says the area of the square is 36 centimeters, what is the measure of each side? So are you disagreeing with me or are you still agreeing? I'm not saying your answer is right or wrong, I'm just wondering how you got it because I wasn't sure.
S: I multiplied $6 \times 6$.
T: Why did you multiply $6 \times 6$?
S: Because there are six rows on the width and six for the height.
T: So there are six rows and then six lines. I can agree with that. Here you kind of showed me how you got the perimeter. I want you to look at it one more time. We're looking for the perimeter of this irregular shape in number 14. Can you explain to me how to get the perimeter of that?
S: Add all the numbers up around it.
T: Tell me the numbers that you would add.
S: 6, 4, 6, 3, and 5.
T: So you're saying that's how you would get. You're not sure that 42 is the right answer, but that's how you get the perimeter. I thank you very much (Student J). I think that's it. You did well.
Student K (treatment 2, level 2, female, class 6)

T: (Student K), I have not interviewed you before at all have I? Okay. But you were in last week's session. How did you feel about last week's class?

S: In your class?

T: Yes.

S: I haven't been in your class yet. I've always been in (teacher's name). I haven't been there yet.

T: You weren't with me last week? Hold up. Okay. So you were with me last week.

S: (Nods head.)

T: Right. Okay. I see now what you mean. You weren't in just a small group with me, but you were all together with me. All right. How did you like those classes when we worked with finding area and perimeter, and used our hands, and we used tiles, and we used geoboards and we used grid paper?

S: I liked it.

T: You liked it. Okay. What was one of your favorite things we did last week?

S: The geoboards.

T: The geoboards. Everybody likes geoboards. Was there any part that you really didn't care for?

S: When we had different kind of shapes and we had to find the perimeter or whatever.

T: Perimeter?

S: Yeah, the perimeter and the area of it. And the five shapes and I couldn't figure out.

T: Okay. So the irregular shapes are hard for you. Did you feel like you learned a lot?

S: Kind of.

T: How did you feel those materials helped you to learn, or do you think having the geoboard and having the grid paper -- But did that help you learn? And how did it if it did?

S: Yeah, kind of.

T: How did it help?

S: It helped me find the fractions in squares and triangles's area and the perimeter. I can't say it.

T: Perimeter is not too bad. Perimeter is how we should pronounce it. Can you tell me a little bit more about how it helped you get it? What
if you didn’t have the tiles or didn’t have the geoboards?

S: It would have been harder because I couldn’t — everybody’s hands are different to go around so it wouldn’t come out exactly the same on all of it.

T: All right.

S: And I don’t know. It just did.

T: It just did. You could see it or something. Were the classes that we had last week pretty much like your usual math classes with (the teacher)?

S: Kind of except we don’t use geoboards and tiles and things like that. But I’m not really used to perimeter.

T: So even what we learned was a little different for you? Okay. What is area?

S: You just measure stuff around the room. Well, perimeter means it goes around.

T: Okay. And what is area?

S: The whole room. This is just to see how far around it goes. The area is the whole room.

T: How do you find the area of a room or a region or space or shape?

S: Well, you can use measures, different kinds of measures. If I was to do this room, I’d probably take all these books and put them all around the room, but just use different measures to get the area.

T: Okay. So what if I wanted to get the area of this room and I would measure a few things for you. I would tell you that it’s 8 feet along that wall and, I would tell you it’s 12 feet along this wall, and I would tell you the ceiling is 7 feet high. Can you get the area of the floor?

S: I would probably have to do some multiplying and dividing.

T: To get area?

S: To get area.

T: What would you multiply and what would you divide?

S: Well, I’d multiply 8 x 8 and that would be the same length and I’d get my answer and then I’d add 12 + 12. I don’t know.

T: You’re not sure what you’d do with that, huh?
S: Huh-uh.

T: What if I told you I would give you $100 if you find the area of this room, what would you do about that?

S: I'd start taking down the books and start doing stuff.

T: Okay. So we were talking about the area of this room and the perimeter of this room. Let me get back to perimeter. What is perimeter? I think you kind of told me.

S: It's the measurement of the way you go all the way around the room like you could start at a certain place and go all the way around the room and that way you'd get perimeter.

T: And how do you find the perimeter? We said all the way around. Why do we learn perimeter?

S: Because if I want to wallpaper my room and I wanted to know how much I need all the way around, I would have to measure it so I need perimeter.

T: Can you think of any other job that you would need to do, or someone else would need to do, where they would need to know perimeter?

S: If I want to put in a bigger window right there, I would need perimeter to see how much window I need to put in. How much I could and would.

T: How about area? What do I need to know area for?

S: To figure out how much room I have. If I want to put in a shelf, I need to know how much area it'll take up and how much I won't have after I put...

T: After you finish up. Okay. How much space. All right. If I told you that in this room, with the same measures I had a while ago, it's 8 feet along that wall and it's 12 feet this way; how would you get the perimeter of this room?

S: I'd add it all up.

T: What would you add?

S: I'd add 12 and 8 and 12 and 8.

T: Okay. All right. Tell me one more time how you would get the area of the floor?

S: I'd take those books and start putting them out and I'd count how many books I put out. I
would get my area, but they have different sizes just like your hands. It wouldn’t come out exact.

T: It wouldn’t be too exact, but it would give you an idea. So if I had to buy carpet for this room, I would go to the store and say, "I need 80 books of carpet." How about that? That’s a good idea, laying books. I like that idea. So what could I do about that?

S: Well, you could see how much a yard is like put all the books together and see how much a yard is and you could go down and measure around the room. And you got that many yards and do it again and do it again and do it again. You can go down and say I need this many yards.

T: You mean square yards of carpet. Okay. That’s good. If I want to buy tile to cover this floor, do I need to know area or perimeter?

S: Probably area.

T: Can you say why?

S: Because area is finding the mass of the room so the tile is not going to be up on the wall, it’ll be on the floor, so you would probably need just the area.

T: Now if I wanted to buy baseboards, baseboards are like those black plastic strips, if I wanted to buy it to go all the way around under all of the shelves, would I have to know area or perimeter?

S: Perimeter.

T: Perimeter. Okay. Could I find an area of the wall? I mean is there an area of the wall?

S: Uh-huh.

T: Okay. How would I do that?

S: That would probably be where you could measure how much that is with like a board or something and you could go around the room like that and you can probably do the same, I mean area, because it was just like taking this room and pushing it over that way. That would be the floor and that would be the wall.

T: Oh. Okay.

S: And you could probably do the same.

T: Okay. So we still could get area and perimeter whether it’s a wall or a floor or a ceiling, right?
S: Uh-huh.
T: Good explanation. I want you to draw a shape on here, any shape, and you give it some lengths and widths or whatever and then I want you to find the area of it and the perimeter. Okay. Show me how you do that.
S: That would be 8. Put 3 here and 3 here. Do you want the area or the perimeter?
T: The perimeter.
S: It would be 22, I think.
T: How did you get 22?
S: I added it all up.
T: Okay. So put \( p = 22 \). You added that very quickly in your head. How did you do that?
S: \( 8 + 8 = 16 \) + 6 = 22.
T: Good. All right. What is the area?
S: That would be longer. I have to do something for that.
T: What do you need to do?
S: I don’t know. I’d have to break it up (Draws grids in the rectangle.). Very crooked. Forty-one units. I guess 41. I might have miscounted.
T: Do you want to count one more time?
S: Yap. 40.
T: Okay. The area equals 40. All right. You were not in that small group that I taught first. In that small group we learned some formulas, and we learned that if I want to get the perimeter of a rectangle, I can take \( 2(l + w) \). So I add the length plus width and I multiply by 2. Can you see any connection — what’s the connection between this formula and finding the perimeter of this rectangle?
S: I don’t see the connection. Well, the 2 could go into my answer, but I don’t think that’s it.
T: Okay, so that wouldn’t.
S: I don’t know.
T: You don’t know why you would add the length plus the width and then double it.
S: Were you talking about like add that and then add that? Add it all together and --
T: It looks like you’d add the length and the width. You add the length and the width.
S: Oh. Okay. That would be 11 into 2.
T: Into 2?
S: Uh-huh. I don’t know if you have to multiply. It would be the length times 2 and that would be 22. So that’s the connection.

T: That’s the connection. In other words, I could add this and then double it. Why would that work? Why could I always add this and then double it and it would work?

S: ’Cause that’s perimeter. Whatever.

T: Whatever that word is. You can’t think of any other reason? It happens to be the perimeter. We also learned that if you want to get the area of a triangle, you can multiply the base times the height. The base is this part, the height is the length of this. I’d multiply those two and then I would take half of that answer I get -- And that would tell me the area of this triangle. Can you think of any connection between that?

S: Do you want me to make up numbers? Like I did on that one?

T: Fine. Yes, if you need to make up numbers. I don’t want you to get the area as much as I want you to tell me what’s the connection. Why would that formula work to get the area of a triangle?

S: I’m trying to do it on that one, with the number there. Half of 15 is -- You have to add it. Half of 8 is 4.

T: Why do you have to add it?

S: I don’t know. I added on that one, so I have to add on this one.

T: Okay. But what does this formula say to do?

S: Maybe I have to multiply because this is area not perimeter, whatever it is.

T: And this formula says to multiply. You were right. You multiply 5 x 3.

S: Fifteen and half of 15 is -- I know what it is but I forgot it months ago.

T: What is half of 14?

S: Seven.

T: Okay. And half of 1?

S: .750.

T: 7.5 or 7½, right? My question is why to get the area of this triangle would I have to take half of 15? Do you know?
S: Well, 15 is probably a big number for just that, so it would have to be -- you would have to add that, but that's area. That's perimeter and that's area.

T: But why wouldn't I take 1/3? If 15 is too big, why wouldn't I take 1/3 instead of 1/2?

S: Well, you don't want to put too much or -- So you just do it.

T: You would just do it. Can you explain why we do it? It would do it, but I wanted you to discover why it would do it. You don't know why it would do it?

S: Huh-uh.

T: Fine. All right. I want to look at your test you took and go over just a few items. I didn't score these yet. I'm not saying they're right or not. You didn't show me exactly how you got your work, and I was curious to know. All right. For example, Number 6. How did you get 19 for that one? Do you remember?

S: I think I did this. I had 3 right here times 6 and that would be 18. Oh, and then, I don't know. I'm not sure how I got that.

T: What if I asked you to get the area of this irregular shape now; could you get it or how would you go about it?

S: That's a hard one. I wasn't sure. I kind of guessed, because I wasn't quite sure how to use the perimeter and area and all that.

T: So you don't know for sure how you would do it now either?

S: Huh-uh.

T: No. Okay. Let's do another one and I want you to explain it. This is perimeter.

S: I think I added all that and that would be 16 and I probably multiplied that and then --

T: Multiplied 5 and 3.

S: And got 15 and then added 4. I don't know. I did some kind of stuff to make it come out, whatever.

T: How would you get the perimeter right now?

S: I would add this all up, not the 5 and the 3, just this; just multiply them.

T: Even if I want the perimeter of the whole thing?
S: Well, then I'd -- I guess I'd have to add that with it or else I'd have to make it where it'd be part of the shape. I would probably turn this around and make it look like that. You know like that. So I think it might be easier for me if I did that, but I don't know.

T: Not real sure how you'd do that. All right. Do you know how you got the area for this rectangle, number 24?

S: I added it all up and got 12.

T: 12, 14, and 16.

S: Uh-huh.

T: Okay. All right. Thank you very much, (Student K).
Student L (treatment 2, level 2, male, class 6)
T: How did you like last week's class that we had?
S: It was fun.
T: You enjoyed it? Okay. Was it like your normal math classes?
S: (Shakes head negatively.)
T: Not like it? Why wasn't it? What was different?
S: The geoboards and all those other shapes we were playing with.
T: Okay. We played with lots of different things. What did you like most about this lesson? What was one of your favorite things? We got area and perimeter with hands, we got it with tiles, we used the geoboards, and we cut out grid paper. What was one of your favorites?
S: Grid paper, I guess.
T: Cutting out grid paper. All right. Was there anything that you did not like in those lessons?
S: Those little cube things.
T: Those little squares?
S: Yeah. They were hard.
T: You found that hard. Do you feel like using those materials, those manipulatives, they're called manipulatives all those things that we handled with our hands, do you feel like those helped you learn math, learn area and perimeter?
S: Uh-huh.
T: Can you say how that kind of helped?
S: Huh-uh.
T: No. You just know that it helped? What is area, (Student L)?
S: The width of something. How much it is, the whole thing.
T: Okay. The whole thing. All right. How do you find the area of a region, of a shape?
S: Times how long this is and how long this is. (Pointing out length and width of desk.)
T: Okay. You multiply those two? Okay. Why do you think we learn about area? Why is that in our math book?
S: It's probably not one of the most important things we need.
T: We probably need it a little bit or else it wouldn't be in the book, huh? Why do you think they teach it? Why do you think they put it in the book?

S: 'Cause whenever you're a carpenter you need it.

T: Do you think anyone who is not a carpenter will ever need it?

S: (Shakes head negatively.)

T: Can you think of anyone else who might need it?

S: Oh yea, for carpet or the floor.

T: Okay. Doing the flooring. Speak up a little bit louder because I want to be sure to catch you.

S: If there's a carpet, you need to know.

T: What about wallpaper; would you need to know area for that?

S: Uh-huh.

T: Possibly painting also. If you're painting the walls outside of a house, would you need to know area?

S: (Nods head.)

T: You guess? Okay. Can you think of anything that your mom does at home where she needs to know about area, besides laying carpet when she's buying carpet or painting a wall or wallpapering? Can you think of anything else? No. Okay. What is perimeter?

S: How long it is all the way around.

T: How do you find the perimeter?

S: Add up all the numbers. (Moving his hand around his desk.)

T: Okay. All of the sides. What kind of jobs need perimeter? Why do we learn perimeter?

S: I don't know.

T: You can't think why we need perimeter? Okay. If I asked you to find the area of this room, could you do that?

S: (Nods head.)

T: How would you do it?

S: I'd find how long it is from that wall and how long it is from that wall and I'd times it.

T: All right. What if I wanted you to get the area of the wall; how would you get the area of the wall?

S: I'd see how long that wall is and how long -- this wall?
T: Let's just say that wall. If I just want you to get the area of that one wall, how would you?
S: Area or perimeter?
T: The area.
S: Measure and see how long the top and the side and I'd times.
T: Okay. To get the area. How could you find the perimeter of that wall?
S: Measure and see how long it is on four sides and add then.
T: Add it up. Okay. If I'm going to buy tile for this room or carpet for this room, do I need to know area or perimeter?
S: Area.
T: How about if I want to buy baseboards? Do you see that black plastic, that's like a baseboard. If I wanted to put it all the way around the room, do I need to know the area or perimeter of the room?
S: Perimeter.
T: Okay. I'd like you to draw a shape, any shape you want, and then I want you to figure out the area and perimeter of it for me. You'll give it measures. Any shape you desire. A triangle. Okay. Now, do you want to give it some measures? Perimeter, what would the perimeter of that triangle be?
S: Eleven.
T: All right. Put $p = 11$ just so we don't forget that. So I have a record. And you simply added up the numbers. Right? What is the area of that triangle?
S: Ten.
T: How did you get 10?
S: I times that times that.
T: Okay. Why don't you put that down. Let me show you. I didn't teach your group this, but the other group, I taught that to get the perimeter there is a formula. Mathematicians say that you can get the perimeter of a rectangle by taking $2(l + w)$ of a region. So you have to add the length and the width and then you double it. I want you to look at that formula, think about a rectangle and tell me
why that would give you the perimeter. What's the connection between them?

S: Because the length is this one and the width is this one. You add these 2 and you just double it.

T: Good. All right. There's also a formula that says for the area of a triangle you take \( \frac{1}{2} \) of the base times the height, the base and the length. The height would be perpendicular to that; going straight up. Now, can you think of any reason why, or any connection between, this formula and the area of a triangle? In a triangle, we didn't have this in your class, this would be the base and this would be the height of this triangle. The formula says that I take one half times the base times the height. Why would we do that?

S: To get the area.

T: Right. But why would that work? What's the connection? You connected this very well. Can you connect these two?

S: 'Cause if you add these.

T: A number next to another one is multiplication, so I'm multiplying.

S: (Shakes head negatively.)

T: That doesn't make any sense to you? Okay. I'd like to look at your test and just ask a couple of questions. This one, you found the area, number 6. The area of this irregular figure, can you tell me how you got it, or what is that number?

S: Four.

T: That's a 4. Okay. Tell me how you got 4. I'm not saying these are right or wrong. I'm just asking. I didn't know how you got it.

S: In this one right here you got a square. You got to see how many squares you got in this. You got 4.

T: Oh. Four squares of this size fit in here. Okay. Good explanation. Number 1 said draw a triangle, label the length 8 and the width 5 and find the area. How did you get the area?

S: I don't know. It should be 40.

T: It should be 40 and you got 20 so something is messed up here. You got 40 by multiplying 8 x 5. I've got one more. Let's see if I marked
one more. This is an interesting one. (# 23)
Another irregular shape and you need to find
the perimeter. Do you remember how you did it?
S: That times that equals...
T: Do you agree that that’s how you get the
perimeter?
S: (Shakes head negatively.)
T: How would you do it today?
S: I would add. This and this would be 10 plus
this added to 10. That would be 20 and this is
10. That’s 30. Then that would be 38. Then
this ...
T: Is there a way you can figure it out you think?
Okay.
S: 2? Is 40.
T: Not 40. Is it 40? Yes, it is 40. And maybe
that’s what you did.
S: I don’t remember.
T: Okay, (Student L). Thank you very much. I
appreciate it.
VITA

Dolores Pesek Simoneaux was born in Shiner, Texas, on April 20, 1942, to Rudolph and Henrietta Pesek. After graduating from St. Ludmila’s Academy in Shiner, Texas, Dolores attended Our Lady of the Lake University, in San Antonio, Texas, where she earned a B.S. degree in elementary education in 1966. After four years of teaching in the primary and junior high school levels, she received a M.Ed. in reading from the University of Southwestern Louisiana in Lafayette, Louisiana, in 1977. In 1967 Dolores married Ernest Simoneaux. They are the proud parents of three daughters: Mimi Michele, Christi Angele, and Niki Daniele. Even though Dolores’ 20 years of teaching experience encompassed primary to adult education, the majority of those years was spent teaching mathematics at the eighth grade level. She pursued the Doctor of Philosophy degree in Curriculum and Instruction with the emphasis in elementary mathematics, awarded in December of 1992 at Louisiana State University.
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Title of Dissertation: The Interference of Non-meaningful Learning on Subsequent Meaningful Learning

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