Regional Comparative Advantage and Inefficiency in Production: Methodological and Empirical Issues.

Barun Kanti Kanjilal
Louisiana State University and Agricultural & Mechanical College

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Regional comparative advantage and inefficiency in production: Methodological and empirical issues

Kanjilal, Barun Kanti, Ph.D.
The Louisiana State University and Agricultural and Mechanical Col., 1992
REGIONAL COMPARATIVE ADVANTAGE AND INEFFICIENCY IN PRODUCTION -- METHODOLOGICAL AND EMPIRICAL ISSUES

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Agricultural Economics and Agribusiness

by

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December, 1992
DEDICATION

This dissertation is dedicated to the memories of

my mother Mrs. Sovarani Kanjilal,

who always wanted me to pay her by this way

and

my father Mr. D.L. Kanjilal,

from whom I could have learnt more about the ways to struggle.
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The general objective of this study was to provide a methodological framework for evaluating stochastic comparative advantage of a crop in a multi-region multi-crop framework and its link with inefficiency at the firm level.

In the first stage, regional comparative advantage, defined in terms of relative profitability, was theoretically analyzed on the basis of a firm's behavior under uncertainty. The empirical application involved derivation of a comparative advantage index for major crops produced in different regions of Louisiana. The results revealed heterogenous survival potential of each crop across regions. Sugarcane was found to have comparative advantage over rice and soybeans in the Sugarcane region and over cotton and corn in the Southwest Rice region. The probable impacts on the comparative advantage due to external shocks were also derived for each crop.

Next, efficiency of selected sugarcane farm-firms was evaluated. Using a panel data of forty-five firms, firm-specific technical and allocative inefficiencies were estimated via alternative model specifications. Statistical results revealed that technical efficiency of each firm has increased over time. No correlation was found between farm-size and efficiency. Allocative inefficiency was found much higher than technical inefficiency. Also, fertilizer is being used over-optimally causing high degree of allocative inefficiency.

Finally, the theoretical structure derived in the first stage was extended to analyze the link between inefficiency and existing resource allocation among firms.
Cost inefficiency (a combination of technical and allocative inefficiency) was estimated directly from the cost function by using the same panel data. A frontier (without inefficiency) index was derived by purging estimated inefficiencies from total cost. Comparison between frontier and observed (with inefficiency) indices revealed that an improvement in efficiency will contribute significantly to firm profitability. However, large firms in general have higher advantage than small firms with or without inefficiency. This supports the hypothesis that the disappearance of small sugarcane firms in Louisiana is not due to lower efficiency, but due to lower income generating capacity.
CHAPTER 1
INTRODUCTION

1.1. Introduction

The study of regional specialization and comparative advantage in agricultural production has contributed to our understanding of the spatial location of producing units and how they respond to changes in resource constraints and other factors. These changes, along with technological progress in agriculture, have caused comparative advantage and efficiency of crop and livestock production to change over time. As a result, the quest for economic intelligence requires frequent reassessment of whether producing units are allocating resources in a way that is consistent with their local economic advantage and in the most economically efficient manner.

The economic literature is abundant with theories that explain and measure comparative advantage and the economic performance of firms. Early in the development of spatial sector programming models in agriculture, for instance, Heady and others applied simple indicators of relative comparative advantage to explain the geographical distribution of farm-firms (henceforth "firm"). These efforts pioneered applications of interregional programming models for agricultural policy formulation over the decades to follow. Little theoretical justification, however, has been presented to justify the adoption of income-cost indicators to measure comparative advantage. One significant aspect of agricultural production
is that regional specialization is not entirely driven by physical factors. Economic incentives trigger supply responses that result in a reallocation of resources. These incentives cause changes in the comparative advantage rankings of various enterprises. It appears, therefore, that the study of comparative advantage should account for stochastic behavior.

Regional comparative advantage analysis is the basic economic force behind the regional specialization one observes for various crops in different areas. Historically, it is often found that a crop, which was considered the most profitable crop in a region, becomes relatively less profitable as a result of adverse changes in supply or demand forces (for example, adverse change in terms-of-trade of the product). Given the uncertainty in the changes of some of these forces, it is one of the important factors for the survival of a firm to acquire and utilize the information of probable impacts of these changes on its profitability and adjust its use of resources or change location of farming accordingly. Regional comparative advantage analysis provides this information on the basis of the historical structure of cost and income of crops in a region.

During the late 1970s and 1980s, agriculture experienced serious financial crises. As a result, decision makers, whether private entrepreneurs or policy makers, have become more concerned of the need to design and study what is commonly referred to as "best management practices." Part of this concern will of necessity deal with firm efficiency in agricultural production. Empirical evidence on the measurement of technical and allocative efficiency is lacking for Louisiana
firms. The generation of such information will be of much value in monitoring firm performance.

The purpose of this research is to develop a theoretical analysis of cost-income relationships as measures of comparative advantage and introduce a procedure for the stochastic analysis of it. The study also establishes a theoretical link between comparative advantage and firm efficiency. The methodology is implemented in two steps. First, evaluation of profitability of a crop produced in a particular region with respect to other crops; and, second, evaluation of the production efficiency and profitability of the firms producing the crop. The first evaluation process is called regional comparative advantage analysis and the second is called inefficiency analysis.

The methodology is of general applicability but is implemented through the study of crop production in Louisiana. Specifically, this study focuses on sugarcane production in this state. Historically, sugarcane occupies an important place in Louisiana's agricultural production. Starting in 1795, the history of sugarcane production in Louisiana has been one of survival and growth against volatile profitability and changing market forces. The growth in this industry is also characterized by stability in acreage, substantial increase in yield, and industry concentration\(^1\). Regionally, it is concentrated in the Sugarcane (lower Mississippi River Delta) area with some acreage in the Southwest and the Central areas.

\(^1\) More details about the structural changes in sugarcane industry are in Section 4.1, Chapter 4.
Although there are several studies on the structural change and economic analysis of the cost structure of this industry\(^2\), no study has yet been made to assess its relative profitability with respect to other crops produced in these regions. Also, the issue of economic efficiency in sugarcane production in Louisiana has not been researched. The current state of knowledge is thus clearly insufficient to evaluate the industry's potential for profitability and its link with internal (i.e., firm-specific) use and allocation of resources. This study is an important step in that direction.

1.2. The problem statement

Regional comparative advantage is closely related to regional resource allocation and profitability of a product. Traditionally, the concept of comparative advantage is defined or explained in terms of some non-random economic (e.g., resource endowment and technological progress) and non-economic (e.g., demographic and sociological) factors. However, there is no theoretical development to analyze the comparative advantage of a product in a stochastic framework by incorporating the contribution of some random factors (e.g., uncontrollable factors, such as, product prices; and, controllable factors, such as, production inefficiency). One can not fully explain the direction of regional specialization in an uncertain world without incorporating these random factors. The problem is thus to recast the neo-classical or modern comparative advantage principle in the context of modeling and evaluating regional specialization under uncertainty.

Since regional economic advantage of a commodity is reflected in the profitability of the firms producing that commodity, the next problem is to assess whether the firms are using and allocating their resources most efficiently. A methodological basis is thus required to identify the contribution of production inefficiency to the relative profitability of a firm. The problem is important because, in the long run, the regional comparative advantage of a product may be adversely affected if the producing units fail to control production inefficiencies. From a research perspective, the problem is thus to use the methodology to generate information on the present and future profitability of the firms due to present pattern of the use and the allocation of resources.

1.3. Problem justification

The study of regional comparative advantage in a stochastic framework helps to identify the present and future directions of resource allocation in a region. Although there are several studies concerning the economics of individual crop production in Louisiana, no significant attempt has yet been made to analyze regional variations in profitability in crop production. The information from this study would be useful to individual producers to restructure production plans, as well as provide analytical input to policy makers and analysts about how and why the regional resource allocation pattern changes.

Higher profitability of a crop does not always imply higher economic efficiency in the production of the crop. This is especially true for smaller firms.
For example, it may be argued that small firms are efficient but have less income generating capacity, or have less capacity to survive adverse economic changes than the larger firms. This may be reflected especially in the sugarcane industry in Louisiana, which experienced rapid growth and increasing concentration. The reason for concentration might be higher income generating capacity of the large firms or greater ability to supply necessary capital and management at different points in time. Whatever the reason, the issue of profitability (i.e, comparative advantage) should be included in the analysis and be linked to inefficiency. This will help individual firms assess the contribution of its production or cost efficiency to its relative profitability ranking. This may also help producers or credit cooperatives to monitor the performance of an individual firm or a group of firms and to restructure their input and credit supply, and to lead producers in the direction of curbing inefficiency, if necessary.

1.4. Objectives

The general objective of this research is to provide a methodological framework for evaluating stochastic comparative advantage of a crop in a multi-region, multi-crop framework and its link with economic efficiency at the firm level.

The specific objectives are:

(1) Build a theoretical framework for analyzing a firm’s behavior under uncertainty in costs and returns and to define comparative advantage of the firm under stochastic conditions.
(2) Develop a methodology for analyzing regional stochastic comparative advantage in crop production in Louisiana.

(3) Discuss methodological issues of inefficiency estimation and analytically derive and estimate firm-specific technical and allocative inefficiencies in sugarcane production in Louisiana.

(4) Extend the theoretical analysis in objective (1) and empirical analysis in objective (3) to analyze firm-specific comparative advantage and its link to cost inefficiency of firms.

1.5. A summary of the methodology

The methodology proposed in this study introduces stochastic behavior in the analysis of comparative advantage which in agricultural production may be explained by (i) profitability of a crop in a region, or (ii) profitability of firms producing a crop in a region. The first one may be defined as regional comparative advantage and the second may be defined as firm-specific comparative advantage.

As illustrated in Figure 1.1, comparative advantage in either sense depends on both random and non-random factors. Traditional theories generally consider only the non-random factors which include both economic and non-economic (non-random factors). Among these, resource endowment is generally considered as the most important determining factor in explaining comparative advantage of a product or a firm (see Heady (1952), chapter-22).
Figure 1.1 Comparative advantage under uncertainty
The methodology proposed in this study treats the concept of comparative advantage in a stochastic framework so that it incorporates the random factors in the definition of comparative advantage. Production inefficiency is a controllable random factor determining comparative advantage and resource allocation. The methodology specifically focuses on the link between comparative advantage and inefficiency in production.

1.6. Review of literature

The analysis of regional comparative advantage was first proposed by Heady (1952) and was further elaborated in Heady and Jensen (1954). The idea was introduced very lucidly:

...The Cornbelt also can grow more potatoes, fruit and vegetables per acre than many regions which specialize in these crops. Many areas in the Northeast can produce 100 bushels of corn per acre, as can many in the Southeast. Why, then, doesn’t the Cornbelt specialize as much in ... potatoes, vegetables and fruit as in corn, hogs and cattle feeding? Why don’t New England farmers produce corn and hogs, and Southeastern farmers go into intensive grain and meat production? The answer...is the law of comparative advantage.

Heady and Jensen (1954, p-33).

After this, the authors (Heady and Jensen) explained the law of comparative advantage by an example. The example may be reproduced here in a nutshell.

Consider two regions for comparison: The Cornbelt and Great Plains, and two crops: wheat and corn. The Cornbelt localities get a margin of $8 per acre on corn and $4 per acre on wheat; the Great Plain localities get only $3 on corn and
$2 on wheat. That means the Cornbelt has an *absolute advantage* over the Great Plains in both crops. In spite of that, "why doesn't the Cornbelt produce both while the Great Plains eliminates them?"

The answer, according to the authors, is given in *comparative* or *percentage* advantage. For appropriate comparison, the true indicator is not the profit per acre, but *percentage* of cost in total income (or, cost per $1 return\(^3\), or, cost-income ratio). The data show that the Cornbelt has $0.88 cost per dollar return from corn and $0.93 cost per dollar return from wheat. Alternatively stated, income for corn in the Cornbelt has a greater percentage above costs than for wheat. Thus, the Cornbelt farmers are going to produce corn because they can make more profit by doing so. On the other hand, in the Great Plains area, the percentage cost above return for corn and wheat are 0.94 and 0.88 respectively, making wheat more profitable than corn. Therefore, the Cornbelt area has comparative advantage in corn and the Great Plains area has comparative advantage in wheat. Finally, the authors concluded that "...if producers want the greatest profit, they should produce those things, considering yields, costs and prices, in which their relative or percentage returns are greatest\(^4\);..."

The above definition, however, is too simplistic to capture the complexities of reality. Note that the definition of *comparative advantage* given above does not

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\(^3\) or, equivalently, return per $1 cost. Heady and Jensen used this (i.e, ratio of return and cost) in their example.

\(^4\) or, alternatively, the cost-income ratio is the lowest.
involve the comparison of two regions in a Ricardian sense. That means, a crop may have comparative advantage in both regions by the above definition (if the percentage of cost over returns is lower in both areas) which is not possible in the Ricardian concept of comparative advantage in a two-country case. Assuming that resources are not perfectly mobile across regions, the above definition is thus just another form of *absolute advantage*. The concept of *regional comparative advantage* becomes meaningful only when the *relative percentage advantage* of one crop derived from one region is compared to the same crop from another region.

Moreover, the conclusion that producers should "swim" in the direction of the highest percentage returns may make sense only when the net returns are more or less the same in both crops. If they differ significantly, then higher net return with lower percentage returns in wheat may overweigh the advantage of corn which has, say, lower net return with higher percentage return. In that case, the advantage derived by the above method may show a distorted picture of reality.

Finally, the concepts of higher profit and higher profitability have been used interchangeably in the above formulation. Although this is acceptable under certainty, the assumption of uncertainty in profit may break their one-to-one correspondence\(^5\). As a result, the definition of comparative advantage used in a non-stochastic framework would be insufficient to explain comparative advantage under uncertainty.

\(^5\) This is shown theoretically in Chapter 2.
The issues concerning relative profitability and regional specialization are discussed by several authors in the context of production in manufacturing and the agricultural sector. Rao (1965) discussed the relative profitability of several crops produced in six typical production regions in India on the basis of the presumption that the farmers' decisions regarding the allocation of area are influenced by the relative profitability of crops at the margin. Using cost and return data in a static non-stochastic framework, he derived relative profitability of the crops on the basis of net-income per acre at the margin from the crops concerned after meeting the variable costs. The role of regional productivity differential on the regional specialization in the context of manufacturing and agribusiness was discussed by Moomaw (1981) where the attraction of the southern states in terms of profitability was attributed to lower wage rate in spite of lower labor productivity. The role of markets in determining the regional specialization was stressed by Perloff et al. (1963).

The efficiency aspect of production and cost has been explored extensively by theoretical and empirical researchers in economics. The possibility that producers might operate inefficiently is typically ignored, and occasionally acknowledged and dismissed, in modern neo-classical production theory. In this literature, as exemplified by the works of Carlson (1939), Hicks (1946), Samuelson (1947), Frisch (1965) and Dano (1966), it is assumed that the producer successfully allocates all resources in a privately efficient manner, efficient relative to the constraints imposed by the structure of production technology and by the structure
of input and output markets, and relative to whatever behavioral goals are attributed to the producer. The possibility of influencing the richness of the resulting testable hypotheses by relaxing the full efficiency assumption has not, however, been explored by any of the above works.

Somewhat outside the mainstream of modern neoclassical production theory, the study of efficiency and its measurement has been undertaken by a number of writers. Early efforts in the investigation of efficiency and its measurement were made by Koopmans (1951,1957) and Debreu (1951). Both, however, studied the concept of technical efficiency. While Koopmans offered a definition and characterization of technical efficiency, it was Debreu who first provided a measure or an index of the degree of technical efficiency with his "coefficient of resource utilization." This coefficient is computed as one minus the maximum equi-proportionate reduction in all inputs consistent with continued production of existing outputs, and from it Debreu obtained measures of the magnitude and the cost of technical inefficiency.

By far the most influential writer on the subject has been Farrell (1957), who first obtained a partial decomposition of private efficiency into technical and allocative components. Farrell begins from a description of a set of firms by plotting them according to inputs per unit of output for the various inputs. In order to get a standard for measuring the efficiency of the firms, he fits a frontier function to the points (as a piece-wise linear function). Farrell calls this "the efficient production function". Next, the efficiency of the various firms is measured by their
location in the input-space relatively to the frontier curve. He then introduces several efficiency concepts. The first, technical efficiency, has been a direct descendent of Debreu's coefficient of resource utilization. It is indicated by the nearness of the input combination point of a firm to the frontier curve measured in the direction towards the origin. The second, price or allocative efficiency is indicated by the degree of correctness in the adaptation of factor proportions to current input prices. Overall efficiency is a combination of the measures of technical efficiency and price efficiency; it indicates (inversely) the savings in costs which could be achieved if the firm were replaced by another which were perfectly efficient, both technically and allocatively.

Leibenstein (1966, 1978), on the other hand, called attention to a source of economic inefficiency which was given the name of X-(in)efficiency. He ascribed increases in X-efficiency to 1) increases in motivational efficiency - workers are stimulated by incentive pay, or management by competition or other adversities; and 2) improvements in the inefficient markets for knowledge. Stigler (1976) contradicted this view and proposed to argue that this type of inefficiency can usefully be assimilated into the traditional theory of allocative inefficiency. In fact, Stigler argued that all perceived inefficiency is just allocative inefficiency.

Recent research efforts have concentrated on the specification and estimation of the efficiency frontier. Four distinct patterns or schools can be identified: 1) deterministic non-parametric, 2) deterministic parametric, 3) deterministic statistical frontiers, and 4) stochastic frontiers. Farrell's approach is
basically deterministic non-parametric, where no explicit model of the frontier has been considered. This approach has been extended and applied by Farrell and Fieldhouse (1962), Seitz (1970), Todd (1971), Afriat (1972) and others. The main disadvantage of this approach, as stated, is its incapability to tackle non-constant returns to scale technology. As a result, Farrell suggested the approach of assuming some specific functional form. This is the deterministic parametric formulation. Following Farrell's suggestion, Aigner and Chu (1968) specified a Cobb-Douglas production frontier to analyze the efficiency issues. More general mathematical specifications have been introduced by Førsund and Jansen (1977). However, like the non-parametric approach, the 'estimated' frontier is supported by a subset of the data and is therefore extremely sensitive to outliers. Further, the 'estimates' which it produces have no statistical properties.

For this reason, some researchers tend towards deterministic statistical frontiers where the frontier has been presented by some statistical model to make the functional forms amenable to statistical analysis. Depending on the choice of different functional forms of one-sided error term, different types of estimation techniques have been proposed by Afriat (1972), Richmond (1974) and Schmidt (1976). These approaches involve assuming some sort of functional form for the frontier and estimating the frontier. The easiest way to estimate the frontier is by using corrected ordinary least square (COLS) technique (where the biased ordinary least square (OLS) estimation technique due to non-zero error mean is corrected). Then the extent of a particular observation's inefficiency is measured by the ratio
of actual output to potential output, with the latter given by the frontier itself. An example is provided by Russell and Young (1983).

However, the deterministic frontiers are difficult to justify empirically. Thus recent works have studied the frontier concept in a stochastic framework where a firm's performance may be affected by outside random factors as well as inside (random) inefficiency factors. The central idea in the stochastic frontier model is that the error term is composed of two parts, two-sided outer random errors, and one-sided inefficiency error. This approach was first proposed by Aigner et al (1977) and Meeusen and Van den Broeck (1977) independently and extended and applied by different researchers throughout the years. While the basic set of econometric estimation techniques has changed relatively little in recent years, there have been some useful combinations and extensions of these basic techniques. Recent research has concentrated on 1) the distribution of the one-sided error term; 2) the measurement of the average inefficiency to the measurement of firm-specific inefficiency; 3) statistical decomposition of the technical and the allocative inefficiencies; and 4) use of panel data. 

The efficiency issue in the field of agricultural production analysis runs parallel with respect to general production analysis. The earlier contribution is attributed to Heady (1952, 1954) who extensively explored the concept of efficiency in agricultural production in a traditional way. He suggested some alternative criteria for measuring economic efficiency, e.g, money income or value productivity, 

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6 A critical review of stochastic frontier models is presented in Chapter 4.
factor price-product price ratio, income-cost ratio. However, most recent contributions center around the application of stochastic and non-stochastic production or cost frontier and developing more sophisticated econometric tools to estimate it. Most works, however, concentrate on technical efficiency. Some of the related empirical work is based on economic engineering or synthetic firm analyses rather than actual firm level data analyses [Carter and Dean (1962), Faris and Armstrong (1963)]. Some studies which used firm-level data assumed the production frontier to be deterministic [Aigner and Chu (1968), Hall and LeVeen (1978)]. Other studies applied stochastic production frontier to firm-level data [Huang (1979), Kalirajan (1981)] to estimate population average technical efficiency. However, some attempts have also been made to estimate the technical efficiency for individual firms in the sample [Bagi and Huang (1983), Huang and Bagi (1984)]. An application of a Ray-homothetic production function in measuring the technical inefficiency can be found in Hassan et al (1987), El-Osta et al (1990) and Grabowski and Belbase (1986). In addition to technical and allocative inefficiencies, scale inefficiencies are also estimated (in a stochastic frontier system) in the context of dairy firms by Bailey et al (1989) and Kumbhakar et al (1989). General application of the stochastic frontier techniques in agricultural production are found in Bravo-Ureta and Rieger (1991), Akridge (1989), Belbase and Garbowski (1985), Dawson and Lingard (1989) among others.
1.7. General procedure

For regional comparative analysis, a theoretical framework was developed by extending classical notion of optimization assuming uncertainty in cost and price. Regional comparative advantage was explained on the basis of a firm’s behavior under uncertainty.

Five crops and seven production regions in Louisiana were selected for the empirical application of regional comparative advantage analysis. Comparative advantage of each crop was estimated on the basis of assumption of randomness in cost and returns. This involved estimation of unconditional and conditional probability density functions (PDF) for an indicator of comparative advantage from (time-series) regional data on cost, yield, and price. A flexible statistical method of estimating PDFs was applied. Absolute and comparative advantage for each crop in all regions were computed on the basis of the definition of comparative advantage.

For inefficiency analysis, panel data on a sample of Louisiana sugarcane firms was selected. Firm-specific technical and allocative inefficiencies were estimated by applying a stochastic frontier methodology. Since this methodology offers a range of alternative model specifications, the selection of an appropriate model is an important issue. Initially five models were selected to estimate the frontier production function and technical inefficiencies. The appropriate econometric model was selected on the basis of a sequential test procedure.
Allocative inefficiency for each firm was computed from the analytically derived cost function.

To investigate the role of inefficiency of a firm in its profitability, the theoretical model of comparative advantage was extended to incorporate efficiency as a source of profitability. For this, a stochastic cost frontier model was specified for the same sugarcane firms. Cost inefficiencies were estimated for each firm. The definition of comparative advantage derived in the first step was slightly modified to fit into the data limitations. Finally, the effects of inefficiency on absolute variation and relative variation in profitability were derived.

1.8. Data

Data on per acre yields, market prices, costs, and direct government payments for five Louisiana crops were collected or estimated from secondary sources for the period 1956 through 1988. This time period was selected because of the availability of data and because by 1956 mechanical harvesters were in common use. Yields and market prices were as reported by the Louisiana Agricultural Statistical Service. Costs were estimated from research conducted in the Department of Agricultural Economics and Agribusiness at Louisiana State University. Direct government payments were obtained from the Louisiana office of the Agricultural Stabilization and Conservation Service, United States Department of Agriculture (USDA).
The data for the individual sugarcane firms in Louisiana were received from a data set provided by First South Production Credit Association at Thibodaux, Louisiana. The data set consists of yields and various components of costs for 45 sugarcane firms (of different sizes) in Louisiana for the years 1986 through 1990.

1.9. Organization of the dissertation

In Chapter 2, the theoretical model of comparative advantage under uncertainty is developed. Chapter 3 is devoted to estimating regional comparative advantage of several crops produced in Louisiana. In Chapter 4, the methodological and empirical issues of firm-specific efficiencies are described. An empirical application on a sample of Louisiana sugarcane firms is also provided in this chapter. Chapter 5 deals with the link between firm-specific inefficiency and comparative advantage. The application in Chapter 4 is extended in this chapter to show the empirical implications. Finally, the summary of the results, data limitations and future research implications are described in Chapter 6.
CHAPTER 2
REGIONAL COMPARATIVE ADVANTAGE: A THEORETICAL FRAMEWORK

2.1. Introduction

There is little doubt that farmers act rationally when attempting to use resources profitably. Evidence of this is the historical and continuing trend in regional specialization one observes for various commodities in different areas (Zapata et al. (1990)). However, adjustments in the directions of relatively higher profitability can never be perfect for all decision-making units in a particular region due to two basic reasons. First, information regarding random market events is not symmetric across all the producing units. As a result, the firms with less access to information are expected to adjust sluggishly in comparison to the firms who have better access to information. Second, even if information were symmetric, the structural flexibility may vary across firms so that the more flexible firms can adjust their decisions more quickly in response to changing economic conditions.

The above problems can be addressed in a general conceptual framework of regional comparative advantage. Traditionally, the concept of comparative advantage is used in the context of international specialization where a country is identified as having comparative advantage in a particular product which is produced with less opportunity cost in comparison to another country. The Law of Comparative Advantage, accordingly, sets the basis for international trade. A country should specialize in producing and exporting the product in which it enjoys
a comparative advantage (i.e., less opportunity cost). In exchange, it should import that commodity in which it has comparative disadvantage (or, high opportunity cost). Under the assumptions of perfect competition, as trade opens up between two countries, the opportunity costs and equivalently the relative domestic prices of the traded products tend to equalize, yielding positive gains to both countries. The causes of differing comparative costs are generally attributed to factor endowments, technology, and taste patterns of individual countries (Chacholiades (1981), p-89). In particular, according to the Hecksher-Ohlin theorem, assuming that technology and tastes are the same in all countries, a country has a comparative advantage in the production of that commodity which uses more intensively the country's more abundant factor (Chacholiades (1981), p-90).

The concept of regional comparative advantage is, in some sense, similar in connotation to the concept of comparative advantage sketched above. In both cases, a product is evaluated in comparison to other products competing for the same kind of resources. However, while the basis of specialization in comparative advantage is absorbing gain through trade among countries, the basis in regional comparative advantage is adjustment toward a relatively more profitable line of production for a better economic prospect. In other words, regional comparative advantage enjoyed by a particular commodity produced in a particular region means the relative profitability of that commodity in that region vis-a-vis the same in other regions within a domestic national boundary. Thus, when used in the context of "within country" specialization, the main focus of comparative advantage is not on
the trade prospect, but on the survival potentiality of a product in a given region in comparison to other regions.

In this context, it is useful to differentiate between the terms "maximum profit" and "maximum profitability" for a particular production process. A firm enjoying maximum profit maximizes total net revenue and this happens if and only if the firm is technically, allocatively, and scale efficient [Førsund et al.(1980)]. On the other hand, "maximum profitability" implies the capability of a firm which has the maximum potential to survive in the long run. Traditional economic theories, which generally assume away any kind of uncertainty, do not distinguish between these two terms as a "maximum profit" earning firm has the maximum potential to survive under complete certainty. As Friedman (1953. p-22) puts it, "...unless the behavior of businessmen in some way or other approximated the behavior consistent with the maximization of returns, it seems unlikely that they would remain in business for long."

However, the above proposition is not necessarily true when uncertainties regarding market forces are assumed in a model. As it will be shown later in this chapter, under the assumptions of uncertainties and decreasing absolute risk aversion, the "survivorship principle" makes a firm move away from the profit-maximizing output. In other words, even without earning maximum profit, a firm may be relatively more profitable than a firm which is earning maximum profit at present. That is, in an uncertain world, a "maximizing profitability" firm reflects better survival capacity than a profit maximizing firm even though it may not be as
efficient as the later, so long as uncertainties regarding prices or cost components prevail.

The concept of regional absolute advantage and regional comparative advantage is based on the principle of "maximization of profitability" rather than on the principle of "maximization of profit". In other words, a firm should be identified as absolutely advantageous in a region if its relative profitability, not profit, is higher than that of any other firm in the same region. It will be comparatively advantageous in one region if its relative profitability is higher in that region in comparison to other regions. The question is: how to identify a relatively profitable firm? A simple way to identify such a firm is to conceive a situation where the firm "feels" more "safe" against all odd uncertainties. Intuitively, such a situation is where the probability of achieving maximum net return per unit is higher than that of other firms (at a particular level of output), because that implies a greater set of choices open to the firm to adjust against uncertainties. More precisely, a firm maximizing profitability may be defined as a firm maximizing the probability of per unit net return under uncertainty, whereas a firm maximizing profit maximizes total net return under certainty. This argument can be elaborated by the following way.

Assume a perfectly competitive market with perfect knowledge (certainty) about product price and costs. A hypothetical situation has been presented in Figure 2.1. Let A be the point where P = MC. That is Q₀ is the profit maximizing output at the given price. Under perfect certainty, as risk is absent, maximum profitability implies maximum profit; therefore, Q₀ is the profit as well as
profitability maximizing output. Now, assume that there is uncertainty in average cost. In that case, profit itself becomes uncertain. Suppose the firm decides to produce less than $Q_0$, say $Q_2$, to guard itself against odd uncertain outcomes. Now consider three firms, firms I, II, and III in a particular region and assume that each of them decides to produce $Q_2$ under uncertainty. As $AC$ is random around the mean (say, $E(AC)$), it may lie anywhere within a range (say, $CG$) at $Q_2$. Now, suppose that firm II has the highest probability to operate at point $D$ whereas the other two firms have the highest probabilities to operate above $D$. In that case,
firm II will have maximum profitability over the other two firms. Note that profit is maximized at output $Q_0$ where per unit net return ($=AB$) is less than the per unit net return at $Q_2$ ($=CD$), which the maximum profitable firm is more probable to achieve. In other words, firm II attains the highest advantage\(^1\) even if it does not achieve maximum profit.

The above argument is also presented more rigorously in Section 2.4. The main implication of the argument is that, for an analysis of regional comparative advantage under uncertainty, the principle criterion of evaluation should be the net return per unit of output, not the absolute amount of profit.

The concept of regional comparative advantage is meaningful in the context of the evaluation of the performance of an industry or a particular crop. Historically, it is often found that the sequence or order of relative profitability of crops produced in a region changes in response to changes in some market or non-market forces. In other words, a particular crop $x$, being considered as the most comparatively advantageous in a region in a particular period of time, may lose this position as a result of adverse changes in supply or demand forces affecting only that crop. A simple example is sudden adverse change in terms-of-trade for an exportable commodity. Other possible sources of changes in market forces are a fall in domestic output and input prices, a change in taste patterns (e.g, use of artificial sweetener in place of sugar), or more rapid cost-reducing technological

\(^1\) Note that in this case absolute and comparative advantage are synonymous as the analysis is confined to the firms in a single region.
changes in competing crops. Possible non-market forces include changes in the world political structure, geographical and climatic changes, and spread of plant disease or insects.

Whatever may be the source of comparative disadvantage, this may lead to the dislocation of a crop (or a firm) from its advantageous position. The extent and degree of damage caused by this dislocation depend on whether the changes are temporary or permanent and whether the signals coming from the changes are received and absorbed by all firms producing the crop. Assuming that the shock is not temporary and the signal (that it is losing comparative advantage) is received in time, the next question is: what can a firm do in this situation? The firm's strategy, in this case, depends on identification of 1) how much of the loss of comparative advantage is attributed to the factors under firm's control, and 2) if the factors under the firm's control contribute not so significantly, what are the external (to the firm) factors responsible for dislocation? The first directly deals with inefficiencies related to production, addressing the question equivalently: how far will the loss in comparative advantage be corrected if the inefficiencies are fully eliminated? The second question helps to identify the exact nature of the requirements to deal with external adversities and to restructure the plans accordingly.

It is one of the important factors for the survival of a firm to acquire and to utilize the information of changing market forces and to adjust quickly in the line of production in which it can gain the highest comparative advantage (or, the least
comparative disadvantage). The sluggishness in the adjustment may, even in the short run, cause significant economic loss and erosion of the financial base of a firm. This is especially true for the small firms where the slightest strategic error may result into significant loss of survival capacity in the long run. Therefore, it is necessary for an economic policy analysis to have as much information as possible about the comparative advantage rankings with respect to firms (producing a single crop) and crops.

In the next section (section 2.2) a simple indicator for measuring profitability is proposed and analyzed. The problem of how to quantify comparative advantage in terms of the indicator will also be addressed. In section 2.3, the firm's behavior under complete certainty and corresponding optimal properties of the indicator under different market structures will be discussed. Section 2.4 deals with the same problem under the assumption of uncertainty from different sources. In this section, some major comparative static results will be derived from the optimal conditions. Finally, in section 2.5, the summary and implications of the theoretical framework will be discussed.

2.2. An indicator of regional comparative advantage

The uncertainties inherent in the supply and demand side make the issue of generating information about regional comparative advantage very important. The relevant problem, in this case, is to identify an indicator which precisely captures manifestations of the random elements both from the supply and the demand side.
In other words, the indicator itself (1) will be stochastic, (2) will reflect the randomness of basic market and non-market forces, and (3) will indicate the location of a firm in the regional comparative advantage hierarchy.

However, the above conditions, though necessary, are not sufficient for the indicator to be a useful tool for analysis. The basic characteristics of a useful indicator, in this context, would be:

1) Simple and understandable to decision-makers.

2) Reflective of maximum necessary information.

3) Reflective of uncertainties associated with the factors under firm’s control.

4) Flexible enough to permit analysis under alternative market structure.

Here an economic indicator is proposed that is expected to satisfy the above conditions and at the same time indicate the relative profitability of a certain crop in a given region. It is simply the ratio of cost to income from farming which is denoted as the Cost-Income Ratio (CIR). This ratio measures the proportion of each dollar in total revenues allocated to total production costs (fixed and variable).

Theoretically, the CIR is a function of the total cost (C) and total revenue (TR) functions; thus the ratio depends on output, output prices, and costs. Using the classical definition of TR, i.e, the product of output (Q) and price (P) per unit of output, the CIR can be defined as the average total cost (AC) divided by output price.
\( r = \frac{C}{TR} = \frac{C}{PQ} = \frac{AC}{P} \)

where \( r \) is the CIR.

The interpretation of the ratio is straightforward; if total cost for soybeans are $100 per acre and gross returns amount to $120, then \( r = \frac{100}{120} = 0.83 \). Equivalently, this is a net return of $20 per acre or $0.17 per $1 of revenue. Equation (2.1) implies that, in a given time period and with output price fixed, the ratio has the same properties as the AC, except by a location and scale component. In other words, when \( P \) is not stochastic, there is strict one-to-one correspondence between \( r \) and AC so that the indicator \( r \) does not add more information than AC. However, if \( P \) is an endogenous variable or stochastic, this one-to-one correspondence breaks down. In that case, \( r \) becomes more informative than AC as \( r \) reflects the randomness of \( P \) as well.

Given the indicator, the next question is: how can this indicator be used to provide information about comparative advantage of a crop in a particular region? As comparative advantage is reflected through relative profitability and as net return per unit of output is argued to be the principle criterion of measuring relative profitability, the indicator seems to be adequate in comparative advantage analysis. Denoting \( \pi \) as per unit net return, the relation between \( \pi \) and \( r \) may be written as

\[
(2.2) \quad \pi = \frac{TR - TC}{Q} = P(1 - r)
\]
Thus \( \pi \) is a decreasing function of \( r \), at a given \( P \) (or, equivalently a decreasing function of \( AC \) at a given \( P \)). In the example provided above, if \( P = $0.50 \), then \( \pi = 0.50(1 - 0.83) = $0.085 \). If, \( P \) itself is endogenous,

\[
(2.3) \quad \pi(Q) = P(Q) - AC(Q),
\]

so that, once again \( \pi \) is a decreasing function of \( r \). Therefore, \( r \) is directly related to the concept of profitability and explicitly takes both \( AC \) and \( P \) into account so that their individual randomness can be separated.

In this context, it is necessary to justify \( r \) against some other possible indicators of profitability. One such indicator is \( AC \) (or, \( 1/AC \)) which is used frequently as an indicator for measuring efficiency. The problem in using this is that it does not contain information on output price. Thus, it can not be used as an indicator when price is stochastic. Another indicator is \( \Pi \) (total profit). It is shown later in this chapter that under the conditions of uncertainty, \( \Pi \) does not qualify as a decision variable of optimization. Lastly, \( \pi \) (per unit profit) can be used as \( r \) is directly related to \( \pi \) and thus has no apparent superiority over \( \pi \) in profitability analysis. But \( r \) has two additional advantages; (i) if efficiency issues are also addressed in investigating the causes behind relative profitability, \( r \) can be used more effectively\(^2\) than \( \pi \); and (ii) as the nature of average or total cost can easily be traced back from the nature of \( r \), the issues regarding economies of size or scale can be addressed in a less complicated way by using \( r \).

\(^2\) This will be discussed in Chapters 4 and 5.
Heady (1954) was one of the first researchers to use the CIR as a measure of regional comparative advantage in a static non-stochastic framework. Hall and Leveen (1978), Miller et al. (1981), Jensen (1984) used this concept in analyzing economies of size and structural change in agriculture. However, no attempt has been made so far to develop a theoretical structure of comparative advantage based on the CIR in a stochastic framework.

As the cost-income ratio is assumed to be a random variable, the simplest way to express the profitability from a crop is to gather historical information on cost and returns from crop production and to make a probability statement about the direction of the cost-income ratio. For example, let

\[ Pr\{CIR \leq 1\} = f_1 \]

That is, the probability of breaking-even in producing a crop is \( f_1 \). Similarly, \( Pr\{0.8 \leq CIR \leq 0.9\} = f_2 \) indicates the probability of achieving a net return between $0.20 and $0.10 per $1 of return. Therefore, probabilities \( f_1 \) and \( f_2 \) may serve as the basis for comparing the profitability of two (or more) crops in a given region at a particular point in time. For example, if there are two crops, say crop I and II, being produced in a particular region, then

\[ Pr\{CIR \leq 0.8\}_I > Pr\{CIR \leq 0.8\}_II \]

implies the probability of making net return = $0.20 per $1 of return is higher for crop I. From this, one may deduce that crop I is in an absolutely advantageous position over crop II.
But that deduction would be too simple to capture reality for two reasons:
(i) if the income generating capacity is not equal for each crop then a higher probability does not necessarily imply higher profitability. For example, if c is a constant and the probabilities for $CIR \leq c$ are the same for both crops, there may be a tendency to deduce equal profitability for both crops. But if it is higher in crop II, then clearly that will be a wrong deduction; even if the probabilities are the same, the crop with higher expected income would be economically more attractive;
(ii) if the true probabilities are unknown, then estimated probabilities from a historical cost and income structure may not reveal recent changes in profitability for a certain crop. In that case, a weight (reflecting the most recent direction of profitability) should be given to the estimated probabilities.

It is thus necessary to develop an indicator measuring the absolute and comparative advantage ranking. The proposed index for crop-specific absolute advantage in a given period $t$ would be

$$ (2.6) \quad AAD_{it} = Pr\{CIR_{it} \leq c\}. m_{it}, $$

where $AAD_{it}$ is the absolute advantage index for the $i$-th crop (in the $t$-th period) in a particular region, and $m_{it}$ is an appropriate weight of expected income and current profitability. Note that equation (2.6) can be expressed alternatively in terms of per unit profit ($\pi$) and total profit ($\Pi$). As $\pi = P(1 - r)$ and $\Pi = TR(1 - r)$, equation (2.6) is expressed equivalently as

$$ (2.7) \quad AAD_{it} = Pr\{\pi_{it} \geq \pi_0\}. m_{it}, \text{ where } \pi_0 = P_0(1 - c), P_0 \text{ being a given output price.} \quad \text{Or,} $$
(2.8) \[ AAD_{it} = P(\Pi_u \geq \Pi_0). \] where \( \Pi_0 = TR_0(1 - c) \), \( TR_0 \) being a given level of total revenue.

All the crops in the region can be ranked according to the index and the crop with higher value of AAD may be denoted as the crop with highest absolute advantage. To compare the profitability in more than one region, the necessary step is to build an index of comparative advantage as

(2.9) \[ CAD_{ijt} = \frac{AAD_{it}}{AAD_{jt}}, \]

where \( CAD_{ijt} \) is the comparative advantage index of crop \( i \) with respect to crop \( j \) (in \( t \)-th period) for a region. Therefore, if there are two regions, say region A and region B and if it is found that

(2.10) \[ (CAD)_A > (CAD)_B, \] i.e, \( (AAD_j/AAD_i)_A > (AAD_j/AAD_i)_B \)

then, crop \( i \) would be identified as having comparative advantage in region A and crop \( j \) having comparative advantage in region B. If there is only one region where both the commodities are produced, absolute advantage will imply comparative advantage. This analysis can also be conceived as a firm-specific advantage analysis where the subscripts \( (i, j) \) indicate particular firms producing the same commodity in a particular region.

In computing the index, the main problem is to find an appropriate expression for \( m \), the weight. The solution to this problem depends on the particular empirical setting and the nature of the data. This is basically a methodological and empirical problem and will be discussed in Chapter 3.
2.3. Cost-income ratio and firm's behavior under certainty

As the CIR (henceforth, r) is considered as the basis for the indicator of comparative advantage, it is necessary to develop a theoretical framework on the basis of optimal properties and comparative static of r. This would help in explaining a firm's behavior and its adjustment towards a relatively profitable position under certainty and uncertainty. The theoretical study begins with r as a function of AC and output price with no uncertainty in either side. From the cost component of the equation, the properties which follow from the usual definition of the cost function $C = C(Q, W_i)$ are adopted, where $W_i$ are the input prices.

1. a. $C(Q, W_i) \geq 0$ for $W_i \geq 0, Q \geq 0$

   b. $\frac{\partial C}{\partial W_i} \geq 0$. That is, the cost function is a non-decreasing function of input prices.

   c. $C(Q, W_i)$ is homogenous of degree one.

   d. $\frac{\partial^2 C(Q, W_i)}{\partial W_i W_j} < 0$

2. No externalities.


   Based upon these properties and assumptions, the correspondence between AC and r in the long run can be shown in both perfectly competitive and imperfectly competitive market structures. Taking natural logarithms on both sides of equation (2.1) and differentiating with respect to $\ln Q$, a familiar elasticity interpretation of the ratio is obtained, which is
which, after substituting for the usual elasticity expressions, becomes

\[(2.12) \quad \epsilon_r = \epsilon_{AC} - \frac{1}{\epsilon_p}\]

where \(\epsilon_r\) is the elasticity of \(r\), \(\epsilon_{AC}\) is the elasticity of \(AC\) and \(\epsilon_p\) is the price elasticity of demand. Further, equation (2.11) gives an interpretation of the relationship between \(AC\) and \(r\) in the following way

\[(2.13) \quad \frac{dr}{dQ} \cdot \frac{Q}{r} = \frac{dAC}{dQ} \cdot \frac{Q}{AC} - \frac{dP}{dQ} \cdot \frac{Q}{P}\]

Rearranging,

\[(2.14) \quad \frac{dr}{dQ} = \frac{1}{P} \left[ \frac{dAC}{dQ} - r \cdot \frac{dP}{dQ} \right]\]

That means, the change in the cost-income ratio is a function of the slope of the average cost and the demand curve. Obviously, in a perfectly competitive market where \(dP/dQ\) is equal to zero for a firm, equation (2.14) becomes

\[(2.15) \quad \frac{dr}{dQ} = \frac{1}{P} \cdot \frac{dAC}{dQ} > 0, \quad \text{as} \quad \frac{dAC}{dQ} < 0\]

and,

\[(2.16) \quad \frac{d^2r}{dQ^2} = \frac{1}{P} \cdot \frac{d^2AC}{dQ^2} > 0, \quad \text{as} \quad \frac{d^2AC}{dQ^2} < 0\]
In other words, in a perfectly competitive output market there is a one-to-one correspondence between the slope and curvature of $r$ and the same of $AC$.

Given the above results, one can investigate the properties and directions of the cost-income ratio under different market structures. Here two basic market structures and corresponding properties of the ratio are considered.

2.3.1. Perfectly competitive market

Assume all the usual characteristics of a perfectly competitive market (including perfect information and mobility). Prices are given to a firm and for each price there is a cost-income ratio curve. The long-run equilibrium condition is depicted in Figure 2.2, where LAC is the long run average cost function and envelopes the short run cost curves. In the upper panel, the long run equilibrium is shown at point $E$ where $P = LAC$ condition is satisfied. For simplicity's sake, assume $P = 1$ in the long-run equilibrium situation.

Given the above properties of the cost-income ratio curve, it is easy to derive the optimum (profit-maximizing) conditions in terms of the ratio. The relation between profit and $r$ can be written as

$$\Pi = TR - C = TR \{ 1 - r \}$$

Differentiating equation (2.17) with respect to $Q$

$$\frac{d\Pi}{dQ} = (1 - r) \frac{dTR}{dQ} - TR \frac{dr}{dQ}$$
Figure 2.2 Cost-income ratio in a perfectly competitive market in the long run.

Applying the profit maximizing condition \( \frac{d\Pi}{dQ} = 0 \) and solving for \( r \), the result is given by:

\[
(2.19) \quad r^* = 1 - Q^* \frac{dr}{dQ}|_{Q=Q^*}
\]

where the superscript * is used to define optimum levels of \( Q \) and \( r \). Note that condition (2.19) is equivalent to the familiar profit-maximizing condition \( P = MC \), because at \( Q^* \),
(2.20) \[ r^* = 1 - Q^* \frac{dr}{dQ} \bigg|_{Q=Q^*} = 1 - \frac{Q^*}{P} \frac{dAC}{dQ} \bigg|_{Q=Q^*} \]

But \( \frac{dAC}{dQ} = 1/Q [MC - AC] \). Thus, substituting this value and simplifying yields

(2.21) \[ r^* = 1 - \frac{MC^*}{P} + r^* \rightarrow P = MC^* \]

Also, the condition can be expressed equivalently in terms of the elasticity of average cost in the following way. At \( Q^* \), equation (2.19) can be restated as

(2.22) \[ r^* = 1 - Q^* \frac{1}{P} \frac{dAC}{dQ} \bigg|_{Q=Q^*} \frac{AC^*}{AC^*} = \frac{1}{1 + \epsilon_{AC^*}} \]

where \( \epsilon_{AC^*} \) is the elasticity of average cost evaluated at \( Q^* \). The conditions (2.19) and (2.22) may be explained as follows. If \( Q^* \) is such that

(i) \( \epsilon_{AC} = 0 \), or equivalently, \( dr/dQ = 0 \), then \( r^* = 1 \),

(ii) \( \epsilon_{AC} > 0 \), or equivalently, \( dr/dQ > 0 \), then \( r^* < 1 \),

(iii) \( \epsilon_{AC} < 0 \), or equivalently, \( dr/dQ < 0 \), then \( r^* > 1 \).

Thus, according to conditions in equation (2.19) and (2.22), the optimum point coincides with the minimum point of \( r \) only when \( r = 1 \), i.e., in the long run. In other cases, when there is excess profit or loss, i.e., when \( r \) is greater than or less than 1, the firm will not operate at the minimum point of the cost-income ratio curve. More specifically, if in the short run the competitive firm enjoys positive
economic profit, the optimum cost-income ratio will be in the rising portion of the curve.

The lower panel of Figure 2.2 shows three cost-income ratio (CIR) curves corresponding to three price situations: \( P < 1 \), \( P = 1 \), and \( P > 1 \). Each curve is the envelope of short run CIRs. That is, \( \text{CIR}_1 \), \( \text{CIR}_2 \), etc. are minimum cost-income ratios at every possible output level. From equations (2.1) and (2.15), it is known that the minimum point of AC and \( r \) correspond at the same level of output. Suppose, initially, the price is \( P > 1 \). The short-run equilibrium point is \( G \) as shown in the upper panel. The corresponding CIR curve is \( \text{CIR}_2 \), with corresponding optimum \( r < 1 \) at point \( H \) in the lower panel. Note that \( H \) is not the minimum point of \( \text{CIR}_2 \). In the long run, as new firms enter (due to supernormal profit) or some old firms exit (due to loss), \( P \) tends to be equal to AC at the latter's minimum point (point \( E \) in the upper panel). Equivalently, the CIR curve shifts up (or down) and the optimum CIR converges to the \( r = 1 \) level (point \( F \) in lower panel). As profit and profitability both are maximized at the profit-maximizing point under certainty, a firm with a lower cost-income ratio at every level of output will be able to enjoy relatively higher profitability or comparative advantage (assuming single region) in the short run. However, in the long run, this profitability disappears so that no firm will enjoy comparative advantage.
2.3.2. Imperfectly competitive market

Now consider an imperfectly competitive market with the possibility of entry in the long run. Such a situation may be conceived when the firm under discussion has some monopoly power in the product market, and/or has better access to information about product price. In such cases, price is an endogenous variable and the one-to-one correspondence between AC and r does not exist any longer. This can be shown as follows.

Assume a monotonically decreasing linear demand curve, so that,

\[ \frac{dP}{dQ} = m < 0, \]

where \( m \) is a constant. Then, according to equation (2.14)

\[ \frac{dr}{dQ} = \frac{1}{P} \cdot \frac{dAC}{dQ} - \frac{r}{P} m \]

At the minimum point of AC, \( dAC/dQ = 0 \). Therefore, corresponding to the minimum point of AC,

\[ \frac{dr}{dQ} = -\frac{r}{P} m > 0 \]

At the minimum point of r, \( dr/dQ = 0 \). Therefore, corresponding to the minimum point of r, the slope of AC from equation (2.14) becomes

\[ \frac{dAC}{dQ} = r.m < 0 \]
Equations (2.25) and (2.26) together imply that, in general, the minimum point of \( r \) is achieved at the falling portion of the AC curve.

The optimal conditions in terms of the CIR curve under imperfect competition will be slightly different than those under perfect competition, as \( P \) becomes an endogenous variable in the former case. Specifically, differentiate (2.17) with respect to \( Q \) on the assumption that \( P \) is an endogenous variable to obtain

\[
(2.27) \quad \frac{d\Pi}{dQ} = -TR \frac{dr}{dQ} + (1 - r) P (1 + \frac{1}{\epsilon_P})
\]

Putting \( d\pi/dQ = 0 \), the optimum condition is

\[
(2.28) \quad r^* = 1 - \frac{Q^* \frac{dr}{dQ}_{|Q=Q^*}}{(1 + \frac{1}{\epsilon_P})}
\]

or, equivalently, by slight manipulation,

\[
(2.29) \quad r^* = \frac{1 + \frac{1}{\epsilon_P}}{1 + \epsilon_{AC^*}}
\]

Equation (2.29) represents the optimality condition in terms of elasticities of demand and average cost curves under imperfect competition. Similar to the competitive market, the condition can be explained in the following way.
As the elasticity of demand under imperfect competition is restricted to be $|\epsilon_p| > 1$ in the operative zone, the numerator is positive. Further, to get a positive $r^*$, the necessary restriction for $\epsilon_{AC}$ is $|\epsilon_{AC}| < 1$, when $\epsilon_{AC} < 0$. Assuming these two restrictions to hold, if $Q^*$ is such that,

(i) $\epsilon_{AC} = 0$, i.e., $\epsilon_r = -(1/\epsilon_p) > 0$, then $r^* = [1 + (1/\epsilon_p)] < 1$.
(ii) $\epsilon_{AC} > 0$, i.e., $\epsilon_r = \epsilon_{AC} - (1/\epsilon_p) > 0$, then $r^* < 1$.
(iii) $\epsilon_{AC} < 0$, then $r^* \leq 1$ as $|\epsilon_{AC}| \leq |1/\epsilon_p|$, and $r^* > 1$ as $|\epsilon_{AC}| > |1/\epsilon_p|$.

(i), (ii), and (iii) together imply that similar to the competitive case, $r$ is not minimized at the profit maximizing point except in the longest run when excess profit disappears due to entry. In the longest run, $r$ also becomes equal to 1 at its minimum point. However, compared to the competitive case, in an imperfectly competitive market the firm operates much closer to the minimum point of $r$ as the multiplicative term $(1 + 1/\epsilon_p)$ is less than one. Note that condition (2.22) is nothing but a special case of condition (2.29) where $\epsilon_p$ is assumed to be equal to $-\infty$.

The behavior and direction of CIR under optimality condition are shown in Figure 2.3. The upper panel shows the adjustment by a firm in the long run against new entry. Assume that the initial demand curve is AR$_2$ and corresponding equilibrium output is $Q_2$ at price $P_2$. The corresponding CIR curve at the lower panel is CIR$_2$, having minimum at point A (which corresponds to falling portion of AC). As expected, minimum $r$ is not achieved at the output where profit is
maximized and it is much below than 1.0 level. The optimum point on the CIR curve is B (corresponding to point F in the upper panel). As new firms enter, the ex ante demand curve gradually shifts down to AR₁. The profit maximizing price accordingly falls to P₁. P₁ is the long run equilibrium price as excess profit at that price is equal to zero.

As the ex ante demand curve shifts down, the CIR curve at each profit-maximizing price shifts up to the left to CIR₁. In the longest run, the optimum point is located at point C on the CIR curve in the lower panel. Note that point
C corresponds to equilibrium point E in the upper panel and at C, \( r^* = 1 \) at the minimum point of CIR curve.

The implications of the above analysis may be explained in terms of price and cost elasticities. Generally, in the presence of excess profit, the point elasticity of demand (\( \epsilon_P \)) is greater than the point elasticity of AC at profit-maximizing output. That means, in the presence of excess profit, \( |\epsilon_P| > |\epsilon_{AC}| \), or equivalently, \( 1/|\epsilon_P| < |\epsilon_{AC}| \). That implies \( r < 1 \). However, as entry takes place, the long run equilibrium tends to hold at lower price and higher average cost, so that \( 1/|\epsilon_P| \to \epsilon_{AC} \) and accordingly, optimum \( r \to 1 \).

As argued earlier, under certainty, the comparative advantage of a particular crop or a particular firm is indicated by the relative profit of (which is equivalent to profitability) a producer. Therefore, in an imperfectly profitable competitive market the comparative advantage will depend on the relative success of the producer to operate in the zone of the demand curve where elasticity is higher, and/or to operate in the zone of the cost curve where elasticity is lower. This is equivalent to operation at a lower optimum cost-income ratio.

Given the above results, the properties of the indicator under uncertainty can be explored. The next section consists of the analysis of the behavior of a perfectly competitive firm under uncertainty when the firm's utility function depends indirectly on the stochastic cost-income ratio.
2.4. Cost-income ratio and firm’s behavior under uncertainty

The role of uncertainty in the theory of the perfectly competitive firm has been analyzed very extensively. For example, uncertainty has been analyzed in output price [Sandmo (1971), Batra and Ullah (1974)], in input prices [Blair (1974)], and in the supply of inputs [Rati and Ullah (1976), Martin (1981)]. In this study, uncertainty in a more general form is introduced so that the uncertainties in both the supply and demand side can be separated in a single framework. For this, consider the cost-income ratio as a pure stochastic variable so that the producers’ decision is based on their beliefs about the ratio which is summarized in a subjective probability distribution of this variable. The randomness in the CIR is caused either by (1) randomness in average cost (which implies randomness in yield or input prices or both), or, (2) randomness in output price, or, (3) both. The basic structure of the model is specified below.

Assumptions: The following assumptions are necessary for this model.

1. This is a one period static model of a competitive firm.

2. The decision on the volume of output to be produced must be taken prior to the purchase of inputs and sales of output.

3. The firm’s beliefs about the cost-income ratio can be summarized in a probability distribution with finite moments.

4. The utility function of the firm is a concave, continuous and differentiable function of profit.
5. The firm is risk-averse.

6. The objective of the firm is to maximize the expected utility of profit.

Given these assumptions the firm’s utility function may be specified as

\[(2.30) \quad U = U(\Pi)\]

where \(\Pi\) is the net profit and \(U\) is the utility derived from \(\Pi\). As increasing \(\Pi\) implies increasing utility, the slope and the curvature of the utility function may be specified as

\[(2.31) \quad U'(\Pi) > 0, \quad U''(\Pi) \leq 0\]

Note that the utility function specified here is a typical Neumann-Morgenstern type utility function. Given the utility function, the attitude toward risk is defined in the following way [Hey (1979), p-47]:

- \(U''(\Pi) < 0\) implies risk aversion
- \(U''(\Pi) = 0\) implies risk neutrality
- \(U''(\Pi) > 0\) implies risk preference.

In this study, only the behavior of a risk averse firm will be considered. According to the definition in equation (2.17), \(\Pi = TR(1 - r)\). Here, \(r\) is a random variable with a known probability distribution \(f(r)\) and an expected value \(E(r) = \mu\). The expected utility of profit can be written as
The firm’s objective is to maximize \( E[U(\Pi)] \). For this, the first order condition is

\[
(2.33) \quad \bar{U}' = E \left[ U'(\Pi) \frac{d\Pi}{dQ} \right]
\]

\[
= E \left[ U'(\Pi) (P(1 - r) - TR \frac{dr}{dQ}) \right] = 0
\]

and the second order condition is

\[
(2.34) \quad \bar{U}'' = E \left[ U''(\Pi) \left( \frac{d\Pi}{dQ} \right)^2 + \frac{d^2\Pi}{dQ^2} U'(\Pi) \right] < 0
\]

Given the above results, it may be checked whether the optimality condition under certainty, derived in equation (2.19) or (2.22), still holds. For this the sources of the randomness are considered one by one in the following sections.

### 2.4.1. AC is random, P is non-random

The first order condition derived in equation (2.33) is

\[
(2.35) \quad E \left[ U'(\Pi) (P(1 - r) - TR \frac{dr}{dQ}) \right] = 0
\]

or,

\[
(2.36) \quad E \left[ U'(\Pi) P (1 - r) \right] = E \left[ U'(\Pi) TR \frac{dr}{dQ} \right]
\]
Solving the first order condition in (2.36), the optimum output \( Q^{**} = Q(\mu, \theta) \) is obtained where, \( \mu \) is the expected cost-income ratio and \( \theta \) is a set of parameters.

Now, assume that \( P \) is non-stochastic, but \( r \) is stochastic and so \( U'(\Pi) \) is also stochastic. Applying the covariance method to the left hand side of equation (2.36) and equating it to the right hand side,

\[
(2.37) \quad E[U'(\Pi)] E[P(1 - r)] + Cov[U'(\Pi), P(1 - r)] = E[U'(\Pi) \, TR \, \frac{dr}{dQ}]
\]

or,

\[
(2.38) \quad [P(1 - \mu) - TR \, \frac{dr}{dQ}] = - \frac{Cov[U'(\Pi), P(1 - r)]}{E[U'(\Pi)]}
\]

As \( U'(\Pi) > 0 \), the sign of the left hand side of equation (2.38) is obviously determined by \( Cov[U'(\Pi), P(1 - r)] \). To determine the sign of the covariance, the following rule may be applied (Hey (1979), p-51):

*If \( g(x) \) and \( h(x) \) are two probability density functions from the same random variable \( x \), then

\( g'(x) > 0, h'(x) > 0 \) implies \( Cov[g(x), h(x)] > 0 \)

\( g'(x) < 0, h'(x) < 0 \) implies \( Cov[g(x), h(x)] > 0 \)

\( g'(x) < 0, h'(x) > 0 \) (or, vice versa) implies \( Cov[g(x), h(x)] < 0 \).

In this case, \( g(x) = g(r) = U'(\Pi) \) and \( h(x) = h(r) = P(1 - r) \). Thus, differentiating both with respect to \( r \)

\[
(2.39) \quad h'(r) = \frac{d[P(1 - r)]}{dr} = - P < 0
\]
and,

\[(2.40) \quad g'(r) = \frac{d[U'(\Pi)]}{dr} = U''(\Pi) \quad (-TR) > 0, \quad as \quad U''(\Pi) < 0\]

From equations (2.39) and (2.40), \(\text{Cov} [U'(\Pi), P(1 - r)]\) is negative. Therefore from equation (2.38),

\[(2.41) \quad P(1 - \mu) - TR \frac{dr}{dQ} > 0\]

which, at the optimum level of output \(Q^{**}\), implies

\[(2.42) \quad \mu^{**} < 1 - Q^{**} \left| \frac{dr}{dQ} \right|_{Q=Q^{**}}\]

According to the same argument presented in (2.21), this leads to

\[(2.43) \quad P > E [MC^{**}]\]

Condition (2.43) automatically implies that if \(E[MC] = MC\), then optimum level of output under cost uncertainty (i.e., \(Q^{**}\)) will be less than output under certainty (i.e., \(Q^{'}\)).

Again, condition (2.42) may be expressed in terms of cost elasticity. Denoting \(E[\epsilon_{AC^{**}}]\) as mathematical expectation of the elasticity of average cost, the condition becomes

\[(2.44) \quad \mu^{**} < \frac{1}{1 + E[\epsilon_{AC^{**}}]}\]

where, \(\mu^{**}\) is the mean of the distribution of optimum CIR at \(Q^{**}\). Note that \(\epsilon_{AC^{**}}\) is stochastic as it entails AC. Therefore, the optimum CIR itself is random and
thus will be different across firms if $E[\epsilon_{AC^*}]$s are different. The exact location of optimum CIR under uncertainty can be identified only if the decided output level $Q^*$ is known. To compare the condition in inequality (2.44) with the condition under certainty, recall the certainty condition derived in equation (2.22).

\[(2.45) \quad r^* = \frac{1}{1 + \epsilon_{AC^*}}\]

To demonstrate the difference between these two conditions and the meaning of comparative advantages in this context, assume that there are three firms in a region - firm 1, firm 2, and firm 3. To consider the certainty condition first, assume that each firm faces a known average cost curve. The situation is depicted in Figure 2.4.

Suppose AC is the average cost curve faced by each firm under certainty. Therefore, G is the optimum point in the upper panel where $P = AC$ and $Q^*$ is the optimum output for each firm. Note that, at $Q^*$, optimum CIR is at point $G'$ ($= 1$) indicating that each firm enjoys normal profit only and hence no comparative advantage is enjoyed by any firm. Now, assume uncertainty in cost so that there are three kinds of PDF of AC (i.e, three different subjective beliefs) with three different means at each level of output. In Figure 2.4, they are $E[AC_1]$, $E[AC_2]$, and $E[AC_3]$ respectively for each firm. Correspondingly, we have $E[r_1]$, $E[r_2]$, and $E[r_3]$ in the lower panel. Note that, for firm 2, the mean of the distribution of AC has coincided with AC in certainty. For firm 1, it is below AC; and for firm 2, it is above AC.
The optimality condition in (2.43) requires that each of the firms would produce at any $P > E[MC]$ point, i.e., anywhere to the left of its $P = E[MC]$ point. Three such points are arbitrarily chosen on respective $E[AC]$ curves where condition (2.43) is satisfied for all firms. They are $E_1$, $E_2$, and $E_3$ for firms 1, 2, and 3 respectively. Corresponding optimum outputs are $Q_1^{**}$, $Q_2^{**}$, and $Q_3^{**}$; optimum CIRs are $m_1^{**}$, $m_2^{**}$, and $m_3^{**}$.
From the analysis, it is clear that each firm is in optimum position, i.e, doing its best to guard against uncertainty. In other words, each firm has absolute advantage over any other firm which does not adjust output in the face of uncertainty (or, acting as if uncertainty does not exist). But, so far as comparative advantage is concerned, the situation is different. According to the definition of relative profitability, it is found that firm 1 has the highest probability to achieve the lowest optimum CIR. In other words, it has maximum probability to achieve maximum per unit net return (= GE₁ in the upper panel). As, corresponding to Q₁**, the probability of incurring loss is relatively small, the risk also is relatively lower against odd uncertainties. That implies that firm 1 has comparative advantage over the other two firms, even if all of them are reacting against cost uncertainties in the proper way.

Several interesting implications are derived from the above analysis. One is that when uncertainty exists in the cost side, a risk-averse firm with high probability of low cost-income ratio comes closer to the optimum condition and thus has a high probability to guard itself against cost uncertainty. For example, if the μ** lies below 0.7, then a firm with probability Pr{0.7 ≤ µ ≤ 0.8} = 0.9 has comparative advantage than a firm with probability Pr{0.7 ≤ µ ≤ 0.8} = 0.7. Another implication is the inverse relationship between optimum CIR (μ**) and optimum output (Q**) when change in Q** is substantially large. From Figure 2.4, it implies that even if firm 1 has the highest probability of operating on E[AC₂], its comparative advantage position may be lost if it tries to over-protect itself by
reducing output too much. For example, if firm 1 decides to produce output even less than $Q_3^{**}$, its $\mu^{**}$ will be much higher and may even exceed that of firm 2. This is one typical example of "over-defense leads sometimes to more exposure to danger". However, if the change in $Q^{**}$ is small, then, in general, its effect on $\mu^{**}$ is uncertain.

2.4.2. Comparative static analysis under uncertainty in cost

Given the optimum results derived above, the next issue is the firm's behavior when the parameters of the distribution of the CIR (defined at optimum levels) change. The issue needs to be addressed as it implies the prediction about the firm's comparative advantage in the face of changing economic conditions. The assumption that price is non-stochastic (but CIR is stochastic) is retained. As CIR captures the randomness in both demand and supply relationships, it is assumed that most of the effects of external or internal random shocks will be reflected in the change in the parameters of the probability distribution of CIR. However, if the shocks do not come from the random factors, then the parameters of the distribution are expected to remain unaffected. Four cases of shocks are considered: two affecting the parameters [A and B] and two not affecting [C and D].
A. Effect of mean preserving spread of CIR

In this case, the focus is on the marginal impact on the firm's output and profitability if the spread of the distribution of the CIR changes with the mean constant. In the vocabulary of the economics of uncertainty, this is called Mean Preserving Spread (MPS) and is often used as a definition for "risk". For this a new random variable \( r' \) is defined, such that

\[
(2.46) \quad r' = \gamma r + \theta(\gamma)
\]

where \( \gamma \) and \( \theta \) are two shift parameters. An increase in \( \gamma \) alone will increase the mean as well as the variance of the distribution. It is assumed that the variance of the distribution is not a function of output; that is, the distribution is homoskedastic for all levels of output. To restore the mean, \( \theta \) will have to be reduced simultaneously, so that

\[
(2.47) \quad dE [\gamma r + \theta] = 0
\]

or,

\[
(2.48) \quad \frac{d\theta}{d\gamma} = -\mu
\]

Thus, the Mean Preserving Spread (MPS) or effect of changing variance keeping mean constant is

\[
(2.49) \quad \frac{dr'}{d\gamma} = r + \frac{dr'}{d\theta} \frac{d\theta}{d\gamma} = r - \mu
\]
Now, to find out the effect of MPS on output, the first order condition derived in equation (2.33) is recalled. At the point $r = r'$ (i.e., $\gamma = 1, \theta = 0$), the condition is

$$U' = E \left[ U'(\Pi) \left( P(1 - r') - TR \frac{dr'}{dQ} \right) \right] = 0$$

Solving the condition, the optimum output under uncertainty, $Q^{\ast\ast} = Q(\mu, \gamma)$, can be obtained. Now differentiate the above condition with respect to $\gamma$ to get

$$\frac{\partial U'}{\partial \gamma} = \frac{\partial Q^{\ast\ast}}{\partial \gamma} |_{Q=Q^{\ast\ast}} + E \left[ U'' \left( -TR \right) (r' - \mu) \right] \left( P(1 - r') \right)$$

$$- TR \left( \frac{dr'}{dQ} \right) |_{Q=Q^{\ast\ast}} - U'(\Pi) \left( P(r' - \mu) \right) = 0$$

Therefore,

$$\frac{\partial Q^{\ast\ast}}{\partial \gamma} = \frac{1}{U''} \left( TR E \left[ U'' (r' - \mu) \right] \left( P(1 - r') \right) - TR \left( \frac{dr'}{dQ} \right) |_{Q=Q^{\ast\ast}} \right)$$

$$+ \frac{1}{U''} P E \left[ U'(\Pi) (r' - \mu) \right]$$

To sign the last term of the R.H.S, consider the following proposition:

**Proposition 1.** For a risk-averse firm, $E \left[ U'(\Pi) (r - \mu) \right] > 0$, for all values of $r$.

**Proof:**

$$E[U'(\Pi) (r - \mu)]$$

$$= E[U'(\Pi)]E[r - \mu] + Cov[U'(\Pi), (r - \mu)]$$

$$= Cov[U'(\Pi), r], \text{ as } E(r - \mu) = 0$$
Define \( h(r) \) as \( U'(\Pi) \) and \( g(r) \) as \( r \). Thus,
\[
    h'(r) = U''(-TR)
\]
\[
    g'(r) = 1
\]
Therefore, \( g'(r) > 0 \), and assuming risk-aversion, i.e., \( U''(\Pi) < 0 \), \( h'(r) > 0 \). So,
\[
    \text{Cov} [U'(\Pi), r] > 0, \forall r
\]
equivalently,
\[
    E[U'(\Pi) (r - \mu)] > 0, \forall r \quad \text{Q.E.D}
\]

Combining the above proposition and the second order condition in equation (2.34), it follows that

\[
    (2.53) \quad \frac{1}{U''} P \ E[U'(\Pi) (r' - \mu)] < 0
\]

Signing the first term in the R.H.S of equation (2.52), however, is not so trivial. For this, consider the following proposition.

**Proposition 2.** Given Decreasing Absolute Risk Aversion (DARA),
\[
    E[U''(r-\mu)[P(1 - r) - TR \frac{dr}{dQ}]] > 0, \text{ for all values of } r.
\]

**Proof:**

\[
    (2.54) \quad E[ U''(\Pi) (r - \mu) \left( P(1 - r) - TR \frac{dr}{dQ} \right) ]
\]
\[
    = - P \ E[U''(\Pi) (r - \mu) \left( r - (1 - Q \frac{dr}{dQ}) \right) ]
\]
The first term of the R.H.S of (2.54) is positive under the assumption of risk-aversion, as \( \{ . \}^2 > 0 \), \( U''(\Pi) < 0 \), and there is a negative sign attached to \( P \). In the second term \( \{(1 - Q \frac{dr}{dQ}) - \mu\} > 0 \) by the first order condition in equation (2.42). Therefore, to prove Proposition 2, it is needed to prove that

\[
= - P \ E[ U''(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \} ]
\]

\[
= - P \ E[ U''(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \}^2 ]
\]

\[
- P \ (1 - Q \frac{dr}{dQ}) - \mu \} E[ U''(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \} ]
\]

The first term of the R.H.S of (2.54) is positive under the assumption of risk-aversion, as \( \{ . \}^2 > 0 \), \( U''(\Pi) < 0 \), and there is a negative sign attached to \( P \). In the second term \( \{(1 - Q \frac{dr}{dQ}) - \mu\} > 0 \) by the first order condition in equation (2.42). Therefore, to prove Proposition 2, it is needed to prove that

\[
(2.55) \quad E[ U''(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \} ] \leq 0
\]

Assume that the CIR where \( \{r - (1 - Q \frac{dr}{dQ})\} = 0 \) is \( r_0 \). Corresponding profit is \( \Pi_0 \). Note that from equation (2.19), \( r_0 \) is the CIR where profit is maximized under certainty. Now, first consider the case where \( r \leq r_0 \).

Case 1: \( r \leq r_0 \)

Note that, \( r \leq r_0 \) implies

\[
(2.56) \quad r - (1 - Q \frac{dr}{dQ}) \leq 0
\]
Now, the Arrow-Pratt measure of absolute risk aversion is used to define risk aversion. By definition, at a particular profit prospect $\Pi$, the measure of risk aversion is [Pratt, 1964]

$$
(2.57) \quad R(\Pi) = \frac{-U''(\Pi)}{U'(\Pi)}
$$

Similarly, at the profit at its maximum point

$$
(2.58) \quad R(\Pi_0) = \frac{-U''(\Pi_0)}{U'(\Pi_0)}
$$

DARA, in the case $r \leq r_0$, would imply $R(\Pi) \leq R(\Pi_0)$, that is,

$$
(2.59) \quad \frac{-U''(\Pi)}{U'(\Pi)} \leq \frac{-U''(\Pi_0)}{U'(\Pi_0)}
$$

multiplying both sides by \{r - (1 - Q \frac{dr}{dQ})\} and reversing the sign (as \{ . \} ≤ 0)

$$
(2.60) \quad \frac{-U''(\Pi)}{U'(\Pi)} U'(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \} \\
\geq \frac{-U''(\Pi_0)}{U'(\Pi_0)} U'(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \}
$$

Equation (2.60) is true for all $r \leq r'$. Now, consider the other possible case, $r > r_0$. 
Case 2: $r > r_0$

Note that $r > r_0$ implies

\[
(2.61) \quad r - (1 - Q \frac{dr}{dQ}) > 0
\]

In this case, i.e., $r > r_0$, DARA implies

\[
(2.62) \quad \frac{-U''(\Pi)}{U'(\Pi)} > \frac{-U''(\Pi_0)}{U'(\Pi_0)}
\]

Multiplying both sides by the same factor as in Case 1,

\[
(2.63) \quad \frac{-U''(\Pi)}{U'(\Pi)} U'(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \} > \frac{-U''(\Pi_0)}{U'(\Pi_0)} U'(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \}
\]

Equation (2.63) is true for all $r > r_0$. Therefore, combining equations (2.60) and (2.63) and taking expectations on both sides it is deduced that

\[
(2.64) \quad -E[U''(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \}] \geq \frac{-U''(\Pi_0)}{U'(\Pi_0)} E[U'(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \}], \quad \forall \ r \leq r'
\]

Now, the first order condition derived in equation (2.33) requires that

\[
(2.65) \quad E[U'(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \}] = 0
\]
That makes the R.H.S of equation (2.64) zero. In other words,

\begin{equation}
(2.66) \quad -E[U''(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \}] \geq 0
\end{equation}

or, equivalently,

\begin{equation}
(2.67) \quad E[U''(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \}] \leq 0
\end{equation}

This completes the process of signing the second term of the R.H.S of equation (2.54). Using the information in equation (2.67), it becomes positive. As shown earlier, the first term is also positive. Therefore, the L.H.S of equation (2.54) is positive. That is, given decreasing absolute risk aversion,

\begin{equation}
(2.68) \quad E[U''(\Pi) (r - \mu) \{ P(1 - r) - TR \frac{dr}{dQ} \}] > 0, \quad \forall \ r
\end{equation}

Q.E.D

The above result leads to

\begin{equation}
(2.69) \quad \frac{1}{U''} E[U''(\Pi) (r' - \mu) \{ P(1 - r') - TR \frac{dr'}{dQ} \mid_{Q=Q''} \}] \leq 0
\end{equation}

Now, combining the information in equations (2.53) and (2.69) and using them in equation (2.52),

\begin{equation}
(2.70) \quad \frac{\partial Q''}{\partial \gamma} < 0
\end{equation}
Thus, according to equation (2.70), given decreasing absolute risk aversion, if the "spread" in the PDF of the CIR increases, the optimum output will decrease. In other words, the more "risky" the indicator becomes, the less will be the output.

B. Supply response with respect to random CIR

The next important question to be addressed is: How the optimum supply decision will respond with respect to a change in CIR itself? But, since the CIR is a random variable, it does not make sense to speak about an "increase in CIR". It seems natural, however, to discuss the closely related problem of an increase in the mathematical expectation of the CIR with higher central moments constant. This can be done in the following way.

Let \( r' = r + \theta \). So, \( \theta \) is again an additive shift parameter. Increasing \( \theta \) is equivalent to moving the probability distribution to the right without changing its shape (or variance). Now, recall the first order condition derived in equation (2.33), which, in terms of \( r' \), is

\[
(2.71) \quad \bar{U}' = E[U'(\Pi) \{ P(1 - r') - TR \frac{dr'}{dQ} \}] = 0
\]

Differentiating the first order condition with respect to \( \theta \),

\[
(2.72) \quad \frac{\partial \bar{U}'}{\partial \theta} = \bar{U}'' \frac{\partial Q}{\partial \theta} \big|_{Q=q^*} - P E[U'(\Pi)] - TR E[U''(\Pi) \{ P(1 - r') - TR \frac{dr'}{dQ} \big|_{Q=q^*} \}] = 0
\]

Therefore, solving for \( \partial Q^*/\partial \theta \),
\[ 2.73 \quad \frac{\partial Q^{**}}{\partial \theta} = \frac{1}{U''} P E[U'(\Pi)] + \frac{1}{U''} TR E[U''(\Pi) \{ P(1 - r') \\ - TR \frac{dr'}{dQ} \vert_{Q=Q^{**}} \}] \]

The first term in the R.H.S of equation (2.73) is obviously negative as \( E[U'(\Pi)] > 0 \), and \( U''(\Pi) < 0 \) by utility maximizing condition. Consider \( E[. \] in the second term (evaluated at \( Q = Q^{**} \)).

\[ 2.74 \quad E[U''(\Pi) \{ P(1 - r') - TR \frac{dr'}{dQ} \}] \]

\[ = - P E[U''(\Pi) \{ r' - (1 - Q \frac{dr'}{dQ}) \}] \]

But, according to equation (2.67), given DARA,

\[ 2.75 \quad E[U''(\Pi) \{ r - (1 - Q \frac{dr}{dQ}) \}] \leq 0 \]

That implies

\[ 2.76 \quad \frac{1}{U''} TR E[U''(\Pi) \{ P(1 - r') - TR \frac{dr'}{dQ} \}] \leq 0 \]

The second term in the R.H.S of equation (2.41) turns out to be non-positive. As the first term has already been proved to be negative, that means

\[ 2.77 \quad \frac{\partial Q^{**}}{\partial \theta} < 0 \]
In other words, given constant variance, a marginal increase in the mean of the distribution of the CIR leads to a fall in output. Note that the sufficient condition for (2.77) is DARA and the necessary condition is the risk-aversion attitude of the producer.

The marginal impact of change in $\theta$ on optimum CIR is straightforward. As $r' = r + \theta$, and $r'$ is defined at the optimum output,

\[
(2.78) \quad \frac{\partial \mu^{**}}{\partial \theta} = 1 > 0
\]

That means, an increase in the mathematical expectation of CIR will lead to a fall in optimum output and an increase in optimum cost-income ratio. This is intuitively justified, as seen in Figure 2.4. For example, if $E[AC_1]$ shifts up to $E[AC_2]$ with same spread, $Q_1^{**}$ will fall to $Q_2^{**}$ and $m_1^{**}$ will rise to $m_2^{**}$. In other words, factors pushing up the mean of the CIR distribution of a firm, *ceteris paribus*, will lead to erosion of comparative advantage of the firm.

**C. Effect of changing output price**

So far the comparative static analysis in terms of changing parameters of the probability distribution of the CIR have been considered. The implicit assumption was that the change in any factor which contributes toward the randomness of the CIR, will be reflected in the changing parameters (mean and variance) of the distribution. Therefore, the impact of any exogenous shock from a random factor
(which contributes toward the randomness of the CIR) may be explained in terms of the previous two comparative static analytical framework.

Now, the effects of a non-random factor on optimum output and the cost-income ratio will be discussed. A natural candidate for this analysis is output price, which has been assumed to be non-stochastic (and, thus does not cause the randomness in the CIR). The question is: is the supply curve of the firm well-behaved if price is non-stochastic but the cost-income ratio is stochastic?

To analyze the impact, differentiate the first order condition in equation (2.33) and solve for $\frac{\partial Q^*}{\partial P}$.

\[
(2.79) \quad \frac{\partial Q^*}{\partial P} = \frac{1}{U''} TR E[U''(II) \{ r - (1 - Q \frac{dr}{dQ}) \}]|_{Q=Q}^* \]

\[
\quad - \frac{1}{U''} (1 - Q \frac{dr}{dQ}) E[U'(II)]|_{Q=Q}^*
\]

Once again, according to equation (2.67), $E[U''(II) \{ r - (1 - Q dr/dQ) \}] \leq 0$ under the sufficient condition of DARA. Therefore the first term in the R.H.S of equation (2.79) is non-negative. To check the sign of the second term, let it be put under scrutiny. From the first order condition given in equation (2.42),

\[
(2.80) \quad 1 - Q \frac{dr}{dQ} > \mu > 0
\]

Also, by assumption, $E[U'(II)] > 0$. Therefore,

\[
(2.81) \quad - \frac{1}{U''} (1 - Q \frac{dr}{dQ}) E[U'(II)] > 0
\]
Thus, the first term in the R.H.S is non-negative and the second term is positive; so,

\[ (2.82) \quad \frac{\partial Q^{**}}{\partial P} > 0 \]

In other words, given the sufficient condition of DARA, the supply elasticity of non-random price is still positive. The impact on optimum CIR also follows simple intuition, that is,

\[ (2.83) \quad \frac{\partial \mu^{**}}{\partial P} = \frac{\partial \left[ \frac{1}{P} E(AC^{**}) \right]}{\partial P} = - \frac{E(AC^{**})}{P^2} < 0 \]

The economic reasoning is that an increase in price reinforces the guard against uncertainty in cost and this leads to a higher supply and a lower cost-income ratio for the relatively profitable firm. In other words, the probability of a higher per unit net return will increase for all firms.

D. Effect of changing government subsidy

This section is concluded by analyzing the effect of a marginal change in government subsidy. It is assumed that:

1. The subsidy is paid on a "per unit" basis. That is, the total amount of subsidy is a proportion of the total amount of output produced.
2. The subsidy to be paid is non-random. That implies that the randomness of the CIR comes from some other source.
Given these assumptions, the post-subsidy CIR, at the optimum point is given as:

\[
(2.84) \quad r_s = \frac{C - sQ^{**}}{TR}
\]

where \( r_s \) is the optimum cost-income ratio after subsidy being paid and \( s \) = per unit subsidy. Differentiating equation (2.84) with respect to \( s \),

\[
(2.85) \quad \frac{\partial r_s}{\partial s} = -\frac{1}{P} < 0
\]

Now, differentiating the first order condition with respect to \( s \) and imposing the value of \( \frac{\partial r_s}{\partial s} \) from equation (2.85), the relevant expression is

\[
(2.86) \quad \frac{\partial Q^{**}}{\partial s} = \frac{1}{U''(Q)} (-Q) \left( E[U''(Q) \{ P(1 - r_s) - TR \frac{\partial r_s}{\partial Q} \}]_{Q=Q^{**}} \right.
\]

\[
\left. - \frac{1}{U''(Q)} E[U''(Q)]_{Q=Q^{**}} \right)
\]

The first term is non-negative under the assumption of DARA. The second term is obviously positive. Therefore,

\[
(2.87) \quad \frac{\partial Q^{**}}{\partial s} > 0
\]

and, from equation (2.85),

\[
(2.88) \quad \frac{\partial \mu^{**}}{\partial s} = -\frac{1}{P} < 0
\]

Thus, increasing the rate of subsidy leads to increase in optimum output and decrease in optimum cost-income ratio. The reasoning is the same as in the case
of change in price. That is, a government subsidy reinforces the strength of a profitable firm to fight cost uncertainty and makes it produce more with a higher probability of lower cost-income ratio. Note that, the qualitative impacts of a subsidy and an increase in price are the same for a firm with comparative advantage.

2.4.3. P is random, AC is non-random

Now, the second possible case is considered, i.e, it is assumed that uncertainty comes from the demand side, while the cost function is known with certainty. Fortunately, this problem and the behavior of a competitive firm under price uncertainty is one of the most thoroughly explored and well-documented in the literature. Instead of reproducing the basic mechanism of optimization, the results and their implications will be summarized on the basis of pioneering works done by Sandmo (1971) and Batra and Ullah (1974).

Based on the assumptions of (i) pure competition (i.e, price is fixed in a probabilistic sense), (ii) maximization of expected utility from profit, (iii) no inventories, and (iv) risk-aversion, Sandmo derived the result that under uncertainty, the optimum condition for a firm is

\[
E [P] > MC
\]
where, $E[P]$ is the mean of the known PDF of $P$, and $MC$ is marginal cost. If the uncertainty were removed, and if the random price $P$ were replaced by a certain price of $E[P]$, the optimal output is given by

\[(2.90) \quad E[P] = MC\]

Comparison of (2.89) and (2.90) shows immediately that, for the risk-averse firm, output under uncertainty is less than output under certainty (where mean price is same in both situations). The same result has been obtained by Batra and Ullah (1974) in a more generalized long-run model, where it has been shown that the expected marginal value product of each input [i.e, $E[P].MP_i$, where $MP_i$ is the marginal product of $i$-th input] exceeds its price under the assumption of diminishing marginal product. That means, under uncertainty, the optimal quantity demanded of each input is lower than the certainty case (for a risk-averse firm). This automatically conforms with the result in inequality (2.89), i.e, the optimal output will also be lower under uncertainty.

The result in (2.89) can be accommodated in the present framework in a straightforward way. Let $AC = C(Q)/Q$. Then differentiating $AC$ with respect to $Q$ yields

\[(2.91) \quad \frac{d(AC)}{dQ} = \frac{C'(Q) Q - C(Q)}{Q^2} = \frac{1}{Q} \left[ MC - AC \right]\]

Rearranging equation (2.91),
Substituting the value of $MC$ and dividing both sides by $E[P]$ in (2.89), the result is

$$\frac{Q}{E[P]} \frac{dAC}{dQ} + \frac{AC}{E[P]} < 1$$

or,

$$\mu^{**} < \frac{1}{1 + \epsilon_{AC^{**}}}$$

Note that the condition derived in (2.94) looks like the same condition derived in the cost uncertainty case in (2.44). However, there is a basic difference between the two. While in (2.44) $\epsilon_{AC^{**}}$ is stochastic, in (2.94) it is non-stochastic. Therefore, even though the results are similar, the graphical demonstration and economic reasoning will be different in this case. For this, consider the diagram in Figure 2.5. To comply with Sandmo's analysis, it is analyzed in terms of the MC curve. For simplicity's sake, assume three different firms producing three different crops in a particular region with the same marginal cost. As depicted, $Q^*$ is the certainty output for each firm, where $P = E[P_2] = MC$. This is achieved at point $E_2$ in the upper panel and correspondingly at point $E_2'$ in the lower panel. Now, assume uncertainty in prices so that there are three probability distributions with three different means - $E[P_1]$, $E[P_2]$, and $E[P_3]$ for firms 1, 2, and 3 respectively.
Figure 2.5 Cost-income ratio under price uncertainty in a competitive market.

Corresponding to these set of mean prices, there are mathematical expectations for CIR - $E[r_1]$, $E[r_2]$, and $E[r_3]$ in the lower panel.

By the optimum condition, i.e, $E[P] > MC$, each firm will produce somewhere to the left of $E[P] = MC$ point. Suppose that such points on the MC curve are $E_1$, $E_2$, and $E_3$ with output $Q_1''$, $Q_2''$, and $Q_3''$ respectively for firms 1, 2,
and 3. Correspondingly, the optimum points on the respective $E[r]$ curves are $E_1'$, $E_2'$, and $E_3'$.

As expected, the optimum CIR for firm 1 ($m_1^{**}$) is the lowest. In other words, the firm with a higher probability to get the highest price has the higher probability to maximize per unit profit and thus is in better comparative advantage situation. This is intuitively simple and falls exactly in line with the arguments presented in the cost-uncertainty case.

The problem is how to explain the case where firms are producing the same crop so that the actual price will be the same for each firm. Suppose $P = E[P_2]$ is the actual price which is realized after output decision (assuming uncertainty) has been made. In that case, firm 1 which has a higher (subjective) probability to achieve the higher price (i.e, over-expectation) has the highest probability to produce $Q^*$, the profit maximizing output. For the other firms, who have equal or under expectation, the tendency to move away leftward from $Q^*$ is more probable under the decision-making process in uncertainty. In other words, under uncertainty, the firm with a higher expectation about price has the comparative advantage (or, maximum profitable) and has a higher probability to achieve maximum profit once price becomes known.

The comparative static analysis in Sandmo's model, though analytically different, produces similar type of results as obtained in the cost-uncertainty case. Since the results are well documented in Sandmo (1971) and Ishii (1977), the results (with extended results about the impact on optimum CIR) are restated here.
(1) Increase in "riskiness" in CIR (i.e., price) leads to a decline in optimum output and optimum CIR.

(2) Upward shift of the mean of the CIR distribution (i.e., downward shift of the mean of the distribution) leads to a decline in optimum output and CIR.

(3) Increase in government cost subsidy leads to a higher optimum output and lower optimum CIR.

(4) Increase in fixed cost leads to a decrease in optimum output and an increase in optimum CIR [Note that under certainty, an increase in fixed cost has no effect on optimum output].

2.4.4. Both P and AC random

Theoretically, assuming uncertainty in both sides does not add any new dimension. It only strengthens the result which are obtained assuming uncertainty in either case. However, analytically it comes closer to the cost-uncertainty case, and is similar to the result we obtained in (2.44), that is,

\[
(2.95) \quad \mu^{**} = \frac{1}{1 + E[e_{AC^{**}}]}
\]

where \( e_{AC^{**}} \) is stochastic. However, in this case, \( \mu^{**} \) is the ratio of mathematical expectations of two random variables, P and AC. Thus (2.95) may be reformulated as

\[
(2.96) \quad E[P] > [E[AC] + Q \frac{dAC}{dQ}]|_{Q-Q^{**}}
\]
Assuming \( \frac{dAC}{dQ} \) non-random, (2.96) implies that, at given \( Q \) and \( \frac{dAC}{dQ} \), the firm must operate in the zone where \( E[P] \) is higher and/or \( E[AC] \) is lower. The situation is a combination of Figure 2.4 and Figure 2.5. Here, there are numerous \( P \) around the \( E[P] \) and numerous \( AC \) curves around \( E[AC] \). Naturally, the ranking of firms according to their comparative advantages will depend on the probabilities of achieving \( P'' \) and \( AC'' \) simultaneously. That automatically implies the probability of achieving \( \mu'' \), which is much lower than the optimum CIR under certainty. In other words, uncertainty from both demand and cost sides reinforces the need for a higher probability to achieve a lower cost-income ratio.

2.5. Summary and implications

This chapter develops the theoretical and analytical basis for discussing regional comparative advantage of agricultural production. Regional comparative advantage has been defined in terms of relative profitability of a crop or a firm in a particular region in comparison to the same in another region. Relative profitability, being itself a function of per unit net revenue, may be expressed in terms of a random indicator - the cost-income ratio. Under certainty, the relative maximum profitability condition coincides with the profit maximizing condition. The optimality condition can be expressed in terms of the indicator and long run adjustment has been derived under different market conditions. Comparative advantage becomes more meaningful under the assumption of uncertainty and it is shown that the optimality condition in this case is that the firm must operate at
lower output and lower cost-income ratio. From the above discussion, the following implications can be derived.

First, a firm's behavior and profitability in uncertainty can be evaluated by the probability of achieving a lower cost-income ratio. But the determination of optimum output (i.e, $Q^{**}$) and optimum CIR (i.e, $\mu^{**}$) is nearly impossible in reality as it requires the evaluation of subjective p.d.f of cost and price for each crop or each firm. Moreover, the inequality sign in the optimal condition is not very informative in determining a particular value of output. However, irrespective of the source of uncertainty, the theoretical framework presented above indicates a simple rule: the more successful a firm is to keep CIR lower than other firms, the more successful it is in guarding itself against uncertainty. Therefore, the objective probabilities of CIR may be an acceptable basis for evaluating the comparative advantage of a firm.

Second, as comparative advantage has been defined in terms of probabilistic statements, a given ranking on the basis of the probabilities may be used as a source of information and prediction of comparative advantage in a particular region, given the fixed value of shift parameters.

Third, CIR is a function of random and non-random elements in supply and demand. Therefore, the impacts of change in these elements may be analyzed in the analytical framework outlined above. For example, the comparative static

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3 For example, in Sandmo's model, $Q^{**} < Q^*$; but that does not help to get a particular value of $Q^{**}$. 
analytical tools used in Section 2.4.2 may be used and modified to analyze the impacts of change in fixed cost, technological parameters, demand parameters, etc.

Finally, the methodology discussed above can be used in two ways: (1) crop-specific comparative advantage, or, comparative advantage of a crop with respect to other crops in all regions; and, (2) firm-specific comparative advantage, or, comparative advantage ranking of firms (producing the same crop) in a particular region or all regions. Analytically, the second analysis is more interesting as in this case the issues of firm-sizes, firm productivity, and firm-specific efficiencies and their relation to comparative advantage can be addressed. In the next chapter, the methodological and empirical issues regarding crop-specific comparative advantage will be addressed. The issue of firm-specific comparative advantage and efficiency will be discussed in Chapters 4 and 5.
CHAPTER 3
REGIONAL COMPARATIVE ADVANTAGE: METHODOLOGICAL AND
EMPIRICAL ISSUES

3.1. Introduction

In this chapter, the methodological and empirical issues regarding the analysis of cropwise regional comparative advantage are discussed on the basis of the theoretical analysis set in Chapter 2. Given a well-behaved probability density function for the cost-income ratio, the CIR satisfying the optimum condition derived in Chapter 2 can be obtained. Further, the comparative static with respect to the random and non-random variables can be analyzed to show the impact on comparative advantage rankings of different crops in a region or across regions. To bring conformity with the theoretical structure (which is based on a firm's behavior) in Chapter 2, the regional data is treated as data experienced by a representative firm.

The major problem in empirical analysis is in identification of the true probability distribution of CIR. The optimal CIRs cannot be derived unless the distribution is known. However, as noted in the last section of Chapter 2, for practical purposes, the objective distribution (based on the historical and present data on costs and returns) of CIR is justified as the objective is to derive the objective basis of comparative advantage. For computational purpose, a series of CIR values (e.g., 1.0, 0.9, 0.8, ...) can be selected as \( \mu \) (where, as derived in
equation (2.44) in Chapter 2, \( \mu'' \) is the optimum CIR under uncertainty). The optimum condition derived under uncertainty (see equation (2.44)) is thus reflected in the estimated probabilities of CIR to attain assumed \( \mu'' \) for each of the value in the series.

In the next section a general procedure of stochastic analysis of the CIR indicator is provided. Section 3.3 discusses the application and data. In Section 3.4, the estimation procedure is discussed. In Section 3.5, results are analyzed. The chapter will be concluded by a summary and conclusion.

3.2. Procedure

In this chapter, numerical expressions for regional comparative advantage of crop production will be derived. For each crop, the CIR is assumed stochastic with an unknown distribution function. This assumption appears reasonable as yields, output prices, or cost (input prices) may be stochastic.

Certain assumptions are needed before introducing the procedure. Let there be \( p \) crops and \( q \) regions and the probability density function (PDF) of the cost-income ratio (CIR) for \( i \)-th crop in \( j \)-th region is denoted as \( f(r_{ij}), i=1,2,...,p, j=1,2,...,q \). The necessary assumptions are:

1) The PDF \( f(r_{ij}) \) for the CIR\(_{ij} \) should be reasonably smooth and non-negative, i.e, \( f(r_{ij}) \geq 0 \).
2) The observed CIR values for a given crop are independent random samples from some unknown density functions. That is, 
\[ f_t(r_{i1}, r_{i2}, \ldots, r_{iq}) = f_t(r_{i1})f_t(r_{i2})\ldots f_t(r_{iq}). \]

3) \( f(r_{ij}) \) is not necessarily equal to \( f(r_{ji}) \). This assumption permits crops in a given area to have values for the CIRs which are random, independent realizations from identical distributions, except possibly for location and scale.

Given these assumptions the first step is to present the information regarding probabilities of CIR in the form of a matrix. Consider a particular level of CIR, say, \( r_0 \). Then the cumulative probability \( F_{ij} = \Pr[\text{CIR} \leq r_0] \) for the \( i \)-th crop in the \( j \)-th region may be derived from the PDF \( f(r_{ij}) \). Therefore, the cumulative density matrix for CIR \( \leq r_0 \) can be simply represented as (Zapata et al (1990)):

Table 3.1 Cumulative density matrix (\( M_{pq} \)) of the CIR \( [F_{ij} = \Pr(\text{CIR} \leq r_0)] \).

| CROPS \( i \) | FARMING AREAS \( j \) |
|---|---|---|---|---|
| \( 1 \) | \( F_{11} \) | \( F_{12} \) | \( F_{13} \) | \( \ldots \) | \( F_{1q} \) |
| \( 2 \) | \( F_{21} \) | \( F_{22} \) | \( F_{23} \) | \( \ldots \) | \( F_{2q} \) |
| \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |
| \( p \) | \( F_{p1} \) | \( F_{p2} \) | \( F_{p3} \) | \( \ldots \) | \( F_{pq} \) |

The assumptions (1) - (3) imply that \( F_{ij} \leq 1 \), \( F_{ij} \)'s are independent, and the matrix is not symmetric. It is to be noted that all \( F_{ij} \)'s are unconditional cumulative...
probabilities. If, however, the distribution of CIR (i.e., shape and moments) is assumed to be affected by an independent variable \( N \), then it is necessary to present the information in the cumulative density matrix in terms of conditional probabilities. In other words, the unconditional probability element \( F_{ij} \) in the matrix presented in Table 3.1 should be replaced by the conditional probabilities \( F_{ij} | N \) (i.e., \( F_{11} | N, F_{12} | N, \ldots, F_{ij} | N, \ldots, F_{pq} | N \)).

The proposed procedure is as follows. First, define the region or state into \( q \) mutually exclusive farming areas. Second, identify the commodities currently produced in these areas. Third, estimate CIRs for all commodities by areas. Fourth, fit unconditional and/or conditional empirical distributions to the CIR for all commodities by areas as in step three and identify the density function that best fit the CIRs. Fifth, estimate the probabilities associated with achieving certain levels of CIR (i.e., net returns per dollar of costs).

The above procedure will lead to an estimated cumulative density matrix \( m_{pq} \) (or, \( m_{pq} | N \)) where each element in the population matrix \( M_{pq} \) (or, \( M_{pq} | N \)) is replaced by its estimates. The estimated probabilities will permit an evaluation of how likely it would be, for farmers in a given area, to obtain certain levels of net return per expense dollars given the current and historical structure of costs. For instance, one could estimate a \( p \times q \) matrix for the probability of the CIR being less than equal to one. This matrix would show by commodity and across farming areas how likely it would be for farmers in that area to break even after variable and fixed factors have been accounted for.
However, as argued in Chapter 2, high probabilities to achieve a given cost-income ratio do not necessarily ensure the comparative advantage of a particular crop. The capacity of income generation by the production of a crop also significantly affects the profitability of the crop. For that reason, estimated probabilities should be weighted by an appropriate weight factor which reflects the recent income generating capacity of respective crops by areas. This will lead to an (conditional or unconditional) Absolute Advantage Matrix. Finally, a Comparative Advantage Matrix will be formed on the basis equation (2.10) in Chapter 2. This will reveal information about whether resources are being used according to comparative advantage, and equally important, identify viable production alternatives to farmers in a given area.

Steps one through three are self-explanatory and flexible enough to accommodate most data situations. For instance, if region level data are not available, the probabilities for the whole state with aggregated data set may be estimated. In that case, the cumulative density matrix would be a column vector. Similarly, one can compare advantages of a crop among regions if the regionwise data set for only a single crop is available. Steps four and five (i.e, fitting empirical distributions and estimating (unconditional and conditional) probabilities) are discussed below. The issue of computing absolute advantages and relevant selection problem of weight factor is then addressed.
3.2.1. Estimation of PDFs

The main objective is to estimate the probabilities of the CIR to be less than some preselected value of $\mu^{**}$ for each crop across the regions. In most cases functional form of CDFs is unknown and should be estimated from the data. Relatively simple methods for fitting probability distributions that are commonly employed in empirical studies generally fall into three categories: (a) free-hand fitting; (b) exponential functions (Dixon and Sonka, 1979); and (c) estimation of the moments of a PDF in the Pearson system of distributions (Day, 1965). However, none of these methods is perfectly satisfactory for most empirical studies. For example, the free-hand method is infinitely flexible, but it has no statistical foundation, and continuous PDFs cannot be obtained from the CDF since the equation of the CDF is unknown. On the other hand, simple exponential functions and Pearson systems, though they have solid theoretical foundation, are quite restrictive in assumptions (Taylor, 1981).

More complex methods of fitting CDFs or PDFs include: (a) spline functions; (b) Fourier series methods which minimize sum-square-errors; and, (c) Fourier methods which maximize a likelihood function less a roughness penalty. The problems with the methods are: (1) they are difficult to use; (2) as the analyst must specify a roughness penalty, the empirical CDFs are approximations to the true but unknown CDF.

A flexible method of fitting empirical CDFs was introduced by Taylor (1981, 1984). It is based on a hyperbolic trigonometric (HT) transformation procedure
which fits a CDF with ordinary least square (OLS) or maximum likelihood (ML) procedure. This method is shown to be flexible enough to closely approximate most theoretical distributions [Taylor(1981), p-4]. The procedure also guarantees to constrain the CDF function to lie between zero and one. Following Taylor (1981) the HT procedure is elaborated below.

3.2.2. Unconditional PDF

Consider a hyperbolic tangent function

\[ \tanh u = \frac{e^u - e^{-u}}{e^u + e^{-u}} \]  

(3.1)

where \(-\infty \leq u \leq \infty\) and \(-1 \leq \tanh u \leq 1\). Graphically, the hyperbolic tangent in (3.1) has the curvature properties similar to a unimodal CDF and its derivative, the square of the hyperbolic secant, has the properties similar to a corresponding PDF. Now, to constrain the CDF to the interval 0-1, the following transformation is considered.

\[ F(X) = 0.5 + 0.5 \tanh [G(X)] \]  

(3.2)

where \(F(X)\) is the CDF of \(X\), and \(G(X)\) is any function of \(X\). For any value of \(G(X)\), transformation (3.2) constrains \(F(X)\) to the interval 0-1. Moreover, the function \(G(X)\) gives the required flexibility to the transformation, permits additional modes to the PDF, and allows for the PDF to be skewed in either direction, or to be symmetrical.
For a more suitable form to estimate the parameters of linear $G(X)$, equation (3.2) needs to be transformed. Using the logarithmic form of the inverse hyperbolic tangent, it can be shown that equation (2) can be expressed equivalently as

\[(3.3)\quad Y = 0.5 \ln \frac{F(X)}{1 - F(X)} = G(X)\]

Initially, each $X_t$ can be assigned a cumulative frequency by ranking the observations on $x$ in ascending order, $X_1 < X_2 < \ldots < X_T$, and then assigning

\[(3.4)\quad F(X_t) = \frac{t}{T}\]

Then all $F(X_t)$ except $F(X_T)$ can be transformed by (3.3) to get a finite $Y_t$. However, at $t = T$, $Y_t = \infty$. To get rid of this problem (i.e., to get a finite value of $Y_T$), a practical approach would be to adjust $F(X_t)$ slightly downward.

Now, OLS can be applied to the transformed data set $(Y_t, X_t)$ to obtain estimates for the parameters of a $G(X)$ linear in parameters and thus of the CDF, equation (3.2). For empirical purposes, various forms of $G(X)$ may be tried to select one which best fits the data. For example, if the data generating process involves a symmetrical distribution of $X$, then the best fit $G(X)$ would include only the odd powers of $X$, i.e.,

\[(3.5)\quad Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_t^3 + \beta_3 X_t^5 + \ldots\]
On the other hand, if the population $X$ is Gamma distributed, following form would be suitable

$$\text{(3.6)} \quad Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_t^2 + \beta_3 X_t^3 + \ldots + \beta_4 (\ln X_t)$$

For the log-normal distribution, the form that would best fit the data is

$$\text{(3.7)} \quad Y_t = \beta_0 + \beta_1 (\ln X_t) + \beta_2 (\ln X_t)^2 + \beta_3 (\ln X_t)^3 + \ldots$$

The problem is that the OLS estimators derived from the above or any other formulations will be biased and inconsistent due to the simple reason that the dependent variable ($Y_t$) is non-stochastic and the independent variable ($X_t$) is stochastic. However, the bias in the OLS estimates may be sufficiently small to ignore in many practical applications [Taylor(1981), p-6].

To avoid bias and inconsistency in the OLS estimates and to achieve estimates with desirable asymptotic properties, a likelihood function based on equation (3.2) should be maximized. Differentiation of equation (3.2) gives the PDF

$$\text{(3.8)} \quad f(X) = 0.5 \ G'(X) \ sech^2 [G(X)]$$

where $f(X)$ is the PDF and $G'(X)$ is $\partial G(X)/\partial X$. From equation (3.8) the log-likelihood function is derived as follows
(3.9) \( \ln L(\beta) = T \ln(0.5) + \sum_{t=1}^{T} \ln[G'(X)] + 2\sum_{t=1}^{T} \ln[\text{sech } G(X)] \)

Taking the partial derivatives of equation (3.9) with respect to the parameter vector, \( \beta \), and setting it to zero gives a set of \( k \) equations that can be solved for the \( k \) parameters

\[
(3.10) \quad \sum_{t=1}^{T} \frac{\partial G(X,\beta)}{\partial \beta_i} - 2 \sum_{t=1}^{T} \frac{\partial G(X,\beta)}{\partial \beta_i} \tanh [G(X,\beta)] = 0
\]

where \( i = 1, 2, ..., k \). As analytical solution of the \( k \) equations in (3.10) for \( \beta \) is nearly impossible, numerical search procedures must be used to solve for the vector \( \beta \). It is suggested that OLS estimates from (3.3) be used as starting values for the ML estimation of \( \beta \). The exact form of \( G(X) \) can be identified by including the transformation of \( X \)’s (e.g, \( X^2, X^3, \ln X \), etc.) for which the estimated coefficients are found significantly different from zero.

3.2.3. Conditional PDF

In Chapter 2 (Section 2.4.2) theoretical solutions for comparative static analysis of CIR have been presented on the basis of assumptions about exogenous variables (i) affecting and (ii) not affecting the parameters (moments) of the distribution. The question is: how this can be addressed empirically? The impact of "not affecting" factors can be derived empirically on the basis of estimates of
unconditional PDF. In that case, a series of unconditional probabilities can be derived at different levels of independent variable.

More important, however, is to derive the impact of variables which affect the moments of the distribution systematically, as this helps to identify the factors that contribute to the randomness of CIR (or, equivalently, profitability). Empirical treatment in this case is an extension of the unconditional case. Specifically, the conditional distribution of a variable \( X \) can be estimated simply by including one or more independent variables in \( G(X) \). In other words, the relevant HT transformation corresponding to equation (3.2) is

\[
(3.11) \quad F(X|N) = 0.5 + 0.5 \tanh [G(X,N)]
\]

where \( N \) is a set of independent variables affecting the shape and moments of the distribution of \( X \). ML estimates can be obtained by using the conditional counterpart to equation (3.9). The statistical significance of the coefficients can be tested by standard test procedures. As before, a practical estimation approach would be to use OLS to obtain the initial estimates of the functional form and parameters of \( G(X,N) \), then apply ML because of its desirable asymptotic properties.

In the present study, \( X \) is \( \text{CIR}_t \); thus, in the context of agricultural production, several variables, such as fertilizer, weather, government farm supports, world prices, etc. may be considered as \( N \) variables. One simple way to incorporate information from variation of all these variables may be to consider \( \text{CIR}_{t-1} \) as \( N \)
variable. This is justified as the production process and lagged supply response in agriculture implies some kind of relationship between the probabilities of CIR in one period with that of the previous period. Further, the individual components captured in lagged CIR can be separated. For example, if direct government support contributes $c$ percent of the total income in the last year, then, *ceteris paribus*, a hypothetical $n$ percent reduction in support in the last period will lead to $(c \times n)$ percent reduction in total income. Correspondingly, CIR increases by $(nc/(1 - nc))$ percent. Thus the increased CIR (hypothetical) in the last period will be $CIR_{t-1}^{H} = CIR_{t-1} \times (1/(1 - nc))$.

To illustrate this point, suppose, in period $t-1$, $c = 30$ percent, total income $(R) = 70$, total cost $(C) = 63$, so that $CIR_{t-1} = 0.90$ and suppose the probability to break even in period $t$ (given CIR in the last period = 0.9) is $Pr\{CIR_t \leq 1 | CIR_{t-1} = 0.9\} = 0.75$. Now consider this question about a hypothetical situation: what would be the probability of breaking even in period $t$ if government support were 15% (instead of 30%) in the last year? For this, calculate the hypothetical CIR:

$$CIR_{t-1}^{H} = CIR_{t-1} \times \frac{1}{1 - nc} = 0.9 \times 1.015 = 0.914$$

The next step is to calculate $Pr\{CIR_t \leq 1 | CIR_{t-1} = 0.914\}$ from the estimated conditional distribution. Note that if cost and other shares of revenue do not change in period $t$, this probability remains same in period $t+1$. 
3.2.4. Selection of weight factor (m_i)

To compute the Absolute Advantage Index (AAD), the estimated probabilities are to be weighted by an indicator \( m_i \) which captures information regarding recent income generating capacity of i-th crop or firm. Several indicators are proposed; the choice of a particular one depends on data availability and the objective of the study.

(1) Expected opportunity cost of investment \( (E(O_{It})) \). As \( E(O_{It}) \) roughly indicates the amount of foregone profit from the best alternative production, it may be represented by \( E(\Pi_{jt}) \) or \( E(\pi_{jt}) \) where the subscript \( j \) refers to the best or next best alternative crop in terms of profit.

(2) Expected net return per unit of output \( (E(\pi_{it})) \).

(3) Expected total net return \( (E(\Pi_{it})) \).

(4) Expected net return per acre \( (E(\Pi^{A}_{it})) \).

Among these, the problem with \( E(O_{It}) \) is that it incorporates information of \( j \)-th crop in computing AAD for \( i \)-th crop; thus, the resulting index does not provide information about absolute advantage in true sense. \( E(\pi_{it}) \) makes better sense and is more useful in the context of firm-specific advantage for a group of firms with homogenous firm size. In that case, the current income generating capacity of each firm is determined solely by current profit generating capacity of output. For the comparison of this capacity between two firms producing the same crop, the appropriate indicator would be current net return per unit of output. This may not indicate true capacity if firms are of unequal size. In that case, the comparison
should be based on total net return \( (E(\Pi_{it})) \) which incorporates the information on firm size. \( E(\Pi_{it}^A) \) is expected to serve better in the context of cropwise (or, regional) advantage as it gives a uniform basis for comparison among crops given technological and measurement differences within the crops. Since regional data on crops are aggregated, a more acceptable basis is acres (which is more or less homogenous), not output or firm size (which are not homogenous).

In this study, \( E(\Pi_{it}^A) \) will be used as \( m_t \). The problem, however, is that the distribution of \( CIR_{it} \) and \( \Pi_{it}^A \) are not independent according to equation (2.8) in Chapter 2. This problem can be empirically solved by assuming a simple expectation rule. Specifically, the realized value of \( \Pi \) (per acre) in the previous period or an average of the same for previous few years may be used as proxy for expected profit per acre. As lagged values are essentially non-stochastic, probabilities and weight may be assumed as independent.

3.3. Data and empirical setting

Regional average data on per acre yields, gross returns, costs, and direct government payments for five Louisiana crops (rice, cotton, soybeans, corn, and sugarcane) were collected from secondary sources for the 1956-1988 period. Some of the more recent years could not be incorporated due to data limitations. The study covers seven production regions in Louisiana: Red River, Central, Delta, Macon Ridge, Southwest, Sugarcane, and Other areas. Regions were defined by the predominant crops and soils, and even though some regions may cut across parish
lines, regions were delineated by parishes for convenience. Yields were regional average yields as reported by the Louisiana Agricultural Statistical Service (LASS). Gross returns were calculated by multiplying yield per acre with the appropriate market price as reported by LASS. Data on direct government payments were obtained from the Agricultural Stabilization and Conservation Service, USDA. Total costs were obtained from budgets developed by the Department of Agricultural Economics and Agribusiness at Louisiana State University. These are enterprise cost and returns projections on a per acre basis and include direct or variable costs, fixed costs, but not land charges, management, or overhead. The production regions in Louisiana are shown in Figure 3.1 where the regions are marked by assigned numbers. Below, a brief geographical and historical introduction of the crops included in this study is presented.

3.3.1. Agricultural production regions in Louisiana

The Red River Area extends along the Red River from the northwest corner to the central part of the state and consists of the parishes of Bossier, Caddo, Grant, Natchitoches, Rapides, and Red River. Crop production is mainly along the bottomlands which are generally fertile and highly productive. Primary crops have been cotton, soybeans, and corn with some areas of rice and sugarcane.

The Central Area is part of the fertile flood plains of the Mississippi and Red Rivers and consists of the parishes of Avoyelles, Point Coupee, and St. Landry. At
Figure 3.1 Agricultural production regions in Louisiana.
important crops are soybeans, rice, and corn, but cotton and sugarcane production have been important in the past.

*The Delta Area* includes the flood plains of the Mississippi and Ouachita Rivers flood plains. It includes the following parishes: Catahoula, Concordia, East Carroll, Madison, and Tensas (Mississippi Delta Area), and Caldwell, Morehouse, and Ouachita (Ouachita River Area). This area is characterized by flat topography with alluvial soils. Primary crops are cotton and soybeans with corn as a minor crop. Rice acreage has been increasing in recent years.

*The Macon Ridge Area* is located in Northeast Louisiana and includes the parishes of Franklin, Richland, and West Carroll. Soils are silty terrace soils, poorly to moderately drained, and range from nearly level to moderately sloping. Major crops are cotton and soybeans.

*The Southwest Area* consists of the parishes of Acadia, Allen, Beauregard, Calcasieu, Cameron, Evangeline, Jefferson Davis, Lafayette, and Vermilion. Soils are generally referred to as coastal prairie soils characterized by low fertility, poor run-off, and poor internal drainage. Rice is the predominant crop followed by soybeans with sugarcane production in the eastern fringe. Cotton, which was a major crop at one time, is rarely grown at present.

*The Sugarcane Area* is located in South Central Louisiana and includes the parishes of Ascension, Assumption, Iberia, Iberville, Lafourche, St. James, St. John the Baptist, St. Martin, St. Mary, Terrebonne, and West Baton Rouge. This area is located in part of the Mississippi flood plain. Soils are generally fertile and
highly productive. Sugarcane is the predominant crop with some acreage devoted to soybeans and rice. Cotton, which was a major crop at one time, is hardly grown at present.

*The Other Areas* include those areas (in Northern and Western Louisiana) which have not been included in the above six regions. The topography is generally hilly or rolling with valleys and flat land between hills. Over the time period studied, there has been a transition from row crops such as cotton to livestock production or forestry. The parishes included in this grouping are Bienville, Claiborne, DeSoto, Lassalle, Lincoln, Sabine, Union, Vernon, and Webster.

3.3.2. Government policies

Within the study period (1956-1988), there have been several government farm policies experienced by the Louisiana agricultural sector. Their primary goals have been to support producer income, to encourage a healthy, competitive domestic agricultural industry, to promote exports, and to ensure a safe, secure, and adequate food supply at reasonable prices. Methods used to accomplish these objectives have been price supports, acreage allotments, diversion payments, deficiency payments, and disaster payments.

All five crops have used a firm level price support program which implies non-recourse loans to farmers, government purchases, and direct payments. All crops, except soybeans, have also used an allotment program by which production

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1 The main source of information provided in this section is McManus (1990)
of a crop is reduced to a level equal to expected consumption, plus exports and a reserve supply. By this program, a farm or operator would be assigned the maximum acres that could be grown in that commodity; the operator could not cultivate more acres than his assigned allotment without penalty.

For this analysis, income was assumed to be market income plus direct government payments. Since indirect price supports and allotments should affect the market price, these measures were not included in determining government payments in this analysis. Direct government payments were assumed to be the sum of direct support payments, disaster payments, diversion payments, and deficiency payments. Direct support payments were provided only to cotton producers in 1964 and 1965. Disaster payments were made to cotton, rice, and corn during the 1970's. Diversion payments were made under a voluntary program for cotton and corn where producers were paid for diverting crop land into conserving uses. Deficiency payments were made to cotton, rice, and corn producers who participated in the farm programs. The amount of the deficiency payment is the product of the program yield per acre times the difference between the target price and the greater of the market price or loan rate. There usually is a set percentage of commodity acreage that must be set aside and a maximum payment to each farm operator.
3.3.3. Crops

Five major Louisiana crops are considered in this analysis: Cotton, Rice, Soybeans, Corn, and Sugarcane.

Cotton: Cotton has been traditionally grown in the Mississippi, Ouachita, and Red River flood plains and on the Macon Ridge. There has been cotton production in the other farming areas, but acreage has declined. Cotton acreage has varied for the time period studied from a high of 693,650 in 1981 to a low of 310,185 in 1975 with average annual acreage of 511,971. State average yields have varied from 399 to 819 pounds per acre with an average of 576 pounds. The Red River, Delta, and Central areas tended to have the highest yields, followed by the Macon Ridge, Southwest, Other, and Sugarcane areas.

Cotton producers in Louisiana have, over the study period, relied heavily on government price stabilization and acreage control programs. Prior to 1966, mandatory allotments were used, but beginning in 1966, cotton farmers were also offered a voluntary diversion program. With the Agricultural Act of 1970, voluntary set-aside payments and cotton loans were available. Under the Agricultural and Consumer Protection Act of 1973 cotton acreage restrictions were not in effect even though there were still allotments, but voluntary diversion programs and price support programs were used. Target and loan prices came into use, but no diversion payments were made in 1974 through 1977. The Food and Agriculture

\footnote{The main source of information provided in this section is McManus (1990).}
Act of 1977 continued the basic concepts of the 1973 Act with modifications. The program was voluntary. Marketing quotas and penalties were not applicable, allotments were discontinued, target and loan prices were used, voluntary diversion payments could be earned. Disaster and low yield payments are still in effect. This program was in effect for 1978 through 1981. Legislation in 1981 and 1985 for cotton included marketing loan provisions similar to the deficiency payments provisions of earlier legislation. Differences were in the amount of target and loan rates and in the method of market price calculation.

Rice: Rice has traditionally been grown in the Southwest Rice Area, and with the relaxation of the allotment programs, acreage has increased in Northeast Louisiana. Harvested rice acreage for Louisiana has varied from a low of 385,000 acres in 1983 to a high of 679,000 with an average of 525,800. State average yields have ranged from 26.5 to 45.5 hundredweight with the higher yields occurring in the Delta area.

Government programs in rice have been similar to the cotton programs. Under the allotment program which ran from 1956 through 1975, rice had two types of allotments. One for farm areas and another for producers where the allotment was tied to the producer rather than to the farm. Beginning in 1976, the rice program changed to a marketing loan concept that with some minor modifications is still in place today. There were no more acreage restrictions, but producers were eligible for deficiency payments.
Soybeans: Soybeans are produced throughout the row crop farming areas, especially in Northeast and Southwest Louisiana. Acreage has increased from 140,000 acres in 1956 to 1,828,400 in 1988 with a peak of 3,300,000 acres in 1979. No government programs providing direct payments have been used in soybeans.

Corn: Corn is produced mainly in the Central and Delta regions. It has been a relatively minor crop in Louisiana with acreage averaging around 150,000 acres. In the late 1950's, acreage included 300,000 acres, decreased in the 60's and 70's, with an increasing trend in 1980's. Yields have ranged from 21 to 118 bushels per acre.

Programs for direct government payments to corn producers began in 1961 (via Agricultural Act of 1961). An emergency feed grain program started in 1961 which provided diversion payments to those corn producers willing to put a percentage of their acreage into conserving uses. This program was extended and in effect through 1973. Under the Agricultural and Consumer Protection Act of 1973, corn was treated similarly to cotton and rice with deficiency and disaster payments for participants. With the Agricultural Act of 1977, disaster payments were still in effect for two more years and voluntary diversion payments became available. With the Food Security Acts of 1981, 1985, and 1990 deficiency, diversion, and disaster payments were continued for corn producers.

Sugarcane: Sugarcane is produced primarily in the Sugarcane area with some acreage in the Southwest and Central Areas. Average acreage have been
consistently in the 200,000 to 300,000 acre range. Yields have also been constant over time ranging from 20 to 29 tons of cane per acre.

The Sugar Act of 1948 covered Louisiana sugarcane production for the years 1956 through 1974. Each year the amount of sugar was determined that would meet U.S. requirements, and a quota was established for each domestic and foreign producing area. When established production was in excess of quotas, farm sugarcane proportionate shares (allotments) were established for all sugar farms. Payments were made to producers if they did not exceed their allotment, and the amount of payments was based upon the quantity of commercially recoverable sugar. With the expiration of the Sugar Act in 1974, producers no longer received direct payments. The Food Security Acts of 1981 and 1985 provided a price support maintained through other means that do not directly impact the producer.

3.4. An overview of direction of the cost-income ratio for five crops

Before computing and discussing the regional comparative advantage, it might be helpful to understand the issues from a common visual demonstration, namely, the plot of cost-income ratios for all of the crops against time. First, descriptive statistics (means and standard deviations) of the cost-income ratios for all crops across seven regions are considered. This is reported in Table 3.2. The minimum mean value observed is 0.53 (soybeans in Red River and Central areas)
Table 3.2 Means and standard deviations* of CIR of five major crops across the agricultural production regions in Louisiana without government (WG) and with government (G) payments, 1956-1988.

<table>
<thead>
<tr>
<th>REGIONS</th>
<th>CROPS</th>
<th>RICE</th>
<th>COTTON</th>
<th>SOYBEANS</th>
<th>CORN</th>
<th>SUGAR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>WG</td>
<td>G</td>
<td>WG</td>
<td>G</td>
<td>WG</td>
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<tr>
<td>RED RIVER</td>
<td></td>
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<td>0.65</td>
<td>0.85</td>
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<tr>
<td></td>
<td>(0.44)</td>
<td>(0.27)</td>
<td>(0.34)</td>
<td>(0.23)</td>
<td>(0.17)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>DELTA</td>
<td></td>
<td>0.64</td>
<td>0.57</td>
<td>0.78</td>
<td>0.66</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(0.23)</td>
<td>(0.29)</td>
<td>(0.19)</td>
<td>(0.17)</td>
<td>(0.17)</td>
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<tr>
<td>RIDGE</td>
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<td>0.94</td>
<td>0.78</td>
<td>0.66</td>
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<tr>
<td></td>
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<td>(0.33)</td>
<td>(0.20)</td>
<td>(0.19)</td>
<td>(0.19)</td>
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<td>0.85</td>
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<td>(0.16)</td>
<td>(0.17)</td>
<td>(0.25)</td>
<td>(0.25)</td>
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<tr>
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<tr>
<td></td>
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<td>(0.26)</td>
<td>(0.19)</td>
<td>(0.22)</td>
<td>(0.25)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>OTHERS</td>
<td></td>
<td>NA</td>
<td>NA</td>
<td>0.90</td>
<td>0.70</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.23)</td>
<td>(0.17)</td>
<td>(0.24)</td>
<td>(0.24)</td>
<td>(0.39)</td>
</tr>
</tbody>
</table>

* Standard deviations are in parenthesis; ** NA = Not available/produced; CIR = Cost-income ratio.
and the maximum value observed is 1.19 (corn in Others area). Soybeans obtained the lowest levels of CIR values (range: 0.53 - 0.73), whereas corn obtained the highest CIRs (range: 0.68 - 0.95) after direct government payment. Cropwise, rice experienced lower mean values in the Delta and the Ridge areas; cotton in the Delta area; soybeans in the Red River, Delta, and the Central areas; corn in the Sugarcane area; and, sugarcane in the Sugarcane and the SW Rice areas. These estimates, however, do not necessarily show the actual profitability of the crops since, due to asymmetry of spread through years, the most recent declines (or, increases) in cost-income ratio might be overwhelmed by very high (or, low) values of the same in initial periods. This will be clear from the time-series plot of cost-income ratio.

The plots are presented in Figures 3.2 and 3.3. For demonstration purposes, cost-income ratio (CIR) of all crops for only two (Red River and Sugarcane) of the seven regions are plotted. The plotted CIRs include direct government payments (except soybeans which did not get any direct payments within the study period). The data on cotton in the Sugarcane area and the data on sugarcane in the Red River area do not cover the whole study period due to the insignificant amount of recent production of these crops in those two respective areas.

As it is clear from the plots, the cost-income ratio of rice and cotton show an upward trend with cyclical fluctuations in both areas. Soybeans also shows a slight upward trend especially in the sugarcane area. On the other hand, sugarcane
Figure 3.2 Time-series plot of the cost-income ratio of five major crops in the Red River area, 1956-1988.
Figure 3.3 Time-series plot of the cost-income ratio of five major crops in the Sugarcane area, 1956-1988.
in the Sugarcane area shows definite recent downward trend in CIR. Corn in the Red River area also shows oscillatory damped path towards CIR much below the 1.0 level. Corn in the Sugarcane area, however, does not show any such definite trend though the cost-income ratio lies well below the 1.0 level during the most recent periods.

So far as these two areas (Red River and Sugarcane) are concerned, sugarcane and corn seem to be moving in a favorable direction with respect to CIR in spite of their high ratios in the initial (study) periods. On the other hand, rice, cotton, and soybeans, in spite of their very low CIR values initially, are found to be moving in adverse direction during the later periods. However, like the descriptive statistics, no inference regarding the comparative advantages can be made from the visual inspection of the time-path.

3.5. Estimation

To estimate the cumulative probabilities of the CIR for each of the 35 scenarios (5 crops × 7 regions), the PDF for each of them must be estimated. For estimation of unconditional probabilities, data for each CIR have been transformed according to the procedure stated in equation (3.3) above. Then suitable combination of X (here, CIR) and its polynomial and logarithmic transformations have been selected on the basis of t-values derived from the step-wise OLS regressions of Y on linear G(X). The t-values may not have any meaning for traditional statistical tests, but they give an indication of the partial contribution of
each term to the regression because the t-ratio is a monotonic transformation of the respective partial correlations. In most of the cases, the first even power ($X^2$) contributed significantly to the regression along with the first three odd powers (i.e., $X, X^3, X^5$). As even powers of $X$ would indicate skewness of the distribution, the primary conclusion from the OLS regression has been that the PDFs of CIR in most of the cases are skewed. The OLS estimates have been used as starting values for ML estimation to maximize the log-likelihood function given in equation (3.9). The estimated set of $\beta$ has been used then in equations (3.1) and (3.2) to get the estimated probabilities.

For estimation of the conditional distribution, the set of values of CIR lagged to one period (i.e, CIR$_{t-1}$) has been considered as an independent variable ($N$) affecting the PDF of CIR. The justification of using the lagged CIR as the conditional variable is given in Section 3.2. The estimation of conditional probabilities have been done in a manner similar to the unconditional one. In this case, various polynomial ($N, N^2, N^3$,...) and interaction ($NX, N^2X, N^3X,...$) terms have been included in the primary estimation. However, in none of the cases higher polynomials and the interaction terms have shown significant t-values in ML estimation. The lack of statistical significance of the interaction term implies that the shape of the PDF is not affected by $N$, whereas the statistical significance of the first polynomial (i.e, $N$) leads to the conclusion that it affects the moments of the PDF. To evaluate how it affects the probabilities, a series of ML estimation were constructed at different hypothetical values of $N$. 
3.6. Results

The probabilities are estimated for both the cases of "with" and "without" government payments. The "without" government CIRs have been calculated simply by subtracting the amount of direct government payments (a total of diversion, deficiency, and disaster payments) from total revenue. It is to be noted that "without" government in this study will always mean "without" direct government payments. In other words, "without" government CIRs do not exclude the effects of other types of government policies such as price supports and acreage allotments. As direct government payments contribute significantly in Louisiana's agricultural production process, the main focus, however, will be on "with" government case.

In Table 3.3 and Table 3.4, the estimated unconditional cumulative probabilities from the fitted PDFs have been tabulated for "without" and "with" government payments respectively. The probability estimates reveal the heterogenous survival potentiality of each crop across regions. For better understanding, three stages of estimates have been presented for three optimum CIR ($\mu^*$) levels -less than equal to 1, less than equal to 0.9, and less than equal to 0.8. Thus, concentration of probability mass being higher in the zone 0.9-0.8, or in the zone "less than equal to 0.8" reveals the high probability of profitability of a crop in a particular region. For example, corn in the Red River and the Delta area without government payment (Table 3.3) shows very poor probabilities of profitability as little probability mass (0.36 and 0.23) is left in the "less than 0.8" area. This should be compared with the corresponding figures in Table 3.4 where
Table 3.3 Estimated unconditional probabilities for selected "less than or equal to" CIR* values: by crops and by regions, Louisiana, 1956-1988 (without government payments).

<table>
<thead>
<tr>
<th>CIR LESS THAN OR EQUAL TO 1.0</th>
<th>FARMING AREAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CROPS</td>
<td>RED RIVER</td>
</tr>
<tr>
<td>RICE</td>
<td>.82</td>
</tr>
<tr>
<td>COTTON</td>
<td>.70</td>
</tr>
<tr>
<td>SOY</td>
<td>1.00</td>
</tr>
<tr>
<td>CORN</td>
<td>.71</td>
</tr>
<tr>
<td>SUGAR</td>
<td>--</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CIR LESS THAN OR EQUAL TO 0.9</th>
<th>FARMING AREAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CROPS</td>
<td>RED RIVER</td>
</tr>
<tr>
<td>RICE</td>
<td>.77</td>
</tr>
<tr>
<td>COTTON</td>
<td>.59</td>
</tr>
<tr>
<td>SOY</td>
<td>1.00</td>
</tr>
<tr>
<td>CORN</td>
<td>.54</td>
</tr>
<tr>
<td>SUGAR</td>
<td>--</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CIR LESS THAN OR EQUAL TO 0.8</th>
<th>FARMING AREAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CROPS</td>
<td>RED RIVER</td>
</tr>
<tr>
<td>RICE</td>
<td>.70</td>
</tr>
<tr>
<td>COTTON</td>
<td>.47</td>
</tr>
<tr>
<td>SOY</td>
<td>.99</td>
</tr>
<tr>
<td>CORN</td>
<td>.36</td>
</tr>
<tr>
<td>SUGAR</td>
<td>--</td>
</tr>
</tbody>
</table>

* CIR = Cost-income ratio.
Table 3.4 Estimated unconditional probabilities for selected "less than or equal to" CIR* values: by crops and by regions, Louisiana, 1956-1988 (with government payments).

<table>
<thead>
<tr>
<th>CIR LESS THAN OR EQUAL TO 1.0</th>
<th>FARMING AREAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CROPS</td>
<td>RED RIVER</td>
</tr>
<tr>
<td>RICE</td>
<td>.83</td>
</tr>
<tr>
<td>COTTON</td>
<td>.89</td>
</tr>
<tr>
<td>SOY</td>
<td>--</td>
</tr>
<tr>
<td>CORN</td>
<td>.90</td>
</tr>
<tr>
<td>SUGAR</td>
<td>--</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CIR LESS THAN OR EQUAL TO 0.9</th>
<th>FARMING AREAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CROPS</td>
<td>RED RIVER</td>
</tr>
<tr>
<td>RICE</td>
<td>.78</td>
</tr>
<tr>
<td>COTTON</td>
<td>.77</td>
</tr>
<tr>
<td>SOY</td>
<td>--</td>
</tr>
<tr>
<td>CORN</td>
<td>.78</td>
</tr>
<tr>
<td>SUGAR</td>
<td>--</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>CIR LESS THAN OR EQUAL TO 0.8</th>
<th>FARMING AREAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CROPS</td>
<td>RED RIVER</td>
</tr>
<tr>
<td>RICE</td>
<td>.71</td>
</tr>
<tr>
<td>COTTON</td>
<td>.64</td>
</tr>
<tr>
<td>SOY</td>
<td>--</td>
</tr>
<tr>
<td>CORN</td>
<td>.60</td>
</tr>
<tr>
<td>SUGAR</td>
<td>--</td>
</tr>
</tbody>
</table>

* CIR = Cost-income ratio.
estimated (with government payments) probabilities show significant improvement (0.60 and 0.53 respectively). Government payments are also important for cotton, particularly at the upper tail of the distribution. Table 3.3 and Table 3.4 classify 30 percent and 11 percent of the observations for CIR > 1.0 without and with government payments respectively in the Central area. Thus for the period 1956-88, cotton producers in this region would not have been able to cover total cash costs 1 out of 5 years had they not participated in government programs. Soybean producers experienced the most symmetric variations in cost-return data. Since there are no government payments for soybeans, the CIRs reflect purely market forces. Even without any direct payments, it experienced the highest probabilities in the Red River, Delta, and the Central areas and this mostly justifies growing concentration of soybean production in these areas. The low probabilities in sugarcane production do not exactly reflect the current concentration in the Sugarcane area as profits showed increasing trend only after 1980. The significant contribution of government payment in raising probabilities is, however, prominent in this case (from 0.57 to 0.81 for "less than or equal to one" case).

Next, the absolute advantage (AAD) indices are computed following equation (2.6) in Chapter 2 and Section 3.2 in Chapter 3. For estimation purposes in this study, \( m_i \) has been represented by expected profit per acre which is proxied by the average of last five years' (1984-1988) per acre net return. This is expected to reveal more information about profitability in recent years. The ultimate objective is to identify comparative advantage of the crops. For this, any two crops
may be selected and their CADs may be computed following equation (2.9) and compared following equation (2.10) in Chapter 2 across all or selected regions. The crop with the highest CAD in a region may be identified as having comparative advantage (or least comparative disadvantage) in that region.

To illustrate the above procedure, consider two crops: rice and corn. To compute the unconditional CAD indices for these two crops in the regions Red River, Delta, Ridge, Central, and Southwest Rice, first the AAD indices are computed by multiplying respective probabilities (given in Table 3.4) with respective net return (per acre) averaging over last five years (1984-1988). The resulting AAD index table for \( \text{Prob(CIR} \leq 1) \) is given in Table 3.5.

<table>
<thead>
<tr>
<th></th>
<th>Red River</th>
<th>Delta</th>
<th>Ridge</th>
<th>Central</th>
<th>SW Rice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rice</td>
<td>19.39</td>
<td>101.94</td>
<td>102.80</td>
<td>7.88</td>
<td>22.01</td>
</tr>
<tr>
<td>Corn</td>
<td>112.29</td>
<td>54.37</td>
<td>77.77</td>
<td>97.92</td>
<td>49.80</td>
</tr>
</tbody>
</table>

* AAD = Absolute advantage index; ** CIR = Cost-income ratio.

Next, the CADs are computed by dividing each AAD value in a column by the other AAD value in the same column (Table 3.6).
Table 3.6  Estimated CAD’s for rice and corn in five production regions of Louisiana [Prob(CIR**<l)].

<table>
<thead>
<tr>
<th></th>
<th>Red River</th>
<th>Delta</th>
<th>Ridge</th>
<th>Central</th>
<th>SW Rice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rice</td>
<td>0.173</td>
<td>1.88</td>
<td>1.32</td>
<td>0.080</td>
<td>0.442</td>
</tr>
<tr>
<td>Corn</td>
<td>5.79</td>
<td>0.533</td>
<td>0.756</td>
<td>12.43</td>
<td>2.26</td>
</tr>
</tbody>
</table>

* CAD = Comparative advantage index; ** CIR = Cost-income ratio.

Finally, comparative advantage for rice and corn are identified by the region where each of the crops has attained the highest CAD value. According to Table 3.6, the highest CAD values for rice and corn are obtained respectively in the Delta (1.88) and the Central (12.43) areas. Thus, rice has comparative advantage (with respect to corn) in the Delta area and corn has comparative advantage (with respect to rice) in the Central area.

Similar pairwise comparison can be conducted for all pairs of crops at a probability level of achieving a particular CIR value (or, hypothetical μ**). The results of pairwise comparisons for all pairs at three different probabilities (Prob(CIR≤1, 0.9, and 0.8)) are summarized in Table 3.7. Comparative advantage for each crop has been assigned as it is done in Table 3.6, i.e, in terms of the region where its CAD is the highest. In the table the labels of the columns and those of the rows have been given in terms of crops; columnwise each crop is treated as crop 1, i.e, the crop who has comparative advantage over the other in a pair. Rowwise,
Table 3.7 Pairwise unconditional comparative advantage of the major crops in the production regions of Louisiana.

<table>
<thead>
<tr>
<th>CROP 2</th>
<th>RICE</th>
<th>COTTON</th>
<th>SOYBEAN</th>
<th>CORN</th>
<th>SUGARCANE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RICE</td>
<td>--</td>
<td>Central</td>
<td>Central</td>
<td>Sugar</td>
<td>Sugar</td>
</tr>
<tr>
<td>COTTON</td>
<td>SW Rice</td>
<td>--</td>
<td>Central</td>
<td>SW Rice</td>
<td>SW Rice</td>
</tr>
<tr>
<td>SOYBEAN</td>
<td>SW Rice</td>
<td>Others</td>
<td>--</td>
<td>Ridge</td>
<td>Sugar</td>
</tr>
<tr>
<td>CORN</td>
<td>Delta</td>
<td>Others</td>
<td>Delta</td>
<td>--</td>
<td>SW Rice</td>
</tr>
<tr>
<td>SUGARCANE</td>
<td>SW Rice</td>
<td>Central</td>
<td>Central</td>
<td>Central</td>
<td>--</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prob [ CIR ≤ 0.9 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>RICE</td>
</tr>
<tr>
<td>COTTON</td>
</tr>
<tr>
<td>SOYBEAN</td>
</tr>
<tr>
<td>CORN</td>
</tr>
<tr>
<td>SUGARCANE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prob [ CIR ≤ 0.8 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>RICE</td>
</tr>
<tr>
<td>COTTON</td>
</tr>
<tr>
<td>SOYBEAN</td>
</tr>
<tr>
<td>CORN</td>
</tr>
<tr>
<td>SUGARCANE</td>
</tr>
</tbody>
</table>

CIR = Cost - income ratio.
Crop 1 = The crop which has comparative advantage over the other in a pair.
Crop 2 = The crop over which the crop 1 has comparative advantage.
SW Rice = The Southwest Rice area.
Sugar = The Sugarcane area.
each crop is treated as crop 2, i.e, the crop over which the crop 1 has comparative advantage. The name of the areas in the table indicates the area where crop 1 has comparative advantage over crop 2. For example, for Prob(CIR≤1) case, SW Rice in column 1 - row 2 indicates that rice has comparative advantage over cotton in the Southwest Rice area, whereas column 2 - row 1 indicates that cotton has comparative advantage over rice in the Central area.

Table 3.7 shows that the comparative advantage for each crop remains the same in three different scenarios (i.e, Prob (CIR ≤ 1, 0.9, 0.8)) except a few cases. Exception is found, for example, in cotton which shows change in CAD ranking from Central to Red River with respect to change in probability from "less than 1.0" to "less than 0.8". That implies that Red River offers comparative advantage when higher profitability is considered.

Analyzing the CADs in scenario 3 (i.e, Prob(CIR ≤ 0.8)), it can be argued that rice maintains its solid strength in the SW Rice area with respect to cotton, soybeans, and sugarcane. Comparative advantage over corn in the Delta area may partly explain the increase of acreage (after relaxation of allotment program) in rice in Northeast Louisiana in 1980's. Cotton shows comparative advantage mainly in the Others area (i.e, parts of north and west Louisiana) and in the Red River and the Central areas. Good performance of Soybeans in the Central area as revealed by its highest ranking in this area over two crops (rice and sugarcane) is consistent with reality as concentration of soybeans production has increased in this area significantly within the study period (from 11,400 acres in 1956 to 366,900 acres in
1956 - almost 100 percent increase per year). It is also interesting to know that, although a minor crop, corn has comparative advantage over soybeans in the Sugarcane area and competes with soybeans regarding advantage over cotton in the Southwest area. Comparative advantage of sugarcane over rice and soybeans in Sugarcane area is prominent. It is also observable that sugarcane, although concentrated mostly in the Sugarcane area seems to have good potential for profitability in Southwest Louisiana.

Next, the estimated conditional probabilities and conditional comparative advantage of the crops across regions are presented. As mentioned earlier, it is assumed that CIR of the previous period \(N = \text{CIR}_{t-1}\) affects the PDF of CIR for each crop in each area. Three scenarios have been considered: \(N = 1\), \(N = 0.9\), and \(N = 0.8\). Conditional PDFs for each crop across regions have been fitted for each case. In Figures 3.4 through 3.8 we present the fitted conditional PDF curves for the Central area.

As it is seen from Figures 3.4 - 3.8, in all cases at least the first two moments (mean and standard deviation) have been affected with different degrees for each crop. For all crops, the fitted distributions show greater skewness to the right at lower \(N\) values indicating higher probability mass in achieving less than mean CIR values (given in Table 3.2). For soybean and sugarcane, the effects on the moments are small indicating that the changes in the immediate past values do not affect the probabilities significantly. In other words, the probabilities of achieving \(\text{CIR} \leq 1\)
Figure 3.4 Fitted conditional PDF of the cost-income ratio (CIR) for rice in the Central area \([N = \text{CIR}_{t-1}]\).
Figure 3.5 Fitted conditional PDF of the cost-income ratio (CIR) for cotton in the Central area [\(N = \text{CIR}_{t+1}\)].
Figure 3.6 Fitted conditional PDF of the cost-income ratio (CIR) for soybeans in the Central area \([N = \text{CIR}_{C}].\)
Figure 3.7 Fitted conditional PDF of the cost-income ratio (CIR) for corn in the Central area \([N = \text{CIR}_{t-1}]\).
Figure 3.8 Fitted conditional PDF of the cost-income ratio (CIR) for sugarcane in the Central area \([N = CIR_{t,i}]\).
(or, equivalently 0.9 or 0.8) do not increase significantly if last year’s CIR would be lower by 10 percent or 20 percent.

Rice, cotton, and corn, however, show significant changes in the moments as N changes. For rice, interestingly, slight bi-modality appears at lower N values which is unexplainable at given level of information. The skewness also changes significantly indicating that if the last year’s CIR would be smaller (by, say, higher government payments) the probabilities of profitability would be much higher. For cotton and corn, the skewness as well as variance increased indicating again the potential responsiveness of probabilities due to exogenous shocks that would reflect in the previous year’s cost-income ratio.

To compute the conditional comparative advantage indices the same procedure as the unconditional case has been followed except that now the unconditional probabilities will be replaced by the conditional probabilities. Here it is assumed that the optimum CIR ($\mu^*$) is less than equal to 0.8. Three scenarios are considered: $N = 1.0$, $N = 0.9$, and $N = 0.8$, where $N$ is CIR$_{t-1}$. Thus, Prob(CIR $\leq$ 0.8 | $N = 1.0$) would give the probability of getting more than 20 percent net return if the net return in the last year is zero. Similarly, Prob(CIR $\leq$ 0.8 | $N = 0.9$) and Prob(CIR $\leq$ 0.8 | $N = 0.8$) would indicate the probability of getting the same percentage of return, given that the net returns in the last year are 10 percent and 20 percent respectively. These probabilities are presented in Table 3.8. The CIR data for all crops (except soybean) used here are CIR "with government payments".
Table 3.8 Estimated *conditional* Probabilities for "less than or equal to 0.8" CIR* values: by crops and by regions, Louisiana, 1956-1988 (with government payments).

<p>| Scenario 1: Prob(CIR ≤ 0.8 | CIR_{t-1} = 1) |
|---------------------------|
| <strong>FARMING AREAS</strong>         |
| <strong>CROPS</strong>                 |
| <strong>RED RIVER</strong>             |
| <strong>DELTA</strong>                 |
| <strong>RIDGE</strong>                 |
| <strong>CENTRAL</strong>               |
| <strong>SUGAR CANE</strong>            |
| <strong>SW RICE</strong>               |
| <strong>OTHERS</strong>                |
| <strong>RICE</strong>                  |
| .17                       |
| .15                       |
| .14                       |
| .21                       |
| .25                       |</p>
<table>
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<tr>
<th>.21</th>
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<tr>
<td><strong>COTTON</strong></td>
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<tr>
<td>.17</td>
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<td>.24</td>
</tr>
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<td>.22</td>
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<td>.33</td>
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<td><strong>SOY</strong></td>
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<td>.79</td>
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<tr>
<td>.48</td>
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<td>.24</td>
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<td>.24</td>
</tr>
<tr>
<td>.24</td>
</tr>
<tr>
<td>.20</td>
</tr>
<tr>
<td><strong>CORN</strong></td>
</tr>
<tr>
<td>.36</td>
</tr>
<tr>
<td>.46</td>
</tr>
<tr>
<td>.45</td>
</tr>
<tr>
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<td>.64</td>
</tr>
<tr>
<td>.41</td>
</tr>
<tr>
<td>.31</td>
</tr>
<tr>
<td><strong>SUGAR</strong></td>
</tr>
<tr>
<td>--</td>
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<td>--</td>
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<tr>
<td>--</td>
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<td>.24</td>
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<td>.26</td>
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<tr>
<td>.30</td>
</tr>
<tr>
<td>--</td>
</tr>
</tbody>
</table>

<p>| Scenario 2: Prob(CIR ≤ 0.8 | CIR_{t-1} = 0.9) |
|---------------------------|
| <strong>CROPS</strong>                 |
| <strong>RED RIVER</strong>             |
| <strong>DELTA</strong>                 |
| <strong>RIDGE</strong>                 |
| <strong>CENTRAL</strong>               |
| <strong>SUGAR CANE</strong>            |
| <strong>SW RICE</strong>               |
| <strong>OTHERS</strong>                |
| <strong>RICE</strong>                  |
| .36                       |
| .32                       |
| .32                       |
| .38                       |
| .42                       |</p>
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<td><strong>SOY</strong></td>
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<td><strong>CORN</strong></td>
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<td><strong>SUGAR</strong></td>
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<p>| Scenario 3: Prob(CIR ≤ 0.8 | CIR_{t-1} = 0.8) |
|---------------------------|
| <strong>CROPS</strong>                 |
| <strong>RED RIVER</strong>             |
| <strong>DELTA</strong>                 |
| <strong>RIDGE</strong>                 |
| <strong>CENTRAL</strong>               |
| <strong>SUGAR CANE</strong>            |
| <strong>SW RICE</strong>               |
| <strong>OTHERS</strong>                |
| <strong>RICE</strong>                  |
| .60                       |
| .56                       |
| .56                       |
| .58                       |
| .57                       |</p>
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<td><strong>COTTON</strong></td>
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<td><strong>SUGAR</strong></td>
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* CIR = Cost-income ratio.
Less flexibility in the conditional PDFs of sugarcane and soybean are prominent in the estimated probabilities. For example, the probabilities for \( N = 1 \), \( N = 0.9 \), and \( N = 0.8 \) for sugarcane in Central area are 0.24, 0.29, and 0.36 respectively. The percent changes in probabilities for 0.1 reduction in \( N \) in this case are approximately 21 percent (from \( N = 1 \) to \( N = 0.9 \)) and 24 percent (from \( N = 0.9 \) to \( N = 0.8 \)). On the contrary, for rice in the Central area, the percent changes for the same reduction in \( N \) are 81 percent and 53 percent respectively. Cotton also shows the same kind of high responsiveness. The corresponding percent changes for cotton in the same area are 82 percent and 65 percent respectively. It is also to be noted that variation of probabilities across regions are low for rice, cotton, and sugarcane and high for corn and soybean. This indicates symmetry of absolute profitability for rice, cotton, and sugarcane across regions and asymmetry of the same for corn and soybean across regions.

Finally, the placements of crops according to conditional comparative advantages are presented in Table 3.9. The procedure is the same as presented in the unconditional case. First, conditional AADs are computed by multiplying estimated probabilities with average of net return of last five years (1984-1988). Then, as in Table 3.6, CADs are computed. Finally, comparative advantage for a crop has been identified in terms of a region where the calculated CAD (for the crop) is the highest. The results in Table 3.9 are computed on the basis of two restricted assumptions for all crops: (i) \( N = \text{CIR}_{t-1} = 1 \) and \( = 0.8 \); and (ii) \( \mu \leq 0.8 \). That is, the regions in the table show the comparative advantage of crop 1 over
crop 2 in a region assuming that both crops broke even in the last period and both firms have the target to maximize \( \Pr(\text{CIR} \leq 0.8) \). Three points should be observed in this context. First, CADs may change if assumption (i) is dropped, i.e., if we compare, for example, rice in scenario 1 (i.e., \( \text{CIR}_{t-1} = 1 \)) with corn in scenario 3 (i.e., \( \text{CIR}_{t-1} = 0.8 \)). Though such a cross-scenario comparison is more realistic, this is not considered here for the sake of simplicity. Second, even if it is assumed that \( \text{CIR}_{t-1} \) is same for all crops, the CAD rankings may change if much higher and lower values of \( \text{CIR}_{t-1} \) are also considered. Thus, to get more definite information regarding the rankings and impacts of declining (or, increasing) CIR in the previous period on them, the CADs should be computed at more alternative scenarios. Third, as the computed CADs are based on hypothetical \( \text{CIR}_{t-1} \) values, the results are more useful in understanding the comparative static derived in Chapter 2 (Section 2.4.2) than the present underlying reality. In other words, the results should be explained in terms of hypothetical "if .. then" condition, e.g., what will be the impact on comparative advantage of a crop in an area if government support increases by certain percentage?

In Table 3.9, two scenarios are compared. (i) Scenario 1: \( \text{CIR}_{t-1} = 1.0, \mu^{**} \leq 0.8 \), and (ii) Scenario 2: \( \text{CIR}_{t-1} = 0.8, \mu^{**} \leq 0.8 \). The table shows more or less same comparative advantage picture as in unconditional case. However, change of \( \text{CIR}_{t-1} \) from 1.0 to 0.8 makes several differences. For example, cotton is comparatively advantageous than rice in the Red River area in the first scenario. But, it becomes advantageous in the Central area when \( \text{CIR}_{t-1} \) is reduced to 0.8.
Table 3.9  Pairwise *conditional* comparative advantage of the major crops in the production regions of Louisiana.

<table>
<thead>
<tr>
<th>CROP 2</th>
<th>CROP 1</th>
<th>CROP 1</th>
<th>CROP 1</th>
<th>CROP 1</th>
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<tr>
<td></td>
<td>RICE</td>
<td>COTTON</td>
<td>SOYBEAN</td>
<td>CORN</td>
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<tr>
<td>RICE</td>
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<td>Central</td>
<td>Central</td>
<td>Sugar</td>
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<tr>
<td>COTTON</td>
<td>SW Rice</td>
<td>--</td>
<td>Central</td>
<td>SW Rice</td>
</tr>
<tr>
<td>SOYBEAN</td>
<td>SW Rice</td>
<td>Others</td>
<td>--</td>
<td>Ridge</td>
</tr>
<tr>
<td>CORN</td>
<td>Delta</td>
<td>Others</td>
<td>Delta</td>
<td>--</td>
</tr>
<tr>
<td>SUGAR CANE</td>
<td>SW Rice</td>
<td>Central</td>
<td>Central</td>
<td>Central</td>
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</table>

Scenario 1: Prob \( \text{CIR} \leq 0.8 \mid \text{CIR}_{t-1} = 1.0 \)

Scenario 2: Prob \( \text{CIR} \leq 0.8 \mid \text{CIR}_{t-1} = 0.8 \)

CIR = Cost - income ratio.
Crop 1 = The crop which has comparative advantage over the other in a pair.
Crop 2 = The crop over which the crop 1 has comparative advantage.
SW Rice = The Southwest Rice area.
Sugar = The Sugarcane area.
This implies that if government support had been increased last year at the same rate for cotton and rice, the producers in the Central area would have been in relatively more advantageous position than the producers in the Red River area (which enjoys advantage without the increase). Similarly, the Ridge area replaces the SW Rice and the Sugarcane area from corn’s comparative advantage (over cotton and soybeans respectively) if, ceteris paribus, cost-income ratio drops by 20 percent. If this drop is caused by, say, government subsidy, then, the result derived in Section 2.4.2(D) (Chapter 2) is substantiated, i.e, \( \mu'' \) will decrease, and thus the probability of achieving "without subsidy" \( \mu'' \) (in this case, \( \mu'' = 1 \) ) will increase. The region with higher cumulative probability of achieving the "without subsidy" CIR will accordingly have the comparative advantage (in this case, the Central area against the Red River area for cotton over rice). Similar explanations hold for comparison between the SW Rice and the Ridge area on the comparative advantage of corn over cotton.

3.7. Summary

In this chapter the empirical issues regarding the evaluation of regional profitability of crops have been discussed. A simple methodological structure has been built to provide information about the comparative advantage of crops across regions on the basis of historical cost and income data information. Unconditional comparative advantage indices have been derived to provide an empirical demonstration of the methodology. As the concept of comparative advantage is
stochastic and dependent on some extraneous (market and non-market) shocks due to similar properties of its indicator (i.e., cost-income ratio), the methodology has been extended to discuss conditional information about comparative advantages based on the theoretical structure built in Chapter 2. The results derived in the above particular application, however, should not be taken as the perfect basis to assess reality due to three reasons: (1) the study period is 1956-1988; thus the data does not include the information about cost and income of the last three years (1989-1991) which may be vital for assessment of current profitability; (2) the aggregation and use of imputed or projected values in some cases may generate significant measurement error problems; this may cast doubt about the desirable properties of the estimates; and, (3) as it was found, a good analysis of conditional comparative advantages requires computation of CADs over a long series of optimum cost-income ratio at large numbers of alternative scenarios. The present study is restricted in this respect.

The analysis of comparative advantage is related to other aspects of the resource allocation—efficiency in production. The efficiency issue should also be dealt with and its linkage to comparative advantage analysis should be explored. However, the issue of efficiency is not very meaningful in the context of regional analysis. It should be addressed by the evaluation of specific firms' performances. For this reason, the analysis needs to be extended to define firm-specific inefficiencies and comparative advantage. The next two chapters focus on some theoretical and empirical issues regarding such an evaluation method.
CHAPTER 4

FIRM-SPECIFIC INEFFICIENCY IN PRODUCTION: METHODOLOGICAL AND EMPIRICAL ISSUES

4.1. Introduction

The subject matter of this chapter is specification and estimation of inefficiency in production. As mentioned in Chapter 1, the theoretical background of efficiency issues in production is well established where, following Farrell, total efficiency has been shown to be decomposed into two parts: technical and allocative efficiencies. Before discussing methods of estimating inefficiency one can ask: why is estimating it important, how useful is it, and who will benefit by it? In other words, what are the motivations behind the analysis of production efficiency in the context of the evaluation of firm’s performance and its profitability? Answers to these questions depend on the behavioral assumption used to characterize the producers. If the objective of all firms in an industry is to minimize cost, one might be interested in knowing the magnitude of such inefficiencies (if they exist), since it is costly to the producers as well as to society. For profit maximizing firms, the study of inefficiency may not have direct implications since under perfect competition (in the long run) only the efficient producers survive. In that case, however, the study of inefficiency is justified on the following grounds:
1) It is important to know why a particular firm is unable to survive. Is it because of technical or allocative inefficiency? If it is underutilization of inputs (technical inefficiency), which inputs are mostly responsible?

2) To analyze the relationship between the size of firm and inefficiency. In other words, it may be interesting to know whether observed increasing tendency of firm-sizes is due to achievement of more efficiency (due to expansion) or due to increasing potentiality of profitability.

3) To characterize the utilization and allocation of resources in a given technology when efficiencies are known or unknown to the firms.

All these have specific policy implications in the context of competitive market conditions. For example, if it is known that correcting inefficiencies (which are under firm's control) will lead to better potentiality to survive for a particular size-class of firms, estimated inefficiencies may help in designing specific policies to correct these inefficiencies through direct or indirect government intervention.

From a more general perspective, the economic function of the firm is to bid resources away from alternative uses and to allocate them in the production of a certain output level. As a result of such resource transfer, aggregate output may be increased or decreased. If inefficiency exists, an increase in output can be achieved by reallocating resources to more efficient uses by adopting proper policy. "Indeed, for no other reason, the importance of such issues and the magnitude of the social and political sensitivities they arouse require that the measurement of
efficiency should be theoretically valid and subject to unambiguous interpretations." (Yotopolous and Nugent (1976, p.87).

Førstund and Hjalmarsson (1974) distinguished inefficiency at the macro, industry and micro levels. Inefficiency at the macro level compares the economic performance of an observed allocation of resources with the result of some ideal allocation (e.g., a Pareto-optimal allocation). At the industry level inefficiency implies potential for an increase in industry output by employing resources in firms using best practice technology. Inefficiency at the micro level concentrates on the utilization of resources within the firm. Here the objectives of a firm have to be specified when characterizing its inefficiency.

In this study attention is focused on cost (or, economic) inefficiency at the micro level since in the real world there is hardly any single industry decision-maker who attempts to maximize profit or minimize costs to allocate resources optimally on the basis of industry production function. Specifically, the estimation methods are applied to data consisting of sample of sugarcane firms in Louisiana over a period of four years (1986-1989). There are two motivations for the selection of sugarcane firms to demonstrate the inefficiency issues: (1) as it is found in regional comparative advantage analysis of crop production in Louisiana, sugarcane holds an important position in the comparative advantage rankings of crops in some important production regions of the state; therefore, following the proposed evaluation methodology, it is important to know whether regional resource allocation in favor of sugarcane is commensurate with optimum resource allocation
within the industry; and the (2) the increasing trend of concentration of production and consequent disappearance of small firms.

The increasing trend in concentration is evident from the decline in the number of firms and increasing market share of larger firms during last 50 years. Between 1939 and 1988 the number of sugarcane growers in the state dropped from 10,918 to 733, an average of 272 growers disappearing annually. The average firm size increased also dramatically within the same time period. In the twenty years prior to 1956, average firm size increased from 28 acres per firm to 60 acres per firm. However, in the period 1956-74, the average firm size increased up to 281 acres per firm. The period 1974-1988 have experienced some years of declining firm size. Yet, the average sugarcane firm size has been estimated to be approximately 380 acres. In other words, in the last 30 years prior to 1988, the average firm size has increased almost sixfold. This has been accompanied by heavy skewness in the distribution toward the lower end of the range. Slightly less than a quarter of all sugarcane firms are less than 150 acres in size. These firms accounted for approximately 3.6 percent of all sugarcane harvested in Louisiana during 1988. Three-quarters of the firms are under 675 acres in size. However, these firms accounted for only 41 percent of all cane harvested. Finally, 90 percent of all growers have firms of less than 1125 acres. These firms accounted for 66 percent of sugarcane harvested. The remaining 10 percent of the population represents the largest 58 plantations, and 34 percent of total cane harvested.
(Chapman (1991)). All these facts lead to the simple conclusion that the production of sugarcane has been concentrated in the hands of a few large growers.

The evidence in favor of concentration raises questions which need to be addressed for the evaluation of intra-industry performance. For example, is it efficiency in resource use that leads the large firms to be economically stronger than smaller firms? If that is true, then which component (technical or allocative) contributed most? What are the sources of inefficiency? To answer these questions, the efficiency level (total and components) for each firm must be quantified or estimated. To our knowledge, this is the first attempt to evaluate firm-specific efficiency of sugarcane industry in Louisiana via estimation of frontier (i.e, no inefficiency) production and cost functions. The generated knowledge will help to identify the relative efficiency structure of each firm. As inefficiency always means extra cost to the society, it will be useful to policy makers as well as the farmers to determine the cost-benefit trade-off associated with neglecting the existing inefficiencies.

In the next section (Section 4.2) the theoretical concepts about production efficiencies are sketched in brief. In Section 4.3, a critical review of efficiency estimation procedures is presented with main focus on stochastic frontier function. This is followed by a discussion on procedures (to be followed in this study) in Section 4.4. In Section 4.5 the empirical model and data are discussed. Section 4.6 discusses the results from the empirical application. The chapter is concluded by a summary given in Section 4.7.
4.2. Inefficiency in production: a review of theoretical concepts

Consider a firm employing \( n \) inputs \( x = (x_1, \ldots, x_n) \), available at fixed prices \( w = (w_1, \ldots, w_n) \) to produce a single output \( y \) that can be sold at fixed price \( p \). The production function which shows the maximum output obtainable from various input vectors is an efficient transformation of inputs into output. This is called frontier production function opposed to the observed production function which lies below the former if inputs are not transformed efficiently. The efficient production technology (for given level of output) can be alternatively represented by the cost function, \( C(w,y) = \min \{ w'x \mid f(x) \geq y, x \geq 0 \} \), which shows the minimum cost of producing the given output level \( y \) at input prices \( w \). A third way of representing production efficiency is given by the profit function \( \pi(p,w) = \max \{ py - w'x \mid f(x) \geq y, x \geq 0, y \geq 0 \} \) which shows the maximum profit available at input prices \( w \) and output price \( p \).

If a firm is observed at a production plan \( (y^0, x^0) \), such a plan is technically efficient if \( y^0 = f(x^0) \) and technically inefficient if \( y^0 < f(x^0) \). Since technical inefficiency arises due to excessive input use, which is costly, \( C(w,y^0) \leq w'x^0 \) and the cost of technical inefficiency is \( w'x^0 - C(w,y^0) \). Again, since cost minimization is a necessary condition for profit maximization, \( \pi(p,w) \leq (py^0 - w'x^0) \), and the cost of technical inefficiency (in terms of foregone profit) is \( py^0 - w'x^0 \). Technical inefficiency can also be measured by the ratio of actual and frontier values.

Technical inefficiency is not the only source of inefficiency. For a cost-minimizing firm, allocative inefficiency arises when inputs are used in the wrong
proportions. The production plan \((y^0, x^0)\) is allocatively inefficient if \(\frac{f_i(x^0)}{f_j(x^0)} \neq \frac{w_i}{w_j}\), where \(f_i(x^0)\) is the marginal product of input \(x_i\).

A combination of technical and allocative inefficiency is necessary but not sufficient for a profit maximizing firm to attain \(\pi(p, w)\). This is so because the firm could still be scale inefficient - which occurs when the firm fails to produce an output level where \(p = C_y(w, y^0)\), \(C_y(w, y^0)\) being the marginal cost at output \(y^0\). Thus \((py^0 - wx^0) = \pi(p, y^0)\) if and only if the firm is technically, allocatively, and scale efficient (see Førsund et al.(1980)).

The preceding analysis points out that at a given output the cost (or, economic) inefficiency of a firm is composed of both technical and allocative inefficiencies. In other words, a cost efficient firm not only utilizes the maximum potential derivable from the existing technology, but also adjusts and allocates its resources according to the changes in the economic forces in the most efficient way.

4.3. Inefficiency estimation - a brief review of stochastic frontier models

Given the theoretical treatment of technical and allocative inefficiency - the task of the econometrician is to recast it for purposes of estimating cost, profit and production frontiers. Among various ways for specifying the frontier, the stochastic frontier models have been gaining more popularity among researchers due to its capability to deal with the inherent randomness of production or cost (not under the control of the firm) as well as the inefficiencies (under the control of the firm). In the deterministic model, on the other hand, the variation in firm performance
relative to the frontier is attributed to inefficiency, thereby ignoring the possibility of variation due to factors not in the control of any firm such as weather variation, machine breakdown, variation in supply of inputs, etc. These are usually referred to as statistical noise (v) and are to be separated from controllable factors (inefficiency) such as managerial inefficiency, quantity and vintage of capital equipment, labor quality, etc. Lumping these together (as in deterministic frontiers) and labeling it as inefficiency is not appropriate. This is especially true for agricultural production where the contribution of uncontrollable factors counts significantly in the yield or cost variation. Thus, the appropriate way of modeling requires that statistical noise be separated from the (controllable) inefficiency factors. That is exactly what is done in stochastic frontier models.

4.3.1. The econometric model of stochastic frontier production function

Consider a stochastic agricultural production function

\[ y_i = F(x_i, B) e^{\epsilon_i} \]

where \( y_i \) is output for the i-th firm, \( x_i \) is a vector of inputs, \( B \) is a vector of parameters, \( \epsilon_i \) is an error term. The stochastic frontier model as specified by Aigner, Lovell, and Schmidt (1977), and Meeusen and Van Den Broeck (1977) postulates that the error term \( \epsilon_i \) is composed of two independent error terms:

\[ \epsilon_i = v_i - u_i \]
where, $v_i$ is the general statistical noise that captures random exogenous shocks not
in the control of any firm, and $u_i$ ($\geq 0$) is the technical inefficiency that may arise
from various possible sources, such as local labor quality, embodied technical
progress, managerial inertia or ignorance, age composition of capital stock, etc. It
is non-positive since by assumption output cannot exceed the frontier output given
by equation (4.1) with $u_i = 0$. Obviously, the deterministic statistical frontier model
is a special case of the stochastic frontier model, in which $v_i = 0$.

The model is complete with the distributional assumptions about the error
components. Since $v_i$ represents uncontrollable random events, it is assumed to be
normally distributed, i.e,

$$v_i \sim N(0, \sigma_v^2)$$

The important problem, however, is to specify an appropriate one-side distribution
for $u_i$. A number of distributions have been suggested or assumed in the literature
for this one-side error. The most commonly assumed distribution (by Aigner et
al.(1977)) has been half-normal, i.e,

$$u_i \sim |N(0, \sigma_u^2)|$$

Several other types of distribution for $u_i$ have been suggested; for example, the
exponential (Meeusen and van den Broeck (1977)), the truncated normal
(Stevenson (1980)), the Pearson family of distributions (Lee (1983)), and gamma
(Greene (1990)). Since the distributional assumption plays a crucial role, especially
in the context of estimating inefficiency, it is important to consider various alternative distributions to see which fits best.

Assuming \( u_i \) to be half-normal, the joint density function of \( \varepsilon_i = v_i - u_i \) as well as the likelihood function of equation (4.1) can be derived. This has been done by Aigner et al. (1977, p.26-27). After estimation of the coefficient and individual variance parameters (i.e, \( \sigma_v^2 \) and \( \sigma_u^2 \)) by maximum likelihood method, one can calculate individual firm measures of technical efficiency as demonstrated by Jondrow et al. (1982). Specifically, they are the expected values of \( u_i \) conditional on \( \varepsilon_i \), that is:

\[
E[u_i|\varepsilon_i] = \frac{\sigma_u \sigma_v}{\sigma} \left[ \frac{f(\varepsilon_i \lambda/\sigma) - \varepsilon_i \lambda}{1 - F(\varepsilon_i \lambda/\sigma)} \right]
\]

where \( \sigma^2 = \sigma_v^2 + \sigma_u^2 \), \( \lambda = \sigma_u / \sigma_v \), \( f(.) \) and \( F(.) \) are the standard normal density function and the standard normal distribution function evaluated at \( (\varepsilon_i \lambda / \sigma) \).

Measures of technical efficiency can then be measured as:

\[
TE_i = e^{-E[u_i|\varepsilon_i]}
\]

so that \( 0 \leq TE_i \leq 1 \).

A firm is said to be allocatively efficient if it equates the marginal rate of substitution between each pair of inputs with the input price ratio. This is modelled by multiplying a random term \( \exp(\varepsilon_j) \) \((j=2,3,...,n)\) to each of the first order conditions of profit maximization or cost minimization. That is,
\[ (4.7) \quad \frac{MP_{x_1}}{MP_{x_1}} = \left( \frac{w_j}{w_1} \right) \exp(e_j), \quad j = 2, 3, ..., n. \]

or, alternatively,

\[ (4.8) \quad \ln(MP_{x_j}) - \ln(MP_{x_1}) = \ln(w_j) - \ln(w_1) + e_j \]

where \( e_j = 0 \) (or, \( \exp(e_j) = 1 \)) implies no allocative inefficiency. Note that unlike technical inefficiency \( (u_j) \), \( e_j \) can be either positive or negative (i.e., \( \exp(e_j) > 1 \) and \( \exp(e_j) < 1 \) both imply allocative inefficiency).

To estimate both technical and allocative efficiencies in a particular optimizing framework, the traditional procedure is to estimate \( n \) equations in (4.1) and (4.7) together as a system via maximum likelihood method. This is a consistent estimation procedure; however, this requires a reliable and exhaustive set of data (especially data on input prices) and high computational efficiency\(^1\). In agricultural economics, where the firm level data are limited in quality, a simpler approach may be followed. One such procedure (followed in this study) will be elaborated in the procedure section.

---

\(^1\) The first one may be one of the reasons why researchers generally apply a systems approach on the data sets of some particular established manufacturing sectors to estimate technical and allocative efficiencies. For example, production and cost data on Metal industries, Class I railroads, steam electric generating plants, etc. have been used recurrently by Kumbhakar (1987,1988,1989), Schimdt(1979,1980,1984), Greene (1980,1990), and others.
4.3.2. Stochastic frontier using panel data

The stochastic production frontier model described in equations (4.1) and (4.2) has some significant shortcomings. Perhaps its most obvious defect is that both the estimation of the model and the subsequent separation of efficiency from random shocks and statistical noise depends on specific and somewhat arbitrary assumptions about the distributions of $v$ and $u$. The evidence of technical inefficiency is reflected in the skewness of the residuals (due to one-sided error term); thus, the skewness in $v$ may be wrongly construed as a symptom of inefficiency. There are other problems with the above formulation. One is that the technical efficiency of a firm can be estimated (by equation (4.6)) but not consistently. As discussed by Jondrow et al (1982), this is because the variability of the conditional distribution of $u$ given $\epsilon$ is independent of sample size (i.e, $\epsilon$ contains only imperfect information about $u$). Another is the implicit assumption that inefficiency is uncorrelated with any of the regressors in the production (or, cost) function. This may seem too naive in some applications; for example, this assumption will be unrealistic if the firms are aware of their inefficiencies to some extent.

All of the above problems are potentially avoidable if one has panel data. A panel, or longitudinal, data set is one that follows a given sample of individuals over time, i.e, say, T observations on each of N firms. Schimdt and Sickles (1984) suggest an alternative way of measuring technical efficiencies from a production function (or, cost efficiency from a cost function) by using panel data. The model
is nothing but an extension of the model considered in equations (4.1) and (4.2). Specifically, in this case, the production function is

\[
y_{i,t} = F(x_{i,t}, B) e^{v_t - u_t} \\
u_i \geq 0; \ i = 1, \ldots, N; \ t = 1, 2, \ldots, T
\]

where subscript \( t \) and \( i \) indicate references to time and firms respectively. Note that the inefficiency term, \( u_i \), has no time subscript; that implies inefficiency is assumed to remain constant over time (but varies across firms). This is a necessary assumption to estimate such a model (See Schimdt and Sickles (1984), p-368). Recent research, however, is concentrated on specifying a general model by which the time-variance property of technical inefficiency can be included in the model (see, for example, Cornwell et al.(1990), Kumbhakar (1990), Battese and Coelli (1991)). Although no general method has been established to incorporate all time-variance properties of inefficiency, it is preferred to use a relatively general model based on the current state of the art. For this reason, a general model proposed by Battese and Coelli (1991) will be followed in this study. The model and estimation procedure are discussed in the next section.

4.4. Procedure

Let us consider a Cobb-Douglas stochastic production function

\[
\ln y_{i,t} = \beta_0 + \sum_{j=1}^{K} \beta_j \ln x_{i,j,t} + v_{i,t} - u_{i,t}, \quad u_{i,t} \geq 0
\]
where \( i \) indexes firms and \( t \) indexes time periods, \( y \) is the output, \( x_j \) is the \( j \)-th input \((j = 1,2,...K)\). Note that this is a more general model as inefficiency has been allowed to vary with respect to time (i.e., time subscript has been added to \( u_i \)).

Assuming that allocative inefficiencies exist, the first order conditions for cost minimization (or, profit maximization) can be specified as

\[
(4.11) \quad \ln x_{i1}, - \ln x_{iJ} = \ln\left(\frac{w_p \beta_1}{w_j \beta_j}\right) + e_j, \quad j = 2,...,K
\]

where \( w_p, w_2,..., w_J \) are the prices of inputs. The parameters of the model (4.10) and (4.11) (including the moments of the disturbances) can be estimated consistently by applying maximum likelihood method in the \( K \)-equations system (4.10) and (4.11). Alternatively, one can derive the input demand functions on the basis of optimizing framework and derive the cost function\(^2\) as follows (see Schmidt and Lovell (1979), p-355).

\[
(4.12) \quad \ln C = k + \frac{1}{r} \ln y + \sum_{j=1}^{K} \frac{\beta_j}{r} \ln w_j - \frac{1}{r}(v - u) + (E - \ln r)
\]

where

\[
(4.13) \quad r = \sum_{j=1}^{K} \beta_j, \quad k = \ln r - \frac{1}{r} \beta_0 - \frac{1}{r} \ln \left[ \prod_{j=1}^{K} \beta_j^{\beta_j} \right]
\]

and

\(^2\) The time and firm subscripts have been dropped for the sake of convenience.
(4.14) \[ E = \sum_{j=2}^{K} \frac{\beta_j}{r} e_j + \ln \left[ \beta_1 + \sum_{j=2}^{K} \beta_j \exp(-e_j) \right] \]

It is to be noted that the allocative (i.e, \( e_j \)) as well as the technical (i.e, \( u \)) inefficiency terms appear in the cost function indicating that cost inefficiency is composed of both technical and allocative inefficiencies. Also note that the underlying production technology can also be identified from the dual function since \( \beta \)'s appear as the parameters in the cost function. Thus, \( \beta \)'s can be estimated consistently if we assume that output is exogenous. However, to estimate technical and allocative inefficiencies separately more information (equations) are needed. Following Greene (1980), or Melfi (1984) one may apply Shepherd's lemma to derive the share equations (or, input demand functions). The problem, however, is that Shepherd's lemma cannot be applied to get the correct share equations if allocative inefficiency exists (see Kumbhakar (1986, p-47)). In that case, input demand functions should be derived directly from the production function as in equation (4.11)\(^3\)

The procedures discussed above, as mentioned earlier, require high data quality of prices and computational resource. A non-conventional but less complicated way to deal with the situation is the following. First, estimate technical efficiencies directly from the production function given in equation (4.10); second,

---

\(^3\) This works nicely for Cobb-Douglas functions as the dual cost functions and direct input demand functions are easily tractable. For more flexible but complicated form (such as Translog functions) this creates problem since production technologies are not tractable from such cost functions.
compute the allocatively efficient share equations from analytically derived dual cost frontier from the production function given in equation (4.10); third, compute the allocatively inefficient share equations directly from the input demand functions; and fourth, compute allocative inefficiency from the results in step (2) and step (3). Step 1 deals with the estimation of technical efficiency through panel data. Steps 2 through 4 are about the estimation of allocative (in)efficiency and as such do not require the panel nature of the data. The steps are discussed below in detail.

4.4.1. Technical efficiency

In the context of panel data the estimation of technical efficiencies can be approached from three different procedures: (1) Fixed effect model (FE); (2) Random effect model without distributional assumption (REGLS); and (3) Random effect model with distributional assumptions (REML). While the first two are straightforward application of standard pooled data estimation procedures, the last one is purely in congruence with the spirit of stochastic frontier estimation methodology. Model (3) can be categorized on the basis of (i) specific distributional assumptions about the one-sided error term (i.e, truncated or half normal), and (ii) time-invariance of inefficiency (i.e, whether or not $u_{it} = u_{is} = u_i$). To specify the categories in terms of model restrictions, let us consider the general model given in equation (4.10). To include all types of categories, following Battese and Coelli (1991) it is assumed that
\begin{equation}
(4.15) \quad v_{it} \sim \text{i.i.d.} (0, \sigma^2_v)
\end{equation}

and

\begin{equation}
(4.16) \quad u_{it} = u_t e^{-\eta(t - r)}
\end{equation}

where $\eta$ is an unknown parameter and $u_t$ are i.i.d. positive truncations of the $N(\mu, \sigma_u^2)$ distribution. Finally, define the parameter $\lambda = \sigma_u / \sigma_v$.

Note that in the above model imposing restrictions on $u_{it}$ and the parameters $\eta$, $\mu$, and $\lambda$ leads to specification of various restricted models. For example, restricting $\eta = 0$ implies that inefficiency is time-invariant. Similarly, $\mu = 0$ implies that the distribution is half-normal and $\lambda = 0$ implies that inefficiency is non-stochastic. The restriction $u_{it} = 0$ implies no inefficiency. The restrictions and the resulting models are summarized in Table 4.1. As it is shown in Table 4.1, imposition of various type of restrictions leads to five types of models (excluding OLS). The models are specified below. Since the fixed (FE) and random effect models without distributional assumption (REGLS) are fully documented (see Judge et al. (1988, pp 468-491), Schmidt and Sickles (1984), pp 368-369), we will just sketch them here.

Model I. Fixed effect model ["Within" estimator].

Consider the model in equation (4.10). The inputs are assumed to be endogenous; however, following the arguments of Zellner, Kmenta, and Dreze
Table 4.1 Specification of frontier models for inefficiency estimation.

<table>
<thead>
<tr>
<th>Models</th>
<th>Restrictions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \eta, \mu, \lambda )</td>
<td></td>
</tr>
<tr>
<td>O.L.S</td>
<td>( \eta = 0, \mu = 0, \lambda = 0 )</td>
<td>( u_{it} = 0 ) No Inefficiency</td>
</tr>
<tr>
<td>FE (Model I)</td>
<td>( \eta = 0, \mu = 0, \lambda = 0 )</td>
<td>( u_{it} = u_{it}(\text{fixed}) ) Inefficiency is time-invariant and fixed for each firm.</td>
</tr>
<tr>
<td>REGLS (Model II)</td>
<td>( \eta = 0, \mu = 0 )</td>
<td>( u_{it} = u_{it}(\text{random}) ) Inefficiency is time-invariant but random for each firm.</td>
</tr>
<tr>
<td>REML_{TINV} (Model III)</td>
<td>( \eta = 0, \mu = 0 )</td>
<td>( u_{it} = u_{it} \sim \text{H.N} ) Inefficiency is time-invariant and distributed as half-normal.</td>
</tr>
<tr>
<td>REML_{TINV} (Model IV)</td>
<td>( \eta = 0 )</td>
<td>( u_{it} = u_{it} \sim \text{T.N} ) Inefficiency is time-invariant and distributed as truncated normal.</td>
</tr>
<tr>
<td>REML_{TV} (Model V)</td>
<td>( \eta = 0 )</td>
<td>( u_{it} \sim \text{T.N} ) Inefficiency is time-invariant and distributed as truncated normal.</td>
</tr>
</tbody>
</table>

\( \mu \) = Distribution parameter; \( \eta \) = time-variance parameter; \( \lambda \) = inefficiency parameter

\( \text{H.N} \) = Half-normal; \( \text{T.N} \) = truncated normal.
(1966) the resulting inconsistency in estimation (due to endogenousness of regressors) can be avoided if one assumes the optimization goal as maximization of expected profit. It is also assumed that the inefficiency \((u_i)\) stays with the firm over time (i.e, time invariant) but it may vary across firms. As \(u_i \geq 0\), it is assumed that \(u_i \sim iid(\mu, \sigma_u^2)\) and independent of the \(v_{it}\). In addition, \(u_i\) may or may not be assumed to be correlated with the regressors. Note that the model differs from the stochastic frontier formulation in imposing no particular distributional form on \(u_i\).

The model in equation (4.10) can be reformulated in the following way:

\[
(4.17) \quad \ln y_{it} = \beta_i + \sum_{j=1}^{K} \beta_j \ln x_{ijt} + v_{it}
\]

where \(\beta_i = \beta_0 - u_i\). This transformation makes the model acceptable for estimation through either a standard fixed or a random effect model.

In the fixed effect model, inefficiency is treated as a fixed effect, so that inefficiency is regarded as being entirely systematic. The coefficients of the model are estimated by the following steps:

1. Estimate \(\beta\)'s by running regression (with no constant term) on

\[
(4.18) \quad (\ln y_{it} - \bar{\ln y_i}) = \sum_{j=1}^{K} \beta_j (\ln x_{ijt} - \bar{\ln x_{ij}}) + v_{it} - \bar{v}_i
\]

2. Estimate \(\beta_i\) with

\[
(4.19) \quad \hat{\beta}_i = \bar{\ln y_i} - \sum_{j=1}^{K} \hat{\beta}_j \bar{\ln x_{ij}}
\]
To separate out the efficiency term (i.e., \( u_i \)) from the overall constant (i.e., \( \beta_0 \)), simply define

\[
\hat{\beta}_0 = \max(\hat{\beta}_i)
\]

and then,

\[
\hat{u}_i = \hat{\beta}_0 - \hat{\beta}_i
\]

Finally, given the logarithmic specification of the production frontier, an index of efficiency can be calculated as

\[
TE_i = 100 \ e^{-\hat{u}_i}
\]

This amounts to treating the most efficient firm in the sample as 100 percent efficient. And as Schmidt and Sickles (1984) point out, this will be true as \( N \to \infty \). However, the consistency of the estimator of \( \beta_i \) requires \( T \to \infty \).

Model II. Time invariant random effect model without distributional assumption

[GLS estimator]

Suppose that the inefficiency (i.e., \( u_i \)) is treated as random and uncorrelated with the regressors. In this case, the appropriate estimator (if no assumption is made about the distribution about \( u_i \)) is the generalized least square (GLS) estimator. The GLS estimator is essentially a weighted average of the within and "between" estimator; the latter being obtained by estimating the model on data
expressed in terms of means over time. The GLS weights are constructed from the covariance matrix$^4$ which is a function of $\sigma_y^2$ and $\sigma_u^2$.

The choice between "Fixed effect" and "Random effect" model is, in practice, constrained by circumstances. In particular, it is not possible to treat inefficiency as a fixed effect if the regressors contain variables that remain constant over time (Schmidt and Sickles (1984), p-369). The reason for this is that the within transformation "sweeps out" not only firm-specific inefficiency effects but also time-invariant firm-specific variables. GLS is also more efficient when T is small in size. The statistical test to select the appropriate model between these two has been discussed in the next section.

Model III and IV. Time invariant random effect models with distributional assumptions [ ML estimator].

If it is assumed that $\nu_i$ is random and uncorrelated with the regressors but one wants to make specific distributional assumptions about $u_i$ and $\nu_i$, traditional stochastic frontier models can be formulated in the context of panel data. In that case, maximum likelihood method produces a feasible and efficient set of estimators. The application of ML method in the context of the general time-invariant model (i.e, Model IV) needs the following assumptions.

(i) $\nu_i$'s are distributed normal with zero mean and variance $\sigma_y^2$.

$^4$ For the covariance matrix and the transformation process, see Hsiao (1986, pp 36-37) or Judge et al.(1988, pp 484-488)
(ii) $u_i$'s are time invariant (i.e, $\eta = 0$) and $u_i$ is distributed as normal with mode $\mu$ and truncated at zero.

(iii) $v_i$ and $u_i$ are independent.

Note that the assumption of $\mu = 0$ leads to Model III. Pitt and Lee (1981) derived the likelihood function for half-normal case (i.e, Model III). This can be extended to generalized case (i.e, Model IV) with the following likelihood function

$$
(4.23) \quad \ln L_i = - \frac{1}{2} \ln 2 - \frac{T}{2} \ln (2\pi) - \frac{T}{2} \ln \sigma^2_v + \frac{1}{2} \ln (\sigma^2_v + T\sigma^2_u) + \frac{1}{2\sigma^2_v} \sum_{t=1}^{T} (e_{it} - \mu/\sigma_u)^2 \left(1 - \frac{\sigma^2_v}{\sigma^2_v + T\sigma^2_u}\right) - \frac{1}{\sigma^2_u} \sum_{t=1}^{T} (e_{it} - \mu)^2 + \ln \Phi \left(\frac{\sigma^2_u}{\sigma^2_v + T\sigma^2_u} \right)^{\frac{1}{2}} \sum_{t=1}^{T} ((e_{it} - \mu/\sigma_u)/\sigma_v) + (\mu/\sigma_u)T \left(1 - \frac{\sigma^2_v}{\sigma^2_u}\right) - \ln \Phi \left(\mu/\sigma_u\right)
$$

From this, $\ln L = \sum_i \ln L_i$. The parameters $\beta$'s, $\sigma^2_v$, $\lambda^2$ ($= \sigma^2_u / \sigma^2_v$), and $\mu$ can be estimated consistently by numerically maximizing $\ln L$. After that $u_i$ can be estimated directly by applying Jondrow et al. estimator (given in equation (4.5)). However, this implies that for each cross-sectional unit, we are computing $T_i$ estimates of the same $u_i$. Battese and Coelli (1988) have obtained an aggregate result which uses the full time series for observation $i$, which is...
where $\phi(.)$ and $\Phi(.)$ are PDF and CDF of the standard normal distribution and $\mu_i^*$ and $\sigma_i$ are as defined in Battese and Coelli (1988, p-389).

Model V. Time-variant random effect models with general distributional assumption.

Model V is the completely unrestricted model with no restriction on parameters $\eta$, $\mu$, and $\lambda$. In this model inefficiency is modelled as a stochastic element which varies across firms and over time. This is justified especially when the production technology is specified in a dynamic setting. According to Schimdt (1985, p-32) "if one is looking for evidence of inefficiency, constancy overtime seems a more reasonable basis for the search...". However, in the context of panel data it is more reasonable to assume that inefficiency today correlates with inefficiency yesterday or tomorrow. More specifically, one might be interested in investigating whether there is any tendency of inefficiency to decay overtime - the firm approaching towards the frontier.

Once the time-varying assumption is made, the general model specified in equation (4.10), (4.15), and (4.16) would be applied with no zero restriction on $\lambda$, $\mu$, and $\eta$. Note that time-variance in the model has been included as a weight to $u_i$ (i.e, time-invariant inefficiency), the weight being a function of $\eta$ and the distance
between observed period (t) and the terminal period (T). However, even if η ≠ 0, the model does not include the time-variance in the most generalized form. This is because, from (4.16), differentiating $u_{it}$ with respect to $t$,

$$\frac{\partial u_{it}}{\partial t} = -\eta u_{it} e^{-\eta t - \eta}$$

which means that if η > 0, as $t$ increases (i.e., as $t \to T$), $u_{it}$ (or, equivalently $e^{u_{it}}$) monotonically decreases. That means as the firm proceeds overtime its inefficiency level monotonically decreases. Alternatively, the firm proceeds steadily towards the frontier function in course of time. Similarly, if η < 0, inefficiency increase monotonically overtime. Therefore the testable hypothesis offered by the model is monotonic decrease (or increase) of inefficiency. While these two cases are highly probable (especially the decay of inefficiency), the hypothesis of fluctuating inefficiency can not be tested in this model.

Given the general formulations of five models, the next question is how to select the appropriate model in a particular empirical problem. This can be done through a sequence of statistical tests. The sequence is outlined below.

Sequence 1. First test the OLS in null against Model I as alternative. That is, test whether there is no significant fixed firm effects. This can be tested by classical F test. High value of F favors fixed effect model (Model I).

Sequence 2. Next test whether the no inefficiency hypothesis is true against random inefficiency. That is, test whether classical homoskedastic non-autocorrelated error model is true against the Error component model. The test
can be carried out by Breusch-Pagan lagrange multiplier test procedure. The test statistic is:

\[
(4.26) \quad LM = \frac{NT}{2(T - 1)} \left[ \frac{\sum_{i}^{N} \left( \sum_{t}^{T} e_{it} \right)^2}{\sum_{i}^{N} \sum_{t}^{T} e_{it}^2} \right] - 1
\]

where \( e_{it} \) are residuals derived from OLS. Large value of LM favor Model II.

Sequence 3. If OLS is rejected in both sequences, then test whether random or fixed effect model is appropriate. As the critical assumption behind the selection of model between these two is the uncorrelatedness of regressors from the one-sided error, the appropriate test procedure would be to test \( H_0 : \) No correlation (i.e, Model II) against \( H_A : \) correlation exists (i.e, Model I). This can be tested by Hausman type test procedure where the test statistic would be

\[
(4.27) \quad H = (\hat{\beta}_H - \hat{\beta}_F)' \left[ Cov(\hat{\beta}_F) - Cov(\hat{\beta}_H) \right]^{-1} (\hat{\beta}_H - \hat{\beta}_F)
\]

where \( H \sim \chi^2 \) with \( K \) degrees of freedom. Low value of \( H \) favors Model II.

Sequence 4. If Model II is not rejected then the next step is to test whether any distributional assumption would be appropriate. To test \( H_0 : \) Model II (i.e, no distributional assumption) against \( H_A : \) Model III (i.e, \( \nu \) normal and \( \mu \) half-normal), the Hausman test can again be applied. High value of \( H \) favors Model III.

Sequence 5. If Model II is rejected, the next step is to test whether an unrestricted error structure for \( \mu \) would be more appropriate. That is, test \( H_0 : \)
Model III (half-normal) against $H_A$: Model IV (error truncated at zero). As both models use distributional assumptions, a likelihood ratio (LR) test would be appropriate in this case. The LR statistic is asymptotically distributed as $\chi^2_1$. High value of LR indicates the appropriateness of Model IV.

Sequence 6. The last step is to test the time-invariance hypothesis. Thus test $H_0$: Model III or IV (time-invariant model) against $H_A$: Model V (time-variant model). Again, LR test may be applied to test the null. High value of $H$ favors Model V.

4.4.2. Allocative efficiency

Consider the cost function in equation (4.12). Assuming that there is no technical or allocative inefficiencies, the stochastic cost frontier is

\[
\ln C^0 = k + \frac{1}{r} \ln y + \sum_{j=1}^{K} \frac{\beta_j}{r} \ln w_j - \frac{1}{r} y
\]

where $\beta_j$s are estimated from the frontier production function given in equation (4.10). Applying Shepherd's lemma to (4.28) the allocatively efficient share equation for each input can be derived, i.e,

\[
S_j = \frac{\partial (\ln C^0)}{\partial (\ln w_j)} = \frac{\beta_j}{r} , \quad j = 1,2,\ldots,K
\]

where $S_j$ is defined as $w_jX_j / C^0$. Note that Shepherd's lemma can be confirmed by deriving share equation directly from the conditional input demand function. In the
case of absence of allocative inefficiency, the share equation derived directly from input demand functions also yield the same result indicating the legitimacy of Shepherd's lemma in this case.

However, this is not true when allocative inefficiency exists. In that case, Shepherd's lemma yield the same result as in equation (4.29). But the share equation derived directly from input demand function (for j-th input) in this case, is (for proof see Kumbhakar (1985))

\[(4.30) \quad S_j = \frac{\beta_j}{r} - B_j\]

where

\[(4.31) \quad B_j = \frac{\beta_j(\beta_1 - r \exp(-e_j\delta) + \sum_{s=2}^{K} \beta_s \exp(-e_s))}{r(\beta_1 + \sum_{s=2}^{K} \beta_s \exp(-e_s))}\]

where \(\delta = 0\) if \(j = 1\), and, \(= 1\) if otherwise

ej are as defined in equation (4.7). Hence the true share equation (when allocative inefficiency exists) can not be determined by Shepherd's lemma as it assumes (wrongly) that the cost function and the share equations are independent (i.e., \(B_1 = 0\)). Equation (4.30), on the other hand, suggests something about the relationship between the errors in the cost function and the share equations.

The above result can be an important basis for computing allocative inefficiency because from equation (4.30), \(B_j\) represents the error in share equations.
which is a function of allocative inefficiency. To compute the allocative inefficiency, we need K-1 share equations (as one equation is always redundant) from which we get

\[ (4.32) \quad -B_j = S_j - \frac{\beta_j}{r}, \quad j = 2, \ldots, K \]

where \( S_j \) is the observed share of input \( j \) and share equation of input 1 is dropped. Now as the \( \beta_j \)'s are estimated from the frontier production, the only unknown in the \( j \)-th equation is \( \exp(e_j) \) (i.e., allocative inefficiency). Solving the K-1 equations the allocative inefficiency for each of K-1 inputs can be solved. The inefficiencies thus computed are to be explained relative to the input whose share equation has been dropped. The procedure to compute allocative efficiency (\( \exp(e_j) \)) can be summarized by the following steps:

(i) Analytically derive the slope coefficients of the stochastic cost frontier by estimated \( \beta_j \)'s from the production frontier. By Shepherd's lemma, these are the efficient shares.

(ii) Compute \( S_j - (\beta_j / r) \).

(iii) Drop one input share and express K-1 equations in (4.32) in terms of \( \exp(e_j) \).

(iv) Solve for K-1 \( \exp(e_j) \)'s from K-1 equations.
4.5. Data and empirical model

Data on yield, harvested acres, and various cash cost components (i.e., cost of hired labor, machinery repair and maintenance, fertilizer, fuel, interest payments, etc.) for forty-five (45) commercial sugarcane firms of south-east Louisiana have been used in this study. The data set, provided by First South Production Credit Association, Thibadeux, Louisiana, consists of the data on 110 firms for the period 1986 through 1990. The firms are the members of the association (identified by the code number) and are located mostly in the Southeast (Sugarcane) region. For this study, only 45 firms are selected on the basis of availability of data for the same firm for at least 3 years. As 1990 was a disaster year for the sugarcane farmers in Louisiana (due to freeze in December, 1989), this year was dropped from the sample period. The firms have been recoded in the scale 1 - 45 on the basis of increasing firm size which is defined as the harvested acres averaged over the years. Among these 45 firms the data for 28 firms covered all four years (1986-89); 17 firms covered three years. Slightly less than 20 percent of these firms are less than 300 acres in size. Approximately 75 percent of the sample firms are under 800 acres in size. Finally, 90 percent of this sample have firms of less than 1000 acres. Only one firm in the sample (2 percent) is more than 2000 acres in size.

There are data limitations worth mentioning: (1) As the firms have at least one common characteristic (i.e., members of the same credit association), the sample can not be considered as true random sample of sugarcane farmers in Louisiana. The selectivity bias may have increased due to further (selective)
elimination of some firms on the basis of data non-availability for all years. However, given the variation in firm-sizes across the sample and concentration of sugarcane firms in the study area, the information from the data set are not expected to be significantly inconsistent with the population characteristics; (2) the data on actual input usage are not available in the data set. They are proxied by dollar values of the inputs used in production. Although this is a standard practice in empirical research, the information is inadequate. For example, the role of family labor (especially on the small firms) can not be included in the modelling of the production process. The degree of specification bias\(^5\), in this case, is unknown.

The empirical model considered for this study is

\[
(4.33) \quad \ln y_{i,t} = \beta_0 + \sum_{j=1}^{4} \beta_j \ln x_{i,j,t} + \nu_{i,t} - u_{i,t}, \quad i=1,\ldots,45; \quad t = 1,\ldots,4,
\]

where \(y\) = yield of raw sugar (in pounds),

\[x_1 = \text{Acres harvested (in acre)},\]

\[x_2 = \text{Hired labor, measured in terms of expenditure on labor (in dollars)},\]

\[x_3 = \text{Annualized flow of capital services from machinery and equipment; it includes repair and maintenance cost and interest charges (in dollars)},\]

\(^5\) Note that to test the existence of specification error (due to, e.g., omitted variables, measurement errors, etc.), a formal treatment of model specification is necessary. However, this requires high precision in data and modification of traditional methodology (because the error term contains inefficiency). Given the data set and insufficient advancement in this area of methodological research, this was not attempted in the present study.
\( x_4 = \) Fertilizer and chemicals, measured in terms of annual expenditure on fertilizer and chemicals (in dollars).

Non-availability of firm-specific input usage and price data imposed restrictions on the model. Fuel has not been included in the model as initial estimation showed extremely small elasticity (and statistically insignificant too). Fertilizer data have been deflated by the index of fertilizer prices used in Vroomen (1989). Labor and capital flow data have not been deflated as the wage and interest rate (on short term loan) have been found to be more or less constant over the study period (see D.A.E Research report (647, 667) and A.E.A Research report (62, 74), Dept. of Agricultural Economics & Agribusiness, Louisiana State University).

The behavioral and other assumptions for the empirical model are:

(i) The farmers are scale efficient; this makes the objective of profit maximization and cost minimization equivalent.

(ii) All firms pay the same price for each input; this is necessary to make the production function valid. However, this assumption is not necessary for estimating allocative inefficiency.

(iii) The same production technology is followed by each firm within the sample.

(iv) Technical and allocative efficiencies are independent.
4.6. Results

The results have been derived in two stages. In the first stage technical efficiency for each firm is estimated from the production function given in (4.33). In the second stage, allocative and cost inefficiencies are derived given the results in the first stage which include parameter estimates, efficiency estimates, model testing, and statistical results on firm-size and efficiency relation. Second stage includes only the derived allocative and cost inefficiencies.

4.6.1. Technical efficiency

First the parameters of the production frontier are estimated by each of Model I through Model V. From the view point of economic analysis of inefficiency, alternative model specifications are important. This is because it allows us to select one which is most appropriate for the given data and production technology. The implication of the assumption about inefficiency on estimation is significant and thus the appropriate way is not to impose restrictions initially. For example, if inefficiency is initially assumed to be time-invariant when it is actually time-variant, the estimates of inefficiency may give a distorted picture of reality. For this reason, from the model-building perspective, it is justified to proceed from various model specifications of inefficiency and then select one which fits the data best.
The results are given in Table 4.2. The first column shows the OLS regression results; the second column is the fixed effect (FE) model; the third column is the random effect model without distributional assumption (REGLS); the fourth column is the time-invariant random effect model with the assumption of half-normality for one-sided error (\( \text{REML}_{1}^{\text{TV}} \)); the fifth column is the same with assumption of generalized (or, truncated at zero) normal error (\( \text{REML}_{2}^{\text{TV}} \)); the last column is the time-variant model with no restriction on any parameter (\( \text{REML}_{\text{TV}} \)).

In all models, land shows the highest output elasticity in the \([0.761, 1.001]\) range with boundaries corresponding to OLS and Model I, respectively. As the summation of estimated coefficients is close to 1.0, it may be argued that the production function experiences approximately constant returns to scale and land contributes the most in percentage changes in output. However, as expected, the OLS model shows the lowest elasticity as the inefficiency in the use of land has not been "purged out" in this model. Surprisingly, labor and capital have very low and insignificant (at 5 percent level) elasticities. Fertilizer, on the other hand, has reasonably high elasticity range (9 percent to 15 percent) across models.

Given the high elasticity of the land input it may be concluded that the major source of inefficiency is in the use of the land resources. In Model I the output elasticity of land appears to be overestimated relative to its estimates in the other models. This is because the fixed firm effects fail to capture the inefficiency
Table 4.2 Estimates for parameters of stochastic frontier production functions for Louisiana's sugarcane farmers under different models*.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model OLS</th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
<th>Model IV</th>
<th>Model V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>7.703</td>
<td>7.783</td>
<td>8.003</td>
<td>8.131</td>
<td>8.331</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.25)</td>
<td>(0.24)</td>
<td>(0.30)</td>
<td>(0.25)</td>
<td></td>
</tr>
<tr>
<td>$\beta_{\text{land}}$</td>
<td>0.761</td>
<td>1.001</td>
<td>0.865</td>
<td>0.889</td>
<td>0.867</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.080)</td>
<td>(0.056)</td>
<td>(0.058)</td>
<td>(0.052)</td>
<td></td>
</tr>
<tr>
<td>$\beta_{\text{labor}}$</td>
<td>0.032</td>
<td>0.014</td>
<td>0.020</td>
<td>0.012</td>
<td>0.020</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.027)</td>
<td>(0.022)</td>
<td>(0.021)</td>
<td>(0.018)</td>
<td></td>
</tr>
<tr>
<td>$\beta_{\text{capital}}$</td>
<td>0.043</td>
<td>0.013</td>
<td>0.018</td>
<td>0.012</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.030)</td>
<td>(0.028)</td>
<td>(0.028)</td>
<td>(0.026)</td>
<td></td>
</tr>
<tr>
<td>$\beta_{\text{fertilizer}}$</td>
<td>0.150</td>
<td>0.097</td>
<td>0.118</td>
<td>0.114</td>
<td>0.093</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.030)</td>
<td>(0.027)</td>
<td>(0.027)</td>
<td>(0.026)</td>
<td></td>
</tr>
<tr>
<td>Ln L</td>
<td>72.21</td>
<td>165.02</td>
<td>95.44</td>
<td>98.19</td>
<td>102.08</td>
<td></td>
</tr>
<tr>
<td>Adj.R$^2$</td>
<td>0.93</td>
<td>0.97</td>
<td>0.93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda^2$</td>
<td>1.28</td>
<td>3.88</td>
<td>1.27</td>
<td>1.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.76)</td>
<td>(0.56)</td>
<td>(0.54)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td></td>
<td></td>
<td></td>
<td>0.344</td>
<td>0.271</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.20)</td>
<td>(0.15)</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.072</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.39)</td>
<td></td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.025</td>
<td>0.011</td>
<td>0.025</td>
<td>0.055</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.022</td>
<td></td>
</tr>
</tbody>
</table>

* Asymptotic standard errors are within parentheses

$\lambda$ = Inefficiency parameter; $\eta$ = time-variance parameter; $\mu$ = distribution parameter.
in the use of land (or, "sweeps out" the firm-specific land effect) as land has been included in the regressors.

The estimates derived from random effect models (Model II - Model V) are close to each other. However, comparing the "average" function estimates (i.e, OLS estimates) with the "frontier" function estimates (i.e, Model III - Model V), it is found that the difference between the intercept and the slope estimates in these models is quite high. This indicates that the frontier function in this case is not a monotonic transformation of the average function and any inference about economies of size and scale on the basis of an average estimated function will lead to misleading conclusion.

The frontier models also show the estimates of inefficiency parameter $\lambda$. In each case, estimated $\lambda$ is greater than unity indicating that the one-sided error sufficiently dominates the white noise. It is also statistically significant (at 5 percent level) in each model. Other parameters ($\eta$, the time-variance parameter and $\mu$, the distribution parameter), however, are not large enough to make substantial differences among the estimates in Model III, IV, and V.

Estimated technical efficiency for each firm for Model I through Model IV (i.e, time-invariant models) are presented in Table 4.3. Due to the computational assumption (i.e, the most efficient firm in the sample is 100 percent efficient) the

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6 In the literature of frontier function this is called "regression fallacy". The typical method of estimating output or cost elasticities from an observed production or cost function is quite unsatisfactory if inefficiency changes the shape as well as the location of the production or cost curves.
Table 4.3 Estimated firm-specific technical efficiencies* of the sugarcane farmers under time-invariant stochastic frontier models.

<table>
<thead>
<tr>
<th>Firm ID</th>
<th>Acre Harvested (average)</th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
<th>Model IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>107</td>
<td>0.8751 (5)</td>
<td>0.7553 (19)</td>
<td>0.8836 (18)</td>
<td>0.7295 (19)</td>
</tr>
<tr>
<td>2</td>
<td>118</td>
<td>0.7703 (17)</td>
<td>0.6818 (31)</td>
<td>0.7994 (31)</td>
<td>0.6719 (31)</td>
</tr>
<tr>
<td>3</td>
<td>162</td>
<td>0.8320 (7)</td>
<td>0.7556 (18)</td>
<td>0.8791 (20)</td>
<td>0.7300 (18)</td>
</tr>
<tr>
<td>4</td>
<td>215</td>
<td>0.8122 (9)</td>
<td>0.7599 (17)</td>
<td>0.8844 (17)</td>
<td>0.7337 (17)</td>
</tr>
<tr>
<td>5</td>
<td>226</td>
<td>0.9583 (2)</td>
<td>0.8904 (2)</td>
<td>0.9603 (3)</td>
<td>0.8304 (2)</td>
</tr>
<tr>
<td>6</td>
<td>287</td>
<td>0.7462 (20)</td>
<td>0.7108 (24)</td>
<td>0.8335 (23)</td>
<td>0.6948 (24)</td>
</tr>
<tr>
<td>7</td>
<td>290</td>
<td>0.7866 (15)</td>
<td>0.7472 (21)</td>
<td>0.8755 (21)</td>
<td>0.7230 (21)</td>
</tr>
<tr>
<td>8</td>
<td>296</td>
<td>0.7183 (24)</td>
<td>0.6873 (29)</td>
<td>0.8042 (29)</td>
<td>0.6747 (29)</td>
</tr>
<tr>
<td>9</td>
<td>316</td>
<td>0.6846 (28)</td>
<td>0.6700 (34)</td>
<td>0.7887 (34)</td>
<td>0.6635 (34)</td>
</tr>
<tr>
<td>10</td>
<td>337</td>
<td>0.6012 (39)</td>
<td>0.5945 (43)</td>
<td>0.7037 (43)</td>
<td>0.6036 (43)</td>
</tr>
<tr>
<td>11</td>
<td>347</td>
<td>0.6841 (29)</td>
<td>0.6775 (33)</td>
<td>0.7989 (32)</td>
<td>0.6693 (33)</td>
</tr>
<tr>
<td>12</td>
<td>348</td>
<td>0.8779 (4)</td>
<td>0.8661 (4)</td>
<td>0.9592 (4)</td>
<td>0.8189 (4)</td>
</tr>
<tr>
<td>13</td>
<td>362</td>
<td>0.9144 (3)</td>
<td>0.8878 (3)</td>
<td>0.9627 (2)</td>
<td>0.8290 (3)</td>
</tr>
<tr>
<td>14</td>
<td>385</td>
<td>0.6737 (31)</td>
<td>0.6672 (36)</td>
<td>0.7856 (36)</td>
<td>0.6612 (36)</td>
</tr>
<tr>
<td>15</td>
<td>420</td>
<td>1.0000 (1)</td>
<td>1.0000 (1)</td>
<td>0.9835 (1)</td>
<td>0.9188 (1)</td>
</tr>
<tr>
<td>16</td>
<td>436</td>
<td>0.5381 (44)</td>
<td>0.5375 (45)</td>
<td>0.6391 (45)</td>
<td>0.5497 (45)</td>
</tr>
<tr>
<td>17</td>
<td>446</td>
<td>0.8650 (6)</td>
<td>0.8601 (5)</td>
<td>0.9584 (5)</td>
<td>0.8144 (5)</td>
</tr>
<tr>
<td>18</td>
<td>474</td>
<td>0.8073 (11)</td>
<td>0.8168 (10)</td>
<td>0.9304 (10)</td>
<td>0.7799 (10)</td>
</tr>
<tr>
<td>19</td>
<td>476</td>
<td>0.6955 (26)</td>
<td>0.7010 (28)</td>
<td>0.8188 (28)</td>
<td>0.6864 (28)</td>
</tr>
<tr>
<td>20</td>
<td>486</td>
<td>0.7984 (12)</td>
<td>0.7966 (12)</td>
<td>0.9176 (12)</td>
<td>0.7611 (12)</td>
</tr>
<tr>
<td>21</td>
<td>490</td>
<td>0.7671 (18)</td>
<td>0.7662 (21)</td>
<td>0.8904 (16)</td>
<td>0.7391 (16)</td>
</tr>
<tr>
<td>22</td>
<td>495</td>
<td>0.7827 (16)</td>
<td>0.7869 (13)</td>
<td>0.9140 (13)</td>
<td>0.7561 (13)</td>
</tr>
<tr>
<td>23</td>
<td>495</td>
<td>0.7915 (13)</td>
<td>0.7849 (14)</td>
<td>0.9078 (14)</td>
<td>0.7542 (14)</td>
</tr>
</tbody>
</table>

* The numbers in parentheses are rankings of the firms with respect to their efficiencies (1 = the most efficient firm, 45 = the least efficient firm).

Model I = Fixed effect model; Model II = random effect model; Model III = inefficiency distributed as half-normal; Model IV = inefficiency distributed as generalized truncated normal.
Table 4.3 (continued) Estimated firm-specific technical efficiencies* of the sugarcane farmers under time-invariant stochastic frontier models.

<table>
<thead>
<tr>
<th>Firm ID</th>
<th>Acre Harvested (Average)</th>
<th>Technical Efficiencies</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Model I</td>
<td>Model II</td>
</tr>
<tr>
<td>24</td>
<td>521</td>
<td>0.6606 (33)</td>
<td>0.6685 (35)</td>
</tr>
<tr>
<td>25</td>
<td>551</td>
<td>0.7569 (19)</td>
<td>0.7772 (15)</td>
</tr>
<tr>
<td>26</td>
<td>553</td>
<td>0.5826 (41)</td>
<td>0.5948 (42)</td>
</tr>
<tr>
<td>27</td>
<td>554</td>
<td>0.8242 (8)</td>
<td>0.8337 (9)</td>
</tr>
<tr>
<td>28</td>
<td>560</td>
<td>0.6501 (34)</td>
<td>0.6614 (38)</td>
</tr>
<tr>
<td>29</td>
<td>574</td>
<td>0.7072 (25)</td>
<td>0.7308 (22)</td>
</tr>
<tr>
<td>30</td>
<td>633</td>
<td>0.7308 (23)</td>
<td>0.7526 (20)</td>
</tr>
<tr>
<td>31</td>
<td>640</td>
<td>0.6761 (30)</td>
<td>0.7040 (27)</td>
</tr>
<tr>
<td>32</td>
<td>642</td>
<td>0.6867 (27)</td>
<td>0.7127 (23)</td>
</tr>
<tr>
<td>33</td>
<td>645</td>
<td>0.6695 (32)</td>
<td>0.7074 (26)</td>
</tr>
<tr>
<td>34</td>
<td>736</td>
<td>0.8116 (10)</td>
<td>0.8532 (6)</td>
</tr>
<tr>
<td>35</td>
<td>800</td>
<td>0.6244 (37)</td>
<td>0.6628 (37)</td>
</tr>
<tr>
<td>36</td>
<td>802</td>
<td>0.5863 (40)</td>
<td>0.6263 (39)</td>
</tr>
<tr>
<td>37</td>
<td>814</td>
<td>0.7906 (14)</td>
<td>0.8404 (7)</td>
</tr>
<tr>
<td>38</td>
<td>865</td>
<td>0.5797 (42)</td>
<td>0.6177 (40)</td>
</tr>
<tr>
<td>39</td>
<td>925</td>
<td>0.5340 (45)</td>
<td>0.5782 (44)</td>
</tr>
<tr>
<td>40</td>
<td>964</td>
<td>0.6218 (38)</td>
<td>0.6805 (32)</td>
</tr>
<tr>
<td>41</td>
<td>969</td>
<td>0.6368 (35)</td>
<td>0.6851 (30)</td>
</tr>
<tr>
<td>42</td>
<td>1034</td>
<td>0.5613 (43)</td>
<td>0.6067 (41)</td>
</tr>
<tr>
<td>43</td>
<td>1116</td>
<td>0.7359 (22)</td>
<td>0.8065 (11)</td>
</tr>
<tr>
<td>44</td>
<td>1441</td>
<td>0.6306 (36)</td>
<td>0.7088 (25)</td>
</tr>
<tr>
<td>45</td>
<td>2170</td>
<td>0.7400 (21)</td>
<td>0.8378 (8)</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>0.7284</td>
<td>0.7344</td>
</tr>
</tbody>
</table>

* The numbers in parentheses are rankings of the firms with respect to their efficiencies (1 = the most efficient firm, 45 = the least efficient firm).

Model I = Fixed effect model; Model II = random effect model; Model III = inefficiency distributed as half-normal; Model IV = inefficiency distributed as generalized truncated normal.
values in the columns of Model I and II show the relative ranking values with respect to the 100 percent efficient firm. The values corresponding to the columns of Model III and IV, however, are absolute measure of efficiencies and thus these values can not be compared directly with those of Model I and II. However, the rankings are comparable as relative locations in the efficiency ranking do not depend upon the absolute values. Ranking of an individual firm (in 1 - 45 rating) is given in the parenthesis after corresponding efficiency estimates.

Table 4.3 suggests that, although individual estimates vary across models, the ranking is invariant to model specification. The most efficient firm in each model is Firm No.15, whereas Firm No.16 shows the least efficiency. Other rankings are also more or less symmetric across the models. The symmetry in ranking is more prominent among random effect models (i.e, Model II, III, and IV) whereas Model I shows slight variation in ranking; the variation may diminish if (say) 6 digits after decimal points are considered. To test whether the rankings are really consistent across models, Spearman's rank correlation coefficients may be considered. The estimated correlation coefficients ($r_s$) are given in Table 4.4.

Extremely high value of the coefficients among Model II, III, and IV indicate that the rankings in any one of them is a good approximation of the same in others. Somewhat lower values (0.90) between Model I and the other models, however, represents a small variation in the rankings between fixed effect and random effect models. The variation between these two types of model are more prominent in
Table 4.4 Spearman's rank correlation coefficients ($r_s$) for estimated technical efficiencies of sugarcane farmers under the time-invariant stochastic frontier models.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>0.9056</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>0.9080</td>
<td>0.9988</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>0.9073</td>
<td>0.9998</td>
<td>0.9989</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The bigger sized firms, where Model I shows relatively high rankings than the other models. This, as argued earlier, may be due to the inability of a fixed effect model to capture the full inefficiency arising from the land input.

The evaluation of technical efficiency of each firm can be conducted on the basis of individual efficiency estimates. This can be done for each model. For Model III and Model IV the estimated efficiency levels [E($e^{ui}$)] of the best practice firm (i.e, Firm no. 15) are 0.9835 and 0.9188. As E($e^{ui}$) = 1 implies zero inefficiency, in both models this firm is still below the estimated production frontier. However, assuming that it is 100 percent efficient (as assumed in Model I and II) a comparative analysis can be done for each firm. For example, in Model I the lowest sized firm (Firm 1) and the highest sized firm (i.e, Firm 45) are 12.5 percent and 26 percent less efficient respectively than the most efficient firm. For Model II, the percentages are 24.5 percent and 16.2 percent respectively. In Model III,
they are 10.1 percent and 4.5 percent (assuming 0.9835 = 100) and in Model IV, they are 20.6 percent and 13.7 percent (assuming 0.9188 = 100) less efficient than the best practice firm. The difference between the most efficient and the least efficient firm are (in percent) 46.6, 46.25, 33.36, and 40.17 respectively for Model I, II, III, and IV. In other words, in this sample of firms, all 45 firms are producing in the 0 percent to 40 percent inefficiency range (assuming that Model IV is true).

Given the results, the next problem is to test which of the time-invariant models should be appropriate in the present context. Also to be tested is whether time-invariant models better fit the data than time-variant models. For this the sequential test procedure sketched in Section 4.4 has been applied in the present study. The test procedures, calculated values and test results are summarized in Table 4.5. As it is found, OLS is rejected against both fixed and random effect least square models (Model I and II) indicating the presence of individual firm effects. However, Model I is rejected against Model II; this implies that the individual effects are more likely to be random. Given the utilization of homogenous type of inputs, it is not unjustified that individual firms will be inefficient due to random factors. Distributional assumption seems to provide better model specification as Model II is rejected against Model III. The test between unrestricted and restricted distribution gives a slightly better edge for Model IV (i.e, unrestricted distribution) as Model III is rejected against Model IV with a probability level lower than 0.05 but higher than 0.01. Finally the test between time-invariant and time-variant models goes in favor of the time-variant
Table 4.5 The sequential tests and selection of the appropriate model for estimating technical efficiency of the sugarcane farmers.

<table>
<thead>
<tr>
<th>Seq</th>
<th>$H_0$ &amp; $H_A$</th>
<th>Test statistic</th>
<th>Distribution</th>
<th>Calculated value</th>
<th>Critical value</th>
<th>Test Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$H_0$: OLS</td>
<td>$H_A$: I</td>
<td>F</td>
<td>$F_{44,114}$</td>
<td>5.501</td>
<td>1.39</td>
</tr>
<tr>
<td>2</td>
<td>$H_0$: OLS</td>
<td>$H_A$: II</td>
<td>Breusch-Pagan LM</td>
<td>$\chi^2_1$</td>
<td>60.06</td>
<td>3.84</td>
</tr>
<tr>
<td>3</td>
<td>$H_0$: II</td>
<td>$H_A$: I</td>
<td>Hausman H</td>
<td>$\chi^2_4$</td>
<td>6.92</td>
<td>9.49</td>
</tr>
<tr>
<td>4</td>
<td>$H_0$: II</td>
<td>$H_A$: III</td>
<td>Hausman H</td>
<td>$\chi^2_4$</td>
<td>18.92</td>
<td>9.49</td>
</tr>
<tr>
<td>5</td>
<td>$H_0$: III</td>
<td>$H_A$: IV</td>
<td>LR</td>
<td>$\chi^2_1$</td>
<td>5.49</td>
<td>3.84</td>
</tr>
<tr>
<td>6</td>
<td>$H_0$: IV</td>
<td>$H_A$: V</td>
<td>LR</td>
<td>$\chi^2_1$</td>
<td>7.79</td>
<td>3.84</td>
</tr>
</tbody>
</table>

Model I = Fixed effect model; Model II = random effect model; Model III = inefficiency distributed as half-normal; Model IV = inefficiency distributed as generalized truncated normal; Model V = time-variant model.

Seq = Sequence; LM = Lagrange multiplier test; LR = Likelihood ratio test
model as the null hypothesis of Model IV is rejected against Model V at any reasonable level of significance.

However, even if the time-invariance property is rejected, frontier functions are not expected to move quite significantly across the study period. This is because the estimated \( \eta \) (given in the last column of Table 4.2) is too low (0.07). It is also reflected in the closeness of the coefficient estimates and estimated \( \lambda \) between Model IV and Model V. Hence it can be argued that even if efficiency increases over time, firm-specific efficiency estimates from model IV will be a good approximated value of the time-wise firm-specific efficiency estimates. More importantly, as Model V forces each firm to be more efficient in the course of time, the efficiency ranking of the firms should not be affected significantly by a time-varying property.

Given the statistical significance of a time-variant model in this context, firm-specific efficiencies are estimated following Battese and Coelli estimation procedure. The results are given in Table 4.6. Note that efficiency estimates steadily increase over the study period. However, as estimated \( \eta \) is small (0.07) the increase in two consecutive years is not very significant. In the final year (1989), \( t - T = 0 \); thus, the estimates in this period indicate \( u_{it} = u_i \). The estimates in other periods are related to the same in the final period by the relation specified in (4.16). For example, the estimate for Firm 1 in period 1986 is related to the estimate for the same firm in 1989 in the following way. Let \( M = e^{-\eta(t - T)} \). Then,

\[
TE(1,1986) = e^{-u(1,1986)} = e^{-u(1,1989)M}
\]
Table 4.6 Estimated firm-specific time-varying technical efficiencies* of the sugarcane farmers under the time-variant inefficiency model. (Year = 1986-1989)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>107</td>
<td>0.7222 (17)</td>
<td>0.7387 (19)</td>
<td>0.7545 (21)</td>
<td>0.7694 (20)</td>
</tr>
<tr>
<td>2</td>
<td>118</td>
<td>NA</td>
<td>0.6560 (36)</td>
<td>0.6756 (38)</td>
<td>0.6943 (37)</td>
</tr>
<tr>
<td>3</td>
<td>162</td>
<td>0.7287 (16)</td>
<td>0.7450 (18)</td>
<td>0.7604 (20)</td>
<td>0.7750 (19)</td>
</tr>
<tr>
<td>4</td>
<td>215</td>
<td>0.7348 (15)</td>
<td>0.7507 (17)</td>
<td>0.7659 (19)</td>
<td>0.7802 (18)</td>
</tr>
<tr>
<td>5</td>
<td>226</td>
<td>NA</td>
<td>0.8627 (3)</td>
<td>0.8715 (3)</td>
<td>0.8800 (3)</td>
</tr>
<tr>
<td>6</td>
<td>287</td>
<td>NA</td>
<td>0.7141 (23)</td>
<td>0.7310 (25)</td>
<td>0.7471 (24)</td>
</tr>
<tr>
<td>7</td>
<td>290</td>
<td>0.7405 (14)</td>
<td>0.7562 (16)</td>
<td>0.7710 (18)</td>
<td>NA</td>
</tr>
<tr>
<td>8</td>
<td>296</td>
<td>0.6757 (24)</td>
<td>0.6944 (28)</td>
<td>0.7123 (30)</td>
<td>0.7293 (29)</td>
</tr>
<tr>
<td>9</td>
<td>316</td>
<td>NA</td>
<td>0.6651 (35)</td>
<td>0.6843 (37)</td>
<td>0.7026 (36)</td>
</tr>
<tr>
<td>10</td>
<td>337</td>
<td>NA</td>
<td>0.5938 (42)</td>
<td>0.6158 (44)</td>
<td>0.6370 (41)</td>
</tr>
<tr>
<td>11</td>
<td>347</td>
<td>NA</td>
<td>0.6724 (33)</td>
<td>0.6913 (35)</td>
<td>0.7093 (34)</td>
</tr>
<tr>
<td>12</td>
<td>348</td>
<td>0.8273 (5)</td>
<td>0.8382 (6)</td>
<td>0.8486 (7)</td>
<td>0.8583 (7)</td>
</tr>
<tr>
<td>13</td>
<td>362</td>
<td>NA</td>
<td>0.8633 (2)</td>
<td>0.8722 (2)</td>
<td>0.8805 (2)</td>
</tr>
<tr>
<td>14</td>
<td>385</td>
<td>NA</td>
<td>0.6660 (34)</td>
<td>0.6851 (36)</td>
<td>0.7034 (35)</td>
</tr>
<tr>
<td>15</td>
<td>420</td>
<td>0.9414 (1)</td>
<td>0.9453 (1)</td>
<td>0.9490 (1)</td>
<td>0.9524 (1)</td>
</tr>
<tr>
<td>16</td>
<td>436</td>
<td>0.5284 (33)</td>
<td>0.5525 (43)</td>
<td>0.5758 (45)</td>
<td>0.5984 (42)</td>
</tr>
<tr>
<td>17</td>
<td>446</td>
<td>0.8396 (2)</td>
<td>0.8498 (4)</td>
<td>0.8595 (4)</td>
<td>0.8686 (4)</td>
</tr>
<tr>
<td>18</td>
<td>474</td>
<td>0.7941 (7)</td>
<td>0.8069 (9)</td>
<td>0.8191 (10)</td>
<td>0.8305 (10)</td>
</tr>
<tr>
<td>19</td>
<td>476</td>
<td>0.6903 (21)</td>
<td>0.7084 (24)</td>
<td>0.7256 (26)</td>
<td>0.7420 (25)</td>
</tr>
<tr>
<td>20</td>
<td>486</td>
<td>NA</td>
<td>0.7729 (14)</td>
<td>0.7869 (16)</td>
<td>0.8001 (16)</td>
</tr>
<tr>
<td>21</td>
<td>490</td>
<td>0.7623 (12)</td>
<td>0.7768 (13)</td>
<td>0.7906 (15)</td>
<td>0.8037 (15)</td>
</tr>
<tr>
<td>22</td>
<td>495</td>
<td>0.7688 (11)</td>
<td>0.7830 (12)</td>
<td>0.7965 (14)</td>
<td>0.8092 (14)</td>
</tr>
<tr>
<td>23</td>
<td>495</td>
<td>0.7733 (9)</td>
<td>0.7872 (10)</td>
<td>0.8004 (12)</td>
<td>0.8129 (12)</td>
</tr>
</tbody>
</table>

* Numbers in parentheses are the rankings of the firms with respect to their efficiencies (1 = the most efficient firm, 45 = the least efficient firm).
Table 4.6 (continued)  Estimated firm-specific time-varying technical efficiencies* of the sugarcane farmers under the time-varying inefficiency model.

(Year = 1986-1989)

<table>
<thead>
<tr>
<th>ID</th>
<th>Acre Harvested (Average)</th>
<th>Technical Efficiencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>521</td>
<td>NA</td>
</tr>
<tr>
<td>25</td>
<td>551</td>
<td>0.7692 (10)</td>
</tr>
<tr>
<td>26</td>
<td>553</td>
<td>0.6069 (30)</td>
</tr>
<tr>
<td>27</td>
<td>554</td>
<td>0.8156 (6)</td>
</tr>
<tr>
<td>28</td>
<td>560</td>
<td>0.6562 (27)</td>
</tr>
<tr>
<td>29</td>
<td>574</td>
<td>0.7174 (18)</td>
</tr>
<tr>
<td>30</td>
<td>633</td>
<td>0.7434 (13)</td>
</tr>
<tr>
<td>31</td>
<td>640</td>
<td>0.6886 (22)</td>
</tr>
<tr>
<td>32</td>
<td>642</td>
<td>0.7030 (20)</td>
</tr>
<tr>
<td>33</td>
<td>645</td>
<td>NA</td>
</tr>
<tr>
<td>34</td>
<td>736</td>
<td>0.8322 (4)</td>
</tr>
<tr>
<td>35</td>
<td>800</td>
<td>0.6598 (26)</td>
</tr>
<tr>
<td>36</td>
<td>802</td>
<td>0.6308 (28)</td>
</tr>
<tr>
<td>37</td>
<td>814</td>
<td>NA</td>
</tr>
<tr>
<td>38</td>
<td>865</td>
<td>0.6226 (29)</td>
</tr>
<tr>
<td>39</td>
<td>925</td>
<td>0.5813 (32)</td>
</tr>
<tr>
<td>40</td>
<td>964</td>
<td>0.6735 (25)</td>
</tr>
<tr>
<td>41</td>
<td>969</td>
<td>0.6816 (23)</td>
</tr>
<tr>
<td>42</td>
<td>1034</td>
<td>0.6040 (31)</td>
</tr>
<tr>
<td>43</td>
<td>1116</td>
<td>0.7925 (8)</td>
</tr>
<tr>
<td>44</td>
<td>1441</td>
<td>0.7100 (19)</td>
</tr>
<tr>
<td>45</td>
<td>2170</td>
<td>0.8353 (3)</td>
</tr>
</tbody>
</table>

Mean 0.7181 0.7345 0.7502 0.7651

* Numbers in parentheses are the rankings of the firms with respect to their efficiencies (1 = the most efficient firm, 45 = the least efficient firm).
Here, $M = 1.2429$ and $-u(1,1989) = \ln(0.7694) = -0.2621$. Hence,

$$TE (1,1986) = e^{(1.2429 \times 0.2621)} = 0.7222.$$  

$M$ decreases as $t$ increases; hence the estimates in the next periods (1987, 1988, 1989) are progressively higher. Note that at $t = 1989$, $M = 1$, so that in 1989 the efficiency estimate is the highest.

The result shows that although efficiencies are increasing for each firm over the study period, still (in the final period) all of them (except one) are at least 10 percent below the frontier. This is evident from the fact that no firm (except Firm 15) lies in the 0.9 - 1.0 range. Thirty-five percent of the firms are found to be in the range 0.8 - 0.9. The maximum number of firms (48 percent) are found to be in the region 0.7 - 0.8 whereas minimum number of firms (15 percent) are in 0.5 - 0.7 range. The rankings are very consistent with the time-invariant results. For example, Spearman's rank correlation coefficient between rankings in Model IV and those in year 1988 of Model V is 0.98 which rejects the null hypothesis of no rank correlation at any reasonable level of significance.

An aggregate analysis of technical efficiency of the firms under study is presented in Table 4.7. A range of efficiencies is observed across the forty-five firms, with lowest spread in Model IV. Nineteen firms are 75 percent or more efficient in Model I. Corresponding numbers (i.e, 75 percent or more efficient) in Model II, III, IV, and V are 20, 38, 26, and 26 respectively. Thus, the stochastic frontier models (Model III, IV, and V) with distributional assumptions show most firms in the higher efficiency range than the other two models. All the models
Table 4.7 Frequency distribution of technical efficiency ratings in each stochastic frontier model of sugarcane production in Louisiana.

<table>
<thead>
<tr>
<th>Efficiency Rating (%)</th>
<th>Number of Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model I</td>
</tr>
<tr>
<td>95-100</td>
<td>2</td>
</tr>
<tr>
<td>90-94.9</td>
<td>1</td>
</tr>
<tr>
<td>85-89.9</td>
<td>3</td>
</tr>
<tr>
<td>80-84.9</td>
<td>5</td>
</tr>
<tr>
<td>75-79.9</td>
<td>8</td>
</tr>
<tr>
<td>70-74.9</td>
<td>6</td>
</tr>
<tr>
<td>65-69.9</td>
<td>9</td>
</tr>
<tr>
<td>60-64.9</td>
<td>5</td>
</tr>
<tr>
<td>55-59.9</td>
<td>4</td>
</tr>
<tr>
<td>0-54.9</td>
<td>2</td>
</tr>
<tr>
<td>Mean</td>
<td>72.84</td>
</tr>
<tr>
<td>S.D</td>
<td>11.04</td>
</tr>
<tr>
<td>Skewness</td>
<td>.284</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.568</td>
</tr>
</tbody>
</table>

* The time variant model produced efficiency estimates for each of the years (1986 through 1989). Here only the estimates for the year 1989 are considered.

Model I = Fixed effect model; Model II = random effect model; Model III = inefficiency distributed as half-normal; Model IV = inefficiency distributed as generalized truncated normal; Model V = time-variant model.
show slightly positive skewness except Model III. All the models show approximately mesokurtic distribution, i.e., neither flat nor peaked with respect to the general appearance of the frequency curve. From the distribution it is also clear that if one is interested in percentage measure of efficiency, model selection is an important issue; it is not so important if one is interested only in relative rankings.

The degree of technical inefficiency on the basis of estimates from Model V is graphically shown in Figure 4.1. Frontier yield (i.e., total yield if no inefficiency exists) for each firm is calculated by dividing the observed yield by estimated efficiency (column 6, Table 4.6). As expected, both frontier and observed yields increase as firm size increases. It clearly shows that the absolute cost of inefficiency in terms of foregone yield is higher for bigger firms (more than 500 acres). However, it is also obvious that all firm operators potentially could either produce more given available resources or produce the same level of output using fewer resources. The first option, given low domestic price and a strong resurgence in Louisiana’s sugarcane production after the freeze damage in 1990-1991 (see Sugar and Sweetner, USDA, p-9), may not benefit the farmers. The second option would have a direct impact on the financial conditions of individual firms. Reduction in input levels would reduce direct and indirect cash costs, without reducing output. The effect should be an increase in firms’ profitability. An economic analysis on this option and the relation between efficiency and profitability are presented in the next chapter.
Figure 4.1 Firm-specific frontier and observed yield of raw sugar for forty-two sugarcane firms in Louisiana (Year = 1989).
Finally, the relation between firm-size and efficiency within the sample observations may be considered. A close inspection of the estimates in all models (Table 4.3 and Table 4.6) reveals that there is no apparent positive correlation between firm-size and efficiency. However, statistical results indicate that random effect models, which assume that the inefficiency and regressors are uncorrelated, fit the data better than fixed effect model. As land is included as a regressor this implies that no correlation between firm-size and inefficiency has been accepted as a basis for the model. Hence, for Model II through Model V, there is no evidence of any correlation between inefficiency and firm-size. In the fixed effect model, however, such correlation is allowed. To test whether such correlation exists, the estimates of efficiency derived from Model I is regressed on firm sizes. The result of this regression (t-value in parenthesis) is

\[
(4.34) \quad TE_i = 0.7958 - 0.0011 \text{ ACRE} \quad R^2 = 0.15
\]

The estimated slope coefficient is negative (and statistically significant) but very small. Thus, the better fit of the random effect model and the very low coefficient in the fixed effect model together seems to imply that no correlation between firm-size and efficiency has been found within the sample data.
4.6.2. Allocative efficiency

The stochastic frontier analytically derived from the stochastic production frontier is a function of logarithms of output, prices of land, labor, capital, and fertilizer, and the error term \( v \). However, land cost in sugarcane farming is typically realized after the realization of production. The cane produced in a particular year goes to the mills and the growers get approximately 60 percent of the raw sugar produced from that amount of cane. The tenant farmers pay a certain percentage (generally 20%) of this raw sugar to the land owners. Thus, from a budgeting point of view, rental cost is not included in calculating total operating cash cost\(^7\). To be consistent with this approach, here, the frontier for stochastic cash-cost frontier (with respect to labor, capital, and fertilizer) will be considered. In other words, the analytically derived cost frontier is explained in terms of costs on these three inputs only. The computed efficient shares for labor, capital, and fertilizer on the basis of the estimates from time-variant production frontier model (i.e, Model V) are

\[
\beta_{Lab} / r = 0.1678 \\
\beta_{Cap} / r = 0.0540 \\
\beta_{fert} / r = 0.7780
\]

Dropping share equation for fertilizer, the share equations are (see equation (4.30))

\[
(4.35) \quad S_{Lab} = 0.1678 - B_{Lab} \\
(4.36) \quad S_{Cap} = 0.0540 - B_{Cap}
\]

\(^7\) It is, however, included in the calculation of net return.
where $B_j$ (j=Lab, Cap) are as defined in equation (4.30). Then, $\exp(e_{Lab})$ and $\exp(e_{Cap})$ are solved from (4.35) and (4.36) for each firm. Finally, the most efficient firm has been assumed as 100 percent efficient and all other firms are ranked according to this condition. The computed (relative) allocative efficiencies of labor and capital (with respect to fertilizer) for the year 1989 are presented in Table 4.8.

Several characteristics of allocative efficiencies are worth mentioning. First, like technical efficiencies the ranking of allocative efficiencies for each input show no definite pattern with respect to increase in firm-size. In other words, allocative efficiencies too are not dependent on firm sizes. Firms "make mistakes" in determining optimum input combinations irrespective of their sizes. Second, in both cases (labor and capital), the absolute value of efficiencies ($\exp(e_j)$) (not presented here) are less than one (i.e., $e_j < 0$). This indicates that both fertilizer/labor and fertilizer/capital ratios are used in excess of the optimal level. That is, given input prices, the observed use of fertilizer and chemical may have contributed in technical efficiency, but it also contributed to allocative inefficiency. Third, the ranking in labor efficiency does not seem to be correlated significantly with that of capital efficiency. Spearman's correlation coefficient is 0.4025 which, even though greater than critical value, cannot be taken as a dependable basis to infer anything about the ranks from one about the other. A small correlation coefficient implies that, in general, a firm is not ranked high in efficiency in allocation of both inputs (except a few firms, e.g., Firm 5, 13). Fourth, the most
Table 4.8 Estimated relative firm specific allocative efficiencies of the sugarcane farmers.

(Year = 1989)

<table>
<thead>
<tr>
<th>Firm ID</th>
<th>Acre Harvested (average)</th>
<th>$AE_{\text{Labor}}$</th>
<th>$AE_{\text{Capital}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>107</td>
<td>0.4665 (17)</td>
<td>1.0000 (1)</td>
</tr>
<tr>
<td>2</td>
<td>118</td>
<td>0.2775 (37)</td>
<td>0.1769 (39)</td>
</tr>
<tr>
<td>3</td>
<td>162</td>
<td>0.3432 (28)</td>
<td>0.1601 (40)</td>
</tr>
<tr>
<td>4</td>
<td>215</td>
<td>0.3349 (30)</td>
<td>0.1947 (37)</td>
</tr>
<tr>
<td>5</td>
<td>226</td>
<td>0.8886 (2)</td>
<td>0.5539 (4)</td>
</tr>
<tr>
<td>6</td>
<td>287</td>
<td>0.7187 (5)</td>
<td>0.2875 (26)</td>
</tr>
<tr>
<td>7</td>
<td>290</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>8</td>
<td>296</td>
<td>0.2259 (38)</td>
<td>0.2095 (36)</td>
</tr>
<tr>
<td>9</td>
<td>316</td>
<td>0.6359 (10)</td>
<td>0.2480 (32)</td>
</tr>
<tr>
<td>10</td>
<td>337</td>
<td>0.3177 (33)</td>
<td>0.1477 (41)</td>
</tr>
<tr>
<td>11</td>
<td>347</td>
<td>0.3576 (26)</td>
<td>0.3108 (24)</td>
</tr>
<tr>
<td>12</td>
<td>348</td>
<td>0.3775 (25)</td>
<td>0.2756 (28)</td>
</tr>
<tr>
<td>13</td>
<td>362</td>
<td>0.6520 (8)</td>
<td>0.4735 (8)</td>
</tr>
<tr>
<td>14</td>
<td>385</td>
<td>0.3009 (36)</td>
<td>0.3533 (17)</td>
</tr>
<tr>
<td>15</td>
<td>420</td>
<td>0.3194 (32)</td>
<td>0.3187 (23)</td>
</tr>
<tr>
<td>16</td>
<td>436</td>
<td>0.5367 (15)</td>
<td>0.2524 (31)</td>
</tr>
<tr>
<td>17</td>
<td>446</td>
<td>0.4837 (16)</td>
<td>0.3399 (21)</td>
</tr>
<tr>
<td>18</td>
<td>474</td>
<td>0.2149 (39)</td>
<td>0.3924 (13)</td>
</tr>
<tr>
<td>19</td>
<td>476</td>
<td>0.4045 (23)</td>
<td>0.3548 (16)</td>
</tr>
<tr>
<td>20</td>
<td>486</td>
<td>0.6790 (7)</td>
<td>0.5106 (7)</td>
</tr>
<tr>
<td>21</td>
<td>490</td>
<td>1.0000 (1)</td>
<td>0.4142 (12)</td>
</tr>
<tr>
<td>22</td>
<td>495</td>
<td>0.1435 (42)</td>
<td>0.2167 (34)</td>
</tr>
<tr>
<td>23</td>
<td>495</td>
<td>0.3161 (34)</td>
<td>0.0975 (42)</td>
</tr>
</tbody>
</table>

$AE_{\text{Labor}} = \text{Allocative efficiency of labor with respect to fertilizer.}$

$AE_{\text{Capital}} = \text{Allocative efficiency of capital with respect to fertilizer.}$
Table 4.8 (continued) Estimated relative firm specific allocative efficiencies of the sugarcane farmers.
(Year = 1989)

<table>
<thead>
<tr>
<th>Firm ID</th>
<th>Acre Harvested (average)</th>
<th>( AE_{\text{Labor}} )</th>
<th>( AE_{\text{Capital}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>521</td>
<td>0.3990 (24)</td>
<td>0.3597 (15)</td>
</tr>
<tr>
<td>25</td>
<td>551</td>
<td>0.2057 (40)</td>
<td>0.2134 (35)</td>
</tr>
<tr>
<td>26</td>
<td>553</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>27</td>
<td>554</td>
<td>0.4159 (20)</td>
<td>0.4432 (9)</td>
</tr>
<tr>
<td>28</td>
<td>560</td>
<td>0.4572 (18)</td>
<td>0.2712 (29)</td>
</tr>
<tr>
<td>29</td>
<td>574</td>
<td>0.7127 (6)</td>
<td>0.2283 (33)</td>
</tr>
<tr>
<td>30</td>
<td>633</td>
<td>0.3441 (29)</td>
<td>0.5390 (6)</td>
</tr>
<tr>
<td>31</td>
<td>640</td>
<td>0.2039 (41)</td>
<td>0.1927 (38)</td>
</tr>
<tr>
<td>32</td>
<td>642</td>
<td>0.6410 (9)</td>
<td>0.4313 (10)</td>
</tr>
<tr>
<td>33</td>
<td>645</td>
<td>0.8837 (3)</td>
<td>0.2569 (30)</td>
</tr>
<tr>
<td>34</td>
<td>736</td>
<td>0.4055 (22)</td>
<td>0.3394 (21)</td>
</tr>
<tr>
<td>35</td>
<td>800</td>
<td>0.5904 (13)</td>
<td>0.5529 (5)</td>
</tr>
<tr>
<td>36</td>
<td>802</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>37</td>
<td>814</td>
<td>0.4364 (19)</td>
<td>0.6996 (2)</td>
</tr>
<tr>
<td>38</td>
<td>865</td>
<td>0.5605 (8)</td>
<td>0.3483 (19)</td>
</tr>
<tr>
<td>39</td>
<td>925</td>
<td>0.4118 (21)</td>
<td>0.3216 (22)</td>
</tr>
<tr>
<td>40</td>
<td>964</td>
<td>0.6201 (11)</td>
<td>0.3518 (18)</td>
</tr>
<tr>
<td>41</td>
<td>969</td>
<td>0.3464 (27)</td>
<td>0.6967 (3)</td>
</tr>
<tr>
<td>42</td>
<td>1034</td>
<td>0.7856 (4)</td>
<td>0.3814 (14)</td>
</tr>
<tr>
<td>43</td>
<td>1116</td>
<td>0.3072 (35)</td>
<td>0.4186 (11)</td>
</tr>
<tr>
<td>44</td>
<td>1441</td>
<td>0.3271 (31)</td>
<td>0.2811 (27)</td>
</tr>
<tr>
<td>45</td>
<td>2170</td>
<td>0.5985 (12)</td>
<td>0.3048 (25)</td>
</tr>
</tbody>
</table>
technically efficient (see the last column in Table 4.6) firm (Firm no. 15) is well below in the allocative efficiency ranking (32 and 23 respectively for labor and capital). On the other hand, the most allocatively efficient in labor is Firm no. 21 which ranks 15 in technical efficiency ranking. Similarly, the most allocatively efficient in capital is Firm no. 1 which ranks 20 in technical efficiency ranking. The only firms which are in a highly ranked position in all efficiency ratings are Firm no. 5 (which ranks 3 in technically efficiency and 2 and 4 respectively in labor and capital allocative efficiency) and Firm no. 13 (which ranks 2 in technical efficiency and 8 in both allocative efficiencies). Given this result it may be concluded that, in general, technical and allocative efficiencies are not correlated for the firms in the sample.

Finally, like technical efficiency, the frequency table for allocative efficiency is presented to get a overall view of allocative efficiencies of the firms. This is presented in Table 4.9. It is clear from the table that so far as allocative efficiencies of labor and capital (with respect to fertilizer) are concerned, firms, in general, are in worse situation than their technical efficiency counterpart. Almost 28 firms (66 percent of all firms) are below 50 percent relative to the most efficient firm with respect to allocation between labor and fertilizer. In the case of capital - fertilizer allocation almost 36 firms (85 percent) firms are below 50 percent relative to the most efficient firm. However, these numbers are to be taken with caution as in our study both capital and labor shares are under-represented due to
Table 4.9  Frequency distribution of allocative efficiency ratings of labor and capital (Year = 1989).

<table>
<thead>
<tr>
<th>Efficiency Rating (%)</th>
<th>Number of firms</th>
<th>$\text{AE}_{\text{Labor}}$</th>
<th>$\text{AE}_{\text{Capital}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>91-100</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>82-91</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>73-82</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>64-73</td>
<td>5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>55-64</td>
<td>5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>46-55</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>37-36</td>
<td>8</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>28-37</td>
<td>11</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>19-28</td>
<td>5</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>10-19</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>0-10</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td><strong>0.4677</strong></td>
<td><strong>0.3551</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td><strong>1.00</strong></td>
<td><strong>1.00</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td><strong>0.1435</strong></td>
<td><strong>0.0973</strong></td>
<td></td>
</tr>
<tr>
<td><strong>S.D</strong></td>
<td><strong>0.2042</strong></td>
<td><strong>0.1707</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td><strong>0.726</strong></td>
<td><strong>1.541</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td><strong>2.797</strong></td>
<td><strong>6.258</strong></td>
<td></td>
</tr>
</tbody>
</table>

$\text{AE}_{\text{Labor}}$ = Allocative efficiency of labor with respect to fertilizer.

$\text{AE}_{\text{Capital}}$ = Allocative efficiency of capital with respect to fertilizer.
two reasons: (1) some of labor and capital costs are not included in the respective shares due to data limitations (e.g., depreciation cost is not included in capital share); and, (2) some of the costs are included in the fertilizer share which should be included in capital and labor cost (e.g., machinery and labor cost in fertilizer application). The ranks (given within parenthesis in Table 4.8), however, are more reliable as the above two limitations are expected to affect all firms uniformly.

4.7. Summary

In this chapter some methodological and empirical issues pertaining to evaluation of cost efficiency have been discussed. As cost (in)efficiency is composed of technical and allocative (in)efficiencies, the performance of firms in an industry should be evaluated on the basis of these components. The concepts of frontier technology and frontier cost function are used in this study to estimate firm specific technical and allocative efficiencies of a sample of Louisiana sugarcane firms. Various issues are addressed here to show how this methodology can be applied in a particular empirical research on intra-industry evaluation of performance.

The results from the empirical study show that there are significant amounts of variations within the sample in estimated efficiencies. No statistical correlation has been found between firm-size and efficiency level, and between technical and allocative efficiency. Also, the firms have been found to perform relatively better in terms of technical efficiency than allocative efficiency (with respect to fertilizer).
In the context of this particular study, it may be concluded that resources are being used relatively more optimally in technological sense than in allocative sense. This points out the possibility that the farmers in this sample are responding more quickly to technological change than changes in input prices. This is justified on economic ground as economic agents often systematically make errors in allocation (or neglects market information) by undertaking production practices which help them reap maximum yield from the current technology. With a richer data set future research may be able to test the hypothesis of systematic allocative inefficiency in Louisiana sugarcane production.

As small-sized firms are not found to be necessarily inefficient, the question remains: why do not they survive? As the issue of survivability is related to the comparative advantage, it is important to know whether a cost efficient firm is necessarily a comparatively advantageous firm. This issue will be addressed in next chapter where the linkage between firm-specific inefficiency and firm-specific comparative advantage will be tested in the context of data and models used in this chapter.
CHAPTER 5

FIRM SPECIFIC INEFFICIENCY AND COMPARATIVE ADVANTAGE

5.1. Introduction

The major economic theory underlying this study has been the theory of a firm's decision in an optimizing framework under uncertainty. The theoretical framework developed in Chapter 2 showed how a firm makes an optimal decision if uncertainty prevails either in cost or in demand or in both sides, and how that decision is changed with changes in random or non-random exogenous shocks. However, in that analysis, the nature or source of the randomness was not explored. In Chapter 4, production inefficiency was identified as one major source of the stochastic behavior of decision variables.

Assuming that the firm produces with some degree of production inefficiency, how can this information may be used to analyze and predict firm behavior and its comparative advantage vis-a-vis other firms in the same industry? More importantly, to what extent efficiency contributes toward the absolute variation (i.e, the spread of average cost or the CIR curve in Figures 2.4 and 2.5 in Chapter 2) and the relative variation (i.e, the rankings in comparative advantage) of the decision variables? Theoretical answers to these problems were provided in Chapter 2 where comparative static analysis was used to show the effect on optimum output and profitability through incremental changes in moments of the CIR distribution. A more comprehensive analytical framework is developed and
applied in this chapter. Since inefficiency is a part of random factors, it is natural to expect that it affects the mean and variance (and possibly higher moments too) of the cost-income ratio. Therefore, the comparative static analysis in terms of "mean preserving spread" and changing mean can be addressed on the basis of inefficiency analysis and its linkage to profitability.

The knowledge generated by such an analysis is extremely important in identifying the role of inefficiency in the existing and future surviving potentiality of the firms in an industry. If the existing linkage between efficiency and comparative advantage within the firms in a particular industry can be identified in terms of quantity of output or amount of cost, that will help producers and policy makers restructure the plans and policies regarding the future direction of a particular firm or a group of firms. It will also shed light on whether the disappearance of small firms is due to higher cost inefficiency or lower income generating capacity, or both.

In the context of sugarcane production in Louisiana, these issues are vital. As illustrated in Chapter 4, technical and allocative inefficiencies are significantly prevalent among a non-statistical sample of sugarcane firms in Southeast Louisiana. Since no correlation has been found between the size of the firms and their inefficiencies, the potential role of inefficiency in the surviving capacity of these firms should be explored. This will generate knowledge about the causes behind

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1 Inefficiency is a controllable random factor. This makes it qualified for an exogenous shift factor in comparative static analysis.
concentration of production and the possible effect on the direction of the concentration if inefficiencies decrease over time.

In the next section, the linkage between inefficiency and comparative advantage is explained on the basis of a simple theoretical framework. Section 5.3 deals with the methodological aspects and procedure of empirical analysis of the link. In Section 5.4 description of data and empirical model, in the context of the same data set used in Chapter 4, are given. The results are discussed in Section 5.5, and the chapter is concluded with a summary in Section 5.6.

5.2. Inefficiency and comparative advantage: theoretical analysis

Define the cost function for the i-th firm as

\[ C_i = C(y_i, w_{ji}, \ldots, w_j, u_i, v_i) \]

where, \( y_i \) is output, \( w_{ji}, \ldots, w_j \) are the given input prices, \( v_i \) is (uncontrollable) random error, and \( u_i (\geq 0) \) is cost inefficiency (controllable). Given the exogenous nature of output, i.e., \( y_i = y^0 \), the cost function \( C_i \) defines the minimum cost (to produce \( y^0 \)) at given technology and input prices.

From (5.1), the corresponding CIR function can be derived as

\[ r_i = r(y_i, w_{ji}, \ldots, w_j, p_i, u_i, v_i, \zeta_i) \]

where \( p_i \) is the output price and \( \zeta_i \) is the (uncontrollable) random error term due to price uncertainty. Note that if \( p \) is non-random, then \( \zeta_i \) is zero so that the randomness in \( r \) is caused by white noise (\( v_i \)) and cost inefficiency (\( u_i \)).
The above specifications leads to important insight about the relationship between relative profitability and inefficiency in production. Note that the difference between actual CIR and the frontier CIR is caused by both inefficiency and the random factors. Thus, at a certain point in time, actual CIR may lie below the stochastic frontier if $v_f + \zeta_i < 0$, and $|v_f + \zeta_i| > u_i$. In other words, the probability of achieving an optimum CIR ($\mu^\ast$) under uncertainty depends on (i) inefficiency of the firm; and (ii) the probability of achieving random shocks in its favor (i.e, Prob ($v_f < 0$)). This point may be illustrated in more detail.

Suppose, there are two identical sized firms in a region producing the same crop with same level of expected net return. Input and output prices are given. However, as illustrated in Chapter 2 (see section 2.4.1) the average costs of two firms may be significantly different; this is due to effects of pure random and inefficiency elements. The differences of CIRs may also be explained by these two factors. Now, two situations may be considered: pure random factors ($v_i$ and $\zeta_i$) which can occur for both firms with (1) equal probability; and (2) with unequal probability. An example of the first case is effect of weather or rainfall, whereas examples of the second case are machine breakdown, uncertainty in supply of certain inputs, etc. Given these categories and regular conditions of optimality of firms' decisions under uncertainty (i.e, decreasing absolute risk aversion), both firms are expected to produce less than they would produce, had certainty prevailed.

---

2 If output price is also random, the variations between CIRs may be even larger, even though cost variations are not affected by this (see Section 2.4.4, Chapter 2).
However, the comparative advantage of the firm 1 requires it to operate on the lowest possible CIR curve; this can happen only if firm 1 has higher probability of achieving

\[(5.3) \quad \nu_1 + \zeta_1 + u_1 < \nu_2 + \zeta_2 + u_2.\]

From the above analysis it should be clear that if the pure random factors have equal probability of occurring for both firms, i.e., if \(\nu_1 + \zeta_1 = \nu_2 + \zeta_2\), then cost efficiency and comparative advantage have strict one-to-one correspondence. In other words, in this situation, a more cost efficient firm (i.e., firm 1) has a higher comparative advantage than firm 2. This one-to-one correspondence may, however, break down if the probabilities are not the same. Specifically, firm 1 may gain comparative advantage (i.e., satisfy the condition given in (5.3)) even if \(u_1 > u_2\).

Thus, in general, the contribution of efficiency in a given CAD ranking can not be determined until the total uncertainties are decomposed into pure and controllable random factors.

The linkage becomes more complicated if market imperfection exists. To explain this, assume that the relation between firm 1 and firm 2 is based on a typical duopolistic leader-follower case, i.e, firm 1 enjoys some monopoly power and sets the price which firm 2 accepts as a follower. This may happen if (1) the market share of firm 1 is substantially larger than for firm 2, or, (2) firm 2 is a new entry and depends on firm 1 regarding absorption of knowledge about technology,
financial credit, etc., or, (3) firm 1 enjoys natural monopoly power. Now, for simplicity, assume a single input production function under certainty, i.e,

\[(5.4) \quad y_i = f(x_i),\]

where \(y_i\) = output of i-th firm and \(x_i\) = amount of input \(x\) employed in the i-th firm (i = 1,2). The equilibrium condition in the input market is

\[(5.5) \quad MRP_x = w_x,\]

where \(MRP_x\) is the marginal revenue product of \(x\) and \(w_x\) is the price of \(x\). Now, equation (5.5) and equation (2.28) in Chapter 2 jointly lead to (with slight manipulation)

\[(5.6) \quad \frac{MP}{AP} \bigg|_{y^*-r^*} = \frac{r^*}{(1 + \frac{1}{\epsilon_p})},\]

where the definitions in Chapter 2 (Section 2.3) hold. As firm 2 is a price-taker, the long-run condition \(MP = AP\) implies \(r^* = 1\) for this firm. But, for firm 1, this implies \(r^* < 1\) (as \(|\epsilon_p| > 1\)). In other words, price-leadership due to any source of market imperfection leads to comparative advantage of a producing unit. This supports the observed phenomenon of less incentive in a monopolistic firm to eliminate production inefficiencies.

The condition stated in (5.3) can be extended to compute CAD indices in multi-firm cases where firms have different expected net returns. As derived in

\[3 \text{ In this case, however, farm 1 is expected to be more cost efficient and that may explain part of comparative advantage.}\]
Chapter 2 (see Section 2.2), in a particular region, firms can be ranked according to their AADs, where the AAD of i-th firm (in t-th period) is

\[ AAD_{it} = Pr\{CIR_u \leq c\} \ m_i \ \ \ i=1,...,N; \ t=1,...,T \]

where \( c \) is a given value of the CIR and \( m_i \) is an appropriate weight which incorporates information about expected income-generating capacity of the firm.

Equation (5.7) can be expressed alternatively in terms of the pure random factors and inefficiency in the CIR function. Let \( \tau = \nu + \zeta \); then

\[ AAD_{it} = Pr\{\tau_{it} + u_{it} \leq C_0\} \ m_i = CAD_{it} , \]

where \( C_0 \) is an arbitrary constant. Equation (5.8) implies that given equal expected net return for all firms in a region, the firm with higher probability of having high cost efficiency and favorable random factors has the highest absolute (or, comparative\(^4\)) advantage with respect to other firms producing the same crop.

The final problem is to show the linkage between inefficiency and comparative advantage, that is, to demonstrate the impact on comparative advantage rankings of firms if inefficiency of each firm is fully eliminated. To illustrate this, assume that inefficiencies have been eliminated so that each firm is on the cost or the CIR frontier. The stochastic CIR frontier function corresponding to equation (5.2) is (for the i-th firm)

\[ r_i^* = r(y_i, w_{il}, ..., w_{ij}, p_i, \tau_i) , \]

---

\(^4\) As argued earlier, AAD = CAD when the analysis is restricted within a single region.
where \( r_i = v_i + \zeta_i \). Obviously, \( r^* \leq r \) as \( u_i \to 0 \). This situation has been analyzed in Chapter 2 (see Section 2.4.2B) where the impact on optimum output and CIR has been shown in terms of a shift parameter \( \theta \) (i.e., \( r^* = r + \theta \)). Here, \( \theta = u \); and recalling the result derived in equation (2.77) in Chapter 2 and putting \( \theta = u \) and \( Q = y \),

\[
\frac{\partial y}{\partial u} \bigg|_{y^*,u} < 0
\]

which implies that given pure random shocks and input usage, an increase in efficiency leads to higher level of output, or, equivalently, lower level of cost at given output. Further, as expected (and derived in equation (2.78)), optimum CIR also declines by the rate when inefficiency decreases by 100 percent.

If the variance of \( u_i \) changes, given a constant mode, the spread of the CIR function (or, equivalently, the cost function) will also change. This is equivalent to "mean preserving spread" discussed in Chapter 2 (Section 2.4.2A). This may happen if the absolute variation of inefficiency across firms decrease due to fast learning of the most inefficient firms or dissemination of technical knowledge (which helps to curb inefficiency) from the relatively efficient to inefficient firms. In that case too, following the result derived in equation (2.70) in Chapter 2, output will increase, or, at a given output, cost will decrease.

Given these results, the CAD index in full efficiency case would be

\[
AAD_{\mu}^* = CAD_{\mu}^* = \text{Pr}\{r_{i} \leq C_{0} | u_{i}=0\} \ m_{i}
\]
Now to compare the CAD rankings, assume that initially $\text{CAD}_{it} > \text{CAD}_{jt}$, i.e., the i-th firm has comparative advantage than j-th firm when inefficiency exists in each firm. If the proportionate change in CAD is defined as

\begin{equation}
(5.12) \quad d_{it} = \frac{\text{CAD}_{it} - \text{CAD}^*_{it}}{\text{CAD}_{it}}
\end{equation}

then $d_{it} > d_{jt}$ would imply that the comparative advantage ranking is unaffected even if inefficiency is eliminated. That means that, in this case, although inefficiency may contribute significantly to increase the probabilities, it does this for both i-th and j-th firms equally, so that their rankings remain unaffected. On the other hand, $d_{it} < d_{jt}$ would imply that inefficiency components are important in the sense that its elimination leads to significant change in the CAD rankings.

5.3. Methodology and procedure

The above theoretical result can be tested using a stochastic frontier concept. Let the cost function in (5.1) be written in terms of a linear regression equation for the i-th firm,

\begin{equation}
(5.13) \quad C_i = \beta_0 + \beta_1 w_{i1} + \ldots + \beta_j w_{ij} + \beta y_i + v_i + u_i
\end{equation}

where the variables are expressed either in raw or in logarithmic terms. Assuming that they are expressed in logarithmic terms (or, equivalently, the production technology is Cobb-Douglas), the corresponding CIR function is specified as
\[(5.14) \quad r_i = \beta_0 + \beta_1 w_{it} + \ldots + \beta_j y_{jt} + \beta_p p_i + (\beta_{j-1} y_i + \tau_i + u_i)\]

where \(\tau_i\) captures the total (pure) random shocks from cost and demand uncertainty. Note that \(u_i\) represents cost inefficiency (i.e., \(u_i > 0\)) and can be decomposed as technical and allocative inefficiency\(^5\). Assuming that \(\tau_i\) is normally distributed with mean 0 and variance \(\sigma_{\tau}^2\) and defining \(\epsilon_i = \tau_i + u_i\), the joint density function of \(\epsilon_i\) can be expressed as

\[(5.15) \quad f(\epsilon_i|\sigma^2, \lambda) = \frac{2}{\sigma} f^* \left( \frac{\epsilon_i}{\sigma} \right) \left[ 1 - F^* \left( \frac{-\epsilon_i}{\sigma} \right) \right]\]

where, \(\sigma^2 = \sigma_{\tau}^2 + \sigma_u^2\), \(\lambda = \sigma_u / \sigma_{\tau}\); and \(f^*\) and \(F^*\) are the standard normal and standard normal cumulative density function, respectively. Given the distribution, the mean and variance of \(\epsilon_i\) are \(\sqrt{2/\pi} \sigma_u\) and \(\sigma_u^2 [(\pi - 2)/\pi] + \sigma_{\tau}^2\), respectively.

The result in equation (5.15) is important as it is to be used in deriving the AAD (or, CAD) index for i-th firm on the basis of equation (5.8). However, as \(\epsilon_i\) and other parameters are unknown, they have to be replaced by their estimates. In other words, the CIR function in (5.14) is to be estimated on the basis of specific assumptions about the individual components of composite error term \(\epsilon_i\).

\(^5\) The exact nature of decomposition for the C-D case is given in equation (4.12), Chapter 4.
The estimation of (5.14) is necessary if the impact of inefficiency on comparative advantage rankings is under investigation. For this, the following steps are to be followed:

(i) estimate the cost function in (5.13) or the CIR function in (5.14);
(ii) estimate firm-specific cost inefficiency by applying Jondrow et al. (1982) estimator (see equation (4.5), Chapter 4);
(iii) derive stochastic CIR frontier \( r^*_i \) by purging estimated inefficiency components from total residuals;
(iv) compute frontier CAD indices \( (CAD^*_i) \) by estimating density function of \( r^*_i \); and
(v) test whether the ranks in observed CAD is significantly different from those of frontier CAD.

5.3.1. Panel data

The major empirical problem in the above procedure is the estimation of the PDF of \( r \) or \( r^*_i \) because this requires a reasonably good number of (time-series) observations for each firm. For regional analysis this is not a serious problem as aggregative data are generally available for a long period of time. In other words, it is relatively more difficult to get a clear picture of historical cost-income structure of a firm than that of a region.

One way to address the problem is to estimate the PDFs from the data on a cross-section of firms in a particular year. But, this is quite unsatisfactory as the
heterogeneity among firms is totally neglected and the estimated probabilities reflect the nature of the industry (or, the group of firms) instead of a firm. A better alternative is to use panel data if available.

However, the problem of estimating probabilities still remains if panel data involves only a few years of data. In that case, a practical approach would be to use average of CIRs across years as an indicator of comparative advantage. This is justified on the grounds that the higher cumulative probabilities of achieving a particular level of CIR, in general, implies a lower average CIR, i.e,

\[ \Pr\{\text{CIR}_{it} \leq c\} \geq \Pr\{\text{CIR}_{jt} \leq c\} \implies E(\text{CIR}_i) \leq E(\text{CIR}_j)^6. \]

---

6 This can also be proved on the basis of second order stochastic dominance (SSD) principle. Suppose the income prospects of two firms (per unit of cost) are \( I_1 \) and \( I_2 \) and corresponding CDFs are \( G_1 \) and \( G_2 \). Assuming risk aversion for both firms, the SSD principle states that the distribution \( G_1 \) is said to dominate \( G_2 \) in the sense of SSD if the sum of the areas where \( G_2 \) lies to the right of the \( G_1 \) must be greater than the sum of the areas where it lies to the left of the dominated choice, i.e, if

\[
\int_0^t [G_1(j) - G_2(j)] \leq 0, \quad \forall t_0
\]

The necessary condition for the above result is \( E(I_1) \geq E(I_2) \) (see Anderson et al.(1977), p-285).

Now, \( r = (1/I) \). Therefore the above result automatically implies

\[
Pr(r_1 \leq c) - Pr(r_2 \leq c) = \int_0^c [F_1(r) - F_2(r)] \geq 0
\]

Then, in terms of corresponding PDFs, \( f_1(r) \) will be SSD over \( f_2(r) \), so that the necessary condition becomes

\[ E(r_1) \leq E(r_2) \]

Q.E.D
Thus, the CAD index (without weight) given in equation (5.7) can be written alternatively as

\[(5.16) \quad CAD_i = 1 - E(r_i),\]

which implies that as \(E(r_i)\) decreases, the \(i\)-th firm gains more probability to attain a given level of the CIR.

Finally, the issue of selecting appropriate weights is also important. If the index is measured on the basis of average CIR (in place of its probability), expected net return (per unit or total) as weight is not appropriate. This is because \(E(\pi)\) or \(E(\Pi)\) are calculated by averaging \(\pi\) or \(\Pi\) over the last few years (say, \(T_1\)); since, for small time-series observations, \(T_1 \approx T\), the same information is being used in calculating \(r_i\) and \(E(\pi)\) or \(E(\Pi)\), making them perfectly dependent on each other. For example, if the average CIR over the last four years of data is \(r_i = 0.75\), that automatically implies a 25 percent positive net return in average. Thus, the average of net return and the CIR carry the same information and cannot be used as independent multiplicative factor.

A better alternative, in this case, would be using an index of firm-size as weights. As the CIR is free of any size-unit (i.e, CIR per acre = CIR total acre) and as firm size is the only differentiating factor in generating income (assuming fixed prices and homogenous technology), an index of acreage seems to provide independent information regarding variations in income generating capacity. Considering the size-index as weight \((m_i)\), the CAD index (with weight) in equation (5.16) would be
where $A_i$ is a size index based on the amount of acres harvested. Note that the above definition implicitly assumes that firm size is also a determinant of CAD. However, that does not imply the equivalence of higher size and higher CAD because, by definition, a large firm with very high cost-income ratio may have less advantage than a small firm with low cost-income ratio.

This section may be summarized as follows: the comparative advantage index (with and without inefficiency) is formed on the basis of the indicator CIR. Depending on the availability of time-series data in the panel, one of the two procedures may be followed: (1) if sufficient amount of time-series data for each firm is available, then the index can be formed by using equation (5.7) or (5.8); i.e., estimating PDFs of the actual and frontier CIRs. Expected net return may be computed from the average of net return over the last few years and be used as weights. (ii) If time-series observations are not sufficient (to estimate the probabilities), then average CIR over the years may be used in place of the respective probability and a size index may be used as weight to compute the CADs.

5.4. Data and empirical model

Since the main objective is to derive comparative advantage and the impact of cost inefficiency on it, estimation of a cost function or a CIR function is necessary. Total cash cost data of each of 45 firms are used, including total cost on
labor, machinery repair and maintenance, interest, fertilizer and chemicals, fuel, supply, seed, insurance, taxes, and other non-itemized costs. As the main focus is on total inefficiency, not the individual components (technical and allocative), the primal cost function considered here is much more exhaustive than the dual cost function considered in Chapter 4.

The data on cost-income ratio is not given; it is computed on the basis of the data on yield and cost for each firm. To compute the CIR for a firm, the procedure followed is:

(i) compute net yield (NY) by deducting the mill's share. The share distribution between mills and the growers is generally 39 - 61 percent. So, \( NY = 0.61 \times Y \), where \( Y \) = total produced raw sugar (in pounds);

(ii) compute disposable yield (DY) by deducting land owner’s share. The share distribution between the tenants and the land-owners is 80 - 20 percent (of net yield). That is, \( DY = 0.8 \times NY \);

(iii) derive total revenue (TR) by multiplying DY of year t (\( DY_t \)) with price of raw sugar in the same year (\( p_t \)). That is, \( TR_t = DY_t \times p_t \); and

(iv) compute \( r_t = (TR/TC)_t \).\(^7\)

The data on domestic raw sugar prices for the years 1986 through 1989 are used as \( p_t \) and are collected from *Sugar and Sweetener*, USDA (1991).

\(^7\) These steps can be carried out by a single step. As \( Y/DY = 2.049 \) is the same for all firms, \( r_t = (Average\ cost/p)_t \times 2.049 \).
Behavioral and other assumptions for the cost (and the CIR) frontier model used in this study are:

(1) all firms have the identical objective to minimize cost;
(2) input and output prices are the same and exogenously given for all firms;
(3) production technology is the same across firms and years; and
(4) inefficiency is time-invariant.

Given these assumptions, the cost function to be estimated is

\[ \ln C_u = \beta_0 + \beta_1 D_{1u} + \beta_2 D_{2u} + \beta_3 D_{3u} + \beta_y \ln y_u + v_u + u_i \]

where, \( D_1 \) = time dummy variable; \( D_1 = 1 \) for 1986, 0 otherwise,
\( D_2 \) = time dummy variable; \( D_2 = 1 \) for 1987, 0 otherwise,
\( D_3 \) = time dummy variable; \( D_3 = 1 \) for 1988, 0 otherwise,
\( C \) = total cash cost measured in dollars, and
\( y \) = total raw sugar yield measured in pounds.

The cost function specified in (5.18) is different from that in (5.13) in the treatment of input prices. Input prices are not included explicitly in (5.18) due to assumption (2) stated above\(^8\). Time dummy variables are included to capture year-

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\(^8\) This assumption is not unrealistic as the firms are spatially close to each other. Variation in input prices over the years could be taken; but, this has not been tried due to two reasons: (i) prices of some inputs (e.g, wage, interest rate) remained more or less constant over the study years (1986-1989); (ii) the variation of the other input prices over only three or four years is not sufficient to make them good candidate for explanatory variables. Moreover, this is not expected to affect the result significantly as this variation is assumed to affect all firms equally.
specific effects on cost. Inefficiency \( (u_i) \) is assumed to be time invariant. Given the small range of time period and the results in Chapter 4, this assumption seems reasonable. Finally, the following assumptions are made about the pure error term \( (v_{it}) \) and the inefficiency term:

(i) \( v_{it} \sim N(0, \sigma_v^2) \) and \( u_i \sim |N(0, \sigma_u^2)| \), and

(ii) \( v_{it} \) and \( u_i \) are independent.

5.5. Results

The results from the estimation of cost function in (5.18) is given in Table 5.1. Two broad categories of models are considered here: (1) "average" or non-frontier model, i.e, OLS; and, (2) "best practice" or stochastic cost frontier (SCF) model. Following the procedures explained in Chapter 4, SCF model can be categorized into various sub-models (such as, models III, IV, and V in Chapter 4). However, since the ranks of efficiency are found to be highly correlated across the models, any one of the models would be sufficient to demonstrate the methodology proposed in this chapter. Model III (i.e, normal - half normal model) is selected here as it is the structure most commonly used.

From the results in Table 5.1, it is obvious that the group of firms under study show close proximity to constant returns to scale. The estimated scale coefficients in the OLS and the SCF models are 1.02 and 0.93 respectively, which are statistically significant at any reasonable level of significance. However, the SCF model shows slight economies of scale/size \((1 - 0.93 = 0.07)\) which implies
Table 5.1  Estimates for the parameters\(^*\) of the stochastic cost function for sugarcane production in Louisiana.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>OLS</th>
<th>Frontier</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_0) (constant)</td>
<td>-2.8562 (5.77)</td>
<td>-1.3033 (1.33)</td>
</tr>
<tr>
<td>(\beta_1) (D(_1))</td>
<td>-0.0731 (1.22)</td>
<td>-0.1014 (1.92)</td>
</tr>
<tr>
<td>(\beta_2) (D(_2))</td>
<td>-0.1360 (2.41)</td>
<td>-0.1535 (3.44)</td>
</tr>
<tr>
<td>(\beta_3) (D(_3))</td>
<td>-0.0651 (1.17)</td>
<td>-0.0754 (1.78)</td>
</tr>
<tr>
<td>(\beta_y) (ln y)</td>
<td>1.0218 (30.78)</td>
<td>0.9336 (14.46)</td>
</tr>
<tr>
<td>Adj.R(^2)</td>
<td>0.8573</td>
<td>--</td>
</tr>
<tr>
<td>ln L</td>
<td>-7.536</td>
<td>-3.2128</td>
</tr>
<tr>
<td>(\lambda^2)</td>
<td>--</td>
<td>2.5247 (2.73)</td>
</tr>
<tr>
<td>(\sigma_u^2)</td>
<td>--</td>
<td>0.0872</td>
</tr>
<tr>
<td>(\sigma^2)</td>
<td>0.0663</td>
<td>0.0843</td>
</tr>
</tbody>
</table>

* t-ratios are in parentheses.

(i) D\(_1\), D\(_2\), and D\(_3\) are time dummy variables for the years 1986, 1987, and 1988 respectively.

(ii) Estimates for Frontier are derived on the basis of the specification of a time-invariant stochastic cost frontier (SCF) function where inefficiency is assumed to be distributed as half-normal.
that if inefficiencies are eliminated from all firms, the firms will experience higher economies of scale\(^9\). The time dummy variable for the year 1987 shows the highest and statistically significant (but small) negative coefficient which implies a neutral downward shift of "average" and "frontier" cost function in the year 1987. The shifts for 1986 and 1988 are not significantly different from zero (at 1 percent level of significance) for the years 1986 and 1988 in each model.

Given the SCF model specification, the next problem is to test the hypothesis about the existence of cost inefficiency. Since the OLS model is nothing but a restricted version of the SCF model, the restriction being \(\lambda^2 = 0\), the appropriate way to test the legitimacy of the SCF model is to test the null hypothesis \(H_0: \lambda^2 = 0\) against \(H_A: \lambda^2 \neq 0\). This can be done either by a t-test or by a LR test (assuming that the errors in the OLS are normally distributed). High t-value (2.73) corresponding to estimated \(\lambda^2\) in the SCF model rejects \(H_0\) at any reasonable level of significance. Further, the LR statistic is 8.64 which is greater than critical value of \(\chi^2_1\) at 1 percent level of significance (=6.63); this also provides a strong statistical evidence in favor of the SCF model.

Since the variables in the cost functions are measured in logarithmic terms (except the dummy variables), the coefficients of the CIR function can be analytically derived from the estimated cost function. This, however, requires \(p\) to be constant. As the variation in \(p\) is zero across firms and insufficient across years,

\(^9\) This also indicates that frontier cost function is not exactly a neutral shift of the "average" cost function.
analytically derived estimates would closely approximate the estimated coefficients from the CIR function given in equation (5.14). The analytically derived CIR elasticity is

\[(5.19) \quad \hat{\epsilon}_r = \hat{\beta}_y^* = \hat{\beta}_y - 1 = 0.9336 - 1 = -0.0664\]

where \(\epsilon_r\) is the elasticity of the CIR with respect to output. Negative sign of \(\epsilon_r\) indicates a falling CIR curve for the group of firms taken as a whole. Since, at a given \(p\), this is equal to elasticity of average cost (AC),

\[(5.20) \quad \hat{\epsilon}_{AC} \approx \hat{\epsilon}_r \approx -0.0664\]

Following the optimality condition derived in Chapter 2 (see equation (2.44), Section 2.4.1),

\[(2.21) \quad \mu^{**} = E(r^{**}) \leq \frac{1}{1 - 0.0664} = 1.071\]

which means that the optimum CIR for the industry is equal or below 1.0 (approximately). It implies that the firm, which has the highest probability to break even, has the highest probability of profitability.

Next, the estimated firm-specific cost inefficiencies and their ranks are presented in Table 5.2. The estimates given in Table 5.2 are the estimated \(\epsilon^{E(u)}\) for each firm. As inefficiency appears in a multiplicative way in a Cobb-Douglas model
Table 5.2 Estimated firm-specific cost inefficiencies* from the stochastic cost frontier model of sugarcane production in Louisiana.

<table>
<thead>
<tr>
<th>ID</th>
<th>Acre Harvested (Average)</th>
<th>Cost Inefficiency</th>
<th>ID</th>
<th>Acre Harvested (Average)</th>
<th>Cost Inefficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>107</td>
<td>1.67 (21)</td>
<td>24</td>
<td>521</td>
<td>1.61 (17)</td>
</tr>
<tr>
<td>2</td>
<td>118</td>
<td>1.13 (3)</td>
<td>25</td>
<td>551</td>
<td>1.56 (15)</td>
</tr>
<tr>
<td>3</td>
<td>162</td>
<td>1.27 (6)</td>
<td>26</td>
<td>553</td>
<td>1.90 (44)</td>
</tr>
<tr>
<td>4</td>
<td>215</td>
<td>1.45 (10)</td>
<td>27</td>
<td>554</td>
<td>1.14 (4)</td>
</tr>
<tr>
<td>5</td>
<td>226</td>
<td>1.07 (2)</td>
<td>28</td>
<td>560</td>
<td>1.79 (35)</td>
</tr>
<tr>
<td>6</td>
<td>287</td>
<td>1.54 (14)</td>
<td>29</td>
<td>574</td>
<td>1.34 (7)</td>
</tr>
<tr>
<td>7</td>
<td>290</td>
<td>1.82 (39)</td>
<td>30</td>
<td>633</td>
<td>1.95 (45)</td>
</tr>
<tr>
<td>8</td>
<td>296</td>
<td>1.40 (8)</td>
<td>31</td>
<td>640</td>
<td>1.70 (25)</td>
</tr>
<tr>
<td>9</td>
<td>316</td>
<td>1.74 (31)</td>
<td>32</td>
<td>642</td>
<td>1.81 (38)</td>
</tr>
<tr>
<td>10</td>
<td>337</td>
<td>1.51 (13)</td>
<td>33</td>
<td>645</td>
<td>1.46 (11)</td>
</tr>
<tr>
<td>11</td>
<td>347</td>
<td>1.71 (27)</td>
<td>34</td>
<td>736</td>
<td>1.65 (20)</td>
</tr>
<tr>
<td>12</td>
<td>348</td>
<td>1.74 (30)</td>
<td>35</td>
<td>800</td>
<td>1.69 (23)</td>
</tr>
<tr>
<td>13</td>
<td>362</td>
<td>1.73 (28)</td>
<td>36</td>
<td>802</td>
<td>1.60 (16)</td>
</tr>
<tr>
<td>14</td>
<td>385</td>
<td>1.79 (36)</td>
<td>37</td>
<td>814</td>
<td>1.68 (22)</td>
</tr>
<tr>
<td>15</td>
<td>420</td>
<td>1.00 (1)</td>
<td>38</td>
<td>865</td>
<td>1.86 (41)</td>
</tr>
<tr>
<td>16</td>
<td>436</td>
<td>1.76 (33)</td>
<td>39</td>
<td>925</td>
<td>1.81 (37)</td>
</tr>
<tr>
<td>17</td>
<td>446</td>
<td>1.49 (12)</td>
<td>40</td>
<td>964</td>
<td>1.61 (18)</td>
</tr>
<tr>
<td>18</td>
<td>474</td>
<td>1.20 (5)</td>
<td>41</td>
<td>969</td>
<td>1.88 (42)</td>
</tr>
<tr>
<td>19</td>
<td>476</td>
<td>1.70 (24)</td>
<td>42</td>
<td>1034</td>
<td>1.43 (9)</td>
</tr>
<tr>
<td>20</td>
<td>486</td>
<td>1.70 (26)</td>
<td>43</td>
<td>1116</td>
<td>1.85 (40)</td>
</tr>
<tr>
<td>21</td>
<td>490</td>
<td>1.74 (29)</td>
<td>44</td>
<td>1441</td>
<td>1.78 (34)</td>
</tr>
<tr>
<td>22</td>
<td>495</td>
<td>1.63 (19)</td>
<td>45</td>
<td>2170</td>
<td>1.76 (32)</td>
</tr>
<tr>
<td>23</td>
<td>495</td>
<td>1.90 (43)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The numbers within parentheses are the rankings of the firms with respect to their inefficiencies (1 = the least inefficient firm, 45 = the most inefficient firm).

ID = Identification numbers of the firms.
(i.e, $C = C(y,...)e^u$), $e^{E(u)} = 1$ implies full cost efficiency (i.e, $E(u)=0$) and $e^{E(u)} > 1$ means inefficiency. The inefficiencies are rated by the same procedure followed in Chapter 4, i.e, assuming the least inefficient firm as the fully efficient (i.e, $\min(e^{E(u)}) \Rightarrow e^{E(u)} = 1$). The inefficiencies of the other firms are rated with respect to the full efficient firm. The most efficient firm is Firm no.15, which was found also to be the most technically efficient firm in Chapter 4 (see Table 4.3). However, high technical efficiency does not always lead to high cost efficiency; this is evident from the comparison of rankings in technical efficiency in Table 4.3 (Chapter 4) and rankings in cost efficiency in Table 5.2. The Spearman's rank correlation coefficient between ranks from technical inefficiencies in Model III (Table 4.3) and those from cost inefficiencies is 0.35 which is too low.

Table 5.3 is presented to show the overall descriptive statistics of cost inefficiencies. Most of the firms (73 percent of total firms) are in 1.5 - 2.0 inefficiency zone which means most of the firms have actual costs 50 - 100 percent higher than their corresponding frontier costs. Alternatively, if inefficiencies are totally eliminated, these firms would have the same yield with 33 to 50 percent less cost\(^{10}\). High negative value of skewness coefficient rightly points out this asymmetry in distribution; i.e, the distribution is skewed to the left. This is also

\[^{10}\text{For example, consider the highest inefficiency in this study, i.e, 1.95. Then actual cost (ACC) = frontier cost (FC) \times 1.95. Therefore, the reduction in cost (if inefficiencies are eliminated) is:}\]

\[
\frac{(ACC - FC)}{ACC} = \frac{(1.95FC - FC)}{1.95FC} = \frac{0.95}{1.95} = 0.487 \approx 49\%
\]
Table 5.3 Frequency distribution of cost inefficiency of sugarcane farms.

<table>
<thead>
<tr>
<th>Efficiency Rating*</th>
<th>Number of Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00-1.10</td>
<td>1</td>
</tr>
<tr>
<td>1.10-1.20</td>
<td>2</td>
</tr>
<tr>
<td>1.20-1.30</td>
<td>2</td>
</tr>
<tr>
<td>1.30-1.40</td>
<td>2</td>
</tr>
<tr>
<td>1.40-1.50</td>
<td>4</td>
</tr>
<tr>
<td>1.50-1.60</td>
<td>3</td>
</tr>
<tr>
<td>1.60-1.70</td>
<td>8</td>
</tr>
<tr>
<td>1.70-1.80</td>
<td>13</td>
</tr>
<tr>
<td>1.80-1.90</td>
<td>8</td>
</tr>
<tr>
<td>1.90-2.00</td>
<td>1</td>
</tr>
<tr>
<td>2.00-</td>
<td>0</td>
</tr>
</tbody>
</table>

Mean 1.6127  
Maximum 1.95  
Minimum 1.00  
S.D 0.2373  
Skewness -0.962  
Kurtosis 3.076

* Efficiency rating = 1.00 ⇒ Cost efficiency (100%)  
> 1.00 ⇒ Cost inefficiency.
clear from the plotted histogram in Figure 5.1. The distribution shows the highest probability (approximately 70 percent) for the 1.6 - 1.9 inefficiency range.

Next, to show the linkage between cost inefficiencies and comparative advantage of the firms, observed \( r \) and frontier CIR \( (r^*) \) s are derived. Both \( r \) and \( r^* \) s are plotted in Figure 5.2. Firms (in increasing sizes) are measured in the horizontal axis. The plot of \( r \) shows significant variations across firms with no definite trend (increasing or decreasing) with respect to increase in firm size. All firms (except one) have cost-income ratio below 1.0 level indicating that in the absolute sense almost all firms gained positive profit (in average) within the study period. \( r^* \), as expected, lies below \( r \), which implies that elimination of inefficiency lowers the cost-income ratio and increases profitability. Slight downward trend (with respect to increase in firm size) is noticed here. The variation in \( r^* \), however, is much less than the variation in \( r \). The estimated \( \lambda^2 \) is 2.52 which implies that \( (\sigma_u^2) \) dominates\(^{11} \) the variance of the pure random term \( (\sigma_v^2) \). This is also confirmed by Figure 5.2 where \( r^* \) is much flatter than \( r \). Thus, it may be deduced that inefficiency contributes much more than the pure random factors in the absolute variations of CIRs across firms.

\(^{11} \) However, \( \sigma_u^2 \) is not exactly equal to \( \text{Var}(u) \). Specifically,

\[
\text{Var}(u) = \left( \frac{\pi}{2} - 1 \right) \sigma_u^2
\]
Figure 5.1 Frequency distribution of cost inefficiency of the sugarcane firms.
Figure 5.2 Observed and frontier cost-income ratios (CIR) of the sugarcane firms.
The question is whether this contributes significantly to the relative variation in the CIR or comparative advantage. In other words, are the rankings of profitability also affected if inefficiencies are eliminated? To address this question, the CAD indices corresponding to both observed and frontier CIRs are to be computed. Table 5.4 shows the computed CADs in both without and with inefficiency cases. The computation procedure is the same as given in equation (5.16). Here, individual acreage (average harvested acres) is selected as the size index \( A_i \). The ranks are computed according to the descending order of CADs (i.e, highest CAD = 1, lowest CAD = 45) and are given within parenthesis next to each CAD. \( \text{CAD}_i \) column gives observed CADs (i.e, with inefficiency) and \( \text{CAD}_i^* \) column gives frontier CADs (i.e, without inefficiency). Since \( \text{CAD}_i \) and \( \text{CAD}_i^* \) are just linear transformations of \( r_i \) and \( r_i^* \), the relative variations are the same in both cases. A quick glance at the rankings reveals that frontier CADs (i.e, \( \text{CAD}_i^* \)'s) are ranked conclusively according to the increasing order of firm size. This pattern, however, is not obvious in the observed CADs. For example, comparison of firms 15 and 43 reveals that the former has comparative advantage although its size (420 acres) is almost one-third of the latter (1116 acres). The issue of higher size becomes important in observed comparative advantage if firms are broadly categorized according to their size, e.g, Category 1: 0 - 500 acres and Category 2: 500 - 2500 acres. In that case, all of the first 10 ranks (except rank 7) are held by the firms under Category 2 (i.e, big and medium sized firms) whereas the last 10
Table 5.4. Estimated observed and frontier comparative advantage index of the sugarcane firms.

<table>
<thead>
<tr>
<th>Firm ID</th>
<th>Acre Harvested (average)</th>
<th>CAD$_1$</th>
<th>CAD$_1^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>107</td>
<td>19.37 (43)</td>
<td>54.57 (45)</td>
</tr>
<tr>
<td>2</td>
<td>118</td>
<td>52.04 (40)</td>
<td>60.18 (44)</td>
</tr>
<tr>
<td>3</td>
<td>162</td>
<td>67.72 (37)</td>
<td>87.80 (43)</td>
</tr>
<tr>
<td>4</td>
<td>215</td>
<td>72.67 (35)</td>
<td>116.74 (42)</td>
</tr>
<tr>
<td>5</td>
<td>226</td>
<td>118.42 (25)</td>
<td>125.43 (41)</td>
</tr>
<tr>
<td>6</td>
<td>287</td>
<td>89.54 (31)</td>
<td>158.71 (39)</td>
</tr>
<tr>
<td>7</td>
<td>290</td>
<td>39.44 (41)</td>
<td>152.25 (40)</td>
</tr>
<tr>
<td>8</td>
<td>296</td>
<td>109.82 (28)</td>
<td>163.09 (38)</td>
</tr>
<tr>
<td>9</td>
<td>316</td>
<td>54.98 (39)</td>
<td>165.90 (37)</td>
</tr>
<tr>
<td>10</td>
<td>337</td>
<td>108.18 (29)</td>
<td>185.35 (36)</td>
</tr>
<tr>
<td>11</td>
<td>347</td>
<td>75.99 (34)</td>
<td>188.42 (35)</td>
</tr>
<tr>
<td>12</td>
<td>348</td>
<td>71.34 (36)</td>
<td>188.96 (34)</td>
</tr>
<tr>
<td>13</td>
<td>362</td>
<td>86.16 (32)</td>
<td>202.72 (33)</td>
</tr>
<tr>
<td>14</td>
<td>385</td>
<td>63.91 (38)</td>
<td>205.59 (32)</td>
</tr>
<tr>
<td>15</td>
<td>420</td>
<td>247.38 (7)</td>
<td>247.38 (30)</td>
</tr>
<tr>
<td>16</td>
<td>436</td>
<td>77.61 (33)</td>
<td>232.39 (31)</td>
</tr>
<tr>
<td>17</td>
<td>446</td>
<td>158.78 (19)</td>
<td>253.33 (29)</td>
</tr>
<tr>
<td>18</td>
<td>474</td>
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<td>273.50 (23)</td>
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<td>263.70 (27)</td>
</tr>
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<td>20</td>
<td>486</td>
<td>119.56 (23)</td>
<td>270.22 (26)</td>
</tr>
<tr>
<td>21</td>
<td>490</td>
<td>112.70 (27)</td>
<td>272.93 (24)</td>
</tr>
<tr>
<td>22</td>
<td>495</td>
<td>142.06 (20)</td>
<td>278.69 (22)</td>
</tr>
<tr>
<td>23</td>
<td>495</td>
<td>37.13 (42)</td>
<td>253.94 (28)</td>
</tr>
</tbody>
</table>

CAD = Comparative advantage index based on observed cost-income ratio;
CAD$^*$ = Comparative advantage index based on frontier cost-income ratio.
Table 5.4 (continued) Estimated observed and frontier comparative advantage index of the sugarcane firms.

<table>
<thead>
<tr>
<th>Firm ID</th>
<th>Acre Harvested (average)</th>
<th>CAD$_i$</th>
<th>CAD$_i^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>521</td>
<td>160.99 (18)</td>
<td>297.49 (20)</td>
</tr>
<tr>
<td>25</td>
<td>551</td>
<td>163.65 (17)</td>
<td>302.50 (19)</td>
</tr>
<tr>
<td>26</td>
<td>553</td>
<td>17.14 (44)</td>
<td>270.97 (25)</td>
</tr>
<tr>
<td>27</td>
<td>554</td>
<td>290.85 (35)</td>
<td>322.98 (17)</td>
</tr>
<tr>
<td>28</td>
<td>560</td>
<td>101.92 (30)</td>
<td>304.08 (18)</td>
</tr>
<tr>
<td>29</td>
<td>574</td>
<td>240.51 (10)</td>
<td>324.88 (16)</td>
</tr>
<tr>
<td>30</td>
<td>633</td>
<td>-41.78 (45)</td>
<td>286.75 (21)</td>
</tr>
<tr>
<td>31</td>
<td>640</td>
<td>165.12 (16)</td>
<td>360.96 (14)</td>
</tr>
<tr>
<td>32</td>
<td>642</td>
<td>119.41 (24)</td>
<td>355.03 (15)</td>
</tr>
<tr>
<td>33</td>
<td>645</td>
<td>246.39 (8)</td>
<td>372.16 (13)</td>
</tr>
<tr>
<td>34</td>
<td>736</td>
<td>216.38 (13)</td>
<td>420.99 (12)</td>
</tr>
<tr>
<td>35</td>
<td>800</td>
<td>222.40 (12)</td>
<td>458.40 (11)</td>
</tr>
<tr>
<td>36</td>
<td>802</td>
<td>267.87 (6)</td>
<td>468.37 (9)</td>
</tr>
<tr>
<td>37</td>
<td>814</td>
<td>243.39 (9)</td>
<td>474.56 (8)</td>
</tr>
<tr>
<td>38</td>
<td>865</td>
<td>125.42 (22)</td>
<td>467.10 (10)</td>
</tr>
<tr>
<td>39</td>
<td>925</td>
<td>185.00 (14)</td>
<td>516.15 (7)</td>
</tr>
<tr>
<td>40</td>
<td>964</td>
<td>315.23 (4)</td>
<td>561.05 (5)</td>
</tr>
<tr>
<td>41</td>
<td>969</td>
<td>136.63 (21)</td>
<td>526.17 (6)</td>
</tr>
<tr>
<td>42</td>
<td>1034</td>
<td>413.60 (2)</td>
<td>599.72 (4)</td>
</tr>
<tr>
<td>43</td>
<td>1116</td>
<td>167.40 (15)</td>
<td>603.76 (3)</td>
</tr>
<tr>
<td>44</td>
<td>1441</td>
<td>368.90 (3)</td>
<td>838.66 (2)</td>
</tr>
<tr>
<td>45</td>
<td>2170</td>
<td>564.20 (1)</td>
<td>1258.6 (1)</td>
</tr>
</tbody>
</table>

CAD = Comparative advantage index based on observed cost-income ratio;

CAD$_i^*$ = Comparative advantage index based on frontier cost-income ratio.
ranks (except ranks 44 and 45) are held by the firms under Category 1 (i.e., small sized firms).

The CADs are plotted in Figure 5.3. Relative smoothness in $CAD_i^*$, similar to $r_i^*$, reflects the dominating role of inefficiency in absolute variations. It is also important to note that $CAD_i^*$ shows a more definite increasing trend with respect to increasing firm size. This implies that if inefficiencies are fully eliminated from all firms under study, a firm with bigger size will have more profitability than a firm with smaller size. This conclusion also holds for observed comparative advantage if the firms are categorized as mentioned in the previous paragraph.

The contribution of inefficiency in relative variation of CADs can easily be derived by comparing the ranks in $CAD_i$ and $CAD_i^*$. Statistically, this can be done by estimating Spearman's rank correlation coefficient ($r_s$). The estimated coefficient is 0.7644. Since $r_s$ precisely estimates the ratio of explained variation to the total variation between the ranks of two variables (i.e., relative variation of the variables), the contribution of inefficiency in CADs may be explained in terms of this coefficient. According to the estimated value of $r_s$, roughly 24 percent ($1 - 0.76$) of the ranks have changed due to shift from observed CAD to frontier CAD. On the other hand, the absolute variation of CAD is determined by the contribution of inefficiency variance in total error variance (approximated by ratio
Figure 5.3 Observed and frontier comparative advantage index (CAD) of the sugarcane firms.
of the two) and is equal to 0.59\(^{12}\). Thus, although the effect of inefficiency on the absolute variation in CADs is significantly high (59 percent), it is not so high in the case of its effect on relative variation (i.e., ranking). Since the other source of variation is pure random error (in cost), the conclusion is that the probability of achieving favorable outcome of uncertainties largely determines the rank of a firm under this study in comparative advantage echelon.

5.6. Summary

In this chapter the methodological and empirical issues regarding the link between cost inefficiency and comparative advantage have been discussed on the basis of a theoretical framework. The proposed methodology involves the separation of inefficiency from the total random error and derivation of frontier cost-income ratio and frontier CAD. An alternative procedure of computing comparative advantage is proposed in the case where sufficient time-series data are not available. For empirical application, the case study in Chapter 4 on several Louisiana sugarcane firms has been further explored from a different and broader perspective. The results indicate that (i) as a whole, the group of firms have slight

\(^{12}\) Since \(\text{Var}(u) = (\pi/2 - 1) \sigma_u^2\), the effect of inefficiency on absolute variation can be derived as follows:

\[
\frac{\text{Var}(u)}{\text{Var}(u) + \text{Var}(v)} = \frac{(\frac{\pi}{2} - 1) \lambda^2}{(\frac{\pi}{2} - 1) \lambda^2 + 1} = \frac{1.44}{2.44} = 0.59
\]
economies of size; (ii) inefficiency is a major contributing factor in the absolute variations of observed CIRs and CADs across firms; (iii) it is not the major factor in relative variations in the comparative advantage ranking. Almost 24 percent of the variation in ranks may be attributed to inefficiency; (iv) medium and large firms (more than 500 acres) as a group have better profitability than the group of small firms (below 500 acres), but individually some firms are ranked lower in CAD than some much smaller firms; (v) if inefficiency is totally eliminated from each firm, the profitability or comparative advantage of larger firms dominates.

Better profitability of the group of medium and large firms in the present situation points out an important implication. This study best supports the explanation that small firms generate low incomes, and these low incomes cause such firms to exit, become part-time units, or expand to increase income whether or not efficiency exists. Such firms can be viable in regions where family income can be supplemented by off-firm income. Otherwise, farmers tend to enlarge their operations in search of higher incomes, rather than to increase cost-efficiency. This, however, does not go without costs to society; expansion without control on inefficiency is most likely to cause the sacrifice of benefits in terms of lower real food costs that consumers could get had inefficiency been controlled.

It may also be argued that in the present situation any policy designed to limit the size of a firm (for example, the 160-acre limit of the Reclamation Act of 1902) may not lead to better performance of the industry. This is because the small firms under this study do not show any conclusive evidence of higher efficiency. If,
however, such policies are supplemented by policies which directly address the inefficiency problem (such as, more extension facilities, training and education offered to farmers or managers), better performance and higher benefits to society may be expected.
6.1. Summary

The general objective of this study was to provide a methodological framework for evaluating stochastic comparative advantage and its link with economic efficiency at the firm level. Theories and analytical techniques from both neo-classical and modern micro-economics were used to accomplish this task. The following is a restatement of the specific objectives. The procedures used and the major results are discussed briefly after each objective.

Objective 1

To build a theoretical framework for analyzing firm's behavior under uncertainty in costs and returns and to define comparative advantage of the firm under stochastic conditions.

The cost-income ratio (CIR) was selected as an indicator of comparative advantage. This indicator was justified on several grounds, the most important being its capability to capture stochastic elements from both cost and revenue sides. Although per unit net return ($\pi$) is directly related to the CIR and may be equally powerful as indicator of profitability, CIR has additional advantages with regard to estimation of cost efficiency and economies of size. The classical theory of firm optimization was recast in terms of the indicator under the assumption of both
certainty and uncertainty in cost and output price. Since comparative advantage under uncertainty is related to relative profitability, the model under uncertainty was elaborated on the basis of different assumptions about the general sources of uncertainty. Comparative static results were derived by using calculus and statistical methods. Graphical tools were also used to explain the concept of comparative advantage.

The derived theoretical results showed that under the assumption of uncertainty and expected utility maximization hypothesis, maximum profitability of a firm implies the highest probability of attaining optimum CIR where optimum CIR was defined the CIR corresponding to optimum level of output. Optimum level of output depends on firm's reaction against uncertainty and it is generally less than output under certainty.

Objective 2

To develop a methodology for analyzing regional stochastic comparative advantage in crop production in Louisiana.

Profitability or comparative advantage was assumed stochastic and a function of the CIR. Due to randomness any evaluation of profitability must be expressed in probabilistic terms. The procedure involved forming a matrix of cumulative probabilities of CIRs for five crops (rice, cotton, soybeans, corn, and sugarcane) in seven mutually exclusive regions (Red River, Ridge, Central, Southwest, Delta, Sugarcane, and Others) in Louisiana. The regional cost and return data for the
period 1956-1988 for each crop (in each region) was used to estimate the cumulative probabilities at three hypothetical CIR level: 1.0, 0.9, and 0.8. A flexible statistical method (hyperbolic trigonometric transformation method) was used to estimate both unconditional and conditional probabilities. For estimation of conditional probabilities, CIR of previous year was assumed to be the conditional variable. After, the unconditional and conditional absolute advantage of a crop in a region were calculated by multiplying the probabilities with respective expected net returns per acre, the latter being proxied by average of net returns per acre over the last five years. Finally, pairwise comparative advantage of crops were derived in terms of the region in which a crop in a pair has comparative advantage over the other crop in the pair.

The results from the estimated probabilities and comparative advantage revealed heterogenous survival potentiality of each crop across regions. Rice was found to have comparative advantage in the SW Rice area with respect to cotton, soybeans, and sugarcane; soybeans is expected to have better performance in the Central area; cotton has comparative advantage in the north and west Louisiana and in the Red River and the Central area; corn has comparative advantage in the Sugarcane area (over soybeans) and in the Southwest (over cotton); sugarcane has comparative advantage in the Sugarcane area over most crops and it also has good potential for profitability in the Southwest area.

The estimated conditional PDFs showed that rice, cotton, and corn have higher response to change in previous year’s CIR. Specifically, if cost and return
remain the same as in the last year, a decrease in the CIR due to shift in exogenous factor (e.g., an increase in government support) will lead to higher profitability for these crops than soybeans or cotton. Computed conditional comparative advantage showed that in some cases a hypothetical shift in the conditional variable changes the comparative advantage ranking implying that change in government supports equally for all crops may lead to change in the comparative advantage ranking of some crops in the next year.

Objective 3

To discuss methodological issues of inefficiency estimation and analytically derive and estimate the firm-specific technical and allocative inefficiencies in sugarcane production in Louisiana.

An unbalanced panel data of 45 sugarcane firms of different size was used to estimate technical and allocative efficiencies. Since distributional assumptions about efficiency play a crucial role, especially in the context of estimating efficiency, it is important to consider various alternative distributions to see which fits best. For this reason, five different models of technical efficiency were initially specified by imposing different restrictions on an unrestricted model: (i) fixed-effect model, (ii) random effect GLS model, (iii) stochastic frontier model with inefficiency distributed as half-normal, (iv) same as (iii) but inefficiency distributed as truncated normal, and (v) time-variant model. While the first four models assume time-invariance of inefficiency, model (v) assumes that inefficiency changes
monotonically over time. Parameters of the production function and technical efficiency for each firm were estimated by each model. To select the appropriate model, a sequential test procedure was followed. For the estimation of allocative inefficiencies, the dual cost function was analytically derived from the estimated production frontier function. The errors in share functions were expressed in terms of allocative inefficiencies of capital and labor with respect to fertilizer. Then inefficiencies for each input (i.e., capital and labor) were derived by solving the set of share equations.

The estimated parameters of the frontier function by different models showed approximately constant returns to scale and the highest output elasticity of land (approximately above 0.76 in each model) followed by fertilizer (above 0.09). Labor and capital were found to have very small elasticities (below 0.03 and 0.04 respectively).

Although the estimated technical efficiencies vary across models, the ranking is quite indifferent to model specification. Assuming that the most efficient firm is 100 per cent efficient, all models showed the rest of the firms to remain in 50 - 100 per cent efficiency range. Since the sequential test procedure favored the time-variant model, the firm-specific efficiencies were also estimated for each year on the basis of unrestricted model. In this model efficiencies are found to increase at very low rate over the study period. The efficiency ranking of firms, however, remained almost the same as in the time-invariant models. This implies that model selection is not a big issue in this particular application when the main interest of research
is ranking of firm-efficiency. Firm-size was not found to be significantly correlated with efficiency; in other words, the hypothesis of higher efficiency of larger firms was rejected.

Estimated allocative efficiencies revealed that fertilizer is used at an over-optimum level with respect to labor and capital. This implies that, given input prices, the use of fertilizer and chemical may have contributed in technical efficiency at the cost of high allocative inefficiency. Also, the rankings of firms according to their technical and allocative efficiencies were not found to be correlated. However, the firms are found more efficient in technical sense than in allocative sense.

Objective 4

To extend the theoretical analysis in objective (1) and empirical analysis in objective (3) to analyze firm-specific comparative advantage and its link to cost inefficiency of firms.

The concept of comparative advantage defined in objective 1 was used to define comparative advantage at the firm level. Specifically, inefficiency was identified as one of the major source of absolute and relative variations of profitability among the firms. Stochastic models of the cost and the CIR frontier of the sugarcane firms were specified and estimated to define firm-specific comparative advantage in terms of the pure random errors and inefficiency. Due to data limitations, an alternative simple procedure of computing comparative
advantage index (CAD) was proposed which involved the use of average CIR in place of cumulative probability of CIR and the use of a firm size index in place of expected net return as weight. Frontier CIRs and CADs for all firms were computed by using the alternative definition. Impact of inefficiency on relative variation (or, ranks) of comparative advantage was estimated from the rank correlation coefficient between observed and frontier CADs.

The estimated frontier cost function parameters showed slight economies of scale which implies that if inefficiencies are eliminated from all firms, the group of firms will experience higher economies of scale. This conforms the result derived from estimation of production frontier where estimated function coefficient was slightly higher than unity.

The estimated cost inefficiencies showed that almost 73 percent of the firms have actual costs 50 - 100 percent higher than their respective frontier costs. In other words, if inefficiencies are totally eliminated, these firms would have the same yield with 33 to 50 percent less cost.

Absolute variation in frontier CIR was much lower than that in observed CIR which implies that inefficiency contributes much more than the pure random factors in the absolute variations of CIRs across firms. The frontier CADs showed more definite pattern in their relation to firm size than did the observed CADs. In other words, we should have more faith in the statement that a firm without inefficiency has more profitability than a smaller firm (without inefficiency) than we should in the case where both firms have inefficiency. If, however, firms are
categorized into two groups: firms below 500 acres and firms above 500 acres, then the second group as a whole has comparative advantage than the first group.

The rank correlation coefficient between observed and frontier CADs was 0.76 which roughly indicates that 24 per cent of the rank variations can be explained by inefficiencies alone. Thus, the contribution of inefficiency in comparative advantage rankings are lower than that of pure random factors.

6.2. Conclusion

The main objective of this study was to develop a methodology by which the overall performance of an industry or a crop can be evaluated under uncertainty. This study supports the intuitive hypothesis that the evaluation of the internal performance of an industry does not necessarily reflect the economic viability of that industry in a region. Similarly, the evaluation of external performance (or, economic viability) of an industry does not necessarily reflect the production performance and viability of a firm.

The advantage of the methodology proposed in this study is in its simplicity and potential for application. The assimilation of traditional comparative advantage theory with the theory of the firm under uncertainty gives an appropriate basis of the decision making process regarding the location of farming. The use of the concept of stochastic frontier also helps identify efficiency as a possible source of variation in profitability. Therefore this methodology is potentially useful in deriving the comparative advantage of any crop in a region and measuring
inefficiencies of the firms producing that crop. This is also useful in deriving the
correlation of inefficiency in the relative profitability of a firm and thus may help
producers choose better ways to survive.

Although comparative advantage for all major crops was derived by regional
comparative analysis, the main focus of empirical demonstration of the methodology
was on sugarcane production in Louisiana. The production of sugarcane is
concentrated mainly in the Sugarcane area (Southeast) of the state. It is also
produced in the Central and the Southwest areas. From this study, it can be
concluded that sugarcane has a higher probability of profitability than rice and
soybeans in the Sugarcane region. Similarly, it is expected to show better
profitability than cotton and corn in the Southwest area. The historical cost and
income structure, however, failed to give sugarcane a comparative advantage over
any crop in the Central area. These results may be considered as an initial step in
the study of production specialization.

From the conditional comparative advantage analysis, it was found that the
comparative advantage of sugarcane in the Sugarcane and the Southwest area is
more or less invariant to exogenous shock (if that shock is equally applied to other
crops). In other words, the pattern of relative profitability of sugarcane remains the
same when the shift of exogenous factors is equal for all crops. This, however, is
not true for all crops. This result helps construct the hypothesis that increasing
government supports would not play a significant role in the relative profitability of
sugarcane.
The inefficiency analysis of the Louisiana sugarcane industry indicated only a moderate level of cost efficiency. This implies that the area internal production performance of sugarcane is not as good as its area external production performance. The main source of cost efficiency was found to be technical efficiency which implies that sugarcane farmers within the sample are more efficient in utilizing the existing technology than utilizing market information. This is accompanied by higher profitability for the group of large firms even if they are equally cost efficient as small firms. Together these facts imply that the large firms are gaining ground in the industry due to three reasons: (i) efficient use of inputs (technical efficiency), (ii) higher adjustability to random shocks, and (iii) higher income generating capacity. In other words, large firms show more economic strength in the Sugarcane region. The impetus to concentration is accompanied by the neglect of total efficiency which implies additional cost to society in terms of higher food prices in the long run and waste of resource.

6.3. Data limitations

The data set used to derive regional comparative advantage was of secondary nature and included cropwise regional cost, yield, and price data for the period 1956-1988. More recent years could not be incorporated due to non-availability of data. The aggregation and use of imputed or projected values in computing costs may generate significant aggregation bias and measurement error. Further, no other regional characteristics such as firm population, irrigation or other input
facilities, local inputs (such as percentage of arable lands, topography, etc.) were incorporated in the data set. This precluded the investigation about sources of comparative advantage. Also, the averaging process did not incorporate any information about intra-regional variation of labor quality, productivity, cost and return.

The data set used for firm specific inefficiency analysis also has limitations. Although the possibility of aggregation bias is eliminated in firm-specific data, the format of the data set was not as useful as it could have been in applying stochastic frontier methodology. The absence of firm-specific input usage, input price, and firm-specific characteristics led to imposition of restrictions on the model. Some input costs were over-represented (for example, fertilizer cost included fertilizer and chemical cost till 1988) and some were under-represented (for example, labor cost did not include imputed family labor cost and capital cost did not include depreciation cost). The data on some important input costs (for example, costs on seed) were available only for a small number of firms.

6.4. Future research directions

The methodology derived and applied in this study can be improved or extended in many directions. The main focus in this study was on random economic factors determining regional comparative advantage and resource use.
The cost-income ratio reflects this economic side of the decision making process\(^1\). However, following Heady (1952) it can be argued that there are other factors which may not be explained purely in terms of economics. For example, "...since Scandinavians settled in northeastern Iowa and Minnesota, dairy production should be concentrated in these areas where the dairying skills of the people were developed in their homelands." (Heady (1952), p-662). Similarly, it is likely that people who place a high value on one particular set of skills for a product, may migrate to areas where that commodity has a high rate of substitution for other products. Although such factors are not prominent in the empirical application of the present study, these should be taken into account in other applications, if necessary. Further, if resource allocation is to be evaluated from the view point of a society as a whole, the present analysis should be extended on the basis of a cost-income ratio which explicitly takes costs and income arising from production externalities into account. One example is the inclusion of environmental costs in total cost.

A second possible direction of future research is measuring absolute and comparative advantage in the presence of interdependence of profitability among crops. Although independence of the PDFs is a reasonable assumption in the present application, it may not be so in the case where one crop is treated as

\(^{1}\) Note that since climate, soil, topography also determine yield and monetary cost of production, they are reflected in the cost-income ratio. Similarly, this ratio captures market locations, transportation and handling costs which are reflected by price. Thus all these factors are included in the economic side of the decision making process.
supplementary or substitute enterprise to another (e.g., sugarcane and sugarbeet). The complexities of the decision-making process in such situations require more flexibility in the assumption of conditional distribution of the CIR.

Another possible area of research is to investigate the causes behind comparative advantage of a crop in a region. Since resource endowment and the efficiency in the use of existing resources are major determinants of comparative advantage, this type of analysis should focus on: (a) region-specific physical and human resource characteristics, such as, topography, soil, climate, farm-nonfarm population ratio, etc., and (b) deriving region-specific production efficiency for each crop and measuring its link toward comparative advantage. For example, the reason behind differences in sugarcane's profitability in the Sugarcane and the Southwest Rice areas may be investigated by analyzing the link between average technical efficiency and absolute advantage in these two regions, and/or the contribution of region-specific characteristics.

Regarding inefficiency analysis, one of the most important directions of future research is to investigate the causes of firm-specific inefficiencies. The research may be directed towards addressing either or both of the questions: (a) which input(s) is (are) most efficiently used? In this study, the notion of technical efficiency encompasses the efficiency of total factor employment. These aggregative measures are incapable of identifying inefficiency of an individual input. "In a sense, these measures treat the contribution of each factor to productive efficiency equally and thereby mask any differences in efficiency that might be attributed to
particular factor inputs. For example, the parsimonious use of fuel and excessive use of capital can yield the same technical efficiency as the reverse pattern of factor use." (Kopp (1981), p.491). Thus the idea of technical inefficiency should be extended to a more disaggregate level, viz., input-specific technical efficiency - inefficiency attributed to each of the inputs used by a firm and develop a method to estimate such inefficiencies in a panel data framework\(^2\); (b) which firm-specific socio-economic characteristics contribute significantly in technical inefficiency? Since technical inefficiency generally arises from managerial ineptitude and tenurial arrangement (Kalirajan (1981), p-289), this type of research would involve information on managers' education, experience, technical knowledge and training, involvement of extension officials, tenurial arrangement and testing the contribution of each of them on estimated technical inefficiency.

Theoretical and empirical research may be conducted on time-variance property of technical inefficiency. This study used a restricted version of time-variance model which does not permit sufficient flexibility in modeling technical efficiency. For example, if inefficiency of one year depends on inefficiency of the previous year, the method used in this study would not be appropriate and estimated inefficiencies would be biased\(^3\). Also, the impact of specification error

\(^2\) See Kumbhakar (1988) for the discussion of one such method.

\(^3\) Although for small time series observations this bias is not expected to be much significant.
on time-variance properties may be derived theoretically; it has significant empirical implications.

Regarding allocative inefficiency, future empirical research may be conducted in the direction of re-estimating allocative inefficiencies in a system framework to compare the results with the findings in this study. The main focus of research would be whether allocative inefficiencies are systematic. Given the productive role of land and fertilizer, it is reasonable to hypothesize that farmers tend to reap maximum advantage from these two inputs by systematically over-utilizing (in allocative sense) these two inputs. This possibility can be allowed by permitting a disturbance with a non-zero mean in the cost minimizing condition. By testing whether these means are zero, it can be tested whether or not there are in fact systematic deviations from the cost minimizing input ratios.

6.5. Implications for sugarcane farms in Louisiana

The implications delineated above are in the direction of future research on the methodology proposed in this study. Since this study used data on sugarcane firms in Louisiana, relevant implications of this study on economics of sugarcane production by these firms may also be traced out. The following is a discussion on the major findings and some directions on how the firms should use the generated information to assess and improve their efficiency and profitability.

Estimates of elasticity of output with respect to land input were found significantly high in each frontier model. Given the statistical superiority of
stochastic frontier models, the estimated elasticity may be roughly approximated as 0.88. This means a 100 per cent increase in land input (harvested acres) results into 88 per cent increase in yield of raw sugar. On the basis of this finding, it can be said that extensive cultivation (i.e., putting more area under harvest) is the best policy to increase yield.

Fertilizer has the second highest elasticity in each model (the range being 0.09 - 0.15). Once again, on the basis of stochastic frontier models, it is deduced that a 100 percent increase in fertilizer (and chemicals) use will result into 10 percent increase in yield (all other inputs remaining the same). In other words, in a given amount of land input, increase in fertilizer (and chemicals) use on average will be more effective than increase in labor or capital input.

Estimated technical and allocative efficiencies showed heterogenous resource use pattern across firms. Most of the firms (almost 80 percent of total firms) were found to be in the technical inefficiency range 0.7 - 1.0. In other words, most firms are getting 0 - 30 percent less than the maximum yield (or, the frontier yield). Alternatively, if technical inefficiencies are eliminated 80 percent of the firms will be able to increase their yield by 30 percent at most. This study also ranked the firms according to their efficiency levels so that this information could easily be used by one firm to assess its performance with respect to the same of another firm.

However, inefficiencies were found to be more significant when the firms allocated their resources in accordance with market information (allocative inefficiency). Almost 66 percent of firms were found 50 percent or more inefficient
than the most efficient firm with respect to allocation between labor and fertilizer (and chemicals). On the other hand, almost 85 percent of the firms were found 50 percent or more inefficient than the most efficient firm with respect to allocation between capital and fertilizer (and chemicals). Moreover, in both cases (labor and capital), the inefficiency came from over-optimal use of fertilizer (and chemicals).

The above findings point out two important implications which may be used as hypotheses for further research. The hypotheses are: (a) sugarcane farmers are neglecting allocative inefficiency (i.e., underallocating labor and capital with respect to fertilizer) to reap the benefits of higher output elasticity of fertilizer; and, (b) the over-optimal use of fertilizer is also caused by technical inefficiency in using it.

While the neglect of allocative efficiency can sometimes be justified on the ground of higher yield and higher cash flow, the neglect of technical efficiency (especially for the smaller firms) cannot be justified on any ground. Thus, on the basis of this study, it is strongly recommended that firms should be more concerned about monitoring technical inefficiency. Given the management structure of individual firms, this would require better understanding of the production technology and more extension assistance.

The inefficiencies were also estimated from a different and wider modelling perspective, i.e., cost inefficiencies for the individual firms were estimated from the cost function. Most of the firms (73 percent) were in the 1.5 - 2.0 inefficiency zone which means if inefficiencies are totally eliminated, these firms would have the same yield with 33 to 50 percent less cost. Since cost inefficiency is a combination
of technical and allocative inefficiency, the high range of inefficiency also supports the existence of high allocative inefficiency within the firms.

Finally, the results showed that the contribution of inefficiency in absolute profitability is much higher than that in relative profitability (or, comparative advantage ranking). In other words, if inefficiencies are totally eliminated from all the firms, return per $1 cost will increase significantly for all firms but relative rankings will not change significantly. That is especially true if firms are grouped into two categories: (i) less than 500 acres and (ii) more than 500 acres. Category (ii) showed conclusive evidence of higher profitability than category (i) with or without inefficiency. However, if firms under category (i) can remove inefficiency substantially while the firms under category (ii) remain inefficient, the relation would be reverse. Thus, the need to address inefficiencies is much greater for firms under 500 acres than the same for the firms above 500 acres. Again, subject to further research on allocative efficiency, the small farmers may want to address the technical inefficiencies first.


VITA

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DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: Barun Kanti Kanjilal

Major Field: Agricultural Economics

Title of Dissertation: Regional Comparative Advantage and Inefficiency in Production—Methodological and Empirical Issues

Approved:

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

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