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Fatigue Failure of Composite Bolted Joints.

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Herrington, Paul Douglas, Ph.D.

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FATIGUE FAILURE OF COMPOSITE BOLTED JOINTS

A Dissertation

**Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy**

in

The Department of Mechanical Engineering

by

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B.S.M.E., University of New Orleans, 1982
M.B.A., Tulane University, 1985
December, 1991**

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ABSTRACT

The objective of this study is to investigate the fatigue behavior of a double-lap, bolted composite joint. The material used for all tests is a quasi-isotropic graphite-epoxy laminate. Prior to fatigue testing, monotonic loading tests were performed in order to establish the important fundamental properties of friction coefficients and the ultimate bearing strength of composite joints.

Proper design of friction type bolted joints for composite materials accounts for the force transmitted through friction between the clamped materials. This consideration requires knowledge of the coefficient of friction between the mating surfaces of the bolt, or washer, and the composite material. In this study, the static and kinetic coefficients of friction between standard washers and the graphite-epoxy composite are found as a function of clamping force and outer ply angle.

When composite materials are bolted together, the relative diameters of the bolt hole, the constraining washer and the bolt itself have a profound effect on the ultimate bearing strength. In this investigation, the effect of the difference between the bolt diameter and the inside diameter of the washer, which provides the lateral constraint at the edge of the hole, is examined. To explain the significant variation of bearing strength with increasing lateral constraint, a model is proposed to explain the mechanics of load transfer and the lateral deformation at the edge of the hole. The test results support the proposed model and provide data for designers.

Fatigue data was generated using constant loading conditions for a double-lap bolted composite joint. The independent variables included six levels of maximum applied stress, four levels of clamping torque, and three different outer ply angles. Analysis of variance, censoring techniques, and goodness-of-fit tests were used to analyze the fatigue data. It was shown that the fatigue life data could adequately be described using the log-normal distribution.

Chapter 1

Introduction

A composite material is one in which two or more different materials are combined in order to create a material system whose properties cannot be provided by a single material. Commonly known materials include plywood, automobile tires, asphalt and concrete. In this work, the composite material system consists of unidirectional continuous strands of graphite fibers embedded in an epoxy matrix.

The load applied to a composite may be transferred, in part, from the matrix to the fibers by means of shear forces acting on the fiber surface, therefore their surface area must be large in relation to their cross sectional area. In other words, an efficient reinforcing element will be much longer in one dimension than it is in the other two. Reinforcing fibers of cotton, rayon, and glass have been used in the past, whereas now "advanced composites" generally refer to composites with fibers of boron, graphite, and Kevlar. Since a group of fibers has little rigidity and as such, very limited structural value, a matrix material is used to bind the fibers. Matrix materials may be metals such as aluminum or magnesium, or as is most commonly used, thermosetting polymers. The matrix not only binds the fibers to create a structural member, but increases the toughness of the system and is mainly responsible for the material properties transverse to the fiber direction.

The properties of an advanced composite are shaped not only by the kind of matrix and reinforcing material, but also by the geometry of the reinforcement,

which is a factor totally distinct from composition. Successive layers can be oriented in different directions to give the composite system stiffness and strength along several axes. Therefore, the internal structure of the composite can be designed to bear the anticipated load in any direction. The ability to specify the internal geometry gives the designer unmatched control over the material. To use this ability most efficiently, it must be possible to predict the behavior of any given laminate knowing the behavior and properties of each individual layer.

In addition to the ability to match the stress distribution to the internal geometry of the material, composites offer other attractive mechanical properties. The specific gravity of a typical fiber reinforced composite is approximately 1.7, compared to 3 for aluminum, and 7.9 for steel. This low specific gravity combined with the relative ease of forming makes high strength and/or stiffness to weight ratios possible. Specific strength (ultimate strength divided by specific gravity) versus specific stiffness (elastic modulus divided by specific gravity) for typical composites and structural materials are shown in Figure 1.1. These factors along with the fact that polymers do not oxidize or corrode in normal moist air make fiber reinforced composites especially attractive for the aerospace, automotive, and chemical industries.

Figure 1.1 Specific strength and specific stiffness for typical structural materials.

The characteristics that allow a designer to tailor a structural part to withstand applied loads with a minimum weight also requires special design criteria due to the anisotropic behavior of composites. The degree of anisotropy can be seen from Figure 1.1 which shows longitudinal (along the fiber length) and transverse (perpendicular to fibers) specific strength.

In order for composites to be used as structural members, joining technologies are required that take into consideration the anisotropic nature of these materials. Typically, failures that develop in structures usually originate at joints. Joints may be designed using adhesive bonding or mechanical fastening, with bonding as the preferred method when maximum joint efficiency is desired. However, the

need for disassembly may outweigh the disadvantages of increased weight and higher stress concentrations associated with mechanical fastening. As shown in Figure 1.2, mechanically fastened joints typically display three principal failure modes: net tension, shear out, and bearing. Combinations of these failure modes are also a possibility.

Figure 1.2 Typical bolted joint failure modes.

Bearing failure in joints may be avoided by the use of thin metal shims located throughout the laminate hold area. The shims are quite effective in enhancing the bearing strength in the joint area. Lateral constraint due to bolt torque can also greatly increase bearing strength. The tensile strength of the material in the reduced cross section can be improved by increasing the spacing between bolts and transferring the load through several rows of bolts, providing a sufficient bolt shear area. Unidirectional composites have low shear strength in the longitudinal direction and hence bolt loading may result in the shear out mode of failure. One way to increase the shear strength is to include the use of off-axis plies, however, this results in an overall lower net tension capability. Another problem unique to composite materials is that the holes in the laminate cause stress concentrations that vary with the fiber orientation relative to the load direction. This problem is multiplied by the fact that composites are generally brittle materials and hence do not plastically deform to redistribute the stresses.

The inhomogeneous nature of fibre composites leads to the generation of widely different stress levels within the material. For example, high local stresses are generated at the edge of a loaded hole, possibly leading to debonding between plies. Another possible region for locally high stresses are the voids and defects produced during the manufacturing process. These local regions of high stress concentrations are potential locations for the initiation of fatigue damage and ultimate failure.

Fatigue failure mechanisms of composite materials must be studied and understood before widespread use can be expected. Generally, the number of loading cycles to fatigue failure increases with a decrease in the applied load. Both polymeric and metallic materials undergo fatigue failure by the initiation and

incremental growth of cracks and are usually characterized by a limiting stress below which fatigue failure does not occur. However, composite laminates fail as a result of the accumulation of many small cracks as opposed to a single dominant crack as in metallic members.

Factors Affecting Mechanically Fastened Joints

There are a number of parameters that affect the strength of mechanically fastened joints. Some are geometric parameters while others are physical material properties. The following lists a number of such parameters.

A. The two geometric parameters which have the most influence on the failure mode of bolted joints are the width of the joint and the distance from the bolt to the edge of the laminate, Figure 1.2. The width of the joint must be sufficient in order to prevent a net tension failure. Likewise, a bolt hole close to an edge will fail in the shear-out mode unless there is sufficient distance from the edge to provide an adequate shear area to support the load. With an adequate width, W , and distance from the edge, E , to prevent failure in shear-out and net tension, failure in the bearing mode becomes important. Figure 1.3 shows that bearing strength is proportional to W/D or E/D at small values, and reaches a constant, beyond a transitional value where bearing failure ensues. This corresponds to a transition in failure mode, from net tension to bearing or from shear out to bearing. Van Scilen [1] and Hart-Smith [2], obtained similar results for the influences of W/D and E/D on bearing strength. Figures 1.3, obtained by Jurf [3], shows the effects of W/D and E/D ratios for quasi-isotropic laminates under static loading.

Figure 1.3. Bearing Stress as a function of width to diameter and edge to diameter ratio. Shown also is effect of lateral constraint. Jurf [3].

B. There often exists a clearance between the fastener and the hole which has an effect on the strength of the joint. Under static loading, Crews and Naik [4] have illustrated the influence of clearance for a pin loaded hole in a graphite epoxy laminate. For a clearance of only 0.8 percent of hole diameter, the peak tangential stress around the hole boundary was 12 percent higher than the snug fitting case. Hyer, Klang, and Cooper [5] have shown that clearance reduces the load capacity of a quasi-isotropic laminate and also changes the location and direction of the maximum stress. It is a difficult task to account for clearance in a stress analysis since it leads to the problem of the contact region at the bolt-hole interface which increases nonlinearly with the bearing load.

C. Another geometrical parameter that may have an effect on the failure mode is the thickness of the laminate. Jurf [3] has stated the assumption that due to the relatively low out-of-plane strength of composites, the ultimate bearing strength is a function of laminate thickness. Due to the lack of restraint from other plies, the exterior plies are the most likely to fail. Thicker laminates have a fewer percentage of the total plies near the exterior of the laminate, and hence the bearing strength should increase. On the other hand, Collins [6] has stated that thickness becomes insignificant when a lateral constraint such as a bolt-washer combination is used.

D. Also shown to have an effect on laminate strength is washer diameter. When small washers were used, bearing failure occurred ahead of the washer rather than just ahead of the bolt. Jurf has shown that as the washer diameter increased so did the bearing strength [3]. At large washer diameters, bearing failure was located totally beneath the washer at the bolt front, and the bearing strength was no longer dependent on the washer size. Similar results were presented by Stockdale and

Matthews [7]. This effect could be due more to an upper limit of washer size than its influence as an independent parameter. Although no reference has been found, it may be expected, as in bolting isotropic materials, an upper limit of effective washer size to be 3 times the hole diameter. Beyond this size, larger washer diameters will not provide a greater contact area between the washer and the laminate.

E. Friction between the washer and laminate should have an effect on joint strength as noted by Crews [4], Stockdale and Matthews [7], and Smith, et al. [8]. Before the bolt comes into contact with the laminate, the applied load must be greater than the normal force between washer and laminate times the coefficient of friction. Values of the coefficient of friction for various lay-ups and surface finishes of a graphite/epoxy composite are limited, Roberts [9], and will be a focus of this study.

F. In considering the effects of out of plane stresses and surface constraint on the bolted joint strength, a question is raised as to the contributions made by free edge interlaminar stresses. Quinn and Matthews [10] performed experiments measuring the bolted joint strength of different stacking sequences. A phenomenon unique to laminated materials is that in a region near a discontinuity, the stresses in each ply must redistribute in order to achieve stress equilibrium. This results in interlaminar normal and shear stresses at the free edge, which if sufficiently large, can initiate failure by edge delamination. There is strong experimental evidence suggesting that laminates subjected to positive (tensile) interlaminar stresses have lower strengths. Stockdale and Matthews [7] have investigated stacking effects and have shown that the strongest laminates have 90

degree plies at or near the surface. These plies near the surface have a tendency to suppress the inner plies, similar to an externally applied lateral constraint.

G Increasing lateral constraint of a loaded composite by applying a clamping torque to the bolt has been shown to greatly increase bearing strength. Crews [11] reported higher bearing strengths by as much as 100 percent when comparing a high clamp up torque to an unconstrained case. Collings [12] noted an increase in bearing strength with increasing lateral constraint, however, it was noted that there exists an upper limit of bolt torque above which no additional increase in strength will be realized. Without exception, greater lateral constraint have resulted in higher bearing strengths, regardless of material type and lay-up.

The factors which influence bolted joint behavior, as discussed above were established under static loading conditions. The purpose of this study is to determine the effect of these variables on the fatigue performance of composite bolted joints.

Mission of the Study

The intention of this work is to advance the body of knowledge concerning the influence of design and geometrical parameters on the fatigue behavior of bolted joints in a laminated graphite-epoxy material. To attain this goal, each of the following chapters represents a building block, establishing a foundation for the subsequent chapters. Chapter 2 is an outline of the composite material and test equipment used throughout this study. Chapter 3 details the results obtained from testing performed to determine the coefficient of friction between the composite plate and typical bolting components. The distribution of planar stresses within the plate is established and analyzed in Chapter 4. Based on the results of Chapters 3 and 4, a model to predict the ultimate bearing strength of the composite bolted joint was derived in Chapter 5. Once the ultimate bearing strength was determined, fatigue testing could begin. Chapter 6 presents and discusses the results obtained from fatigue testing. A summary of the major findings of this study is outlined in Chapter 7.

Chapter 2

Material and Test Equipment

Test Specimens

The material used for all test samples is AS-3501 graphite fibers in combination with the AS-4 epoxy resin system, both manufactured by the aerospace division of Hercules Corporation. Individual lamina properties for the material system used are well known and are shown in Table 2.1. This material is available in a unidirectional prepreg tape form (fibers impregnated with resin) which may be oriented in any direction to suit the designers needs. Different layers may be added and oriented in order to obtain the required laminate strength and thickness. When placed in an autoclave and heated, these individual layers coalesce to form a composite sheet. From a sheet such as this, samples were cut and drilled into test specimens.

Hercules 3501-6/AS-4

<u>Property</u>	<u>GPa</u>	<u>(psi)</u>
E_{11}	138	(20.02 x 10⁶)
E_{22}	8.96	(1.3 x 10⁶)
G_{12}	7.1	(1.03 x 10⁶)
ν_{12}	0.3	
Tensile Strength		
(Longitudinal)	1.447	(210 x 10³)
Tensile Strength		
(Transverse)	0.206	(29.9 x 10³)
Shear Strength	0.093	(13.5 x 10³)

Table 2.1 Material properties for individual laminae of 3501-6/AS-4.

The composite sheet used is oriented $[0/90/45/-45]_s$, denoting a quasi-isotropic laminate. This orientation, one of the most widely used layups, is designed to approximate isotropic material behavior in a composite specimen. Figure 2.1 shows the different plies used to form the laminate.

Figure 2.1. Plies of 0, 90, +45, and -45 degrees used to form quasi-isotropic laminate.

Test coupons were cut from the laminated sheet in the three different angles as shown in Figure 2.2.

Figure 2.2. Test specimens cut from the quasi-isotropic laminate at angles of 0, 45, and 90 degrees.

The purpose for cutting the samples as shown is to investigate the performance of the material system under loading at three different laminate orientations. When placed in the testing machine, samples cut at 0 degrees resulted in a test specimen orientation of $[0,90,+45,-45]_s$, while those cut at 45 and 90 degrees resulted in orientations of $[+45,-45,90,0]_s$, and $[90,0,-45,+45]_s$, respectively.

Machining of the test samples was also provided by Hercules. A discussion of machining of composite materials can be found in the works of Ho-Cheng and

Dharan [13], Radhakrishnan [14], and Collings [15]. Proper machining, especially during drilling procedures, is critical to assure uniformity in test specimens and reproducibility in test results. Drilling induced damage and delamination can reduce the structural integrity of the laminate. Proper drill speed, thrust and tool geometry are essential to avoid this damage during drilling. After drilling, the holes were then reamed to the specified dimension. To prevent any edge defects which could cause high localized stresses, all edges of the test coupons were cut using a diamond tipped saw.

As discussed in Chapter 1, two important parameters of bolted joint behavior are the edge and width to diameter ratios. For quasi-isotropic laminates it was shown in Figure 1.3 that at edge and width dimensions of approximately 5 times the hole diameter, a strength plateau was reached. Therefore to test for ultimate bearing strength, the specimens should have edge and width dimensions greater than 5 times the hole diameter. The test sample dimensions are given in Figure 2.3.

Sample dimensions;

L = 177.8 mm (7 in.)

W = 38.1 mm (1.5 in.)

E = 38.1 mm (1.5 in.)

D = 6.35 mm (0.25 in.)

Figure 2.3 Test sample dimensions

Wedge-type grips were used to mount the lower portion of the sample to the testing machine. End tabs were not used since it was determined that direct contact between the grip faces and the composite was sufficient to prevent slip of the

sample relative to the grip. The upper end of the samples were mounted as shown in Figure 2.4.

Figure 2.4 Arrangement used to mount specimens in the testing machine.

Equipment Used

An Instron Model 1332 closed loop servohydraulic testing machine was used for all tests involving the bolted composite specimens. This testing system provides the most accurate and efficient method of materials testing to date. The basic components of the system include a loading frame with an attached load cell for monitoring the applied load and a servo-valve controlled hydraulic actuator. A component of the hydraulic actuator is an internal LVDT (linear variable differential transformer) which is used to determine the position of the actuator. A positive displacement pump is used to continuously supply hydraulic fluid to the actuator. Command input and data acquisition may be performed through a computer interfaced with the main CPU or the supplied control panel. The main CPU is the hardware necessary for servo control of the testing machine. A summary of the servo control system is given in the following paragraph.

The amplitude, frequency, and shape of the desired waveform is input to the control panel. From this input, the function generator then creates a digital waveform and sends this information to the servo-controllers. The servo-controller then instructs the hydraulic actuator to apply the load to the specimen. The feedback signal from the load transducer is compared to the desired waveform. Any differences between the actual and desired waveform is then corrected by the servo-controller.

In the past, fatigue test programs were generally conducted in which the sample was cycled at a constant deflection. Using constant deflection cycling, the actual applied stress is known only at the beginning of the test. Since the specimen

deflection remains constant, as testing continues, the formation and propagation of cracks causes a decrease in the applied stress. Due to advances in digital control technologies, the application of constant load test programs have become possible. This constant load control feature is available on the Instron 1332 and was used in this study for all fatigue tests. Therefore, the applied load on any particular sample was known throughout the test.

Prior to the fatigue testing program, a set of calibration weights were used to investigate the accuracy of the load cell. A calibration was also performed at the end of the test program to determine if any changes had occurred in the load cell output. In both calibration tests, the performance of the load cell was well within the rated accuracy of $\pm 0.5\%$ of the indicated load.

Chapter 3

Experimental Determination of Friction Coefficients

Proper design of friction-type bolted joints for composite materials accounts for the force transmitted through friction between the clamped materials. This consideration requires knowledge of the coefficient of friction between the mating surfaces of the bolted joint, namely the washer and the composite material. In this Chapter, the static and kinetic coefficients of friction between standard washers and a graphite-epoxy composite are found as a function of clamping force and outer ply angle.

In order for composites to be used as structural members, design and analysis of composite bolted joints are required. General design and empirical analysis of bolted joints are provided by Hart-Smith [2] and Oplinger [16]. Chang [17], De Jong [18], and Quinn and Matthews [10], have reported experimental data concerning the failure of pin loaded joints and discussed the mode of failure of such joints. The bolted joints, although similar to the pin joints, differ in that the friction force created as a result of applying a torque to the threaded fastener, contributes to the load bearing capacity of the joint. Crews [11], Stockdale and Matthews [7], and Collings [6], have all reported an increase in the bearing strength of a bolted joint with increasing clamping force. This increase in bearing

strength can be attributed to a number of factors, among them, the friction force acting between the washers and the composite surface. Friction also acts between the bolt and the composite bearing surface. This frictional force also affects bearing strength and will be discussed in Chapter 4. Another parameter shown to have an effect on the load bearing capacity of the joint is the washer diameter, as reported by Hart-Smith [2], Jurf [3], and Smith, Ashby, and Pascoe [8].

Few published works exist on the subject of friction coefficients between graphite-epoxy composites and metallic surfaces. An in-depth discussion of friction in polymer based materials is provided by Bely, et al. [18]. Sung [19] investigated the friction and wear characteristics of graphite-epoxy composites with ply angles oriented parallel and perpendicular to the sliding direction. Sung reports a maximum friction coefficient when the orientation of the fibers is transverse to the direction of sliding. Roberts [9] conducted tests on graphite-epoxy composites and reported a generally decreasing friction coefficient with an increasing normal force.

The purpose of this chapter is to establish the friction coefficients between the surface of the composite sample and typical metallic washers. The primary test objective in this chapter is the determination of both static and kinetic coefficients of friction under the influence of independent variables such as, 1) applied clamping force, 2), orientation of the outer ply of the composite, and 3), the type of washer material in contact with the composite.

Experimental Program for Determining Friction Coefficients

Samples were cut from the laminated plate at three different angles as described in Chapter 2, in order to test for an influence of outer ply angle on friction coefficients. Cuts made at the three different angles resulted in outer ply angles of 0, 45, and 90 degrees, relative to the direction of loading.

The Instron servohydraulic testing machine was used to determine the force required to cause slip between the washer and composite. The loading rate for all tests was held constant at 0.2 cm./minute. Knowing the clamping force and the force required to start slippage between the composite and the washers, the static coefficient of friction may be determined. A free-body diagram of the loading arrangement is shown in Figure 3.1.

Figure 3.1 Arrangement for loading sample.

The clamp up force, as shown in Figure 3.1, was measured using a 10000 N load cell. Clamping forces of 2220, 4440, 6660, and 8880 N (500, 1000, 1500, and 2000 lbs.) were used to investigate the variation of friction coefficient with increasing normal loads. Tests were conducted using brass, stainless steel, standard SAE, galvanized, and fender type washers. All washers had a nominal inside diameter of 6.35 mm, while outside diameters varied as shown in Table 3.1. Using the four clamping forces, in conjunction with the different surface areas

shown in Table 3.1, one can find the specific pressures. The specific pressures (Clamping force/washer area) for these tests were between 2.9 and 55.2 MPa.

<u>Washer</u>	<u>O. D. (cm)</u>	<u>Surface Area (cm²)</u>
Large Fender	3.18	7.59
Small Fender	2.54	4.73
Galvanized	1.89	2.34
Stainless Steel	1.89	2.34
Brass	1.76	2.03
SAE (Steel)	1.59	1.61

Table 3.1 Washer surface area and outside diameter.

Figure 3.2 shows the typical variation of axial load versus displacement for a given clamp up force. In Figure 3.2, the portion of the curve in which the axial load increases with increasing displacement represents elongation of the test specimen, that is, no slip occurs between the washer and the composite. Just after the load reaches a maximum, the curve becomes horizontal which signifies slip between the washer and the composite. Slippage of the washers relative to the clamping bracket was prevented by affixing the washer to the bracket via a permanent adhesive. (Appendix 1 contains copies of the plotter output for all friction tests).

Figure 3.2 Typical load-deflection data. Shown are three experimental tests of a small fender washer in contact with the composite at an outer ply angle of 0 degrees and at a clamping force of 4440 N.

A total of seventy-two (72) specimens were tested, each corresponding to a given level of the three outer ply angles, four values of clamping torque, and the six different types of washers. Three replications of each test were carried out using the same sample, however, care was taken such that the region of contact between the washer and the composite did not coincide with a previous test. Figure 3.2 is an example of the three trials at a given level of clamp up force, outer ply angle, and washer type.

Coefficients of friction were obtained using the following formula,

$$\mu_{s,k} = \frac{F_f}{2N} \quad (3.1)$$

where N is the normal, or clamp up force, and F_f is the force required to initiate slip between the washer and the composite. The factor of two in the denominator is the result of the two washer surfaces being in contact with the composite. From Figure 3.2, the average peak load of the three experimental trials is used to obtain the static coefficient of friction. The kinetic coefficient is obtained using the horizontal portion of the load-position curve.

Effect of Washer Type

Table 3.2 lists the overall means for the six different washer types, at all levels of clamping force and outer ply angle. It can be seen that the washer material has a significant influence on the coefficient of friction. This was found in all cases with the exception of contrasting the difference between the small and large fender washers. The experimentally determined friction coefficients for the six washer types are shown in Figures 3.3 through 3.8.

<u>Washer Type</u>	<u>Coefficient of Friction</u>
Galvanized	0.10
Stainless Steel	0.10
Brass	0.11
SAE	0.12
Large Fender	0.13
Small Fender	0.13

Table 3.2 Static coefficient of friction data for washer type.

Figure 3.3 shows the variation of friction coefficients versus clamping force for a galvanized washer in contact with the graphite-epoxy composite. Outer ply angles of 0, 45, and 90 degrees are designated on the plot. Solid and dotted lines

represent static and kinetic coefficients, respectively. As can be seen from the plots, and as in most friction pairs, the kinetic coefficient is slightly lower than the static coefficient of friction. In comparison with the five other washer types, the galvanized washer resulted in both the lowest overall coefficient of friction and the minimum scatter between tests. The low coefficient of friction results from the relatively soft galvanized surface coating applied to the washer.

Figure 3.3 Friction Coefficients as a function of clamping force for a galvanized washer in contact with a graphite-epoxy composite.

Results from testing a standard SAE type washer in contact with the graphite-epoxy composite are shown in Figure 3.4. The SAE washer had the smallest surface area of the washers tested (1.6 cm^2), which resulted in a maximum specific pressure of 55.2 MPa. At these high specific pressures, a

strong relationship between friction and either clamping force or outer ply angle was not found. It is noted that in Figure 3.4, the line connecting data points for the 45 degree outer ply angle is discontinued after a clamping force of 6660N. The reason is that 'plowing', or localized destruction of the resin surface had occurred at the clamp-up force of 8880N. Once this surface failure occurs and individual fibers are exposed, the coefficient of friction rises sharply. (The asterisk shown is the mean value obtained for the coefficient of friction when plowing was evident.) It is important to isolate the effect of plowing from the actual effect of clamping force on friction coefficients. Therefore experimental trials in which plowing occurred were not included when calculating the overall means.

Figure 3.4 Friction Coefficients as a function of clamping force for a SAE washer in contact with a graphite-epoxy composite.

Figures 3.5 and 3.6 represents data obtained from 'small' and 'large' fender washers, respectively. In this case, the washer material and heat treatments are the same yet the surface area of the large fender washer is 1.6 times the area of the small fender washer. Observing the test data, it appears that the different surface areas of the washers do not significantly affect friction coefficients. However, the test results showed that a greater amount of composite surface damage was present using the small versus the large fender washer. Namely, for the small fender washer, plowing occurred at three experimental levels of outer ply angle and clamping force (0 degrees - 6660 and 8880N, 90 degrees - 8880N), and for the large fender washer, plowing occurred at two experimental levels, (0 degrees - 6660 and 8880N).

Figure 3.5 Friction Coefficients as a function of clamping force for a small fender washer in contact with a graphite-epoxy composite.

Figure 3.6 Friction Coefficients as a function of clamping force for a large fender washer in contact with a graphite-epoxy composite.

Figure 3.7 shows the experimental data for the case of a stainless steel washer in contact with the graphite-epoxy composite. It is noted that plowing again occurred at a clamping force of 6660 and 8880N for the 0 degree outer ply angle. Figure 3.8 shows a similar result for the brass washer.

Figure 3.7 Friction Coefficients as a function of clamping force for a stainless steel washer in contact with a graphite-epoxy composite.

Figure 3.8 Friction Coefficients as a function of clamping force for a brass washer in contact with a graphite-epoxy composite.

Effect of Outer Ply Angle

Sung [19] observed a strong relationship between the coefficient of friction and the loading direction relative to the direction of the outer plies. It was shown that when the indenter motion was parallel to the outer fibers, friction was significantly lower than in the case where the direction of load is perpendicular to the fibers. Jurf [3] and Roberts [9] have also suggested a relationship between the coefficient of friction and the outer ply angle. However, these results were not supported by the data obtained in this study.

The dependence between friction coefficients and outer ply angle as found in this study can be inferred from Figures 3.3 through 3.8. Table 3.3 summarizes the overall means for the three different outer ply angles, at all levels of clamping force and washer type. The observations made based on these figures and the tabulated data do not substantiate the claim that the coefficient of friction reaches a maximum when the direction of the applied force is transverse to the outer plies. Likewise, the previous conclusion that the minimum friction coefficient occurs at an outer ply angle of 0 degrees cannot be supported. These differences in results can be attributed to the processes used to manufacture the composite. In previous studies, room temperature curing resins may have been used. When the resins cure at room temperature, the surface profile of the composite follows the outline of the outer ply angle. This surface profile may also exist when composites are made using a high fiber volume fraction. The composite material used for this investigation was cycle cured in an autoclave, resulting in a smooth surface. Hence, the friction coefficients found in this study are mainly attributable to the surface topography which is a direct result of the manufacturing process.

<u>Outer Ply Angle</u>	<u>Coefficient of Friction</u>
0	0.11
45	0.11
90	0.12

Table 3.3 Static coefficient of friction data for outer ply angle (Data shown is mean of all washer types and clamping torques.)

Effect of Clamping Force

Table 3.4 lists the overall means for the four levels of clamping force, at all levels of washer types and outer ply angle. Analysis of variance testing showed that some clamping force effects were present. However, pairwise comparisons of clamping force means showed significant differences only when comparing the cases 8880N vs. 4440N, and 8880N vs. 2220N. All other clamping force comparisons resulted in an insignificant difference in the coefficient of friction. While the effect of clamping force on friction coefficients is minimal, the possibility of plowing is of course, greater with increasing clamping force.

<u>Clamp up Force(N)</u>	<u>Coefficient of Friction</u>
2220	0.11
4440	0.11
6660	0.11
8880	0.12

Table 3.4 Static coefficient of friction data for the tested clamp-up forces.
(Data shown is the mean of all washer types and orientations.)

Effect of Surface Preparation

Prior to testing reported thus far, all washers were cleaned using a degreasing solvent (trichloroethane). A complete set of tests was also performed using as-received washers (without degreasing). Figure 3.9 is a plot of friction coefficients versus clamping force using the same stainless steel washer that was used to produce the results shown in Figure 3.7. Comparing Figure 3.9 with Figure 3.7, it is clear that significantly lower friction coefficients will result if proper cleaning of the washer surfaces is not performed. With the exception of the galvanized washer, the tests conducted using all other washer types points out that an as-received washer will have a significantly lower friction coefficient, as much as 50% lower, when in contact with an autoclave cured graphite epoxy composite. In the case of the galvanized washer, only a slight increase in the coefficient of friction occurred after degreasing. It is suggested that the reason for this exception is possibly due to the relatively soft surface coating of the galvanized washer. The

substantial reduction in friction resulting from routine handling of the washer is due to a lubricating film, formed on the washer and composite surface. This film, which has a considerably lower shearing resistance than the washer or the composite, reduces the force required to cause slip between the contact surfaces. While a lubricant introduced between the contact surfaces of two polymers may not greatly reduce the coefficient of friction [12], the results obtained here support the conclusion that a lubricant used between contact surfaces of polymers and metals will significantly reduce friction.

Figure 3.9 Friction Coefficients as a function of clamping force for a stainless steel washer in contact with a graphite-epoxy composite. note; as received, prior to degreasing.

Surface Damage

As discussed previously, several experimental trials resulted in surface damage to the composite. While these trials were not included in the data analysis, the failure mechanism caused by the washer sliding over the composite is of interest. Samples which exhibited plowing were observed microscopically and are shown in Figures 3.10 through 3.13.

Figure 3.10 is a scanning electron photomicrograph of the damage resulting after testing the small fender washer under an applied clamping load of 8880N. The outer ply angle is 90 degrees relative to the direction of washer motion. The photograph shows the initial stage of resin surface damage. In the upper left corner of the photograph, the smooth surface finish of the composite can be seen. Further relative motion between the washer and composite is shown in Figure 3.11, which demonstrates increased local resin surface disintegration, hence exposing individual fibers. Under high clamping forces, the stress-strain state of the composite which is in contact with the washer, exceeds the surface strength of the composite at local contact points. Once this surface failure occurs and individual fibers are exposed, the coefficient of friction rises sharply. Figure 3.12 shows composite surface damage when the washer motion is parallel to the outer fibers of the composite. High clamping forces and relative motion between the washer and the composite has caused resin surface disintegration and fiber breakage. Surface damage resulting from testing the SAE washer at an outer ply angle of 45 degrees is shown in Figure 3.13.

Figure 3.10 Photomicrograph of initial stage of resin surface damage under a clamping force of 8880 N. Load was applied perpendicular to the direction of outer plies. Magnification = 500X.

Figure 3.11 Damage caused by further relative motion between washer and composite. Load was applied perpendicular to the direction of outer plies. Magnification = 500X.

Figure 3.12 Photomicrograph showing resin surface disintegration, fiber breakage, and fiber dislocations. Load was applied perpendicular to the direction of outer plies. Magnification = 500X.

Figure 3.13 Surface damage resulting from high (8880N) clamp-up force. Outer ply angle is 45 degrees relative to the direction of the applied load. Magnification = 500X.

Conclusions

To summarize the major conclusions of this chapter, friction coefficients were shown to be dominated by the surface properties of both washer and composite. Surface contamination caused by routine handling of as-received washers, resulted in coefficients of friction that were typically 50% lower than the same washer that was carefully degreased.

The washer material was shown to have a significant effect on the coefficient of friction between the composite material and the metallic washer. Increasing normal force was shown to have little effect on the coefficient of friction. However, increasing normal force raises the likelihood of plowing, which when occurs, causes a transition from a friction phenomenon to that of a wear phenomenon.

The direction of outer fibers showed a smaller influence on the coefficient of friction than previously reported data. Friction coefficients are more dependent upon resin surface characteristics than the direction of the outermost layer of fibers.

Chapter 4

Stress Analysis

The purpose of this chapter is to determine the stress distribution in the composite plate around the loaded hole. The planar stresses are a required input to the bearing strength model which is described in the following chapter.

Development of Stress Components

The theory of elasticity was used to develop the expressions for the planar stresses. Finite element techniques could likewise have been applied to this problem, however, finite element solutions are generally not as 'portable' as elasticity solutions. Using the elasticity approach, once the expressions for the stresses are obtained, the effects the independent variables are easily determined.

Figure 4.1 shows the coordinate system, geometry and nomenclature of the problem. Excluding the situation of a snug or interference fit, there will exist a clearance between the bolt and the hole in the composite plate. Therefore the pin radius is denoted as r_0 and the hole radius as a . Terms r and θ represents the polar coordinates of any arbitrary point.

Figure 4.1 Model geometry of pin and composite plate.

The pin is considered fixed and rigid, with the load applied to the composite in the minus x direction. Figure 4.2 shows the applied load L , and the corresponding angle of contact between the composite and the pin, where $\theta_{\max}$ is the angle at the threshold of contact.

Figure 4.2 Position of the pin in the hole as the load is applied.

The outline of the elasticity solution follows. In polar coordinates, the equilibrium equations are given [21] as,

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \quad (4.1)$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{2\sigma_{r\theta}}{r} = 0 \quad (4.2)$$

An Airy stress function ϕ is defined such that the equilibrium equations are automatically satisfied,

$$\sigma_{rr} = \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} \quad (4.3)$$

$$\sigma_{\theta\theta} = \frac{\partial^2 \phi}{\partial r^2} \quad (4.4)$$

$$\sigma_{r\theta} = -\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \phi}{\partial \theta} \right) \quad (4.5)$$

The strain-displacement relations in the same coordinate system are given as,

$$\epsilon_r = \frac{\partial u}{\partial r}$$

$$\epsilon_{\theta} = \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial \theta} \quad (4.6)$$

$$\gamma_{r\theta} = \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r}$$

Eliminating u and v from Equation (4.6) gives the compatibility equation,

$$\frac{\partial^2 \epsilon_{\theta}}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \epsilon_r}{\partial \theta^2} + \frac{2}{r} \frac{\partial \epsilon_{\theta}}{\partial r} - \frac{1}{r} \frac{\partial \epsilon_r}{\partial r} = \frac{1}{r} \frac{\partial^2 \gamma_{r\theta}}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial \gamma_{r\theta}}{\partial \theta} \quad (4.7)$$

For the case of plane stress, the stress-strain relations in radial and tangential coordinates are,

$$\begin{aligned}\epsilon_r &= \frac{1}{E} (\sigma_r - \nu \sigma_\theta) \\ \epsilon_\theta &= \frac{1}{E} (\sigma_\theta - \nu \sigma_r) \\ \gamma_{r\theta} &= \frac{2}{E} (1 + \nu) \tau_{r\theta}\end{aligned}\tag{4.8}$$

Substituting the stress-strain relations into the compatibility equation (Equation 4.7), gives,

$$\begin{aligned}\frac{\partial^2}{\partial r^2} (\sigma_\theta - \nu \sigma_r) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} (\sigma_r - \nu \sigma_\theta) + \frac{2}{r} \frac{\partial}{\partial r} (\sigma_\theta - \nu \sigma_r) - \frac{1}{r} \frac{\partial}{\partial r} (\sigma_r - \nu \sigma_\theta) \\ = \frac{2}{r} (1 + \nu) \left[\frac{\partial^2 \tau_{r\theta}}{\partial r \partial \theta} + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} \right]\end{aligned}\tag{4.9}$$

Writing the above equation in terms of the stress function ϕ , gives,

$$\nabla^4 \phi = 0\tag{4.10}$$

where ∇^2 is the Laplacian operator in radial and transverse coordinates,

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \quad (4.11)$$

From Bickley [21], a general form of the solution to Equation (4.10) is,

$$\begin{aligned} \phi = & a^2 \left[A_0 \log\left(\frac{r}{a}\right) + B_0 + A_1 \left\{ \frac{r}{a} \theta \sin \theta - 0.5(1 - \nu) \frac{r}{a} \log\left(\frac{r}{a}\right) \cos \theta \right\} \right. \\ & + B_1 \left\{ \frac{r}{a} \theta \cos \theta + 0.5(1 - \nu) \frac{r}{a} \log\left(\frac{r}{a}\right) \sin \theta \right\} + C_1 a \cos \theta + D_1 a \sin \frac{\theta}{r} \\ & \left. + \sum_{m=2}^{\infty} \left\{ \left(\frac{a}{r}\right)^{m-2} (A_m \cos m\theta + B_m \sin m\theta) + \left(\frac{a}{r}\right)^m (C_m \cos m\theta + D_m \sin m\theta) \right\} \right] \end{aligned} \quad (4.12)$$

where r is the radial distance from the hole center, a is the radius of the hole, ν is the in-plane Poisson's ratio, and θ is measured from the x -axis as shown in Figure 4.1. A_m , B_m , C_m , and D_m are unknown coefficients to be determined from the boundary conditions.

To find the unknown coefficients, one must first prescribe the boundary conditions along the hole edge. Assuming that the angle of contact, $\theta_{\max}$, is known, the boundary condition for σ_{rr} as suggested by Bickley [21], may be written as;

$$\sigma_{rr} = -p \left(1 - \left(\frac{\theta}{\theta_{\max}} \right)^2 \right)^{0.5} \quad \text{at: } r = a, \quad -\theta_{\max} \leq \theta \leq \theta_{\max} \quad (4.13)$$

$$\sigma_{rr} = 0 \quad \text{at: } r = a, \quad \theta_{\max} \leq \theta \leq 2\pi - \theta_{\max}$$

In order to account for shear stresses caused by friction between the bolt and composite, the boundary condition for $\sigma_{\theta\theta}$ is assumed to follow the form given below.

$$\sigma_{r\theta} = A \sin \left(\frac{\pi\theta}{\theta_{\max}} \right)^2 \quad \text{at: } r = a, \quad -\theta_{\max} \leq \theta \leq \theta_{\max} \quad (4.14)$$

$$\sigma_{r\theta} = 0 \quad \text{at: } r = a, \quad \theta_{\max} \leq \theta \leq 2\pi - \theta_{\max}$$

where,

$$A = \mu p \cos \left(\frac{\theta_{\max}}{2} \right)$$

and μ is the coefficient of friction between the bolt bearing surface and the composite plate. The term p appearing in the boundary condition equations is the

maximum interface pressure at the point of symmetry. It can be obtained by summing the resultant force per unit thickness in the x-direction,

$$F_x = \frac{L}{t} = 2 \int_0^{\pi} \left(-\sigma_{rr}(\text{edge}) \cos \theta + \sigma_{r\theta}(\text{edge}) \sin \theta \right) a \, d\theta \quad (4.15)$$

where L is the applied load as shown in Figure 4.2, and t is the thickness of the composite. Substituting for σ_{rr} and $\sigma_{r\theta}$ from Equations (4.13) and (4.14), respectively, gives

$$\frac{L}{t} = 2a \int_0^{\theta_{\max}} p \left(1 - \left(\frac{\theta}{\theta_{\max}} \right)^2 \right)^{0.5} \cos \theta + A \sin \left(\frac{\pi \theta}{\theta_{\max}} \right)^2 \sin \theta \, d\theta \quad (4.16)$$

Performing the integration,

$$\int_0^{\theta_{\max}} p \left(1 - \left(\frac{\theta}{\theta_{\max}} \right)^2 \right)^{0.5} \cos \theta \, d\theta = p \pi J_1(\theta_{\max}) \quad (4.17)$$

$$\begin{aligned}
\int_0^{\theta_{\max}} A \sin\left(\frac{\pi\theta}{\theta_{\max}}\right)^2 \sin \theta \, d\theta &= \mu p \cos\left(\frac{\theta_{\max}}{2}\right) \left\{ \frac{(1-2n)[\cos(2n+1)\theta_{\max}]}{4(1-2n)(1+2n)} + \right. \\
&\quad \left. + \frac{(1+2n)[\cos(1-2n)\theta_{\max} - 2(1-2n)\cos\theta_{\max}] - 4n^2}{4(1-2n)(1+2n)} \right\}
\end{aligned}
\tag{4.18}$$

Using the results of Equations (4.17) and (4.18), the expression for p , can be obtained from Equation (4.16),

$$\begin{aligned}
p &= L \left[t a \left\{ \pi J_1(\theta_{\max}) + 2\mu \cos\left(\frac{\theta_{\max}}{2}\right) * \right. \right. \\
&\quad \left. \left\{ \frac{(1-2n)[\cos(2n+1)\theta_{\max}]}{4(1-2n)(1+2n)} + \right. \right. \\
&\quad \left. \left. \frac{(1+2n)[\cos(1-2n)\theta_{\max} - 2(1-2n)\cos\theta_{\max}] - 4n^2}{4(1-2n)(1+2n)} \right\} \right]^{-1}
\end{aligned}
\tag{4.19}$$

where,

$$n = \frac{\pi \theta}{\theta_{\max}}$$

and, J_1 is Bessel's function of the first order, and $\theta_{\max}$ is the maximum angle of contact. Due to clearance between the bolt and the bolt hole, the contact angle will

vary nonlinearly as the applied load is increased. To obtain the contact angle, the results obtained by Eshwar [22] were utilized through a curve fit of the form,

$$\theta_{\max} = 49.74 + 33.67 \text{ Log } (x), \quad x > 0 \quad (4.20)$$

where,

$$x = \frac{L}{Ea\lambda}$$

and λ is a measure of the clearance between the bolt and hole,

$$\text{Pin Diameter} = 2r_o = 2a(1 + \lambda).$$

From Bickley [21], writing the stress components, Equations (4.3), (4.4), and (4.5) in terms of the general solution, Equation (4.12), gives,

$$\begin{aligned} \sigma_{rr} = & A_0 \left(\frac{a}{r}\right)^2 + 0.5(3 + \nu) \left(A_1 \cos \theta - B_1 \sin \theta \right) - 2 \left(\frac{a}{r}\right)^3 \left(C_1 \cos \theta + D_1 \sin \theta \right) \\ & - \sum_{m=2}^{\infty} \{ (m+2)(m-1) \left(\frac{a}{r}\right)^m \left(A_m \cos m\theta + B_m \sin m\theta \right) \\ & + m(m+1) \left(\frac{a}{r}\right)^{m+2} \left(C_m \cos m\theta + D_m \sin m\theta \right) \} \end{aligned} \quad (4.21)$$

$$\begin{aligned}
\sigma_{\theta\theta} = & -A_0\left(\frac{a}{r}\right)^2 - 0.5(1-\nu)\left(A_1 \cos \theta - B_1 \sin \theta\right) + 2\left(\frac{a}{r}\right)^3\left(C_1 \cos \theta + D_1 \sin \theta\right) \\
& + \sum_{m=2}^{\infty} \left\{ (m-2)(m-1)\left(\frac{a}{r}\right)^m \left(A_m \cos m\theta + B_m \sin m\theta\right) \right. \\
& \left. + m(m+1)\left(\frac{a}{r}\right)^{m+2} \left(C_m \cos m\theta + D_m \sin m\theta\right) \right\}
\end{aligned} \tag{4.22}$$

$$\begin{aligned}
\sigma_{r\theta} = & -B_0\left(\frac{a}{r}\right)^2 - 0.5(1-\nu)\left(A_1 \sin \theta - B_1 \cos \theta\right) - 2\left(\frac{a}{r}\right)^3\left(C_1 \sin \theta - D_1 \cos \theta\right) \\
& + \sum_{m=2}^{\infty} \left\{ m(m-1)\left(\frac{a}{r}\right)^m \left(-A_m \sin m\theta + B_m \cos m\theta\right) \right. \\
& \left. + m(m+1)\left(\frac{a}{r}\right)^{m+2} \left(-C_m \sin m\theta + D_m \cos m\theta\right) \right\}
\end{aligned} \tag{4.23}$$

Expanding the boundary conditions, Equations (4.13) and (4.14), in terms of Fourier series,

$$\begin{aligned}
\sigma_{rr} = & \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos n\theta + b_n \sin n\theta \right) \\
\sigma_{r\theta} = & \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos n\theta + b_n \sin n\theta \right)
\end{aligned} \tag{4.24}$$

where,

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx \, dx \quad n = 0, 1, 2, \dots$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx \, dx \quad n = 1, 2, 3, \dots$$
(4.25)

gives,

$$a_0(\sigma_\pi) = 0.25 p \theta_{\max}, \quad a_n(\sigma_\pi) = \frac{p}{n} J_1(n \theta_{\max})$$

$$b_n(\sigma_\pi) = 0$$
(4.26)

and,

$$a_0(\sigma_{r\theta}) = 0$$

$$a_n(\sigma_{r\theta}) = -\frac{\sin(2c - n)\theta_{\max}}{2(2c - n)} + \frac{\sin n\theta}{n} - \frac{\sin(2c + n)\theta_{\max}}{2(2c + n)}$$

$$b_n(\sigma_{r\theta}) = 0$$
(4.27)

Equating the coefficients of like terms of $\cos(m\theta)$ and $\sin(m\theta)$, one can then determine the unknown coefficients A_m , B_m , C_m , and D_m of Equation (4.21),

(4.22), and (4.23). These coefficients can be written as,

$$\begin{aligned}
 A_m &= \frac{a_n(\sigma_{rr})}{2(m-1)} \\
 B_m &= \frac{a_n(\sigma_{r\theta})}{2(m-1)} \\
 C_m &= \frac{-m a_n(\sigma_{rr})}{2m(m+1)} \\
 D_m &= \frac{-(m+2) a_n(\sigma_{r\theta})}{2m(m+1)}
 \end{aligned}
 \tag{4.28}$$

Substituting the coefficients into the stress expressions (Equations (4.21), (4.22) and (4.23)), gives the resulting stresses as,

$$\begin{aligned}
 \sigma_{rr} &= -\frac{p}{2} \sum_{m=1}^{\infty} \frac{a^m}{r^m} \cos(m\theta) J_1(m\theta_{\max}) \left\{ \frac{2}{m} + \left(1 - \frac{a^2}{r^2} \right) \right\} \\
 &\quad - \frac{1}{4} p \theta_{\max} \left(\frac{a}{r} \right)^2 + \frac{3-\nu}{4} p J_1(\theta_{\max}) \left(1 - \frac{a^2}{r^2} \right) \frac{a}{r} \cos \theta \\
 &\quad - \frac{1}{2} \sum_{m=1}^{\infty} \frac{a^m}{r^m} \sin(m\theta) (T_m) (2+m) \left(1 - \frac{a^2}{r^2} \right) \\
 &\quad + \frac{3-\nu}{4} \left(1 - \frac{a^2}{r^2} \right) T_1 \frac{a}{r} \sin \theta
 \end{aligned}
 \tag{4.29}$$

$$\begin{aligned}
\sigma_{\theta\theta} = & \frac{p}{2} \sum_{m=1}^{\infty} \frac{a^m}{r^m} \cos(m\theta) J_1(m\theta_{\max}) \left\{ -\frac{2}{m} + \left(1 - \frac{a^2}{r^2} \right) \right\} \\
& + \frac{1}{4} p \theta_{\max} \left(\frac{a}{r} \right)^2 + \frac{3-\nu}{4} p J_1(\theta_{\max}) \left(1 + \frac{a^2}{r^2} \right) \frac{a}{r} \cos \theta
\end{aligned} \tag{4.30}$$

$$\begin{aligned}
& + \frac{1}{2} \sum_{m=1}^{\infty} \frac{a^m}{r^m} \sin(m\theta) (T_m) \left\{ -2 \left(1 + \frac{a^2}{r^2} \right) + m \left(1 - \frac{a^2}{r^2} \right) \right\} \\
& + \frac{(3-\nu)}{4} \left[1 + \frac{a^2}{r^2} \right] T_1 \frac{a}{r} \sin \theta
\end{aligned}$$

$$\begin{aligned}
\sigma_{r\theta} = & -\frac{p}{2} \sum_{m=1}^{\infty} \frac{a^m}{r^m} \sin(m\theta) J_1(m\theta_{\max}) \left(1 - \frac{a^2}{r^2} \right) \\
& + \frac{3-\nu}{4} p J_1(\theta_{\max}) \left(1 - \frac{a^2}{r^2} \right) \frac{a}{r} \sin \theta
\end{aligned} \tag{4.31}$$

$$\begin{aligned}
& + \frac{1}{2} \sum_{m=1}^{\infty} \frac{a^m}{r^m} \cos(m\theta) (T_m) \left\{ -2 \frac{a^2}{r^2} + m \left(1 - \frac{a^2}{r^2} \right) \right\} \\
& - T_0 \frac{a^2}{r^2} - \frac{(3-\nu)}{4} \frac{a}{r} \left[1 - \frac{a^2}{r^2} \right] T_1 \cos \theta
\end{aligned}$$

where,

$$T_m = a_m(\sigma_{r\theta}) \quad m = 0, 1, 2, \dots$$

The program listing of the stress resultants σ_{rr} , $\sigma_{\theta\theta}$, and $\sigma_{r\theta}$ are given in Appendix B.

Stress Analysis Results

The results of the stress analysis are shown in Figures 4.3 through 4.8.

Figure 4.3 shows the distribution of stresses around the loaded hole using a perfect-fit pin, ($|\lambda| = 0$). On this and Figures 4.4 and 4.5, the x-axis is measured in degrees and the y-axis is denoted $\frac{\sigma}{\sigma_b}$, where σ_b is the bearing stress.

(Bearing stress is defined as $L/2a^*t$). The coefficient of friction for this case as well as in the following figures is equal to 0.1, as obtained from the results of Chapter 3. For Figure 4.3, the applied load equals 1776 N, although the normalized results would be the same under any applied load.

Figure 4.3 Distribution of stresses around loaded hole with $\lambda = 0$.

Figure 4.4 illustrates the stress distribution around the loaded hole for the case of an imperfect-fit pin, $|\lambda| = 0.004$. The applied load is again 1776 N. For the case of a perfect-fit pin, the stresses around the hole are linearly related to the applied load. However, for the case of $|\lambda| > 0$, the stresses are nonlinearly related to the applied load due to the resulting contact angle. As seen in Figure 4.4 for $|\lambda| = 0.004$ and an applied load of 1776 N, σ_{π} becomes zero at approximately 60 degrees. At this same amount of clearance, a lower applied load would decrease the contact angle and would change the relative magnitudes of the resulting stresses.

From Figure 4.4, it can be seen that with clearance, the peak tangential stress changes from $\theta = 90$ degrees to $\theta = \theta_{\max}$. Not only does the location of the peak stress change, but the relative magnitude of $\sigma_{\theta\theta}$ changes as well. For the case of a perfect-fit pin, the peak tangential stress (normalized with respect to the bearing stress) equals 0.8. Under the same load with $|\lambda| = 0.004$, the peak value of $\sigma_{\theta\theta}$ becomes 1.3. A similar situation exists for the case of the radial stresses. Although the peak value of σ_{π} occurs at the same location for both cases of $|\lambda| = 0$ and $|\lambda| > 0$, the relative magnitude increases with increasing $|\lambda|$. For the perfect-fit case, $\frac{\sigma_{\pi}}{\sigma_b}$ equals -1.09, while $\frac{\sigma_{\pi}}{\sigma_b}$ equals -1.5 for $|\lambda| = 0.004$.

Figure 4.4 Distribution of stresses around loaded hole with $\lambda = 0.004$ and applied load = 1776 N.

Figure 4.5 illustrates the distribution of stresses obtained when $|\lambda| = 0.016$. At an applied load of 1776 N, the peak tangential stress, $\frac{\sigma_{\theta\theta}}{\sigma_b} = 1.5$, occurs at $\theta = \theta_{\max}$. With a clearance of $|\lambda| = 0.016$, the peak radial stress, $\frac{\sigma_{rr}}{\sigma_b}$ equals -

2.2. The general conclusion which may be drawn from Figures 4.3 through 4.5 is that clearance has a significant effect on the stress distribution around the loaded hole. This clearance would create a corresponding decrease in the load carrying capacity of the bolted joint. This conclusion is supported by other theoretical and experimental studies made by DeJong [18], Hyer, Klang, and Cooper [5], and Tsai and Morton [23]. Support of this conclusion is further illustrated by Figure 4.6 which shows the normalized radial stress versus the distance from the hole center along the x-axis. The effect of clearance can be seen by noting that increasing clearance causes an increase in the peak value of $\frac{\sigma_{rr}}{\sigma_b}$.

Figure 4.5 Distribution of stresses around loaded hole with $\lambda = 0.016$ and applied load = 1776 N.

Figure 4.6 Radial stress distribution along the line $\theta = 0$.

Figure 4.7 is a contour plot of the normalized radial stresses as a function of x and y . Due to symmetry, the plot shows one-quarter of the composite plate which is in contact with the pin. The hole center is located at $(0,0)$, with the x and y coordinates given in terms of r/a , where r is the distance from the hole center and the term a equals the hole radius. Figure 4.7 illustrates the radial stress distribution for the case of $|\lambda| = 0.004$, and under an applied load of 1776 N. Figure 4.8 shows the normalized tangential stress distribution under the same loading and clearance conditions as in Figure 4.7.

Once the expressions for the planar stresses have been determined, a bearing strength model is developed to predict the ultimate bearing strength of a bolted joint. The development of this model is outlined in the following chapter.

SIGMA-R

Figure 4.7 Contour plot of σ_{π} on the loaded region of the hole. $\lambda = 0.004$, and applied load = 1776 N.

SIGMA-THETA

Figure 4.8 Contour plot of $\sigma_{\theta\theta}$ on the loaded region of the hole. $\lambda = 0.004$, and applied load = 1776 N.

Chapter 5

Laminate Bearing Strength

Two major topics are covered in this Chapter - 1) Experimental determination of the static ultimate bearing strength of the composite laminate, 2) Investigation of the influence of various bolting parameters on the strength of composite bolted joints. This information will be used as a basis for the selection of stress amplitudes in the following Chapter.

When composite materials are bolted together, the relative diameters of the bolt hole, the constraining washer and the bolt itself have a profound effect on the bearing strength. In this Chapter, the effect of the difference between the bolt diameter and the inside diameter of the washer, which provides the lateral constraint at the edge of the hole, is examined. To explain the significant variation of bearing strength, a model is proposed which describes the mechanics of load transfer and the lateral deformation at the edge of the hole. The experimentally determined results of ultimate bearing strength tests, support the proposed model and provide data for designers.

Background

A major goal of the research conducted on composite bolted joints has been to determine the effect of various bolting parameters on the bearing strength of the joint. Several investigators have reported an increase in the bearing strength of the composite joint with an increasing lateral constraint. Crews [11], and Stockdale and Matthews [7] reported an increase in bearing strength of up to 100 percent when compared to a pin bearing case. Jurf [3,24] found similar results when testing bolted joints of both Kevlar and graphite composites. Experiments performed by Eriksson [25] have shown that the bearing strength of a laterally constrained graphite-epoxy composite is as much as 2.4 times the strength of a similar pin-loaded sample. Collings [6] suggests that the increase in bearing strength with increasing lateral constraint is due to the effect of washers in preventing the 'brooming' type failure associated with pin loaded bearing tests. In an effort to increase the bearing strength of composite joints by preventing the brooming type failures, Nilsson [26] investigated the use of bonded caps around the hole edge. The use of bonded caps resulted in significantly stiffer and stronger joints.

Collings [6] and Jurf [3] have shown that geometric factors such as laminate thickness, hole diameter, width/diameter, and edge/diameter ratios have a strong influence on bearing strength. Other parameters such as stacking sequence, ply orientation, and the coefficient of friction can influence the bearing strength. Quinn and Matthews [10] have reported that the stacking sequence affects both bearing strength and the failure mode of the joint. Collings [6] shows that ply orientation, especially the outer ply, affects bearing strength. Higher bearing

strengths can be realized with higher coefficients of friction due to increased load transfer through frictional forces. Roberts [9] and Sung [20] have provided experimental data on the friction coefficients of various composites.

Due to the mathematical complexities, a very limited number of three dimensional analyses of bolted joint behavior are reported. Matthews, Wong, and Chryssafitis [27], and Marshall, Arnold, and Wood [28] have used three dimensional finite element analyses to model composite bolted joints. Both papers have shown that an increase in lateral constraint has a significant influence on the interlaminar normal and shear stresses acting at the hole edge. Based on a classical elasticity approach, Smith, Ashby, and Pascoe [8] modeled the important clampup effects of a composite bolted joint. Satisfactory agreement was found between the Smith, Ashby, and Pascoe model prediction and experimental results.

In a typical bolted joint, there is clearance between the bolt and the bolt hole in the composite plate. Stockdale and Matthews [7], and Hyer, Klang, and Cooper [5] have shown that this clearance is an important parameter in determining the ultimate bearing strength. In addition to clearance between the bolt and the bolt hole, often there is a radial clearance of a different magnitude between the bolt and the washer, normally independent of the clearance between the bolt and the bolt hole in the composite plate. It is this bolt-washer clearance that is being investigated and reported in this chapter.

Proposed Model

A predictive model was developed in order to show the effects of important parameters such as friction and the relative geometry of the bolt and washer. The model is made up of various elements, each contributing to the bearing strength of the composite joint. The total load carrying capacity of the composite joint (at the point of failure) can be written as,

$$\sigma_{bf} = \sigma_p + \sigma_0^* + \frac{2\mu N_c}{dt} + \frac{2\mu N_r}{dt} \quad (5.1)$$

where σ_p is the bearing strength of a pin-loaded composite plate defined in terms of load per projected area of the bolt hole (load/td). The following three terms of the model represent the increase in bearing strength due to an applied lateral constraint. The term σ_0^* represents the increase in the allowable in-plane forces applied to the composite due to an out-of-plane constraint. That is, a compressive stress σ_z alters the distribution of the stress around the hole and helps inhibit the relatively large interlaminar stresses, and consequently the delamination due to in-plane loading. An estimate of σ_0^* (which was found experimentally) is given in the results section of this Chapter. It will be shown that σ_0^* is highly dependent on the washer/bolt diameter ratio. The final two terms of the model represented in Equation (5.1) are due to frictional forces generated between the surfaces of the composite and the washer. The term $2\mu N_c$, in which μ is the coefficient of friction between the plate and the washer, and N_c is the normal force applied to the washers

due to the clamp up torque, represents the increase in bearing strength due to a portion of the applied load being carried by frictional forces. The multiple 2 is used since in the case of a double lap joint as used here, both sides of the plate contribute to the load transfer through friction. This term is also divided by the product dt (hole diameter times plate thickness), in order to normalize this effect with respect to bearing stress. The final term represents the increase in frictional transfer of load due to the increase in axial force in the bolt. The increase in the bolt axial force is N_r , and is a result of the washers resisting lateral expansion at the edge of the hole. N_r is both a function of σ_0^* and washer/bolt diameter ratio, as will be shown later. As before, this term is normalized with respect to the projected area of the bolt hole to convert it to the convenient units of bearing stress.

Smith, Ashby, and Pascoe [8] proposed the theory that the lateral strain (ϵ_z) of the composite in the region ahead of the bolt causes an increase in the axial force on the bolt. This increase in axial force generates a larger normal force (the contact force between the washer and the composite surface), hence increasing the load transfer through the mechanism of friction. The method proposed here is to find N_r and is similar to the method proposed by Smith, et al. [8], but with some important differences. The stress analysis used by Smith to obtain ϵ_z , is that of Bickley [21], for the case of a tight fit pin, a cosine radial pressure boundary condition, and frictionless contact between the pin and the composite. The stress analysis in this Chapter incorporates clearance between the pin and the hole, and the tangential stress boundary conditions due to friction. Also included is the effect of the difference between the bolt diameter and the washer inside diameter.

To determine the additional normal force N_r , it is required to know the lateral strain around the hole, ϵ_z . The lateral strain is obtained from

$$\epsilon_z = -\frac{\nu}{E}(\sigma_{rr} + \sigma_{\theta\theta}) \quad (5.2)$$

where ν is the in-plane Poisson's ratio, E is the in-plane modulus of elasticity, and σ_{rr} , $\sigma_{\theta\theta}$ are the radial and tangential components of stress, respectively. The force required to bring this lateral expansion back to zero is then,

$$N_r = E_t \epsilon_z A_c \quad (5.3)$$

where E_t is the through-thickness modulus, and A_c is the area of contact between the washer and the composite. Throughout this study, the area of contact between the washer and the composite was taken over the region which exhibited positive values for ϵ_z . This provides an upper limit to the estimate of the maximum force required to bring ϵ_z back to zero. Due to the elasticity of the bolt and washer, ϵ_z is not completely brought back to zero.

The in-plane stresses, σ_{rr} and $\sigma_{\theta\theta}$, required for Equation (5.2) were obtained in Chapter 4, using the theory of elasticity. The lateral strain, ϵ_z , may now be found by substituting the stresses σ_{rr} and $\sigma_{\theta\theta}$ into Equation (5.2). A plot showing the positive values of ϵ_z on the loaded portion of the composite plate is

given in Figure 5.1. Values for N_r may be obtained through the use of Equation (5.3).

Figure 5.1. Plot of positive values of ϵ_z around loaded portion of the hole. Hole center located at $(x=0, y=0)$.

Experimental Procedure

Prior to assembly of the bolt and composite, Fel-Pro C5A thread lubricant was applied to the bolt threads. This procedure was performed in order to reduce variation in the preload applied to the bolted joint. Bickford [29], and Kulak [30], have shown that lubricating bolt threads substantially reduces torque-preload scatter. Obtaining the proper preload is critical since the preload has a direct effect on the frictional force carried by the joint. In a friction type joint, the axial force in the bolt and the coefficient of friction comprise a frictional force which acts to prevent sliding between the mating surfaces. In this case, no shear loads are applied to the bolt. Joints designed such that mating surfaces slide over each other until the bolt carries the external load in shear are known as 'bearing joints'. In this study, preloads were selected such that fatigue loading was performed under pure bearing and under conditions which are typical of both friction and bearing joint designs.

All bolts and nuts used were 1/4 - 20 UNC steel with a shank outside diameter of 6.25 mm (0.246 in). This results in a hole - bolt clearance of 0.1 mm (0.004 in.), similar to that stated by Crews [31] as being typical of aircraft joints. It should be noted that in all tests, the smooth shank portion, as opposed to the threaded portion of the bolt came in contact with the laminate. Torque was applied to the nuts using a dial indicator torque wrench with a tool accuracy of $\pm 2\%$ of full scale (F. S. = 28.25 N-m). To obtain the torque required for a given axial preload, the following equation is used,

$$T = cdF \quad (5.4)$$

where T is the applied torque (N-m), c is the nut factor, d is the nominal bolt diameter (m) and F is the axial force (N) within the bolt. Bickford [29] reports a mean nut factor c , of 0.132 for a steel fastener when used with a Fel-Pro type lubricant.

Bearing strength tests were performed on a total of five different washers, all having an outside diameter of 1.905 cm, and inside diameters of 0.702 cm, 0.665 cm, 0.639 cm, 0.632 cm, and 0.628 cm. These washer inside diameters divided by the bolt diameter gives ratios of washer I.D. to bolt O.D. of 1.124, 1.065, 1.024, 1.012, and 1.005, respectively. Initially, four ratios of washer inside diameter to bolt diameter were tested; 1.124, 1.065, 1.024, and 1.005. To validate the model and to update predicted parameters, another series of tests were performed using the diameter ratio of 1.012. It is important to note that off-the-shelf bolting components have washer I.D. to bolt O.D. ratios typically on the order of 1.04 and greater. Washers with diameter ratios smaller than 1.04 were tested in order to investigate the effect on bearing strength of this important parameter.

The specimens used for bearing strength tests were described in Chapter 2. The specimen dimensions have sufficient edge and width to diameter ratios such that failure will occur only in the bearing mode, and not in net tension or shear-out. All tests were performed using the Instron servohydraulic testing machine at a constant loading rate of 0.127 cm/min. (0.05 in/min.). Real time plots of load versus displacement were obtained for each test (Appendix C). A plot typical of the results obtained is shown in Figure 5.2. The load-displacement curves showed linear behavior up to a point, followed by an unstable, nonlinear behavior due to fiber crushing and delamination near the edge of the hole. Throughout this study,

bearing failure is defined as the maximum load attained just prior to unstable, nonlinear behavior. The maximum load was then divided by the product of hole diameter and the composite thickness to obtain a system of units consistent with previous bearing studies.

Figure 5.2. Typical load-displacement curve for bearing test. Clamping torque = 3.76 N-m.

Results

Table 5.1 presents the static ultimate bearing strength data for the composite laminate at a clamping torque of 0 N-m (pin-loaded case). Shown are the three experimental trials at each outer ply angle. Tables 5.2 through 5.5 present the data obtained when a clamp-up force is applied to the bolt. Table 5.2 shows the effect of clamp-up torques of 3.76, 7.53, and 11.3 N-m on the bearing strength using an ID/OD ratio of 1.142. The results obtained using washer diameter ratios of 1.065, 1.024, and 1.005 are given in Tables 5.3, 5.4, and 5.5, respectively. A copy of the plotter output for each test is given in Appendix C.

<u>Load Bearing Data</u>			
<u>Pin Bearing Case</u>			
<u>Clamping Torque(Nm)</u>	<u>Outer Ply Angle</u>	<u>Bearing Strength (MPa)</u>	<u>Mean Bearing Strength (MPa)</u>
0	0	296.31	288.9
		285.19	
		285.21	
0	45	291.37	279.02
		251.86	
		293.84	
0	90	269.15	276.56
		276.56	
		283.97	

Table 5.1. Static ultimate bearing strength for pin-loaded case.

Load Bearing Data
Washer I.D./Bolt O.D. = 1.142

<u>Clamping Torque(Nm)</u>	<u>Outer Ply Angle</u>	<u>Bearing Strength (MPa)</u>	<u>Mean Bearing Strength(MPa)</u>
3.76	0	342.56	332.67
		317.83	
		337.62	
7.53	0	321.54	347.95
		351.29	
		371.01	
11.3	0	465.00	489.73
		484.79	
		519.41	

Table 5.2. Static ultimate bearing strength for washer diameter ratio of 1.142.

Load Bearing Data
Washer I.D./Bolt O.D. = 1.065

<u>Clamping Torque(Nm)</u>	<u>Outer Ply Angle</u>	<u>Bearing Strength (MPa)</u>	<u>Mean Bearing Strength (MPa)</u>
3.76	0	430.37	
		375.96	427.07
		474.89	
7.53	0	593.62	
		568.88	580.42
		578.78	
11.3	0	692.55	
		712.02	700.69
		697.50	

Table 5.3. Static ultimate bearing strength for washer diameter ratio of 1.065.

Load Bearing Data
Washer I.D./Bolt O.D. = 1.024

<u>Clamping Torque(Nm)</u>	<u>Outer Ply Angle</u>	<u>Bearing Strength (MPa)</u>	<u>Mean Bearing Strength(MPa)</u>
3.76	0	494.68	554.73
		613.21	
		556.29	
	45	584.38	576.16
		609.08	
		535.01	
	90	592.61	611.82
		648.46	
		584.38	
7.53	0	618.35	725.54
		742.02	
		816.25	
	45	707.85	722.94
		720.19	
		740.77	
	90	699.62	720.19
		720.19	
		740.77	
11.3	0	828.39	832.06
		783.67	
		914.12	
	45	691.38	784.67
		806.62	
		856.01	
	90	707.85	786.11
		786.25	
		864.24	

Table 5.4. Static ultimate bearing strength for washer diameter ratio of 1.024.

Load Bearing Data
Washer I.D./Bolt O.D. = 1.005

<u>Clamping Torque(Nm)</u>	<u>Outer Ply Angle</u>	<u>Bearing Strength (MPa)</u>	<u>Mean Bearing Strength (MPa)</u>
3.76	0	720.96	729.21
		710.98	
		755.73	
7.53	0	902.24	909.61
		894.88	
		931.71	
11.3	0	1119.52	1091.28
		1093.74	
		1060.59	

Table 5.5. Static ultimate bearing strength for washer diameter ratio of 1.005.

A plot of bearing strength versus clamping torque for four different bolt to washer diameter ratios is shown in Figure 5.3. Each point shown is the mean value of three experimental trials. The point at which clamping torque equals zero corresponds to the pin-loaded bearing strength which equalled a mean value of 288.9 MPa. Also shown in Figure 5.3 is a dashed line which corresponds to the load transfer through friction. This line was obtained through the use of Equation (5.4), normalized to the units of bearing stress. A value of 0.1 as obtained from Chapter 3 was used as the coefficient of friction. The origin of the friction line was placed at the pin-loaded bearing strength in order to show the increase in bearing strength due to friction alone. From Figure 5.3, it can be seen that for the bolt to washer diameter ratios of 1.005, 1.024, and 1.065, an increase in bearing strength over that due to friction alone occurs. However, there is sufficient clearance in the case of the diameter ratio of 1.142 that the bearing strength is below the amount due to friction. This is consistent with the observation made by Stockdale and Matthews [7], that is, the effect of loose fit washers was to cause failure at a load approximately equal to that obtained at zero clamping force. The reason for this result is that there is sufficient clearance between the bolt and washer such that at the edge of the hole, the composite experiences a pin-loaded or 'brooming' type of failure. When there is only a small amount of clearance, this brooming failure cannot occur due to the restraint of the washers. Therefore, it can be implied from Figure 5.3 that in order to obtain an increase in the bearing strength with increasing clamping torque, a conservative value for the bolt to washer diameter ratio would be 1.065 or less.

Figure 5.3. Bearing strength versus clamping torque for various bolt to washer diameter ratios.

Shown in Figures 5.4, 5.5, and 5.6 are the bearing strength versus clamping torque for bolt to washer diameter ratios of 1.065, 1.024, and 1.005, respectively. Again, the origin of the friction curve was placed at the pin-bearing strength in order to show the increase in bearing strength due to friction.

Considering the difference between the bearing strength curve and the friction curve as an estimate of σ_0^* , one can then find N_r at any value of clamping force. For example, in Figure 5.5, at a clamping force of 11.3 N-m, the bearing strength minus the load transfer due to friction equals 204.22 MPa. If σ_0^* is estimated to be 204.22 MPa, then the load on the composite used to find N_r equals $(288.9 + 204.22) \cdot dt$. Knowing the load on the bearing surface of the composite, N_r can be found using Equation (5.3). The difference between the N_r curve and the friction curve represents the increase in bearing strength due to the resisted lateral expansion and can be obtained from Figures 5.4, 5.5, and 5.6.

Figure 5.4. Variation of bearing strength versus clamping torque for a bolt to washer diameter ratio of 1.065.

Figure 5.5. Variation of bearing strength versus clamping torque for a bolt to washer diameter ratio of 1.024.

Figure 5.6. Variation of bearing strength versus clamping torque for a bolt to washer diameter ratio of 1.005.

Figure 5.7 illustrates the resisted force N_r versus bearing strength for the bolt to washer diameter ratios of 1.005, 1.024, and 1.065. Values for the x-axis of Figure 5.7 are the load on the composite divided by the projected area of the bolt hole. The values for N_r are given in terms of stress (MPa). This is essentially the final term in Equation (5.1). It is seen that N_r is slightly nonlinear, due to the nonlinear nature of the contact surface, and is also a function of washer diameter ratio.

Figure 5.7. Frictional load transfer due to suppressed lateral expansion versus bearing stress for various washer/hole diameter ratios.

Knowing the load transfer due to friction, the pin-bearing strength, and N_f , σ_0^* may be obtained from the experimental data shown in Figures 5.4, 5.5, and 5.6. Values of σ_0^* for washer diameter ratios of 1.005, 1.024, and 1.065 are plotted versus clamping torque in Figure 5.8. From Figure 5.8, it is observed that σ_0^* is strongly dependent upon the washer diameter ratio. For both diameter ratios and for clamping torques of less than 3.76 N-m, σ_0^* is assumed to vary in an approximately linear fashion. At clamping torques greater than 3.76 N-m, a linear relationship between σ_0^* and clamping torque is again assumed, however, this relationship is not as strongly dependent upon the washer diameter ratio as in cases where the clamping torque is less than 3.76 N-m.

Figure 5.8. Increase in the in-plane bearing strength due to an out-of-plane constraint (σ_0^*), versus clamping torque.

For the two regions of clamping torque discussed above, a plot of the slope of Figure 5.8, $\frac{d\sigma_0^*}{dT}$, versus diameter ratio is shown in Figure 5.9. Using a least squares curve fit through the data shown in Figure 5.9, an estimate of the slope may be obtained,

$$0 < T \leq 3.76 \text{ N-m}, \quad \frac{d\sigma_0^*}{dT} = 90.25 \left(\frac{ID_{\text{washer}}}{OD_{\text{bolt}}} \right)^{-48.03} \quad (5.5)$$

$$3.76 < T \leq 11.3 \text{ N-m}, \quad \frac{d\sigma_0^*}{dT} = 25.95 \left(\frac{ID_{\text{washer}}}{OD_{\text{bolt}}} \right)^{-32.98}$$

Knowing $\frac{d\sigma_0^*}{dT}$, and that σ_0^* varies linearly with T (in the two regions given above, Figure 5.8), an estimate of σ_0^* can be made for any given diameter ratio and clamping torque,

$$\sigma_0^* = \frac{d\sigma_0^*}{dT} T \quad (5.6)$$

where T is the clamping torque in N-m.

Figure 5.9. Slope of σ_0^* versus diameter ratio for the tested ranges of clamping torque.

Using the results given above, a prediction of the bearing strength to failure may now be made. At a washer diameter ratio of 1.012, the predicted bearing

strength from Equation (5.1) for clamp-up torques of 3.76, 7.53, and 11.3 N-m are 616.63, 803.36, and 986.22 MPa, respectively. The results of a final experimental trial with a diameter ratio of 1.012 at clamp-up torques of 3.76, 7.53, and 11.3 N-m, were 641.65, 795.18, and 1051.28 MPa, respectively. These results versus predicted results are given in Figure 5.10. Excellent agreement is observed between the predicted and experimental results. Figure 5.10 also shows the magnitudes of the different components contributing to the bearing strength.

As additional experiments are performed at varying diameter ratios, updates to the graph of $\frac{d\sigma_o^*}{dT}$ versus diameter ratio (Figure 5.9) are possible. Additional data points to Figure 5.9 would allow a more accurate fit to approximate σ_o^* .

Figure 5.10. Predicted bearing strength versus experimental results for a bolt to washer diameter ratio of 1.012.

Conclusions

The bolt to washer diameter ratio was found to be a significant parameter in determining the bearing strength of a bolted composite joint. It was shown that a close fit between the bolt diameter and the washer inside diameter provides conditions for maximum bearing strength. Bolt-washer combinations which have sufficiently large radial clearances were found to have a bearing strength approaching that of the pin-loaded case, even under a relatively large clamping force.

From the results obtained, it may be concluded that the increase in bearing strength with increasing clamp-up torque is not due to the load transfer from friction alone. Even considering the increase in the axial load in the bolt due to lateral expansion does not account for the total increase in bearing strength. The remaining increase in bearing strength is due to the out-of-plane constraint which prohibits failure due to 'brooming'. The effectiveness of this constraint was shown to be strongly related to the bolt/washer diameter ratio.

A model has been proposed which predicts the bearing strength of a double lap bolted joint. This model incorporates the major parameters of the joint, namely the pin-bearing strength, friction, resisted lateral expansion, and the increase in the in-plane strength due to an out-of-plane constraint. Satisfactory results were obtained when the model was used to predict the bearing strength of a bolted composite joint.

Chapter 6

Fatigue Testing and Results

Introduction

The purpose of this study is to investigate the fatigue characteristics of a bolted joint in a graphite epoxy composite laminate. In particular, the primary objective of this research is to study the influence of three independent parameters; applied stress level, the orientation of outer layer reinforcing filaments and the bolt torque level on the fatigue life characteristics of a graphite epoxy composite. The previous chapters were necessary steps in achieving this goal. Static ultimate bearing strength, which is required as a basis for selecting alternating stress amplitudes, was discussed in Chapter 4. The ultimate bearing strength of a bolted joint is directly related to the friction coefficients between mating surfaces. Friction coefficients between the composite samples and various metallic washers were obtained as described in Chapter 2. The following sections of this chapter will outline the previous work performed in this field, the fatigue test program carried out, and the techniques used for the analysis of the obtained results.

General Aspects of Testing Composite Materials Under Fatigue Loading

It is well established that under repeated loading, most materials, including fiber reinforced composites, exhibit a decay in strength. In comparisons with metallic isotropic materials, composites display a relatively good resistance to the effects of fatigue cycling. Argarwall [32] and Xian [33] have shown that under repeated loading, the endurance limit for composites is approximately 60% of their ultimate static strength. This can be compared with the endurance limit for metallic materials, which is generally accepted as 28-40% of their ultimate static strength.

The behavior of composites under fatigue loading is generally more complex than the behavior shown by metallic materials. While fatigue damage in metallic materials is typically characterized by a single crack propagating through the material, composites exhibit damage via several different modes, namely delamination, fiber breakage and debonding, matrix crazing, and development of matrix and interface micro-cracks. In the case of composites, this damage may develop early in the fatigue life of the material. A relatively large change in stiffness during the early stages of fatigue cycling of composites has been demonstrated by Highsmith and Reifsnider [34].

The effects of loading frequency is a major concern when fatigue testing composites. Testing performed at low frequencies involves extensive test durations which may create complicating conditions such as creep of the material, while testing at high frequencies (high frequencies generally imply test frequencies greater than 10 Hz) can cause high specimen temperatures which could result in softening of the matrix material. Several studies have focused on the effect of

frequency on the fatigue life of composite materials [35,36,37]. For graphite epoxy, the composite material used in this study, Saff [35] has found that loading frequency significantly affected the fatigue life. The general trend found (with tests conducted at frequencies ranging from 0.1 to 10 Hz) was that as the frequency of load cycling increased, the number of cycles to failure also increased. This effect is particularly prominent at low frequencies, those below 1 Hz. It was also found that matrix dominated layups are much more sensitive to frequency than fiber dominated layups. Tsai et al., [36] also using a composite fabricated with the AS4/3501-6 prepreg, found that the effect of frequency can be measured through the use of the initial dynamic modulus. Tsai then modelled the damage process with a frequency effect modification to a power law relationship. Under the different levels of frequency tested, Tsai found that up to a point at which matrix softening occurs, higher frequencies resulted in longer fatigue lives. Hahn and Kim [37] found a similar trend while testing a glass-epoxy composite within the range of 4 Hz to 10 Hz. Hahn and Kim also noted that while the temperature increase in the composite samples undergoing testing was directly related to the frequency, they concluded that within the above range of frequencies tested, the effect of this increase in temperature was insignificant.

The design of bolted composite structures requires regions of holes, notches, and other discontinuities which provide favorable locations for the initiation and propagation of cracks. Understanding the fatigue behavior of composite materials around notches was the goal of Morton et al. [38], Whitcomb [39], and Tsangarakis [40]. Morton studied the effect of temperature and moisture on the damage developed around notches and found that fatigue behavior around notches is most highly dependent upon lay-up. Whitcomb provided fatigue data on a graphite/epoxy

composite containing a circular hole and found that damage was primarily due to delamination and ply cracking at regions near the hole edge. Based upon the average stress model of Whitney and Nuismer [41], Tsangarakis [40] predicted the endurance limit of notched specimens, given that the endurance limit of unnotched specimens is known. Although the model was verified by testing a metal-matrix composite, it is reasonable to assume that the model would likewise apply for polymer matrix composites.

The laminate ply orientation has been shown to have a significant effect on the fatigue life of composites. The reason is that under loading, high interlaminar stresses may develop at the free edges of the sample. Depending on the layup of the laminate, these interlaminar stresses may be tensile or compressive. If such interlaminar stresses are tensile, they provide a favorable location for the formation of cracks. Rotem [42] has shown that interlaminar normal stresses are the major cause of crack initiation and propagation. While unidirectional laminates do not experience high interlaminar stresses under in-plane loading, Awerbuch and Hahn [43] have shown that they often exhibit a variety of failure modes ranging from fiber fracture to matrix failure, depending on the direction of loading.

Using a graphite-epoxy composite, Rotem [44] investigated the effect of fatigue loading using different values of the applied mean stress. Fatigue failure envelopes are presented for various loadings including tension-tension, compression-compression, and tension-compression. These failure envelopes allow a prediction of the number of cycles endured, given the values of maximum applied stress and the corresponding mean stress level.

Statistical Considerations

A variation exists in fiber and composite strengths under static loading and much more so during fatigue cycling. Therefore, when analyzing fatigue results, some measure of the underlying variability must be provided. To satisfy this requirement, the use of statistical techniques is used. Statistical analysis of fatigue data can be found in the work of Whitney [45,46], Sendekyj [47], and Little [48]. In general, the first step in the statistical analysis of fatigue data is to assume the distributional form of the fatigue life data. Once an assumed distribution has been chosen and experimental tests have been completed, the analysis of the data should include goodness-of-fit tests to investigate the appropriateness of the distribution. If the distribution is deemed appropriate, then a functional form of the relationship between the applied stress and number of cycles can be postulated.

The two most widely used distributions for describing fatigue life data are the log-normal and the two parameter Weibull [49]. The reasons for their popularity include their flexible density functions and previous use in characterizing ultimate tensile strength under static loading in metals. In composites, researchers who have assumed static strength to follow a two-parameter Weibull distribution include Hahn and Kim [37], Chou and Croman [50], Rich and Mass [51], Tenn [52], and others [53-61]. Studies that have assumed composite material's fatigue life data to likewise follow a Weibull distribution include Whitney [45,46], Sendekyj [47], Park and Kim [53,54], and several other investigators [37,43,50,55-61]. In mathematical form, this distribution is given as,

$$R(N) = \exp \left[- \left(\frac{N}{N_0} \right)^\alpha \right] \quad (6.1)$$

where $R(N)$ is the reliability after N number of cycles. N_0 and α represent the two parameters, the characteristic life parameter and the shape parameter of the Weibull distribution, respectively. Although many researchers have used this distribution for all load levels, Shimokawa [62] has shown that while the Weibull distribution provides a better fit at low applied loads, the log-normal distribution gives a better fit for high to medium applied loads. His final conclusion is that the log-normal distribution provides a better overall fit of fatigue data. Another important finding is that the variance of log-life over the range of stress levels tested was found to be constant. Therefore, the underlying distribution of log-life is assumed to be normal, or, fatigue life follows a log-normal distribution. Whitney [45] and Little [63] have also successfully used the log-normal distribution to characterize composite fatigue data.

In general, the distribution of fatigue life is unsymmetric and varies with the applied stress. Weibull [49] concluded that in regions relatively close to the mean, $\log N$ may be assumed to be normally distributed. He also noted that the variance generally increases with decreasing applied stress. Weibull [49] states that experience with fatigue testing of traditional metallic materials has shown that $N^{-\alpha}$ has a distribution independent of stress level. This implies that it may be possible to pool the data resulting from tests conducted at different stress levels.

From the results obtained from fatigue testing a quasi-isotropic composite, Hahn and Kim [37] concluded that the primary failure mechanism in fatigue is a wear-out process that was followed by a "chance" failure. The authors pooled their fatigue data, and fit this data to a two-parameter power law relationship. The parameters were then used to estimate the scatter in fatigue life. Hahn and Kim made the basic intuitive assumption that a specimen with a higher static strength will experience a longer fatigue life. Investigations of this "strength-life equal rank assumption" were conducted by Chou and Croman [50], as well as Kim and Park [54], who established a power law relationship between static strength and fatigue life. The implications of this assumption is that knowing the scatter in static strength, it may be possible to predict the scatter that would be observed during fatigue testing.

Damage

Much effort has been directed toward the characterization of damage developed in composites during fatigue cycling. The term damage in this context generally refers to matrix cracking and crazing, delamination, and fiber breakage. Monitoring crack formation is one method of damage detection, however, since the failure mode of composites is not characterized by a single crack, other methods to monitor damage have been developed. Development and progress of fatigue damage causes continuous changes in the stiffness and strength of the composite. Hence, many investigations focus on the decreases in strength or modulus during fatigue testing [33,64-68]. Residual strength and stiffness of composites after fatigue cycling has been studied by Yang, Whitney, Whitworth, and others [46,55,68,69]. These researchers have proposed residual stiffness or strength degradation models by fitting residual strength data to Weibull distributions or by strength/modulus degradation laws. These degradation laws generally follow power law relationships of the form,

$$\frac{d E (N)}{d N} = \frac{- C f [S, E(0), R, \omega, \text{etc.}]}{a E (N)^{a - 1}} \quad (6.2)$$

where

$E(N)$ = residual stiffness (strength) after N cycles

$E(0)$ = initial stiffness (strength)

$f[]$ = function of applied stress, initial modulus, stress ratio, frequency of loading, and other possible variables such as layup and waveform.

a, C = constants obtained from experimental data.

While these methods require destruction of the sample, methods of nondestructive testing such as Moire interferometry [70], variations in damping [71], and embedded optical fibers [72] have also been used to monitor fatigue damage. Developments in the various ultrasonic techniques [71] have led to the wide use of such techniques for both quality control and inspection as well as fatigue damage detection.

Damage hypothesis regarding the cumulative effects of different load levels applied to a component has been the topic of study of several investigators, among them, Hwang and Han [66], Yang and Jones [56], Yang and Du [57], and Hashin [73]. The objective of these studies is to use the fatigue data obtained from testing a specimen at a single stress level, in order to predict the material's behavior under a wide spectrum of applied loads, as would likely be encountered in use. This is a complex task and much work remains to be done. Most of the models proposed to date are generally extensions of the classical Miner's rule as developed for metals. An extensive review of cumulative damage models has been provided by Hwang and Han [65].

Bolted Joints

Relatively few investigations have been performed on the topic of fatigue of mechanically fastened joints. Studies performed in this area includes that of Crews [74], Mallick, et al. [75], Little, et al. [76], Kam [77], Ramkumar [78] and Schutz, et al. [79]. Crews [74] tested graphite-epoxy laminates under various environmental conditions (testing in air versus testing in water) and clamping torques. The major conclusion of this work is that bolt torque has a significant effect on the strength and life of a composite joint. Mallick and Little [75,76] have investigated the fatigue properties of bolted joints in sheet molding compound composites. In their studies, the joint parameters investigated included clamp-up torque and various specimen geometry variables such as edge and width to diameter ratios. These geometry variables are critical to the performance of bolted joints since they are the major factors in determining the failure mode (bearing, net tension, shear). Schutz, et al. [79] have compared the fatigue behavior of single versus double lap graphite-epoxy bolted joints. Schutz presented data which concluded that due to the excessive hole deformation of single lap joints, double lap joints experienced significantly longer fatigue lives. The study performed by Kam [77] investigated bolt hole deformation in graphite-epoxy laminates under fatigue loading. The overall conclusion, which may also be inferred from the results of Chapter 5 of this study, was that an increase in fatigue life is realized as the bolt diameter to hole diameter ratio approaches unity.

Fatigue Testing Program

The general testing technique used throughout this study follows, as closely as possible, the ASTM Standard E 466-82 [80], "Standard Practice for Conducting Constant Amplitude Axial Tests of Metallic Materials". While this standard is written for metallic materials, many of its guidelines are applicable to composite materials. Details of the testing techniques which are more specific to this study are described below.

Typically, short term relaxation occurs in a bolted joint due to creep and local yielding of high spots on thread mating surfaces. Shivakumar and Crews [81] investigated bolt clamp-up relaxation in graphite-epoxy laminates and found that clamp-up forces may decrease by as much as 8% over a period of 24 hours. Likewise, Kulak, Fisher, and Struik [30] showed a rapid loss in bolt tension immediately following tightening, then a decrease in tension at a much slower rate. New bolts were shown to be more affected by short term relaxation than previously tightened bolts. For this reason, the bolts used were torqued and untorqued several times prior to use.

In a bolted joint, the direct coupling of a metallic bolt-washer combination with the graphite epoxy composite can create a galvanic corrosion couple. Tucker, et al. [82], performed corrosion tests on graphite composites directly coupled with various metals and found that the composite was cathodic with respect to the metals. Therefore in electrolytic atmospheres, it is desirable to isolate the bolt and washer from direct contact with the composite through the use of rubber inserts or gaskets. However, no isolation was provided throughout the test series since the maximum

time of contact between the bolt and any particular composite specimen is approximately 30 hours.

All bolts used during fatigue testing were 1/4 - 28 UNC stainless steel, as described in Chapter 2. The washer used throughout fatigue testing was a stainless steel washer having a inside diameter to bolt diameter ratio of 1.024 and an outside diameter of 1.75 cm. Prior to each test, the washer surfaces were cleaned using a degreasing agent as described in Chapter 3. While the threaded portion of the bolt was lubricated, the shank portion which contacts the composite bearing area was also degreased to comply with ASTM Standard E 238-84 [83] ("Standard Test Methods for Pin-Type Bearing Strength of Metallic Materials"), which notes that the presence of incidental lubricants on the bearing surfaces could significantly lower bearing yield strength.

Once the upper mount and composite joint were assembled on the testing machine, the lower grip was positioned such that proper alignment was assured. The alignment of the fixed load cell and cross-head to that of the lower grip is crucial in order to avoid any undesirable bending moments being applied to the specimen during testing.

All fatigue tests were performed under the load control function of the Instron testing machine. The load control function assures that a constant amplitude alternating load will be applied to the specimen throughout its life. Therefore the independent variable for a given bolted joint is the applied stress, σ_a , and the dependent variable is the number of cycles endured, N (fatigue life). The loading waveform selected was sinusoidal in shape and repeated itself at a frequency of 10 Hz. The applied varying stress is defined by the paired component σ_{max} and the stress ratio R , where R is defined as $\sigma_{min}/\sigma_{max}$, Figure 6.1. The stress ratio used

for all fatigue testing was $R \approx 0.1$, therefore the mean stress is equal to 55% of the maximum applied stress.

Figure 6.1. Load-controlled waveform applied to test specimens.

Once the fatigue test had begun, the test was continued until failure or a terminating value of the number of cycles was reached. As previously mentioned, a failure criteria must be defined when testing composite materials. Although a bolted joint may fail due to one of several different modes (shear-out, net tension, bearing, etc.), the samples used in this study had edge and width to diameter ratios sufficiently large to avoid a shear-out or net-tension failure. Hence, a failure due to bearing is the expected failure mode. Knowing the failure mode, it is now possible to define when a specimen has failed. The definition used for fatigue failure in this study is a bolt hole deformation equal to 4% of the original hole diameter. This definition of failure is consistent with ASTM Standard D953 [84] (Standard Test Method for Bearing Strength of Plastics) which defines the static ultimate bearing strength as the stress that causes a 4% elongation in the hole diameter, d .

The failure criteria of 4% elongation was measured relative to the position transducer of the testing machine. Upon loading, the minimum position of the actuator was noted. Adding $0.04 \cdot d$ to the minimum position resulted in the value used to signal a shut-off of the applied waveform.

If a test piece did not fail (hole elongation $< 0.04 \cdot d$) after undergoing 10^6 stress reversals, then the test was suspended and that specimen was considered a run-out. When run-outs occur, the data for that sample is considered censored. The fact that a sample is considered censored does not imply that it is not otherwise counted. In statistical terminology, the term censored refers to situations in which the sampled specimens must lie within a given interval in order to perform an unrestricted analysis of the data. Cases in which sample results are outside of the given limits are called censored data points and special consideration of these points are required in order to perform a valid analysis of the data. In the case of fatigue testing, time and resource restrictions often require an upper limit of the amount of time a single specimen may be tested. This upper limit is often given as a maximum number of cycles a specimen may undergo before the test is suspended. As mentioned, the upper limit of 10^6 cycles was selected for this test program. Imposing the condition that the test be suspended after a sample withstands 10^6 cycles without failure has important implications upon the analysis of the data [85]. Namely, the use of 10^6 cycles as the fatigue life of a test-suspended sample when determining the sample mean and the sample variance is neither appropriate nor correct. What can be said about these suspended tests is that the specimen had a fatigue life greater than 10^6 cycles and that the residual strength of the sample was greater than the applied stress. However, statistical techniques are available which provide maximum likelihood estimates of the sample mean and the sample

variance when a limited number of specimens are censored. The technique used throughout this study to determine the sample mean and variance is discussed in the Data Analysis section of this chapter.

The independent variables in this test program are outer ply angle/layup, clamping torque, and the applied stress level. Three outer ply angles, (0, 45, and 90 degrees), four clamping torques, (0, 3.76, 7.53, and 11.3 N-m), and six stress levels were used for testing. While the three outer ply angles and four clamping torques remained the same throughout all tests, the applied stress levels depended upon which applied clamping torque was being tested. The maximum applied stress levels chosen for the pin-loaded case ranged from 73 to 94 percent of the ultimate static strength. For the clamped joint case, the stress levels were 86%, 84%, 82%, 80%, and 78% of the ultimate static bearing strength. Because of the relatively large scatter in fatigue life, three fatigue tests were performed under identical conditions of the independent variables (applied clamping torque, maximum applied stress, and outer ply angle). Four levels of applied clamping torque, six stress levels, three outer ply angles and two replications of each test led to a total of 216 experiments. During testing, several test malfunctions occurred, (such as accidental interruption of testing) however, sufficient additional samples were available to repeat these tests. Therefore under each condition of clamping torque, stress level, and outer ply angle, three experimental trials were performed.

The guidelines for percent replication in fatigue testing given by Little [86] and accepted as an ASTM Standard are as follows. Percent replication for research and development testing of components and specimens is a minimum of 33 to 50%

where the definition of percent replication is given as,

$$\% \text{ Replication} = 100 \left[1 - \frac{\text{total number of stress levels used in testing}}{\text{total number of specimens tested}} \right] \quad (6.4)$$

Percent replication for design allowables is given as 50 to 75%, minimum. For this study, under a given clamping torque and outer ply angle, six stress levels and eighteen specimens tested result in a percent replication of 66.67. As will be discussed later, grouping of the outer ply angle data resulted in a percent replication of 88.89.

Data Analysis

In general, the objective of fatigue testing is the determination of the distribution of the number of cycles to failure, N , by means of determining the parameters of a chosen distribution function. In this case, the distribution function has been chosen, and the objective is to determine the parameters which characterize this distribution. The following section describes the statistical foundation upon which the fatigue data is analyzed.

Comparisons between main factor effects (whether a significant difference exists between maximum applied stress level, outer ply orientation, and applied torque) were made according to the multifactor ANOVA model given as,

$$Y_{ijkm} = \mu_{...} + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \varepsilon_{ijkm} \quad (6.5)$$

where Y_{ijkm} is the m -th observation for the i -th applied stress level ($i = 1-5$), the j -th level of laminate orientation ($j = 1-3$), and the k -th level of clamping torque ($k = 1-4$). The term $\mu_{...}$ is the overall mean, α_i is the effect of the i stress levels, β_j is the effect of the j orientations, and γ_k is the effect of the k torque levels. Interaction effects are taken into account through the terms $(\alpha\beta)_{ij}$, $(\alpha\gamma)_{ik}$, $(\beta\gamma)_{jk}$, and $(\alpha\beta\gamma)_{ijk}$ and ε_{ijkm} are the independent error terms.

Significance of main factor effects and interactions were determined by using hypothesis testing and the F -test statistic. If the hypothesis of equal factor level means are rejected, that is, a significant difference exists in the fatigue life due to

either clamping torque, applied stress level, or outer ply angle, then further tests will be made concerning the effects of the individual factor levels.

At each level of clamping torque, comparisons between the effects of outer ply orientation (whether a significant difference in the bearing strength exists between the three outer ply orientations) were made according to the single factor ANOVA model given as,

$$Y_{ij} = \mu_i + \varepsilon_{ij} \quad (6.6)$$

where Y_{ij} is the value of bearing strength for the j -th trial under the i -th condition of outer ply orientation. The term μ_i is the sample mean for the i -th level of outer ply orientation and ε_{ij} are independent error terms which follow $N(0, s^2)$. The analysis of variance model partitions the total variability of the observed data into deviations within the groups and the deviations between the groups. This is accomplished via the sum of the squares of these deviations. The sum of the squares of deviations between groups divided by their degrees of freedom is termed the treatment mean square while the sum of squares of deviations within groups divided by the degrees of freedom is termed error mean square. To test whether a significant difference in the bearing strength exists when comparing results from the three outer ply orientations, hypothesis testing is used. The possible conclusions are,

$$H_0 : \mu_1 = \mu_2 = \mu_3 \quad (6.7)$$

$$H_a : \text{not all } \mu_i \text{ are equal}$$

The test statistic for this comparison is,

$$F^* = \frac{\text{treatment mean square}}{\text{error mean square}} \quad (6.8)$$

A large value of F^* implies that the deviations between the groups is sufficiently large compared to the deviations within the groups. Therefore it is reasonable to assume that a significant difference exists in the means between groups. Hence, this reasoning supports H_a . On the other hand, a small value of F^* supports H_o , since the deviations between groups are small relative to deviations within groups.

If all μ_i 's are equal, H_o is true, then F^* is distributed as $F(v_1, v_2)$. The terms v_1 and v_2 are the two parameters of the F distribution, namely, the numerator degrees of freedom and the denominator degrees of freedom, respectively. The decision rule with a level of significance α is given by,

$$F^* \leq F(1 - \alpha; r - 1, n - r), \quad \text{Conclude } H_o \quad (6.9)$$

$$F^* > F(1 - \alpha; r - 1, n - r), \quad \text{Conclude } H_a$$

where r = number of treatment levels ($r = 3$), and n = number of observations.

Tabulated values of $F(1 - \alpha; v_1, v_2)$ can be found in Neter, et al. [87].

Probability Functions

The fatigue data obtained in this study will be characterized by the use of probability distribution functions. The following section briefly outlines the form of the assumed distribution and the technique used to determine its parameters.

The probability density function of the random variable X , which follows the normal distribution is given as,

$$f(X) = \frac{1}{\sqrt{2\pi} s_o} \exp \left\{ -\frac{1}{2} \left[\frac{X - \mu_o}{s_o} \right]^2 \right\}, \quad -\infty < X < +\infty \quad (6.10)$$

Where μ_o and s_o^2 are the two parameters of the normal distribution, namely the mean and variance of the normal random variable X , given as,

$$\mu_o = E(X) \quad (6.11)$$

$$s_o^2 = E(X^2) - \mu_o^2$$

$E[u(X)]$ is the mathematical expectation of $u(X)$ defined as,

$$E[u(X)] = \int_{-\infty}^{\infty} u(x) f(x) dx \quad (6.12)$$

where $f(x)$ is the probability density function. The cumulative distribution function $F(y)$, which gives the probability that a continuous random variable Y is less than or equal to the point y is defined as,

$$F(y) = \Pr(Y \leq y) = \int_{-\infty}^y f(x) dx \quad (6.13)$$

For populations following the normal distribution, the cumulative distribution function is defined as,

$$F(y) = \int_{-\infty}^y \frac{1}{\sqrt{2\pi} s_0} \exp\left\{-\frac{1}{2} \left[\frac{x - \mu_0}{s_0}\right]^2\right\} dx. \quad (6.14)$$

In this study it is assumed that the static ultimate bearing strength follows a log-normal distribution. The test for the validity of this assumption will be discussed in a later section. Assuming that the static strength data, $(\sigma(0))$, follows a log-normal distribution, a transformation is made to the normal cumulative distribution function. Making the transformation $x = \log y$, the log-normal cumulative distribution function $F(y)$ is given as,

$$F(\sigma_{u,j}) = \int_0^{\sigma_{u,j}} \frac{\beta_o^{-1}}{\sqrt{2\pi} y} \exp \left\{ \frac{-1}{2\beta_o^2} [\log(y) - \alpha_o]^2 \right\} dy. \quad (6.15)$$

where α_o is the log-mean of the population, β_o^2 is the log-variance of the population, and $\sigma_{u,j}$ is the ultimate static bearing strength under a clamping torque j. The solution of Equation (6.15) follows.

Letting $\alpha_o = \log(a)$, the above integral can be written as,

$$F(\sigma_{u,j}) = \frac{\beta_o^{-1}}{\sqrt{2\pi}} \int_0^{\sigma_{u,j}} \frac{1}{y} \exp \left\{ \frac{-1}{2\beta_o^2} \left[\log\left(\frac{y}{a}\right) \right]^2 \right\} dy. \quad (6.16)$$

Letting

$$c = \frac{\beta_o^{-2}}{2}, \text{ and } x = \log\left(\frac{y}{a}\right),$$

the transformed integral becomes,

$$F(\sigma_{u,j}) = \frac{\beta_o^{-1}}{\sqrt{2\pi}} \int_{-\infty}^{\log\left(\frac{\sigma_{u,j}}{a}\right)} \exp \{-cx^2\} dx. \quad (6.17)$$

$$F(\sigma_{u,j}) = 0.5 + \frac{\beta_o^{-1}}{\sqrt{2\pi}} \left(\frac{1}{2} \sqrt{\frac{\pi}{c}} \right) \operatorname{erf} \left[\log \left(\frac{\sigma_{u,j}}{a} \right) \sqrt{c} \right], \quad (6.18)$$

where $\operatorname{erf}(x)$ designates the error function. The error function can be written as,

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k (x)^{2k+1}}{k! (2k+1)} \quad (6.19)$$

Therefore, the solution to Equation (6.15) becomes,

$$F(\sigma_{u,j}) = 0.5 + \frac{\beta_o^{-1}}{\sqrt{2\pi c}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\log \left(\frac{\sigma_{u,j}}{a} \right) \sqrt{c} \right)^{2k+1}}{k! (2k+1)} \quad (6.20)$$

In terms of the two parameters, α_o and β_o , Equation (6.20) becomes

$$F(\sigma_{u,j}) = 0.5 + \frac{\beta_o^{-1}}{\sqrt{2\pi c}} \sum_{k=0}^{\infty} \frac{(-1)^k [(\log \sigma_{u,j} - \alpha_o) \sqrt{c}]^{2k+1}}{k! (2k+1)} \quad (6.21)$$

Differentiating Equation (6.15), we obtain the probability density function for the random variable Y ,

$$f_{\alpha_0}(Y) = \frac{\beta_0^{-1}}{\sqrt{2\pi} Y} \exp \left\{ \frac{-1}{2\beta_0^2} [\log(Y) - \alpha_0]^2 \right\}, \quad Y > 0. \quad (6.22)$$

The above equation is called the log-normal probability density function for the random variable Y . Therefore the log-normal distribution function is described by the two parameters, α_0 , and β_0 . For fully populated data, the maximum likelihood estimator $\hat{\alpha}_0$ of α_0 is given by [88],

$$\hat{\alpha}_0 = \frac{1}{n} \sum_{i=1}^n \log y_i \quad (6.23)$$

and the maximum likelihood estimator $\hat{\beta}_0$ of β_0 is given as,

$$\hat{\beta}_0 = \left\{ \frac{1}{n} \left[\sum_{i=1}^n [\log(y_i)]^2 - n \left[\sum_{i=1}^n \frac{\log(y_i)}{n} \right]^2 \right] \right\}^{-0.5} \quad (6.24)$$

For the case of bearing strength data, the estimators $\hat{\alpha}_0$ and $\hat{\beta}_0$ represent the characteristic strength ($\bar{\sigma}_{u,j}$), and the standard deviation ($\bar{\beta}_{0,j}$), at a given clamping torque j , as,

$$\bar{\sigma}_{u,j} = \frac{1}{n_j} \sum_{i=1}^{n_j} \log \sigma_{ui,j} \quad (6.25)$$

$$\bar{\beta}_{oj} = \left\{ \frac{1}{n_j} \left[\sum_{i=1}^{n_j} [\log(\sigma_{ui,j})]^2 - n_j \left[\sum_{i=1}^{n_j} \frac{\log(\sigma_{ui,j})}{n_j} \right]^2 \right] \right\}^{-0.5} \quad (6.26)$$

where

$\sigma_{ui,j}$ = static strength of specimen i at clamping torque j ,

n_j = number of specimens tested under static loading at clamping torque j .

The values of these parameters for both static and fatigue data can be found in the summary statistics section of Tables 6.1 through 6.24.

Assuming that fatigue life also follows a two-parameter log-normal distribution, then the probability that a test specimen survives less than N_j cycles is given as,

$$F(N_j) = \int_0^{N_j} \frac{\beta_{of}^{-1}}{\sqrt{2\pi} y} \exp \left\{ -\frac{1}{2\beta_{of}^2} [\log(y) - \alpha_{of}]^2 \right\} dy. \quad (6.27)$$

or,

$$F(N_j) = 0.5 + \frac{\beta_{of}^{-1}}{\sqrt{2\pi c}} \sum_{k=0}^{\infty} \frac{(-1)^k [(\log N_j - \alpha_{of}) \sqrt{c}]^{2k+1}}{k! (2k+1)}. \quad (6.28)$$

For fully populated fatigue data, the maximum likelihood estimator for the characteristic life at a clamping torque j , $\bar{N}_j$, is given by,

$$\bar{N}_j = \alpha_{of} = \frac{1}{n_{fj}} \sum_{i=1}^{n_{fj}} \log (N_{ij}) \quad (6.29)$$

and the maximum likelihood estimator $\bar{\beta}_{ofj}$, of β_{ofj} at a clamping torque j , is given as,

$$\bar{\beta}_{ofj} = \left\{ \frac{1}{n_{fj}} \left[\sum_{i=1}^{n_{fj}} [\log (N_{ij})]^2 - n_{fj} \left[\sum_{i=1}^{n_{fj}} \frac{\log (N_{ij})}{n_{fj}} \right]^2 \right] \right\}^{-0.5} \quad (6.30)$$

where

$N_{i,j}$ = number of cycles to failure of specimen i , under a clamping torque j

n_{fj} = number of specimens tested under fatigue loading and at a clamping torque j .

Maximum Likelihood Estimator for Censored Data

As discussed in the test program section of this chapter, fatigue testing was continued until failure of the test specimen occurred or until a terminating value of 10^6 cycles was reached. The termination of testing has important implications upon the statistical analysis of the data. Namely, the maximum likelihood estimators for complete samples as given above, do not hold for censored samples. What is needed is a correction of the restricted sample mean and variance involving a function of the terminating value.

In the statistical literature, this type of censoring is termed Type I. It is characterized by the fact that the number of failures occurring at the terminating value is a random variable. More complete discussions on the topic of censoring may be found in the works of Gupta [89], Cohen [90,91], Nelson [92], and Bain [93]. The method of maximum likelihood estimation for this type of censoring is demonstrated below.

The log likelihood for a sample of M experimental runs with r failures, ($M - r$) = number of censored tests, is given by,

$$L = \sum_{i=1}^r \ln \left\{ \left(\frac{1}{s_0} \right) f \left(\frac{y_i - \mu_0}{s_0} \right) \right\} + \sum_{i=1}^{M-r} \ln \left\{ 1 - F \left(\frac{y_i - \mu_0}{s_0} \right) \right\} \quad (6.31)$$

where,

$f\left(\frac{y_i - \mu_o}{s_o}\right)$ = normal probability density function as given in Equation (6.10),

$F\left(\frac{y_i - \mu_o}{s_o}\right)$ = cumulative distribution function given by Equation (6.14).

The maximum likelihood estimate is obtained by maximizing Equation (6.31), which is equivalent to the solutions of the likelihood equations,

$$\frac{\partial L}{\partial \mu_o} = \left(\frac{1}{s_o}\right) \sum_{i=1}^{M-r} \frac{f\left(\frac{y_i - \mu_o}{s_o}\right)}{1 - F\left(\frac{y_i - \mu_o}{s_o}\right)} + \frac{r}{s_o} \left(\frac{\bar{y} - \mu_o}{s_o}\right) = 0 \quad (6.32)$$

$$\begin{aligned} \frac{\partial L}{\partial s_o} = & \left(\frac{1}{s_o}\right) \sum_{i=1}^{M-r} \left(\frac{y_i - \mu_o}{s_o}\right) \frac{f\left(\frac{y_i - \mu_o}{s_o}\right)}{1 - F\left(\frac{y_i - \mu_o}{s_o}\right)} + \frac{r}{s_o} \\ & + \frac{r}{(s_o)^3} \left(\sum_{i=1}^r \frac{(y_i - \bar{y})^2}{r} + (\bar{y} - \mu_o)^2 \right) = 0 \end{aligned} \quad (6.33)$$

Several methods may be used to obtain the estimators which include maximization of the log likelihood equation, or solution of the system of partial differential equations via numerical methods. Within this study, the method for obtaining the maximum likelihood estimators was the use of the technique provided by Cohen [90,91]. The

use of Cohen's method provides directly the estimates without the iterative technique of numerical methods. An example of how the estimators are found using Cohen's method is given below. The mean and variance for Type I singly censored samples are given as,

$$\hat{\mu} = \bar{x} - \hat{\lambda}(\bar{x} - x_0) \quad (6.34)$$

$$\hat{s}^2 = s^2 + \hat{\lambda}(\bar{x} - x_0)^2 \quad (6.35)$$

where $\bar{x}$ and s^2 are the restricted sample mean and variance, respectively (restricted sample mean and variance are the parameters of those samples which were tested until failure, without run-outs). The term x_0 is the terminating value, in this case, $\log(10^6)$ cycles. $\hat{\lambda}$ is obtained by linear interpolation of tables which are a function of γ and h ,

$$\gamma = \frac{1 - Y(Y - \xi)}{(Y - \xi)^2}, \quad h = \frac{M - n}{M}, \quad (6.36)$$

where

$$Y = \frac{[h/(1-h)\varphi(\xi)]}{F(\xi)},$$

$$F(\xi) = \int_{-\infty}^{\xi} \varphi(t) dt,$$

$$\varphi(t) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right),$$
(6.37)

and

$$\xi = \frac{x_0 - \mu}{s}.$$

The data obtained from the tests at a clamping torque of 11.3 N-m and at a maximum applied stress level of 621 MPa will be used to illustrate the above technique. The number of cycles under these loading conditions for outer ply angles of 0, 45, and 90 degrees combined are (343188, 409734, 685405), (117652, 517521, 755204), and (219524, 1000000, 1000000), respectively. After taking the log of the data, the restricted mean and standard deviation are, $\bar{x} = 5.569$ and $s = 0.286$. The proportion of censored samples, $h = (M - n)/M = 2/9$. Using $x_0 = \log(10^6)$, and two-way interpolation of the Cohen tables, λ was found to be 0.32827. Applying this value to Equations (6.34) and (6.35), resulted in a change of the sample mean and standard deviation to the values $\bar{x} = 5.71$ and $s = 0.384$. The results of maximum likelihood estimators for both censored and uncensored data can be found in the summary statistics section of Tables 6.5 through 6.24.

Fatigue Results

Table 6.1 lists the results for the bearing strength tests conducted at a clamping torque of 0 N-m. Shown in the table are the three trials conducted at each of the three outer ply angles (0, 45, and 90 degrees), along with the corresponding mean value of the ultimate bearing strength. The next section of Table 6.1 provides the summary statistics for the above data. The first value listed is F^* . This is the calculated value (Equation 6.8), as given by [87],

$$F^* = \frac{\text{Treatment Mean Square}}{\text{Error Mean Square}}$$

$$\text{Treatment Mean Square} = \frac{\sum_i n_i (\bar{Y}_i - \bar{Y}_{..})^2}{r - 1} \quad (6.38)$$

$$\text{Error Mean Square} = \frac{\frac{1}{n_t - r} \sum_i \sum_j (Y_{ij} - \bar{Y}_i)^2}{n_t - r} \quad (6.39)$$

where,

n_t = total number of observations,

n_i = number of observations of the i -th outer ply angle,

r = number of groups (3 - 0, 45, 90),

Y_{ij} = ultimate static bearing strength for the i -th outer ply angle under the j -th trial,

$\bar{Y}_i = \frac{\sum_{j=1}^{n_i} Y_{ij}}{n_i}$, the sample mean for the i -th outer ply angle and,

$$\bar{Y}_{..} = \frac{\sum_i \sum_j Y_{ij}}{n_t}, \text{ the overall mean for all observations.}$$

This value, F^* , is compared with tabulated values of the F distribution via the decision rule, Equation (6.9). For the case of 3 groups, 3 trials within each group, and a level of significance of 0.05, the tabulated value of F is 5.14. Since the calculated value F^* was found to be 0.57, less than the tabulated value of 5.14, one cannot reject the null hypothesis at the 0.05 level of significance. In other words, in a comparison of the three outer ply angles, it cannot be said that a significant difference exists in the ultimate bearing strength. Also given in Table 6.1 is p , the probability that the tabulated value with the corresponding degrees of freedom is greater than the calculated F^* , that is, $P[F(2,6) > F^* = 0.57]$.

<u>Clamping Torque (N-m)</u>	<u>Outer Ply Angle</u>	<u>Bearing Strength (MPa)</u>	<u>Mean Bearing Strength (MPa)</u>
0	0	296.31	
	0	285.19	288.9
	0	285.21	
0	45	291.37	
	45	251.86	279.02
	45	293.84	
0	90	269.15	
	90	276.56	276.56
	90	283.97	

Summary statistics of grouped data for clamping torque = 0 N-m;

$$F^*[\log(\sigma_j)] = 0.57 \quad p = 0.59$$

$$\mu[\log(\sigma_j)] = 2.454 \quad s[\log(\sigma_j)] = 0.0217$$

$$\text{Skewness} = -1.14 \quad \text{Kurtosis} = -0.42$$

Table 6.1. Bearing data and summary statistics for a clamping torque = 0 N-m.

From the above results, it may be concluded that outer ply angle exerts an insignificant effect on the ultimate bearing strength. Therefore, the data obtained for the three groups of outer ply angle may be combined as a set of nine bearing strength tests under a clamping load of 0 N-m.

The values μ and s of Table 6.1 represent the log-mean and the log-standard deviation of the grouped data, respectively. As the mean and standard deviation are obtained from the first and second moments of the distribution, the values of skewness and kurtosis are obtained from the third and fourth moments, respectively. Skewness is a measure of the departure from symmetry of a distribution. Negative values of skewness refers to distributions which are skewed to the left (the density curve shows a longer tail to the left of the central maximum), and positive skewness refers to distributions skewed to the right. Kurtosis refers to the "peakedness" or "flatness" of the distribution, with a positive value representing a distribution more peaked than the normal, and negative kurtosis represents a distribution flatter than the normal. These values of skewness and kurtosis at the given clamping torque are likewise given in Table 6.1 and the tables which follow. Tables 6.2 through 6.4 provides the results obtained for static bearing strength tests under clamping torques of 3.76, 7.53, and 11.3 N-m, respectively.

<u>Clamping Torque (N-m)</u>	<u>Outer Ply Angle</u>	<u>Bearing Strength (MPa)</u>	<u>Mean Bearing Strength (MPa)</u>
3.76	0	494.68	
	0	613.21	554.73
	0	556.29	
3.76	45	584.38	
	45	609.08	576.16
	45	535.01	
3.76	90	592.61	
	90	648.46	611.82
	90	584.38	

Summary statistics of grouped data for clamping torque = 3.76 N-m;

$$F^*[\log(\sigma_j)] = 1.14 \quad p = 0.38$$

$$\mu[\log(\sigma_j)] = 2.762 \quad s[\log(\sigma_j)] = 0.035$$

$$\text{Skewness} = -0.46 \quad \text{Kurtosis} = -0.25$$

Table 6.2. Bearing data and summary statistics for a clamping torque = 3.76 N-m.

<u>Clamping Torque (N-m)</u>	<u>Outer Ply Angle</u>	<u>Bearing Strength (MPa)</u>	<u>Mean Bearing Strength (MPa)</u>
7.53	0	618.35	
	0	742.02	725.54
	0	816.25	
7.53	45	707.85	
	45	720.19	722.94
	45	740.77	
7.53	90	699.62	
	90	720.19	720.19
	90	740.77	

Summary statistics of grouped data for clamping torque = 7.53 N-m;

$$F^*[\log(\sigma_j)] = 0.002 \quad p = 0.99$$

$$\mu[\log(\sigma_j)] = 2.857 \quad s[\log(\sigma_j)] = 0.031$$

$$\text{Skewness} = -0.59 \quad \text{Kurtosis} = 0.97$$

Table 6.3. Bearing data and summary statistics for a clamping torque = 7.53 N-m.

<u>Clamping Torque (N-m)</u>	<u>Outer Ply Angle</u>	<u>Bearing Strength (MPa)</u>	<u>Mean Bearing Strength (MPa)</u>
11.3	0	828.39	
	0	783.67	832.06
	0	914.12	
11.3	45	691.38	
	45	806.62	784.67
	45	856.01	
11.3	90	707.85	
	90	786.25	786.11
	90	864.24	

Summary statistics of grouped data for clamping torque = 11.3 N-m;

$$F^*[\log(\sigma_j)] = 0.54 \quad p = 0.61$$

$$\mu[\log(\sigma_j)] = 2.904 \quad s[\log(\sigma_j)] = 0.039$$

$$\text{Skewness} = -0.37 \quad \text{Kurtosis} = -0.80$$

Table 6.4. Bearing data and summary statistics for a clamping torque = 11.3 N-m.

Table 6.5 is a summary of the output obtained from the three factor analysis of variance model, Equation (6.5). The data used for this model is the fatigue life obtained from the clamped joint case (Torque = 3.76, 7.53, and 11.3 N-m). The fatigue life data from the pin-loaded case was not included here since different ratios of applied stress were used. From Table 6.5, it can be seen that neither three factor (stress*ply*clamp), nor two factor (stress*ply, stress*clamp, ply*clamp) interactions were present. Since interactions do not exist, it is possible to investigate main factor effects. For the case of outer ply angle, it is seen that the calculated value of F^* is less than the critical value of F , hence the null hypothesis may not be rejected. Therefore, it is unreasonable to say that the outer ply angle exerts a significant difference on the fatigue life of the composite joint. A similar result is obtained for factor B, the effect of clamping torque. This result implies that clamping torque does not influence the fatigue life of the bolted joint. However, it must be noted that the applied stress level is normalized with respect to the ultimate bearing strength. As shown in Chapter 5, clamping torque is directly related to the ultimate bearing strength. Therefore, the analysis of variance test yields an interesting result. Irregardless of clamping torque, the critical factor in fatigue life tests is the ratio of applied stress to ultimate strength. High clamping torques yield high values of ultimate bearing strength while low clamping torques yield relatively low values of ultimate bearing strength. However, when the cyclic load is applied as a percentage of ultimate bearing strength, the difference in fatigue life at the various clamping torques was found to be insignificant. Another conclusion which may be drawn from Table 6.5 is that factor C, the maximum applied stress level does exert a significant influence on the fatigue life of the bolted

joint. This was an expected result, that an increase in the applied cyclic stress corresponded to a decrease in fatigue life.

<u>Source</u>	<u>D.F.</u>	<u>Mean Square</u>	<u>F*</u>	<u>p-value</u>
Outer Ply (A)	2	0.043	0.366	0.695
Clamp (B)	2	0.077	0.656	0.521
Stress (C)	4	26.449	223.984	0.0001
A*B	4	0.171	1.446	0.225
A*C	8	0.174	1.472	0.179
B*C	8	0.076	0.642	0.740
A*B*C	16	0.145	1.224	0.266

Table 6.5 Three-factor analysis of variance model output.

The pin-loaded results were not included in the three-factor analysis of variance model since its behavior, and the required applied stress levels, were vastly different than the results obtained from the clamped (Torque > 0) case. For the pin-loaded case, a two factor analysis of variance model was applied. The two independent factors in this case are the outer ply orientation and the applied stress. Table 6.6 shows the model results. As in the three-factor case, the outer ply orientation did not significantly affect fatigue life. As expected, the effect of applied stress did again influence the fatigue life.

<u>Source</u>	<u>D.F.</u>	<u>Mean Square</u>	<u>F*</u>	<u>p-value</u>
Outer Ply (A)	2	0.129	0.397	0.675
Stress (B)	4	16.445	50.606	0.0001
A*B	8	0.229	0.705	0.685

Table 6.6 Two-factor analysis of variance model output.

The results of the fatigue tests are shown in Tables 6.7 through 6.26, which list the number of cycles endured while undergoing a given clamping torque and applied stress. Summary statistics for the assumed log-normal distribution are also provided for the nine samples (by grouping the outer ply angle data). These tables are of the same form as Tables 6.1 through 6.4, yet instead of static ultimate bearing strength, they present the results of the various fatigue tests. The test of hypothesis and the calculated F^* values now are used to test whether the outer ply angle exerts a significant effect on the number of cycles to failure.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
265	0	98		
	0	7	1.85	0.94
	0	509		
265	45	976		
	45	788	2.70	0.43
	45	159		
265	90	273		
	90	915	2.11	1.04
	90	9		

Summary statistics of grouped data for clamping torque = 0 N-m;

$$F^*[\log(N)] = 0.78 \quad p = 0.51$$

$$\mu[\log(N)] = 2.22 \quad s[\log(N)] = 0.82$$

$$\text{Skewness} = -0.79 \quad \text{Kurtosis} = -0.84$$

$$\text{Median} = 2.44$$

Table 6.7. Fatigue data for maximum applied stress level = 265 MPa ($R = 0.1$), and summary statistics for a clamping torque = 0 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
250	0	3510		
	0	564	3.36	0.54
	0	6168		
250	45	503		
	45	553	2.88	0.28
	45	1572		
250	90	984		
	90	3659	3.22	0.30
	90	1258		

Summary statistics of grouped data for clamping torque = 0 N-m;

$$F^*[\log(N)] = 1.192 \quad p = 0.37$$

$$\mu[\log(N)] = 3.15 \quad s[\log(N)] = 0.402$$

$$\text{Skewness} = -0.13 \quad \text{Kurtosis} = -1.11$$

$$\text{Median} = 3.10$$

Table 6.8. Fatigue data for maximum applied stress level = 250 MPa (R = 0.1), and summary statistics for a clamping torque = 0 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
235	0	60229	4.45	0.72
	0	4199		
	0	85631		
235	45	45672	4.41	0.25
	45	25971		
	45	14690		
235	90	63574	4.27	0.46
	90	11627		
	90	9041		

Summary statistics of grouped data for clamping torque = 0 N-m;

$$F^*[\log(N)] = 0.09 \quad p = 0.92$$

$$\mu[\log(N)] = 4.38 \quad s[\log(N)] = 0.45$$

$$\text{Skewness} = -0.31 \quad \text{Kurtosis} = -1.18$$

$$\text{Median} = 4.41$$

Table 6.9. Fatigue data for maximum applied stress level = 235 MPa ($R = 0.1$), and summary statistics for a clamping torque = 0 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
220	0	451768	5.19	0.55
	0	206814		
	0	38646		
220	45	51560	4.89	0.25
	45	151412		
	45	63412		
220	90	12722	4.82	0.62
	90	165845		
	90	138260		

Summary statistics of grouped data for clamping torque = 0 N-m;

$$F^*[\log(N)] = 0.44 \quad p = 0.66$$

$$\mu[\log(N)] = 4.97 \quad s[\log(N)] = 0.46$$

$$\text{Skewness} = -0.44 \quad \text{Kurtosis} = -0.47$$

$$\text{Median} = 5.14$$

Table 6.10. Fatigue data for maximum applied stress level = 220 MPa (R = 0.1), and summary statistics for a clamping torque = 0 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
205	0	489571		
	0	1000000	5.76	0.21
	0	394475		
205	45	829045		
	45	1000000	5.55	0.71
	45	53051		
205	90	251253		
	90	68594	5.29	0.42
	90	449360		

Summary statistics of grouped data for clamping torque = 0 N-m;

$$F^*[\log(N)] = 0.67 \quad p = 0.55$$

$$\mu[\log(N)] = 5.61 \quad s[\log(N)] = 0.68$$

$$\text{Skewness} = -0.77 \quad \text{Kurtosis} = -0.77$$

$$\text{Median} = 5.65$$

Table 6.11. Fatigue data for maximum applied stress level = 205 MPa (R = 0.1), and summary statistics for a clamping torque = 0 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
500	0	6801		
	0	1548	3.25	0.55
	0	547		
500	45	1445		
	45	913	3.15	0.18
	45	2085		
500	90	2166		
	90	2890	3.45	0.11
	90	3545		

Summary statistics of grouped data for clamping torque = 3.76 N-m;

$$F^*[\log(N)] = 0.61 \quad p = 0.57$$

$$\mu[\log(N)] = 3.28 \quad s[\log(N)] = 0.32$$

$$\text{Skewness} = -0.028 \quad \text{Kurtosis} = -0.454$$

$$\text{Median} = 3.32$$

Table 6.12. Fatigue data for maximum applied stress level = 500 MPa (R = 0.1), and summary statistics for a clamping torque = 3.76 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
487	0	31156	4.07	0.38
	0	9057		
	0	5684		
487	45	4870	3.96	0.29
	45	18967		
	45	8096		
487	90	17680	4.25	0.03
	90	16545		
	90	18654		

Summary statistics of grouped data for clamping torque = 3.76 N-m;

$$F^*[\log(N)] = 0.81 \quad p = 0.49$$

$$\mu[\log(N)] = 4.09 \quad s[\log(N)] = 0.27$$

$$\text{Skewness} = -0.18 \quad \text{Kurtosis} = -1.19$$

$$\text{Median} = 4.22$$

Table 6.13. Fatigue data for maximum applied stress level = 487 MPa (R = 0.1), and summary statistics for a clamping torque = 3.76 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
475	0	193010		
	0	109387	4.97	0.36
	0	38435		
475	45	35458		
	45	53901	4.65	0.09
	45	45380		
475	90	28464		
	90	33541	4.69	0.36
	90	128345		

Summary statistics of grouped data for clamping torque = 3.76 N-m;

$$F^*[\log(N)] = 1.037 \quad p = 0.41$$

$$\mu[\log(N)] = 4.77 \quad s[\log(N)] = 0.30$$

$$\text{Skewness} = -0.62 \quad \text{Kurtosis} = -1.11$$

$$\text{Median} = 4.66$$

Table 6.14. Fatigue data for maximum applied stress level = 475 MPa (R = 0.1), and summary statistics for a clamping torque = 3.76 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
462	0	468105		
	0	638438	5.48	0.44
	0	94621		
462	45	163176		
	45	112361	5.21	0.16
	45	230520		
462	90	350155		
	90	350805	5.65	0.18
	90	723607		

Summary statistics of grouped data for clamping torque = 3.76 N-m;

$$F^*[\log(N)] = 1.75 \quad p = 0.25$$

$$\mu[\log(N)] = 5.45 \quad s[\log(N)] = 0.318$$

$$\text{Skewness} = -0.215 \quad \text{Kurtosis} = -1.27$$

$$\text{Median} = 5.55$$

Table 6.15. Fatigue data for maximum applied stress level = 462 MPa (R = 0.1), and summary statistics for a clamping torque = 3.76 N-m.

Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
450	0	1000000		
	0	746395	5.77	0.29
	0	274368		
450	45	380692		
	45	96830	5.46	0.43
	45	658428		
450	90	910565		
	90	546845	5.65	0.35
	90	184932		

Summary statistics of grouped data for clamping torque = 3.76 N-m;

$$F^*[\log(N)] = 0.55 \quad p = 0.60$$

$$\mu[\log(N)] = 5.65 \quad s[\log(N)] = 0.38$$

$$\text{Skewness} = -0.70 \quad \text{Kurtosis} = -0.69$$

$$\text{Median} = 5.74$$

Table 6.16. Fatigue data for maximum applied stress level = 450 MPa (R = 0.1), and summary statistics for a clamping torque = 3.76 N-m.
Maximum

Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
625	0	841		
	0	1014	2.94	0.058
	0	785		
625	45	2933		
	45	10681	3.69	0.296
	45	3840		
625	90	3581		
	90	1309	3.25	0.27
	90	1168		

Summary statistics of grouped data for clamping torque = 7.53 N-m;

$$F^*[\log(N)] = 7.905$$

$$p = 0.021$$

$$\mu[\log(N)] = 3.294$$

$$s[\log(N)] = 0.385$$

$$\text{Skewness} = 0.675$$

$$\text{Kurtosis} = -0.694$$

$$\text{Median} = 3.12$$

Table 6.17. Fatigue data for maximum applied stress level = 625 MPa ($R = 0.1$), and summary statistics for a clamping torque = 7.53N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
609	0	9106		
	0	3681	3.97	0.41
	0	24518		
609	45	3688		
	45	3537	3.70	0.25
	45	9773		
609	90	29076		
	90	3447	4.33	0.73
	90	96770		

Summary statistics of grouped data for clamping torque = 7.53 N-m;

$$F^*[\log(N)] = 1.16 \quad p = 0.38$$

$$\mu[\log(N)] = 4.01 \quad s[\log(N)] = 0.52$$

$$\text{Skewness} = 0.73 \quad \text{Kurtosis} = -0.68$$

$$\text{Median} = 3.96$$

Table 6.18. Fatigue data for maximum applied stress level = 609 MPa (R = 0.1), and summary statistics for a clamping torque = 7.53 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
594	0	78490		
	0	105683	4.94	0.07
	0	80197		
594	45	489344		
	45	84851	5.05	0.58
	45	35076		
594	90	215607		
	90	165487	5.19	0.15
	90	108501		

Summary statistics of grouped data for clamping torque = 7.53 N-m;

$$F^*[\log(N)] = 0.39 \quad p = 0.69$$

$$\mu[\log(N)] = 5.06 \quad s[\log(N)] = 0.32$$

$$\text{Skewness} = 0.47 \quad \text{Kurtosis} = 0.03$$

$$\text{Median} = 5.03$$

Table 6.19. Fatigue data for maximum applied stress level = 594MPa (R = 0.1), and summary statistics for a clamping torque = 7.53 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
578	0	284830		
	0	295042	5.48	0.04
	0	340415		
578	45	620882		
	45	589022	5.48	0.52
	45	76851		
578	90	365181		
	90	90584	5.47	0.47
	90	768035		

Summary statistics of grouped data for clamping torque = 7.53 N-m;

$$F^*[\log(N)] = .002 \quad p = 0.99$$

$$\mu[\log(N)] = 5.48 \quad s[\log(N)] = 0.35$$

$$\text{Skewness} = -0.69 \quad \text{Kurtosis} = -0.745$$

$$\text{Median} = 5.53$$

Table 6.20. Fatigue data for maximum applied stress level = 578 MPa (R = 0.1), and summary statistics for a clamping torque = 7.53 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
562	0	837483		
	0	1000000	5.90	0.11
	0	608134		
562	45	1000000		
	45	830133	5.77	0.32
	45	257402		
562	90	78051		
	90	560380	5.47	0.50
	90	597772		

Summary statistics of grouped data for clamping torque = 7.53 N-m;

$$F^*[\log(N)] = 1.20 \quad p = 0.36$$

$$\mu[\log(N)] = 5.77 \quad s[\log(N)] = 0.43$$

$$\text{Skewness} = -1.52 \quad \text{Kurtosis} = 1.17$$

$$\text{Median} = 5.78$$

Table 6.21. Fatigue data for maximum applied stress level = 562 MPa (R = 0.1), and summary statistics for a clamping torque = 7.53 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
690	0	1808		
	0	671	2.84	0.40
	0	284		
690	45	2067		
	45	4815	3.54	0.20
	45	4280		
690	90	11133		
	90	1069	3.44	0.54
	90	1674		

Summary statistics of grouped data for clamping torque = 11.3 N-m;

$$F^*[\log(N)] = -2.56 \quad p = 0.16$$

$$\mu[\log(N)] = 3.27 \quad s[\log(N)] = 0.48$$

$$\text{Skewness} = -0.09 \quad \text{Kurtosis} = -0.59$$

$$\text{Median} = 3.26$$

Table 6.22. Fatigue data for maximum applied stress level = 690 MPa (R = 0.1), and summary statistics for a clamping torque = 11.3 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
673	0	35184		
	0	26181	4.31	0.30
	0	9285		
673	45	12841		
	45	22186	4.38	0.29
	45	49804		
673	90	5813		
	90	18642	3.98	0.26
	90	8149		

Summary statistics of grouped data for clamping torque = 11.3 N-m;

$$F^*[\log(N)] = 1.66 \quad p = 0.27$$

$$\mu[\log(N)] = 4.22 \quad s[\log(N)] = 0.31$$

$$\text{Skewness} = -0.004 \quad \text{Kurtosis} = -1.16$$

$$\text{Median} = 4.27$$

Table 6.23. Fatigue data for maximum applied stress level = 673 MPa (R = 0.1), and summary statistics for a clamping torque = 11.3 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
655	0	281669		
	0	122109	5.07	0.39
	0	46928		
655	45	37456		
	45	103916	4.85	0.24
	45	89322		
655	90	205211		
	90	176530	5.18	0.17
	90	98033		

Summary statistics of grouped data for clamping torque = 11.3 N-m;

$$F^*[\log(N)] = 1.11 \quad p = 0.39$$

$$\mu[\log(N)] = 5.03 \quad s[\log(N)] = 0.28$$

$$\text{Skewness} = -0.27 \quad \text{Kurtosis} = -0.87$$

$$\text{Median} = 5.02$$

Table 6.24. Fatigue data for maximum applied stress level = 655 MPa (R = 0.1), and summary statistics for a clamping torque = 11.3 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
638	0	1000000		
	0	303739	5.45	0.57
	0	73199		
638	45	258068		
	45	641127	5.57	0.21
	45	304148		
638	90	95644		
	90	183307	5.19	0.18
	90	213451		

Summary statistics of grouped data for clamping torque = 11.3 N-m;

$$F^*[\log(N)] = 0.83 \quad p = 0.48$$

$$\mu[\log(N)] = 5.42 \quad s[\log(N)] = 0.39$$

$$\text{Skewness} = 0.135 \quad \text{Kurtosis} = -0.69$$

$$\text{Median} = 5.41$$

Table 6.25. Fatigue data for maximum applied stress level = 638 MPa (R = 0.1), and summary statistics for a clamping torque = 11.3 N-m.

Maximum Applied Stress (MPa)	Outer Ply Angle	Number of Cycles	$\mu[\log(N)]$	$s[\log(N)]$
621	0	409734		
	0	343188	5.66	0.16
	0	685405		
621	45	517521		
	45	117654	5.55	0.43
	45	755204		
621	90	219524		
	90	1000000	5.78	0.38
	90	1000000		

Summary statistics of grouped data for clamping torque = 11.3 N-m;

$$F^*[\log(N)] = 0.33 \quad p = 0.73$$

$$\mu[\log(N)] = 5.71 \quad s[\log(N)] = 0.38$$

$$\text{Skewness} = -0.68 \quad \text{Kurtosis} = -0.56$$

$$\text{Median} = 5.72$$

Table 6.26. Fatigue data for maximum applied stress level = 621 MPa ($R = 0.1$), and summary statistics for a clamping torque = 11.3 N-m.

The fatigue data is also represented by a paired value of fluctuating stress amplitude and the corresponding number of cycles to failure (S,N), denoted as a single point on the graph as shown in Figure 6.2. The value N, number of cycles to failure is defined as the number of load cycles when a permanent deformation (change in the bolt hole diameter) of $0.04d$ is reached. This is the classical form of stress-life (S-N) curves. The semi-log S-N curves are plotted with the values of S plotted on a linear scale and the values of N plotted to a logarithmic scale. This will present the values of both large and small N with the same relative accuracy. These S-N curves yield an estimate of the mean number of cycles to failure as a function of the maximum applied alternating stress. For ease of comparisons of results with other fatigue data, the use of relative values of maximum applied stress to the mean ultimate stress is used throughout Figures 6.2 through 6.5.

Figure 6.2. Maximum applied stress versus number of cycles to failure for a clamping torque = 0 N-m.

Figure 6.3. Maximum applied stress versus number of cycles to failure for a clamping torque = 3.76 N-m.

Figure 6.4. Maximum applied stress versus number of cycles to failure for a clamping torque = 7.53 N-m.

Figure 6.5. Maximum applied stress versus number of cycles to failure for a clamping torque = 11.3 N-m.

Figures 6.2 through 6.5 are the S-log N curves for a composite bolted joint under clamping torques of 0, 3.76, 7.53, and 11.3 N-m, respectively. Plotted on the ordinate axis of these figures is the term $\frac{\sigma_{a,j}}{\bar{\sigma}_{u,j}}$. The numerator of this term represents the maximum applied stress at the clamping torque j, and the denominator is the mean ultimate bearing strength under a clamping torque j. For the Figures that follow, j = 1, 2, 3, 4, corresponds to clamping torques 0, 3.76, 7.53, and 11.3 N-m, respectively. The mean ultimate bearing strength, $\bar{\sigma}_{u,j}$, in these figures were 284.45, 578.09, 719.45, and 803.53 Mpa for j = 1, 2, 3, and 4.

At each of the stress levels tested, the effect of outer ply angle on the fatigue life was desired. From a casual observation of Figures 6.2 through 6.5, it is difficult to make a concise statement concerning the effect of outer ply angle. To make such a statement, the technique of analysis of variance, as previously discussed, was applied. The overall conclusion drawn from this analysis (Values of F* are given in Tables 6.7 through 6.26) was that the laminate outer ply angle has an insignificant effect on the fatigue life of the composite bolted joint.

Probability Plots

As previously mentioned, data from both static bearing strength tests and fatigue tests were assumed to follow a log-normal distribution. The validity of this assumption will be reviewed in this section.

Beginning with the ultimate bearing strength data, Figure 6.6 shows a plot of the grouped experimental data (combining the three outer ply angles) imposed on the cumulative log-normal distribution function. The term plotted on the abscissa, $\frac{\sigma_{i,j}}{\bar{\sigma}_{u,j}}$, represents normalized strength of the joint where, $\sigma_{i,j}$ is the static ultimate bearing strength of the i -th specimen at the j -th level of clamping torque. The term $\bar{\sigma}_{u,j}$ is the mean value of ultimate bearing strength at the j -th level of clamping torque. The legend on the ordinate axis, $F(\sigma_{i,j})$, is the cumulative log-normal distribution function of the normalized strength. The symbols shown correspond to the experimental data of the three outer play angles. These experimental data points are plotted as probabilities of failure through the use of the median rank, which is given as,

$$P(\text{failure}) = \frac{i - 0.3}{n + 0.4} \quad (6.40)$$

where i is the rank order number of the grouped data and n is the number of specimens tested. The continuous curve of Figure 6.6 is a plot of the cumulative log-normal distribution, Equation (6.21), versus normalized strength. The two parameters used in Equation (6.21) were found using Equations (6.25) and (6.26).

Therefore, the smooth curve represents the results which would be obtained if an infinite number of tests were performed providing that the underlying population was log-normal with its parameters given by Equations (6.25) and (6.26).

Presenting results in this manner can be of considerable use if a warranty of the strength of the joint is required. For example, it may be necessary to determine the probability of joint failure given that the applied load is 95% of the mean ultimate bearing strength. The point 0.95 on the abscissa of Figure 6.6 corresponds to 0.17 on the ordinate axis. Therefore there is a 17% chance that the joint will fail given the applied load of 0.95 times the mean bearing strength. Figures 6.7 through 6.9 provide similar plots of the static bearing strength tests under clamping torques of 3.76, 7.53, and 11.3 N-m, respectively.

Figure 6.6. Cumulative lognormal distribution function versus normalized ultimate static bearing strength for a clamping torque = 0 N-m.

Figure 6.7. Cumulative lognormal distribution function versus normalized ultimate static bearing strength for a clamping torque = 3.76 N-m.

Figure 6.8. Cumulative lognormal distribution function versus normalized ultimate static bearing strength for a clamping torque = 7.53 N-m.

Figure 6.9. Cumulative lognormal distribution function versus normalized ultimate static bearing strength for a clamping torque = 11.3 N-m.

Results obtained from the fatigue tests may likewise be presented in the form of Figures 6.6 through 6.9. Figure 6.10 shows the results obtained from grouping the nine fatigue tests performed under a clamping torque of 0 N-m and a maximum applied stress of 265 MPa. In this figure, as well as those which follow, the abscissa legend is of the form $\text{Log} \left(\frac{N_{i,j,k}}{\bar{N}_{j,k}} \right)$, where $N_{i,j,k}$ represents the number of cycles to failure of the i -th specimen under the j -th clamping torque and the k -th level of maximum applied stress. $\bar{N}_{j,k}$ is the mean value of number of cycles to failure for the three orientations under the j -th clamping torque and k -th level of maximum applied stress. Plotted on the ordinate axis is $F(N_{i,j,k})$, the cumulative log-normal distribution as given by Equations (6.28) through (6.30). These figures can likewise be of considerable use to designers. For example, the log-mean number of cycles to failure for a clamping torque of 0 N-m and at a maximum applied stress of 220 MPa was found to be 93526. It may be necessary to determine the probability that fatigue life under these loading conditions will be greater than 200,000 cycles. This information may be obtained from Figure 6.13. For this case, $N_{i,1,2}$ is equal to 200,000 and $\bar{N}_{1,2}$ equals 93,526. The log of the ratio $\left(\frac{N_{i,1,2}}{\bar{N}_{1,2}} \right)$ is 0.33. From Figure 6.13 it is seen that with an x-coordinate of 0.33, the corresponding y-coordinate, $F(N_{i,1,2})$ is approximately equal to 0.78. Hence there is approximately a 22% chance that a tested specimen will survive after 200,000 cycles. Figures 6.11 through 6.29 are plots showing the results of the fatigue tests performed under the remaining values of clamping torque and applied load.

Figure 6.10. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 265 MPa, clamping torque = 0 N-m.

Figure 6.11. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 250 MPa, clamping torque = 0 N-m.

Figure 6.12. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 235 MPa, clamping torque = 0 N-m.

Figure 6.13. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 220 MPa, clamping torque = 0 N-m.

Figure 6.14. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 205 MPa, clamping torque = 0 N-m.

Figure 6.15. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 500 MPa, clamping torque = 3.76 N-m.

Figure 6.16. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 487.5 MPa, clamping torque = 3.76 N-m.

Figure 6.17. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 475 MPa, clamping torque = 3.76 N-m.

Figure 6.18. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 462.5 MPa, clamping torque = 3.76 N-m.

Figure 6.19. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 450 MPa, clamping torque = 3.76 N-m.

Figure 6.20. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 625 MPa, clamping torque = 7.53 N-m.

Figure 6.21. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 609 MPa, clamping torque = 7.53 N-m.

Figure 6.22. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 594 MPa, clamping torque = 7.53 N-m.

Figure 6.23. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 578.5 MPa, clamping torque = 7.53 N-m.

Figure 6.24. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 562.5 MPa, clamping torque = 7.53 N-m.

Figure 6.25. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 690 MPa, clamping torque = 11.3 N-m.

Figure 6.26. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 673 MPa, clamping torque = 11.3 N-m.

Figure 6.27. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 655.5 MPa, clamping torque = 11.3 N-m.

Figure 6.28. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 638 MPa, clamping torque = 11.3 N-m.

Figure 6.22. Cumulative lognormal distribution function versus normalized fatigue life. Max applied stress = 621 MPa, clamping torque = 11.3 N-m.

Figures 6.14, 6.19, 6.24, 6.28, and 6.29 require special consideration. These figures are representative of the cases in which the test specimen survived one million cycles. As previously discussed, tests which were suspended at 10^6 cycles require the use of censoring techniques to determine the two parameters which characterize the log-normal distribution. The continuous cumulative distribution curve shown in these figures results from Equation 6.28, however, the parameters used in this equation were found through the use of Cohen's method, which was outlined in the data analysis section of this Chapter.

Test for Normality

The deviation of the experimental data points from the continuous curve of Figures 6.6 through 6.29 indicate the departure from the assumed log-normal distribution. Instead of making a casual statement such as "the assumption that fatigue life data follows a log-normal distribution appears justified," a test was performed so that a more concise statement could be made. At each stress level, the deviation of the data from the log-normal distribution was investigated to determine if these deviations were statistically significant. If the deviations were found to be statistically significant, this would imply that the assumption that fatigue life data follows a log-normal distribution is invalid. The following section discusses the test used and the resulting conclusions.

As discussed, the form of the distribution for both static ultimate bearing strength and fatigue life was assumed to be log-normal. To test this assumption, a

statistical goodness of fit test was applied to the strength and life data. The test chosen was that of Lilliefors [94], which is similar to the classical Kolmogorov-Smirnov test of normality. Under the null hypothesis, Lilliefors test assumes normality of the distribution with an unknown mean and variance, whereas the Kolmogorov-Smirnov test assumes that the mean and variance is known. Lilliefors' test statistic is given as,

$$D_n = \max_{1 \leq i \leq n} (\delta_i) \quad (6.44)$$

$$\delta_i = \max \left[\Phi(Y_i) - \frac{i-1}{n}, \frac{i}{n} - \Phi(Y_i) \right] \quad (6.45)$$

where n is the number of samples, $\Phi(Y_i)$ is the cumulative standard normal, and $Y_i = (X_i - \bar{X})/s$. The mean of the data is given as $\bar{X}$, and s equals the standard deviation. Given a significance level α , the hypothesis that the log of the data follows a normal distribution will be rejected if the computed value D_n , exceeds a critical value given by tables found in Mann [88] or Lilliefors [94]. For a sample size $n = 9$, the critical value of the test statistic D_n at a significance level of 0.05, is equal to 0.271. With one exception, all computed values of D_n were less than the critical value of 0.271. Therefore at a significance level of 0.05, the hypothesis of the data following a log-normal distribution was not rejected. The single exception to the above occurred at a clamping torque of 7.53 N-m and at a maximum applied stress level of 562 MPa (Table 6.21). Observing the data obtained under these

loading conditions, a rejection of normality of log-failure times is not surprising. Grouping the 0, 45, and 90 degree outer ply orientations, the number of cycles obtained under the above loading conditions were (608134, 837483, 1000000), (78051, 560380, 597772), and (257402, 830133, 1000000). Taking the logarithm of the above data gives (5.784, 5.923, 6), (4.892, 5.748, 5.776) and (5.411, 5.919, 6), respectively. The low number of cycles obtained in one of the 45 degree specimens caused the rejection of a normally distributed sample. The reason for such few cycles endured is likely due to experimental error such as improper applied stress, clamping torque, or alignment of the sample. A faulty specimen is another possibility, however, the more likely cause is experimental error. With the exception of this single loading condition, the conclusions of the tests performed on Figures 6.10 through 6.29 were that the assumption of normality was not rejected.

Stress-Independent Results

Figures 6.30 through 6.33 present the results of combining the fatigue data obtained from the four levels of clamping torque, independent of the various stress levels tested. Figures 6.30 through 6.33 represent the data obtained under clamping torques of 0, 3.76, 7.53, and 11.3 N-m, respectively. The symbols represent experimental data plotted against the probability of failure, and the smooth curve shown is the underlying log-normal distribution. The legend on the abscissa axis of these figures is of the form $\text{Log} \left(\frac{N_{i,j}}{\bar{N}_j} \right)$, where $N_{i,j}$ is the number of cycles to failure of the i -th specimen under a clamping torque j . $\bar{N}_j$ is the log-mean

number of cycles to failure under the clamping torque j . The legend on the ordinate axis is of the form $F(N_{i,j})$, which is the log-normal cumulative distribution function, Equation 6.28, evaluated at $N_{i,j}$.

Figure 6.30. Cumulative lognormal distribution function versus pooled normalized fatigue life under a clamping torque = 0 N-m.

Figure 6.31. Cumulative lognormal distribution function versus pooled normalized fatigue life under a clamping torque = 3.76 N-m.

Figure 6.32. Cumulative lognormal distribution function versus pooled normalized fatigue life under a clamping torque = 7.53 N-m.

Figure 6.33. Cumulative lognormal distribution function versus pooled normalized fatigue life under a clamping torque = 11.3 N-m.

Curve Fitting Results

An analysis of the fatigue data was performed in order to determine the relationship between the applied stress and number of cycles to failure. The first technique used was to approximate the S-N (S - log(N)) relationship as a straight line. At each level of clamping torque, the linear model was fit to the fatigue data as,

$$\frac{\sigma_a}{\bar{\sigma}_{uj}} = m \log(N_j) + b \quad (6.41)$$

where,

σ_a = maximum applied cyclic stress,

$\bar{\sigma}_{uj}$ = mean ultimate bearing strength under clamping torque j,

N = number of cycles to failure,

m, b = parameters which are found via the method of least squares. Values for the ultimate bearing strength may be found plotted in Figure 6.34, which presents the raw data of maximum applied stress versus the median number of cycles to failure.

Figure 6.34. Maximum applied stress versus median number of cycles to failure of the four clamping torques tested.

The classical S-N power law relationship was also applied to the fatigue data,

$$K \left(\frac{\sigma_a}{\sigma_{uj}} \right)^b N_j = 1 \quad (6.42)$$

where K and b are the parameters of the power law relation.

Taking the logarithms of both sides yields,

$$\log N_j = -b \log \left(\frac{\sigma_a}{\sigma_{uj}} \right) - \log K \quad (6.43)$$

Plots of the Equations (6.41) and (6.43), fitted to the median fatigue data are shown in Figures 6.35 and 6.36. The two parameters and the correlation coefficients of the above equations are given in Tables 6.28 and 6.29. As seen in the plots and from the correlation coefficients, a slightly better fit is found using Equation (6.43) instead of (6.41). Equation (6.43) was then used to characterize the complete set of fatigue data (from the S-N curves) for each value of clamping torque. Table 6.28 lists the parameters obtained and Figures 6.37 through 6.40 shows the fitted equation in relation to the experimental data. The overall conclusions which can be inferred from the fatigue tests of the composite bolted joints are discussed in the following section.

Figure 6.35. Median number of cycles to failure fit to the equation $\frac{\sigma_a}{\sigma_{uj}} = m \log(N_j) + b$.

Figure 6.36. Median number of cycles to failure fit to the equation

$$K \left(\frac{\sigma_a}{\sigma_{uj}} \right)^b N = 1.$$

Clamping Torque (N·m)	b	K	R ²
0	34.505	15.577	0.836
3.76	53.412	0.841	0.918
7.53	55.573	0.475	0.828
11.3	52.176	1.046	0.926

Table 6.28. Parameters and correlation coefficients of the equation,
 $K \left(\frac{\sigma_a}{\sigma_{ult}} \right)^b N = 1$ for fatigue life data.

Clamping Torque (N·m)	m	b	R ²
0	-4.944e-2	1.018	0.968
3.76	-3.818e-2	0.995	0.989
7.53	-3.565e-2	0.988	0.967
11.3	-3.842e-2	0.996	0.988
all	-3.555e-2	0.984	0.963

Table 6.29. Parameters and correlation coefficients of the equation,
 $\left(\frac{\sigma_a}{\sigma_{ult}} \right) = m \ln (N) + b$ for median fatigue life.

Figure 6.37. Fit of the equation $K \left(\frac{\sigma_a}{\sigma_{uj}} \right)^b N = 1$ for a clamping torque = 0 N-m.

Figure 6.38. Fit of the equation $K \left(\frac{\sigma_a}{\sigma_{uj}} \right)^b N = 1$ for a clamping torque = 3.76 N-m.

Figure 6.39. Fit of the equation $K \left(\frac{\sigma_a}{\sigma_{uj}} \right)^b N = 1$ for a clamping torque = 7.53 N-m.

Figure 6.40. Fit of the equation $K \left(\frac{\sigma_a}{\sigma_{uj}} \right)^b N = 1$ for a clamping torque = 11.3 N-m.

Summary of Fatigue Results

A major conclusion resulting from the fatigue testing program is that the angle of the outer ply with respect to the loading direction has an insignificant effect on the fatigue life of the bolted joint. Analysis of variance tests were performed in order to establish this result. While this result was obtained using a single composite material, it is expected to hold true for other composites. However, this conclusion may not be applicable to composites that are room temperature cured, or composites in which the surface topography follows that of the outer ply fibers. If the surface of the composite follows the outer ply fibers, then the coefficient of friction would vary with the direction of loading. While the change in friction coefficient with the direction of loading would affect the bearing strength, it is unlikely that the fatigue life would differ given an applied stress that is normalized by the mean bearing strength.

Once the effect of outer ply angle was determined to be insignificant, it was possible to group the data so that a total of nine specimens which were tested at each stress level could be analyzed. After grouping the data, the results were investigated to determine if the assumed distribution was valid. Through the use of a statistical goodness-of-fit test, both the static bearing strength and the fatigue life data were found to be adequately characterized by the log-normal distribution. The two parameters of the log-normal distribution were found for each level of applied stress and clamping torque, as well as the parameters for the stress independent pooled data.

The effect of clamping torque on the fatigue life of bolted joints should be classified into two groups; the response of the pin-loaded case (clamping torque =

0), and the response of the clamped joint (torque > 0). The behavior of the pin-loaded joint differed from that of the clamped joint due to the difference in the stiffness of the joints. The clamped joints constitute much stiffer joints than the pin-loaded case. The overall response of the pin joint is that its fatigue life is significantly shorter than the case of clamped joints. For the case of clamped joints, and in absolute terms, an increase in clamping torque caused an increase in the fatigue life. However, in terms of the maximum applied stress relative to the static bearing strength at each clamping torque, the fatigue life is essentially the same. Therefore knowing the fatigue response at one level of clamping torque, it is possible to estimate the fatigue life at a different level of clamping torque.

A two parameter power law equation was used to estimate the relationship between the applied stress and the number of cycles to failure. Knowing these parameters and the static bearing strength at a given level of clamping torque, within the range of the stress levels tested, it is possible to predict the number of cycles to failure at any applied stress level.

Chapter 7

Summary and Conclusions

The major findings of this work may be grouped into three interrelated areas - results of friction, bearing strength, and fatigue testing. A summary of the interpretation of test data for each group follows.

Static and kinetic coefficients of friction were experimentally obtained for six different metallic washers in contact with a graphite-epoxy composite. The independent variables chosen also include four levels of applied clamping torque and different outer ply angles (0, 45, and 90 degrees). The data obtained and presented can be of considerable use to designers of composite structures.

Through experimental testing, it was found that the washer material has a significant influence on the coefficients of friction. Both the applied clamping force and the direction of outer ply fibers relative to the direction of loading have little effect on friction coefficients.

Surface preparation, prior to assembly, of both the composite and the mating washer was found to be critical to achieve consistent and reliable test data. Surface

contamination caused by routine handling may reduce the friction coefficients by as much as 50% over samples which were carefully degreased.

The results obtained from ultimate bearing strength tests confirmed previous studies which showed that increasing lateral constraint resulted in an increase in bearing strength. By providing sufficient lateral constraint, it was found that the bearing strength of a clamped joint was greater than twice the strength of a comparable pin-loaded joint.

While other studies have discussed the effect of bolt-hole clearance on bearing strength, none has yet discussed the effect of the bolt to washer diameter ratio. This ratio was found to substantially influence the ultimate bearing strength. In fact, if the bolt to washer diameter ratio is sufficiently large, the increase in strength due to the provision of a lateral constraint can be nullified. As in the case of bolt-hole clearance, a close fit between the bolt diameter and the washer diameter provides the conditions needed for maximum bearing strength. The cause of this increase in bearing strength due to a close fit between the bolt and the washer is that the out-of-plane failure known as 'brooming', is restricted.

Realizing the effect of diameter ratios on composite bolted joints, a model was proposed which incorporates this important parameter. The model was developed to predict the static ultimate bearing strength of a composite joint at various levels of applied clamping torque and bolt-washer diameter ratios. The proposed model includes the effects of pin bearing strength, and frictional load transfer due to clamp-up forces. The model also considers the influence of a

suppressed lateral expansion, and the increase in in-plane strength due to an out-of-plane constraint. Satisfactory results were obtained using the model to predict bolted joint performance.

A required input to the bearing strength model is the in-plane stress distribution of the composite material. A stress analysis based on elasticity techniques was performed which incorporated the effects due to an imperfect-fit pin, a nonlinear contact region, and friction acting between the bolt and the composite.

A fatigue test program was carried out in order to investigate the performance of a double-lap bolted composite joint. The independent variables chosen were the maximum applied cyclic stress, the amount of clamping torque, and the outer ply angle of the composite sample. Tests were performed under constant load, frequency, and stress ratio conditions. Test results were presented in graphical and tabular form, and in the form of log-normal probability distribution plots.

The effect of the maximum applied stress is consistent with other published reports. As expected, decreasing the applied cyclic stress resulted in a corresponding increase in the number of cycles to failure. Using the obtained test data, classical stress-life relations were used to describe the relationship between the maximum applied stress and the resulting number of cycles to failure.

The effect of clamping force on the fatigue life of composite bolted joints should be classified into two distinct groups. One group is the fatigue response of pin-loaded joints, and the other is the response of the clamped joints. It was found that for failure to occur at a given number of cycles, the clamped joints required a higher ratio of applied stress to static ultimate strength. This result may be attributed to the large stiffness of the clamped joint, relative to the pin-bearing case.

An important point which may be inferred from the fatigue data is that (for clamped joints) irregardless of the applied clamping torque, the median number of cycles to failure at a given ratio of applied stress to ultimate strength was essentially constant. Therefore, knowing the fatigue response at one level of clamping torque, it is possible to estimate the fatigue life at different levels of clamping torque.

Another conclusion which may be drawn from the fatigue testing program, and similar to the results derived from testing friction coefficients, is that the angle of the outer ply with respect to the loading direction, has an insignificant effect on the fatigue life of the bolted joint. Once this result was established, the data obtained for the three groups of outer ply angle could be combined, which allowed a more complete analysis of the data at each level of applied stress and clamping torque.

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APPENDICIES

Appendix A

Experimental data obtained during friction testing.

A.

B.

C.

D.

A1. Friction Tests - SAE Washer

A- 0 degrees, 2220, 4440 N

B- 0 degrees, 6660, 8880 N

C- 45 degrees

D- 90 degrees

A.

B.

C.

D.

A2. Friction Tests - Small Fender Washer

A- 0 degrees, 2220, 4440, 6660 N

B- 0 degrees, 8880 N

C- 45 degrees, 2220, 4440, 6660 N

D- 45 degrees, 8880 N

A3. Friction Tests - Large Fender Washer
 A- 0 degrees, 2220, 4440, 6660 N
 B- 0 degrees, 8880 N
 C- 45 degrees
 D- 90 degrees

A4. Friction Tests - Brass Washer

A- 0 degrees

B- 45 degrees

C- 90 degrees

A5. Friction Tests - Stainless Steel Washer

A- 0 degrees

B- 45 degrees

C- 90 degrees

A6. Friction Tests - Galvanized Washer

A- 0 degrees, 2220, 4440 N

B- 0 degrees, 6660, 8880 N

C- 45 degrees, 2220, 4440 N

D- 45 degrees, 6660, 8880 N

A7. Friction Tests - Galvanized Washer

A- 90 degrees, 2220, 4440 N

B- 90 degrees, 6660, 8880 N

C- Small Fender Washer, 90 degrees

Appendix B

Source code listing for stress analysis program.

```

PROGRAM stress

Finds sigmar and sigmatheta given
Load      :load in pounds
a         :radius of hole
r         :upper limit of integration
v         :Poisson's ratio
u         :coefficient of friction
modB      :modulus of elasticity of bolt
modC      :modulus of elasticity of composite in the plane
modT      :modulus of elasticity of composite in the z direction
dia       :diameter of hole
bltdia    :bolt diameter
thick     :thickness of plate
dr        :increase in r direction
dtheta    :increase in theta direction
rmax      :find epz until r = rmax

REAL mpa
Open(unit = 10,file = 'data',status = 'unknown')
Load = 400.
a = 0.125
pi = acos(-1.)
v = 0.3
u = 0.1
modB = 30.e6
modC = 10.e6
modT = 2.e6
dia = 0.25
bltdia = 0.246
thick = 0.047

write(6,*)'Input dr,dtheta,rmax,load,r'
read(6,*)dr,dtheta,rmax,load,r

del = -0.5*(bltdia - 2.*a)/a
x = Load/(thick*modC*a*del)
tmxdeg = 8.8422+102.32*x-103.5*x**2.+59.055*x**3.-17.002*
+      x**4.+1.9010*x**5.
write(6,*)tmxdeg
thetmx = tmxdeg*pi/180.
c = pi/thetmx

term=(cos((2.*c+1)*thetmx)*(1.-2.*c)+(1.+2.*c)*(cos((1.-2.*c)
+      *thetmx) - 2.*(1.-2.*c)*cos(thetmx)) - 4.*c**2.)/
+      ((4.-8.*c)*(1.+2.*c))

x = thetmx
p = Load/(a*thick*(pi*Bessjl(x) + 2.*u*cos(thetmx/2.)*term))

DO WHILE (r .le. rmax)
epz = 0.
theta = 0.
term1 = 1. - (a/r)**2.
term2 = 1. + (a/r)**2.

```

```

DO WHILE (theta .le. pi)
  yr = 0.
  yt = 0.
  yrt = 0.

  DO 10, i=1,150
    x = i*thetmx
    y = ((a/r)**i)*cos(i*theta)*Bessj1(x)
    yr = yr + y*(2./i + term1)
    yt = yt + y*(-2./i + term1)
    yrt = yrt + term1*((a/r)**i)*sin(i*theta)*Bessj1(x)
10  CONTINUE

  y1r = -0.5*p*yr
  y1t = 0.5*p*yt
  y1rt = -0.5*p*yrt

  y2r = -0.25*thetmx*p*(a/r)**2.
  y2t = -y2r
  y2rt = 0.5*u*p*cos(thetmx/2.)*(thetmx/pi)*(a/r)**2.

  x = thetmx
  term3 = 0.25*(3.-v)*p*Bessj1(x)*(a/r)*cos(theta)
  y3r = term1*term3
  y3t = term2*term3
  y3rt = 0.25*(3.-v)*p*Bessj1(x)*(a/r)*sin(theta)*term1

  yr2 = 0.
  yt2 = 0.
  yrt2 = 0.

  DO 20 i=1,150
    CALL Coeff(i,theta,pi,c,u,p,thetmx,tm)
    term4 = ((a/r)**i)*sin(i*theta)*tm
    termrt = ((a/r)**i)*cos(i*theta)*tm
    yr2 = yr2 + term4*term1*(2.+i)
    yt2 = yt2 + term4*(i*term1 - 2.*term2)
    yrt2 = yrt2 + termrt*(i*term1 - 2.*(a/r)**2.)
20  CONTINUE

  y4r = -0.5*yr2
  y4t = 0.5*yt2
  y4rt = 0.5*yrt2

  i = 1
  CALL Coeff(i,theta,pi,c,u,p,thetmx,tm)
  term4 = 0.25*(3-v)*(a/r)*tm*sin(theta)
  y5r = term4*term1
  y5t = term4*term2
  y5rt = -term1*0.25*(3-v)*(a/r)*tm*cos(theta)

  sigr = y1r+y2r+y3r+y4r+y5r
  sigt = y1t+y2t+y3t+y4t+y5t
  sigrt = y1rt+y2rt+y3rt+y4rt+y5rt

  write(10,*)r,theta,sigr,sigt,sigrt
  theta = theta + dtheta
END DO

```



```

r = r + dr
END DO

STOP
END

SUBROUTINE Coeff(i,theta,pi,c,u,p,thetmx,tm)

  temp1 = 2.*c - i
  temp2 = 2.*c + i

  tm = -(-0.5*sin(temp1*thetmx)/temp1+sin(i*thetmx)/i-0.5*sin(
+   temp2*thetmx)/temp2)*u*p*cos(thetmx/2.)/pi

RETURN
END

FUNCTION Bessj1(x)
  DOUBLE PRECISION y,p1,p2,p3,p4,p5,q1,q2,q3,q4,q5,r1,r2,r3,
+   r4,r5,r6,s1,s2,s3,s4,s5,s6
  DATA r1,r2,r3,r4,r5,r6/72362614232.D0,-7895059235.D0,
+   242396853.1D0,-2972611.439D0,15704.48260D0,
+   -30.16036606D0/
  DATA s1,s2,s3,s4,s5,s6/144725228442.D0,2300535178.D0,
+   18583304.74D0,99447.43394D0,376.9991397D0,1.D0/
  DATA p1,p2,p3,p4,p5/1.D0,.183105D-2,-.3516396496D-4,
+   .2457520174D-5,-.240337019D-6/
  DATA q1,q2,q3,q4,q5/.04687499995D0,-.2002690873D-3,
+   .8449199096D-5,-.88228987D-6,.105787412D-6/

  IF(ABS(X) .LT. 8.) THEN
    y = x**2.
    Bessj1 = x*(r1+y*(r2+y*(r3+y*(r4+y*(r5+y*r6))))
+   /(s1+y*(s2+y*(s3+y*(s4+y*(s5+y*s6))))
  ELSE
    ax = ABS(x)
    z = 8./ax
    y = z**2.
    xx = ax - 2.356194491
    Bessj1 = SQRT(.636619772/ax)*(COS(xx)*(p1+y*(p2
+   +y*(p3+y*(p4+y*p5))))-z*SIN(xx)*(q1+y*(q2+y*(q3+y*(q4
+   +y*q5)))))*SIGN(1.,x)
  ENDIF
RETURN
END

```

Appendix C

Experimental data obtained from static ultimate bearing strength tests.

C1. Bearing Test - Pin Loaded
 A- 0 degrees
 B- 45 degrees

C2. Bearing Test - Pin Loaded

A- 45 degrees

B- 45 degrees

C3. Bearing Test - Pin Loaded

A- 90 degrees

B- 90 degrees

C- 90 degrees

C4. Bearing Test - ID/OD = 1.142, 0 degrees

A- Torque = 3.76 N-m

B- Torque = 7.53 N-m

C- Torque = 11.3 N-m

C5. Bearing Test - ID/OD = 1.065, 0 degrees

A- Torque = 3.76 N-m

B- Torque = 7.53 N-m

C6. Bearing Test - ID/OD = 1.065, 0 degrees
torque = 11.3 N-m

C7. Bearing Test - ID/OD = 1.024, Torque = 3.76 N-m

A- 0 degrees

B- 45 degrees

C- 90 degrees

C8. Bearing Test - ID/OD = 1.024, Torque = 7.53 N-m

- A- 0 degrees
- B- 45 degrees
- C- 90 degrees

C9. Bearing Test - ID/OD = 1.024, Torque = 11.3 N-m
 A- 0 degrees
 B- 45 degrees

C10. Bearing Test - ID/OD = 1.024, Torque = 11.3 N-m
90 degrees

C11. Bearing Test - ID/OD = 1.005, 0 degrees

A- Torque = 3.76 N-m

B- Torque = 7.53 N-m

C12. Bearing Test - ID/OD = 1.005, 0 degrees
Torque = 11.3 N-m

VITA

Paul Douglas Herrington was born April 3, 1959, in New Orleans, Louisiana. In May, 1982, he received the Bachelor of Science Degree in Mechanical Engineering from the University of New Orleans. In June, 1985, he was employed as a Mechanical Engineer for the Westinghouse Electric Corporation. He began graduate work at Tulane University where he obtained the MBA degree in May, 1985. He is currently a candidate for the degree of Doctor of Philosophy in Mechanical Engineering to be awarded at the Fall commencement, 1991.

DOCTORAL EXAMINATION AND DISSERTATION REPORT

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Major Field: Mechanical Engineering

Title of Dissertation: Fatigue Failure of Composite Bolted Joints.

Approved:

Major Professor and Chairman

Dean of the Graduate School

EXAMINING COMMITTEE:

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