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Elizabeth Devalcourt Gray

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An analysis of syntactic skills used in translating natural language sentences into algebraic equations

Gray, Elizabeth deValcourt, Ph.D.
The Louisiana State University and Agricultural and Mechanical Col., 1991
AN ANALYSIS OF SYNTACTIC SKILLS USED IN TRANSLATING
NATURAL LANGUAGE SENTENCES INTO ALGEBRAIC EQUATIONS

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
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requirements for the degree of
Doctor of Philosophy

in

The Department of Curriculum and Instruction

by

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ABSTRACT

The purpose of this study is to determine how a competent person translates a natural language sentence into an algebraic equation. A current theory of how translation occurs is espoused by a group of cognitive scientists who propose that the competent translator uses only conceptual strategies in the translation process. I propose and investigate an alternate theory that the translation skills of a competent student are based on a syntactic analysis and syntactic manipulation of the sentence. In most cases such direct, syntactic translation processes will suffice, but for a small minority of sentence types a decision must be made to abandon the syntactic processes and use conceptual strategies. The detailed syntactic model includes this decision process. The theory was tested by embedding the two alternate theories in classroom treatments using a repeated measures control group design.

Subjects included five college algebra classes. Two groups were taught the translation process as outlined in the syntactic theory, whereas another group was taught the translation process stressing conceptual strategies. A pretest was given prior to instruction; a posttest one week after a three day treatment period, and a retention
test twelve weeks after the posttest.

Results indicated that the students who received the syntactic treatments had significantly better mean scores on translation tasks on the posttest and on the retention test than students who received either the conceptual treatment or no treatment. On the retention test, the conceptual treatment students' scores decreased so that they were no longer significantly different from those of the control group.

These results suggest that the knowledge components stressed in the syntactic treatments more closely parallel the knowledge components of a competent translator. Thus, this study provides evidence that competence in translation of a natural language sentence into an algebraic equation involves a syntactic manipulation of the sentence, and that instruction that includes syntactic strategies has pedagogical merit.
CHAPTER ONE

STATEMENT OF THE PROBLEM

The mathematics curriculum covers a wide range of problem types including the novel, puzzle-type problem, mathematical modeling of the real world, and the routine word problems of algebra. Students tend to dislike algebra word problems; in fact "it is well known that word problems have traditionally been the nemesis of many mathematics students" (Lochhead & Mestre, 1988, p. 134). Perhaps this is because algebra word problems are twice the work; not only must the students solve an algebraic equation, but they must first write a sentence in algebraic notation which is inferred from their natural language.

How does one who is competent perform this translation task? What knowledge structures underlie this capability? What kind of educational experiences can best support the acquisition of this skill? In order to address these questions, it is necessary to look first at problem solving in general.

Background on Problem Solving

For at least the past ten years research in mathematics education has been dominated by an interest in problem solving. In 1980, the National Council of
Teachers of Mathematics (NCTM) issued *An Agenda for Action: Recommendations for School Mathematics of the 1980s* (NCTM, 1980). The *Agenda* made eight recommendations for school mathematics, and problem solving was the top priority.

The *Agenda* stimulated problem solving classes, in-service workshops, and research during the 1980s, but by the end of the decade the dismal picture of mathematics performance by students had not changed (Silver et al., 1988). In 1989, *The Curriculum and Evaluation Standards for School Mathematics*, (NCTM, 1989) was published. The overall theme of the *Standards* was that students should learn more mathematics and better mathematics, that they should learn to reason and communicate and value mathematics, and above all, that they should become mathematical problem solvers. In each curriculum division, K-4, 5-8, and 9-12, problem solving was listed as the first standard to be used in training students in mathematical literacy.

Although the mathematics curriculum covers a wide range of problem types, most of current algebra curriculum centers on the typical algebra word problem (e.g., coin, digit, and rate problems). But pedagogical imperatives like the *Agenda* and the *Standards* have called for a decrease in this type of problem solving. The premise is that routine problem solving does nothing but
train in syntactic routines, whereas problem solving ought to be a semantic and conceptual activity. The current problem solving emphasis on semantics and concepts uses the four steps to problem solving promoted by the famous teacher of problem solving, George Polya (1962), who stated that in problem solving "our first and most obvious duty is to understand the problem, its meaning, its purpose" (p. 27).

Recently, in the NCTM's call for a research agenda in the 1990's, Carpenter (1989) noted that "a major focus of recent cognitive science research in problem solving has been on problem solving in semantically rich content domains, and a primary concern has been with the analysis and representation of the knowledge and problems within these content domains" (p.192). As these agendas begin to affect the mathematics education community it is important for algebra courses to address translation. There are researchers (Mestre, Gerace, & Lochhead, 1982) who believe that students need translation skills, because "achievement in a technical field requires that students be able to translate between verbal statements and mathematical equations" (p. 399).

Research on algebra word problem solving (e.g., Paige & Simon, 1966; Hinsley, Hayes, & Simon, 1977; Mayer, 1980) suggests that the process of solving one of these problems involves three parts:
(a) translation: the process of expressing the problem as an algebraic equation, (b) manipulation: the process of manipulating the algebraic equation to arrive at an answer to the problem, and (c) verification: the process of checking to see if the solution fits the situation.

It is well documented (Mayer, 1980; Berger & Wilde, 1984), that a major source of difficulty for students in problem solving lies in the translation process. This translation process is the translation of the entire problem, and involves (a) categorizing the problem into a type, (b) using background information (facts and formulas), (c) expressing given statements of variable relationships as algebraic equations (sentence translation), and sometimes (d) construction of the overall problem equation.

This model for algebra problem translation (Figure 1) shows that sentence translation is only one of several steps in problem translation. The focus of this study is on sentence translation.

Sentence Translation Skills

A review of the literature (Chapter 2) suggests that although sentence translation is poorly represented in the literature, there are two basic approaches to sentence translation skills.
Syntactic Model of Sentence Translation

One approach to sentence translation, discussed by Paige and Simon (1966), suggests that the translation process involves a syntactical analysis of the sentence. This process, which they call "direct translation" (p. 66), is suggested to be not unlike what happens when one translates from one natural language to another. Phrases of one language are matched with phrases of another language, and certain rearrangements are made to match the syntax of the two languages.

Daniel Bobrow (1964/1968) wrote a computer program called STUDENT which "accepts as input a comfortable but restricted subset of English which can be used to express a wide variety of algebra story problems" (Bobrow, 1968, p. 146). In examining the STUDENT program, Paige and
Simon (1966) noted that "what is chiefly interesting about STUDENT in this report is that a rather elementary scheme of syntactical analysis is in fact sufficient for handling an interesting range of algebra word problems" (p. 66).

In a review of Paige and Simon's work, Hinsley, Hayes and Simon (1977) interpret the categorization of problems into type (e.g., rate, work, coin) as a semantic approach, recognizing the value of STUDENT only for uncategorizable sentences. They observe that STUDENT can do syntactic sentence translation on a wide range of sentences, but it has "a very narrow semantic base" (Bobrow, 1968, p. 201). Furthermore, in cases where STUDENT was unable to detect an inconsistency in a stated problem, the researchers critiqued the program for not having the semantic abilities needed in problem translation instead of noting its powers in direct translation processes. In this analysis, Hinsley, Hayes, and Simon (1977) appear to not see sentence translation as a separable subcomponent of problem translation.

**Conceptual Model of Translation**

In contrast to the syntactic model seen in the work with the STUDENT program, another focus on sentence translation is taken by a different group of researchers (e.g., Clement, 1982; Clement, Lochhead, & Monk, 1981;
Clement, Narode, & Rosnick, 1981; Caldwell & Goldin, 1987). This group of researchers contends that the syntactic approach of Paige and Simon (1966) is fundamentally misconceived, as Caldwell and Goldin (1987) found:

Many mathematics teachers (unfortunately, in our view) rely on 'direct translation' to teach students how to solve problems. They use, for example, the 'key word' approach, in which English words are identified with their most common mathematical meanings. More recent information-processing models of problem-solving competence reject reliance on direct translation and suggest that a crucial stage in insightful problem solving is the construction of a 'nonverbal' or 'imagistic' internal representation of the problem. Such models suggest that students must learn to visualize the problem situation or otherwise represent it nonverbally, rather than rely on direct translation. (p. 188)

Much of the empirical support for decreased emphasis on routine word problem solving comes from the research of this group that has observed and studied a particular translation error pattern called the reversal error. The reversal error is seen in the now famous student and professor (S & P) problem. When asked to translate the
problem, "At this university there are six times as many students as professors," students who are told to use $S$ for the number of students and $P$ for the number of professors, often write "$6S = P$" instead of the correct "$S = 6P$." Many studies (Clement, 1982; Clement, Lochhead, & Monk, 1981; Clement, Narode, & Rosnick, 1981; Cooper, 1986; Fisher, 1988; Hasty, 1987; Lochhead, 1980; Lochhead & Mestre, 1988; Mestre & Gerace, 1986; Mestre, Gerace, & Lochhead, 1982; Mestre & Lochhead, 1983; Schrader, 1985; Seeger, 1990; Soloway, Lochhead, & Clement, 1982; and Wollman, 1983) have demonstrated this result for a wide range of age groups.

Another problem (Mindy's) that was used by this group and also produced high reversal error rates is stated "At Mindy's restaurant, for every four people who ordered cheesecake, there were five who ordered strudel."

In an attempt to understand the reversal error, Clement (1982) set up audiotaped and videotaped interviews with fifteen freshman engineering students. From his analyses of these think-aloud protocols, Clement characterized three types of solvers, only one of which was competent.

The essence of the position on sentence translation of Clement and other cognitive scientists (the cognitivist position) is that an expert does a conceptual analysis of the sentence in which he understands the
relation between the variables in the sentence. The expert sees the equation \( S = 6P \) "as representing an operation (e.g., the coefficient 6 as a multiplier) on a variable quantity (e.g., an unspecified number of professors) to produce a number equal to another unspecified quantity (e.g., the number of students)" (Clement, 1982, p. 21). The novice, however, "simply assumes that the order of key words in the problem statement will map directly into the order of symbols appearing in the equation" (Clement, 1982, p. 18).

A third type of student seems to be in an intermediate state between the expert and the novice.

The tendency toward syntactic interference of those who are unsuccessful, according to Clement, is the result of inappropriate instruction, since many textbooks explicitly teach a word-order matching strategy (e.g., Dolciani, Swanson, & Graham, 1986, p. 35-42) similar to direct translation. Clement (1982) acknowledges that word order matching techniques are viable for many translation tasks, but finds the deployment of these techniques ultimately misleading for students. He and his colleagues (Soloway, Lochhead, & Clement, 1982) believe that syntactic translation has a role only in contrived problems such as those that "are constructed so that they can be solved thorough a trivial word-to-symbol matching algorithm" (p. 174). In summary,
the sentence translation process for experts according to the cognitivist theory of Clement and his colleagues (Figure 2) does not involve direct translation.

**Figure 2. Translation Process - Cognitivist Theory**

**Methodological objections**

Methodological objections can be raised to Clement's (1982) study. His theory of how a competent student translates a natural language sentence into an algebraic equation is based on think-aloud protocols. These protocols deny at the outset the possible relevance of syntactic knowledge, since such knowledge is well known to be unconscious and introspectively inaccessible (Foss and Hakes, 1978, p. 13).

Think-aloud protocols are a primary tool that cognitive scientists use to determine the strategies that are employed in solving logical problems. But such introspective reports can only reflect the elements of conscious, rational knowledge available to the subject.
(Ericsson & Simon, 1984, pp. 14-15). Thus, Clement's use of such methods in this study is bound to support his assumption that the knowledge structures to be tapped are entirely rational in character. For this reason the think-aloud methodology employed by Clement (1982) is not a fitting basis for rejecting a syntactic approach to sentence translation.

Statement of the Problem

The literature on algebra word problem solving states that translation is the most important part of the process, and that translation of a problem involves sentence translation. Two models of how sentence translation occurs have been examined. The work of Paige and Simon (1966), which examined the STUDENT program, suggests that syntactic processes are sufficient for translating a broad range of sentence types, but are unable to handle others. The work of Clement and his colleagues has examined the conceptual methods needed to translate S & P type sentences that are not directly translatable (through syntax). But there has been no attention to the possibility that competence in translation is essentially syntactic and that the expert responds to syntactic cues to abandon syntax for the small minority of sentences for which conceptual methods
are needed (see Figure 3). In Chapter 3, I explicate such a model.

Figure 3. Translation Process - Syntactic Theory

Organization of the Dissertation

A review of the literature is presented in Chapter 2, and examines the two approaches to sentence translation, suggesting that the syntactic theory of translation has been prematurely dismissed by current researchers.

A syntactic theory of sentence translation is proposed in Chapter 3. The components of knowledge necessary for competence in translation are also presented in that chapter.

The two contrasting theories were embedded in
instructional programs and used in experimental treatments. The design of the study, along with a description of the subjects, instruments, and procedures used in treatments, testing, and analysis of the data are listed in Chapter 4.

Instructional materials aimed at remediating deficiencies identified in the syntactic theory are used to test the following experimental hypothesis:

*An instructional treatment designed according to the Syntactic Theory will be more effective in promoting competence in algebraic sentence translation than an instructional approach designed on the Cognitivist Theory.*

The basic design includes an attempt to make students into experts as defined by each of the different translation models. An evaluation of the students' translation skills made after the instructional treatments is reported in Chapter 5. The inference is that the students who perform more like experts, possess more of the cognitive tools underlying expertise in translation.

A limitation of this design approach is that students may perform more like experts without being more like experts, i.e., the students may have learned a trick
that produces a correct translation. A defense of this design is presented in Chapter 6.

Final conclusions, discussions, and limitations of the study are presented in Chapter 6. Also included are recommendations for future research and practice.

Importance of the Study

Problem solving is the most important learning task of the algebra student according to the National Council of Teachers of Mathematics' *An Agenda for Action* (1980), and the more recent *Curriculum and Evaluation Standards for School Mathematics* (1989). The most critical part of problem solving lies in the translation rather than the manipulation of the problem (Berger & Wilde, 1984; Mayer, 1980). Yet, the *Standards* (1989, p. 126) calls for reduced attention to standard algebra word problems.

It is likely that the S & P research of Clement and his colleagues was an important factor in the call to decrease emphasis on routine word problem solving. Since the think-aloud protocol analysis methods of Clement and his colleagues do not accommodate the possibility of syntactic processes in translation, the issue has been prejudged, and there is an imperative to further assess if their analyses are correct.

Another imperative to further assess the nature of expertise in translation skills comes from pedagogical
considerations. Clement (1982) suggested that "the identification of these conceptual stumbling blocks...should make it easier to design instructional strategies to overcome them" (p. 29). But several studies (Clement, Narode, & Rosnick, 1981; Fisher, 1988; Rosnick & Clement, 1980) designed to test such instructional strategies, instead showed that the reversal error is extremely resistant to change. Other studies (Fisher, 1988; Schrader, 1985; Hasty, 1987; Rosnick & Clement, 1980) which also have attempted pedagogical approaches based on the premises of the Cognitivist Theory, were poorly conceived or inconclusive (see Chapter 2). If the syntactic theory is correct, then the recent recommendations to reduce the syntactic translation emphases of the curriculum (NCTM, 1989, p. 150) may be retrogressive.

The present study can shed light on syntactically-based instructional methods that have been part of the curriculum for centuries, but are neither understood nor appreciated within the cognitive science frameworks which dominate educational psychology today.
CHAPTER TWO
REVIEW OF THE LITERATURE

As indicated in Chapter 1 (Figure 1) I distinguish between problem translation and sentence translation as follows: Problem translation involves (a) categorizing a problem into type, (b) using background information, (c) expressing natural language statements of variable relationships as algebraic equations, and (d) constructing the problem equation. The third step in problem translation is called sentence translation.

This review of the literature describes two approaches to sentence translation skill. The review is organized in the following manner:
(1) Introduction
(2) Linguistic approaches to sentence translation skill.
   a) Syntactic
   b) Others
(3) Conceptual approaches to sentence translation skill.
(4) History of sentence translation curriculum.
(5) Pedagogical studies
(6) Summary

Introduction

The focus of this review is on the sentence translation skill used in solving algebra word problems.
In this study, translation skill is defined as the ability to represent the quantitative relations expressed in a natural language sentence as an algebraic equation. The importance of this skill is attested to by Lochhead (1980) who notes that "If mathematics instruction at all levels were to place greater emphasis on developing translation skills, perhaps we might see a time when the power of mathematical language is available to all" (p. 35).

Research into algebra problem solving covers many different areas. Although many different traditions in problem solving research incorporate translation they do not all clearly distinguish between problem translation and sentence translation.

Mayer (1980), who is concerned about why algebra word problems are so difficult, suggests that the answer to this question "would provide the basis for a psychological theory of human problem solving as well as a pedagogy of mathematical learning" (p. 2).

Mayer is one researcher who seems to make a distinction between problem translation and sentence translation. His work led him to conclude that translation is the most difficult part of problem solving. He gives evidence (Mayer, 1980, p. 3) that translation skill involves both the ability to exploit the structural properties of the propositions in the
sentence, and to see schematic representations (schemas) of problems.

Mayer also states that most algebra problems contain four types of propositions: (a) assignments, (b) relations between variables, (c) questions, and (d) relevant facts (Mayer, 1980, p.6). He hypothesized that the relation between two variables is the most difficult to translate of the four propositions.

Most of Mayer's own work dealt with students' categorizing problems into types, and their ability to recall various types of problems. Still, he states that "special attention should be paid to teaching children how to translate among relational propositions (in English), relational equations, and concrete manipulatives or pictures" (1980, p. 28). It is precisely the translation of this type of relational proposition (relating two variables) in a sentence that is focused on in this review of the literature on translation skill.

Linguistic Approaches to Sentence Translation Skill

Research using syntactic approaches to translation skill can be traced back to a study done by Paige and Simon (1966) in which they claimed that "almost all the thinking and problem solving that people do requires that
they handle natural language" (p. 51). In their study, the researchers used Daniel Bobrow's (1968) computer program STUDENT that was written to translate algebra word problems. They were interested in "detecting the extent to which the human subjects make use of direct processes like those incorporated in the STUDENT program" (p. 116). For certain sentences, the human subjects used direct translation, and "the general translational approach of STUDENT gives a pretty good explanation of what the student is doing in solving the problem" (p. 69).

After observing several human subjects, the researchers noted that competent translators often need to rely on auxiliary cues (e.g., "He must know that the value of a quantity of coins equals the number of coins times the value per coin" (p. 84)), and the internal structure of the given situation (e.g., detecting impossible situations). Note that these are aspects of problem translation other than sentence translation.

Other researchers have mentioned the importance of syntactic skills in their work as well. In a study of the relation between language structure and algebra word problems, Hinsley, Hayes and Simon (1977) found that if one recognizes that a sentence matches a certain prototype, one may categorize it and then apply certain heuristic techniques useful to problems in that category.
(e.g., formulas for work or distance problems). But if the student does not recognize a category for a certain problem, the student reverts to a procedure resembling the direct translation of the STUDENT program.

The observation that a sentence that does not fit into a category must be directly translatable underestimates the role of direct translation. Even after a problem is categorized, it may still contain propositions that must be translated. The sentence: "Train A travelled twice as far as Train B," is easily categorized as a part of a distance problem, but it is still necessary to translate the relational proposition expressed in the sentence.

These researchers used think-aloud protocols with a group of students. They found one student who was a good problem solver relying almost completely on direct translation, and another who used auxiliary cues and physical representations (see p. 116). They stated that these "results have forced us to conclude that people use more than a single approach in comprehending algebra word problems" (p. 105).

In evaluating the STUDENT program they note that for certain sentences it used "a simple syntactical parsing scheme" (p. 91). But because the program makes no reference to semantic relations in these sentences, STUDENT "cannot account for human solution processes".
which rely on semantic knowledge" (p. 91).

This analysis of the translation process used by STUDENT is deficient in two respects. Firstly, the critics fault STUDENT because it was not a good semantic translator, unable to detect ambiguities even though its author made no claim that it was a semantic translator (Bobrow, 1968, p.201). Secondly, they did not fully evaluate the syntactic limitations of STUDENT since they failed to observe (or report on) its attempts to translate sentences of the S & P or Mindy's type which do not respond to direct translation.

Other researchers have mentioned linguistic skills in the translation process. Betty Travis (1981), observed 84 students in a college intermediate algebra class as they translated word problems. She finds value in the early information processing techniques of Paige and Simon (1966), but like Hinsley, Hayes, and Simon (1977) suggests that "a direct translation scheme in its purest form has to be augmented by specific semantic knowledge to insure full understanding of the problem" (p. 3). She suggests that: (a) students should be encouraged to set up equations in phrases, (b) teachers should stress phrase structures of word problems, and (c) students should use auxiliary cues, including pictures, diagrams and flowcharts, with strong emphasis on phrases and phrase structure. This emphasis on phrase
structure in problems suggests the necessity of a syntactic analysis in the translation process.

Mathematics educator, Thomas (1988) states that "word problems make special linguistic demands on the reader" (p. 245). These demands include both the technical terms in the problem and the language in which it is stated. Besides understanding the mathematical terms, formulas and rules, he observes that a problem solver must have an "ordinary linguistic knowledge," (p. 246) and suggests that "the role of reading in problem solving is important enough that teachers of science and mathematics should not dismiss the linguistic aspects of problem solving as trivial" (p. 244).

Martha Burton (1988), who directs a mathematics laboratory and conducts research on how students solve problems, sees the trouble with sentence translation as based in the natural language. She claims that current strategies that require the students to make immediate variable assignments (e.g., $S = \# \text{ of students}$, $P = \# \text{ of professors}$), take the students away from the structure of the natural language sentence where the verb is. She says that "Until the student has a verb for the problem statement, possession of algebraic words for all the appropriate nominals cannot lead to an equation" (p. 5). It is interesting to note that the students in Clement's (1982) study who used direct translation did
not locate the verb in the sentence, but translated the word "as" for "=" in the algebraic equation.

**Conceptual Approaches to Sentence Translation Skill**

There is a body of literature that takes the approach that the competent sentence translator uses only conceptual strategies as outlined in the Cognitivist Theory (Figure 2). Several researchers (Clement, 1982; Clement, Lochhead, & Monk, 1981; Clement, Narode, & Rosnick, 1981) have studied the reversal error (Chapter 1, p. 7) that occurs in sentence translation.

Evidence of the reversal error was first alluded to in a letter to the editor of a mathematical journal (Kaput & Clement, 1979). A study by Rosnick and Clement (1980) indicated that a significant number of otherwise competent students make the reversal error when translating algebra word problems. In fact, of 150 college freshman engineering majors who were tested, 37% of them missed the student and professor problem, and two-thirds of the errors were the reversal error (see p. 7).

Another problem used in the study was "At Mindy's restaurant, for every four people who ordered cheesecake, there were five who ordered strudel." The error rate for this problem was 73%, and again, two-thirds of the errors
were the reversal error (Rosnick & Clement, 1980, p. 5). The students translated the sentence as "4C = 5S", instead of the correct "5C = 4S".

In an attempt to discover why students had such difficulty in translating these sentences, Clement (1982) set up audiotaped and videotaped interviews with fifteen of the freshman engineering students. From his analyses of these think-aloud protocols, Clement characterized three types of solvers.

A Type I student uses a "word order matching" (p. 18) approach. The student wrote "6S = P" corresponding to the order of the words in the sentence: Six times as many students as professors. This type of student was unsuccessful.

A typical Type II student attempted word order matching, but was dissatisfied with the resulting equation "6S = P". The student reversed the equation to "S = 6P" but found that it did not match the conceptual image of six students for every one professor. The student drew a model of the situation that had one circle with a P in it, and six circles with S in them. The student compared the circles and assumed that the six students in some way matched with the one professor. Although the resulting equation was not correct, such students recognize the relevance of the semantic relations expressed in the sentence. Clement (1982)
calls the semantic strategy used by a Type II student a "static comparison approach" (p. 19).

The successful, Type III, student thought that since there were more students than professors, the number of professors should be multiplied by six so it would equal the number of students. This type seems to understand that translating involves a relating of two quantities, and Clement calls this method an "operative approach" (p. 20).

Clement also suggests that the competent translator's success lies in "(a) remembering that variables stand for numbers rather than objects in these problems, and (b) being able to invent a hypothetical operation on the variables that creates an equivalence" (p. 28).

Lochhead (1980) investigated "whether it [the reversal error] is so persistent that it remains even among faculty" (p. 30). He gave 200 university faculty (at physics or engineering departmental seminars and a university-wide seminar), and 150 high school science teachers the equation "A = 7S". They were to write one sentence in English to express this information. They were told that A is the number of assemblers in a factory and S is the number of solderers in a factory. Lochhead divided the faculty into groups (a) in the physical sciences; (b) in natural, behavioral and social sciences;
and (c) others. He found that outside of the physical sciences, "the overall success rate was about 50%" (p. 34). He found (p. 33) that in the physical sciences, the success rate of university faculty was 80%, and for high school faculty it was a dismal 44%. He did not categorize the types of errors as reversals or other, but showed several examples of the types of interpretations made and they often included the reversal error. It is important to note that this study is actually concerned with writing an equation as a sentence rather than translating a sentence into an equation.

Later studies by Mestre, Gerace, and Lochhead (1982), and Mestre and Lochhead (1983), showed that the reversal error occurs in bilingual and other cultural groups as well.

In a short study to determine students' concepts of variable, Rosnick (1981) wrote the equation S = 6P, and asked the students in his college statistics course what the letter S and the letter P represented. He was amazed that over 40% were incapable of answering his questions correctly. This led to his conclusion that students need to develop better understandings of the concepts of variable and equation.

Davis (1980) offers an information processing approach to explaining the reversal error. He postulates certain "frames" of knowledge (p. 170) that students can
retrieve from memory. He assumes that students have two separate "frames" that they learn in mathematics classes that act like schemas. For example, the "label frame" is used to state that twelve inches represents one foot, as in

\[ 12i = 1f. \]

The "numerical variables frame" is used to state that if \( I \) represents the number of inches and \( F \) represents the number of feet, then

\[ I = 12F. \]

In interviews with mathematicians who are presumably competent translators, Davis discovered that they are aware of the two different frames and usually check to be sure they are selecting the correct frame. This led him to suggest that instruction should alert students to the presence of the two frames and the necessity of checking to see if the correct one has been selected.

This checking strategy may sound like a third, alternate theory as to how the competent translator operates, but it seems more probable that it is a part of a conceptual strategy. Translators who have had the experience of being weak in conceptual strategies, probably have learned to use checking as a backup strategy for additional support. If they check the equation using numbers they may catch the reversal error, but if they check the equation against their image of
sets they may not detect the error.

Another study (Caldwell & Goldin, 1987) examined factors that affect the difficulty of a word problem. They divided word problems into four types: abstract factual (AF), abstract hypothetical (AH), concrete factual (CF), and concrete hypothetical (CH), and tested students to see which types were more difficult.

Problems were given a syntactic complexity coefficient (Goldin & Caldwell, 1984, p. 256) based on counting the number of words, the number of sentences, and the number of numbers in a sentence. This measure of syntactic complexity resembles techniques commonly used in assessing text difficulty in arithmetic word problems. But it is not sensitive to grammatical characteristics of the reversal error problems. For instance, Caldwell and Goldin (1987, p. 189) assign a higher syntactic complexity coefficient to an AF problem than to a CF problem, even though the CF problem contains a sentence that does not yield to direct translation.

History of Sentence Translation Curriculum

Some past approaches

A review of some algebra textbooks show that even a hundred years ago the translation of a word problem was considered to be very difficult. In Elements of Algebra, Davis (1857) defined the statement of the problem as "the
operation of expressing, algebraically, the relations between the known and unknown quantities which enter it" (p. 80). He said that the statement of the problem cannot be defined by a rule.

In *New Elementary Algebra*, Ray (1866) paraphrased the same idea and reiterated that some conditions are made explicit by the sentence, while others are implicit. These texts show no particular reference to syntactic processes used in translation.

An algebra text by the School Mathematics Study Group (1962) used one entire chapter to slowly develop the translation of phrases and sentences into algebraic notation. This development is very syntactical (see p. 156) in its direct translation approach.

**Some current approaches**

A current algebra text (Dolciani, Swanson, & Graham, 1986) uses a syntactic approach when it states that in order to translate a word phrase into a numerical or variable expression "you must be able to translate each part of the word phrase into an appropriate mathematical symbol" (p. 35). The translation of a word sentence into a mathematical sentence is handled in the same way, with many examples showing literal translation. It is interesting that a sentence of the S & P type, "the team won five times as many games as it lost" (p. 42), appears
in the exercises with no instruction to aid the student with intended syntactic methods.

**Some proposed approaches**

The proposed pedagogical approaches as found in *An Agenda for Action* (NCTM, 1980), *The Curriculum and Evaluation Standards for School Mathematics*, (NCTM, 1989), or as suggested by the predominant cognitivist research, downplay translation skills as they are currently presented in the algebra curriculum.

The *Agenda* (1980) suggests that the mathematics curriculum be expanded to include a wide range of non-routine problems and "not be limited to the conventional 'word problem' mode" (p. 3). The *Standards* (1989) suggests decreased attention be given to typical word problems and suggests problem solving that helps students "apply the process of mathematical modeling to real-world problem situations" (p. 137). Such proposals are consistent with current recommendations from the research community (see the next section) that advocate conceptual control of the translation process with no apparent role for syntactic methods.

**Pedagogical Studies**

Clement (1982) noted that syntactic interference caused some students to be unsuccessful at sentence
translation, and he suggested that "the identification of these conceptual stumbling blocks using protocol analysis should make it easier to design instructional strategies to overcome them" (p. 29).

Some of the general approaches taken to remediating the reversal error illustrate the underlying cognitive analysis, i.e., that translation is accomplished using conceptual strategies only. For instance, the suggestions of Clement, Lochhead and Monk (1981) for halting the reversal error include: (a) spending more time on translation skills, (b) assigning more problems of the type that cannot be solved by non-operative approaches, (c) showing the shortcomings of the non-operative approaches, and (d) emphasizing the operative approach.

In trying to explain how fairly competent students could make translation errors Soloway, Lochhead, and Clement (1982) found these reasons to downplay syntactic processes:

It appears that these students have developed special purpose translation algorithms which work for many textbook problems, but which do not involve anything that could reasonably be called a semantic understanding of algebra. Many word problems are constructed so that they can be solved through a trivial word-to-symbol matching algorithm...While
these techniques may be partially successful in many classroom situations, they are too primitive and unreliable to be trusted in any but the most routine applications. (p. 174)

Several studies (Rosnick and Clement, 1980; Rosnick, 1981; Soloway, Lochhead, & Clement, 1982; Fisher, 1988; Wollman, 1983; Schrader, 1985; Cooper, 1986; Hasty, 1987; Seeger, 1990) have tested instructional strategies aimed at remediating the reversal error in translation. These studies were all based on the conceptual approach.

The first study by Rosnick and Clement (1980) tried tutoring strategies to halt the reversal error. They suggested that the teaching unit used was not ideal, but that what they "were interested in knowing was whether a fairly simple, traditional, algorithmic approach to teaching would be sufficient to help the students with the reversal error" (p. 16). The lesson stressed, (a) understanding the sentence, (b) finding numbers that would fit the relationship, and (c) checking those numbers in the algebraic equation.

Six students were interviewed and taped during the instruction. The researchers found that even when some of the students' behavior changed, i.e., they wrote the correct equation, they still had no conceptual understanding of the problem. They wrote the equation so that the numbers worked, but they were dissatisfied with
the order of the variables. The results of this study led to the conclusion that the reversal error is due to a resilient misconception about equations. Blame for this problem was laid on a school system that "focuses primarily on manipulation skills" (p. 23).

A series of experiments by Soloway, Lochhead, and Clement (1982), was used to study the effects of computer programming on the problem of translation. They had determined from previous analyses of student protocols, in attempts to translate the S & P problem, that "the key to fully understanding the correct translation lies in viewing the number six as an operator which transforms the number of professors into the number of students" (p. 177).

In one experiment using 17 professional engineers, 47% missed a Mindy's type problem. After a day of writing and running programs that used assignment, conditional, and for-next statements, all of the group answered a Mindy's type problem correctly. Since no retention time was used with this group, and they were tested on one sentence only, it is highly likely that the engineers' success was due to the "practice effect" (p. 179) mentioned by the researchers.

In another experiment the same researchers used freshmen and sophomores in an assembly language class. The students were all given a Mindy's type problem. Half
were asked to write an equation for the sentence, and the other half were asked to write a computer program to indicate a relation between the variables in the sentence. The students who wrote the equation had a 55% error rate, while those who wrote the computer program had a 31% error rate.

A final experiment by the same researchers had freshman engineering students interpret an S & P type equation and a computer program that related two variables. The number of students "who answered the computer problem correctly, but the equation problem incorrectly was more than 3 times as large as the group who answered the equation problem correctly, but missed the computer problem" (p. 180).

The researchers noted that programming helped the students focus on an understanding of the semantics of the expression they were translating. The step by step process of writing a program and the debugging process also helped the students. This led to the conclusion that "if students were placed in an environment which could induce them to take a more active, procedural view of equations, then the error rate on these problems should go down"(p. 178). Five hypotheses about why students had more success in a programming environment were listed (p. 181) and are paraphrased here: (a) The symbol '==' is defined as an act of replacement, (b) one
must be explicit in writing '6*S' instead of '6S',
(c) the right side of the equation is operated on to get
the left side, (d) students are encouraged to debug
programs, and (e) students get partial results along the
way.

The data from this study point to a connection
between skills in programming that could halt the
reversal error. But the reasons for the effect are
unclear. Computer languages are not algebraic language.
Variables and equations have a different function in
computing, as in \( x = x + 1 \) which means that the value on
the right will replace the value on the left.
Perhaps the computer language syntax is not amenable to
direct translation approaches, and so subjects in their
study have had to attend more carefully to developing
conceptual translation methods.

Cooper (1986) tried to determine, using high school
students, if the incidence of the reversal phenomenon was
affected by the use of letters other than the initial
letter of the object (x and y instead of S and P), or if
the insertion of a multiplication sign \( S = 6 \times P \)
affected the interpretation of an equation. He found
that the insertion of a multiplication sign in an
equation tends to lower the incidence of reversals, but
that the letters made no difference. But this study
involved equation interpretation rather than sentence
A short study by Fisher (1988) also tested the theory that students have difficulty with the idea that a letter represents a number. She tried the technique of changing the variables from S for students and P for professor, to Ns for "number of students" and Np for "number of professors", but this method had negative results. Whereas 40% of the students were incorrect using S and P, 57% failed using Ns and Np. The results led to the conjecture that "the new notation does reduce literal mapping errors but that these represent just a small part of many students' difficulties" (p. 261).

Wollman (1983) conducted a series of six studies using 180 female elementary education majors. He combined a search for theory and pedagogical methods for halting the reversal error. The first study tested the students' understanding of the natural language sentence. It was found that they did understand the meaning of the sentence. The second study tested the students' understanding of the equation \[ y = mx \]. The results were that the students understood the equation also. The third study asked the students to translate a sentence into algebra. The sentence was similar to the S & P problem, but some of the directions were different. The students were told to translate the sentence "A store sells six times as many volleyballs as footballs." They
were also told "The equation should relate $F$, the number of footballs, to $V$, the number of volleyballs. The total number of footballs and volleyballs has nothing to do with the question" (Wollman, 1983, p. 173).

Wollman also conducted two pilot studies. In one study he found that contextual knowledge, like the number of students is usually larger than the number of professors, did not affect success rate. In another study he discovered that a large number of errors were due to a misunderstanding that the equation should contain a total. Over half (52%) of the students were not successful.

Interviews of the students to discover if they had considered reversing the equation led to a fourth study on spontaneous checking in which the students were asked to justify their equations. The results showed that the unsuccessful students could rarely justify their equations. The fifth study involved tacit cueing. Before the students were asked to translate the sentence, they were asked to compute with numbers or compare the sizes of the variables. This method seemed to have some effect, and led to the sixth study on the effect of explicit cueing. Here the student was asked "According to your equation, which letter stands for the larger number, $F$ or $V$? If your answer doesn't agree with your answer to [the comparison question based on the
sentence], change your equation so that it does" (Wollman, p. 178).

Wollman claimed that after a ten minute classroom intervention that focused on (a) the operative approach, (b) the need for studying the meaning of the sentence, and (c) checking the resulting equation, he attained immediate success in nearly 100% of the cases.

In a heated critique of the Wollman study (Kaput, Sims-Knight, & Clement, 1985), his behavioral approach is attacked for three reasons summarized as: (a) no cognitive model was used to explain the phenomena under consideration; (b) a failure to discuss other knowledge structures of the students' performance; and (c) no description of how relevant concepts were developed (p. 57).

Two recent studies (Schrader, 1985; Hasty, 1987) involved designing instructional strategies to halt the reversal error. Both were doctoral dissertations done within two years, under the same director. Schrader (1985), tried a five-day instructional method with two classes of high school algebra students. This study used the pretest, posttest, retention test design, but the retention time was only three weeks.

Hasty (1987) used a cognitive approach that stressed the following seven steps:
1. Statement of the problem.
2. Draw a representation of the problem (This should be a diagram or picture.)
3. Develop a table of information.
4. Develop an algorithm for the procedure to find one variable, given the other.
5. Convert the algorithm to an algebraic equation.
6. Test the equation using numbers from the table.
7. Review the procedure to ensure understanding. (p. 23)

The subjects were two groups of seventh graders, two high school algebra classes, one of Algebra I and one of Algebra II, and two university classes, one a beginning calculus class and the other an engineering class. They were given a pretest. The next day they had one fifty-minute instruction period. The day following the instruction, a posttest was administered. Two weeks later a retention test was given.

Results showed a significant reduction in reversal errors for the seventh grade groups and the university groups from pretest to posttest and from pretest to retention test. A longer retention time was suggested in future research. The Schrader and Hasty studies attempted to find a "quick fix" for the reversal error, with inadequate attention to a long term change in students' behavior.
Seeger (1990) conducted a study in which students who made the reversal error seemed to have no trouble getting a correct arithmetical result from the faulty equation. This reinforces the notion that the students have a poor concept of variable. For example, if the students are using the variables to represent sets of students and professors, then they will interpret the equation $6S = P$ as correct because as they would say, a set of six students matches with a set of one professor. These students seem to be using the static comparison approach identified by Clement (1982).

Summary

Ever since the reversal error phenomenon was noted in 1979 there have been attempts to discover if various groups do in fact make the error, and why. The research has been done by a variety of researchers based on a cognitivist theory of translation (see p. 10). Several studies used a traditional, algorithmic approach (Rosnick and Clement, 1980; Rosnick, 1981; Schrader, 1985; Hasty, 1987). One used computer programming as an aid in understanding variables and operations on variables (Soloway, Lochhead, & Clement, 1982). Some tried the trick of changing the notation in the equation (Fisher, 1988; Cooper, 1986; Seeger, 1990). And another series of studies tried to use operative approaches together with
checking the resulting equation (Wollman, 1983). Since these theorists are convinced that the competent translator is using a conceptual strategy, all attempts at remediation have stressed some form of conceptual strategy. Generally these treatments have not produced improved performance, except in cases where the retention times have been very short, or entirely absent. Thus the cognitivist assumptions have not yielded effective pedagogical methods.

Research that stands in contrast to the conceptual approach is virtually non-existent. There are a few suggestions that it is necessary to investigate an alternate theory that incorporates a syntactic analysis into the translation process (Paige & Simon, 1966; and Hinsley, Hayes, and Simon, 1977), but this research agenda which appeared to focus on sentence translation, was actually more concerned with problem translation.

If a natural language sentence is stated with a simple structure like [quantified noun phrase] [is equal to] [quantified noun phrase], it is easily translated because its syntax parallels the syntax of the algebraic equation. But no one has attempted to address the linguistic differences between these S & P and Mindy's type problems which have a more difficult construction.
In the next chapter, I propose a syntactic oriented theory of translation that encompasses all possible sentence types.
In Chapter One, I argued for the need of a theory of competence in algebraic translation that involves syntactic knowledge. Such a theory would conflict with current theoretical frameworks in which conceptual knowledge alone is seen as the foundation for competence. In this chapter, I outline such a theory, and delineate its components of knowledge (particularly the syntactic components), in sufficient detail so that instruction leading to acquisition of the (purportedly relevant) knowledge structures can be designed. Such instruction, in competition with instruction based on the cognitivist theory, can provide evidence as to which theory is more adequate.

The power of cognitive think-aloud methods (where appropriate) used in the Cognitivist Theory is that they focus a great deal of attention on the mind engaged in a particular act of reasoning. The price for such a snapshot of the mind in action may be a lack of attention to the cognitive context of the problem.

The cognitive context of the S & P type problems is that the typical student already has substantial
experience in translating sentences that are not of the S & P type, and which do yield to syntactic methods.

Four Types of Translation Sentences

The current method of classroom instruction in translation involves chopping the natural language sentence into phrases and translating the phrases sequentially. But instruction in such direct translation techniques is generally haphazard and unsystematic, and the typical student has not been taught about sentences for which this method may not apply. Indeed no consideration is raised that different types of sentences may require different approaches.

Four types of translatable sentences can be identified in standard algebra word problems. Three of these sentence types can be translated successfully by using direct phrase-by-phrase translation processes. The fourth type, though rare, requires conceptual strategies as well. The four sentence types are:

1) Phrase Order Matched (POM) - The noun phrases are quantified, and all phrases of the sentence are in a one-to-one correspondence with algebraic symbols, so that the process of direct translation can occur immediately. Such sentences that are written in this form will be called phrase order matched (POM). An example of a POM sentence is:
1-a) [The number] [squared] [is equal to] [the number] [plus] [six.]
Let $x$ represent "the number."
Translation: $x^2 = x + 6$
Note that each algebraic symbol replaces a bracketed word or phrase.

2) Within Phrase Adjustment (WPA) - Other sentences are not POM, but the noun phrases are quantified, so that the sentence can be transformed to a POM sentence using simple lexical or phrase-level adjustments. We will call these sentences (WPA) because they require within-phrase adjustments. In this case, after the adjustments are made, the sentence is POM and can be translated directly.

Following is an example of a WPA sentence as it is transformed to POM form. The transformations involve only lexical or phrase-level adjustments or substitutions.

2-a) [The number of cookies] [is equal to] [four less than the number of biscuits.] [WPA]
   i) [The number of cookies] [is equal to] [the number of biscuits] [minus] [four]. [POM]
Let $C$ represent "the number of cookies" and $B$ represent "the number of biscuits".
Translation: $C = B - 4$
Note that adjustments to arrive at POM form occur within phrases which do not include the main verb of the
sentence.

3) Whole Sentence Transformation (WST) - Another type of sentence also yields to syntactic methods, but the noun phrases are not quantified. These sentences will be called (WST) because they require whole sentence transformation. In such cases, it is the verb phrase that contains the reference to quantity, and the sentence must be radically transformed to achieve WPA form. Such transformations involve distributing the quantitative aspects of the verb to the noun phrases. Once the noun phrases are quantified, the sentence is in WPA form and may still need adjustments at the lexical or phrase level to reach the final POM form.

Following is an example of a WST sentence as it is transformed to POM form. The transformations involve distributing quantity to the noun phrases, followed by lexical or phrase-level adjustments or substitutions.

3-a) John weighs three pounds more than Mary. [WST]
   i) [John's weight] [is] [three pounds more than] [Mary's weight]. [WPA]
   ii) [John's weight in pounds] [is equal to] [Mary's weight in pounds] [plus] [three]. [POM]

Let J represent "John's weight in pounds" and M represent "Mary's weight in pounds".

Translation: $J = M + 3$
4) Not Phrase Order Matchable (NPOM) - The final sentence type also has non-quantitative noun phrases, but it cannot be translated using phrase order matching because transformations do not leave the sentence in POM form or because the requisite adjustments cannot be made. This type will be called not phrase order matchable (NPOM).

Following are several examples of NPOM sentences:

4-a) There are five times as many dimes as nickels.
4-b) Six fewer apartments were available than condos.
4-c) We have three spoons for every two forks.

The NPOM category includes the full range of sentences which Clement and his colleagues have identified as problematic for students with the addition of some additive (and subtractive) cases. This sentence type is called NPOM because lexical and phrase-level adjustments, and transformations that quantify the noun phrases, cannot result in a POM sentence. Either the quantified noun phrases end up on the same side of the "is equal to" (e.g., 4-a and 4-c), or the lexical adjustments cannot be made (e.g., 4-b and 4-c). For example:

4-a) There are five times as many dimes as nickels.
   i) [There are] [five] [times] [the number of dimes] [as] [the number of nickels].
ii) [There is equal to] [five] [times] [the number of dimes] [as] [the number of nickels].

The syntactic process is abandoned because the POM form, [a quantified noun phrase] [is equal to] [a quantified noun phrase], cannot result from the adjustments and transformations.

4-b) Six fewer apartments were available than condos.

   i) [Six] [fewer] [the number of apartments] [were available than] [the number of condos].

   ii) [Six] [fewer] [the number of apartments] [is equal to] [than] [the number of condos].

The syntactic process is abandoned because there is no adjustment for "fewer [something] than".

4-c) We have three spoons for every two forks.

   i) [We have] [three] [the number of spoons] [for] [two] [the number of forks].

   ii) [There is equal to] [three] [the number of spoons] [for] [two] [the number of forks].

The syntactic process is abandoned because the POM form of [a quantified noun phrase] [is equal to] [a quantified noun phrase] cannot result from the adjustments and transformations.

It should be noted that there are sentences in POM form that are equivalent to these given above:

4-a) The number of dimes is equal to five times the number of nickels.
4-b) The number of apartments is equal to the number of condos less six.

4-c) Three times the number of forks is equal to two times the number of spoons.

But crucially, these equivalent sentence forms cannot be achieved through the syntactic methods of transformation and adjustment described above for the WPA and WST sentences. Attempts at transformations will founder, and consequently, for these sentences, syntactic methods of translation must be abandoned in favor of conceptual strategies.

A Syntactic Theory of Sentence Translation

These considerations suggest two different sorts of hypotheses that can be entertained concerning experts (i.e., those who regularly succeed with all four sentence types): (a) Either they ignore the possibilities for phrase ordered translation of POM, WPA, and WST sentences, and always deal with the conceptual content of the sentence; or (b) they normally use syntactic methods of transformation on POM, WPA, and WST sentences but recognize the inappropriateness of these syntactic methods for NPOM sentences, adopting conceptual strategies only for this type. This syntactical alternative can stand only if there are purely syntactic means available to identify
NPOM sentences. In the next section I present a model of this discriminatory function.

The Discriminatory Function

In principle, there are two sorts of mechanisms that could be the basis for discriminating NPOM sentences from directly translatable sentences. The first method would involve some syntactic cue that reliably identifies NPOM sentences at the outset. The second method would involve an initial classification of NPOM sentences as phrase order matchable until unsuccessful attempts at lexical and phrase-level transformation force the abandonment of syntactic methods in favor of conceptual methods.

An examination of the sentences in the last section suggests that there is no obvious syntactic feature that reliably discriminates types. For instance, all of the sentences that are POM, WPA, and WST are in [noun phrase] [verb] [noun phrase] order, but one of the NPOM types (4-b) is also. None of the NPOM sentences contain quantified noun phrases, but the WST sentence shares this feature. One of the NPOM sentences (4-b) has a "split" in the expression "fewer than" (six fewer condos than), but this does not occur in all NPOM sentences.

Of course it is possible that competence in discriminating NPOM types is the result of knowing a certain taxonomy of sentence structures including,
a) those that begin with an existential there: "there are", b) those of the form "For every p x's there are k y's", and c) those that contain a "split" in the words "more than" or "less than." But this possibility is not pursued for two reasons. The first reason is because of its ad hoc and adventitious nature. The taxonomy given above was constructed from available examples, and it is entirely possible that further investigation would yield an expanded list of features. Thus the taxonomy theory is open-ended and non-principled.

The second problem with the taxonomy theory stems from acquisition considerations. If a student knows the taxonomy, he must have induced it unconsciously from his experience with NPOM sentences (such taxonomies are not taught). But such unconscious induction would require the trigger of a persistent problem situation with regular feedback about the efficacy of the choices made (Holland et al., 1986). This is implausible, since the NPOM sentences rarely appear in the curriculum and feedback on errors is sporadic.

Turning away from the first alternative of a taxonomy of salient characteristics, we consider instead the second: that NPOM sentences are identified by provisionally classifying them as WST until unsuccessful attempts at transformation force a reclassification. A characteristic of all POM sentences is the presence of
noun phrases quantified by "number" or "the number of".
I propose that WPA, WST and NPOM sentences may be readily and reliably distinguished by attempts to transform them into POM sentences using the adjustments and transformations as illustrated in the previous section. WPA and WST sentences can be transformed into POM sentences using simple adjustments and transformations; NPOM sentences cannot be transformed into POM sentences using these same adjustments and transformations. Of course such processes of transformation and evaluation are presumed to be unconscious and inaccessible to introspection.

To summarize, my hypothesis concerning the expert translator (Figure 4) is as follows: After reading the sentence, a syntactic analysis is done to determine sentence type. If the noun phrases are quantified and no lexical adjustments are needed the sentence is identified as POM and the translation is performed. If the noun phrases are quantified, but the sentence is not POM, then lexical and phrase-level substitutions and adjustments are performed. If the noun phrases are not quantified, a search is made for a verb phrase source for distributing quantity to the nouns. The nouns are quantified and the substitutions are attempted. If the sentence is POM it is translated, if not the syntactic process is abandoned.
READ SENTENCE AND IDENTIFY THE VARIABLES (1,4)

ARE THE VARIABLES EXPRESSED AS (4) QUANTITIES?

NO → FIND THE SOURCE OF QUANTITY AND DISTRIBUTE (4) QUANTITY TO THE Nouns

YES → ARE LEXICAL ADJUSTMENTS (3) NEEDED?

NO → CAN LEXICAL ADJUSTMENTS (3) BE PERFORMED?

NO → ABANDON THE SYNTACTIC METHOD (1,2)

YES → DO LEXICAL (2,3) ADJUSTMENTS

DO A CONCEPTUAL ANALYSIS (5)

USE A CONCEPTUAL STRATEGY (6) (COMPENSATION, PROPORTION)

IS THE SENTENCE POM?

YES → TRANSLATE, USING PHRASE-BY-PHRASe (2) SUBSTITUTION

Figure 4. Translation Process - Syntactic Theory
According to this theory, it is not some difficult conceptual strategy that makes the NPOM sentence difficult to translate, but knowing when to abandon the syntactic process. In either of the two cases in which the syntactic process is abandoned (position of the verb or inability to perform an adjustment), the translator resorts to a conceptual analysis and strategy. This theory is consistent with Clement's (1982) data, since the preliminary, unconscious, syntactic processes would be inaccessible to introspective probes.

Components of Knowledge

From this analysis we can identify six components of knowledge that competent translators have available to them for translating a natural language sentence into an algebraic equation. The numbers in Figure 4 locate these components of knowledge within the processes of translation. A characterization of each component is given here.

1. **Variable Identification.** The first component of knowledge is the ability to identify the two variable terms after reading the sentence. This may involve more than one reading because the variables are not always immediately obvious. For example, in the sentence, John has twice as many apples as bananas, the variables are the number of apples John has and the number of bananas John has, whereas, in the sentence, John has twice as
many apples as Fred, the variables are the number of apples John has and the number of apples Fred has.

2. **Symbol Reference.** The second component, symbol reference, is knowledge that every elementary algebra student is taught, and most assimilate easily. A student with this skill knows correspondences between natural language terms and mathematical symbols. Specifically, this component incorporates a) knowledge that a quantified noun can be represented by a variable, b) knowledge that the expression "is equal to" can be represented by the equal sign, and c) knowledge about various operational words: *plus*, *minus*, *times*, and *divided by*, that can be represented by operational symbols: +, −, ×, and ÷. The student who understands symbol reference can now translate POM sentences like [the train's time] [is equal to] [two] [times] [the plane's time] as \( T = 2P \).

3. **Adjustments.** The third component is knowledge about lexical and phrase-level adjustments. These adjustments involve expressing the verb "to be" as "is equal to." Adjustments may possibly involve rearranging order within a phrase (e.g., "six less than a number" becomes "a number less six." The expressions "n more than," "n greater than," "n increased by," become "plus." "n less than," "n decreased by," "n diminished by," or "n younger than," become "minus." Expressions
such as "twice" or "k times as many," or "1/nth of," become "two times," or "k times," or "1/nth times." "The quotient of," "the ratio of," or "a over b," become "divided by," and any form of the natural verb "to be" is written "is equal to". "Exceeds [something] by k" becomes "is equal to [something] plus k").

4. **Transformations.** If the noun phrases that are identified as the variables are not quantified, a search must be made for a reference to quantity. If a verb phrase makes reference to being older than, or going faster than, or being longer than, or weighing more than, or being warmer than, the student must distribute this to the noun by quantifying the noun to include age, speed, time, distance, length, volume, weight or temperature. Sometimes the reference is simply to a number of things or to having a number of things, and the nouns must be represented as a number of things that are had or used in some capacity (e.g., things that are eaten, sold, collected, available, bought, had, etc.).

If transformations on the sentence do not produce a POM sentence, the syntactic process is abandoned. The student then uses the last two components of knowledge.

5. **Conceptual Analysis.** The fifth component, conceptual analysis, is only needed if the translator abandons the syntactic process. For a large majority of sentences (POM, WPA, and WST), this analysis and the
subsequent employment of a conceptual strategy are unnecessary. The conceptual analysis involves determining the relationship between the two variables in the NPOM sentence. Wollman (1983) found that students do not have difficulty understanding the meaning of the sentence. They can determine from the NPOM sentence that one of the numbers is larger than the other. In the second example: "There are five times as many nickels as dimes," it is obvious after a conceptual analysis, that there are more nickels than dimes. This leads to the last component.

6. **Conceptual Strategies.** The sixth component involves the use of a conceptual strategy and is employed whenever the sentence is NPOM. In this case the translator has made a conceptual analysis and determined which of the variables is less than the other. I have identified three strategies that could be employed in such a case:

1) **Ratio-Proportion:** Since a ratio of the two variables can be determined from the conceptual analysis, a proportion is set up and the principle of proportion is used to make the translation. This can only be done in a multiplicative case.

2) **Compensation:** The two variables are placed on either side of the equal sign and the variable that has been determined as the smaller quantity is compensated
with multiplication or addition, to create an equality with the larger quantity.

3) **Interrogative method:** The sentence is queried for information about a variable. This method is used to lift one of the quantified nouns (say, N) from the sentence, by asking "How many N's are there." The question "how many" quantifies N, so the answer to the question is a quantified noun. The answer to the question will be found in the altered sentence. This method does not seem to work in the Mindy's type.

Here is an example of how each of these strategies would be used on the sentence "there are five times as many nickels as dimes."

**Ratio-Proportion:** \( \frac{N}{5} = \frac{D}{1} \) which leads to \( N = 5D \).

**Compensation:** Since D is less than N, D must be compensated by a multiplicative increase.

This leads to \( N = 5D \).

**Interrogative method:** Lift the word "nickels" from the sentence and ask "How many nickels are there"? The altered sentence is "there are five times as many as dimes." This leads to \( N = 5D \).

Here is an example of how each strategy would be used on an additive sentence of the S & P type. "Central High has five hundred more freshmen than seniors."

**Ratio-Proportion:** Not applicable in additive cases.

**Compensation:** Since S is less than F, S must be
compensated by an additive increase.

This leads to \( F = S + 500 \).

**Interrogative method**: Lift the word "freshmen" from the sentence and ask "How many freshmen are there"? The altered sentence is "Central High has five hundred more than seniors."

This leads to "the number of freshmen is five hundred more than the number of seniors," which is translated as \( F = S + 500 \).

It is a curious feature of the interrogative method, that, though conceptual in nature, it can sometimes be employed non-reflectively. Particularly for additive sentences, this can result in a sense of syntactic transforming.

**The Translation Process Illustrated**

Since the translation process outlined in Figure 4 is a hypothesis about how a competent translator operates, four sentences will be illustrated as they pass through the process. Figure 5 illustrates the sentence: The number of cats is three less than the number of dogs; Figure 6 illustrates: The boat is twice as long as the dock; Figure 7 illustrates: There are six times as many students as professors; and Figure 8 illustrates the sentence: Three forks are used for every four spoons.
The sentence is read and the variables are identified as the number of cats and the number of dogs.

[The number of cats] [is three less than the number of dogs].

Since the noun phrases are already quantified the lexical adjustments and substitutions are made.

[The number of cats] [is equal to] [the number of dogs] [minus] [three].

Since the sentence is POM it is translated.

Translation: \( C = D - 3 \).

**Figure 5.** Translation Process for the WPA sentence: The number of cats is three less than the number of dogs.
The sentence is read and the variables are identified as the length of the boat and the length of the dock.

[The boat] [is twice as long as the dock].

Since the noun phrases are not quantified, length is distributed to the noun phrases.

[The length of the boat] [is] [twice] [the length of the dock].

The lexical adjustments are made.

[The length of the boat] [is equal to] [two] [times] [the length of the dock].

Since the sentence is POM it is translated.

Translation:  B = 2D

Figure 6. Translation Process for the WST sentence: The boat is twice as long as the dock.
The sentence is read and the variables are identified as the number of students and the number of professors.

There are six times as many students as professors.

Since the noun phrases are not quantified, "the number of" is distributed to the noun phrases.

[There are] [six] [times] [the number of students] [as] [the number of professors].

The lexical adjustments are made.

[There is equal to] [six] [times] [the number of students] [as] [the number of professors].

Since the two quantified noun phrases are both on the same side of the equal sign, the sentence is NPOM and the syntactic process is abandoned.

A conceptual analysis indicates that there are more students than professors, so by the compensation strategy the number of professors must be multiplied by six.

Or a proportion is set up: $S/6 = P/1$.

Or we ask how many students are there, and the sentence indicates six times as many as professors.

Translation: $S = 6P$.

**Figure 7.** Translation Process for the NPOM sentence: There are six times as many students as professors.
The sentence is read and the variables are identified as the number of forks and the number of spoons.

Three [forks] are used for every four [spoons].

Since the noun phrases are not quantified, "the number used" is distributed to the noun phrases.

[Three] [the number of forks used] [is equal to] [four] [the number of spoons used].

The two quantified noun phrases are on either side of the equal sign, but there is no operational word between three and the number of forks or between four and the number of spoons, so the adjustments cannot be performed and the syntactic process is abandoned.

A conceptual analysis indicates that there are more spoons than forks, so the number of forks must be multiplied by the larger number.

Or a proportion is set up: \( \frac{3}{F} = \frac{4}{S} \)

Either strategy produces:

Translation: \( 3S = 4F \).

Figure 8. Translation Process for the NPOM sentence:
Three forks are used for every four spoons.
Consistency of the Syntactic Theory

with Previous Data

The above theory is concerned with the translation processes for all four sentence types, and can be applied to analyze the observations that Clement (1982) made of translation of NPOM sentences. According to this Syntactic Theory, the Type I student who does "word order matching" (recall page 24), failed to recognize the need for transformations and simply translated words and phrases as they occurred in sequence. This points to deficiencies in the first four components of the model.

The Type II student who does a static comparison, either: (a) has a deficiency in the use of conceptual strategies and chooses the faulty conceptual strategy of putting the large number (six) by the larger group (students), indicating a deficiency in knowledge of conceptual strategies (component 6), or, (b) lets S stand for students rather than the number of students, and also behaves like the Type I student in verifying his results with the sequence of words in the sentence. This points to poor variable identification (component 1), a poor reference system for symbols and quantities (component 2 and 3), and to a deficiency in knowledge of conceptual strategies (component 6). The type III student possesses all of the necessary knowledge components.
Summary

The Syntactic Theory of translation skills proposes several components of knowledge that are needed in translating a natural language sentence into an algebraic equation.

If students can be taught the relation between the syntax of a natural sentence and the syntax of an algebraic equation, then they may be able to learn the process outlined in the Syntactic Theory. This process will involve a decision to abandon syntactic methods for sentences of the S & P or Mindy's type, which are unacceptable for direct translation into algebraic language because they do not yield to the syntactic methods.

As outlined in this chapter, the Syntactic Theory stands in direct contrast to the Cognitivist Theory which suggests that a conceptual analysis and subsequent conceptual strategies should suffice for competent translation of a sentence. These two theories were embedded in instructional strategies and used to teach different treatment groups. The design and methodology used in the treatments are found in Chapter 4.
CHAPTER FOUR  
METHODOLOGY

This chapter contains the purpose, hypothesis, design, subjects, instruments, and the classroom procedures of the study.

Purpose of the Study

In Chapter 3, I proposed a theory of competence in algebraic translation involving six components of knowledge, including some related to syntactic knowledge. This theory conflicts with previous theoretical frameworks in which conceptual knowledge alone is seen as the foundation for competence. More specifically, the question to be researched and analyzed is:

Will a syntactic instructional treatment produce fewer instances of the reversal error than a conceptual instructional treatment?

Hypothesis

The hypothesis to be tested in this study is:
An instructional treatment designed on the Syntactic Theory will be more effective in promoting competence in algebraic sentence translation than an instructional treatment designed on the Cognitivist Theory.
Design of the Study

This study involved a teaching experiment with three experimental treatment groups (SYN-1, SYN-2, and CONCEP) and two control treatment groups (CONT-1 and CONT-2). A description of the treatments given to the groups is given in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Description of the Treatments</th>
</tr>
</thead>
<tbody>
<tr>
<td>SYN-1 Syntactic treatment - translation process based on Syntactic Theory</td>
</tr>
<tr>
<td>SYN-2 Syntactic treatment - more emphasis on the decision to abandon syntactic processes</td>
</tr>
<tr>
<td>CONCEP Conceptual treatment - translation based on Cognitivist Theory</td>
</tr>
<tr>
<td>CONT-1 Problem solving with no emphasis on translation</td>
</tr>
<tr>
<td>CONT-2 No problem solving at all</td>
</tr>
</tbody>
</table>

A pretest was administered on the first day of class. The researcher then presented an instructional treatment of fifty minutes for three consecutive regular class days. Instruction consisted of a combination of lecture and discussion and is on audio-tape. A graduate psychology student was an observer for detecting bias. (See Appendix H for observer notes). The first two groups (SYN-1 and SYN-2) received a treatment (see Procedures below) based on the Syntactic Theory. The third group (CONCEP) received a treatment based on the Cognitivist Theory. A posttest was given one week after
the treatment and a retention test was given twelve weeks after the posttest. (See the tests in Appendices E, F, and G).

The first control group (CONT-1), had a pretest. Three days of regular textbook problem solving was conducted. Instruction consisted of the teacher lecturing and the students working in groups with some assistance from the teacher. A posttest was given one week after the treatment and a retention test was given twelve weeks after the posttest.

The second control group (CONT-2), had no pretest. The instructor followed the required syllabus for the course with no instruction in problem solving. A posttest was given at the same time it was given to the other groups and a retention test was given twelve weeks after the retention test. This group was used to see if the pretest had an effect on CONT-1.

Subjects

The reversal error was first detected in a group of college freshman engineering students (Kaput & Clement, 1979), but it has also been reported across many other grade levels (Hasty, 1987; Lochhead, 1980; Schrader, 1985; Seeger, 1990; Wollman, 1983). Since students in an intermediate algebra class could translate POM, WPA, and WST sentences, I used the next level of college algebra
students so that I would not have to teach them all of the components of knowledge needed for translation.

Five regular college algebra classes (250 students) enrolled at Southeastern Louisiana University were used in this study. A pre-requisite for the course is either two years of high school algebra or two years of intermediate algebra at the post-secondary level. Four of the classes were used in the fall semester and randomly assigned to be SYN-1, CONCEP, CONT-1, and CONT-2. One class in the spring semester was assigned to be SYN-2. A profile of the classes is given in Table 2.

Table 2
Descriptive Statistics Based on Pre-Assessments

<table>
<thead>
<tr>
<th>Group</th>
<th>SYN-1</th>
<th>SYN-2</th>
<th>CONCEP</th>
<th>CONT-1</th>
<th>CONT-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean gender</td>
<td>1.689</td>
<td>1.444</td>
<td>1.571</td>
<td>1.700</td>
<td>1.576</td>
</tr>
<tr>
<td>Mean ACT(Math)</td>
<td>17.333</td>
<td>18.667</td>
<td>20.029</td>
<td>17.967</td>
<td>16.939</td>
</tr>
<tr>
<td>Number of years in algebra</td>
<td>2.444</td>
<td>2.694</td>
<td>2.343</td>
<td>2.600</td>
<td>2.061</td>
</tr>
<tr>
<td>Proportional reasoning</td>
<td>1.822</td>
<td>1.833</td>
<td>2.114</td>
<td>1.867</td>
<td>-</td>
</tr>
<tr>
<td>Language</td>
<td>1.022</td>
<td>1.028</td>
<td>1.057</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Note. SYN-1 n=45; SYN-2 n=36 CONCEP n=35; CONT-1 n=30; C-2 n=33

*Male = 1  Female = 2

bNumber of correct proportion problems out of three.

'Native language English = 1  Other = 2
Students who were failing the class at mid-term withdrew from the classes, so they did not take the retention test. Seventy percent of the original students finished the study. This was a drop from 250 students to 179 students. This parallels average drop rates for all college algebra classes at this university. During the fall semester 49% of all students enrolled in college algebra either withdrew or failed the course.

Analysis of these data shows that there was no significant difference in age, gender, number of years studying algebra, proportional reasoning skills, or in native language between any of the groups. There was a significant difference in ACT score for group CONCEP over SYN-1 AND CONT-2, but when it was determined that CONCEP did not score as high as SYN-1 on the posttest, and CONT-2 was not used in the final analysis of data, it was decided that ignoring such differences would not reduce the explanatory power of the study.

Description of the Instruments

The development of the pretest, posttest, and retention test was based on my classification of sentence types. The tests have questions similar to those on the test used by Clement (1982), but they also include other types so that translation skills on all types of sentences could be investigated. Each test had one POM
sentence, one sentence that was POM but of complex syntax, one WST, one S & P type, one S & P type that is additive, and one Mindy's type (see p. 7). On the pretest (Appendix E) these sentences were included in an assessment that asked the students for background data and also tested for proportional reasoning skills (See Chapter 6 for a discussion). On the posttest (Appendix F) these sentences were included on a regular semester test. On the retention test (Appendix G) these sentences were included with a list of other sentences to be translated. The validity of the tests did not seem to be problematic in this study.

Reliability on the tests was determined using a Kuder-Richardson method of rational equivalence to determine a reliability coefficient "between one experimental form of a test and a hypothetically equivalent form" (Richard & Kuder, 1939, p. 681). Formula 21 (p. 682) was used because it tends to underestimate the reliability coefficient. The calculated reliability coefficient was .797.

Procedures

The First Syntactic Treatment

The goal of the syntactic instructional program was to produce students who are competent translators according to the Syntactic Theory. This section contains
the treatment given to the first syntactic treatment group (SYN-1). A modification of this treatment that was used with group SYN-2 will be explained in the next section.

Since the syntactic treatment aimed at mechanical aspects of translation, the processes were spelled out in detail. The students were taught a) to recognize and translate POM sentences, b) to understand correspondences between symbols and mathematical ideas that are necessary for direct translation, c) to transform sentences so that the noun phrases are quantified, and d) to perform a conceptual strategy when an analysis leads in that direction (See Figure 2 in Chapter 3 for an outline of the syntactic process).

A description of the treatment for the three days is given here. (See Appendix A for the complete lesson plans for the syntactic treatment.)

Day One

The first day of treatment laid the ground work for translation that should have been covered in any elementary algebra class.

In a brief introduction to problem solving, the students were told that problem solving involves translating, manipulating and checking, but that this session would concentrate on translating English sentences into algebraic equations. A class discussion
led to an agreement that an equation would be referred to as \([\text{an algebraic expression}] = [\text{an algebraic expression}]\). Students also agreed that an algebraic expression is composed of numbers and variables connected by operational symbols. Since an English sentence has to be translated into one of these equations, the English sentence would have to have the specific form \([\text{a phrase about quantities or numbers}] \text{ is equal to } [\text{a phrase about quantities or numbers}]\).

The students had an opportunity to suggest various algebraic expressions and equations to ensure that they had the right idea. They were told that in the simplest case the sentence is ready to be translated piece by piece and that such a sentence was to be called phrase order matched. The students were guided through several simple POM sentences.

After the students practiced these basic concepts, they were given the example "One number is equal to five times a second number plus eight" which is ambiguously translated as \(x = 5(y + 8)\) or \(x = 5y + 8\). They could see the necessity for rewriting this sentence so that it was not ambiguous. This lead to phrases which are not POM but which can be adjusted to POM, (e.g., six less than a number) The students were then given a long list of English phrases and adjustments that make them POM. They spent some time doing adjustments and translations on POM
and WPA phrases.

At the end of the first class, the students were told that in the translation process one of four things can happen: (a) the sentence is already POM and it can be translated, (b) the sentence is almost POM but it needs some adjustments before it can be translated, (c) the sentence needs to be completely rephrased before it can be translated, and (d) the sentence needs a totally different strategy for translation.

Day Two

On the second day the students were told that in some sentences the noun phrase does not state the quantity. The sentence "John is four inches taller than Mary" was put on the screen and the following dialogue ensued:

(R = Researcher, S = Students)

(Transcription Excerpts)

1 R: What quantities is this sentence about?
2 S: Height.
3 R: What variables would you like to use to represent height?
4 S: J = John's height, M = Mary's height.
5 R: What was it about the sentence that told you that it was about height?
6 S: Is taller than.
7 R: So this sentence has to be transformed so that the
noun phrases are about quantities. We rephrase the sentence, or we distribute height to John and to Mary so that the rephrased sentence is "John's height is four inches more than Mary's height."

After this sentence was recognized as WPA, the adjustments were made and the sentence was translated.

In the next example, "The train took twice as long as the plane," there was discussion about whether took twice as long as referred to time or speed. One student asked if it was necessary to rewrite the sentence each time, and it was suggested that the rephrasing can be done mentally. The following sequence was stressed as the students tried to translate several WST sentences: (a) what quantities is the sentence about?, (b) Are the nouns expressed as quantities?, (c) What variables would you like to use?, and (d) distribute the quantity to the nouns.

The students were given an NPOM sentence "I have twice as many shirts as pants" and the following discussion ensued:

(R = Researcher, S = Student)

(Transcript Excerpts)

1 R: What quantities is this sentence about?
2 S: Number of shirts and pants.
3 R: If we distribute "the number of" to the two nouns we have "I have twice as many the number of shirts
as the number of pants." Do we have [a phrase about numbers] is equal to [a phrase about numbers]?

If you think so, then write the equation.

4 S: The number of pants is equal to two times the number of shirts.

5 R: Where is the verb in the sentence?

See, I'm not asking if you can make it work, I'm asking "Is this sentence ready to translate?" "As" is not "is equal to." The sentence is not POM...This sentence is a troubling type of sentence that cannot be translated piece by piece, so we need a new strategy. Probably ninety percent of the sentences you will see will be easily translated. But every once in a while you will come across a sentence like this. You will need a totally different strategy because you tried to get it into this POM form and it doesn't work.

Day Three

On the third day of the treatment the students were reminded again of the translation process. If the sentence is already POM, then translate it. If the sentence is almost POM, make the adjustments and then translate it. If the phrases are not about quantities, determine what the quantities are and rephrase the
sentence so that it is POM. If it is still not POM, then choose a new strategy. The rest of the day was spent learning two conceptual strategies.

The strategy of compensation was explained and an example given in the following transcript:

(R = Researcher, S = Student)

(Transcript Excerpts)

1 R: Look at the sentence "Central High has three times as many freshmen as seniors." Can you tell which of the quantities is larger?

2 S: Yes. (All agreed it was the number of freshmen.)

3 R: So you put the two variables on either side of an equal sign and compensate the smaller. Which is smaller?

4 S: Seniors.

5 R: So what do you do?

6 S: Multiply S by three.

In the sentence "Five more bushes were seen than saplings" the students had to determine when to abandon the POMing process. They wanted to add five to the number of bushes, but they were reminded that the sentence does not say five more than the number of bushes. The proper translation for the addition sign is "more than" as in "five more than a number". In this case the adjustment cannot be made and the process of POMing must be abandoned.
Instruction on the proportion strategy involved defining a proportion, setting up a proportion, talking about the principle of proportions, and reminding the students that they could only use a proportion in a multiplicative relationship, never in an additive relationship.

The Second Syntactic Treatment

For the second treatment group (SYN-2), there was a modification of the treatment used with (SYN-1). The first day of treatment was identical in the two groups. The second day the first group saw two NPOM sentences, but the decision to abandon the POMing process was vague and not stressed. The decision process was stressed even more with this second syntactic treatment. The two reasons for abandoning POMing were (1) not finding a POM sentence, and (2) the inability to make an adjustment. On the third day for the SYN-1 group it had been presumed that the students would know when to abandon POMing, and they were quickly led into the conceptual strategies to be used on NPOM sentences. With the SYN-2 group the entire decision making process was reiterated in examples before learning the conceptual strategies.

The Conceptual Instructional Treatment

The goal of the conceptual instruction treatment was to produce students who are competent translators.
according to the Cognitivist Theory.

The instructional program for the CONCEP group is based on the one used by Rosnick and Clement (1980), but contained more instruction on the meaning of variable, the meaning of an equation, and on conceptual strategies.

The students were taught to use three steps in the translation process: a) to do a conceptual analysis to understand the natural language sentence; b) to make a table of data that fits the situation; and c) to write the equation after doing some conceptual strategy. These students heard nothing about syntactic analyses or transformations. A description of the treatment for the three days is given here. (See Appendix B for the complete lesson plans for the cognitive treatment.)

Day One

After a brief introduction to problem solving, the students were told that a situation that arises in the real world often presents a problem. If the problem situation is stated in words, then it is necessary to translate the words into algebra before the problem can be solved. Since the quantities in the problem are represented by variables, a thorough development of the concept of variable was conducted.

The situation of a static variable was presented. The idea that a variable can only stand for a quantity or a number of things was stressed, as indicated in the
following transcript. (R = Researcher, S = Student)

(Transcript Excerpts)

1 R: If I said "I have twice as many quarters as nickels?" is this sentence about quantities?
2 S: Yes.
3 R: What kinds? What does Q stand for?
4 S: Quarters and nickels.
5 R: No. Q has to stand for a quantity. One of the mistakes we make is to let the variable stand for things (like quarters), but Q stands for the number of quarters and N stands for the number of nickels. If you think of nothing else, think that the variable stands for a number of things.

After the student discussed several examples and realized that they must determine what letter to use and what quantity it will represent, some examples were done with variables that are not static. A situation concerning a luxury liner operation was sketched with the variables P for the number of passengers, X for the number of pounds of potatoes, and G for the number of pounds of garbage produced in a day. There was a group discussion about the relationships between the variables and suggestions were given for sentences relating two or more of the variables. Every time the students talked about a variable as a thing, they were reminded that a
variable represents a number of that thing.

The problem of painting a fence was suggested to the group. The students were to represent the situation using variables, and they suggested the variables X for the number of square feet of fencing, Y for the number of gallons of paint, W for the number of workers, B for the number of brushes, and T for the number of hours to complete the job. After a discussion about the variable choices, the class was reminded that when a problem situation is presented you must ask what quantities the problem is about and how each of them will be represented. The class broke into small groups and each group was to find a unique situation with three or more variables. Some of the examples discussed were producing a rock concert, taking a trip to Florida, and carpeting a room.

After the students constructed variables, they were reminded that variables and numerals are connected in equations with various mathematical symbols. The English phrases that represent the basic operations were reviewed and the students seemed to have little trouble with these.

Day Two

On the second day of treatment the students were presented with an equation \( B = 3G \) and asked for a real situation to interpret it. When one student suggested
"balls equals three times the number of players", they were reminded that only numbers or expressions about numbers or quantities can be represented by the variables in the equation. The student then suggested "the number of balls equals three times the number of players."

The students were reminded that the current task was to find a strategy to translate sentences. The strategy would involve taking the sentence as it is presented in written form and trying to represent it in a table of data. The students were instructed to put their pencils down so they would not be tempted to follow the order of the words in the sentence. The sentence "Joe ate four more cookies than donuts" was put on the overhead projector. After the students completed the table of data, the sentence was removed from the screen and only the table of data was visible during the translation process. The dialogue in the next transcript indicates what happened.

(R = Researcher, S = Student)

(Transcript Excerpts)

1  R: What are the quantities we are talking about here?

2  S: Cookies and donuts.

3  R: No. We are not talking about cookies and donuts. We are talking about a number of cookies and a number of donuts. I'm being picky because that's
the reason we have difficulty translating. We try to translate words into words instead of words into numbers. Now what are two numbers that fit this situation?

4 S: One donut and five cookies.

5 R: O.K. We're going to put that into a table. Another example?

6 S: Six cookies and two donuts.

7 R: (Filling in the table under the columns D and C) You are building a table of data. One more example?

8 S: Three donuts and seven cookies.

9 R: So if the number D is one, the number C is five; if the number D is 2, the number C is six; if the number D is three, the number C is seven. Can you write an equation using these variables to represent this data? What does D represent?

10 S: The number of donuts.

11 R: And what does C represent?

12 S: The number of cookies.

13 R: So can someone volunteer a sentence that relates these numbers?

14 S: D = C + 4

15 R: Is that what you meant to say?

16 S: No, the number of cookies is the number of donuts plus four.
17 R: Now, can you say that in algebra?

18 S: C = 4 + D

19 R: Now, you just translated the sentence without really looking at the words of the sentence. Sometimes people are distracted by the words of the sentence and what they do is let the words lead them the wrong way, but you weren't looking at the words when you actually made the sentence. You were looking at the data just like the scientist would do. And the scientist is the one who knows how to write algebra. And we want you to know how to write algebra.

As is seen in the transcript (14) the students still made the reversal error when translating from the table. They were told that it is common to make this error, but that they should continue to ask what is the relation between the two quantities in the table. This type of discussion and translation from a table of data continued for several other examples.

For the example "Southeastern enrolled ten times as many freshmen as seniors" the students made the correct table of data, but suggested all of these equations: \( F = 10S, \ 10F = S, \) and \( S = F + 10. \) The majority agreed with the correct sentence, \( F = 10S. \)

The students had some difficulty with the Mindy's type problem "We have three spoons for every two forks."
The table of data was made correctly, but they had some difficulty with the sentence. They finally agreed on $F = (2/3)S$.

The class ended with the students practicing several examples on their own.

**Day Three**

On the third day several students said that they got the variables reversed almost every time. It was suggested that they consider the variable as a **number** of things. It was also suggested that one of the strategies that would be taught on this day might help.

The compensation strategy was introduced by drawing a balance and explaining that if two different quantities are on either side of the balance, something must be done to achieve a balance. Compensation is the strategy of increasing the smaller quantity by adding or multiplying it by a positive number. The strategy is described in the next transcript. (R = Researcher, S = Student)

(Transcript Excerpts)

1 R: Let's try "Joe spends four times as much on rent as he does on food." What are the variables?

2 S: R and F

3 R: Which is the larger number?

4 S: R

5 R: So what do you do? Right, you compensate the side
with F and get R = 4F. So we read "the number he spends on rent is four times the number he spends on food". Do you see the relation between this and the table of data? Which one is easier?

6 S: This one.

A majority of the students seemed to like compensation better than the table of data, and they said no one had ever shown them compensation before. In the Mindy's type example "Three dollars were collected for every two quarters" the students had difficulty again because of the two numbers and the fractional relationship they created. This led to a discussion of the proportion strategy.

The students were told that for any problem they could use the table of data, compensation, or a proportion. The proportion was defined and it was emphasized that this method works only if the problem is multiplicative. For the problem "Three dollars were collected for every two quarters" the students set up the proportion D/Q = 3/2 and compared this equation with the one they made using the compensation strategy. When asked which strategy they liked best, over half of the students said compensation. They were reminded again that compensation works most of the time, but that in a multiplicative case proportion can be better. This class
ended with the students translating several sentences. They noted which strategy they used each time.

**Instruction for the Control Group**

The control group (CONT-1) lessons during the three days were designed to teach problem solving as it is done in textbooks, with no special emphasis on translation. The three days are outlined below. (See Appendix D for the complete lesson plans for the control group.)

**Day One**

The students were given an overview of problem solving. They were told that after reading the problem they should pick a variable and then write the problem in terms of that variable. The first day was spent on number, age and coin problems. The instructor worked one of each type on the board and then the students worked alone or in groups while the instructor circulated around the room giving assistance where needed.

**Day Two**

On the second day the students were told that some problems require outside information, as in formulas. A review of the formulas for area, perimeter, angle sums, and complementary and supplementary angles were reviewed. The instructor worked one of each type on the board and then the students worked alone or in groups while the instructor circulated around the room giving assistance.
where needed.

Day Three

On the third day the students were given formulas for interest and distance. The instructor worked one of each type on the board and then the students worked alone or in groups while the instructor circulated around the room giving assistance where needed.

Scoring

All items on the instruments were objective, so there was little danger of interference from subjective interpretation. All tests were scored by three independent graders to reduce the possibility of bias. Since each item was either correct or incorrect, the third grader looked for discrepancies between the other two and found twenty discrepancies out of over three thousand items. This gave an inter-grader reliability rating greater than 99%.

Method of Analysis

The hypothesis was tested by computing an analysis of variance for repeated measures on the mean scores on the pretest, posttest, and retention test. Tables and results of all of the analyses are given in Chapter 5.
CHAPTER FIVE
ANALYSIS OF THE DATA

The purpose of this study was to compare syntactic and conceptual skills in translating natural language sentences into algebraic notation. The major research question examined was:

Will a predominantly syntactic instructional treatment produce fewer instances of sentence translation errors than a purely conceptual instructional treatment?

This chapter provides an examination of the descriptive statistics and tests used to answer this question. The analyses, results, and other related questions follow. A summary of the results concludes the chapter.

Disposition of One Control Group

Since CONT-2 was only used to see if the pretest had an effect on CONT-1, a simple t-test was used to detect if there was a significant difference in the scores of the two control groups on the posttest. The results of the posttest for the control groups are given in Table 3.
Table 3

**Mean scores of the Control Groups for the Posttest**

<table>
<thead>
<tr>
<th>Group</th>
<th>CONT-1</th>
<th>CONT-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>.433</td>
<td>.359</td>
</tr>
</tbody>
</table>

*Note.* CONT-1 n=30; CONT-2 n=33

The results of the t-test on the posttest scores for the two control groups can be found in Table 4.

Table 4

**t-test Results of the Control Groups for the Posttest**

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>Sum of Square</th>
<th>Mean Square</th>
<th>F Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>1</td>
<td>.0878</td>
<td>.0878</td>
<td>.9206</td>
</tr>
<tr>
<td>Subject within Treatment</td>
<td>61</td>
<td>5.8178</td>
<td>.0954</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>62</td>
<td>5.9056</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since the t-test showed no significant difference between the posttest scores given within one week of the treatment to the two control groups, it was determined that the pretest did not affect CONT-1, so CONT-2 was not used in the analysis of the data after this point.

**Pretest Results**

An item-by-item analysis of the students' equations on the pretest was performed because reversal errors on problems other than the S & P and Mindy's type have not
previously been reported in the literature. The analysis of items on the pretest showed that, not only did these students make the reversal error on S & P and Mindy's problems, but they made reversals on POM sentences as well. An analysis of some errors made on the pretest is given in Table 5. For the POM sentence 63% of the errors were reversals, for the S & P sentence, 66% of the errors were reversals, and for the Mindy's problem 47% of the errors were reversals. The rate of failure on the POM sentences was higher than expected. The rate of reversals on the S & P sentence agreed with the rate of 68% noted by Clement (1982, p. 17), but the rate on the Mindy's problem does not agree with the 68% noted by Clement (1982, p. 17). This discrepancy is probably due to the fact that Clement's population was freshman engineering majors, while this population was freshman from a normal college algebra class that contained students from across the curriculum. The typical error made on the NPOM sentences was not even an equation (e.g., 6S + P or 6S - P).
Table 5

**Performance of Four Groups on Three Items of the PreTest**

<table>
<thead>
<tr>
<th>Questions</th>
<th>Answer</th>
<th>% Correct</th>
<th>% Reversals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. In America, the number of cats is twice the number of dogs. Let C stand for the number of cats and D for the number of dogs.</td>
<td>C = 2D</td>
<td>54</td>
<td>63</td>
</tr>
<tr>
<td>2. At this college, there are six times as many students as professors. Let S stand for the number of students and P for the number of professors.</td>
<td>S = 6P</td>
<td>48</td>
<td>66</td>
</tr>
<tr>
<td>3. At Mindy’s restaurant, for every four people who ordered cheesecake there were five who ordered strudel. Let C stand for the number of orders of cheesecake and S for the number of orders of strudel.</td>
<td>5C = 4S</td>
<td>10</td>
<td>47</td>
</tr>
</tbody>
</table>

**Note.** n=146

* Percent of the incorrect that were reversals.

**Analysis and Results**

The main research question asked if there was more achievement on translation tasks for students receiving the syntactic treatment than for students receiving the conceptual treatment. This question was answered by doing an analysis of variance for repeated measures. The analysis uses a 4 x 3 mixed design with four groups:
SYN-1, SYN-2, CONCEP, and CONT-1 over three times: pretest, posttest, and retention test.

The twelve cell means for the four groups on the three tests are given in Table 6.

### Table 6

<table>
<thead>
<tr>
<th>Group</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Retention</th>
</tr>
</thead>
<tbody>
<tr>
<td>SYN-1</td>
<td>.374</td>
<td>.896</td>
<td>.896</td>
</tr>
<tr>
<td>SYN-2</td>
<td>.384</td>
<td>.926</td>
<td>.894</td>
</tr>
<tr>
<td>CONCEP</td>
<td>.319</td>
<td>.781</td>
<td>.652</td>
</tr>
<tr>
<td>CONT-1</td>
<td>.494</td>
<td>.433</td>
<td>.594</td>
</tr>
</tbody>
</table>

**Note.** SYN-1 n=45; SYN-2 N=36; CONCEP n=35; CONT-1 n=30;

The graph of the twelve cell means for all groups is given in Figure 9.

![Graph of the twelve cell means](image-url)
The graph indicates that there is some interaction between the groups and the times of the tests, so the expanded summary ANOVA with tests for simple effects is given in Table 7.

Table 7

Expanded Summary ANOVA Including Tests of Simple Effects

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rows (Treatment)</td>
<td>3.76</td>
<td>3</td>
<td>1.25</td>
<td>13.88*</td>
</tr>
<tr>
<td>Rows (pre)</td>
<td>.52</td>
<td>3</td>
<td>.17</td>
<td>1.88</td>
</tr>
<tr>
<td>Rows (post)</td>
<td>4.98</td>
<td>3</td>
<td>1.66</td>
<td>18.44†</td>
</tr>
<tr>
<td>Rows (ret)</td>
<td>2.67</td>
<td>3</td>
<td>.89</td>
<td>9.88†</td>
</tr>
<tr>
<td>Within cell</td>
<td>13.49</td>
<td>142</td>
<td>.09</td>
<td></td>
</tr>
<tr>
<td>Columns (Test)</td>
<td>12.78</td>
<td>2</td>
<td>6.39</td>
<td>159.47*</td>
</tr>
<tr>
<td>Columns (SYN-1)</td>
<td>8.18</td>
<td>2</td>
<td>4.09</td>
<td>102.25†</td>
</tr>
<tr>
<td>Columns (SYN-2)</td>
<td>6.66</td>
<td>2</td>
<td>3.33</td>
<td>83.25†</td>
</tr>
<tr>
<td>Columns (CONCEP)</td>
<td>3.98</td>
<td>2</td>
<td>1.99</td>
<td>49.75†</td>
</tr>
<tr>
<td>Columns (CONT-1)</td>
<td>.40</td>
<td>2</td>
<td>.20</td>
<td>5.00</td>
</tr>
<tr>
<td>Interaction</td>
<td>4.41</td>
<td>6</td>
<td>.73</td>
<td>18.25†</td>
</tr>
<tr>
<td>Within cell</td>
<td>11.98</td>
<td>284</td>
<td>.04</td>
<td></td>
</tr>
</tbody>
</table>

Note. *Significant at .05 level
†Significant at .05/3 level
‡Significant at .05/4 level

The results of the analysis of the simple effects for the rows at each column indicate that, between the groups there was a difference in mean scores at different times of the tests. There was no significant difference between any two groups on the pretest, but there was a significant difference between groups on the posttest and
on the retention test.

Since the ANOVA showed a significant difference in the posttest scores, a Tukey post hoc multiple comparison test was run to determine which groups showed a difference. The Tukey comparisons in Table 8 give a Q score. The Q scores show that CONT-1 differs significantly from all of the treatment groups, and CONCEP differs significantly from SYN-1 and SYN-2. There is no significant difference between the syntactic treatment groups and the conceptual group when the pretest is used as a covariate. But the p-value (.056) is marginal in that case.

Table 8

Tukey's Pairwise Comparisons for Mean Number of Correct Responses on Posttest for all Groups

<table>
<thead>
<tr>
<th>Mean</th>
<th>Group</th>
<th>CONT-1</th>
<th>CONCEP</th>
<th>SYN-1</th>
<th>SYN-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>.433</td>
<td>CONT-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.781</td>
<td>CONCEP</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.896</td>
<td>SYN-1</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.926</td>
<td>SYN-2</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Difference is significant at .05/3 level

The results of the Tukey post hoc comparisons for the retention test are given in Table 9. They show that CONT-1 and CONCEP differ significantly from the two syntactic treatment groups SYN-1 and SYN-2.
Table 9

**Tukey's Pairwise Comparisons for Mean Number of Correct Responses on Retention test for all Groups**

<table>
<thead>
<tr>
<th>Mean</th>
<th>Group</th>
<th>CONT-1</th>
<th>CONCEP</th>
<th>SYN-1</th>
<th>SYN-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>.594</td>
<td>CONT-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.652</td>
<td>CONCEP</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.896</td>
<td>SYN-1</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.894</td>
<td>SYN-2</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Difference is significant at .05/3 level

The results of the analysis of the simple effects for the columns at each row indicate that within treatment groups there was a difference in gain or loss on mean scores across the three test times. There was no significant difference in gains on mean scores for the control group across time.

Since the ANOVA showed a significant difference in gains for all treatment groups, a Tukey post hoc multiple comparison test was run to determine where were the significant gains. The Tukey comparisons in Table 10 show significant gains for SYN-1 and SYN-2 between the pretest and the posttest and between the pretest and the retention test, with no significant drop between the posttest and the retention test.
Table 10

Tukey's Pairwise Comparisons for Mean Gain on Tests Groups (SYN-1 and SYN-2)

<table>
<thead>
<tr>
<th>Test</th>
<th>Pre</th>
<th>Post</th>
<th>Retention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Post</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retention</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Difference is significant at .05/4 level

The Tukey comparisons in Table 11 show significant gains for CONCEP between the pretest and the posttest and between the pretest and the retention test, but there is also a significant drop between the posttest and the retention test.

Table 11

Tukey's Pairwise Comparisons for Mean Gain on Tests (CONCEP)

<table>
<thead>
<tr>
<th>Test</th>
<th>Pre</th>
<th>Post</th>
<th>Retention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Post</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retention</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Gain is significant at .05/4 level
◆ Drop is significant at .05/4 level
The ANOVA for repeated measures was also run for the two items that correspond to the S & P and Mindy's type problems. The pattern of results was the same as for the full six-item tests. Data are shown in Appendix I.

Summary

The analyses described in this chapter show that there were no significant differences between any of the four groups on the pretest scores. An item analysis of the equations written by the students indicated that they made the reversal error on the S & P problem at the same rate as the students in previous studies (Clement, 1982). They did not make the reversal error on the Mindy's problem at the same rate, because they were usually not able to write an equation in that case. One unpredicted result was that 46% of the students could not even translate the POM sentence. This will be discussed in Chapter 6.

The analyses also indicate that students (SYN-1 or SYN-2) who received either of the syntactic treatments scored higher on the posttest (even though the gains in scores were similar) and on the retention test than students (CONCEP) who received a purely conceptual treatment. An analysis of variance for repeated measures showed that there was a significant difference between each of the treatments (syntactic or conceptual) and the
control group on the posttest. After a twelve week retention time however, there was no significant drop in retention for either of the syntactic groups, but there was a significant drop in the performance of the conceptual group. At the retention time, the conceptual group was no longer significantly different from the control group. Conclusions and discussions of these results is found in Chapter 6.
CHAPTER SIX
CONCLUSIONS, DISCUSSION, LIMITATIONS
AND IMPLICATIONS

The principal purpose of this study was to determine whether expertise in translating natural language sentences into algebraic equations is a conceptual or a predominantly syntactic skill. In order to address the issue, I proposed a theory that the competent sentence translator approaches the translation task syntactically, abandoning syntactic methods in favor of conceptual strategies only for sentences that do not yield to syntactic translation. This theory stands in contrast to a theory, implicit in the work of many cognitive psychologists, that the competent sentence translator uses conceptual strategies only.

Curricula were designed to model the kinds of knowledge components proposed in the sentence translation process according to the two different theories. A predominantly syntactic and a conceptual treatment were administered to different groups, and tests were used to determine the effectiveness of the treatments.

The results indicated significant improvements due to conceptual and syntactic treatments, with the syntactic groups scoring significantly better than those
receiving conceptual instruction. After three months, the syntactic groups remained significantly more able than the conceptual and control groups on a retention test, but more importantly, there were no longer significant differences between the conceptual and control groups. This chapter contains conclusions formed on the basis of these results, a discussion of some observations, limitations of the study, and recommendations for future mathematics education research and practice.

Conclusions

Statistical analyses of the mean scores on all tests show that students who received the syntactic treatments scored significantly better on translation tasks over a period of time than students who received the conceptual treatment. Since students who received the syntactic treatments were taught the knowledge components contained in the Syntactic Theory, it is possible that competent translation of a natural language sentence into an algebraic equation involves the syntactical processes outlined in the theory.

The significant drop in scores on the NPOM items of the retention test (see Appendix I) for students in the conceptual group suggests that the problem of the novice is not the absence of conceptual knowledge, but rather
insensitivity to the syntactic cues that should trigger abandoning syntactic methods and adopting conceptual methods. Scores for this group on the posttest for NPOM items were high, but their scores on the NPOM items on the retention test dropped off, suggesting that conceptual methods are not sufficient for long term improvement.

The syntactic groups, on the other hand, retained their gains because they had learned to be discerning about when to abandon syntactic methods for conceptual methods. The drop off in scores for the conceptual treatment group speaks clearly to the idea that correct translation depends on the decision process. Since the decision process was taught to the syntactic groups, they seem to know better when to abandon syntactic methods.

The design and results of the study also hold implications for educational practice: Syntactic approaches to algebraic translation can promote more successful and longer-lasting achievement than conceptual instruction.

Finally, it should be noted that the think-aloud protocols used by the cognitive scientists were inappropriate. Such introspective reports, as noted in Chapter 1, can only reflect conscious, rational knowledge that is readily available to the subject. They cannot uncover linguistic knowledge which is largely
unconscious. Researchers should be wary of using such techniques in areas where the use of linguistic processes is in dispute.

Discussion

Several questions deserving of further discussion emerge from the study and will be addressed here:

The first question to be discussed addresses arguments that the tendency to reverse variables in translation is not something that can be "taught away" (Rosnick & Clement, 1980, p.3) in just three days. This study showed that students who received the conceptual treatment did not have the problem taught away, whereas the long term performance of students who received the syntactic treatment gives evidence that they were significantly changed.

Still, it might be argued that perhaps the students who received the syntactic treatment acquired a "trick" that is not really indicative of expert knowledge, but can produce the correct equation. But it cannot be argued that syntactic skills are totally irrelevant. For certain sentences with complex syntax (e.g. Four less than the square of three less than Bill's age is eighteen more than the square root of Mary's age three years ago.) there seems to be no other way to translate them than using syntactic processes, so competent translators must
have access to these methods.

The reversal error is an empirical phenomenon that has been focused on by those who espouse a conceptual theory of translation. Long term improvements in students' behavior, such as those achieved in the current study cannot be lightly dismissed. On the contrary, it is the conceptual treatment that produced transitory and decaying improvement in performance.

A recent study (Kirshner et al., in press), which used a computerized timer as college mathematics instructors and graduate students attempted sentence translations, also lends support to the theory that successful translators approach sentence translation tasks syntactically, only abandoning them in the case of NPOM sentences. It was reasoned that if the syntactic theory is correct, it should take longer to translate a sentence that does not yield to direct translation, than one that does. Measurement of response latencies in expert mathematicians' translations showed the predicted gap. In fact, significant differences in response times were also recorded for POM and WPA versus WST sentences, thus strengthening the argument that syntactic distinctions underlie expert performance.

The second question concerns the accuracy of the Syntactic Theory that is embedded in the syntactic treatments. The syntactic treatments involved teaching
students to do a syntactic analysis of a sentence, including within-phrase adjustments and transformations, and more importantly, teaching them when to abandon the syntactic process for conceptual strategies. According to the Syntactic Theory the decision to abandon the syntactic process is triggered by syntactic cues that a competent translator recognizes during the syntactic analysis. The mechanism for abandoning syntax in this study is that the translator is either unable to perform within phrase adjustments, or strict adherence to rules of adjustments and transformations produce a sentence that is not POM.

But another mechanism could be postulated. Perhaps the translator recognizes that the attempt to transform the sentence to a POM sentence has produced a sentence that is not grammatical (e.g., [There is equal to] [six] [times] [the number of students] [as] [the number of professors]). This possibility was not pursued in the syntactic treatment because there was concern about the ability to teach students when a sentence is not grammatical. Still, it is possible that subtle syntactic sensitivity to the transformed sentences plays a role in the process.

The third question concerns the high rate (63%) of reversals in POM sentences on the pretest. In Table 5 (Chapter 5), alarming error rates were reported. For the
POM sentence on the pretest 46% of the answers were errors of which 63% were reversals. On the retention test 17% of the answers were errors with 43% reversals. None of the POM reversals were made by students in the syntactic treatment groups. The other types of errors were various types of non-equations.

Previous studies of translation have focused on the reversal error only in NPOM sentences. Error rates on POM sentences had not been recorded until a very recent study (MacGregor, 1991) in which students in Melbourne, Australia were asked to write sentences in mathematical symbols. In a sample of 216 students in Year 9 of the school system, for the sentence The number y is eight times the number z, 56% of those who attempted the equation wrote a reversal. For the sentence s is eight more than t, of those who attempted the equation, 46% wrote reversals (p. 96). Although these sentences used letters rather than words, the rates of reversals are in line with those of this study.

There are two possible explanations of these data. The first is conceptual in nature. It holds that reversal is the result of linking together in algebraic notation the larger variable with the larger number. This interpretation would tend to suggest that translation is initially approached as a conceptual task, but without adequate conceptual tools, and that syntactic
translation approaches are acquired through school experiences.

The second explanation hold that the primary approach to translation is syntactic, but that the knowledge of when to abandon syntax is so weak that students have overgeneralized it. As a result, they opt for conceptual approaches (which also are deficient) in many circumstances where they need not. Further research is needed to evaluate these two possible explanations.

The fourth question concerns the types of errors made by the students on S & P type problems, and whether they persisted over time. As was reported in Table 5, on the pretest 52% of the answers were errors with 66% of the errors reversals. On the retention test 38% of the answers were errors with 81% of the errors reversals. Even though the number of students in the syntactic treatment groups was 55% of the total, only one-fourth of the reversal errors were made by students in the syntactic treatment groups.

For the Mindy's type problem, as was reported in Table 5, on the pretest 90% of the answers were errors of which 47% were reversals. On the retention test 58% of the answers were errors of which 54% were reversals. About 33% of the reversal errors were made by students in the syntactic treatment groups even though they were 55% of the total number of students. One reason why the
percent of reversal errors is smaller for the Mindy's type problem is that the students tended to write an expression for this sentence which is not even an equation (e.g., $6S + P$, or $6S - P$). The data suggest that reversal errors persist more for students who have not had the syntactic treatment.

The fifth question to be discussed considers the fact that this study examines translation skills used in writing natural language sentences as algebraic equations, but only uses English sentences. The researcher was limited to the English language because it is native to the population used in the study. The actual adjustments and transformations that would be made on sentences written in other natural languages would depend on the syntax of the language, and would have to be studied by a speaker of that natural language. Whether or not there are general principles of language that determine translation into algebra is beyond the scope of this study.

The sixth question considers whether a student's proportional reasoning skills are correlated with the ability to translate NPOM sentences that contain integral ($S:P = 6:1$) or rational ($C:S = 4:5$) proportion statements. Much research during the past twenty years (Karplus, Karplus, & Wollman, 1974; Karplus & Peterson, 1970; Karplus, Adi, & Lawson, 1980; Karplus, Pulos, &
Stage, 1983) has focused on strategies that young adolescents use when doing proportional problems. Although there may be some relationship between formal reasoning, information processing capacity, and proportional reasoning (Karplus, Pulos, & Stage, 1983, p. 220) the work of these researchers does not focus on translation of proportional type problems like the S & P and Mindy's type problems.

Students in this study were given three missing value type proportional problems on the pretest (see Appendix E). Since there was no significant difference between any of the groups in proportional reasoning, the issues of correlation or prediction between proportional reasoning and translation skills was not pursued in this study.

Limitations

This section discusses limitations of the design which affect the usefulness of this dissertation to other researchers and mathematics educators. The limitations are:

1. **Researcher bias.** The researcher was the instructor for the treatments. An effort was made to present each treatment as effectively as possible; however, there is the possibility of treatment bias. As a partial control, an unbiased observer was hired to
attend all sessions and report possible instances of bias. None were reported (See Appendix H).

Practical considerations mediated against training other instructors to conduct the treatments. This training would have introduced further design problems.

2. **Length of the Experiment.** Results on the retention test after a twelve week period provide some confidence that the performance of those who received the syntactic treatment had been significantly improved. In order to ascertain that the treatment really did contribute to the development of expertise in translation it would have been most desirable to do a longitudinal study of students' abilities over a greater period. Still, this study compares very favorably with the short retention times of other studies (Rosnick & Clement, 1980; Schrader, 1985, Hasty, 1987).

**Recommendations for Research**

This is the first study that has used pedagogical treatments to support a syntactic theory of competence in translation. The results of this study have generated several recommendations for future research. They are discussed here:

1. Since the study suggests that it is possible that the competent translator uses a syntactic analysis of the sentence, it is recommended that more detailed and
extensive linguistic analyses of sentence types should be conducted to discover if there are sentence types other than those identified in this study, or to detect if there are linguistic features of NPOM sentences which readily distinguish them from other sentence types.

2. Other frameworks should be sought for the model of syntactic skills used in translation. The discussion section describes one other possibility. Such attempts could investigate other possible syntactic theories.

3. This study used students who had on the average two years of previous study of algebra. It would be interesting to do this study with beginning algebra students so they would not be influenced by the translation techniques which are presently used in the algebra curriculum. Such a treatment, however, would have to be more elaborate since one could not presume that the students had previously acquired any of the hypothesized knowledge components of either of the translation theories.

4. As discussed in the third question, research should be conducted to explain the high occurrence of reversals in POM sentences.

5. As discussed in the limitations section, a longitudinal study should be attempted to obtain a retention period of longer duration.
6. There were very few students in this study who had English as a second language. A study should be conducted in a cross-linguistic environment to discover the effect that working in a second language has on sentence translation skills. It may be that interference would be created by a conflict of syntactic rules of the mother tongue and the newly acquired English language.

7. Studies using languages other than English should be attempted to determine the specific syntactic skills needed in each of these languages, and whether there are general principles which underlie all languages or specific groups of languages.

8. The conceptual treatment was based on suggestions of Clement and Kaput (telephone conversations, August, 1990). Other refinements of the conceptual treatment should be tested against the Syntactic Theory for replications of the results of this study.

Recommendations for Practice

In Chapter 1, it was proposed that if the syntactic analysis of competence in translation is correct, then instructional strategies aimed at remedying these deficiencies should be more successful than the methods advanced by previous research which focused almost exclusively on the conceptual strategies. The results of
this study have generated two recommendations for pedagogical practice.

1. This research on translation skills of algebra students adds evidence to other studies which showed that the current method of teaching translation skills is not sufficient. Since neither of the current theories (syntactic or cognitivist) is incorporated in standard curricular practices, textbook writers should revise instruction on sentence translation.

2. Since the results of this study show that a treatment based on the Cognitivist Theory does not produce the same level of competence in students as treatments based on the Syntactic Theory, instruction should be based on syntactic as well as conceptual aspects of translation. Current pedagogical innovations stemming from documents like the Standards (NCTM, 1989) which recommend less emphasis on solving routine algebra word problems are not in accord with this recommendation.
REFERENCES


APPENDICES
Appendix A

Lesson Plan for SYN-1

Objective: The students will be able to translate an English sentence into an algebraic equation according to the linguistic model, i.e. as a mechanized process that does not reference the conceptual structure of the sentence. The students will be taught to approach all sentences syntactically, abandoning this method only in the special NPOM cases.

Day One

I. Overview of word problem solving

Give the students a broad outline of problem solving by stating that in general solving word problems in algebra involves three steps:

1) translating from English to an algebraic equation
2) manipulating the equation and solving it
3) checking

(State that translating will be the main focus for the beginning of the course because it is the most crucial part of translation. Tell the students that translation becomes even more important in later work in mathematics. Do not refer to their competence in translation on the pre-test).
II. Broad overview of translation

A. The algebraic equation

Tell the students that if translation is the process of writing an English sentence as an algebraic equation it is important to know what an equation is. Ask "what is an equation?" and try to get the students to see that it is an algebraic sentence in the precise form of [expression] = [expression]. (Be sure that their definition includes the equal sign in the middle, with algebraic expressions on either side. Ask for examples of expressions and have them provide the correct definition of an expression. (Be sure that the definition includes numbers, variables, and operations on numbers and variables in the proper order.)

B. The English sentence

Get the students to see that not every English sentence can be translated into an algebraic equation. The sentence can only be translated if it is in the form of [a phrase about quantities or numbers] is equal to [a phrase about quantities or numbers]. Let them see that in such a case the English phrases make reference piece by piece to mathematical symbols, but the verb must be "is equal to" and the two phrases on either side of the verb must be about numbers. (Do an example like: "The number of dogs is equal to the number of cats," to show piece by piece translation).
III. Symbol reference

Tell the students that certain English phrases translate directly to mathematical symbols and that eventually the translation process will involve breaking the sentence into such phrases. The mathematical symbols that appear in an equation might be any or all of the following: (a) letters or variables representing phrases about numbers (e.g. a certain number, the number of pounds, or the length of the boat, where the phrase makes reference to a quantity directly or indirectly); (b) numbers representing number names; (c) the equal sign representing "is equal to"; and (d) operational symbols (+, −, x, /, exponents and radicals) representing plus, minus, times, divided by, raising to a power, and taking a root).

IV. POM sentences

Tell the students that the translation process ordinarily involves deciding on the necessary quantities in the situation and then translating the sentence piece by piece. Sometimes the sentence to be translated is already in the correct form (we will call these word order matched) and the process occurs piece by piece. Do these examples:

The number of apples is equal to two times the number of pears.
John's weight is equal to Mary's weight minus four.
Six times a certain number is two times the square root of a second number.

V. WPA sentences

A. Rationale for why some sentences cannot be written in POM form.
Ask the students to try to translate:
One number is equal to five times a second number plus eight.
Have a discussion about the ambiguity of this sentence.
Ask if it should be $A = 5B + 8$ or $A = 5(B + 8)$.
Tell the students that in order to avoid this type of ambiguity the sentence could be written
One number is equal to five times the sum of a second number plus eight. or
One number is equal to the sum of five times a second number and eight.
Tell the students that in order to avoid such ambiguous sentences a sentence is often written in a form that is not POM. Very often, some slight adjustments will be needed, because the operations are not in exact wording that is needed for translation.

B. Adjustments to WPA sentences
Tell the students that in cases where the sentence is not in POM form, it is important to know that certain
phrases of a sentence may contain references to mathematical operations of addition, subtraction, multiplication, division, raising to a power or taking a root without being explicit. These phrases must be adjusted to include the explicit names for the operations (e.g. plus, minus, times, divided by, squared, or square root of).

Exhaust the list of elementary algebra phrases. Write the phrase and have the students give the adjustment:

- four more than C means C plus 4
- four less than C means C minus 4
  (Say that the meaning is that you have a number and that number must be reduced by four).
- twice C means 2 times C
- the product of C and 4 means 4 times C
- the difference between C and 4 (if C > 4) means C minus 4
- the quotient of C and 4 means C divided by 4
- the ratio of C and 4 means C divided by 4
- twice the sum of C and 4 means 2 times (C plus 4)
- the square of C means C squared

Have the students look at a sentence that is not in POM form, but needs some adjustment. Take as an example: The number of frogs is four less than the number of eels.

Have the students try to translate this sentence and
see if they make any common errors. If a student writes correctly E - 4, ask why did she write it that way, and reiterate the importance of the adjustment.

C. Practice

Have the students make the adjustments and do the translations on each of these phrases and sentences:

1. three less than twice the second number
2. eight more than twice the sum of two and the number
3. three less than the square of the length
4. half the square of four more than a number
5. The number of Jo's cats is twice the number of Pat's dogs.
6. The number of pounds in John's sack is five pounds less than the number of pounds in Mary's sack.
7. Three less than two times the square of a number is five more than half another number.
8. Five times a number is eight times the sum of three and the square of another number.
9. The square of the length of Paul's boat is three feet less than twice the length of Bill's boat.
10. Twice the square of Joe's age is four less than three times his father's age five years ago.

VI. Summary and Overview
Tell the students that today they have seen that translation is the process of writing an English sentence as an algebraic equation. They have seen two cases:

1) The POM sentence in which the sentence can be translated piece by piece, and

2) The WPA sentence in which minor adjustments must be made before the sentence is translated piece by piece.

Tell the students that in the next class they will see other sentences in which the phrases are not explicitly about quantities. In such cases the sentence will have to be rephrased in order to get it into POM form. In even more difficult sentences the rephrasing of the sentence may not work and they will have to learn other strategies for translation.

Day Two

VII. Review of the translation process

Ask if there are any questions about the sentences that were looked at on Day One. (Depending on questions or comments, some review may have to be done). Recall that there are four types of sentences that can be translated into algebra and also that they looked at the two easiest types during the last class.

VIII. Transformations on WST sentences

Tell the students that the sentences they will see
now are in slightly more difficult form. They will need to determine what quantities the sentence is about, because the noun phrases are not explicitly quantified. Usually this can be done just by rephrasing according to a certain rule.

A. The transformation process

Now take a case in which the English sentence does not contain explicitly stated quantified noun phrases. Show how the verb or verb phrase will give a hint about the type of quantities that are being compared. Use the sentence: John is four inches taller than Mary. Ask the students the following sequence of sentences:

1) What quantities is the sentence about?
   (If they say John and Mary, say that names are not quantities. Get them to say John's and Mary's height).

2) Are the quantities expressed explicitly?
   (Are they a number, a number of things, or reference to a number? They should say no).

3) What variables would you like to create?
   (Hopefully they will say something like let \( J = \) John's height and \( M = \) Mary's height).

4) How did you know what the quantities were?
   (When they say because of the word "taller," ask what "taller" means and try to help them see that it means "more height").
The sentence is now:

**John is four inches more height than Mary.**

Instruct the students that once they have determined the quantities, they should be expressed explicitly in each of the noun phrases to read:

**John's height is four inches more than Mary's height.**

Explain that this process is done by "parcelling out" or distributing "height" to the two nouns, thus transforming the noun phrases into quantified noun phrases and leaving "more than" behind.

Tell the students that this is the way they are allowed to rephrase the sentence so that it contains phrases about quantities.

After this transformation is made, other adjustments must be made. Ask for the simple adjustments to transform the sentence:

**John's height is Mary's height plus four.**

**John's height is equal to Mary's height plus four.**

Which can be translated as $J = M + 4$.

Use the sentence: **John weighs twice as much as Mary.**

Ask what quantities the sentence is about.

Ask if they are expressed explicitly.

Ask what variables they would like to create?

Ask how they knew what the quantities are?

Get the students to derive the transformed sentence:

**John's weight is twice as much as Mary's weight.**
Do the adjustments:

John's weight is equal to two times Mary's weight.
And the translation: \( J = 2M \).

B. Practice with the transformation process on WST sentences. The students should be asked each of the four questions needed to make the transformations of these WST sentences. (There are two NPOM sentence shown here so they know what is coming later).

1. The train took twice as long as the plane.
2. The dentist saw three fewer patients than the chiropractor.
3. Patty eats 100 more calories a day than her friend.
4. The boy is five years younger than the girl.
5. Oak trees produce 5000 more leaves than twice the square of the number of leaves produced by maple trees.
6. Ace ran five miles farther than Barry.
7. Mrs. Smith sells five less than twice the number of pies as Mrs. Jones.
8. Jo's boat is three feet less than twice as long as Bill's boat.
9. I have twice as many shirts as pants.
10. At the party, two rolls were eaten for every three donuts.

IX. Summary overview

Summarize the translation process for all sentences:
Tell the students that in general they should do the following things when attempting a translation:

1. Read the sentence and determine what type of sentence it is.
2. If the sentence is POM, then do the translation.
3. If the sentence is WPA but needs some adjustments, make the necessary adjustments and then do the translation.
4. If the noun phrases are not quantified, do the transformation that quantifies the noun phrases, make necessary adjustments, and then translate.
5. If in some difficult cases, the transformations do not work, then do a different strategy. (Tell the students they will see this type of sentence in the next class).

Day Three

X. Determining when a sentence is NPOM

Tell the students that whenever the phrases of the sentence are not explicitly about numbers or quantities they must attempt the transformation learned in the last class. If the transformation does not work they must not try to force the translation, but they should abandon the piece by piece translation process.

After questions are answered, present an NPOM sentence for translation.
Have the students look at the sentence:

**Central has three times as many freshmen as seniors.**

Attempt the transformations that were learned yesterday by asking the students the following sequence of sentences:

1) What quantities is the sentence about?
   (They should say freshmen and seniors, but remind them that names are not quantities. Get them to say numbers of freshmen and seniors).

2) Are the quantities expressed explicitly?
   (Are they a number, a number of things or reference to a number? They should say no).

3) What variables would you like to create?
   (Hopefully they will say something like let \( F \) = number of freshmen and \( S \) = number of seniors)

4) How did you know what the quantities were?
   (They will probably say because of a reference to "many").

**Attempt the transformation:**

**There are three times as many the number of freshmen as the number of seniors.**

And the adjustments:

**There is equal to three times as many the number of**
fresmen as the number of seniors.

Read the sentence several times and ask the students what is wrong with the sentence. Try to get the students to determine that in this case, after all the transformations are made, the sentence is not in the form \([\text{phrase about numbers}] \text{ is equal to } [\text{phrase about numbers}]\). Tell them that this is a case in which they must not try to force the translation. Instead they must stop the translation process and realize that this is a type of sentence that is not easily translated. Tell the students that in such a case they will use a different strategy.

(Re-do the process with a Mindy's type problem if there is time). Five more bushes were seen than saplings.

What are the quantities? Numbers of bushes and saplings. Are the quantities expressed explicitly? No.

How did you know? (At this point the student must be led to see that the verb "were seen" does not refer to a quantity, so the noun phrases cannot be quantified. The process should be abandoned).

Use these examples to have the students decide when the sentence is NPOM:

1. Tony weighs five pounds less than Bill.
2. The boat is three feet longer than the dock.
3. Four more apples were eaten than pears.
4. The Indians ran 50 more miles than twice the number of miles run by the Braves.
5. The sailboat took twice as long as the yacht.
6. Roger built six fewer apartments than Bill did.
7. Carol has twice as many pencils as erasers.
8. The pool is five feet longer than the tennis court.
9. Six fewer A's were made than B's.
10. My height is two inches less than twice my son's height.

XI. Do a conceptual analysis

In each of the sentences in which the students decided to abandon POMing, ask if they can tell from the sentence which of the quantities is smaller? (This should be fairly obvious. Do each of the previous examples).

   Four more apples were eaten than pears.
   Carol has twice as many pencils as erasers.
   Six fewer A's were made than B's.

Tell the students that whenever the transformations do not lead to a sentence in translatable form they should stop the translation process and try to figure out which of the quantities is smaller than the other.

XII. Do a conceptual strategy

   A. Compensation
Once the students have determined the two variables, they should be placed on either side of an equation. The students must see that the variable that represents the smaller quantity will have to be compensated by addition or multiplication in order to get an equivalence with the variable representing the larger quantity. In some cases there are two numbers mentioned and the smaller quantity must be multiplied by the larger number and vice-versa. See that they can tell whether to use addition or multiplication from the context of the sentence.

Draw a balance to help the students understand the concept of compensation.

Do these examples:

I have five more apples than bananas.

Joe spends four times as much on rent as he does on food.

Three dollars were collected for every two quarters.

B. Proportion

Remind the students that a proportion is the equality of two ratios. Tell them that they can set up a proportion using the ratio of the two variables on one side and the numbers of each that are represented in the sentence to get a proportion of the form \( \frac{a}{b} = \frac{c}{d} \). Remind the students that the proportion can only be used in the multiplicative case, not in the additive case.
Do the above examples.

Practice using the strategies learned

1. Ann bought three times as many hats as belts.
2. The can of nuts contains one cashew for every ten peanuts.
3. Joe took twice as long as Tony to finish the test.
4. Six fewer Chevy trucks were bought than vans.
5. Donald stands four inches taller than Greg.
6. For every two textbooks, we use up five notebooks.
7. Clark High enrolled three times as many freshman as seniors.
8. The width is eight feet less than the length.
9. Eight more goals were made by Team A than by Team B.
10. Hats were three times as expensive as belts.
Appendix B

Lesson Plan for SYN-2

Objective: The students will be able to translate an English sentence into algebraic notation according to a strong conceptual approach. They will be given a strong conceptual treatment of variable and equation, and many conceptual strategies that will be applied to all sentence types. Studies by Rosnick and Clement (1980) suggested that students need a stronger treatment on variables and equations. In phone conversations with Clement and Kaput they suggested that the strongest strategy that can be used in S & P-type problems is to have the students make a table of data.

Day One

I. Overview of problem solving

Tell the students that word problem solving involves three parts: translation, manipulation, and checking.

Tell the students that translation of word problems involves going from a situation in the world involving quantities to representation as an algebraic equation. Often this process is quite difficult and it is worth spending a few days on.

Tell the students that in higher mathematics and scientific applications the setting up is often the only difficulty in problem solving. The equation solving part where the manipulation is done is often routine and can
be done by computers. But the problem is that computers can't think and to set up a problem as an equation you must think.

Tell the students that they will receive direction in their thinking in the translation phase of problem solving.

First, problems in the world come in all kinds of forms - sometimes an architect has a problem embedded in his diagrams or a scientist may have a problem embedded in a table of data. Because the school curriculum stresses problems stated verbally, that is where our focus will be, but it is often a good idea to re-represent a problem in a diagram form or a table of data to help you think. We will look at some of these strategies.

II. Variables - Static

Tell the students that the first thing to keep in mind when starting on a translation is that algebra is a language of quantities, so they have to identify what are the quantities the problem is about. If there are unknown quantities then they will use a letter to represent them in algebra.

Tell them for instance, if I have twice as many quarters as nickels, what are the quantities involved? (Insist that they say "number of").

How should we represent these quantities in algebra?
(Get them to say by Q and D).

Keep in mind however that Q and D are the number of quarters and the number of dimes, not labels for quarters and dimes.

Take another example: I have twice the value of quarters in my pocket as I do of dimes.

(What are the quantities involved? How should we represent them?)

Tell them to notice that the letter itself tells you nothing on its own. You have to keep in mind what quantities the letters are standing for.

III. Variables - Dynamic

Tell the students that often in real applications variables are intricately related to each other over time.

Give the students the example of running a luxury liner. Explain that there are many variables involved (use for example, number of passengers, number of pounds of potatoes eaten, number of pounds of garbage produced.) Explain how these variables are linked in a dynamic way: as one variable changes, the others changes in relation to it. See if the students can understand that there is an implied temporal dimension in this case. For example: 

**Eight hundred pounds of garbage is produced for every twelve passengers.**

Have the students give a sentence that relates all
three variables.

Pose the problem of painting a fence. What are the variables? How are they linked?

Have the students work in pairs to come up with examples of situations that contain two or more dynamically linked variables. Tell them to be prepared to discuss the nature of the linkages.

Call on several students to give their examples and point out how the variables are dynamically linked. In each case be sure that the student refers to a quantity and specifically refers to a "number of things."

IV. Symbol Reference

Tell the students that they have covered variables that represent unknown quantities, but the relationship between these quantities needs to be symbolized as mathematical operations. (This is mostly review but still important enough to go over briefly).

Explain that the mathematical operations of addition, subtraction, multiplication, division, raising to a power and taking a root are occasionally represented by the words plus, minus, times, divided by, squared or square root. But that is only in the simplest problems. Usually you have to figure out what the operation is from the situation depicted. For instance if we've decided that Q is the number of quarters in my pocket, then how would you represent "four less than the number of quarters in
"my pocket?"

(If they say 4 - Q, stop and say sternly, what did I say about thinking---never translate word for word - THINK! Stress the semantics of the situation - you must start with the number and take four less).

The number of frogs is four less than the number of eels.

Review certain expressions from elementary algebra that are needed to represent less explicit forms. For example, exhaust the list of elementary algebra phrases:

- four more than C means C plus 4
- four less than C means C minus 4
- the product of 4 and C means 4 times C
- the difference between C and 4 (if C>4) means C minus 4
- the quotient of C and 4 means C divided by 4
- twice the sum of C and 4 means 2 times (C plus 4)

Give the students some phrases to translate using numbers.

For example: What does each of these mean:

1. Four less than seven
2. Twice the square of nineteen
3. Two less than three times eighteen
4. Eight more than twice the square of three
5. The square of twice nine
6. Twice the sum of four and eight
Day Two

I. Review anything that the students want to see from the previous day, but do it quickly.

II. Conceptual Strategy

A. Table of Data

Explain that a very useful strategy in translating a word problem is the table of data. (Have the students put their pencils down so they don't try to translate from the words). Teach the table strategy by suggesting that the following sentence is given:

Joe ate four more cookies than donuts.

Have the students determine that the two quantities referred to are the number of cookies and the number of donuts. Have the students fill in the table by asking what they can say about how the number of nickels is related to the number of dimes:

\[
\begin{array}{cc}
c & d \\
5 & 1 \\
6 & 2 \\
7 & 3 \\
\end{array}
\]

Try to get the students to say that the relationship is that the number of cookies is always four more than the number of donuts, and therefore they should write \( c = d + 4 \).

Do another example using the sentence:

John's age is twice the square of Bill's age.
Have the students see that the two quantities referred to are John's age and Bill's age. The table would be

<table>
<thead>
<tr>
<th>j</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
</tr>
</tbody>
</table>

Try to get the students to say that John's age is always two times Bill's age squared.

Do the table of data strategy on each of the problems for day two but follow this method exactly:

1. Tell the students to put their pencils down for oral work.

2. Put one sentence (Item) at the top of an overhead transparency with room for a table and an equation as shown here:

   ITEM

   =

3. Have the students produce the table of data by asking them for pairs of numbers that would satisfy the situation.

4. After the table of data is obtained, slide the item off the top of the overhead screen and have the students write the equation from the table of data only.
Have the students practice this method with the following problems:

1. There are two more boys than girls on the playground.
2. Joe has five times as many nickels as dimes.
3. The bus took twice as long as the train.
4. Three less jets were available than tanks.
5. Joe weighs thirty pounds less than Tom.
6. The tank traveled two miles for every mile traveled by the cannon.
7. The cops outnumber the robbers by four.
8. We have three spoons for every two forks.
9. Carol is five inches shorter than Betsy.
10. Six donuts were sold for every two rolls.

Day Three

I. Review the previous days work.

II. More conceptual strategies.

Have the students learn more conceptual strategies.

A. The compensation strategy.

Tell the students that if they simply place the two variables on either side of an equal sign, then the variable that represents the smaller number will have to be compensated either by addition or multiplication in order to get an equivalence with the larger variable. The choice of whether to use addition or multiplication
will be made according to the meaning of the sentence.

Draw a balance to help the students understand the concept of compensation.

Do these examples:

* I have five more apples than oranges.
* Joe spends four times as much on rent as he does on food.
* Toby ran five miles more than Chad ran.
* Three dollars were collected for every two quarters.

B. The proportion strategy.

Remind the students that a proportion is the equality of two ratios. Tell them that they can set up a proportion using the ratio of the two variables on one side and the numbers of each that are represented in the sentence to get a proportion of the form $a/b = c/d$. Tell the students that this strategy does not work in the additive case.

Look at the example:

* Joe has five times as many nickels as dimes.

Joe has five nickels for every one dime. Show the students how to write $n/d = 5/1$.

Stress that these important strategies can be used on any type of example, except that the proportion strategy works only for the multiplicative case.

Continue to stress the meaning of variable and the meaning of equation and that the variable represents a
number of things.

Do these examples with the strategy:

1. Ann bought three times as many hats as belts.
2. The can of nuts contains one cashew for every ten peanuts.
3. Joe took twice as long as Tony to finish the test.
4. Six fewer Chevy trucks were bought than vans.
5. Donald stands four inches taller than Greg.
6. For every two textbooks, we use up five notebooks.
7. Clark High enrolled three times as many freshman as seniors.
8. The width is eight feet less than the length.
9. Eight more goals were made by Team A than by Team B.
10. Hats were twice as expensive as belts.
Appendix C
Lesson Plan for CONCEP

Objective: The students will be able to translate an English sentence into an algebraic equation according to the linguistic model, i.e. as a mechanized process that does not reference the conceptual structure of the sentence. The students will be taught to approach all sentences syntactically, abandoning this method only in the special NPOM cases.

Day One

I. Overview of word problem solving

Give the students a broad outline of problem solving by stating that in general solving word problems in algebra involves three steps:

1) translating from English to an algebraic equation
2) manipulating the equation and solving it
3) checking

(State that translating will be the main focus for the beginning of the course because it is the most crucial part of translation. Tell the students that translation becomes even more important in later work in mathematics. Do not refer to how they did in translation on the pre-test).

II. Broad overview of translation

A. The algebraic equation

Tell the students that if translation is the process
of writing an English sentence as an algebraic equation it is important to know what an equation is. Ask "what is an equation?" and try to get the students to see that it is an algebraic sentence in the precise form of [expression] = [expression]. (Be sure that there definition includes the equal sign in the middle, with algebraic expressions on either side. Ask for examples of expressions and have them give the correct definition of an expression. (Be sure that the definition includes numbers, variables, and operations on numbers and variables in the proper order.)

B. The English sentence

Get the students to see that not every English sentence can be translated to an algebraic equation. The sentence can only be translated if it is in the form of [a phrase about quantities or numbers] is equal to [a phrase about quantities or numbers]. Let them see that in such a case the English phrases make reference piece by piece to mathematical symbols, but the verb must be "is equal to" and the two phrases on either side of the verb must be about numbers. (Do an example like: "The number of dogs is equal to the number of cats" to show piece by piece translation).

III. Symbol reference

Tell the students that certain English phrases translate directly to mathematical symbols and that
eventually the translation process will involve breaking the sentence into such phrases. The mathematical symbols that appear in an equation might be any or all of the following: letters or variables representing phrases about numbers (e.g. a certain number, the number of pounds, or the length of the boat, where the phrase makes reference to a quantity directly or indirectly); numbers representing number names, the equal sign representing "is equal to", and operational symbols (+, -, x, /, exponents and radicals) representing plus, minus, times, divided by, raising to a power, and taking a root). 

IV. POM sentences

Tell the students that the translation process ordinarily involves deciding on the necessary quantities in the situation and then translating the sentence piece by piece. Sometimes the sentence to be translated is already in the correct form (we will call these word order matched) and the process occurs piece by piece. Do these examples:

The number of apples is equal to two times the number of pears.

John's weight is equal to Mary's weight minus four pounds.

Six times a certain number is two times the square root of a second number.
V. WPA sentences

A. Rationale for why some sentences cannot be written in POM form.

Ask the students to try to translate:

One number is equal to five times a second number plus eight.

Have a discussion about the ambiguity of this sentence. Ask if it should be \( A = 5B + 8 \) or \( A = 5(B + 8) \).

Tell the students that in order to avoid this type of ambiguity the sentence could be written

One number is equal to five times the sum of a second number and eight. or

One number is equal to the sum of five times a second number and eight.

Tell the students that in order to avoid such ambiguous sentences the form in which a sentence is written is usually not POM. Very often, some slight adjustments will be needed, because the operations are not in the exact wording that is needed for translation.

B. Adjustments to WPA sentences

Tell the students that in cases where the sentence is not in POM form, it is important to know that certain phrases of a sentence may contain references to mathematical operations of addition, subtraction, multiplication, division, raising to a power or taking a root without being explicit. These phrases must be
adjusted to include the explicit names for the operations: e.g. plus, minus, times, divided by, squared, or square root of.

Exhaust the list of elementary algebra phrases.
Write the phrase and have the students give the adjustment:

- four more than C means C plus 4
- four less than C means C minus 4
  (Say that the meaning is that you have a number and that number must be reduced by four).
- twice C means 2 times C
- the product of 4 and C means 4 times C
- the difference between C and 4 (if C > 4) means C minus 4
- the quotient of C and 4 means C divided by 4
- the ratio of C and 4 means C divided by 4
- twice the sum of C and 4 means 2 times (C plus 4)
- the square of C means C squared

Have the students look at a sentence that is not in POM form, but needs some adjustment. Take as an example:

The number of frogs is four less than the number of eels.

Have the students try to translate this sentence and see if they make any common errors. If a student writes correctly E - 4, ask why did she write it that way, and reiterate the importance of the adjustment.

C. Practice
Have the students make the adjustments and do the translations on each of these phrases and sentences:

1. three less than twice the second number
2. eight more than twice the sum of two and the number
3. three less than the square of the length
4. half the square of four more than a number
5. The number of Jo's cats is twice the number of Pat's dogs.
6. The number of pounds in John's sack is five pounds less than the number of pounds in Mary's sack.
7. Three less than two times the square of a number is five more than half another number.
8. Five times a number is eight times the sum of three and the square of another number.
9. The square of the length of Paul's boat is three feet less than twice the length of Bill's boat.
10. Twice the square of Joe's age is four less than three times his father's age five years ago.

VI. Summary and Overview

Tell the students that today they have seen that translation is the process of writing an English sentence as an algebraic equation. They have seen two cases:

1) The POM sentence in which the sentence can be translated piece by piece, and
2) The WPA sentence in which minor adjustments must be made before the sentence is translated piece by piece.

Tell the students that in the next class they will see other sentences in which the phrases are not explicitly about quantities. In such cases the sentence will have to be rephrased in order to get it into POM form. In even more difficult sentences the rephrasing of the sentence may not work and they will have to learn other strategies for translation.

**Day Two**

VII. Review of the translation process

Ask if there are any questions about the sentences that were looked at on Day One. (Depending on questions or comments, some review may have to be done). Tell the students to recall that there are four types of sentences that can be translated into algebra and also that they looked at the two easiest types during the last class.

VIII. Transformations on WST sentences

Tell the students that the sentences they will see now are in slightly more difficult form. They will need to determine what quantities the sentence is about, because the noun phrases are not explicitly quantified. Usually this can be done just by rephrasing according to a certain rule.

A. The transformation process

Now take a case in which the English sentence does
not contain explicitly stated quantified noun phrases. Show how the verb or verb phrase will give a hint about the type of quantities that are being compared. Use the sentence: John is four inches taller than Mary. Ask the students the following sequence of sentences:

1) What quantities is the sentence about?
   (If they say John and Mary, say that names are not quantities. Get them to say John's and Mary's height).

2) Are the quantities expressed explicitly?
   (Are they a number, a number of things, or reference to a number? They should say no).

3) What variables would you like to create?
   (Hopefully they will say something like let J = John's height and M = Mary's height).

4) How did you know what the quantities were?
   (When they say because of the word "taller," ask what "taller" means and try to get them see that it means "more height").

Instruct the students that once they have determined the quantities, they should be expressed explicitly in each of the noun phrases to read:

John's height in inches is four more than Mary's height in inches.

Explain that this process is done by "parcelling out" or distributing "height" to the two nouns, thus
transforming the noun phrases into quantified noun phrases and leaving "more than" behind.

Tell the students that this is the way they are allowed to rephrase the sentence so that it contains phrases about quantities.

After this transformation is made other adjustments must be made. Ask for the simple adjustments to derive the sentence:

*John's height is Mary's height plus four.*

*John's height is equal to Mary's height plus four.*

Which can be translated as \( J = M + 4 \).

Use the sentence: *John weighs twice as much as Mary.*

Ask what quantities the sentence is about.

Ask if they are expressed explicitly.

Ask what variables they would like to create?

Ask how they knew what the quantities are?

Get the students to derive the transformed sentence:

*John's weight is twice as much as Mary's weight.*

Do the adjustments:

*John's weight is equal to two times Mary's weight.*

And the translation: \( J = 2M \).

B. Practice with the transformation process on WST sentences. The students should be asked each of the four questions needed to make the transformations of these WST sentences.
1. The train took twice as long as the plane.
2. The dentist saw three fewer patients than the chiropractor.
3. John has three more marbles than Bill.

Have the students do four examples on their own and then discuss with the class while summarizing the steps.
1. Joe is five years younger than Meg.
2. Holden High has fifty more seniors than Walker High.
3. The canoe took three times as long as the boat.
4. Paul served five fewer hotdogs than Sue did.

IX. Transformations on NPOM sentences

Tell the students that in some difficult cases, the transformations do not work and the present method must be abandoned. The attempt to turn the sentence into a POM sentence fails. Do these examples:
1. I have twice as many shirts as pants.
2. At the party, two rolls were eaten for every three donuts.
3. Joe has four more cars than trucks.

IX. Summary overview

Summarize the translation process for all sentences:
Tell the students that in general they should do the following things when attempting a translation:
1. Read the sentence and determine what type of sentence it is.
2. If the sentence is POM, then do the translation.

3. If the sentence is almost POM but needs some adjustments, make the necessary adjustments and then do the translation.

4. If the noun phrases are not quantified, do the transformation that quantifies the noun phrases, make necessary adjustments and then translate.

5. If in some difficult cases, the transformations do not work, then do a different strategy. You will have to learn to figure out which ones work and which ones don't. You will also have to learn what to do if the transformations do not work. During the next class you will learn to identify the types that do not work and you will learn other strategies than can be used in such cases.

Day Three
X. Determining when a sentence is NPOM

Tell the students then whenever the phrases of the sentence are not explicitly about numbers or quantities they must attempt the transformation learned in the last class. If the transformations do not work they must not try to force the translation, but they should abandon the piece by piece translation process.

After questions are answered, present an NPOM sentence for translation.
Have the students look at the sentence:

**Central High has three times as many freshmen as seniors.**

Attempt the transformations that were learned yesterday by asking the students the following sequence of sentences:

1) **What quantities is the sentence about?**
   
   (They should say freshmen and seniors, but remind them that names are not quantities. Get them to say numbers of freshmen and seniors).

2) **Are the quantities expressed explicitly?**
   
   (Are they a number, a number of things or reference to a number? They should say no).

3) **What variables would you like to create?**
   
   (Hopefully they will say something like let $F =$ number of freshmen and $S =$ number of seniors).

4) **How did you know what the quantities were?**
   
   (They will probably say because of a reference to many).

Attempt the transformations:

**There are three times the number of freshmen as the number of seniors.**

And the adjustments:

**There is equal to three times the number of freshmen as the number of seniors.**
Read the sentence several times and ask the students what is wrong with the sentence. Try to get the students to determine that in this case, after all the transformations are made, the sentence is not in the form [phrase about numbers] is equal to [phrase about numbers]. Tell them that this is a case in which they must not try to force the translation. Instead they must stop the translation process and realize that this is a type of sentence that is not easily translated. Tell the students that in such a case they will use a different strategy.

(Redo the process with a Mindy's type problem).

**Five more bushes were seen than saplings.**

What are the quantities? Numbers of bushes and saplings. Are the quantities expressed explicitly? No. How did you know what the quantities were? Seeing things. Attempt the translation:

**Five more the number of bushes were seen than the number of saplings.**

(Five more b cannot be adjusted to five more than b, so the process must be abandoned).

Use these examples to have the students decide when the sentence is NPOM. Have them mark each one:

1. Tony weighs five pounds less than Bill.
2. The boat is three feet longer than the dock.
3. Four more apples were eaten than pears.
4. Four apples were eaten for every three pears.
5. There is twice as much sauce as gravy.
6. Roger built six fewer apartments than Bill did.
7. Carol has twice as many pencils as erasers.
8. Carol has twice as many pencils as Mary.
9. Six fewer A's were made than B's.
10. My height is two inches less than twice my son's height.

XI. Do a conceptual analysis

Tell the students that in each of the cases that they marked as NPOM they will have to use a different strategy to get the proper translation.

In each of the sentences in which the students decided to abandon POMing, ask if they can tell from the sentence which of the quantities is smaller? (This should be fairly obvious. Do each of the previous examples).

Four more apples were eaten than pears.
Four apples were eaten for every three pears.
There is twice as much sauce as gravy.
Carol has twice as many pencils as erasers.
Six fewer A's were made than B's.

Tell the students that whenever the transformations do not lead to a sentence in translatable form they should stop the translation process and try to figure out which of the quantities is smaller than the
other.

XII. Do a conceptual strategy

A. Compensation

Once the student has determined that the sentence is NPOM, he should place the variables on either side of an equal sign. The student must see that the variable that represents the smaller quantity will have to be compensated by addition or multiplication in order to get an equivalence with the variable representing the larger quantity. In some cases there are two numbers mentioned and the smaller quantity must be multiplied by the larger number and vice-versa. See that they can tell whether to use addition or multiplication from the context of the sentence.

Draw a balance to help the students understand the concept of compensation. Do these examples:

I have five more apples than bananas.

Joe spends four times as much on rent as he does on food.

Three dollars were collected for every two quarters.

B. Proportion

Remind the students that a proportion is the equality of two ratios. Tell them that they can set up a proportion using the ratio of the two variables on one side and the numbers of each that are represented in the sentence to get a proportion of the form \( \frac{a}{b} = \frac{c}{d} \).
Remind the students that the proportion can only be used in the multiplicative case, not in the additive case.

Do the above examples and practice using the strategies learned:

1. Ann bought three times as many hats as belts.
2. The can of nuts contains one cashew for every ten peanuts.
3. Joe took twice as long as Tony to finish the test.
4. Six fewer Chevy trucks were bought than vans.
5. Donald stands four inches taller than Greg.
6. For every two textbooks, we use up five notebooks.
7. Clark High enrolled three times as many freshman as seniors.
8. The width is eight feet less than the length.
9. Eight more goals were made by Team A than by Team B.
10. Hats were three times as expensive as belts.
Appendix D
Lesson Plan for CONT-1
Since these students will not receive a treatment, they will not have any special instruction in translation. The algebra curriculum, usually driven by the textbook, consists of an overview of problem solving with examples shown by the teacher, followed by student practice. Translation is dealt with as needed in the course of particular problems.

Day One

I. Overview of solving word problems.

A. Tell the students that there are several types of word problems that they will learn to solve. They will be divided as follows:

Day One- Simple types (number, age, coin)
Day Two- Formula types (geometry)
Day Three -Formula types (Interest and Distance)
Later - Harder types (Mixture and Work)

B. Remind the students that solving a word problem usually involves (paraphrased from Lial & Miller, 1989):

1) Deciding on an unknown.
2) Drawing a sketch or making a table.
3) Deciding on a variable expression to represent any other unknowns in the problem (e.g. if W represents the width and L the length of a rectangle, and you know
that the length is one more than twice the width, write
down \( L = 1 + 2W \) (sic)).

4) Writing an equation.

5) Solving the equation.

Work this example using two variables. (Instruct the students in POMing techniques whenever necessary): A certain number is three more than another number. The sum of the two numbers is 9. What are the numbers?

II. Solve simple word problems using two variables.

A. Remind the students that for consecutive integer problems they will need to know the relation between integers on the number line. Draw a number line to show the relation between consecutive integers, consecutive even integers and consecutive odd integers.

Work this example: The sum of three consecutive integers is 99. Find them.

B. Do age problems. Show the necessity of a chart to keep track of ages past, ages in the future, and ages in the present.

Work this example: Mary is 8 years older than Jimmy. In two years she will be twice as old as Jimmy. Find their present ages.

C. Do coin problems and show how to represent money.

Work this example: A collection of 40 nickels and dimes is worth $3.05. How many of each kind of coin are in the collection?
Work this example: A collection of nickels, dimes and quarters has a total value of $4.35. If there are three more quarters than dimes, and twice as many nickels as dimes, how many of each kind of coin are in the collection?

Word problems - Day One

1. The sum of two numbers is 102, and the larger number is five times the smaller. Find the numbers.
2. Six times a number decreased by 5 times the number is equal to 10. Find the number.
3. Joe ate four more donuts than Bill. If they ate 16 donuts, how many did Joe eat?
4. The number of nickels that Carol has is five less than the number of dimes she has. How many dimes does she have if the value of these coins is $1.70.
5. Tony is 5 years older than Kate. Two years ago Tony was twice as old as Kate. Find the present ages of Tony and Kate.
6. Five apartments are available for every dorm room at this college. If 3600 living spaces are available, how many are dorm rooms?
7. Two consecutive odd integers have a sum of 256. Find the integers.
8. A train took three times as long as a plane to get from New Orleans to Lafayette. Write the time of the train in terms of the time of the plane.
9. Polly ate four fewer oranges than Bill. If together they ate twenty oranges, how many did Polly eat?

10. Ellen is one third as old as her brother. In 9 years she will be the same age her brother was 13 years ago. Find the present ages of Ellen and her brother.

Day Two

I. Review problems from day one.

II. State that in the problems that were looked at yesterday, all the information was given directly. But remind the students that there are many types of problems that require outside information like formulas or vocabulary from geometry.

   Review the ideas of area, perimeter, angle sums, complementary and supplementary angles.

   Draw examples of geometric figures with proper labeling.

III. Solve examples of geometry problems.

   Work this example pointing out all of the steps listed on Day One: If the length of a side of a square is increased by 3 cm., the perimeter of the new square is 40 cm. more than twice the length of a side of the original square. Find the dimensions of the original square.
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Word Problems - Day Two

1. The length of a rectangle is 1 m. more than twice its width. The perimeter is 110 m. Find the dimensions of the rectangle.

2. Two angles are complementary if the sum of their measures is 90. If $A$ and $B$ are complementary angles, and $A$ is twice as large as $B$, find the number of degrees in both $A$ and $B$.

3. The length of a playing field is 30 yards less than twice its width. If the perimeter of the field is 390 yards, find its dimensions.

4. A triangle has 5 more degrees in the second angle than in the first angle, and the third angle is 65 degrees less than the first angle. What are the measures of the three angles?

5. One side of a triangle is 5 meters longer than twice the shortest side. The third side is three times as long as the shortest side. Find the length of the shortest side if the perimeter of the triangle is 71 meters.

6. Find the measures of the three angles of a triangle if the second angle is 4 degrees less than three times the first and the third angle is 38 degrees less than three times the second.

7. The length of a rectangle is 3 less than twice the width. The perimeter of the rectangle is 42 feet.
Find the length and width of the rectangle.

8. A rectangular field is enclosed by 350 feet of fence. If the field has 2 feet of width for every 3 feet of length, find its length and width.

9. The sum of the three angles in a triangle is 180 degrees. The second angle is 20 degrees more than the first and the third angle is twice the first. What is the measure of each of the angles in the triangle?

10. The length of a rectangle is three times its width. If the perimeter is 160 inches, determine the area of the rectangle.

Day Three

I. Review the problems of the previous two days.

II. Discuss the fact that some problems involve formulas that must be recalled. Discuss the simple formula for I=PRT.

   Discuss the fact that in a word problem you often have to add the interest that is earned in more than one account. In each case the interest must be expressed as a product of principle, rate and time.

   Work this example: Candy invests $20,000 in two ways, some at 10% and some at 6%. If she makes $1560 interest in one year, how much was invested at each rate?
III. Discuss common situations of distance problems that involve either two distances that are equal or two distances that sum to a given value. In such cases distance must be represented according to the formula, $d = rt$. Draw pictures of the different cases.

Show how a chart is helpful in keeping track of the given information.

Work this example: On a bicycle trip from A to B, Josie averages 15 mph going there and 10 mph on the return trip. What is the distance between A and B if her total traveling time was 5 hours?

Word Problems-Day Three

1. Anna invests $2300, part at 6% and the remainder at 7%. Her total interest from the two investments is $153. How much did she invest at each rate?

2. Jackie and Winona are 190 miles apart. They begin to drive toward each other. Winona travels 15 mph faster than Jackie. If they meet in 2 hours, how fast will they be going?

3. One train is 30 km/h faster than another. If both leave A at the same time in opposite directions, they will be 800 km apart after 4 hours. How fast is each train traveling?

4. Bob gets a 12% commission on each new magazine subscription he sells and a 3% commission on each renewal and he got $1350 on total sales of $26,250.
How much did he collect in new subscriptions?

5. Two cars head towards each other at the same time from two points 360 miles apart. If one car averages 40 mph and the other 50 mph, how much time will pass before they meet?

6. Two planes start traveling towards each other from points 1400 miles apart. There is a 120 mph increase in the speed of the first plane over the second plane. The planes meet in $2\frac{1}{2}$ hours. Find the speed of each plane.

7. Sue invested some money in two accounts. For every ten dollars she invested in an account earning 15% interest she invested three dollars in an account earning 12% interest. If she earned $744 in interest after one year, how much did she invest in each account?

8. Todd starts out on a trip traveling 45 mph. A half hour later his wife leaves from the same point and travels the same route at 50 mph. How long does it take his wife to overtake Todd?

9. How can $7500 be invested in two accounts yielding 8% and 12% interest, so that the interest will be the same on each account?

10. If $5000 is invested at 5%, how much additional money must be invested at 7% so that the total yearly interest from both investments is $510?
Appendix E

Pretest

This test will not affect your grade, but you may get bonus points by doing well on it.

Name: _______________  Soc. Sec. # ________________

Birthdate: ___  Gender: (M or F)_______________

ACT Math Score ______________

Native Language: English, Spanish, Other

(specify)____________________

Number of previous Algebra courses taken: 1 2 3 4 5

******************************************************************************

FOR EACH OF THESE PROBLEMS, SHOW ALL OF YOUR WORK:

[The following three items were used to give a score on the students' ability to do proportional reasoning.]

3. A certain recipe calls for 2 teaspoons of salt and 5 cups of flour. If twenty cups of flour are used, how much salt is needed?

4. John can paint 9 rooms using 6 cans of paint. If he paints 15 rooms he will use (?)_______ cans of paint.

(Assume the same size of rooms and the same conditions).

5. On a map, the scale is $\frac{1}{4}$" represents 10 miles. If Troy is 12" from Athens on the map, what is the actual distance between the two cities?
For questions 7-12: Write an equation for each sentence using the indicated variables. (There is no numerical solution required for these).

7. In America, the number of cats is twice the number of dogs. Let C stand for the number of cats and D for the number of dogs.

8. At this college, there are six times as many students as professors. Let S stand for the number of students and P for the number of professors.

9. The first number is three more than twice the square of the second number. Let A stand for the first number and B for the second number.

10. The XYZ company produced 782 more mayrods than widgets. Let M stand for the number of mayrods and W for the number of widgets.

11. Last night at the movies, the feature film took 72 minutes longer than the cartoon. Let F stand for the length of the feature film and C for the length of the cartoon.

12. At Mindy's restaurant, for every four people who ordered cheesecake, there were five who ordered strudel. Let C stand for the number of orders of cheesecake and S for the number of orders of strudel.
Appendix F

Posttest (This was an insert in a regularly scheduled test).

For questions 5-10: Write an equation for each sentence using the indicated variables: (There is no numerical solution required for these).

5. In Franklin, the number of working men is ten times the number of unemployed. Let W stand for the number of working men and U for the number of unemployed.

6. In Louisiana, there are twenty times as many baseballs sold as soccer balls. Let B stand for the number of baseballs and S for the number of soccer balls.

7. The first number increased by two is seven less than ten times the square of the second number. Let P stand for the first number and Q for the second number.

8. The hunters shot 117 more zooks than dillers. Let Z stand for the number of zooks and D for the number of dillers.

9. At Paul's bakery, three more donuts were eaten than rolls. Let D stand for the number of donuts and R for the number of rolls.

10. In this salad dressing, for every three cups of vinegar we use two cups of corn oil. Let V stand for the number of cups of vinegar and C for the number of cups of corn oil.
Appendix G

Retention Test

Name ________________

Write an equation for each sentence using the indicated variables. (There is no numerical solution required for these).

1. Three more than the square of a number is the square of eight less than another number. Let A stand for the first number and B for the second number.

2. The length of the letter is one inch less than the length of the envelope. Let L stand for the length of the letter in inches and E for the length of the envelope in inches.

3. Four less than William's age in years is eight less than twice Mary's age in years. Let W stand for William's age in years and M for Mary's age in years.

4. John is six inches taller than Mary. Let J stand for John's height in inches and M for Mary's height in inches.

5. The number of votes that John received in the election was equal to forty plus the number of votes Mary received. Let J stand for the number of votes John received and M for the number of votes Mary received.

6. Paulo's height is six inches less than the minimum height needed to be on the basketball team. Let P stand for Paulo's height in inches and M for the minimum height in inches.

7. If the rate of the speed boat is increased by six miles per hour it would be three miles per hour faster than the square of the rate of the pirogue. Let S stand for the rate of the speed boat in miles per hour and P for the rate of the pirogue in miles per hour.

8. John is now three times as old as Dick. Let J stand for John's present age in years and D for Dick's present age in years.
age in years.

9. The square of five less than six times a number is one less than the product of two and the square root of another number. Let A stand for the first number and B for the other number.

10. Twice a certain number is nine less than double another number. Let V stand for the first number and W for the other number.

11. When ten more than 5 times the square of a number is doubled the result is two less than a second number cubed. Let P stand for the first number and R for the second number.

12. In the landscaping business there are eight times as many azalea bushes planted as rose bushes. Let A stand for the number of azalea bushes and R for the number of rose bushes.

13. Tony's Novelty factory produced 324 more spepoops than whirligigs. Let S stand for the number of spepoops and W for the number of whirligigs.

14. The height of a triangle is two inches less than half the base. Let H stand for the height of the triangle in inches and B for the base of the triangle in inches.

15. At Wally's pizza parlor, eight Hawaiian pizzas were sold for every three Combos. Let H stand for the number of Hawaiian pizzas and C for the number of combos.
Appendix H
Observer Notes

I, Gary Griffin, a graduate student in psychology at Southeastern Louisiana University was asked to observe three classes for experimenter bias. The classes were conducted in the fall semester of 1990 by Elizabeth Gray. The experiment was testing the conditions of a conceptual orientation versus a syntactic orientation in the translating of algebra word problems.

The first group was an experimental group using the conceptual orientation, and consisted of undergraduate Math 161 students. The group was observed for the week of treatment at 8:00 a.m. on Monday, Wednesday, and Friday. Mrs. Gray consistently arrived at class on time on the days of observation. Mrs. Gray greeted the class in a friendly manner, took roll, and began class. She gave clear, concise instructions and illustrations on how to translate sentences into algebra using the conceptual method, which followed the lesson plan in detail. Mrs. Gray was patient in fielding questions from the class in a genuine pleasant and helpful manner. Mrs. Gray worked at the classes' pace and proceeded only when she felt the majority of the class had understood the previous example. Mrs. Gray would start each class session with a review of the previous session and a short question and answer period.
The second group was a treatment group using the syntactic orientation. This class consisted of undergraduate Math 161 students, who met at 10:00 a.m. on Monday, Wednesday, and Friday. This group met in the same room as the first treatment group. Mrs. Gray arrived on time for class. Again, she greeted the class in a friendly manner, took roll, and began class. Mrs. Gray, once again, gave clear, concise instructions and illustrations on how to translate sentences using the syntactic method, and followed the lesson plan in detail. Mrs. Gray fielded questions from the class in her usual friendly and helpful manner. She worked at the classes' pace and proceeded only when she felt the majority of the class had understood the previous example given. Mrs. Gray followed her same procedure with a review of the previous session, and a short question and answer period for this group also.

The third group was the control group. It was taught how to solve algebra word problems from the technique illustrated in the textbook, *College Algebra* by Dennis T. Christy. This group also consisted of undergraduate Math 161 students. This group met at 12:00 p.m. in a different room from the other two groups. This groups was observed on Monday, Wednesday, and Friday. Mrs. Gray again arrived on time for class on the days of observation. She greeted the class in her usual friendly
manner, took roll, and began class. Mrs. Gray worked from the book with this group. She once again worked at the classes' pace and proceeded when she felt the majority of the class had understood the previous example. Mrs. Gray, as with the other groups, started each class with a review session and a short question and answer period. Again, let it be noted that, Mrs. Gray followed her lesson plan for the control group in teaching them from the book.

For all three groups, the sessions were recorded. The recordings of the sessions for the week of treatment were not kept from the awareness of the students. The subjects in all three groups were aware that Mrs. Gray was recording the sessions for experimental purposes. The subjects, however, were not aware of what the experiment was testing.

On an unscheduled and unannounced visit approximately one month later, I observed Mrs. Gray interacting with her three classes in the same friendly and unbiased manner as when she was conducting her experiment. It is therefore the conclusion of this observer that Mrs. Gray taught each group with the same enthusiasm and friendly manner. She did not bias one of the groups against the other, but taught all of them rather in the same manner.

Submitted: November 28, 1990
During the first week of the Spring semester of 1991, I was again contacted to observe another class taught by Mrs. Gray. This group was also given a syntactic treatment. The class consisted of undergraduate Math 161 students. This group was observed during the treatment on Monday, Wednesday, and Friday. This group met in a different room than any of the three groups in the fall. Mrs. Gray arrived on time for class on the days of the experiment. As with the other groups, she greeted the class in a friendly manner, took roll, and began class. Mrs. Gray gave clear, concise instructions and illustrations on how to translate sentences using the syntactic method which followed the lesson plan in detail. Mrs. Gray fielded questions from the class in a friendly and helpful manner. She worked at the classes' pace and proceeded only when she felt the majority of the class had understood the previous example given. Mrs. Gray, as with the other three groups, started each class with a review of the previous session and a short question and answer period.

On an unscheduled and unannounced visit one and one half months later, I observed Mrs. Gray interacting with her class in the same manner as before. She conducted herself in the same friendly and unbiased manner as she had when she was conducting the experiment. It is still therefore the conclusion of this observer that Mrs. Gray
taught each group with the same enthusiasm and friendly manner. She did not bias this group against any of the other groups, but taught all of them rather in the same unbiased manner.

Submitted March 26, 1991

[Signature]

Gary L. Griffin
Appendix I

Tables of Data for 2-item Test
(S & P and Mindy's type only)

Table 12

Twelve cell means (2 items) for all Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Retention</th>
</tr>
</thead>
<tbody>
<tr>
<td>SYN-1</td>
<td>.189</td>
<td>.800</td>
<td>.800</td>
</tr>
<tr>
<td>SYN-2</td>
<td>.278</td>
<td>.889</td>
<td>.833</td>
</tr>
<tr>
<td>CONCEP</td>
<td>.157</td>
<td>.600</td>
<td>.343</td>
</tr>
<tr>
<td>CONT-1</td>
<td>.283</td>
<td>.283</td>
<td>.283</td>
</tr>
</tbody>
</table>

Note. SYN-1 n=45; SYN-2 N=36; CONCEP n=35; CONT-1 n=30;

The graph of the twelve cell means for the two-item test is given in Figure 10.

![Graph of the twelve cells means (2-items)](image)
The graph indicates that there is some interaction between the groups and the times of the tests, so the expanded summary ANOVA with tests for simple effects for the two-item test is given in Table 13.

Table 13

Expanded Summary ANOVA including Tests of Simple Effects (2-items)

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rows (Treatment)</td>
<td>10.33</td>
<td>3</td>
<td>3.44</td>
<td>24.57*</td>
</tr>
<tr>
<td>Rows (pre)</td>
<td>.42</td>
<td>3</td>
<td>.14</td>
<td>1.00</td>
</tr>
<tr>
<td>Rows (post)</td>
<td>7.14</td>
<td>3</td>
<td>2.38</td>
<td>17.00*</td>
</tr>
<tr>
<td>Rows (ret)</td>
<td>9.07</td>
<td>3</td>
<td>3.02</td>
<td>21.57*</td>
</tr>
<tr>
<td>Within cell</td>
<td>20.57</td>
<td>142</td>
<td>.14</td>
<td></td>
</tr>
<tr>
<td>Columns (Test)</td>
<td>14.00</td>
<td>2</td>
<td>7.00</td>
<td>100.00*</td>
</tr>
<tr>
<td>Columns (SYN-1)</td>
<td>11.20</td>
<td>2</td>
<td>5.60</td>
<td>80.00*</td>
</tr>
<tr>
<td>Columns (SYN-2)</td>
<td>8.21</td>
<td>2</td>
<td>4.11</td>
<td>58.71*</td>
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<tr>
<td>Columns (CONCEP)</td>
<td>3.46</td>
<td>2</td>
<td>1.73</td>
<td>24.71*</td>
</tr>
<tr>
<td>Columns (CONT-1)</td>
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<td>2</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Interaction</td>
<td>6.31</td>
<td>6</td>
<td>1.05</td>
<td>15.00*</td>
</tr>
<tr>
<td>Within cell</td>
<td>19.45</td>
<td>284</td>
<td>.07</td>
<td></td>
</tr>
</tbody>
</table>

Note. 'Significant at .05 level
* Significant at .05/3 level
† Significant at .05/4 level
VITA

Elizabeth deValcourt Gray was born in New Iberia, Louisiana, on March 25, 1939, the daughter of John and Rosa deValcourt. After graduating as Valedictorian from Mount Carmel Academy in 1957, she attended St. Mary's Dominican College in New Orleans, Louisiana. She received the A.B. degree cum laude, in 1963, with a major in mathematics and a minor in education. After seven years of high school teaching in New Orleans, she attended the University of Notre Dame in Notre Dame, Indiana on a National Science Foundation grant, and received the M.S. in mathematics in 1970. In 1970, she married Larry Allen Gray. They are the proud parents of Ian Christopher Gray born in 1972 and Juliet Elizabeth Gray born in 1974. After teaching part-time and full-time at Southeastern Louisiana University, she pursued the Doctor of Philosophy degree in Curriculum and Instruction with the emphasis in Mathematics Education, awarded in December of 1991 at Louisiana State University.
DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: ELIZABETH deVALCOURT GRAY

Major Field: EDUCATION

Title of Dissertation: AN ANALYSIS OF SYNTACTIC SKILLS USED IN TRANSLATING NATURAL LANGUAGE SENTENCES INTO ALGEBRAIC EQUATIONS

Approved:

[Signature]
Major Professor and Chairman

[Signature]
Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

OCTOBER 23, 1991