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The field equation from Newton's law of motion and absence of magnetic monopole

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Abstract

By requiring the linear differential operator in Newton's law of motion to be self adjoint, we obtain the field equation for the linear theory, which is the classical electrodynamics. In the process, we are also led to a fundamental universal chiral relation between electric and magnetic monopoles which implies that the two are related. Thus there could just exist only one kind of charge which is conventionally called electric.

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1 Introduction

The equations of motion for both particles and classical fields are second order differential equations. There exist both the cases of the equation being linear as well as non-linear. The former is the case of the Maxwell field and motion of charged particle in it while the latter is the case of Einstein's

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theory of gravitation with the geodesic motion in curved spacetime for particles. In Einstein's theory, general relativity (GR), the particle equation is derivable from the field equation. Intuitively it could be simply understood as follows, since the field is described by the curvature of spacetime and it no longer remains an external field, the motion of particle under gravity would naturally be free motion relative to curved spacetime which is given by the geodesic equation. Thus for GR, it can be said that the particle equation is contained in the field equation. Note that both the equations are non linear. It raises an interesting question, does there also exist some relation between the equation of motion of particle and field in the case of the linear equation? The only linear theory for a classical field is the Maxwell's theory of electromagnetism. That is to probe for a connection between the equations of motion of the charged particle and the Maxwell electromagnetic field. Since the equation is linear, one cannot contain the other as was the case for Einsteinian gravitation. However, could one lead to the other and under what conditions? This is precisely the question we wish to address in this paper.

Let us then ask what does a linear differential equation allow us to do which a non linear equation does not? It allows us to construct adjoint of the differential operator, which can be done only for the linear operator. How about asking for the self adjointness of the operator? This property is most effectively used in the quantum theory to ensure reality of the eigenvalues. In the classical mechanics, it is used in identifying the velocity dependent potential which could be included in the Lagrangian¹. In general, it is not possible to incorporate dissipative forces in a Lagrangian (in some cases introduction of the Lagrange multipliers facilitates), however a particular kind of velocity dependent potential is permitted which is picked up by the self adjointness of the operator. The linearity of the equation implies that force on the right of the Newton's second law equation should involve the velocity linearly and so should be the case for the velocity dependent potential in the Lagrangian as well. That means that the potential in the Lagrangian would in addition to a scalar have a scalar term involving velocity linearly. More elegantly, all this is ensured simply by asking that the linear differential operator in the Newtonian equation of motion for particle is self adjoint. First, it would ensure existence of Lagrangian, second, it would determine the force law (in the Lorentz force form) involving two vector fields, one polar and the other axial and then their derivation in terms of the scalar and vector potentials given by the homogeneous source free set of the Maxwell

equations. Note that without any reference to the Maxwell theory, purely from the general mechanics considerations follow the force law and half of the Maxwell's equations. All this would be true for any linear field theory and the Maxwell electrodynamics happens to be one such theory.

Following Dyson's paper² discussing Feynman's derivation of the homogeneous Maxwell equations, there has been spurt of activity in recent times in this direction. Feynman used commutation relations between coordinates and velocities rather than canonical momenta. It was soon realized that the problem is related to existence of an *Action/Lagrangian* for a given equation of motion. Since then a lot of effort has gone into building a relativistic generalization of Feynman's proof and its extension to non-Abelian gauge theories and to curved space³⁻⁵. All these attempts (with the sole exception of a recent paper⁶ in which it has been shown that by incorporating magnetic monopoles in Feynman's formalism it is possible to derive the complete set of generalized Maxwell equations, though certain questions remain unanswered) refer only to the homogeneous set which we have simply got in by demanding self adjointness of the Newton's second law. Our main concern is thus to obtain the remaining two Maxwell equations which describe the dynamics of the field. This we address in the more general context of seeking relationship between the equations of motion for particle and field for the linear equation.

It may be noted that all these attempts involved commutation relations and quantum theory considerations. We would however like to stick to the classical mechanics and some simple general considerations. The main question is, could we do something imaginative to the homogeneous set which involves two vector fields, one each of polar and axial kind. In a field theory, field is produced by a source which is generally called charge. Without prejudice to the one or the other, let us consider the corresponding monopole charges for the both polar (scalar) and axial (pseudo scalar) fields. This will allow us to write each vector field in terms of a polar and axial (new) vectors by involving scalar and pseudo scalar charges. That is we expand our system from two to four fields and the two kinds of charges. Substitute this in the two homogeneous equations. We are led to to four equations which in parts look like the Maxwell equations in four vectors. It thus becomes highly under determined system. To proceed any further we have to contract the system back to the two fields which we do by postulating linear relations between the two pairs of polar and axial vector fields. These proportionality relations

give rise to a constant which has the dimension of velocity. That is how an invariant speed has come up. A polar field is produced when a charge is at rest and an axial field is produced when it is in motion. Similarly a pseudo charge at rest would produce axial field and polar when moving. That means a polar/axial field could be produced by a stationary scalar/pseudo charge as well as by a moving pseudo/scalar charge. However to a test charge, it is simply a polar/axial field irrespective of its source. The field produced in these two different ways must be indistinguishable and hence there must exist a (chiral) universal relation between scalar and pseudo scalar charges. Then the set of equations in question becomes the complete set of the Maxwell's equations and the force law, the Lorentz force of the electrodynamics. We have thus obtained the equation of motion for the field corresponding to self adjoint Newton's law of motion. This is the complete set of the Maxwell's equations of classical electrodynamics. Most importantly, our method also leads to an important and profound result that electric and magnetic monopole cannot exist independently thereby implying that there could occur only one kind of charge, call it electric or magnetic. The linear theory consistent with the Newton's law could thus have only one kind of monopole charge.

The paper is organized as follows. In the next Sec., we briefly recall the discussion of self adjointness of the second order differential operator and the inverse problem in classical mechanics, which lead to the Lorentz-like force with the homogeneous set of two equations. In Sec. III, we derive the intermediate set which is Galilean invariant followed by in Sec. IV the derivation of the entire set of the Maxwell equations and the fundamental relation between electric and magnetic charges. We conclude with a discussion of general issues and the ones to be taken up in future.

2 Self adjointness and the inverse problem

The inverse problem in classical mechanics deals with the demand of a Lagrangian for a given equation of motion. It turns out that the sufficient condition for the existence of a Lagrangian is that the equation of motion is self adjoint . Let \mathcal{F}_i be a system of second order linear differential equations,

$$\mathcal{F}_i(t, q, \dot{q}, \ddot{q}) = 0. \quad i = 1, 2, \dots, n. \quad (1)$$

If $M(u)$ is a linear differential expression then the adjoint of $M(u)$, which we denote by $\bar{M}(u)$, satisfies the Lagrange Identity

$$\bar{v}M(u) - u\bar{M}(\bar{v}) = \frac{d}{dt}Q(u, \bar{v}). \quad (2)$$

Let $M(u)$ be of the form

$$M_i(u) = a_{ik}u^k + b_{ik}\frac{du^k}{dt} + c_{ik}\frac{d^2u^k}{dt^2} \quad (3)$$

where $a_{ik} = \frac{\partial \mathcal{F}_i}{\partial q^k}$, $b_{ik} = \frac{\partial \mathcal{F}_i}{\partial \dot{q}^k}$, and $c_{ik} = \frac{\partial \mathcal{F}_i}{\partial \ddot{q}^k}$. It is then straightforward to check using the Lagrange identity that

$$\bar{M}_i(\bar{v}) = a_{ik}\bar{v}^k - \frac{d}{dt}(b_{ik}\bar{v}^k) + \frac{d^2}{dt^2}(c_{ik}\bar{v}^k) \quad (4)$$

$$Q(u, \bar{v}) = \left[\bar{v}^i b_{ik} u^k + \bar{v}^i a_{ik} \frac{du^k}{dt} - u^k \frac{d}{dt}(\bar{v}^i a_{ik}) \right]. \quad (5)$$

In the case when $M(u) = \bar{M}(u)$ for all values of u , then $M(u)$ is termed as self adjoint. It turns out that necessary and sufficient conditions for eq.(1) to be self adjoint and hence for Lagrangian to exist are ^{4,7}

$$\frac{\partial \mathcal{F}_i}{\partial \ddot{q}^j} = \frac{\partial \mathcal{F}_j}{\partial \ddot{q}^i} \quad (6)$$

$$\frac{\partial \mathcal{F}_i}{\partial \dot{q}^j} + \frac{\partial \mathcal{F}_j}{\partial \dot{q}^i} = \frac{d}{dt} \left(\frac{\partial \mathcal{F}_i}{\partial \ddot{q}^j} + \frac{\partial \mathcal{F}_j}{\partial \ddot{q}^i} \right) \quad (7)$$

$$\frac{\partial \mathcal{F}_i}{\partial q^j} - \frac{\partial \mathcal{F}_j}{\partial q^i} = \frac{1}{2} \frac{d}{dt} \left(\frac{\partial \mathcal{F}_i}{\partial \dot{q}^j} - \frac{\partial \mathcal{F}_j}{\partial \dot{q}^i} \right). \quad (8)$$

Eqs. (6 - 8) are known as the Helmholtz conditions and for the Newtonian equation of motion, we write \mathcal{F}_i as

$$m\ddot{q}_i - F_i(t, q, \dot{q}) = 0 \quad (9)$$

where F_i is the force experienced by a test particle. Substitution of eq.(9) in eq.(7) yields

$$\frac{\partial F_i}{\partial \dot{q}^j} + \frac{\partial F_j}{\partial \dot{q}^i} = 0. \quad (10)$$

If we substitute eq.(9) in eq.(8) we obtain

$$\begin{aligned} \frac{\partial F_i}{\partial q^j} - \frac{\partial F_j}{\partial q^i} &= \frac{1}{2} \left(\frac{\partial}{\partial t} + \dot{q}^k \frac{\partial}{\partial q^k} + \ddot{q}^k \frac{\partial}{\partial \dot{q}^k} \right) \left[\frac{\partial F_i}{\partial \dot{q}^j} - \frac{\partial F_j}{\partial \dot{q}^i} \right] \\ &= \frac{1}{2} \left(\frac{\partial}{\partial t} + \dot{q}^k \frac{\partial}{\partial q^k} \right) \left[\frac{\partial F_i}{\partial \dot{q}^j} - \frac{\partial F_j}{\partial \dot{q}^i} \right] \\ &+ \frac{1}{2} \ddot{q}^k \left[\frac{\partial^2 F_i}{\partial \dot{q}^j \partial \dot{q}^k} - \frac{\partial^2 F_j}{\partial \dot{q}^i \partial \dot{q}^k} \right]. \end{aligned} \quad (11)$$

Since, the left hand side of the above equation is independent of accelerations, hence, for it to hold we must have

$$\frac{\partial^2 F_i}{\partial \dot{q}^j \partial \dot{q}^k} - \frac{\partial^2 F_j}{\partial \dot{q}^i \partial \dot{q}^k} = 0 \quad (12)$$

$$\frac{\partial F_i}{\partial q^j} - \frac{\partial F_j}{\partial q^i} = \frac{1}{2} \left(\frac{\partial}{\partial t} + \dot{q}^k \frac{\partial}{\partial q^k} \right) \left[\frac{\partial F_i}{\partial \dot{q}^j} - \frac{\partial F_j}{\partial \dot{q}^i} \right]. \quad (13)$$

From eqs.(10 & 12), it is easily seen that

$$m\ddot{q}_i = \lambda_i(t, q) + \xi_{ij}(t, q)\dot{q}^j \quad (14)$$

which when substituted in eqs.(10, 12 & 13) leads to

$$\xi_{ij} + \xi_{ji} = 0 \quad (15)$$

$$\frac{\partial \xi_{ij}}{\partial q^k} + \frac{\partial \xi_{jk}}{\partial q^i} + \frac{\partial \xi_{ki}}{\partial q^j} = 0 \quad (16)$$

$$\frac{\partial \xi_{ij}}{\partial t} = \frac{\partial \lambda_i}{\partial q^j} - \frac{\partial \lambda_j}{\partial q^i}. \quad (17)$$

Eqs.(14 - 17) are the necessary and sufficient conditions for existence of a Lagrangian for the Newtonian equation of motion. If we define

$$\lambda_i \equiv \mathcal{X}_i \quad (18)$$

and

$$\xi_{ij} \equiv \epsilon_{ijk} \mathcal{Y}^k, \quad (19)$$

then eqs.(14, 16 & 17) can be written in the vector form as

$$\vec{F} = \vec{\mathcal{X}} + \vec{v} \times \vec{\mathcal{Y}} \quad (20)$$

$$\vec{\nabla} \times \vec{\mathcal{X}} = -\frac{\partial \vec{\mathcal{Y}}}{\partial t} \quad (21)$$

$$\vec{\nabla} \cdot \vec{\mathcal{Y}} = 0. \quad (22)$$

It is important to note here that $\vec{\mathcal{X}}$ and $\vec{\mathcal{Y}}$ are any arbitrary fields experienced by a test particle and eqs.(20 - 22) will hold for *any* Newtonian force which has self adjoint equation of motion. These are the equations which were derived by Feynman in 1948 by assuming the commutation relation between coordinates and velocities. However, these equations can also be obtained by assuming the similar Poisson bracket relations ^{5,8}.

3 The Galilean invariant intermediate set of equations

In eqs.(20 - 22), we have the Lorentz force and the homogeneous set of the Maxwell equations for the two fields involved. For derivation of the complete set of the Maxwell equations, we need only to bring the remaining two equations. This we shall do by first splitting the two vector fields into four and then recombining them. We note that \vec{F} is a polar vector and so is $\vec{\mathcal{X}}$ while $\vec{\mathcal{Y}}$ is axial. We further decompose the vectors $\vec{\mathcal{X}}$ and $\vec{\mathcal{Y}}$ in terms of two polar $\vec{\mathcal{E}}$ & $\vec{\mathcal{D}}$ and two axial $\vec{\mathcal{B}}$ & $\vec{\mathcal{H}}$ vector fields as follows.

$$\vec{\mathcal{X}} = q_s \vec{\mathcal{E}} + q_p \vec{\mathcal{H}} \quad (23)$$

$$\vec{\mathcal{Y}} = q_s \vec{\mathcal{B}} - q_p \vec{\mathcal{D}} \quad (24)$$

where q_s indicates a constant scalar charge and q_p the constant pseudo-scalar charge.

Substituting them in eqs.(20 - 22), we obtain

$$\vec{F} = q_s(\vec{\mathcal{E}} + \vec{v} \times \vec{\mathcal{B}}) + q_p(\vec{\mathcal{H}} - \vec{v} \times \vec{\mathcal{D}}) \quad (25)$$

$$\vec{\nabla} \times \vec{\mathcal{E}} = -\frac{\partial \vec{\mathcal{B}}}{\partial t} \quad (26)$$

$$\vec{\nabla} \cdot \vec{\mathcal{B}} = 0 \quad (27)$$

$$\vec{\nabla} \times \vec{\mathcal{H}} = \frac{\partial \vec{\mathcal{D}}}{\partial t} \quad (28)$$

$$\vec{\nabla} \cdot \vec{\mathcal{D}} = 0. \quad (29)$$

This is the intermediate set which is Maxwellian like but not quite as it involves four independent vector fields. It can be easily checked that this set is invariant under the Galilean transformation because

$$\vec{\nabla}' = \vec{\nabla} \quad (30)$$

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla}. \quad (31)$$

The covariance of the force law (20) determines the following laws of transformations for the vector fields involved.

$$\vec{\mathcal{E}}' = \vec{\mathcal{E}} + \vec{V} \times \vec{\mathcal{B}} \quad (32)$$

$$\vec{\mathcal{B}}' = \vec{\mathcal{B}} \quad (33)$$

$$\vec{\mathcal{H}}' = \vec{\mathcal{H}} - \vec{V} \times \vec{\mathcal{D}} \quad (34)$$

$$\vec{\mathcal{D}}' = \vec{\mathcal{D}}. \quad (35)$$

4 The Maxwell equations and the fundamental relation

Clearly we cannot proceed further from the intermediate set (26-29) because it is under determined, four differential relations for four vector fields. In fact twice as many would be required for the system to be solvable. For determining a vector field both its divergence and curl must be given. Thus the integrability condition for the system requires that there must exist the linear relations between the two polar and two axial vectors. Secondly, field produced by a stationary scalar charge or a moving pseudo scalar charge must be indistinguishable for a test particle. This would also demand a chiral relation between the charges. Thus we write

$$\vec{\mathcal{D}} = \epsilon \vec{\mathcal{E}} \quad (36)$$

$$\vec{\mathcal{B}} = \mu \vec{\mathcal{H}} \quad (37)$$

and

$$q_p = \sqrt{\mu/\epsilon} q_s \tan \theta. \quad (38)$$

Here ϵ and μ are constants and $(\mu\epsilon)^{1/2}$ has dimensions of velocity. It is to be noted that in the context of charge quantization Schwinger⁹ proposed a similar relation for dyons (particles carrying both electric and magnetic charge). Clearly, our context is entirely different from that of Schwinger. Substituting the above relations in the intermediate set (25-29), we obtain

$$\vec{F} = q_s(\vec{\mathcal{E}} + \vec{v} \times \vec{\mathcal{B}}) + \frac{q_p}{\mu}(\vec{\mathcal{B}} - \mu\epsilon\vec{v} \times \vec{\mathcal{E}}) \quad (39)$$

$$\vec{\nabla} \times \vec{\mathcal{E}} = -\frac{\partial \vec{\mathcal{B}}}{\partial t} \quad (40)$$

$$\vec{\nabla} \cdot \vec{\mathcal{B}} = 0 \quad (41)$$

$$\vec{\nabla} \times \vec{\mathcal{B}} = \mu\epsilon \frac{\partial \vec{\mathcal{E}}}{\partial t} \quad (42)$$

$$\vec{\nabla} \cdot \vec{\mathcal{E}} = 0. \quad (43)$$

Using eq.(38) the force law can now be rewritten as

$$\vec{F} = q_s(\vec{\mathcal{E}} + \vec{v} \times \vec{\mathcal{B}}) + (\mu\epsilon)^{-1/2} q_s \tan \theta (\vec{\mathcal{B}} - \mu\epsilon\vec{v} \times \vec{\mathcal{E}}) \quad (44)$$

or

$$\vec{F} = q_s(\vec{\mathcal{E}} + (\mu\epsilon)^{-1/2} \tan \theta \vec{\mathcal{B}}) + q_s(\vec{v} \times (\vec{\mathcal{B}} - (\mu\epsilon)^{1/2} \tan \theta \vec{\mathcal{E}})). \quad (45)$$

We now define two fields \vec{E} & \vec{B} such that

$$\vec{E} \equiv \vec{\mathcal{E}} + (\mu\epsilon)^{-1/2} \tan \theta \vec{\mathcal{B}} \quad (46)$$

$$\vec{B} \equiv \vec{\mathcal{B}} - (\mu\epsilon)^{1/2} \tan \theta \vec{\mathcal{E}} \quad (47)$$

and hence

$$\vec{F} = q_s(\vec{E} + \vec{v} \times \vec{B}). \quad (48)$$

Inversely, one can rewrite eqns(46 & 47) as

$$\vec{\mathcal{E}} = \cos^2 \theta \vec{E} - (\mu\epsilon)^{-1/2} \cos \theta \sin \theta \vec{B} \quad (49)$$

$$\vec{\mathcal{B}} = \cos^2 \theta \vec{B} + (\mu\epsilon)^{1/2} \cos \theta \sin \theta \vec{E}. \quad (50)$$

Substituting these identifications of $\vec{\mathcal{E}}$ & $\vec{\mathcal{B}}$ in eqs.(40 & 41), we obtain

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = \tan \theta \left[(\mu\epsilon)^{-1/2} \vec{\nabla} \times \vec{B} - (\mu\epsilon)^{1/2} \frac{\partial \vec{E}}{\partial t} \right] \quad (51)$$

$$\vec{\nabla} \cdot \vec{B} = -(\mu\epsilon)^{1/2} \tan \theta \vec{\nabla} \cdot \vec{E} \quad . \quad (52)$$

Similarly by substituting eqs.(49 & 50) in eqs.(42 & 43) we obtain

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = -\cot \theta \left[(\mu\epsilon)^{-1/2} \vec{\nabla} \times \vec{B} - (\mu\epsilon)^{1/2} \frac{\partial \vec{E}}{\partial t} \right] \quad (53)$$

$$\vec{\nabla} \cdot \vec{B} = (\mu\epsilon)^{1/2} \cot \theta \vec{\nabla} \cdot \vec{E} \quad . \quad (54)$$

The consistency of eqs.(51 - 54) demands

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (55)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (56)$$

$$\vec{\nabla} \times \vec{B} = \mu\epsilon \frac{\partial \vec{E}}{\partial t} \quad (57)$$

$$\vec{\nabla} \cdot \vec{E} = 0 \quad (58)$$

which is the complete Maxwell set for the electric and magnetic fields, \vec{E} and \vec{B} . Further, note that the Lorentz force law we have obtained could have equally been written down in terms of the pseudo-scalar charge as,

$$\vec{F} = \frac{q_p}{\mu} (\vec{B} - \mu\epsilon \vec{v} \times \vec{E}) \quad (59)$$

and the physics would have remained unchanged. That is, once q_s and q_p are related through eq.(38), it is only a matter of convention how does one write the force law and identifies ‘electric’ and ‘magnetic’ fields. This relation means that if electric and magnetic charges exist, then they must be

related and ultimately there is only one independent charge, call it electric or magnetic. Further, since q_s and q_p are constants, we immediately deduce from eq.(38) that

$$\left(\frac{\mu}{\epsilon}\right)^{1/2} \tan \theta = K \quad (60)$$

where K is a fundamental constant. Hence, we predict that the product of the characteristic impedance $\sqrt{\mu/\epsilon}$ times $\tan \theta$ is a fundamental constant.

5 Discussion

We have used the self-adjointness of the differential operator in the Newton's law and have obtained the half (homogenous) of the Maxwell equations. In more familiar terms, alternatively this is equivalent to demanding that the force is linear in velocity and is derivable from an appropriate velocity dependent potential. To get the other half of the equations, we resort to the generality that there is a priori no reason to prefer one kind of charge over the other. Since there occur polar and axial vector fields, they would be produced by the corresponding scalar and pseudo scalar charges. That would lead to the two sets of the homogenous equations in four vector fields; a pair of polar and axial vectors corresponding to the each kind of charge. This means that there are four equations for four vector fields. It is therefore under determined and consequently unsolvable. Finally the solvability of the system requires that the polar and axial vectors must bear a linear relation between them. Then the other set reduces to the other half of the Maxwell equations for free space. Further it would also lead to the chiral relation between the scalar and pseudo scalar charges implying that only one kind of charge could exist.

Magnetic monopole was first introduced by Dirac^{10,11} and it was envisioned as one end of an infinite string of dipoles or a solenoid. It did a wonderful job of quantizing electric charge even if one such entity existed in the whole Universe. The idea soon became famous and even found a rightful place in college textbooks¹². However, if on one side Dirac's monopoles had a strong support and were eagerly sought by experimentalists, on the other side it was shown that a theory containing them cannot be derived from an action principle^{13,14}. Further they led to singularity problems related to strings¹⁵ and nor do they fit well in the quantum electrodynamics^{16,17}. Thus

magnetic monopole has become an enigma, for it is required for the charge quantization but it could not successfully be accommodated in the existing theories.

The idea that a charged particle can be envisioned as carrying both electric and magnetic charge was already there in classical physics¹², however in contrary to what is prevalent in literature¹², we have convincingly shown that both of these charges can not exist independently. One is simply the dual of the other and what is being observed is conventionally called ‘electric’. Hence, there is now little doubt on why the search for Dirac’s monopole has been futile for last 70 years.

Of course the question of quantization remains. For that we have to appeal to some quantum principle or relation. By using the Dirac quantization condition and the fine structure constant, it is straight forward to write our fundamental relation, eq.(38). In SI units the Dirac quantization condition is

$$q_s q_p = n h. \quad (61)$$

where n is an integer. Using eq.(61) in eq.(60), we find the impedance K as

$$K = n \frac{h}{q_s^2} = 2.5883... \times 10^4 n \text{ ohms}. \quad (62)$$

Using eq.(61) in the fine structure constant relation

$$\alpha = \frac{q_s^2}{4\pi\epsilon\hbar c} \quad (63)$$

we get

$$\frac{q_p}{\mu} = \frac{n}{2\alpha} q_s c. \quad (64)$$

Defining

$$\tan \theta = \frac{n}{2\alpha} \quad (65)$$

leads to our relation $q_p/\mu = q_s c \tan \theta$. Conversely, let us begin with the above relation and write it as $q_s q_p = \mu q_s^2 c \tan \theta$, divide both sides by $4\pi\epsilon\hbar c$ and choose $\tan \theta = n/2\alpha$ to obtain the Dirac quantization condition (61). Hence, if we take some relation from the quantum theory, then the charge quantization readily follows.

A natural extension of this formalism would be for the case of particles having internal spin. It is interesting to note that by taking the limit of massive wave equation for a spin-1 object, a Maxwellian theory can be obtained^{18,19}. Further, it has been shown that the linearity of wave equations is directly related to the spin for massive particles²⁰. It would be worth while to probe whether all these issues can be reconciled with the technique proposed in this work.

The first instance of relation between the particle and field equations was in GR where the former followed from the latter. Here we have followed the reverse route and have obtained the latter from the former under certain general and reasonable assumptions. The question is, could for GR as well this path be followed ? The Maxwell field equations followed by demanding the force law to be linear in velocity and derivable from a potential. Let us first implement this for the relativistic particle equation, then the Maxwell equations follow very elegantly and cogently. Now ask what field equation would follow from a quadratic in velocity force law ? It would turn out to be the Einstein's equations for gravitation. This would be published soon separately²¹.

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