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1-1-2004

Big crunch avoidance in k = 1 semiclassical loop quantum cosmology

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ter with $\omega \geq 0$ [10] and the transition to a contracting phase proceeds. We should also notice that a wide family of potentials without a local minima invoked to explain issues of dark energy (see [11, 12, 13] for reviews) lead to eternal expansion even in the case of k = 1, and such models do not face the big crunch problem. This is also true for some potentials with local minima as has been shown in [14] & [15]. In this work we will, however only consider potentials with a local mimina in a framework which leads classically to a big crunch.

An important question which thus arises is whether a collapsing universe with a scalar field experiences a bounce or ends in a big crunch. Answer to this depends not only on potential $V(\phi)$, but also on a particular phase trajectory chosen. Bounce is indeed possible for potentials which are not very steep, but detailed investigations have shown, that this in general requires a severe fine-tuning of initial conditions. Moreover, the revealed chaoticity of such cosmological dynamics [9] indicates, that a bounce in the end of one cosmological cycle reflects nothing about a possibility to have a bounce during the next contraction stage of the universe. To ensure two consecutive bounces it is therefore necessary to further constrain the initial conditions. As a result only a zero measure set of initial conditions gives us a possibility to have infinite number of bounces, and then to escape a singularity for an infinite time interval. This leads to a conclusion that the future singularity problem can not be solved in classical cosmology with a massive scalar field.

Occurrence of singularities in classical cosmology has always been thought of as a signature of a domain where general relativity ceases to be operational and must be replaced by a quantum theory of gravity. In the present paper we try to understand dynamics leading to big crunch in light of one of the approaches to quantize general relativity known as loop quantum gravity. It is based on canonical quantization of general relativity and its salient features include background independence, non-perturbativeness and prediction of discrete spectrum for geometrical operators [16]. This theory has recently been applied in cosmological scenarios and has yielded various novel results which include absence of big bang singularity [17], insights on initial conditions of the universe

[18], possibility of inflation [19] with right e-foldings [20], suppression of power at large scales in CMB [20] etc. For related works in this direction see for example [21].

One of the key features of loop quantum gravity is that it predicts a modification in the behavior of geometrical density a^{-3} at short distances. Unlike in the conventional quantum cosmology where the geometrical density blows up when $a \longrightarrow 0$, in loop quantum cosmology the eigenvalues of the corresponding operator become zero at $a = 0$. We should note that near $a \sim 0$ the picture of a continuous spacetime breaks down and evolution is described by difference equations, however, there exists a domain where the continuous spacetime picture is valid and quantum gravity effects are manifest. The eigenvalues of geometrical density operator in loop quantum cosmology are given as [17],

$$d_j(a) = D(a^2/a_*^2)a^{-3}\,,\ \ a_*^2 = \gamma \ell_{\mathrm{pl}}^2\, j/3\,, \tag{1}$$

where a is the scale factor, ℓ_{pl} is the Planck length, γ is the Barbero-Immirzi parameter and j is a half-integer greater than unity. Here the function $D(a^2/a_*^2)$ is derived from quantum theory and is given by,

$$
\begin{aligned}
D(q) &= (8/77)^6 q^{3/2}\{7[(q+1)^{11/4} - |q-1|^{11/4}] \\
&\quad - 11q[(q+1)^{7/4} - \mathrm{sgn}\,(q-1)|q-1|^{7/4}]\}^6,
\end{aligned} \tag{2}
$$

where $q := a^2/a_*^2$. It should be noted that the scale where quantum effects become significant is defined by a_* and though the Barbero-Immirzi parameter can be fixed to $\gamma = \ln 2/(\sqrt{3}\pi)$ by black hole thermodynamics, the parameter j is arbitrary and can be used to set an effective quantum gravity scale in the theory which may even be bigger than ℓ_{pl}.

The density operator in eq.(1) has various peculiar features. In the quantum domain when $a \ll a_*$ it behaves as

$$d_j \approx (12/7)^6 (a/a_*)^{15} a^{-3}\,. \tag{3}$$

In transition to the classical regime when a increases, d_j effectively becomes proportional to lower powers of a and eventually becomes $d_j \approx a^{-3}$ when $a >> a_*$, which signals onset of a classical regime.

The Hamiltonian of a massive scalar field ϕ with a potential $V(\phi)$ in loop quantum cosmology can be written as

$$\mathcal{H} = d_j \frac{p_\phi^2}{2} + a^3 V(\phi)\,, \tag{4}$$

where p_ϕ is the canonically conjugate momentum to ϕ and satisfies $p_\phi = d_j^{-1}\dot{\phi}$. For the case of k = 1 FRW model the effective Friedmann equation becomes

$$H^2 = \frac{8\pi}{3M_{\mathrm{pl}}^2}\left(\frac{1}{D}\frac{\dot{\phi}^2}{2} + V(\phi)\right) - \frac{1}{a^2} \tag{5}$$

together with the modified Klein-Gordon equation

$$
\begin{aligned}
\ddot{\phi} &= a\, H\, \dot{\phi}\, \frac{d}{da}\ln d_j - a^3\, d_j\, V_{,\phi} \\
&= \left(-3H + \frac{\dot{D}}{D}\right)\dot{\phi} - D\, V_{,\phi}
\end{aligned} \tag{6}
$$

which yield the Raychaudhuri equation with quantum gravity correction as

$$
\begin{aligned}
\frac{\ddot{a}}{a} &= -\frac{2\pi}{3M_{\mathrm{pl}}^2}\left(\frac{1}{D}\dot{\phi}^2 - \frac{1}{a^2}\dot{\phi}^2\frac{d}{da}d_j\right) + \frac{8\pi}{3M_{\mathrm{pl}}^2}\, V(\phi) \\
&= -\frac{8\pi}{3M_{\mathrm{pl}}^2 D}\dot{\phi}^2\left(1 - \frac{\dot{D}}{4HD}\right) + \frac{8\pi}{3M_{\mathrm{pl}}^2}\, V(\phi)\,. \tag{7}
\end{aligned}
$$

Before we study some interesting phenomenon due to quantum gravity modifications in above dynamical equations, it is important to emphasize the semi-classical nature of these equations. As stressed earlier, the picture of a continuous spacetime breaksdown near $a \sim 0$, where the description is entirely in terms of spin network states. Hence, the dynamical equations (5,6,7) are not valid very close to $a \sim 0$. This is the regime where full quantum gravity is functional and the evolution can be described only in terms of quantum difference equations. The dynamical equations are thus semi-classical in nature which are valid in the domain around a_*, where we have a continuous spacetime but with quantum gravity modifications. It is also interesting to note that the full theory of loop quantum gravity does not have a notion of external time which is present at the semi-classical level, so the notion of trajectories which are solutions of the dynamical equations is invalid near $a \sim 0$. Thus if we wish to resolve the initial big bang singularity then one has to do a full quantum gravity treatment based on discrete form of the Wheeler-DeWitt equation [17]. However, as we would show, to avoid a big crunch singularity, the modifications of dynamical equations at the above semi-classical level are sufficient.

Using eq.(1) it is easy to see that the $\dot{\phi}$ term in Klein-Gordon equation changes its sign as the universe evolves from a quantum regime to a classical regime or vice versa. This behavior is crucial in establishing various key results in loop quantum cosmology. For example, if the universe is expanding from a quantum regime the usual friction term in the Klein-Gordon equation is replaced by an anti-friction term which is responsible for an inflaton to climb the potential hill, even if it starts from its bottom, as has been shown for the flat model [19, 20] and the close model [22]. It is important to note that the potential term in Klein-Gordon equation becomes sub-dominant in quantum domain, since the function $D \ll 1$ for $a \ll a_*$.

If we use the conventional Klein-Gordon equation without quantum correction in the case when universe is collapsing, then the $\dot{\phi}$ term in classical regime acts like an anti-friction term and the scalar field acquires very high values just before big crunch. However, in loop quantum

cosmology this situation will be prevented since when a collapsing universe reaches a size comparable or less than a_*, the otherwise anti-friction term becomes a frictional term with a higher magnitude which stops the scalar field and leads to a bounce. In the quantum regime, the strong dominance of the kinetic term over the potential term means not only that the mechanism is robust to a change in $V(\phi)$, but also that it dominates over gradient terms in scalar field.

To demonstrate the effect of loop quantum gravity on big crunch let us first consider the case of a quadratic potential

$$V(\phi) = \frac{1}{2}\, m_\phi^2\, \phi^2 \ . \tag{8}$$

In the classical regime the equation of a massive scalar field (6) describes the behavior of a harmonic oscillator. A detailed analysis of cosmological solutions in this case can be found for example in [4]. It is important to note that in the classical regime when the $\dot\phi$ term is anti-frictional during collapse, almost all initial conditions lead to (possibly after a period of growing scalar field oscillations) a regime where the potential term in Friedmann equation (5) is negligible in comparison to the kinetic term. In this domain,

$$H = \frac{1}{3(t_0 - t)}, \qquad |\phi| = \frac{M_{\rm pl}}{\sqrt{12\pi}} \ln \frac{t_0 - t}{c} \tag{9}$$

and a singularity is reached at some moment $t = t_0$, where t_0 and c are constants of integration. Some trajectories can avoid this regime and experience bounce, giving rise to an interesting fractal structure of the phase space (see [6, 7, 9] for details), but their measure is very small. Such a possibility is approximately inversely proportional to $a_{max}/\ell_{\rm pl}$, where a_{max} is the scale factor at the point of maximal expansion [4].

Solving eqs.(5 - 7) along with eq.(1) numerically we can understand the change in dynamics due to quantum gravity effects and as expected, the change from the classical equations is very drastic. The evolution of scale factor for the quadratic potential is shown in Fig. [1]. We consider initial conditions such that the universe encounters a big crunch classically (as shown by a dashed curve). The evolution of scalar field with respect to scale factor is shown in Fig. [2]. As can be seen near the big crunch the field ϕ takes very large values. It is clear from both of these figures that this situation is averted in quantum gravity.

When the size of the universe becomes comparable to a_* and less, the quantum corrections become important in dynamical equations. As mentioned earlier, in the collapsing case the anti-friction term which was $-3H\dot\phi$ (with negative H) in classical regime, changes sign, becomes frictional and even goes as $12H\dot\phi$ for $a \ll a_*$. In this process, quantum gravity essentially applies breaks to the motion of ϕ through a large friction term in Klein-Gordon equation and the scalar field almost completely
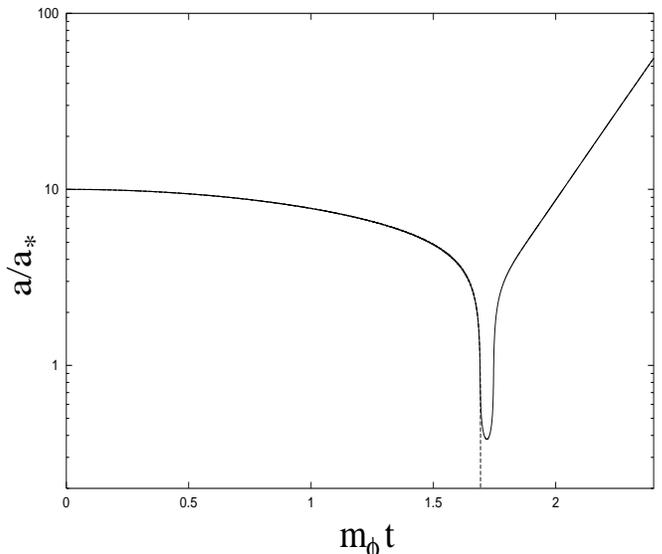

FIG. 1: Evolution of scale factor for a closed universe in classical cosmology is shown by the dashed curve and in loop quantum cosmology is shown by the solid curve for the quadratic potential with $j = 100$, $m_\phi = 0.1 M_{\rm pl}$ and initial conditions $a_i = 10\, a_*, H_i = 0, \phi_i = 0$ and $\dot\phi_i$ determined from eq.(5). A collapsing closed universe in classical cosmology encounters a big crunch whereas in loop quantum cosmology it bounces into an expanding phase when $a = 0.38\, a_*$ and avoids big crunch.

"freezes" at some high value ϕ_f. At the instant when the motion of ϕ ceases and $\dot\phi = 0$, all quantum gravity signatures are excluded from the model and equation of state becomes -1. Such equation of state in the case of positive spatial curvature ultimately leads to a bounce. So, the contracting universe bounces and turns to expansion.

After the bounce when the universe is still smaller than a_*, the $\dot\phi$ term in eq.(6) acts like an anti-friction term (since, the Hubble parameter H changes sign to positive in expanding phase). The scale factor in this phase grows very rapidly and eventually as $a \gg a_*$, the quantum corrections become negligible and the $\dot\phi$ term becomes frictional. The universe then enters a standard inflationary stage. An example for this behavior can be seen in variation of scale factor in Fig. [1]. After checking a large range of initial conditions and parameters j and m_ϕ we found that a bounce always occurs and the behavior of dynamics is qualitatively the same as shown in Figs. [1 & 2].

With the occurrence of bounce established for the quadratic potential in a generic way we now examine the case for a steep potential,

$$V(\phi) = 2 \left(\cosh(\phi^2/M_{\rm pl}^2) - 1 \right) \ . \tag{10}$$

The dynamics of a scalar field in classical cosmology is rather different for potentials less and more steep than the exponential one. A steep potential never becomes
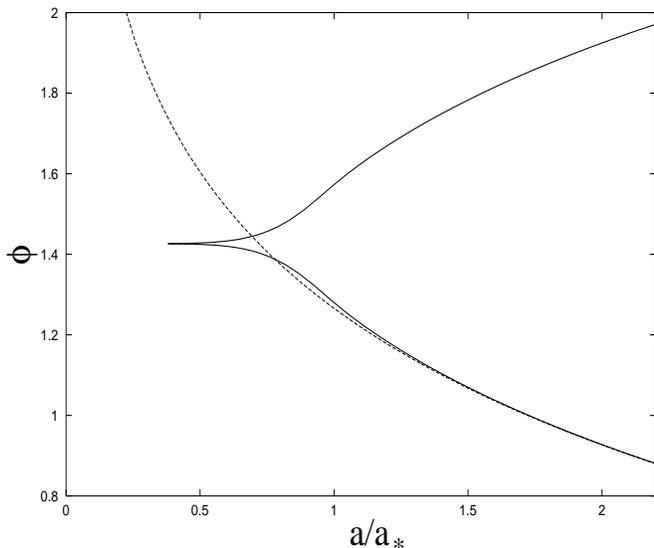
FIG. 2: Plot of ϕ in Planck units and a/a_* in the domain where the transition from classical to quantum and then again to classical occurs in loop quantum cosmology for ϕ^2 potential. The dashed curve is the one for classical cosmology and the field approaches infinity as universe encounters big crunch. The solid curve is for loop quantum cosmology evolution. In this case it is seen that the field freezes near the bounce. Initial conditions, j and m_ϕ are same as in Fig. [1].
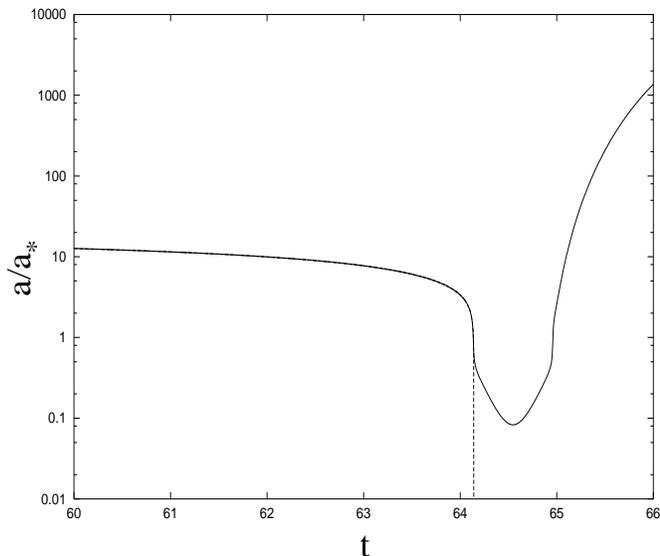

FIG. 3: Scale factor for the case of steep potential (10) at late times (in Planck units) of collapse. Initial conditions are $a_i = 100\,a_*$, $H_i = 0$ and $\phi_i = 0$ (which also determine $\dot{\phi}_i$) with $j = 100$. The evolution in loop quantum cosmology is depicted by solid curve which avoid big crunch when $a = 0.082 a_*$ whereas classical cosmology shown by dashed curve encounters a singularity.

dynamically unimportant at a contraction stage, and a

scalar field oscillates infinitely while falling into a singularity [23]. The bounce for steep potentials as classically understood is impossible and a thus a singularity is inevitable [8].

These features, however, do not change the dynamics in quantum regime. The quantum gravity corrections are significant mainly due to their influence on $\dot{\phi}$ terms in dynamical equations and thus in the quantum regime they become dominant over the potential terms. The numerical results for the potential (10) are shown in Figs. [3 & 4] which clearly show a bounce. The behavior of the universe near bounce is very similar to one shown in Figs. [1 & 2]. In Fig. [4] we have shown how in classical regime the field ϕ undergoes oscillations and then in quantum regime it first freezes and bounces back. As for the quadratic potential we found that the bounce for steep potential (10) always occurs for arbitrary choice of initial conditions and j.
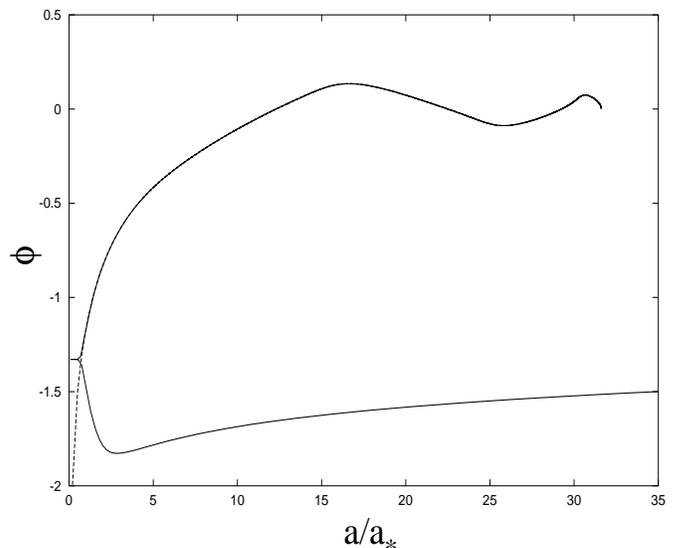

FIG. 4: Dynamics of ϕ (in Planck units) and a/a_* for potential (10) with initial conditions same as in Fig. [3]. In the classical regime the field undergoes oscillations. In loop quantum cosmology the field freezes near bounce (shown by solid curve) whereas in classical cosmology (shown by dashed curve) it takes large values near big crunch.

Our results are in a sharp contrast to earlier results in classical cosmology, where a occurrence of a bounce requires a very special initial condition to be chosen. As the quantum bounce is independent of initial conditions, it eventually occurs also in the second, third and consecutive cycles of contractions giving rise to some kind of aperiodic cyclic universe. It is interesting that the bounce via loop quantum gravity is rather similar to an artificial bounce put by hand into the classical picture in [24] ($\dot{a} \to -\dot{a}$ and $\dot{\phi} \to \dot{\phi}$ at some scale factor of the order of the Planck length). This suggests that analysis of effects of "hysteresis" of the scalar field evolution and time asymmetry which have been done in [24] are also

applicable to our model.

We should stress that for both considered potentials, quadratic (8) and steep (10), we found that the bounce occurs when size of the universe is not very small compared to a_*. For the quadratic potential a closed universe bounces when $a \sim 0.2 - 0.5\,a_*$ and for a steep potential it bounces when $a \sim 0.05 - 0.3\,a_*$. We also found that the role of j on the nature of dynamics near big crunch is very weak. It is important to note that the big crunch singularity has been avoided purely at the semi-classical level of effective dynamical equations with quantum gravity modifications. This is in distinction to the earlier work in loop quantum cosmology [17], where the singularity avoidance was accomplished using discrete quantum evolution near $a = 0$, in which case one can not use the picture of a continuous spacetime and the dynamical equations (5 - 7) are invalid.

Thus, we establish that for a closed universe with a minimally coupled massive scalar field a bounce will oc-cur for any choice of initial conditions as soon as universe becomes of the size that its dynamics is governed by equations with loop quantum gravity modifications. Hence a big crunch or a cosmic doomsday will always be avoided. In future, it will be important to study dynamics of bounce in anisotropic case and also the role of matter and radiation near bounce. Apart from loop quantum cosmology, bouncing universe scenarios have been in discussion in various string models (for example see [25]). It will be interesting to investigate their common features with loop quantum effects for various values of geometrical curvature.

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