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Effective State Metamorphosis in Semi-Classical Loop Quantum Cosmology

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Modification to the behavior of geometrical density at short scales is a key result of loop quantum cosmology, responsible for an interesting phenomenology in the very early universe. We demonstrate the way matter with arbitrary scale factor dependence in Hamiltonian incorporates this change in its effective dynamics in the loop modified phase. For generic matter, the equation of state starts varying near a critical scale factor, becomes negative below it and violates strong energy condition. This opens a new avenue to generalize various phenomenological applications in loop quantum cosmology. We show that different ways to define energy density may yield radically different results, especially for the case corresponding to classical dust. We also discuss implications for frequency dispersion induced by modification to geometric density at small scales.

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I. INTRODUCTION

Quantum gravity is expected to radically modify our classical intuition of space-time and matter. Progress in loop quantum gravity (LQG), one of the background independent and non-perturbative candidates for quantization of gravity, suggests that at the quantum level classical spacetime continuum is replaced by discrete quantum geometry [1, 2]. The continuum spacetime emerges from quantum geometry in a large eigenvalue limit. An important question in this setting is the way behavior of the ordinary matter is modified at small scales. Answering this question in complete generality is difficult since we lack a full theory of non-perturbative quantum gravity including matter. However, valuable insights can be obtained if we work in a simpler symmetry reduced setting like quantum cosmology.

Loop quantum cosmology (LQC) is the quantization of homogeneous and isotropic mini-superspaces based on LQG whose applications include a resolution of the big-bang singularity in homogeneous and isotropic mini-superspace setting [3, 4, 5] (for a recent review see [6], also see Refs. [7, 8] for critical discussions). One of its key result is that eigenvalues of geometrical density operator (or positive powers of inverse scale factor in general) become proportional to positive powers of the scale factor [9] below a critical value, $a_* = \sqrt{j\gamma/3}\ell_{\rm P}$. Here j is a half integer greater than unity, $\ell_{\rm P}$ is the Planck length and $\gamma \approx 0.2375$ is the Barbero-Immirzi parameter whose value is set by black hole thermodynamics [10]. Also as, $a \longrightarrow 0$ the spectrum of the inverse scale factor operator remains bounded and curvature does not diverge.

At the fundamental level the evolution in LQC is governed by quantum difference equations. However above the scale factor $a_i \approx \sqrt{\gamma}\ell_{\rm P}$, dynamics can be approximated by Friedmann equations with non-perturbative modifications [11]. The spacetime does not immediately become classical as we go from low to high eigenvalues of the scale factor and the regime $a_i \lesssim a \lesssim a_*$ is very interesting to explore new physical effects. The dynamics in this semi-classical regime has been studied extensively for a scalar field and various interesting results have been obtained, like dynamical initial conditions for the universe [12], naturalness of inflation [13, 14, 15, 16, 17, 18, 19, 20] , avoidance of big crunch in closed models [21, 22, 23], non-singular brane bounce in cyclic models [24], emergent universe scenario [25], possibility of signatures in cosmic microwave background (CMB) [15] and discrete corrections to classical trajectories [26, 27]. The non-perturbative modifications to Friedmann equations have also been shown to match well with underlying quantum difference equations till very small scales [27, 28]. Investigations to the issues pertaining to perturbations have also been initiated [29, 30]. LQC techniques have also been applied to scalar field collapse models and it has been shown that black hole and naked singularities can be avoided [31, 32].

Though LQC has yielded various interesting results in scalar field dynamics, phenomenological applications for arbitrary matter remain an open issue. This problem is not only important by itself to gain insights in behavior of matter at scales near and below a_*, but also to have understanding of a more complete model of the universe with loop modifications. It should be noted that even in most of the scalar field applications considered in LQC, various properties of the scalar field are just assumed as in standard cosmology. Therefore, it is highly desirable to look for alternatives to scalar field phenomenology. We shall recall that in classical cosmology scalar field is a very attractive entity since it can easily violate the strong energy condition in the presence of a potential, which can lead to various interesting consequences. Our interest here is to explore the possibilities for generic matter when scale factor is smaller than a_*. If equation of state of matter like dust or radiation behaves in a radically different way below some scale factor, then the results obtained in LQC using scalar field might be generalized to other matter. In particular any such result is

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particularly useful to study the last stages of a contracting universe or gravitational collapse scenarios in LQC. Further profound insights might be obtained for some phenomena in the very early universe, thus making the LQC phenomenology for generic matter very important.

Since we are working in a symmetry reduced framework, incorporating generic matter configurations is a difficult issue. This is due to the reason that various forms of matter in classical cosmology, like perfect fluids, are not as fundamental as the scalar field. Further, consistent analysis of perfect fluids at all scales (including discrete quantum regime) may require a full non-perturbative treatment of quantum gravity with matter. Since such a theory is still under development, our treatment here would be very phenomenological.

In classical cosmology matter coupling to gravity is identified via the stress-energy tensor of a perfect fluid or the energy density and pressure. Latter are related via the thermodynamic relation of equation of state and obey the conservation law derived from the adiabatic expansion of the universe (which is same as the divergence free property of stress-energy tensor). The conservation law provides us with the proportionality of energy density with the scale factor. Matter components with constant but different equation of states are proportional to different powers of the scale factor. Since we can not include such classical fluids in current treatment, we would investigate the modifications to Hamiltonian with suitable scale factor dependence such that the corresponding energy density at classical volumes ($a \gg a_*$) has the same behavior in scale factor as the energy density of a classical matter component like dust, radiation (or relativistic gas of particles) etc. In this way we can obtain insights on the way energy density for matter shall modify as we approach semi-classical scales. In Ref. [28], it was shown that inverse scale factor modification to energy density of matter behaving as classical dust is sufficient to mimic the underlying quantum dynamics to the scales of the order of a_i. Further at large volumes ($a \gg a_*$), we recover back classical form of densities and standard dynamics. Thus, dynamical equations with modified energy density make a good effective theory to the fundamental quantum difference equations in the semi-classical domain and we can study the variation from classical behavior, like for the equation of state, in the effective description without referring to the underlying quantum dynamics. Our analysis would be based on this effective phenomenological picture.

We emphasize that we do not include perfect fluids in current framework of LQC but we investigate matter Hamiltonians whose energy density at classical scales mimic those of fluids like dust, radiation or stiff matter. We further assume as in Ref. [26] that time scales of loop modified cosmological dynamics are large compared to those which establish thermodynamic equilibrium in matter processes such that notion of equation of state is well defined and conservation law can be used. It is possible that in deep quantum regime near a_i, above assumption may break down. In any case, in the full quantum zone our phenomenological picture would be invalid and thus insights gained from this work can be trusted only for scales not much below a_* with the latter chosen large enough compared to ℓ_P.

Our first result is to show that there are different ways to define energy density given a matter Hamiltonian, depending on whether we construct a corresponding quantum operator or not. This is also related to the way we can obtain the effective Friedmann equation in the semi-classical regime, either as an extension of the classical Friedmann equation or the semi-classical limit of the corresponding quantum construction. We demonstrate that the distinction between energy densities become important especially in the case for matter Hamiltonian independent of the scale factor. Phenomenologically this would correspond to the coupling resembling classical dust. This can have important consequences for the perturbations in the semi-classical regime. However, irrespective of the choice of definition of the energy density, we further show that equation of state shows variation from the behavior at classical volumes near and below a_*. This would effectively correspond to existence of a negative pressure by using the conservation law. Thus matter which couples with classical gravity as pressureless or positive pressure component, may transform into a negative pressure form at scales below a_*. Interestingly this is true even when the equation of state is constant in classical theory, thus giving an indication that equation of state may vary in semi-classical regime. As discussed earlier this can have profound implications for scales $a \lesssim a_*$ with the possibility that matter like dust and radiation can provide a viable alternative to scalar field like in gravitational collapse scenarios [31, 32]. This result is also important for multi-component models in semi-classical LQC. Our results are also immediately applicable to models with more than one scalar field where one of the scalar field behaves effectively as dust or radiation with a suitable choice of potential. Then the variation of equation of state to negative values for such matter component suggests that may be it is possible to successfully generalize previous results of LQC. We also show that for matter coupling corresponding to radiation, the frequency experiences dispersion at scales smaller than a_*. Interestingly, dispersion is similar to the results obtained earlier using trans-Planckian cut offs [33, 34].

II. MODIFIED DYNAMICS

The root of modification of dynamics in LQC can be traced back to the operator representing the quantum inverse scale factor. Since the scale factor in LQC has discrete eigenvalues including zero, one begins with an identity on the Ashtekar-Barbero phase space [35]. In terms of the basic phase space variables, connection c

and the triad p, a^{-1} is given by [36]

$$a^{-1} = \left[\frac{3}{8\pi G \gamma l} \{ c, |p|^l \} \right]^{1/(2-2l)} \tag{1}$$

where l is a quantum ambiguity parameter with $0 < l < 1$. The triad p is related to the scale factor a via $|p| = a^2$ and on classical solutions (of general relativity) c is given by $c = (k - \gamma \dot{a})/2$, k being curvature index which we take to be zero in this work. We can then quantize this identity and obtain the eigenvalues of inverse scale factor. It turns out that the eigenvalue spectrum is bounded on the entire Hilbert space and evolution through $a = 0$ is non-singular [3, 4, 5]. The eigenvalues of $\widehat{(1/a)}$ below a_* become proportional to the positive powers of scale factor and are not inverse of those of $\hat{a}$.

The geometrical density operator $\widehat{(1/a^3)}$ can be similarly constructed and its eigenvalues for large j are approximated as [13]

$$d_{j,l}(a) = D_l(q) a^{-3}, \quad q := a^2/a_*^2, \quad a_* := \sqrt{j\gamma/3}\, \ell_{\mathrm{P}} \tag{2}$$

where

$$
\begin{aligned}
D_l(q) = & \left\{ \frac{3}{2l} q^{1-l} \left[\frac{1}{2+l} \left((q+1)^{l+2} - |q-1|^{l+2} \right) \right. \right. \\
& \left. \left. - \frac{q}{1+l} \left((q+1)^{l+1} - \mathrm{sgn}(q-1)|q-1|^{l+1} \right) \right] \right\}^{\frac{3}{(2-2l)}} .
\end{aligned}
\tag{3}
$$

Radical modifications to the behavior of geometrical density become obvious if $a \ll a_*$ when

$$d_{j,l}(a) \approx \left(\frac{3}{1+l} \right)^{3/(2-2l)} \left(\frac{a}{a_*} \right)^{3(2-l)/(1-l)} a^{-3} . \tag{4}$$

At classical scales $a \gg a_*$, $d_{j,l} \approx a^{-3}$ and we recover back the classical description.

For a matter specified by Hamiltonian $\mathcal{H}_M$, dynamics can be obtained from total Hamiltonian constraint which on quantization leads to the following difference equation [5]

$$
\begin{aligned}
& \left(V_{\mu+5\mu_0} - V_{\mu+3\mu_0} \right) \psi_{\mu+4\mu_0} - 2 \left(V_{\mu+\mu_0} - V_{\mu-\mu_0} \right) \psi_\mu \\
& + \left(V_{\mu-3\mu_0} - V_{\mu-5\mu_0} \right) \psi_{\mu-4\mu_0} = -\frac{8\pi G}{3} \gamma^3 \ell_{\mathrm{P}}^2\, \hat{\mathcal{H}}_M(\mu)\psi_\mu
\end{aligned}
\tag{5}
$$

where V_μ are eigenvalues of volume operator and are related to eigenvalues (μ) of triad as $V_\mu = (\gamma|\mu|/6)^{3/2}\ell_{\mathrm{P}}^3$. In the semi-classical limit it can be shown that above equation can be approximated by the following differential equation [11, 13]

$$-3 \dot{a}^2 a + 8 \pi G E_M(a, \phi) = 0 . \tag{6}$$

Here $E_M(a, \phi)$ are eigenvalues of matter Hamiltonian operator, assumed to correspond to a matter field ϕ. It is important to note that if $\mathcal{H}_M$ depends on the inverse scale factor, then $E_M(a, \phi)$ inherits the modifications to

the eigenvalues of $\widehat{(1/a)}$ upon quantization. Thus, dynamics in the semi-classical regime is distinct from its classical counterpart [37]. A useful example is the case of a massive scalar field whose classical Hamiltonian

$$\mathcal{H}_M = \frac{1}{a^3} \frac{p_\phi^2}{2} + a^3 V(\phi) \tag{7}$$

on loop quantization yields modified eigenvalues as [13, 14, 15]

$$E_M(a, \phi) = d_{j,l}(a) \frac{p_\phi^2}{2} + a^3 V(\phi) . \tag{8}$$

Dynamics in this case is then completely determined by further using the modified Klein-Gordon equation [13, 14, 15]

$$\ddot{\phi} + \left(3\frac{\dot{a}}{a} - \frac{\dot{D}_l(q)}{D_l(q)} \right) \dot{\phi} + D_l(q) V_{,\phi}(\phi) = 0 , \tag{9}$$

where we have used the Hamilton's equations $\dot{\phi} = d_{j,l}(a)p_\phi$ and $\dot{p}_\phi = -a^3 V_{,\phi}(\phi)$. In the regime $a \lesssim a_*$, $D_l(q) \ll 1$ and the modified dynamics becomes independent of the potential. Also, the Klein-Gordon equation (9) can be approximated as

$$\ddot{\phi} - 3 \left[\frac{2-l}{1-l} - 1 \right] \frac{\dot{a}}{a} \dot{\phi} \approx 0 \tag{10}$$

where we have used eq.(4). Since $0 < l < 1$ the coefficient of $\dot{\phi}$ in eq.(9) changes sign compared to its classical value and the scalar field experiences anti-friction (friction) instead of friction (anti-friction) for an expanding (contracting) universe. Such a peculiar change is responsible for various interesting physical effects for example super-inflation [13, 14, 15, 16, 17, 18, 19, 20], avoidance of big crunch [21, 22, 23] and resolution of black hole and naked singularities with possible observable signatures [31, 32].

Let us now determine the modifications to the energy density. In classical theory once we know $E_{M_{\mathrm{cl}}}(a, \phi)$, it is straightforward to evaluate classical energy density which is defined as $\rho_{\mathrm{cl}} = E_{M_{\mathrm{cl}}}(a, \phi)/a^3$. Since geometric density is modified in the semi-classical regime, the notion of energy density becomes subtle. To obtain the semi-classical density one way is to define a density operator $\hat{\rho}_{\mathrm{q}} = \widehat{\mathcal{H}_M/a^3}$ and then take the semi-classical limit. Another way is to define semi-classical density as the ratio of modified energy density to the volume i.e. $\rho_{\mathrm{sc}} = E_M(a, \phi)/a^3$ (where $E_M(a, \phi)$ includes the appropriate modifications to inverse scale factor) [13]. Note that since we so far lack the understanding of physical inner product in LQC, these definitions are considered with the caveat that they may not correspond to the expectation values taken with respect to physical semi-classical states. It may turn out that energy density obtained via expectation values is different than ρ_{q} or ρ_{sc}, which may require further analysis of results presented

here. There are other possible quantization ambiguities in defining energy density, like we can classically write $a^{-3} = a^{-3\beta} a^{3(\beta-1)}$ for $\beta > 0$, quantize it and obtain the energy density which would now depend on β. Note that all such quantization ambiguities lead to same classical energy density and in a quantum theory a choice has to be made, either on the ground of natural value of parameter β (which would be $\beta = 1$) or the connection with full theory of LQG. The origin of such an ambiguity parameter is similar to the parameter l which is present in eq.(1) and is discussed in detail in Ref. [16] where it was shown that it is the natural choice of parameter which may arise from LQG. This augurs well with the general expectation that natural values of such quantization ambiguity parameters would be favored by the full theory of LQG. We would focus our discussion on the natural value of the β parameter and discuss the implications for other possible values in the concluding section. It would turn out that physics and conclusions of this work are very robust to arbitrary choices of β.

The eigenvalues for $\widehat{\rho_q}$ can be obtained in the way described above for $\widehat{\mathcal{H}}_M$ and $\widehat{1/a^3}$ and they are

$$\rho_q = d_{j,l}(a)\, E_M(a,\phi) = D_l(q)\, a^{-3}\, E_M(a,\phi) = D_l(q)\, \rho_{sc} \ . \tag{11}$$

For $a \lesssim a_*$, $D_l(q) \lesssim 1$ implies $\rho_q \lesssim \rho_{sc}$. It is to be noted that ρ_q incorporates modifications both in energy and geometric density eigenvalues, whereas ρ_{sc} does not receive any contribution from modifications in behavior of $1/a^3$. Though dynamics does not depend on the choice of the density, there are important distinguishing features which we discuss in next section.

Modifications to energy and density eigenvalues of matter Hamiltonians with arbitrary scale factor dependence can be determined in a similar way. In the semi-classical regime, at the effective level, we shall first replace the inverse scale factors in Hamiltonian with the appropriate powers of $d_{j,l}$ and then obtain energy density ρ_q (or ρ_{sc}). For example, if $\mathcal{H}_M$ is independent of the scale factor then its energy eigenvalues would not get modified for scales less than a_*. The energy density defined via ρ_q is modified. At classical scales energy density becomes proportional to a^{-3}. Thus, this form of matter coupling would resemble that of dust at large volumes. Similarly, the Hamiltonian whose energy density would classically couple as that of radiation is $\mathcal{H}_M(a) \propto 1/a$. The energy eigenvalues in this case get modified below a_*, which is equivalent to modification to behavior of frequency at small volumes (see Ref. [30] for a similar discussion).

The semi-classical dynamics can be obtained directly from the Hamiltonian constraint eq.(6) and since the latter does not depend on ρ_q or ρ_{sc}, the trajectories like $a(t)$ would be identical for both choices of energy density. However, if instead of using Hamiltonian constraint we wish to use effective Friedmann equation to determine dynamics then there are some subtleties. One way to obtain the effective Friedmann equation is as done clas-

sically, that is dividing the Hamiltonian constraint by a^3. This yields the gravitational energy density as $\dot{a}^2/a^2$ which is identified with the square of the classical Hubble rate ($H_{cl} := \dot{a}/a$) and is equal to the classical matter energy density (up to numerical factor of $8\pi G/3$). With this algorithm, the division of eq.(6) by a^3 yields the modified Friedmann equation with unmodified gravitational energy density, and thus effective Hubble rate which is same as the classical one, i.e. $H_{sc}^2 = H_{cl}^2 := \dot{a}^2/a^2$, and is proportional to ρ_{sc}.

An alternative method to obtain semi-classical effective Friedmann equation is to first quantize the classical Friedmann equation and then obtain its semi-classical limit [38]. As explained for the construction of $\widehat{\rho_q}$, this method would imply that the modified Friedmann equation in the regime $a \lesssim a_*$ is given by $d_{j,l}$ multiplied with eq.(6). This means that the gravitational energy density also gets modified below a_* and is given by $D_l\, \dot{a}^2/a^2$. That is the effective Hubble rate in semi-classical regime is given by $H_q := D_l^{1/2} H_{sc} = D_l^{1/2} \dot{a}/a$, whose square is proportional to the energy density ρ_q. For $a > a_*$, both H_q and ρ_q approach H_{cl} and ρ_{cl} respectively, and the classical dynamics is recovered. Interestingly the modified Friedmann equation for H_q can be further divided by D_l to obtain a $\dot{a}^2/a^2$ equation which resembles the modified Friedmann equation obtained from the first method, however that does not imply that either $\dot{a}^2/a^2$ or ρ_{sc} can be identified with the actual gravitational and matter energy densities ($D_l\, \dot{a}^2/a^2$ and ρ_q).

The point to note is that different gravitational and matter densities appear in modified Friedmann equation depending on the way latter is obtained, either by extending classical theory to semi-classical regime which leads to an unmodified Hubble rate or taking the semi-classical limit of the quantum theory which modifies classical Hubble rate to H_q. As we mentioned above, these differences (or ambiguities to obtain effective Friedmann equation) do not affect dynamical trajectories and the phenomenological applications like inflation and bounces in LQC are robust to such ambiguities. However they do lead to different effective Hubble rates and energy densities in the semi-classical regime $a \lesssim a_*$. This can be important for example in the investigations of constraints on the value of loop parameters to yield viable initial conditions for conventional inflation [16]. We recall that in Ref. [16], $H_{sc}^{-1} > \sqrt{\gamma}\ell_P$ was used to obtain constraint on parameter j. The phenomenological constraint on j is thus expected to change if instead we use H_q. Further, in Ref. [27] estimates on the scale below which discrete quantum geometry corrections become significant to the dynamics have been obtained. This scale which is related to a critical density obtained using ρ_{sc} may also be affected if we use ρ_q. These issues will important for future investigations in this direction.

III. VARIATION OF EQUATION OF STATE

Given classical matter with a constant equation of state ($w_{\rm cl}$), matter density evolves as

$$\rho_{\rm cl} = \rho_0\, a^{-3(1+w_{\rm cl})} \tag{12}$$

where ρ_0 is a constant. At the effective phenomenological level the modifications to energy density of matter as for $\rho_{\rm q}$ corresponds to replacing inverse powers of a in eq.(12) with appropriate powers of $d_{j,l}$. Thus

$$\rho_{\rm q} = \rho_0\, d_{j,l}^{(1+w_{\rm cl})} = D_l^{(1+w_{\rm cl})}\, \rho_{\rm cl}\ . \tag{13}$$

Further, on using eq.(11) we obtain

$$\rho_{\rm sc} = D_l^{-1}\, \rho_{\rm q} = D_l^{w_{\rm cl}}\, \rho_{\rm cl}\ . \tag{14}$$

Hence both prescriptions ($\rho_{\rm q}$ and $\rho_{\rm sc}$) to obtain semi-classical density lead to the modifications from classical density. The energy conservation law obeyed by matter immediately ensures that pressure must change to accommodate any modification in energy density. The rate of change of energy density $\rho_{\rm q}$ with respect to the scale factor is

$$a\,\frac{d}{da}\rho_{\rm q} = (1+w_{\rm cl})\,\rho_{\rm q}\left[\frac{d\ln D_l}{d\ln a} - 3\right]\ . \tag{15}$$

Using this in energy conservation equation

$$a\,\frac{d}{da}\rho = -3\,(\rho + p) \tag{16}$$

we easily obtain the expression for modified pressure $p_{\rm q}$,

$$p_{\rm q} = w_{\rm cl}\,\rho_{\rm q} - (1+w_{\rm cl})\frac{d\ln D_l}{d\ln a}\frac{\rho_{\rm q}}{3} \tag{17}$$

which leads to an effective equation of state ($w_{\rm q}$) defined as the ratio $p_{\rm q}/\rho_{\rm q}$,

$$w_{\rm q} = \left[w_{\rm cl} - \frac{(1+w_{\rm cl})}{3}\frac{d\ln D_l}{d\ln a}\right]\ . \tag{18}$$

Similar derivation can be done by starting from energy density $\rho_{\rm sc}$ and the resulting effective equation of state $w_{\rm sc} = p_{\rm sc}/\rho_{\rm sc}$ is

$$w_{\rm sc} = w_{\rm cl}\left[1 - \frac{1}{3}\frac{d\ln D_l}{d\ln a}\right]\ . \tag{19}$$

At classical scales $a \gg a_*$, $D_l(q) = 1$. However at scales near and below a_*, $D_l(q)$ starts varying and becomes much smaller than unity. This leads to the variation of equation of state for matter for $a \lesssim a_*$. If our phenomenological picture is allowed to be trusted even for scales $a \sim a_i$, then fixing the parameter l to its natural value of 3/4 [16], it is straightforward to see from
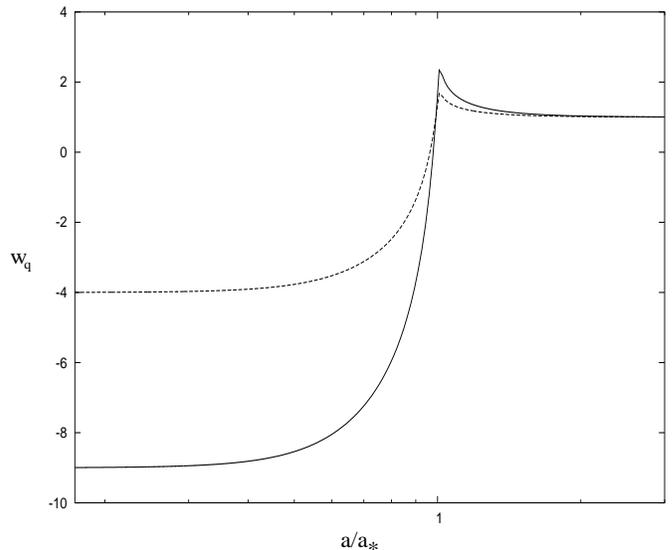

FIG. 1: Behavior of effective equation of state for matter whose energy density at classical volumes has the behavior of that of stiff matter. The solid curve shows $w_{\rm q}$ and dashed curve shows $w_{\rm sc}$. Parameters are $j = 100$ and $l = 3/4$. Same variation holds for a massive scalar field (for details see text).

eq.(4) that for $a \sim a_i \approx \sqrt{\gamma}\ell_{\rm P}$, $d\ln D_{3/4}/d\ln a \approx 15$ and thus

$$w_{\rm q} \approx -4w_{\rm cl} - 5 \tag{20}$$

and

$$w_{\rm sc} \approx -4w_{\rm cl}\ . \tag{21}$$

As discussed earlier we do not expect this behavior to be valid at scales so close to Planck length, however we do expect that for scales close to a_*, equation of state would start varying even if it is constant for $a \gg a_*$. We illustrate this with examples of matter Hamiltonians of the form C_1/a^3, C_2/a and C_3 where C_i are some constants. Since the energy density of these Hamiltonians resemble that of stiff matter, radiation and dust respectively in scale factor dependence at classical volumes, we would refer to examples with this correspondence. We would also review the case of massive scalar field and study the effect of using $\rho_{\rm q}$ instead of $\rho_{\rm sc}$ as done earlier [13, 15, 16, 17, 19].

Stiff Matter: For matter Hamiltonian which is of the form C_1/a^3, both $\rho_{\rm q}$ and $\rho_{\rm sc}$ approach $\rho_{\rm cl} \propto a^{-6}$ as for stiff matter at scales $a \gg a_*$. As expected the equation of state $w_{\rm q}$ or $w_{\rm sc}$ are equal to unity at large volumes. However, for scales $a \sim a_*$, it starts varying and becomes negative for $a < a_*$. The increase in equation of state from its classical value for a small domain near a_* is due to the corresponding peak in $D_l(q)$ for $a \sim a_*$. The variation of effective equation of state is shown in Fig. 1. As can be seen, the equation of state quickly becomes less than $-1/3$ after the peak at $a \sim a_*$, that is it violates
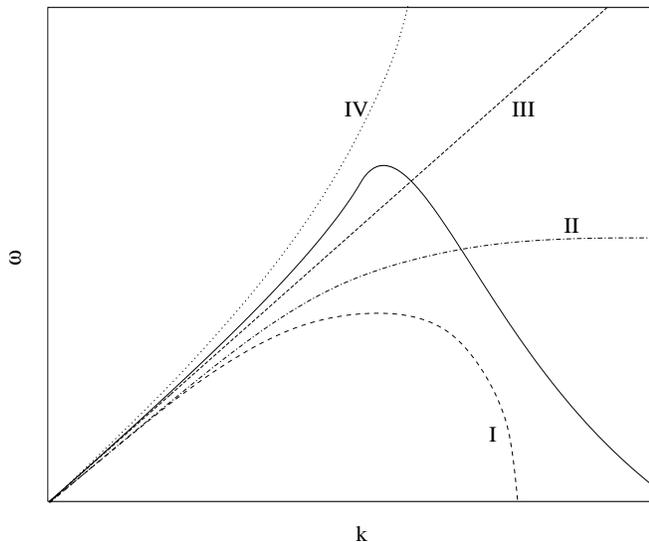
FIG. 2: Evolution of frequency ω (solid curve) with k. We choose quantum gravity parameters as $l = 3/4$ and $j = 100$. Behavior for other choices is similar. Classical curve denoted by III is not modified at any value of scale factor. The curves I,IV and II are obtained respectively in Ref.[34] and Ref.[33] by cutoffs to dispersion relation at short scales. Quantum gravity naturally produces a modification to frequency at short scales without introduction of any cutoff.

FIG. 3: Evolution of $w_{\rm q}$ (solid curve) and $w_{\rm sc}$ (dashed curve) for $\mathcal{H}_M \propto a^{-1}$ with loop parameters as in Fig. 1. If phenomenological picture is valid till $a \sim a_i$, matter which couples to gravity at classical volumes as radiation transforms into one with super-negative pressure. Inset shows the variation of $D_l(q)$ with its peak at $a \sim a_*$.

strong energy condition. It also becomes less than -1 for values of scale factor which are of the order of $0.8a_*$, thus violating weak energy condition. This behavior is independent of choice of loop quantum parameters and suggests that matter with a constant stiff equation of state at classical volumes may transform effectively into a form with an equation of state which is negative. Though of little validity for our phenomenological picture we see that for $a \sim a_i$, $w_{\rm q} \approx -9$ and $w_{\rm sc} \approx -4$.

Radiation: Matter Hamiltonians which are proportional to inverse scale factor lead to energy density of the form of radiation at classical volumes. The Hamiltonian gets modified by $d_{j,l}^{1/3}$ in the semi-classical regime. Effectively, it implies that frequency corresponding to classical radiation component gets modified from the standard behavior below a_*. In this case classically we have $E_{M_{\rm cl}} = E_0 a_0/a$ (with E_0 and a_0 being constants) which gets modified to $E_M = d_{j,l}^{1/3} a_0 E_0 = D_l^{1/3} a^{-1} a_0 E_0$. Thus for $a \lesssim a_*$, E_M would become proportional to positive powers of scale factor. Since for radiation E_M is linearly related to frequency via Planck law, this modification implies a change in behavior of frequency at scales below a_* which would be given by $\omega = \omega_0 a_0 d_{j,l}^{1/3} = \omega_0 a_0 D_l^{1/3} a^{-1}$. Modifications to the behavior of frequency at short scales have been expected and desired from quantum gravity models. However, most of the times these are introduced by introduction of a short scale cut off [33, 34]. Here we see that such a cut off is provided by the scale below which behavior of density changes. We have plotted the
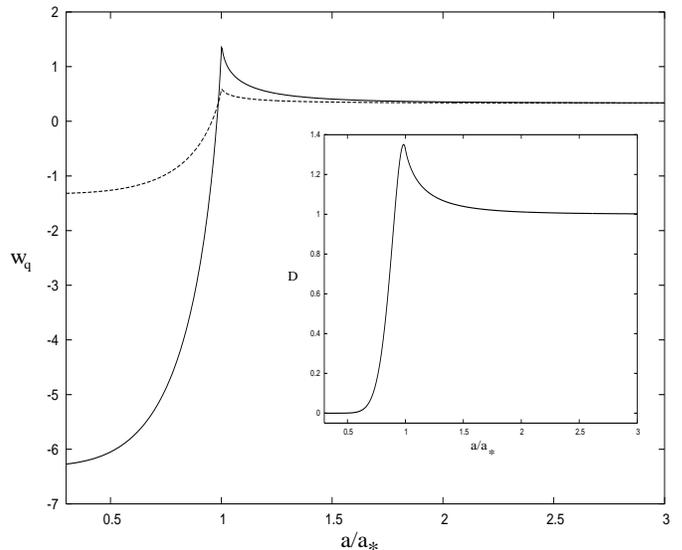
behavior of ω with $k = 2\pi/\lambda$ in Fig. 2. As can be seen, the modification to frequency is similar to those expected earlier and inspires further investigations to understand modification to dispersion relation using quantum gravity models as we have done here. These issues will be addressed elsewhere.

The equation of state for matter coupling like radiation via $\rho_{\rm q}$ construction becomes

$$w_{\rm q} = \frac{1}{3} - \frac{4}{9} \frac{d\ln D_l}{d\ln a} \ . \tag{22}$$

At classical scales, $D_l(q) = 1$ and thus $w_{\rm q} = w_{\rm sc} = 1/3$. However for scales less than a_*, $D_l(q)$ starts varying which leads to variation of $w_{\rm q}$ and $w_{\rm sc}$. In Fig. 3, we have shown the evolution of $w_{\rm q}$ and $w_{\rm sc}$ with scale factor. Inset shows evolution of $D_l(q)$ with its peak at $a \sim a_*$. As in the the case of stiff matter, the variation in $w_{\rm q}$ is more rapid than $w_{\rm sc}$. Though both variations suggest that strong and weak energy conditions are violated for scales near a_*, these violations occur at slightly larger scale factors for $w_{\rm q}$ than for $w_{\rm sc}$.

Dust: Hamiltonian which is independent of the scale factor does not receive any modifications due to $d_{j,l}$ for scales less than a_*. The energy density $\rho_{\rm sc}$ is equal to $\rho_{\rm cl}$ at all scales and thus diverges for small scale factors. However, on using density operator we obtain $\rho_{\rm q} = D_l \rho_{\rm cl}$ and

$$w_{\rm q} = -\frac{1}{3} \frac{d\ln D_l}{d\ln a}. \tag{23}$$

We have shown the variation of effective equation of state in Fig. 4. As for the case of stiff matter and radiation,

w_q varies for $a \sim a_*$ and becomes negative below the critical scale factor. It also violates weak energy condition for $a \sim 0.9a_*$. On the other hand w_{sc} remains constant (equal to zero) all through the period of evolution. The differences between ρ_q and ρ_{sc} become very evident in this case. As $a \longrightarrow 0$, ρ_{sc} blows up whereas ρ_q remains finite. It is important to note that in ρ_q effects of geometric density regulate the diverging energy density at small scale factors.

Massive Scalar Field: The case of massive scalar field has been studied in detail in LQC [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30]. The period of super-acceleration in the regime $a \lesssim a_*$ plays a dominant role in most of the interesting effects like setting up right conditions for inflation or preventing singularities. This phenomena also indicate that effective equation of state becomes less than -1 and thus weak energy condition is violated. The energy density of a massive scalar field ϕ with potential $V(\phi)$ via density operator construction is given by

$$\rho_q = d_{j,l}\, E_M(a,\phi) = \frac{\dot{\phi}^2}{2} + D_l(q)\, V(\phi) \qquad (24)$$

where we have used eqs.(11) and (8). The expression for effective pressure can be obtained by using eq.(16) and Klein-Gordon equation (eq.(9)). It turns out to be

$$p_q = \left[1 - \frac{2}{3}\frac{d\ln D_l}{d\ln a}\right]\frac{\dot{\phi}^2}{2} - D_l(q)\, V(\phi) - \frac{1}{3}\frac{d\ln D_l}{d\ln a}\, V(\phi) \qquad (25)$$

and thus effective equation of state can be obtained by using $w_q = p_q/\rho_q$. It shall be noted that effective negative pressure obtained from ρ_q is much stronger than that obtained from ρ_{sc} leading to [13, 17]

$$w_{sc} = -1 + \frac{2\dot{\phi}^2}{\dot{\phi}^2 + 2D_l(q)V(\phi)}\left[1 - \frac{1}{6}\frac{d\ln D_l}{d\ln a}\right]. \qquad (26)$$

Since $D_l(q) \ll 1$ for $a \ll a_*$, the potential terms become negligible compared to kinetic terms and it is easy to verify that both w_q and w_{sc} behave in the same way as for stiff matter. The variation of equation of state and the super-negative pressure are thus as shown in Fig. 1. It is important to note that use of ρ_q leads to stronger violation of energy conditions than ρ_{sc}. As we discussed in the previous section this issue might be linked to the problem of stability of perturbations in the regime $a \lesssim a_*$ [30]. Hence choice of ρ_q or ρ_{sc} is bound to play an important role in obtaining viable density perturbations in loop quantum modified regime. We would leave such an investigation for future work.

IV. CONCLUSIONS

Working in the semi-classical limit of LQC we have studied the behavior of matter Hamiltonians with arbitrary scale factor dependence, in particular those whose
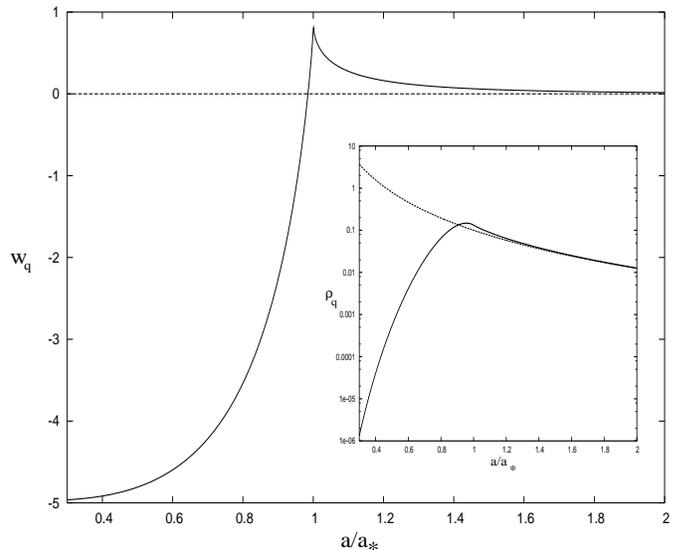
FIG. 4: Variation of effective equation of state for matter satisfying $w_{cl} = 0$ in the classical regime. Solid curve depicts w_q and dashed curve shows w_{sc}. Parameters are same as in Fig. 1. Inset shows evolution of ρ_q with scale factor which is well regulated as $a \longrightarrow 0$ whereas the energy density without $d_{j,l}$ modification blows up (shown by dashed curve).

energy density scales as that of dust, radiation and stiff matter at classical volumes. Through our phenomenological effective treatment we are able to gain useful insights on what shall be expected to be the modified behavior of matter at scales near and less than a_*. Our first result is to show that there are at least two ways to define energy density and thus equation of state. Both of them lead to same classical behavior for $a \gg a_*$, but for $a \lesssim a_*$, there are significant distinctions between them. For example in the case of matter Hamiltonian with no scale factor dependence (classically corresponding to dust), at small scale factors energy density defined via ρ_{sc} blows up whereas ρ_q remains regulated and finite.

The effective equation of state for matter mimics the classical behavior for $a \gg a_*$. However we have shown that near a_* it starts varying even if it is classically constant. It increases initially for $a \sim a_*$ and then rapidly decreases leading to violation of energy conditions. This violation is independent of the choice of w_q or w_{sc}. It suggests that classical matter may effectively metamorphose itself to various forms at short scales and may serve as viable alternative to scalar field phenomenology. The case of radiation offers a new insight on the trans-Planckian modifications to the frequency dispersion. It is intriguing that inverse scale factor modifications may provide a natural explanation to much sought frequency dispersion at short scales, however this requires a detailed analysis which is beyond the scope of present discussion.

As we discussed in Sec. II, there is a new quantization ambiguity which may arise in construction of energy density via quantum operator. It can be checked that

if instead of taking $\beta = 1$, we keep it arbitrary positive value then ρ_q becomes $\rho_q = D_l^{\beta(1+w_{cl})} \rho_{cl}$ and the factor $(1 + w_{cl})$ in eq.(18) gets multiplied by β. Since β is positive, its multiplication with $(1 + w_{cl})$ in eq.(18) does not affect the qualitative behavior of equation of state w_q. For any choice of β, the effective metamorphosis of equation of state at scales smaller than a_* would occur. The parameter β only affects the magnitude of variation of w_q. For example, if we fix $l = 3/4$ then for the case of dust all values of $\beta > 1/15$ would imply violation of strong energy condition for $a \sim a_i$. By taking the same value of l and $\beta > 1/5$, ρ_q for dust like matter would scale as positive power of scale factor for $a \sim a_i$ and energy density does not blow up at small scale factors. Similarly for radiation and stiff matter, an arbitrarily chosen β results in a different magnitude of variation of the energy density and equation of state and the qualitative picture does not change. The result of modification to frequency dispersion for radiation is independent of the choice of energy density and is unaffected by this ambiguity parameter. This leads us to the conclusion that phenomenological effects discussed in this work are very robust and the qualitative picture does not depend on different choices of the quantization ambiguity parameter β. We recall that in Ref. [16], it was demonstrated that phenomenological results are qualitatively independent of the choice of parameter l. We have earlier discussed that parameter β originates in a very similar way as l. Now we see that both parameters also have a very similar effect towards phenomenological description and the underlying physical predictions are quite robust to the choices of these parameters.

Future investigations with full LQG techniques would be able to confirm or rule out the expectations of metamorphosis of equation of state and natural modifications to frequency dispersion. This opens a novel avenue to explore phenomenology at $a \lesssim a_*$, with matter like dust and radiation. Since the equation of state for matter coupling as classical dust or radiation to gravity becomes negative for $a < a_*$, it implies that multicomponent models of scalar field interacting with various matter components would also yield similar qualitative results like super-acceleration and bounce in semi-classical LQC. Of course the results obtained here are directly applicable to loop quantum cosmological models with two or more scalar fields where at least one of them behaves as classical matter component like dust or radiation with appropriate choice of potential. In Ref. [31] and [32] gravitational collapse scenarios with scalar field were studied and possibilities of interesting astrophysical signatures have been reported. However, in a more realistic scenario inclusion of matter which behaves as dust, radiation or stiff matter is important to investigate the last stages of astrophysical collapse. Our results would be particularly useful in this arena and may open the possibility to link LQC phenomenology with astrophysical observations.

Though the choice of energy density does not affect the dynamical trajectories, it may however change some of the phenomenological constraints imposed on loop parameters using Hubble rate (or energy density). This issue should be investigated further which may give us useful insights. However, a more fundamental understanding of matter in LQC and detailed analysis of physical semiclassical states might also guide us towards resolving this ambiguity. Our result of the effective state metamorphosis may also have some interesting implications for the problem of dark energy where some of the scenarios require effective equation of state to become less than -1. In particular recently considerable attention has focused on crossing the -1 divide in equation of state (which separates domains of validity of weak energy condition) and the role of quantum gravity effects [40]. In semiclassical LQC such a behavior is observed very naturally for various forms of matter. Investigation on the relation of LQC dynamics with that of standard cosmology using a scale factor duality has been done earlier [18]. On a speculative side such a duality might provide a valuable link between quantum gravity effects at short scales with dark energy at large scale factors of the universe.

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