Unidirectional and Nonreciprocal Nanophotonic Devices Based on Graphene and Magneto-Optical Materials

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UNIDIRECTIONAL AND NONRECIPROCAL NANOPHOTONIC DEVICES BASED ON GRAPHENE AND MAGNETO-OPTICAL MATERIALS

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

Division of Electrical and Computer Engineering.

by

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December 2019
Acknowledgments

Above all else, I would like to thank my advisor Professor Georgios Veronis for his support and guidance throughout the past five years of my career as a graduate student at LSU. I would also like to thank Professor Jonathan P. Dowling, Professor Kidong Park, and Professor Manas R. Gartia for serving on my dissertation committee and providing me with their thoughtful comments and suggestions. There are so many students and researchers that have helped me along the way. Specifically, I would like to thank Rod Tohid, Farthang Bayat, Amirreza Mahigir, Ali Haddadpour, Kasra Fattah-Hesari, Sahar Marefat Navaz, Shahrzad Shirzad, and Alireza Kheirkhahan. I would also like to express my love to my little friend Damon Shirzad. I am grateful for all the support I received from my amazing cousin, Ahmad Ranaee, who always believed in me. A truly amazing person with a great heart and spirit.
# Table of Contents

ACKNOWLEDGMENTS ......................................................................................... ii
ABSTRACT ........................................................................................................... v

CHAPTER

1 INTRODUCTION .............................................................................................. 1
   1.1 Introduction ............................................................................................... 1
   1.2 Channeling SPPs at optical wavelengths .................................................. 2
   1.3 SPPs on graphene ...................................................................................... 3
   1.4 Non-reciprocity .......................................................................................... 6

2 TUNABLE SPATIAL MODE CONVERTERS AND OPTICAL DIODES FOR GRAPHENE PARALLEL PLATE WAVEGUIDES ................................................................................................................... 11
   2.1 Introduction ............................................................................................... 11
   2.2 Mode converter ......................................................................................... 12
   2.3 Optical diode ............................................................................................ 24

3 NANOSCALE PLASMONIC ISOLATOR ................................................................. 30
   3.1 Introduction ............................................................................................... 30
   3.2 Results ....................................................................................................... 31
   3.3 Effect of MO activity on the cavity modes ............................................... 36
   3.4 Effect of the MDM waveguide on the cavity modes .................................. 37
   3.5 The proposed design ................................................................................. 41

4 THEORETICAL INVESTIGATION OF PLASMONIC MULTILAYERED CYLINDRICAL CAVITIES FOR ENHANCING THE MAGNETO-OPTICAL EFFECT ......................................................... 48
   4.1 Introduction ............................................................................................... 48
   4.2 Theoretical analysis ................................................................................... 50
   4.3 The effect of $\epsilon_r$ .................................................................................... 51
   4.4 The effect of $N$ .......................................................................................... 56
   4.5 Application ............................................................................................... 58
   4.6 The effect of loss ....................................................................................... 60

5 CONCLUSIONS ................................................................................................ 64

APPENDIX ........................................................................................................... 67
   A SUPPLEMENTARY INFORMATION FOR CHAPTER 2 ................................ 67
   B COPYRIGHT INFORMATION FOR CHAPTER 2 ........................................ 69
Abstract

In this dissertation, we first introduce compact tunable spatial mode converters for graphene parallel plate (GPP) waveguides. The converters are reciprocal and based on spatial modulation of graphene’s conductivity. The wavelength of operation of the mode converters is tunable in the mid-infrared wavelength range by adjusting the chemical potential of a strip on one of the graphene layers of the GPP waveguides. We also introduce optical diodes for GPP waveguides based on a spatial mode converter and a coupler, which consists of a single layer of graphene placed in the middle between the two plates of two GPP waveguides.

Next, we propose an ultracompact isolator for optical frequencies employing the potential of plasmonics for miniaturizing optical structures along with the magneto-optical (MO) effect of Bismuth Iron Garnet (BIG) to achieve non-reciprocity. The proposed structure consists of a cylindrical cavity placed in the proximity of a metal-dielectric-metal (MDM) waveguide. The isolation occurs due to the frequency splitting phenomenon along with the difference in transmitted power for waves incident on the device from different directions, in the presence of MO activity. The first order perturbation theory is used to explain the frequency splitting phenomenon. The difference between the transmission spectra is described via studying the nature of the resonant modes inside the cavity. The performance of the device can be tuned via changing the parameters of the isolator such as the radius of the cavity and the distance between the MDM and the cavity.

Next, we theoretically explore the notion of employing plasmonic multilayered cylindrical cavities to enhance the MO effect at optical frequencies. The frequency splitting observed in the presence of MO activity can be greatly increased in the multilayered cylindrical cavities with alternating layers of BIG and metal. We also investigated the effect of different parameters of the structures on enhancing the MO activity using transfer matrix
approach as well as numerical investigations. We showed that, provided that the material
and the design are chosen carefully, the MO activity can be noticeably increased. Finally,
we designed an all-optical circulator based on the layered structure.
Chapter 1
Introduction

1.1 Introduction

The field of photonics has been growing exponentially due to the unprecedented data bandwidth and low power consumption of photonic devices, as well as the low cost of on-chip photonic designs. However, decreasing the size of optical elements down to subwavelength scales is hindered by the diffraction limit [1, 2]. Surface plasmon polaritons (SPPs), which are the waves propagating at the interface between a metal and a dielectric due to the coupling between electromagnetic waves and free electrons in metals, are one of the ways to overcome the diffraction limit [1, 3]. The ability to overcome the diffraction limit has made SPPs an excellent candidate for further miniaturizing optical components down to the subwavelength regime. In order to be able to unlock the full potential of SPPs in optical designs, it is imperative to have a platform that enables the propagation of these waves. There are several basic designs that can support the propagation of SPPs. Among these designs, metal-dielectric-metal (MDM) waveguides are one of the most efficient ways to channel SPPs [4].

While MDM waveguides offer a platform for channeling SPPs at optical wavelengths, at larger wavelengths (infrared and far infrared), designs based on graphene are preferable. Graphene is a two-dimensional version of graphite which is capable of supporting the propagation of SPPs at infrared wavelengths. In comparison with conventional plasmonic materials, such as silver and gold, the SPPs in graphene have several advantages, including tunability, low-loss, and extreme mode confinement [5–7]. In addition, the ability to control the properties of graphene layers via external magnetic and electric fields provides a road map for developing active magneto- and electro-optical devices [8, 9]. In the following sections, a brief introduction to channeling SPPs at optical and infrared wavelengths is presented.
channeling SPPs at optical wavelengths

Figure 1.1(a) shows the schematic of a MDM waveguide. The two layers of metal with permittivity $\epsilon_m$ are separated by a layer of dielectric with thickness $d$ and permittivity $\epsilon_1$. The structure is assumed to be infinite in the $y$ direction. The SPPs propagate at the interface between two materials whose dielectric permittivities are of the opposite sign. In this case, the materials with positive and negative permittivities are dielectrics and metals, respectively. The MDM waveguides have two metal-dielectric interfaces, and they are both capable of supporting the propagation of SPPs. There are therefore two sets of SPPs propagating at the lower and upper interface. The waveguide modes propagating within the waveguide are the result of the interaction between the SPPs propagating at the lower and upper interface. The interaction between these two SPPs creates two separate modes; the even (symmetric) and, the odd (antisymmetric) modes. Figure 1.1(b) shows the dispersion relation for the symmetric and the antisymmetric mode denoted as $S_b$, and $a_b$, respectively. One can observe that the antisymmetric mode, if the gap between the two metal layers is small enough, will vanish. In other words, this mode has a cut-off frequency which prevents the propagation of SPPs for frequencies that fall below the cut-off frequency. On the other hand, the behavior of the symmetric mode is completely different. One can see that this mode does not have any cut-off frequency and is a propagating mode regardless of the size of the gap $d$. This characteristic allows MDM waveguides to go beyond the diffraction limit, making the design of extremely miniaturized optical devices possible. In addition, the red curves in Fig. 1.1(c) display the typical field profiles of these two modes. One can observe that the symmetric mode has a symmetrical field profile along the cross section of the waveguide peaking at the center of the waveguide. Figure 3.1(c) shows the power distribution within the waveguide for the two modes. One can observe that the SPPs are mostly confined within the dielectric layer of the waveguide.
Figure 1.1: (a) The schematic of a MDM waveguide. The structure is assumed to be infinite in the $y$ direction. (b) The dispersion relation for the symmetric ($S_b$) and antisymmetric ($a_b$) modes. (c) The typical field profile of the symmetric and antisymmetric modes, displayed in solid and dashed lines, respectively. The blue curve shows the phase distribution of the modes. (d) The power distribution of the symmetric and antisymmetric modes within the waveguide, shown in solid and dashed lines, respectively.

1.3 SPPs on graphene

The field of graphene plasmonics has been growing over the past decade. In spite of graphene’s extremely small thickness, graphene is about 200 times stronger than the strongest steel and can efficiently conduct heat and electricity. Graphene is nearly transparent [10]. Furthermore, it supports the flow of SPPs in the infrared wavelength range [11]. One of the most important features of graphene is the fact that its optical properties are easily tunable via applying an external electric or magnetic field [5]. Exposing graphene to an external electric (or magnetic) field alters the number of carriers (electrons and holes) of a graphene layer which in turn changes the chemical potential of the layer exposed to the external field. The change in chemical potential consequently changes the permittivity of the layer [8, 9, 12]. The change of the optical properties of graphene in the presence
of external fields enables the active control of the flow of SPPs on graphene. Figure 1.2 shows a single layer of graphene placed in the proximity of a substrate. As can be seen, the middle part of the substrate (the part with width $W_2$) is closer to the graphene layer compared to the rest (the parts with width $W_1$). The bias voltage ($V_{bias}$) is set in such a way that only the part of the layer above the raised middle section of the substrate has the ability to support the propagation of SPPs. The smaller gap between the middle part of the dielectric and the graphene layer compared to the rest of the dielectric, causes the intensity of the field to be enhanced in the middle section. In addition, the chemical potential of graphene can be modified in a way that it would stop supporting the propagation of SPPs at a certain wavelength range. As is shown in Fig. 1.2, a single layer of graphene is capable of supporting the propagation of SPPs.

Figure 1.2: The schematic of a graphene layer suspended in air on top of a dielectric substrate with step like geometrical shape. The $V_{bias}$ is the voltage difference between the graphene layer and the dielectric substrate [13].
Figure 1.3: (a) The schematic of a GPPW with two layers of graphene placed in parallel with a coupling distance \(d\) between them. The graphene layers are assumed to be infinite along the \(y\) axis. (b) The symmetric and antisymmetric modes of the GPPW.

However, similar to MDM plasmonic waveguides, using several sheets of graphene and exploiting the coupling between them can increase the confinement of the modes. In addition, the coupling between layers can decrease the effect of loss [14–18]. Graphene parallel plate waveguides (GPPWs) are a class of graphene waveguides consisting of two or more parallel layers of graphene placed in the proximity of each other. Figure 1.3(a) shows the schematic of a GPPW with two layers of graphene. Placing two layers of graphene, each capable of supporting SPPs, in close proximity to each other allows the coupling between the layers, resulting in the appearance of two modes. Depending on the phase difference between the SPPs, the propagating mode can be symmetric (or even) when the SPPs are completely out of phase, i.e. the phase difference is \(\Delta \phi = \pi\) [Fig. 1.3(b)]. Similarly, if the SPPs are completely in phase \((\Delta \phi = \pi)\), the mode within the waveguide would be antisymmetric (or odd) [Fig. 1.3(b)]. Any wave propagating within the GPPWs with two graphene layers
can be categorized as a superposition of the two aforementioned modes. GPPWs or similar graphene structures, which are capable of supporting several modes, are commonly used in the design of optical devices ranging from waveguides [19, 20] to optical logic gates [21]. It is therefore imperative to have the ability to convert one mode to another. This is the reason that I designed a mode converter for GGPWs. The details of the structure will be discussed in the following chapter.

1.4 Non-reciprocity

Realization of fully functional optical integrated circuits requires nonreciprocal elements such as isolators and circulators. Despite the emergence of new designs based on materials with time-varying characteristics [22], or nonlinear materials [23, 24], the magneto-optical (MO) effect remains the main way to achieve non-reciprocity. However, the weak MO response of natural materials at optical wavelengths has limited the design and fabrication of nonreciprocal elements [25]. In the following section a basic introduction to the MO effect and its application in optics is presented.

1.4.1 Magneto-optical effect

The magneto-optical effect occurs when light travels through a gyrotropic or gyromagnetic medium that is exposed to an external quasi-static magnetic field. The presence of the magnetic field breaks the time reversal symmetry enabling the design of non-reciprocal elements such as isolators and circulators.

In gyrotropic materials the exposure to an external magnetic field alters the permittivity tensor of the material. The change in permittivity manifests itself as a nonzero off-diagonal imaginary component. The permittivity of a gyrotropic material can be expressed as follows:

\[
\epsilon = \begin{bmatrix}
\epsilon_{xx} & \epsilon_{xy} + ig_z & \epsilon_{xz} - ig_y \\
\epsilon_{yx} - ig_z & \epsilon_{yy} & \epsilon_{yz} + ig_x \\
\epsilon_{zx} - ig_y & \epsilon_{zy} - ig_x & \epsilon_{zz}
\end{bmatrix}.
\]  

(1.1)

Using the permittivity tensor, as well as the relation between the displacement field \(D\) and
The MO material employed in the design is BIG whose permittivity tensor is displayed in Eq. (1.5). The permittivity of the metal is assumed to be -10.

electric field $E$, one can deduce the following equation:

$$D = \varepsilon_0 \varepsilon_r E + iE \times g,$$

(1.2)

where

$$\varepsilon_r = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix},$$

(1.3)

and $g=(g_x, g_y, g_z)$ is the gyration vector. In comparison to $\varepsilon_r$, the components of the
gyration vector are very small. The components of the gyration vector are a function of the external magnetic field. Neglecting higher order terms, the gyration to first order is linearly proportional to the applied field:

\[ g = \epsilon_0 \chi^m H, \]

(1.4)

where \( \chi^m \) is the magneto-optical susceptibility, which is scalar for isotropic materials but
in more general is a tensor. However, upon increasing the intensity of the external magnetic field the linear relation does not hold any more and the material becomes saturated. The saturation causes the components of the gyration tensor to reach a maximum saturated value which depends on the material [26]. Due to the small value of the component of the gyration tensor, especially at optical wavelengths, the MO materials employed in the design of optical elements are usually exposed to an external magnetic field that causes the material to saturate. The required external field is usually in the order of mT, and it depends on the material [26]. One of the most commonly used MO materials in optics is bismuth iron garnet (BIG) whose permittivity can be expressed as follows:

\[
\varepsilon = \varepsilon_0 \begin{bmatrix}
\varepsilon_r & i\alpha & 0 \\
-i\alpha & \varepsilon_r & 0 \\
0 & 0 & \varepsilon_{\perp}
\end{bmatrix},
\]

where, \( \varepsilon_r = 6.25 \) and \( \alpha = 0.06 \) [27, 28], which saturates for an externally applied static magnetic field of \( B = 150 \text{mT} \) [26]. One observes that the ratio \( g/\varepsilon_r = 0.0096 \), as mentioned above, is very small. This weak MO response has limited the design and fabrication of nonreciprocal elements. Nevertheless, various designs based on the aforementioned method were proposed by researchers [29–33]. For instance, Fig. 1.4 displays the theoretical design for a circulator proposed by Davoyan and Engheta [28]. The principle of operation is based on MO activity in the vicinity of the resonance frequency [34]. The presence of MO activity leads to a split in the resonant frequencies of the resonator, which in turn enables the non-reciprocity observed in Fig. 1.5. Frequency splitting will be discussed in detail in Chapter 3. Figure 1.5 displays the operating principle of the proposed design. One observes that, in spite of the fact that part of the incident power escapes from the waveguides, the overall performance of the design as a circulator is satisfactory. Figure 1.5(a) displays the operation of the device when there is no external magnetic field. It is observed that in this case, the incident power from port 1 is equally transmitted to ports 2 and 3. However,
in Fig. 1.5, which shows the performance of the device in the presence of MO activity, the incident power is no longer equally transmitted to ports 2 and 3. Instead, the device demonstrates a nonreciprocal behavior, favoring one port over the other, depending on the direction of the incoming wave.
Chapter 2
Tunable Spatial Mode Converters And Optical Diodes for Graphene Parallel Plate Waveguides

2.1 Introduction

Graphene, which is a two-dimensional version of graphite, has been shown to have many interesting electronic and optical properties [6, 35–38]. The ability of layers of graphene to support surface plasmon polaritons has opened a new path for researchers to manipulate light at subwavelength scales [11, 39, 40]. Compared to conventional plasmonic materials, such as silver or gold, surface plasmons on graphene exhibit several important features including tunability [5], low-loss [6], and extreme mode confinement [7]. In addition, the conductivity and transport characteristics of graphene can be tuned using either electrostatic or magnetostatic gating, or via chemical doping [8, 9, 12]. Because of these properties, graphene is an excellent candidate for designing novel subwavelength plasmonic devices. Various structures exploiting plasmons on graphene have been proposed including waveguides [19, 20], modulators [41], filters [42, 43], photodetectors [44], and sensors [45]. In addition, the coupling between graphene plates has been investigated [14–18], and it has been demonstrated that coupled graphene plates, otherwise known as graphene parallel plate (GPP) waveguides, could be employed in designing different structures such as logic gates [21], phase shifters [46], and optical switches [47, 48]. Since GPP waveguides support multiple modes [16], it is important to develop spatial mode converters for such waveguides. The ability to manipulate optical spatial modes in integrated photonic circuits has significant importance due to its potential applications in mode-division multiplexing, efficient waveguide coupling, and all-optical logic gates and diodes [49–52]. Several different mode converters based on photonic crystal, silicon, metal-insulator-metal, and

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1 The materials used in this chapter previously appeared in a somewhat different format in journal of Optics Express. (Vahid Foroughi Nezhad, Ali Haddadpour, and Georgios Veronis. Tunable spatial mode converters and optical diodes for graphene parallel plate waveguides. Optics Express 24.21 (2016): 23883-23897)
nanowire waveguides have been proposed, and their connection to multiplexing and all-optical diodes have been investigated \([52–60]\). Recently, an all-optical modal isolator based on a multimode silicon waveguide was designed and fabricated by Feng et al. \([61]\). In addition, Frandsen et al. designed and experimentally verified a topology optimized mode converter, which converts the fundamental even mode of a dispersion engineered photonic crystal waveguide to the higher order odd mode \([62]\).

In this chapter, we first introduce a spatial mode converter between the even and odd modes of a GPP waveguide, based on modifying the chemical potential of a strip on one of the graphene layers of the waveguide. We find that using a chemical potential profile that corresponds to a piecewise approximation of a triangular envelope reduces reflection in the mode converter to almost zero in a broad wavelength range. In addition, modifying the chemical potential of a strip on both the upper and lower graphene plates leads to larger maximum conversion efficiency. We show that the wavelength of operation of the mode converter can be tuned in the mid-infrared wavelength range simply by adjusting the chemical potential of the graphene strip. We also introduce an optical diode for GPP waveguides based on the mode converter and a coupler, which consists of a single layer of graphene placed in the middle between the two plates of two GPP waveguides. We find that for both the spatial mode converter and the optical diode the device functionality is preserved in the presence of loss.

2.2 Mode converter

Figure 2.1 shows the schematic of a typical mode converter along with its scattering matrix. We use A and B to denote the even and odd modes, respectively, to the left of the converter. Similarly, we use C and D to denote the even and odd modes, respectively, to the right of the converter. The scattering matrix \(S\) of the converter relates the amplitudes of the outgoing modes to the amplitudes of the ingoing modes \([52]\). Since we are interested in the power conversion between different modes, we also define \(T_{ij} \equiv |S_{ij}|^2\). Thus, the elements of matrix \(T\) represent the power coupling efficiency between different modes. For
Figure 2.1: (a) The schematic of a typical mode converter. We use the letters A, B, C, and D to denote different modes. The blue and red arrows correspond to input and output in each mode, respectively. (b) The scattering matrix $S$ of the converter relates the amplitudes of the outgoing modes (denoted in red) to the amplitudes of the ingoing modes (denoted in blue).

We first briefly review the material properties of graphene. Based on the Kubo formula the surface conductivity of graphene in the absence of magnetic field can be written as follows [63]

\[
\sigma(\omega, \mu_c, \tau, T) = \sigma_{\text{intra}} + \sigma_{\text{inter}},
\]

where

\[
\sigma_{\text{intra}} = \frac{\tau e^2 k_B T}{\pi \hbar^2 (j\omega \tau + 1)} \left\{ \frac{\mu_c}{k_B T} + 2 \ln \left[ \exp \left( -\frac{\mu_c}{k_B T} \right) + 1 \right] \right\},
\]

an ideal converter $T_{AD} = T_{DA} = T_{BC} = T_{CB} = 1$, while all other matrix elements are equal to zero. Thus, the scattering matrix of an ideal converter is an anti-diagonal matrix with entries that are all equal to one. Due to fact that the device is reciprocal, one only needs to specify the first two rows of matrix $T$ to completely determine this matrix.

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\[
\sigma(\omega, \mu_c, \tau, T) = \sigma_{\text{intra}} + \sigma_{\text{inter}},
\]

where

\[
\sigma_{\text{intra}} = \frac{\tau e^2 k_B T}{\pi \hbar^2 (j\omega \tau + 1)} \left\{ \frac{\mu_c}{k_B T} + 2 \ln \left[ \exp \left( -\frac{\mu_c}{k_B T} \right) + 1 \right] \right\},
\]
and
\[
\sigma_{\text{inter}} = \frac{-je^2}{4\pi \hbar} \ln \left[ \frac{2|\mu_c| - \hbar(\omega - j\tau^{-1})}{2|\mu_c| + \hbar(\omega - j\tau^{-1})} \right].
\] (2.3)

The first and second terms in Eq. (2.1) are associated with intraband and interband contributions, respectively. The parameters \(\mu_c, \omega, \tau, k_B, \hbar, -e\) are the chemical potential, angular frequency, phenomenological carrier relaxation time, Boltzmann constant, reduced Planck constant, temperature, and electron charge, respectively. For the frequency range of interest in this chapter we have \(\hbar \omega \gg k_B T\) so that \(\sigma_{\text{intra}} \gg \sigma_{\text{inter}}\). The dependence of the carrier density of undoped graphene \(n_s\) on the external voltage \(V_g\) is given by [63]

\[
n_s = \frac{V_g \epsilon_0 \epsilon_r}{ed},
\] (2.4)

where \(\epsilon_0\) is the dielectric permittivity of free space, \(\epsilon_r\) is the relative dielectric permittivity of the intermediate layer (the layer between graphene and the electrode where the voltage \(V_g\) is applied), and \(d\) is the thickness of the intermediate layer. The chemical potential can be calculated via [63]

\[
n_s = \frac{2}{\pi \hbar^2 v_f^2} \int_0^\infty \epsilon [f_d(\epsilon) - f_d(\epsilon + 2\mu_c)] d\epsilon.
\] (2.5)

The function \(f_d(\epsilon) = [1 + \exp(\frac{\epsilon - \mu_c}{k_B T})]^{-1}\) is the Fermi function, and \(v_f\) is the Fermi velocity. Thus, based on Eqs. (2.4) and (2.5), the chemical potential of graphene can be easily adjusted by applying an external voltage. The negative imaginary part of \(\sigma\) in the infrared wavelength range, enables graphene to support surface plasmons.

The dispersion relation of the TM modes supported by a single-layer graphene sheet is given by [11]

\[
\frac{\epsilon_r 1}{k_1} + \frac{\epsilon_r 2}{k_2} = j \frac{\sigma}{\omega \varepsilon_0},
\] (2.6)
where

\[ k_i = \sqrt{\beta^2 - k_0^2\epsilon_{ri}}, \]  

\( (2.7) \)

\( \epsilon_{r1} \) and \( \epsilon_{r2} \) are the relative dielectric permittivities of the media surrounding the graphene layer, \( k_0 = \omega / c \) is the free space wavenumber, and \( \beta \) is the propagation constant of the single-layer graphene. Here, all graphene sheets are modeled via a two-dimensional surface boundary condition with surface conductivity \( \sigma = \sigma_{\text{intra}} \) using the two-dimensional finite-difference time-domain (2D-FDTD) method [64, 65]. GPP waveguides consist of two layers of graphene that are brought close together. Due to the proximity of the two layers, their modes couple. GPP waveguides therefore support a symmetric (even) and an antisymmetric (odd) mode [16]. Here we propose a structure based on GPP waveguides that operates as a mode converter between the even and odd modes of GPP waveguides. The principle of the device operation is based on the fact that the odd and even modes can be thought of as in-phase and \( \pi \) out-of-phase interactions, respectively, between the surface plasmons propagating on the upper and lower graphene layers. Assuming that the coupling between the two layers is weak, to convert one mode into the other, one needs to create an odd multiple of \( \pi \) phase shift between them. To achieve the required phase shift, we modify the chemical potential \( \mu_c \) on a strip on one of the graphene layers by applying an external voltage or by chemically doping it. The schematic of the converter is shown in Fig. 2.2. The strip with the different chemical potential is shown in red. If the coupling between the two graphene layers is weak, the structure can be treated as two weakly coupled single-layer waveguides and the condition for the required phase shift can be approximated as

\[ \beta_sL - \beta_gL = (2m - 1)\pi, \]  

\( (2.8) \)

where \( \beta_s \) and \( \beta_g \) are the propagation constants of surface plasmons propagating on single-layer graphene waveguides with chemical potential \( \mu_{cs} \) and \( \mu_{cg} \), respectively, which can be calculated using Eqs. (2.6) and (2.7). \( L \) is the length of the strip, and \( m \) is an integer.
Figure 2.2: Schematic of a spatial mode converter consisting of two parallel layers of graphene with chemical potential $\mu_{cg}$, and a strip of length $L$ with chemical potential $\mu_{cs}$, shown in red.

number. Therefore, for a given $\mu_{cg}$, by appropriate choices of $\mu_{cs}$ and $L$ one can obtain the phase shift required for the conversion process. To simplify the design procedure, we first assume that the graphene layers are lossless ($\tau \to \infty$). The effect of loss will be considered later. We also assume that all graphene layers are suspended in air and that the temperature is $T = 300$ K. Fig. 2.3(a) shows the profile of the chemical potential of the upper graphene layer. For such a profile Fig. 2.3(b) shows the transmission spectra from mode A to modes C and D, as well as the reflection spectra to modes A and B calculated with FDTD. Even though we only show here the conversion efficiency from mode A to mode D, the other three conversion processes (D to A, B to C, and C to B) have identical spectra due to reciprocity and the symmetry of the structure [52]. We observe that for the given structure parameters, the calculated conversion efficiency from mode A to mode D is maximized at $\lambda = 10.35 \, \mu$m. In addition, the condition for the required phase shift for maximum conversion efficiency [Eq. (2.8)] with $m=1$ is satisfied for $\lambda = 10.25 \, \mu$m, which is in good agreement with the numerical simulation result. We also observe that,
due to the abrupt change in the chemical potential on the upper plate, the conversion from mode A to mode D at the optimum wavelength $\lambda = 10.35 \, \mu m$ is not complete. One of the methods to reduce the reflection in such a structure is to modify the chemical potential profile of the strip [46]. Here, we use a chemical potential profile that corresponds to a piecewise approximation of a triangular envelope as shown in Fig. 2.4(a) with minimum potential of $\mu_{cm}$. Modifying the chemical potential profile of the strip reduces the reflection coefficients of the mode converter to almost zero in a broad wavelength range [Fig. 2.4(b)]. The profile of $H_z$ shown in the inset of Fig. 2.4(b), corresponds to mode conversion with $\sim 99\%$ efficiency at a wavelength of $\lambda = 10.1 \, \mu m$. The condition for the optimum mode conversion for the structure with modified chemical potential profile can be approximated as

$$\sum_{i=1}^{N} \beta_i d_i - \beta_g L = (2m - 1)\pi, \quad (2.9)$$

where $d_i$ is the length of the $i^{th}$ segment of the strip with corresponding chemical potential $\mu_{ci}$, $N$ is the number of segments, and $L = \sum_{i=1}^{N} d_i$ is the total length of the strip. In addition, $\beta_i$ is the propagation constant of the surface plasmon mode propagating on a single-layer graphene waveguide with chemical potential $\mu_{ci}$, which can be calculated using Eqs. (2.6) and (2.7). For the profile shown in Fig. 2.4(a), $d_i = 50 \, nm$ for $i \neq 4$, and $d_4 = 100 \, nm$. Using Eq. (2.9), the wavelength which corresponds to maximum conversion efficiency is calculated to be $\lambda_{opt} = 10.24 \, \mu m$, which is in good agreement with the optimum wavelength obtained from the FDTD numerical simulations ($\lambda = 10.1 \, \mu m$).

The wavelength of operation of the mode converter can be tuned by adjusting the chemical potential of the graphene strip. Figure 2.5(a) shows the conversion efficiency for different values of the minimum potential $\mu_{cm}$ of the triangular envelope [Fig. 2.4(a)]. The profile applied here is as in Fig. 2.4(a). We observe that the wavelength which corresponds to maximum conversion efficiency can be tuned simply by adjusting the minimum value $\mu_{cm}$ of the applied profile of the chemical potential. As mentioned above, the chemical potential of graphene can in turn be adjusted through the external applied voltage. We observe
Figure 2.3: (a) Profile of the chemical potential of the upper graphene layer in the spatial mode converter shown in Fig. 2.2. (b) Transmission spectra from mode A to modes C and D, and reflection spectra to modes A and B calculated with FDTD for the mode converter shown in Fig. 2.2 with chemical potential profile as in Fig. 2.3(a), and $\mu_{cg} = 0.3$ eV, $\mu_{cs} = 0.205$ eV, $h = 150$ nm, $L = 400$ nm. The graphene layers are assumed to be lossless.

that, as $\mu_{cm}$ decreases, the wavelength of maximum conversion efficiency $\lambda_{opt}$ increases [Fig. 2.5(a)]. As before, we use Eq. (2.9) to calculate the wavelength which corresponds
Figure 2.4: (a) Chemical potential profile of the graphene strip in the spatial mode converter shown in Fig. 2.2. The profile corresponds to a piecewise approximation with a step size of 50 nm of a triangular envelope with minimum potential of $\mu_{cm}$. (b) Transmission spectra from mode A to modes C and D, and reflection spectra to modes A and B calculated with FDTD for the mode converter shown in Fig. 2.2 with chemical potential profile as in Fig. 2.4(a), and $\mu_{cm} = 0.127$ eV. All other parameters are as in Fig. 2.3(b). The magnetic field profile at $\lambda = 10.1 \mu m$ shown in the inset demonstrates the complete conversion of the even mode A incident from the left into the odd mode D propagating to the right, and vice versa.
Figure 2.5: (a) Conversion efficiency spectra calculated with FDTD for the mode converter shown in Fig. 2.2 with chemical potential profile as in Fig. 2.4(a) for different values of the minimum potential $\mu_{cm}$ of the triangular envelope. Results are shown for $\mu_{cm} = 0.145$ eV, $\mu_{cm} = 0.115$ eV, and $\mu_{cm} = 0.08$ eV. All other parameters are as in Fig. 2.3(b). (b) The wavelength which corresponds to maximum conversion efficiency $\lambda_{opt}$ as a function of $\mu_{cm}$ calculated using Eq. (2.9) (blue line), and FDTD (red line). All other parameters are as in Fig. 2.5(a).

to maximum conversion efficiency $\lambda_{opt}$ as a function of $\mu_{cm}$, and find that it is in good agreement with the optimum wavelength obtained from the FDTD numerical simulations.
Figure 2.6: Schematic of the double strip spatial mode converter. It consists of two parallel layers of graphene with strips with modified chemical potential on both the upper and lower graphene plates, shown in red and blue, respectively.

[Fig. 2.5(b)]. We also observe that the maximum conversion efficiency slightly decreases, as $\mu_{cm}$ decreases [Fig. 2.5(a)]. The decrease in the maximum conversion efficiency is due to stronger coupling between the graphene plates. Several factors could lead to increased coupling between the sheets including increasing the wavelength, decreasing the distance between the plates, and increasing the chemical potential of the plates [16]. Under the strong coupling condition, the system can no longer be treated as two weakly coupled single-layer waveguides. To overcome this issue, we modify the chemical potential of a strip on both the upper and lower graphene plates (Fig. 2.6). As the chemical potential of the lower plate is reduced, the coupling between the modes supported by the two plates decreases. The condition for the optimum mode conversion for the structure with strips with modified chemical potential on both the upper and lower graphene plates (Fig. 2.6), assuming similar profiles on both plates, can be approximated as

$$\sum_{i=1}^{N} (\beta_{ui} - \beta_{di})d_i = (2m - 1)\pi,$$

(2.10)
Figure 2.7: (a) Conversion efficiency spectra calculated with FDTD for the single strip (Fig. 2.2) and double strip (Fig. 2.6) mode converters with chemical potential profile as in Fig. 2.4(a). The minimum potential for the single strip converter is set to be $\mu_{cm} = 0.08$ eV. For the double strip case the minimum potentials of the upper and lower strips are set to be $\mu_{cmu} = 0.05$ eV and $\mu_{cmd} = 0.19$ eV, respectively. The strip length is $L = 400$ nm and the plate separation is $h = 150$ nm in both cases. All other parameters are as in Fig. 2.3. (b) Same as in (a), except that $L = 500$ nm, $\mu_{cg} = 0.5$ eV, the minimum potential for the single strip converter is set to be $\mu_{cm} = 0.16$ eV, and for the double strip case the minimum potentials of the upper and lower strips are set to be $\mu_{cm} = 0.16$ eV and $\mu_{cmd} = 0.3$ eV, respectively. (c) Same as in (a), except that $L = 500$ nm, $h = 100$ nm, the minimum potential for the single strip converter is set to be $\mu_{cmd} = 0.15$ eV, and for the double strip case the minimum potentials of the upper and lower strips are set to be $\mu_{cmu} = 0.05$ eV and $\mu_{cmd} = 0.098$ eV, respectively.

where $d_i$ is the length of the $i^{th}$ segment of the strips with corresponding chemical potentials $\mu_{cui}$ and $\mu_{cli}$ for the upper and lower plates, respectively. In addition, $\beta_{ui}$ ($\beta_{li}$) is the propagation constant of the surface plasmon mode propagating on a single-layer graphene
Figure 2.8: (a) Conversion efficiency spectra calculated with FDTD for the mode converter shown in Fig. 2.2 with chemical potential profile as in Fig. 2.4(a) for different values of the minimum potential $\mu_{cm}$ of the triangular envelope, when the effect of material loss in graphene is included. All other parameters are as in Fig. 2.5(a). (b) Magnetic field profiles for the converters with $\mu_{cm} = 0.145$ eV (top figure), $\mu_{cm} = 0.115$ eV (middle figure), and $\mu_{cm} = 0.08$ eV (bottom figure). In each case the profile is shown at the wavelength which corresponds to maximum conversion efficiency $\lambda_{opt}$. 
waveguide with chemical potential $\mu_{\text{cui}}$ ($\mu_{\text{cli}}$), which can be calculated using Eqs. (2.6) and (2.7). In order to compare the performance of the double strip converter with the single strip converter, the chemical potential profile parameters are chosen so that the wavelength which corresponds to maximum conversion efficiency $\lambda_{\text{opt}}$ is the same in both cases. The minimum potential for the single strip converter is set to be $\mu_{cm} = 0.08$ eV. For the double strip case the minimum potentials of the upper and lower strips are set to be $\mu_{cmu} = 0.05$ eV and $\mu_{cmd} = 0.19$ eV, respectively. The strip length is $L = 400$ nm and the plate separation is $h = 150$ nm in both cases. We observe that the double strip converter achieves larger maximum conversion efficiency compared to the single strip converter [Fig. 2.7(a)]. Thus, in the case of strong coupling between the graphene plates, which limits the maximum conversion efficiency, the double strip converter can be used to increase the efficiency. Finally, the effect of the loss on the structure is investigated by setting the phenomenological carrier relaxation time $\tau$ [Eqs. (2.1)-(2.3)] to be $\tau = \mu \mu_c / (e v_f^2)$, where $\mu = 10^4$ cm$^2$V$^{-1}$s$^{-1}$ is the carrier mobility, and $v_f = 10^6$ ms$^{-1}$ is the Fermi velocity [11]. The lossless case results [Fig. 2.5(a)] can be compared to the lossy case results shown in Fig. 2.8. When the effect of loss is included, there is absorption in the mode converter, which causes the transmission and consequently the conversion efficiency to decrease. It should be noted that the reduction of the conversion efficiency from mode A to mode D is solely due to absorption. We found that the maxima of the transmission spectra from mode A to mode C, and of the reflection spectra from mode A to modes A and B are less than 1%. The $H_z$ profiles shown in Fig. 2.8(b) demonstrate that, even in the presence of loss, mode A, which is even, is converted to mode D, which is odd, with negligible transmission to mode C and reflection to modes A and B.

2.3 Optical diode

An ideal optical diode is a device which allows the complete transmission of one mode in one direction, while entirely reflects all other modes entering the device from the same direction. If the same mode is sent from the other direction, it is completely reflected [52].
Figure 2.9: Schematic of an optical diode consisting of two GPP waveguides, which includes a mode converter on the left GPP waveguide (shown in red color), and a coupler, which consists of a single layer of graphene placed in the middle between the two plates of the two GPP waveguides.

Figure 2.9 shows the schematic of the proposed optical diode. The device includes a mode converter as the one analyzed in the previous section (shown in red color), and a coupler, which consists of a single layer of graphene placed in the middle between the two plates of two GPP waveguides. This structure is an ideal optical diode if mode A from the left (Fig. 2.1) is completely transmitted to mode D on the right, while mode B from the left is completely reflected into the same mode B on the left. In addition, mode D from the right is completely transmitted to mode A on the left, while mode C from the right is completely reflected into the same mode C on the right [52]. Similar to the analysis of the mode converter, we first assume that all graphene layers are lossless ($\tau \to \infty$), while the effect of loss is considered later. The coupling between the GPP waveguides and the middle single graphene layer enables the transmission of light from one side of the structure to the other. The single graphene layer supports an antisymmetric (odd) mode. As mentioned above, GPP waveguides support a symmetric (even) and an anti-symmetric (odd) mode. Thus, if the even mode of the GPP waveguide is incident on the coupler, the coupling between the GPP waveguide and the single graphene layer is zero, due to the complete field profile mismatch. In contrast, there is strong coupling, if the odd mode of the GPP waveguide is incident on the coupler. This different coupling behavior along with the existence of the mode converter only on side of the device enable it to perform as an optical diode. When the even mode of the GPP waveguide is incident from the left, it is converted into the odd mode.
of the GPP waveguide through the mode converter. The odd mode of the GPP waveguide on the left is in turn strongly coupled to the mode of the single graphene layer, which then couples to the odd mode of the GPP waveguide on the right. On the other hand, when the odd mode of the GPP waveguide is incident from the left, it is converted into the even mode of the GPP waveguide through the mode converter. However, as mentioned above, the even mode of the GPP waveguide on the left cannot couple to the mode of the single graphene layer, and is therefore reflected. Since the right GPP waveguide does not include a mode converter, when the even (odd) mode of the GPP waveguide is incident from the right, it will be reflected (transmitted). The distance between the mode converter on the left GPP waveguide and the coupler $L_f$ (Fig. 2.9) should be large enough, so that the mode on the output of the converter is fully formed. On the other hand, increasing the size of the device results in increased insertion loss. The length of the coupler $L_c$ (Fig. 2.9) has to be chosen so that the coupling, when the odd mode of the GPP waveguide is incident on the coupler, is maximized at the wavelength $\lambda_{\text{opt}}$ which corresponds to maximum conversion efficiency of the mode converter. The coupler length $L_c$ which leads to maximum coupling can be estimated as $L_c = \pi/(2|\beta_{\text{even}} - \beta_{\text{odd}}|)$ [47], where $\beta_{\text{even}}$ and $\beta_{\text{odd}}$ are the propagation constants of the even and odd modes of the GPP waveguide with width $h/2$ equal to the distance between the plates of the GPP waveguides of the diode and the middle single graphene layer (Fig. 2.9). Using this equation, we obtain $L_c = 173$ nm which results in transmission calculated with FDTD of $\sim 0.99$ at $\lambda_{\text{opt}} = 9.85$ $\mu$m. However, in our design we use $L_c = 130$ nm, since we found that such a coupler length leads to transmission of more than 0.93 in a broad wavelength range extending from $\lambda_1 = 9.4$ $\mu$m to $\lambda_2 = 13.4$ $\mu$m. Using a smaller $L_c$ also decreases the insertion loss. Finally, the distance $L_s$ between the two GPP waveguides of the optical diode (Fig. 2.9) should be large enough, so that the direct coupling between the modes of the two GPP waveguides is negligible. The profile for the chemical potential of the mode converter in the optical diode is as the one in Fig. 2.4(a) with $\mu_{\text{em}} = 0.135$ eV. Figure 2.10(a) shows the transmission spectra for the lossless
Figure 2.10: (a) Transmission spectra from mode A to mode D, and reflection spectra from mode B to mode B and from mode C to mode C calculated with FDTD for the optical diode shown in Fig. 2.9 with $L = 400$ nm, $L_f = 200$ nm, $L_c = 130$ nm, $L_s = 400$ nm, and $h = 150$ nm. The mode converter of the diode has a chemical potential profile as in Fig. 2.4(a) with $\mu_{cm} = 0.135$ eV. The graphene layers are assumed to be lossless. (b) Magnetic field profiles for the optical diode of Fig. 2.9 for even and odd modes entering from the left and right directions. The red and black arrows indicate the direction of incidence of the even and odd modes, respectively. The profiles are shown at the wavelength which corresponds to maximum conversion efficiency $\lambda_{opt} = 9.85$ $\mu$m. The graphene layers are assumed to be lossless. It is observed that when the even mode enters the device from the right, it is reflected, whereas, when it enters the device from the left, it is transmitted. In contrast, the odd mode is transmitted, when it enters the device from the right, and reflected, when it enters the device from the left.
Figure 2.11: (a) Transmission spectra and (b) the magnetic field profiles for the optical diode shown in Fig. 2.9, when the effect of loss is included. All other parameters are as in Fig. 2.10.

case from mode A to mode D, as well as the reflection spectra from mode B to mode B and from mode C to mode C calculated with FDTD. Even though the transmission from mode D to mode A is not shown, we note that it has identical spectra with the transmission from mode A to mode D due to reciprocity [52]. We observe that for the lossless case all coefficients (A to D, B to B, and C to C) are close to 1 at the wavelength at which the diode was designed ($\lambda_{opt} = 9.85 \mu m$). Figure 2.11(a) shows the same spectra when the effect of loss is included. When loss is included, there is absorption in the optical diode,
which causes the transmission from mode A to mode D to decrease. However, we found that, despite the loss, the structure still acts as an optical diode at the design wavelength $\lambda_{opt}$, in the sense that even mode A from the left is only transmitted to odd mode D on the right, while even mode C from the right is only reflected into the same mode C on the right. All other coefficients (A to A, A to B, and A to C) remain less than 0.005 at $\lambda_{opt}$. The $H_z$ profiles at $\lambda_{opt} = 9.85 \, \mu m$ for the lossless and lossy cases are shown in Fig. 2.10(b), and Fig. 2.11(b), respectively. We observe that, when the even mode enters the device from the left, it is transmitted, whereas, when it enters the device from the right, it is reflected. In contrast, the odd mode is transmitted, when it enters the device from the right, and reflected, when it enters the device from the left. This functionality is preserved in the presence of loss [Fig. 2.10(b), and Fig. 2.11(b)].
Chapter 3
Nanoscale Plasmonic Isolator

3.1 Introduction

The progress in the field of plasmonics and nanophotonics has made the design of ultra-compact on-chip optical elements possible [66, 67]. Different optical elements such as filters, sensors, beam splitters, and switches have been demonstrated [68]. Meanwhile, complete realization of optical integrated circuits requires the design of nonreciprocal elements such as circulators and isolators. Designing nonreciprocal components necessitates breaking time reversal symmetry [69, 70], which can be achieved via different methods such as using nonlinear materials [23, 24], materials with time dependent properties [22], and magneto-optical (MO) materials [71]. However, due to the weak MO response of naturally occurring materials at optical wavelengths, designing nonreciprocal devices employing MO activity results in bulky structures that are much larger than the wavelength [25]. The advent of silicon photonics and magneto-photonic crystals has reduced the size down to wavelength scale and facilitated the design of compact nonreciprocal optical components [72–74]. However, to further decrease the size down to subwavelength scales (< λ), one needs to overcome the diffraction limit. The problem of diffraction limit can be surmounted by exploiting surface plasmon polaritons (SPPs) [1]. SPPs are the waves propagating at the interface between metals and dielectrics due to coupling between electromagnetic waves and free electrons in metals [1, 75]. Their ability to overcome the diffraction limit has made SPPs excellent candidates to make optical circuits more miniaturized and compact [3]. The combination of plasmonic with MO materials can therefore pave the way for fabrication of highly-compact nonreciprocal subwavelength-sized elements. Various groups have proposed different methods and structures exploiting the nonreciprocal characteristics of MO materials along with the compactness of plasmonic elements [29–33]. Recently, a nanoscale plasmonic circulator based on metallic nanorod structures immersed in MO media was pro-
posed [28]. The authors demonstrated the possibility of enhancing the MO response in the deep subwavelength regime via plasmonic mode engineering. One of the promising ways to engineer on-chip integrated optical circuits based on SPPs is employing metal-dielectric-metal (MDM) waveguides [76]. In the last decade, an increasing amount of research has been allotted to the development of optical structures using the potential of MDM waveguides for channeling SPPs. Among these structures, one can mention filters [2, 77], couplers [78, 79], sensors [80, 81], switches [82], and isolators [83].

In this chapter, we propose an extremely compact isolator based on nanoscale MDM waveguides. The structure is composed of a cylindrical cavity filled with a MO dielectric placed in the proximity of a MDM waveguide. In the presence of MO activity, the resonant modes of the cavity exhibit an azimuthal symmetry breaking which consequently causes the resonant frequencies of the cavity to split, a phenomenon similar to the Zeeman frequency splitting in quantum mechanics [34]. An analytical formula based on first order perturbation theory is derived to explain the nature of the resonance splitting. It is demonstrated that, in the absence of MO activity, the resonant modes of the cavity are standing waves. On the other hand, in the presence of MO activity, the cavity becomes a traveling-wave resonator and since the cavity modes decay rates for traveling-wave modes are direction dependent due to momentum matching [84], the device exhibits different transmission spectra for the SPPs incident from different directions. The conversion of standing-wave modes into traveling-wave modes due to MO activity along with the mode splitting phenomenon enable the device to operate as an isolator. The proposed structure is ultracompact and the wavelength of interest can be tuned via adjusting the parameters of the structure including the radius of the cavity and the distance between the cavity and the MDM waveguide.

3.2 Results

Figure 3.1 shows the schematic of the proposed nanoplasmonic isolator. The device consists of a cylindrical resonator placed in the proximity of a MDM waveguide. The structure is subject to an externally applied static magnetic field in the z direction.
metal is silver. We use the Drude model to describe the dielectric permittivity of silver $\epsilon_m$ with dielectric constant at infinite frequency $\epsilon_\infty = 3.7$, bulk plasmon frequency $\omega_p = 9.1$ eV, and collision frequency $\gamma = 0.018$ eV [85]. The waveguide and resonator are filled with a MO material. In the presence of a static magnetic field in the $z$ direction, the dielectric permittivity of the MO material is described by a tensor

$$
\epsilon = \epsilon_0 \begin{bmatrix}
\epsilon_r & i\alpha & 0 \\
-i\alpha & \epsilon_r & 0 \\
0 & 0 & \epsilon_\perp
\end{bmatrix},
$$

(3.1)

where $\alpha$ is the strength of MO activity. The MO material used here is bismuth iron garnet (BIG) with $\epsilon_r = 6.25$, and $\alpha = 0.06$ [27, 28], which saturates for an externally applied static magnetic field of $B = 150$ mT [26]. However, for our analytical investigation, the value of $\alpha$ is varied from 0 to 0.5. We first investigate the resonance condition and modes of the cylindrical resonator exposed to an external static magnetic field in the absence of the MDM waveguide. We then investigate the effect of the waveguide on the resonance condition of the cavity and the performance of the device as an isolator. By applying the boundary conditions at the metal-dielectric interface at $\rho = R$, one can derive the following resonance condition for the modes supported by the cavity [86]

$$
\frac{m\alpha}{\epsilon_r \epsilon_e R} + \frac{k_1 J'_m(k_1 R)}{\epsilon_e J_m(k_1 R)} = \frac{k_2 H^{(2)'}_m(k_2 R)}{\epsilon_m H^{(2)}_m(k_2 R)},
$$

(3.2)

where $m$ is the azimuthal mode number, $J_m$ is the $m^{th}$-order Bessel function of the first kind, $H^{(2)}_m$ is the $m^{th}$-order Hankel function of the second kind, $k_1 = k_0 \sqrt{\epsilon_r}$, $k_2 = k_0 \sqrt{\epsilon_m}$, $k_0 = \omega/c$ is the free-space wave number, and $\epsilon_e = \epsilon_r - \alpha^2/\epsilon_r$. In the presence of material loss in the metal, the wavelengths which satisfy Eq. (3.2) are complex. Here we only consider the fundamental cavity mode which has relatively low loss, so that its resonant wavelength has small imaginary part. Each cavity mode is characterized by two mode numbers, the
Figure 3.1: The schematic of the nanoplasmonic isolator consisting of a cylindrical cavity with radius $R$ placed in the proximity of a MDM waveguide with width $W$. The blue and orange colors correspond to metal and MO material, respectively. The structure is subject to an externally applied static magnetic field in the $z$ direction.

radial mode number $n$, and the azimuthal mode number $m$. The fundamental mode of the cavity has $m = n = 1$. We observe that, when the off-diagonal elements of the dielectric permittivity tensor in Eq. (3.1) are set equal to zero, Eq. (3.2) reduces to the resonance condition for an isotropic cylindrical resonator [85]. There are two differences between Eq. (3.2) and the resonance condition for isotropic cylindrical resonators: First, when the resonator is filled with a MO material, the first term on the left side of Eq. (3.2) appears. Second, the wavenumber $k_1$ is modified since $\epsilon_e$ is different than $\epsilon_r$. In the absence of the first term on the left side of Eq. (3.2), the $\pm m$ cavity modes are degenerate with the same resonant wavelength. In the presence of a MO material, the first term on the left side of Eq. (3.2) breaks the symmetry between the $\pm m$ cavity modes. These modes are no longer degenerate, since they have different resonant wavelengths. The cavity mode with resonant wavelength $\lambda_c$ in the absence of MO activity ($\alpha = 0$), splits into a red-shifted and a blue-shifted mode with resonant wavelengths $\lambda_{\pm} = \lambda_{cm} \pm \Delta \lambda / 2$, in the presence of MO activity ($\alpha \neq 0$). We calculate analytically using Eq. (3.2) the resonant wavelengths of the cylindrical resonator in the absence of the MDM waveguide ($d = \infty$ in Fig. 3.1) as a function of the radius $R$ of the cylindrical cavity in the presence and absence of
Figure 3.2: Resonant wavelengths of the cylindrical resonator in the absence of the MDM waveguide \(d = \infty\) in Fig. 3.1) as a function of the radius \(R\) of the cylindrical cavity. The metal is silver and the MO material is bismuth iron garnet (BIG). The resonant wavelength in the absence of MO activity \((\alpha = 0)\) calculated with FDFD is shown with black circles. The red-shifted \((\lambda_+\) and blue-shifted \((\lambda_-)\) resonant wavelengths in the presence of MO activity \((\alpha = 0.5)\) calculated with FDFD are shown with red and blue circles, respectively. The black, red, and blue solid lines correspond to the analytically calculated resonant wavelengths of the cylindrical resonator in the absence of the MDM waveguide [Eq. (3.2)].

MO activity (Fig. 3.2). As expected, increasing \(R\) increases the resonant wavelengths of the structure (Fig. 3.2). The dependence of the resonant wavelengths on the radius \(R\) is approximately linear. We also calculate the resonant wavelengths of the cylindrical resonator in the absence of the MDM waveguide using the two-dimensional finite-difference-frequency-domain (FDFD) method (Fig. 3.2). We observe that there is excellent agreement between the analytical results and the numerical results obtained using the FDFD method. Since the functionality of the structure of Fig. 3.1 as an isolator is based on the cavity mode splitting in the presence of MO activity, we investigate the dependence of the wavelength splitting in the presence of MO activity \((\Delta \lambda)\) on the parameters of the structure. More specifically, using first order perturbation theory, we find the following expression for the
Figure 3.3: The difference between the red-shifted ($\lambda_+$) and blue-shifted ($\lambda_-$) resonant wavelengths ($\Delta \lambda = \lambda_+ - \lambda_-$) of the cylindrical resonator in the absence of the MDM waveguide ($d = \infty$ in Fig. 3.1) as a function of the strength of magneto-optical activity $\alpha$ calculated with FDFD (red dots) for $R = 120$ nm. Also shown are the analytically calculated results [Eq. (3.2)] (solid line), and the results obtained with perturbation theory [Eq. (3.3)] (open circles) for the cylindrical resonator in the absence of the MDM waveguide.

Wavelength splitting $\Delta \lambda$ [87]

$$|\Delta \lambda| \approx \frac{\Delta \omega \lambda_c^2}{2\pi c} \approx \frac{m\alpha \lambda_c^3 |J_m(k_1'R)|^2}{\int_0^R |J_m(k'\rho)|^2 \rho d\rho + \int_R^\infty \frac{|J_m(k'R)|^2}{|H_m^{(2)}(k_2R)|^2} \int_0^\infty |H_m^{(2)}(k_2\rho)|^2 \rho d\rho} \epsilon_r \epsilon_e \pi^2. \quad (3.3)$$

It should be noted that Eq. (3.3) is derived under the assumption that $\frac{\alpha}{\epsilon_r} \ll 1$. Figure 3.3, shows the calculated values of $\Delta \lambda$ for different values of $\alpha$. We observe that the results obtained with perturbation theory [Eq. (3.3)] for the wavelength splitting ($\Delta \lambda$) in the presence of MO activity and in the absence of the MDM waveguide are in good agreement with the analytical results [Eq. (3.2)] confirming the validity of the perturbation theory in describing the properties of the structure. We also observe that for large $\alpha$ the results...
obtained with the perturbation theory slightly deviate from the exact analytical results, since $\frac{\alpha}{\epsilon_r} \ll 1$ no longer holds.

### 3.3 Effect of MO activity on the cavity modes

We now consider how the off-diagonal elements in the dielectric permittivity tensor of the MO material affect the cavity modes. In the absence of MO activity ($\alpha = 0$), the $\hat{\phi}$ component of the total Poynting vector, which is the sum of the $\hat{\phi}$ components of the Poynting vectors of the $\pm m$ modes, is zero. The resonant modes inside the cavity are therefore standing modes. This can be seen by calculating the $\hat{\phi}$ component of the Poynting vector inside the cavity. The $H_z$ and $E_\rho$ field components inside the cavity can be written as follows

\[ H_z = AJ_m(k_1 \rho)e^{im\phi}, \quad (3.4a) \]

\[ E_\rho = \frac{1}{\rho \omega \epsilon_0 \epsilon_r \epsilon_e} \left( \alpha \rho \frac{\partial H_z}{\partial \rho} - i \epsilon_r \frac{\partial H_z}{\partial \phi} \right) = \frac{A}{\rho \omega \epsilon_0 \epsilon_r \epsilon_e} \left[ \alpha k_1 \rho J'_m(k_1 \rho) + m \epsilon_r J_m(k_1 \rho) \right] e^{im\phi}. \quad (3.4b) \]

Using Eqs. (3.4.a) and (3.4.b), the $\hat{\phi}$ component of the Poynting vector inside the cavity can be calculated as follows

\[ S_\phi = -Re\left\{ \frac{1}{2} \hat{E}_\rho H_z a_\phi \right\} = Re\left\{ -\frac{|A|^2 J'_m(k_1 \rho)}{2 \rho \omega \epsilon_0 \epsilon_r \epsilon_e} \left[ m \epsilon_r J_m(k_1 \rho) + \alpha k_1 \rho J'_m(k_1 \rho) \right] \right\}. \quad (3.5) \]

Based on Eq. (3.5), when $\alpha = 0$, the $\hat{\phi}$ component of the Poynting vector $S_\phi$ is an odd function of the azimuthal mode number $m$. The $\hat{\phi}$ component of the total Poynting vector at the resonant wavelength $\lambda_c$, which is the sum of the $\hat{\phi}$ components of the Poynting vectors of the $+m$ and $-m$ modes, is therefore zero. Thus, the resonant modes of the cavity are standing waves. On the other hand, in the presence of MO activity ($\alpha \neq 0$) the $\hat{\phi}$ component of the Poynting vector $S_\phi$ is no longer an odd function of the mode number $m$, because of the second term in Eq. (3.5). In addition, the $\pm m$ modes are no longer degenerate. Thus, when $\alpha \neq 0$, the $\hat{\phi}$ component of the total Poynting vector is not zero. Consequently, the modes inside the cavity are no longer standing modes and the cavity
3.4 Effect of the MDM waveguide on the cavity modes

So far, we investigated the modes of the cylindrical cavity in the absence of the MDM waveguide. We demonstrated that when \( \alpha \neq 0 \) the cavity mode splits into two modes with resonant wavelengths which can be calculated using Eq. (3.2). In addition, we showed that the presence of MO activity converts the modes of the cavity from standing modes to traveling modes. The resonant wavelengths of the structure of Fig. 3.1 are affected when the cavity is placed in close proximity to the MDM waveguide. We will first investigate the coupling between the MDM waveguide and the cavity and its effect on the resonant wavelengths of the modes of the structure in the absence of MO activity. We will next investigate the effect of the waveguide in the presence of MO activity. To investigate the effect of the coupling between the waveguide and the cavity on the resonant wavelengths of the structure, we calculate the transmission, reflection and absorption spectra in the absence of MO activity (\( \alpha = 0 \)) for the structure of Fig. 3.1. Results are shown in Fig. 3.4 for different values of the distance \( d \) between the cavity and the waveguide.

We observe that the presence of the MDM waveguide leads to a second resonance in the response of the structure [Figs. 3.4(a) and (b)]. Similar to the case where MO activity is present, the two resonant wavelengths are denoted as \( \lambda_+ \) and \( \lambda_- \). We also observe that decreasing the distance \( d \) between the cavity and the waveguide shifts \( \lambda_+ \) towards longer wavelengths and increases the splitting between the two modes. On the other hand, increasing the distance \( d \), as expected, reduces the effect of the MDM waveguide, and brings the resonant wavelengths closer to the resonant wavelengths of the cavity. Figure 3.4(c) shows the transmission, reflection, and absorption spectra of the structure of Fig. 3.1 with \( d = 30 \) nm. We observe that the structure appears to have only one resonant mode. One could conclude that increasing \( d \) reduces the effect of the waveguide to the extent that the two modes coincide. However, in order to demonstrate the effect of loss, we show the transmission spectra of the same structure (\( d = 30 \) nm) in the absence of loss
Figure 3.4: (a) Transmission $T$, reflection $R$, and absorption $A$ spectra in the absence of MO activity ($\alpha = 0$) for the structure of Fig. 3.1 calculated with FDFD for $R = 120$ nm, $W = 50$ nm and $d = 10$ nm. (b) Same as (a), except $d = 15$ nm. (c) Same as (a), except $d = 30$ nm. (d) Transmission $T$ spectra for $\alpha = 0$ and $d = 30$ nm. Also shown are the transmission spectra in the absence of loss in the metal ($\gamma = 0$).

in Fig. 3.4(d). We observe that in the lossless case the presence of the MDM waveguide leads to a second resonance in the response of the structure. The reason that the second resonance is not observed in the lossy structure [Fig. 3.4(c)] is that the presence of loss in the system increases the bandwidth of the modes as well as the transmitted power at the resonant wavelengths. As a result, if $d$ is large enough, the two modes become
indistinguishable. One can therefore conclude that the presence of the waveguide in the proximity of the cavity leads to a new resonant mode, which causes the structure to resonate at two different wavelengths. The appearance of the new resonant mode is due to the geometrical asymmetry in the structure caused by the existence of the waveguide. In the absence of the waveguide, the structure has spatial rotational symmetry. However, the presence of the waveguide breaks the symmetry of the structure and causes the cavity modes to have different resonant wavelengths. One can compare the geometrical asymmetry here to a similar structure with a rectangular resonator, where the difference between the width and the length of the resonator results in different resonant wavelengths for the cavity modes. In addition, we observe that the reflection and absorption at the two resonance wavelengths are different [Figs. 3.4(a-b)] with the incident power being mostly absorbed (reflected) at $\lambda_-$ ($\lambda_+$). This difference is due to the fact that the proximity of the waveguide to the cavity affects the effective refractive index experienced by light, which in turn introduces a resonant wavelength at $\lambda_+$ at which light is mostly reflected. On the other hand, at $\lambda = \lambda_-$ the drop in transmission is mostly due to absorption. In addition, the resonant wavelength $\lambda_-$ is almost independent of the distance $d$ between the cavity and the waveguide, whereas the resonant wavelength $\lambda_+$ is strongly dependent on $d$. In the absence of the MDM waveguide, any line through the center of the cylindrical cavity is an axis of symmetry. However, in the presence of the waveguide this no longer holds. The MDM waveguide breaks the symmetry with respect to the $x$ axis. At the same time, the structure is still symmetric with respect to the $y$ axis. As a result, only one mode, the mode at $\lambda = \lambda_+$, is affected. The analogy between rectangular resonators can be used again to explain the physics behind this phenomenon. The effect of decreasing the distance between the cavity and the waveguide is similar to changing the length of a rectangular resonator while keeping its width constant.

We next investigate the effect of MO activity on the performance of the device. As discussed above, the cavity mode in the absence of MO activity splits into a red-shifted and
Figure 3.5: The difference between the red-shifted ($\lambda_+$) and blue-shifted ($\lambda_-$) resonant wavelengths ($\Delta \lambda = \lambda_+ - \lambda_-$) of the structure shown in Fig. 3.1 with $W = 50$ nm and $R = 120$ nm as a function of the distance $d$ between the cavity and the waveguide. Results are shown for different strengths of magneto-optical activity ($\alpha = 0, 0.05, 0.15, 0.2$) and for lossless metal ($\gamma = 0$).

A blue-shifted mode in the presence of MO activity. The presence of the MDM waveguide also leads to splitting of the modes of the structure by breaking its geometrical symmetry. Here, we investigate the performance of the device in the presence of both the MDM waveguide and MO activity. In this case, the mode splitting is due to both geometrical asymmetry and MO activity. As discussed above, when the distance $d$ between the cavity and the waveguide is large enough ($d > 20$ nm), the presence of loss makes the wavelength splitting due to the MDM waveguide unobservable. Therefore, here, in order to analyze the effect of the MDM waveguide in the presence of MO activity on the wavelength splitting $\Delta \lambda$, the metal (silver) is assumed to be lossless. It should be noted that including the effect of loss does not significantly change the value of the calculated $\Delta \lambda$. However, in the presence of loss the splitting becomes unobservable for large distances $d$ between the cavity...
and the waveguide. Figure 3.5 shows the wavelength splitting $\Delta \lambda$ as a function of the distance $d$ between the cavity and the waveguide for different strengths of magneto-optical activity ($\alpha = 0, 0.05, 0.15, 0.2$) and for lossless metal ($\gamma = 0$). We observe that, as expected, decreasing the distance $d$ between the cavity and the waveguide, increases the splitting $\Delta \lambda$. In addition, as the distance $d$ between the cavity and the waveguide is increased, the effect of the MDM waveguide becomes less significant, and the wavelength splitting is mostly due to the presence of MO activity. Thus, as the distance $d$ between the cavity and the waveguide increases, the wavelength splitting $\delta \lambda$ approaches the value corresponding to the absence of the waveguide [Eq. (3.2)].

3.5 The proposed design

After investigating the cavity in the absence and presence of the MDM waveguide and MO activity, we propose an all-optical isolator based on the structure shown in the schematic in Fig. 3.1. As discussed above, the presence of the MDM waveguide and MO activity lead into mode splitting. In addition, the resonant cavity modes in the presence of MO activity become traveling modes. These modes do not decay equally into the forward and backward propagating waveguide mode due to momentum matching [84]. This property along with the observed frequency splitting enables the device to operate as an isolator. Figs. 3.6(a-c) show the transmission, reflection, and absorption spectra, respectively, for a structure with $R = 120$ nm, $d = 15$ nm, and $W = 50$ nm, in the presence (blue and red curves) and absence (black curve) of MO activity. The blue and red curves in Fig. 3.6(a) show the transmission spectra in the presence of MO activity when the MDM waveguide mode is incident from the left and right directions, respectively. Compared to the case where there is no MO activity (black curve), the presence of MO activity shifts the resonant wavelengths $\lambda_-$ and $\lambda_+$ towards shorter and longer wavelengths, respectively. In addition, we observe that in the presence of MO activity the resonant mode at $\lambda = \lambda_-$ is not excited when the MDM waveguide mode is incident from the right direction (red curve). On the other hand, the resonant mode at $\lambda = \lambda_+$ is excited in both cases with a small difference in
Figure 3.6: (a) Transmission spectra for the structure of Fig. 3.1 calculated with FDFD for $R = 120$ nm, $d = 15$ nm, $W = 50$ nm, and in the presence of MO activity ($\alpha = 0.06$), when the MDM waveguide mode is incident from the left (blue curve) and right (red curve) direction. Also shown are the transmission spectra in the absence of MO activity ($\alpha = 0$) (black curve). (b) Reflection spectra for the structure of Fig. 3.1. All other parameters are as in (a). (c) Absorption spectra for the structure of Fig. 3.1. All other parameters are as in (a).
Figure 3.7: Magnetic field profiles at $\lambda = \lambda_- = 1310$ nm in the presence of MO activity ($\alpha = 0.06$), when the MDM waveguide mode is incident from the left (bottom figure) and right (top figure) direction. All other parameters are as in (Fig. 3.6).

The value of the transmission depending on whether the MDM waveguide mode is incident from the left (blue curve) or right (red curve) direction [Fig. 3.6(a)]. The small difference in transmission for modes incident from different directions at $\lambda = \lambda_+$ is due to the difference in absorption [Fig. 3.6(c)].

As mentioned above, the presence of MO activity converts the modes of the structure from standing modes to traveling modes. Since traveling-wave cavities exhibit different decay behavior into the forward and backward propagating waveguide mode, the transmission spectra depend on the direction of the incident MDM waveguide mode. As shown in Fig. 3.6(b), in the presence of MO activity, the reflection spectra of the structure for the MDM waveguide mode incident from the left (blue curve) and right (red curve) directions are identical. However, the absorption spectra in the presence of MO activity [Fig. 3.6(c)]
Figure 3.8: The resonant wavelength $\lambda_-$ of the blue-shifted cavity mode in the presence of MO activity for the structure of Fig. 3.1 as a function of the radius $R$ of the cylindrical cavity and the distance $d$ between the cavity and the waveguide calculated with FDFD. All other parameters are as in Fig. 3.6.

depend on whether the MDM waveguide mode is incident from the left (blue curve) or right (red curve) direction. The difference in the transmission spectra in the presence of MO activity, when the MDM waveguide mode is incident from the left and right directions, is due to this difference in the absorption spectra. We also observe that at $\lambda = \lambda_-$ in the presence of MO activity, when light is incident from the left (blue curve), it is mostly absorbed [Fig. 3.6(c)], whereas, when light is incident from the right (red curve), it is mostly transmitted [Fig. 3.6(a)]. The proposed structure therefore operates as an optical isolator at $\lambda = \lambda_-$ in the presence of an external static magnetic field. Figure 3.7 shows the
Figure 3.9: Insertion loss, defined as $-10 \log_{10}(T_{on})$, for the structure of Fig. 3.1 as a function of the radius $R$ of the cylindrical cavity and the distance $d$ between the cavity and the waveguide calculated with FDFD. Here, $T_{on}$ is the transmission for light incident from the right direction. The insertion loss is calculated at the resonant wavelength of the blue-shifted cavity mode in the presence of MO activity ($\lambda = \lambda_-$), which is a function of $R$ and $d$.

magnetic field profile at $\lambda = \lambda_-$ = 1310 nm when the MDM waveguide mode is incident from the left (bottom) and right (top) direction, confirming that the device operates as an isolator in the presence of MO activity.

We next investigate in detail the effect of the geometrical parameters of the structure on its performance. As mentioned above, the structure is used as an optical isolator at the resonant wavelength of the blue-shifted cavity mode in the presence of MO activity ($\lambda = \lambda_-$). The blue-shifted resonant wavelength $\lambda_-$ is itself a function of the parameters of the structure (Fig. 3.2). In Fig. 3.8 we show the blue-shifted resonant wavelength
Figure 3.10: Isolation ratio for the structure of Fig. 3.1 as a function of the radius $R$ of the cylindrical cavity and the distance $d$ between the cavity and the waveguide calculated with FDFD. The isolation ratio is defined as the ratio of the transmission for light incident from the right direction $T_{\text{on}}$ to the transmission for light incident from the left direction $T_{\text{off}}$. The isolation ratio is calculated at the resonant wavelength of the blue-shifted cavity mode in the presence of MO activity ($\lambda = \lambda_-$), which is a function of $R$ and $d$.

$\lambda_-$ as a function of the radius $R$ of the cylindrical cavity and the distance $d$ between the cavity and the waveguide. As also discussed above, the resonant wavelength $\lambda_-$ increases approximately linearly with the radius $R$ of the cylindrical cavity. In addition, the presence of the MDM waveguide shifts the cavity resonance towards longer wavelengths, and the resonant wavelength $\lambda_-$ increases as the distance $d$ between the cavity and the waveguide decreases (Fig. 3.8). In Fig. 3.9 we show the insertion loss of the isolator, defined as $-10 \log_1 0(T_{\text{on}})$, as a function of the radius $R$ of the cylindrical cavity and the distance
$d$ between the cavity and the waveguide. Here, $T_{\text{on}}$ is the transmission for light incident from the right direction. The insertion loss is calculated at the resonant wavelength of the blue-shifted cavity mode in the presence of MO activity $\lambda_-$, which, as mentioned above, is a function of $R$ and $d$ [$\lambda = \lambda_-(R, d)$]. We observe that increasing the distance $d$ between the cavity and the waveguide decreases the insertion loss. This is expected since increasing $d$ decreases the coupling between the MDM waveguide and the cavity [84], so that more power is transmitted on resonance. Finally, in Fig. 3.10 we show the isolation ratio of the structure, defined as the ratio of the transmission for light incident from the right direction $T_{\text{on}}$ to the transmission for light incident from the left direction $T_{\text{off}}$ (Fig. 3.7). Similar to the insertion loss, the isolation ratio is also calculated at the resonant wavelength of the blue-shifted cavity mode in the presence of MO activity $\lambda_-$ [$\lambda = \lambda_-(R, d)$]. We observe that for a given radius $R$ of the cylindrical cavity, the isolation ratio is maximized for a specific value of the distance $d$ between the cavity and the waveguide. Further increasing the distance $d$, decreases the insertion loss but also decreases the isolation ratio. In other words, there is a tradeoff between the isolation ratio and the insertion loss, as the geometrical parameters of the structure are varied.
Chapter 4
Theoretical Investigation of Plasmonic Multilayered Cylindrical Cavities for Enhancing the Magneto-optical Effect

4.1 Introduction

Nonreciprocal components are crucial for developing fully functional integrated optical circuits. At the same time, the realization of nonreciprocal systems requires breaking the time reversal symmetry [71]. Despite emerging new paradigms, such as dynamic modulation [88] and optical nonlinearity [89], the magneto-optical (MO) effect continues to be the most efficient approach to break the time reversal symmetry [90, 91]. The ability to control the performance of MO-based nonreciprocal components, such as circulators and isolators, provides these designs with the tunability desired in optical systems [92, 93]. The MO activity strength of a magnetized medium can be described by the ratio of the imaginary off-diagonal component of its permittivity to the diagonal component [31]. Meanwhile, the materials found in nature exhibit weak MO response at the optical frequency regime, making the realization of non-reciprocal optical structures challenging [94]. A significant amount of research has been dedicated to the incorporation of MO materials with plasmonic nanostructures owing to the possible enhancement of MO activity in highly localized fields [95, 96]. Different schemes based on the combination of ferromagnetic materials and plasmonic nanostructures were employed to enhance the non-reciprocity [97–99]. The enhancement of the MO effect by different combinations of metallic nanoparticles with MO dielectrics was reported by several groups [100–103]. In addition, a theoretical design, offering enhanced nonreciprocal response in the deep subwavelength regime via employing plasmonic nanorods embedded in a MO host, was reported [104]. Recently, the possibility of boosting the non-reciprocity by directly engineering the dielectric tensor was theoretically investigated [31, 105]. Despite the significant progress achieved by the combination of MO materials with plasmonic nanostructures, the intrinsic loss of metals remains the biggest
Figure 4.1: Schematic of the multilayered cylindrical cavity consisting of $2k - 1$ concentric layers of bismuth iron garnet (BIG) with thickness $d_1$, shown in orange color, and a general isotropic material with permittivity $\epsilon_r$ and thickness $d_2$, shown in yellow color. The background medium, shown in dark turquoise, is silver.

challenge limiting the performance of the proposed designs [106, 107]. Nonetheless, these approaches have provided the roadmap for designing various ultracompact nonreciprocal optical components, such as isolators [108, 109], circulators [28], and switches [110].

In this chapter, we theoretically explore the possibility of enhancing the nonreciprocity in the nanoscale regime by employing multilayered cylindrical cavities. The proposed multilayered designs are composed of alternating layers of bismuth iron garnet (the employed MO material) and an arbitrary general isotopic material (GIM). In the presence of an external magnetic field, the resonant wavelengths of a circular cylinder filled with a MO dielectric experience a splitting $\Delta \lambda$ [86]. The amplitude of $\Delta \lambda$ can be used to characterize the strength of the MO activity. For our theoretical investigations, the permittivity of the GIM used $\epsilon_r$ is considered as a parametric variable and the values of $\Delta \lambda$ corresponding to different values of $\epsilon_r$ are calculated. The investigation of the structures is carried out ana-
lytically and numerically using the transfer matrix method and the finite element method (the commercial software COMSOL), respectively. We demonstrate that for negative values of $\epsilon_r$, the proposed designs exhibit larger $\Delta \lambda$ compared to equivalent cavities filled entirely with BIG. In addition, we investigate the effect of different parameters of the structures on $\Delta \lambda$. We also propose an ultracompact circulator, based on the approach that we introduce, operating at $\lambda = 1550$ nm. The study of the effect of loss on the performance of the circulator reveals that incorporating loss does not alter the functionality of the device.

4.2 Theoretical analysis

The schematic of the multilayered cylindrical structure is shown in Fig. 4.1. It is composed of $N = 2k - 1$ concentric cylindrical layers of BIG (shown in orange color) and GIM (shown in yellow color) surrounded by silver (shown in dark turquoise color). The dielectric constant of silver is modeled via the well-known Drude model with parameters $(\epsilon_\infty, \omega_p, \gamma) = (3.7, 9.1 \text{ eV}, 0.018 \text{ eV})$ [2]. The permittivity of BIG is

$$
\epsilon = \epsilon_0 \begin{bmatrix} \epsilon_1 & i\alpha \\ -i\alpha & \epsilon_1 \end{bmatrix},
$$

where $\epsilon_1 = 6.25$ and $\alpha = 0.06$ [28]. The structure is assumed to be infinite in the $z$ direction. For simplicity, the thicknesses of the layers are set to be equal i.e. $d_1 = d_2 = d = \frac{R}{N}$, and the permittivity of the GIM is assumed to be non-dispersive.

Since we investigate the structure at its resonance wavelengths, it is beneficial to review some aspects of the resonance condition of cylindrical cavities. The resonant modes of a cylindrical cavity can be obtained by solving Maxwell’s equations and imposing the boundary conditions in cylindrical coordinates. The solution of Maxwell’s equations shows that the field profiles, satisfying the resonance condition for each mode, are specified by two numbers, the radial and azimuthal number denoted as $n$ and $m$, respectively. In this chapter, we only consider the fundamental mode, i.e. $n = m = 1$. Based on the resonance condition for cylindrical resonators filled with a MO material, in the absence of
MO activity, i.e. $\alpha = 0$, the resonant modes of the cavity with azimuthal mode numbers $\pm m$ are degenerate. In other words, the resonant conditions for the $\pm m$ modes are satisfied at the same wavelength. However, in the presence of MO activity, the degeneracy of the $\pm m$ modes is broken and the device resonates at different wavelengths [86]. Thus, assuming that $\lambda_1$ and $\lambda_2$ are the wavelengths of resonance corresponding to the $+m$ and $-m$ modes, respectively, in the absence of MO activity $\Delta \lambda = |\lambda_1 - \lambda_2| = 0$. Whereas in the presence of MO activity, i.e. $\alpha \neq 0$, $\Delta \lambda = |\lambda_1 - \lambda_2| \neq 0$, which implies the broken degeneracy of the modes. Thus, the value of $\Delta \lambda$ can be used as a parameter characterizing the strength of the MO activity present in the structure. This value for nanoscale cylindrical resonators is small, due to the weak MO response of materials found in nature. In this chapter, we demonstrate that, with proper design of the proposed structure (Fig. 4.1), the MO activity can be enhanced and consequently $\Delta \lambda$ can be increased. To evaluate the effect of the multilayered structure on the enhancement of non-reciprocity, the parameter $\sigma$ is defined as follows

$$\sigma = \frac{\Delta \lambda}{\Delta \lambda_0}, \quad (4.2)$$

where $\Delta \lambda$ and $\Delta \lambda_0$ are the values of the wavelength splitting observed in the multilayered cavity and the equivalent cavity filled entirely with BIG, respectively. The radius of the equivalent cavity, denoted as $R_0$, is adjusted in such a way that its wavelength of resonance coincides with the resonant wavelength $\lambda_c$ of the multilayered structure in the absence of MO activity ($\alpha = 0$). The introduced parameter $\sigma$ acts as a figure of merit for assessing the efficiency of the proposed designs in enhancing the non-reciprocity of the system. In the following subsections, we investigate the effect of different parameters on the efficiency of the device.

4.3 The effect of $\varepsilon_r$

Here, we investigate the effect that the relative permittivity of the GIM has on the efficiency of the structure. To this end, the parameters of the layered cavity are set to be $R = 200$ nm (the outer radius of the last layer), $d = 50$ nm (the thickness of the layers), and
finally $N = 5 \ (k = 3)$ (the number of layers). In order to calculate $\Delta \lambda$ analytically, the transfer matrix method is used in which the resonance condition of the structure is obtained.

Figure 4.2: The investigation of the effect of $\epsilon_r$ on $\sigma$, when the number of layers, the outer radius of the last layer, and the thicknesses of the layers are fixed to be $N = 5 \ (k = 3)$, $R = 200$ nm, and $d_1 = d_2 = 50$ nm, respectively. (a) The calculated values of $\sigma$ corresponding to different values of $\epsilon_r$, ranging from 1 to 13. (b) The corresponding wavelengths of resonance $\lambda_c$ evaluated for the same range of $\epsilon_r$ as in part (a).
via solving Maxwell’s equations in cylindrical coordinates and imposing the boundary conditions at the $2k - 1$ boundaries. Figures 4.2(a) and (b) show the calculated values of $\sigma$ and $\lambda_c$, respectively, when the GIM has positive permittivity $\epsilon_r$ ranging from 1 to 13. We observe that the calculated values of $\sigma$, for the given range of $\epsilon_r$, are all smaller than 1, implying that using GIMs with $\epsilon_r > 0$ has a negative impact on the nonreciprocal response of the system. In addition, since $R$ is kept constant, increasing the amplitude of $\epsilon_r$ shifts the wavelengths of resonance towards longer wavelengths. Figure 4.3 shows the radii $R_0$ of the equivalent cavities filled entirely with BIG as a function of $\lambda_c$. Here, since the dielectric (BIG) filling the cavity is the same in all cases, increasing $\lambda_c$ increases the value of $R_0$. The continuous blue curve and the discrete red circles depict the analytical and numerical results, respectively. The same procedure is repeated for GIM with $\epsilon_r$ ranging from -15 to -6 and the results are shown in Figs. 4.4(a) and (b), and in Fig. 4.5. In this case, as shown in Fig. 4.4(a), we observe that using GIMs with $\epsilon_r < 0$ enhances the non-reciprocity of
Figure 4.4: (a) The calculated values of $\sigma$ corresponding to different values of $\epsilon_r$ ranging from -15 to -6. (b) The corresponding wavelengths of resonance $\lambda_c$ evaluated for the same range of $\epsilon_r$ as in part (a). All other parameters are the same as in Fig. 4.2.
Figure 4.5: The radii $R_0$ of the equivalent cylinders filled entirely with BIG resonating at $\lambda_c$ calculated for the same range of $\epsilon_r$ as in Fig. 4.4.

the system. Furthermore, the calculated values of $\sigma$ exhibit a small fluctuation when $\epsilon_r$ is varied. However, the minimum value of $\sigma$ for the given range of $\epsilon_r$ is more than 3 implying an enhancement by a factor of at least 3. In addition, the calculated values of $\lambda_c$ and $R_0$ [Fig. 4.4(b) and Fig. 4.5] exhibit similar behavior as the one observed in Fig. 4.2(b) and Fig. 4.3.

In order to shed some light on the physics behind the enhancement of nonreciprocity, we aim to qualitatively explain the rationale behind the observed enhancement. The use of alternating layers with negative and positive dielectric constants in the layout displayed in Fig. 4.1, causes the excitation of surface plasmon polaritons (SPPs) at the interfaces between the materials [1]. Since all BIG layers, with the exception of the central layer, are sandwiched between two layers of material with negative permittivity, the combination of any three consecutive layers, consisting of two layers of GIM (or silver) and a layer of BIG, forms a metal-dielectric-metal (MDM) waveguide [1]. These MDM waveguides
are capable of channeling the excited SPPs in the azimuthal direction. In addition, due to the proximity of the layers, there is strong coupling between the MDM waveguides. Provided that $\epsilon_r < 0$, a structure with $2k - 1$ layers therefore consists of $k - 1$ coupled MDM waveguides. In the absence of MO activity and under the resonance condition, the clockwise and counter-clockwise rotating SPPs have the same propagation constants $\beta$, implying the degeneracy of the $\pm m$ modes [111]. Whereas in the presence of MO activity, the propagation constants of SPPs rotating clockwise and counter-clockwise are different, causing the degeneracy of the $\pm m$ modes to break. In other words, the cavity modes experience frequency splitting. This explanation for the frequency splitting behavior applies to both layered and non-layered cavities. However, the difference between the propagation constants of clockwise and counter-clockwise rotating SPPs ($\Delta \beta$) is larger in the layered structures. The enhancement of nonreciprocity in the multilayer structures is due to the increased effective index which originates from the presence of the MDM waveguides. This increased effective index leads to larger propagation constants ($\beta$), and consequently larger $\Delta \beta$.

4.4 The effect of $N$

Next, the effect of the number of layers $N$ on the efficiency of the structure ($\sigma$) is investigated. To this end, the wavelength of resonance of the device is assumed to be fixed at $\lambda_c = 1550$ nm and the corresponding values of $\sigma$ are evaluated for different numbers of layers. To keep the wavelength of resonance constant, the outer radius of the last layer ($R$) is adjusted in such a way that, even though the number of layers $N$ is varied, $\lambda_c$ remains constant. Figure 4.6(a) displays the calculated values of $\sigma$ corresponding to different values of $k$. In addition, to show the combined effect of $N$ and $\epsilon_r$, the process is repeated for three different values of $\epsilon_r$. We observe that increasing the number of layers, increases $\sigma$. Furthermore, the efficiency of the device is enhanced when the absolute value of $\epsilon_r$ is increased. Figure 4.6(b) depicts the calculated values of $R$, evaluated so that the wavelength of resonance remains constant at $\lambda_c = 1550$ nm, for the corresponding values of
Figure 4.6: The effect of the number of layers $N = 2k - 1$ on $\sigma$. The continuous curves are obtained analytically and the discrete circles show the simulation results. (a) The red (discrete turquoise circles), blue (discrete green circles), and black (discrete purple circles) display the calculated values of $\sigma$ for structures with $\epsilon_r = -7$, -8, and -9, respectively. (b) The radii of the outer layer $R$ evaluated for the same values of $k$ and $\epsilon_r$ as in part (a). The colors represent the same structures shown in part (a).
We conclude that to keep the wavelength of resonance constant, upon increasing the number of layers, the size of the cavity should be increased as well. Additionally, increasing the absolute value of the permittivity \( \epsilon_r \) and the number of layers \( N \) has a similar effect on \( \sigma \); they both enhance \( \sigma \), while increasing the size of the structure required to have a resonance at the wavelength of interest.

To qualitatively explain these results, we need to consider what happens when the number of layers is increased. As mentioned above, in order to compare the performance of devices with different number of layers, the value of \( R \) is adjusted in such a way that the cavities are resonating at the same wavelength \( \lambda_c \). Even though increasing the number of layers \( N \) increases the size of the resonator, since the relative increase of \( R \) is smaller than the one of \( N \), the ratio \( d = R/N \), which corresponds to the thickness of each layer, decreases. Since \( d \) decreases, as the number of layers \( N \) is increased, the width of the azimuthally oriented MDM waveguides decreases. The reduction of the width of a MDM waveguide leads to an increase of its effective index [1], which consequently results in larger difference between the propagation constants of the clockwise and counter-clockwise rotating SPPs (\( \Delta \beta \)) in the cavity [30, 110]. As discussed in the previous section, \( \Delta \beta \) is the origin of the observed nonreciprocal response. Thus, increasing the number of layers enhances the efficiency of the device.

4.5 Application

In this section, we introduce a circulator based on the layered structure proposed in the previous section. The schematic of the device is displayed in Fig. 4.7. It consists of three MDM waveguides with width of \( W = 30 \) nm placed in the radial direction with relative angle of \( \theta = 120^\circ \) with respect to each other. The cavity has three layers of BIG, shown in orange color, two layers of GIM with \( \epsilon_r = -10 \), shown in yellow color, and the surrounding medium is silver. The radius of the outer layer and the thickness of the layers are fixed to be \( R = 212.5 \) nm and \( d = \frac{R}{5} \), respectively. The performance of the device as a circulator is directly dependent on the strength of the MO activity observed in the structure. As was
Figure 4.7: Schematic of the proposed circulator consisting of three MDM waveguides with width of \( W = 30 \) nm placed symmetrically in the radial direction with relative angles of \( \theta = 120^\circ \). The cylindrical cavity has five concentric layers with equal thicknesses \( (d_1 = d_2 = R/5) \), and the outer radius of the last layer is \( R = 212.5 \) nm. The orange, yellow, and turquoise colors correspond to BIG, GIM with \( \epsilon_r = -10 \), and silver, respectively.

demonstrated in the previous section, employing GIM with negative permittivity enhances the value of \( \sigma \), which consequently, provides the enhanced non-reciprocity required for the operation of the circulator. An ideal three-port circulator requires an input port which channels the SPPs into the structure, an output port where the normalized output power is expected to be "1", and an isolated port where the normalized transmitted power is expected to be "0". The transmission spectra to the output and isolated ports for the structure displayed in Fig. 4.7 are calculated when \( \alpha = 0.06 \) and shown in Fig 4.8. In addition, the transmission spectra to the unbiased ports \( (\alpha = 0) \) are also calculated and displayed in Fig. 4.8. We observe that when \( \alpha = 0 \), due to the symmetry of the structure, the transmission spectra to each port are equal, whereas when \( \alpha = 0.06 \), the transmission spectra to the isolated and output ports are different. The circulator is designed to have an operating wavelength at \( \lambda = 1550 \) nm. At this wavelength the difference between the transmission to the output and isolated ports is maximum. It should be noted that the
Figure 4.8: Calculated transmission spectra to the isolated and output ports in the presence of MO activity ($\alpha = 0.06$) for the circulator displayed in Fig. 4.7. Also shown are the transmission spectra to the unbiased ports in the absence of MO activity ($\alpha = 0$).

The presence of the waveguides disturbs the resonance condition of the cavity, so that the previously derived analytical resonance condition for an isolated cavity is not accurate in this case. However, the wavelength of operation can still be tuned by adjusting the parameters of the cavity such as $R$ or $\epsilon_r$. The magnetic field $H_z$ profile at $\lambda = 1550$ nm when SPPs are incident at ports 1, 2, and 3 are calculated and displayed in Figs. 4.9(a), (b), and (c), respectively. The black, blue, and red arrows indicate the propagation direction of the SPPs at the input, output, and isolated ports, respectively. We observe that SPPs that enter the device from port 1 (2, 3) are transmitted to port 2 (3, 1) which shows that the device operates as a circulator.

4.6 The effect of loss

Next, we study the effect of loss of the GIM on the performance of the proposed circulator. For the structure shown in Fig. 4.7, the GIM was assumed to be lossless. However, here we investigate the effect of material loss in the GIM. To incorporate loss in
Figure 4.9: Magnetic field $H_z$ profile at $\lambda = 1550$ nm for the circulator shown in Fig. 4.7 when the incoming SPPs are incident at ports 1 (a), 2 (b), and 3 (c). The black arrow depicts the direction of propagation of the incoming SPPs while the blue and red arrows indicate the propagation direction of SPPs at the output and isolated ports, respectively.

the GIM, we assume that the permittivity of the GIM $\epsilon_r$ has a nonzero imaginary part. Figure 4.10 shows the transmission to the output and isolated ports when $\epsilon_r = -10 - 0.01i$. We observe that despite the decrease (increase) of transmission into the output (isolated) port, the functionality of the device as a circulator is preserved. To further investigate the effect of loss on the performance of the circulator, the absolute value of the imaginary part of $\epsilon_r$ is varied from 0 to 0.1 and the transmission to the output and isolated ports at
Figure 4.10: Transmission spectra to the output (red curve) and isolated (blue curve) ports when $\epsilon_r = -10 - 0.01i$.

Figure 4.11: Transmission to the output and isolated ports at the wavelength of operation ($\lambda = 1550$ nm), when the imaginary part of $\epsilon_r$ is varied from 0 to 0.1 while its real part is kept constant at -10. All other parameters are as in Fig. 4.7.
$\lambda = 1550$ nm is evaluated and shown in Fig. 4.11. We observe that increasing the loss deteriorates the performance of the circulator. However the functionality of the device is preserved even when the amount of material loss is increased.
Chapter 5
Conclusions

In the first chapter we first briefly reviewed the importance of plasmonic structures for subwavelength guiding and enhanced light matter interactions. We also briefly reviewed nanophotonics devices based on graphene and magneto-optical materials. A few previous works related to the aforementioned topics were presented and discussed.

In the second chapter, we first introduced a structure that operates as a mode converter between the even and odd modes of GPP waveguides. The converter is reciprocal and is based on spatial modulation of graphene's conductivity. Assuming that the coupling between the two graphene layers of the waveguide is weak, to convert one mode into the other, one needs to create an odd multiple of $\pi$ phase shift between them. To achieve the required phase shift, we modified the chemical potential on a strip on one of the graphene layers. We first assumed that all graphene layers are lossless. We found that, if the change in the chemical potential on the upper plate is abrupt, the conversion from the even mode on the left to the odd mode on the right at the optimum wavelength is not complete. To reduce the reflection in such a structure, we used a chemical potential profile that corresponds to a piecewise approximation of a triangular envelope, and found that modifying the chemical potential profile reduces the reflection coefficients of the mode converter to almost zero in a broad wavelength range. In addition, the wavelength of operation of the mode converter can be tuned simply by adjusting the chemical potential of the graphene strip. We also found that the maximum conversion efficiency decreases, when the coupling between the graphene plates becomes stronger. To overcome this issue, we modified the chemical potential of a strip on both the upper and lower graphene plates and found that the double strip converter achieves larger maximum conversion efficiency compared to the single strip converter. When the effect of loss is included, there is absorption in the mode converter, which causes the conversion efficiency to decrease. We found, however, that the
functionality of the mode converter is preserved in the presence of loss. We then introduced an optical diode for GPP waveguides based on a mode converter, and a coupler, which consists of a single layer of graphene placed in the middle between the two plates of two GPP waveguides. As before, we first assumed that all graphene layers are lossless. We found that the existence of the mode converter only on one side of the device enables it to perform as an optical diode. When the even mode enters the device from the left, it is transmitted, whereas, when it enters the device from the right, it is reflected. In contrast, the odd mode is transmitted, when it enters the device from the right, and reflected, when it enters the device from the left. When loss is included, there is absorption in the optical diode, which causes the transmission from the even mode on the left to the odd mode on the right to decrease. We found, however, that the device functionality is preserved in the presence of loss.

Next, we introduced all optical isolators based on MDM waveguides via employing the magneto-optical (MO) effect. The proposed design consists of a MDM waveguide placed in the proximity of a cylindrical resonator filled with MO material. For the MO material we employed BIG which has an off-diagonal permittivity component of $\alpha = 0.06$ when saturated. We first investigated the effect of having nonzero off-diagonal permittivity component on the resonant condition of the resonator. Our study revealed that exposing the resonator to an external magnetic field causes the resonant frequency of the resonator to split. In addition, the study of the rotating Poynting vector within the cavity showed that in the presence of MO activity, the resonant modes of the resonators have nonzero rotation along the center of the cavity. Next, we studied the effect of placing the MDM waveguide in the proximity of the resonator. We found that the existence of the MDM waveguide within the coupling distance of the cavity, regardless of the presence of MO activity, causes a different type of split associated with the geometrical asymmetry of the whole structure. The study of the effect of loss revealed that, when the coupling distance between the resonator and the waveguide is large enough, the quality factor of each resonance decreases.
making the distinction between the two different resonances impossible. Finally, we took advantage of the splitting phenomena caused by the MDM waveguide and MO activity and proposed a design that can act as an all-optical isolator at optical wavelengths. The isolation was made possible due to the fact that the traveling waves within the resonator couple differently to incoming waves from different directions due to momentum matching.

Finally, in chapter four we theoretically explored the possibility of enhancing the non-reciprocity in the nanoscale regime via introducing designs based on multilayered cylindrical cavities. It was shown that the frequency splitting phenomenon observed in the presence of MO activity, can be largely enhanced by incorporating BIG and GIMs with negative permittivity in the proposed layout. The operation of the designs was analyzed via employing the transfer matrix method and the obtained results were verified numerically. Furthermore, a qualitative physical explanation for the observed enhancement based on the behavior of the SPPs in MDM waveguides was provided. As a possible application, an ultra-compact circulator based on the introduced multilayered structure was proposed. Furthermore, it was demonstrated that the incorporation of loss does not limit the performance of the device. Additionally, the evaluation of the operation of the circulator for different values of loss, which was included as the imaginary part of the GIM’s permittivity, confirmed the robust performance of the device under different lossy conditions.
Appendix A
Supplementary Information for Chapter 2

Since all graphene layers are suspended in air, we have

\[ \epsilon_{r1} = \epsilon_{r2} = 1, \]  
(A.1)

\[ k_1 = k_2 = k = \sqrt{\beta^2 - k_0^2}. \]  
(A.2)

Substituting Eqs. (A.1) and (A.2) in Eq. (2.6) we obtain

\[ \beta = k_0 \sqrt{1 - \left( \frac{2}{\eta_0 \sigma} \right)^2}, \]  
(A.3)

where \( \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \) is the free space impedance. The surface conductivity of a lossless graphene sheet can be written as follows

\[ \sigma = \sigma_{\text{intra}} = \frac{e^2 k_B T}{\pi \hbar^2 j \omega} \left( \frac{\mu_c}{k_B T} + 2 \ln \left[ \exp \left( - \frac{\mu_c}{k_B T} \right) + 1 \right] \right). \]  
(A.4)

At room temperature \((T = 300 \text{ K})\), \(k_B T \approx 26 \text{ meV}\) and, since the applied chemical potentials in this dissertation are larger than 100 meV, we have \(2 \ln \left[ \exp \left( - \frac{\mu_c}{k_B T} \right) + 1 \right] \ll \frac{\mu_c}{k_B T}\). Consequently, we obtain

\[ \sigma \approx -\frac{je^2 \mu_c}{\pi \hbar^2 \omega}. \]  
(A.5)

Substituting Eq. (A.5) into Eq. (A.3) we obtain

\[ \beta = k_0 \sqrt{1 + \frac{2\pi \hbar^2 \omega}{\eta_0 e^2 \mu_c}^2}. \]  
(A.6)

For the range of wavelengths considered in this dissertation we have \(\left( \frac{2\pi \hbar^2 \omega}{\eta_0 e^2 \mu_c} \right)^2 \gg 1\). Therefore, Eq. (A.6) can be simplified as follows
\[
\beta = k_0 \left( \frac{2\pi h^2 \omega}{\eta_0 e^2 \mu_c} \right) = \frac{2\pi h^2}{\mu_0 e^2 \mu_c} \frac{1}{\lambda^2}.
\]  
(A.7)

Using Eq. (A.7), the propagation constants for the \(i^{th}\) segment of the upper and lower plates are

\[
\beta_{ui} = \frac{2\pi h^2}{\mu_0 e^2} \frac{1}{\mu_{cui} \lambda^2},
\]  
(A.8)

\[
\beta_{di} = \frac{2\pi h^2}{\mu_0 e^2} \frac{1}{\mu_{cdi} \lambda^2},
\]  
(A.9)

where \(\mu_{cui}\) and \(\mu_{cdi}\) are the chemical potentials of the \(i^{th}\) segment of the upper and lower plates, respectively. Substituting Eqs. (A.8) and (A.9) into Eq. (2.10), the wavelength of maximum conversion efficiency \(\lambda_{opt}\) can be calculated as follows

\[
\lambda_{opt} = \sqrt{\frac{2h^2}{(2m - 1)\mu_0 e^2} \left[ \sum_{i=1}^{N} d_i \left( \frac{1}{\mu_{cui}} - \frac{1}{\mu_{cdi}} \right) \right]}.
\]  
(A.10)

The values of \(\mu_{cui}\), and \(\mu_{cdi}\) depend on the the minimum potentials \(\mu_{cnu}\, and \mu_{cnd}\) of the corresponding triangular envelopes [Fig. 2.4(a)]. We verified that Eq. (A.10) gives results indistinguishable from those obtained using Eq. (2.9).
### Appendix B

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Figure B.1: The materials used in chapter 2 previously appeared in a somewhat different format in journal of Optics Express. (Vahid Foroughi Nezhad, Ali Haddadpour, and Georgios Veronis. *Tunable spatial mode converters and optical diodes for graphene parallel plate waveguides*. Optics Express 24.21 (2016): 23883-23897). This figure displays the copyright policy of the aforementioned journal.
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VITA

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