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Daniel E. Sheehy
University of Illinois Urbana-Champaign

Paul M. Goldbart
University of Illinois Urbana-Champaign

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Intrinsic resistivity and the SO(5) theory of high-temperature superconductors

Daniel E. Sheehy and Paul M. Goldbart

*Department of Physics and Materials Research Laboratory,
University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*
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The topological structure of the order parameter in Zhang's SO(5) theory of superconductivity allows for an unusual type of dissipation mechanism via which current-carrying states can decay. The resistivity due to this mechanism, which involves orientation rather than amplitude order-parameter fluctuations, is calculated for the case of a thin superconducting wire. The approach is a suitably modified version of that pioneered by Langer and Ambegaokar for conventional superconductors.

74.40+k, 74.72-h, 74.80-g

Zhang's approach to the physics of the high-temperature superconductors [1] is rooted in the observation that the phase diagram of these materials contains nearby regions of antiferromagnetism and superconductivity. This has led Zhang to propose a low-energy effective description of these materials that combines the U(1) symmetry sector associated with the superconductivity with the SO(3) symmetry sector associated with the antiferromagnetism, SO(5) being the minimal symmetry group that admits this combination. In the resulting description, the state of the system is characterized, locally, by a five-dimensional "superspin," subject to a symmetry-reducing term that favors one or other of the sectors, U(1) or SO(3). The superspin is constrained to have unit magnitude, which is appropriate for temperatures sufficiently low that fluctuations in the magnitude of the superspin are negligible. The possibility of rotations between the U(1) and SO(3) sectors is where a number of the unusual consequences of Zhang's approach lie [2-4].

It has long been appreciated [5,6] that in conventional superconductors topologically accessible fluctuations in the *amplitude* of the superconducting order parameter provide an intrinsic mechanism via which supercurrent can be dissipated in a thin wire. A uniform current-carrying state characterized by a specific uniform phase-gradient along the wire is only *metastable*, thermodynamically, so that by undergoing amplitude-reducing thermal fluctuations the system can decrease its current. The purpose of the present Paper is to investigate a related intrinsic dissipation mechanism appropriate for the superconducting state of the SO(5) model of high-temperature superconducting materials. In this case, the relevant dissipative process is a fluctuation in the *orientation* of the superspin, during which the system temporarily becomes antiferromagnetic along a small segment of the wire, allowing the analogue of a phase-slip process to occur. Such fluctuations are equivalent to the passing of superconducting vortices with antiferromagnetic cores [2] across the wire. Related dissipative processes have been addressed in the context of superfluid ³He-A [7] and (for appropriate values of the gradient coupling constants) thin tubes of nematic liquid crystal [8]. By constructing a

version of the approach to intrinsic dissipation pioneered by Langer and Ambegaokar for conventional superconductors [6], suitably modified for the SO(5) model of superconductivity, we shall estimate the free-energy barrier for this type of fluctuation and, hence, arrive at an estimate of the current-voltage relationship for a sufficiently thin wire.

In Zhang's theory of high-temperature superconductivity and antiferromagnetism, then, the local state of the system at the spatial position \mathbf{r} is determined by a five-component unit-vector field $\mathbf{n}(\mathbf{r})$ such that the components n^1, n^2 and n^3 together specify the antiferromagnetic Néel vector, and the components n^4 and n^5 together specify the amplitude and phase of the superconducting order. The constraint on the magnitude of \mathbf{n} implements the notion that the enhancement of antiferromagnetic order is necessarily accompanied by the diminution of superconducting order, and vice versa. The free-energy density f comprises an isotropic gradient term along with a symmetry-reducing term:

$$f = \frac{\rho}{2} \sum_{a=1}^5 \partial_{\mu} n^a \partial_{\mu} n^a - \frac{g}{2} \sum_{a=1}^3 (n^a)^2. \quad (1)$$

Here, the (real-space) subscript μ runs from 1 to 3 (or x, y, z), repeated indices being summed over. The (chemical-potential dependent) parameter g governs whether the stable homogeneous state is superconducting ($g < 0$), as we select here, or antiferromagnetic ($g > 0$). We parametrize \mathbf{n} via the angles $\{\tilde{\theta}, \tilde{\phi}, \tilde{\psi}, \tilde{\chi}\}$ such that

$$\begin{aligned} n^1 &= \sin \tilde{\theta} \cos \tilde{\psi} \cos \tilde{\chi}, & n^4 &= \cos \tilde{\theta} \cos \tilde{\phi}, \\ n^2 &= \sin \tilde{\theta} \cos \tilde{\psi} \sin \tilde{\chi}, & n^5 &= \cos \tilde{\theta} \sin \tilde{\phi}, \\ n^3 &= \sin \tilde{\theta} \sin \tilde{\psi}, \end{aligned}$$

The angle $\tilde{\theta}$ measures the relative amount of antiferromagnetic versus superconducting order (without regard for the orientation of the antiferromagnetism or the phase of the superconductivity), with $\tilde{\theta} \equiv 0$ in the purely superconducting state. Furthermore, $\tilde{\phi}$ is the phase of the superconductivity, and $\tilde{\psi}$ and $\tilde{\chi}$ specify the orientation of the antiferromagnetic Néel vector.

Let us suppose that the system consists of a wire of length L , sufficiently long and narrow that we may assume that $\{\tilde{\theta}, \tilde{\phi}, \tilde{\psi}, \tilde{\chi}\}$ do not vary in the directions transverse to the wire (i.e. in the y or z directions). Under these circumstances f becomes

$$\begin{aligned} f &= (\rho/2)\{(\partial_x \tilde{\theta})^2 + (\partial_x \tilde{\phi})^2 \cos^2 \tilde{\theta} + (\partial_x \tilde{\psi})^2 \sin^2 \tilde{\theta} \\ &\quad + (\partial_x \tilde{\chi})^2 \sin^2 \tilde{\theta} \cos^2 \tilde{\psi}\} + (|g|/2) \sin^2 \tilde{\theta} \quad (2a) \\ &= (|g|/2)\{(\partial_\tau \theta)^2 + (\partial_\tau \phi)^2 \cos^2 \theta + (\partial_\tau \psi)^2 \sin^2 \theta \\ &\quad + (\partial_\tau \chi)^2 \sin^2 \theta \cos^2 \psi + \sin^2 \theta\}. \quad (2b) \end{aligned}$$

To obtain Eq. (2b) we have exchanged the independent variable x for its dimensionless counterpart τ , defined via $\tau \equiv x/\xi_\pi$, where $\xi_\pi \equiv \sqrt{\rho/|g|}$ is the correlation length for antiferromagnetic fluctuations, which sets the length-scale for dissipative events. Furthermore, we have defined the function θ such that $\theta(\tau) \equiv \tilde{\theta}(x)$, and similarly for ϕ , ψ and χ . The quantity ℓ will denote the dimensionless length of the wire, i.e. $\ell \equiv L/\xi_\pi$.

In order to estimate the rate at which current-dissipating processes occur we compute the height of the free-energy barrier opposing them. To do this we follow Langer and Ambegaokar [6] and seek the metastable (i.e. uniform, current-carrying) states between which the system fluctuates, and the transition (i.e. unstable saddle-point) states through which the system passes as current is dissipated. Both classes of states, metastable and transition, are stationary configurations of the free energy, and therefore obey the corresponding Euler-Lagrange equations. These equations may be simplified, however, due to the homogeneity (i.e. τ -independence) and gauge-invariance in the superconducting sector (i.e. ϕ -independence) of f . The former symmetry leads to the existence of the first integral

$$\dot{\theta}^2 + \dot{\phi}^2 \cos^2 \theta + \dot{\psi}^2 \sin^2 \theta + \dot{\chi}^2 \sin^2 \theta \cos^2 \psi - \sin^2 \theta \equiv \epsilon, \quad (3a)$$

where overdots denote derivatives with respect to τ , and ϵ is constant. The latter symmetry leads to the existence of a cyclic coordinate, so that

$$\dot{\phi} \cos^2 \theta \equiv I. \quad (3b)$$

Here, the dimensionless supercurrent density I , in terms of which the dimensionful supercurrent density J is given by $J = 2e\rho I/\hbar\xi_\pi$, is constant. By replacing ϕ in Eq. (3a) with I , using Eq. (3b), we obtain

$$\dot{\theta}^2 + \frac{I^2}{\cos^2 \theta} + \dot{\psi}^2 \sin^2 \theta + \dot{\chi}^2 \sin^2 \theta \cos^2 \psi - \sin^2 \theta = \epsilon. \quad (4)$$

The relevant supercurrent-carrying metastable states are solutions of Eqs. (3) for which $\theta(\tau) \equiv 0$ (and therefore ψ and χ play no role) and $\dot{\phi} = I$. We refer to these states as ‘‘uniformly winding’’ because in them ϕ increases linearly with position along the sample. As for the transition states, we assume that they possess appreciable antiferromagnetic order only within a small

segment of the wire, and that elsewhere θ is negligibly small. Furthermore, the orientation of the antiferromagnetic order generated during the fluctuation is uniform, because transition states with inhomogeneous antiferromagnetic orientation would have higher free energy, and would correspondingly occur less frequently. Therefore we only consider transition states for which $\dot{\psi} \equiv \dot{\chi} \equiv 0$. Thus, by applying Eq. (4) far from the antiferromagnetic region we determine that

$$I^2 = \epsilon. \quad (5)$$

Let us denote the extreme value of $\theta(\tau)$ by θ_0 , and let us suppose that it occurs at the location $\tau = \tau_0$. At this location $\dot{\theta} = 0$, so that Eq. (4), combined with Eq. (5), gives

$$\dot{\theta}^2 = \sin^2 \theta - I_t^2 \tan^2 \theta, \quad (6a)$$

$$I_t^2 = \cos^2 \theta_0. \quad (6b)$$

By integrating Eq. (6a) we find

$$\sin \theta(\tau) = \sin \theta_0 \sqrt{1 - \tanh^2[(\tau - \tau_0) \sin \theta_0]}. \quad (7)$$

Finally, by using Eq. (3b), along with Eq. (7), we obtain

$$\begin{aligned} \phi(\tau) - \phi(\tau_0) &= I_t \int_{\tau_0}^{\tau} d\tau' / \cos^2 \theta(\tau') \quad (8) \\ &= I_t(\tau - \tau_0) + \arctan \{ \tan \theta_0 \tanh [(\tau - \tau_0) \sin \theta_0] \}. \end{aligned}$$

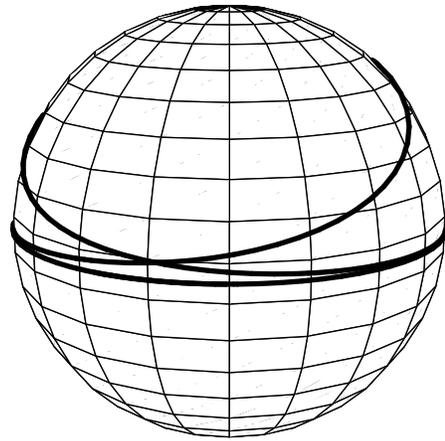


FIG. 1. Parametric plot of a typical (partially antiferromagnetic) transition state (full curve) on the surface of the unit 2-sphere in the space spanned by $\{n^4, n^5, \sin \theta\}$. Trajectories of the (purely superconducting) metastable states lie on the equator of this sphere, for which $\theta = 0$.

Thus we see that far from τ_0 (i.e. far from the center of the fluctuation) the transition states wind uniformly, as do the metastable states. However, in the transition states the order parameter undergoes an orientational distortion over a length of order ξ_π that takes θ out of the $\theta = 0$ plane. Thus, a given transition state possesses a

region of antiferromagnetic order in a fixed but arbitrary direction. Indeed, there is a family of symmetry-related transition states generated by changing this direction, just as there is a family of transition states generated by the (arbitrary) location of τ_0 and the (arbitrary) over-all phase. An example of a transition state is shown in Fig. 1. Hence we see that the relevant fluctuation for SO(5) superconductors is one in which supercurrent can be dissipated via a thermally activated process in which a “loop” of order parameter passes over the order-parameter sphere, so that the total phase difference (and hence supercurrent in the resulting metastable state) is reduced by 2π . Intuitively, it seems reasonable that if the “anisotropy” g is not too large (it being adjustable by changing the chemical potential) then this type of fluctuation should be less energetically costly than amplitude fluctuations, and should hence provide the dominant pathway for dissipation. Having determined the form of the metastable and transition states, we now identify the particular transition state through which the system passes as current is dissipated from a given metastable state. In passing from a metastable state to the corresponding transition state, en route to diminishing the current, the superspin nowhere becomes entirely antiferromagnetic (i.e. $|\theta| < \pi/2$). [The transition state represents a configuration in which a loop is about to pass over the order parameter sphere, but has not yet done so.] Thus, if we consider a transition from one metastable state (which we refer to as the “upper metastable state”) to a metastable state with 2π less total phase difference (the “lower metastable state”) then the actual loss of the phase will occur in passing from the transition state to the lower metastable state. Therefore, the total phase-difference across the sample $\Delta\phi$ is the same in the transition state and the upper metastable state, so that

$$\int_0^\ell d\tau \dot{\phi}_m = \Delta\phi = \int_0^\ell d\tau \dot{\phi}_t, \quad (9)$$

where $\phi_m(\tau)$ is the phase of the uniform metastable state and $\phi_t(\tau)$ is the phase of the transition state. This formula provides a connection between the current in the upper metastable state I_m^+ and that in the transition state I_t . The left-most term in this equation is given by $I_m^+ \ell$, which follows readily from the uniformly twisting character of the metastable states. The right-most term may be evaluated via Eq. (8), which gives $I_t \ell + 2\theta_0$, or equivalently, using Eq. (6b), $I_t \ell + 2 \arccos I_t$. Thus we arrive at an implicit equation for I_t in terms of I_m^+ :

$$I_m^+ \ell - I_t \ell = 2 \arccos I_t \geq 0. \quad (10)$$

Next we use Arrhenius rate-law considerations to develop the transition rate. To do this, we need expressions for the free energies of the upper (+) and lower (-) metastable states, as well as of the transition state connecting them. From Eq. (2b) and the form of the upper and lower metastable states we integrate over the volume

of the wire to obtain expressions for the free energies F_m^\pm of these states:

$$F_m^\pm = (|g|/2) A \xi_\pi \ell (I_m^\pm)^2. \quad (11)$$

Here, $I_m^- (= I_m^+ - 2\pi/\ell)$ is the current in the lower metastable state and A is the cross-sectional area of the wire. Similarly, the free-energy density of the transition states f_t may be obtained using Eqs. (2b), (3b) and (6a):

$$f_t = (|g|/2) (I^2 + 2 \sin^2\theta). \quad (12)$$

By integrating Eq. (12) over the volume of the wire we arrive at an expression for the free energy F_t of the transition state:

$$F_t = (|g|/2) A \xi_\pi \left\{ \ell I_t^2 + 2 \int_0^\ell d\tau \sin^2\theta(\tau) \right\} \quad (13a)$$

$$= (|g|/2) A \xi_\pi \left\{ \ell I_t^2 + 4\sqrt{1 - I_t^2} \right\}. \quad (13b)$$

By using Eqs. (11) and (13b) we obtain an expression for the free energy barrier ΔF for current dissipation,

$$\Delta F \equiv F_t - F_m^+ = \frac{|g|}{2} A \xi_\pi \left\{ \ell I_t^2 - \ell (I_m^+)^2 + 4\sqrt{1 - I_t^2} \right\},$$

which may be simplified by using the relation (10) between I_t and I_m^+ . By restricting our attention to states in which the phase winds many times along the wire (i.e. $2 \arccos I_t \ll I_t \ell$) we obtain $I_t^2 - (I_m^+)^2 \simeq -4I_t \arccos I_t$, and thus ΔF becomes

$$\Delta F = |g| A \xi_\pi \left\{ -2 I_t \arccos I_t + 2\sqrt{1 - I_t^2} \right\}. \quad (14)$$

It should be noted that $I_t = 1$ is the critical current, in the sense that uniformly twisted states with larger values of the current are unstable rather than metastable. For later use, we note that for currents slightly smaller than critical ΔF in the SO(5) model is given by

$$\Delta F \simeq 2\sqrt{2} |g| A \xi_\pi (1 - I_t)^{3/2}, \quad (15)$$

whereas the corresponding expression for conventional superconductors [6] is

$$\Delta F \simeq \sqrt{2} (8/3)^{5/4} A \xi (g_n - g_s) (1 - I)^{5/4}. \quad (16)$$

Here, $(g_n - g_s)$ represents the free-energy cost of amplitude fluctuations, and is the analogue of the parameter g within the present theory, and ξ is the superconducting fluctuation correlation length.

To compute the rates of current-decreasing and current-increasing fluctuations we follow LA by assuming that these rates depend exponentially on the free-energy barrier heights. Specifically, the rate $\Gamma(I_m^+ \rightarrow I_m^-)$ at which current-decreasing fluctuations occur is given by

$$\Gamma(I_m^+ \rightarrow I_m^-) = \Gamma_0 \exp\{-\beta \Delta F\} \quad (17a)$$

$$= \Gamma_0 \exp \left\{ -\beta |g| A \xi_\pi \left(-2I_t \arccos I_t + 2\sqrt{1 - I_t^2} \right) \right\},$$

where $\beta = 1/k_{\text{B}}T$ measures the inverse temperature and Γ_0 is an attempt frequency for dissipative fluctuations. Current-increasing fluctuations, in which the system passes from the lower metastable state to the upper metastable state, also occur. However, these fluctuations have a higher barrier opposing them (i.e. for them $\Delta F \rightarrow \Delta F + F_{\text{m}}^+ - F_{\text{m}}^-$), and thus occur at a lower rate. By using Eq. (11) we therefore obtain

$$\Gamma(I_{\text{m}}^+ \leftarrow I_{\text{m}}^-) = \Gamma_0 \exp \left\{ -\beta |g| A \xi_{\pi} \left(-2I_{\text{t}} \arccos I_{\text{t}} + 2\sqrt{1 - I_{\text{t}}^2} + 2\pi I_{\text{m}}^+ \right) \right\}. \quad (17\text{b})$$

The application of a voltage difference ΔV between the ends of the sample causes $\Delta\phi$ to increase linearly with time [i.e. $d(\Delta\phi)/dt = 4\pi e \Delta V/h$, where h is Planck's constant and $-e$ is the electronic charge] [9]. In steady state, this will be balanced by the net rate Γ_{net} at which current-decreasing fluctuations occur, i.e.,

$$\Gamma_{\text{net}} = \Gamma(I_{\text{m}}^+ \rightarrow I_{\text{m}}^-) - \Gamma(I_{\text{m}}^+ \leftarrow I_{\text{m}}^-) = \frac{1}{2\pi} \frac{d(\Delta\phi)}{dt}. \quad (18)$$

The difference between I_{t} and I_{m}^+ is of order ℓ^{-1} , and we shall henceforth neglect it; thus we simply refer to the current I . This leads to the following expression for the voltage between the ends of the wire in terms of the current along it:

$$\Delta V = (h/e) \Gamma_0 \sinh\{\pi\beta|g|A\xi_{\pi}I\} \times \exp \left\{ -\beta |g| A \xi_{\pi} \left(-2I \arccos I + 2\sqrt{1 - I^2} + \pi I \right) \right\}. \quad (19)$$

The corresponding expression for conventional superconductors may be found in Eq. (2.12) of Ref. [6]. As the length-scale of the fluctuation is ξ_{π} it is natural for ΔV to depend exponentially on $\beta|g|A\xi_{\pi}$ (i.e. the condensation energy in a correlation length per $k_{\text{B}}T$). Following McCumber and Halperin [10], we estimate Γ_0 to be of order N/τ , where $N \equiv 4\pi L/\xi_{\pi}$ reflects the range of possible locations along the wire at which the fluctuation may occur (L/ξ_{π}), as well as the variety of possible Néel-vector orientations (4π). For conventional superconductors τ diverges as $T \rightarrow T_{\text{c}}$. For the SO(5) model we expect τ to diverge as the chemical potential approaches its critical value.

It may be useful to compare expression (19) for ΔV with that obtained in Refs. [6,10] for the case of conventional superconductors. Within the SO(5) model the expression for ΔV in terms of the dimensionful supercurrent density J is given, for $J \simeq J_{\text{c}}$, by

$$\Delta V \simeq (h/e) \Gamma_0 \sinh\{\beta AhJ/4e\} \times \exp \left\{ -\beta A \left((hJ/4e) + 2\sqrt{2} |g| \xi_{\pi} (1 - J/J_{\text{c}})^{\frac{3}{2}} \right) \right\}, \quad (20)$$

whereas the corresponding expression obtained in Refs. [6,10] is given, for $J \simeq J_{\text{c}}$, by

$$\Delta V \simeq (h/e) \Omega_0 \sinh\{\beta AhJ/4e\} \times \exp \left\{ -\beta A \left((hJ/4e) + \sqrt{2}(8/3)^{\frac{5}{4}} \xi (g_{\text{n}} - g_{\text{s}}) (1 - J/J_{\text{c}})^{\frac{5}{4}} \right) \right\}. \quad (21)$$

(We have ignored the current dependence of the attempt frequency Ω_0 obtained in Ref. [10].) We see that in this regime the primary qualitative distinction between formulas (20) and (21) comes from the distinction between the nature of the fluctuations, and the resulting difference between the barrier heights.

We conclude by noting that for conventional superconductors LA [6] have obtained, as a condition for the validity of their approach, the constraint that the wire be thinner than the temperature-dependent superconducting correlation length ξ . (To accomplish this they consider the second variation of the free energy, and require that the transition state be unstable in only one direction in configuration space.) We emphasize that the corresponding condition in the context of the SO(5) theory of high-temperature superconductivity is that the wire be thinner than the correlation length for orientation fluctuations of the superspin, i.e., ξ_{π} . This length ξ_{π} is expected to diverge when the chemical potential approaches the superconductor-to-antiferromagnet phase boundary.

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- [1] S.-C. Zhang, *Science* **275**, 1089 (1997).
 - [2] D. P. Arovas, A. J. Berlinsky, C. Kallin and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997).
 - [3] E. Demler, A. J. Berlinsky, C. Kallin, G. B. Arnold and M. R. Beasley, *Proximity effect and Josephson coupling in the SO(5) theory of high- T_{c} superconductivity*; cond-mat/9707030.
 - [4] P. M. Goldbart, *Antiferromagnetic hedgehogs with superconducting cores*; UIUC Preprint P-97-10-030-i.
 - [5] W. A. Little, *Phys. Rev.* **156**, 108 (1967).
 - [6] J. S. Langer and V. Ambegaokar, *Phys. Rev.* **164**, 498 (1967).
 - [7] P. W. Anderson and G. Toulouse, *Phys. Rev. Lett.* **38**, 508 (1977); G. E. Volovik, *Pis'ma ZhETF* **27**, 605 (1978) [*JETP Lett.* **27**, 573 (1978)].
 - [8] P. M. Goldbart and P. Ao, *Phys. Rev. Lett.* **64**, 910 (1990).
 - [9] B. D. Josephson, *Adv. Phys.* **14**, 419 (1965).
 - [10] D. McCumber and B. I. Halperin, *Phys. Rev. B* **1**, 1054 (1969).