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Spin-selective scatterers as a probe of pairing in a one-dimensional interacting fermion gas

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Abstract. We study two species of attractively interacting fermion confined to a quasi-one-dimensional geometry, in the presence of a strong scattering potential that can couple, selectively, to one or both species. We show that the fermion density distribution in the presence of such a spin-selective scattering potential reflects the pairing spin gap of the fermions.

Contents

1. Introduction

In recent years, there has been much excitement surrounding the achievement of correlated phases of matter (like those found in condensed-matter systems) in cold-atom experiments [1, 2]. Part of the excitement stems from the extreme controllability of such cold-atom experiments, with parameters such as the interactions, particle densities and even spatial geometry being experimentally adjustable. This controllability provides a large parameter space of interacting systems to study, enhancing the comparison between theory and experiment.

In this paper we study the properties of two species of fermion confined to a quasi-one-dimensional (1D) geometry (achievable via a highly prolate trapping potential), with attractive

short-ranged interactions that lead to pairing correlations. Recent experiments have studied attractive fermions in the 1D limit [3], in the case of an imposed population imbalance that can have an imbalanced superfluid phase that is predicted [4]–[9] to be a 1D analogue of the Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) state [10, 11], in which the population imbalance induces a spatial modulation in the local pairing correlations. One open question concerns how the FFLO pairing correlations in imbalanced 1D gases would be observed experimentally.

Here, we focus on a balanced gas, but consider the case of an imposed potential that couples differently to the two fermion species, and which can create a local population imbalance. The motivation for studying this system follows from the fact that, in recent years, experimentalists have begun to explore spin-dependent trapping potentials [12, 13], and it is natural to consider how generic spin-dependent potentials can probe interacting Fermi gases. Recent theoretical work has investigated the properties of fermions in spin-dependent optical lattices [14]–[18] and in the case of separate harmonic trapping potentials for the two spin states [19], predicted to locally induce FFLO-like oscillatory pairing correlations. In this paper, we consider the case of a spatially inhomogeneous trapping potential that forms a local 'bump' or impurity at the spatial origin. Such systems have been studied for many years [20]–[24], mainly in the context of a local impurity or barrier in a 1D electron gas. We also note recent theoretical work studying the effect of impurity potentials on 1D cold atomic gases [25, 26]. In this paper, we study a spin-selective local scattering center that one can imagine suppresses local singlet pairing in the 1D gas, thereby probing such pairing correlations.

2. Model Hamiltonian

Our model Hamiltonian involves two species of interacting fermion (ψ_σ) confined to a quasi-1D limit:

$$H = \frac{\hbar^2}{2m} \sum_\sigma \int dx \left[|\partial_x \psi_\sigma|^2 - k_F^2 \rho_\sigma(x) \right] + \lambda \int dx \rho_\uparrow(x) \rho_\downarrow(x) + \sum_\sigma \int dx \, V_\sigma(x) \rho_\sigma(x), \qquad (1)$$

where $\lambda < 0$ measures the strength of the short-range attractive interspecies interactions, $\rho_\sigma = |\psi_\sigma|^2$, k_F is the Fermi wavevector, and $V_\sigma(x)$ represents a short-ranged, possibly spin-dependent, scattering potential centered at the origin $x = 0$. In a cold-atom experiment involving a highly prolate trapping geometry, λ can be related to an underlying effective 1D scattering length a_{1D} via

$$\lambda = -2\hbar^2 / m a_{1D}, \qquad (2)$$

which, in turn, can be related to the (experimentally controllable) 3D scattering length and the trap oscillator length [27].

We will analyze this system using the bosonization approach, which has the advantage of being technically straightforward while still retaining nontrivial interaction effects. We thus express the fermions in terms of bosonic fields θ_σ and ϕ_σ as [23, 24, 28, 29]

$$\psi_\sigma(x) \simeq \psi_{R\sigma}(x) + \psi_{L\sigma}(x), \qquad (3)$$

$$\simeq \frac{1}{\sqrt{2}} \sqrt{\rho_0/2 - \frac{1}{\pi} \partial_x \phi_\sigma} \left[e^{ik_F x} e^{i(\theta_\sigma - \phi_\sigma)} + e^{-ik_F x} e^{i(\theta_\sigma + \phi_\sigma)} \right], \qquad (4)$$

where ρ_0 is the mean fermion density, related to the Fermi wavevector via $k_F = \pi \rho_0 / 2$. In the first line, the subscripts R and L refer to right- and left-moving fermions. As these expressions

involve expansions for momenta near the Fermi surface, they only apply at sufficiently low energies, below a scale Λ of the order of the Fermi energy.

The bosonized form of the fermion densities are given by

$$\rho_\sigma(x) = \frac{1}{2}\rho_0 - \frac{1}{\pi}\partial_x\phi_\sigma + \frac{1}{2}\rho_0\cos(2k_F x + 2\phi_\sigma). \tag{5}$$

It is standard to introduce charge (ϕ_c) and spin (ϕ_s) versions of $\phi_{\uparrow,\downarrow}$, defined by

$$\phi_c = \frac{1}{\sqrt{2}}(\phi_\uparrow + \phi_\downarrow), \tag{6}$$

$$\phi_s = \frac{1}{\sqrt{2}}(\phi_\uparrow - \phi_\downarrow). \tag{7}$$

Using analogous expressions to define θ_c and θ_s, and switching to an effective action for the bosonic fields, we find

$$S = \frac{1}{2\pi}\sum_{i=c,s}\int dx\,d\tau\left[2i\partial_x\phi_i\partial_\tau\theta_i + \frac{u_i}{K_i}(\partial_x\phi_i)^2 + u_i K_i(\partial_x\theta_i)^2\right] + g\int dx\,d\tau\,\cos[2\sqrt{2}\phi_s] + S_{imp}, \tag{8}$$

where S_{imp} is the impurity action and the Luttinger interaction parameters and velocities take the well-known form

$$K_{c,s} = \frac{1}{\sqrt{1 \pm \lambda/(\pi v_F)}}, \tag{9}$$

$$u_{c,s} = v_F\sqrt{1 \pm \lambda/(\pi v_F)}, \tag{10}$$

with the $+$ $(-)$ being associated with c (s) in each line, so that $K_s < 1$, $K_c > 1$, and $u_s > u_c$ for the present case of attractive interactions ($\lambda < 0$), reflecting spin–charge separation. Here, $g = \frac{1}{2}\lambda\rho_0^2$ and $v_F = \hbar k_F/m$ is the Fermi velocity.

In the bulk, the attractive interactions lead to a 1D paired superfluid state, pinning $\phi_s \approx 0$ (representing the spin gap). Our aim is to see how the impurity, localized at $x = 0$, alters the local fermion densities and pairing correlations. We consider two kinds of impurity: symmetric (which couples to both species equally) and asymmetric (which couples to to only one species, that we will always take to be the $\downarrow$ species). While an symmetric impurity can be realized by locally modifying the prolate trapping potential (e.g. a local constriction), an asymmetric impurity would require a spin-dependent trapping potential. We also restrict attention to the strong-scattering limit, in which the impurity potential V_σ effectively pushes $\rho_\sigma \to 0$. In the symmetric impurity case, this constrains $\phi_\sigma(0) = \pi/2$ at the impurity center ($x = 0$) for both $\sigma = \uparrow$ and $\sigma = \downarrow$, whereas for the asymmetric case we have $\phi_\downarrow(0) = \pi/2$ with $\phi_\uparrow$ unconstrained.

3. Symmetric scattering potential

We begin with the case of a strong symmetric scattering potential that couples equally to the spins-$\uparrow$ and the spins-$\downarrow$. In the language of the spin and charge fields, this constrains $\phi_c(0) = \alpha_c = \pi/\sqrt{2}$ and $\phi_s(0) = \alpha_s = 0$. In the bulk, the field $\phi_s(x, \tau)$ is expected to exhibit only small fluctuations around the minimum value arising from the cosine term in equation (8). Given this, we can approximate this term by an effective gap Δ^2 representing the energy penalty

towards pair breaking, i.e. the underlying spin gap. An estimate for Δ is provided below in section 3.

Our aim is to calculate the local density and magnetization near the scattering center, given by (neglecting the derivative term)

$$\rho(x) = \rho_0 \Big[1 + \langle \cos[2k_F x + \sqrt{2}\phi_c] \cos \sqrt{2}\phi_s \rangle \Big], \tag{11}$$

$$M(x) = -\rho_0 \langle \sin[2k_F x + \sqrt{2}\phi_c] \sin \sqrt{2}\phi_s \rangle, \tag{12}$$

with the angle brackets representing the average with respect to the action S. Our approach, following [21, 22], involves implementing the above constraints with Lagrange multiplier fields $\lambda_i(\tau)$ (with $i = c, s$) that pin ϕ_c and ϕ_s to the values α_c and α_s consistent with the impurity potential. Integrating out the θ_i fields, we have the effective action $S = S_c + S_s + S_\lambda$ with

$$S_c = \frac{1}{2\pi K_c} \int dx d\tau \Big[u_c(\partial_x \phi_c)^2 + \frac{1}{u_c}(\partial_\tau \phi_c)^2 \Big], \tag{13}$$

$$S_s = \frac{1}{2\pi K_s} \int dx d\tau \Big[u_s(\partial_x \phi_s)^2 + \frac{1}{u_s}(\partial_\tau \phi_s)^2 + \frac{1}{u_s}\Delta^2 \phi_s^2 \Big], \tag{14}$$

$$S_\lambda = i \int dx d\tau \delta(x) \Big(\lambda_c(\tau)[\phi_c(x, \tau) - \alpha_c] + \lambda_s(\tau)[\phi_s(x, \tau) - \alpha_s] \Big). \tag{15}$$

It is sufficient to consider the averages $\langle e^{i\beta\phi_i(x)} \rangle$ with $i = c, s$. These decouple since, in the present case, the symmetric scatterer preserves spin–charge separation. Beginning with the ϕ_c average, we have

$$\langle e^{i\beta\phi_c(x)} \rangle = \frac{1}{Z} \int D\phi_c D\lambda_c \exp[-S_c - S_{\lambda_c} + i\beta\phi_c(x, 0)], \tag{16}$$

with S_{λ_c} being the part of S_λ containing λ_c and the denominator Z being the other factor with $\beta = 0$. Evaluating the functional integral over ϕ_c introduces the corresponding Green function (note we always take temperature $T \to 0$)

$$G_c(x, \omega) = K_c \int \frac{d\omega}{2\pi} \int \frac{dq}{2\pi} \frac{1}{u_c q^2 + \omega^2/u_c} e^{iqx} = \frac{K_c}{2|\omega|} \exp\Big[-\frac{|\omega x|}{u_c} \Big]. \tag{17}$$

We then obtain

$$\langle e^{i\beta\phi_c(x)} \rangle = \frac{1}{Z} \int D\lambda_c \exp[-S_{\text{eff.},\lambda_c}], \tag{18}$$

with the effective action for the Lagrange multiplier field

$$S_{\text{eff.},\lambda_c} = \frac{1}{4} \int d\omega [G_c(0, \omega)(\lambda_c(\omega)\lambda_c(-\omega) + \beta^2) - 2\beta G_c(x, \omega)\lambda_c(-\omega) - 4i\alpha_c\lambda_c(\omega)\delta(\omega)]. \tag{19}$$

Here, we have used the fact that $G_c(x, \omega)$ is even in its arguments. The remaining functional integral can be evaluated by considering the equation of motion for $\lambda_c(\omega)$:

$$\lambda_c(\omega) = \frac{1}{G_c(0, \omega)}[\beta G_c(x, \omega) + 2i\alpha\delta(\omega)], \tag{20}$$

which, when inserted back into equation (19) and using equation (18), yields

$$\langle e^{i\beta\phi_c(x)} \rangle = \exp\Big[i\alpha_c\beta \frac{G_c(x, 0)}{G_c(0, 0)} + \frac{1}{4}\beta^2 \int_{-\infty}^{\infty} d\omega \frac{G_c^2(x, \omega) - G_c^2(0, 0)}{G_c(0, \omega)} \Big], \tag{21}$$

where the first term in square brackets should be interpreted as $\lim_{\omega\to 0} G_c(x, \omega)/G_c(0, \omega) = 1$. The second integral is formally divergent, but should be regularized [21, 22] at the ultraviolet (UV) scale by introducing the cutoff Λ (by multiplying the integrand by $\exp[-\omega/\Lambda]$). This gives

$$\langle e^{i\beta\phi_c(x)} \rangle = \frac{e^{i\alpha_c\beta}}{[1 + 2x\Lambda/u_c]^{K_c\beta^2/4}}, \tag{22}$$

which has the correct limiting behavior for $x \to 0$ (consistent with $\phi_c(0) = \alpha_c$).

The second average we need is for the spin field, $\langle e^{i\beta\phi_s(x)} \rangle$, subject to the constraint $\phi_s(0) = \alpha_s$. However, examining equation (14) it is clear that the only difference relative to the preceding calculation is the fact that the ϕ_s action now has a mass due to the presence of pairing. Thus, the result must be the same but with the Green function G_c replaced by the corresponding spin Green function

$$G_s(x, \omega) = K_s \int \frac{d\omega}{2\pi} \int \frac{dq}{2\pi} \frac{1}{u_s q^2 + \omega^2/u_s + \Delta^2/u_s} e^{iqx}, \tag{23}$$

$$= \frac{K_s}{2\sqrt{\omega^2 + \Delta^2}} \exp\left[-\frac{|x|}{u_s}\sqrt{\omega^2 + \Delta^2}\right], \tag{24}$$

sensitive to the pairing scale Δ. With this, we have the effective action for λ_s:

$$S_{\text{eff.},\lambda_s} = \frac{1}{4} \int d\omega[G_s(0, \omega)(\lambda_s(\omega)\lambda_s(-\omega) + \beta^2) - 2\beta G_s(x, \omega)\lambda_s(-\omega) - 4i\alpha_s\lambda_s(\omega)\delta(\omega)], \tag{25}$$

and, using $\lim_{\omega\to 0} G_s(x, \omega)/G_s(0, \omega) = \exp[-|x|\Delta/u_s]$, we find for the necessary average:

$$\langle e^{i\beta\phi_s(x)} \rangle = \exp\left[i\alpha_s\beta \exp(-|x|\Delta/u_s) + \frac{1}{4}\beta^2 \int_{-\infty}^{\infty} d\omega \frac{G_s^2(x, \omega) - G_s^2(0, 0)}{G_s(0, \omega)}\right], \tag{26}$$

which again is consistent with the constraint $\phi_s(0) = \alpha_s$.

We now proceed to use these averages, along with equations (11) and (12), to determine the density response in the presence of such a strong symmetric scatterer. For the correlator involving the spin field, we clearly need equation (26) with $\alpha_s = 0$ and $\beta = \pm\sqrt{2}$; however, since $\langle e^{i\sqrt{2}\phi_s(x)} \rangle = \langle e^{-i\sqrt{2}\phi_s(x)} \rangle$, we immediately have $M(x) = 0$ everywhere, as expected, since the scatterer couples equally to the two fermion species. Turning to the correlator involving the charge field, examining equation (22) with $\alpha_c = \pi/\sqrt{2}$ and $\beta = \pm\sqrt{2}$, and using equation (26), we find for the density

$$\frac{\rho(x)}{\rho_0} = 1 - \cos(2k_F x)\frac{\exp[\frac{1}{2}K_s I(x)]}{[1 + 2x\Lambda/u_c]^{K_c/2}}, \tag{27}$$

where $I(x)$ is proportional to the integral in equation (26), and given by

$$I(x) = \int_0^{\infty} d\omega \frac{1}{\sqrt{\omega^2 + \Delta^2}}[e^{-2|x|\sqrt{\omega^2 + \Delta^2}/u_s} - 1]e^{-\omega/\Lambda}. \tag{28}$$

Although $I(x)$ cannot be easily evaluated analytically, it basic behavior is straightforward. For $x \to 0$, $I(x)$ vanishes (consistent with $\phi_s(0) = 0$), while, for $x \to \infty$, $I(x) \simeq \ln \Delta/\Lambda$ (reflecting the small Gaussian fluctuations of ϕ_s around zero), so that we finally have

$$\rho(x) = \rho_0 \left(1 + \cos(2k_F x) \frac{(\Delta/\Lambda)^{K_s/2}}{[1 + 2|x|\Lambda/u_c]^{K_c/2}} \right), \tag{29}$$

valid for $x \to \infty$, describing the asymptotic Friedel oscillations in this 1D paired-fermion gas [21, 22, 30] The power law of the envelope reflects the superfluid stiffness Luttinger parameter K_c, implying that the long- distance response to such a symmetric scatterer is due to low-energy phase fluctuations of the superfluid field θ_c. The result equation (29) agrees with well-established literature predicting that a strong impurity should lead to Friedel oscillations in the local density [21, 22, 30, 31] of a 1D gas. We note that the precise exponent $-K_c/2$ follows from the results of [30] upon taking the spin Luttinger parameter (K_s in our notation) to vanish to account for the spin gap.

4. Magnetic scattering potential

In the preceding section, we found that a strong scattering potential that couples equally to the two fermion species leads to a suppression of $\rho(x)$ that oscillates with wavevector $2k_F$, along with an overall envelope that decays away from the origin as a power law $\sim |x|^{-K_c/2}$. In contrast, in the present section, we show that an asymmetric or 'magnetic' scattering potential, that couples preferentially to one spin species, shows quite a different spatial profile that is sensitive to the underlying spin gap Δ of the system. Once again, we consider the case of a strong scattering potential, but assume it couples only to the spins-$\downarrow$, effectively constraining $\phi_\downarrow(0) = \pi/2$ (so that, near $x \to 0$, $\rho_\downarrow \to 0$). Then we have (once again integrating out θ_c and θ_s) the effective action

$$S = S_c + S_s + i \int dx d\tau \delta(x) \lambda(\tau) \left[\frac{1}{\sqrt{2}} \phi_c(x, \tau) - \frac{1}{\sqrt{2}} \phi_s(x, \tau) - \alpha \right], \tag{30}$$

with the parameter $\alpha = \pi/2$.

To obtain the spin-$\uparrow$ and spin-$\downarrow$ densities, we need to compute the average

$$\langle e^{i\beta_c\phi_c(x)} e^{i\beta_s\phi_s(x)} \rangle = \frac{1}{Z} \int D\phi_c D\lambda_c D\phi_s D\lambda_s \exp[-S + i(\beta_c\phi_c(x, 0) + \beta_s\phi_s(x, 0))]. \tag{31}$$

The difference relative to the preceding section is clear. Previously, we had separate Lagrange multiplier fields for charge (λ_c) and spin (λ_s), and we separately integrated out the ϕ_c and ϕ_s fields to obtain effective actions for λ_c and λ_s. In the present case, we have only one Lagrange multiplier field λ that couples to both ϕ_c and ϕ_s in a way similar to the preceding section but with the rescalings $\lambda_c \to \lambda/\sqrt{2}$ and $\lambda_s \to -\lambda/\sqrt{2}$. This implies, upon integrating out the ϕ_c and ϕ_s fields, an effective action for λ (by analogy with equations (19) and (25))

$$S_{\text{eff.},\lambda} = \frac{1}{4} \int d\omega \left[G_c(0, \omega) \left(\frac{1}{2} \lambda(\omega)\lambda(-\omega) + \beta_c^2 \right) - \sqrt{2}\beta_c G_c(x, \omega)\lambda(-\omega) \right.$$
$$\left. + G_s(0, \omega) \left(\frac{1}{2} \lambda(\omega)\lambda(-\omega) + \beta_s^2 \right) + \sqrt{2}\beta_s G_s(x, \omega)\lambda(-\omega) - 4i\alpha\lambda(\omega)\delta(\omega) \right]. \tag{32}$$

Following the procedure of the preceding section, the equation of motion for $\lambda(\omega)$ leads to

$$\lambda(\omega) = \frac{4i\alpha\delta(\omega) + \sqrt{2}(\beta_c G_c(x, \omega) - \beta_s G_s(x, \omega))}{G_c(0, \omega) + G_s(0, \omega)}, \tag{33}$$

which we must insert into equation (32) to get the final result for equation (31). At this point, we focus on the cases of $\beta_s = \pm\beta_c$, which amounts to considering the corresponding averages $\langle e^{i\sqrt{2}\beta\phi_{\uparrow,\downarrow}}\rangle$ for $\phi_{\uparrow,\downarrow}$. We obtain

$$\langle e^{i\sqrt{2}\beta\phi_\sigma}\rangle = \exp\left[\sqrt{2}i\alpha\beta + \frac{\beta^2}{4}I_\sigma(x)\right], \tag{34}$$

with $I_\sigma(x)$ given by

$$I_\sigma(x) = \int_{-\infty}^{\infty} d\omega \frac{[G_c(x, \omega) - \sigma G_s(x, \omega)]^2 - [G_c(0, \omega) + G_s(0, \omega)]^2}{G_c(0, \omega) + G_s(0, \omega)}, \tag{35}$$

where $\uparrow = +$ and $\downarrow = -$ on the right side. Equation (35) satisfies $I_\downarrow(0) = 0$, consistent with the constraint $\phi_\downarrow(0) = \alpha$ when inserted in equation (34).

Using this result for I_σ along with equations (11) and (12), we find

$$\frac{\rho(x)}{\rho_0} = 1 - \cos(2k_F x)\delta\rho(x), \tag{36}$$

$$\frac{M(x)}{\rho_0} = \cos(2k_F x)\delta M(x), \tag{37}$$

where $\delta\rho(x)$ and $\delta M(x)$ represent the envelope functions of the local density and magnetization, given by

$$\delta\rho(x) = e^{1/4(I_\uparrow(x)+I_\downarrow(x))} \cosh\left[\tfrac{1}{4}(I_\uparrow(x) - I_\downarrow(x))\right], \tag{38}$$

$$\delta M(x) = -e^{1/4(I_\uparrow(x)+I_\downarrow(x))} \sinh\left[\tfrac{1}{4}(I_\uparrow(x) - I_\downarrow(x))\right]. \tag{39}$$

Although the integrals $I_\uparrow(x)$ and $I_\downarrow(x)$ must be calculated numerically, it is straightforward to determine the asymptotic large x behavior of these functions, and thus of $\rho(x)$ and $M(x)$, showing how the spin-gap energy scale Δ would emerge in an experiment. We find that the sum $I_\uparrow(x) + I_\downarrow(x) \simeq 2K_c \ln\frac{u_c}{|x|}$, whereas the difference satisfies $I_\uparrow(x) - I_\downarrow(x) \propto \exp(-|x|/L_\Delta)/(|x|/L_\Delta)$, decaying exponentially over a length scale $L_\Delta \approx v_F/\Delta$ that directly reflects the underlying spin gap in this 1D Fermi gas (in this expression, we have assumed $u_s \simeq u_c \simeq v_F$ for simplicity). These limiting results, plugged into equations (38) and (39), imply a rapid vanishing of $\delta M(x)$ beyond the length scale L_Δ, but a slow power-law variation of $\delta\rho(x)$ with increasing x

$$\delta\rho(x) \sim \frac{1}{|x|^{K_c/2}}, \tag{40}$$

$$\delta M(x) \sim \frac{1}{|x|^{K_c/2+1}} \exp(-|x|/L_\Delta). \tag{41}$$

This qualitative behavior is seen in the numerical calculations of the dimensionless quantities $\delta\rho(x)$ and $\delta M(x)$, equations (38) and (39), which we plot in figure 1. For this we chose the
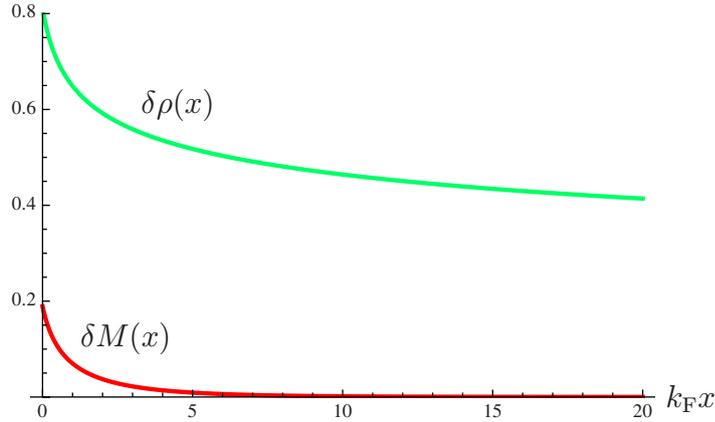
Figure 1. The density deviation from uniformity, $\delta\rho(x)$, and magnetization deviation, $\delta M(x)$, according to equations (38) and (39), due to a spin-selective scattering potential, with parameters given in the text. While $\delta\rho(x)$ exhibits a slow power-law variation at large x, $\delta M(x)$ decays to zero over a length scale $L_\Delta \simeq v_F/\Delta$ reflecting the underlying spin gap. Note that the maximum of $\delta\rho(x)$ at $x \to 0$ represents a *suppression* of $\rho(x)$ near the scatterer.

UV scale Λ to be equal to the Fermi energy $\epsilon_f = \hbar^2 k_F^2/2m$, the spin gap $\Delta \simeq 0.3\epsilon_f$, and the coupling parameter entering equations (9) and (10) to be $\lambda/\pi v_F \simeq -0.5$.

We can try to estimate the magnitude of Δ in present-day experiments by using the standard Bardeen–Cooper–Schrieffer mean-field theory, which gives $\Delta \approx \epsilon_f \exp\left[\frac{\sqrt{2\epsilon_f}\hbar\pi}{\sqrt{m}\lambda}\right]$. Plugging in equation (2) gives the argument of the exponential function to be $-\pi k_F a_{1D}/2$. Using the reported value of $a_{1D} \approx 2099 a_0$ in [3] (with a_0 the Bohr radius), and estimating $k_F = 5.5 \times 10^6 \, \mathrm{m}^{-1}$ (from the reported Fermi energy), we find this exponent to be ≈ -0.96, implying $\Delta/\epsilon_f \simeq 0.38$, a strong pairing gap with the associated length scale $L_\Delta \simeq 0.95 \, \mu\mathrm{m}$

To conclude, we have shown that, in response to a strong spin-selective scattering potential, the total density responds analogously to the case of a symmetric impurity (i.e. with the same power law as in equation (29)), while the local magnetization or population imbalance returns to its equilibrium value over a length scale governed by the pairing spin gap parameter Δ. There are several remaining questions for future research, including the behavior away from the asymptotic strong scattering limit, as well as the generalization to the case of an imbalanced gas. Regarding the latter problem, one may imagine that such a spin-selective scattering potential may pin or enhance the oscillatory pairing correlations in the 1D FFLO phase (that have been detected in computational theory [7, 8] but not yet in experiments), making them easier to observe. Finally, there is the issue of the harmonic trapping potential that is always present in experiments but ignored here. We expect that, as long as the spatial variation of the atom density due to the trap (i.e. the Thomas–Fermi length) is large compared to the density variations induced by the scatterer, our principal conclusions will remain intact. However, an important direction for future research would be to study this claim in detail and examine how the trapping potential impacts the Friedel oscillations induced by local scattering potentials.

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