Optimal and Efficient Decision-Making for Power System Expansion Planning

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OPTIMAL AND EFFICIENT DECISION-MAKING FOR POWER SYSTEM EXPANSION PLANNING

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy in

The Department of Electrical and Computer Engineering

by

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August 2019
This work is dedicated to
my mother
Mina
for her kindness, devotion, and endless support
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ABSTRACT

A typical power system consists of three major sectors: generation, transmission, and distribution. Due to ever increasing electricity consumption and aging of the existing components, generation, transmission, and distribution systems and equipment must be analyzed frequently and if needed be replaced and/or expanded timely. By definition, the process of power system expansion planning aims to decide on new as well as upgrading existing system components in order to adequately satisfy the load for a foreseen future.

In this dissertation, multiple economically optimal and computationally efficient methods are proposed for expanding power generation, transmission, and distribution systems. First, a computationally efficient model is proposed for transmission expansion planning (TEP). While the existing TEP models use bus voltage angles, the proposed TEP takes advantages of linear sensitivity factors to omit voltage angles from the formulation and replace all nodal power balance constraints by one equivalent constraint. Simulation results show that the proposed model provides the same results as the classical angle-based model while being much faster. Second, a distributed collaborative TEP algorithm for interconnected multi-regional power systems is proposed. The information privacy is respected as each local planner shares limited information related to cross-border tie-lines with its neighboring planners. To coordinate the local planners, a two-level distributed optimization algorithm is proposed based on the concept of analytical target cascading for multidisciplinary design optimization. Third, a security-constrained generation and transmission expansion planning (G&TEP) model with respect to the risk of possible N-1 contingencies is proposed. Using the concept of risk, non-identical probability and severity of individual contingencies are modeled in the proposed G&TEP model. Finally, a mixed-integer linear programming model is proposed for resilient feeder routing in power distribution systems.
Geographical information system (GIS) data is used in the proposed model. As it is proven, having GIS facilities will lead to a more cost-efficient and resilient feeder routing scheme than the scheme obtained using electrical points. The proposed model and solution algorithm are comprehensive from several practical aspects such as economic objectives (installation cost, power losses, resiliency), technical constraints (voltage drops, radially constraint, reliability), and geographical constraints (obstacles, right-of-ways).
CHAPTER 1
AN INTRODUCTION TO POWER SYSTEM EXPANSION PLANNING

1.1. Introduction and Definition

1.1.1. Transmission Expansion Planning

In electric power systems, transmission networks must have adequate capacity and appropriate design for reliable, secure, and fair electricity delivery from generating units to consumers. New transmission lines might be required to support load growth, remove transmission congestion, support integration of distributed renewable energy sources, provide nondiscriminatory transmission access for all market participants, and support system reliability [1, 2]. The location, number, and installation time of new transmission lines are determined by a long-term transmission expansion planning (TEP) problem. The TEP problem is solved by planning entities (i.e., regional transmission organization (RTO) in the United States and transmission system operator (TSO) in Europe) to optimally expand the network topology with the least investment costs. This is a very expensive, lengthy, and burdensome procedure [3]. It is proven that TEP is an NP-hard problem [4], and thus obtaining the optimal solution of this problem is difficult, especially for large-scale systems.

1.1.2. Generation and Transmission Expansion Planning

The main objective of a generation and transmission expansion planning (G&TEP) problem is to determine the optimal investment in new generation and/or new transmission sectors to ensure a certain reliability level for the forecasted demand. Usually, a G&TEP model is tackle either in a market-based framework or a centralized framework. In the market-based framework, profit-oriented agents determine their own G&TEP plans with the aim of maximizing their expected profits. On the other hand, in the centralized approach, a central planner, e.g., the independent system operator (ISO), determines the G&TEP plan that has the highest profit for the whole
system. Then, the central planner encourages private entities, by different incentives, to expand their system accordingly [5]. According to the North American Electric Reliability Corporation (NERC), standard 51 [6], a planned network must be able to supply demands in the case of outage of a single element (N-1 security criterion). Therefore, considering N-1 security criterion for a G&TEP model is essential.

1.1.3. Distribution Expansion Planning

Power distribution networks are key elements of an electric infrastructure system. A distribution network must have adequate capacity and appropriate design for reliable, secure, and high quality electricity delivery to consumers. Designing new distribution systems and upgrading existing networks are required to support load growth. An important step in distribution expansion planning is feeder routing (FR). By definition, FR refers to a model for finding the optimum radial routes and conductors’ size from a medium voltage substation to residential, commercial, or industrial load points. Multiple factors such as system planning cost, amount of losses, reliability, voltage quality, and grid resiliency are directly related to the configuration of the system. The financial justification of a new expansion planning rely on all these factors, and therefore, they should be considered in an integral evaluation.

1.2. Motivation and Literature Review

1.2.1. Modeling Candidate Lines’ Flow

The main challenge of the TEP problem is in the modeling of power flow constraints corresponding to candidate transmission lines. Depending on modeling of these constraints, the TEP problem is either mixed-integer nonlinear programming (MINLP) or mixed-integer linear programming (MIP) [7]. Heuristic or evolutionary algorithms are widely deployed to solve MINLP models [8], while Benders decomposition and branch and bound methods are used to solve MIP
models [9]. Since the MIP models are based on a disjunctive technique to linearize power flow constraints, these models are also known as disjunctive models. Although the disjunctive TEP is more efficient than the MINLP model, it is still computationally expensive. As proven in [4], the computational complexity of the basic DC TEP problem is NP-hard. By increasing the size of the system and consequently increasing the number of variables and constraints, the computational burden increases exponentially. The TEP problem becomes even more complex when realistic constraints, such as N-1 security criterion, are taken into account. TEP with realistic constraints might be an intractable problem, especially for large systems [10]. The size of the problem depends on the number of continuous variables, binary variable, and constraints. That is, reducing the number of variables (even continuous variables) and constraints significantly decreases computational burden of TEP. This motivates us to investigate reformulation of the DC TEP model with power system linear sensitivity factors instead of using bus voltage angles. The linear sensitivity factors, such as shift factor (SF) and line outage distribution factor (LODF), determine line flow changes with respect to changes in nodal power injections [11]. SFs and LODFs can be deployed to model the DC power flow equations with no need for bus voltage angles. This reduces the number of variables and size of an optimization problem. The sensitivity factors have been used for DC optimal power flow and security-constrained unit commitment problems [12, 13]. The SF values are dependent on the grid topology, i.e., SFs change with any alterations in the grid topology. However, the grid topology is not known before solving the TEP problem, since candidate lines might or might not be installed in the network. Thus, it is very challenging to model the linear sensitivity factors for the TEP problem in which the grid topology changes over the course of optimization.

Another problem that makes TEP more challenging is N-1 security criterion that is an essential
constraint in the long-term planning problem. The system must be reliable and stable not only under normal conditions but also after occurrence of contingencies. Based on North American Electric Reliability Corporation (NERC) standards, a planned network must be able to operate in a way that outage of a single component does not interrupt supplying demands [6]. However, taking into account the N-1 security criterion drastically increases the size of TEP and might make the problem intractable, in particular, for large systems.

Moreover, renewable energy integration, load uncertainties, and component outages should be taken into account in long term planning [14]. Several methods, such as stochastic programming [9] and robust optimization [15], have been proposed in the literature to model uncertainties in TEP. However, the assumption of known probabilistic distribution functions (PDFs) of uncertain parameters in stochastic optimization and dealing with a worst-case scenario in robust optimization (which leads to a too conservative solution) are the main drawbacks of these two approaches. Data-driven optimization approaches are potential solutions to address these drawbacks [16]. In a data-driven optimization approach, by learning from historical data, an unknown probability distribution is considered for an uncertain parameter, and a confidence set with a certain confidence level is constructed. The worst-case distribution in the confidence set, which leads to the worst cost, is selected for the optimization model [17]. It is theoretically proven that by increasing the size of historical data, the conservativeness of the model decreases [18].

1.2.2. TEP in Interconnected Multi-Regional Power Systems

TEP is a more challenging problem in interconnected multi-regional power systems including several independent networks. Each power system could have its own local transmission planner [3]. If the planners separately solve their TEP problem, the grid topology might not be optimal from the perspective of the whole grid. This separate planning imposes unnecessary investment
costs and reduces social welfare. Most of the existing literature ignores interactions between the regions when solving for regional TEPs [19, 20]. Several papers have dealt with TEP in the interconnected power systems, while most of them assume an entity that has all grid information and formulates a centralized TEP problem for the whole grid [3]. Such a TEP framework potentially enhances system performance and reduces overall investment/operational costs. However, in the privatized power sector, planners (or operators) are not willing to share their commercially sensitive information with other parties, and each transmission planner (or operator) seeks to find its optimal TEP solution [21]. On the other hand, any decision made by a transmission planner affects TEP results of other planners, as the whole system is an interconnected grid. Therefore, implementing an individual TEP by a planner regardless of TEP results in other areas might cause higher transmission planning costs and lower system reliability.

A limited number of papers has been published on TEP in multi-regional power systems. In previous studies, two different approaches have been proposed for multi-regional TEP: cooperative approach and non-cooperative approach. In the cooperative approach, all planners work together to achieve the highest overall social welfare. One TEP problem can be formulated for the whole system. While, in the non-cooperative approach, each transmission planner seeks to maximize its social welfare considering the planning decisions of other planners [22]. In the non-cooperative approach, the social welfare of the whole interconnected system might not be obtained due to competition between different transmission planners [23]. The concept of cooperative and non-cooperative solutions for TEP in the multilateral context is discussed in [24]. Research works that deal with the non-cooperative TEP consider, usually, the cooperative TEP’s solution as benchmark results since it maximizes the overall social welfare. Most of the existing works in the field of cooperative/non-cooperative are based on either a centralized optimization or game theory.
While several studies have been done on the distributed optimal power flow [25] and distributed unit commitment [26], due to the complex nature of TEP, no reference is reported on the domain of distributed transmission expansion planning (DTEP) for interconnected power systems.

Another problem that makes TEP more challenging is N-1 security criterion that is an essential constraint in the long-term planning problem. The N-1 criterion must be taken into consideration in a cooperative/non-cooperative TEP framework.

1.2.3. Modeling Risk of N-1 Contingencies in G&TEP

The authors of [27] proposed a systematic method to identify the most important contingencies for the N-1 security-constrained transmission expansion planning (TEP) problem. However, this model is not tractable for stochastic TEP due to accumulative effect of considering all scenarios at the beginning of decision-making process. New identification indices were proposed to integrate the necessary contingencies gradually for each iteration of the stochastic TEP model [28]. A TEP model with probabilistic reliability criteria was presented in [29]. The suitability of the proposed TEP model subject to future practical uncertainties was demonstrated. A mixed-integer linear programming (MILP) model of G&TEP was proposed in [30]. The model considers probabilistic reliability criteria for random generator and line outages with known historical outage rates. However, all of these references (and many other works) assume identical severity for all possible contingencies. It should be noted that the probability and severity of contingencies are not the same in practice. Ignoring this concept may degrade the quality and effectiveness of the planning results.

1.2.4. Modeling a Resilient Feeder Routing Problem

One important aspect of FR is resiliency of the planned grid. The resiliency of the distribution network is defined as “ability of the grid to continue operating and delivering power even in the low probability events causing high-consequence disruptions, such as hurricanes, floods,
earthquakes, and cyber-attacks” [31]. A grid with low resiliency is vulnerable and difficult-to-recover when an event occurs. In other words, the cost of a system with low resiliency is high in the case of an extreme event. On the other hand, designing a highly resilient grid might be very expensive. Hence, a tradeoff should be made between grid resiliency and investment costs.

Recently, applications of geographical information system (GIS) facilities are extended for distribution system operation and maintenance [32]. Power distribution systems can be represented in more details by taking advantage of GIS facilities. The existing tools of GIS facilities can be further extended to cover distribution systems expansion planning and feeder routing. The presence of GIS facilities results in more convenient resiliency analysis since they, for instance, help to incorporate the exact geographical location of each component into the expansion planning model. As another advantage, GIS facilities can be used to model the level of accessibility and repair time of each feeder after an extreme event.

Despite traditional FR models, few works have been done on the GIS-based feeder routing. For the first time, the GIS-based FR was modeled by [33]. A dynamic programming technique was proposed to solve the model. Afterward, a mixed-integer nonlinear programming model for spatial power system planning with respect to GIS facilities was proposed in [34]. In [35], a GIS-based methodology for transmission line routing with respect to analytic hierarchy process was proposed. However, uncertainties of the demand and renewable energy resources and resiliency of the planned network were ignored in the aforementioned references.

1.3. Contribution and Organization

In chapter 2 of this dissertation, entitled A Sensitivity Factor-Based Transmission Expansion Planning Model with N-1 Security Criteria, a realistic shift factor-based transmission expansion planning (SF-based TEP) model is proposed. The power flow constraints are reformulated using
the SF values, and the bus voltage angles are omitted from TEP to reduce the size of the optimization problem. Candidate transmission lines and N-1 security criterion (outage of a unit, an existing line, or a candidate line) are modeled with the concept of flow cancellation. Power balance constraints are reformulated to ensure the validity of the SF-based TEP model in the case of occurrence of islanding under contingency. The proposed model utilizes the SF matrix of the complete network in which it is assumed that all candidate lines are installed. This is an important feature of the proposed TEP model as there is no need to calculate and use the SF matrix for each possible grid topology after installation of new lines or contingencies. We account for the load and generation uncertainties by applying a data-driven technique to construct confidence sets of probability distributions for these uncertain parameters based on their historical data. The proposed SF-based TEP has several advantages over the conventional angle-based TEP model: 1) efficient modeling of the existing/candidate transmission lines with less number of decision variables. 2) Efficient modeling of N-1 security constraints without adding many new decision variables to TEP (compared with the angle-based model in which many new variables need to be introduced for each contingency). 3) The capability of monitoring a limited subset of lines instead of the entire network. 4) Providing a formulation for modeling single candidate line within a corridor with multiple parallel lines. 5) Reducing the size of the problem and its solution time. These features make the SF-based TEP more computation ally efficient and tractable than the classical angle-based TEP, while both models provide the same level of accuracy. Thus, the proposed TEP is an alternative scalable model for planning of medium- and large-scale power systems.

In chapter 3 of this dissertation, entitled Distributed Collaborative Security-Constrained Transmission Expansion Planning with Analytical Target Cascading, a DTEP algorithm is proposed for interconnected multi-regional power systems in a collaborative framework taking
into account N-1 security criterion for each region and uncertainties. The proposed DTEP is a multi-agent-based TEP. Each region has its own independent planning entity. Interactions (i.e., power exchange) between the regions through tie-lines are modeled by a set of pseudo generations and a set of pseudo loads, and the power balance equation at border buses are modified accordingly. Taking into account the information privacy of the planning entities and their mutual interactions, a local TEP problem is formulated for each region. Each local planner handles the TEP problem of its network while having access to the information of cross-border tie-lines connecting that network to its neighbors. Based on the concept of analytical target cascading (ATC) technique for multidisciplinary design optimization, a two-level distributed optimization algorithm is developed. While the classical ATC is a sequential procedure, the developed ATC allows the parallel solution of local TEP problems in level one with the use of a central coordinator in level two. Since the level two’s problem is a convex optimization, it is further replaced in the level one’s TEP problems by the Karush-Kuhn-Tucker (KKT) conditions. With this procedure, we eliminate the need for a central coordinator by introducing a set of coordinating variables and enforcing a set of constraints in the local TEP of each region. This makes the proposed algorithm fully parallelized. An initialization strategy is suggested to reduce the number of iterations of the DTEP algorithm. The proposed collaborative DTEP is applied to the IEEE 24-bus and 118-bus test systems, and promising results are obtained.

In chapter 4 of this dissertation, entitled *Security-Constrained Generation and Transmission Expansion Planning with Risk of Contingencies*, a security-constrained G&TEP model taking into account the risk of possible N-1 contingencies is proposed. Using the concept of risk, which is the product of probability and severity, non-identical consequences of different contingencies are modeled. The risk index of each contingency is added to the objective function of the G&TEP
model. That is, in the proposed G&TEP model, high-risk contingencies are penalized more dominantly. As a result, the expenses for keeping the reliability of the power system during an expansion planning will be spent more wisely. Modeling the developed risk index in the objective function makes the problem nonlinear. Therefore, a linearization technique is developed to convert the problem into the MILP format.

In chapter 5 of this dissertation, entitled *A Mixed-Integer Linear Model for GIS-Based Resilient Feeder Routing*, an MILP model is proposed for the resilient feeder routing problem using geographical information system (GIS) facilities. It is proven in the paper that having GIS facilities will lead to a better feeder routing scheme than the scheme obtained using electrical points. The uncertainty of rooftop solar generations and demand forecasting errors are taken into account in the proposed model, and a stochastic programming-based solution algorithm is developed. The proposed model and solution algorithm are comprehensive from several practical aspects such as economic objectives (installation cost, power losses, resiliency), technical constraints (voltage drops, radially constraint, reliability), and geographical constraints (obstacles, right-of-ways, high-cost passages). The efficiency of the algorithm is elaborated for a small-scale system, and it is further illustrated for a realistic large-scale system.

Finally, concluding remarks and suggestions of the future works are provided in chapter 6 of this dissertation.

### 1.4. References


CHAPTER 2  
A SENSITIVITY FACTOR-BASED TRANSMISSION EXPANSION PLANNING MODEL WITH N-1 SECURITY CRITERIA

2.1. Introduction

In this chapter, a fundamentally novel model for the transmission expansion planning (TEP) problem is presented. While the existing DC power flow based TEP models use bus voltage angles, the proposed TEP takes advantages of shift factor (SF) and line outage distribution factor (LODF) matrices to formulate the planning problem. The operational costs and N-1 security criteria are taken into account. All equations are formulated using the SF and LODF values of the original network (i.e., the grid topology prior to implementation of planning and occurrence of a contingency). The bus voltage angles are omitted from the formulation, and nodal power balance constraints are replaced by one equivalent constraint. Thus, the proposed TEP model include less number of variables and constraints compared with the classical disjunctive model. In addition, an operator has the flexibility for monitoring only a few subsets of important lines. These features significantly reduce computational costs of the planning problem and enhance its scalability, especially for large-scale systems. Simulation results show that the proposed model provides the same results as the conventional angle-based model while being much faster (more than 58% based on our case studies) and computationally more efficient.

2.2. Symbols

A. Indices and Sets:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>Index for load blocks.</td>
</tr>
<tr>
<td>c</td>
<td>Index for contingency.</td>
</tr>
<tr>
<td>d</td>
<td>Index for demand.</td>
</tr>
<tr>
<td>g</td>
<td>Index for generating units.</td>
</tr>
<tr>
<td>l, k</td>
<td>Index for transmission lines.</td>
</tr>
<tr>
<td>i, j, s, t</td>
<td>Index for buses.</td>
</tr>
<tr>
<td>w</td>
<td>Index for wind farms.</td>
</tr>
</tbody>
</table>
\( r(l) \) Receiving-end node of transmission line \( l \).
\( s(l) \) Sending-end node of transmission line \( l \).
\( \Omega_i^D \) Set of all demands located at node \( i \).
\( \Omega_i^G \) Set of all generating units located at node \( i \).
\( \Omega_i^W \) Set of all wind farms connected to node \( i \).
\( \Omega_i^L \) Set of all existing transmission lines.
\( \Omega_i^{L+} \) Set of all candidate transmission lines.
\( \rho \) Set of all possible islands during N-1 contingency.
\( \Gamma \) Set of decision variables in SF-based TEP model.
\( \Delta_{g}^{\text{max}} \) Maximum decision capability of unit \( g \).

B. Parameters:
\( A_g^C \) Parameter that is equal to 0 if unit \( g \) is unavailable under contingency \( C \) and 1 otherwise.
\( A_l^C \) Parameter that is equal to 0 if line \( l \) is unavailable under contingency \( C \) and 1 otherwise.
\( B_l \) Susceptance of transmission line \( l \).
\( C_d \) Load-shedding cost of demand \( d \).
\( C_d^C \) Load-shedding cost of demand \( d \) in contingency \( C \).
\( C_g \) Production cost of generating unit \( g \).
\( F_l^{\text{max}} \) Maximum capacity of transmission line \( l \).
\( I_l \) Investment cost of candidate transmission line \( l \).
\( f_{\text{max}} \) Investment budget for building candidate lines.
\( LODF_{l,k} \) Line outage distribution factor of line \( l \) when line \( k \) is out.
\( M \) Large enough number, called big-M.
\( PTDF_{i,j}^l \) Power transfer distribution factor of line \( l \) with respect to injection node \( i \) and consumption node \( j \).
\( P_d \) Power of demand \( d \).
\( P_g^{\text{max}} \) Production capacity of generating unit \( g \).
\( SF_{l,i} \) Shift factor of line \( l \) with respect to bus \( i \).
\( SF_{l,k} \) Shift factor of line \( l \) with respect to bus \( i \) when line \( k \) is out.
\( Z_{s,i} \) Element \((s,i)\) of the inductance matrix, which is inverse of the admittance matrix.
\( z_l \) Inductance of line \( l \).

C. Variables:
\( x_l \) Binary decision variable to indicate whether candidate line \( l \) is constructed.
\( P_{g,b} \) Power produced by generating unit \( g \) for load block \( b \).
\( P_{w,b} \) Power produced by wind farm \( w \) for load block \( b \).
\( P_{i,b} \) Net power injection at node \( i \), i.e., generation minus load for load block \( b \).
\( f_{l,b} \) Power flow through transmission line \( l \) for load block \( b \).
\( P_{d,s} \) Load shedding of demand \( d \) for load block \( b \).
\( \bar{P}_{st,b} \)  Virtual power injection at bus \( s \), withdrawal from bus \( t \) for load block \( b \).
\( \theta_{i,b} \)  Voltage angle at node \( i \) for load block \( b \).

2.3. Contributions

We present a realistic shift factor-based transmission expansion planning (SF-based TEP) model. The power flow constraints are reformulated using the SF values, and the bus voltage angles are omitted from TEP to reduce the size of the optimization problem. Candidate transmission lines and N-1 security criterion (outage of a unit, an existing line, or a candidate line) are modeled with the concept of flow cancellation. Power balance constraints are reformulated to ensure the validity of the SF-based TEP model in the case of occurrence of islanding under contingency. The proposed model utilizes the SF matrix of the complete network in which it is assumed that all candidate lines are installed. This is an important feature of the proposed TEP model as there is no need to calculate and use the SF matrix for each possible grid topology after installation of new lines or contingencies. We account for the load and generation uncertainties by applying a data-driven technique to construct confidence sets of probability distributions for these uncertain parameters based on their historical data. The proposed SF-based TEP has several advantages over the conventional angle-based TEP model:

- Efficient modeling of the existing/candidate transmission lines with less number of decision variables.
- Efficient modeling of N-1 security constraints without adding many new decision variables to TEP (compared with the angle-based model in which many new variables need to be introduced for each contingency).
- The capability of monitoring a limited subset of lines instead of the entire network.
- Providing a formulation for modeling single candidate line within a corridor with multiple
parallel lines.

- Reducing the size of the problem and its solution time.

These features make the SF-based TEP more computationally efficient and tractable than the classical angle-based TEP, while both models provide the same level of accuracy. Thus, the proposed TEP is an alternative scalable model for planning of medium- and large-scale power systems.

In a nutshell, as a continuation of [1], this chapter contributes to the field by considering realistic constraints and objectives. As compared to [1], the considered realistic constraints and objectives are: the operational cost of generating units, cost of load shedding, planners’ budget constraint, N-1 security criterion for existing/candidate lines as well as generating units, islanding situation after an N-1 contingency, and system uncertainties.

2.4. Power Flow With SF and LODF Matrices

The shift factor matrix projects the power flow model from a system of equations with three sets of variables (i.e., \( P_g \), \( f_i \), and \( \theta \)) to a system of equations with two sets of variables (i.e., \( P_g \) and \( f_i \)) [2]. The variable \( \theta \) can be interpreted as a set of intermediate variables that makes a relationship between \( P_g \) and \( f_i \). SF values omit the need for having such intermediate variables for line flow and nodal power balance modeling. SF and line outage distribution factor (LODF) have also been applied for power systems studies under contingency when a set of predetermined contingencies is given. In the voltage angle-based model, three sets of new variables (i.e., \( P_g^c \), \( f_i^c \), and \( \theta^c \)) need to be introduced for each contingency. The existence of \( \theta^c \) increases the number of variables, especially in medium- and large-scale systems with many buses and possible contingencies. In the SF-based model, no \( \theta^c \) (\( \forall c \)) exists. This significantly reduces the number of decision variables of SF-based model compared to the angle-based model.
Line flow and nodal power balance calculations are among the most important modules of many energy management functions. Using the SF matrix enhances the computational efficiency of these modules on power system optimization problems, such as optimal power flow and security-constrained unit commitment. In addition, a set of important lines can be selected and monitored in the SF-based model, whereas all lines and angles need to be included in the angle-based model. Because of these advantages, most system operators, such as MISO and CAISO, and commercial software packages, such as PLEXOS and PROMOD, use SFs for power flow and post-contingency analysis [3, 4]. We briefly explain SF and LODF:

1) SF represents the sensitivity of the flow in a specific line to a change in power transfer from an injection node to the slack bus (consumption node), which defines a voltage angle reference for the whole system.

\[
SF_I^l = \frac{(Z_{s,t} - Z_{t,l})}{z_l}; \quad s(l) = s, r(l) = t
\]

where \(Z_{s,t}\) is element \((s, i)\) of the inductance matrix, and \(z_l\) is the inductance of line \(l\).

2) LODF represents changes in flow in a specific line with respect to the outage of another line.

\[
LODF_{lk} = \frac{SF_I^l - SF_J^l}{1 - (SF_I^k - SF_J^k)}; \quad s(k) = i, r(k) = j
\]

Remark 1: Although bus voltage angles are omitted in the SF-based model, they can easily be recovered from \(P_g, f_l\), and the system admittance matrix.

Remark 2: The SF values depend on the topology. That is, to construct the SF matrix, the grid topology must not change.

2.5. Mathematical Formulation for TEP

2.5.1. Conventional Angle-Based TEP Model

The SF values have not been adopted for the transmission expansion planning problem because
of grid topology alterations over the course of optimization. In the classical DC TEP, voltage angles of buses are used to calculate power flow in existing and candidate lines. The main challenge in DC TEP is to calculate power flow in candidate transmission lines. This is modeled through bilinear equations, where integer variables representing candidate lines are multiplied to bus voltage angles. Bilinear equations can be transformed to linear equations using a Big-M method as presented in [5]. This TEP model is computationally expensive. By increasing the size of the system, the TEP’s computational burden exponentially increases, in particular when N-1 security criterion is taken into account [6]. Decomposition techniques might be needed to solve security-constrained TEP models for medium and large systems [7].

2.5.2. Proposed SF-Based TEP Model

If we take advantage of SFs for DC power flow calculations, we can significantly reduce the TEP computational costs and enhance its scalability. Here, an SF-based TEP is proposed taking into account realistic long-term planning constraints, e.g., generation cost, load shedding cost, budget constraints, power system uncertainties, and N-1 security criterion (as the most important condition for reliable planning).

A. Concept of Flow Cancellation for TEP

We adopt the concept of flow cancellation. Consider a typical line of the network located in corridor $st$ that transfers $\bar{P}_{st}$ MW from bus $s$ to bus $t$ (see Fig. 2.1a). If the line goes out for any reason, it cannot transfer power from bus $s$ to $t$ (Fig. 2.1b). The absence of line $st$ will change the topology of the network and consequently the SF matrix. To keep the topology and SF matrix unchanged, we can model the line outage with a virtual generation located in the sending bus $s$ and a virtual load located in the receiving bus $t$ (see Fig. 2.1c).
This means that the power injection to bus $s$ and withdrawal from bus $t$ cancel out each other with no impact on other lines. Therefore, we can ignore the outage of the line and consider the system topology unchanged. We use the concept of flow cancellation, and reformulate power flow in candidate lines and lines that are disconnected because of contingencies. Equation (3), which is based on the voltage angles, is replaced by expressions (4) and (5), which are based on the shift factor values:

$$f_{l,b} = B_l \cdot (\theta_{i,b} - \theta_{j,b}); \quad \forall \ b, l, s(l) = i, r(l) = j$$

$$f_{l,b} = \sum_i S F_i^l \cdot P_{i,b} + \sum_{(s,t) \in \Omega^L} (S F_s^l - S F_t^l) \cdot \tilde{P}_{st,b};$$

$$\forall \ b, l \in \Omega^L, s(l) = i, r(l) = j$$

$$f_{l,b} = \tilde{p}_{ij,b} - \sum_i S F_i^l \cdot P_{i,b} - \sum_{(s,t) \in \Omega^L} (S F_s^l - S F_t^l) \cdot \tilde{P}_{st,b};$$

$$\forall \ b, l \in \Omega^{L^+}, s(l) = i, r(l) = j$$
Equation (4) models power flow in the existing lines, whereas expression (5) models power flow in the candidate lines. Variations of the power flow in a line with respect to variations of power in a specific bus can be obtained using the system’s SF matrix. Thus, the total variation of power flow in a line with respect to variations of power in all buses is derived by superposition. This is shown in the first right-hand side term of (4). Moreover, post-contingency analysis of the outage of a line is expressed by a virtual power injection/withdrawal from the corresponding sending/receiving terminal of that line. This concept is modeled in the second right-hand side term of (4).

For instance, consider that, in Fig. 2.1a the line st transfers 1 MW from bus s to bus t. In this situation, since the line is connected, and the amount of virtual generation (load) on bus s (t) is zero, the second right-hand side term of (4) is zero. Therefore, using the concept of SF, power flow in a typical existing line l can be calculated only by the first right-hand side term of (4). If line st is disconnected from the network (Fig. 2.1b), instead of deleting it from the network graph, we assume injecting a virtual generation equal to 1 MW on bus s and withdrawing a virtual load equal to 1 MW on bus t (Fig. 2.1c). In this situation, the network topology and the SF matrix are the same as before. However, the nodal power of buses s and t has been changed, and as a result, the power flow of other lines is changed. Using the concept of SF, the impact of virtual generation (load) on bus s (t) to the power flow of other lines can be modeled by the second right-hand side term of (4).

Note that to formulate (4) and (5), we only need the SF values of the complete network configuration, i.e., the graph in which it is assumed that all candidate lines are installed to the original grid topology. That is, we do not need to determine the SF values corresponding to all possible grid topologies after installation of a candidate line(s). An explanation of constructing (4)
and (5) from (3) using the SF concept is presented in our previous work [1].

B. Proposed TEP Formulation

The proposed SF-based TEP is formulated in (6)-(25). The objective function (6) consists of the costs of installing new transmission lines and expected operational costs $Q$, which depends on the uncertainties ($\xi$). The operational costs include generation cost and load shedding cost under normal and contingency conditions. The load shedding cost is a penalty term to minimize the post-contingency load shedding possibility and is not a payable cost in normal operation. Parameter $C^c_d$ is specified based on the probability of each contingency. The set of decision variables is shown in (7), where $x_i$ is binary, $\{f_{l,b}, P_{g,b}, P_{d,b}^{LS}, f_{l,b}^c, P_{g,b}^c, P_{d,b}^{LS,c}\}$ are bounded, and $\{\bar{P}_{st,b}, \bar{P}_{st,b}^c\}$ are free variables.

$$
\min \Gamma \sum_{l \in \Omega_L^+} \bar{l}_i \cdot x_i + E\left[Q\left(P_{g,b}, P_{d,b}^{LS}, P_{d,b}^{LS,c}, \xi^n\right)\right];
$$

$$
Q = \sum_{b \in B} \sum_{g \in \Omega_B^G} C_g \cdot P_{g,b} + \sum_{b \in B} \sum_{d \in \Omega_D} C_d \cdot P_{d,b}^{LS} + \sum_{b \in B} \sum_{c \in \Omega_C^c} \sum_{d \in \Omega_D} C^c_d \cdot P_{d,b}^{LS,c}
$$

$$
\Gamma = \{x_i, f_{l,b}, P_{g,b}, P_{d,b}^{LS}, \bar{P}_{st,b}, f_{l,b}^c, P_{g,b}^c, P_{d,b}^{LS,c}, \bar{P}_{st,b}^c\}
$$

The proposed SF-based planning constraints under normal condition are formulated in (8)-(16). Constraint (8) accounts for the investment budget of an entity responsible for installing new transmission lines. The total power balance equation and net power formula are respectively represented in (10) and (11). In the SF-based TEP, the power balance constraint (10) is modeled by one equality constraint between total loads and generations of the entire system; however, in the angle-based TEP model, an equality constraint is needed to balance power demand and generation of each node of the system. That is, the classical angle-based model needs $n$ equality power balance constraints, where $n$ is the number of nodes. The SF-based power flow constraints
for the existing lines are modeled by (12). In addition, SF-based power flow constraints for the candidate lines are formulated by (13) and (14). Constraints (15) and (16) impose limits of generating units and load shedding.

\[
\sum_{l \in \Omega^L} \bar{I}_l. x_l \leq I^{max}
\]  (8)

\[x_l = \{0, 1\}; \quad \forall l \in \Omega^L^+
\]  (9)

\[
\sum_{l} P_{l,b} = 0 \quad \forall b
\]  (10)

\[
P_{l,b} = \sum_{g \in \Omega_b^G} P_{g,b} + \sum_{w \in \Omega_b^W} P_{w,b} - \sum_{d \in \Omega_b^D} (P_{d,b} - P_{d,b}^{LS}); \quad \forall i, b
\]  (11)

\[-F_{l}^{max} \leq \sum_{l} SF_{l,b} P_{l,b} + \sum_{(s,t) \in \Omega^L^+} (SF_{s,t} - SF_{l,b}) \bar{P}_{st,b} \leq F_{l}^{max};
\]  (12)

\[\forall b, l \in \Omega^L, s(l) = i, r(l) = j
\]

\[-x_l. F_{l}^{max} \leq \bar{P}_{ij,b} - \sum_{l} SF_{l,b} P_{l,b} - \sum_{(s,t) \in \Omega^L^+} (SF_{s,t} - SF_{l,b}) \bar{P}_{st,b}
\]

\[\leq x_l. F_{l}^{max} ; \quad \forall b, l \in \Omega^L^+, s(l) = i, r(l) = j
\]

\[-(1 - x_l). M \leq \bar{P}_{ij,b} \leq (1 - x_l). M ;
\]  (14)

\[\forall b, l \in \Omega^L^+, s(l) = i, r(l) = j
\]

\[0 \leq P_{g,b} \leq P_{g}^{max} ; \quad \forall g, b
\]  (15)

\[0 \leq P_{d,b}^{LS} \leq P_{d,b} ; \quad \forall g, b
\]  (16)

The planning constraints for the proposed model under a contingency situation (i.e., N-1 security constraints) are modeled in (17)-(25). In this model, outages of the existing lines, candidate lines, and generating units are considered as possible contingencies. We derive (17) to determine the
system’s new SFs matrix under contingency based on the concept of LODF.

\[
SF_{l,k}^I = SF_{l}^I + LODF_{l,k}SF_{l}^I; \quad \forall i, k \in C
\]  

(17)

Note, we need SF of the complete network graph without the need to find the SF matrix of the new grid topology after installation of a candidate line or the outage of an existing or candidate line. \(SF_{l,k}^I\) denotes shift factor of line \(l\) with respect to the bus \(i\) when line \(k\) is out. Power balance constraints for each contingency are represented by (18) and (19).

\[
\sum_{\forall i} P_{i,b}^c = 0; \quad \forall C, b, \forall \rho
\]  

(18)

\[
P_{i,b}^c = \sum_{g \in \Omega_g^c} P_{g,b}^c + \sum_{w \in \Omega_w^c} P_{w,b} - \sum_{d \in \Omega_d^c} (P_{d,b} - P_{d,b}^{LS}) \quad \forall i, b, C, \forall \rho
\]  

(19)

Note that when a line outage occurs, a part of the grid might become islanded. In the conventional disjunctive TEP model, since power balance constraints are formulated for each node, these constraints will be automatically satisfied for each possible island. However, in the SF-based TEP, only one power balance constraint, i.e., (18), exists for the whole system. Thus, this power balance constraint must be enforced for each possible island \(\rho\) after a contingency to ensure the system feasibility in each island and the whole system. Note that one equality constraint is added to the SF-based TEP model for each island. This additional constraint has a negligible impact on the size of the TEP optimization problem.

Outages that cause system islanding are distinguished either by input system data or by SF of the original system. The outage of a line with the SF value of \(\pm 1\) leads to system islanding [2]. Note that we use the shift factor of the complete network, and it is not required to calculate the SF matrix for each possible island \(\rho\) after occurrence of contingencies. A sub-matrix of the SF matrix corresponding to an inland \(\rho\) is selected for modeling the constraints in that island without
the need to set a reference bus for each island (see (17)).

Constraints (20)-(24) need to be fulfilled for each contingency. Here, $A_l^C$ is a parameter that is equal to 0 if line $l$ is unavailable under contingency $C$ and 1 otherwise. The same meaning is used for unavailability of generating units under contingencies by $A_g^C$. In addition, adjustment capabilities of generating units to provide either preventive or corrective action in response to a contingency are modeled by (25).

$$-A_l^C. F_l^{max} \leq \sum_{vl} SF_{l,v}^c.P_{i,b}^c + \sum_{(s,t) \in \Omega^L} (SF_{s,t}^c - SF_{t,v}^c)\tilde{P}_{st,b}^c$$

$$\leq A_l^C. F_l^{max}; \forall C,b,\rho, l \in \Omega^L, s(l) = i, r(1) = j$$

$$-A_l^C. x_i. F_l^{max} \leq \tilde{P}_{i,j,b}^c - \sum_{vl} SF_{l,v}^c.P_{i,b}^c - \sum_{(s,t) \in \Omega^L} (SF_{s,t}^c - SF_{t,v}^c)\tilde{P}_{st,b}^c$$

$$\leq A_l^C. x_i. F_l^{max}; \forall C,b,\rho, l \in \Omega^L, s(l) = i, r(1) = j$$

$$-(1 - x_i). M \leq \tilde{P}_{i,j,b}^c \leq (1 - x_i). M;$$

$$\forall C,b,\rho, l \in \Omega^L, s(l) = i, r(1) = j$$

$$0 \leq P_{g,b}^c \leq P_{g}^{max}; \forall g, b, C, \rho$$

$$0 \leq P_{d,b}^{L,S,c} \leq P_{d,b}; \forall d, b, C, \rho$$

$$A_g^C. (P_{g,b} - \Delta_{g}^{max}) \leq P_{g,b}^c \leq A_g^C. (P_{g,b} + \Delta_{g}^{max}); \forall g, b, C, \rho$$

C. Data-Driven Uncertainty Modeling

The available production capacity of generating units ($P_{g}^{max}$), forecasted power demand ($P_{d,b}$), and forecasted generation of wind farms ($P_{w,b}$) are uncertain parameters in the TEP model (6)-(25). We utilize a data-driven two-stage TEP approach presented in [8] to take into account the
impact of these uncertain parameters in the long-term planning. We impose no assumption on probability distributions of the uncertain parameters and apply a distribution-based approach to construct a confidence set for each uncertain parameter. Consider that the true distribution $X$ is unknown. The confidence set $D$ is defined in a way to minimize the tolerance level of the distance between the reference distribution $\hat{X}$ and the true distribution $X$ [8].

$$D = \{ \forall X: d(X, \hat{X}) \leq \alpha \} \quad (26)$$

where parameter $\alpha$ represents a tolerance level of the distance, and $d(X, \hat{X})$ is the predefined probability distance between $X$ and $\hat{X}$. A histogram of historical data is used as the reference distribution. The data domain is divided into $N$ bins. The probability distribution of each bin is $\hat{\chi}^n = S_n/S$, where $S$ is the total data size, and $S_n$ is the size of data in bin $n = 1, 2, \ldots, N$. The reference distribution is $\hat{X} = (\hat{\chi}_1, \hat{\chi}_2, \ldots, \hat{\chi}_N)$.

Generally, having more information on the true distribution leads to a more accurate estimation of reference distribution that is closer to the true PDF. Two probability metrics are selected [9], norm one as the probability distance and (27) as a tolerance level. The probability distance depends on the number of bins $N$, the total size of data $S$, and the confidence level $\beta$ [10]. Finally, the confidence set is constructed as (28).

$$\alpha_1 = \left(\frac{N}{2S}\right) \cdot \log \left(\frac{2N}{1 - \beta}\right) \quad (27)$$

$$D = \left\{ X \in \mathbb{R}^N : \sum_{n=1}^N |\chi^n - \hat{\chi}^n| \leq \alpha_1 \right\} \quad (28)$$

After constructing the confidence set $D$ for each uncertain parameter (i.e., $P_{g,b}^{max}$, $P_{d,b}$, and $P_{w,b}$), the objective function (6) is minimized subject to (7)-(25), as a two-stage problem and under the worst-case distribution in $D$ [8].
D. Solution Algorithm

Considering the confidence set \( D \), the proposed TEP model is decomposed into a master problem and a subproblem associated with uncertainties. The deterministic master problem (29) includes costs of transmission lines investment and a decision variable \( \vartheta \) to model the worst-case operational costs. Ignoring uncertainties in generation and loads, constraints (7)-(25) are considered along with Benders’ optimality cuts in the master problem.

\[
\min \sum_{l \in \Omega^L} \bar{I}_l \cdot x_l + \vartheta ; \quad \text{s.t.} \quad (7) - (25) \text{ and Optimality cuts} \quad (29)
\]

The subproblem \( \omega(x_l) \) is the dual of constraints (10)-(25). In addition, constraints (30)-(32) are considered in the subproblem, where \( x_{i,b}^n \) and \( \bar{x}_{i,b}^n \) represent the true and the reference distributions, respectively. Slack variable \( K_{i,b}^n \) represents \( |x_{i,b}^n - \bar{x}_{i,b}^n| \).

\[
\begin{align*}
\dot{x}_{i,b}^n - K_{i,b}^n & \leq \bar{x}_{i,b}^n ; \quad \forall i, b, n \quad (30) \\
\dot{x}_{i,b}^n + K_{i,b}^n & \geq \bar{x}_{i,b}^n ; \quad \forall i, b, n \quad (31) \\
\sum_{n=1}^{N} K_{i,b}^n & \leq \alpha_1 ; \quad \forall i, b, n \quad (32)
\end{align*}
\]

Benders’ decomposition technique is applied to solve the two-stage TEP model. Since the load shedding option is considered in the proposed TEP model, the problem is always feasible. That is, no Benders’ feasibility cut is required. However, a Benders’ optimality cut is needed to penalize the master problem in each iteration if the worst-case operation cost obtained by the master problem \( \vartheta \) is less the cost obtained by the subproblem \( \omega(x_l) \). Figure 2.2 depicts the flowchart of the solution algorithm, in which no uncertainty is considered in the master problem and the binary decision variable \( x_l \) is fixed in the subproblem. More details of the applied data-driven two-stage solution algorithm is presented in [8].
2.5.3. Comparison of Angle-Based and SF-Based TEP Models

The conventional angle-based and the proposed SF-based TEP models are mixed-integer linear programming problems. The main difference between these models is in the formulation of power flow equations of existing/candidate transmission lines. In the SF-based TEP, instead of voltage angles, the flow cancellation concept and SFs are utilized. In the angle-based TEP, a set of new voltage angle variables is required to model constraints corresponding to each contingency, whereas the proposed SF-based TEP uses LODF values to formulate $N$-1 security constraints. It will be illustrated in the case study section that this modification significantly reduces the computational burden of TEP, especially for large systems with $N$-1 security criterion.

One of the main advantages of the SF-based TEP is its capability of taking into account constraints of only a few selected monitored lines (i.e., important and sensitive lines, and lines that
historically operate near their capacity). The angle-based TEP does not have this capability since voltage angles of all buses are required to be calculated. Another feature that significantly benefits the proposed TEP is its capability of pruning the SF matrix. A cut-off point can be defined to set small sensitivity factors to zero. The two mentioned features significantly enhance the scalability of the proposed TEP model compared to the angle-based model. Thus, the proposed TEP is an alternative for the conventional angle-based model, especially in situations that the computational burden leads to an intractable angle-based TEP problem.

Remark 3: The proposed model does not need to compute a new SF matrix for each possible grid topology after installation of a line or a combination of lines, or after contingency on a line.

2.6. Numerical Results

Three popular test systems for TEP studies, including the Garver System, the IEEE RTS 24-bus system, and the IEEE 118-bus system, are used. All simulations are carried out using GAMS and ILOG CPLEX’s MIP solver [11]. A personal computer with an Intel(R) Xeon(R) CPU @2.6 GHz, including eight cores and 16 GB of RAM, is used.

2.6.1. Garver System

The Garver system is used as an illustrative example [12]. The topology of the system is depicted in Fig. 2.3. Existing lines are shown with solid lines, while dashed lines show candidate lines. The system includes six buses, three generating units, six existing lines, six candidate lines, and five load points. For the sake of explanation, one load block is considered. The total expected load of the system is 760 MW, and the total expected generation capacity is 1110 MW. Bus data and branch data are given in Tables 2.1 and 2.2. The maximum adjustment capability of units during a contingency is assumed to be 50 MW (the same number is considered for all cases).
Table 2.1. Bus data for Garver system

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Maximum generation capacity (MW)</th>
<th>Generation cost ($/MWh)</th>
<th>Load (MW)</th>
<th>Load shedding penalty cost ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150</td>
<td>20</td>
<td>80</td>
<td>9000</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>240</td>
<td>9000</td>
</tr>
<tr>
<td>3</td>
<td>360</td>
<td>30</td>
<td>40</td>
<td>9000</td>
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<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>160</td>
<td>9000</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>240</td>
<td>9000</td>
</tr>
<tr>
<td>6</td>
<td>600</td>
<td>40</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.2. Branch data for Garver system

<table>
<thead>
<tr>
<th>From bus</th>
<th>To bus</th>
<th>Maximum capacity per line (MW)</th>
<th>Inductance per line (p.u.)</th>
<th>No. of candidate lines</th>
<th>Annual investment cost per line (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>100</td>
<td>0.40</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>80</td>
<td>0.60</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>100</td>
<td>0.20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>100</td>
<td>0.20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>100</td>
<td>0.40</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>100</td>
<td>0.20</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>100</td>
<td>0.48</td>
<td>2</td>
<td>48</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>100</td>
<td>0.30</td>
<td>3</td>
<td>30</td>
</tr>
</tbody>
</table>

Fig. 2.3. One-line diagram of Garver system.
While the angle-based TEP model without/with considering N-1 security criteria takes 0.22/0.41 seconds, the proposed SF-based TEP without/with considering N-1 security criteria converges after 0.13/0.19 seconds (more details of the mathematical optimization model of SF-based TEP are provided in section 2.8. Appendix). Table 2.3 shows the planning results. The power generation and line flows in the normal operating condition are presented in Tables 2.4 and 2.5. The results shown in these Tables are the same for both TEP models. This means that the proposed TEP model provides the optimal planning results when being 54% faster than the conventional TEP model.

Note that according to the results for the case of considering N-1 security criterion, the expected generation and the number of candidate lines are not sufficient. Therefore, a large amount of nodal loads (200.73 MW of load on bus 2, 93.412 MW of load on bus 4, and 161.46 MW of load on bus 5) cannot be supplied in case of specific outages. This is understandable as the Graver system is small with few components, and when an outage occurs, the rest of the system may not be able to ensure system reliability with no load shedding. For instance, units 1 and 2 (considering their ramping capability) might not be able to support the load after the outage of unit 3, which is a large unit.

Table 2.3. Simulation results obtained by both angle-based and SF-Based models for the Garver system

<table>
<thead>
<tr>
<th></th>
<th>Obj. function (M$)</th>
<th>Annual cost of candidate lines (M$)</th>
<th>Candidate line No. to be installed</th>
<th>Annual cost of units (M$)</th>
<th>Annual cost of load shedding (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Contingency</td>
<td>322.675</td>
<td>110</td>
<td>3-5, (4-6)x3</td>
<td>212.675</td>
<td>0</td>
</tr>
<tr>
<td>N-1 Security criterion</td>
<td>36201.5</td>
<td>206</td>
<td>3-5, (3-6)x2, (4-6)x3</td>
<td>75.785</td>
<td>35919.7</td>
</tr>
</tbody>
</table>

Table 2.4. Power generation (MW) values for the Garver system

<table>
<thead>
<tr>
<th></th>
<th>Unit 1 (Bus 1)</th>
<th>Unit 2 (Bus 3)</th>
<th>Unit 3 (bus 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Contingency</td>
<td>150</td>
<td>312.2</td>
<td>297.8</td>
</tr>
<tr>
<td>N-1 Security criterion</td>
<td>108.8</td>
<td>134.86</td>
<td>60.73</td>
</tr>
</tbody>
</table>
Table 2.5. Power flow (MW) values for the Garver system

<table>
<thead>
<tr>
<th>Corridor No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Contingency</td>
<td>40.9</td>
<td>-38.79</td>
<td>67.82</td>
<td>-100</td>
<td>-99.08</td>
<td>86.04x2</td>
<td>0</td>
<td>-99.26x3</td>
</tr>
<tr>
<td>N-1 Security criterion</td>
<td>7.85</td>
<td>5.24</td>
<td>15.71</td>
<td>-31.42</td>
<td>0</td>
<td>31.42x2</td>
<td>0.62x2</td>
<td>-61.35x3</td>
</tr>
</tbody>
</table>

2.6.2. IEEE 24-Bus System

Ruiz and Conejo modified the IEEE RTS 24-bus system for TEP studies. Characteristics of generating units, loads, and existing transmission lines are provided in [13]. The system consists of 18 generating units, 17 demand loads, 34 existing transmission lines, and seven candidate lines. The peak demand is 840 MW. Four seasonal load blocks equal to \{0.9, 1, 0.85, 0.95\} of the peak demand are considered. The candidate lines’ data is given in Table 2.6.

Table 2.6. Candidate lines’ parameters for the IEEE 24-Bus system

<table>
<thead>
<tr>
<th>Line No.</th>
<th>From bus</th>
<th>To bus</th>
<th>Maximum capacity (MW)</th>
<th>Susceptance (per Ohm)</th>
<th>Annual investment cost (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>14</td>
<td>100</td>
<td>500</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>15</td>
<td>100</td>
<td>500</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>20</td>
<td>100</td>
<td>500</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>18</td>
<td>100</td>
<td>500</td>
<td>1.2</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>22</td>
<td>100</td>
<td>500</td>
<td>1.3</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>23</td>
<td>100</td>
<td>500</td>
<td>1.1</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>19</td>
<td>100</td>
<td>500</td>
<td>0.9</td>
</tr>
</tbody>
</table>

To simulate the impact of wind uncertainty, half of the conventional generation capacity of units connected to bus 7 and 22 is replaced by wind farms. Parameters of each wind farm are given in [14]. The possibility of contingency in any existing and candidate transmission lines as well as generating units is considered in the N-1 security criterion. Bus 7 of the IEEE RTS 24-bus system is an ending bus that is connected to the rest of the system only by line 7-8. An outage on this line leads to islanding of bus 7. Therefore, two power balance equations, i.e., (18), are needed, one for bus 7 (i.e., the islanded zone) and another one for the rest of the interconnected system. That is, if a contingency occurs on line 7-8, the generating unit at bus 7 needs to produce exactly equal to
demand at this bus (note that load shedding is also considered). The planning entity’s annualized investment budget for building new transmission lines is $5M.

Table 2.7 shows the number of decision variables and constraints for the classical angle-based and the proposed SF-based TEP models.

<table>
<thead>
<tr>
<th></th>
<th>No. of continuous decision variables</th>
<th>No. of discrete decision variables</th>
<th>No. of constraints (single equations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>angles-based model</td>
<td>24,008</td>
<td>7</td>
<td>29,382</td>
</tr>
<tr>
<td>SF-based model</td>
<td>19,928</td>
<td>7</td>
<td>23,862</td>
</tr>
<tr>
<td>Difference between two models</td>
<td>4,080</td>
<td>0</td>
<td>5,520</td>
</tr>
</tbody>
</table>

While both models have the same number of discrete decision variables, which correspond to the candidate lines’ status, the number of continuous decision variables of the models is different. The SF-based model has 4,080 continuous decision variables less than the classical angle-based TEP. In addition, the SF-based TEP has 23,862 constraints that are 5,520 constraints less than that for the angle-based TEP. It should be noted that fewer numbers of decision variables and constraints lead to a smaller TEP model; however, it does not necessarily lead to a more tractable or faster-to-solve model. Thus, observing the solution process and solving time is also necessary for investigating the computational burden of the model. The solution progresses for both angle-based and SF-based TEP models with $N$-1 security criterion are depicted in Fig. 2.4. The angle-based model converges after 22,302 iterations (this is the number of iterations that CPLEX needs to solve the optimization model [11]) within 71.69 seconds, whereas the SF-based TEP model converges after 10,109 iterations within 29.1 seconds. The reported times are total simulation times, including SF computation time and other pre/post-processing times. The SF-based TEP needs almost 60% less time and 55% fewer number of iterations to converge to the zero optimality gap. This demonstrates that less computational burden is imposed on the solver by the SF-based
model in comparison to the angle-based model while both models provide the same level of accuracy.

Fig. 2.4. Solution progress of the angle-based (disjunctive) and proposed TEP models for the IEEE 24-bus test system.

The proposed SF-based TEP converges to the solution of the classical angle-based TEP model. Table 2.8 shows the planning results for four scenarios: ignoring contingencies and ignoring uncertainties, ignoring contingencies and considering uncertainties, considering N-1 security criterion and ignoring uncertainties, and considering both N-1 security criterion and uncertainties. Due to low uncertainty penetration (only half of the generation capacity of units connected to bus 7 and 22 is replaced by wind farms), the planning decisions are the same before and after considering uncertainty of wind generations. However, the presence of wind power uncertainty results in additional operation cost. Considering uncertainties, the total cost without and with N-1 security criterion are $120.15M and $133.45M, respectively. This $13.30M cost increment is indeed the cost that the operator pays to ensure the system reliability after the occurrence of a contingency on a line or a generating unit.

While two candidate lines (i.e., lines 1 and 2) are selected for the case with no N − 1 constraints, one more candidate lines, (i.e., line 4) must be installed to satisfy the system security after the
occurrence of a contingency. The annual investment, generation, and load shedding costs are also increased by taking into account N-1 security criterion.

Table 2.8. Simulation results obtained by both angle-based and SF-based models for the IEEE 24-Bus system

<table>
<thead>
<tr>
<th>Condition</th>
<th>Obj. function (M$)</th>
<th>Annual cost of candidate lines (M$)</th>
<th>Candidate line No. to be installed</th>
<th>Annual cost of units (M$)</th>
<th>Annual cost of load shedding (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No contingency/ No uncertainty</td>
<td>117.08</td>
<td>1.5</td>
<td>1, 2</td>
<td>109.14</td>
<td>6.44</td>
</tr>
<tr>
<td>No contingency/ With uncertainty</td>
<td>120.15</td>
<td>1.5</td>
<td>1, 2</td>
<td>112.20</td>
<td>6.45</td>
</tr>
<tr>
<td>N-1 Security criterion/ No uncertainty</td>
<td>131.21</td>
<td>2.7</td>
<td>1, 2, 4</td>
<td>110.23</td>
<td>18.28</td>
</tr>
<tr>
<td>N-1 Security criterion/ With uncertainty</td>
<td>133.45</td>
<td>2.7</td>
<td>1, 2, 4</td>
<td>112.47</td>
<td>18.28</td>
</tr>
</tbody>
</table>

2.6.3. IEEE 118-Bus System

The system information is provided in [15]. The system is modified by adding three wind farms on buses 36, 69, and 77. The output power of wind farms is given in [16]. The annual investment budget of the planning entity for building candidate transmission lines is assumed $10M. The total capacity of the generating units is 7470 MW, and the peak demand is 5567 MW. Twelve monthly load blocks equal to \{0.9, 0.92, 0.93, 0.95, 0.97, 1, 0.99, 0.94, 0.91, 0.87, 0.85, 0.88\} of the peak demand are considered. Note that an outage of each of the existing lines 7, 9, 113, 133, 176, 177, 183, 184 causes islanding. It means that eight possible islanding situations exist for the 118-bus system. Two power balance equations are needed if either of these lines goes out, one equation for the islanded zone and another one for the rest of the interconnected system. To illustrate the effect of number of candidate lines on the effectiveness of the proposed SF-based TEP model, two scenarios are studied. Ten and 30 candidate lines are respectively considered in scenarios 1 and 2. The ten candidate lines in scenario 1 are obtained from a PSERC report [17]. For scenario 2, we have considered a set of candidate lines that geographically make sense and decrease the operational costs if installed. Candidate lines’ parameters for the scenarios are presented in Tables
2.9 and 2.10. In scenario 2, to force TEP to install more candidate lines, the parameters of the system are modified. The total load is increased by 3%, and the capacity of existing lines is decreased by 50%.

Table 2.9. Candidate lines’ parameters (scenario 1)

<table>
<thead>
<tr>
<th>Line No.</th>
<th>From bus</th>
<th>To bus</th>
<th>Maximum capacity (MW)</th>
<th>Susceptance (per Ohm)</th>
<th>Annual investment cost (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>4</td>
<td>390</td>
<td>30</td>
<td>0.406</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>18</td>
<td>390</td>
<td>30</td>
<td>0.324</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>115</td>
<td>390</td>
<td>30</td>
<td>0.406</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
<td>6</td>
<td>390</td>
<td>30</td>
<td>0.507</td>
</tr>
<tr>
<td>5</td>
<td>36</td>
<td>34</td>
<td>390</td>
<td>30</td>
<td>0.284</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td>77</td>
<td>390</td>
<td>30</td>
<td>0.446</td>
</tr>
<tr>
<td>7</td>
<td>70</td>
<td>25</td>
<td>390</td>
<td>30</td>
<td>0.974</td>
</tr>
<tr>
<td>8</td>
<td>86</td>
<td>82</td>
<td>390</td>
<td>30</td>
<td>0.365</td>
</tr>
<tr>
<td>9</td>
<td>87</td>
<td>106</td>
<td>390</td>
<td>30</td>
<td>0.629</td>
</tr>
<tr>
<td>10</td>
<td>87</td>
<td>108</td>
<td>390</td>
<td>30</td>
<td>0.527</td>
</tr>
</tbody>
</table>

Table 2.10. Candidate lines’ parameters (scenario 2)

<table>
<thead>
<tr>
<th>Line No.</th>
<th>From bus</th>
<th>To bus</th>
<th>Annual investment cost (M$)</th>
<th>Line No.</th>
<th>From bus</th>
<th>To bus</th>
<th>Annual investment cost (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>12</td>
<td>0.406</td>
<td>16</td>
<td>61</td>
<td>58</td>
<td>0.446</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>117</td>
<td>0.324</td>
<td>17</td>
<td>65</td>
<td>81</td>
<td>0.324</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>16</td>
<td>0.406</td>
<td>18</td>
<td>65</td>
<td>68</td>
<td>0.365</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>16</td>
<td>0.507</td>
<td>19</td>
<td>69</td>
<td>66</td>
<td>0.324</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>29</td>
<td>0.284</td>
<td>20</td>
<td>116</td>
<td>62</td>
<td>0.527</td>
</tr>
<tr>
<td>6</td>
<td>22</td>
<td>26</td>
<td>0.446</td>
<td>21</td>
<td>90</td>
<td>84</td>
<td>0.104</td>
</tr>
<tr>
<td>7</td>
<td>24</td>
<td>71</td>
<td>0.324</td>
<td>22</td>
<td>91</td>
<td>93</td>
<td>0.104</td>
</tr>
<tr>
<td>8</td>
<td>113</td>
<td>115</td>
<td>0.365</td>
<td>23</td>
<td>90</td>
<td>93</td>
<td>0.324</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td>24</td>
<td>0.629</td>
<td>24</td>
<td>103</td>
<td>110</td>
<td>0.104</td>
</tr>
<tr>
<td>10</td>
<td>24</td>
<td>25</td>
<td>0.527</td>
<td>25</td>
<td>111</td>
<td>93</td>
<td>0.284</td>
</tr>
<tr>
<td>11</td>
<td>52</td>
<td>54</td>
<td>0.406</td>
<td>26</td>
<td>111</td>
<td>94</td>
<td>0.106</td>
</tr>
<tr>
<td>12</td>
<td>59</td>
<td>62</td>
<td>0.324</td>
<td>27</td>
<td>107</td>
<td>93</td>
<td>0.324</td>
</tr>
<tr>
<td>13</td>
<td>99</td>
<td>63</td>
<td>0.406</td>
<td>28</td>
<td>107</td>
<td>94</td>
<td>0.105</td>
</tr>
<tr>
<td>14</td>
<td>66</td>
<td>49</td>
<td>0.507</td>
<td>29</td>
<td>87</td>
<td>84</td>
<td>0.324</td>
</tr>
<tr>
<td>15</td>
<td>61</td>
<td>51</td>
<td>0.284</td>
<td>30</td>
<td>103</td>
<td>84</td>
<td>0.104</td>
</tr>
</tbody>
</table>

The proposed TEP and the classical angel-based TEP model provide the same results as given in Tables 1.11 and 2.12. For scenario 2, while nine new lines are installed for the case without considering contingencies, ten new lines must be installed to satisfy N-1 security. Modeling N-1
security increases the planning costs and the annual costs of the generating units, which leads to a total cost increment of $104.45 (the same trend observed for scenario 1). This cost can be interpreted as annual reliability costs.

Table 2.11. Simulation results obtained by both angle-based and SF-based models for the IEEE 118-Bus system (scenario 1)

<table>
<thead>
<tr>
<th></th>
<th>Obj. function (M$)</th>
<th>Annual cost for candidate lines (M$)</th>
<th>Candidate line No. to be installed</th>
<th>Annual cost of units (M$)</th>
<th>Annual cost of load shedding (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Contingency</td>
<td>488.38</td>
<td>0</td>
<td>-</td>
<td>488.38</td>
<td>0</td>
</tr>
<tr>
<td>N-1 Security criteria</td>
<td>498.02</td>
<td>1.299</td>
<td>1, 8, 10</td>
<td>496.72</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.12. Simulation results obtained by both angle-based and SF-based models for the IEEE 118-Bus system (scenario 2)

<table>
<thead>
<tr>
<th></th>
<th>Obj. function (M$)</th>
<th>Annual cost of candidate lines (M$)</th>
<th>Candidate line No. to be installed</th>
<th>Annual cost of generating units (M$)</th>
<th>Annual cost of load shedding (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Contingency</td>
<td>585.93</td>
<td>2.865</td>
<td>1, 3, 5, 7, 13, 15, 17, 29, 30</td>
<td>583.07</td>
<td>0</td>
</tr>
<tr>
<td>N-1 Security criterion</td>
<td>690.38</td>
<td>3.49</td>
<td>1, 3, 5, 6, 13, 14, 15, 17, 29, 30</td>
<td>686.89</td>
<td>0</td>
</tr>
</tbody>
</table>

The number of variables and constraints for scenario 2 is presented in Table 2.13. The number of binary decision variables for both TEP models is the same. The proposed SF-based TEP contains 967,692 continuous variables and 1,946,671 constraints that are, respectively, 221,760 and 279,720 less than that for the angle-based model. Thus, the size of the SF-based TEP is smaller. Note that we have considered a complete set of N-1 contingency. The number of possible N-1 contingencies is equal to the number of existing lines (186) plus the number of generating units (54) plus the number of candidate lines. Therefore, the number of possible contingencies is at least 250 for scenario 1 and 270 for scenario 2. One may consider a selected set of high-risk of high-probability contingencies. Since the proposed SF-based TEP omits the voltage angle variables, its size is less than that for the classical angle-based TEP if one considers either a complete set of
N-1 contingency or a subset of contingencies. Although the size of SF-based TEP is smaller, decomposition strategies might be needed to solve both SF-based and angle-based models if the size of the system and the number of considered contingencies are very large. Even for such cases, the proposed SF-based TEP is more efficient as it contains fewer variables and constraints.

Table 2.13. Statistics of the TEP model of the IEEE 118-Bus system (scenario 2)

<table>
<thead>
<tr>
<th></th>
<th>No. of decision variables</th>
<th>No. of discrete decision variables</th>
<th>No. of constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle-based model</td>
<td>1,189,452</td>
<td>30</td>
<td>2,226,391</td>
</tr>
<tr>
<td>SF-based model</td>
<td>967,692</td>
<td>30</td>
<td>1,946,671</td>
</tr>
<tr>
<td>Difference between to models</td>
<td>221,760</td>
<td>0</td>
<td>279,720</td>
</tr>
</tbody>
</table>

Fig. 2.5. Solution progress of the angle-based (disjunctive) and proposed TEP models for the IEEE 118-bus test system, a) scenario 1, and b) scenario 2.
Figure 2.5 shows the solution progresses of both scenarios considering N-1 security criterion. As shown in Fig. 2.5b, the SF-based model decreases the solution time by more than 58%. This indicates a significant reduction in the computational burden. Comparing Figs. 2.5a and 2.5b shows that the number of candidate lines (i.e., the number of binary variables) does not affect the timesaving that we obtain from the proposed SF-based TEP model. The time differences between the angle-based and SF-based TEP models in both scenario 1 (ten candidate lines=ten binary variables) and scenario 2 (30 candidate lines=30 binary variables) are considerably large (more than 56%).

2.7. Conclusion

In this chapter, a computationally efficient SF-based transmission expansion planning model was proposed. Realistic planning objectives and constraints, such as operational costs, budget constraints, system uncertainties, and N-1 security criterion were taken into account. Using the SF values of the complete network (i.e., the network assuming all candidate lines are installed) and the flow cancellation concept, the SF-based planning model was developed. The proposed model is capable of handling possible islanding condition after the occurrence of a contingency.

The proposed SF-based TEP contains less number of constraints and variables than the angle-based TEP. This makes the proposed TEP computationally less expensive and more tractable, especially for large systems. Simulation results show the proposed TEP provides the same results as the angle-based model while being much faster and computationally more efficient. For the Garver system, IEEE 24-bus system, and 118-bus system, the SF-based TEP is, respectively, 54%, 60%, and 58% faster than the angle-based model. The studied cases showed that increasing the size of the system and the number of candidate lines (binary variables) had no negative effect on the advantageous performance of the proposed TEP as compared to the angle-based TEP.
2.8. Appendix

In this Appendix, we show how to calculate the SF values for the Garver system and use them to formulate the SF-based TEP model. To calculate the required SF matrix, we assume that all six candidate lines are connected (i.e., complete network graph). First, the equivalent inductance of parallel lines is calculated. For instance, according to Table 2.2, the equivalent inductance for corridor 4-6 is 0.1 p.u. Then, the inductance matrix is formed, and the SF matrix is constructed according to (1). Note that based on the definition of SF, the calculated SF values are for the equivalent of parallel lines in each corridor. To calculate the SF value for each line in a corridor, the corridor’s SF is decomposed to lines’ SFs proportionally to one over inductance of the lines. For instance, the SF value for corridor 4-6 with respect to bus four is 0.3429, and the SF value for each candidate line in this corridor is 0.1143, which is one-third of the SF of the corridor. The SF values for the lines of each corridor are shown in Table 2.14 (note that since the inductance of parallel lines is assumed to be the same, lines in a corridor have the same SF values).

<table>
<thead>
<tr>
<th>Corridors</th>
<th>Bus 1</th>
<th>Bus 2</th>
<th>Bus 3</th>
<th>Bus 4</th>
<th>Bus 5</th>
<th>Bus 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>0.0000</td>
<td>-0.4545</td>
<td>-0.2727</td>
<td>-0.2727</td>
<td>-0.1818</td>
<td>-0.2727</td>
</tr>
<tr>
<td>1-4</td>
<td>0.0000</td>
<td>-0.1818</td>
<td>-0.1777</td>
<td>-0.3719</td>
<td>-0.1184</td>
<td>-0.3148</td>
</tr>
<tr>
<td>1-5</td>
<td>0.0000</td>
<td>-0.3636</td>
<td>-0.5496</td>
<td>-0.3553</td>
<td>-0.6997</td>
<td>-0.4125</td>
</tr>
<tr>
<td>2-3</td>
<td>0.0000</td>
<td>0.3636</td>
<td>-0.2790</td>
<td>0.0125</td>
<td>-0.1860</td>
<td>-0.0732</td>
</tr>
<tr>
<td>2-4</td>
<td>0.0000</td>
<td>0.1818</td>
<td>0.0062</td>
<td>-0.2852</td>
<td>0.0042</td>
<td>-0.1995</td>
</tr>
<tr>
<td>3-5</td>
<td>0.0000</td>
<td>0.1818</td>
<td>0.2748</td>
<td>0.1777</td>
<td>-0.1501</td>
<td>0.2062</td>
</tr>
<tr>
<td>3-6</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0857</td>
<td>-0.1714</td>
<td>0.0571</td>
<td>-0.2429</td>
</tr>
<tr>
<td>4-6</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0571</td>
<td>0.1143</td>
<td>-0.0381</td>
<td>-0.1714</td>
</tr>
</tbody>
</table>

The TEP objective function is formulated by (A.1). The coefficients are annual (8760 h) investment and operation costs per Million dollars (refer to Tables 2.1 and 2.2). Constraints include
power balance, line flow limits, and generation limits (uncertainties, load shedding, and N-1 security criteria are ignored). The system power balance equations are expanded in (A.2).

\[
\begin{align*}
\min & \quad (20x_{35}^1 + 48x_{36}^1 + 48x_{36}^2 + 30x_{46}^1 + 30x_{46}^2 + 30x_{46}^3 \\
& \quad + 0.1752P_{g1} + 0.2628P_{g2} + 0.3504P_{g3}) \\
& \quad P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 0
\end{align*}
\]

(A.1)

\[
P_1 = P_{g1} - 80; \quad P_2 = -240; \quad P_3 = P_{g2} - 40; \quad P_4 = -160; \quad P_5 = -240; \quad P_6 = P_{g3};
\]

(A.2)

The system has six existing line, and a line flow constraint is formulated for each existing line. For instance, the line flow constraints for existing line 2-3 is expanded in (A.3). Coefficients are calculated using SF values given in Table 2.14. A similar expansion can be obtained for the rest of five existing lines according to (12).

\[
-100 \leq -0.3636P_2 + 0.279P_3 - 0.0125P_4 + 0.186P_5 + 0.0732P_6 + 0.093\tilde{P}_{35}^1 \\
+ 0.2058\tilde{P}_{36}^1 + 0.2058\tilde{P}_{36}^2 - 0.0857\tilde{P}_{46}^1 - 0.0857\tilde{P}_{46}^2 - 0.0857\tilde{P}_{46}^3 \leq 100
\]

(A.3)

The system has six candidate lines, and two constraints are formulated for each candidate line. For instance, the line flow constraint for candidate line 3-5 is expanded in (A.4) and (A.5). The coefficients are obtained from Table 2.14. For example, according to (13), the coefficient of \(\tilde{P}_{46}^3\) in (A.5) is calculated as SF of candidate line 3-5 with respect to bus 6 minus SF with respect to bus 4, i.e., 0.2062-0.1777=0.0285. The same expansion can be obtained for the rest of the candidate lines according to (13) and (14).

\[
-100 \cdot x_{35}^1 \leq 0.1818 + 0.2748P_3 + 0.1777P_4 - 0.1501 + 0.2062P_6 - 0.5751\tilde{P}_{35}^1 \\
+ 0.0686\tilde{P}_{36}^1 + 0.0686\tilde{P}_{36}^2 - 0.0285\tilde{P}_{46}^1 - 0.0285\tilde{P}_{46}^2 - 0.0285\tilde{P}_{46}^3 \leq 100x_{35}^1
\]

(A.4)

\[
-(1 - x_{35}^1) \cdot M \leq \tilde{P}_{35}^1 \leq (1 - x_{35}^1) \cdot M
\]

(A.5)
2.9. References


CHAPTER 3
DISTRIBUTED COLLABORATIVE SECURITY-CONSTRAINED TRANSMISSION EXPANSION PLANNING WITH ANALYTICAL TARGET CASCADING

3.1. Introduction

This chapter presents a distributed collaborative transmission expansion planning (TEP) algorithm for interconnected multi-regional power systems. A local TEP is formulated for each region with respect to the region’s local characteristic and interactions (i.e., tie-line flows) with its neighbors. Nodal power balances at border buses are modified to model the interactions. Realistic planning constraints and objectives, such as budget constraints, operational costs, and N-1 security criterion, are modeled in the local TEPs. The information privacy is respected as each local planner needs to share only limited information related to cross-border tie-lines with other planners. To coordinate local planners, a two-level distributed algorithm is proposed based on the concept of analytical target cascading (ATC) for multidisciplinary optimization. While the upper-level solves the local TEPs in parallel, the lower-level seeks to coordinate neighboring regions. The lower-level problem is further replaced in the upper-level optimization by KKT conditions to relax the need for any form of central coordinator. This makes the proposed ATC-based TEP a fully parallelized distributed algorithm that is potentially less vulnerable to cyber-attacks and communication failures than the distributed methods utilizing a coordinator.

3.2. Symbols

A. Indices and Sets:
\begin{itemize}
  \item \textit{b} \quad \text{Index for load blocks.}
  \item \textit{C} \quad \text{Index for contingencies.}
  \item \textit{d} \quad \text{Index for demand.}
  \item \textit{g} \quad \text{Index for generating units.}
  \item \textit{k} \quad \text{Index for iterations.}
  \item \textit{l} \quad \text{Index for transmission lines.}
  \item \textit{i,j} \quad \text{Index for buses.}
\end{itemize}
\( m, n \) Index for regions (networks).
\( w \) Index for wind farms.
\( \tau \) Index for time intervals in the planning horizon.
\( r(l) \) Receiving-end node of transmission line \( l \).
\( s(l) \) Sending-end node of transmission line \( l \).
\( \Omega_i^D \) Set of all demands located at node \( i \).
\( \Omega_i^W \) Set of all wind farms connected to node \( i \).
\( \Omega_i^L \) Set of all existing transmission lines.
\( \Omega_i^{L+} \) Set of all candidate transmission lines.
\( \Omega_i^m \) Set of all buses of region \( m \).
\( \Omega_i^{m+} \) Set of border buses of region \( m \).
\( \Gamma_i \) Set of decision variables in disjunctive TEP model.
\( \Delta_{g}^{max} \) Maximum adjustment capability of unit \( g \).

B. Parameters:
\( A_g^C \) Parameter that is equal to 0 if unit \( g \) is unavailable under contingency \( C \) and 1 otherwise.
\( A_l^C \) Parameter that is equal to 0 if line \( l \) is unavailable under contingency \( C \) and 1 otherwise.
\( B_l \) Susceptance of transmission line \( l \).
\( C_{d,\tau} \) Load-shedding cost of demand \( d \) at period \( \tau \).
\( C_{d,\tau}^C \) Load-shedding cost of demand \( d \) in contingency \( C \) at period \( \tau \).
\( C_{g,\tau} \) Production cost of generating unit \( g \) at period \( \tau \).
\( F_l^{max} \) Capacity of transmission line \( l \).
\( I_l^{max} \) Investment cost of candidate transmission line \( l \) at period \( \tau \).
\( I_t^{max} \) Investment budget for building candidate lines at period \( \tau \).
\( M \) Large enough number, called big-M.
\( P_{d,b,\tau} \) Expected power demand \( d \) for load block \( b \) at period \( \tau \).
\( P_{w,b,\tau} \) Expected power produced by wind farm \( w \) for load block \( b \) at period \( \tau \).
\( P_{g,\tau}^{max} \) Expected production capacity of generating unit \( g \).
\( \pi^m \) Penalty function corresponding to consistency constraints of area \( m \).
\( \alpha, \beta \) Penalty multipliers.
\( \lambda \) Tuning parameter.

C. Variables:
\( x_{l,\tau} \) Binary decision variable to indicate whether candidate line \( l \) is constructed at period \( \tau \).
\( P_{g,b,\tau} \) Power produced by generating unit \( g \) for load block \( b \) at period \( \tau \).
\( f_{l,b,\tau} \) Power flow through transmission line \( l \) for load block \( b \) at period \( \tau \).
\( P_{d,b,\tau}^{LS} \) Load shedding of demand \( d \) for load block \( b \) at period \( \tau \).
\( f_{ji}^{cm,k} \) Coordinating (or response) variables corresponding to tie-line \( ji \) sent by the central coordinator to region \( m \) in iteration \( k \).
\( \theta_{i,b,\tau} \) Voltage angle at node \( i \) for load block \( b \) at period \( \tau \).
3.3. Contributions

In this chapter, a DTEP algorithm for interconnected multi-regional power systems in a collaborative framework taking into account $N - 1$ security criterion for each region is presented. Each region has its own independent planning entity. Interactions (i.e., power exchange) between the regions through tie-lines are modeled by a set of pseudo generations and a set of pseudo loads, and the power balance equation at border buses are modified accordingly. Taking into account information privacy of planning entities and their mutual interactions, a local TEP problem is formulated for each region. A local planner handles the TEP problem of its network while having access to the information of cross-border tie-lines connecting that network to its neighbors. Based on the concept of analytical target cascading (ATC) technique for multidisciplinary optimization, a two-level distributed optimization algorithm is developed. While the classical ATC is a sequential procedure, the developed ATC allows the parallel solution of local TEP problems in level one with the use of a central coordinator in level two. Since the level two’s problem is a convex optimization, it is further replaced in the level one’s TEP problems by the Karush-Kuhn-Tucker (KKT) conditions. With this procedure, we eliminate the need for a central coordinator by introducing a set of coordinating variables and enforcing a set of constraints in the local TEP of each region. Having no need for any form of central coordinator, the proposed algorithm is fully parallelized and is potentially less vulnerable to cyber-attacks and communication failures than the distributed methods utilizing a coordinator. An initialization strategy is suggested to reduce the number of iterations of the DTEP algorithm.

The remainder of this chapter is organized as follows. The mathematical formulation of DTEP is explained in section 3.4. The ATC-based distributed solution algorithm is presented in section 3.5. Section 3.6 provides numerical results. Concluding remarks are discussed in section 3.7.
3.4. Mathematical Formulation of DTEP

The TEP problem is mostly modeled with DC power flow in the literature [1-5]. Voltage angles of buses are used to calculate power flow in existing and candidate lines. Power flow of candidate lines is, usually, modeled through bilinear equations, where integer variables representing candidate lines are multiplied by bus voltage angles. Bilinear equations can be transformed into linear equations using disjunctive techniques [6]. The resultant TEP formulation is mixed-integer linear programming, which is called the disjunctive TEP model. A complete formulation of the disjunctive TEP model considering N-1 security criterion is given in Appendix.

In this section, we present the DTEP model for the system shown in Fig. 3.1, which can be extended for interconnected systems with multiple regions. The regions exchange power with each other through the tie-lines. Consider that in a given time, the power in the tie-line \( ij \) flows from network \( n \) toward the network \( m \), and the direction of power flow in tie-line \( i'j' \) is toward the network \( n \). We model the tie-line \( ij \) as a controllable pseudo generation in the network \( m \) and as a controllable pseudo load in the network \( n \). The same concept is used for the tie-line \( i'j' \) as shown in Fig. 3.2. Values of the pseudo generations and loads are equal to the line flows. Following this approach, we can virtually disconnect networks \( m \) and \( n \). As the pseudo generations and loads appear in both networks, we call them coupling (shared) variables between networks \( m \) and \( n \).

Remark1: In most applications of distributed optimization on power system problems, e.g., distributed optimal power flow [7] and distributed unit commitment [8], voltage angles of cross-border buses (i.e., \( i, i', j, \) and \( j' \)) are considered as coupling variables. That is, two coupling variables should be assumed for each tie-line. We directly use tie-line flows as coupling variables to have less number of variables in the model. It is obvious that in the DC power flow, bus voltage angles are proportional quantities, which depend on the reference bus location. However, power
flow through each line, which depends on the difference between voltage angles of sending and receiving buses (see (a.6) in the section 3.8. Appendix), is independent from the location of reference bus. Therefore, it is better to select power flow of tie-lines as coupling variables. This enhances the convergence performance of the DTEP algorithm.

Remark 2: In the proposed method, the reference bus is selected for one region (e.g., region $m$). After convergence, the values of voltage angles for other regions can be obtained from the values of voltage angles of cross-border buses of region $m$ and power flow through tie-lines (see (a.6)).

![Fig. 3.1. Two interconnected networks.](image1)

![Fig. 3.2. Modeling power exchange between interconnected networks $m$ and $n$.](image2)
Now, each planning entity can formulate a local TEP problem with respect to its local parameters/variables and coupling variables (i.e., the pseudo generations and loads).

3.4.1. Local objective and constraints

The objective function of the local TEP problem of network \( m \), i.e., (1a), is to minimize the investment and operational costs of region \( m \). The sets of equality constraints (1b) and inequality constrains (1c) are local constraints that only include local variables and parameters of region \( m \). These constraints are the same as constraints of the disjunctive TEP model given in section 3.8.

Appendix.

\[
\begin{align*}
\min \ & F^m(\Gamma) \\
\text{s.t.} \ & g^m(\Gamma) = 0 \\
\ & h^m(\Gamma) \leq 0
\end{align*}
\] (1a) (1b) (1c)

3.4.2. Constraints of border buses

In addition to constraints (1b) and (1c), we formulate constraints (2a)-(2e) that not only consist of local variables/parameters but also include coupling variables between network \( m \) and its neighbors. Constraint (2a) indicates that if bus \( i \) of network \( m \) is connected to a tie-line between this network and its neighbors, and if the tie-line is modeled as a pseudo generation from the perspective of network \( m \), power injected to bus \( i \) is equal to power output of the pseudo generation plus power generated by actual units on this bus. A similar discussion is valid for (2b) where the tie-line is modeled as a pseudo load. Inequalities (2c) and (2d) ensure that the pseudo generations and loads are within their limits. These limits are defined according to capacity of the tie-lines. Note that \( f^m_{ji} \) and \( f^m_{i'j'} \) can be negative. In this case, \( f^m_{ji} \) becomes a pseudo load and \( f^m_{i'j'} \) becomes a pseudo generation. The nodal power balance at the border buses is modeled by equality (2e).
Consider tie-line $ij$ in Fig. 3.1, which is modeled by a pseudo generation and a pseudo load in Fig. 3.2. If planner $m$ separately solves its local TEP regardless of the TEP of planner $n$, the solution procedure might result in different values for $f^m_{ji}$ and $f^n_{ji}$ that is not a feasible solution. As $f^m_{ji}$ and $f^n_{ji}$ refer to the tie-line flow, their values need to be the same to reach a consistent and feasible solution for the whole grid. We introduce new sets of equality constraints (3a) and (3b), named consistency constraints, and enforce them in local TEPs $m$ and $n$. The consistency constraints enforce pseudo generations to be equal to their corresponding pseudo loads. In other words, the consistency constraints ensure that the values of coupling variables are the same in the local TEP problems.

$$CC1: \quad f^m_{ji} - f^n_{ji} = 0; \quad i \in \Omega^+_i, j \in \Omega^+_j$$  

$$CC2: \quad f^{i'}_{i'j'} - f^{i'}_{i'j'} = 0; \quad i' \in \Omega^+_i, j' \in \Omega^+_j$$

Finally, in the DTEP model, the local TEP for region $m$ is modeled as follows:

$$\min F^m(\Gamma); \quad s. t. \ (1b), (1c), (2a) - (2e), (3a), (3b)$$
3.5. Solution Algorithm for DTEP

Although the consistency constraints ensure feasibility of TEP results, these hard constraints are barriers for the separate solution of local TEPs. A strategy is needed to coordinate the planners’ decisions for the values of pseudo generations and loads.

3.5.1. ATC Technique for DTEP Implementation

Based on the concept of analytical target cascading (ATC), we develop an algorithm to coordinate the planners’ decisions and solve the local TEPs in a distributed manner. ATC is a model-based method for multilevel hierarchical optimization problems [9-11]. The general concept of ATC is similar to other popular algorithms that are based on augmented Lagrangian relaxation, such as the auxiliary problem principle (APP) and the alternating direction method of multipliers (ADMM) [12]. However, ATC has a hierarchical structure and works based on propagating target values from upper-level systems toward the lower-level systems, and passing the response variables from lower-levels to upper-levels. In addition, we have flexibility in the choice of penalty function in the ATC method and can select, for instance, a quadratic function or an exponential function [11]. Consider the local TEP for region \( m \). We penalize violations of the consistency constraints (3a) and (3b) into the objective function using two penalty functions. Here, we applied second-order penalty functions for \( \pi_1^m(\cdot) \) and \( \pi_2^m(\cdot) \). Accordingly, the objective function of TEP \( m \) is rewritten as follows:

\[
\min_{\Gamma} F^m(\Gamma) + \pi_1^m(CC1) + \pi_2^m(CC2)
\]

\[
\min_{\Gamma} F^m(\Gamma) + \sum_{\forall r} \sum_{ij \in \Omega_{ir}^t} \{ \alpha_{ji}^m(f_{ji}^m - f_{ji}^n) + \| \beta_{ji}^m \circ (f_{ji}^m - f_{ji}^n) \|_2^2 \}
\]

\[
+ \alpha_{i'j'}^m(f_{i'j'}^m - f_{i'j'}^n) + \| \beta_{i'j'}^m \circ (f_{i'j'}^m - f_{i'j'}^n) \|_2^2 \}
\]
where the symbol \( \circ \) represents the Hadamard product. Parameter \( \alpha \) is the Lagrangian multiplier and \( \beta \) is a penalty parameter. A similar relaxed TEP problem is formulated for network \( n \), and accordingly, local TEP problems of all interconnected networks will be formulated. Note that in the ATC-based solution algorithm, decision variables of TEP \( m \) include local variables of network \( m \) and the coupling variables (i.e., pseudo generations and loads) denoted by superscript \( m \). The pseudo generations and loads denoted by superscript \( n \) are constants received from neighbor \( n \).

The local TEPs can be assumed to be connected hierarchically, and a sequential, iterative procedure can be applied to coordinate the optimization problems (see [9, 10] for more details). While a planner is solving its local TEP, other planners should stay idle. This increases the overall computational time of the solution process. We will further propose a fully parallel, scalable solution procedure.

### 3.5.2. Decentralized Parallel TEP Implementation

#### A. Partially Parallel Solution Algorithm

We introduce a coordinator to enable a parallel solution of the local TEPs. This coordinator virtually disconnects the TEPs of neighboring planners as shown in Fig. 3.3. The local TEPs are at the upper level, and the coordinator is at the lower level. Instead of direct interaction between the neighboring planners, each planner only communicates with the coordinator. Therefore, regions can solve their local TEPs in parallel. Note that there is no need to have an entity to play the role of a coordinator. Any of local planners may handle the role of the coordinator (we will relax the need for such a virtual coordinator in the next section).

According to the concept of ATC, values of variables sent by local regions to the central coordinator are named targets, and the values sent by the coordinator toward the regions are called responses. We indicate the communication direction in the superscripts. For instance, \( f_{ji}^{mc} \) denotes
the target variable that is calculated in region $m$ and sent to the coordinator. We introduce *coordinating variables* $r_{ji}^{cm}$ that are the responses sent from the central coordinator to region $m$. A set of consistency constraints is formulated to make the solution of the central coordinator and the local regions consistent. These constraints are relaxed in the objective function by augmented Lagrangian penalty functions (see [13] for more details). This procedure leads to the following TEP problem for region $m$ in iteration $k$:

$$
\min_{(\Gamma, f^{mc})} F^{m,k}(\Gamma) + \sum_{vt} \sum_{ij \in \Omega_i^t} \left\{ \alpha_{ij}^{m,k} (r_{ji}^{cm,k-1} - f_{ji}^{mc,k}) + \|\beta_{ji}^{m,k} \circ (r_{ji}^{cm,k-1} - f_{ji}^{mc,k})\|_2^2 \right\}
$$

$$
+ \alpha_{i'j'}^{m,k} \left( r_{i'j'}^{cm,k-1} - f_{i'j'}^{mc,k} \right) + \|\beta_{i'j'}^{m,k} \circ (r_{i'j'}^{cm,k-1} - f_{i'j'}^{mc,k})\|_2^2
$$

s.t. (1b), (1c), (2a)-(2e)

A similar local TEP problem is formulated for each region. The optimization problem is formulated for the central coordinator as shown in (7a)-(7c). The coordinator has no role except coordinating local TEPs. Thus, its objective function includes only a set of penalty functions for relaxing the consistency constraints. For the system shown in Fig. 3.2, the coordinator's objective
function is formulated as (7a). Four quadratic augmented Lagrangian penalty functions appear in (7a). The first two functions are related to relaxation of the consistency constraints \((r_{ij}^{cm} - f_{ji}^{mc})\) and \((r_{ij'}^{cm} - f_{ij'}^{mc})\) between the coordinator and TEP of region \(m\), and the last two penalty functions relax the consistency constraints \((r_{ij}^{cn} - f_{ji}^{nc})\) and \((r_{ij'}^{cn} - f_{ij'}^{nc})\) between the coordinator and TEP of region \(n\).

The response values determined by the central coordinator must be consistent for both regions \(m\) and \(n\). This is reflected in equations (7b) and (7c). With the above procedure, the local TEPs become virtually separated, and a two-level ATC can be applied to solve TEPs in parallel. However, the main drawback is the need for a central coordinator that is responsible for coordinating the coupling variables between the regions.

\[
\begin{align*}
\sum_{\forall \tau} \sum_{i,j \in \Omega_{I}^{\tau}} \{ & a_{ji}^{m,k} (r_{ji}^{cm,k} - f_{ji}^{mc,k}) + \| \beta_{ji}^{m,k} \circ (r_{ji}^{cm,k} - f_{ji}^{mc,k}) \|^2_2 + \\
& a_{i'j'}^{m,k} (r_{i'j'}^{cm,k} - f_{i'j'}^{mc,k}) + \| \beta_{i'j'}^{m,k} \circ (r_{i'j'}^{cm,k} - f_{i'j'}^{mc,k}) \|^2_2 \} \\
& + a_{ji}^{n,k} (r_{ji}^{cn,k} - f_{ji}^{nc,k}) + \| \beta_{ji}^{n,k} \circ (r_{ji}^{cn,k} - f_{ji}^{nc,k}) \|^2_2 + \\
& a_{i'j'}^{n,k} (r_{i'j'}^{cn,k} - f_{i'j'}^{nc,k}) + \| \beta_{i'j'}^{n,k} \circ (r_{i'j'}^{cn,k} - f_{i'j'}^{nc,k}) \|^2_2 \}
\end{align*}
\]

\[
(r_{ji}^{cm,k} = r_{ji}^{cn,k}; \ i \in \Omega_{I}^{m+}, \ j \in \Omega_{I}^{n+}) \quad (7b)
\]

\[
(r_{i'j'}^{cm,k} = r_{i'j'}^{cn,k}; \ i' \in \Omega_{I}^{m+}, \ j' \in \Omega_{I}^{n+}) \quad (7c)
\]

B. Fully Parallel Solution Algorithm

Assume that region \(m\) and the coordinator are interacting as shown in Fig. 3.3. We can form a bi-level optimization problem to model their interactions in which TEP \(m\) is the leader’s problem.
and the coordinator’s optimization is the follower’s problem. The leader’s problem includes (1a)-(1c) and (2a)-(2e), and the follower’s problem consists of (7a)-(7c). Note that in iteration \( k \), region \( m \) uses values of the coordinated shared variables determined by the coordinator in iteration \( k - 1 \). Thus, while the leader’s problem deals with variables of iteration \( k \), we use the coordinating variable \( r^{k-1} \) in the follower’s problem. Note that we use only the coordinating values of the coordinator that are needed for region \( m \) (i.e., \( r_{cm,k-1}^{m} \)) and ignore other coordinating values (that is, any coupling variables that have no direct impact on region \( m \) do not appear in the bi-level optimization of this region). The bi-level model of TEP \( m \) is formulated as follows:

Upper-level (leader):  
\[
\begin{align*}
\min_{(T, f_{mc}^m)} & \quad (6) \\
\text{s.t.} & \quad (1b), (1c), (2a) - (2e)
\end{align*}
\]  
(8a)

Lower-level (follower):  
\[
\begin{align*}
\min_{r_{cm,k-1}^m} & \quad (7a) \\
\text{s.t.} & \quad (7b), (7c)
\end{align*}
\]  
(8b)

The follower problem includes only a set of convex quadratic penalty functions and linear constraints. Therefore, it is a convex optimization problem that can be replaced in the leader problem by applying KKT conditions and solving the resulting mathematical programming problem with equilibrium constraints. Consider penalty terms corresponding to a shared variable between regions \( m \) and \( n \) (i.e., power flow in tie-line \( ji \)) shown in (9). From the constraints of the follower problem, we have \( r_{cm,k-1}^{cm} = r_{cn,k-1}^{cn} \). Taking the partial derivative of \( \pi \) with respect to \( r_{cm,k-1}^{cm} \) results in the formulation (10) for \( r_{cm,k-1}^{cm} \).

\[
\pi = \alpha_{ji}^{m,k-1}(r_{ji}^{cm,k-1} - f_{ji}^{mc,k-1}) + \beta_{ji}^{m,k-1} \circ (r_{ji}^{cm,k-1} - f_{ji}^{mc,k-1})^2 + \alpha_{ji}^{n,k-1}(r_{ji}^{cn,k-1} - f_{ji}^{nc,k-1}) + \beta_{ji}^{n,k-1} \circ (r_{ji}^{cn,k-1} - f_{ji}^{nc,k-1})^2
\]  
(9)

\[
r_{ji}^{cm,k-1} = \frac{2\beta_{ji}^{m,k-1} f_{ji}^{mc,k-1} + 2\beta_{ji}^{n,k-1} f_{ji}^{nc,k-1} - \alpha_{ji}^{m,k-1} - \alpha_{ji}^{n,k-1}}{2\beta_{ji}^{m,k-1} + 2\beta_{ji}^{n,k-1}}
\]  
(10)
If each region directly calculates values of central coordinator’s coordinating variables \( r_{ji}^{cm,k-1} \), the coordinator can be omitted. However, to calculate \( r_{ji}^{cm,k-1} \), the equivalents of \( a_{ji}^{m,k-1} \), \( \alpha_{ji}^{n,k-1} \), \( \beta_{ji}^{m,k-1} \), \( \beta_{ji}^{n,k-1} \), \( f_{ji}^{mc,k-1} \), and \( f_{ji}^{nc,k-1} \) are needed. If regions \( m \) and \( n \) directly exchange information without a central coordinator, and multipliers \( \alpha \) and \( \beta \) are updated as (11a) and (11b), then we can replace \( \alpha_{ji}^{m,k} \), \( \alpha_{ji}^{n,k} \), \( \beta_{ji}^{m,k} \), \( \beta_{ji}^{n,k} \) by their equivalents \( \alpha_{ji}^{mn,k} \), \( \alpha_{ji}^{mn,k} \), \( \beta_{ji}^{mn,k} \), \( \beta_{ji}^{mn,k} \), respectively.

\[
a^k = a^{k-1} + 2(\beta^{k-1})^2(r^{k-1} - f^{k-1}) \quad \text{(11a)}
\]

\[
\beta^k = \lambda.\beta^{k-1} \quad \text{(11b)}
\]

The value of \( r_{i'j'}^{cm,k-1} \) can be obtained in a similar manner. Finally, by replacing the follower optimization with its KKT conditions in the leader optimization, the decentralized formulation for local TEP of region \( m \) is modeled as shown by (12).

\[
\text{min}_{(\Gamma', f_{mn,k}^{mn,k})} F_{mn,k}^{m,k}(\Gamma)
\]

\[
+ \sum_{\forall \tau} \sum_{ij \in \Omega_i^+} \{ a_{ji}^{mn,k} (r_{ji}^{cm,k-1} - f_{ji}^{mn,k}) + \| \beta_{ji}^{mn,k} \circ (r_{ji}^{cm,k-1} - f_{ji}^{mn,k}) \|^2 \}
\]

\[
+ \alpha_{i'j'}^{mn,k} (r_{i'j'}^{cm,k-1} - f_{i'j'}^{mn,k}) + \| \beta_{i'j'}^{mn,k} \circ (r_{i'j'}^{cm,k-1} - f_{i'j'}^{mn,k}) \|^2 \}
\]

\[\text{s.t. (1b), (1c), (2a) - (2e)}\]

\[
r_{ji}^{cm,k-1} = \frac{2\beta_{ji}^{mn,k-1} \cdot f_{ji}^{mn,k-1} + 2\beta_{ji}^{nm,k-1} \cdot f_{ji}^{nm,k-1} - a_{ji}^{nm,k-1} - a_{ji}^{mn,k-1}}{2\beta_{ji}^{mn,k-1} + 2\beta_{ji}^{nm,k-1}}
\]

\[
r_{i'j'}^{cm,k-1} = \frac{2\beta_{i'j'}^{mn,k-1} \cdot f_{i'j'}^{mn,k-1} + 2\beta_{i'j'}^{nm,k-1} \cdot f_{i'j'}^{nm,k-1} - a_{i'j'}^{nm,k-1} - a_{i'j'}^{mn,k-1}}{2\beta_{i'j'}^{mn,k-1} + 2\beta_{i'j'}^{nm,k-1}}
\]

Variables \( f_{ji}^{mc,k} \) and \( f_{i'j'}^{mc,k} \) are replaced by \( f_{ji}^{mn,k} \) and \( f_{i'j'}^{mn,k} \), which shows that region \( m \) exchanges values of coupling variables directly with its neighbors instead of the central coordinator. Therefore, the local TEP of region \( m \) only relies on the local information of region \( m \).
and coupling variables between this region and its immediate neighboring regions. Meaning, it can be solved in a decentralized manner with no need for a central coordinator. Moreover, in iteration \( k \), each region needs values of coupling variables calculated by its neighbors in iteration \( k - 1 \). That is, the regions do not need to stay idle and can solve their local TEP problems concurrently. In other words, the local TEPs can be solved in a parallel way with peer-to-peer communications only among the immediate neighboring regions. We propose the iterative coordination strategy shown in Algorithm 1, which is a fully parallel ATC technique, to solve the TEPs of the regions:

**Algorithm 1.** The proposed parallel ATC-based coordination strategy with no coordinator for decentralized TEP implementation

1. Initialize coupling variables, \( \alpha, \beta \), and set \( \lambda, k = 0 \)
2. **While** {not converged} **do**
   
   \( k = k + 1 \)

3. Solve (12) for all regions in parallel and determine the optimal values of coupling variables
4. Exchange the values of coupling variables between neighboring regions
5. Update \( \alpha^k = \alpha^{k-1} + 2(\beta^{k-1})^2 (r^{k-1} - f^{k-1}) \)
6. Update \( \beta^k = \lambda \cdot \beta^{k-1} \)
7. **If** \( |f_{mn}^k - f_{nm}^k| \leq \epsilon_1 \) and \( \frac{|E_{mk}(\Gamma) - E_{m,k-1}(\Gamma)|}{E_{m,k}(\Gamma)} \leq \epsilon_2 \) **then**
8. Declare convergence
9. **End if**
10. **End while**

### 3.6. Numerical Results

Two popular test systems for TEP studies, including the IEEE RTS 24-bus system and the IEEE 118-bus system, are used to evaluate the performance of the proposed method. All simulations are carried out using GAMS and ILOG CPLEX MIQP solver [14]. A personal computer with an Intel(R) Xeon(R) CPU @2.6 GHz, including eight cores and 16 GB of RAM, is used.
3.6.1. IEEE 24-Bus System

The one-line diagram of the modified 24-bus system is shown in Fig. 3.4. This system is modified by Ruiz and Conejo for TEP studies, and the list of candidate lines are suggested accordingly [15]. We replace half of the conventional generation capacity of units connected to buses seven and 22 by wind farms with parameters given in [16]. The system includes two regions, which are connected via tie-lines 3-24, 9-11, and 10-12. Each region contains five candidate lines listed in Table 3.1.

Fig 3.4. One-line diagram of modified 24-bus system
Table 3.1. Candidate lines’ parameters for the IEEE 24-Bus system

<table>
<thead>
<tr>
<th>From bus</th>
<th>To bus</th>
<th>Region</th>
<th>Annual investment cost (Million $)</th>
<th>From bus</th>
<th>To bus</th>
<th>Region</th>
<th>Annual investment cost (Million $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>17</td>
<td>1</td>
<td>0.7</td>
<td>20</td>
<td>22</td>
<td>2</td>
<td>1.1</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>1</td>
<td>0.8</td>
<td>14</td>
<td>15</td>
<td>2</td>
<td>0.9</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1</td>
<td>1.0</td>
<td>15</td>
<td>19</td>
<td>2</td>
<td>0.8</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>1.2</td>
<td>13</td>
<td>14</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>1</td>
<td>1.3</td>
<td>19</td>
<td>21</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: All candidate lines are assumed to have a susceptance of 500 (per Ohm) and a capacity of 500 (MW).

Three scenarios are studied as follows:

- Scenario 1: each region implements its local TEP regardless of its interactions with its neighboring regions.
- Scenario 2: it is assumed that a centralized planning entity exists and gathers the information of all regions and aims at maximizing the social welfare (or minimizing the total costs) for the whole power system. Since this fictitious centralized entity has information of all regions, it can provide the optimal TEP solution from the perspective of the whole system.
- Scenario 3: the proposed collaborative DTEP is implemented taking into account interactions between the regions and the information privacy.

We consider results of scenario 2 (that provides the best results from the perspective of the whole grid) as the benchmark and use the following convergence measure to evaluate the performance of the proposed DTEP.

\[
rel = \frac{|f^* - f^d|}{f^*}
\]  

(13)

where \(f^*\) is the cost function of the centralized TEP, and \(f^d\) is the cost function determined by the proposed collaborative DTEP. For scenario 3, the initial value of penalty multipliers and tuning parameter \(\lambda\) are set to one, and the convergence thresholds \(\epsilon_1\) and \(\epsilon_2\) are 0.05 and 0.01, respectively. Simulation results of all scenarios are summarized in Table 3.2.
### Table 3.2. Simulation results for the IEEE 24-Bus system

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Cost function (M$)</th>
<th>Cost of candidate lines (M$)</th>
<th>Candidate lines for installation</th>
<th>Annual cost of units (M$)</th>
<th>Annual cost of load shedding (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>553.12</td>
<td>2.5</td>
<td>1, 8, 9</td>
<td>216.45</td>
<td>334.17</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>231.53</td>
<td>1.5</td>
<td>1, 8</td>
<td>230.03</td>
<td>0</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>231.53</td>
<td>1.5</td>
<td>1, 8</td>
<td>230.03</td>
<td>0</td>
</tr>
</tbody>
</table>

When each local TEP is solved individually regardless of its interactions with its neighboring regions, the total cost is $553.12M (cost of regions 1 and 2 is $ 43.58M and $ 172.87M, respectively), and candidate lines 1, 8, and 9 are selected to be installed. The annual investment cost of installing new lines is higher than the two other scenarios. Moreover, region 2 cannot support the forecasted load and has to shed 74.8 MW of the load at bus 15. For the centralized TEP, the total cost goes down to $231.53M, and candidate lines 1 and 8 are selected to be installed. Although the cost decreases because of interactions between the regions, the information privacy of the regions is not preserved. The proposed collaborative DTEP algorithm converges after 27 iterations. Figures 3.5 and 3.6 show the coordinating variables \( r_{ij}^{cm,k} \) and the \( rel \) index over the course of iterations. Upon convergence, the \( rel \) index and differences between the share variables are within an acceptable threshold. The \( rel \) index is roughly \( 1e - 6 \). The total cost of DTEP is the same as the centralized TEP. The cost of regions 1 and 2 is $ 58.86M and $ 171.17M, respectively, and candidate lines 1 and 8 are planned for installation. The proposed collaborative DTEP could provide the benchmark results obtained in scenario 2 while preserving the information privacy of the local planning entities.

One may terminate the DTEP algorithm before iteration 27th (e.g., 19) and obtain acceptable results (\( rel \) is around \( 1e - 5 \)). However, a trade-off should be considered between the optimality (i.e., the \( rel \) index), feasibility (i.e., \( |f_{mn}^k - f_{nm}^k| \)), and the number of iterations. Although the convergence measure \( rel \) decreases in overall, in several iterations it goes up. This is because of
the feasibility criterion. In those iterations, for instance in transition from iteration 19 to iteration 19, the distributed algorithm wants to reduce the feasibility gap $|f^k_{mn} - f^k_{nm}|$, and this may lead to a slightly larger $rel$ index.

Fig. 3.5. Coordinating variables (response values) in scenario 3.

Fig. 3.6. The $rel$ index obtained by DTEP (scenario 3) for 24-bus system.
3.6.2. IEEE 118-Bus System

The system information is provided in [17]. For this system, we have considered a set of candidate lines that geographically make sense and decrease the operational costs if installed. The system is modified by adding three wind farms on buses 36, 69, and 77 [18]. The system includes three regions. Seven coupling variables exist. Regions one and two are connected through five tie-lines (15-33, 19-34, 30-38, 75-77, and 75-188), and regions two and three are connected through two tie-lines (77-82 and 80-96). Each region includes ten candidate lines as listed in Table 3.3.

Table 3.3. Candidate lines’ parameters for the IEEE 118-Bus system.

<table>
<thead>
<tr>
<th># of lines</th>
<th>From bus</th>
<th>To bus</th>
<th>Annual cost (M$)</th>
<th>Region</th>
<th># of lines</th>
<th>From bus</th>
<th>To bus</th>
<th>Annual cost (M$)</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>12</td>
<td>0.406</td>
<td>1</td>
<td>16</td>
<td>61</td>
<td>58</td>
<td>0.446</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>117</td>
<td>0.324</td>
<td>1</td>
<td>17</td>
<td>65</td>
<td>81</td>
<td>0.324</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>16</td>
<td>0.406</td>
<td>1</td>
<td>18</td>
<td>65</td>
<td>68</td>
<td>0.365</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>16</td>
<td>0.507</td>
<td>1</td>
<td>19</td>
<td>69</td>
<td>66</td>
<td>0.324</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>29</td>
<td>0.284</td>
<td>1</td>
<td>20</td>
<td>116</td>
<td>62</td>
<td>0.527</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>22</td>
<td>26</td>
<td>0.446</td>
<td>1</td>
<td>21</td>
<td>90</td>
<td>84</td>
<td>0.104</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>24</td>
<td>71</td>
<td>0.324</td>
<td>1</td>
<td>22</td>
<td>91</td>
<td>93</td>
<td>0.054</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>113</td>
<td>115</td>
<td>0.365</td>
<td>1</td>
<td>23</td>
<td>90</td>
<td>93</td>
<td>0.104</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td>24</td>
<td>0.629</td>
<td>1</td>
<td>24</td>
<td>103</td>
<td>110</td>
<td>0.104</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>24</td>
<td>25</td>
<td>0.527</td>
<td>1</td>
<td>25</td>
<td>111</td>
<td>93</td>
<td>0.054</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>52</td>
<td>54</td>
<td>0.406</td>
<td>2</td>
<td>26</td>
<td>111</td>
<td>94</td>
<td>0.104</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>59</td>
<td>62</td>
<td>0.324</td>
<td>2</td>
<td>27</td>
<td>107</td>
<td>93</td>
<td>0.104</td>
<td>3</td>
</tr>
<tr>
<td>13</td>
<td>99</td>
<td>63</td>
<td>0.406</td>
<td>2</td>
<td>28</td>
<td>107</td>
<td>94</td>
<td>0.104</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>66</td>
<td>49</td>
<td>0.507</td>
<td>2</td>
<td>29</td>
<td>87</td>
<td>84</td>
<td>0.104</td>
<td>3</td>
</tr>
<tr>
<td>15</td>
<td>61</td>
<td>51</td>
<td>0.284</td>
<td>2</td>
<td>30</td>
<td>103</td>
<td>84</td>
<td>0.054</td>
<td>3</td>
</tr>
</tbody>
</table>

Note: All candidate lines are assumed to have a susceptibility of 30 (per Ohm) and a capacity of 390 (MW).

The same three scenarios as in case 1 are considered. Simulation results are summarized in Table 3.4. In scenario 1, the operation costs of regions one, two, and three are $236.28M, $298.30M, and $214.07M, respectively. Two more candidate lines (lines eight and 27) are selected to be installed compared with the benchmark results, and the total cost is larger than the benchmark cost. The proposed collaborative DTEP of scenario 3 converges after 129 iterations. Figure 3.7
shows the $rel$ index, which is small enough (almost $1e-4$) upon the convergence. The annual planning cost of installing new lines is the same as that obtained in scenario 2. The proposed DTEP provides planning decisions similar to the benchmark results while respecting the information privacy of planning entities. Note that in TEP, the cost of generation units (operation cost) might not realize in the real time operation. The main goal of solving TEP problem is to decide about installing new transmission lines. Therefore, the slight difference (this 0.015% error is because of the considered acceptable gap for the distributed algorithm) between operational costs of scenarios 2 and 3 has no effect on planning decisions.

DTEP with Initialization

In the previous DTEP scenario, the initial values of coupling variables were set to zero. To reduce the number of iterations, the initial value of coupling variables in the decentralized ATC technique can be selected wisely. For this purpose, we fixed the decision variable of installing new lines to zero ($x_l = 0$) and simplify DTEP to a distributed OPF problem, which is a convex problem. The solution of the distributed OPF is selected to initialize the coupling variables in DTEP. In this case, DTEP converges after 62 iterations, which almost 50% less than that for DTEP with flat start (i.e., initializing the coupling variables to zero). The $rel$ index value is depicted in Fig. 3.7, and the planning results are the same as them for scenario 3 of Table 3.4.

Table 3.4. Simulation results for the IEEE 118-Bus system.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Cost function (M$)</th>
<th>Annual cost of candidate lines (M$)</th>
<th># of Candidate lines for installation</th>
<th>Annual cost of units (M$)</th>
<th>Annual cost of load shedding (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>751.24</td>
<td>2.59</td>
<td>5, 7, 8, 9, 13, 17, 22, 27, 29</td>
<td>748.65</td>
<td>0</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>735.43</td>
<td>2.13</td>
<td>5, 7, 9, 13, 17, 22, 29</td>
<td>733.30</td>
<td>0</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>735.54</td>
<td>2.13</td>
<td>5, 7, 9, 13, 17, 22, 29</td>
<td>733.41</td>
<td>0</td>
</tr>
</tbody>
</table>
Fig. 3.7. The rel index in scenario 3 with a flat start and with the suggested initialization strategy (for the IEEE 118-bus system).

3.7. Conclusion

A distributed collaborative transmission expansion planning algorithm was presented for interconnected multi-regional power systems. Realistic planning constraints and objectives, such as budget constraints, operational costs, and $N - 1$ security criterion were taken into account. A data-driven approach was applied to account for uncertainties of power demand, the capacity of generating units, and wind power generation. While each region handles its local planning problem, it collaborates with other regions to achieve the optimal and feasible planning scheme for the whole interconnected grid. A two-level decentralized solution algorithm was developed based on the concept of analytical target cascading. The proposed distributed algorithm allowed parallel implementation of TEP subproblems with no need for a central coordinator.

Three scenarios were simulated. If each region solves its local TEP regardless of interactions with its neighbors (scenario 1), the overall investment planning (and operational) costs go up as compared to those costs obtained by taking into account the coordination of local TEPs (scenarios
2 and 3). Although the centralized TEP with the coordination of planners provided the optimal results, information of the local planners must be gathered in a center. The proposed collaborative DTEP algorithm provided the benchmark results as the centralized TEP while respecting the information privacy of the independent planners. The suggested initialization strategy reduced the number of iterations of DTEP by 50%.

3.8. Appendix

The disjunctive TEP model with N-1 security criterion is formulated by (a.1)-(a.23). The objective function (a.1) consists of costs of installing new lines and expected operational costs $Q$. Set of decision variables $\Gamma$ is expressed in (a.2), where $x_{l,t}$ is a binary variable and the rests are bounded decision variables. Constraints of normal operation and N-1 security criterion are respectively shown by (a.3)-(a.13) and (a.14)-(a.23). Budget constraints (a.4), nodal power balance constraints (a.5), and flow constraints of the existing and candidate lines (a.6)-(a.9) must be satisfied in both normal and contingency conditions. In addition, adjustment capabilities of generating units to provide either preventive or corrective action in response to a contingency are expressed by (a.23). More details of the disjunctive TEP model are presented in [6].

\[
\min_{\Gamma} F(\Gamma); \\
F(\Gamma) = \sum_{\forall \tau} \left( \sum_{l \in \Omega^L} \bar{I}_{l,t} \cdot x_{l,t} + E\left[Q(p_{g,}p_{d}^{LS},p_{d}^{LS,c})\right] \right); \\
Q = \sum_{b \in B} \sum_{g \in \Omega^D} C_{g,t} \cdot p_{g,b,t} + \sum_{b \in B} \sum_{d \in \Omega^D} C_{d,t} \cdot p_{d,b,t}^{LS} + \sum_{b \in B} \sum_{c \in \Omega^C} \sum_{d \in \Omega^D} C_{c,d,t} \cdot p_{d,b,t}^{LS,c} \\
\Gamma = \{x_{l,t},f_{l,b,t},p_{g,b,t}^{LS},p_{d,b,t}^{LS,c},\theta_{l,b,t},f_{c,b,t},p_{g,b,t}^{LS,c},p_{d,b,t}^{LS,c},\theta_{c,b,t}\} \\
x_l = \{0, 1\}; \quad \forall l \Omega^L+ 
\]
\[
\sum_{l \in \Omega_{l^+}} I_{l, \tau} x_{l, \tau} \leq I_{\tau}^{\max}; \quad \forall \tau, l \Omega_{l^+} \tag{a.4}
\]

\[
\sum_{g \in \Omega_g^L} P_{g,b,\tau} + \sum_{w \in \Omega_w^W} P_{w,b,\tau} - \sum_{l|s(l) = i} f_{l,b,\tau} + \sum_{l|r(l) = i} f_{l,b,\tau}
= \sum_{d \in \Omega_d^L} (P_{d,b,\tau} - P_{d,b,\tau}^{LS}); \quad \forall i, b, \tau \tag{a.5}
\]

\[
f_{l,b,\tau} = B_{l}.(\theta_{l,b,\tau} - \theta_{j,b,\tau}); \quad \forall b, \tau, l \in \Omega_{l^+}, s(l) = i, r(l) = j \tag{a.6}
\]

\[-F_{l}^{\max} \leq f_{l,b,\tau} \leq F_{l}^{\max}; \quad \forall b, \tau, l \in \Omega_{l^+} \tag{a.7}
\]

\[-(1 - x_{l,\tau}).M \leq f_{l,b,\tau} - B_{l}.(\theta_{l,b,\tau} - \theta_{j,b,\tau}) \leq (1 - x_{l,\tau}).M \tag{a.8}
\]

\[\forall b, \tau, l \in \Omega_{l^+}, s(l) = i, r(l) = j \]

\[-x_{l,\tau}.F_{l}^{\max} \leq f_{l,b,\tau} \leq x_{l,\tau}.F_{l}^{\max}; \tag{a.9}
\]

\[\forall b, \tau, l \in \Omega_{l^+}, s(l) = i, r(l) = j \]

\[0 \leq P_{g,b,\tau} \leq P_{g}^{\max}; \quad \forall g, b, \tau \tag{a.10}
\]

\[0 \leq P_{d,b,\tau}^{LS} \leq P_{d,b,\tau}; \quad \forall g, b, \tau \tag{a.11}
\]

\[-\pi \leq \theta_{l,b,\tau} \leq \pi; \quad \forall i, b, \tau \tag{a.12}
\]

\[\theta_{l,b,\tau} = 0; \quad \forall b, \tau, i = \text{ref.} \tag{a.13}
\]

\[
\sum_{g \in \Omega_g^L} P_{g,b,\tau}^c + \sum_{w \in \Omega_w^W} P_{w,b,\tau}^c - \sum_{l|s(l) = i} f_{l,b,\tau}^c + \sum_{l|r(l) = i} f_{l,b,\tau}^c
= \sum_{d \in \Omega_d^L} (P_{d,b,\tau} - P_{d,b,\tau}^{LS,c}); \quad \forall i, b, \tau, c \tag{a.14}
\]

\[f_{l,b,\tau}^c = A_{l}^{c}.B_{l}.(\theta_{l,b,\tau}^c - \theta_{j,b,\tau}^c); \tag{a.15}
\]

\[\forall b, \tau, c, l \in \Omega_{l^+}, s(l) = i, r(l) = j \]
\[-F_l^{\text{max}} \leq f_{l,b,\tau}^c \leq F_l^{\text{max}}, \quad \forall b, \tau, C, l \in \Omega^L \]  
(a.16)

\[-(1 - A_{l,i, \tau}^c) \cdot M \leq f_{l,b,\tau}^c - B_l \cdot (\theta_{l,b,\tau}^c - \theta_{j,b,\tau}^c) \leq (1 - A_{l,i, \tau}^c) \cdot M \]  
\forall b, \tau, C, l \in \Omega^L, s(l) = i, r(l) = j  
(a.17)

\[-A_{l,i, \tau}^c \cdot x_{l,\tau}, F_l^{\text{max}} \leq f_{l,b,\tau}^c \leq A_{l,i, \tau}^c \cdot x_{l,\tau} \cdot F_l^{\text{max}} \]  
\forall b, \tau, C, l \in \Omega^L, s(l) = i, r(l) = j  
(a.18)

\[0 \leq P_{g,b,\tau}^c \leq P_{g}^{\text{max}}, \quad \forall g, b, \tau, C \]  
(a.19)

\[0 \leq P_{d,b,\tau}^{LS,c} \leq P_{d,b,\tau}, \quad \forall d, b, \tau, C \]  
(a.20)

\[-\pi \leq \theta_{l,b,\tau}^c \leq \pi; \quad \forall i, b, \tau, C \]  
(a.21)

\[\theta_{l,b,\tau}^c = 0; \quad \forall b, \tau, C, i = \text{ref}. \]  
(a.22)

\[A_{g}^c \cdot (P_{g,b,\tau} - \Delta_{g}^{\text{max}}) \leq P_{g,b,\tau}^c \leq A_{g}^c \cdot (P_{g,b,\tau} + \Delta_{g}^{\text{max}}); \quad \forall g, b, \tau, C \]  
(a.23)

3.9. References


CHAPTER 4
SECURITY-CONSTRAINED GENERATION AND TRANSMISSION EXPANSION PLANNING WITH RISK OF CONTINGENCIES

4.1. Introduction

In a centralized generation and transmission expansion planning (G&TEP) framework, a central planner, e.g., the independent system operator, determines the plan that has the highest profit for the whole system. In this chapter, we propose a security-constrained G&TEP model taking into account the risk of possible $N - 1$ contingencies. Using the concept of risk, which is the product of probability and severity, non-identical consequences of different contingencies are modeled. The risk index of each individual contingency is added to the objective function of the G&TEP model. That is, in the proposed G&TEP model, high-risk contingencies are penalized more dominantly. As a result, the expenses for keeping the reliability of the power system during an expansion planning will be spent more wisely. Modeling the developed risk index in the objective function make the problem nonlinear. A linearization technique is developed to convert the problem into the MILP format.

4.2. Symbols

A. Indices and Sets:
- $C$: Index for contingencies.
- $d$: Index for demand.
- $g$: Index for generating units.
- $l$: Index for transmission lines.
- $n$: Index for buses.
- $o$: Index for operating conditions.
- $r(l)$: Receiving-end node of transmission line $l$.
- $s(l)$: Sending-end node of transmission line $l$.
- $\Omega^L$: Set of all existing transmission lines.
- $\Omega^{L+}$: Set of all candidate transmission lines.
- $\Omega^G$: Set of all existing generating units.
- $\Omega^{G+}$: Set of all candidate generating units.
\( \Omega_c^L \) Set of all contingencies in existing lines.
\( \Omega_c^{L+} \) Set of all contingencies in candidate lines.
\( \Omega_c^G \) Set of all contingencies in existing units.
\( \Omega_c^{G+} \) Set of all contingencies in candidate units.

B. Parameters:
\( A_g^c, A_l^c \) Parameter that is equal to 0 if unit \( g \) / line \( l \) is unavailable under contingency \( c \), and 1 otherwise.
\( B_l \) Susceptance of transmission line \( l \).
\( C_d \) Load-shedding cost of demand \( d \).
\( C_d^c \) Load-shedding cost of demand \( d \) in contingency \( c \).
\( C_g \) Production cost of generating unit \( g \).
\( F_l^{max} \) Capacity of transmission line \( l \).
\( I_l \) Investment cost of candidate transmission line \( l \).
\( I_g \) Investment cost of candidate generating unit \( g \).
\( M \) Large enough number, called big-M.
\( P_{do} \) Value of demand \( d \) in operating condition \( o \).
\( P_g^{Emax} \) Maximum capacity of existing generating unit \( g \).
\( P_g^{Cmax} \) Maximum capacity of candidate generating unit \( g \).
\( \alpha_l, \alpha_g \) Amortization rate for investment cost of transmission line \( l \) / generating unit \( g \).
\( \rho_o \) Weight of operating condition \( o \) [hours].
\( P_{IMW} \) Megawatt performance index.
\( P_{IVQ} \) Voltage-reactive power performance index.
\( \Delta_g^{max} \) Maximum adjustment capability of unit \( g \).

C. Variables:
\( x_l \) Binary decision variable to indicate whether candidate line \( l \) is constructed.
\( x_g \) Binary decision variable to indicate whether candidate generating unit \( g \) is constructed.
\( p_{go} \) Generation of unit \( g \) in operating condition \( o \).
\( p_{go}^c \) Generation of unit \( g \) in operating condition \( o \) during contingency \( c \).
\( f_{lo} \) Power flow through line \( l \) in operating condition \( o \).
\( f_{lo}^c \) Power flow through line \( l \) in operating condition \( o \) during contingency \( c \).
\( \theta_{n,o} \) Voltage angle at node \( n \) in operating condition \( o \).
\( \theta_{n,o}^c \) Voltage angle at node \( n \) in operating condition \( o \) during contingency \( c \).
\( \Gamma_{do} \) EENS for demand \( d \) in operating condition \( o \).
\( \Gamma_{do}^c \) EENS for demand \( d \) in operating condition \( o \) during contingency \( c \).
\( \gamma_{do}^c \) Auxiliary variable defined for linearization.

4.3. Security-Constrained G&TEP Formulation
Because of the computational complexity of the AC power flow model that makes G&TEP intractable for large-scale (even medium-sized) systems, the DC power flow is the most popular model among researchers [1]. In addition, the main purpose of G&TEP problem is to find the best decision for installing new generating units and transmission lines to optimize active power dispatch in the system. That is, reactive power dispatch and voltage regulation, which are local issues, can be compensated afterward. Therefore, in this research, the widely used DC power flow model is applied.

4.3.1. G&TEP formulation

A typical DC G&TEP model is presented is (1)-(13). The objective of the model is to minimize the investment cost (the first line of (1)) as well as the operation cost of existing and candidate generating units (the second line of (1)). The objective function (1) should be minimized over the set of decision variables Δ, which is expressed in (2). To make the model more realistic, different operating conditions (i.e., load blocks) are considered. The operating conditions represent demand realizations and have a weight of $\rho_o$ hours per year. In addition, a load shedding option is considered for each operating condition, which includes the penalty cost $C_d$ equal to the value of lost load (VOLL). The nodal power balance constraint is presented by (3). Note, having the option of load shedding for each operating conditions, $\Gamma_{do}$, the optimization problem is always feasible. Meaning, the applied G&TEP problem is feasible for any operating condition. Power flow constraints for the existing lines are presented by (4) and (5). Using a disjunctive technique [2], the linearized power flow constraints for candidate lines are modeled by (6) and (7). The limits of existing and candidate generating units are imposed by (8) and (9), respectively. The limitation of load shedding, voltage angle boundaries, and fixing the voltage angle of the reference bus to zero are modeled by (10)-(12), respectively. The binary variables are distinguished from the continuous
ones as shown in (13).

\[
\min \Delta \sum_{l \in \Omega^{L+}} I_l \cdot x_l + a_g \sum_{g \in \Omega^{G+}} I_g \cdot x_g + \sum_{o} \rho_o \left( \sum_{g} C_g \cdot P_{go} + \sum_{d} C_d \cdot \Gamma_{do} \right) 
\]

(1)

\[
\Delta = \{x_l, x_g, P_{go}, \Gamma_{do}, f_{lo}, \theta_{no} \}
\]

(2)

\[
\sum_{g \in \Omega_n^{G,G+}} P_{go} - \sum_{l \mid s(l) = n} f_{lo} + \sum_{l \mid r(l) = n} f_{lo} = \sum_{d \in \Omega_n^{D}} (P_{do} - \Gamma_{do}) \quad \forall n, o
\]

(3)

\[-F_{l_{\text{max}}} \leq f_{lo} \leq F_{l_{\text{max}}} \quad \forall l \in \Omega^{L}, o \]

(4)

\[f_{lo} = B_l \cdot (\theta_{s(l),o} - \theta_{r(l),o}) \quad \forall l \in \Omega^{L}, o \]

(5)

\[-x_l \cdot F_{l_{\text{max}}} \leq f_{lo} \leq x_l \cdot F_{l_{\text{max}}} \quad \forall l \in \Omega^{L+}, o \]

(6)

\[-(1 - x_l) \cdot M \leq f_{lo} - B_l \cdot (\theta_{s(l),o} - \theta_{r(l),o}) \leq (1 - x_l) \cdot M \quad \forall l \in \Omega^{L+}, o \]

(7)

\[0 \leq P_{go} \leq P_{g_{\text{max}}^{E}} \quad \forall g \in \Omega^{G}, o \]

(8)

\[0 \leq P_{go} \leq P_{g_{\text{max}}^{C}} \cdot x_g \quad \forall g \in \Omega^{G+}, o \]

(9)

\[0 \leq \Gamma_{do} \leq P_{do} \quad \forall d, o \]

(10)

\[-\pi \leq \theta_{no} \leq \pi \quad \forall n, o \]

(11)

\[\theta_{\text{ref},o} = 0 \quad \forall o \]

(12)

\[x_g, x_l \in \{0, 1\} \quad \forall g \in \Omega^{G+}, l \in \Omega^{L+} \]

(13)

4.3.2. N-1 security criterion modeling
To model the effect of \(N - 1\) security criterion, two penalty terms are added to the objective function (14) to improve the reliability of the planned system. These two new terms model the expected energy not supplied (EENS), \(\Gamma_{do}^c\), for each demand in the respective operation condition under contingency in a generating unit or a transmission line. Note that the objective function (14) should be minimized over the set of decision variables \(\Delta\) and \(\Delta^c = \{p_{go}^c, \Gamma_{do}^c, f_{lo}^c, \theta_{no}^c\}\). In addition, to consider \(N - 1\) security criterion, equations (15)-(24) must be valid for each contingency in the contingency set \(C\). The adjustment capability of generating units for preventive/corrective actions is modeled by (25).

\[
\begin{align*}
\min_{\Delta, \Delta^c} & \quad a_t \sum_{l \in \Omega^L} \bar{I}_l \cdot x_l + a_g \sum_{g \in \Omega^G} \bar{I}_g \cdot x_g + \\
& \quad \sum_{\forall o} \rho_o \left\{ \sum_{\forall g} C_g \cdot P_{go} + \sum_{\forall d} C_d \cdot \Gamma_{do} + \sum_{\forall c(\Omega^L, \Omega^G)} \sum_{\forall d} C_{do} \cdot x_g \cdot \Gamma_{do}^c \right\} \\
& \quad \sum_{\forall g \in \Omega^G} p_{go}^c - \sum_{\forall l|s(l) = n} f_{lo}^c + \sum_{\forall l|r(l) = n} f_{lo}^c = \sum_{d \in \Omega^G} (P_{do} - \Gamma_{do}^c); \\
& \forall n, o, c
\end{align*}
\]
\[-(1 - x_l \cdot A_c^l) \cdot M \leq f^c_{i_0} - B_i \cdot (\theta^c_{s(l),o} - \theta^c_{r(l),o}) \leq (1 - x_l \cdot A_c^l) \cdot M;\]

\[\forall \, l \in \Omega^{L+}, o, c\] (19)

\[0 \leq P^c_{go} \leq P_g^{Emax} \cdot A_g^c; \quad \forall \, g \in \Omega^c, o, c\] (20)

\[0 \leq P^c_{go} \leq P_g^{Cmax} \cdot x_g \cdot A_g^c; \quad \forall \, g \in \Omega^{G+}, o, c\] (21)

\[0 \leq \Gamma^c_{do} \leq P_{do}; \quad \forall \, d, o, c\] (22)

\[-\pi \leq \theta^c_{no} \leq \pi; \quad \forall \, n, o, c\] (23)

\[\theta^c_{ref,o} = 0; \quad \forall \, o, c\] (24)

\[A_g^c \cdot (P_{go} - \Delta_g^{max}) \leq P^c_{go} \leq A_g^c \cdot (P_{go} + \Delta_g^{max}); \quad \forall \, o, c\] (25)

### 4.3.3. Proposed mixed-integer linear model

Note that the security-constrained G&TEP model presented in (14)-(25) is a mixed-integer nonlinear programming (MINLP) problem. The nonlinearity is because of the product of binary variables \(x\) and continuous variables \(\Gamma^c_{do}\) in the last two terms of the objective function (14). Since MINLP problems are computationally expensive and their optimality gap is not guaranteed, it is valuable to linearize the model [3].

We define a continuous auxiliary variable \(\gamma^c_{do}\), which is equal to the product of the binary variable \(x\) and the continuous variable \(\Gamma^c_{do}\). Then, the set of contingencies is divided into four subsets, \(\{\Omega^c_L, \Omega^c_{L+}, \Omega^c_G, \Omega^c_{G+}\}\), representing contingencies in existing lines, candidate lines, existing units, and candidate units, respectively. Replacing the objective function (14) by (26), and adding constraints (27)-(32), the linear model of the problem is obtained.
\[
\min_{\Delta, \Delta^c} a_l \sum_{l \in \Omega^L} \tilde{l}_l \cdot x_l + a_g \sum_{g \in \Omega^G} \tilde{g}_g \cdot x_g + \\
\sum_{\nu o} \rho_o \left\{ \sum_{v g} C_g \cdot P_{g o} + \sum_{v d} C_d \cdot \Gamma_{d o} + \right. \\
\sum_{v c \in (\Omega^L_{c}, \Omega^L_{c}+)} \sum_{v d} C^{c}_{d o} \cdot \gamma^{c,l}_{d o} + \sum_{v c \in (\Omega^G_{c}, \Omega^G_{c}+)} \sum_{v d} C^{c}_{d o} \cdot \gamma^{c,g}_{d o} \left\} \\
\gamma^{c,l}_{d o} = \Gamma^{c}_{d o} ; \forall d, o, c \in \Omega^L_{c} \\
-x_i \cdot M \leq \gamma^{c,l}_{d o} \leq x_i \cdot M; \forall d, o, c \in \Omega^L_{c}+ \\
-(1-x_i) \cdot M + \Gamma^{c}_{d o} \leq \gamma^{c,l}_{d o} \leq (1-x_i) \cdot M + \Gamma^{c}_{d o}; \\
\forall d, o, c \in \Omega^L_{c}+ \\
\gamma^{c,g}_{d o} = \Gamma^{c}_{d o} ; \forall d, o, c \in \Omega^G_{c} \\
-x_i \cdot M \leq \gamma^{c,g}_{d o} \leq x_i \cdot M; \forall d, o, c \in \Omega^G_{c}+ \\
-(1-x_g) \cdot M + \Gamma^{c}_{d o} \leq \gamma^{c,g}_{d o} \leq (1-x_g) \cdot M + \Gamma^{c}_{d o}; \\
\forall d, o, c \in \Omega^G_{c}+ 
\]

Note that the outage of existing generating units and transmission lines are modeled by (27) and (30), respectively. For outage of a candidate line that is selected for installation \((x = 1)\), according to (29), variable \(\gamma^{c,l}_{d o}\) must be equal to the amount of load shedding under that contingency \((\Gamma^{c}_{d o})\), while no restriction will be imposed on the amount of \(\gamma^{c,l}_{d o}\) based on (28). On the other hand, for the outage of a candidate line that is not selected for installation \((x = 0)\), according to (28), the value of \(\gamma^{c,l}_{d o}\) must be equal to zero. Meaning, such outage is not possible, and therefore, its cost does not penalize the objective function. The same logic can be interpreted for outages in candidate
generating units according to (31) and (32). In summary, the MILP model of the security-constrained G&TEP problem is to minimize (26) subject to (3)-(13), (15)-(25), and (27)-(32).

4.4. Proposed Model For Risk of Contingencies

The developed G&TEP is an MILP model that can be efficiently solved by standard solvers [4]. However, one question should be answered: “what would be the value of load shedding penalty cost under a contingency ($C_{do}$) in the objective function (26)?” In this section, we propose to consider risk of each contingency for this purpose.

Concretely, it can be observed that the impact of each contingency on the power system operation is not the same. The probability and severity of outages are different. Therefore, the value of penalty coefficient $C_{do}$ should not be assumed identical for all contingencies. The concept of risk is proposed for the value of $C_{do}$ to cover the probability and severity of contingencies. Consequently, the expenses for keeping the reliability of the power system during an expansion will be planned more wisely.

4.4.1. Contingency probability

The probability of unscheduled outage of an individual component, mainly, depends on the failure rate of the component and the installation environmental condition. Conventionally, the Markov chain model has been used for estimating the probability of a contingency [5]. In this chapter, we assumed that the system has been observed over a long enough period, and therefore, historical data of outages are available. Using recorded historical data, a forced outage rate (FOR), $\lambda$, can be assigned to each individual component. For instance, if the number of forced outages for a specific component during the past five years are equal to \{1,2,0,3,1\}, the FOR of the component will be ($\lambda = 1.4$). The advantage of using the historical data of an operating component
for λ estimation is that both component failure rate and environmental condition are considered in
the estimation. By increasing the size of historical data and observing horizon, the estimated λ will
be closer to its true value.

After estimating λ, the probability of having at least one outage during the next planning year is
obtained using Poisson cumulative probability function (33), where x is the number of outages.

\[
P(x, \lambda) = \sum_{i=1}^{x} \frac{e^{-\lambda} \lambda^i}{i!}
\]  

(33)

4.4.2. Contingency severity

To model the severity of each contingency, two performance indices are proposed: MW
performance index and voltage-reactive power performance index. Note that for analyzing the
contingency severity using the performance indices, the annual growth rate of the peak load needs
to be considered. The value of demand \(d\) for operating condition \(o\) in year \(t\) is obtained by (34),
where \(P_{do1}\) is the amount of demand in the first year and \(P_{dot}\) is the amount of demand in year \(t\).

\[
P_{dot} = P_{do1}(1 + rate)^{t-1}; \forall d, o, t
\]  

(34)

The proposed MW performance index represents the change of active power of transmission
lines due to outage of another line or a generating unit. The MW performance index for operating
condition \(o\) under contingency \(c\) is calculated by (35), where \(f_{lo}^c\) is the change of power in line \(l\)
due to contingency \(c\), \(f_{lo}^{max}\) is the maximum thermal capacity of line \(l\), \(w_l\) is a desired weight
coefficient, and \(m\) is a specified exponent (e.g., \(m = 1\)). The change in real power of line \(l\) after
the occurrence of contingency \(c\) can be either obtained from the recorded historical data or
calculated using sensitivity factors of the system by (36) and (37), where \(F_0^c(0)\) and \(P_{g0}^c(0)\) are the
pre-fault values of line flow and unit production, respectively. Line outage distribution factor (LODF) and shift factor (SF) are sensitivity factors that can be directly calculated using the inverse of system admittance matrix [6].

\[ PI_{MW}(c, o) = \sum_{\forall l} \frac{w_l}{2m} \left( \frac{f_{lo}^c}{f_{lo}^{max}} \right)^{2n}; \forall c, o \]  

\[ f_{lo}^c = LODF_l^c \cdot F_{lo}^c(0); \forall l, c \epsilon \{ \Omega_L^c, \Omega_L^{L+} \} \]  

\[ f_{lo}^c = SF_g^c \cdot P_{go}^c(0); \forall l, c \epsilon \{ \Omega_G^c, \Omega_G^{G+} \} \]

This performance index is proposed to model the effect of an outage on post-contingency voltage violation and reactive power deficiency of certain buses. Violation in voltage of a certain bus is, usually, because of lack of enough reactive power transferred to that bus. This issue is more critical in weakly connected buses and outages that might lead to a system islanding [7]. Hence, considering this performance index in the planning step is advantageous. The voltage-reactive power performance index for operating condition \( o \) under contingency \( c \) is presented by (38), where \( V_{n,o}^c \) is the post-contingency voltage magnitude at bus \( n \), \( V_{n,o}^{rated} \) is the rated voltage magnitude at bus \( n \), \( \Delta V_{n,o}^{lim} \) is the voltage deviation limit at bus \( n \), \( Q_g^c \) is the reactive power produced by unit \( g \) after occurrence of contingency \( c \), and \( Q_g^{max} \) is the reactive power capacity limit of unit \( g \). Post-contingency voltages and reactive power of generation buses can be either obtained from the recorded historical data or calculated from AC load flow analysis for each contingency considering annual growing rate of the peak load (see (34)). Here, we assume that the historical data (e.g., obtained from PMUs) are available.

\[ PI_{VQ}(c, o) = \sum_{\forall n} w_n \left( \frac{|V_{n,o}^c| - |V_{n,o}^{rated}|}{\Delta V_{n,o}^{lim}} \right)^{2n} + \sum_{\forall g} w_g \left( \frac{Q_g^c}{Q_g^{max}} \right)^{2n}; \forall c, o \]
4.4.3. Risk indices

Having the performance indices for each operating condition and FOR of each component, the proposed risk index is calculated by (39), which includes both probability and severity of the contingency $c$. The proposed post-contingency load shedding penalty cost is defined based on the risk indices and VOLL as shown in (40). Note that VOLL ($C_d$) depends on the importance of the forecasted demand and is a fixed value for each demand $d$ in operating condition $o$.

$$RI(c,o) = P(x \geq 1, \lambda) \left( P_{IMW}(c,o) + P_{IVQ}(c,o) \right); \forall c,o$$ (39)

$$C^c_{d,o} = RI(c,o) \times C_d; \forall c,d,o$$ (40)

4.5. Case Study

The IEEE RTS 24-bus system is used to evaluate the performance of the proposed security-constrained G&TEP model. All simulations are carried out using GAMS and CPLEX 12.7 solver with default options [4]. A personal computer with an Intel(R) Xeon(R) CPU @2.6 GHz, including eight cores and 16 GB of RAM, is used.

A one-line diagram of the system is shown in Fig. 4.1. The system consists of 18 existing generating units, 17 loads, and 34 existing transmission lines. The system parameters are given in [2]. The peak load is assumed to be 880 MW. Five operating conditions with the load factors equal to $\{0.5, 0.65, 0.8, 0.9, 1\}$ and weights (hours per year) of $\{1510, 2800, 2720, 1120, 610\}$ are assumed [3]. Ten candidate lines and 18 candidate units are considered. The candidate lines’ data is given in Table 4.1.
Fig. 4.1. One-line diagram of the IEEE 24-bus system [2]

Table 4.1. Candidate lines’ parameters for the IEEE 24-Bus system

<table>
<thead>
<tr>
<th>Line No.</th>
<th>From bus</th>
<th>To bus</th>
<th>Capacity (MW)</th>
<th>Susceptance (1/Ohm)</th>
<th>Investment cost (M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>14</td>
<td>100</td>
<td>500</td>
<td>0.7</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>15</td>
<td>100</td>
<td>500</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>20</td>
<td>100</td>
<td>500</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>18</td>
<td>100</td>
<td>500</td>
<td>1.2</td>
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<tr>
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<td>1</td>
<td>22</td>
<td>100</td>
<td>500</td>
<td>1.3</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>23</td>
<td>100</td>
<td>500</td>
<td>1.1</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>19</td>
<td>100</td>
<td>500</td>
<td>0.9</td>
</tr>
<tr>
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<td>7</td>
<td>8</td>
<td>100</td>
<td>500</td>
<td>0.5</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>1</td>
<td>100</td>
<td>500</td>
<td>0.4</td>
</tr>
<tr>
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<td>7</td>
<td>2</td>
<td>100</td>
<td>500</td>
<td>0.6</td>
</tr>
</tbody>
</table>
Generation capacity, operation cost, and location of the candidate units are assumed to be the same as the existing units. The investment cost of each candidate unit is assumed to be 50 Million dollars. The amortization rate for both candidate transmission lines and generating units is 20% ($a_l = a_g = 0.2$).

For the studied system, 80 single outages are considered, which are 34 existing lines, ten candidate lines, 18 existing units, and 18 candidate units. Three scenarios are studied: G&TEP without contingency modeling, security-constrained G&TEP with identical contingency modeling, and the proposed security-constrained G&TEP.

Scenario 1: The $N - 1$ security criterion is ignored. That is, load shedding penalty cost under a contingency ($C^c_{do}$) in the objective function (26) is set to zero, and all contingency constraints are eliminated. The objective function is 91.536 Million dollars. Candidate lines 1, 2, 5, 9 and candidate units 1-3 and 5 are selected for installation.

Scenario 2: In this scenario, the $N - 1$ security criterion with identical post-contingency penalty costs is considered. Meaning, contingencies are assumed to have the same probability and severity. All risk indices are set to 1 over the number of contingencies (e.g., 1/80). The objective function is 94.497 Million dollars. Candidate lines 1, 2, 4, 5, 10 and candidate units 1-3 and 5 are selected for installation. The additional cost of installing extra candidate lines in this scenario is, indeed, the reliability cost, which must be paid to keep the reliability of the system within an acceptable level.

Scenario 3: In this scenario, the proposed risk indices are applied to the security-constrained G&TEP problem.

Figure 4.2 depicts the values of the 80 risk indices calculated for the RTS 24-bus system. Five critical contingencies exist, which are associated with the outage of existing lines 7-8, 11-13,
15-24, and the outage of existing and candidate units located on bus 13. This means that the value of the load shedding penalty cost under a contingency (C_{do}^c) is dominant for these five outages. The objective function is 93.996 Million dollars. As compared to scenario 2, one additional candidate line, i.e., candidate line 8, is selected for installation in this scenario. Note, the selection of candidate line 8 in scenario 3 is because of the high-risk index of existing line 7-8. An example of a high-risk contingency is the outage of existing line 7-8 that causes islanding of bus 7 (see Fig. 1). Consequently, installation of candidate line 8, which is a new path between buses 7 and 8, is an appropriate decision to avoid this high-risk contingency.

![Risk Indices Chart](image)

**Fig. 4.2.** The values of 80 risk indices of the IEEE 24-bus system

A comparison between results of the three scenarios is presented in Table 4.2. By installing candidate line 8, the annual cost of post-contingency load shedding will reduce considerably. This illustrates the advantageous performance of the proposed risk index-based model for G&TEP. Note that in Table 4.2, the annual investment cost for candidate lines is the planning decision that
must be paid. The annual cost of generating units will be paid if the forecasted demand is realized in the real-time operation, and the annual penalty cost of post-contingency load shedding is just a quantity which shows the reliability and robustness of the planned system.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Obj. function [M$]</th>
<th>Investment cost for candidate lines [M$]</th>
<th>Candidate lines to be installed</th>
<th>Investment cost for units [M$]</th>
<th>Units to be installed</th>
<th>Cost of generating units [M$]</th>
<th>Cost of load shedding in normal operation [M$]</th>
<th>Cost of post-contingency load shedding [M$]</th>
<th>CPU time [Min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>91.536</td>
<td>0.64</td>
<td>1, 2, 5, 9</td>
<td>40</td>
<td>1-3, 5</td>
<td>50.896</td>
<td>0</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>94.497</td>
<td>0.92</td>
<td>1, 2, 4, 5, 10</td>
<td>40</td>
<td>1-3, 5</td>
<td>51.257</td>
<td>0</td>
<td><strong>2.320</strong></td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>93.996</td>
<td>1.02</td>
<td>1, 2, 4, 5, 8, 10</td>
<td>40</td>
<td>1-3, 5</td>
<td>51.303</td>
<td>0</td>
<td><strong>1.673</strong></td>
<td>25</td>
</tr>
</tbody>
</table>

4.6. Conclusion

A security-constrained G&TEP model with respect to the risk of possible N-1 contingencies was proposed in this paper. A linearization technique was developed to make the model MILP. The proposed model took advantage of the concept of risk indices to consider the non-identical probability and severity of individual contingencies. The numerical analysis of the proposed model on the IEEE RTS 24-bus system shown the advantages of reducing post-contingency load shedding cost after considering the severity and probability of dominant contingencies.

4.7. References


CHAPTER 5
A MIXED-INTEGER LINEAR MODEL FOR GIS-BASED RESILIENT FEEDER ROUTING

5.1. Introduction

One important task in power distribution system expansion planning is feeder routing. The feeder routing problem is to determine the optimum route from a medium voltage substation to load points and the optimum size of conductors to be installed. This chapter proposes a mixed-integer linear programming model for the resilient feeder routing problem using geographical information system (GIS) facilities. It is proven in the chapter that having GIS facilities will lead to a better feeder routing scheme than the scheme obtained using electrical points. The uncertainty of rooftop solar generations and demand forecasting errors are taken into account in the proposed model, and a stochastic programming-based solution algorithm is developed. The proposed model and solution algorithm are comprehensive from several practical aspects such as economic objectives (installation cost, power losses, resiliency), technical constraints (voltage drops, radially constraint, reliability), and geographical constraints (obstacles, right-of-ways, high-cost passages). The efficiency of the algorithm is elaborated for a small-scale system, and it is further illustrated for a realistic large-scale system.

The chapter contributes to the field from the following standpoints:

- A mathematical proof is provided to demonstrate that the optimality of the FR solution is enhanced with the GIS-based model.
- Based on a proposed representing graph, a stochastic mixed-integer programming (MIP) model is formulated for the GIS-based FR taking into consideration the uncertainty of rooftop solar generations and demand forecasting errors. The solution of the model is optimal from multiple aspects, i.e., investments, power losses, and resiliency, while it is feasible from the voltage drop
and reliability aspects.

- With GIS data, a cost model of distribution feeder resiliency is formulated. The proposed cost is appended to the FR problem’s objective function to keep the solution optimality from the resiliency perspective.
- Finally, a solution algorithm based on stochastic programming is proposed to solve the GIS-based resilient FR model.

The remainder of the chapter is organized as follows. Explanations of the GIS-based FR model and its advantage over the conventional FR model are given in section 5.2. The proposed representing graph for the FR optimization is discussed in section 5.3. The mathematical MIP formulation and the solution algorithm are presented in sections 5.4 and 5.5, respectively. Numerical results are discussed in section 5.6. Concluding remarks are provided in section 5.7.

5.2. GIS-Based Feeder Routing

The GIS-based FR model has several advantages over the conventional models ignoring GIS nodes. For instance:

- The GIS-based FR model is more realistic due to the capability of modeling barriers and obstacles that are mapped with the geographical situation of the environment. As will be shown later, this benefit of GIS-based model is very compatible with studying resiliency aspect of the planned distribution system.
- If only electrical nodes are considered in the model, laterals (branches with lower thermal limits that are separated from the mainline by fuses) are restricted to start from an electrical node. Meaning, the lateral nodes are forced to be located on an electrical node. This might degrade the optimality level of the final FR solution. For instance, consider the simple illustration depicted in Fig. 5.1.
Fig. 5.1. Illustration of lateral nodes spot; (a) without GIS nodes, (b) with GIS nodes.

Two load points, L1 and L2, must be connected to the substation nodes, which is shown by a square-shape node. In case no GIS nodes exist, the optimal solution is obtained in the case that the lateral node is located in either electrical node L1 or L2 (Fig. 5.1(a)). However, with GIS nodes, the optimal solution is obtained when the lateral node “M” is located in a GIS node (Fig. 5.1(b)). Considering the distances between nodes as shown in Fig. 5.1, it can be observed that the solution (we assume that solution only depends on distance of nodes) of the GIS-based model \(1 + 2\sqrt{2} \approx 3.83\) is better than the case without GIS nodes \(2 + \sqrt{5} \approx 4.24\).

- Multiple right-of-ways between two adjacent electrical nodes can be defined by having GIS nodes. That is, there will be several options to connect two adjacent electrical nodes each of which has a specific cost related to geographical condition (see Fig. 5.1). However, ignoring GIS nodes, there is only one candidate path (right-of-way) between two adjacent electrical nodes, and the feasible design space of the optimization problem is more limited. By knowing that there will be geographical obstacles, high-cost passages, and ownership issue of lands, having alternative paths to connect two electrical nodes is more realistic and gives more freedom to the optimization algorithm.

- Finally yet importantly, when the GIS nodes are ignored, the cost of the FR problem can be as high as two times of the GIS-based FR optimum cost. This is proven in Proposition 1.

**Definition:** Given an undirected graph \(G = (V, E)\), a cost function for each edge, and a partition of \(V\) into two sets \(O\) and \(S\), the problem of finding a minimum cost tree that contains all vertices
in \( \mathcal{O} \) and any subset of the vertices in \( \mathcal{S} \) is called Steiner tree problem. If the set of vertices \( \mathcal{S} \) is empty, and \( \mathcal{O} = \mathcal{V} \), it is a minimum spanning tree problem.

**Proposition 1:** Given that cost for each feeder depends only on the length of that feeder, the FR solution without GIS nodes is within a factor of 2 from the GIS-based FR optimal solution.

**Proof:** Considering radiality constraints of the planned network, the GIS-based FR problem is, indeed, the Steiner tree problem [1], while the FR problem without GIS nodes is the approximated minimum spanning tree problem of the corresponding Steiner tree problem. Let us consider the graph in Fig. 5.2 (a) as the optimum Steiner tree (GIS-based FR) solution, where \( \mathcal{O} = \{1, 2, 3, 4\} \) and \( \mathcal{S} = \{M, N\} \) is the set of GIS nodes. If we double each edge, we get an Eulerian graph (a connected graph including even degree vertices). According to the Eulerian graph property, there exists an Eulerian tour \((1 \rightarrow M \rightarrow 2 \rightarrow M \rightarrow N \rightarrow 3 \rightarrow N \rightarrow 4 \rightarrow N \rightarrow M \rightarrow 1)\) that its cost is 2 times the optimum Steiner tree cost. The Eulerian tour, which includes both vertices of \( \mathcal{O} \) and \( \mathcal{S} \), is a Hamiltonian cycle on \( \mathcal{O} \) (see Fig. 5.2 (b)). Note that the cost of path \((1 \rightarrow 2)\) in Fig. 5.2(b) is equal to the cost of path \((1 \rightarrow M \rightarrow 2)\) in Fig. 5.2(a). Removing one optional edge from the Hamiltonian cycle results in a minimum spanning tree of \( \mathcal{O} \) with the cost at most 2 times the optimum Steiner tree cost (see Fig. 5.2(c)). Thus, the cost of minimum spanning tree on \( \mathcal{O} \) (i.e., FR solution without GIS nodes) is within a factor of 2 from the cost of the Steiner tree (i.e., GIS-based FR optimal solution). This concludes the proof of Proposition 1.  

\[ \square \]

![Fig. 5.2. Illustrative proof for optimality of GIS-based FR solution.](image_url)
5.3. MIP Model With Representing Directed Graph

Proposition 2: Given that the cost of each feeder depends on the length and energy loss of that feeder, the GIS-based FR problem is NP-hard.

Proof: First, assume that the cost of each feeder (edge) depends only on its length. As mentioned before, in this case, the GIS-based FR problem is the Steiner tree problem. It is proven in 1972 by Richard Karp that the Steiner tree problem is NP-hard [2]. Meaning, if $P \neq NP$, there is no polynomial time algorithm to find the solution of the Steiner tree problem. Knowing that the dependency of cost of each edge to the energy loss will increase the computational complexity of the problem, it holds that the GIS-based FR problem is NP-hard. □

Because of the NP-hardness of the GIS-based FR problem and the recent progress in integer programming, we propose an MIP model for the GIS-based FR problem. The proposed MIP model considers power system constraints, while the accuracy of the solution can be measured by the reported duality gap.

It is worth mentioning that dynamic programming is not a good option for the GIS-based FR problem for two main reasons. First, the principle of optimality does not hold for the GIS-based FR (Steiner tree) problem [2]. By definition, “a problem is said to satisfy the principle of optimality if the subsolutions of an optimal solution of the problem are themselves optimal solutions for their subproblems [3].” More details about this property and its role in combinatorial optimization are presented in [3]. Therefore, the solution obtained from dynamic programming for the GIS-based FR problem is just an approximation. Second, as presented in [4], the accuracy of this approximation cannot be measured.

To construct an MIP model for the GIS-based FR problem, we propose a representing directed graph in Euclidean plane as shown in Fig. 5.3(a). The substation node is located in the center of
the graph named as stage zero \((k = 0)\). Dashed red lines distinguish other stages \((k = 1, 2, \ldots)\).

The specific property of this directed graph is that there is no path from farther stages to closed stages. Having this feature and knowing that the optimal solution is a subset of the representing graph, the radially constraint of the solution is guaranteed. Meaning, there will be no cycle in the optimal solution. Each edge in the representing graph includes parallel candidate paths each of which representing a candidate conductor size (see Fig. 5.3(b)). The mathematical formulation of the proposed MIP GIS-based FR model constructed based on the representing graph is presented in the next section.

![Graph representation](image)

Fig. 5.3. Representing directed graph in Euclidean plane for the GIS-based FR problem: a) network graph and b) possible edges (different conductor size) between two adjacent nodes.
5.4. Mathematical MIP Formulation

A. Cost of Feeder:

The total cost of a conductor is divided into three components, i.e., initial installation cost, annual operation and maintenance cost, and losses. Initial installation cost $c_i$ is a constant cost that should be paid at the time of building the line. This cost differs for different conductor sizes since larger conductors need heavier hardware\(^5\). Annual operation and maintenance cost $c_{O&M}$ is also higher for larger conductors since their failure affects a larger number of customers. The present worth of the operation and maintenance cost with a discount rate of $d$ over a period of $n$ years is calculated by the present worth factor $\omega_1$ as shown in (1). Thus, the fixed part of the cost per length ($$/mile) is modeled as (2).

\[ \omega_1 = \frac{(1 + d)^n - 1}{d(1 + d)^n} \]  

(1)

\[ a_1 = c_i + (\omega_1 \times c_{O&M}) \]  

(2)

The variable cost depends on energy losses, which are a function of load. This cost is increased yearly because of load growth. For a specific load point for which the peak load in the first year of operation is $P$ MW, the power factor is $PF$, and the line-to-line voltage is $V$ kV, the required current is calculated by (3). Hence, for a three-phase line with a resistance of $r$ ohm and an annual loss factor of $L_f$, the total energy loss per mile for the first year is calculated by (4).

\[ I = \frac{10^3 \times P}{\sqrt{3} \times V \cdot PF} \]  

(A)  

(3)

\[ \ell = 3 \times r \cdot I^2 = 8760 \times \frac{10^3 \times L_f \cdot r \cdot P^2}{(V \cdot PF)^2} \text{ (kWh/mile)} \]  

(4)

Note that having a fixed power factor, the annual loss factor, which is the ratio of the average loss to the loss at the peak load, can be obtained directly from the conductor resistance and the
annual load profile [6].

To obtain the cost of energy loss in $/mile for the first year, (4) is multiplied by the cost of energy \( e_c \) ($/kWh). Considering an annual increasing rate \( \sigma \) for the peak load, energy loss will increase at a rate of \( \dot{\sigma} = \sigma^2 + 2\sigma \). Therefore, to derive the present worth of the losses with a discount rate of \( d \) over a period of \( n \) years, the value of energy loss for the first year is multiplied by a present worth factor \( \omega_2 \) [7].

\[
\omega_2 = \frac{1 - \left(\frac{1 + \dot{\sigma}}{1 + d}\right)^n}{d - \dot{\sigma}} \tag{5}
\]

Finally, the total present worth cost per mile for a specific conductor is calculated by (1)-(7) considering installation cost and the present worth of operation, maintenance, and losses.

\[
b = 8760 \times \frac{10^3 \times L_f \cdot r \cdot e_c \cdot \omega_2}{(V \cdot PF)^2} \tag{6}
\]

\[
\omega_c = a_1 + bP^2 \quad ($/mile) \tag{7}
\]

It can be observed in (7) that the total cost for each conductor in terms of peak load is a quadratic function.

Fig. 5.4. Economic characteristics of four conductors.
As shown in Fig. 5.4, four quadratic curves, corresponding to different conductors, intersect with each other. That is, for a specific peak load, there is only one type of conductor with the minimum cost [8]. In other words, the economic loading limit of a conductor, which is always less than its thermal loading limit, is at the intersecting point of its curve with the curve of the next higher conductor (see Fig. 5.4). For instance, ACSR-16 is the optimum conductor size for peak loads less than 0.61 MW, and ACSR-95 is the optimum conductor size for peak loads higher than 1.97 MW.

B. Concept of Reach for Voltage Drop

To meet the permissible voltage drop for the GIS-based FR solution, the concept of *economic load reach* is used. The economic load reach of a conductor is defined as the maximum distance that the conductor can carrying power equal to its maximum economic loading limit without violating the voltage drop limits [8]. Having the approximate voltage drop per mile for a distribution feeder as (8), the reach ($\mathcal{R}$) of a conductor can be calculated by (9).

$$\frac{\%V_{\text{drop}}}{\text{mile}} = 100 \times \frac{\sqrt{3} \times I(r.\cos \theta + x.\sin \theta)}{1000 \times V} \quad (8)$$

$$\mathcal{R} = \frac{1000 \times V(\Delta V_{100})}{\sqrt{3} \times I(r.\cos \theta + x.\sin \theta)} \quad (\text{mile}) \quad (9)$$

where $\cos \theta$ is the power factor, $x$ represents the per mile inductive reactance of the conductor, and $\Delta V$ is the percentage of permissible voltage drop. If we set $I$ to the current carrying by a conductor at the point of its thermal/economical loading limit, this reach is the thermal/economical loading reach of the conductor. It is common in the distribution system design to set the reach of all conductors to a selected value. By changing the value of the desired reach, the economical curves of conductors are rescaled align with the axes of Fig. 5.4, and the economic loading points (intersecting points) are moved. This changes the cost function, and as a result, the solution of the
FR problem. Therefore, by adjusting the value of reach, it is possible to obtain an optimal FR solution that is not violating voltage drop limits. Note, after rescaling, the quadratic form of the curves and the existing of intersecting points are still valid [5]. Thus, a mixed-integer quadratic programming (MIQP) model can be applied.

C. Stochastic MIQP Model for GIS-based FR

Having economic curves of different conductors and the proposed representing graph as shown in Fig. 5.3, the deterministic MIQP model of the GIS-based FR problem can be constructed. The objective function is modeled as (10).

\[
\min \sum_{(i,j) \in E} d_{ij} (a_{ij} I_{ij} + b_{ij} P_{ij}^2)
\]

where indices \(i\) and \(j\) represent GIS nodes (centers of GIS pixels). An edge \((i, j)\) belongs to the set of all possible edges \(E\). Parameter \(d_{ij}\) is the distance between GIS nodes \(i\) and \(j\). Note that the distances between GIS nodes are not the same since the geographical elevations of GIS nodes are different. Parameter \(b_{ij}\) is the variable cost of edge \((i, j)\) which can be obtained by (6). Parameter \(a_{ij}\) is the fixed cost of edge \((i, j)\) which can be calculated by (2) plus other penalizing (positive) costs or incentive (negative) costs (i.e., \(a = a_1 + a_2 + a_3 + \cdots\)). These penalizing/incentive costs can be defined according to environmental situation for each edge. For instance, an edge that is close to a road should be incentivized since it is more accessible for operation and maintenance. On the other hand, an edge close to a commonly flooded area should be penalized since it deteriorates the reliability of the FR solution. More information for how to set penalizing/incentive costs can be found in [9]. For edges that are geographically impossible for installing a line, the parameter \(a_{ij}\) must be set to a large enough value in order to force the model not to select it. For all other edges, both \(a_{ij}\) and \(b_{ij}\) should be normalize. It will help the solution algorithm to handle
the model more efficiently. Continuous variable $P_{ij}$ is the decision variable representing the amount of power that should be carried out if the line installed at edge $(i,j)$. Binary variable $l_{ij}$ is the decision variable representing the installation status of a candidate conductor at edge $(i,j)$. It is equal to one if the conductor is going to be installed at edge $(i,j)$, and zero otherwise. In addition, for all existing feeders $l_{ij}$ must be fixed to one.

However, the FR problem is among steps of long-term distribution network expansion planning, and it requires long-term forecasting of loads and solar generations connected to each node. Having the forecasting errors, a stochastic framework is necessary to cope with the deviations from the forecasted values. There are two types of decisions that need to be made in the FR problem; i.e., the *here and now* decision ($l_{ij}$) and the *wait and see* decision ($P_{ij}$), which depends on the realization of a series of random scenarios. Usually, a two-stage stochastic optimization is applied in this situation. Note, $PD_i$, the peak value of net loads connected to vertex $i$, is the only uncertain parameter in the model. In fact, the value of $PD_i$ is the realization of the peak demand of node $i$ minus realization of the solar generations connected to that node. Since there is only one uncertain parameter in the model ($PD_i$), the two-stage stochastic optimization model can be simplified in one stage as presented by (11) (see [10] for more details of the one-stage equivalent of a two-stage stochastic optimization problem with only one uncertain parameter and a limited number of realizations). The second term of the objective function is replaced by the expected value of the variable (operational) cost, which is a sum over all possible realizations ($P_{ij}^s$). Here, a finite number of realizations with associated weight $\rho_s(x)$ is assumed. Note that the summation of weights over all scenarios must be equal to one (i.e., $\sum_{s \in S} \rho_s = 1$).

Constraints of the proposed model, which are presented in (12)-(16), must be satisfied for all possible realizations. The nodal power balance for each vertex (GIS node) is modeled by (12).
min \sum_{(i,j) \in E} \{d_{ij} \cdot a_{ij} \cdot I_{ij} + E_s[Q(P_{ij}, S)]\} \\
E_s[Q(P_{ij}, S)] = \sum_{(i,j) \in E} \sum_{s \in S} \rho_s \cdot d_{ij} \cdot b_{ij} \cdot (P_{ij}^s)^2 \\
\sum_{j|(j,i) \in E} P_{ij}^s - \sum_{j|(i,j) \in E} P_{ij}^s = PD_i^s ; \quad i \in V, s \in S \\
0 \leq P_{ij}^s \leq P_{ij}^{max} \cdot I_{ij} ; \quad (i,j) \in E, s \in S \\
\sum_{(i,j) \in E} I_{ij} \geq F ; \quad i = \text{Substation vertex} \\
I_{ij} \in \{0,1\} ; \quad (i,j) \in E \\
\sum_{i \in V} PD_i^s = 0 ; \quad s \in S \\

Parameter \( PD_i^s \) is the forecasted peak value of net demands connected to vertex \( i \) in scenario \( s \).

As modeled by (13), the maximum thermal limit for each conductor \( P_{ij}^{max} \) is used to limit the power. The minimum number of feeders starting from substation node is restricted by (14). The setting parameter \( F \) is used to obtain the required reliability of the FR solution. Definition of the binary variables is considered in (15). Note that the value of \( PD_i^s \) is negative for substation node as restricted by (16). Since we have binary variables \( (I_{ij}) \) in the model, and the model includes a quadratic term in the objective function \((P_{ij}^s)^2\), the proposed model is MIQP.

D. Stochastic MIP Model for GIS-based FR

The proposed MIQP model includes a large number of binary variables. The number of binary variables for the proposed model is equal to the number of defined edges \(|E|\). As will be shown by numerical studies, solving an MIQP model including a large number of binary variables by standard solvers might be time consuming. Therefore, proposing a more tractable linearized
version of the GIS-based FR problem is desired. It will be numerically illustrate in the Numerical Results section that the proposed MIP model is solved much faster in compare with the MIQP model.

Here, we applied a piecewise linear approximation of the economic cost curves of conductors using equal segmentation technique. It is obvious that other linearization techniques are conceivable for this purpose [11]. The proposed MIP model for GIS-based FR is presented by (17)-(22).

\[
\begin{align*}
\min & \sum_{(i,j) \in E} \{d_{ij} \cdot a_{ij} \cdot I_{ij} + \sum_{s \in S} \sum_{\forall \gamma} p_s \cdot d_{ij} \cdot m_{ij,\gamma} \cdot P_{ij,\gamma}^s \} \\
\sum_{j|(j,i) \in E} \forall \gamma & P_{ji,\gamma}^s - \sum_{j|(i,j) \in E} \forall \gamma P_{ij,\gamma}^s = PD_{i}^s \; ; i \in V, s \in S \\
0 & \leq P_{ij,\gamma}^s \leq P_{c,\gamma}^{max} \cdot I_{ij} ; \; (i,j) \in E, s \in S, \forall \gamma \\
\sum_{(i,j) \in E} I_{ij} & \geq F \; ; \; i = \text{Substation vertex} \\
I_{ij} & \in \{0,1\} \; ; \; (i,j) \in E \\
\sum_{i \in V} PD_{i}^s & = 0 \; ; \; s \in S
\end{align*}
\]

Where \(m_{ij,\gamma}\) is the slope of segment \(\gamma\) of the cost curve of edge \((i,j)\), and \(P_{ij,\gamma}^s\) is the continuous variable for power flow of edge \((i,j)\) corresponds to segment \(\gamma\) in scenario \(s\).

E. Resiliency Modelling for GIS-based FR

The main objective of grid resiliency is to reduce the magnitude of events and to alleviate consequences that occur as a result of disruptions. The consequences are closely related to power delivery and grid operation. Therefore, most of the performance metrics defined for resiliency modeling are consequence-based metrics [12]. A simple illustration of three stages of system
operation during an event (i.e., normal operation, disruption, and restoration) is depicted in Fig. 5.5. As it can be observed in Fig. 5.5, the grid resiliency is improved by reducing magnitude of event and restoration time.

![Illustrative of resiliency concept](image-url)

Fig. 5.5. Illustrative of resiliency concept.

In this chapter, we introduce an economic resiliency metric that can be included in the proposed GIS-based FR model. The consistency of exact geographical location of power system components with the GIS nodes is applied in the metric definition. Knowing the GIS nodes correspond to the geographical location of the repair and maintenance centers (RMCs), the restoration time for edge \((i, j)\) is obtained by (23). The restoration time depends on the distance of edge \((i, j)\) from closest RMC, average speed of repair team, and required time to repair edge \((i, j)\) as well as nearby components if necessary. Eventually, the total resiliency cost of the system can be calculated by (24). The magnitude of outages is modeled by MW flow carried by edge \((i, j)\) if installed \((P^S_{ij,y})\).

Note that the restoration time for each edge is known, and the value of lost load (VoLL), which has a unit of dollars per MWh, is assumed to be known [13]. Adding the resiliency cost to the MIP model, the objective function of the GIS-based FR problem including resiliency cost is modeled by (25).
\[
\tau_{ij} = \frac{\text{Distance from the closest RMC}}{\text{Average speed of repair team}} + \text{Repair time of edge (ij)} \quad (23)
\]

\[
C_{res} = \sum_{(i,j) \in E} \sum_{s \in S} \sum_{\forall y} \rho_s \cdot \tau_{ij} \cdot VoLL \cdot P_{ij,y}^s \quad (24)
\]

Finally, to obtain the optimal and resilient GIS-based FR solution, (25) is minimized subject to (18)-(22).

\[
\min \sum_{(i,j) \in E} \{d_{ij} \cdot a_{ij} \cdot I_{ij} + \sum_{s \in S} \sum_{\forall y} \rho_s \cdot (d_{ij} \cdot m_{ij,y} + \tau_{ij} \cdot VoLL) \cdot P_{ij,y}^s \} \quad (25)
\]

5.5. Solution Algorithm

A. Solution Steps

The flowchart of the proposed algorithm for the GIS-based FR problem is shown in Fig. 5.6.

Step 1: The first step is to map the representing graph to the GIS image of the system with the desired resolution (number of stages in representing graph), and read available historical data of load and PV generations correspond to each load point.

Steps 2-4 (uncertainty modeling): A probabilistic load forecasting technique is applied to find a PDF that is the best fit to the probabilistic behavior of the net demands (i.e., load minus PV generation). As experimentally demonstrated in [14], the daily peak loads of distribution feeders follow the power law distribution. The parameters of the power law distribution can be obtained using maximum likelihood estimation (MLE) and Kolmogorov-Smirnov test, while the uncertainty of the model is quantified by bootstrapping. Having the null hypothesis of the test as “the observed data follow the power law distribution”, the \( p \)-value will be greater than the significance level of the test. Meaning that the null hypothesis cannot be rejected. However, for other distributions, e.g., exponential, gamma, and lognormal, the \( p \)-value will be less than the
significance level, and the null hypothesis is rejected [15]. We applied the approach presented in [14] for this study. After estimating the power law distribution of each demand, a large enough number of scenarios can be generated. A multivariate K-means algorithm is applied to reduce the number of generated scenarios. Using K-means, the large number of initial scenarios is clustered into several groups. This leads to reducing the computational complexity of the problem while keeping an acceptable accuracy level of the results. Because, with a large number of scenarios, only a limited number of them are unique, and redundant scenarios have insignificant impact on the model [16]. To implement scenario clustering with K-means, a specific number of centroids is assumed. Then, each scenario is assigned to its closest centroid to form a cluster. Finally, the means of clusters are selected as new centroids, and the loop is continued until centroids do not move between two iterations. To alleviate the dependency of the K-means algorithm to the initial centroids, we applied a smart initialization approach, called K-means++, that is proposed by [17]. Introducing $D$ as the shortest distance of a data point from the closest centroid, the K-means++ algorithm selects initial centroids one at a time according to the $D^2$-weighting. It was proved that the solution of K-means++ is generally better than the conventional K-means approach [17].

Step 5 (solving MIP model): The proposed stochastic MIP model of the GIS-based FR problem can be solved, which is minimization of (25) subject to (18)-(22).

Steps 6-7 (voltage drop calculation): After obtaining the optimal solution, the voltage drop of each feeder is calculated by (8). If any impermissible voltage drop is observed, the value of reach must be increased in the next iteration of step five (see (9)).

Steps 8-9 (reliability evaluation): The reliability violation of the solution is corrected by increasing the minimum degree of the substation node (see Fig. 5.6).

Steps 10: Finally, an optimal solution that is within the permissible voltage drop and reliability
limits is obtain in the last step.

Fig. 5.6. Flowchart of the solution algorithm.
B. Discussion on Computational Complexity

One might be concerned that the proposed GIS-based MIP model includes a significant number of binary variables, and it is impractical to solve. We provide following discussions on computational complexity of the proposed method:

- First, as it was proved earlier in this paper, the GIS-based FR problem is NP-hard. That is, like any other NP-hard problem, there is no guarantee that it can be solved with reasonable solution time when the size of the problem grows up dramatically. However, as it is illustrated in the Numerical Results section, the solution time is acceptable for the real world systems.

- Second, in realistic cases, a considerable number of binary variables is known before solving the problem due to existence of the edges and geographical barriers. This is more dominant in an urban area.

- Third, parallel processing would be a practical solution, like for any other large-scale optimization problems.

- Finally, several adjusting parameters can be used to provide an acceptable tradeoff between solution time and accuracy of the results, i.e., setting the optimality gap of the solver, selecting the number of sections for the piecewise linear approximation, and the number of GIS stages (GIS resolution). These adjusting parameters can be set wisely to improve the solution time for large-scale problems.

5.6. Numerical Results

The proposed algorithm is tested on a small test system and a realistic large-scale system. A nominal voltage of \( V = 20 \, kV \) is assumed, and the permissible voltage drop is between 0.95 and 1.05 per unit. Four types of conductors with economic characteristics shown in Fig. 5.4 are selected. The economic loading reach of conductors is initially set to 4.7 miles with a step size of 0.1 miles. All simulations are carried out using ILOG CPLEX’s MIP solver and BARON’s NLP
solver in GAMS environment [18]. A computer with two Intel(R) Xeon(R) CPU @2.1 GHz, including 32 cores and 512 GB of RAM, is used for simulations.

A. Small Test system

This case is studied to illustrate the effect of having GIS nodes and to evaluate the presence of uncertainties in the FR problem. The test system is shown in Fig. 5.7(a). The annual peak load of each load point is depicted in the diagram, and the cost of resiliency is ignored [4]. It is assumed that each load point includes a solar photovoltaic system with a size (kWp) of 10% of the annual peak load. The data of solar generations are acquired from Ausgrid study on 300 solar homes from July 2010 to June 2013 [19]. For the sake of simplicity and without loss of generality, the elevation of all GIS nodes is assumed to be the same. Four cases are studied:

Case 1: The nonlinear FR model without GIS nodes is solved in GAMS by BARON. Ignoring GIS nodes, only eight labeled electrical nodes remain, i.e., seven load points plus substation node. We implement a complete enumeration to find the global optimum planning result. The total number of possible trees on $|\mathcal{V}|$ labeled vertices is calculated by Cayley’s formula ($|\mathcal{V}|(|\mathcal{V}|−1)$). Therefore, $8^6 = 262,144$ possible radial configurations exist. Given the system configuration, the problem is simplified to a minimum flow with nonlinear cost problem for each possible configuration. The nonlinear cost function for each edge is, in fact, the lowest part of the combined economic characteristics of all conductors (see Fig. 5.4). The optimum planning configuration shown in Fig. 5.7(b) is obtained with the objective function of $0.4275M$.

Case 2: The MIQP GIS-based feeder routing model is solved by BARON to be comparable with the results of the previous case. BARON’s options, i.e., the number of outer approximators of convex univariant functions (Nouter1) and number of rounds of cutting plane generation at node relaxation (NoutIter), are adjusted by trial and error to obtain the minimum solution time. The optimum planning configuration is shown in Fig. 5.7(c), and the objective function is $0.4114M$. 
Fig. 5.7. Small-scale test system: (a) GIS map, (b) Case 1, and (c) Case 2-4.
Case 3: The MIP GIS-based FR model is solved with CPLEX. Economic characteristics for each conductor, shown in Fig. 5.4, are linearized into five equal segments. The optimum planning configuration is the same as of that for case 2.

Case 4: The stochastic MIP model of the GIS-based FR problem is solved. The power law distribution expressed by (26) is applied for probabilistic net load (demand minus solar generation) forecasting. MLE and Kolmogorov-Smirnov tests presented in [14] are utilized for parameter estimation ($x_{\text{min}} = 1564, \alpha = 37.11$). After obtaining the distribution function, 10,000 initial scenarios are generated. The K-means++ method is applied to cluster scenarios into ten final scenarios [17]. The optimum planning configuration is the same as case 3. However, the objective function is increased to $0.4201M$ due to the presence of uncertainties.

$$p(x) = \frac{\alpha - 1}{x_{\text{min}}} \left( \frac{x}{x_{\text{min}}} \right)^{-\alpha} \text{ for } x \geq x_{\text{min}} \quad (26)$$

Comparison between results: The objective functions of all four cases and the total simulation times are presented in Table 5.1. It can be observed that the objective function of case 2 is less than the objective function of case 1. That is, considering GIS nodes reduces the FR cost. Furthermore, linearizing the model improves the solution time significantly as observed in the total simulation time of case 2 and case 3. Finally, modeling the presence of load forecasting errors and uncertainty in rooftop solar generations in case 4 leads to higher variable cost (cost of losses), while the investment cost remains unchanged due to insignificant size of the system.

<table>
<thead>
<tr>
<th>Case</th>
<th>Objective Function (M$)</th>
<th>Total simulation time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: FR without GIS nodes (MIQP)</td>
<td>0.4275</td>
<td>361</td>
</tr>
<tr>
<td>Case 2: GIS-based FR (MIQP)</td>
<td>0.4114</td>
<td>76</td>
</tr>
<tr>
<td>Case 3: GIS-based FR (MIP)</td>
<td>0.4114</td>
<td>4.2</td>
</tr>
<tr>
<td>Case 4: Stochastic GIS-based FR (MIP)</td>
<td>0.4201</td>
<td>9.6</td>
</tr>
</tbody>
</table>
B. Large-Scale Test system

A realistic large case study presented in [4] is considered to illustrate the effect of modeling resiliency and the presence of geographical obstacles in the FR problem. The forecast annual peak demand for each load point is given in Fig. 5.8(a). It is assumed that each load point includes a solar photovoltaic system with the size (kWp) of 10% of the annual peak load and with the same data used for the previous test system. Three cases are studied considering the same linear economic characteristics and the same procedure for finding the final clusters of scenarios discussed in case 4 of the small test system. According to simulation results, to keep the system reliable, the minimum number of feeders started from the substation node needs to be two (i.e., \( F = 2 \)) for all three cases.

While the presence of obstacles is ignored is case 1, these obstacles are considered in case 2. The objective functions and total simulation times are summarized in Table 5.2. The obtained objective functions for cases 1 and 2 are $1.499M and $1.515M, and the optimum planning configurations are shown in Figs. 5.8(b) and (c) with solid lines. The investment cost in the presence of obstacles is higher because of more restricted geographical positions. The simulation time of case 2 is less since this case includes 25% fewer binary variables as compared to case 1.

In case 3, both costs of resiliency and the presence of obstacles are considered. The assumed values of average repair time, average speed of repair team, and VoLL are 5 hours, 20 miles per hours, and 10 $/kWh, respectively. The optimum planning configuration is different than case 2 as illustrated in Fig. 5.8(c). To enhance the network resiliency, the feeders marked by dashed line are installed instead of the feeders marked by dashed cross. The objective function is $2.049M, from which $1.524M is the investment cost and $0.525M is cost of resiliency. In case 3, the proposed algorithm prefers to install feeders that are closer to the RMC node to take into account the system resiliency. This changes the system configuration and increases the investment cost.
Fig. 5.8. Large-scale test system: (a) GIS map, (b) Case 1, and (c) Case 2 (solid lines)/ Case 3 (dashed lines).
Table 5.2. Large-scale test system results

<table>
<thead>
<tr>
<th>Stochastic MIP models</th>
<th>Objective Function (M$)</th>
<th>Total simulation time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: GIS-based FR without obstacles</td>
<td>1.499</td>
<td>37</td>
</tr>
<tr>
<td>Case 2: GIS-based FR with obstacles</td>
<td>1.515</td>
<td>11</td>
</tr>
<tr>
<td>Case 3: Resilient GIS-based FR with obstacles</td>
<td>1.524+0.525 =2.049</td>
<td>9.3</td>
</tr>
</tbody>
</table>

5.7. Conclusion

A new MIP model for resilient FR using GIS facilities is proposed in this paper. Economic objectives, technical constraints, and geographical restrictions of FR are considered in the proposed model. Additionally, a new model for the cost of distribution feeder resiliency is presented using available GIS data. The uncertainty of rooftop solar generations and demand forecasting errors are considered. A stochastic programming-based solution algorithm is developed to solve the formulated FR problem. It is proven and illustrated by numerical results that having GIS facilities leads to a better (less expensive) FR solution. While the solution of the FR without GIS nodes is calculated $0.4275M, it is calculated $0.4114M for the GIS-based model. It is illustrated by numerical results that the linearized version of the proposed model can be solved within reasonable simulation time even for real-size systems (e.g., less than 10 min). Furthermore, it is observed that the presence of geographical obstacles leads to extra routing cost (by 1.06% in our case study) while it reduces the required simulation time (by 70% in our case study). Finally, considering cost of resiliency, the proposed algorithm prefers to install feeders that are closer to the RMC node to take into account the system resiliency.

5.8. References


CHAPTER 6
CONCLUDING REMARKS AND FUTURE WORKS

6.1. Concluding Remarks

In this dissertation, several novel models and solution algorithms aimed at optimal and efficient decision-making for power system expansion planning were proposed.

First, a computationally efficient SF-based transmission expansion planning model was proposed. Realistic planning objectives and constraints, such as operational costs, budget constraints, system uncertainties, and N-1 security criterion were taken into account. The proposed SF-based TEP contains less number of constraints and variables than the classical angle-based TEP. This makes the proposed TEP computationally less expensive and more tractable, especially for large systems. Simulation results show the proposed TEP provides the same results as the classical angle-based model while being much faster and computationally more efficient. For the Garver system, IEEE 24-bus system, and 118-bus system, the SF-based TEP is, respectively, 54%, 60%, and 58% faster than the angle-based model. The studied cases showed that increasing the size of the system and the number of candidate lines (binary variables) had no negative effect on the advantageous performance of the proposed TEP as compared to the classical angle-based TEP.

Second, a distributed collaborative transmission expansion planning algorithm was presented for interconnected multi-regional power systems. Realistic planning constraints and objectives, such as budget constraints, operational costs, and N-1 security criterion were taken into account. While each region handles its local planning problem, it collaborates with other regions to achieve the optimal and feasible planning scheme for the whole interconnected grid. A two-level decentralized solution algorithm was developed based on the concept of analytical target cascading. The proposed distributed algorithm allowed parallel implementation of TEP subproblems with no need for a central coordinator. Three scenarios were simulated. If each region
solves its local TEP regardless of interactions with its neighbors (scenario 1), the overall investment planning (and operational) costs go up as compared to those costs obtained by taking into account the coordination of local TEPs (scenarios 2 and 3). Although the centralized TEP with the coordination of planners provided the optimal results, information of the local planners must be gathered in a center. The proposed collaborative DTEP algorithm provided the benchmark results as the centralized TEP while respecting the information privacy of the independent planners. The suggested initialization strategy reduced the number of iterations of DTEP by 50%.

Third, a security-constrained generation and transmission expansion planning model with respect to the risk of possible $N - 1$ contingencies was proposed. A linearization technique was developed to make the model MIP. The proposed model took advantage of the concept of risk indices to consider the non-identical probability and severity of individual contingencies. The numerical analysis of the proposed model on the IEEE RTS 24-bus system shown the advantages of reducing post-contingency load shedding cost after considering the severity and probability of dominant contingencies.

Finally, a new MIP model for resilient feeder routing using GIS facilities is proposed in this dissertation. Economic objectives, technical constraints, and geographical restrictions of FR are considered in the proposed model. Additionally, a new model for the cost of distribution feeder resiliency is presented using available GIS data. The uncertainty of rooftop solar generations and demand forecasting errors are considered. A stochastic programming-based solution algorithm is developed to solve the formulated FR problem. It is proven and illustrated by numerical results that having GIS facilities leads to a better (less expensive) FR solution. While the solution of the FR without GIS nodes is calculated $0.4275M$, it is calculated $0.4114M$ for the GIS-based model. It is illustrated by numerical results that the linearized version of the proposed model can be solved
within reasonable simulation time even for real-size systems (e.g., less than 10 min). Furthermore, it is observed that the presence of geographical obstacles leads to extra routing cost (by 1.06% in our case study) while it reduces the required simulation time (by 70% in our case study). Finally, considering cost of resiliency, the proposed algorithm prefers to install feeders that are closer to the RMC node to take into account the system resiliency.

6.2. Future Works

Following suggestions can be considered as extensions of the proposed methods:

- Considering the option of installing tie-lines in the proposed distributed collaborative security-constrained TEP for multi-regional systems.
- Extending the proposed distributed collaborative TEP model to joint transmission and distribution expansion planning.
- Extending the proposed security-constrained generation and transmission expansion planning to multistage (dynamic) planning.
- Considering the convex version of AC flow constraints, e.g., second-order cone programming (SOCP), in the proposed generation and transmission expansion planning and the proposed GIS-based resilient feeder routing.
- Expanding the proposed GIS-based resilient feeder routing to have multiple substations and incorporating the cost and location of required switches.
VITA

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