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Oscar Edwin Pruet
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Particle production characteristics in central nucleus-nucleus collisions at 14.6, 60 and 200 GeV/nucleon

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The Louisiana State University and Agricultural and Mechanical Col., 1990
Particle Production Characteristics in Central Nucleus-Nucleus Collisions at 14.6, 60 and 200 GeV/Nucleon

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the Requirements for the Degree of Doctor of Philosophy in The Department of Physics and Astronomy

by
Oscar Edwin Pruet
B.S., Auburn University, (1977)
M.S., Louisiana State University (1979)
May 1990
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I would never have completed this task without an enormous amount of trust and understanding from my wife Judy, love from my daughter Alison, and patient faith from my mother.

This modest work is dedicated to the memory of my father and my brother, Judge George Wendell Pruett, Sr. and Dr. George Wendell Pruett, Jr., and to the memory of my Uncle Booge, Mr. Oscar Byron Pruett, and my grandfather, Mr.
Oscar Leonidas Pruet. One thing I know that never dies: respect for the dead.
## CONTENTS

I. Introduction ................................................................. 1

II. Nuclear Emulsions and the KLM Heavy Ion Experiments . 6
   1. Emulsion Characteristics ..................................................... 6
   2. Heavy Ion Exposures ........................................................... 9
      a. BNL $^{16}$O Exposure ........................................................ 9
      b. BNL $^{28}$Si Exposure .......................................................... 13
   3. Development and Scanning ................................................. 14
   4. Measuring Procedure ........................................................ 17
   5. Central Collision Selection Criteria ...................................... 22

III. Charged Particle Multiplicities ......................................... 23

IV. Superposition and the Wounded Nucleon Model ............... 32
   1. Wounded Nucleon Calculation ............................................ 33
   2. Impact Parameter Determination .......................................... 40
   3. Test of Superposition ....................................................... 44

V. Angular Distributions ..................................................... 53
   1. Regions of Pseudo-rapidity ................................................. 53
   2. Observed Distributions ...................................................... 57
   3. Particle Correlations ....................................................... 67

VI. Fluctuations ............................................................. 68
   1. The Method of Moments ................................................... 68
   2. Intermittency ................................................................. 72

VII. Conclusion ................................................................. 82

References ................................................................. 84
Vita .................................................. 87
TABLES

I. KLM Exposures ................................................. 5
II. BR-2 Emulsion ................................................. 8
III. Scanning Results ............................................. 16
IV. Central Collision multiplicities ......................... 26
V. Wounded Nucleons .......................................... 43
VI. Gaussian Fit Parameters ................................. 60
VII. Energy Density .............................................. 61
VIII. Intermittency Slope Parameters, $\phi_i$ ............. 76
FIGURES

1. Schematic diagram of a typical horizontal exposure configuration. ........................................ 10

2. Schematic diagram of a typical inelastic event as it would appear in the microscope. ........................................ 19

3. Dispersion $D(N)$ vs the mean production multiplicity $N$ for $^{16}$O ($\triangle$), $^{28}$Si ($\bigcirc$) and $^{32}$S ($\square$) primaries. ........................................ 27

4a. Normalized shower-particle multiplicity distributions for $^{16}$O primaries at 14.6, 60, and 200 GeV/nucleon. ........................................ 28

4b. Normalized shower-particle multiplicity distributions for $^{28}$Si primaries at 14.6 GeV/nucleon and $^{32}$S primaries at 200 GeV/nucleon. (The scale is identical to Fig. 4a to facilitate comparison.) ........................................ 29

5. Dependence of multiplicity on projectile mass for A-AgBr central interactions. The lower line represents $A = 16$ ($\triangle$) and 28 ($\bigcirc$) at 14.6 GeV/nucleon. The upper line corresponds to $A = 1$ (dark $\triangle$), 16 ($\triangle$), and 32 ($\square$) at 200 GeV/nucleon. ........................................ 30

6. Dependence of multiplicity on the primary energy for protons (dark $\triangle$'s) and $^{16}$O ($\triangle$'s). The points for $^{28}$Si ($\bigcirc$) and $^{32}$S ($\square$) are also plotted for comparison. ........................................ 31

7. Dependence of multiplicity in A-AgBr central collisions on the multiplicity in inclusive p-Emulsion interactions at equivalent energies for $A = 1, 16, and 28$. ........................................ 34

8a. Number of wounded nucleons $W$ vs $b_{max}$ for $^{16}$O projectiles. ........................................ 37

8b. Number of wounded nucleons $W$ vs $b_{max}$ for $^{28}$Si projectiles. ........................................ 38

8c. Number of wounded nucleons $W$ vs $b_{max}$ for $^{32}$S projectiles. ........................................ 39
9. Produced particle multiplicities \( N \) vs \( W \). Solid \( \triangle \)'s: inclusive proton-Emulsion data at 14.6 and 60 GeV/nucleon and both inclusive proton-Emulsion and central proton-AgBr data at 200 GeV/nucleon. Open symbols: central A-AgBr collisions for \(^{16}\text{O} (\triangle \)'s), \(^{28}\text{Si} (\bigcirc)\) and \(^{32}\text{S} (\square)\) projectiles. ................................................................. 45

10. Produced particles per wounded nucleon \( N/W \) vs mean production multiplicity \( n_{pp} \) in proton-proton interactions at equivalent energies. ......................................................................................................................... 46

11a. \( N/W \) vs mean \( N_{h} \) for \(^{16}\text{O} \) projectiles. ................................. 49

11b. \( N/W \) vs mean \( N_{h} \) for \(^{28}\text{Si} \) projectiles. ................................. 50

11c. \( N/W \) vs mean \( N_{h} \) for \(^{32}\text{S} \) projectiles. ................................. 51

11d. Produced particles per wounded nucleon \( N/W \) for events with \( N_{h} > 30 \) and \( N_{f} = 0 \) vs mean production multiplicity \( n_{pp} \) in proton-proton interactions at equivalent energies. ................................................................. 52

12a. A schematic pseudo-rapidity (\( \eta \)) distribution: target fragmentation region (a), central production region (b), and projectile fragmentation region (c). ................................................................. 55

12b. Track characteristics corresponding to the distribution in Fig. 12a. . 56

13a. Normalized pseudo-rapidity distribution for \(^{16}\text{O} \) at 14.6 GeV/nucleon. 62

13b. Normalized pseudo-rapidity distribution for \(^{28}\text{Si} \) at 14.6 GeV/nucleon. 63

13c. Normalized pseudo-rapidity distribution for \(^{16}\text{O} \) at 60 GeV/nucleon. 64

13d. Normalized pseudo-rapidity distribution for \(^{16}\text{O} \) at 200 GeV/nucleon. 65

13e. Normalized pseudo-rapidity distribution for \(^{32}\text{S} \) at 200 GeV/nucleon. 66

14a. Normalized factorial moments for \(^{16}\text{O} \) at 14.6 GeV/nucleon. ....... 77
14b. Normalized factorial moments for $^{28}$Si at 14.6 GeV/nucleon. . . . . . 78
14c. Normalized factorial moments for $^{16}$O at 60 GeV/nucleon. . . . . . 79
14d. Normalized factorial moments for $^{16}$O at 200 GeV/nucleon. . . . . 80
14e. Normalized factorial moments for $^{32}$S at 200 GeV/nucleon. . . . . 81
SYMBOLS

$b$  collision impact parameter

$C_i$  scaled regular moment of order $i$

$\epsilon$  energy density

$\eta$  pseudo-rapidity $\eta = -\ln \left[ \tan \left( \frac{\theta}{2} \right) \right]$

$F_i$  scaled factorial moment of order $i$

$N$  number of produced particles ($N = N_s - Z_p$)

$N_b$  number of slow target fragments (black tracks)

$N_{rv}$  number of measured events

$N_f$  number of projectile fragments with $Z > 1$

$N_g$  number of fast target fragments (grey tracks)

$N_h$  total number of target fragments ("heavy" tracks)

$N_s$  number of shower particles

$r(\eta)$  pseudo-rapidity distribution $r(\eta) = \frac{1}{N_{ev}} \frac{dN}{d\eta}$

$R_i$  correction factor of order $i$ for non-flat $r(\eta)$

$R_2$  two-particle pseudo-rapidity correlation function

$\sigma$  total inelastic cross section

$\theta$  polar angle with respect to the beam axis

$W$  number of interacting or "wounded" nucleons

$y$  rapidity $y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right)$

$Z_i$  normalized scaled factorial moment of order $i$
ABSTRACT

Nuclear emulsions have been exposed to a series of relativistic heavy ion beams at Brookhaven National Laboratory ($^{16}\text{O}$ and $^{28}\text{Si}$ nuclei at 14.6 GeV/nucleon) and CERN ($^{16}\text{O}$ nuclei at 60 and 200 GeV/nucleon and $^{32}\text{S}$ nuclei at 200 GeV/nucleon). These beams represent the highest energies currently available for heavy projectiles. Central collisions between the beam nuclei and the heavy emulsion components $^{108}\text{Ag}$ and $^{80}\text{Br}$ have been compared with proton-emulsion data at equivalent energies. The multiplicities of produced charged particles are generally consistent with a conservative superposition model of nucleus-nucleus interactions.

The pseudo-rapidity distributions of produced particles are gaussian in their broad features and do not exhibit the plateau structure associated with collective behavior. The method of scaled factorial moments has been used to examine the fine structure of the pseudo-rapidity distributions. An intermittent power-law growth of the moments with decreasing scales in pseudo-rapidity is observed. However, the strength of this characteristic signature of intermittency declines, for a given energy, as the number of nucleons participating in the central collisions increases. Since this decline is contrary to the expectations associated with collective phenomena, intermittency may be a general characteristic of particle production rather than an unambiguous signature of quark-gluon plasma formation.
I. INTRODUCTION

Current descriptions of the structure and behavior of strongly interacting particles are built around the successful and attractive quark model of hadron construction coupled with the quantum chromodynamical theory of strong interactions (QCD). In this scheme baryons are composed of three quarks while mesons are made from quark-antiquark pairs. The appropriate combinations of quarks and/or antiquarks required to construct a particular particle are bound together within a bag corresponding to the particle's volume by the color force described by QCD. Outside the particle, or bag, lies the QCD vacuum. The color force between quarks is transmitted by particles called gluons which form chains (or strings) linking (or "gluing") the quarks together. The gluons themselves may also interact via the color force which operates in such a way that the hadron itself remains colorless. The quarks so bound are permitted to move about inside the colorless particle they collectively define so long as their chains remain slack. They are, however, restrained by these chains from breaking away from their encompassing hadron and appearing as independent particles. Thus, individual quarks are unavailable for observation within the standard parametric conditions describing actual hadrons.

Normal nuclear matter, for example, is characterized by a quark density of about 0.5 quarks/fm$^3$ and an energy density of about 0.15 GeV/fm$^3$. However, under extreme conditions of quark density and/or energy density, the nature of the QCD vacuum should be altered so that the color sources (gluons and quarks) can freely move throughout extended volumes as long as global color neutrality is maintained. This process is called deconfinement and should result in a "spe-
cials” state of matter, a quark-gluon plasma [Shuryak 1979 and Gross et al. 1981]. Plasma formation is a fundamental issue in QCD and is expected to occur at quark densities on the order of 5 quarks/fm$^3$ and energy densities of several GeV/fm$^3$ [Jacob 1984]. Such conditions are believed to have characterized the early universe until $10^{-5}$ seconds after the Big Bang. The process of quark-gluon plasma (QGP) formation and subsequent condensation into hadronic matter is, therefore, an important aspect of astrophysical and cosmological theories of the universe. The conditions necessary for plasma formation may be approached terrestrially during central collisions between relativistic heavy nuclei [Shuryak 1984, Satz 1985, Cleymans et al. 1986 and Shuryak 1988].

Prior to the availability of such collisions, systematic studies of high energy interactions were limited to hadron beams or to heavy ions at less than 5 GeV/nucleon. Heavy ion energies were too low and the high energy ions ($\alpha$ particles at CERN for example) were too small for a reasonable expectation of QGP formation. Particle production multiplicities in hadron-nucleus or nucleus-nucleus interactions were typically explicable in terms of the superposition of many individual hadron-hadron interactions [Babecki et al. 1974b, Elias et al. 1980 and Otterlund 1984], and signatures of plasma formation were not observed [Bamberger et al. 1987].

The successful acceleration of heavy ions to very high energies, by the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory (BNL) in the U.S. and by the modified CERN Super Proton Synchrotron (SPS) in Geneva, has made possible the exploration of new regimes of temperature and energy density in which something "unusual" may occur.
The KLM collaboration has exposed emulsions containing heavy target nuclei to the series of relativistic heavy ion beams accelerated at both BNL and CERN. Member institutions involved in this collaboration include the Institute of Nuclear Physics (INP), Krakow, Poland, Louisiana State University (LSU), and the University of Minnesota (UM). The author supervised two exposures at BNL (\(^{16}\)O and \(^{28}\)Si ions at 14.6 GeV/nucleon) and other KLM personnel performed exposures at CERN (\(^{16}\)O at 60 and 200 GeV/nucleon and \(^{32}\)S at 200 GeV/nucleon). Table I contains a summary of the five exposures, which were some of the first experiments to use the very high energy BNL and CERN heavy ion beams. The present work presents results from this series of experiments, in particular particle production characteristics in central collisions between the beam nuclei and the heavy emulsion components \(^{110}\)Ag and \(^{80}\)Br.

The experimental exposure and measurement procedures are described in Chapter II, and the resulting meson production multiplicities are presented in Chapter III. Collisions between heavy nuclei at BNL and CERN energies may still involve mainly many individual nucleon-nucleon interactions without substantial collective behavior such as plasma formation [Barbier et al. 1988b]. Therefore, the nucleus-nucleus production multiplicities are examined within the framework of a superposition model in Chapter IV. Large scale failure of this model would suggest the presence of collective phenomena.

In addition to substantial deviations from superposition, another widely anticipated signature of something "special" involves non-random, or non-statistical, fluctuations in the angular distribution of secondary particles [Jacob 1989]. The
angular distributions of produced mesons are presented in Chapter V in terms of the pseudo-rapidity scaling variable. A method of examining the pseudo-rapidity distributions for non-statistical fluctuations is described in Chapter VI. Overall conclusions are summarized in the final chapter.

In addition to the data obtained from the present set of experiments, the author has obtained a set of low energy (4.5 GeV/nucleon) $^{28}$Si events from Dubna in the USSR, kindly provided by Dr. Roman Holynski of INP. These data are used in the following multiplicity presentations for comparative purposes. Proton-nucleus data discussed below have been obtained from the literature and from the KLM databank [Babecki et al. 1973 and 1974, Alma-Ata-Leningrad-Moscow-Tashkent Collaboration 1975, Boos et al. 1978, Babecki et al. 1978, Abduzhamilov et al. 1987 and 1989, and Barbier et al. 1988a].
<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Lab</th>
<th>Beam</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-808</td>
<td>BNL</td>
<td>$^{16}$O</td>
<td>14.6 GeV/nucleon</td>
</tr>
<tr>
<td>EMU-07</td>
<td>CERN</td>
<td>$^{16}$O</td>
<td>60 GeV/nucleon</td>
</tr>
<tr>
<td>EMU-07</td>
<td>CERN</td>
<td>$^{16}$O</td>
<td>200 GeV/nucleon</td>
</tr>
<tr>
<td>E-808</td>
<td>BNL</td>
<td>$^{28}$Si</td>
<td>14.6 GeV/nucleon</td>
</tr>
<tr>
<td>EMU-07</td>
<td>CERN</td>
<td>$^{32}$S</td>
<td>200 GeV/nucleon</td>
</tr>
</tbody>
</table>
II. NUCLEAR EMULSIONS AND THE KLM HEAVY ION EXPERIMENTS

Nuclear emulsions have a long history of success in trajectory measurement and identification of charged particles passing through the emulsion medium [Powell, Fowler and Perkins 1959]. The original identification of heavy atomic nuclei in the cosmic rays [Freier et al. 1948] was done with emulsion, and the emulsion detector remains valuable as both a target and detection apparatus for cosmic ray studies in balloon-borne [Jones et al. 1987] as well as satellite-borne [Parnell et al. 1989] experiments. This is due to the emulsion's unique combination of simplicity and superb particle resolution. Since the techniques of measuring nucleus-nucleus interactions occurring within emulsion have been developed and refined through the study of collisions involving cosmic ray nuclei, emulsions were a natural choice for measuring similar interactions between emulsion constituents and heavy ions accelerated artificially by particle accelerators.

1. Emulsion Characteristics

Nuclear emulsion consists of silver-bromide (AgBr) crystals, or grains, embedded in a gelatin composed mainly of carbon, hydrogen, nitrogen, and oxygen atoms, but also containing other elements depending on the formulation used. The emulsions used in this series of experiments were of Russian type BR-2. Table II gives a breakdown of BR-2 emulsion into its constituent parts and their corresponding contribution in atoms per cubic centimeter. As can be seen, a wide variety of possible targets is available. Passage of charged particles through the emulsion alters the structure of the AgBr crystals (i.e., they become ionized) and is revealed as
darkened grains after the exposed emulsion is developed. Paths of charged particles can therefore be observed by following the trails of dark grains in the processed emulsion. Emulsion cannot be used to observe neutral particles directly, although charged particles from a neutral parent can often be used to determine characteristics of the parent particle. The density of altered grains (grain density or ionization density) depends on the charge and velocity of the penetrating particle and can be used for particle identification and/or classification. For example, differentiation between fast produced particles and slow target fragments is straightforward because of obvious differences in grain density.

The emulsion detector permits measurement of particle trajectories within a full $4\pi$ solid angle so that an interaction between a beam ion and a constituent component of the emulsion can be fully analyzed with regard to charged particles. Angular resolution of individual tracks down to about 1 milliradian can be achieved routinely, and this permits the fine structure of angular distributions to be studied.
<table>
<thead>
<tr>
<th>Component</th>
<th>A</th>
<th>atoms/cm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag</td>
<td>107.8</td>
<td>$1.0360 \times 10^{22}$</td>
</tr>
<tr>
<td>Br</td>
<td>79.9</td>
<td>$1.0310 \times 10^{22}$</td>
</tr>
<tr>
<td>I</td>
<td>126.9</td>
<td>$0.0020 \times 10^{22}$</td>
</tr>
<tr>
<td>S</td>
<td>32.0</td>
<td>$0.0040 \times 10^{22}$</td>
</tr>
<tr>
<td>H</td>
<td>1.0</td>
<td>$3.1480 \times 10^{22}$</td>
</tr>
<tr>
<td>C</td>
<td>12.0</td>
<td>$1.4120 \times 10^{22}$</td>
</tr>
<tr>
<td>N</td>
<td>14.0</td>
<td>$0.3960 \times 10^{22}$</td>
</tr>
<tr>
<td>O</td>
<td>16.0</td>
<td>$0.9560 \times 10^{22}$</td>
</tr>
</tbody>
</table>
2. Heavy Ion Exposures

In contrast to cosmic rays which enter a detector at unpredictable angles, the well-collimated accelerator beams allow emulsion strips (or pellicles) to be exposed parallel to the beam. This horizontal orientation ensures that the jet, or cone of produced particles, will be contained within the pellicle in which the interaction takes place, and it greatly facilitates measurement of the event. Figure 1 presents a schematic diagram of a typical exposure configuration.

The pellicles used in the KLM exposures were of dimensions 10 cm x 5 cm in length and width, respectively, and were 600 μm thick. The emulsion material was sliced into such strips so that uniform absorption of the fluids used in development would occur. A number of such strips were placed one on top of the other to form an emulsion stack, which was then inserted into a sealed container and placed in the path of the accelerator beam. The exposure time was chosen to obtain a sufficiently high density of incident (primary) ion collisions with the component nuclear targets within the strips. This required a typical exposure intensity on the order of $10^3$ ions per cm$^2$. The two exposures performed by the author at BNL are described in some detail below.

2.a. BNL $^{16}$O Exposure

Four stacks of horizontal pellicles were prepared for exposure to the 14.6 GeV/nuclide $^{16}$O beam and were mounted on a traveling cart assembly set up in the experimental area. This cart ran along a horizontal track 9 feet in length mounted perpendicular to the beam line, and 8.22 inches below the surveyed
Figure 1. Schematic diagram of a typical horizontal exposure configuration.
beam center line. Nylon cord connected the cart to an electric motor which pulled it down the track at an adjustable speed of up to 80 cm/sec. An experimenter positioned a safe distance from the beam could then control the entrance and exit of stacks secured to the cart. The rotational orientation and vertical and horizontal position of a stack on the cart were adjustable relative to the beam line.

Two overlapping scintillation counters were placed in the beam line directly behind the exposure position and were used to monitor the counting rate. The beam profile was measured by changing the overlap region of these two counters to sample different areas of the beam spot.

The Brookhaven AGS was delivering an 85% pure beam of $^{16}$O nuclei to the experimental area at the time of the exposures. Contaminants were $\alpha$ particles and deuterons. The beam intensity was 3000 ions/pulse with a pulse length of 500 milliseconds and a frequency of 1 pulse/3 seconds. The beam profile consisted of a 100 cm$^2$ beam spot with 80% of the counts/pulse distributed over a 25 cm$^2$ rectangular central region.

This corresponded to $0.80 \times 3000 \text{ ions/pulse/25cm}^2 = 96 \text{ ions/pulse/cm}^2$ in the central beam spot. The desired exposure intensity was $4.5 \times 10^1 \text{ ions/cm}^2$. Since $5000 \text{ ions/cm}^2 \div 96 \text{ ions/cm}^2/\text{pulse} = 52.1 \text{ pulses}$, 156 seconds of exposure were required. The exposure time uncertainties associated with the brief entry and exit times (approximately 1 second each) required to move a stack in and out of the exposure position were acceptable, so the beam was left on during the entire exposure sequence. This avoided both beam quality discrepancies and lost time during beam shut-down and start-up.
Each stack was carefully attached to the traveling cart and propelled into the $^{16}$O beam in such a way that the center of the front surface of the stacked emulsion pellicles would be presented perpendicularly to the beam center line when in the exposure position. A switch, tripped by the leading edge of the cart, stopped the stack at the proper place. Incident ions traversed the stack parallel to the horizontal pellicles. The front surface of each stack presented a cross sectional area of $1.5 \text{ cm} \times 4 \text{ cm} = 6 \text{ cm}^2$ to the beam. Therefore an exposure level of 5000 ions/cm$^2$ would yield $5000 \text{ ions/cm}^2 \times 6 \text{ cm}^2 = 30,000$ ions impinging on the front surface. The distribution of ions within the central core of the beam was sampled by adjusting the overlap region of the two scintillators and judged to be reasonably uniform.

Fifty-seven percent of the incident oxygen nuclei should, on average, suffer collisions with target nuclei within the emulsion pellicles, leaving about 43% to emerge from the rear of the stack to be recorded by the counters, i.e., $0.43 \times 30,000 = 12,900$ ions should be counted by the portion of the scintillators directly behind the stacks. Assuming that the incident particles outside the stack passed undisturbed into the counters, $5000 \text{ ions/cm}^2 \times (25 - 6)\text{cm}^2 = 95,000$ more ions were added to the total count within the 25 cm$^2$ central region, for a total of $12,900 + 95,000 = 107,900$ ions. Since this region received only 80% of the total flux, $107,900 \div 0.80 = 134,875$ counts should produce a satisfactory exposure of roughly 5000 ions/cm$^2$. An actual count of $1.2 \times 10^5$ ions was requested for each exposure.
2.b. BNL $^{28}\text{Si}$ Exposure

Four more horizontal stacks were prepared for exposure to the beam of $^{28}\text{Si}$ nuclei. The traveling cart assembly was again set up in the experimental area prior to exposure. As before, the cart ran along a horizontal track 9 feet long mounted perpendicular to the beam line. The surveyed beam center line ran 8.40 inches above the track.

Two pairs of overlapping scintillation counters were mounted in the beam line to monitor beam intensity. One of these pairs, placed upstream of the stack position, was removed just before exposure. The downstream set measured actual counting rates. Two multiwire proportional counters installed behind the exposure position provided vertical and horizontal profiles of the beam spot. As an added check on beam spot location, two small scintillators forming a 4 cm$^2$ overlap region were secured to the cart in the position normally occupied by a stack. The cart, with attached scintillators, was placed in the exposure area and used to determine an accurate map of the beam spot.

At the time the exposures were made, the AGS was producing a stable beam of 500 ions/cm$^2$/pulse in the central region of the beam spot as measured with the small scintillators. The center of this central region was determined to correspond precisely to the actual surveyed height above the reference level but to be translated 1.5 cm horizontally from the survey mark. The purity of the new 14.6 GeV/nucleon $^{28}\text{Si}$ beam had been improved to 95 %, with $\alpha$ particles and deuterons again forming the chief contaminants. The AGS was sending beam bursts to the exposure area with a frequency of 1 pulse/5 seconds. An exposure of $4-5 \times 10^3$ ions/cm$^2$ was again
chosen as optimal.

Immediately prior to exposure the cart with attached small scintillators was propelled along the track into the exposure position within the silicon beam and the following counting rates were obtained: 1869 ions/pulse registered by the small counters on the cart and 6308 ions/pulse registered by the large downstream counters. Since the area presented to the beam by the small scintillators was 4 cm$^2$, $\frac{1869 \text{ ions/pulse}}{4 \text{ cm}^2} = 467 \text{ ions/cm}^2/\text{pulse}$. An exposure to 10 pulses would produce a total exposure of 4670 ions/cm$^2$. This would correspond to the desired exposure level of $4.5 \times 10^3$ ions/cm$^2$.

The small counters were removed from the cart and replaced with the actual stacks. As before, these stacks were fastened so as to present the front edge of the pellicles perpendicularly to the beam. The plane of the pellicles was parallel to the beam line and to the horizontal, so the incident $^{28}\text{Si}$ nuclei would traverse the stack parallel to the pellicles. Each stack was carefully pushed into position during an interval between beam bursts and left in the beam for 10 pulses. Stacks were hauled out of the beam line during intervals between pulses.

3. Development and Scanning

With the exception of half of the 14.6 GeV/nucleon $^{16}\text{O}$ stacks which were developed at the University of Minnesota, the stacks were shipped, following exposure, to Dubna in the USSR for processing. This involved removing the pellicles from their stacks, attaching them to glass slides and developing them with the proper chemicals. The processed pellicles, called plates, were then distributed in approximately equal shares to INP, UM, and LSU.
At each laboratory a process of scanning and measuring began soon after receipt of the plates. A representative plate was first examined carefully with an optical microscope to determine the beam exposure profile. Next the microscope operator (called a scanner) performed a careful along-the-track scan at medium (530X) magnification. The latter involved following each primary track in the central region of the beam profile for 5 cm or until an interaction between the primary and an emulsion component was observed. Coordinates of each interaction location were recorded along with the interaction type (classification). After scanning many plates, the interaction mean free path ($\lambda$) of the incident nuclei was determined from the total number, $N_I$, of interactions found along the total length of track scanned. The experimental total inelastic cross section is related to $\lambda$ according to the equation

$$\sigma_{exp} = \frac{1}{\lambda \rho_{emul}}$$

where $\rho_{emul}$ is the density of emulsion targets.

This experimentally determined cross section can be compared to the accepted theoretical total inelastic cross section for the primary nucleus incident on emulsion material. Agreement between the predicted [Westfall et al. 1979 or Hagen 1976] and experimentally measured cross sections substantiates the accuracy of the scanning process. Table III, which shows the scanning results for the three projectiles used in this analysis, indicates a reasonably unbiased scan.
TABLE III. Scanning Results

<table>
<thead>
<tr>
<th>Beam</th>
<th>Events</th>
<th>$\lambda$(cm)</th>
<th>$\sigma_{exp}$(mb)</th>
<th>$\sigma_{calc}^{11.ests/14all}$(mb)</th>
<th>$\sigma_{calc}^{Hagen}$(mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O</td>
<td>2061</td>
<td>12.0±0.3</td>
<td>1052±26</td>
<td>1009</td>
<td>1043</td>
</tr>
<tr>
<td>$^{28}$Si</td>
<td>669</td>
<td>10.3±0.4</td>
<td>1230±70</td>
<td>1253</td>
<td>1328</td>
</tr>
<tr>
<td>$^{32}$S</td>
<td>690</td>
<td>8.5±0.3</td>
<td>1492±60</td>
<td>1323</td>
<td>1407</td>
</tr>
</tbody>
</table>
4. Measuring Procedure

After scanning the plates to locate and classify the interactions (events), selected events were measured under high (1000X) magnification. The measurement process included classifying and counting each track emanating from the interaction vertex, as well as measuring the trajectory of each track. Figure 2 illustrates a typical inelastic event as it would appear in the microscope. In this schematic the primary represents an $^{16}$O nucleus moving from left to right, as indicated by the heavy ionization trail in the emulsion. After striking a nuclear target the primary breaks up into fragments that move away from the interaction vertex within a small cone centered about the event axis, i.e., the direction or trajectory of the incoming nucleus. The total number of primary fragments with charge greater than unity is denoted as $N_f$.

If the interaction were inelastic, fragments would typically also be knocked out of the target nucleus. The fast target fragments leave trails that appear grey to the eye, so they have historically been termed "grey tracks" since the early days of emulsion research. The total number of such grey tracks in an event was counted and recorded as $N_g$.

The residual target nucleus is generally highly excited after an inelastic collision, so it would quickly emit slowly moving fragments that travel at most a few hundred microns, typically much less, from the interaction vertex before stopping. These slow fragments would leave tracks that appear black, and their total number in an event was counted and recorded as $N_b$. The total number of target fragments is given by $N_g + N_b = N_h$, which is sometimes used to represent the number of target
fragments irrespective of velocity and size.

The remaining tracks correspond to secondary particles, i.e., those created in the collision process from the energy of the primary. Such particles are generally singly charged, relativistic, and appear to "burst out" from the interaction vertex in a shower or cone about the primary axis. They were counted and recorded as $N_s$.

The track classification scheme used to discriminate between these various types of tracks was as follows:

1. If a track had a grain density greater than 1.4 times the minimum density $g_{min}$ for the emulsion in use, if it was in the forward direction close to the primary axis, and if it extended at least 10,000 $\mu m$ without scattering from a straight line, the track was identified as a projectile fragment of charge greater than one unit and labeled $f$. Note that a projectile fragment with unit charge was difficult to distinguish from a singly charged produced particle and was thus initially labeled as a shower track. See classification 4 below.

2. If a track had a grain density greater than 1.4 times $g_{min}$ and extended farther than 3000 $\mu m$ from the interaction vertex but deviated from a straight path within 10,000 $\mu m$, the track was labeled a grey ($g$) track.
Figure 2. Schematic diagram of a typical inelastic event as it would appear in the microscope.
3. If a track had a grain density greater than 1.4 times $g_{min}$ and a range less than 3000 $\mu m$ from the vertex, the track was labeled a black ($b$) track.

4. If a track had a grain density less than 1.4 times $g_{min}$, the track was labeled a produced or shower ($s$) track.

These classification criteria were developed empirically over the years as scanners learned to calibrate themselves and the emulsion detector. The tracks classified as shower particles are typically associated with relativistic mesons (mostly pions but also some kaons and other mesons) with $\beta > 0.7$.

Polar and azimuthal emission angles for the shower tracks, as well as for the heavy tracks emerging from the interaction vertex, were measured with the microscope at high magnification. The polar angle $\theta$ was defined relative to the direction of motion of the incoming primary, while the azimuthal angle $\phi$ was defined relative to an orthogonal axis constrained to lie within the plane of the emulsion pellicle.

At LSU a system of three linear digital gauges interfaced to a Digital Equipment Corporation PDP-11/23 mini-computer has been developed to facilitate the emission angle measurements. Two of the gauges are mounted to the microscope in such a way that translation of the microscope stage parallel to the plane of the stage (i.e. along the axes defined as $X$ and $Y$) is digitally encoded and recorded in a computer file. A third gauge is mounted perpendicularly to the stage and registers vertical ($Z$) displacement. Thus, a microscope operator would digitally record the trajectory of a particle through an emulsion pellicle by attaching the pellicle to the microscope stage and visually following the track of the particle.
The digital gauges are accurate to within one micron.

The $X, Y, Z$ coordinate system was set to $(0,0,0)$ at the vertex of an interaction to be measured. Then several sets of $X, Y, Z$ values along each track were recorded relative to this origin. The path of the incoming projectile was measured to determine the reference axis for the polar angle. Tracks emerging from the vertex close to this projected reference axis had to be measured relative to a reference track (typically a nearby beam track) to reduce measurement error associated with possible distortions in the developed emulsion. This procedure was necessary since tracks with small polar angles had to be followed for perhaps several thousand microns from the vertex to get a displacement of more than one micron perpendicular to the reference axis. A linear regression analysis was used to fit the measured sets of $X, Y, Z$ coordinates for each track to a straight line and to determine the reference axis. The emission angles were then calculated in terms of $\theta$ and $\phi$. After the tracks in an event were classified and counted and the emission angles determined, the measurement process was complete.
5. Central Collision Selection Criteria

The thrust of the present study involves analysis of central collisions between heavy nuclei. Therefore, it was necessary to screen the complete set of events found in scanning and to select for measurement those events involving central collisions of the beam ions with the heavy emulsion constituents. In BR-2 emulsion, the latter include \(^{32}\text{S},^{80}\text{Br},^{108}\text{Ag}\) and \(^{127}\text{I}\). Sulphur and iodine are present only in negligibly small quantities (see Table II), so the heavy targets were effectively supplied by the AgBr crystals. An \(N_h > 15\) cut can therefore ensure interactions with either Ag or Br, since only these targets have the capacity to boil off more than 15 heavily ionizing tracks (i.e. have more than 15 protons). Centrality was obtained by further requiring complete fragmentation of the projectile (beam ion) into its component nucleons (i.e. no projectile fragments with charge greater than one, \(N_f = 0\)).

In summary, the selection criteria

1. \(N_h > 15\)
2. \(N_f = 0\)

operationally defined central interactions between the heavy beam nuclei and the AgBr targets.
III. CHARGED PARTICLE MULTIPlicITIES

The emulsion detector provided complete coverage over the full solid angle of $4\pi$ steradians and thus allowed the total number of charged secondary particles to be counted for each event measured. The multiplicity of produced particles is therefore an experimentally well-defined quantity.

The working definition of a central collision for this analysis included the requirement for complete breakup of the projectile into $Z_p$ proton fragments (neutrally charged particles were not measurable), where $Z_p$ is the projectile charge. As mentioned earlier, these fast protons were initially labeled as shower tracks during the measurement process, so they must be subtracted from the total number of observed shower tracks $N_s$ to obtain the true number of produced particles, $N$, which is given by:

$$N = N_s - Z_p .$$

Table IV summarizes, for each combination of beam species and laboratory energy, the observed mean production multiplicities $N$ of produced particles and their dispersions $D(N)$ for the events meeting the central collision selection criteria. The dispersion,

$$D = \sqrt{\frac{1}{N_{ee}} \sum_{i=1}^{N_{ee}} (N - N_i)^2} ,$$

generally obeys the following linear dependence on $N$ for all combinations of beam
mass and energy:

\[ D = a \, N + b. \]  \hspace{1cm} (4)

Figure 3 shows that the slope of this relationship is $0.27 \pm 0.02$ in contrast to an expected value of around unity for interactions averaged over all impact parameters [Wosiek 1986]. This indicates that the central event (small impact parameter) selection rules are indeed extracting a subset of measured events much more narrowly spread about $N$ than would be the case for a totally inclusive dataset.

The shower-particle multiplicity distributions are presented in Figs. 4a and 4b for oxygen, silicon, and sulphur primaries. For a fixed primary mass, the increase in mean multiplicity as the beam energy is raised to provide more energy for secondary particle production produces a broadening and flattening of the distribution. Similar changes are observed for a fixed beam energy as the mass number $A$ is increased, due to the increasing numbers of interacting nucleons.

The dependence of average multiplicity on projectile mass $A$ is shown in Fig. 5 for central $A$-$\text{AgBr}$ interactions at $E = 14.6 \text{ GeV/nucleon}$ ($A = 16$ and 28) and $200 \text{ GeV/nucleon}$ ($A = 1$, 16, and 32). The rise in $N$ as $A$ increases reflects the increasing number of nucleons participating in the collision process. (The slopes of the lines are $1.06 \pm 0.05$ and $0.74 \pm 0.01$ at 14.6 and 200 GeV/nucleon, respectively.) The dependence of multiplicity on the primary energy for central $^{16}\text{O}$-$\text{AgBr}$ collisions ($\triangle$'s) is shown in Fig. 6, along with the same dependence for central $p$-$\text{AgBr}$ collisions (dark $\triangle$'s). (The $^{28}\text{Si}$ and $^{32}\text{S}$ data are also plotted for comparison.) The observed power law behavior displays the same relationship as
in the case of p-p interactions [Carruthers and Duong-Van 1972] for which

\[ N = \text{const} \, E^\alpha. \]  

The production multiplicities in \(^{16}\text{O}-\text{AgBr}\) collisions are substantially greater than those in p-AgBr interactions and show a substantial increase with increasing projectile energy. However, since both the p-AgBr and \(^{16}\text{O}-\text{AgBr}\) results (\(\alpha = 0.33 \pm 0.04\) and \(0.52 \pm 0.04\), respectively) display the same character as the p-p dependence (\(\alpha = 0.25\)), they invite an attempt to explain the nucleus-nucleus production multiplicities by simply superimposing many individual nucleon-nucleon collisions. The following chapter describes such an analysis using the wounded nucleon superposition model.
### TABLE IV. Central Collision Multiplicities

<table>
<thead>
<tr>
<th>Beam</th>
<th>E(GeV/n)</th>
<th>Events</th>
<th>$N$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}\text{O}$</td>
<td>14.6</td>
<td>215</td>
<td>43.2 ± 1.0</td>
<td>14.7</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>14.6</td>
<td>154</td>
<td>78.3 ± 2.1</td>
<td>26.1</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>60.0</td>
<td>226</td>
<td>98.2 ± 2.3</td>
<td>34.6</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>200.0</td>
<td>131</td>
<td>164.8 ± 4.2</td>
<td>48.1</td>
</tr>
<tr>
<td>$^{32}\text{S}$</td>
<td>200.0</td>
<td>135</td>
<td>284.4 ± 6.4</td>
<td>74.4</td>
</tr>
</tbody>
</table>
Figure 3. Dispersion $D(N)$ vs the mean production multiplicity $N$ for $^{16}$O ($\triangle$), $^{28}$Si (○) and $^{32}$S (□) primaries.
Figure 4a. Normalized shower-particle multiplicity distributions for $^{16}O$ primaries at 14.6, 60, and 200 GeV/nucleon.
Figure 4b. Normalized shower-particle multiplicity distributions for $^{28}$Si primaries at 14.6 GeV/nucleon and $^{32}$S primaries at 200 GeV/nucleon. (The scale is identical to Fig. 4a to facilitate comparison.)
Figure 5. Dependence of multiplicity on projectile mass for A-AgBr central interactions. The lower line represents $A = 16 (\triangle)$ and $28 (\bigcirc)$ at 14.6 GeV/nucleon. The upper line corresponds to $A = 1$ (dark $\triangle$), 16 ($\triangle$), and 32 ($\square$) at 200 GeV/nucleon.
Figure 6. Dependence of multiplicity on the primary energy for protons (dark \( \Delta \)'s) and \(^{16}\text{O} (\Delta \text{'s}). \) The points for \(^{28}\text{Si} (\bigcirc)\) and \(^{32}\text{S} (\square)\) are also plotted for comparison.
IV. SUPERPOSITION AND THE WOUNDED NUCLEON MODEL

The wounded nucleon model of hadronic interactions [Bialas, Bleszynski and Czyz 1976] describes hadron-nucleus (p-A) or nucleus-nucleus (A-A) interactions as an incoherent superposition of collisions of individual nucleons. The average multiplicity of particles produced in a p-A or A-A interaction is therefore taken to be proportional to the number, $W$, of nucleons participating in the collision and the mean production multiplicity $n_{pp}$ in proton-proton (p-p) interactions at an equivalent energy.

$$N = \frac{1}{2} W n_{pp}$$

Equation (6) reflects the following fundamental assumption in a superposition interaction picture:

$$\frac{N}{W} = \frac{n_{pp}}{2},$$

i.e., for a given bombardment energy the average multiplicity per interacting (or "wounded") nucleon in complicated p-A or A-A interactions is equal to the same ratio for the (relatively) simpler p-p collisions.

As long as superposition holds, this ratio should remain true for both central (small impact parameter) and inclusive (including all possible impact parameters) collisions, so that Eq. (7) may be extended to

$$\left(\frac{N}{W}\right)_{central} = \left(\frac{N}{W}\right)_{inclusive}.$$  

In Fig. 7 the dependence of the average multiplicity in A-AgBr central collisions on the average multiplicity in inclusive p-Emulsion interactions is presented for $A$
The linearity observed is consistent with a superposition model of nuclear interactions. The slopes represent the ratio $W_{central}/W_{inclusive}$.

1. Wounded Nucleon Calculation

Unlike $N$ and $n_{pp}$, the number of wounded nucleons is not directly measurable. Therefore examination of production multiplicities within the superposition framework involves calculating $W$. Since collective effects are omitted by definition from the model, this calculation can be done by integrating the probability of a single nucleon-nucleon collision over the distribution of nucleons in projectile A and target B. The average number of wounded nucleons obtained in this manner [Bialas, Bleszynski and Czyz 1976] averaged over all impact parameters is

$$W = A \frac{\sigma_{pH}}{\sigma_{AH}} + B \frac{\sigma_{pA}}{\sigma_{AH}}. \tag{9}$$

The first term in Eq. (9) corresponds to the number of nucleons in the projectile that participate in the interaction, while the second term represents the number of interacting target nucleons. The inelastic cross sections $\sigma_{pA}$ and $\sigma_{pH}$ represent an interaction between a single hadron (p) and nucleus A or B, respectively, while $\sigma_{AH}$ is the total A-B inelastic cross section.

The value of $W$ can then be easily determined for inclusive interactions by using a semiempirical formula to calculate the inelastic cross sections in Eq. (9). These semiempirical models include the previously mentioned Westfall and Hagen equations for the A-B cross sections and the Letaw formula for p-A cross sections [Letaw et al. 1983].
Figure 7. Dependence of multiplicity in A-AgBr central collisions on the multiplicity in inclusive p-Emulsion interactions at equivalent energies for A = 1, 16, and 28.
However, applications involving central, or semi-central, collisions require \( W \) to be calculable as a function of the impact parameter \( b \). This has been done in the general case by Sumiyoshi [1983] and for the particular case of AgBr targets by Atwater [1986].

As before, \( W \) is the sum of nucleons wounded in the projectile and target nuclei, and the mathematical form of the two contributions is symmetric. For example, the average number \( W_A \) of interacting projectile nucleons in a set of collisions with impact parameters up to a maximum impact parameter \( b_{\text{max}} \) is given by

\[
W_A(b_{\text{max}}) = A \frac{\sigma_{pB}(b_{\text{max}})}{\sigma_{A\beta}(b_{\text{max}})},
\]  

where

\[
\sigma_{pB}(b_{\text{max}}) = \int_{b_0}^{b_{\text{max}}} d^2 b \left\{ 1 - \frac{d^2 b'}{d^2 b} \left[ 1 - \sigma_{pp} t_B(b') \right] t_A(b - b') \right\},
\]

\[
\sigma_{A\beta}(b_{\text{max}}) = \int_{b_0}^{b_{\text{max}}} d^2 b \left\{ 1 - [1 - \sigma_{pp} \int d^2 b' t_B(b - b') t_A(b')] A^H \right\},
\]

\( \sigma_{pp} \) is the inelastic p-p cross section, and \( t_A \) and \( t_B \) are the nuclear density functions of the projectile and target nuclei, respectively. These density functions are obtained by transforming the density in polar coordinates [Negele 1970]

\[
\rho(r) = \frac{\rho_0}{1 - \exp \left( \frac{r - c}{a} \right)}
\]

into cylindrical coordinates, assuming azimuthal symmetry, and integrating out the \( z \) (beam axis) dependence. Here

\[
a = 0.54 \text{ fm} ,
\]

\[
c = (0.978 + 0.0206A^{1/3})A^{1/3} \text{ fm} ,
\]
A semiempirical formula was derived for \( \sigma_{pp} \) by Atwater [1986] using data published by the Particle Data Group [1984].

The probable number, \( W_B \), of interacting target nucleons takes the same form and is given by

\[
W_B(b_{\text{max}}) = B \frac{\sigma_{pA}(b_{\text{max}})}{\sigma_{AB}(b_{\text{max}})},
\]

so that the form of Eq. (9)

\[
W = W_A + W_B
\]

remains unchanged, although \( W \) is now a function of the maximum impact parameter. Figures 8a, b and c illustrate the functional dependence of \( W \) on \( b_{\text{max}} \) for each of the three projectiles used in this study. Curves are presented separately for Ag and Br. The total \( W \) is a weighted sum of the two individual components.
Figure 8a. Number of wounded nucleons $W$ vs $b_{\text{max}}$ for $^{16}\text{O}$ projectiles.
Figure 8b. Number of wounded nucleons $W$ vs $b_{\text{max}}$ for $^{28}\text{Si}$ projectiles.
Figure 8c. Number of wounded nucleons $W$ vs $b_{\text{max}}$ for $^{32}\text{S}$ projectiles.
2. Impact Parameter Determination

In order to calculate $W$ for comparison with our experimental central collision dataset, the value of $b_{\text{max}}$ that accurately reflected the data had to be determined. Recall that during the careful and unbiased scan (see Chap. II) of the processed emulsion, the total number, $N_J$, of interactions (involving all emulsion components and impact parameters) found along the projectile tracks was recorded. By imposing the appropriate selection criteria, a subset of central collisions with heavy targets was selected for each of the $^{16}$O, $^{28}$Si and $^{32}$S projectiles. The selected events correspond to highly excited Ag or Br targets ($N_h > 15$) and totally fragmented projectiles ($N_f = 0$), so the subset will be designated $N_{^{117}\text{AgBr}}^c$. The ratio

$$\frac{\sigma_{^{117}\text{AgBr}}}{{\sigma}_{^{117}\text{AgBr}}} = \frac{N_{^{117}\text{AgBr}}}{{N}_{^{117}\text{AgBr}}}$$

is assumed to hold, where $\sigma_{^{117}\text{AgBr}}$ is the inelastic cross section corresponding to the $N_{^{117}\text{AgBr}}^c$ central AgBr events. Similarly, $\sigma_{^{117}\text{AgBr}}$ and $N_{^{117}\text{AgBr}}$ are the cross section and number of events, respectively, which involve AgBr as a target regardless of impact parameter.

The value of $b_{\text{max}}$ used in the calculation of $W$ was found by regarding $\sigma_{^{117}\text{AgBr}}$ as a black disk with radius $b_{\text{max}}$, i.e.,

$$\sigma_{^{117}\text{AgBr}} = \pi b_{\text{max}}^2$$

Determination of $\sigma_{^{117}\text{AgBr}}$ then specifies $b_{\text{max}}$.

Table II shows that the elements $^{108}$Ag and $^{80}$Br are the most abundant components of BR-2 emulsion, which also includes $^{127}$I, $^{32}$S, $^1$H, $^{12}$C, $^{11}$N and $^{16}$O. The
total inelastic cross section for emulsion contains a contribution \( a(A)\sigma_A \) for each of the components: \( A = 1, 12, 14, 16, 32, 80, 108 \) and 127, i.e.,

\[
\sigma_{emul} = a(1)\sigma_1 + a(12)\sigma_{12} + \cdots + a(127)\sigma_{127}.
\]  

(21)

The \( a(i) \) are the ratios of the number of targets with \( A = i \) to the total number of targets for a unit volume of emulsion, and the \( \sigma_i \) are the inelastic cross sections for individual emulsion components computed from the Westfall [1979] formalism

\[
\sigma_i = 10\pi(1.35)^2[A_{\text{projectile}}^{1/3} + i^{1/3} - 0.89]^2.
\]  

(22)

The relative contribution of the heavy constituents Ag and Br to \( \sigma_{emul} \) is given by

\[
\frac{a(108)\sigma_{118} + a(80)\sigma_{80}}{\sigma_{emul}},
\]  

(23)

and \( N_{AgBr} \) is then

\[
N_{AgBr} = \frac{a(108)\sigma_{118} + a(80)\sigma_{80}}{\sigma_{emul}} N_T.
\]  

(24)

Equation (14) can also be used to evaluate \( \sigma_{AgBr} \) :

\[
\sigma_{AgBr} = b(108)\sigma_{118} + b(80)\sigma_{80},
\]  

(25)

where the \( b(i) \) are the ratios of the number of Ag or Br targets to the total number of Ag and Br targets per unit volume of emulsion. Once \( N_{AgBr} \), \( N_{AgBr} \) and \( \sigma_{AgBr} \) are known, \( \sigma_{AgBr} \) can be determined from Eq. (19), and \( b_{max} \) follows from Eq. (20). The value of \( b_{max} \), which is directly connected with the selection criteria that determined the \( N_{AgBr} \), then determines the number of interacting nucleons through the calculation of

\[
W(b_{max}) = W_A(b_{max}) + W_B(b_{max}).
\]  

(26)
Table V presents the values of $b_{\text{max}}$ resulting from the $N_h > 15$ and $N_f = 0$ selection criteria, and the corresponding $W$ values, for the A-AgBr central collisions. The value of $b_{\text{max}}$ required to boil off more than fifteen evaporation tracks from the AgBr targets decreases, perhaps surprisingly, as projectile mass (and size) increases. This trend is probably due to the $N_f = 0$ requirement, since smaller impact parameters should be necessary for total fragmentation of the increasingly massive projectiles.
<table>
<thead>
<tr>
<th>Beam</th>
<th>( b_{\text{max}} \text{ (fm)} )</th>
<th>( W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{16}\text{O})</td>
<td>4.68 ± 0.16</td>
<td>38.5 ± 3.0</td>
</tr>
<tr>
<td>(^{28}\text{Si})</td>
<td>4.27 ± 0.23</td>
<td>60.8 ± 3.9</td>
</tr>
<tr>
<td>(^{32}\text{S})</td>
<td>4.08 ± 0.22</td>
<td>68.4 ± 3.9</td>
</tr>
</tbody>
</table>
3. Test of Superposition

The measured particle production multiplicities for the present set of heavy ion data [Jurak et al. 1989] are plotted in Fig. 9 as a function of the calculated $W$ values. Earlier hadron-nucleus studies have shown that proton-nucleus particle production can be described by a superposition of elementary interactions. Consequently, lines with unit slope have been drawn through the proton-nucleus data points at each energy. The lines may therefore be taken to represent the superposition principle. The $^{16}$O, $^{28}$Si and $^{32}$S central collisions generally follow the same dependence on the number of wounded nucleons as the proton data, indicating agreement with a superposition picture.

Recalling Eq. (7), the relationship between $N/W$ and $n_{pp}$ is explored in Fig. 10 for the $^{16}$O, $^{28}$Si and $^{32}$S central interactions. The average multiplicity per wounded nucleon in the central collisions is plotted as a function of the multiplicity in elementary proton-proton collisions at equivalent energies. The solid line represents a fit to the inclusive proton-Emulsion and central proton-AgBr data and is flanked by dashed lines which indicate the limits of the fit. The proton-nucleus data describe a straight line as expected from Eqs. (6) and (7), so the dashed lines may be considered to bound a region consistent with superposition. Results of the central interactions fall within this region and are, therefore, not inconsistent with superposition over laboratory beam energies from 14.6 to 200 GeV/nucleon.
Figure 9. Produced particle multiplicities $N$ vs $W$. Solid $\triangle$'s: inclusive proton-Emulsion data at 14.6 and 60 GeV/nucleon and both inclusive proton-Emulsion and central proton-AgBr data at 200 GeV/nucleon. Open symbols: central A-AgBr collisions for $^{16}$O ($\triangle$'s), $^{28}$Si (○) and $^{32}$S (□) projectiles.
Figure 10. Produced particles per wounded nucleon $N/W$ vs mean production multiplicity $n_{pp}$ in proton-proton interactions at equivalent energies.
Recall that the requirement of a completely fragmented projectile, i.e., no projectile fragments with charge greater than one \( N_f = 0 \), ensured central collisions. The value of \( b_{mar} \), and hence \( W \), also depends on the effect of applying the chosen \( N_h > 15 \) cut, which effectively ensures interactions with either Ag or Br. Possible systematic uncertainties introduced by this somewhat arbitrary choice can be addressed by analyzing events with \( N_f = 0 \), but with different \( N_h \) cuts.

The ratio \( N/W \) is plotted as a function of mean \( N_h \) (corresponding to \( N_h > 15, 20, 25 \) and 30) for the \(^{16}\text{O},^{28}\text{Si}\) and \(^{32}\text{S}\) primaries in Figs. 11a, b and c, respectively. As mean \( N_h \) increases, \( N/W \) slowly decreases toward an apparent asymptotic value. This tendency is somewhat puzzling within a superposition interaction picture and may indicate the presence of a systematic problem associated with linking an \( N_h \) cut to \( b_{mar} \) and \( W \). This systematic effect may be moderated, as mean \( N_h \) increases and more nearly direct hits \( (b_{mar}=0) \) are selected, as the range of permissible impact parameters \( (0 \leq b \leq b_{mar}) \) shrinks. However, it does not remove any data point from the region of superposition in Fig. 10, and therefore does not change the preliminary conclusion: production multiplicities in central A-AgBr interactions may be adequately described by the simple superposition of many individual nucleon-nucleon collisions. In fact, as mean \( N_h \) increases, the resulting \( N/W \) ratios tend to support the superposition model more precisely.

If Fig. 10 is redone using an \( N_h > 30 \) cut along with the \( N_f = 0 \) requirement, the result strongly supports a superposition interpretation of nucleus-nucleus interactions. Such a graph is shown in Fig. 11d. Here, rather than reproducing the fit to proton-nucleus data, a line has been drawn through the origin with a
slope of $\frac{1}{2}$; exactly as prescribed by Eq. (7), the fundamental equation in the wounded nucleon model. The data follow this line of superposition with consistency and strongly suggest a final conclusion: particle production multiplicities in the present set of central nucleus-nucleus collisions can be accurately described by superimposing many individual hadron-hadron interactions. Collective contributions to the measured mean multiplicities appear to be minimal.
Figure 11a. $N/W$ vs mean $N_h$ for $^{16}$O projectiles.
Figure 11b. $N/W$ vs mean $N_h$ for $^{28}\text{Si}$ projectiles.
Figure 11c. $N/W$ vs mean $N_h$ for $^{32}\text{S}$ projectiles.
Figure 11d. Produced particles per wounded nucleon $N/W$ for events with $N_h > 30$ and $N_f = 0$ vs mean production multiplicity $n_{pp}$ in proton-proton interactions at equivalent energies.
1. Regions of Pseudo-rapidity

The longitudinal momentum distributions of particles produced in high energy hadron-hadron interactions are customarily presented in terms of the rapidity scaling variable, \( y \), where \( y \) is defined as

\[
y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z},
\]

where \( E \) is the total energy of a particle in question, and \( p_z \) is the component of the particle's momentum parallel to the designated \( z \) axis, usually the beam direction. The particle's rapidity in any other Lorentz reference frame takes the form \( y' = y + (\text{constant}) \). This implies that the rapidity distribution is Lorentz invariant, so a transformation shifts the position of the distribution in rapidity space but leaves the structure of the distribution unchanged. Notice also that \( y(p_z = 0) = 0 \), which means that a particle with only transverse momentum has zero rapidity.

Measurement of \( p_z \) is extremely difficult in nuclear emulsion. However, if \( p^2 = p_t^2 + p_z^2 \gg m^2 \) then \( E = \sqrt{p^2 + m^2} \rightarrow p \) and

\[
y \rightarrow \frac{1}{2} \ln \left[ \frac{p + p_z}{p - p_z} \right] = -\ln \left[ \tan \left( \frac{\theta}{2} \right) \right] \equiv \eta
\]

Since \( \theta \), the polar angle with respect to the beam axis, is measured directly and easily in emulsions, the quantity \( \eta \) (called the pseudo-rapidity) is generally used in place of \( y \) in emulsion experiments. The substitution is valid because most of the particles produced in high energy nucleus-nucleus collisions are pions with mass \( m \) around 0.14 GeV/c\(^2\), and the difference between \( \eta \) and \( y \) is typically
less than the experimental uncertainty associated with measuring $\eta$. For the case of heavier particles, such as kaons, protons and anti-protons, the approximation becomes more problematic, but fortunately the cross section for pion production is so dominant over the other possible channels that $\eta$ remains a viable parameter.

A schematic pseudo-rapidity distribution is presented in Fig. 12a. The distribution is divided into three regions. The broad central peak (b) contains the multiplicity of particles (mostly pi mesons) produced during the interaction. These relativistic ($\beta \geq 0.7$) particles leave light ionization trails ($g \leq 1.4 g_{\text{min}}$) in the developed emulsion. For illustrative purposes they are shown to be distributed about an $\eta$ value of 3.0, although the particular value of the centroid in $\eta$ is determined by the kinematics involved. Figure 12b shows this shower of produced particles to be emitted in a cone about the centroid $\eta$ value.

The small peak (c) located at large $\eta$ (small $\theta$) contains the remains of the projectile and is labeled the projectile fragmentation region. These projectile fragments emerge from the interaction vertex as relativistic (lightly ionizing) tracks narrowly collimated about the beam axis and, hence, at large $\eta$. They are shown as the narrow cone in Fig. 12b.

The third peak (a) in Fig. 12a is called the target fragmentation region, and corresponds to target fragments (both grey knockout protons and black evaporation-track particles) that emerge from the interaction region at large angles. This distribution is typically centered around $\eta = 0.0$ ($\theta = 90$ degrees.) In practice, the target fragments and produced particles are not generally displayed together on the same graph.
Figure 12a. A schematic pseudo-rapidity ($\eta$) distribution: target fragmentation region (a), central production region (b), and projectile fragmentation region (c).
Figure 12b. Track characteristics corresponding to the distribution in Fig. 12a.
2. Observed Distributions

The measured pseudo-rapidity distributions for the charged particles produced at each combination of beam species and energy are presented in Figs. 13a, b, c, d and e. The data points are represented by crosses. The solid curves correspond to gaussian distributions which fit the central pseudo-rapidity regions reasonably well but fail to reproduce the tails. The fit parameters are given in Table VI. The forward tails include fragments from the projectile since these particles are relativistic and have unit charge. (Recall that we required complete disintegration of the projectile into its constituent nucleons in order to ensure that the selected collisions were central. It is, in practice, very difficult to disentangle these minimum ionizing projectile fragments from the particles produced by the deposition of energy in the interaction volume.) The distributions are normalized to the number of events measured, so the area under each curve corresponds to the average production multiplicity, after allowance for the projectile fragments (whose number is known: 8 for \(^{16}\text{O}\), etc.). Recall that neutral particles are not observed in emulsion and that target fragments do not present a background for produced particles because of their distinct track characteristics.

It should be noted that particle production is expected to become independent of pseudo-rapidity at energies high enough to produce a quark-gluon plasma [Bjorken 1983, Blaizot and Ollitrault 1990]. This would correspond to a plateau-like structure in the central (production) region of the \(\eta\) distributions. No plateau has been observed in the present data, although some flattening occurs in the central region of the \(^{32}\text{S}\) distribution. Bjorken's formula for calculating the density of
energy deposited in the interaction volume is given below,

$$\epsilon = \frac{3}{2} \left[ \frac{1}{N_{ev}} \frac{dN}{d\eta} \right] \sqrt{p_t^2 + m^2} \left( \frac{1}{\pi T_0 R^2} \right).$$  \hfill (29)

The last term corresponds to the interaction volume, which is taken to be a cylinder of length $T_0 = 1$ fm and radius $R = R_0 A_0^{1/3}$ where $R_0 = 1.2$ fm and $A_0$ is the atomic mass number of the projectile. The second term represents the average transverse energy deposited per produced meson, and the first term gives the number of mesons involved in the interaction ($\frac{1}{N_{ev}} \frac{dN}{d\eta}$ is essentially a multiplicity distribution in $\eta$ space.) The factor $3/2$ takes into account the produced neutral particles (unseen in nuclear emulsion).

Assuming that most of the produced particles are pi mesons of mass 0.14 GeV/c$^2$ and using the value of $\frac{1}{N_{ev}} \frac{dN}{d\eta}$ in the flattest (central) region of each distribution results in the values of $\epsilon$ given in Table VII. Bjorken and others have speculated that quark-gluon plasma formation may occur if energy densities on the order of 2-3 GeV/fm$^3$ are realized. As can be seen, this level of energy deposition has not been achieved in the present set of data. On the other hand, it should be remembered that all these values of $\epsilon$ are approximate in nature rather than definitive.

Since the quantity $\sqrt{p_t^2 + m^2}$ remains roughly constant at high energy, the most direct way to increase $\epsilon$ is to increase $\frac{1}{N_{ev}} \frac{dN}{d\eta}$, i.e., to increase the production multiplicity. This can be accomplished either by using more massive projectiles (and/or targets) to increase the number of nucleons participating in the collisions or by increasing the energy/nucleon available for the production of new particles (i.e. $\sqrt{S}$). The first approach is being planned by CERN in Geneva, where $^{207}$Pb
nuclei are scheduled for acceleration about 1991. The second method is planned for implementation at Brookhaven National Laboratory through the development of a Relativistic Heavy Ion Collider (RHIC), which will provide center of mass energies on the order of 200 GeV/nucleon. Both of these approaches will greatly increase the density of produced particles and, therefore, contribute substantially to larger values of energy density. At present, we must conclude that the broad features of the $\eta$ distributions do not suggest any unusual phenomena, although it should be remembered that at least one very high energy cosmic ray event ($\text{Ca + C at } E = 100 \text{ TeV/nucleon}$, [Burnett et al. 1983]) has been observed to exhibit a plateau in $\eta$ over many units of pseudo-rapidity.
TABLE VI. Gaussian Fit Parameters

<table>
<thead>
<tr>
<th>Beam</th>
<th>E (GeV/n)</th>
<th>$\eta_0$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O</td>
<td>14.6</td>
<td>1.695±0.008</td>
<td>1.047±0.010</td>
</tr>
<tr>
<td>$^{28}$Si</td>
<td>14.6</td>
<td>1.767±0.003</td>
<td>1.067±0.004</td>
</tr>
<tr>
<td>$^{16}$O</td>
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<td>2.207±0.004</td>
<td>1.311±0.005</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>200.0</td>
<td>2.884±0.003</td>
<td>1.623±0.004</td>
</tr>
<tr>
<td>$^{32}$S</td>
<td>200.0</td>
<td>2.990±0.001</td>
<td>1.635±0.002</td>
</tr>
</tbody>
</table>
TABLE VII. Energy Density

<table>
<thead>
<tr>
<th>Beam</th>
<th>E (GeV/n)</th>
<th>$\epsilon$ (GeV/fm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.4</td>
</tr>
<tr>
<td>$^{28}$Si</td>
<td>14.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>60.0</td>
<td>0.6</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>200.0</td>
<td>1.1</td>
</tr>
<tr>
<td>$^{32}$S</td>
<td>200.0</td>
<td>1.3</td>
</tr>
</tbody>
</table>
Figure 13a. Normalized pseudo-rapidity distribution for $^{16}$O at 14.6 GeV/nucleon.
Figure 13b. Normalized pseudo-rapidity distribution for $^{28}\text{Si}$ at 14.6 GeV/nucleon.
Figure 13c. Normalized pseudo-rapidity distribution for $^{16}$O at 60 GeV/nucleon.
Figure 13d. Normalized pseudo-rapidity distribution for $^{16}$O at 200 GeV/nucleon.
Figure 13e. Normalized pseudo-rapidity distribution for $^{32}$S at 200 GeV/nucleon.
3. Particle Correlations

The resolution of the emulsion detector has also permitted study of the fine structure of the pseudo-rapidity distributions. The nature of possible structure can be addressed, for example, through calculation of the two-particle correlation function [Berger 1975]

\[ R_2(\eta_1, \eta_2) = \frac{r_2(\eta_1, \eta_2)}{r_1(\eta_1)r_1(\eta_2)} - 1.0, \]  

where \( r_1 \) and \( r_2 \) are the single-particle and two-particle pseudo-rapidity densities, respectively. The value of this function is constrained to lie between 1.0 (perfect correlation) and -1.0 (perfect anti-correlation). A value of zero implies no correlation between particles produced at \( \eta_1 \) and \( \eta_2 \). Results of such analysis performed on p-A data suggest non-independent production, i.e., that particles emerge from the interaction region in clusters [Barbier et al. 1988a]. However, similar calculations with the present set of A-A data have essentially shown a null result, i.e., no correlations [Von Gersdorff et al. 1989]. This difference is perhaps not surprising, because substantially increased production multiplicities in collisions involving large numbers of participating nucleons may make individual clusters more difficult to resolve. In other words, actual correlations may be masked by many (supposed) clusters superimposed on each other in \( \eta \) space.

An alternate approach involving analysis of the lengths (or gaps) in pseudo-rapidity space between produced particles, and the number of particles in those gaps, has also indicated no significant cluster formation in the A-A data [Von Gersdorff et al. 1989].
VI. FLUCTUATIONS

1. The Method of Moments

Since the standard correlation techniques seem inadequate to examine structure in the \( \eta \) distributions for the typical heavy ion collisions found in the present study, a different approach is required. A time-honored method for extracting useful information from complex distributions involves the calculation of moments of the distribution in question. For example, the second standard moment about the mean value of a set of \( N \) measurements of the quantity \( x \) is the familiar variance, or square of the standard deviation of the distribution of measured values of \( x \). The variance specifies the spread about the mean value of the individual measurements, i.e.,

\[
\sigma^2 = (x_i - \bar{x})^2. \tag{31}
\]

Bialas and Peschanski [1986, 1988a and 1988b] in a series of papers have developed a method of calculating scaled moments of the single particle rapidity (or pseudo-rapidity) distribution

\[
\tau(\eta) = \frac{1}{N_{ev}} \frac{dN}{d\eta}. \tag{32}
\]

Given a set of measured events satisfying some selection criteria, the method involves calculating the single particle \( \eta \) distribution for each event and for the set of events. A region \( \Delta \eta = \eta_{\text{max}} - \eta_{\text{min}} \) of the kinematically available \( \eta \) space is chosen for examination and subdivided into \( M \) bins, each of length (or scale)

\[
\delta\eta = \Delta\eta/M. \tag{33}
\]
The measured distribution is a manifestation of the underlying or parent distribution

\[ P(p_1, p_2, \ldots, p_M) \]  

(34)
of probabilities

\[ p_1, p_2, \ldots, p_M \]  

(35)
for finding particles in each of the \( m = 1, \ldots, M \) bins. This probability distribution follows the usual normalization requirements of

\[ p_1 + p_2 + \cdots + p_M = 1 \]  

(36)
and

\[ \int dp_1 \cdots dp_M P(p_1, p_2, \ldots, p_M) = 1. \]  

(37)
The scaled moments of order \( i \) of this distribution are given by

\[ \langle C_i \rangle = \int dp_1 \cdots dp_M P(p_1, p_2, \ldots, p_M) \frac{1}{M} \sum_{m=1}^{M} (M p_m)^i. \]  

(38)

Consequently, these moments depend on the number of bins, \( M \), into which the \( \eta \) region of interest has been subdivided. Equivalently, they are functions of the bin size \( \delta \eta \), and their behavior as the bin size is varied should reveal information on the scale or size of structure in pseudo-rapidity.

However, it is difficult to obtain the complicated multi-dimensional distribution \( P(p_1, p_2, \ldots, p_M) \). Using the \( \langle C_i \rangle \) to examine the \( \eta \) distributions would remain problematical if not for the important discovery that scaled moments of the parent distribution can be accurately approximated by calculating scaled factorial moments of the experimentally measured distribution [Bialas and Peschanski 1986].
These scaled factorial moments of order \( i \) are given for a set of events of non-fixed multiplicity by

\[
F_i = \frac{M^{i-1}}{\langle K \rangle^i} \sum_{m=1}^{N} k_m (k_m - 1) \cdots (k_m - i + 1),
\]  

(39)

where \( k_m \) is the number of particles in the \( m^{th} \) bin and \( \langle K \rangle \) is the average multiplicity of the set of events. For the case of fixed multiplicity \( N \) the \( F_i \) are given by

\[
F_i = \frac{M^{i-1}}{N(N-1) \cdots (N-i+1)} \sum_{m=1}^{N} k_m (k_m - 1) \cdots (k_m - i + 1).
\]  

(40)

For both fixed and non-fixed multiplicities, Bialas and Peschanski show that the factorial moments become equal to the scaled regular moments if the number of particles in the measured sample is large, i.e., \( \langle F_i \rangle = \langle C_i \rangle \) for a large dataset. The operational procedure is simply to calculate the \( F_i \) for each event in the sample and then average over all the events. Most importantly, the statistical fluctuations due to a large, but still finite, number of particles are removed by calculating the factorial moments, which reproduce the results of the regular calculation on the true parent distribution. Therefore, use of the factorial moments facilitates a search for non-statistical fluctuations of particle density in the bins, i.e., the method enables a search for fluctuations of physical origin over many scales of pseudo-rapidity.

Before proceeding further we should examine the hypothetical case in which only statistical fluctuations are present in a dataset. If the experimental distribution and the parent distribution are identical except for statistical fluctuations in the measured numbers of particles in the bins, the probabilities \( p_1, ..., p_M \) are given by
the average experimental values \( \langle p_m \rangle \), so that

\[
p_m = \langle p_m \rangle = \frac{\int_{\Delta \eta} r(\eta) d\eta}{\int_{\Delta \eta} r(\eta) d\eta},
\]

(41)

and \( P(p_1, \ldots, p_M) \) becomes a product of Dirac delta functions

\[
P(p_1, \ldots, p_M) = \delta(p_1 - \langle p_1 \rangle) \cdots \delta(p_M - \langle p_M \rangle).
\]

(42)

Using Eq. (42) in Eq. (38), the moments \( \langle C_i \rangle \) are now given by

\[
\langle C_i \rangle = \frac{1}{M} \sum_{m=1}^{M} (M \langle p_m \rangle)^i.
\]

(43)

Recalling that \( M = \frac{\Delta \eta}{\delta \eta} \), one may write

\[
\langle C_i \rangle = \frac{1}{\Delta \eta} \sum_{m=1}^{M} \int_{\Delta \eta} [\frac{1}{\delta \eta} \int_{\Delta \eta} r(\eta) d\eta]^{i-1} \delta \eta.
\]

(44)

In the limit of small bin sizes

\[
\delta \eta \rightarrow d\eta,
\]

(45)

and Eq. (44) becomes

\[
\langle C_i \rangle = \frac{1}{\Delta \eta} \int_{\Delta \eta} [r(\eta)]^i d\eta \equiv R_i,
\]

(46)

which is independent of \( \delta \eta \). We thus find that moments \( R_i \) for distributions containing only statistical fluctuations become independent of bin size as the bin size becomes smaller than a typical scale in pseudo-rapidity over which the distribution changes substantially, and the substitution of \( d\eta \) for \( \delta \eta \) is appropriate. Since the \( R_i \) are independent of \( \delta \eta \) only for a flat \( r(\eta) \), even a distribution with only statistical fluctuations will generate moments with some dependence on the binning, provided the overall shape of \( r(\eta) \) is not flat. This shape contribution corresponds to the
bin-to-bin changes (or "fluctuations") in average particle density that occur as the broad features of the distribution change shape. An analogous shape contribution may be expected in the factorial moments. Equation (46), however, provides a means of correcting the \( F_i \) if the normalized moments are defined [Fialkowski et al. 1989] as

\[
\langle Z_i \rangle = \frac{\langle C_i \rangle}{R_i} = \frac{\langle F_i \rangle}{R_i}.
\]  

(47)

Structure in the experimental \( \eta \) distributions can be examined by replacing the \( \langle C_i \rangle \) with the scaled factorial moments \( \langle F_i \rangle \) and adjusting for distribution shape by dividing the \( \langle F_i \rangle \) by the correction factor \( R_i \) [Holynski et al. 1989a]. The factorial moments \( F_i \) are calculated event-by-event and then averaged to obtain \( \langle F_i \rangle \), while \( R_i \) is calculated from the average pseudo-rapidity distribution for the set of events under investigation. Thus \( R_i \) reflects the broad features of the average distribution shape.

2. Intermittency

Large fluctuations of particle density in the pseudo-rapidity bins have been widely proposed [Jacob 1989] as a signal of collective behavior in high energy heavy ion interactions. Such behavior may be linked to a phase transition from a quark-gluon plasma to normal hadronic matter during the expansion and cooling of the plasma [Van Hove 1984]. Calculation of the normalized moments provides a systematic method for evaluating these fluctuations.

A form of pseudo-rapidity density fluctuation over all scales, called intermittency, is characterized by a power law dependence of the scaled moments on bin
size [Holynski et al. 1989a and 1989b]:

$$\langle Z_i \rangle = M^{\phi_i} = \left( \frac{\Delta \eta}{\delta \eta} \right)^{\phi_i}.$$  \hspace{1cm} (48)

This type of dependence takes its name from the term used to describe the transition between turbulent and laminar flow in fluid hydrodynamics. In the case of hadronization of the QGP, intermittency may correspond to a cascade process during the hydrodynamical expansion in which the expanding and cooling plasma breaks up into segments of pseudo-rapidity that fluctuate independently and, in turn, break into smaller independently fluctuating segments [Bialas and Peschanski 1986]. The intermittent pattern of fluctuating pseudo-rapidity density is consistent with the disorder existing during a phase transition.

The present set of heavy ion interactions provides a range of energies and volumes of interacting nucleons, and hence a range of energy densities, over which to test for intermittent behavior in pseudo-rapidity. Equation (48) can be written

$$\ln \langle Z_i \rangle = \phi_i [ - \ln(\delta \eta) ] + \text{Intercept},$$  \hspace{1cm} (49)

so that intermittent behavior in the moments will show up as a linear rise in \( \ln \langle Z_i \rangle \) as the bin size \( \delta \eta \) decreases and the variable \( - \ln(\delta \eta) \) increases. The bin size may be decreased until limited by the experimental measuring error. This limit, in practice, occurs below a bin size of 0.1 units in \( \eta \).

Intermittency is predicted to be a phenomenon of the fine structure in \( \eta \) and is not an appropriate interpretation of moment behavior at large bin sizes. Therefore, bins larger than one unit in \( \eta \) were omitted from the intermittency analysis. The five datasets corresponding to the five combinations of beam energy and projectile
mass in the present experiment have been examined for intermittent behavior for resolution in pseudo-rapidity (or bin size) in the range of scales from 1.0 down to 0.1 \( \eta \) units. The pseudo-rapidity range \( \Delta \eta \) was chosen in each case to be the full width at half maximum (FWHM) from the gaussian fits (Table VI). This range was then binned as closely as possible into cells of width 1.0 down to 0.1 units of \( \eta \). The results of the analysis, for moments of order \( i = 2, 3, 4, 5 \) and 6, are shown graphically in Figs. 14a, b, c, d and e. A linear relationship between \( \ln(Z_i) \) and \(-\ln(\delta \eta)\) is apparent in each case. This means that non-statistical fluctuations are indeed occurring in the pseudo-rapidity distributions over all scales sampled in the binning process.

The slopes, \( \phi_i \), corresponding to the observed linear relationships are given in Table VIII. The number in parenthesis following a quoted slope value corresponds to the uncertainty in the last quoted digit. The slopes generally tend to increase with increasing moment order for a given combination of projectile and beam energy.

The intermittency effect in the present data is most pronounced (and most reliably observed) for the combination of smallest number of interacting nucleons and highest energy per nucleon (200 Gev/nucleon \(^{16}\)O primaries). An even stronger signature has been observed in p-AgBr central collisions at the same energy [Holynski et al. 1989b]. Increasing the primary to \(^{32}\)S produces the weakest signature at 200 GeV/nucleon. The same pattern is observed at 14.6 GeV/nucleon, where intermittency is fairly well pronounced for \(^{16}\)O but not for \(^{28}\)Si (which has the smallest ratio of energy per interacting nucleon to total number of interacting nucleons, and
the weakest intermittency signature, in the current data). Intermittent behavior
may therefore be correlated with a large value of energy per interacting nucleon.
Conversely stated, intermittent behavior seems to diminish for a given (high) en-
ergy if the number of nucleons participating in the collision increases. This is
contrary to the association of intermittency with quark-gluon plasma formation,
since collective effects, such as plasma formation, are expected to occur when large
numbers of energetic nucleons interact.
TABLE VIII. Intermittency Slope Parameters, $\phi_i$

<table>
<thead>
<tr>
<th></th>
<th>$\phi_i$</th>
<th>$\phi_i$</th>
<th>$\phi_i$</th>
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<tbody>
<tr>
<td>$^{16}\text{O}$</td>
<td>14.6 GeV/n</td>
<td>60 GeV/n</td>
<td>200 GeV/n</td>
</tr>
<tr>
<td>2</td>
<td>0.006 (2)</td>
<td>0.010 (1)</td>
<td>0.007 (1)</td>
</tr>
<tr>
<td>3</td>
<td>0.018 (4)</td>
<td>0.025 (3)</td>
<td>0.024 (2)</td>
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<tr>
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<td>0.038 (11)</td>
<td>0.037 (7)</td>
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<td>5</td>
<td>0.064 (24)</td>
<td>0.040 (13)</td>
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</tr>
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<td>6</td>
<td>0.079 (43)</td>
<td>0.032 (23)</td>
<td>0.144 (12)</td>
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<table>
<thead>
<tr>
<th>$^{28}\text{Si}$</th>
<th>$^{32}\text{S}$</th>
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<tbody>
<tr>
<td>14.6 GeV/n</td>
<td>200 GeV/n</td>
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<td>2</td>
<td>0.009 (1)</td>
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<td>3</td>
<td>0.024 (4)</td>
</tr>
<tr>
<td>4</td>
<td>0.036 (8)</td>
</tr>
<tr>
<td>5</td>
<td>0.031 (15)</td>
</tr>
<tr>
<td>6</td>
<td>-0.013 (26)</td>
</tr>
</tbody>
</table>
Figure 14a. Normalized factorial moments for $^{16}$O at 14.6 GeV/nucleon.
Figure 14b. Normalized factorial moments for $^{28}$Si at 14.6 GeV/nucleon.
Figure 14c. Normalized factorial moments for $^{16}$O at 60 GeV/nucleon.
Figure 14d. Normalized factorial moments for $^{16}$O at 200 GeV/nucleon.
Figure 14e. Normalized factorial moments for $^{32}$S at 200 GeV/nucleon.
Central collisions of $^{16}$O, $^{28}$Si and $^{32}$S nuclei with the $^{108}$Ag and $^{80}$Br targets in nuclear emulsion have been measured at the highest currently available beam energies for heavy projectiles. The observed particle production multiplicities are generally consistent with an incoherent superposition of many elementary nucleon-nucleon interactions, as expected from an extrapolation based on results from studies of hadron-nucleus collisions. For the chosen $N_h > 15$ cut, the number of produced particles per interacting nucleon, $N/W$, rises somewhat faster as a function of $n_{pn}$ than a strict superposition interpretation would suggest. This may be due to some cascading during the more complex nucleus-nucleus interactions, although as shown in Fig. 10, the nucleus-nucleus $N/W$ values fall within the boundaries of the fit to proton-nucleus data (which do not exhibit cascading).

Overall, the agreement between nucleus-nucleus and proton-nucleus data suggests that the nucleus-nucleus results can be adequately explained by the superposition principle over the range of energies available at present. Events corresponding to larger values of the $N_h$ cutoff follow the superposition model even more closely and strongly reinforce this conservative picture. Further substantiation comes from the multiplicity distributions of produced particles, which have been examined in pseudo-rapidity ($\eta$) space and exhibit a gaussian shape in the central production regions rather than the plateau associated with QGP formation.

The fine structure of the pseudo-rapidity distributions was examined by calculation of normalized scaled factorial moments, which were found to be functions of the bin width, or scale, into which the $\eta$ distributions were subdivided. A power
law dependence of the moments on the bin size is apparent over all experimentally measurable scales in $\eta$. This form of dependence over all scales, called intermittency, may be associated with collective behavior, although the strength of the intermittent effect declines at a given energy as the number of interacting nucleons increases, contrary to the expectations associated with a transition to a quark-gluon plasma phase. Since intermittent behavior has been observed in p-A and A-A collisions at all energies considered, it may be a general property of particle production rather than an unambiguous signature of collective behavior. This conclusion is supported by the association of intermittent behavior with collimated emission of particles (pencil jets) in hadron cascade models [Ochs and Wosiek 1988], and with the hadronic final states following electron-positron annihilation [Fialkowski et al. 1989].

The energy densities achieved in the present set of nucleus-nucleus collisions may simply be too small to enable an adequate examination of production multiplicities for behavior associated unambiguously with quark matter. The proposed development of a Relativistic Heavy Ion Collider (RHIC) at BNL provides encouragement for future research. Meanwhile, the imminent acceleration of $^{208}$Pb to 200 GeV/nucleon at CERN should provide an order of magnitude increase in energy density and perhaps provide answers to the questions surrounding the existence of the quark-gluon plasma.
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