Modeling of Fluid Injection for Hydraulic Fracturing Initiation in Porous Rock Formations

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MODELING OF FLUID INJECTION FOR HYDRAULIC FRACTURING INITIATION IN POROUS ROCK FORMATIONS

A Thesis

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering in

The Department of Civil and Environmental Engineering

by

Krishna Adhikari
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ABSTRACT

Hydraulic fracturing has been widely opted in recent times to simulate the unconventional reservoirs and thus, has become key a subject of interest in petroleum engineering. The stress concentration around the borehole affects the breakdown pressure and fracture reopening pressure during the hydraulic fracturing treatments. Thus, the state of stress around borehole wall and its surrounding rocks due to instantaneous drilling and a fluid injection at borehole surface needs to be determined accurately to initiate the hydraulic fractures. This research intends to derive the analytical stress solution of inclined borehole subjected to time-dependent fluid injection and in-situ stress, verify the solution and develop the simulator tool for its implementation. Upon drilling a borehole in a fluid saturated porous medium, it is assumed that pressurized fluid is injected with different flow rate in a finite section of borehole wall. The variation of pore pressure, effective tangential stress and radial stress have been calculated analytically using linear theory of poroelasticity and then, numerically with the use of finite element analysis program ABAQUS. The comparison is made between them to verify the accuracy of formulated analytical solution. Finally, after confirming the validity of analytical solution, a Matlab based simulator tool with a user friendly interface is developed to ease and simplify the procedure of determining the time-dependent stress and pore pressure induced due to in-situ stress and fluid injection during the hydraulic fracturing process.
CHAPTER 1. INTRODUCTION

1.1. Overview

The process of creating fractures into the rocks by injecting the high pressured fluid to extract oil and gas trapped inside the fluid-saturated formation is referred as hydraulic fracturing. It is also well known as fracking. Fracking is considered to be highly productive due to advanced production process and also being cost effective at the same time. The state of stress of the formation during the fluid injection process is the most critical parameters required for the analysis, design and initiation of fractures. The breakdown occurs when the effective tangential stress exceeds the tensile strength of the formation. Since rocks fractured in this process are a porous medium, general theory linear poroelasticity which was given by Biot (1941) can be idealize and use to solve problems in hydraulic fracturing. It can also be used to solve problems in other engineering fields such as geotechnical engineering, geomechanical engineering etc. For instance, borehole stability analysis can be performed based on theory of poroelasticity. The instability of wellbore has been another problem that petroleum industry often face. If the stress distribution after drilling of wellbore are not accurately estimated, required mud pressure may not be calculated correctly. This may lead to wellbore collapse. Hence, it is required to fully understand the poroelastic behavior of materials for a successful drilling a borehole, its stability analysis and initiation of hydraulic fractures.

Theory of linear poroelasticity can be used to formulate the stress solution of an inclined borehole subjected to time-dependent fluid injection on a finite section in addition to the in-situ stress. It can then, determine the direction and location of maximum and minimum principal stress. The concept of poroelasticity comes from the fact that when an external loading is imposed on a
material that is formed of both solid and liquid, the behavior wouldn’t be same as in perfectly solid or non-porous medium. It is because of the coupling effect that occurs between solid and the fluid in a porous material. Since, linear theory of elasticity does not account the coupling effects between stresses and pore fluid pressure induced due to the application of external loading, theory of poroelastic is thought to be more rigorous and accurate when solving problems related with porous mediums filled with fluid or gas. Hence, the linear theory of poroelasticity has been adopted in solving problems in geotechnical engineering, geomechanics and hydrogeology. Poroelastic behavior of a material can be primarily categorized into two types in terms of the underlying phenomena. They are solid to fluid coupling and fluid to solid coupling (Wang, 2000). The change occurred in the fluid pressure or mass of a fluid due to the change in stress applied is known as solid to fluid coupling. Similarly, the change in volume of a material caused by the change in applied fluid pressure or fluid mass is known as fluid to solid coupling. Some of the examples of these coupling includes change in water level in a well when train passes nearby F.H. King (1892), Water levels rise in Wells Near a Pumping Well Verruijt (1969), subsidence of land area after extraction of huge amount of oil, gas or any other fluid etc. As described by Biot (1941), stress, strain, pore pressure and fluid content increment are the four basic variables associated with the poroelasticity. Other five bulk materials constants as stated by Detournay & Cheng (1988) includes shear Modulus, drained Poisson’s ratio, undrained Poisson’s ratio, Skempton’s pore pressure coefficient and hydraulic diffusivity. All these poroelastic constants are required to fully describe a poroelastic phenomenon of an isotropic fluid filled porous media. A proper and accurate formulation of stress solution will aid in calculating mud pressure to stabilize the wellbore in addition to the required fluid injection rate to initiate and propagate the fractures in rocks.
1.2. Objectives and Scope

When a borehole is drilled in porous formation such as sandstone or a shale, assuming that the drilling would be instantaneous, there will be a change in state of stress due to the release of some portion of rock from the formation. Along with that, when fluid is discharged in or out of the system, the change in state of stress in the borehole wall and its vicinity will be even more significant. One of the objectives of this research is to formulate the analytical stress solution of a borehole subjected to the far-field in-situ stress and various time-dependent fluid injection at a finite section of borehole wall. The three different types of fluid injection boundary conditions considered in this research are (a) gradually applied unit step fluid injection (b) linearly decreasing fluid injection and (c) square wave fluid injection. The solutions are obtained for in-situ stress boundary problem and fluid injection boundary problem separately as a decomposed problem and then, superposed at the end to get the final solution. After deriving the analytical stress solution, the problem is modeled in finite element program ABAQUS and simulations are carried out for all of those three fluid injection boundary cases to check the accuracy of the derived analytical stress solution.

Another objective of this research is to develop a Matlab based tool with a user-friendly interface which will provide the analytical results for those above described fluid injection boundary cases. The analytical stress solution is somewhat complex and requires several Matlab or Mathematica coding to get the final result. So, in an effort to save time and simplify the complexity, a user friendly simulator tool is developed. The results can be obtained in few minutes using the simulator tool while the same problem takes hours or even days if we opted the numerical approach.
The actual reservoir stimulation, fracture geometry and design were not studied in this. But the derived analytical solution is the basis to determine or to estimate the fluid injection rate and fluid injection volume required to initiate the fractures. Breakdown of the formation occurs when, the pressure induced by the fluid injection exceeds the tensile strength of the formation. If the tensile strength of the formation is known, this solution can be used to predict whether or not fracture initiates with certain flow rate.

1.3. Thesis outline

There are six chapters in this thesis. The first chapter starts with a brief overview on hydraulic fracturing, poroelasticity and wellbore stability followed by scope and objectives of this research. Detailed literature review on these related topics are included in chapter 2. Chapter 3 provides the detail formulation of analytical stress solution of inclined borehole which is subjected different time-dependent fluid injection and in-situ stress. The results of numerical simulation performed using ABAQUS are presented in chapter 4. The comparison between analytical and numerical solution are also presented in the same chapter. Chapter 5 describes the developed Matlab based simulator tool and its functions. It also includes a guideline on how to use the tool properly with an illustration. Conclusion and summary of this thesis are presented in chapter 6. Some recommendations are given for the future study and further improvements on this topic.
CHAPTER 2.
LITERATURE REVIEW

2.1. Hydraulic Fracturing

Even though the concept of injecting fluid to simulate the reservoirs was successfully applied around 1930s, the major development started after mid-20th centuries. Nearly 2.5 million fracturing have been already completed throughout the world since the introduction of hydraulic fracturing in 1949 (Montgomery and Smith, 2010). One of the massive fracking was performed in 1968 in Oklahoma, USA. As petroleum industry grew rapidly in past few decades, intensive research in maximizing the productivity, reducing the cost and environmental impacts started peaking up. Fluid injection rate and volume are directly associated with the stability of wellbore, possible seismic impacts, higher production cost etc. Injecting the fluid at higher rate or pressure may cause wellbore collapse and may induce small earthquakes. The risk of earthquakes can be reduced by reducing the fluid injection rate (Ellsworth, 2013). Hydraulic fracturing experimental study (Bohloli and Pater, 2006) showed that short fractures were induced at high stress level with branches where straight and longer were obtained at lower stress level. Ji et al. (2009) developed a fully coupled model which considered the poroelastic effects, fracture growth, variation of stress and pressures during the fracturing process as well as the conductivity and volume of the fracture simultaneously. The fracture model analyzed 2D and 3D fracture geometry corresponding to the injection history. Several studies on hydraulic fracturing and its modeling gave emphasis on the relationship between the injection rate and injection volume with the fracture geometry. Being able to analyze the stress variation on the borehole wall and its vicinity due to fluid injection will be beneficial for the further improvements on such models and eventually aiding to an effective fracture design.
In order to determine the appropriate fluid injection rate into the rocks and other porous medium, it is essential to know the initial state of stress of such formations. Injection rate required depends on several factors such as type of rocks, depth of the formation, desired fracture geometry, chemical properties of fluid etc. Hubbert (1957) concluded that the fractures initiated are required to be perpendicular to the axis of minimum stress. Author also stated that initial in-situ stress, geometry of wellbore and characteristics of injecting fluid are the major factors influencing the breakdown pressure and fracture initiation. Another important conclusion from his work was that it seemed mechanically impossible to have horizontal fractures if overburden pressure in the formation is larger than the pressure induced by injecting the fluid. (Scheidegger, 1962; Kehle, 1964; Haimson and Fairhurst, 1968) all studied about the determining the in-situ stress using the data from hydraulic fracturing. They concluded that the obtained data throughout the hydraulic fracturing process can be used to back calculate the initial in-situ stress for boreholes with larger depth. The required injection pressure also depends on the type or characteristics of fluid used during the process. Several studies have also been conducted to determine the appropriate or an optimum characteristics of fracturing fluids to perform an effective hydraulic fracturing treatment. Howard and Fast (1957) concluded that fracturing fluid coefficients which is indirectly proportional to the effectiveness of fracturing fluid, can be reduced by raising the fluid viscosity or by using additives to decrease the fluid loss. In general, it appears that the hydraulic fracturing using pressurized fluid has several aspects to be considered and the fluid injection design is one of the critical part of the process.

2.2. Poroelasticity and its application in Geomechanics

In early days, studies were conducted without giving considerations to the porosity of rocks. Simple theory of linear elasticity were used to analyze the problems in petroleum and
geomechanical engineering. It is obvious that the breakdown pressure and fracture closure pressure estimated using elastic model will be much lower than the one predicted using poroelastic model (Detournay et al., 1989). Porous elastic constants of the rock, horizontal principal stress, formation pore fluid pressure and rock’s tensile strength are the factors influencing the required wellbore fluid pressure in order to initiate the fractures (Hamison and Fairhurst 1967). In fact, Hamison and Fairhurst (1967) were also the first to consider the effects of fluid flow in analyzing the hydraulic fracturing initiation and propagation. They made a comparison between the lab test results of hydraulic fracturing with the field data to check the influence of injected fluid flow into the rocks. As time progresses on, research works started considering rock as a porous formation. Researchers started using poroelasticity theory to perform the analysis and design of hydraulic fracturing, borehole stability analysis and other related geomechanical problems. Notable works in wellbore drilling and hydraulic fracturing based upon theory of linear poroelasticity started more significantly after 1980s. Since then, poroelastic theory has been consistently used to solve various problems in the field of mining, petroleum and geotechnical engineering. Earlier, Cleary (1976) gave the fundamental analytical solution of a fluid saturated porous solid. Detournay & Cheng (1988) gave the analytical displacement, stress and pore pressure solution in non-hydrostatic in-situ-stress field condition. That solution was useful to analyze the state of stress around the borehole wall and estimate the direction of minimum and maximum horizontal principal stress. The technique of Laplace transformation was used to derive the solution in Laplace domain first and numerically inverted using the Stehfest (1970) algorithm to transform back to time domain. The solution of vertically drilled wellbore was later extended to inclined borehole by Cui et.al (1997) by decomposing the loading schemes to plane-strain, antilpane and uni-axial elastic problems. However, 2-D solutions were not accurate enough to simulate the real field condition.
The three dimensional solution was proposed by Rajapakse (1993) but without considering the initial in-situ stress in the formation. Another wrong assumption on his work was that the radial stress at the borehole wall would be zero. The stress and pore pressure solution for various time-dependent pore pressure and flux boundary were given by Ekbote et al. (2004) for the plane strain problem. Their study included the poroelastic 2-D solution of the inclined borehole for three different types of pore pressure boundaries. Similarly, solutions for additional two types of flux boundary i.e. linearly reducing fluid discharge boundary and square wave fluid discharge boundary were also given. The shortcomings of solution given by Rajapakse (1993) were later rectified by Abousleiman and Chen (2010). The initial in-situ stress and radial stress were considered in their work by imposing more accurate boundary condition for the fluid injection boundary problem.

However, the solution given by Abousleiman and Chen (2010) only included the constant fluid injection boundary. It may not be always feasible or desirable to have constant injection pressure due to different circumstances and fractures design limitations. In such cases, it may require to consider time-dependent fluid injection rather than a steady fluid injection to simulate the real field conditions. Gradually applied unit step fluid injection, linearly decreasing fluid injection and square wave fluid injection boundary are some common time-dependent fluid injection patterns applicable and encountered in hydraulic fracturing initiation process. Gradually increasing fluid injection could be applied in formations with risk of borehole instability due to immediate injection of very high pressurized fluids in a recently drilled borehole. Gradually decreasing fluid injection can simulate the state of filter cake development and formation being completely impermeable after certain time. Similarly, square wave type fluid injection could be applied to create fractures from fatigue failure. Additionally, the solution of time-dependent flux given by Ekbote et al. (2004) was formulated for a plane strain case and may not be very accurate to match with the field
condition. Thus, both of the exiting works (Ekbote et al., 2004; Abousleiman and Chen, 2010) needs to be extended to derive the analytical 3-D stress solution for time-dependent fluid injection boundary using linear theory of poroelasticity. It would provide some valuable insights to the engineers to perform effective and efficient fracture initiation in addition to maintaining the stability of borehole.

2.3. Hydraulic Fracturing Simulator

For the purpose of making the analysis procedure of hydraulic fracturing treatments easier, quicker and less complex, computer application and simulators were widely in practice since early days. Both the numerical and analytical simulators were developed in the past for different hydraulic fracturing design and treatments. The simulators were mostly focused on analyzing geometry of fractures, fracture growth and critical break down pressure of the formation. Clearly et al. (1983) developed the three dimensional simulator called 3DHDRAC which could find the opening of cracks with respect to pressure distribution and rate of propagation of a tip. Crack opening was found by using surface integral method and the flow of fluid along the cracks were modeled by finite elements method. The simulator was tested and validated with the analytical and laboratory experimental results. It was further improved by Lam et al. (1986) with more added features. Clifton and Wang (1991) developed the hydraulic fracturing simulator tool called the TerraFrac Code considering the poroelastic effects. They concluded that, in a formation with high leak off, the poroelastic effects are higher. Thus, the change in pore pressure and in-situ stress is also significantly higher which could be the effect of longer injection time and fluid losses. The simulator was useful on calculating the back stress using the approximate crack opening time histories and approximate leak off. Clifton et al. (1991) also noted the effects of thermal stress on
initial in-situ stress change while injecting the cold fluid into the formation. As time progresses, 3-D simulation tools were developed. Most of those tools were based on numerical method.

3-D Numerical hydraulic fracturing simulator for fracture propagation was developed by Vandamme and Churran (1989) based on displacement discontinuity (DD) method. Another three dimensional numerical fracturing simulator capable of solving the hydraulically induced fracture in 3-D heterogeneous formation based on discontinuity displacement method and elastic theory was developed by Yamamoto et al. (1999). Linear elastic fracture mechanics, proppant transportation and non-Newtonian fracture fluid behavior were used to model fracture propagation. 2-D hydraulic fracturing simulator based on linear theory of poroelasticity were developed by Vandamme and Roegiers (1990) to study the effects of fluids leak off on variation of pore pressure, fracture width and opening. The conclusion was that, for the small normalized time less than 0.1, the poroelastic effects are lower and vice versa. Similar, analysis was carried out based on 3-D Displacement Continuity Method by Zhou and Ghassemi (2011). It was concluded the fracture aperture increases as the applied pressure exceeds the initial in-situ stress. Their simulation showed that slip or dilation would occur upon increasing the pressure on a joint that is critically stressed or pre-stressed. They concluded that such behavior can be observed in a discontinuous fluid pressured injected into the formation. The simulator created by Dean and Schmidt (2009) incorporated the hydraulic fracture extension, multiphase flow in a porous medium, heat transfer and solid deposition as well as poroelastic/plastic deformations. The simulator tool calculated the fracture growth based on critical SIF or cohesive elements exhibiting strain-softening behavior. A tool to simulate shale fracturing was designed by Xu et al. (2010) considering the influence of fluid injection that was able to predict the fracture growth and networks. The tool had with features such as support of different fluid types, proppant types, pumping schedules, etc. From these literatures,
it is clear that the developed simulator tools were more concerned on fracture geometry and leak-off test.

Even though many researchers pointed out that the fluid injection and stress distribution around the borehole is one of the most important parameter in designing and initiating the fractures, tools to estimate such stress variation due to fluid injection were very rare. Especially, the tools based on analytical poroelastic solution to estimate the stress and pore pressure developed due to fluid injection were not found. Most of the developed simulators found in literature were focused on the geometry of fractures, fracture growth and critical break down pressure of the formation. Successful development of such tool would facilitate the implementation of analytical solution that is derived in chapter 3. It can also be combined with other fracture design application and simulators for developing more advanced tool.

2.4. Summary

A thorough literature review was conducted on hydraulic fracturing, poroelasticity and hydraulic fracturing related simulator tools. The research studies on effects of time-dependent fluid injection during the hydraulic fracturing process were found to be very limited. Among the majority of literatures reviewed, the pressure generated due to injection was mentioned to be a dominant factor in hydraulic fracturing. Time-dependent flux and pore pressure were considered by Ekbote et al. (2005) for analyzing the wellbore stability but was only valid for a 2-D model. The solution of Abousleiman and Chen (2010) failed to include time-dependent fluid injection scenarios which prompted this research motivation. Similarly, simulators based on poroelastic solution to estimate the stress variation due to the application of time-dependent fluid injection
were not available in literature. Development of such simulators will certainly simplify the task of fluid injection design for hydraulic fracturing initiation.
CHAPTER 3.
DERIVATION OF ANALYTICAL SOLUTION

3.1. Introduction

This chapter presents the derivation and discussion of analytical stress solution of time-depended fluid injection in a finite section of infinitely long inclined borehole. The formulated stress solution is based on linear theory of poroelasticity. The primary purpose of fluid injection is to initiate and propagate fractures. Sometimes, it is used to stabilize the borehole also known as mud pressure during the drilling of a borehole. In this study, it is assumed that the borehole is drilled instantaneously and fluid injection is started to initiate the fractures. Borehole considered here is assumed to be in initial in-situ stress condition and drilled in a fluid saturated porous formation. In porous formation, the interaction between the solid matrix and time dependent fluid should be accounted to obtain the accurate and rigorous solution (Cui et al., 1997). The exact problem statement of a borehole subjected to time-dependent fluid injection and in-situ stress is described in details in the following section.

3.2. Problem Statement

An infinitely long inclined borehole as shown in figure 3.1a, drilled in fluid saturated porous medium has a radius R. Initially, the compressive in situ stress \( S_x', S_y', \text{ and } S_z' \) are acting on the formation whose virgin pore pressure is represented by \( p_0 \). The borehole can be rotated by zenith angle \( \varphi_y \) about z-axis to align vertically with the formation as shown in figure 3.1b (Cui et al., 1997; Abousleiman and Chen, 2010) to simplify the solution procedure. The length 2b is the fluid injection section length and \( Q_0 \) is the fluid flow rate. This problem can be decomposed into two
different problems as shown in figure 3.1c and 3.1d, solve them individually and simply use the rule of superposition to obtain the final solution for the problem (Abousleiman and Chen, 2010)

Figure 3.1. Geometry of inclined borehole a) In the state of initial compressive in-situ stress b) Equivalent far field stress (Abousleiman and Chen, 2010) c) Fluid injection boundary d) Stress boundary

3.3. Governing equations

The deformation of such inclined boreholes on a porous and isotropic formation based on theory of linear poroelasticity is governed by the following equations given in polar co-ordinate system (Biot, 1941; Rice and Clearly, 1976; Wang 2000).

\[
\nabla^2 u_r + \frac{1}{1 - 2v} \frac{1}{r} \frac{\partial e_v}{\partial r} - \frac{1}{r} \left( \frac{2}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \right) - \frac{2B(1 + v_u)}{3(1 - 2v_u)} \frac{\partial e_v}{\partial r} = 0
\]

(3.1)

\[
\nabla^2 u_\theta + \frac{1}{1 - 2v} \frac{1}{\partial \theta} - \frac{1}{r} \left( \frac{2}{r} \frac{\partial u_r}{\partial \theta} + \frac{u_\theta}{r} \right) - \frac{2B(1 + v_u)}{3(1 - 2v_u)} \frac{\partial e_v}{r \partial \theta} = 0
\]

(3.2)
\[ \nabla^2 u_z + \frac{1}{1-2v} \frac{\partial e_v}{\partial z} - \frac{2B(1 + v_u)}{3(1 - 2v_u)} \frac{\partial e_v}{\partial z} = 0 \] (3.3)

\[ \nabla^2 \varepsilon_v = \frac{1}{c} \frac{\partial \varepsilon_v}{\partial t} \] (3.4)

Similarly, constitutive equation can be written as:

\[ \sigma_{ij} = 2G e_{ij} + \frac{2G \nu}{1-2v} \delta_{ij} e_v - \alpha \delta_{ij} p \text{ for } (i, j = r, \theta, z) \] (3.5)

\[ p = -\frac{2GB(1 + v_u)}{3(1 - 2v_u)} e_v - \frac{2GB^2(1 - v)(1 + v_u)^2}{9(v - v_u)(1 - 2v_u)} e_v \] (3.6)

Here \( u_r, u_\theta \text{ and } u_z \) represents the displacement of solid matrix in radial, circumferential and vertical direction respectively. Matrix dilation \( e_v \) and Laplacian operator \( \nabla^2 \) can be expressed respectively as:

\[ e_v = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} \]

\[ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \]

Total stress and total strain is represented by \( \sigma_{ij} \) and \( e_{ij} \) respectively, excess pore water pressure is denoted by \( p \), Kronecker delta is denoted by \( \delta_{ij} \), \( e_v \) represents fluid content variation, Biot’s effective stress coefficient \( \alpha \) and diffusion coefficient denoted by \( c \) can be written as follows:

\[ \alpha = \frac{3(v_u - v)}{B(1-2v)(1+v_u)} \]

\[ c = \frac{2G\kappa B^2(1-v)(1+v_u)^2}{9(1-v_u)(v_u-v)} \]
where $G$ denotes the shear modulus, Skempton’s pore pressure coefficient is represented by $B$, $v$ is drained Poisson’s ratio and $v_u$ is undrained Poisson’s ratio. $\kappa \left( \frac{k}{\mu} \right)$ is the ratio of permeability to the fluid viscosity.

The transformation matrix (Jaeger and Cook, 1969) can be used to transform three principal stresses $S_x', S_y', S_z'$ into a stress tensor with six components $S_x', S_y', S_z', S_{xy}', S_{yz}', S_{xz}'$, and then apply as a boundary condition for an initial in-situ stress at the far field in polar coordinate system as follows (Abousleiman and Chen, 2010):

$$
\sigma_{xx} = -S_x, \sigma_{yy} = -S_y, \sigma_{zz} = -S_z, \sigma_{xy} = -S_{xy}, \sigma_{yz} = -S_{yz}, \sigma_{xz} = -S_{xz}, p = p_0
$$

(3.7a)

And the boundary condition on the borehole wall can be expressed as: (Abousleiman and Chen, 2010)

$$
\sigma_{rr} = \begin{cases} 
-p & 0 \leq |z| \leq b \\
0 & 0 < |z| < \infty
\end{cases}
$$

(3.7b)

$$
\sigma_{r\theta} = 0
$$

(3.7c)

$$
\sigma_{rz} = 0
$$

(3.7d)

$$
q(t,z) = \begin{cases} 
\frac{Q_0}{4\pi R_b} & 0 \leq |z| \leq b \\
0 & 0 < |z| < \infty
\end{cases}
$$

(3.8)

where $\sigma_{rr}$ denotes the radial stress, $p$ is the time dependent pore pressure, $q$ is the fluid flow rate.

### 3.4. Time-dependent Fluid Injection Boundary

The boundary fluid injection condition shown in equation (3.8) applies for a steady fluid injection at the borehole wall. Solving the governing equation with that boundary condition gives the stress solution for a constant fluid injection rate as given by used by Abousleiman and Chen (2010). However, the objective of this research is to apply various time-dependent fluid injection
boundary instead of a constant fluid injection. The three time-dependent fluid injection boundary considered in this research study are described below.

### 3.4.1. Gradually Increasing Unit Step Fluid Injection

![Gradually applied unit step fluid injection boundary](image)

The gradually applied unit step fluid injection boundary as shown in Figure 3.2 starts with the fluid injection at time $t = 0$ and linearly increases until it reaches certain time $t_0$. It is often called as a ramp type loading. After attaining its maximum value at time $t_0$, it stays constant for the rest of the time. This type of time-dependent fluid injection will be suitable for the case where maximum fluid flow can’t cannot be achieved immediately after starting injection due to the nature of formation, fracture design limitation and other technical difficulties. Such boundary condition can be expressed in mathematical term as follow (Ekbote et al., 1998; 2004)

$$q(t) = q \times \left\{ \left[1 - H(t - t_0)\right] \times \frac{t}{t_0} + H(t - t_0) \right\} \quad (3.9)$$

Substituting this term into equation (3.8), fluid injection at borehole wall for gradually applied step load can be written as:

$$q(t, z) = \begin{cases} \left( \frac{Q_0}{4\pi R_b} \right) \times \left\{ \left[1 - H(t - t_0)\right] \times \frac{t}{t_0} + H(t - t_0) \right\} & 0 \leq |z| \leq b, t \geq t_0 > 0 \\ 0 & 0 < |z| < \infty, t \geq t_0 > 0 \end{cases}$$
Equations (3.7), (3.8) and (3.10) describes the boundary condition of an inclined borehole subjected to gradually applied unit step fluid injection in a finite length and initial in-situ stress. Solving the deformation governing partial differential equations (3.1-3.4) of poroelasticity with above boundary conditions together with the constitutive equations yields the stress and pore pressure solution. The full solution procedure is shown in section 3.5.

3.4.2. Linearly Decreasing Fluid Injection

The second type of time-dependent fluid injection boundary considered in this research study is the linearly decreasing fluid injection boundary. This type of loading as shown in figure 3.3 starts at certain initial value at time $t = 0$ and linearly decreases to zero at time $\tau_0$. This fluid discharge case is suitable when filter cake is developed during the injection process and the injection section becomes completely impermeable at time $\tau_0$ due to the formation of filter cake (Ekbote et al., 2004). This type of injection can also be modeled during the production phase. This boundary condition can be expressed in mathematical terms as follows (Ekbote et al., 1998):

\[
q(t) = q \ast \{ H(t) - H(t - \tau_0) - [1 - H(t - \tau_0)] \ast t/\tau_0 \} \quad (3.11)
\]
Substituting this expression into equation (3.8), fluid injection boundary at borehole wall for linearly decreasing fluid injection can be written as:

\[
q(t, z) = \begin{cases} 
\left(\frac{Q_0}{4\pi R b}\right) \times \left\{ H(t) - H(t - \tau_0) - [1 - H(t - \tau_0)] \times \frac{t}{\tau_0} \right\} & 0 \leq |z| \leq b, t \geq \tau_0 > 0 \\
0 & 0 < |z| < \infty, t \geq \tau_0 > 0
\end{cases}
\]  

(3.12)

Equations (3.7), (3.8) and (3.12) are the boundary condition describing inclined borehole subjected to linearly decreasing fluid injection in a finite length and initial in-situ stress. These three equations together with governing equations (3.1-3.4) describes the problem for linearly decreasing fluid injection in a finite section. The solution procedure for this problem will be shown in section 3.5 together with the time-dependent fluid injection boundary conditions described in section 3.4.1.

### 3.4.3. Square Wave Fluid Injection

The third type of fluid injection boundary is square wave flux and is shown in figure 3.4. This loading starts at certain initial value at time \( t = 0 \) and remains constants until \( \tau_1 \), suddenly drops to zero and stays on zero until time \( \tau_2 \) completes the cycle 1 and repeats again. This type of loading is also known as cyclic loading as the cycle repeats itself at certain interval. This type of loading is suitable when fracture design prohibits very high injection pressure at once and for a long period of time. This type of loading can be applied to create fatigue failure. The mathematical expression for such loading can be written as (Ekbote et al, 1998):

\[
q(t) = q \times \{ H(t) - H(t - \tau_1) + H(t - \tau_2) - H(t - \tau_3) + \cdots - H(t - \tau_{2n-1}) \} 
\]

(3.13)

where \( \tau_1 < \tau_2 < \tau_3 < \cdots < \tau_{2n-1} < t \) and \( n \) is the number of cycles.
After substituting above expression into equation (3.8), fluid injection boundary at borehole wall for square wave flux becomes:

\[
q(t, z) = \begin{cases} \left( \frac{Q_0}{4\pi R b} \right) \ast \left\{ H(t) - H(t - \tau_1) + H(t - \tau_2) - \cdots - H(t - \tau_{2n-1}) \right\} & 0 \leq |z| \leq b, \ t \geq \tau_1 > 0 \\ 0 & 0 < |z| < \infty, \ t \geq \tau_1 > 0 \end{cases}
\]

(3.14)

As in equation (3.13), this expression is valid for \( \tau_1 < \tau_2 < \tau_3 < \cdots \ \tau_{2n-1} < t \).

Figure 3.4. Square wave fluid injection boundary

Equations (3.7), (3.8) and (3.14) are the boundary condition representing inclined borehole subjected to square wave flux type fluid injection in a finite length and initial in-situ stress. As in other two fluid boundary types, the solution of governing equations (3.1-3.4) together with boundary condition (3.7),(3.8) and (3.14) gives the stress and pore pressure solution for a square wave fluid injection in a finite section. The solution procedure for this problem and other two time-dependent fluid injection cases are shown in detail in the proceeding section.
3.5. Solution Procedure

The problem of inclined borehole subjected to a time-dependent fluid injection and in-situ stress can be solved by decomposing into two different problems in term of boundary conditions. Those boundary problem are 1) fluid injection boundary and 2) stress boundary. First, they can be solved individually then, use the rule of superposition to obtain the final solution. The solution procedure for time-dependent fluid injection boundary is similar to the constant boundary problem given (Abousleiman and Chen, 2010). The solution is presented for each of three time-dependent fluid injection problem below in section 3.5.1. and the solution of stress boundary problem is shown in section 3.5.2.

**CASE 1** (*Gradually applied unit step fluid injection*)

After decomposition of the problem, boundary condition for gradually applied unit step fluid injection case becomes:

In far field ($r \to \infty$),

\[
\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yz} = \sigma_{xz} = p = 0
\]  
(3.15)

At the borehole surface,

\[
\sigma_{r\theta} = \sigma_{rz} = 0, \quad \text{and} \quad \sigma_{rr} = \begin{cases} 
-p & 0 \leq |z| \leq b \\
0 & 0 < |z| < \infty 
\end{cases}
\]  
(3.16)

\[
q(t, z) = \frac{Q_0}{4\pi Rb} \left\{ [1 - H(t - \tau_0)] \frac{t}{\tau_0} + H(t - \tau_0) \right\} \quad 0 \leq |z| \leq b, \, t \geq \tau_0 > 0 \\
0 \quad 0 < |z| < \infty, \, t \geq \tau_0 > 0
\]  
(3.17)

**CASE 2** (*Linearly decreasing fluid injection*)

Similarly, for linearly decreasing fluid injection the following boundary condition applies.
In far field \((r \to \infty)\),
\[
\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yz} = \sigma_{xz} = p = 0
\]  
(3.18)

At the borehole surface,
\[
\sigma_{r\theta} = \sigma_{rz} = 0, \quad \text{and} \quad \sigma_{rr} = \begin{cases} -p & 0 \leq |z| \leq b \\ 0 & 0 < |z| < \infty \end{cases}
\]  
(3.19)

\[
q(t, z) = \begin{cases} \frac{Q_0}{4\pi Rb} * \left\{ H(t) - H(t - \tau_0) - [1 - H(t - \tau_0)] * \frac{t}{\tau_0} \right\} & 0 \leq |z| \leq b, t \geq \tau_0 > 0 \\ 0 & 0 < |z| < \infty, t \geq \tau_0 > 0 \end{cases}
\]  
(3.20)

**CASE 3** *(Square wave fluid injection)*

And for the third and final case of fluid injection, the boundary condition becomes:

In far field \((r \to \infty)\),
\[
\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yz} = \sigma_{xz} = p = 0
\]  
(3.21)

At the borehole surface,
\[
\sigma_{r\theta} = \sigma_{rz} = 0, \quad \text{and} \quad \sigma_{rr} = \begin{cases} -p & 0 \leq |z| \leq b \\ 0 & 0 < |z| < \infty \end{cases}
\]  
(3.22)

\[
q(t, z) = \begin{cases} \frac{Q_0}{4\pi Rb} * \left\{ \frac{H(t) - H(t - \tau_1) + H(t - \tau_2) - \ldots + H(t - \tau_{2n-1})}{\tau} \right\} & 0 \leq |z| \leq b, t \geq 0 \\ 0 & 0 < |z| < \infty, t \geq 0 \end{cases}
\]  
(3.23)

After defining the boundary condition for three types of fluid discharge cases, the solution procedure can be started by solving deformation governing equations (3.1-3.4) to obtain the general solution first. Four partial differential equations with four independent variables \(r, \theta, z, t\) are to be solved. Because the problem is axially symmetric, governing equations will be independent of \(\theta\) variable.
The technique of Laplace transformation can be employed to transform time variable \( t \) and Fourier transformation to transform axial co-ordinate \( z \). It will change the partial differential equations to simple ordinary differential equations with only one independent variable \( r \) which can be easily solved. Inverse Laplace and Fourier transform yields the general solution to these four governing equation (3.1-3.4) in original domain.

Laplace transform is a technique which transforms differential and integral equations into simple algebraic equations. In other words, it transforms a variable \( t \) from time domain into frequency domain \( s \). This transformation technique is quite helpful in solving various complicated ordinary and partial differential equation. After transformation into frequency domain to solve any problem, it can be inverted to obtain the solution in original time domain. Laplace transformation can be expressed as:

\[
F(s) = \int_{0}^{\infty} f(t)e^{-st} \, dt
\]  
(3.24)

And its inverse can be expressed as:

\[
f(t) = \frac{1}{2\pi j} \int_{\gamma-j\infty}^{\gamma+j\infty} F(s)e^{st} \, ds
\]  
(3.25)

However, due to the complex nature of the problem, it is difficult to transform back to original time domain by using equation (3.25). So, numerical inversion methods can be opted for such cases.

Similarly, Fourier transform is another integral transformation technique given by Joseph Fourier. Just like a Laplace transformation, it can be used to solve complicated ordinary and partial differential equations. In solving the governing equations, Fourier transform is taken with respect to \( z \) co-ordinates. The basic Fourier transformation is given by
\[ F(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\xi x} \, dx \]  
\( (3.26) \)

And its inverse can be expressed as:

\[ f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\xi) e^{i\xi x} \, d\xi \]  
\( (3.27) \)

**3.5.1. Solution Formulation for a Fluid Injection Boundary**

Those two integral transform techniques will be employed to solve governing equations (3.1-3.4). The general solution for those equation were originally given by Abousleiman and Chen (2010), but more detailed derivation with explanations will be presented here. As we ignore the term \( \theta \) from our governing equation (3.1-3.4) because of axially symmetric problem, it can be rewritten as follows:

\[ \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_r}{\partial z^2} + \frac{1}{1-2v} \frac{\partial e_v}{\partial r} - \frac{2B(1 + v_u)}{3(1-2v_u)} \frac{\partial e_v}{\partial r} = 0 \]  
\( (3.28) \)

\[ \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial z^2} + \frac{1}{1-2v} \frac{\partial e_v}{\partial z} - \frac{2B(1 + v_u)}{3(1-2v_u)} \frac{\partial e_v}{\partial z} = 0 \]  
\( (3.29) \)

\[ \frac{\partial^2 e_v}{\partial r^2} + \frac{1}{r} \frac{\partial e_v}{\partial r} + \frac{\partial^2 e_v}{\partial z^2} = \frac{1}{c} \frac{\partial e_v}{\partial t} \]  
\( (3.30) \)

And from constitutive equations, the stress components and pore pressure can be expanded as follows:

\[ p = -\frac{2GB(1 + v_u)}{3(1-2v_u)} e_v - \frac{2GB^2(1 - v)(1 + v_u)^2}{9(v - v_u)(1 - 2v_u)} e_v \]  
\( (3.31) \)
\[
\sigma_{rr} = 2G \frac{\partial u_r}{\partial r} + \frac{2Gv}{1-2v} e_v - \frac{3(v_u - v)}{B(1-2v)(1 + v_u)} p
\]  \hspace{1cm} (3.32)

\[
\sigma_{zz} = 2G \frac{\partial u_z}{\partial z} + \frac{2Gv}{1-2v} e_v - \frac{3(v_u - v)}{B(1-2v)(1 + v_u)} p
\]  \hspace{1cm} (3.33)

\[
\sigma_{\theta\theta} = 2G \frac{u_r}{r} + \frac{2Gv}{1-2v} e_v - \frac{3(v_u - v)}{B(1-2v)(1 + v_u)} p
\]  \hspace{1cm} (3.34)

\[
\sigma_{rz} = G \left[ \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right]
\]  \hspace{1cm} (3.35)

After taking Laplace transform of equation (3.30) with respect to \( t \), we get:

\[
\frac{\partial^2 \varepsilon_v}{\partial r^2} + \frac{1}{r} \frac{\partial \varepsilon_v}{\partial r} + \frac{\partial^2 \varepsilon_v}{\partial z^2} = \frac{s}{c} \varepsilon_v
\]  \hspace{1cm} (3.36)

Again taking Fourier transform of equation (3.36) with respect to \( z \),

\[
\frac{\partial^2 \varepsilon_v}{\partial r^2} + \frac{1}{r} \frac{\partial \varepsilon_v}{\partial r} - \xi^2 \varepsilon_v = \frac{s}{c} \varepsilon_v
\]  \hspace{1cm} (3.37)

where \( s \) and \( \xi \) are Laplace and Fourier transformation parameters respectively. Equation (3.37) becomes an ordinary differential equation with only \( r \) as an independent variable which can be further simplified and written as:

\[
\frac{d^2 \varepsilon_v}{dr^2} + \frac{1}{r} \frac{d \varepsilon_v}{dr} - \xi^2 \varepsilon_v = \frac{s}{c} \varepsilon_v
\]

\[
\frac{d^2 \varepsilon_v}{dr^2} + \frac{1}{r} \frac{d \varepsilon_v}{dr} - (\xi^2 + \frac{s}{c}) \varepsilon_v = 0
\]

\[
\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - (\xi^2 + \frac{s}{c}) \right) \varepsilon_v = 0
\]  \hspace{1cm} (3.38)
The solution to equation (3.38) can be obtained by direct comparison with Bessel differential equation and its solution is:

\[ \varepsilon_v = A(\xi, s)K_0(\eta r) \]  
(3.39)

The term \((\xi^2 + s/c)\) can be represented by single variable \(\eta\) for convenience in such a way that

\[ \eta = + \sqrt{\xi^2 + \frac{s}{c}} \]

Taking Fourier transform with respect to \(z\) coordinates on equation (3.28) makes it ordinary differential equations with single independent variable \(r\).

\[
\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} \frac{u_r}{r^2} - \xi^2 u_r + \frac{1}{1 - 2\nu} \frac{\partial \varepsilon_v}{\partial r} - \frac{2B(1 + \nu_u)}{3(1 - 2\nu_u)} \frac{\partial \varepsilon_v}{\partial r} = 0 
\]  
(3.40a)

\[
\frac{d^2 u_r}{dr^2} + \frac{1}{r} \frac{du_r}{dr} - \xi^2 u_r + \frac{1}{1 - 2\nu} \frac{de_v}{dr} - \frac{2B(1 + \nu_u)}{3(1 - 2\nu_u)} \frac{de_v}{dr} = 0 
\]  
(3.40b)

Similarly, equation (3.29) becomes as follows

\[
\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} - \xi^2 u_z + \frac{1}{1 - 2\nu} (-i\xi) e_v - \frac{2B(1 + \nu_u)}{3(1 - 2\nu_u)} \frac{\partial \varepsilon_v}{\partial z} = 0 
\]  
(3.41a)

\[
\frac{d^2 u_z}{dr^2} + \frac{1}{r} \frac{du_z}{dr} - \xi^2 u_z + \frac{1}{1 - 2\nu} (-i\xi) e_v - \frac{2B(1 + \nu_u)}{3(1 - 2\nu_u)} (-i\xi) \varepsilon_v = 0 
\]  
(3.41b)

Adding equations (3.40) and (3.41) and simplifying yields,

\[
\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \xi^2 \right) e_v = \frac{B(1 + \nu_u)}{3(1 - 2\nu_u)} \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \xi^2 \right) \varepsilon_v 
\]  
(3.42)

The solution of equation (3.42) is given by Bessel differential equation as in equation (3.39) and substitution of equation (3.39) on to it gives the solution as:
\[ e_v = C(\xi, s)|\xi|K_0(\rho) + \frac{B(1 + v_u)}{3(1 - v_u)} A(\xi, s)K_0(\eta r) \] (3.43)

In above equation (3.43), \( \rho = |\xi|r \) and \( C(\xi, s) \) is another new function. Since, solution for \( e_v \) and \( \varepsilon_v \) are obtained, it can be substituted back to pore pressure equation (3.31) to find solution of \( p \) in terms of two unknown functions \( A(\xi, s) \) and \( C(\xi, s) \) in Laplace and Fourier transformed domain as below:

\[ p = -\frac{2GB(1 + v_u)}{3(1 - 2v_u)} C(\xi, s)|\xi|K_0(\rho) - \frac{2GB^2(1 - v)(1 + v_u)^2}{9(v - v_u)(1 - v_u)} A(\xi, s)K_0(\eta r) \] (3.44)

Similarly, the flux \( q = -\kappa dp/dr \) in transformed domain can be written as:

\[ q = -\frac{2G\kappa B(1 + v_u)}{3(1 - 2v_u)} \xi^2 K_1(\rho) C(\xi, s) - \frac{2GB^2(1 - v)(1 + v_u)^2}{9(v - v_u)(1 - v_u)} \eta K_1(\eta r) A(\xi, s) \] (3.45)

Again if \( e_v \) and \( \varepsilon_v \) are substituted into equation (3.40) and simplified, the following displacement component in radial direction in terms of unknown functions \( A(\xi, s) \) and \( C(\xi, s) \) can be obtained.

\[ \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} - \xi^2 \right) u_r = \frac{2B(1 + v_u)}{B(1 - 2v_u)} \frac{d\varepsilon_v}{dr} - \frac{1}{1 - 2v_u} \frac{d\varepsilon_v}{dr} \] (3.46a)

\[ \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} - \xi^2 \right) u_r = \frac{2B(1 + v_u)}{B(1 - 2v_u)} \frac{d}{dr}\{A(\xi, s)K_0(\eta r)\} \]

\[ - \frac{1}{1 - 2v_u} \frac{d}{dr}\{C(\xi, s)|\xi|K_0(\rho) \} \]

\[ + \frac{B(1 + v_u)}{3(1 - v_u)} A(\xi, s)K_0(\eta r) \] (3.46b)
\[
\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} - \xi^2
= \frac{2B(1 + v_u)}{B(1 - 2v_u)} \left\{ -\eta K_1(\eta r) A(\xi, s) \right\}
- \frac{1}{1 - 2v_u} \left\{ C(\xi, s) \xi | K_1(\rho) \right\}
+ \frac{B(1 + v_u)}{3(1 - v_u)} (\eta K_1(\eta r) A(\xi, s))
\]  
(3.46c)

\[
\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} - \xi^2 \right) u_r
= -\frac{B(1 + v_u)}{3(1 - v_u)} A(\xi, s) K_1(\eta r) + \frac{\xi^2 C(\xi, s) K_1(\rho)}{1 - 2v_u}
\]  
(3.46d)

Similarly, if we substitute \( e_v \) and \( \varepsilon_v \) into equations (3.41) and simplify, the following displacement component in axial direction in terms of unknown functions \( A(\xi, s) \) and \( C(\xi, s) \) can be obtained.

\[
\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} - \xi^2 \right) u_r
= i\xi \frac{e_v}{1 - 2v_u} - i\xi \varepsilon_v \frac{2B(1 + v_u)}{3(1 - 2v_u)}
\]  
(3.47a)

\[
\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} - \xi^2 \right) u_z
= \frac{i\xi}{1 - 2v_u} \left( C(\xi, s) | K_0(\rho) + \frac{B(1 + v_u)}{3(1 - v_u)} A(\xi, s) K_0(\eta r) \right)
+ i\xi A(\xi, s) K_0(\eta r)
\]  
(3.47b)

\[
\left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} - \xi^2 \right) u_z
= -\frac{i\xi B(1 + v_u)}{3(1 - 2v_u)} A(\xi, s) K_0(\eta r) + \frac{i\xi | K_0(\rho)}{1 - 2v_u} C(\xi, s)
\]  
(3.47c)

The two ordinary differential equations (3.46) and (3.47) can be solved by trial and error methods taking numerous iterations of Bessel function. Its solution can be expressed in transformed domain as given by Abousleiman and Chen (2010) with additional new function \( D(\xi, s) \) as:

\[
u_r = \frac{B(1 + v_u)\eta c}{3s(1 - v_u)} A(\xi, s) K_1(\eta r) - \left( K_1(\rho) + \frac{\rho K_2(\rho)}{2(1 - 2v_u)} \right) C(\xi, s) - i\xi K_1(\rho) D(\xi, s)
\]  
(3.48)
\[ u_z = -\frac{i\xi B(1 + v_u)c}{3s(1 - v_u)} A(\xi, s)K_0(\eta r) - \frac{i\xi r}{2(1 - 2v_u)} K_1(\rho)C(\xi, s) + D(\xi, s)|\xi|K_0(\rho) \] (3.49)

After the solutions for displacement components and pore pressure are obtained, the next step is to substitute them back to constitutive equations (3.31-3.36) to get the stress components.

The first term of radial stress from equation (3.32) can be simplified as:

\[
2G \frac{\partial u_r}{\partial r} = 2G \frac{\partial}{\partial r} \left( -\frac{B(1 + v_u)\eta c}{3s(1 - v_u)} A(\xi, s)K_1(\eta r) - \left( K_1(\rho) + \frac{\rho K_2(\rho)}{2(1 - 2v_u)} \right) C(\xi, s) \right. \\
- \left. i\xi K_1(\rho)D(\xi, s) \right) 
\] (3.50a)

\[
\frac{\sigma_{rr}}{2G} = \frac{\partial}{\partial r} \left( -\frac{B(1 + v_u)\eta c}{3s(1 - v_u)} A(\xi, s)K_1(\eta r) \right. \\
- \left. \frac{\partial}{\partial r} \left( K_1(\rho) + \frac{\rho K_2(\rho)}{2(1 - 2v_u)} \right) C(\xi, s) - i\xi K_1(\rho)D(\xi, s) \right) 
\] (3.50b)

Similarly, the second term can be obtained simply taking partial derivate with respect to \(r\). Combining both terms of radial stress components on transformed domain yields;

\[
\frac{\sigma_{rr}}{2G} = \frac{B(1 + v_u)}{3(1 - v_u)} \left\{ \left( \frac{c\eta^2}{s} - 1 \right) K_0(\eta r) + \frac{c\eta}{sr} K_1(\eta r) \right\} A(\xi, s) + i\xi \left\{ |\xi|K_0(\rho) + \frac{K_1(\rho)}{r} \right\} D(\xi, s) \\
+ \left\{ \frac{1 - v_u}{1 - 2v_u} |\xi|K_0(\rho) + \left( \frac{1}{r} + \rho|\xi| \right) K_1(\rho) + \frac{|\xi|}{2(1 - 2v_u)} K_2(\rho) \right\} C(\xi, s) 
\] (3.50c)

The first term of tangential stress from equation (3.34) can be simplified as:

\[
2G \frac{u_t}{r} = \frac{2G}{r} - \frac{B(1 + v_u)\eta c}{3s(1 - v_u)} A(\xi, s)K_1(\eta r) - \left( K_1(\rho) + \frac{\rho K_2(\rho)}{2(1 - 2v_u)} \right) C(\xi, s) \\
- i\xi K_1(\rho)D(\xi, s) 
\] (3.51a)

The second term of tangential stress from equation (3.34) can be simplified as:
\[ 2Gv \frac{e_v}{1-2v} = \frac{v}{1-2v} \left\{ C(\xi,s)|\xi|K_0(\rho) + \frac{B(1+v_u)}{3(1-v_u)} A(\xi,s)K_0(\eta r) \right\} \]  

(3.51b)

Similarly, the third term of tangential stress from equation (3.34) can be simplified as:

\[ \frac{3(v_u-v)}{B(1-2v)(1+v_u)} p = \frac{2GB(1+v_u)}{3(1-2v_u)} C(\xi,s)|\xi|K_0(\rho) \]

\[- \frac{2GB^2(1-v)(1+v_u)^2}{9(v-v_u)(1-v_u)} A(\xi,s)K_0(\eta r) \]  

(3.51c)

Thus, finally combining all three terms and simplifying of results the following:

\[ \frac{\sigma_{\theta\theta}}{2G} = \frac{B(1+v_u)}{3(1-v_u)} \left\{ -K_0(\eta r) + \frac{c\eta}{sr} K_1(\eta r) \right\} A(\xi,s) \]

\[ + \left\{ \frac{v_u}{1-2v_u} |\xi|K_0(\rho) - \frac{K_1(\rho)}{r} - \frac{|\xi|K_2(\rho)}{2(1-2v_u)} \right\} C(\xi,s) - i\frac{\xi}{r} K_1(\rho)D(\xi,s) \]  

(3.51d)

In similar fashion, the other stress components can be derived in transformed domain and written as follows:

\[ \frac{\sigma_{zz}}{2G} = \frac{B(1+v_u)}{3(1-v_u)} \left( -\frac{c\xi^2}{s} - 1 \right) K_0(\eta r)A(\xi,s) + \left\{ \frac{v_u}{1-2v_u} |\xi|K_0(\rho) - \frac{\xi^2 r K_1(\rho)}{2(1-2v_u)} \right\} C(\xi,s) \]

\[- i\xi |\xi|K_0(\rho)D(\xi,s) \]  

(3.52)

\[ \frac{\sigma_{rz}}{2G} = \frac{i\xi B(1+v_u) c}{3s(1-v_u)} \eta K_1(\eta r)A(\xi,s) + \left\{ \frac{i\xi \rho}{4(1-2v_u)} (K_0(\rho) + K_2(\rho)) + \left( \frac{i\xi}{2} K_1(\rho) \right) \right\} C(\xi,s) \]

\[- \xi^2 K_1(\rho)D(\xi,s) \]  

(3.53)

So, equations (3.50), (3.51), (3.52), (3.53) are the general solutions of stress components in terms of three unknown functions \( A(\xi,s), B(\xi,s), C(\xi,s) \) which needs to be determined from the fluid injection boundary conditions. Since, these solutions are in Laplace and Fourier domain, the boundary conditions are also needed to be transformed. Equation (3.10) can be transformed into Laplace and Fourier transform as below:
\[ \sigma_{rz} = 0 \]  \hspace{1cm} (3.54a)

\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[ \sigma_{rr} + p \right] e^{-i\xi z} d\xi = 0 \quad 0 \leq |z| \leq b
\]

\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma_{rr} e^{-i\xi z} d\xi = 0 \quad 0 \leq |z| \leq \infty
\]  \hspace{1cm} (3.54b)

In above, \( \sigma_{rr} \) and \( p \) are the functions of \( r, s \) and \( \xi \) since they are transformed in Laplace and Fourier domain.

**CASE 1** *(Gradually applied unit step fluid injection)*

The gradually applied unit step fluid injection boundary type is given in equation (3.10) can be transformed as below:

\[
q(t, z) = \begin{cases}
\frac{Q_0}{4\pi Rb} * \left\{ [1 - H(t - \tau_0)] * \frac{t}{\tau_0} + H(t - \tau_0) \right\} & 0 \leq |z| \leq b, t \geq \tau_0 > 0 \\
0 & 0 < |z| < \infty, t \geq \tau_0 > 0
\end{cases}
\]

*Laplace transform*

\[
q(s, z) = \begin{cases}
\frac{Q_0}{4\pi Rb} * \left\{ \frac{1 - e^{-s\tau_0}(1 + s\tau_0)}{s^2\tau_0} \right\} + \frac{e^{-s\tau_0}}{s} & 0 \leq |z| \leq b \\
0 & 0 < |z| < \infty
\end{cases}
\]

*Fourier Transform*

\[
q(s, \xi) = \frac{\sin(\xi b)}{2\sqrt{2\pi}} \frac{Q_0}{\xi \pi Rb} * \left\{ \frac{1 - e^{-s\tau_0}}{s^2\tau_0} \right\}
\]

\[= \frac{2q_0 \sin(\xi b)}{\sqrt{2\pi} \xi} * \left\{ \frac{1 - e^{-s\tau_0}}{s^2\tau_0} \right\}
\]  \hspace{1cm} (3.56)

where, \( q_0 = \frac{Q_0}{4\pi Rb} \)

**CASE 2** *(Linearly decreasing fluid injection)*
Similarly, the linearly decreasing fluid injection boundary type given by equation (3.12) can be transformed as below:

\[
q(t, z) = \begin{cases} 
\frac{Q_0}{4\pi R_b} \left\{ H(t) - H(t - \tau_0) - \left[1 - H(t - \tau_0)\right] \frac{t}{\tau_0} \right\} & 0 \leq |z| \leq b, t \geq \tau_0 > 0 \\
0 & 0 < |z| < \infty, t \geq \tau_0 > 0
\end{cases}
\]

Laplace transform

\[
q(s, z) = \begin{cases} 
\frac{Q_0}{4\pi R_b} \left\{ \frac{1}{s} - \frac{e^{-s\tau_0}}{s} - \left(\frac{1 - e^{-s\tau_0}(1 + s\tau_0)}{s^2\tau_0}\right) \right\} & \\
0 & \end{cases}
\]

\[
q(s, z) = \frac{Q_0}{4\pi R_b} \left( \frac{s\tau_0 - 1 + e^{-s\tau_0}}{s^2\tau_0} \right)
\]

Fourier Transform

\[
q(s, \xi) = \frac{\sin(\xi b)}{2\sqrt{2\pi}} \frac{Q_0}{\xi \pi R_b} \left( \frac{s\tau_0 - 1 + e^{-s\tau_0}}{s^2\tau_0} \right)
\]

\[
q(s, \xi) = \frac{2q_0\sin(\xi b)}{\sqrt{2\pi\xi}} \left( \frac{s\tau_0 - 1 + e^{-s\tau_0}}{s^2\tau_0} \right)
\]

where, \( q_0 = \frac{Q_0}{4\pi R_b} \)

**CASE 3 (Square wave fluid injection)**

The final type of boundary square wave fluid injection boundary type given by equation (3.14) can be transformed as below:

\[
q(t, z) = \begin{cases} 
\frac{Q_0}{4\pi R_b} \left\{ H(t) - H(t - \tau_1) + H(t - \tau_2) - H(t - \tau_3) + \cdots - H(t - \tau_{2n-1}) \right\} & \\
0 & 0 \leq |z| \leq b, t \geq \tau_3 > \tau_2 > \tau_1 > 0
\end{cases}
\]

Laplace transform

\[
q(s, z) = \begin{cases} 
\frac{Q_0}{4\pi R_b} \left\{ \frac{1}{s} - \frac{e^{-\tau_1 s}}{s} + \frac{e^{-\tau_2 s}}{s} - \frac{e^{-\tau_3 s}}{s} + \cdots - \frac{e^{-\tau_{2n-1} s}}{s} \right\} & \\
0 & \end{cases}
\]

\[ (3.59) \]
\[ q(s, \xi) = \frac{\sin(\xi b)}{2\sqrt{2\pi} \xi \pi R b} \left( \frac{1}{s} - \frac{e^{-\tau_1 s}}{s} + \frac{e^{-\tau_2 s}}{s} - \frac{e^{-\tau_3 s}}{s} + \cdots - \frac{e^{-\tau_{2n-1} s}}{s} \right) \]

\[ q(s, \xi) = \frac{2q_0 \sin(\xi b)}{\sqrt{2\pi} \xi} \left( \frac{1}{s} - \frac{e^{-\tau_1 s}}{s} + \frac{e^{-\tau_2 s}}{s} - \frac{e^{-\tau_3 s}}{s} + \cdots - \frac{e^{-\tau_{2n-1} s}}{s} \right) \]  \hspace{1cm} (3.60)

where, \( q_0 = \frac{Q_0}{4\pi R b} \)

With the transformed version of fluid injection boundary conditions, equations (3.45), (3.50), (3.53) and (3.54a), unknown functions \( A(s, \xi) \), \( C(s, \xi) \) and \( D(s, \xi) \) are be found in terms of \( \sigma_{rr} \).

(Abousleiman and Chen, 2010) as below:

\[ A(s, \xi) = \frac{1}{\mathbf{\nabla}} \left( -\alpha_{23}\alpha_{32}\sigma_{rr} + (\alpha_{12}\alpha_{23} - \alpha_{22}\alpha_{13}) \ast q(s, z) \right) \]  \hspace{1cm} (3.61a)

\[ C(s, \xi) = \frac{1}{\mathbf{\nabla}} \left( \alpha_{23}\alpha_{31}\sigma_{rr} + (\alpha_{21}\alpha_{13} - \alpha_{11}\alpha_{23}) \ast q(s, z) \right) \]  \hspace{1cm} (3.61b)

\[ D(s, \xi) = \frac{1}{\mathbf{\nabla}} \left( \alpha_{21}\alpha_{32} - \alpha_{31}\alpha_{22} \right) \sigma_{rr} + (\alpha_{11}\alpha_{22} - \alpha_{21}\alpha_{12}) \ast q(s, z) \)  \hspace{1cm} (3.61c)

where,

\[ \mathbf{\nabla} = \alpha_{12}\alpha_{23}\alpha_{31} + \alpha_{13}\alpha_{21}\alpha_{32} - \alpha_{11}\alpha_{23}\alpha_{32} - \alpha_{13}\alpha_{22}\alpha_{31} \]

\[ \alpha_{11} = 2G \frac{B(1+v_u)}{3(1-v_u)} \left\{ \left( \frac{c\eta^2}{s} - 1 \right) K_0(\eta R) + \frac{c\eta}{sR} K_1(\eta R) \right\} \]

\[ \alpha_{12} = 2G \left\{ \frac{1 - v_u}{1 - 2v_u} |\xi| K_0(|\xi| R) + \left[ \frac{1}{R} + \frac{\xi^2 R}{2(1 - 2v_u)} \right] K_1(|\xi| R) + \frac{|\xi|}{2(1 - 2v_u)} K_2(|\xi| R) \right\} \]

\[ \alpha_{13} = 2Gi\xi \left\{ |\xi| K_0(|\xi| R) + \frac{K_1(|\xi| R)}{R} \right\} \]

\[ \alpha_{21} = 2G \frac{i\xi B(1 + v_u)c}{3(1 - v_u)s} \eta K_1(\eta R) \]

\[ \alpha_{22} = 2G \left\{ \frac{i\xi |\xi| R}{4(1 - 2v_u)} [K_0(|\xi| R) + K_2(|\xi| R)] + \left[ \frac{1}{R} + \frac{\xi^2 R}{2(1 - 2v_u)} \right] K_1(|\xi| R) + \frac{i\xi}{2} K_1(|\xi| R) \right\} \]

33
\[
\alpha_{31} = \frac{2G\kappa B^2(1 - \nu)(1 + v_u)^2}{9(1 - 2v_u)(1 - 2v_u)} \eta K_1(\eta R)
\]
\[
\alpha_{32} = \frac{2G\kappa B^2(1 + v_u)}{3(1 - 2v_u)} \xi^2 K_1(|\xi| R)
\]

It is to be noted that above equations are in transformed domain. Now that we have defined three unknown functions in terms of \(\sigma_{rr}\) in transformed domain, \(\sigma_{rr}\) needs to determine the numerically since \(\sigma_{rr} \neq 0\) (as assumed by Rajapakse, 1993). Substituting, equation (3.41a) and (3.41b) into transformed pore pressure equation (3.44), we get:

\[
p = -\frac{2GB}{3(1 - 2v_u)} K_0(\rho) \left\{ \frac{1}{\sqrt{V}} \left( \alpha_{23} \alpha_{31} \sigma_{rr} + (\alpha_{21} \alpha_{13} - \alpha_{11} \alpha_{23}) * q(s, z) \right) \right\}
\]
\[
- \frac{2GB^2(1 - \nu)(1 + v_u)^2}{9(v - v_u)(1 - v_u)} K_0(\eta r) \left\{ \frac{1}{\sqrt{V}} (-\alpha_{23} \alpha_{32} \sigma_{rr})
\]
\[
+ (\alpha_{12} \alpha_{23} - \alpha_{22} \alpha_{13}) * q(s, z) \right) \right\}
\]

(3.62)

If we let,

\[
- \frac{2GB^2(1 - \nu)(1 + v_u)^2}{9(v - v_u)(1 - v_u)} K_0(\eta r) = \alpha_{31p}
\]
\[
- \frac{2GB(1 + v_u)}{3(1 - 2v_u)} K_0(\rho) = \alpha_{32p}
\]

\[
\frac{1}{\sqrt{V}} (-\alpha_{23} \alpha_{32} \alpha_{31p} + \alpha_{23} \alpha_{31p}) = f_1(s, \xi) \text{ and}
\]
\[
\frac{1}{\sqrt{V}} ((\alpha_{21} \alpha_{13} - \alpha_{11} \alpha_{23}) \alpha_{32p} + (\alpha_{12} \alpha_{23} - \alpha_{22} \alpha_{13}) \alpha_{31p}) = f_2(s, \xi),
\]

Equation (3.62) becomes,

\[
p(R, s, \xi) = f_1(s, \xi) \sigma_{rr} + f_2(s, \xi) q(s, \xi)
\]

Substituting, equation (3.62) in equation (3.54b) and solving further two steps gives,
\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [\sigma_{rr} + p] e^{-i\xi z} d\xi = 0 \quad 0 \leq |z| \leq b
\]

\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [\sigma_{rr} + (f_1(s, \xi)\sigma_{rr} + f_2(s, \xi)q(s, \xi))] e^{-i\xi z} d\xi = 0
\]

\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [1 + f_1(s, \xi)\sigma_{rr}] e^{-i\xi z} d\xi = \int_{-\infty}^{\infty} f_2(s, \xi)q(s, \xi)) e^{-i\xi z} d\xi \quad 0 \leq |z| \leq b \quad (3.63a)
\]

and,

\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma_{rr} e^{-i\xi z} d\xi = 0 \quad 0 \leq |z| \leq \infty \quad (3.63b)
\]

Equation (3.63) can be further simplified as below since \(f_1, f_2\) and \(\sigma_{rr}\) are turns out to be real and even functions of \(\xi\) (Abousleiman and Chen, 2010).

\[
\int_{0}^{\infty} \frac{1}{\xi^2} [1 + f_1(s, \xi)] \sigma_{rr} J_{-1/2}(\xi z) d\xi = \sqrt{2} \int_{0}^{\infty} g(z, s) \cos \frac{\pi z}{s} \cos (\pi \xi z) \xi d\xi \quad 0 \leq |z| \leq b \quad (3.64a)
\]

\[
\int_{0}^{\infty} \frac{1}{\xi^2} \sigma_{rr} J_{-1/2}(\xi z) d\xi = 0 \quad 0 \leq |z| \leq b \quad (3.64b)
\]

where,

\[
g(z, s) = \int_{0}^{\infty} f_2(s, \xi)q(s, \xi) \cos \xi z \ d\xi \text{ and } \lim_{\xi \to \infty} f_1(\xi, s) = 0
\]

A new function by Noble (1963), \(\theta(x, s)\) is introduced to give the following:

\[
\sigma_{rr}(R, \xi, s) = \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} \frac{1}{\sqrt{\xi}} \theta(s, \xi) \frac{2}{\pi^x} \cos (\pi x \xi) dx
\]

\[
= \frac{2}{\pi} \int_{0}^{b} \frac{1}{x^2} \theta(x, \xi) \cos (\pi x \xi) dx
\]
\[
\sigma_{rr}(R, \xi, s) = \frac{2}{\pi} \int_{0}^{b} \theta(x, s) \cos(x \xi) dx
\]  

(3.65)

Since, equation (3.64a) and (3.64b) are equivalent to second kind Fredholm integral equation, \( \theta(x, s) \) can be found from:

\[
\theta(x, s) + \frac{2}{\pi} \int_{0}^{b} M(x, y, s) \theta(y, \xi) dy = g(x, s)
\]  

(3.66)

where,

\[
M(x, y, s) = \int_{0}^{\infty} f_{1}(s, \xi) \cos(x \xi) \cos(y \xi) d\xi
\]

(3.67)

Equation (3.65) along with (3.66) and (3.67) can determine \( \sigma_{rr}(R, \xi, s) \) in transformed domain which can be substituted back to equations (3.61a-3.61c) to obtain values of \( A(s, \xi), C(s, \xi) \) and \( D(s, \xi) \) in Laplace-Fourier domain. It can be then numerically inverted to get the final solution in physical domain. The inversion of Laplace and Fourier transform is complicated because of integrands. Thus, the stress components has to be determined by inverting the transformed solution numerically. Inverse Fourier transform can be inverted by numerically integrating it using Matlab with large enough upper bound of semi-infinite integral. And then Laplace transform can be inversed numerically using Stehfest (1970) algorithm.

The following equation represents the approximate inversion of Laplace transform given by Stehfest.

\[
f(t) = \ln 2 \frac{2}{t} \sum_{n=1}^{N} X_{n} \tilde{f} \left( \frac{n \ln 2}{t} \right),
\]

where \( X_{n} = (-1)^{n} \frac{N}{2} \sum_{j=\left[\frac{n+1}{2}\right]}^{\min(n, N/2)} \frac{N}{2j} \frac{N}{(\frac{N}{2} - j)! j! (j - 1)! (n - j)! (2j - n)!}
\]

(3.68)
represents number of terms of the series and it must be even. The values of $N$ 8 or 10 will give the satisfactory results.

3.5.2. Solution Procedure for Stress Boundary Problem

After obtaining the solution of fluid injection boundary, the solution of stress boundary needs to be determined to obtain the final stress solution of the borehole subjected to fluid injection and in-situ stress. For impermeable borehole wall, pore pressure is way less significant than that of flux loading and thus radial stress at borehole wall can be taken as $\sigma_{rr} = 0$. This assumption will create a generalized plane strain problem whose approximate solution is given by Cui et al. (1997). The problem can be decomposed into three different problems, solve individually and then superposed to get the final solution.

The boundary condition for stress problem is represented as follow (Cui et al., 1997):

After transformation of in-situ stress, the boundary condition at the far failed ($r \to \infty$), as in equation (6) can be written as:

\[ \sigma_{xx} = -S_x \]  
\[ \sigma_{yy} = -S_y \]  
\[ \sigma_{zz} = -v(S_x + S_y) - \alpha(1 - 2v)p_0 \]  
\[ \sigma_{xy} = -S_{xy} \]  
\[ \sigma_{yz} = \sigma_{xz} = 0 \]  
\[ p = p_0 \]

Similarly, at the borehole surface ($r = R$), the following boundary condition applies:

\[ \sigma_{rr} = -S_r H(-t) \]  
\[ \sigma_{r\theta} = -S_{r\theta} H(-t) \]  
\[ \sigma_{rz} = 0 \]
\( p = p_0 H(-t) \) \hspace{1cm} (3.70d)

The boundary condition for three modes and their respective solution presented by Cui et al. (1997) are presented below. The term \( p_0 \) denotes the mean compressive stress, \( S_0 \) represents the deviatoric stress and \( \theta_r \) denotes the rotation angle. They can be expressed as following respectively:

\[
P_0 = \frac{S_x + S_y}{2} \tag{3.71a}
\]

\[
S_0 = \sqrt{\left(\frac{S_x + S_y}{2}\right)^2 + S_{xy}^2} \tag{3.71b}
\]

\[
\theta_r = \frac{1}{2} \tan^{-1} \frac{2S_{xy}}{S_x - S_y} \tag{3.71c}
\]

In following boundary conditions and solutions, subscripts 1, 2, 3 denotes the corresponding loading modes.

**Mode 1**

Boundary conditions:

\[
\sigma_{rr}^{(1)} = p_0 H(t) \tag{3.72a}
\]

\[
\sigma_{r\theta}^{(1)} = 0 \tag{3.72b}
\]

\[
p^{(1)} = 0 \tag{3.72c}
\]

And its solution is given by:

\[
\sigma_{rr}^{(1)} = H(t) \frac{R^2}{r^2} \tag{3.73a}
\]

\[
\sigma_{\theta\theta}^{(1)} = -H(t) \frac{R^2}{r^2} \tag{3.73a}
\]

**Mode 2**

Boundary conditions:
\( \sigma_{rr}^{(2)} = 0 \)  
\( \sigma_{r\theta}^{(2)} = 0 \)  
\( p^{(2)} = p_0 H(-t) \)  

And its solution in Laplace domain is given by

\[
\sigma_{rr}^{2} = -\frac{2\eta p_0}{s} \left[ \frac{R K_1(\xi)}{r \beta K_0(\beta)} - \frac{R^2 K_1(\beta)}{r^2 \beta K_0(\beta)} \right] \tag{3.75a}
\]

Similarly,

\[
\sigma_{\theta\theta}^{2} = \frac{2\eta p_0}{s} \left[ \frac{R K_1(\xi)}{r \beta K_0(\beta)} - \frac{R^2 K_1(\beta)}{r^2 \beta K_0(\beta)} + \frac{K_1(\xi)}{K_0(\beta)} \right] \tag{3.75b}
\]

\[
p^{(2)} = \frac{p_0 K_0(\xi)}{s K_0(\beta)} \tag{3.75c}
\]

In equations (75), \( K_1 \) and \( K_2 \) denotes Bessel function of second kind order one and two respectively.

**Mode 3**

Boundary conditions:

\[
\sigma_{rr}^{(3)} = -S_0 H(t) \cos 2(\theta - \theta_r) \tag{3.76a}
\]

\[
\sigma_{r\theta}^{(3)} = S_0 H(t) \sin 2(\theta - \theta_r) \tag{3.76b}
\]

\[
p^{(3)} = 0 \tag{3.76c}
\]

And its solution in Laplace domain is given by

\[
\sigma_{rr}^{(3)} = \frac{S_0}{s} \left\{ \frac{B(1 + v_u)}{3(1 - v_u)} C_1 \left[ \frac{1}{\xi} K_1(\xi) + \frac{6}{\xi^2} K_2(\xi) \right] - \frac{1}{1 - v_u} C_2 \frac{R^2}{r^2} \right. \\
\left. - 3C_3 \frac{R^4}{r^4} \right\} \cos 2(\theta - \theta_r) \tag{3.77a}
\]

Similarly,
\[
\sigma_{r\theta}^{(3)} = \frac{S_0}{s} \left\{ -\frac{B(1 + v_u)}{3(1 - v_u)} C_1 \left[ \frac{1}{\xi} K_1(\xi) + \left(1 + \frac{6}{\xi^2}\right) K_2(\xi) \right] \\
+ 3C_3 \frac{R^4}{r^4} \right\} \cos 2(\theta - \theta_r) 
\]

\[
\sigma_r^{(3)} = \frac{S_0}{s} \left\{ 2B(1 + v_u) \left[ \frac{1}{\xi} K_1(\xi) + \frac{3}{\xi^2} K_2(\xi) \right] - \frac{1}{2(1 - v_u)} C_2 \frac{R^2}{r^2} \\
- 3C_3 \frac{R^4}{r^4} \right\} \sin 2(\theta - \theta_r) 
\]

\[
p^{(3)} = \frac{S_0}{s} \left\{ -\frac{B^2(1 - v_u)(1 + v_u)^2}{9(1 - v_u)(v_u - v)} C_1 K_2(\xi) \\
+ \frac{B(1 + v_u)}{3(1 - v_u)} C_2 \frac{R^2}{r^2} \right\} \cos 2(\theta - \theta_r) 
\]

where,

\[
C_1 = -\frac{12\beta(1 - v_u)(v_u - v)}{B(1 + v_u)(D_2 - D_1)} 
\]

\[
C_2 = \frac{4(1 - v_u)D_2}{(D_2 - D_1)} 
\]

\[
C_3 = -\frac{\beta(D_2 + D_1) + 8(v_u - v)K_2(\beta)}{\beta(D_2 - D_1)} 
\]

and,

\[
D_1 = 2(v_u - v)K_1(\beta) 
\]

\[
D_2 = \beta(1 - v)K_2(\beta) 
\]

The pore pressure is obtained by adding the pore pressure generated from mode 2 and 3 with the virgin pore pressure. Similarly, all the final solution of stress components can be determined by superposing as shown in equation (3.80)

\[
p = p_0 + p^{(2)} + p^{(3)} 
\]
\[
\sigma_{rr} = -P_0 + S_0 \cos 2(\theta - \theta_r) + \sigma_{rr}^{(2)} + \sigma_{rr}^{(3)} \\
\sigma_{\theta\theta} = -P_0 - S_0 \cos 2(\theta - \theta_r) + \sigma_{\theta\theta}^{(1)} + \sigma_{\theta\theta}^{(2)} + \sigma_{\theta\theta}^{(3)} \\
\sigma_{zz} = v \left[ \sigma_{rr}^{(1)} + \sigma_{\theta\theta}^{(1)} \right] - \alpha (1 - 2v) p' \]
\[
\sigma_{r\theta} = -S_0 \sin 2(\theta - \theta_r) + \sigma_{r\theta}^{(3)} \\
\sigma_{rz} = \sigma_{r\theta} = 0
\]

(3.80b) - (3.80f)

### 3.6. Parametric study of Analytical Solution

The derived analytical solution was used to perform some numerical examples to illustrate its implementation in obtaining the stress distribution around the borehole wall. Matlab was utilized for the convenience for performing the calculations including the inversion of Laplace and Fourier transform. The analysis was conducted on Ruhr sandstone. Its properties and parameter were taken directly from Abousleiman and Chen (2010) which are as follows:

**Ruhr Sandstone Properties**

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>G (MPa)</th>
<th>v</th>
<th>(v_u)</th>
<th>B</th>
<th>(\alpha)</th>
<th>(c \text{ (m}^2/\text{day)})</th>
<th>K (m/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ruhr Sandstone</td>
<td>13000</td>
<td>0.12</td>
<td>0.3</td>
<td>0.849</td>
<td>0.645</td>
<td>424</td>
<td>0.02</td>
</tr>
</tbody>
</table>

where,

\[G = \text{Shear Modulus}\]
\[v = \text{Drained Poisson’s ratio}\]
\[v_u = \text{Undrained Poisson’s ratio}\]
\[B = \text{Skempton’s pore pressure coefficient}\]
\( \alpha = \) Biot coefficient \\
c = Diffusion coefficient \\
k = Permeability of the formation

**In-situ Stress**

Tensile strength of sandstone = \( 4 - 25 \) MPa

\( S_{x'} = 20 \) MPa  \\
\( S_{y'} = 18 \) MPa  \\
\( S_{z'} = 25 \) MPa  \\
\( p_0 = 9.8 \) MPa

Initial Fluid Flow rate \( 1.41 \) l/min

**Borehole Dimension and Orientation**

Borehole radius (R) = 0.1 m  \\
Azimuth Angle (\( \varphi_{z'} \)) = 0°  \\
Zenith Angle (\( \varphi_{y'} \)) = 0°

Further parametric studies can be conducted for other rock types using the varying parameters found in the literature such as Abousleiman and Chen (2010). These properties can be directly inserted in the simulator tool developed and described in later part of this thesis.

Table 3.2. Material Properties for further Parametric studies

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>G (MPa)</th>
<th>( v )</th>
<th>( v_u )</th>
<th>B</th>
<th>( \alpha )</th>
<th>C (m(^2)/day)</th>
<th>K (m/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Danian chalk</td>
<td>2200</td>
<td>0.227</td>
<td>0.354</td>
<td>0.709</td>
<td>0.725</td>
<td>3.67</td>
<td>0.01</td>
</tr>
<tr>
<td>Gulf of Mexico Shale</td>
<td>760</td>
<td>0.219</td>
<td>0.447</td>
<td>0.868</td>
<td>0.968</td>
<td>0.00143</td>
<td>0.0001</td>
</tr>
</tbody>
</table>
Using the Matlab codes, the solution was used to solve the problem with above parameters as an example. Only Terzaghi’s effective tangential stress and pore pressure variations were studied as they are the most important stress components controlling the fracturing and stability of borehole. Effective tangential stress and pore pressure variations of each of three fluid discharge boundary are presented.

3.6.1. Example of Gradually Increasing Unit Step Fluid Injection

In this section, the results obtained from derived solutions for gradually increasing fluid injection was used to conduct the parametric study with the parameters shown in the table 3.1 and results are discussed. The results are presented for three different time intervals to analyze the effects of time interval of fluid injection. The two major components pore pressure and effective tangential stress are analyzed.

![Figure 3.5. Pore pressure variation of gradually applied unit step fluid injection](image-url)
Figures 3.5 and 3.6 are the results of pore pressure and stress distribution of numerical example for gradually applied fluid injection obtained from the derived analytical solution respectively. The fluid injection initially starts at zero at time $t = 0$ and linearly increases and gains its maximum value at $t_0 = 0.001$ day then remains constant for the remainder of the time. Both the pore pressures and effective tangential increases as time increases. However, it can be seen that both the pore pressure and effective tangential stress doesn’t change much as time increases from 0.1 day to 1 day. As expected both value decreases gradually as distance away from the borehole increases. Figure 3.7 shows the effects of increasing discharge section length on effective tangential stress. The stress and pore pressure values are higher for smaller discharge section and vice versa.
Figure 3.7. Effective tangential stress variation with discharge section length of gradually applied unit step fluid injection

3.6.2. Example of Linearly Decreasing Fluid Injection

Following figures 3.8 and 3.9 shows the pore pressure and stress distribution for linearly decreasing fluid injection case. It is obvious that after $t_0$ (0.001 day), the pore pressure and effective tangential stress value generated due to fluid discharge would be very small as discharge has been already stopped at $t_0$. So, in longer time such as $t = 0.02$ day the effects of fluid discharge would be insignificance and may even produce some small negative pore pressure effects and will eventually reach the value close to virgin pore pressure value. The effects of increasing discharge section length on effective tangential stress is shown in figure 3.8 for this boundary type. The effect of fluid injection seems to extremely small for larger discharge length to radius ratio.
Figure 3.8. Pore pressure variation of linearly decreasing fluid injection

Figure 3.9. Effective tangential stress variation of linearly decreasing fluid injection
3.6.3 Example of Square Wave Fluid Injection

Similarly, the pore pressure variation and effective stress variation for square wave fluid discharge boundary are shown in figure 3.11 and 3.12 respectively. Two cycles of square waves were considered in this case. Each cycle is injected for 0.05 day with and 0.5 day interval between the cycles. The variations of pore pressure and effective tangential stress shows that after the end of injection cycles, both the values can be considered somewhat significant for early time \( t = 0.2 \)
and becomes less and less significance as time moves on and reaches the in-situ stress state.

Figure 3.11. Pore pressure variation of square wave fluid injection

Figure 3.12. Effective tangential stress variation of square wave fluid injection
Furthermore, the comparison between the effects of different number of fluid injection cycle were also studied. Figures 3.11 and 3.12 shows the longer and less cycles of injection gives higher pore pressure and tangential stress values than shorter and more cycles of fluid injection. However, this could be just for our specific case with fewer numbers of cycles. More number of cycles and longer time interval cases should be studied again before confirming this to be the general trend for such square wave flux. Figure 3.15 shows the effects of discharge length on effective tangential stress due to square wave flux. The results follows the general pattern of inverse relation with the discharge section length. The tangential stress decreases as the ratio of discharge length to radius increases.

![Pore Pressure vs. r/R](image)

Figure 3.13. Effects of injection cycles for a constant time period on pore pressure
Figure 3.14. Effects of injection cycles for a constant time on tangential stress

Figure 3.15. Effects of discharge length on tangential stress of square wave fluid injection
Figures 3.16 and 3.17 show the pore pressure variation and effective tangential stress variation for various number of cycles of fluid injection with constant injection rate. The total injection volume was kept constant. The analysis were conducted for 2 cycles, 4 cycles and 8 cycles of fluid injection with total injection volume of 0.004 m$^3$. The total time of analysis was 0.003 day and flow rate was 1.4 l/min. It can be seen that the pore pressure and effective tangential stress decreases with increasing number of cycles. It is because, with the increase in cycles the pause period also increases. Having longer pause period will eventually induce lower pore pressure and the stress. In terms of fluid volume, square wave fluid injection is not economical. Nevertheless, this type of injection can be used to create fractures from fatigue failure. It can also be applied in geotechnical drilling where fractures needs to be avoided. In such cases, square wave injection is better than a constant fluid injection.

![Figure 3.16](image)

**Figure 3.16.** Effects of injection cycles for a constant volume of fluid injection on pore pressure
Figure 3.17. Effects of injection cycles for a constant volume of fluid injection on effective tangential stress

3.7. Fracture Initiation

The sandstone rock chosen for this parametric study has the tensile strength ranging from 4-25 MPa. The effective tangential stress induced by the fluid injection exceeds this tensile strength of the rock at some point and fractures will be initiated. In this particular study, the maximum effective tangential stress was found to be 78 MPa for gradually applied unit step fluid injection and in the similar range for the square wave fluid injection which are much greater than the strength of the sandstone. Fluid injection rate or volume can be adjusted to optimum level using this solution. Increasing the discharge section length or increasing the radius of borehole is another way to bring the effective stress to optimum level. In geotechnical engineering in other drilling instances, the drilling fluid may initiates unwanted fracture causing leak off, seepage and creating
instability of the structures. Further study can be conducted to investigate the applicability of this solution in such problems.

3.8. Summary

The analytical solution was derived for the three different time dependent fluid injection boundary conditions of an inclined borehole by solving the governing equations of deformations based on theory of linear poroelasticity. Problem was decomposed into the fluid injection boundary case at the borehole wall and in-situ stress boundary at far-filed. Laplace transformation was used to solve the stress boundary part while both the Laplace and Fourrier transformation technique were required for solving the time dependent fluid injection boundary. A parametric study was conducted on a Ruhr sandstone with materials properties found in literature and results were presented for each cases.
CHAPTER 4.
NUMERICAL SIMULATION

4.1. Introduction

This chapter presents the results of numerical simulation of a borehole in a porous formations which is subjected to various time-dependent fluid injection and in-situ stress. Finite element modeling and analysis is the most common numerical technique for solving science and engineering problems. Finite elements model discretizes the whole problem domain into small finer elements, calculates the displacements, strains, stress etc. individually and combines them together to provide the final results. ABAQUS FEM software was used to conduct the numerical simulation. The main objective of this numerical simulation was to verify the analytical solution derived in chapter 3. Thus, the comparison with analytical solution are shown in this chapter.

4.2. Finite Element Modeling of Borehole in ABAQUS

The process of developing a finite element model includes creating geometric design of the problem, defining material properties associated with the problem, assigning appropriate material sections and its corresponding properties, applying the loading and boundary conditions and discretizing the model. One of the very useful feature of ABAQUS is to support user subroutines created for specific analysis not available as a built-in feature. The borehole problem considered here requires to employ user subroutine to simulate some condition not available in ABAQUS built-in interface. It will be explained in later section of this chapter. All the steps and details of establishing the numerical model for a vertical borehole are discussed in brief below.

4.2.1. Borehole Geometry
A circular borehole drilled in infinitely large formation as shown in figure 4.1 is axis symmetric. For convenience, the borehole is assumed to be drilled vertically with principal stresses aligning with the x y and z coordinates. In doing so, rotation angle should be taken as zero in analytical solution in order create the exact comparable model. The length of fluid injection section is 0.1 meter. The radius of borehole was taken as 0.1 meter as in analytical parametric study. The geometry modeled was same for both the fluid injection boundary and stress boundary. The results corresponding to stress boundary and fluid injection boundary were obtained separately and added together to form the final result.
Figure 4.2. Borehole geometry in ABAQUS

The geometry modeled for the current analysis is shown in figure 4.2. The closer view of the the finite length discharge section is shown on the right side of the figure. The radius length used is 30 meters and height is 25 meters. Those number are good enough to model as infinitely large medium as the injection section is only 0.1 meters. To reduce the time taken to conduct numerical calculations, only one quadrant of the model was used since the problem was axially symmetric. Furthermore, the domain which is in square shape can be changed in circular section as the distance increases further away from the center of borehole, the effects will not be significant. It will reduces the number of elements while discretizing the model. Hence, it reduces the overall time needed to perform the numerical simulation. The length $b$ on right side of figure 4.2 is the length of fluid injection section length which is 0.1 meters.
4.2.2. Material Properties

The next step of creating a numerical model is to input material properties of different sections that we are considering. The material properties and parameters needed for this analysis can be obtained through various lab test, in-situ test, empirical co-relations etc. Since our model is a poroelastic, all the poroelastic constant in addition to the elastic properties are required. Young’s Modulus and Poisson’s ratio are the two elastic parameters needed to define elastic model. Void ratio, specific weight of the fluid and permeability of the formation are other three parameters defining the characteristic of porous medium. In addition, bulk modulus of solid grain and fluids are two properties associated with the poroelastic model. It should be noted that in current analytical solution, Skempton’s pore pressure coefficient \( (B) \), shear modulus \( (G) \), drained Poisson ratio \( (\nu) \) and undrained Poisson ratio \( (\nu_u) \). However, these properties are not included in material properties of numerical model. ABAQUS uses the relationship between Void ratio, bulk modulus of solid and fluid to determine those properties as given by following relationship.

\[
B = 1 - \frac{\phi K (K_s - K_f)}{K_f (K_s - K) + \phi K (K_s - K_f)} \tag{4.1}
\]

where,

\( \phi = \) Porosity

\( K = \) Bulk Modulus

\( K_s = \) Bulk Modulus of the solid grain

\( K_f = \) Bulk Modulus of the fluid

The values of these material properties were taken same as in analytical solution example problem. Required conversion were made to make same values in both analytical example problem and this numerical model. Material properties and their respective sections were assigned. Table 4.1 shows the values for each the material properties used in this simulation.
Table 4.1. Material properties used for the numerical simulation

<table>
<thead>
<tr>
<th>S.N</th>
<th>Material Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Young’s Modulus</td>
<td>29120</td>
</tr>
<tr>
<td>2.</td>
<td>Poisson’s ratio</td>
<td>0.12</td>
</tr>
<tr>
<td>3.</td>
<td>Permeability</td>
<td>$1 \times 10^{-9}$</td>
</tr>
<tr>
<td>4.</td>
<td>Void Ratio</td>
<td>0.020408</td>
</tr>
<tr>
<td>5.</td>
<td>Bulk Modulus of Grains</td>
<td>36000</td>
</tr>
<tr>
<td>6.</td>
<td>Bulk Modulus of Fluid</td>
<td>2100</td>
</tr>
<tr>
<td>7.</td>
<td>Specific weight of wetting liquid</td>
<td>0.0098</td>
</tr>
</tbody>
</table>

4.2.3. Analysis Steps

For time dependent analysis, the analysis steps should be clearly defined to let the program know the correct order of calculation process. Since the model the combination of both the borehole drilling and fluid injection, it is needed to include both the drilling step and time associated with it as well as add the fluid injection and time period of injection in correct order. The initial step includes all the initial conditions such as in-situ stress, initial void ratio, pore pressure etc. The second step also known as Geostatic is added to obtain the initial equilibrium. After creating equilibrium, the next step is to create a drilling in which the boundary will be released. As per our instantaneous borehole drilling assumption, the time taken for this step would be extremely small. Once the drilling step is created the final step is to start fluid injection on the finite section (length = b = 0.1m). The time period of this step depends on the total fluid injection time.
4.2.4. Boundary Conditions and Loading

Another important aspect of numerical modeling is to assign the appropriate boundary condition and loading scheme. Boundary condition can be defined at the borehole wall, at fluid injection section and at the far field. The boundary conditions can be the displacement or rotation, connection type such as pinned or free to rotate about some axis etc. Similarly, the loading condition needs to be applied if there exists any external loading acting on the model. For current analysis, the only external loading is the fluid injection in addition to initial in-situ stress. A distributed pore surface fluid pressure was applied on the injection section. The magnitude of this pressure depends upon the fluid boundary injection chosen. Three different time-dependent fluid injection scenario were studied. The time-dependent fluid pressure can be applied in ABAQUS using amplitude function. Amplitude can be defined to apply the transient fluid flow.

4.2.5. User Subroutine

Due to the complex nature of the problem, user subroutine was needed to model the current problem and run this simulation. User sub-routine allows user to program their own calculation or analysis criteria written in supported programing language and link it to its interface. ABAQUS supports the user subroutine created in Fortran Programing language. In analytical solution, as per the boundary for fluid injection (equation 3.7), the radial stress the borehole wall is equal to the negative pore pressure. But it was not possible to model this condition directly in ABAQUS interface. A radial load was initially assigned as a boundary condition where magnitude was chosen to be given by the user subroutine. Before running the analysis, subroutine was attached. Then, a user subroutine was called automatically by ABAQUS at the end of each time increment during the simulation. The subroutine code with URDFIL and DLOAD written in FORTRAN retrieves pore pressure on each element at the end of each increment. The pore pressure was
recorded at the centroid of each elements and interpolated across the element. The obtained value was applied as a radial stress by the program. This procedure allowed to simulate the boundary condition equivalent to the analytical solution.

**4.2.6. Discretizing and Meshing**

The final step before running the model for analysis is to discretize the model and creating the finite element mesh. Generally, higher the number of elements, higher the accuracy of results will be. However, increasing the number of elements also increases the calculation time. So, it is important to make sure that appropriate number of elements are created. The meshing of the current numerical model can be seen in figure 4.3 below.

![Figure 4.3. Mesh creation of model in ABAQUS](image)

It can be seen that in creating the mesh, number of elements at the injection zone is comparatively denser than at the distance away from the borehole center. Since the injection section and around
its vicinity are the region to have more stress concentration, those area is more important and main subject of concern. Having more elements in those region increases the accuracy of results.

4.3. Numerical simulation in ABAQUS and Results

In this section the results of numerical simulation for three different time-dependent fluid injection are presented and discussed. As in analytical solution, the problem was decomposed into two different boundary problems i.e. stress boundary and fluid injection boundary. Simulations were conducted separately for each boundary problem and add them to obtain the final solution. Some of the example results of pore pressure distribution and tangential stress distribution counterplot are presented here.

4.3.1. Stress Boundary Results

The following figures show the counterplot of variation of pore pressure, effective tangential stress and effective radial stress due to the drilling of borehole in a formation which is subjected to initial in-situ stress after 0.1 day. The pore pressure, radial stress and effective tangential stress are in the unit of Mega Pascal. It should be noted that it doesn’t include the fluid injection phase yet. The contour plot is transformed from default Cartesian coordinate to polar coordinate system in ABAQUS. The effective tangential stress increases from \( \theta = 0^\circ \) and has maximum negative value of -36.73 MPa at \( \theta = 90^\circ \) the borehole wall (\( r = R \)). The solution is axially symmetric. Similarly, the pore pressure is constant and equal to the virgin pore pressure. The radial stress is also maximum at the borehole wall. These values together with the values obtained form the fluid injection boundary are superposed to obtain the final stress and pore pressure variations.
Figure 4.4. Contour plot of effective tangential stress variation due to borhole drilling

Figure 4.5. Contour plot of pore pressure variation due to borhole drilling
4.3.2. Fluid Discharge Boundary Results

In this section, the results of variation of pore pressure and effective tangential stress obtained from numerical simulations for the fluid injection problem are presented and discussed. Contour plots are presented for each cases. In this boundary problem, the initial in-situ stress was removed. The simulation starts from injecting the fluid without considering the drilling of borehole which is already taken into account in stress boundary simulation. Simulation is conducted for each of the three time dependent fluid injection cases by applying the corresponding discharge. User subroutines were used to run the simulation of this fluid injection boundary. The pore pressure and effective tangential stress are expressed in the unit of Mega Pascal as in stress boundary.
Figure 4.7 shows the pore pressure distribution of gradually applied step load type fluid injection. In this type of fluid injection, initially the loading rate starts from zero at time zero seconds. It gradually increases linearly until time t equals to 0.86 seconds and stays constant afterwards. The results after 8.64 seconds of continuous fluid injection shows that highest pore pressure (80MPa) is at the wellbore wall and reduces gradually as distance from wellbore wall increases as we expect.

Similarly, the Tarzaghi’s effective stress variation contour plot is shown in figure 4.8. The effective tangentials stress is higher (84.2MPa) at the borehole surface and decreases while moving away the the borehole wall. The time period of this injection is 8.64 seconds as well. The main reason to run simulation for such short span is to show that there is not a big difference in stress between 0.0001 day and 0.001 day.
The following figure 4.9 and 4.10 shows the pore pressure and effective tangential variation caused by the linearly decreasing fluid injection respectively. Initially, the fluid flow injection rate is 1.41 l/min and linearly decreases to zero at time $t = 86.4$ seconds. It can be seen that after injection rate drops to zero the tangential stress and pore pressure becomes less significant. It can also be observed in the figure that the pore pressure and effective tangential stress do not drop immediately as the distance from the borehole wall increases but rather stays somewhat same as in the borehole wall and then gradually declines. The maximum pore pressure value is 5.54 MPa and maximum tangential stress is 8.97 MPa at the borehole wall.
Figure 4.9. Contour plot of pore pressure variation due to linearly decreasing fluid injection

Figure 4.10. Contour plot of effective tangential stress variation due to linearly decreasing fluid injection
Figure 4.11. Pore pressure comparison after 1st and 2nd cycle of square wave fluid injection

Figure 4.12. Effective tangential stress comparison after 1st and 2nd cycle of square wave fluid injection
The comparison of pore pressure and stress variation after first and second cycle of square wave fluid injection is shown in above figures 4.11 and 4.12 respectively. The first cycle of fluid injection starts from certain value of flux (1.4 l/min in this case) and drops to zero at \( t = 86.4 \text{ seconds} \) and remains at zero for another 86.4 seconds before starting the new cycle. The square wave flux applied with total of two cycles with equal cycle timing shows that the stress and pore pressure values increase at the end of each cycle. For example, after first cycle in figure 4.11, the pore pressure at the borehole wall is 2.47 MPa and after second cycle 3.84 MPa which shows the value increases with the increase in number of cycles. Similarly, the effective tangential stress is increased from 2.84 Mpa to 3.56 MPa from first to second cycle in similar manner as pore pressure plot shown in figure 4.12. Such cyclic loading can be useful to create the fatigue failure to initiate the fractures. The main advantage of such type of fluid injection is that it is not required to apply high fluid injection pressure continuously to initiate the fractures which may cause the instability of borehole and causing a potential borehole collapse.

4.4. Comparison of Analytical Solution with the Numerical simulation

In this section, the comparison between the analytical results obtained from the analytical solution of chapter 3 and the results given by numerical simulation using ABAQUS are compared. Just like in other sections above, the comparison for all of the three fluid injection cases are presented and discussed. The results in this section accounts for both the stress variation due to borehole drilling in a formation with initial in-situ stress and fluid injection at the borehole surface except figure 4.13 which shows the results of stress boundary only. Just like as in analytical solution, the superposition scheme was used to get the final results.
4.4.1. Comparison of Stress Boundary only

The following figure 4.13 shows the comparison of effective tangential stress of the borehole that is subjected to the initial in-situ stress only. Since, the superposition method was opted to get the final solution of problem, the stress variation results due to in-situ stress was obtained first and then added the stress due to the fluid injection. At depth $z = 0$ and time $t = 0.1$ day the analytical solution given by Cui et al. (1997) seems to be in very good agreement with our ABAQUS simulation results at both angles ($\theta = 0^\circ$ and $\theta = 90^\circ$). The negative effective tangential stress (tension) at $\theta = 90^\circ$ is lower than that of $\theta = 0^\circ$. The value increases as the normalized distance $r/R$ increases. Analysis were conditioned for different time interval such as 0.001 day, 0.01 day, 1 day etc. It was noticed during the analysis that the effect of time duration on pore pressure and effective tangential stress due to in-situ stress upon borehole drilling was minimal and was mainly concentrated around the borehole. The influence of borehole orientation was also found to be smaller and smaller as the normalized distance $r/R$ increased.

![Figure 4.13](image_url)  

Figure 4.13. Comparison of Effective tangential stress variation due to insitu stress only in ABAQUS simulation vs analytical solution.
4.4.2. Comparison of Gradually Applied Unit Step Fluid Injection

The following two figures 4.14 and 4.15 shows the comparison between the analytical solutions and numerical simulation. The pore pressure and Terzaghi’s effective tangential stress for the case of gradually applied unit step fluid injection are presented. For this specific example, all parameters same as the analytical solution, linearly increasing until \( \tau_0 = 0.001 \) day and total injection time \( t = 0.0015 \) day are considered. The ABAQUS simulated solutions are slightly smaller than the constant fluid injection results by Abousleiman and Chen (2010). But the difference could be large for higher injection rate. The analytical and numerical results are in good agreement thus, the accuracy of our analytical solution can be verified for the case of gradually applied unit step fluid injection. This particular case can be used to model the fluid injection scenario where the maximum fluid injection rate cannot be achieved due to several limitations during hydraulic fracturing. Such limitations could also be associated with the stability of borehole.

\[
\begin{align*}
z &= 0, \quad \theta = 0^\circ, \quad \varphi_y = 0^\circ, \quad \tau_0 = 0.001 \text{ day} \\
t &= 0.0015 \text{ day}, \quad Q = 1.4 \text{ l/min}
\end{align*}
\]

Figure 4.14. Comparison of pore pressure variation for gradually applied unit step fluid injection in ABAQUS simulation vs analytical solution.
4.4.3. Comparison of Linearly Decreasing Fluid Injection

The comparison between the analytical solution and numerical simulation of pore pressure and Terzaghi’s effective tangential stress for the case of linearly decreasing fluid injection are shown is figure 4.16 and 4.17 respectively. Just like an example for gradually applied unit step fluid injection, all the parameters are same as analytical solution. Fluid injection rate was linearly decreasing to 0 at $\tau_0 = 0.001$ day. The total time studied was $t = 0.0015$ day and initial flow rate was 1.4 l/min. The results of numerical simulation are in good agreement with the analytical solution derived in previous chapter. From those comparison plots, it can be seen that analytical solution is slightly smaller (less than 5% difference) than the ABAQUS solution.
Figure 4.16. Comparison of pore pressure variation for linearly decreasing fluid injection in ABAQUS simulation vs analytical solution.

$z = 0, \theta = 0^\circ, \phi_y = 0^\circ, \tau_0 = 0.001 \text{ day}$

$t = 0.0015 \text{ day}, Q = 1.41 \text{ l/min}$

Figure 4.17. Comparison of effective tangential stress variation for linearly decreasing fluid injection in ABAQUS simulation vs analytical solution.

$z = 0, \theta = 0^\circ, \phi_y = 0^\circ, \tau_0 = 0.001 \text{ day}$

$t = 0.0015 \text{ day}, Q = 1.4 \text{ l/min}$
4.4.3 Comparison of Square Wave Fluid Injection

Figures 4.18 and 4.19 shows the comparison between the analytical solutions and numerical simulation of pore pressure and Terzaghi’s effective tangential stress for the case of square wave fluid injection respectively. The comparison results are for the 2 cycles of square wave fluid injection. The first cycle of injection is stopped at 0.001 day and second cycle is stopped at 0.003 day. All the material properties were same as other analytical solution example problems. The initial fluid flow rate starts at of 1.4 l/min. The results of numerical simulation was found be in good agreement with solution given by the analytical solution. The most likely cause for very small discrepancy at the borehole wall could be the sudden drop of fluid injection while applying Heaviside function in the numerical model. A separate user routine could be used in the numerical model of the square wave fluid injection discontinuous function to obtain exact same results.

Figure 4.18. Comparison of effective tangential stress variation for 1 cycle of square wave fluid injection in ABAQUS simulation vs analytical solution
Figure 4.19  Comparison of effective tangential stress variation for 2 cycle of square wave fluid injection in ABAQUS simulation vs analytical solution

4.5. Summary

This chapter presented the results of numerical simulation using ABAQUS of borehole drilled in saturated formation subjected to initial in-situ stress and various time-dependent fluid injection to model the realistic problems encountered in hydraulic fracturing initiation and borehole stability analysis. Different counter plots of the numerical simulation were included. The comparison of numerical simulation with formulated analytical solution for all of the three different cases of fluid injection were also shown. The analytical solution were found to be in good agreement with the numerical simulation.
CHAPTER 5.
MATLAB-BASED SIMULATOR TOOL

5.1. Introduction

The derived analytical solution for three different time-dependent fluid injection boundary needs to be evaluated using some calculation programs due to the complexity involved in inverting the Laplace transformation and Fourier transformation. Such programs can be Mathematica, Matlab etc. Matlab is used to obtain the solutions presented in this thesis. As the problem consists of many parameters and different variations of results, it is extremely important to simplify the results obtaining procedures. A user with very limited knowledge might find confusing since Matlab code are written with four different sub codes and the procedure is not straight forward. Thus, to overcome this problem, an effort is made to develop a user friendly and less complex Matlab-based user interface to simulate the problem analytically. This chapter covers the scope, procedure/methods of such tool development and user guide of the developed simulator tool. This tool can be used to estimate the pore pressure and stress components at various distance from borehole center for all dimension of boreholes during the design of hydraulic fracturing initiation. The analytical solution can be obtained using Matlab in very short time unlike the numerical simulation. The numerical simulation takes much longer time especially if the number of finer elements are very high and time duration (t) of study is longer. This tool can be used a as standalone desktop application without needing to have Matlab installed in the computer. In this sense, this tool can be very useful to those without Matlab or its full license.

5.2. Scope

This simulator tool can be used to calculate the pore pressure, effective tangential stress and effective radial stress at different time intervals and borehole inclination/rotation angles for the
inclined boreholes subjected to initial in-situ stress and four different types of fluid injection. Those four type of fluid discharge boundary includes gradually applied unit step fluid injection at finite section of borehole wall, linearly decreasing fluid injection, square wave fluid injection and a constant fluid injection case as in Abousleiman and Chen (2010). The effective stress mentioned here is Terzaghi’s effective stress. The Matlab plotted graphs generated by this tools can be exported and analyzed to facilitate the hydraulic fracturing design and treatment. The major problem encountered in the phase two of this project is that the Matlab App designer has a limited feature and it was difficult to add more functionality in the interface to make it more attractive in terms of appearance and features.

5.3. Methodology and Procedure

This task can be divided into two phase’s development process. The first phase of this project involved writing the Matlab code to find the solution for pore pressure, effective tangential stress and radial stress based upon the analytical solution derived in chapter 3. The second phase was to develop the user interface using the Matlab and link the Matlab code of first phase into it. As the problem is a combination of stress boundary problem and fluid discharge problem, the first phase of this simulator development consisted of two different Matlab codes for each boundary type and another one to input parameters and superpose the two solutions of stress and fluid injection boundary.

5.4. User Interface and Guidelines

The simulator can be installed as an add-on on Matlab app or can be used as a standalone desktop app without having to have Matlab installed. After running the application, user interface window as shown in figure 5.1 will be displayed as the homepage for the program.
There are menu options available at the top of the window. On the left of the home page, help section is located. The standard template picture of original problem statement is represented as central homepage icon. The result window where the results are displayed once the analysis is completed is situated at the bottom section the window. The functions of each button are explained below:

**NEW**: Once the program starts, **NEW** button can be pushed to start the analysis process. An exit dialogue as shown in following figure 5.2 will appear. Choosing ‘**Yes**’ will start new analysis window and ‘**No**’ will cancel the process.
Figure 5.2. New Analysis Dialogue

**View:** The second menu feature **View** after expanding provides option to display plots, input parameters and analysis Time.

**Options:** The third menu button **Options** can be expanded to choose sample parameter mode to run the trial analysis. The default values are those used in chapter 3 and chapter 4 for numerical example problem of Ruhr sandstone. The clear option below the default parameters clears all the default values and changes everything back to zero.

**Analysis:** The fourth button on top menu of the window is Analysis button. It allows users to select the stress components of interest for analysis. Radial Stress or Tangential stress can be accessed form run option and analysis can be aborted selecting stop option below it.

**Close:** To return to homepage from any point during the analysis process, **Close** button can be pushed and similar dialogue box as figure 5.2 will appear for confirmation.

**Quit:** The program can be closed using the **Quit** button.

The result window at the bottom will display the results as the analysis is completed. The four text box includes the radial distance \((r/R)\), pore pressure, effective tangential stress and effective radial stress respectively while the total time elapsed during the analysis is shown on the right. A window as shown in figure 5.3 will be displayed if new analysis is selected. Then, it is required to
input all the requested parameters. The input parameters panel contains six different input parameter tab. The default value for all the space are set to 0 initially. It is important to note that the analysis error will occur if all parameters are not entered. Each of those six parameters tab in the panel are defined below.

Figure 5.3. New Analysis Window

**In-situ Stress**: This tab contains the input parameters of initial in-situ stress in the formation. The $S_x$, $S_y$, and $S_z$ are the stresses in x, y and z direction respectively and $p_0$ is the virgin pore pressure of the formation. The unit is fixed to MPa which is often used in practice. By default these values are set to zero. The clear button at the button can be pushed to reset all the value to defaults again. These parameters can be obtained from various lab testing, in-situ testing and small trial fracturing process.
**Dimension:** The dimension tab has all the required dimensions of the borehole in context. The radius of borehole and length of injection section are required to be entered in meter. The azimuth angle expressed in degree is related to the inclination of borehole with respect to principal stress direction. Setting it at the default value zero models the vertical borehole. The theta angle is the angle of interest in effective tangential stress. It could vary from zero to ninety degrees.

**Poroelastic Parameters:** This tab contains six different poroelastic properties of the formation at which the borehole is drilled. Drained and undrained Poisson’s ratio are dimensionless. The shear Modulus is expressed in Mega Pascal. Diffusion and permeability are other two coefficient expressed in meter square per day are required to enter along with the other dimensionless Skempton’s Coefficient. The clear button at the bottom of tab resets all these parameters to zero.

**Time:** This tab is to enter the total time period of simulation we are interested it. The unit of time is day. Like other, clear resets the time value to zero.

**Fluid Flow:** The fifth tab contains the information related to the fluid injection. The type of time-dependent fluid injection and initial fluid flow rate can be selected here. The default injection type is the constant fluid injection from start to end. If gradually applied unit step or linearly decreasing fluid boundary is selected, corresponding $t_0$ value is also required to enter. Similarly, if square wave type injection is selected, the corresponding values of $t_1$, $t_2$ and $t_3$ are also required to enter. The initial fluid flow rate is expressed in cubic meter per day.

**Normalized Distance (r/R):** The final input parameter section is where the normalized distance from borehole center is situated. The radial distance is adjustable. ‘From’ is the start point and normally is taken as one. ‘To’ is the last point of our interest and ‘Spacing’ field is the distance between of points along the path. The clear button clears all the above values.
5.5. Sample Analysis and Results

In this section, a short description of an example simulation with is presented including the sample result window and plot of the analysis. The following window as shown in figure 5.4 will appear after successfully running the analysis. On the right, the pore pressure and stress components plot is located. The default plot will be displayed for pore pressure. The plot can be switched to effective tangential stress and effective radial stress depending upon our analysis type. The exporting or closing options for the generated plots are positioned below the plot at the right side of the window. The results or the analysis and the data used in the plot can be seen on the result section situated at the bottom of the window. The radial stress section at the bottom is empty because the sample analysis was performed for the effective tangential stress in this particular example. On the right, the total time taken for this analysis is displayed. The time taken was 167 seconds (2.78 minutes) which is much faster than the numerical simulation time it would have taken for the same analysis.
5.6. Summary

A description on the Matlab-based simulator tool development procedure, user guidelines and sample analysis were presented in this chapter. This simulator tool seems to simplify the process of obtaining the analytical stress solutions of time dependent fluid injection in a finite section of an inclined borehole under in-situ stress condition. From an example analysis, it was shown that the simulation process takes just few minutes depending upon the range of normalized radial distance. This tool is an effective means of solving and estimating the stress and pore pressures at the borehole wall and its vicinity, generated due to several fluid injection scenarios together with
the in-situ stress of the formation. The development of this tool shall be considered as a success of this overall of study and research.
CHAPTER 6.
CONCLUSIONS AND RECOMMENDATIONS

5.1. Conclusions

This research study derives the analytical stress solution of several time dependent fluid injection cases that could be applied during the hydraulic fracturing initiation process. The state of stress and pore pressure generated after the application of fluid injection are regarded as one of the critical parameters of hydraulic fracturing initiation and overall fracturing treatment. Thus, an accurate approximation of stress caused by drilling followed by fluid injection could contribute to the effective and efficient hydraulic fracturing initiation process. The stress solution given by Abousleiman and Chen (2010) for a steady fluid injection at the finite section of borehole wall is extended to three different time dependent fluid injection boundary. Similar technique of Laplace and Fourier integral transformation was used to solve the deformation governing partial differential equations in a poroelastic medium and corresponding boundary conditions. The obtained solution in transformed domain was inverted by using numerical integration and Stehfest inversion algorithm. Among the three time dependent fluid injection scenarios, the gradually applied unit step fluid injection and square wave fluid injection seems to be more applicable in real field while the linearly decreasing type fluid injection could be used for the cases where development of filter cake could occur due to the nature of formation or chemicals used in the process. The problem was decomposed into two different problem as a stress boundary problem and fluid injection boundary problem. The approximate stress solution given by Cui et al. (1997) for the stress boundary problem was superposed with the formulated stress solution for a time dependent fluid injection boundary to obtain the final solution of inclined borehole in a porous medium.
Upon formulating the analytical solution for three time dependent fluid injection cases, finite element numerical modeling and analysis on ABAQUS was conducted and the results were compared to each other. An additional user defined subroutine was required to simulate the problem in ABAQUS software. The obtained results from those analysis were found to be in good agreement with analytical solution. A very small but acceptable difference on some specific cases were observed. Number of meshes generated, seeding approximation, number of iteration controls etc. are some of the factors that could have attributed to the minor discrepancies in the comparison results.

The derived stress solution was coded in Matlab to facilitate the process obtaining stresses and pore pressure around the borehole wall and its surroundings required during the hydraulic fracturing initiation. The results can be obtained in few minutes using this analytical solution method while the numerical simulation takes hours and even days depending upon the time period used in analysis. Furthermore, to make the process even simpler and convenient, a Matlab based simulator tool was developed using Matlab program with a user friendly interface and options to use the tool as a standalone desktop application as well. User with limited knowledge or no knowledge could still use this tool to simulate a constant fluid injection case and three time dependent fluid injection case encountered in hydraulic fracturing initiation and design.

5.2. Recommendations

Based on this research study, several recommendations can be made for the further study and improvements in this topic and simulator tool development. These recommendations are listed below:
1. The three time dependent boundary conditions are just few of the cases that may be applied during the fluid injections. Several other time dependent model can be established and formulate the solution for them in similar manner. Triangular wave, modulated wave, spectrum and actuator are some of the possible time dependent fluid injection scenarios.

2. The stress boundary solution given by Cui et al. (1997) is an approximate estimation of stress and pore pressure due to the in-situ stress. A more rigorous and exact solution for stress boundary problem was given by Chen (2019). Thus, it is recommended to superpose the fluid injection boundary solution with that stress to derive the most accurate solution.

3. Borehole stability analysis and horizontal directional drilling are two other potential application of this stress solution. However, further studies and investigations are necessary before confirming the accuracy of this solution of such scenarios. The geotechnical drilling fluid may create unwanted fractures. Currently, empirical equations are used to evaluate such problems. Rigorous solution may provide great benefits to geotechnical industries.

4. Although, Matlab is a simple and convenient platform to create basic simulator tools, the design, appearance and functionality has lot of limitations. It is recommended to use other programs such as FORTRAN, C++, etc. to develop more advanced simulator tool.

5. This tool can be combined with other fracture design and treatment applications and tools to developed more advance simulator which does the multiple operations such as fracture design, fracture initiation and fracture propagation.
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VITA

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