Quantitative Evaluation of a Lightweight Sediment for a Physical Model of the Lower Mississippi River

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QUANTITATIVE EVALUATION OF A LIGHTWEIGHT SEDIMENT FOR A PHYSICAL MODEL OF THE LOWER MISSISSIPPI RIVER

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

in

The Department of Civil and Environmental Engineering

by

Mauricio Hooper
B.S., Technological University of Panama, 2010
M.S., Technological University of Panama, 2015
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This thesis was possible with the love, help and support of my wife Karen, my son Lucas, my mother Edy, my brother Lincoln and the rest of my family members. To my father Lincoln, who, although is no longer with us, always believed in me. Thanks to the Fullbright-Laspau scholarship and the Technological University of Panama to allow me to study this degree. I want to thanks to all my colleagues and friends that helped me in one way or another, specially to Ron Rodi and his family who were always there to support me. Special thanks to my advisor Dr. Clinton Willson who supported me from the very beginning and encouraged me to continue learning every day.
Table of Contents

Acknowledgment .................................................. ii
Abstract ............................................................ iv
List of Tables ....................................................... v
List of Figures ....................................................... vi
Chapter 1. Introduction ............................................ 1
  1.1 Problem Statement ........................................ 1
  1.2 Objectives and Research Questions ....................... 2
  1.3 Organization of Dissertation ............................. 3
  1.4 References .................................................. 4
Chapter 2. Literature Review ..................................... 6
  2.1 Background and Scaling .................................. 6
  2.2 Distorted River Physical Models ......................... 7
  2.3 Design Methodologies ..................................... 9
  2.4 LMRPM Design ............................................. 27
  2.5 References ................................................ 38
Chapter 3. Incipient Motion and Subaqueous Dune Formation of a Lightweight
  Physical Model Sediment of the Lower Mississippi River .... 40
  3.1 Introduction ............................................... 40
  3.2 Experimental Setup ...................................... 45
  3.3 Results ...................................................... 47
  3.4 Discussion .................................................. 53
  3.5 Conclusion .................................................. 57
  3.6 References .................................................. 58
Chapter 4. Determination of the Sediment Time Scale for a Distorted, Mobile Bed
  River Physical Model of the Lower Mississippi River ........ 62
  4.1 Introduction ............................................... 62
  4.2 Flume Experiment ......................................... 67
  4.3 Sediment Time Scale 6600 at the LMRPM ................ 73
  4.4 Conclusion .................................................. 77
  4.5 References ................................................ 79
Chapter 5. Discussion and Conclusions .......................... 81
  5.1 References .................................................. 85
Appendix A. Rational Methodologies Calculations ...................... 86
Appendix B. Additional Photographs from Experiments at the LSU Flume and the
  LMRPM ........................................................... 96
Vita ................................................................. 103
Abstract

The Lower Mississippi River Physical Model (LMRPM), housed at the LSU Center for River Studies on the Baton Rouge, LA Water Campus, is a distorted, movable bed model comprising the lower 195 miles of the Mississippi River from Donaldsonville through the Head of Passes into the Gulf of Mexico. Since the LMRPM was designed to replicate the hydraulics (i.e., flow and river stages) and bulk non-cohesive sediment transport, the model lightweight sediment must replicate both the incipient motion and two-dimensional dune characteristics (height and length). In addition, the model scale and distortion require that the sediment time scale be determined empirically.

A series of flume experiments were conducted to evaluate the model sediment response to prototype scaled conditions in terms of the incipient motion, dune formation and sediment time scale. Results showed that the stresses in terms of Shields parameter well satisfied the Shields criterion which suggests that overall the local turbulence generated at the initiation of model particles motion is consistent with the theoretical approaches for bed evolution in rivers with sand. Scales dunes in the experiments reflected dune lengths and heights similar to those found in the prototype reach. Finally, flume and model results demonstrate that a model sediment time scale of 6600 provides good replication of prototype (bulk) bed morphodynamics.
## List of Tables

2.1 Scaling ratios for Froude and Reynolds similitudes (Chanson, 2004) .................. 8
2.2 Scaling ratios for mobile bed river physical models (Julien, 2002) ....................... 11
2.3 Proposed LMRPM geometric scales ................................................................. 29
2.4 Design characteristic sediment sizes for both prototype and model .................... 32
2.5 LMRPM design utilizing different rational methodologies. All ratio scales are defined as prototype to model ................................................................. 36
3.1 Observations for the three experiments .............................................................. 47
3.2 Frequency of collected data in terms of prototype values. Prototype values from El Kheiashy et al. (2007). fp is prototype frequency and fexp is experimental frequency ............................................................................................................................. 56
4.1 Results from the two sediment time scales ......................................................... 70
4.2 Measured dunes celerities at the flume experiments in terms of prototype scale. Discharge from top to bottom is the sequence they were run. .......................... 72
List of Figures

2.1 Lower Mississippi River Physical Model domain. Flow is from top to bottom. Upper boundary is Donaldsonville, LA (top-left) and the lower boundary is the Gulf of Mexico (bottom-right). ......................................................... 28
2.2 LMRPM model sediment made of unexpanded polystyrene with a density of 1.05 g m$^{-3}$. Particles ranging from 0.1-1 mm. ................................................................. 31
3.1 Lower Mississippi River Physical Model domain. Flow is from top to bottom. Upper boundary is Donaldsonville, LA (top-left) and the lower boundary is the Gulf of Mexico (bottom-right). ......................................................... 42
3.2 Representative grain size distributions of model material sampled before being placed into the flume. Prototype design gradation ($D_{10}$, $D_{50}$ and $D_{90}$) from available data at Belle Chasse. ................................................................. 44
3.3 Schematic of the experimental setup. ................................................................. 45
3.4 Profile view: Example of four dunes observed during the experiments. ............ 48
3.5 Near bed shear stresses calculated from the TKE method utilizing the ADV collected velocity fluctuations for all three experiments. ................................................................. 49
3.6 Shields parameters calculated from the near bed shear stresses combining all three experiment data. Average values with one standard deviation. Dash line is the critical Shields parameter from Parker et al. (2003). ................. 51
3.7 Defect observed during the experiments: Small amount of sediment accumulates to develop a dune. ................................................................. 52
3.8 Dunes lengths and heights collected from all three experiments. Dunes steepness values were calculated from dunes lengths and heights measurements. ............ 53
3.9 Dunes characteristics grouped from all three experiments. .............................. 54
3.10 Dunes characteristics from all three experiments in terms of prototype scale. From top to bottom: height versus length, height versus water depth and length versus water depth. ................................................................. 55
4.1 Lower Mississippi River Physical Model domain. Flow is from top to bottom. Upper boundary is Donaldsonville, LA (top-left) and the lower boundary is the Gulf of Mexico (bottom-right). ......................................................... 64
4.2 Representative grain size distribution of model material sampled before being placed into the flume. Prototype design gradation ($D_{10}$, $D_{50}$ and $D_{90}$) from available data at Belle Chasse. ................................................................. 65
4.3 Schematic of the experiment setup. ................................................................. 67
4.4 Average hydrograph. Circles are the design hydrograph for the LMRPM. Dash lines are the five constant discharges utilized in the experiments to simulate the design hydrograph.

4.5 Initial and final beds for all three sediment time scales experiments. From top to bottom: $T_{sr} = 9900$, $T_{sr} = 3840$ and $T_{sr} = 6600$. The first two showed symmetrical, rounded dunes, whereas the last one showed asymmetrical dunes.

4.6 Example of four dunes developed during the experiment for the sediment time scale $T_{sr} = 6600$.

4.7 Bed measurements taken at the model in terms of the prototype. Left is for $T_{sr} = 9900$ and right is $T_{sr} = 6600$. From top to bottom: Reserve, Carrollton, Alliance and Empire. Solid lines are the model without sediment and dash lines are the design target values.

4.8 Maximum stage levels recorded at maximum water discharge. Left is for $T_{sr} = 9900$ and right is for $T_{sr} = 6600$. Dash lines are design target values.

4.9 Rating curves produced for the model in terms of the prototype. Left is for $T_{sr} = 9900$ and right is for $T_{sr} = 6600$. From top to bottom: Reserve, Carrollton, Alliance and Empire. Dash lines are the design target values.

4.10 Example of dunes observed after finishing 29 runs for the sediment time scale $T_{sr} = 9900$.

4.11 All dunes lengths and heights measured at the end of each experiment. Left is for $T_{sr} = 9900$ and right is for $T_{sr} = 6600$.

B.1 LSU Armfield S6 MKII flume where both flume experiments were conducted (Chapter 3 and 4).

B.2 Preparing initial bed before conducting experiment for incipient motion and dune formation at the LSU flume.

B.3 SonTek 16-MHz MicroADV utilized to measure velocity fluctuations at the flume.

B.4 Running experiment for the sediment time scale $T_{sr} = 3840$ at the LSU flume.

B.5 Running the LMRPM for the series with $T_{sr} = 9900$.

B.6 Taking measurements at the LMRPM with a caliper at the end of the series with $T_{sr} = 9900$.

B.7 Plan view of dunes after finishing experiments for the sediment time scale $T_{sr} = 6600$. 
Chapter 1. Introduction

1.1 Problem Statement

The Lower Mississippi River Physical Model (LMRPM), housed at the LSU Center for River Studies on the Baton Rouge, LA Water Campus, is a distorted, mobile-bed model comprising the lower 195 miles of the Mississippi River from Donaldsonville through the Head of Passes into the Gulf of Mexico. The model utilizes a lightweight sediment to replicate prototype hydraulics (i.e., flow and river stages) and bulk non-cohesive sediment transport. Understanding how well the LMRPM lightweight sediment replicates water and bed processes is important for the model to succeed in the study of the flooding, navigation and coastal restoration of the Lower Mississippi River.

Prototype sediment was scaled utilizing two similitude parameters critical Shields parameter ($\tau^* \kappa$) and critical particle Reynolds number parameter ($R^*_ec$). The equality of these two parameters between model and prototype along with river sediment density of 2.65 g cm$^{-3}$ and a model sediment density of 1.05 g cm$^{-3}$, resulted in model particle diameters 3.2 times larger than those found in the prototype. The sediment used is a fine synthetic sediment made of unexpanded polystyrene with diameters that vary from 0.10 to 1.00 mm with a mean particle diameter $d_{50} = 0.35$ mm. Lightweight material, such as walnut shells, silica sand, cation resin, coal, apricot pip and polystyrene has been widely used in mobile-bed physical models for replicating bed behavior (Kantoush et al., 2006; Shaw et al., 2008). These materials have been proven to be non-cohesive materials, able to replicate entrainment conditions from the bed (Sequeiros et al., 2009) in a much quicker time when compared with natural sand without compromising the Froude scale (Vermeulen et al., 2014).

Rational and empirical methods are two approaches when designing mobile-bed physical models (Graf, 1971). The rational method emphasizes in the dimensionless similitude criteria between model and prototype while keeping model distortion as low as possible (Maynord, 2006). The empirical method focuses on replicating the bed movement of the model as in
the prototype while relaxing similitude criteria (Stevens, 1942; Warnock, 1950). The LMRPM falls within those two methods because it was designed to follow three dimensionless similitude criteria Froude, Shields parameter and particle Reynolds number, while relaxing the Reynolds number criterion, but its distortion is higher than recommended in rational methods (Julien, 2002; Shen, 2012). Although the sediment was designed utilizing the aforementioned parameters, the time scale ratio was based on an empirical equation that needs to be verified through model experimentation (BCG Engineering & Consulting, Inc., 2015).

It is because of this unique mobile-bed physical model design that understanding how well the lightweight sediment is representing typical prototype bed is of utmost importance for the model to succeed. Because of the large domain and size of the model, it is difficult and expensive to evaluate the sediment response to those scaled prototype typical conditions. It is because of these limitations that before any change in design criteria is proposed, the sediment needs to be tested in a series of flume experiments, testing incipient motion, dune formation and their typical characteristics as well as the time scale ratio at which the sediment better replicates prototype bed typical characteristics. Once these evaluations are done, it is important to verify what the model bed is finally replicating to ensure that the model is capable of the typical conditions of the Lower Mississippi River.

1.2 Objectives and Research Questions

The LMRPM follows the most important similitude criteria for distorted, mobile-bed physical models the Froude number, Shields parameter and particle Reynolds number. These criteria set the basis for the model to replicate prototype hydraulics and bulk non-cohesive sediment transport. Because of the high distortion resulted from design, the model sediment needs to be verified that it is replicating the main characteristics found in the actual prototype. The objectives of this research is to evaluate the sediment behavior under scaled prototype typical conditions. The three main questions to answer are:

1. Is the model sediment fulfilling the design similitude criterion of critical Shields pa-
rameter?

2. Are dunes formed in the model typical of those found in the prototype?

3. Is the sediment time scale from the empirical equation sufficient to mobilize model sediment in such way that resembles prototype sediment?

1.3 Organization of Dissertation

This dissertation consists of five chapters. All chapters, except for the introduction (chapter one) and discussion and conclusions (chapter five), are written based on papers that have been under review or are to be submitted to peer-reviewed journals, and are constructed using the journal paper format that is approved by the Graduate School of Louisiana State University. Therefore, each chapter is relatively independent, though some information of the reviews and references may be repeated in certain chapters for completeness and clarity. All chapters of the dissertation document the research work of the Ph.D. candidate under the guidance of the major advisor and committee members.

Chapter two summarizes a literature review for river physical models. Two approaches are presented: rational and empirical. The rational approach includes rigorous, scientific-based design methodologies from five researchers: Pierre Julien, M. Selim Yalin, Tetsuro Tsujimoto, Hubert Chanson and Alan L. Prasuhn. These methodologies are explained step by step highlighting the basic principles as well as how to calculate the most important scales. The empirical approach includes work from three researchers: Walter Hans Graf, John Franco and Robert Davinroy. These methodologies are a summary of the most important recommendations from each author when modeling river physical models. At the end of the chapter, the LMRPM design scales are explained and reviewed under the five rational approaches to compare similarities and differences between methodologies.

Chapter three presents the results of a flume experiment conducted to investigate the sediment incipient motion condition and dune formation. This chapter answers question
one regarding the sediment incipient motion condition utilizing an micro ADV to measure instantaneous velocity vectors which are further converted into shear stresses using the turbulent kinetic energy method (TKE). These stresses are non-dimensionalized into Shields parameters to be evaluated with the critical Shields criterion. Also, this chapter answers question two regarding dunes observed at the flume where the main characteristics length and height were measured with a millimetric-graduated ruler and compared against river dune data measured at the Lower Mississippi River reach. Further information about dune characteristics measured in the model is included in the next chapter.

Chapter four proposes a methodology to calculate sediment time scale for mobile bed river physical models. First, it is presented the results of a flume experiment where different sediment time scales were tested to evaluated the sediment response to time. From the flume experiment results, the suggested sediment time scale was evaluated at the model to see the response in terms of bed changes to time after 29 model cycles. This chapter answers question three on how changes in the sediment time scale influences the model sediment behavior and how the bed can better replicate river characteristics with a different sediment time scale.

Chapter five concludes this research discussing the results from all the experiments both at the flume and the model. Flume experiments are a great tool to investigate sediment responses to different parameters when modeling river physical models. A different sediment time scale was tested, improving the model similarity to scaled conditions experienced at the river.

1.4 References


Chapter 2. Literature Review

2.1 Background and Scaling

River physical models have existed at least as early as the 19th century when Louis Jerome Fargue built a model of the Garonne River, Bordeaux in France by 1875 (Julien, 2002). Since then, many researchers have further investigated these type of physical models such as Einstein and Chien (1956), Yalin (1971), Parker (1978), proposing different approaches to better replicate natural rivers. This type of models allows the study of rivers under controlled laboratory conditions to predict different scenarios at a low cost when compared with field studies. In order to differentiate prototype to model, a subscript ‘p’ is given to prototype, a subscript ‘m’ to model and a subscript ‘r’ to the ratio prototype to model, i.e., \( \lambda_r = \lambda_p \lambda_m^{-1} \). River physical models need to meet three similitude criteria in order to properly replicate prototype conditions: geometric, kinematic and dynamic. Geometric similitude involves parameters related to geometrical dimensions where \( x_r \) are downstream, \( y_r \) lateral and \( z_r \) vertical ratios. In the cases that \( x_r = y_r \), they are also known as the horizontal scales. A complete geometric similitude is achieved when all three dimensional ratios between model and prototype are the same, i.e., \( x_r = y_r = z_r \). Models that are designed with incomplete geometric similitude are either distorted, i.e., \( y_r \neq z_r \) or tilted, i.e., \( x_r \neq z_r \) (Julien, 2002). Kinematic similitude involves parameters related to length and time, i.e, velocity, acceleration and kinematic viscosity. Models that are designed with kinematic similitude follow the Froude similitude criterion, i.e., \( F_r = 1 \). Dynamic similitude involves parameters related to mass, i.e., density and dynamic viscosity. This similitude is achieved when the Froude and Reynolds number ratios are the same. However; in river physical models this similitude is only achieved at full scale, which is impossible, unpractical and unrealistic.

There are two types of river physical models: rigid and mobile bed physical models. Rigid models are used when river flow conditions are not capable of transporting sediment. These
models could be designed following either exact Froude similitude or Froude similitude for tilted models (Julien, 2002). Mobile bed physical models are used when there is sediment transport in the river. This type of models is a small representation of rivers that is adjusted to replicate the natural characteristics of those rivers to solve sedimentation problems (Franco, 1978). Whether a rigid or a mobile bed physical model is used, river physical models can help to understand complex river systems providing a tool that allows researchers to investigate the better solutions when answering the natural phenomena that these rivers present.

2.2 Distorted River Physical Models

River physical models can be built as either undistorted or distorted models. Theoretically, undistorted models meet all the three similitude criteria: geometric, kinematic and dynamic similitude. Since river models use water as the same fluid as in the prototype, complete undistorted models cannot be designed due to the impossibility to meet both Froude and Reynolds similitude criteria. The term undistorted model is then usually referred to geometric undistorted models where models are scaled with the same ratio in their three x, y and z components. For this type of model, kinematic and dynamic similitude are achieved from choosing the criterion that dominates in the model either Froude or Reynolds. Chanson (2004) proposed equations for scaling river models based on the aforementioned similitude criteria (Table 2.1). When undistorted models cannot be built due to space, sediment material or equipment restrictions, distorted models are used. Distortion in mobile bed physical models is achieved by varying any one or several of five different aspects of the models (Ettema et al., 2000): 1) geometric (vertical dimension, particle, slope), 2) densimetric (density, fall velocity), 3) flow (velocity), 4) time, and 5) sediment transport rate. Geometric distortion is one of the first aspects typically evaluated when distortion in models is needed. The distortion of the vertical dimension in models increases turbulence, improves accuracy in measurements and provides larger Reynolds number (Chanson, 2004; Ettema et al., 2000).
Table 2.1. Scaling ratios for Froude and Reynolds similitudes (Chanson, 2004).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Scale ratios with</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Froude</td>
<td>Froude (distorted)</td>
</tr>
<tr>
<td>Geometric properties</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>m</td>
<td>$L_r$</td>
<td>$x_r z_r$</td>
</tr>
<tr>
<td>Area</td>
<td>m$^2$</td>
<td>$L_r^2$</td>
<td>$-$</td>
</tr>
<tr>
<td>Kinematic properties</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velocity</td>
<td>m s$^{-1}$</td>
<td>$L_r^{1/2}$</td>
<td>$z_r^{1/2}$</td>
</tr>
<tr>
<td>Discharge per width</td>
<td>m$^2$ s$^{-1}$</td>
<td>$L_r^{3/2}$</td>
<td>$z_r^{3/2} x_r$</td>
</tr>
<tr>
<td>Discharge</td>
<td>m$^3$ s$^{-1}$</td>
<td>$L_r^{5/2}$</td>
<td>$z_r^{3/2} x_r$</td>
</tr>
<tr>
<td>Time</td>
<td>s</td>
<td>$L_r^{1/2}$</td>
<td>$x_r z_r^{-1/2}$</td>
</tr>
<tr>
<td>Dynamic properties</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Force</td>
<td>N</td>
<td>$\rho_r L_r^3$</td>
<td>$-$</td>
</tr>
<tr>
<td>Pressure</td>
<td>Pa</td>
<td>$\rho_r L_r$</td>
<td>$\rho_r z_r$</td>
</tr>
<tr>
<td>Density</td>
<td>kg m$^{-3}$</td>
<td>$\rho_r$</td>
<td>$\rho_r$</td>
</tr>
<tr>
<td>Dynamic viscosity</td>
<td>Pa s</td>
<td>$L_r^{3/2} \rho_r^{1/2}$</td>
<td>$-$</td>
</tr>
<tr>
<td>Surface Tension</td>
<td>N m$^{-1}$</td>
<td>$L_r^2$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

$L_r =$ lengths scale (undistorted), $x_r =$ horizontal scale (distorted), $z_r =$ vertical scale (distorted), $\rho_r =$ fluid density scale, $\mu_r =$ fluid viscosity scale.

The natural geometric slope of models is achieved from the dimensional scale factors of models; however, when this slope is not sufficient for transporting sediments, a supplementary slope is required (Franco, 1978). Sediment particles cannot be scaled with the same vertical scale ratio because it would result in particles so small that cohesion forces take a more important role in the interaction between particles (Garcia, 2008; Green, 2014). The use of the same material found in rivers would imply the exaggeration of other aspects of river physical models in order to properly move the material. It is then when the densimetric distortion is applied to scale sediment particles with a lower density and larger sizes than those found in prototypes (Garcia, 2008). Flow, time and sediment transport rate are inherently related to geometric and densimetric distortions (Ettema et al., 2000) and they should be changed accordingly; however, since the replication of the river bed is of the great importance, these parameters may be adjusted independently until models satisfy prototypes’ similar bed conditions (Franco, 1978).
2.3 Design Methodologies

Rational and empirical methods are two approaches when designing mobile bed physical models (Graf, 1971). The rational method emphasizes in the dimensionless similitude criteria including Froude number and Shields parameter. For this type of models, it is recommended to keep distortion as low as possible (Maynord, 2006). Several researchers have proposed different rational methods including Julien, Yalin, Tsujimoto, Chanson and Prasuhn. The empirical method focuses on replicating the bed movement of the model as in the prototype when relaxing the similitude criteria (Warnock, 1950). This method is well summarized from the statement that if a model is capable of replicating prototype known events, then it should replicate events that may happen in the prototype (Stevens, 1942). Franco (1978) documented a guideline for modeling mobile bed physical models using the empirical method concept based on his experience at the U.S. Army Engineer Waterway Experiment Station. Another example of an empirical method is micro-modeling or Hydraulic Sediment Response method (HRS) where models are of small size with depths and widths as low as 1 cm and 4 cm, respectively. There are some models that do not fall into any of the two categories aforementioned and may be classified as “other type of mobile bed models” such as the study of the Mersey Estuary in England by Reynolds (Maynord, 2006).

2.3.1 Rational Methods

Rational methods are based on the establishment of similitude between model and prototype. Each of the methods has its own set of equations derived from different similitude criteria. Five methods are presented including a brief summary of the principles that the methods are based on, the equations and their limitations.

2.3.1.1 Julien Method

Pierre Julien derived his method based on four similitude criteria including Froude, resistance, dimensionless grain diameter and Shields parameter (Julien, 2002). Depending on
whether or not models meet all criteria, models may be either complete or incomplete mobile bed similitude. Complete mobile bed physical models require the similitude of all the four criteria; however, this is not always possible when the model vertical scale is larger than 25 as model sediment that results from calculations is not possible to find in reality. In the case that complete mobile bed physical model design is not achievable, incomplete mobile bed physical model is selected sacrificing one of the four criteria. The author presents a set of equations for two incomplete mobile bed physical model cases relaxing either the Froude (non-Froudian) or dimensionless grain diameter criterion (quasi similitude in sediment transport). Non-Froudian models are selected when the same sediment material is used in a river with fairly constant Froude number. This type of model properly models sediment transport; however, it cannot simulate vertical and lateral flow acceleration. On the other hand, quasi similitude in sediment transport is selected for models where the main transport mode is bed load since the suspended transport may not be well replicated in this type of model. The set of equations is presented in Table (2.2) where scale ratios are defined as prototype to model.

### 2.3.1.2 Yalin Method

M. Selim Yalin stated that a model is valid as long as the characteristic of interest is proportionally related to the counterpart in the prototype by a constant (Martins, 2012; Shen, 2012). Based on this concept, physical models are designed using dimensionless analysis. One of the advantages of the method is that model scales do not depend on prototype’s characteristics (depth, slope, etc.). The main restriction that physical models have is they are built under the same effect of gravity \( g_r = 1 \) and for economical purposes they operate with the same fluid \( \rho_r = 1 \). In the specific case of modeling river flow with bed covered by sand waves (ripples and/or dunes), Yalin developed his iterative method based on Froude similitude, geometric distortion, similarity of energy losses or also called friction factor and cohesionless sediment. The four important scale ratios (model to prototype) to model are:
Table 2.2. Scaling ratios for mobile bed river physical models (Julien, 2002).

<table>
<thead>
<tr>
<th>Complete</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>$d_{sr}$</td>
</tr>
</tbody>
</table>

Geometric

- **Depth**
  - $z_r$
  - $z_r$
  - $z_r$

- **Width**
  - $y_r$
  - $y_r$

- **Length**
  - $z_r(1+4m)(1+m)^{-1}$
  - $z_r^{1+2m}d_{sr}^{-2m}$
  - $z_r^2d_{sr}^2$

- **Particle diameter**
  - $z_r(2m-1)(2+2m)^{-1}$
  - $d_{sr}$
  - $d_{sr}$

Kinematic

- **Time (flow)**
  - $z_r^{1/2+2m}d_{sr}^{-2m}$
  - $z_r^{3/2+3m}d_{sr}^{-1-3m}$
  - $z_r^3d_{sr}^2$

- **Time (bed)**
  - $z_r^{1/2}$
  - $z_r^{1/2}$
  - $z_r^{m}d_{sr}^{-1-m}$

- **Velocity**
  - $1$
  - $z_r^{1/2}d_{sr}^{-1+m}$
  - $1$

- **Unit bed load discharge**
  - $1$
  - $z_r^{1/2}d_{sr}^{-1+m}$
  - $1$

Dimensionless

- **Froude**
  - $1$
  - $1$
  - $z_r^{m-1/2}d_{sr}^{-1-m}$

- **Reynolds**
  - $z_r^{3/2}$
  - $z_r^{3/2}$
  - $z_r^{1+m}d_{sr}^{-1-m}$

- **Shields**
  - $1$
  - $1$
  - $1$

- **Grain Reynolds**
  - $1$
  - $z_r^{3/2-m}d_{sr}^{-1+m}$
  - $1$

- **Dimensionless diameter**
  - $1$
  - $z_r^{1/3-2m/3}d_{sr}^{2/3+2m/3}$
  - $1$

- **Sediment Density**
  - $z_r(3-6m)(2+2m)^{-1}$
  - $z_r^{1-2m}d_{sr}^{2m-1}$
  - $d_{sr}^{-3}$

---

flow depth, flow slope, submerged specific gravity of the sediment and sediment particle size. From these ratios two equations are to be satisfied:

\[
Y_r = Y_{crr} \neq 1 \quad (2.1)
\]

\[
C_r = \frac{1}{\sqrt{S_r}} \neq 1 \quad (2.2)
\]

where $Y_r = \text{mobility factor scale}$, $Y_{crr} = \text{critical mobility factor scale}$, $C_r = \text{friction factor scale}$, $S_r = \text{energy slope scale}$. Yalin stated that there may be many solutions that satisfy the similitude criteria; however, from the practical design point of view, some recommendations when scaling mobile bed physical models can be enclosed in what he called the “three
dimensional box”:

\[ 0.03 \leq \left( \frac{\rho_r}{\rho} - 1 \right)_r \leq 1 \]  \hspace{1cm} (2.3)

\[ 1 \leq S_r \leq z_r^{-1/2} \]  \hspace{1cm} (2.4)

\[ z_r \leq d_{sr} \leq 3.22 \]  \hspace{1cm} (2.5)

The set of equations and procedure are briefly presented below where scale ratios are defined as model to prototype.

1. Calculate all parameters for the prototype:

\[ X = \frac{R_s}{2} = \frac{d_s}{\nu} \sqrt{g S_z} \]  \hspace{1cm} (2.6)

\[ Y = \frac{\gamma S_z}{\gamma_s D} \]  \hspace{1cm} (2.7)

\[ Y = \frac{z}{D} \]  \hspace{1cm} (2.8)

\[ \xi = \left( \frac{X^2}{Y} \right)^{1/3} \]  \hspace{1cm} (2.9)

\[ Y_{cr} = 0.135 \xi^{-0.392} e^{-0.02\xi^2} + 0.05 \left( 1 - e^{-0.068\xi} \right) \]  \hspace{1cm} (2.10)

\[ \sigma = \frac{Y}{Y_{cr}} - 1 \]  \hspace{1cm} (2.11)

\[ B = 8.5 + [2.5 \ln(R_s) - 3] e^{-0.217[\ln(R_s)]^2} \]  \hspace{1cm} (2.12)

\[ b = e^{0.4B-1} \]  \hspace{1cm} (2.13)
\[ A_f = \left[ 2.5 \ln \left( \frac{b \cdot z}{2d_s} \right) \right]^{-2} \]  \hspace{1cm} (2.14)

\[ f_d(X) = \frac{1}{2} \left[ 1 - \cos \left( \frac{\pi X - 10}{20} \right) \right] \text{ if } X \in (10, 30) \]

and \[ \begin{cases} f_d(X) = 0 \text{ if } X < 10 \\ f_d(X) = 1 \text{ if } X > 30 \end{cases} \]  \hspace{1cm} (2.15)

\[ f_r(X) = 1 - f_d(X) \]  \hspace{1cm} (2.16)

\[ \left( \frac{\Delta}{\Lambda} \right)_d = f_d(X)0.0127\sigma e^{-0.078\sigma} \frac{1 - e^{-0.01Z}}{1} \]  \hspace{1cm} (2.17)

\[ \left( \frac{\Delta}{\Lambda} \right)_r = f_r(X)0.147\sigma e^{-0.3\sigma + 0.007\xi^3} \]  \hspace{1cm} (2.18)

\[ \frac{1}{C^2} = A_f + \frac{1}{2} \left( \frac{\Delta}{\Lambda} \right)_d^2 2\pi + \frac{1}{2} \left( \frac{\Delta}{\Lambda} \right)_r^2 \frac{1000}{Z} \]  \hspace{1cm} (2.19)

2. Select a convenient model depth and calculate the flow depth ratio \((z_r)\). Yalin suggested the model flow depth ranging from 15 cm to 35 cm.

3. Assume a submerged density of the sediment ratio \((\rho_r\rho^{-1} - 1)_r\) from (2.3).

4. Calculate the first approximation for the flow slope ratio and sediment particle size ratio based on the prototype parameters:

\[ S_r = \frac{C^2}{2.5 \ln \left( \frac{5.5z_rZ}{d_{sr}} \right)} \left[ \frac{1}{2} \left( \frac{\Delta}{\Lambda} \right)_d^2 2\pi + \frac{1}{2} \left( \frac{\Delta}{\Lambda} \right)_r^2 \frac{1000}{Z} \right] \]  \hspace{1cm} (2.20)

\[ d_{sr} = \frac{S_r z_r \left( \frac{\rho_s}{\rho} - 1 \right)_r}{(\rho_s\rho^{-1} - 1)_r} \]  \hspace{1cm} (2.21)

5. From the results of step 4, calculate the first approximation of the model flow slope and sediment particle size.

6. Calculate all parameters for the model from equations (2.6) to (2.19).
7. Calculate the second approximation for \((S_r)\) and \((d_{sr})\):

\[
S_r = \frac{(C_{\text{model}})^2}{(C_{\text{prototype}})^2}
\]

(2.22)

\[
d_{sr} = \frac{S_r z_r}{\left(\frac{\rho_s}{\rho} - 1\right) Y_{cr}}
\]

(2.23)

8. Calculate the second approximation of the model flow slope and sediment particle size.

9. Repeat step 6 and 7 until \(S_r\) and \(d_{sr}\) do not considerably vary from one approximation to the other.

10. If any of the resultant ratio scales do not fit into the three dimensional box, select a new submerged specific gravity of the sediment ratio \((\gamma_r)\) from (2.3) and repeat all the calculations starting at step 3. Another consideration may be changing the model flow depth.

11. Calculate the bed load rate scale using prototype values:

\[
q_{sr} = \sqrt{z_r Y_{cr}} \frac{A_f C_r K - 1}{K - 1}
\]

(2.24)

\[
K = \frac{\tau_o}{\tau_{ocr}} A_f C^2
\]

(2.25)

12. Calculate the time of formation of either an erosion or deposition:

\[
T_t = \frac{\left(\frac{\rho_s}{\rho} - 1\right) z_r^2}{S_r q_{sr}}
\]

(2.26)

2.3.1.3 Tsujimoto Method

Tetsuro Tsujimoto derived a rational method for distorted models applied to fluvial processes when he studied local scour effects around bridge piers (Shen, 2012). The three
basic scales to model are: the sediment particle size, the flow depth and the horizontal area. According to the author, the sediment particle size scale governs the bed roughness and sediment transport, the flow depth scale provides the required turbulence in the model and the horizontal scale is restricted to available physical space for the model. It is based on these concepts that Tsujimoto stated that distorting a model is reasonable as long as it is in the turbulent flow regime and the similitude in sediment motion exists. The method is based on three main criteria: Froude number \((F)\), transport intensity \((\Phi_B)\) and sediment diameter to depth ratio \((d_sz^{-1})\). The model dimensions result from the selection of a horizontal scale \((x_r)\), a vertical scale \((z_r)\) and a model sediment material. The equations and procedure are briefly described below where scale ratios are defined as model to prototype.

1. Select the horizontal \((x_r)\), vertical scale \((z_r)\) and the model sediment material.

2. Calculate the submerged specific gravity ratio \((\rho_s \rho^{-1} - 1)_r\).

3. Calculate the distortion ratio:

   \[
   \epsilon_d = \left( \frac{\rho_s}{\rho} - 1 \right)_{r}^{-3/2} \quad (2.27)
   \]

4. Calculate the sediment particle size ratio:

   \[
   d_{sr} = \epsilon_d z_r \quad (2.28)
   \]

5. Calculate the mean flow velocity:

   \[
   V_r = z_r^{1/2} \quad (2.29)
   \]

6. Calculate the sediment transport rate scale (substantial volume per unit width):

   \[
   q_{br} = \epsilon_d^{7/6} z_r^{3/2} \quad (2.30)
   \]
7. Calculate the hydraulic time scale:

\[ T_r = \frac{x_r}{z_r^{1/2}} \]  

(2.31)

8. Calculate the sediment time scale:

\[ T_{sr} = \frac{x_r}{z_r^{1/2} \epsilon_d^{7/6}} \]  

(2.32)

### 2.3.1.4 Chanson Method

Hubert Chanson recognized that mobile bed physical models are difficult to model and they quite often produce unsatisfactory results (Chanson, 2004). In despite of these limitations, the author developed a method for distorted, mobile bed physical models based on the similitude of the Froude number and the Shields parameter, deriving his equations from the definition of the boundary shear stress:

\[ \tau_o = \frac{f}{8} \rho V^2 \]  

(2.33)

where \( f = \) Darcy friction coefficient, \( \rho = \) water density and \( V = \) mean flow velocity. Once the model is designed, it is highly recommended that it needs to be verified and calibrated for their further use as a prediction tool.

The equations and procedure are briefly described below where scale ratios are defined as model to prototype.

1. Select the horizontal scale, vertical scale and the model sediment material.

2. Calculate the sediment particles size ratio:

\[ d_{sr} = \frac{z_r^2}{x_r \left( \frac{\rho_s}{\rho} - 1 \right)_r} \]  

(2.34)
3. Calculate the bed load transport rate per unit width ratio:

\[ q_{sr} = \frac{z_r}{\sqrt{x_r}} \]  

(2.35)

4. Calculate the hydraulic time scale ratio:

\[ T_r = x_r\sqrt{z_r} \]  

(2.36)

5. Calculate the sediment time scale ratio:

\[ T_{sr} = (1 - P_o) \frac{x_r z_r}{q_{tsr}} \]  

(2.37)

where \( P_o = \) sediment bed porosity and \( q_{tsr} = \) total sediment transport rate.

2.3.1.5 Prasuhn Method

Alan L. Prasuhn presented a set of equations to design mobile bed physical models based on the similitude criteria between river models and prototypes (Prasuhn, 1987). In the special case of mobile bed physical models, accuracy in modeling is hard to achieve since the bed roughness in dependent of flow conditions. Another problem, according to the author, is that the time scale for flow is different than the time scale for the bed. Moreover, the time scale for the bed is different for suspended motion, bed movement, bed development, scouring and other bed processes, complicating even more the similitude of the model. In despite of these limitations, the method aims to achieve at least a lower degree of similitude as long as there are similitude in Shields parameter and particle Reynolds number. The key component is to assume a model sediment particle specific gravity, and then based on the specific gravity ratio, the method’s equations are evaluated. Another assumption of the model is that the prototype is wide enough that the hydraulic radius is approximately equal to the depth.
The set of equations is presented below where scale ratios are defined as model to prototype.

1. Select the model sediment material.

2. Calculate the submerged specific gravity ratio:

\[ \alpha_r = \left( \frac{\rho_s}{\rho} - 1 \right)_r \]  
(2.38)

3. Calculate the sediment particle size ratio:

\[ d_{sr} = \alpha_r^{-1/3} \]  
(2.39)

4. Calculate the vertical scale ratio:

\[ z_r = \alpha_r^{7/6} \]  
(2.40)

5. Calculate the horizontal scale ratio:

\[ x_r = \alpha_r^{5/3} \]  
(2.41)

6. Calculate the sediment time ratio:

\[ T_{sr} = \frac{x_r z_r}{q_{sr}} \]  
(2.42)

where \( q_{sr} = 1 \).

Although these equations describe the most important model parameters, models need to be verified reproducing documented events. If models still require additional adjustments, tilting the models beyond their natural distorted slope may be the solution. This is justified as long as the Froude number is less than 0.5 (Prasuhn, 1987) and the particle Reynolds
number is higher than 100 (Bogardi, 1959).

2.3.2 Empirical Methods

2.3.2.1 Graf Method

Walter Hans Graf developed an empirical method for the design of mobile bed models based on Manning formula and model verification with known past events (Graf, 1971). The author stated that dynamic similarity is not achievable once the model is distorted since the longitudinal slope is increased and therefore, the velocity profile, which at the same time influences the sediment transport. This distortion benefits bed material transport since the shear stress is as well increased as a result of the slope distortion. The method requires the use of lightweight particles such as coal dust or plastic since models have low velocities and shallow depths. Once the longitudinal scales are determined utilizing the Manning formula, the model needs to be verified with prototype known events when comparing both bed configurations. In the case beds are different, further adjustments are required. This can be done either tilting the model or adjusting the sediment time scale. The latter is adjusted lengthening the time scale for low flows when shortening the same for high flows (Stevens, 1942). Based on all these distortions, the author emphasized that this type of models gives qualitative results, and that for quantitative results rational methods should be used instead. The method is presented below where scale ratios are defined as prototype to model.

1. Select a model bed lightweight material.

2. Estimate the Manning coefficients for both model and prototype, and then calculate the coefficient ratio \( n_r \).

3. Select a vertical scale ratio \( z_r \).

4. Calculate the velocity scale ratio based on the Froude similitude:

\[
V_r = \sqrt{z_r} \tag{2.43}
\]
5. Calculate the slope ratio based on Manning formula:

\[ S_r = \frac{n_r^2}{z_r^{1/3}} \]  

6. Calculate the horizontal scale:

\[ x_r = \frac{z_r}{S_r} \]  

If the model does not replicate past known events, the model should be tilted or the sediment time scale adjusted until the model experience similar bed configuration.

### 2.3.2.2 Franco Method

John Franco developed a general guide for modeling sediment transport in alluvial streams (Franco, 1978). This document collected experiences from different engineers at the U.S. Army Engineer Waterways Experiment Station (WES). The main idea of this type of models is to reproduce specific characteristics of interest of the river under study, adjusting the model until a satisfactory development of the channel is achieved. One of the most important requirements is the adequate simulation of bed material movement; therefore, it is recommended appropriate selection of the type of bed material since larger and heavier particles require larger forces to move the material. When modeling, these forces are obtained via model scales distortion (larger model depth and slope), supplementary slope (larger discharges) or a combination of both which further results in velocity changes that satisfactory reproduce bed movement. The author mentioned that model distortion and supplementary slope produce exaggeration of river banks and structures which affects local flow conditions, channel development and distribution of energy within the channel. In despite of these differences between model and prototype, models can still reproduce satisfactory results relying on model verification rather than mathematical analysis, involving the adjustment of hydraulic forces, time scale, rate of sediment input and operation technique until the model is capable.
of reproduce previous known prototype events.

The method is described below divided into nine steps, summarizing the most important recommendations. Here are summarized the main point of each step that should be followed when designing a river physical model.

1. Preliminary evaluation:
   
   (a) Prototype data is collected as much as required to meet the purposes of the model including water flow and sediment characteristics.

   (b) The model scales are empirically selected based on the purpose of the study, keeping in mind the possible sites for the model.

   (c) Model costs include design, construction, contingencies, operation and final reports. It is important to include contingencies for increases in any cost of the project, from the beginning during the design step up to presenting the final report.

2. Design of model:

   (a) Establish the accuracy of the required results based on the purpose of the model, including solutions to better replicate the areas of interest.

   (b) Areas of both fixed and mobile bed are defined providing flexibility for additional adjustments.

   (c) Determine the required elevation of troughs for bed material.

   (d) Controls for draining and flooding the model are recommended.

   (e) Template layouts are prepared based on the prototype channel alignment and banks, providing smooth edges of the fixed banks.

   (f) Model gauges are located based on gauges found in the prototype, changes in water surface slope or changes expected in the channel alignment in future conditions.
(g) Provide sufficient water supply for running the model for current and possible future conditions.

3. Inspection of model construction

(a) Knowledge of prototype’s behavior is recommended as well as of river modeling.

(b) Daily advances in the model construction should be tracked including working people and areas that require special attention.

(c) Verify with aerial photographs that all model parts are aligned as planned, including fixed parts, banks, depressions and ridges.

4. Preparation of data:

(a) Maps of the prototype topography are required for molding the bed and comparison of model results.

(b) Keep records of measurements that can be used for the final report.

(c) Stage-discharge relationship for the prototype are developed for model verification.

(d) Based on experience from previous models, discharge and sediment scales are selected.

(e) Model operation is based on the resulting scales relation curves previously defined.

5. Molding mobile bed:

(a) An engineer or technician should supervise the molding process.

(b) Bed topography is molded using contour maps including channel elevations and sandbars. Special attention between model templates should be taken when changes in bed occurs in these zones.

(c) Verification of the fixed elements is required as well as the transitional sections between fixed and movable beds.
(d) Observe changes in bed elevation when the model is initially flooded.

6. Preliminary adjustment:

(a) Gage elevations are referenced to bench mark elevations.

(b) The initial model water level is as high as the stage elevations before introducing water flow into the model.

(c) In the case of sand-bed models, it is recommended that the initial discharge should be the highest possible to allow bed developments such as ripples.

(d) Discharge increments are done before varying tailgates to obtain the required stages.

(e) Satisfactory bed movement is the ultimate goal of the model operation. In the case that adequate bed movement is achieved, either the water level is maintained or the water discharge is decreased to provide the correct stage while maintaining the same bed movement. On the other hand, unsatisfactory bed movement is adjusted either lowering the stage or increasing the water discharge while adjusting stages as well.

(f) For stages changing from lower to higher levels, the tailgate is raised before increasing the water discharge, whereas from higher to lower stage the discharge is decreased before the tailgate is adjusted.

7. Model verification:

(a) Bed is surveyed at the end of each annual hydrograph for comparison with prototype bed. In the case that the model bed is excessively disturbed, it should be properly remolded.

(b) Model parameters are adjusted for correcting differences between model and prototype. It is recommended to adjust one parameter at a time to measure its influence on bed response.
(c) After any adjustment made, run the model again and verify those areas of interest, comparing with prototype, previous model results and previous model operations.

(d) Bed developments that are not representative of the prototype should be corrected. It is recommended that during the annual hydrograph, special attention should be paid in these spots to determine what specific conditions are driving these unrealistic river sedimentation processes.

8. Base test

(a) Model bed is molded with the available bathymetric data.

(b) Run typical hydrograph until bed is stable and it reproduces general river trends.

(c) Bed is required to be surveyed at the end of each run.

(d) Rating curves are developed at control gages for testing improvement plans.

9. Test of improvement plans

(a) Improvement plans should be installed and bed remolded if the results from the base test are not satisfactory.

(b) Model is operated with the typical hydrograph, and records of developments at low, medium and high flows are recommended.

(c) Prepare results for the final report including the description of the plan and its final results.

2.3.2.3 Micro Modeling Method

Another empirical methodology for modeling mobile bed physical models is known as micro modeling. It consists of extremely small river physical models fitted in a table top size flume that can be operated by one person. The main principle of micro modeling is to replicate sediment transport when relaxing the laws of similitude through individual adjustment of parameters such as slope, flow discharge and sediment input. Robert Davinroy
developed a micro model of a reach of the Mississippi River and it was compared to a larger model of the same reach conducted by the United State Army Waterways Experiment Station (Davinroy, 1994). The results showed good agreement between the micro model and prototype, providing realistic sediment transport simulations and usable velocity distribution data. Micro models follow the principles of the empirical model methodology; however, due to the small size of the model, some variations of the methodology are applied. The method is described below divided into three steps including micro model theory, model appurtenances and calibration.

1. Micro model theory:

1.1. Scale: Geometric scales should not be selected too small that useful measurements of bed and velocities cannot be taken. One recommendation is that the horizontal scales can be the same as available aerial photography of the model. The vertical scale is obtained from testing the model. Its methodology is explained later.

1.2. State of the flow: The state of the flow in the model may be different than in prototype. Even though both model and prototype may exhibit subcritical flow, its Froude ratio is possible to be different than unity. The Reynolds number are quite different, resulting in a turbulent condition in the prototype, whereas it is not necessary true for the model where it can even exhibit a transitional flow condition. In despite of these differences, the methodology lies on the similitude of the state of sediment transport rather than in the state of the flow.

1.3. Sediment material: Lightweight material allows the transport of sediment faster than regular sand, reducing the amount of time required to simulate a yearly material of the prototype. The lighter the material, the shorter the time to complete a simulation. Materials such as coal and plastic are recommended. It is also recommended that to obtain sediment transport as in the prototype, large relative model particles should be used.
2. Model appurtenances:

2.1. Design, materials and construction: A list of elements that models require includes a flume, pumps, water outlet control, alignment insert, sediment input bay, tailbay and drainage reservoir.

2.2. Operation: Three model components are adjusted including discharge, bed material input and slope. Discharge is controlled appropriately until reproducing the required water level stages. Bed material introduced to the model is found by trial and error, placing sufficient material to simulate a year hydrograph. Slope is properly adjusted depending on the sediment transport requirements.

2.3. Data collection and output scheme: It is recommended to utilize high precision devices for measuring cross sectional data point in all x, y and z directions. Contours maps of bed configuration can be obtained from software such as AUTOCAD once the data is collected after simulations.

3. Calibration:

3.1. Geometric scale: The vertical scale is obtained from the model calibration according to following procedure:

(a) Evenly fill the model with sediment up to half of the depth.

(b) Run a number of different steady discharges.

(c) Compare the model bed to the prototype bed at different selected control sections and define the vertical scale. Theoretically, there is one vertical scale that satisfactorily replicates the prototype bed configuration.

3.2. Discharge relation curve: The curve is empirically developed from prototype known discharges and computed average discharges. The curve is a straight line when plotted in semi-logarithmic scale.
3.3. Slope: The slope is empirically adjusted until satisfactory sediment transport is obtained in the model conducting a trial and error process. Extreme distortion of the slope is acceptable as long as the water surface is not being modeled.

3.4. Average annual hydrograph: The hydrograph is developed from average monthly discharges from past known records for a period of time, e.g., 10 years. Utilizing a hydraulic software, the stage levels for average hydrograph are known, and then these levels can be converted into model stage levels, resulting into the model hydrograph.

3.5. Time scale and sediment input: The sediment time scale is empirically found through a trial and error procedure testing different times until appropriate sediment transport occurs. Through experimental procedure it is also found the amount of sediment required to model an annual hydrograph which is added and the beginning of each simulation. Since the sediment time scale is empirically found, this time is not interpreted by any means as the actual time required to accomplish the measured changes in the bed topography.

3.6. Development of starting conditions: Bed topography is not pre-molded but formed through consecutive runs until point bars are developed at bends. In the case that models do not adequately form bed topography, further adjustment of the slope and discharge are required.

2.4 LMRPM Design

The Lower Mississippi River Physical Model (LMRPM), housed at the LSU Center for River Studies on the Baton Rouge, LA Water Campus (Figure 2.1), is a mobile bed physical model comprising the lower 195 miles of the Mississippi River from Donaldsonville through the Head of Passes into the Gulf of Mexico. This model is the continuation of the Small-Scale Physical Model (SSPM), a physical model comprising the lower 60 miles of the Mississippi River as a part of the extensive studies promoted by the Coastal Protection and Restoration
Authority of Louisiana to investigate various management strategies and their effects on flooding, navigation and coastal restoration. The design and construction of the LMRPM was led by Ardurra Group (formerly BCG Engineering & Consulting, Inc.), an engineering consulting group. The LMRPM design philosophy is a combination between rational and empirical methodologies where selection of some of the parameters follow complete similarity such as Froude, critical Shields parameter and critical particle Reynolds number, while empirically calibrating parameters such as sediment time scale. The general design scales are presented below including geometric, dynamic, model sediment, time scales and sediment discharge.
2.4.1 Geometric Scales

Geometric scales were selected based on two main factors including large domain and a minimum model Reynolds number ($R_{em}$) for turbulent conditions, i.e., higher than 2000 for turbulent conditions and 10000 for fully turbulent conditions.

The Reynolds number is defined by the following equation:

$$R_e = \frac{4RV}{\nu}$$

(2.46)

where $R =$ hydraulic radius, $V =$ water velocity and $\nu =$ kinematic water viscosity (BCG Engineering & Consulting, Inc., 2011).

During the model design several geometric scales were proposed including horizontal scale ratios ($x_r = y_r$) 12000, 9000, 6000 that were combined with vertical scale ratios ($z_r$) 600, 500 and 400. From the results in Table (2.3), the selected horizontal and vertical scale were 6000 and 400, respectively. These scales provided an appropriate model size while ensuring high enough Reynolds numbers for the required model discharges.

Table 2.3. Proposed LMRPM geometric scales.

<table>
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<tr>
<th>Discharge (cfs)</th>
<th>$z_r$</th>
<th>$x_r$</th>
<th>Distortion</th>
<th>$R_{ep}(1x10^9)$</th>
<th>$R_{em}$</th>
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</table>

$R_{ep} =$ prototype Reynolds number, $R_{em} =$ model Reynolds number
2.4.2 Dynamic Scales

Two of the most important dynamic scale to model for a mobile bed river model are
the Froude and Reynolds number scales. Simultaneous similitude of both parameters is
impossible to achieve when the river models use the same fluid as in the prototype (i.e.
water). One of the two parameters needs to be relaxed when modeling physical models. It
is recommended that for rivers the Froude similitude is required while relaxing the Reynolds
number similitude (Green, 2014). The LMRPM was designed following the Froude similitude
criterion between model and prototype $F_{rp} = F_{rm}$ while the Reynolds numbers are not equal
$R_{ep} \neq R_{em}$. This relaxation is allowed since turbulent and laminar channels are governed by
the same dynamics using the same dimensionless equations (Graveleau et al., 2011).

The model velocity ratio and water discharge ratio are calculated from the Froude similitude:

$$V_r = \sqrt{z_r} \quad (2.47)$$
$$Q_r = z_r^{3/2} x_r \quad (2.48)$$

2.4.3 Model Sediment

Model sediment was selected based on the similarity of both critical particle Reynolds
number and critical Shields parameter. Setting both parameter ratios to unity, the size of
the model material was scaled as a function of the prototype diameter and the sediment
specific gravity ratio. The derivation of the sediment size ratio is presented as follows (BCG

$$(R_{ec*})_p = \frac{(u_{ec})_p d_p}{\nu}$$

$$(R_{ec*})_m = \frac{(u_{ec})_m d_m}{\nu}$$
Figure 2.2. LMRPM model sediment made of unexpanded polystyrene with a density of 1.05 g m$^{-3}$. Particles ranging from 0.1-1 mm.

\[(R_{ccs})_p = (R_{ccs})_m,\]

\[\frac{(u_{sc})_p}{(u_{sc})_m} = \frac{d_p}{d_m}\]  \hspace{1cm} (2.49)

\[(\tau_{sc})_p = \frac{(u_{sc})_p^2}{g(S_p - 1)d_p}\]

\[(\tau_{sc})_m = \frac{(u_{sc})_m^2}{g(S_m - 1)d_m}\]
\[(\tau_{sc})_p = (\tau_{sc})_m\]

\[\left(\frac{u_{sc}}{u_{sc}}\right)_p^2 = \frac{(S_p - 1)d_p}{(S_m - 1)d_m}\] (2.50)

Combining equations (2.49) and (2.50) results in:

\[d_m = d_p[(S - 1)_r]^{1/3}\] (2.51)

The selected bed material was ground unexpanded polystyrene (Figure 2.2) with a density of 1.05 g m\(^{-3}\) which is a widely used lightweight sediment in physical modeling (Frostick, McLelland, & Mercer, 2011). Setting the specific gravity of the material into equation (2.51) results into the following sediment size equation:

\[d_m = 3.2d_p\] (2.52)

Sediment in the Lower Mississippi River changes seasonally depending on whether the flow is high or low. Based on a survey at a river region right below New Orleans (BCG Engineering & Consulting, Inc., 2015), it was found that the prototype sediment is mainly composed by very fine to fine sand. The model sediment is properly scaled utilizing equation (2.52) and summarized in Table (2.4).

<table>
<thead>
<tr>
<th>Type</th>
<th>(D_{10}) (mm)</th>
<th>(D_{50}) (mm)</th>
<th>(D_{90}) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prototype (Mississippi)</td>
<td>0.08</td>
<td>0.12-0.14</td>
<td>0.25</td>
</tr>
<tr>
<td>Model</td>
<td>0.25</td>
<td>0.40-0.45</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 2.4. Design characteristic sediment sizes for both prototype and model.
2.4.4 Time Scales

The two main model time scales are the hydraulic time scale ($T_r$) and the sediment time scale ($T_s$). The hydraulic time scale ratio is the scale that relates the fluid transport and it is based on the Froude similitude:

$$T_r = \frac{x_r}{\sqrt{z_r}} \quad (2.53)$$

The sediment time scale is the scale that relates the sediment transport and it varies depending on the direction of interest (Peakall, Ashworth, & Best, 1996). The model is run with the sediment time scale. This time was calculated based on previous studies conducted by SOGREAH Consultants, a French company specialized in physical models of rivers. SOGREAH Consultants (2004) proposed a sediment time scale based on a similarity law that resulted from a study of the Seine Estuary as a function of the hydraulic time scale and the sediment specific gravity ratio:

$$T_{sr} = T_r (\gamma_s - 1)_r \quad (2.54)$$

The model time scale ratios are calculated from equations (2.53) and (2.54) resulting in $T_r = 300$ and $T_{sr} = 9900$, respectively. According to Allen Teeter (BCG Engineering & Consulting, Inc., 2015) the model time scale is empirically found through model testing, varying from the hydraulic time scale to the sediment time scale suggested by SOGREAH Consultants.

2.4.5 Sediment Discharge

The upstream boundary for the LMRPM is located at Donaldsonville at RM 175.4. Observed sand concentration values were not found at this specific location. A computer model was developed by Mike Trawle and analyzed by William A. Thomas, utilizing the computer
program HEC-6T, to estimate very fine, fine, medium and coarse sand concentration values at Donaldsonville (Thomas, 2014). The model reach was defined from Tarbert Landing at RM 306.3 to the Mouth of Southwest Pass at RM -18 below Head of Passes. Tarbert Landing was chosen because it was the region of the Lower Mississippi River where reliable sediment data was found.

In order to verify the model results, sediment concentration measurements at Belle Chasse (RM 76) were calculated and compared with USGS measured suspended sediment concentration at the same location. Good agreement was found from model estimations which resulted in five polynomial regression equations for four different sands at Donaldsonville classified by their diameter (d):

very fine sand (0.062\(\text{mm} > d > 0.125\text{mm}\)) for \(Q_w < 1,400,000\text{cfs}\),

\[
C_{vfs} = 1.09795e^{-27}Q^5 - 4.85532e^{-21}Q^4 + 8.08841e^{-15}Q^3 - 6.19504e^{-9}Q^2 + 2.15307e^{-3}Q - 2.48510e^2 \tag{2.55}
\]

very fine sand (0.062\(\text{mm} > d > 0.125\text{mm}\)) for \(Q_w > 1,400,000\text{cfs}\),

\[
C_{vfs} = 2.95178e^{-28}Q^5 - 1.49439e^{-21}Q^4 + 2.83220e^{-15}Q^3 - 2.41405e^{-9}Q^2 + 9.21024e^{-4}Q - 1.06620e^2 \tag{2.56}
\]

fine sand (0.125\(\text{mm} > d > 0.250\text{mm}\)),

\[
C_{fs} = 9.71804e^{-17}Q^3 - 4.94401e^{-11}Q^2 + 3.53669e^{-5}Q - 6.85551 \tag{2.57}
\]
medium sand \((0.250 mm > d > 0.500 mm)\),

\[ C_{ms} = 1.124977e^{-17}Q^3 - 1.182845e^{-11}Q^2 + 1.063810e^{-5}Q - 1.462548 \] (2.58)

crude sand \((0.500 mm > d > 1.000 mm)\),

\[ C_{cs} = 1.24007e^{-13}Q^2 - 4.070498e^{-8}Q + 4.646504e^{-3} \] (2.59)

where \(C = \) concentration in \(mg l^{-1}\) and \(Q = \) water discharge in \(cfs\).

Estimation of the model sediment inputs was based on:

\[ q_{bvr} = \frac{x_r z_r}{t_{sr} C_{vbr}} \] (2.60)

where \(q_{bvr} = \) volumetric transport rate per unit width scale, \(x_r = \) horizontal scale, \(z_r = \) vertical scale, \(t_{sr} = \) sediment time scale and \(C_{vbr} = \) volume concentration of sediment in the bed scale. The prototype total volumetric transport rates \((Q_{bvp})\) are calculated utilizing the total concentration summing all five regression concentration equations (2.55 to 2.59). The total volumetric transport rate scale \((Q_{bvr})\) is calculated by multiplying the volumetric transport rate per unit width scale \((q_{bvr})\) by the horizontal scale \((x_r)\) which is used then to calculate the model total volumetric transport rate \((Q_{bvm})\).

2.4.6 LMRPM Under Different Rational Design Methodologies.

The LMRPM was evaluated utilizing the five rational methodologies: Julien, Yalin, Tsujimoto, Chanson and Prasuhn. These more rigorous methodologies allow selection of some of the scales while calculating the remaining scales. All calculations are presented in appendix A. The results are summarized in Table (2.5).

1. Geometric scales: The geometric scales, in most of the methodologies, are designed choosing one of either the horizontal or the vertical scales. For comparison purposes,
Table 2.5. LMRPM design utilizing different rational methodologies. All ratio scales are defined as prototype to model.

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Julien</th>
<th>Yalin</th>
<th>Tsujimoto</th>
<th>Chanson</th>
<th>Prasuhn</th>
<th>LMRPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>z&lt;sub&gt;r&lt;/sub&gt;</td>
<td>x&lt;sub&gt;r&lt;/sub&gt;</td>
<td>d&lt;sub&gt;s&lt;/sub&gt;r</td>
<td>T&lt;sub&gt;r&lt;/sub&gt;</td>
<td>T&lt;sub&gt;s&lt;/sub&gt;r</td>
<td>V&lt;sub&gt;r&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>1264</td>
<td>0.089</td>
<td>63</td>
<td>1855</td>
<td>20</td>
</tr>
</tbody>
</table>

The vertical scale was fixed to be the same as the LMRPM, i.e., \(z_r = 400\) for all methodologies but Prasuhn, where both geometric scales resulted from the selection of the same material as the LMRPM. Tsujimoto and Chanson allowed the selection of the horizontal scale, whereas the others resulted from the set of equations from each methodology. When the horizontal scale was calculated, it is seen that large distortion is not allowed. Even though geometric distortion should be kept as low as possible, models with a small distortion are rather difficult to be built when the model represents a large domain such as in the LMRPM.

2. Model sediment: Four of the methodologies utilized the same type of lightweight sediment used in the LMRPM, i.e., ground unexpanded polystyrene with a density of \(1.05 \text{g m}^{-3}\). It was only Julien’s methodology that the material was a result of the equations. Each methodology presented different equations for calculating the model diameter sizes but Prasuhn, where the same equation was derived as the LMRPM to calculate the diameter sizes. It is seen that four of the methodologies suggested increasing the particle diameters comparing to prototype’s particles. It was only Yalin that suggested the opposite, i.e., decreasing the particle diameters. This resulted in
model particle 238 times smaller than their similar in the prototype which may suggest such small particles that cohesion effects may appear during model simulation.

3. Time scales: The two scales modeled were hydraulic time scale and sediment time scale. The hydraulic time scale for all the five methodologies was calculated from the Froude similitude. The sediment time scale ratios vary considerably from one methodology to the other. Yalin, Chanson and Prasuhn suggested that this parameter is the ratio of the geometric scales to the sediment discharge per unit width. The remaining methods presented their own equations. Four of the authors, except Julien, suggested sediment time scale ratios larger than 100000 which is impractical in reality since this would mean that a prototype year is run in few minutes. These results may be biased from practical scales due to the large geometric scales of the model; however, empirical calibration of this parameter may be needed as suggested from the empirical methodologies.

4. Sediment discharge: There is no good agreement for the calculation of this scale. Some of the methodologies suggested that the sediment discharge ratio should be equal to unity including Julien and Prasuhn, whereas Tsujimoto and Chanson suggested a lower model sediment discharge when compared with prototype sediment discharge. It was only Yalin that suggested the opposite, i.e., a larger model sediment discharge when compare to its counterpart in the prototype.

5. Froude, Shields parameter and particle Reynolds number similitude: There is good agreement that Froude similitude is suggested when modeling rivers with mobile bed. Most of the methodologies recommended Shields parameter similitude except Yalin. This is because Yalin based his methodology the similitude of the ratio of the mobility number to the critical mobility number, rather than similitude of the critical mobility numbers. In terms of the the particle Reynolds number, relaxation of this similitude is allowed in three of the methodologies including Yalin, Tsujiimoto and Chanson, the
remaining methodologies must keep similitude for this parameter.

2.5 References


Chapter 3. Incipient Motion and Subaqueous Dune Formation of a Lightweight Physical Model Sediment of the Lower Mississippi River

3.1 Introduction

Incipient motion of stream beds is a fundamental process widely utilized to investigate river engineering problems such as in canal design (Lane, 1955), bed protection near weirs and spillways (Vollmer & Kleinhans, 2007), river restoration and habitat (Buffington, Montgomery, & Greenberg, 2004) and bed load prediction (Recking, 2009). In a river channel with a movable bed, a threshold or incipient motion condition is established when the flow intensity is barely enough to move the particles in the bed (Kanellopoulos, 1998). Initial particle movement occurs when the attacking forces overcome the resisting forces in the bed (Vollmer & Kleinhans, 2007). This balance between forces are translated into shear stresses that exceed the critical shear stress of the bed, allowing particle movement initiation. The Shields parameter \( \tau^* \) is commonly used in river engineering practices (Julien, 2002) for estimating incipient motion:

\[
\tau^* = \frac{\tau}{(\gamma_s - \gamma)d_s},
\]

where \( \tau \) = near bed shear stress, \( \gamma_s \) = specific weight of sediment \( \gamma \) = specific weight of water, \( d_s \) = particle size.

For river physical models with mobile beds, similitude in Shields parameter ensures similitude in sediment transport (Ho, Coonrod, Gill, & Mefford, 2010; Julien, 2002; Pugh & Dodge, 1991). In the case of the LMRPM, the critical Shields parameter \( (\tau^*_c) \) was utilized to scale the model sediment diameters. The critical Shields parameter is the value at which particles initiate mobilization at the bed surface. Proper replication of the incipient motion condition validates that model sediment diameter sizes are scaled correctly, and also that the model is mobilizing sediment at the correct scaled hydraulic condition.

Subaqueous dunes play an important role in rivers where the bed is composed of loose material such as sand. Once dunes have formed, they produce an additional form resistance
that is larger than the grain resistance of the bed material (van Rijn, 1984). Form resistance in river beds that are composed of dunes is related to the turbulence occurring at the lee sides of the dunes which further determines the sizes and shapes of the dunes (Wilbers, 2004). This dynamic process modifies the hydrodynamics of rivers, promoting turbulence and enhancing bed load transport.

River physical models are required to reproduce bed morphology to some extent depending on the objective of the model (Franco, 1978). In the case of the Lower Mississippi River, its bed develops dunes during both low and high flow discharge (Nittrouer, Mohrig, & Allison, 2011). El Kheiashy, Mccorquodale, Georgiou, and Meselhe (2007) analyzed a bathymetric survey collected by the Army Corps of Engineers in the Lower Mississippi River from June 2003 to August 2003 for a range of water discharges varying from 7141 to 28 764 m$^3$s$^{-1}$, where mean dunes lengths and mean dunes height were measured to be $108.10 \pm 112.29$ m and $2.35 \pm 1.78$ m, respectively. Replication of these bed features is important for modeling this reach of the river where fluvial sand fluxes and depositions patterns are responsible for shaping the complex river bed (Nittrouer, Allison, & Campanella, 2008), parameters to consider when studying problems such as river sediment diversions and dredging requirements for navigation.

3.1.1 The Lower Mississippi River Physical Model

The LMRPM, housed at the LSU Center for River Studies on the Baton Rouge, LA Water Campus, is a distorted, movable bed model comprising the lower 195 miles of the Mississippi River from Donaldsonville through the Head of Passes into the Gulf of Mexico (Figure 3.1). The model replicates the prototype hydraulics (i.e., flow and river stages) and, through the use of a lightweight sediment, bulk non-cohesive sediment transport. Understanding how well the LMRPM lightweight sediment replicates near bed processes is important for validation and for demonstrating how the model can be used to study how the Lower Mississippi River will respond to future natural and man made (or anthropogenic) processes and features and
Figure 3.1. Lower Mississippi River Physical Model domain. Flow is from top to bottom. Upper boundary is Donaldsonville, LA (top-left) and the lower boundary is the Gulf of Mexico (bottom-right).
the impact on coastal restoration, navigation and flooding.

The LMRPM geometric scales are horizontal 1:6000 and vertical 1:400, thus having a distortion of 15. These scales were determined using an iterative process, based on the domain to be modeled, physical laboratory space, and the objectives of the model experiments. The model mean flow is designed to maintain Froude number similarity between the prototype and the model, while the flow Reynolds number is relaxed, but high enough that the model operates under the turbulent regime. From Froude similitude the velocity and discharge scales are set to 1:20 and 1:48,000,000.

Rational and empirical methods are two approaches when designing mobile bed river physical models (Graf, 1984). The rational method emphasizes the dimensionless similitude criteria between model and prototype while keeping model dimensional distortion as low as possible (Maynord, 2006). The empirical method focuses on replicating the bed movement of the model as in the prototype while relaxing similitude criteria (Stevens, 1942; Warnock, 1950). The LMRPM falls within those two methods because it was designed to follow three dimensionless similitude criteria: Froude, critical Shields parameter and critical particle Reynolds number, while relaxing the Reynolds number criterion. In addition, its distortion is higher than recommended in rational methods (Julien, 2002; Shen, 2012).

Lightweight sediment, such as walnut shells, silica sand, cation resin, coal and polystyrene has been widely used in movable bed river physical models for replicating bed behavior (Kantoush, Bollaert, Boillat, Schleiss, & Uijttewaal, 2006; Shaw, Makhlouf, Walter, & Parlange, 2008). These materials have been proven to be non-cohesive materials, able to replicate entrainment conditions from the bed (Sequeiros, Naruse, Endo, Garcia, & Parker, 2009) in a much quicker time when compared with natural sand, without compromising the Froude scale (Vermeulen et al., 2014). LMRPM sediment properties were chosen through model and prototype similitude in critical Shields parameter and critical particle Reynolds number, along with a river sediment density of 2.65 g cm$^{-3}$ and a model sediment density of 1.05 g cm$^{-3}$. This approach resulted in model particle diameters 3.2 times larger than those
found in the prototype. The sediment used is a fine synthetic sediment made of unexpanded polystyrene with diameters that vary from 0.10 to 1.00 mm with a mean particle diameter $d_{50} = 0.35$ mm (Figure 3.2).

Since the LMRPM was designed to replicate the hydraulics (i.e., flow and river stages) and bulk non-cohesive sediment transport, the model sediment must replicate both the incipient motion and two-dimensional dunes characteristics (height and length). Understanding how the LMRPM lightweight sediment behaves under different hydraulic conditions is a critical step in quantifying how useful the model will be in studying the impact of future anthropogenic and natural processes on coastal restoration, flood risk and navigation. Due to the challenge in quantitatively studying the model sediment processes in the physical model, a series of flume experiments were conducted in which the incipient motion and dune formation and migration could be observed and quantified, and then compared to the Mississippi River prototype sand transport processes.
3.2 Experimental Setup

The experiments were run using an Armfield S6 MKII flume, 0.30 m wide, 0.45 m deep and 5 m long. Scaling conditions imposed modifications in the flume (Figure 3.3). A plexiglass wall was used to reduce the flume width to 13 cm to replicate typical river widths (600 m to 1000 m) and a downstream weir was set to maintain a constant water depth of 6 cm, in order to mimic typical river depths (22 m to 25 m) (Nittrouer, Shaw, Lamb, & Mohrig, 2012). At the upstream region of the flume, nine 4 cm diameter circular tubes were set to align the flow. These tubes also promoted sufficient initial turbulence to pick up sediment from the bed to form defects once the mean flow velocity was high enough to mobilize particles. Both flume and bed slopes were set horizontal. Prior to each experiment, the bed was screeded flat with a 2 cm thickness in order to allow dunes to develop while minimizing scouring pools. For all runs, water was recirculated and additional sediment was not added since the total volume of sediment at the bed compared with the mobilized volume was large enough to be considered an inexhaustible supply (Shvidchenko & Pender, 2000) and experiments were
stopped once scour holes appeared (Baptist, 2003).

The flume width/depth ratio (B/h) was greater than 2 and the depth/particle size ratio (h/D) was greater than 67. These ratios were sufficient to ensure two-dimensional dunes (Wren, Kuhnle, & Wilson, 2007) since the walls did not distort the crest of the dunes and no lateral movement of dunes was observed (Abraham, Kuhnle, & Odgaard, 2011). Bed sediment levels were measured with a millimeter-graduated ruler from the bottom of the flume through one of the transparent lateral walls of the flume. The accuracy of all measurements made using the millimeter-graduated ruler was ±2 mm.

A 16 MHz SonTek acoustic Doppler velocimeter (ADV) was used to take instantaneous velocity measurements at a rate of 25 Hz for 60 seconds and at a distance from the bed surface ranging from 0.5 to 1 cm. Analysis of the ADV measurements showed that 93 percent of the signal-to-noise ratio (SNR) values were higher than twenty, thus ensuring good data quality (Gordon & Oltman, 2000). Average correlation values for all the three experiments were found to be higher than 70 percent, which was considered satisfactory (Sontek, 2001).

The near bed shear stresses were determined utilizing the turbulent kinetic energy (TKE) method. The bed shear stress is proportional to the TKE density (Soulsby, 1983; Stapleton & Huntley, 1995), following:

$$\tau = 0.19 TKE.$$  \hspace{1cm} (3.2)

Three-dimensional instantaneous velocity measurements collected from the ADV can be separated into two parts: the mean velocity ($\overline{U}$, $\overline{V}$ and $\overline{W}$) and the fluctuations ($u'$, $v'$ and $w'$). The TKE is the sum of the variance of all the three-dimensional components (Pope, Widdows, & Brinsley, 2006) and defined as:

$$TKE = \frac{1}{2} \rho (u'^2 + v'^2 + w'^2).$$  \hspace{1cm} (3.3)

The experiments were run from low mean flow velocities, where no particles moved, to high enough mean velocity to produce a change in the bed. For all experiments, the
Table 3.1. Observations for the three experiments.

<table>
<thead>
<tr>
<th>Vm (cm s(^{-1}))</th>
<th>Vp (cm s(^{-1}))</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><strong>Experiment # 1: t = 180 min, T = 540 min</strong></td>
</tr>
<tr>
<td>4.0</td>
<td>80</td>
<td>No sediment movement.</td>
</tr>
<tr>
<td>4.6</td>
<td>92</td>
<td>Sediment movement without forming defects.</td>
</tr>
<tr>
<td>6.4</td>
<td>128</td>
<td>Defects were observed, later forming a total of four dunes. A scour hole formed by the end of the test.</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Experiment # 2: t = 60 min, T = 240 min</strong></td>
</tr>
<tr>
<td>4.5</td>
<td>90</td>
<td>No sediment movement.</td>
</tr>
<tr>
<td>5.4</td>
<td>108</td>
<td>Few particles moved the first 10 minutes, then the bed became stable.</td>
</tr>
<tr>
<td>5.9</td>
<td>118</td>
<td>A thin layer of particles constantly moved, forming defects all along the bed.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>After the first 30 minutes, only upstream defects remained, and then the bed became stable.</td>
</tr>
<tr>
<td>7.4</td>
<td>148</td>
<td>A total of four dunes formed moving downstream without splitting into new dunes. No scour holes were observed.</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Experiment # 3: t = 30 min, T = 120 min</strong></td>
</tr>
<tr>
<td>4.5</td>
<td>90</td>
<td>No sediment movement.</td>
</tr>
<tr>
<td>4.9</td>
<td>98</td>
<td>Few particles moved the first 15 minutes, then the bed became stable.</td>
</tr>
<tr>
<td>6.4</td>
<td>128</td>
<td>Defects were formed all along the bed. Bed became stable during the last 20 minutes.</td>
</tr>
<tr>
<td>6.9</td>
<td>138</td>
<td>Four dunes formed upstream, two of them reached downstream by the end of the run. No scour holes were observed.</td>
</tr>
</tbody>
</table>

Vm = model mean flow velocity, Vp = prototype mean flow velocity, t = time interval, T = time duration

Discharges were kept constant at each time step and then increased until a new bed condition was observed. For experiment # 1 the duration was 540 min with 3 time steps of 180 min. Experiment # 2 duration was 240 min with 4 time steps of 60 min. Experiment # 3 was the shortest with time duration of 120 min with 4 time steps of 30 min.

### 3.3 Results

Four conditions were defined to classify the bed responses (Table 3.1): no motion, when no or very few particles move in some spots of the bed; incipient motion, as when few particles start to continuously move along the bed; general motion, when a thin layer of sediment continuously moves along the bed; and dune formation, when sediment accumulates...
and forms asymmetrical dunes with steep lee sides and gentle stoss sides. No sediment movement was observed for model mean flow velocity lower than 4.5 cm s$^{-1}$. Incipient motion occurred when the mean flow velocity was between 4.5 cm s$^{-1}$ to 5.4 cm s$^{-1}$. More pronounced movement or general motion of particles occurred when the mean flow velocity was between 5.4 cm s$^{-1}$ to 6.4 cm s$^{-1}$. Dunes were developed at a mean flow velocity higher than 6.4 cm s$^{-1}$ (Figure 3.4).

Dunes formed after 20 minutes in experiment # 1 and after 10 minutes in experiments # 2 and # 3, which suggests that the bed, comprised of lightweight sediment, responds rather quickly to changes in mean flow velocity. Similar flume experiments utilizing natural sand, with bedform developments that include dunes, may require several hours to develop these features (Venditti, Church, & Bennett, 2005; Wren et al., 2007). It was observed for all the three experiments that once dunes formed they were similar in heights, lengths and...
Figure 3.5. Near bed shear stresses calculated from the TKE method utilizing the ADV collected velocity fluctuations for all three experiments.

steepness and that the time at which they developed was within a similar time frame, e.g., 10-20 min. Therefore, it is acceptable that the data collected during all three experiments can be combined to obtain a better understanding of the bed characteristics.

3.3.1 Near Bed Shear Stresses

Instantaneous velocity measurements were made in the longitudinal direction of the flume to estimate near bed shear stresses. A total of seventy six ADV measurements were conducted for the three experiments. The measured average near bed shear stresses and their standard deviations (Figure 3.5) were $\tau = 0.0069 \pm 0.0011$ Pa for no motion, $\tau = 0.0088 \pm 0.0022$ Pa for incipient motion, $\tau = 0.0139 \pm 0.0023$ Pa for general motion and $\tau = 0.0152 \pm 0.0044$ Pa for dune formation. Along the dunes, the near bed shear stresses varied depending on the measurement location, with higher values on the lee side of the dunes when compared with the crests and stoss sides. The values on the lee side ranged from 0.0172 to 0.1190 Pa, resulting in values up to seven-fold the average value found in the dunes crests and stoss sides.
Shields parameter is utilized for identifying initiation of non-cohesive sediment motion in flows when the non-dimensional bed shear stress is larger than a threshold (Frostick, McLelland, & Mercer, 2011). Modified Shields diagrams have been developed to relate the Shields parameter in terms of a non-dimensional particle diameter rather than the original particle Reynolds number dependency. The calculated incipient motion criterion is defined by the analytical form (Parker, Toro-Escobar, Ramey, & Beck, 2003):

\[
\tau_c^* = \frac{1}{2} \left( 0.22 R_p^{-0.6} + 0.06 \times 10^{-7.7 R_p^{0.6}} \right),
\]

(3.4)

\[
R_p = \sqrt{\frac{\left( \frac{\rho_s}{\rho} - 1 \right) g d_{50} d_{50}}{\nu}},
\]

(3.5)

where \( \rho_s \) = sediment density, \( \rho \) = water density, \( g \) = gravitational acceleration, \( d_{50} \) = median particle diameter and \( \nu \) = kinematic viscosity. Calculations using (3.4) and (3.5) and the lightweight model sediment resulted in incipient motion criterion equals to 0.0441. A plot of all of the Shields parameters, calculated from the average shear stresses, versus mean flow velocity (Figure 3.6) highlights two main points. First, the Shields parameters for no motion and incipient motion are below and just above the critical Shields parameter and second, as expected, the Shields parameters increase for the general motion and dune formation conditions.

### 3.3.2 Dunes Development

A total of 12 dunes were observed among the three experiments. Each dune was measured several times as they migrated downstream at different time intervals to collect sufficient data. Dunes initiation started from defects during general motion condition. These defects generally had heights lower than 0.5 cm and lengths lower than 10 cm. Rather than well developed dune shapes, defects appeared as small sediment accumulations (Figure 3.7) that later developed into dunes once the mean flow velocity and bed shear stresses were high.
Figure 3.6. Shields parameters calculated from the near bed shear stresses combining all three experiment data. Average values with one standard deviation. Dash line is the critical Shields parameter from Parker et al. (2003).

enough to initiate the bed erosion and deposition process.

Dunes characteristics from the experiments are shown in Figure 3.8. Dunes length (L) measured during the three experiments showed values ranging from 10 to 50 cm. Measurements showed that dunes lengths were not directly related to the experiments time steps. Although it was observed that the majority of dunes length were between 10 and 30 cm, longer time steps such as experiments # 1 and # 2 allowed formation of longer dunes lengths (40 to 50 cm), whereas in experiment # 3 these longer dunes lengths were not observed. Dunes height (H) among the three experiments ranged from 0.5 to 1.6 cm. Dunes height measurements showed variability among the three experiments suggesting no direct dependency with the time steps. Most of the dunes heights were observed between 0.5 to 0.9 cm, except for experiment # 3 where the majority of dunes ranged from 0.7 to 1.3 cm.

The Dunes steepness (H/L) parameter was calculated from the dune lengths and heights. All the three experiments were consistent in dunes steepness (H/L), with a majority in the range of 0.025 to 0.040 cm cm\(^{-1}\). Calculated steepness values in terms of degrees suggest low
Figure 3.7. Defect observed during the experiments: Small amount of sediment accumulates to develop a dune.
Figure 3.8. Dunes lengths and heights collected from all three experiments. Dunes steepness values were calculated from dunes lengths and heights measurements.

Stoss slopes angles ranging from 0.60 to 4°. For the three experiments, the lee slopes angles ranged from 45 to 90°, suggesting steeper lee slopes than typical asymmetrical sand dunes found in nature 25-35° (Best, Kostaschuk, & Hardy, 2004). Lee slopes are related to the angle of repose of the particles (Hendershot et al., 2016). The angle of repose for the model sediment was found to be about 60° (BCG Engineering & Consulting, Inc., 2015) which lies within the observed range in all the three experiments.

Dune properties from all three experiments were combined and plotted to identify the major features developed in this type of lightweight sediment (Figure 3.9). Data suggested that the typical dunes length measurements were between 10 to 20 cm, dunes height between 0.70 to 0.90 cm and dunes steepness between 0.025 to 0.040 cm cm⁻¹.

3.4 Discussion

Experimental results showed that the lightweight model particles started to move (i.e., incipient motion) when the Shields parameter was $\tau_{c,exp}^* = 0.0511 \pm 0.0127$. There is good agreement between measured values when compared with the calculated theoretical value of
Figure 3.9. Dunes characteristics grouped from all the three experiments.

$\tau_c^* = 0.0441$. It is important to note that the measured values also fall within the range of theoretical Shields parameter values (Julien, 2002) for modeled (prototype) river sands. The theoretical Shields parameter for very fine (0.062-0.125 mm) and fine sands (0.125-0.250 mm) vary from $\tau_c^* = 0.0545-0.1099$ and from $\tau_c^* = 0.0356-0.0712$, respectively, confirming that the model sediment is in scale similitude for the critical Shields parameter criterion.

It was observed that dunes developed from defects. The change in flow over the defects created a separation zone in front of the defects that promoted local scour further developing the lee side of the dunes. Dunes over the three experiments developed quickly, within minutes, once the mean flow velocity was higher than 6.4 cm s$^{-1}$. Experiment # 1 developed well-formed dunes during the first 20 minutes, whereas experiment # 2 and # 3 within 10 minutes. The experiments that had longer time durations allowed erosion of defects from the bed which ultimately returned to planar when the mean flow velocity was lower than 6.4 cm s$^{-1}$. A shorter time duration allowed defects to form and remain, promoting dune formation from these defects. This confirms that changes in lightweight sediment model bed morphology occur within 10-20 min rather than hours, as in actual sand material.

Data collected by the Army Corps of Engineer in the Lower Mississippi River and ana-
Figure 3.10. Dunes characteristics from all three experiments in terms of prototype scale. From top to bottom: height versus length, height versus water depth and length versus water depth.

Dunes data has been collected by different researchers who have proposed mathematical relationships to characterized how lengths and heights are related, and how these characteristics are related to water depth. Flemming (2000) analyzed data collected from a variety of experiments and compared the results with those observed in the prototype. The analysis showed good agreement between the model and prototype dune parameters. Dunes lengths observed in the experiments are comparable with those in the prototype where the majority of the observations were about 36.50 m. The majority of the dunes height observations are slightly larger than those observed in the prototype where experiments values peaked at 3.16 m, whereas prototype values peaked at 2.00 m. This difference was considered satisfactory because of the experiments measurement accuracy which in terms of the prototype is ±0.80 m. Larger dunes height also produced larger steepness values. Experiments values showed that the majority of the steepness values are 0.03825 cm cm⁻¹, whereas prototype values were at 0.01666 m m⁻¹. This difference is influenced by the slightly higher heights measured during the experiments.
Table 3.2. Frequency of collected data in terms of prototype values. Prototype values from El Kheiashy et al. (2007). fp is prototype frequency and fexp is experimental frequency.

<table>
<thead>
<tr>
<th>Range</th>
<th>fp</th>
<th>fexp</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dunes length (m)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-130.03</td>
<td>254</td>
<td>39</td>
<td>Prototype and model showed high frequency in lengths lower than 251.08 m.</td>
</tr>
<tr>
<td>130.03-251.08</td>
<td>63</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>251.08-372.14</td>
<td>17</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>372.14-493.19</td>
<td>8</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>&gt;493.19</td>
<td>4</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Dunes height (m)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-1.42</td>
<td>107</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>1.42-2.58</td>
<td>142</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2.58-3.74</td>
<td>53</td>
<td>21</td>
<td>Prototype high frequency was observed for heights lower than 2.58 m, whereas in the model high frequency was found in the ranges between 2.58 to 4.89 m.</td>
</tr>
<tr>
<td>3.74-4.89</td>
<td>16</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>4.89-6.05</td>
<td>9</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6.05-7.21</td>
<td>7</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>&gt;7.21</td>
<td>11</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Dunes steepness (m/m)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-0.01164</td>
<td>38</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0.01164-0.02230</td>
<td>111</td>
<td>1</td>
<td>Prototype highest frequency was observed for the ranges between 0.01164 to 0.03293 m/m, whereas in the model high frequency was observed for a wider range between 0.02230 to 0.05421 m/m.</td>
</tr>
<tr>
<td>0.02230-0.03293</td>
<td>84</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>0.03293-0.04357</td>
<td>49</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>0.04357-0.05421</td>
<td>40</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>0.05421-0.06845</td>
<td>18</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>0.06485-0.07550</td>
<td>7</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>&gt;0.07550</td>
<td>11</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
of natural environments and flume experiments where dunes were developed. The analysis of dunes lengths and heights resulted in a linear log/log relationship \( H_{\text{mean}} = 0.0677L^{0.8098} \) with an upper limit relationship \( H_{\text{max}} = 0.16L^{0.84} \). Allen (1968) also proposed a similar relationship from field data \( H_{\text{mean}} = 0.074L^{0.77} \). Jackson (1976) and Yalin et al. (1992) found relationships to predict dunes characteristics in terms of water depth. Experimental dunes lengths and heights were upscaled to prototype conditions utilizing the 1:400 vertical scale for both characteristics since the dunes length is depth dependent. The experimental dunes characteristics fit within the range of those found in nature when scaled to prototype dimensions both when (Figure 3.10).

### 3.5 Conclusions

Near bed shear stresses over a bed containing model sediment were calculated over a wide range of mean flow velocities. Stresses, in terms of Shields parameter, and the resultant motion were well described by comparison to the critical Shields parameter (\( \tau_* < 0.0441 \)): the average values (\( \tau_* < 0.0445 \)) were below the critical value for no motion condition; within the curve (0.0445 < \( \tau_* < 0.0667 \)) for incipient motion to general motion condition; and above the curve (\( \tau_* > 0.0821 \)) for dune formation condition, which suggests that overall the local turbulence generated during each stage is consistent with the theoretical approaches for bed evolution in rivers with sand.

Fully developed dunes were formed during all the three experiments with sizes and shapes similar to what is found in the Mississippi River and reported by El Kheiashy et al. (2007). Dunes length ranged from 10 to 50 cm which translates into prototypes dunes of 40.00 to 200.00 m, whereas measured data has found that dunes in the Lower Mississippi River ranges from 8.98 to 1227.44 m. Despite the wider range is the data found in the river, both model and prototype showed that the majority of the dunes length are found to be closed to 69.50 m. Dunes heights showed a slight difference between model and prototype measurements. Model dunes height ranged from 0.5 to 1.6 cm which converted in prototype scale results in dunes
height from 2.0 to 6.4 m, whereas measured data ranged from 0.84 to 10.98 m. The majority of the model dunes height measurements were close to 3.16 m, whereas measured heights were at 2.0 m. This difference was considered satisfactory due to the model measurement accuracy of ±2 mm (0.80 m in prototype scale). Larger model dunes height resulted in larger model steepness values. Model steepness ranged from 0.0100 to 0.0700 cm cm$^{-1}$, whereas prototype steepness ranged from 0.0063 to 0.1021 m m$^{-1}$. The majority of the model steepness were found to be larger than 0.0382 cm cm$^{-1}$, whereas prototype steepness were found smaller than the value which suggests than in the model it is likely to find steeper dunes than those in the prototype due to the utilized lightweight material.

It was found that dune formation with lightweight sediments can occur very quickly when the mean flow velocity (and corresponding bed shear stresses) is increased. Experimental dunes developed 10-20 min after the initiation of general motion. Further evaluation of shorter time intervals (less than 30 min used in this study) are suggested to investigate whether or not fully developed dunes can form under such hydraulic conditions.

3.6 References


Flemming, B. (2000). The role of grain size, water depth and flow velocity as scaling factors controlling the size of subaqueous dunes. In *Marine sandwave dynamics, international workshop* (pp. 23–24).


Chapter 4. Determination of the Sediment Time Scale for a Distorted, Mobile Bed River Physical Model of the Lower Mississippi River

4.1 Introduction

One of the important parameters in river modeling is time scale. Time scale is defined as the time step involved in the variation of flow quantities such as, sediment concentration, discharge, bed changes (Cao, Li, & Yue, 2007; Kuai & Tsai, 2012). Understanding time scale is essential in river modeling as different parameters exhibit different time scales. For example, within a river flow, discharge occurs at a different time scale from sediment transport, and within the sediment transport, erosion-accretion time scale is different from horizontal displacement of granular material (Yalin, 1971).

In order to achieve similitude between model (scaled phenomenon) and prototype (actual phenomenon), proper time scale ratio should be used. Incorrect time scale may lead to inappropriate modeling, compromising the model capacity to replicate prototype conditions. It is important to understand that flow time scale is different from sediment time scales (Henderson, 1966). Flow time scale, or also named hydraulic time scale, is the time that characterizes water flow changes such as in a flood event, whereas the sediment time scales include the bed morphology changes. The hydraulic time scale ratio is based on the Froude similitude:

\[ T_r = \frac{x_r}{\sqrt{z_r}} \]  \hspace{1cm} (4.1)

For mobile bed river physical models, sediment time scale should be used (Graf, 1971; Prasuhn, 1987). Different authors have proposed approaches on how to calculate sediment time scale for mobile bed river physical models with low model distortion. Yalin (1971) developed different formulations for sediment time scale depending on the analyzed bed direction based on the principle that the characteristic length which drives dynamic similarity near bed is the
characteristic grain diameter ratio. Julien (2002) developed his formulations based on the type of similitude achieved between model and prototype, either complete (Froude, dimensionless grain diameter, Shields parameter and resistance ratios) or incomplete similitude (quasi or Non-Froudian similitude). Zwamborn, J. A. (1966) stated that when data is not suitable to be collected, the sediment time scale ratio could be approximated as ten times the hydraulic time scale ratio based on observations at different river physical models.

On the other hand, high distorted models results in sediment time scales that have to be empirically calculated (Lee & Liu, 2004). Henderson (1966) proposed an empirical method based on running a known hydrograph through the model, changing slope and sediment time scale until the mobile model bed configuration fits the prototype's conditions for the same hydrograph. Waldron (2008) reported that the LSU's Small Scale Physical Model (SSPM) of a reach of the Mississippi River used an approach developed by SOGREAH, a French company specializing in physical models, where they proposed an equation based on the similarity law investigated with a model of the Seine Estuary (SOGREAH Consultants, 2004):

$$T_{sr} = T_r (S_s - 1)_r$$  \hspace{1cm} (4.2)

where $S_s = $ sediment specific gravity.

The Lower Mississippi River Physical Model (LMRPM), housed at the LSU Center for River Studies on the Baton Rouge, LA Water Campus (Figure 4.1), is a distorted, movable bed model comprising the lower 195 miles of the Mississippi River from Donaldsonville through the Head of Passes into the Gulf of Mexico. The model utilizes a lightweight sediment to replicate prototype hydraulics (i.e., flow and river stages) and bulk non-cohesive sediment transport. The dimensional scales were set to horizontal $x_r = 6000$ and vertical $z_r = 400$ based on the LMRPM domain and its objectives, resulting in a distortion of 15. Similarity in Froude number (Fr) resulted in a velocity scale $V_r = 20$. Relaxation of flow
Figure 4.1. Lower Mississippi River Physical Model domain. Flow is from top to bottom. Upper boundary is Donaldsonville, LA (top-left) and the lower boundary is the Gulf of Mexico (bottom-right).
Reynolds number (Re) similitude was a design requirement, still high enough turbulence was allowed to operate the model in the turbulent regime. The utilized sediment was a fine synthetic material made of unexpanded polystyrene with a mean particle diameter $d_{50} = 0.35 \text{ mm}$ and a density of $1.05 \text{ g cm}^{-3}$ (Figure 4.2). Two similitude parameters the critical Shields parameter and critical particle Reynolds number parameter were chosen to scale the sediment. The equality of these two parameters between model and prototype along with river sediment density of $2.65 \text{ g cm}^{-3}$, resulted in model particle diameters 3.2 times larger than those found in the prototype.

The sediment time scale for the LMRPM was designed utilizing SOGREGA's approach (equation 4.2). This equation inputs are the hydraulic time scale (equation 4.1), which is based on Froude scale for distorted river physical models, and the specific gravity of the sediment for both model and prototype, providing a first approach in determining the sediment time scale.

Utilizing the LMRPM dimensional scales and equation (4.1) into (4.2), the model sediment
time scale is calculated:

\[ T_{sr} = \frac{x_r z_r}{\sqrt{z_r}} (S_s - 1) \]

\[ T_{sr} = 9900 \]

where \( x_r = 6000, z_r = 400, S_{sp} = 2.65 \) and \( S_{sm} = 1.05 \).

The sediment time scale \( T_{sr} = 9900 \) needs to be tested in the model to verify that the model is replicating both the hydraulic and the main bed morphology features. Initial LMRPM calibration has shown satisfactory results utilizing the design sediment time scale \( T_{sr} = 9900 \). However, it was noticed that more deposition of sediment than expected was observed at the upstream region of the model, suggesting that the duration of the runs should be longer, i.e., reducing the sediment time scale ratio.

Based on previous experiences from the SSPM, it was suggested that since both models SSPM and LMRPM have the same sediment gradation and density, a new sediment time scale ratio can be calculated for the LMRPM assuming the same volumetric sediment discharge ratio per unit width \( q_{sr} \) as the SSPM. In order to calculate the volumetric sediment discharge ratio per unit width for the SSPM, first here is defined the sediment time scale based on the time to fill a unit-width volume by \( q_{sr} \):

\[ T_{sr} = \frac{x_r z_r}{q_{sr} C_{bvr}} \]  \hspace{1cm} (4.3)

Utilizing equation (4.3) and the SSPM scales, the volumetric sediment discharge ratio per unit width is calculated:

\[ q_{sr} = \frac{x_r z_r}{T_{sr} C_{bvr}} \]

\[ q_{sr} = 469 \]
where $x_r = 12000$, $z_r = 500$, $T_{sr} = 9600$ and $C_{bvr}$ = volume concentration of sediment in the bed to account for model to prototype differences in bed porosity estimated to $C_{bvp} = 0.60$ and $C_{bvm} = 0.45$. The new sediment time scale for the LMRPM is calculated using $q_{sr} = 469$, the model dimensional scales ($x_r$ and $z_r$) and equation (4.3), resulting in $T_{sr} = 3840$.

Finding the appropriate LMRPM sediment time scale is important for the model to succeed in the study of how the Lower Mississippi River will respond to future natural and man made (or anthropogenic) processes and features and the impact on coastal restoration, navigation and flooding. Two experiments were conducted both at a flume laboratory and the model in which different sediment time scales were evaluated to observe the sediment response to the sediment time scale.

### 4.2 Flume Experiment

A flume experiment was conducted to compare the sediment response to the design sediment time scale $T_{sr} = 9900$ with the recommended sediment time scale $T_{sr} = 3840$. 
The experiment was run using a flume Armfield S6 MKII which was 0.30 m wide, 0.45 m deep and 5 m long (Figure B.4). The LMRPM scales were utilized to represent the Lower Mississippi River reach in the experiments. Representative prototype widths and depths vary from 600 m to 1000 m and 22 m to 25 m, respectively (Nittrouer, Shaw, Lamb, & Mohrig, 2012). Taking the averages of these values along with the LMRPM dimensional scales, the flume dimensions were set to be 0.13 m in width and 0.06 m in water depth. These scaled conditions imposed modifications in the flume that were achieved with an acrylic wall to reduce the flume width (Figure 4.3). At the upstream region of the flume, six circular tubes of 0.04 m diameter were set to align the flow and a downstream weir was set to control the water depth constant. Both flume and initial bed slopes were set horizontal.

The flume experiment hydrograph was a modification of the LMRPM design hydrograph. The model hydrograph is a yearly average based on USACE/USGS discharge data from 1992 to 2013 at Tarbert Landing, Louisiana (BCG Engineering & Consulting, Inc., 2015). For the experiment, it was further averaged into five constant discharges for control convenience (Figure 4.4). Water was pumped into the flume with a bilge pump and manually controlled through a D/C power supply.

Bed levels were measured with a millimeter-graduated ruler through one of the transparent flume walls. The main bed measurements were done once dunes were formed: at dunes tails, middle point of the stoss sides, crests and the lowest point of the lee side. The accuracy of all measurements with the millimeter-graduated ruler was ±0.002 m (0.81 m prototype).

Prior to the experiment, the bed was screeded flat with a 2 cm thickness in order to allow dunes to develop while minimizing scouring pools. Additional sediment was manually added upstream according to the LMRPM design sediment input. This model input was designed to meet the yearly sand discharge at Donaldsonville, LA, summing a total of 2.2 L of saturated sediment (BCG Engineering & Consulting, Inc., 2015). Two continuous hydrographs were run to evaluate the bed response after dunes were developed and the influence that the sediment time scale has on them. The first hydrograph was run to initiate dune formation.
4.2.1 Bed Development

Bed development was evaluated utilizing three criteria: total erosion volume, dunes characteristics (length and height) and dunes celerity. Results are shown in Table 4.1. Total erosion volume was quantified measuring both initial and final bed volume for each experiment and then subtracting both final values. This parameter was set to assess whether or not a change in sediment time scale would produce more movement of material downstream. It was found that the longer the time duration, the more material was eroded from the flume. This finding suggests that utilizing the sediment time scale $T_{sr} = 3840$ would transport more material downstream over accumulated years in the model of about 55% when comparing $T_{sr} = 3840$ to $T_{sr} = 9900$.

Dunes characteristics were evaluated measuring dunes length and height. Dunes lengths were measured as the horizontal distance from the dunes crest up to tails. Dunes heights were measured from crests up to average elevation between the lowest point of lee side and
Table 4.1. Results from the two sediment time scales.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sediment time scale ((T_{sr}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9900</td>
</tr>
<tr>
<td>V (L)</td>
<td>0.9 ± 0.2</td>
</tr>
<tr>
<td>L (cm)</td>
<td>20 ± 6</td>
</tr>
<tr>
<td>H (cm)</td>
<td>1.2 ± 0.7</td>
</tr>
<tr>
<td>C (cm/min)</td>
<td>0-3.42</td>
</tr>
</tbody>
</table>

V = eroded volume plus 0.2 L due to measurements accuracy, 
L = mean length and H = mean height plus one standard deviation, 
C = celerity range of values

tail of the next dune. Dunes lengths were found to be about the same range of values with similar mean lengths for both sediment time scales. Dunes heights also ranged about the same values with similar mean heights for both sediment time scales with location of crests in the middle of the dunes for both sediment time scales, resulting in symmetrical dunes. These finding suggests that dunes lengths and heights dimensions are independent of the sediment time scale, but rather vertical scale dependent, but the sediment time scale could affect the shape of dunes. Dunes in the prototype are expected to be asymmetrical dunes, as expected in typical rivers.

Dunes celerity was measured tracking dunes crest displacements after time periods of about 2 minutes. It is important to mention that even though the celerities are slower for the sediment time scale \(T_{sr}=3840\) than \(T_{sr} = 9900\), when upscaled this pattern is not the same because the experiments were conducted with different sediment time scales.

### 4.2.2 Sediment Time Scales 9900 and 3840 Observations

Bed measurements at the end of the experiment showed that as the total time duration increases (smaller sediment time scale), more material is transported downstream. This finding suggests that over long period of modeling time, the sediment is more likely to reach further downstream reaches faster. This is confirmed from comparing upstream volume with downstream volume. For the sediment time scale \(T_{sr} = 9900\), the upstream volume was
found to be 55.7%, whereas $T_{sr} = 3840$ was 49.4%.

Bed development was found to be dependent of the sediment time scale. Each sediment time scale showed particularly different bed morphology evolution. The LMRPM sediment time scale $T_{sr} = 9900$ developed dunes that initially resembled those found in natural rivers (asymmetrical dunes); however, near the peak discharge, dunes started to lose two dimensionality due to accumulation of material in the stoss sides of dunes. Due to the short duration of this sediment time scale, sediment rather than being transported downstream, it was deposited near the crest of dunes, forming dunes that were rounded with crests close to the middle portion of the dunes (symmetrical dunes). Also, it was observed that some dunes were split which suggests that sediment time scale did not allow them to grow and move downstream as the water discharge was increasing.

The other sediment time scale $T_{sr} = 3840$ showed differences when compared to the 1:9900. Initially it formed dunes fairly similar to those found in natural rivers as well; however, by the end of the year one, the started to shape rounded. Dunes also were split and more importantly, excessive scouring was observed, reaching a point where the flume surface was exposed. This finding suggests that this sediment time scale was too long to develop natural bed transport.

Dunes celerity was evaluated in terms of prototype values. These values were converted utilizing the horizontal scale $x_r = 6000$ and the respective sediment time scale. The results showed that the longer the duration (smaller sediment time scale) the faster the dunes celerity prediction. Dunes celerity magnitudes were found to be in accordance to those dunes celerities measured in the Lower Mississippi River. Nittrouer, Mohrig, and Allison (2011) measured dunes celerities at Empire reach, finding that dunes celerity varied from 0.12 to 0.19 m h$^{-1}$ and 0.64 to 2.47 m h$^{-1}$ when when the river discharge was roughly 11723 and 38 369 m$^3$ s$^{-1}$, respectively. Estimated values from all the experiments were within those ranges, suggesting that both sediment time scales can fairly predict prototype dunes celerity (Table 4.2).
Table 4.2. Measured dunes celerities at the flume experiments in terms of prototype scale. Discharge from top to bottom is the sequence that they were run.

<table>
<thead>
<tr>
<th>Discharge (m³ s⁻¹)</th>
<th>Celerity (m h⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tₚₛ = 9900</td>
<td>Tₛₚ = 3840</td>
</tr>
<tr>
<td>13960</td>
<td>-</td>
</tr>
<tr>
<td>20162</td>
<td>0.35</td>
</tr>
<tr>
<td>24012</td>
<td>0.65</td>
</tr>
<tr>
<td>26702</td>
<td>0.80</td>
</tr>
<tr>
<td>16707</td>
<td>0.23</td>
</tr>
</tbody>
</table>

4.2.3 Sediment Time Scale 6600

A new sediment time scale was suggested after finding that the sediment time scale 3840 produced symmetrical dunes and excessive erosion. An intermediate sediment time scale between $T_{sp} = 9900$ and $T_{sp} = 3840$ was suggested, resulting in a sediment time scale $T_{sp} = 6600$. For the new sediment time scale, the flume was prepared the same as the previous time scales and the water and sediment injection rates were adjusted accordingly.

The total erosion volume was found to be 33% more when compared to the design sediment time scale $T_{sp} = 9900$, but lower than sediment time scale $T_{sp} = 3840$ as expected for an intermediate sediment time scale. Dunes lengths and heights on average were found to be the same dimensions as for the other two sediment time scales, suggesting, one more time, independence in sediment time scale for the dunes dimensions. However, the locations of the crests were found towards one side of the dunes (lee side), resulting in asymmetrical dunes as expected for this model when compared with the symmetrical dunes found in the other two sediment time scales (Figure 4.6). Bed development was found to be different from both previous sediment time scales. Once dunes formed in asymmetrical shape (Figure 4.5), they kept this form as were transported downstream for the rest of the experiment without evidence of excessive erosion as observed in the sediment time scale $T_{sp} = 3840$. Dunes celerity was also found within the same ranges found in the previous sediment time scales, showing the same trend that the longer the duration the slower the dunes celerity.
when compared with the design sediment time scale $T_{sr} = 9900$.

### 4.3 Sediment Time Scale 6600 at the LMRPM

The observations of the intermediate sediment time scale $T_{sr} = 6600$ at the flume showed promising results that were tested at the model. For comparison, the model was first run at the design sediment time scale $T_{sr} = 9900$ (Figure B.5). Once this experiment was finished, the model was cleaned and the same initial conditions were set, only changing the water and sediment scale ratios resulted from the intermediate sediment time scale $T_{sr} = 6600$.

The LMRPM upstream region was initially pre-loaded from river mile 173 to 80, approximately. The added sediment is at the crossings with roughly 50% of the expected volume to be stored in these regions. The model is then run utilizing the design water and sediment yearly hydrographs until the bed stabilizes to the design bathymetry. For comparison, 29 runs were tested for each sediment time scale. It was observed that for the sediment time scale $T_{sr} = 9900$ took 24 runs to fill the bed along the model, whereas for the sediment time scale $T_{sr} = 6600$ took 18 runs. Consecutive runs after filling the model resulted in
Figure 4.6. Example of four dunes developed during the experiment for the sediment time scale $T_{sr} = 6600$
Figure 4.7. Bed measurements taken at the model in terms of the prototype. Left is for $T_{sr} = 9900$ and right is for $T_{sr} = 6600$. From top to bottom: Reserve, Carrollton, Alliance and Empire. Solid lines are the model without sediment and dash lines are the design target values.

continuous accumulation of material at the crossings, specifically at the upstream region for the sediment time scale $T_{sr} = 9900$, whereas for the $T_{sr} = 6600$ this accumulation pattern was not observed but rather a periodical erosion-deposition pattern.

Bed measurements were conducted along the model domain at 4 different locations including the areas of Reserve, Carrollton, Alliance and Empire. A fifth location in the area of South West Pass was monitored to observe when the water depth was too shallow that dredging was required. The measurements were conducted at the end of two consecutive runs and were done with a caliper from the model surface to the bed surface (Figure B.6). These measurements were compared with the design bathymetry (target) to evaluate the bed progression. The results showed good agreement for the sediment time scale $T_{sr} = 6600$. All areas quickly stabilized after 22 runs and South West Pass required dredging after run 18, which is part of the regular yearly operation of this area. All these observations occurred much quicker when compared to the sediment time scale $T_{sr} = 9900$ where after 29 run the bed elevations continued to grow (Figure 4.7).
Maximum river stage levels were collected from the model sensors that simulate the river gauges located along the river. The results showed agreement for the sediment time scale $T_{sr} = 6600$. From the middle to downstream, the model reached the target values (averaged from actual data collected at the river by the USGS from 1992 to 2013) after 18 runs and stabilized after 23 runs. At the upstream region, the values at the model are slightly lower than the expected values which suggests some model modifications may be needed to rise the water levels in this region. The sediment time scale $T_{sr} = 9900$ showed a continuous rising in stage levels which suggests that this sediment time scale is not mobilizing enough material downstream but rather accumulating more sediment than expected (Figure 4.8).

Rating curves were plotted utilizing the collected data for the four analyzed areas. These curves showed better agreement for the sediment time scale $T_{sr} = 6600$ when compared with the $T_{sr} = 9900$. All locations showed closer values to the targets which are data collected from the USGS available during the years 1992 to 2013. Figure 4.9 shows the analyzed areas where runs from 23-29 showed good improvements when targeting the actual data.

Dunes lengths and heights were collected after 29 runs were completed to observe in the
Figure 4.9. Rating curves produced for the model in terms of the prototype. Left is for $T_{sr} = 9900$ and right is for $T_{sr} = 6600$. From top to bottom: Reserve, Carrollton, Alliance and Empire. Dash lines are the design target values.

long terms if the change in sediment time scale would impact these dunes characteristics (Figures 4.10 and B.7). The results showed no significant difference in dunes lengths and heights sizes since overall the range of values were about the same in both cases (Figure 4.11).

4.4 Conclusions

The equation proposed by SOGREAH provides a good first approximation for estimating the sediment time scale for distorted river physical models. This simple equation only requires the model dimensional scales and the model material properties. It is suggested to test the resulted sediment time scale to evaluate the model response in comparison to the prototype design targets, including both water and bed response. Various sediment time scales were tested to estimate Lower Mississippi River Physical Model. Even though the sediment time scale $T_{sr} = 9900$ produce good results, the sediment time scale $T_{sr} = 6600$ showed improvements when compared to the original sediment time scale. It is recommended to utilize the sediment time scale $T_{sr} = 6600$ at the model for all future scenarios.
Figure 4.10. Example of dunes observed after finishing 29 runs for the sediment time scale $T_{sr} = 9900$
Figure 4.11. All dunes lengths and heights measured at the end of each experiment. Left is for $T_{sr} = 9900$ and right is for $T_{sr} = 6600$.

4.5 References


Chapter 5. Discussion and Conclusions

The LMRPM is a distorted, mobile bed model that was designed utilizing a particular combination of scaling parameters, based on the experience of different experts that participated in the model design. Four main scaling criteria were chosen to design the model:

1. Froude scale: This scale is of utmost importance for river physical models since it is the main parameter for correctly replicating water-related processes, including water discharges, average velocities and river stages. A review of the literature found that this is the only parameter that all five rational scaling methodologies agreed must be utilized for physical models. To date, LMRPM calibration and validation experiments confirm that ability of the model to replicate prototype flow, average velocities and stages.

2. Shields parameter and particle Reynolds number: Following Prasuhn (1987), both parameters were utilized to scale the model sediment. The equality of these two parameters, along with known prototype sediment density and sizes, and specified model sediment density, resulted in model sediment particle sizes that are 3.2 time larger than the prototype sediment.

3. Reynolds numbers: Based on the Reynolds number calculated with the hydraulic diameter as the length scale, the minimum Reynolds numbers allowed in the model was set to 2000 for turbulent flow (low water discharge) and 10000 for fully turbulent flow (high water discharge). In order to satisfy these criteria and ensure that the model size is not too large, both dimensional scales were set to 1:6000 horizontal and 1:400 vertical. In many cases one of the two dimensional scales is free and the other results from equations that ensure similarity. In the case of the LMRPM, the horizontal length scale was chosen to define the physical model size and the vertical scale adjusted to ensure Reynolds number criterion.
4. Sediment time scale: Due to the model geometric lengths scales and relatively high distortion, none of the rational approaches could be used to determine the sediment time scale. Initial model design utilized a sediment time scale derived from an equation proposed by SOGREAH Consultants (2004) for use in a movable bed physical model, using a lightweight sediment, of the Seine River. This equation provided a good initial estimation of the sediment time scale; however, further testing at the model suggested that additional modification of this parameter was required to better replicate bed processes.

The LMRPM design is a combination of both rational and empirical methodologies, which sets the model in a different category. Even though there are different methodologies for design of river physical models, a model with a large domain and distortion, requires more degrees of freedom as proposed by rational methodologies when selecting its scales.

The objective of this research was to quantitatively evaluate the design model sediment and provide insights into how that sediment behaves under prototype scaled conditions. Quantitative results from the flume experiments demonstrated that the lightweight sediment followed the Shields criterion for no motion, incipient motion and general movement. Thus, there is a high degree of confidence that the model sediment will begin to move in the LMRPM under similar hydraulic conditions as the prototype river sand.

It was also observed that the sizes of the dunes formed with the sediment are similar when scaled and compared with those found in the Lower Mississippi River. Dunes formed in the flume were quickly formed from bed defects, which suggests that any bed irregularity or sediment accumulation promotes dune formation. Dunes at the flume formed within 10-20 minutes after the mean flow velocity was higher than 6.4 cm s\(^{-1}\). This quick formation of models dunes, following short time interval changes in model flow velocity, confirms that the use of lightweight sediment accelerates river studies since many scenarios can be modeled in a shorter period of time than studies that utilize natural sediments.

The distribution of measured dune heights in the flume were slightly higher when scaled
and compared to those found in the prototype. There are several possible explanations for this finding. 1. The flume experiments were performed under constant depth conditions, while the prototype flow depth is obviously variable. 2. The angular model sediment has a measured angle of repose of about 60 degrees, compared to a prototype sand value of approximately 30 degrees. This could lead to a higher level of accumulation of model sediment near the dune crests. 3. The flume dune measurements were made when the dunes achieved their maximum height, whereas at the prototype were measured during a specific time frame (June to August 2003) without necessarily being at their maximum.

The design sediment time scale, proposed by SOGREAH Consultants (2004), provided a good initial estimation of the parameter; however, further testing both at the flume and the model revealed that a new sediment time scale, 1:6600, provided better results both in terms of water and bed processes. Flume experiments showed that different sediment time scales provide different bed responses. Long time duration promotes excessive sediment mobilization at the point where local model bed can be exposed, specially in front of dunes (lee side). Short time duration, on the other hand, promotes excessive accumulation, specially at upstream region of the model, since not enough time is allowed for the correct downstream bed mobilization. The appropriate sediment time scale is found when the model bed replicates the scaled prototype conditions and it could be found starting from the equation proposed by SOGREAH and then modified accordingly based on model observations and data.

In the flume, there was little or no significant movement of sediment particles at a scaled of 15 000 m$^3$s$^{-1}$. This finding supports an operational assumption of the model that any five-day period with a water discharge lower than 11 000 m$^3$s$^{-1}$ does not need to be simulated, as long as the water depth is above the bed level.

The sediment time scale of 1:6600 provided better results when tested at the model. Water levels were found closer to the design levels and the bed stabilized after approximately 20-25 runs using an average annual hydrograph. The next 6 runs, using actual prototype
hydrographs, provided a pattern of erosion-deposition at the model bed without affecting the water level. The time required for the bed to come to equilibrium was found to be quicker when using this new sediment time scale, accelerating the initial model preparation of the model. It is recommended to use the sediment time scale of 1:6600 for all the future scenarios run at the model.

Future model testing is recommended at the model. This research was focused on the initial movement of the sediment and dune formation along with an estimation of the sediment time scale. Future research should be focused on the suspended sand transport. The amount of sand transported in suspension is important for the Mississippi River as well as for the LMRPM. Limitations to making these measurements is the difficulty in properly measuring the suspended volume while running the tests both at the flume and the model. It is suggested to find a device such as an acoustic backscatter system to estimate suspended material. Quantifying the amount of volume transported in suspension would help to understand how much material could be diverted for all the different scenarios that the model will test, not only over the long simulations times (25 or 50 years), but yearly changes. It is also suggested when testing the suspended sand transport, to investigate whether or not the sediment time scale of 1:6600 is enough to properly replicate transport in suspension as the sediment time scale for the LMRPM was calibrated observing bed surface changes.

Measuring the model bed provides valuable information. To date, bed measurements have been conducted utilizing a caliper to take bed measurement at the end of simulated years, providing better understanding how the model bed responses to water discharge changes. It is recommended to investigate a non-intrusive bed measurements device such as an acoustic sensor that could take bed level measurements during a simulated year. Instantaneous bed measurements would provide information not only about dune formation but also to estimate sediment bedload transport such as in Abraham and Kuhnle (2006).
5.1 References


Appendix A. Rational Methodologies Calculations

The design of the LMRPM utilizing different rational methodologies are shown below.

1. Julien (complete mobile bed)

\( z_p = 24 \text{ m} \) and \( d_{sp} = 0.13 \text{ mm} \)

Assumptions: \( z_r = 400 \) and \( y_r = x_r \).

\[
m = \frac{1}{\ln \left( \frac{12.2 \cdot z_p}{d_{sp}} \right)}
\]

\[
m = \frac{1}{\ln \left( \frac{12.2 \cdot 24}{0.00013} \right)}
\]

\[
m = 0.0684
\]

\[
x_r = z_r \left( \frac{1 + 4m}{1 + m} \right)
\]

\[
x_r = 400 \left( \frac{1 + 4 \cdot 0.0684}{1 + 0.0684} \right)
\]

\[
x_r = 1264.21
\]

\[
\Delta = \frac{y_r}{z_r}
\]

\[
\Delta = \frac{1264}{400}
\]

\[
\Delta = 3.16
\]
\[
\begin{align*}
    d_{sr} &= z_r \frac{2m - 1}{2 + 2m} \\
    &= 400 \frac{2}{2 + 2 \times 0.0684} \\
    &= 0.0889
\end{align*}
\]

\[
\begin{align*}
    T_r &= z_r \frac{1 + 7m}{2 + 2m} \\
    &= z_r \frac{1 + 7 \times 0.0684}{2 + 2 \times 0.0684} \\
    &= 63.21
\end{align*}
\]

\[
\begin{align*}
    T_{sr} &= z_r \frac{1 + 5 \times 0.0684}{1 + 0.0684} \\
    &= 1855.26
\end{align*}
\]

\[
\begin{align*}
    V_r &= z_r^{1/2} \\
    &= 400^{1/2} \\
    &= 20
\end{align*}
\]

\[
\begin{align*}
    Q_r &= x_r z_r^{3/2} = 1264.21 \times 400^{3/2} \\
    &= 10113680
\end{align*}
\]
\[
\left( \frac{\rho_s}{\rho} - 1 \right)_r = \frac{3 - 6m}{z_r^2 + 2m} \\
\left( \frac{\rho_s}{\rho} - 1 \right)_r = 400^2 + 2 \times 0.0684 \\
\left( \frac{\rho_s}{\rho} - 1 \right)_r = 1423.81 \\
q_{sr} = 1
\]
2. Yalin

Here is presented the last 4 iterations that satisfy the method’s criteria.

\[ z_r = \frac{1}{400}, \left( \frac{\rho_s}{\rho} - 1 \right)_m = 0.05, \quad z_p = 24 \text{ m}, \quad S_p = 1.2 \times 10^{-5} \text{ m/m}, \quad d_{sp} = 0.13 \text{ mm} \quad \text{and} \quad \tau_{srcp} = 0.072. \]

<table>
<thead>
<tr>
<th>Model Iteration</th>
<th>Prototype</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>6.91</td>
<td>0.05</td>
<td>0.0019</td>
<td>0.0017</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>1.34</td>
<td>1.34</td>
<td>16.11</td>
<td>17.15</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>184615</td>
<td>4615</td>
<td>92308</td>
<td>101280</td>
</tr>
<tr>
<td></td>
<td>( \epsilon )</td>
<td>3.29</td>
<td>0.12</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>( Y_{cr} )</td>
<td>0.08</td>
<td>0.31</td>
<td>1.00</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>( \sigma )</td>
<td>16.17</td>
<td>3.34</td>
<td>15.12</td>
<td>15.55</td>
</tr>
<tr>
<td></td>
<td>( B )</td>
<td>9.30</td>
<td>5.77</td>
<td>8.48</td>
<td>8.48</td>
</tr>
<tr>
<td></td>
<td>( b )</td>
<td>15.17</td>
<td>3.71</td>
<td>10.94</td>
<td>10.96</td>
</tr>
<tr>
<td></td>
<td>( A_f )</td>
<td>0.0008</td>
<td>0.0020</td>
<td>0.0009</td>
<td>0.0009</td>
</tr>
<tr>
<td></td>
<td>( f_d(X) )</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>( f_r(X) )</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>( \left( \frac{\Delta A}{A} \right)_d )</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>( \left( \frac{\Delta A}{A} \right)_r )</td>
<td>0.015</td>
<td>0.180</td>
<td>0.024</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>( C^{-2} )</td>
<td>0.0008</td>
<td>0.0055</td>
<td>0.0009</td>
<td>0.0009</td>
</tr>
<tr>
<td></td>
<td>( S_r )</td>
<td>1.200</td>
<td>1.164</td>
<td>1.147</td>
<td>1.147</td>
</tr>
<tr>
<td></td>
<td>( d_{sr} )</td>
<td>0.0050</td>
<td>0.0046</td>
<td>0.0043</td>
<td>0.0043</td>
</tr>
</tbody>
</table>

\[
X_r = \frac{z_r}{S_r} = \frac{1}{400 \times 1.137} = \frac{1}{454.8}
\]

\[
u_{sp}^2 = gS_p z_p
\]

\[
u_{sp}^2 = 9.81 \times 0.000012 \times 24
\]

\[
u_{sp}^2 = 0.00282 \frac{m^2}{s^2}
\]
\[ \tau_{op} = \rho u_s^2 \]
\[ \tau_{op} = 1000 \times 0.00282 \]
\[ \tau_{op} = 2.82 \text{ Pa} \]

\[ \tau_{ocrp} = \tau_{crp}(\gamma_s - \gamma) \rho d_p \]
\[ \tau_{ocrp} = 0.072 \times (25996.5 - 9810) \times 0.00013 \]
\[ \tau_{ocrp} = 0.15 \text{ Pa} \]

\[ K_p = \frac{\tau_{op}}{\tau_{ocrp}} A_f \rho C_p^2 \]
\[ K_p = \frac{2.82}{0.15} \times 0.0008 \times \frac{1}{0.0008} \]
\[ K_p = 18.84 \]

\[ q_{sr} = z_r^{1/2} Y_{cr} A_f \rho C_r K_p - 1 \]
\[ q_{sr} = \left( \frac{0.06}{24} \right)^{1/2} \times 1.06 \times \frac{0.0009}{0.08} \times \frac{\left( \frac{0.0009}{0.0009} \right)^{1/2}}{18.84 - 1} \]
\[ q_{sr} = \frac{1}{1.42} \]

\[ T_{sr} = \left( \frac{\rho_s - 1}{\rho} \right) z_r^2 \]
\[ T_{sr} = \frac{1}{33} \times \frac{1}{400^2} \]
\[ T_{sr} = \frac{1.137 \times 1}{1.42} \]
\[ T_{sr} = \frac{1}{4219409.28} \]
3. Tsujimoto

\[ x_r = \frac{1}{6000}, \; z_r = \frac{1}{400} \; \text{and} \; \left( \frac{\rho_s}{\rho} - 1 \right)_m = 0.05 \]

\[
\left( \frac{\rho_s}{\rho} - 1 \right)_r = \frac{1.05 - 1}{2.65 - 1}
\]

\[
\left( \frac{\rho_s}{\rho} - 1 \right)_r = \frac{1}{33}
\]

\[
\epsilon = \left( \frac{\rho_s}{\rho} - 1 \right)_r^{-3/2}
\]

\[
\epsilon = \left( \frac{1}{33} \right)^{-3/2}
\]

\[
\epsilon = 189.57
\]

\[
d_{sr} = \epsilon d z_r
\]

\[
d_{sr} = 189.57 \times \frac{1}{400}
\]

\[
d_{sr} = \frac{1}{2.11}
\]

\[
V_r = z_r^{1/2}
\]

\[
V_r = (400)^{1/2}
\]

\[
V_r = 20
\]
\[ T_r = \frac{x_r}{z_r^{1/2}} \]
\[ T_r = \frac{6000}{1^{1/2}} \]
\[ T_r = \frac{1}{400} \]
\[ T_r = \frac{1}{300} \]

\[ T_{sr} = \frac{x_r}{z_r^{1/2} \epsilon^{7/6}} \]
\[ T_{sr} = \frac{1}{6000} \]
\[ \left( \frac{1}{400} \right)^{1/2} \ast (189.57)^{7/6} \]
\[ T_{sr} = \frac{1}{136307.31} \]
4. Chanson

\[ x_r = 6000, \; z_r = 400 \; \text{and} \; \left( \frac{\rho_s}{\rho} - 1 \right)_m = 0.05 \]

\[
d_{sr} = \frac{z_r^2}{x_r \left( \frac{\rho_s}{\rho} - 1 \right)_r}
\]

\[ d_{sr} = \frac{400^2}{6000 \cdot 33} \]

\[ d_{sr} = 0.81 \]

\[ q_{sr} = \frac{z_r}{x_r^{1/2}} \]

\[ q_{sr} = \frac{400}{(6000)^{1/2}} \]

\[ q_{sr} = 5.16 \]

\[ T_r = \frac{x_r}{z_r^{1/2}} \]

\[ T_r = \frac{6000}{(400)^{1/2}} \]

\[ T_r = 300 \]

\[ q_{tsr} = q_{sr} \]

\[ T_{sr} = (1 - P_o) \frac{x_r z_r}{q_{tsr}} \]

\[ T_{sr} = \left( \frac{1 - 0.3}{1 - 0.5} \right) \cdot \frac{6000 \cdot 400}{5.16} \]

\[ T_{sr} = 651162.79 \]
5. Prasuhn

\[
\left( \frac{\rho_s}{\rho} - 1 \right)_m = 0.05
\]

\[
\alpha_r = \left( \frac{\rho_s}{\rho} - 1 \right)_r
\]

\[
\alpha_r = \frac{0.05}{1.65}
\]

\[
\alpha_r = \frac{1}{33}
\]

\[
 d_{sr} = \alpha_r^{-1/3}
\]

\[
 d_{sr} = \left( \frac{1}{33} \right)^{-1/3}
\]

\[
 d_{sr} = 3.2
\]

\[
 z_r = \alpha_r^{7/6}
\]

\[
 z_r = \left( \frac{1}{33} \right)^{7/6}
\]

\[
 z_r = \frac{1}{59.10}
\]

\[
 x_r = \alpha_r^{5/3}
\]

\[
 x_r = \left( \frac{1}{33} \right)^{5/3}
\]

\[
 x_r = \frac{1}{339.51}
\]

\[
 q_{sr} = 1
\]
\[T_{sr} = \frac{x_r z_r}{q_{sr}}\]

\[T_{sr} = \frac{1}{6000 \times 400}\]

\[T_{sr} = \frac{1}{2400000}\]
Appendix B. Additional Photographs from Experiments at the LSU Flume and the LMRPM

Figure B.1. LSU Armfield S6 MKII flume where both flume experiments were conducted (Chapter 3 and 4).
Figure B.2. Preparing initial bed before conducting experiment for incipient motion and dune formation at the LSU flume.
Figure B.3. SonTek 16-MHz MicroADV utilized to measure velocity fluctuations at the flume.
Figure B.4. Running experiment for the sediment time scale $T_{sr} = 3840$ at the LSU flume.
Figure B.5. Running the LMRPM for the series with $T_{sr} = 9900$. 
Figure B.6. Taking measurements at the LMRPM with a caliper at the end of the series with $T_{sr} = 9900$. 
Figure B.7. Plan view of dunes after finishing experiment for sediment time scale $T_{sr} = 6600$. 
Vita

Mauricio Jesus Hooper De Leon was born in Santiago, Panama, in 1986. He received his Bachelor’s degree in Civil Engineering from the Technological University of Panama, in September 2010. He then worked at the Technological University of Panama from 2011 to 2013 as a assistant professor of the faculty of Civil Engineering. In September 2011, he started his Master’s degree in Structural Engineering from the Technological University of Panama. He then won a Fulbright-Laspau scholarship and entered the doctoral program in the Department of Civil and Environmental Engineering at Louisiana State University in 2014 where he conducted research in the Lower Mississippi River Physical Model, focusing on the sediment transport processes of the model. He finished his Masters degree in March 2015. He is expected to earn the degree of Doctor of Philosophy in May 2019.