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Analytical Studies of Wireline Formation Testing and Pressure Losses Across Gravel Packs.

Turhan Yildiz

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Analytical studies of wireline formation testing and pressure losses across gravel packs

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A Dissertation

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in

The Department of Petroleum Engineering

by

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Abstract

A three-dimensional (3-D) analytical model was developed to improve the analyses of the transient pressure response of a reservoir to wireline formation testing (WFT). Available analysis techniques of the WFT data oversimplify the flow geometry by assuming either a radial or a spherical flow pattern. The proposed model simulates the exact flow geometry of the WFT flow pattern. The model which assumes a constant drawdown rate was derived by solving the 3-D diffusivity equation coupled with the boundary conditions prevailing during the test. The Laplace transformation and the separation of variables technique were used to solve the boundary value problem. The solution is expressed in terms of the infinite Fourier-Bessel series in the Laplace space and inverted into the real space by means of Stehfest algorithm. The reduced versions of the 3-D transient model were compared to published 2-D and 1-D models in the literature to verify the solution. Excellent agreement was obtained between the models. The mathematical model was used to evaluate the validity of current interpretation techniques and to investigate the sensitivity of transient pressure behavior to wellbore parameters. A new interpretation technique and a proposed new design for the tool resulted from this study.

The study was extended to obtain an analytical model describing laminar flow through a gravel pack in the annular space between a
perforation and the gravel screen. An electrical analog was built to verify the mathematical solution. The mathematical solution compared favorably with the experimental data. The analytical model was then used to investigate the effects of several wellbore parameters on the productivity of a gravel packed well. The sensitivity of pressure losses across the pack to perforation size, perforation density, phasing angle, and gravel anisotropy were examined. The perforation size and perforation shot density were identified as the most important parameters.
CHAPTER I

Fluid Flow in Porous Media

In this section the differential equation governing fluid flow in a porous media will be reviewed. The differential equation is derived from the mass balance, empirical momentum equation for flow in a porous media and an appropriate equation of state. In deriving the differential equation, some assumptions are made:

1. A single phase is flowing;
2. formation is homogeneous;
3. fluid has constant and low compressibility;
4. fluid viscosity is constant;
5. flow is laminar;
6. fluid does not chemically react with the formation.

Under these assumptions, the differential equation describing the flow of fluids in a porous media is the well-known diffusivity equation. Due to the geometry of a wellbore and the reservoir, the diffusivity equation in cylindrical coordinates is suitable for many reservoir engineering problems.

If the physical problem is time-dependent, the differential equation is:¹,²
The symbols in Eq. (1.1) are defined in the nomenclature section.

For steady-state problems, the right hand side of Eq. (1.1) is zero.

\[
\frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial P}{\partial r} \right] + \frac{1}{r^2} \frac{\partial^2 P}{\partial \Theta^2} + \frac{k_z}{k_r} \frac{\partial^2 P}{\partial z^2} = \frac{\Phi \mu c_t}{\alpha k_r} \frac{\partial P}{\partial t} \tag{1.1}
\]

Even though most of the physical problems in reservoir engineering are multidimensional, these problems are usually reduced to simple one dimensional counterparts due to the difficulties in solving the multidimensional problems. For example, the transient flow into a perforated well is a three dimensional physical phenomenon, but the pressure behavior of such a well is analysed as if the flow is completely radial. Other examples of treating multidimensional problems as simple one dimensional problems are calculation of pressure losses across a gravel pack, transient pressure analysis of gravel-packed wells, well testing of partially-penetrating wells, and wells completed with a slotted liner, pressure analysis of wireline formation testing, and modeling of electrical heating and well logging of oil wells.

Although multidimensional problems mentioned above are physically different and belong to different branches of petroleum engineering, the mathematics of these problems are similar. In this study, a general mathematical approach is taken to solve the multidimensional
problem. One transient problem (wireline formation testing) and one steady-state problem (flow of fluids across a gravel pack) were chosen to show the mathematical techniques to solve the multidimensional problems. The solution for the other physical phenomena mentioned above can be obtained similarly by choosing the proper boundary condition and momentum equation. For example, the solution for transient pressure analysis of wireline formation testing data can be easily modified to model the transient flow into the perforated well or a partially penetrating well.
CHAPTER II

Wireline Formation Testing

2.1. Introduction

Wireline formation testing (WFT) is an open-hole logging technique. WFT data has been used to determine initial reservoir pressure, vertical pressure distribution along an open-hole, fluid contacts, and formation permeability. Formation permeability is deduced from the pressure behavior observed during the test.\textsuperscript{3,4}

The test consists of one or two drawdown periods and a buildup period. A WFT tool typically has a probe, a pressure gage and one or two pretest chambers. A schematic of the WFT tool with two chambers is given in Figure 2.1. The test is performed after the packer on the tool squeezes the mud cake out and properly isolates the formation. The probe is inserted into the formation through the mud cake, and formation fluid is withdrawn until the first chamber is full; the second chamber is filled at a higher flow rate. Then, the fluid withdrawal is stopped.

During the drawdown periods and subsequent buildup period, the pressure at the probe is recorded by a pressure transducer. Figure 2.2 illustrates the flow-rate schedule and expected pressure behavior at the probe for a WFT tool with two pretest chambers.
Figure 2.1 - Wireline formation testing tool with two chambers.
Figure 2.2 - Flow rate schedule and expected pressure response to WFT tool.
2.2. Literature Review

2.2.1. Permeability Estimation from WFT Data

In the present analysis of the WFT test data, the flow geometry is oversimplified. Also, the drawdown and buildup portions of the test data are evaluated separately. The drawdown data is analyzed by using the steady state spherical flow with a correction factor for simplifying the flow geometry. Since there are two drawdown periods, this gives two different drawdown permeabilities (k_{d1} and k_{d2}). If the drawdown pressure reaches atmospheric pressure, an integration scheme on the WFT test data is used to determine the formation permeability (k_{dint}). The buildup portion of the test is analyzed by means of transient flow models. A radial or a spherical flow configuration is assumed. The use of transient spherical and radial flow models give another permeability value, spherical or radial permeability (k_{bs} or k_{br}). Therefore, four different permeabilities, which often differ significantly, are deduced from only one set of WFT test data. Also radial and vertical permeabilities cannot be determined from the spherical permeability.

2.2.1.1. Drawdown Analysis

It has been common practice to use the drawdown portion of test data to determine a quantitative value for formation permeability.\textsuperscript{3,4} Usually, the pressure at the probe stabilizes by the end of each drawdown period. The stabilized pressure is used to determine permeability. The
model for drawdown analysis assumes 1-D spherical flow during the test. It also includes a correction factor to compensate for the simplifying assumption of the flow pattern. The drawdown analysis offers spherical permeability. The spherical permeability in terms of the drawdown parameters is given by:\(^4,5\)

\[
k_s = 5660 \frac{q_d \mu}{(\Delta P)_d}
\]  

(2.1)

where \( k_s, \mu, q_d, \) and \((\Delta P)_d\) are the spherical formation permeability (md), fluid viscosity (cp), flow rate (cc/sec), and the stabilized pressure drop (psi) at the end of the drawdown period, respectively.

The spherical permeability is related to the radial and vertical permeabilities as follows:\(^6\)

\[
k_s = k_r^{2/3} k_z^{1/3}
\]  

(2.2)

where \( k_r \) and \( k_z \) are the radial and vertical permeabilities, respectively.

Both sets of drawdown data can be used in the equations above. Often, the permeability calculated from the first drawdown \( k_{d1} \) differ significantly from the permeability calculated from the second drawdown \( k_{d2} \). It is believed that \( k_{d2} \) is a better representation for a formation permeability value because some clean-up may have occurred during the first drawdown. It has also been proposed to take the average of the two drawdown permeabilities \( k_{avr} \).\(^7\)
Since Eq. (2.1) is semi-empirical, the spherical permeability determined from Eq. (2.1) is a qualitative estimate of true formation permeability. Even if Eq. (2.1) gives the actual spherical formation permeability, there is no unique solution for radial and vertical permeabilities given by the relationship in Eq. 2.2.

2.2.1.2 Integration Technique

Drawdown or buildup analysis of WFT data requires certain conditions to be satisfied. The conditions and assumptions in these models may not be met in some WFT tests.

In low-pressure and low-permeability reservoirs, the pressure at the probe may reduce to atmospheric pressure during both drawdowns; hence, the flow rate during the drawdown periods is not constant. Sometimes, steady state or even semi-steady state may not be reached at the end of the drawdown periods. In these cases, Eq. (2.1) cannot be used. The buildup analysis is also not applicable since the transient-flow solution is based on the constant flow-rate assumption.

An integration technique based on Darcy's equation for 1-D spherical flow has been developed to evaluate the permeability for WFT in which the difficulties mentioned above arise. It also has a correction factor compensating non-spherical flow geometry during the WFT test. In this technique, total fluid withdrawn needs to be known. The final equation is:
where $V$ is the total volume of the fluid withdrawn (cc); $P_i$ is the initial reservoir pressure (psi); $P_{wf}$ is the pressure at the probe (psi); $C_F$ is the correction factor.

A numerical technique is needed to compute the integral in the denominator of Eq. 2.3.

This integration technique also gives the spherical permeability, from which no unique solution can be obtained for radial and vertical permeabilities.

2.2.1.3. Buildup Analysis

Use of the buildup portion of WFT data (buildup analysis) to determine the formation permeability is a more elaborate technique. Basically, two flow models, spherical and radial geometry, are used to analyze the buildup portion of the test. Usually, both spherical and radial buildup plots of the test data are constructed. The plot showing a better straight line is considered to be the prevailing flow geometry during the test.
The equation for spherical buildup is as follows:\textsuperscript{3,4,8}

\[ P = P_i - m_s f_s(t) \]  \hspace{1cm} (2.4)

where \( m_s \) and \( f_s \) are the slope of the spherical straight line and the spherical time function, respectively. The equation for the slope of the spherical straight line is:

\[ m_s = \frac{q_1 \mu}{4 \pi k_s^{3/2}} \sqrt{\frac{\Phi \mu c_t}{\pi}} \]  \hspace{1cm} (2.5)

If the tool has only one chamber then the spherical time function is:

\[ f_s(t) = \frac{1}{\sqrt{t - t_1}} - \frac{1}{\sqrt{t}} \]  \hspace{1cm} (2.6)

For two-stage drawdown, superposition principle is used and the spherical time function becomes:

\[ f_s(t) = \frac{1}{\sqrt{t - t_1}} - \frac{1}{\sqrt{t}} + \frac{q_2}{q_1} \left[ \frac{1}{\sqrt{t - t_2}} - \frac{1}{\sqrt{t - t_1}} \right] \]  \hspace{1cm} (2.7)

If the radial pressure propagation is reached during the test, a conventional or modified version of the Horner plot is constructed to analyze the buildup test data.\textsuperscript{3,4} The radial buildup equation is:

\[ P = P_i - m_r f_r(t) \]  \hspace{1cm} (2.8)

where \( m_r \) and \( f_r \) are the slope of the radial straight line and radial time
function, respectively.

\[ m_r = \frac{q_1 \mu}{4 \pi k_f h} \]  

(2.9)

For the single chamber test tool, the radial time function is:

\[ f_r(t) = \ln \left( \frac{t}{t - t_1} \right) \]  

(2.10)

For the double chamber test tool, the radial time function becomes:

\[ f_r(t) = \ln \left( \frac{t}{t - t_1} \right) + \frac{q_2}{q_1} \ln \left( \frac{t - t_1}{t - t_2} \right) \]  

(2.11)

Buildup analysis offers another formation permeability value, spherical or radial permeability (\( k_{bs} \) or \( k_{br} \)).

If the buildup pattern is spherical, there is no unique solution for radial and vertical permeabilities.

2.2.2 Other Uses of WFT Data

In conjunction with well log evaluations, the fluid samples and pressure data obtained from the WFT can be used to determine initial formation pressure, to establish vertical pressure gradient. The recovered samples also provide information on fluid type, fluid density and viscosity, and water cut. The WFT is also used to identify
the production potential of the formation.

Recently, Desbrandes and co-workers\textsuperscript{12,13} developed an interpretation technique to determine in-situ wettability of the formation by using WFT data.

\subsection*{2.2.3. Problems Associated With WFT}

First of all, there is ambiguity and uncertainty in the interpretation techniques since the current interpretation methods are based on simplified flow geometries such as spherical or radial. Due to the oversimplification of the flow geometry, some errors will be inherent in the analysis.

The drawdown, integration, and spherical buildup analysis yields the spherical permeability. The spherical permeability is related to the horizontal and vertical permeabilities in such a way that a unique solution cannot be obtained for horizontal and vertical permeabilities.

When radial buildup prevails during the test, formation capacity can be determined from the slope of the straight line on the pressure vs radial time function plot. Formation thickness should be known in order to deduce the formation permeability from the formation capacity. Since only a small portion of the reservoir responds to the WFT tool, an effective thickness needs to be assigned to calculate the formation permeability from the formation capacity. Generally, the effective thickness is chosen to be 0.5 ft, and no justification for this other than
match with core analysis data has been given.

In low permeability reservoirs, the pressure at the probe declines to atmospheric pressure during the test; and the flow rate changes continuously. Since the tool cannot measure the changes in flow rate, the analysis of the test data becomes more ambiguous.

Up to five permeability values can be determined for a single set of WFT data. Very often, these permeabilities differ significantly. Usually, the first drawdown permeability is lower than the second drawdown permeability. The buildup permeability is an order of magnitude smaller than the drawdown permeabilities. An order of magnitude difference between the permeability derived from WFT and permeability from core analysis and production tests has also been reported.

There are also mechanical problems with the WFT tool. Quite often, it is difficult to keep a constant flow rate during the drawdown periods even in moderate and high permeability reservoirs.

2.3. Mathematical Modeling

2.3.1. Dimensionless Variables

To have a universal model, the solution was constructed in terms of dimensionless variables. The dimensionless parameters are defined as the same as those in well testing literature. \(^1,2\)
Dimensionless Pressure:

\[ P_D = \alpha_1 \frac{2\pi k_r h}{q \mu} [P_i - P] \]  \hspace{1cm} (2.12)

where \( \alpha_1 \) is 1 for Darcy units and 1.127 \( 10^{-3} \) for field units.

Dimensionless time:

\[ t_D = \alpha_2 \frac{k_r t}{\phi \mu c_t r_w^2} \]  \hspace{1cm} (2.13)

where \( \alpha_2 \) is 1 for Darcy units and 2.63679 \( 10^{-4} \) for field units.

\[ r_D = \frac{r}{r_w} \]  \hspace{1cm} (2.14)

\[ b_D = \frac{b}{r_w} \]  \hspace{1cm} (2.15)

\[ a_D = \frac{a}{r_w} \]  \hspace{1cm} (2.16)

\[ z_D = \frac{z}{r_w} \]  \hspace{1cm} (2.17)

\[ w_D = \frac{w}{r_w} \]  \hspace{1cm} (2.18)

\[ h_D = \frac{h}{r_w} \]  \hspace{1cm} (2.19)
2.3.2. Assumptions

The flow into the WFT probe is a 3-D physical phenomenon and the flow pattern is convergent. Neither 1-D spherical nor 1-D radial geometry is a true representative of the convergent flow geometry during the test. A schematic of the flow pattern during the test is illustrated in Figure 2.3. The formulation of the 3-D convergent flow into the probe required the following assumptions:

1. Darcy's law is valid;
2. there is only one phase flowing;
3. the fluid has constant and small compressibility;
4. the formation is homogeneous;
5. there is no supercharge effect;
6. the WFT probe has a square shape;
7. the flow rates during the drawdowns are constant but can be different.

Under these assumptions, the 3-D unsteady state diffusivity equation in cylindrical coordinates describes the convergent flow into the WFT probe. The derivation of the differential equation can be found in classical text books.
2.3.3. Differential Equation and Boundary Conditions

The governing differential equation is:

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left[ r_D \frac{\partial P_D}{\partial r_D} \right] + \frac{1}{r_D^2} \frac{\partial^2 P_D}{\partial \theta^2} + k_D \frac{\partial^2 P_D}{\partial z_D^2} = \frac{\partial P_D}{\partial t_D}$$

(2.21)

One initial condition and six boundary conditions are needed to solve this differential equation. The initial and boundary conditions existing during the WFT are as follows:

Initial condition:

$$t_D = 0 \quad P_D[r_D, \theta, z_D, 0] = 0$$

(2.22)

Boundary conditions:

1. $$r_D \to \infty \quad \lim_{t_D \to \infty} P_D[r_D, \theta, z_D, t_D] = 0$$

(2.23)

2. $$r_D = 1 \quad \left[ r_D \frac{\partial P_D}{\partial r_D} \right]_{r_D=1} = -\frac{2 \pi h_D}{a_D w_D}$$ at probe
Figure 2.3 - Flow geometry during the wireline formation testing.
Figure 2.4 - The model used in this study.
\[
\begin{bmatrix}
\begin{array}{c}
\partial P_D \\
\partial r_D
\end{array}
\end{bmatrix}_{r_D=1} = 0 \\
\text{elsewhere}
\] (2.24)

3. \( P_D[r_D, \theta, z_D, t_D] = P_D[r_D, \theta + 2\pi, z_D, t_D] \) (2.25)

4. \( \frac{\partial P_D}{\partial r_D}[r_D, \theta, z_D, t_D] = \frac{\partial P_D}{\partial r_D}[r_D, \theta + 2\pi, z_D, t_D] \) (2.26)

5. \( z_D = 0 \quad \frac{\partial P_D}{\partial z_D}[r_D, \theta, 0, t_D] = 0 \) (2.27)

6. \( z_D = h_D \quad \frac{\partial P_D}{\partial z_D}[r_D, \theta, h_D, t_D] = 0 \) (2.28)

The initial condition describes constant and initial pressure throughout the reservoir before the test. The first boundary condition is the constant and initial pressure in the undisturbed portion of the reservoir. The second boundary condition represents uniform flux at the probe and no-flux elsewhere. The third and fourth boundary conditions result from the periodicity of the problem in the angular direction. The fifth and sixth boundary conditions describe the no-flow boundaries at the top and bottom of the reservoir.
2.3.4. The Solution for Drawdown

The Laplace transformation was applied to the differential equation and the boundary conditions in the time domain. This reduced the differential equation to the Helmutz equation in the Laplace space. Then, the separation of variables technique was used to solve the Helmutz equation. The final solution in the Laplace domain is:

\[
\bar{P}_D(s) = \frac{1}{s^{3/2}} \frac{K_0(\sqrt{\lambda_m})}{K_1(\sqrt{\lambda_m})} + \frac{1}{s} \frac{2}{w_D^2} \sum_{m=1}^{\infty} \frac{BO}{m^2 \pi^2} TP + \\
\frac{1}{s} \frac{2}{a_D^2} \sum_{v=1}^{\infty} \frac{BV}{v^2} TS + \frac{4}{s} \frac{2}{w_D^2} \sum_{m=1}^{\infty} \sum_{v=1}^{\infty} \frac{BB}{m^2 \pi^2} \frac{TP}{v^2} \frac{TS}{v^2}
\]

(2.29)

where

\[
BO = \frac{K_0(\sqrt{\lambda_m})}{\sqrt{\lambda_m} K_1(\sqrt{\lambda_m})} \tag{2.30}
\]

\[
BV = \frac{K_v(\sqrt{s})}{\sqrt{s} K_{v+1}(\sqrt{s}) - v K_v(\sqrt{s})} \tag{2.31}
\]
The solution given above needs to be taken back into the real domain. This was accomplished by using the numerical Stehfest algorithm.\textsuperscript{15}

2.3.5. The Solution for Buildup

Once the solution for constant flow rate is obtained, the superposition principle is applied to Eq. (2.29) to model the buildup portion of the WFT test. For single drawdown, the buildup pressure is given as:

\begin{align}
BB &= \frac{K_v (\sqrt{\lambda_m})}{\sqrt{\lambda_m} K_{v+1} (\sqrt{\lambda_m}) - v K_v (\sqrt{\lambda_m})} \\
TP &= [\sin m\pi (b_D + w_D) - \sin m\pi b_D]^2 \\
TS &= \sin^2 v a_D + (1 - \cos a_D)^2 \\
\lambda_m &= \sqrt{s + \xi_m} \\
\xi_m &= m \pi \sqrt{k_D}
\end{align}

\textsuperscript{(2.32) (2.33) (2.34) (2.35) (2.36)}
\[ P_{Db} = q_{D1}[P_{D}(t_{D}) - P_{D}(t_{D}-t_{D1})] \]  \hspace{1cm} (2.37)

If two different drawdowns are applied before the shut-in then the pressure buildup response is:

\[ P_{Db} = q_{D1}[P_{D}(t_{D})+S] + U_{t_{D1}}(q_{D2}-q_{D1})[P_{D}(t_{D}-t_{D1})+S] - \\
U_{t_{D2}} q_{D2}[P_{D}(t_{D}-t_{D2})+S] \]  \hspace{1cm} (2.38)

where \( U_{t} \) is the unit step function defined as:

\[ U_{t_{Dj}} = \begin{cases} 0 & 0 \leq t_{D} \leq t_{Dj} \\ 1 & t_{D} \geq t_{Dj} \end{cases} \]  \hspace{1cm} (2.39)

and

\[ q_{Dj} = \frac{q_{i}}{q_{r}} \]  \hspace{1cm} (2.40)

where \( q_{r} \) is the reference flow rate. Any flow rate can be chosen as the reference flow rate. Here, the flow rate at the first drawdown was taken as the reference.

### 2.3.6. Proof of The Solution

The solution presented above was tested for validity by several means. First, the solution was substituted into the differential equation
Figure 2.5 - Comparison of the 3-D model with the partially penetrating well model
This study

Figure 2.6 - Comparison of the 3-D model with the slotted liner models.
and the boundary conditions. This has indicated that the solution given by Eq. (2.29) indeed satisfies the differential equation and the boundary conditions.

Separate solutions were derived for 2-D problems in (r,z) (partially penetrating well) and (r,θ) (slotted liner). When the partially-penetrating and slotted liner flow geometries were imposed on the final 3-D solution, the solution degenerates to both of 2-D solutions, increasing the confidence in the solution. Also, the 3-D solution was compared with 2-D solutions in the petroleum engineering literature. Figure 2.5 shows the results from the 3-D solution and the 2-D partially penetrating well model presented by Kuchuk and Kirwan.\(^\text{16}\) The solutions are compared for two different wellbore storage coefficients (C_D). The model by Kuchuk and Kirwan treats the wellbore as a line source. The 3-D solution assumes a cylindrical source well. The small deviation between the models at the very early time period is due to the different representation of the wellbore. In Figure 2.6, the pseudo skin for a well completed with a slotted liner is illustrated. The pseudo skin was computed from the large-time expansion of Eq. (2.29) and from the equations presented in Refs. 17 and 18. Very good agreement was established between the 3-D solution and the model of Ref. 18. The solution given in Ref. 17 is an approximation and it is valid for small values of open fraction. Hence, the model of Ref. 17 deviates from the others at high values of open fraction. Additionally, when the radial flow geometry was imposed on the 3-D solution, the second, third, and fourth terms in Eq. (2.29) reduce to zero, and the solution collapses to the well-known form of the van Everd ingen-Hurst solution for cylindrical source wells.\(^\text{19}\)
2.3.7. Computational Procedure

The rigorous solution given by Eq. (2.29) was successfully computed using a floating point system computer. This recent system was necessary to compute the numerical values of the modified Bessel functions in the series solution. The fixed point computer proved unable to calculate sufficient terms in a reasonable time to have a converged solution.

The computation of the solution is more difficult in anisotropic formations. An accelerating scheme was used to speed up the convergence of the series solution for anisotropic problems.

To have a stabilized and converged solution, 100 terms on $v$ and 600 terms on $m$ were computed in the summation term.

The numerical value of the modified Bessel function was determined using the package developed by Amos.$^{20}$

Once a converged and stabilized solution was accomplished in the Laplace space, the numerical Stehfest algorithm$^{15}$ was used to obtain the solution in real time domain.

2.4. Evaluation of Current WFT Analysis Techniques

An evaluation of the methods currently being used for determining the permeability from WFT data was made by using the solution obtained in the study. After being programmed into a floating-point system
computer, the model was used to simulate the pressure behavior of a well undergoing WFT. For a given set of hypothetical reservoir data listed in Table 2.1, the pressure-time relationship was generated using the computer. Then, computer-generated pressure behavior was analyzed using current interpretation techniques.

**Table 2.1 - Reservoir data for the simulated tests.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st drawdown flow rate</td>
<td>0.454 cc/sec</td>
</tr>
<tr>
<td>2nd drawdown flow rate</td>
<td>0.625 cc/sec</td>
</tr>
<tr>
<td>Formation thickness</td>
<td>20 ft</td>
</tr>
<tr>
<td>Viscosity</td>
<td>2 cp</td>
</tr>
<tr>
<td>Permeability</td>
<td>100 md</td>
</tr>
<tr>
<td>Formation thickness</td>
<td>20 ft</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.2</td>
</tr>
<tr>
<td>Compressibility</td>
<td>5. E -06</td>
</tr>
<tr>
<td>Wellbore radius</td>
<td>0.5 ft</td>
</tr>
<tr>
<td>Probe radius</td>
<td>0.5 inches</td>
</tr>
</tbody>
</table>

Eq. (2.1) was used to determine the drawdown permeabilities. The spherical and radial buildup analyses were done by using Eqs. (2.4) - (2.7) and Eqs. (2.8) -(2.11).
Figure 2.7 - Simulated reservoir response to WFT tool.
2.4.1. Isotropic Media

The computer generated pressure-time relationship in an isotropic porous medium of 100 md is given in Figure 2.7 for different wellbore sizes. It seems that, for the wellbore sizes studied, the pressure response of the reservoir to the WFT tool is not affected by the wellbore size.

The spherical and radial buildup plots of the test in the hundred millidarcy reservoir are also illustrated in Figures 2.8 and 2.9. The same plots for other permeability values are shown in Figures 2.10 - 2.13. It appears that both the radial and spherical buildup plots give straight lines. However, it is not clear at that point if these are true straight lines or not. The shape of the curve on the plots suggests that what seems to be a straight line on the buildup plots may be the result of an inflection point.

At this point, further investigation is needed to validate the legitimacy of the straight lines. First of all, if the buildup portion of the test is radial or spherical then the drawdown portion of the test should be radial or spherical. In Figures 2.14 and 2.15, the drawdown portion of the simulated test in the hundred millidarcy reservoir is analyzed. As can be seen, it gets harder to find a straight line on the radial and spherical drawdown plots. This raises more doubts on the legitimacy of the straight lines on the buildup plots.

In order to prove the existence of the straight lines, the pressure derivative behavior of the simulated test should be examined. If the straight lines on the buildup plots are true then the pressure derivative with respect to radial and spherical time functions should be constant in
Figure 2.8 - Buildup pressure vs radial time function for Test 1.

$k_r = 100 \text{ md}$

$k_D = 1.0$
Figure 2.9 - Buildup pressure vs spherical time function for Test 1.

$kr = 100 \text{ md}$

$k_D = 1.0$
Figure 2.10 - Buildup pressure vs radial time function for Test 2.

\begin{align*}
    k_r &= 50 \text{ md} \\
    k_D &= 1.0
\end{align*}
Figure 2.11 - Buildup pressure vs spherical time function for Test 2.

$k_r = 50 \text{ md}$

$k_D = 1.0$
Figure 2.12 - Buildup pressure vs radial time function for Test 3.

$k_r = 25$ md
$k_D = 1.0$
Figure 2.13 - Buildup pressure vs spherical time function for Test 3.

$kr = 25$ md

$kd = 1.0$
Figure 2.14 - Drawdown pressure vs radial time function for Test 1.
Figure 2.15 - Drawdown pressure vs spherical time function for Test 1.
Figure 2.16 - Buildup pressure and pressure derivative for Test 1.
Figure 2.17 - Buildup pressure and pressure derivative for Test 1.
Table 2.2 - Evaluation of current interpretation techniques in isotropic formations.

<table>
<thead>
<tr>
<th>Well radius (ft)</th>
<th>Assigned Permeability</th>
<th>1st DD</th>
<th>2nd DD</th>
<th>Spherical Buildup</th>
<th>Radial Buildup</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>100.0</td>
<td>195.5</td>
<td>195.4</td>
<td>70.7</td>
<td>37.80</td>
</tr>
<tr>
<td>0.50</td>
<td>10.0</td>
<td>19.6</td>
<td>19.6</td>
<td>7.1</td>
<td>3.90</td>
</tr>
<tr>
<td>0.50</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
<td>0.7</td>
<td>0.38</td>
</tr>
<tr>
<td>0.50</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.04</td>
</tr>
<tr>
<td>0.75</td>
<td>100.0</td>
<td>194.3</td>
<td>194.2</td>
<td>68.2</td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>100.0</td>
<td>193.1</td>
<td>193.1</td>
<td>67.3</td>
<td></td>
</tr>
</tbody>
</table>
the range that the straight lines exist. If the straight lines are result of an inflection point then the pressure derivative would make a minimum or a maximum in the range that buildup plots display straight lines. The pressure derivative with respect to radial and spherical time functions were calculated and plotted in Figures 2.16 and 2.17 along the pressure itself. As can be seen, the pressure derivative indeed has a minimum on both buildup plots in the range that the buildup pressure vs time function plot displays a straight line. Therefore, the straight lines on the buildup plots are not real; rather, they are result of the inflection points. Hence, the permeabilities calculated from the slope of false straight lines would be in error.

Regardless of the questions on the legitimacy of the straight lines, the best possible straight lines were drawn on the buildup plots. The permeabilities were calculated from the slope of straight lines using Eqs. (2.5) and (2.9). The results from the buildup plots for the isotropic porous media are given in Table 2.2.

A comparison of the calculated spherical to the assigned spherical permeabilities shows that the spherical buildup analysis underestimates the assigned permeabilities by a factor of about 0.7, regardless of the value of the permeability.

In radial buildup analysis, an effective thickness of 0.5 ft was considered. The radial buildup underestimates the permeability by a factor of about 0.4.

The drawdown analysis technique was also applied to the simulated
tests. The permeabilities were determined from Eq. (2.1). Almost the same permeability values are obtained from the first and second drawdown periods. But the drawdown analysis consistently overestimates the assigned permeability values by a factor of about two, regardless of the assigned permeability.

2.4.2. Anisotropic Media

In most of the subsurface formations, the horizontal permeability of the formation is greater than the vertical permeability. Due to this permeability difference, hydrocarbon bearing formations are anisotropic. In multidimensional flow problems, the formation anisotropy affects the response of the formation to the external disturbances.

Here, the formation anisotropy is expressed in terms of the permeability ratio defined below.

\[ k_D = \frac{k_z}{k_r} \]  

(2.41)

Computation of the solution is more difficult for anisotropic cases. An accelerating scheme \(^{21}\) was used to speed up the convergence of the series solution for anisotropic problems.

As in isotropic cases, the pressure-time relationship was computed, and the current analysis techniques were applied to the generated pressure behavior. Figure 2.18 displays the series of the tests simulated in the anisotropic porous media. As can be seen, the pressure behavior
Figure 2.18 - Simulated pressure response of anisotropic reservoirs to WFT tool.
during WFT is a strong function of formation anisotropy.

The radial and spherical buildup plots of the tests in anisotropic formations are shown in Figures 2.19 - 2.26. The permeabilities from the buildup plots are listed in Table 2.3.

In the buildup analysis of anisotropic formations, the straight line shortens with increasing anisotropy. The more the anisotropy, the harder it becomes to draw a straight line on both the radial and spherical plots.

The drawdown permeabilities were determined from Eqs. (2.1) and (2.2). Contrary to that in an isotropic formation, the first and second drawdown permeabilities are not the same. The second drawdown permeability is consistently lower than the first one. A comparison of the assigned permeabilities to the calculated ones reveals that the permeability overprediction from the drawdown analysis grows with increasing anisotropy of the media. Therefore, the difference in drawdown permeabilities is not necessarily an indication of formation cleanup.

The results of the anisotropic cases are summarized in Table 2.3.

2.5. Maximum Flow Rate

As mentioned earlier, the pressure at the WFT probe may decrease to atmospheric pressure during the test. Since the flow rate varies during the test, it is difficult to analyze the test data in such a case.
Figure 2.19 - Simulated radial pressure buildup in an anisotropic formation for Test A.1.

kr = 100 md
kD = 0.75
Figure 2.20 - Spherical pressure buildup in an anisotropic formation (Test A.1).

$k_r = 100 \text{ md}$

$k_D = 0.75$
Figure 2.21 - Radial pressure buildup in an anisotropic formation (Test A.2).

\[ k_r = 100 \text{ md} \]
\[ k_D = 0.50 \]
Figure 2.22 - Spherical pressure buildup in an anisotropic formation (Test A.2).

\[ k_r = 100 \text{ md} \]
\[ k_D = 0.50 \]
Figure 2.23 - Radial pressure buildup in an anisotropic formation (Test A.3).

$k_r = 100 \text{ md}$

$k_D = 0.25$
Figure 2.24 - Spherical buildup pressure in an anisotropic formation (Test A.3).

Test A.3

$k_r = 100 \text{ md}$

$K_D = 0.25$
Figure 2.25 - Radial pressure buildup in an anisotropic formation (Test A.4).
Figure 2.26 - Spherical pressure buildup in an anisotropic formation (Test A.4).

$k_f = 100 \text{ md}$

$k_D = 0.10$
Table 2.3 - Evaluation of current interpretation techniques in anisotropic formations.

<table>
<thead>
<tr>
<th>Assigned Permeability, md</th>
<th>Calculated Permeability, md</th>
<th>1st DD</th>
<th>2nd DD</th>
<th>Spherical Buildup</th>
<th>Radial Buildup</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_D 1.00</td>
<td>k_S 100.0</td>
<td>195.5</td>
<td>195.4</td>
<td>70.7</td>
<td>37.80</td>
</tr>
<tr>
<td>0.75</td>
<td>90.8</td>
<td>180.0</td>
<td>130.7</td>
<td>65.1</td>
<td>37.2</td>
</tr>
<tr>
<td>0.50</td>
<td>79.4</td>
<td>164.6</td>
<td>119.9</td>
<td>57.4</td>
<td>33.2</td>
</tr>
<tr>
<td>0.25</td>
<td>63.0</td>
<td>142.2</td>
<td>103.3</td>
<td>46.4</td>
<td>28.8</td>
</tr>
<tr>
<td>0.10</td>
<td>46.4</td>
<td>120.3</td>
<td>87.4</td>
<td>34.0</td>
<td>25.6</td>
</tr>
</tbody>
</table>
Therefore, the drawdown flow rates should be regulated to avoid atmospheric pressure at the probe.

In this study, an equation for estimating the maximum flow rate to avoid atmospheric pressure was derived:

$$ q_{\text{max}} = \frac{k_r}{141.22 \mu C} \left[ P_i - P_{\text{min}} \right] $$  \hspace{1cm} (2.42)

where $q_{\text{max}}$, $P_i$, $P_{\text{min}}$, and $C$, are, respectively, the maximum drawdown flow rate (bbl/day), initial reservoir pressure (psi), minimum pressure desired at the probe (psi), and a constant which is a function of formation anisotropy.

The numerical values for $C$ are given in Table 2.4.

Table 2.4 - Numerical values of the coefficient $C$

<table>
<thead>
<tr>
<th>$k_D$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>37.5</td>
</tr>
<tr>
<td>0.75</td>
<td>56.0</td>
</tr>
<tr>
<td>0.50</td>
<td>61.3</td>
</tr>
<tr>
<td>0.25</td>
<td>70.7</td>
</tr>
<tr>
<td>0.10</td>
<td>83.6</td>
</tr>
</tbody>
</table>
2.6. New Design for WFT Tool

The ultimate purpose of this project was to develop an interpretation technique to determine horizontal and vertical permeabilities, a local skin factor, and the initial reservoir pressure. Different combinations of these parameters may give the same pressure behavior at the probe, resulting in a non-unique solution. To avoid non-unique solutions, a new WFT tool with two pads is needed. The tool is equipped with a main pad and an observation pad. At the main pad, the formation fluid will be produced and pressure will be recorded. At the second pad, which will be some distance away from the main pad, only pressure measurements will be done. A schematic of the new tool is given in Figure 2.27.

A feasibility study of the new tool was conducted for isotropic and anisotropic formations.

2.6.1. Isotropic Media

The pressure behavior at the main pad and at the observation pad were simulated for different reservoir permeabilities. Figure 2.28 shows the pressure behavior at the main pad and observation pad at distances of 1.5, 2, 3, and 4 inches. The observation pad at 1.5-in. away from the main pad feels the pressure wave only 0.00013 seconds after the beginning of the test. The pressure drop at the main probe is 26.15 psi at the end of the first drawdown and 35.96 psi at the end of second drawdown. The pressure drop at the observation pad is 6.93 psi at the
Figure 2.27 - Recommended design for new WFT tool (Double pad tool).
Figure 2.28 - Pressure response of an isotropic formation to new WFT tool.
Table 2.5 - Response to the new double pad tool in isotropic formations.

<table>
<thead>
<tr>
<th>Interval (inches)</th>
<th>Time to feel pressure wave, sec</th>
<th>ΔP₁, psi</th>
<th>ΔP₂, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.00013</td>
<td>6.93</td>
<td>9.53</td>
</tr>
<tr>
<td>2.0</td>
<td>0.00026</td>
<td>4.54</td>
<td>6.26</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0007</td>
<td>2.64</td>
<td>3.65</td>
</tr>
<tr>
<td>4.0</td>
<td>0.0025</td>
<td>1.79</td>
<td>2.47</td>
</tr>
<tr>
<td>Probe</td>
<td>26.15</td>
<td></td>
<td>35.96</td>
</tr>
</tbody>
</table>
end of first drawdown and 9.59 psi at the end of second drawdown. If the observation pad is 4-in. away, pressure drops of 1.77 and 2.48 psi are monitored at the end of the first and second drawdown periods, respectively. A summary of the results is given in Table 2.5.

As can be seen, the pressure wave travels fast in the vertical direction, but the amplitude of the wave is low farther away from the main pad. The pressure drop at 1.5-inches away from the main pad is about 25 percent of the pressure drop at the main pad. The pressure drop at 4-inches away is only about 7 percent of the pressure drop at the main pad. This suggests that only a very small portion of the reservoir responds to fluid withdrawal created by the WFT tool, and the permeabilities determined from the WFT represents the formation about 1 ft. above and below the main pad.

2.6.2. Anisotropic Media

The response of the anisotropic formations to the double pad tool was investigated. The results are presented in Figure 2.29 and Table 2.6. The pressure drop in the observation pad is lower in anisotropic formations than that in isotropic formations due to low vertical permeability. In the case of 0.1 formation anisotropy (the ratio of vertical to horizontal permeability), the pressure drop at the 4-in. interval is only 2% of the pressure drop at the main pad. This implies that the portion of the reservoir responding to the WFT tool is even smaller in anisotropic formations.
Figure 2.29 - Pressure response of an anisotropic formation to new WFT tool.
Table 2.6 - Response to the new double pad tool in anisotropic formations.

<table>
<thead>
<tr>
<th>Formation Anisotropy</th>
<th>Time to feel pressure wave, sec</th>
<th>$\Delta P_1$, psi</th>
<th>$\Delta P_2$, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 1/2&quot;</td>
<td>4&quot;</td>
<td>1 1/2&quot;</td>
</tr>
<tr>
<td>1.0</td>
<td>0.00013</td>
<td>0.0025</td>
<td>6.93</td>
</tr>
<tr>
<td>0.5</td>
<td>0.00040</td>
<td>0.0040</td>
<td>4.60</td>
</tr>
<tr>
<td>0.1</td>
<td>0.00260</td>
<td>0.0145</td>
<td>4.05</td>
</tr>
</tbody>
</table>
As expected, the observation pad feels the pressure wave at a later time in the anisotropic media because of low vertical permeability.

**2.7. Pressure Derivative Analysis**

Several combinations of horizontal and vertical permeabilities and local skin factor may produce the same pressure behavior at the WFT tool, resulting in a non-unique solution. To avoid the non-unique solution, another source of information is necessary. Even if the pressure behavior is not unique, the time rate of change pressure (pressure derivative) may be more characteristic. In order to enhance the interpretation of transient pressure data and to find a unique solution, the pressure derivative has been used in the well testing literature. The pressure derivative function is defined as follows:

\[
\frac{\partial P_D}{\partial (\ln t_D)} = t_D \frac{\partial P_D}{\partial t_D}
\]  

(2.43)

The pressure derivative behavior during WFT was investigated. The pressure derivatives were computed in the Laplace space and converted into the real space by means of the Stehfest algorithm.

Several pressure derivative groups and time functions were tried to construct the pressure derivative plot. Buildup pressure derivative group versus buildup time has resulted in the most distinctive curve. The curves for different formation anisotropies have more character in this plot. Figure 2.30 shows the simulated buildup pressure derivative group as a
Figure 2.30 - Drawdown and buildup pressure derivative behavior for different formation anisotropy.
function of the buildup time. The buildup time is defined as the time from the beginning of the test minus drawdown time. The buildup pressure derivative group is the derivative of the pressure with respect to logarithm of the buildup time.

Formation anisotropy and permeability may be uniquely calculated by using the pressure derivative analysis of the field data if the pressure derivative of the field data can be computed. Usually the pressure derivative is calculated numerically, and is very sensitive to the noise in the measured data. Therefore, steps should be taken to minimize the noise in the pressure measurements.

2.8. Conclusions

1. A 3-D analytical model for the convergent flow geometry of the WFT tool has been developed. The model regenerates 2-D and 1-D solutions.

2. The model was used to evaluate the validity of the current interpretation techniques. Analysis of current interpretation techniques has indicated that both spherical and radial buildup plots may result in a straight line contrary to the belief that only one plot shows a straight line for a given test data.

3. The pressure derivative analysis has shown that the straight lines usually identified on the buildup plots are not true straight lines. They are the result of the inflection points.
4. Drawdown analysis overestimates permeability by a factor of 2 in isotropic formations and by a factor of about 1.5 in anisotropic formations.

5. Spherical buildup plots of the test data underestimates permeability by factor of 0.7.

6. If a 0.5 feet effective formation thickness is assumed then radial buildup analysis underestimates the permeability by a factor of 0.4.

7. A new interpretation technique is needed. Formation anisotropy must be explicitly included in the interpretation method since the pressure response to the WFT tool is a strong function of the formation anisotropy.

8. Wellbore size does not have any significant effect on pressure behavior during WFT.

9. A double pad tool and the pressure derivative analysis will enhance the interpretation of the WFT data, providing unique solution for the radial and vertical permeabilities.

2.9. Recommendations and Future Work

1. The model presented here does not consider the supercharging effect resulting from the difference between hydrostatic wellbore pressure and formation pressure. The model should be extended to
include the supercharging effect.

2. All the current interpretation methods are qualitative and inherent errors exist due to the oversimplification of the flow geometry. Hence, a new interpretation technique based on type-curve matching should be developed.

3. A new test tool with double pads will provide more information about the formation being tested. Therefore, the WFT tools currently in use should be modified.

4. To avoid the difficulties of keeping a constant flow rate during the test, the test should be conducted applying a constant pressure at the WFT probe.
CHAPTER III

Pressure Losses Across Gravel Packs

3.1. Introduction

Gravel packing is a common sand control technique used in many wells in the Gulf of Mexico as well as many other areas of the world. Much time and effort have been devoted to improving mechanical placement of the gravel to prevent screen collapse, reduce pressure losses, increase packing efficiency, and minimize formation damage. But, no model has been presented to calculate the pressure drop across the gravel pack.

The motivation for this study was the fact that, in system analysis, the contribution of each component to total pressure drop must be known, including the contribution of the gravel packed casing/screen annulus. Also, in the efficiency evaluation of production operations such as perforating, acidizing, and injection, the pressure drop across the gravel pack should be known.

Today, the contribution of the gravel packed annulus to total pressure drop is either included in the skin factor or determined by using a linear or radial flow model, both of which are not accurate.
Figure 3.1 - Near wellbore schematic of a gravel-packed well.
Figure 3.2 - Cross-section of a gravel-packed well.
A near wellbore schematic of a gravel packed well is given in Figures 3.1 and 3.2. The flow in the perforation tunnel surrounded by casing and cement is linear. But, flow across the gravel pack is neither linear nor radial. As soon as the fluid enters the casing/screen annulus, a divergent flow pattern is formed. Therefore, the objective in this study is to develop a predictive model to simulate the divergent flow across the gravel pack.

3.2. Literature Review

Sand production in oil and gas wells is one of the oldest oilfield problems. Today, in major oil-producing areas like the Gulf of Mexico, Alaska, and Venezuela, many unconsolidated formations are encountered. There are basically three sand control mechanisms; decreasing flow rate, bridging the sand mechanically (gravel pack), and increasing formation strength. Mechanical methods of sand control are the most successful and widely used. In this method, a mass of gravel is placed between the formation and screen to hold formation sand in place.

Many investigators have studied the design parameters that control the success of an effective gravel pack. Among the important parameters are formation sand size, gravel size, well inclination, carrying capacity of completion fluids and operation procedures. The basic problem in gravel packing is to pack the gravel effectively and uniformly without cavities. Many experimental studies have been conducted on how to accomplish a compacted gravel pack. Based upon experimental studies,
many rules of thumb have been proposed. There are also several mathematical studies to simulate the gravel packing operation. It has been claimed that mathematical models can estimate the gravel distribution and location of cavities in the gravel body. Also, several operational techniques have been proposed to repair the deficiencies.

3.2.1. Linear Flow Model

In the linear flow model, the gravel packed section is assumed to be an extension of the perforation tunnel. For Darcy flow, the linear flow model is formulated as follows:

\[ \Delta P = \frac{q \mu L}{kA} \]  

(3.1)

where

\( \Delta P \) : Pressure losses across gravel pack, atm
\( q \) : Flow rate per perforation, cc/sec
\( \mu \) : Viscosity, cp
\( k \) : Gravel permeability, darcy
\( A \) : Cross sectional area of perforation, sq cm
\( L \) : Gravel thickness, cm

If the flow rate is relatively high, the flow is non-Darcy, and the pressure drop across the pack is given as
Here $\rho$ is the fluid density in gm/cc, and $\beta$ is the non-Darcy flow coefficient. Several empirical equations are available in the literature to calculate $\beta$.

The linear flow model does not consider the divergence of the fluid in the gravel packed annulus. Therefore, in linear flow, the area open to flow is assumed to be smaller than it actually is. Hence, it is expected that the linear flow model would overestimate the pressure drop across the pack.

3.2.2. Radial Flow Model

In the radial flow model, a complete ring along perforations around the casing is assumed to be open to flow.\textsuperscript{31} In other words, the area open to flow is much larger in radial model than it actually is in gravel pack flow geometry. As the linear flow model, the radial flow model does not have the ability to include the flow divergence across the pack. Therefore, it is expected that the radial flow model would underestimate the pressure losses across the gravel pack.

For low velocity radial flow, the pressure losses is given by:

$$\Delta P = \frac{q \mu L}{k A} + \frac{q^2 \rho \beta L}{A^2}$$

(3.2)
where

\[ h : \text{Perforation diameter, cm} \]
\[ r_c : \text{Casing radius, cm} \]
\[ r_s : \text{Screen radius, cm} \]

Under non-Darcy flow conditions, the radial flow equation becomes

\[
\Delta P = \frac{q \mu}{2 \pi k h} \ln \frac{r_c}{r_s} + \frac{q^2 \rho \beta}{4 \pi^2 h^2} \left( \frac{1}{r_s} - \frac{1}{r_c} \right) \tag{3.4}
\]

### 3.2.3. Pseudo-Skin Models

It is also common to include the contribution of the gravel packed annulus to the total pressure drop in terms of a skin factor. Details of these methods have been presented by Beggs\textsuperscript{30}, Jones et al.\textsuperscript{32}, Jones and Thorp\textsuperscript{33}, Himmatramka\textsuperscript{34}, and Buell and Crafton\textsuperscript{35}.

### 3.3. Experimental Work

The calculation of the pressure drop across a gravel pack does not yield to mathematical modeling very readily; consequently, electrical analogs for a modeling technique was considered.
3.3.1. Electrical Analogy to Fluid Flow in Porous Media

The differential equation for electrical current flow is identical to the differential equation governing the flow of fluids in porous media.\textsuperscript{36,37} However, the particular equations for electrical and fluid flow are different but analogous. The particular equation for radial fluid flow in porous media is the Darcy's equation.

\[
\Delta P = \frac{q \mu}{2 \pi k h} \ln \frac{r_c}{r_s} \tag{3.5}
\]

The particular equation for electrical current flow is the Ohm's law.

\[
\Delta E = \frac{I}{2 \pi C h} \ln \frac{r_c}{r_s} \tag{3.6}
\]

where

\[\begin{align*}
\Delta E & : \text{Electrical potential, volts} \\
I & : \text{Current, amps} \\
C & : \text{Electrical conductivity}
\end{align*}\]

If we define a set of dimensionless variables, even the particular equations for electrical and fluid flow can be made identical. Let us define the dimensionless pressure and dimensionless potential as follows;
Dimensionless pressure:

\[ \Delta P_D = \frac{2 \pi k h}{q \mu} \Delta P \]  

(3.7)

Dimensionless potential:

\[ \Delta E_D = \frac{2 \pi C h}{I} \Delta E \]  

(3.8)

If we write the particular equations in terms of new variables, Eqs. (3.5) and (3.6) become

\[ \Delta P_D = \ln \frac{r_c}{r_s} \]  

(3.9)

\[ \Delta E_D = \ln \frac{r_c}{r_s} \]  

(3.10)

As can be seen from Eqs. (3.9) and (3.10), in dimensionless form, even the particular equations for fluid flow in porous media and electrical flow are identical.

Since an analogy results between electrical and fluid flow and it is easier to control the current flow, an electrical analogy apparatus was constructed to carry out the experiments.
Figure 3.3 - Experimental apparatus.
3.3.2. Experimental Apparatus

A schematic of the apparatus is given in Figure 3.3. The apparatus basically consists of two concentric cylinders, inner and outer. The concentric cylinders were fitted with a non-conductive, flat bottom. The inner cylinder is electrically conductive and analogous to the screen in gravel packed well. To simulate complete radial flow, an electrically conductive outer cylinder was chosen. A fluid of known salinity, representing a gravel packed annulus, was introduced into the annular space between the two electrodes to a depth of 3 inches.

First, the apparatus was checked by measuring the resistance between the electrodes in the above described configuration and calculating the conductivity of the fluid. When calculations yielded acceptable agreement with published correlations of water resistivity as a function of salinity and temperature, the apparatus was assumed to be calibrated. To avoid hysteresis, an A-C source was used and experiments were repeated with different salinities. The conductance measured above between the cylindrical electrodes is analogous to true radial flow, and is the standard to which other experiments were compared.

To simulate a gravel-packed well, the electrically conductive outer cylinder was replaced by a non-conductive cylinder representing the well casing. A thin circular copper electrode representing the perforation was placed at the non-conductive outer cylinder. Measurement of conductance between the inner electrode and perforation electrodes are analogous to the flow of fluid exiting a perforation and traveling to the screen through the gravel pack.
To investigate the effect of perforation size, the size of the circular electrode on non-conductive outer cylinder was changed from 1/4 to 1 1/8 inches. Sensitivity to screen size was examined by changing the diameter of the inner conductive cylinder.

The electrical potential used was a variable A-C voltage source with a range of 0-12 volts. Readings were taken at several values of voltage for each case to insure accuracy. The A-C source was used to eliminate polarization and hysteresis effects in the water.

3.3.3. Productivity Ratio

In oil production engineering literature, it is customary to evaluate well performance relative to the productivity of an open hole which completely penetrates the formation. The productivity ratio (PR) is the ratio of the productivity index of a well in any condition to the productivity index of open completion well. The productivity index is expressed as the ratio of the production rate to the pressure drawdown.27

\[ \text{PR} = \frac{(PI)_{\text{packed}}}{(PI)_{\text{open}}} \]  

(3.11)

In case of current flow, the productivity ratio is given as follows;

\[ \text{PR} = \frac{C_{\text{packed}}}{C_{\text{open}}} \]  

(3.12)
3.3.4. Experimental Results

A total of 16 experiments was performed on the apparatus described in section 3.3.2. The results are condensed in terms of productivity ratio and listed in Table 3.1. A plot of productivity ratio vs the ratio of casing/screen radii is shown in Figure 3.4. This plot indicates that the productivity of a gravel pack installed inside casing is greatly reduced relative to a standard open hole completion.

<table>
<thead>
<tr>
<th>Screen Radius, cm</th>
<th>2.93</th>
<th>1.93</th>
<th>1.30</th>
<th>0.66</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.645</td>
<td>0.250</td>
<td>0.186</td>
<td>0.141</td>
<td>0.084</td>
</tr>
<tr>
<td>1.960</td>
<td>0.168</td>
<td>0.124</td>
<td>0.091</td>
<td>0.051</td>
</tr>
<tr>
<td>3.870</td>
<td>0.111</td>
<td>0.080</td>
<td>0.059</td>
<td>0.032</td>
</tr>
<tr>
<td>6.780</td>
<td>0.058</td>
<td>0.034</td>
<td>0.022</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Table 3.1. Experimental Results.
Figure 3.4 - Result of the experiments on electrical analogy apparatus.
3.4. Mathematical Modeling

To have a universal solution, a model was constructed in terms of dimensionless variables.

3.4.1. Dimensionless Variables

The dimensionless variables are defined as follows:

Dimensionless pressure:

\[ P_D = \alpha_1 \frac{2 \pi k_r h}{q \mu} [P - P_{wf}] \] (3.13)

where \( \alpha_1 \) is 1 for Darcy units, and \( 1.127 \times 10^{-3} \) for field units.

Dimensionless radius:

\[ r_D = \frac{r}{r_s} \] (3.14)

Dimensionless elevation:

\[ z_D = \frac{z}{h} \] (3.15)
Dimensionless permeability (anisotropy):

\[ k_D = \frac{k_z}{k_r} \quad (3.16) \]
\[ a_D = \frac{a}{r_s} \quad (3.17) \]
\[ w_D = \frac{w}{h} \quad (3.18) \]
\[ A_D = \sum_{j=1}^{np} a_{Dj} w_{Dj} \quad (3.19) \]

**3.4.2. Assumptions**

The flow across the gravel pack is three dimensional (3-D) and it has a divergent flow pattern. The modeling of 3-D divergent flow required the following assumptions.

1. Darcy type flow,
2. fluid has small and constant compressibility,
3. fluid viscosity is constant,
4. the annulus between casing and screen is completely gravel-filled,
5. the screen is concentrically placed in the casing,
6. and the most important assumption, the perforations have a square shape of area equal to circular perforations.
Under these assumptions, the 3-D diffusivity equation in cylindrical coordinates describes the divergent flow of fluid from the perforations to the screen across the gravel pack.

A schematics of the model is given in Figure 3.5.

3.4.3. Differential Equation and Boundary Conditions

The non-dimensional form of the diffusivity equation and its boundary conditions for flow across the gravel pack are as follows;

The differential equation;
\[
\frac{1}{r_D} \frac{\partial}{\partial r_D} \left[ r_D \frac{\partial P_D}{\partial r_D} \right] + \frac{1}{r_D^2} \frac{\partial^2 P_D}{\partial \theta^2} + \frac{r^2}{h^2 k_r} \frac{\partial^2 P_D}{\partial z^2} = 0 \quad (3.20)
\]

The boundary conditions;

1. \( r_D = 1 \) \quad \[ P_D \left[ 1, \theta, z_D \right] = 0 \quad (3.21) \]

2. \( r_D = r_{De} \) \quad \[ \frac{\partial P_D}{\partial r_D} = - \frac{2 \pi}{A_D} \] at perforation faces \quad (3.22)

\[ \frac{\partial P_D}{\partial r_D} = 0 \] elsewhere
Figure 3.5 - The model used in this study.
3. \[ P_D[r_D, \theta, z_D] = P_D[r_D, \theta + 2\pi, z_D] \] (3.23)

4. \[ \frac{\partial P_D}{\partial r_D}[r_D, \theta, z_D] = \frac{\partial P_D}{\partial r_D}[r_D, \theta + 2\pi, z_D] \] (3.24)

5. \[ z_D = 0 \quad \frac{\partial P_D}{\partial z_D}[r_D, \theta, 0] = 0 \] (3.25)

6. \[ z_D = 1 \quad \frac{\partial P_D}{\partial z_D}[r_D, \theta, 1] = 0 \] (3.26)

The first boundary condition is a constant pressure at the wellbore. The second represents the constant flow rate at the perforation face and no-flow at the casing face. The third and fourth boundary conditions exist due to the periodicity in the angular (\(\theta\)) direction. The fifth and sixth boundary conditions describe the no-flow boundaries at the top and bottom of the gravel-packed annulus.

### 3.4.4. Solution Method

When the 3-D diffusivity equation is coupled with the boundary conditions prevailing in flow across the gravel pack, a boundary value problem is obtained. There exist several mathematical techniques to solve the boundary value problems. Among these methods are Green's function, finite differences, integral transformations, and separation of
variables. Here, separation of variables technique was used to solve the boundary value problem. The solution to Eq.(3.20) is expressed as:

\[ P_D[r_D, \theta, z_D] = R(r_D) \psi(\theta) Z(z_D) \] (3.27)

where \( R(r_D), \psi(\theta), \) and \( Z(z_D) \) are non-zero functions.

When the definition (3.27) was substituted into the boundary value problem, three ordinary differential equations resulted. Using the ordinary differential equations and their boundary conditions, expressions for \( R(r_D), \psi(\theta), \) and \( Z(z_D) \) were obtained. The substitution of these expressions into Eq.(4.27) yielded the solution for \( P_D[r_D, \theta, z_D] \). The final solution has the form of an infinite Fourier-Bessel series.

\[
P_D[r_D, \theta, z_D] = \ln r_{Dc} + \frac{2}{A_D} \sum_{j=1}^{np} \frac{r_{Dc}}{a_{Dj}} \sum_{v=1}^{\infty} \frac{1}{v^3} TR * SS + \]

\[
\frac{2}{A_D} \sum_{j=1}^{np} \frac{1}{w_{Dj}} \sum_{m=1}^{\infty} \frac{BO}{m^2 \pi^2 \lambda_m} \left[ \sum_{i=1}^{np} TT_i a_{Dj} \right] TT_j + \]

\[
\frac{4}{A_D} \sum_{j=1}^{np} \frac{TT_j}{a_{Dj} w_{Dj}} \sum_{m=1}^{\infty} \sum_{v=1}^{\infty} \frac{BB}{m^2 \pi^2 v^2} GG \] (3.28)
The rest of the terms in Eq.(3.28) are defined as follows;

\[
\begin{align*}
TR &= \frac{[r_D - r_D^{-\nu}]}{[r_D^{\nu-1} + r_D^{-\nu-1}]} \quad (3.29) \\
SS &= \left[ \sum_{i=1}^{np} TS_i w_{Di} \right] TS_j + \left[ \sum_{i=1}^{np} TC_i w_{Di} \right] TC_j \quad (3.30) \\
GG &= \left[ \sum_{i=1}^{np} TS_i TT_i \right] TS_j + \left[ \sum_{i=1}^{np} TC_i TT_i \right] TC_j \quad (3.31) \\
TT_i &= \sin m\pi [b_{Di} + w_{Di}] - \sin m\pi b_{Di} \quad (3.32) \\
TS_i &= \sin v [c_{Di} + a_{Di}] - \sin v c_{Di} \quad (3.33) \\
TC_i &= \cos v c_{Di} - \cos v [c_{Di} + a_{Di}] \quad (3.34) \\
BO &= \left[ \begin{array}{cc} I_0 (\lambda_m r_{De}) - K_0 (\lambda_m r_{De}) \\ I_0 (\lambda_m) & K_0 (\lambda_m) \\ I_1 (\lambda_m r_{De}) - K_1 (\lambda_m r_{De}) \\ I_0 (\lambda_m) & K_0 (\lambda_m) \end{array} \right] \quad (3.35)
\end{align*}
\]
\[ BB = \frac{BB_1}{BB_2 + BB_3} \quad (3.36) \]

\[ BB_1 = \frac{I_v (\lambda_m r_{DC})}{I_v (\lambda_m)} - \frac{K_v (\lambda_m r_{DC})}{K_v (\lambda_m)} \quad (3.37) \]

\[ BB_2 = \frac{\lambda_m I_{v-1} (\lambda_m r_{DC}) - \frac{V}{r_{DC}} I_v (\lambda_m r_{DC})}{I_v (\lambda_m)} \quad (3.38) \]

\[ BB_3 = \frac{\lambda_m K_{v-1} (\lambda_m r_{DC}) + \frac{V}{r_{DC}} K_v (\lambda_m r_{DC})}{K_v (\lambda_m)} \quad (3.39) \]

\[ \lambda_m \] is the eigenvalue of the problem.

3.4.5. Proof of The Solution

The accuracy of the solution was checked by several means. First, the solution was substituted into the governing differential equation and into the boundary conditions. It was observed that the differential equation and boundary conditions were indeed satisfied.
Figure 3.6 - The comparison of the model with the experimental data.
Separate solutions were obtained for the two dimensional (2-D) problems in \((r,z)\) only and \((r,\theta)\) only. The final 3-D solution degenerates to both of 2-D solutions. Additionally, when the open hole situation was imposed on the 3-D analytical model, the second, third, and fourth terms in Eq. (3.28) reduce to zero and the solution collapses to the well-known form of the Darcy's equation for true radial flow.\(^{36}\)

Next, the solution was compared with the experimental data obtained from the electrical analogy apparatus. Results here are shown in Figure 3.6, comparing calculated values with measured values. Very good agreement was obtained. The small deviations can be attributed to experimental error and the assumption of a square perforation area.

### 3.4.6. Laminar Gas Flow Across Gravel Pack

The governing differential equation for low-velocity gas flow in porous media is non-linear. Non-linearity is caused by pressure-dependent viscosity and gas compressibility factor of the natural gases. But, by using proper pseudo variables, the differential equation can be linearized. Let us define the following pseudo variables.

Pseudo pressure:

\[
P_{pn} = \frac{\mu_i z_i}{P_i} \int_0^P \frac{P}{\mu(P) z(P)} dP \tag{3.40}
\]
Dimensionless pseudo pressure:

$$P_{pnD} = \alpha_1 \left[ \frac{2\pi k_r h}{\frac{P_{sc}}{Q_{sc}} \frac{\mu_i z_i}{P_i} \frac{T_i}{T_{sc}}} \right] \left[ P_{pni} - P_{pn} \right]$$  \hspace{1cm} (3.41)

Replacement of the dimensionless pressure by dimensionless pseudo pressure will linearize the gas flow equation very effectively. Therefore, the solution given by Eq. (3.28) can be easily adopted for gas flow.

3.5. Investigation of Sensitivity of Pressure Losses

The sensitivity of the pressure drop was investigated by making multiple calculations, allowing a particular variable to change. For example, the dependence of $P_D$ on the casing size to screen size ratio is shown in Figure 3.7. Figure 3.7 shows that the pressure drop across the gravel pack is essentially independent of screen diameter.

The effect of phasing angle was also looked at. An example of the dependency of $P_D$ to perforation phasing angle is given in Figure 3.8. For all the cases studied, $P_D$ did not show any sensitivity to phasing angle, that is, the pressure drop expected is independent of perforation location.

As would be expected, the sensitive variables were found to be the perforation diameter and the number of perforations per foot, both of which is a measure of the total area open to flow. The effects of perforation size and shot density are illustrated in Figures 3.9 and 3.10.
Figure 3.7 - Dimensionless pressure as a function of casing/screen radius ratio.
Figure 3.8 - Dimensionless Pressure as a function of perforation phasing angle.
Casing Size = 7 5/8"
Screen Size = 4"
Permeability Ratio = 1.0

Figure 3.9 - The effect of the perforation size on the pressure losses.
Figure 3.10 - The effect of the perforation shot density on the pressure losses.
Figure 3.11 - Dimensionless pressure as a function of formation anisotropy.
The dependency of $P_D$ on perforation diameter is almost linear. However, for 8 and 12 shots per foot (see Figure 3.10), the relationship is no longer linear. This implies that for lower perforation shot densities, the streamlines of flow from a particular perforation do not affect those of nearby perforations. However, at high perforation densities, the streamlines from adjacent perforations interfere.

The effect of the media anisotropy on the pressure drop was also investigated to see the possible effects of differences in vertical and horizontal gravel permeabilities (see Figure 3.11). This shows that a vertical to horizontal permeability ratio of 0.1 can result in twofold increase in the pressure losses across the gravel pack. This implies that the majority of the divergent flow is in the radial (horizontal) direction.

### 3.6. The Validity of Linear and Radial Flow Models

The accuracy of the linear and radial flow models were investigated by comparing them with the results from the 3-D analytical solution. To see the magnitude of the pressure losses in real terms, a hypothetical set of data was chosen. The data is listed in Table 3.2. The pressure drop across the gravel pack was calculated for the data of Table 3.2, using both the linear and radial flow models. Then, for the same data set, the pressure drop was computed from the 3-D analytical solution. The results are shown in Table 3.3. As expected, the linear flow model overestimates pressure losses. The pressure drop computed from the linear flow model is about nine to twenty times greater than the new model prediction. On the other hand, the radial flow model underestimates the pressure losses,
as expected. The pressure drop estimated from the radial flow model is about ten to twenty five times less.

Table 3.2 - The data set for example problem.

<table>
<thead>
<tr>
<th>Flow Rate = 2000 BOPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity = 2 cp</td>
</tr>
<tr>
<td>Gravel Column = 10 ft</td>
</tr>
<tr>
<td>Gravel Permeability = 40 Darcies</td>
</tr>
<tr>
<td>Casing Diameter = 7 5/8 &quot;</td>
</tr>
<tr>
<td>Screen Diameter = 4 &quot;</td>
</tr>
<tr>
<td>Perforation Size = 0.5 &quot;</td>
</tr>
<tr>
<td>Shot Density = 4 SPF</td>
</tr>
<tr>
<td>Phasing Angle = 0 degrees</td>
</tr>
</tbody>
</table>

Table 3.3 - The validity of the linear and radial flow models

<table>
<thead>
<tr>
<th>Screen Size</th>
<th>Pressure Drop, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>This Study</td>
</tr>
<tr>
<td>4</td>
<td>22.17</td>
</tr>
<tr>
<td>3</td>
<td>22.80</td>
</tr>
<tr>
<td>2 1/16</td>
<td>23.75</td>
</tr>
<tr>
<td>1 1/2</td>
<td>24.32</td>
</tr>
</tbody>
</table>
3.7. Conclusions

1. A 3-D analytical solution was derived to model the pressure losses across a gravel pack. The solution regenerates the 1-D and 2-D solutions available in the literature and agrees very closely with the experimental data.

2. The pressure drop across a gravel pack is rather insensitive to the annular clearance between the casing and screen.

3. The pressure drop across the pack is also independent of the perforation phasing angle, indicating that flow from nearby perforations does not interfere.

4. The greatest influence on the pressure drop was found to depend on shot density and perforation size.

5. The gravel anisotropy does affect the total pressure drop, but to a lesser extent than shot density and perforation size.
**Nomenclature**

- **a**: Width of square probe, or width of square perforation
- **b**: Distance between probe location and bottom of reservoir or distance between perforation location and bottom of reservoir
- **c**: Angle between vertical axis and probe location or perforation phasing angle
- **CD**: Dimensionless wellbore storage
- **c_t**: Total compressibility
- **h**: Reservoir thickness
- **k**: Permeability
- **k_{br}**: Permeability from radial buildup analysis
- **k_{bs}**: Permeability from spherical buildup analysis
- **k_{d1}**: Permeability from first drawdown
- **k_{d2}**: Permeability from second drawdown
- **k_{int}**: Permeability from integration technique
- **k_r**: Horizontal permeability
- **k_s**: Spherical permeability
- **k_z**: Vertical permeability
- **q**: Flow rate
- **q_{ref}**: Reference flow rate
- **q_D**: Dimensionless flow rate
- **r**: Radius
- **r_w**: Wellbore radius
- **P**: Pressure
- **P_i**: Initial reservoir pressure

- 101 -
$P_{wf}$ : Constant wellbore flowing pressure

$P_D$ : Dimensionless pressure

$P_{Db}$ : Dimensionless buildup pressure at the WFT probe

$s$ : Laplace space variable

$S$ : Mechanical skin due to damage or stimulation

$U$ : Unit step function

$t$ : Time

$t_D$ : Dimensionless time

$w$ : Height of square probe or height of square perforation

$z$ : Vertical distance

$\mu$ : Viscosity

$\phi$ : Porosity

$\alpha$ : Unit conversion factor; 1 for Darcy units, $2.63679 \times 10^{-4}$ for field units

$\alpha_1$ : Unit conversion factor; 1 for Darcy units, $1.127 \times 10^{-3}$ for field units

$\alpha_2$ : Unit conversion factor; 1 for Darcy units, $2.63679 \times 10^{-4}$ for field units

**Subscripts**

$b$ : Buildup

d : Drawdown

$D$ : Dimensionless

$i$ : Initial

$r$ : Radial

$s$ : Spherical
wb : Wellbore
wf : Wellbore flowing
z : Vertical direction
1 : First drawdown
2 : Second drawdown
Bibliography


Appendix A. Computer Programs

A.1. Wireline Formation Testing

The computer program for Eq. (2.29) is listed below. A short description of input parameters is also given.

A.1.1. Description of Input Parameters

H : Formation thickness, ft
B : Distance between the bottom of reservoir and probe location, ft
DPIN : Probe diameter, inches
RW : Wellbore radius, ft
VK : Vertical permeability, md
HK : Horizontal permeability, md
T1 : Duration of first drawdown, sec
T2 : Duration of first plus second drawdowns, sec
VOL1 : Volume of first chamber, cc
VOL2 : Volume of first chamber, cc
POR : Porosity of formation, fraction
VIS : Viscosity of the fluid, cp
COMP : Total formation compressibility, psi\(^{-1}\)
PI : Pi number, 3.1415926
NM : Number of terms in first summation term in Eq. (2.29)
NNQ : Number of terms in second summation term in Eq. (2.29)
NTM : Number of terms in third summation term in Eq. (2.29)
NTNQ : Number of terms in third summation term in Eq. (2.29)
COEF : Unit conversion factor, $2.63679 \times 10^{-4}$
NTIME : Number of times
TIME : The time at which pressure to be calculated.

A.1.2. Program Listings

C

C MEMBER WFT

C IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON /Cl/ HD, BD, WD, SD, DK, PI, NM, NNQ, NTM, NTNQ
DIMENSION TIME (500)

C OPEN (UNIT=5, FILE='WFT1.DAT', STATUS='UNKNOWN')

C READ (5,*) H, B, DPIN
READ (5,*) RW, VK, HK
READ (5,*) T1, T2
READ (5,*) VOL1, VOL2
READ (5,*) POR, VIS, COMP
READ (5,*) PI
READ (5,*) NM, NNQ
READ (5,*) NTM, NTNQ
READ (5,*) COEF
READ (5,*) NTIME

C
DO 2 J=1, NTIME
READ (5,*) TIME(J)
2 CONTINUE

C
CLOSE(UNIT=5)

C
DP=DPIN/12.
AREA=PI*(DP**2/4)
S=AREA**0.5
W=S
SD=S/RW
WD=W/RW
BD=B/RW
HD=H/RW
DK=VK/HK
T1=T1/3600
T2=T2/3600
Q1=VOL1/T1
Q2=VOL2/T2
QD2=Q2/Q1
CTD=COEF*HK/POIUVIS/COMP/RW/RW
TX1=T1*CTD
TX2=(T1+T2)*CTD

C
C
OPEN(UNIT=8,FILE='WFT1.OUT',STATUS='UNKNOWN')
C
WRITE(8,100)
100 FORMAT(/,17X,'WIRELINE FORMATION TESTING')
WRITE(8,101)
101 FORMAT(15X,'__________________________')
WRITE(8,102) H,B,DPIN,RW
102 FORMAT(//,4X,'HEIGHT=',F5.1,3X,'PROB.LOC.=',F5.1,3X,
   $ 'PROB.DIA.=',F5.3,3X,'WELL RAD.=',F4.2)
WRITE(8,106) TX1,TX2,QD2
106 FORMAT(/,4X,'TX1=',D10.4,5X,'TX2=',D10.4,5X,'QD2=',F8.4,//)
WRITE(8,103)
103 FORMAT(23X,'TD',14X,'PD')
WRITE(8,104)
104 FORMAT(19X,'________',6X,'________')
     WRITE(*,110)
110 FORMAT(//,17X,'WIRELINE FORMATION TESTING')
     WRITE(*,111)
111 FORMAT(15X,'______________________________')
     WRITE(*,112) H,B,DPIN,RW
112 FORMAT(//,4X,'HEIGHT=',F5.1,3X,'PROB.LOC.=',F5.1,3X,
   $ 'PROB.DIA.=',F5.3,3X,'WELL RAD.=',F4.2)
     WRITE(*,116) TX1,TX2,QD2
116 FORMAT(/,4X,'TX1=',D10.4,5X,'TX2=',D10.4,5X,'QD2=',F8.4,//)
     WRITE(*,113)
113 FORMAT(23X,'TD',14X,'PD')
     WRITE(*,114)
114 FORMAT(19X,'________',6X,'________')
     WRITE(*,110)
110 FORMAT(//,17X,'WIRELINE FORMATION TESTING')
     WRITE(*,111)
111 FORMAT(15X,'______________________________')
     WRITE(*,112) H,B,DPIN,RW
112 FORMAT(//,4X,'HEIGHT=',F5.1,3X,'PROB.LOC.=',F5.1,3X,
   $ 'PROB.DIA.=',F5.3,3X,'WELL RAD.=',F4.2)
     WRITE(*,116) TX1,TX2,QD2
116 FORMAT(/,4X,'TX1=',D10.4,5X,'TX2=',D10.4,5X,'QD2=',F8.4,//)
     WRITE(*,113)
113 FORMAT(23X,'TD',14X,'PD')
     WRITE(*,114)
114 FORMAT(19X,'________',6X,'________')
     DO 11 I=1, NTIME
PTD1=0.
PTD2=0.
N=8
M=0
TD=TIME(I)
CALL LAPINV(TD,PTD,N,M)
IF(TD.LT.TX1) GO TO 120
N=8
M=0
TD1=TD-TX1
CALL LAPINV(TD1,PTD1,N,M)
IF(TD.LT.TX2) GO TO 120
N=8
M=0
TD2=TD-TX2
CALL LAPINV(TD2,PTD2,N,M)
120 PDBUL=PTD+(QD2-1.)*PTD1-QD2*PTD2
WRITE(8,105) TD,PDBUL
105 FORMAT(18X,F10.4,7X,F10.5)
WRITE(*,115) TD,PDBUL
115 FORMAT(18X,F10.4,7X,F10.5)
1 CONTINUE
C
CLOSE(UNIT=8)
C
STOP
END
C
C
SUBROUTINE LAPINV(T,FA,N,M)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)

**********************************************************************

* This subroutine inverts the Laplace space function into the *
* real time domain, numerically. It is based on the algorithm *
* by Stefhest. *

CALL SOLLAP(PLAP,ARG)
RETURN
END

C
C
SUBROUTINE SOLLAP(PDIM,P)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
**This subroutine computes the function in Laplace space. The Laplace space solution is given as Eq. (2.29) in the text.**

```
COMMON/C1/HD,BD,WD,SD,DK,PI,NM,NNQ,NTM,NTNQ

STH=P**0.5
ORK=0.
CALL DBESK(STH,ORK,1,1,VHO,NZ)
ORD=1.
CALL DBESK(STH,ORD,1,1,VH1,NZ1)
RAT=VH0/VH1/(P**1.5)

STERM=0.0

DO 2 M=1,NM
XN=M*PI/HD
EPS=(DK)**0.5*XN
XLAN=(P+EPS**2)**0.5
ORP=0.
CALL DBESK(XLAN,ORP,1,1,XKO,NZIO)
ORD=1.
CALL DBESK(XLAN,ORD,1,1,XK1,NZI1)
IF(XK1.EQ.0..OR.XKO.EQ.0.) GO TO 3
CEK=XK1/XK0
TEP=XK0/XK1/XLAN
IF(CEK.LT.1.00000000001) NM=M
IF(CEK.LT.1.00000000001) GO TO 3
TT=DSIN(XN*(BD+WD))-DSIN(XN*BD)
TERM=TEP*TT*TT/XN/XN
STERM=STERM+TERM
```
C

2 CONTINUE

C

3 SPAR=2*STERM/WD/WD/P
   WRITE(8,*) M

C

C

TANG=0.0

C

DO 5 NQ=1,NNQ
   SQP=(P)**0.5
   XNP=1.*DBLE(NQ)
   CALL DBESK(SQP,XNP,1,1,XKN,NZNQ)
   NP1=NQ+1
   XP1=1.*DBLE(NP1)
   CALL DBESK(SQP,XP1,1,1,XKNP1,NZNP)
   IF(XKNP1.EQ.0..OR.XKN.EQ.0.) GO TO 15
   CRI=XKNP1/XKN
   IF(CRI.LT.1.00000000001) NNQ=NQ
   IF(CRI.LT.1.00000000001) GO TO 15
   DON=SQP*XKNP1/XKN-NQ
   SON=1/DON

C

TS=DSIN(NQ*SD)
   TC=1-DCOS(NQ*SD)
   TRIG=TS*TS+TC*TC

C

TSUM=SON*TRIG/NQ/NQ
   TANG=TANG+TSUM

5 CONTINUE

C

15 SANG=2*TANG/SD/SD/P
   WRITE(8,*) NQ

C

C

TOP=0.0
DO 12 NQ=1,NTNQ
STIM=0.0

DO 11 M=1,NTM
XN=M*PI/HD
EPS=(DK)**0.5*XN
XLAN=(P+EPS**2)**0.5
FNQ=1.*DBLE(NQ)
CALL DBESK(XLAN,FNQ,1,1,XBES0,NZQ)
NP1=NQ+1
FNP=1.*DBLE(NP1)
CALL DBESK(XLAN,FNP,1,1,XBES1,NZP)
IF(XBES1.EQ.0..OR.XBES0.EQ.0.) GO TO 25
CRA=XBES1/XBES0
IF(CRA.LT.1.0000000001) NTNQ=NQ
IF(CRA.LT.1.0000000001) GO TO 25
COP=XLAN*XBES1/XBES0-FNQ
TAK=1/COP
TT=DSIN(XN*(BD+WD))-DSIN(XN*BD)
TT2=TT*TT
TS=DSIN(NQ*SD)
TC=1-DCOS(NQ*SD)
TRIG=TS*TS+TC*TC
TIM=TAK*TRIG*TT2/XN/XN/NQ/NQ
STIM=STIM+TIM
11 CONTINUE

25 TOP=TOP+STIM
12 CONTINUE
WRITE(8,*) NQ,M
SCOM=4*TOP/SD/SD/WD/WD/P
PDIM=RAT+SANG+SPAR+SCOM
RETURN
END
A.2. Pressure Losses Across Gravel Packs

To determine pressure losses across a gravel pack, a computer program for Eq. (3.28) is constructed. Input parameters are described.

A.2.1. Description of Input Parameters

NSPF : Number of shots per foot
PERD : Perforation diameter, inches
PHAN : Phsing angle, degrees
RI : Screen radius, inches
RO : Casing radius, inches
H : Height of gravel column, ft
LH : Height of gravel column, ft
HORK : Horizontal permeability, md
VERK : Vertical permeability, md
ZN : The location where pressure to be calculated, ft
PI : Pi number, 3.1415926
A.2.2. Program Listings

C MEMBER GRAVEL

C IMPLICIT REAL*8 (A-H,O-Z)
C DIMENSION SD(90),WD(90),CD(90),TD(90)
C DIMENSION SN(90),WN(90),CN(90),TN(90)
C DIMENSION TT(90),TS(90),TC(90)

C OPEN(UNIT=5,FILE='GRA.DAT',STATUS='UNKNOWN')

C READ(5,*) NSPF
C READ(5,*) PERD,PHAN
C READ(5,*) RI,RO,H,LH
C READ(5,*) HORK,VERK
C READ(5,*) ZN
C READ(5,*) PI

C CLOSE(UNIT=5)

C WRITE(*,301) NSPF
C 301 FORMAT(10X,'Number of Shots Per Foot=',3X,F5.0)
C WRITE(*,302) PERD
C 302 FORMAT(10X,'Perforation Diameter,inches=',3X,F5.2)
C WRITE(*,303) PHAN
C 303 FORMAT(10X,'Perforation Phasing Angle,degrees=',3X,F5.0)
C WRITE(*,304) RI
C 304 FORMAT(10X,'Screen Radius,inches=',3X,F7.2)
C WRITE(*,305) RO
C 305 FORMAT(10X,'Wellbore Radius,inches=',3X,F7.2)
C WRITE(*,306) H
C 306 FORMAT(10X,'Height of Gravel Pack, ft=',3X,F7.2)
WRITE(*,307) HORK
307 FORMAT(10X,'Horizontal Permeability, md=',3X,F7.2)
WRITE(*,308) VERK
308 FORMAT(10X,'Vertical Permeability, md=',3X,F7.2)
WRITE(*,309) ZN
309 FORMAT(10X,'Location where Pres. Calc.ed,ft=',3X,F7.2)

C
OPEN(UNIT=8,FILE='GRA.OUT',STATUS='UNKNOWN')

C
WRITE(8,401) NSPF
401 FORMAT(10X,'Number of Shots Per Foot=',3X,F5.0)
WRITE(8,402) PERD
402 FORMAT(10X,'Perforation Diameter,inches=',3X,F5.2)
WRITE(8,403) PHAN
403 FORMAT(10X,'Perforation Phasing Angle,degrees=',3X,F5.0)
WRITE(8,404) RI
404 FORMAT(10X,'Screen Radius,inches=',3X,F7.2)
WRITE(8,405) RO
405 FORMAT(10X,'Wellbore Radius,inches=',3X,F7.2)
WRITE(8,406) H
406 FORMAT(10X,'Height of Gravel Pack, ft=',3X,F7.2)
WRITE(8,407) HORK
407 FORMAT(10X,'Horizontal Permeability, md=',3X,F7.2)
WRITE(8,408) VERK
408 FORMAT(10X,'Vertical Permeability, md=',3X,F7.2)
WRITE(8,409) ZN
409 FORMAT(10X,'Location where Pres. Calc.ed,ft=',3X,F7.2)

C

DLKS=(VERK/HORK)**0.5
NPERF=LH*NSPF
WRITE(8,*) RI,RO,H,NPERF

C

DISP=1.*LH/(NPERF-1)
DO 13 L=1,NPERF
CN(L)=(L-1)*PHAN
SN(L)=(PI*PERD**2/4.)**.5
WN(L)=SN(L)
TN(L)=(L-1)
13 CONTINUE

ATOT=0.

DO 93 L=1,NPERF
CD(L)=(CN(L)/180.)*PI*RO/RI
SD(L)=SN(L)/RI
TD(L)=TN(L)/H
WD(L)=WN(L)/H
ATOT=ATOT+SD(L)*WD(L)
93 CONTINUE

RDO=RO/RI

REST=0.0

DO 61 J=1,NPERF
SUMT=0.0
1 DO 1 NU=1,50
SSAM=0.0
SCAM=0.0

DO 11 I=1,NPERF
TS(I)=DSIN(NU*(CD(I)+SD(I))/RDO)-DSIN(NU*CD(I)/RDO)
TC(I)=DCOS(NU*CD(I)/RDO)-DCOS(NU*(CD(I)+SD(I))/RDO)
TSAM=TS(I)*WD(I)
TCAM=TC(I)*WD(I)
SSAM=SSAM+TSAM
SCAM=SCAM+TCAM
CONTINUE

ST1=SSAM*TS(J)+SCAM*TC(J)
ST3=(RDO**NU-RDO**(-NU))/(RDO**NU-1)+RDO**(-NU-1))
ST3=ST3/NU/NU/NU
ST4=ST1*ST3
SUMT1=SUMT
SUMT=SUMT+ST4
DF1=DABS(SUMT-SUMT1)*100./DABS(SUMT)

CONTINUE

SEST=2.*SUMT*RDO/ATOT/SD(J)
REST=REST+SEST

DO 62 J=1,NPERF
SRZ=0.

DO 2 M1=1,30
XN=M1*PI
XLAM=DLKS*RI*XN/H
XLRDO=XLAM*RDO
CALL MODBES (0, XLRDO, XI0RDO, XK0RDO, SIBRDO, Y0RDO, $SCRDO)
CALL MODBES (0, XLAM, XI0, XK0, SIB0, Y0, S0)
CALL MODBES (1, XLRDO, XI1RDO, XK1RDO, SIB1RD, Y1RDO, $S1RDO)
R1=SIB0/SIBRDO
R2=SIB0/SIB1RD
Q1=Y0/Y0RDO
Q2=Y0/Y1RDO
YRDO=XLRDO
YAM=XLAM
IF(XLRDO.LE.3.5) YRDO=0.
IF(XLAM.LE.3.5) YAM=0.
IF(M1.GT.20) SZ12=0.
IF(M1.GT.20) SZ22=0.
IF(M1.GT.20) GO TO 51
RAT1=DEXP(-YRDO/2.)/DEXP(-YAM/2.)
RAT2=SO/SCRDO
RAT3=SO/S1RDO
SZ12=(XK0RDO/XK0)*RAT1*RAT1*RAT2
SZ22=(XK1RDO/XK0)*RAT1*RAT1*RAT3
51 SZ11=(X10RDO/X10)*R1*Q1*R1*Q1*R1*Q1*R1*Q1*R1*Q1
SZ1=SZ11-SZ12
SZ21=(X11RDO/X10)*R1*Q1*R1*Q1*R1*Q1*R1*Q1*R1*Q1
SZ2=SZ21+SZ22
TTS=0.0
C
DO 21 MX=1,NPERF
TT(MX)=DSIN(XN*(TD(MX)+WD(MX)))-DSIN(XN*TD(MX))
TTS=TTS+TT(MX)*SD(MX)/RDO
21 CONTINUE
C
SZ3=TTS*TT(J)
SZ4=SZ1*SZ3/SZ2/XN/XN/XLAM
SRZ1=SRZ
SRZ=SRZ+SZ4
DF2=ABS(SRZ-SRZ1)*100./ABS(SRZ)
2 CONTINUE
C
SESZ=2.*SRZ/ATOT/WD(J)
RESZ=RESZ+SESZ
62 CONTINUE

DRES=0.0
C
DO 63 J=1,NPERF
   DTOT=0.
C
DO 3 NN=1,30
   DOBR=0.
C
DO 4 M2=1,30
   XN=M2*PI
   DT4=0.
   ST3=0.
   ST4=0.
C
DO 31 I=1,NPERF
   TT(I)=DSIN(XN*(TD(I)+WD(I)))-DSIN(XN*TD(I))
   TS(I)=DSIN(NN*(CD(I)+SD(I))/RDO)-DSIN(NN*CD(I)/RDO)
   TC(I)=DCOS(NN*CD(I)/RDO)-DCOS(NN*(CD(I)+SD(I))/RDO)
   DT3=TS(I)*TT(I)
   DT4=TC(I)*TT(I)
   ST3=ST3+DT3
   ST4=ST4+DT4
31 CONTINUE
C
   DTT=ST3*TS(J)+ST4*TC(J)
   DTOP=DTT*TT(J)/XN/XN/NN/NN
   XLAM=DLKS*RI*XN/H
   XLRDO=XLAM*RDO
   NMV=NN-1
   CALL MODBES (NN, XLAM, XIVLAM, XKVLAM, SIBLAM, ZVLAM, $   SVLAM)
   CALL MODBES (NN, XLRDO, XILRDO, XKLRDO, SIBRDO, ZIBRDO, $   SVRDO)
   CALL MODBES (NMV, XLRDO, XINMVO, XKNMVO,SIBNMV,ZNMV, $   SCNMV)
   RIB1=SIBLAM/SIBRDO
RIB2 = SIBLAM/SIBNMV
QIB1 = ZVLM/ZIBRDO
QIB2 = ZVLM/ZNMV
RQ1 = RIB1*QIB1
RQ2 = RIB2*QIB2
ZRDO = XLRDO
ZAM = XLAM
IF (XLRDO.LE.3.5) ZRDO = 0.
IF (XLAM.LE.3.5) ZAM = 0.
IF (NN.GT.20) DB221 = 0.
IF (NN.GT.20) DB222 = 0.
IF (NN.GT.20) GO TO 52
RAM1 = DEXP(-ZRDO/2.)/DEXP(-ZAM/2.)
RAM2 = SVLAM/SVRDO
RAM3 = SVLAM/SCNMV
DB12 = (XKLRODO/XKVLAM)*RAM1*RAM1*RAM2
DB221 = XLAM*(XKNMVO/XKVLAM)*RAM1*RAM1*RAM3
DB222 = (NN/RDO)*(XKLRODO/XKVLAM)*RAM1*RAM1*RAM2
52 DB11 = RQ1*RQ1*RQ1*XLRDO/XIVLAM*RQ1*RQ1
DB1 = DB11 - DB12
DB21 = RQ2*RQ2*RQ2*XLAM*XINMVO/XIVLAM*RQ2*RQ2
$ -RQ1*RQ1*(NN/RDO)*RQ1*XLRDO/XIVLAM*RQ1*RQ1
DB22 = DB221 + DB222
DFG = DB21 + DB22
DB3 = DB1/DFG
DTER = DB3*DTOP
DOBR1 = DOBR
DOBR = DOBR + DTER
DF3 = DABS(DOBR-DOBR1)*100./DABS(DOBR)
4 CONTINUE
C
DTOT1 = DTOT
DTOT = DTOT + DOBR
DF4 = DABS(DTOT-DTOT1)*100./DABS(DTOT)
3 CONTINUE
C
SRES = 4. * DTOT * RDO / ATOT / SD(J) / WD(J)
DRES = DRES + SRES
63 CONTINUE
C
PD = DLOG(RDO) + REST + RESZ + DRES
PDOV = 1. / PD
RDOV = 1. / RDO
C
WRITE(*,310) RDO
310 FORMAT(///,10X,'Dimensionless Radius=',3x,F10.6)
WRITE(*,311) PD
311 FORMAT(///,10X,'Dimensionless Pressure Drop=',3x,F12.6)
C
WRITE(8,410) RDO
410 FORMAT(///,10X,'Dimensionless Radius=',3x,F10.6)
WRITE(8,411) PD
411 FORMAT(///,10X,'Dimensionless Pressure Drop=',3x,F12.6)
C
CLOSE(UNIT=8)
C
STOP
END
C
C
SUBROUTINE MODBESCNV(NV,X,BESIV,BESKV,XKK,XKM,SCL)
IMPLICIT REAL*8 (A-H,O-Z)
SBIV = 0.
SBIV1 = 0.
K = 0
20 XJK = 8. ** (1. * K / 5. + 1. * NV / 10.)
XKK = 1.
IF(X.GT.120.) XKK = 1.D-08
IF(NV.GT.40.AND.X.GT.80.) XKK = 1.D-10
IF(NV.GT.40.AND.X.LT.3.5) XKK = 1.D+02
IF(NV.GT.60.AND.X.LT.3.5) XKK=1.D+10
XKM=1.
IF(NV.GT.20.AND.X.LT.7.) XKM=1.D+10
IF(NV.GT.40.AND.X.LT.7.) XKM=1.D+18
IF(NV.GT.60.AND.X.LT.1.) XKM=1.D+40
XNUM=(X/16.)**(1.*K/5.+1.*NV/10.)
CALL FACT (K, SK1, SK2, SK3, SK4, SK5, SK6, SK7, SK8, SK9, SK10, SK11, SK12, SK13, SK14, SK15, SK16, SK17, SK18, SK19, SK20)
$N1=K+NV$
CALL FACT (N1, SG1, SG2, SG3, SG4, SG5, SG6, SG7, SG8, SG9, SG10, SG11, SG12, SG13, SG14, SG15, SG16, SG17, SG18, SG19, SG20)
$BN1=XKK*XNUM/SG1/SG20$
$BN2=XKK*XNUM/SG2/SG19$
$BN3=XKK*XNUM/SG3/SG18$
$BN4=XKK*XNUM/SG4/SG17$
$BN5=XKK*XNUM/SG5/SG16$
$BN6=XJK/SG6/SG15$
$BN7=XJK/SG7/SG14$
$BN8=XJK/SG8/SG13$
$BN9=XJK/SG9/SG12$
$BN10=XJK/SG10/SG11$
$BIN=BN1*BN10*BN3*BN9*BN5*BN8*BN7*BN6*BN4*BN2$
$BM1=XKM*XNUM/SG1/SG20$
$BM2=XKM*XNUM/SG2/SG19$
$BM3=XKM*XNUM/SG3/SG18$
$BM4=XKM*XNUM/SG4/SG17$
$BM5=XKM*XNUM/SG5/SG16$
$BM6=XJK/SG6/SG15$
$BM7=XJK/SG7/SG14$
$BM8=XJK/SG8/SG13$
$BM9=XJK/SG9/SG12$
$BM10=XJK/SG10/SG11$
$BIM=BM1*BM8*BM3*BM9*BM5*BM6*BM7*BM2*BM4*BM10$
$BIV=BIN*BIM$
SBIV = SBIV + BIV  
SDIF = DABS(SBIV - SBIV1) / DABS(SBIV)  
IF(SDIF.LT.0.001) GO TO 60  
SBIV1 = SBIV  
K = K + 1  
GO TO 20  
60  BESIV = SBIV  
IF(NV.GT.20) BESKV = 0.  
IF(NV.GT.20) SCL = 1.  
IF(NV.GT.20) RETURN  
IF(X.GT.3.5) GO TO 210  
XKST1 = 0.  
GAMA = .577215665  
IF(NV.EQ.0) GO TO 133  
DO 1 KK1 = 1, NV  
K = KK1 - 1  
N1 = NV - K - 1  
CALL FACT (N1, SN1, SN2, SN3, SN4, SN5, SN6, SN7, SN8,  
$ \text{SN12, SN13, SN14, SN15, SN16, SN17, SN18, SN19, SN20})  
$ CALL FACT(K, XKF1, XKF2, XKF3, XKF4, XKF5, XKF6, XKF7,  
$ XKF8, XKF9, XKF10, XKF11, XKF12, XKF13, XKF14, XKF15, XKF16, XKF17, XKF18, XKF19, XKF20)  
T1 = (X/2.)**(2*K-NV)  
ST1 = (-1)**K * SN1 * T1/XKF1  
ST1 = ST1 * SN2 / XKF2  
ST1 = ST1 * SN3 / XKF3  
ST1 = ST1 * SN4 / XKF4  
ST1 = ST1 * SN5 / XKF5  
ST1 = ST1 * SN6 / XKF6  
ST1 = ST1 * SN7 / XKF7  
ST1 = ST1 * SN8 / XKF8  
ST1 = ST1 * SN9 / XKF9  
ST1 = ST1 * SN10 / XKF10  
ST1 = ST1 * SN11 / XKF11
ST1=ST1*SN12/XKF12
ST1=ST1*SN13/XKF13
ST1=ST1*SN14/XKF14
ST1=ST1*SN15/XKF15
ST1=ST1*SN16/XKF16
ST1=ST1*SN17/XKF17
ST1=ST1*SN18/XKF18
ST1=ST1*SN19/XKF19
ST1=ST1*SN20/XKF20
XKST1=XKST1+ST1
1 CONTINUE
133 XKSUM=0.
IF(NV.GT.30) GO TO 3
IF(X.LT.1.AND.NV.GT.25) GO TO 3
K=0.
130 XKSUM1=XKSUM
CALL FACT(K, SK51, SK52, SK53, SK54, SK55, SK56, SK57,
$     SK58, SK59, SK510, SK511, SK512, SK513, SK514,
$     SK515, SK516, SK517, SK518, SK519, SK520)
N1=NV+K
CALL FACT(N1, VKF1, VKF2, VKF3, VKF4, VKF5, VKF6,
$     VKF7, VKF8, VKF9, VKF10, VKF11, VKF12, VKF13,
$     VKF14, VKF15, VKF16, VKF17, VKF18, VKF19, VKF20)
T2=(X/2.)**(1.*K+NV/2.)
CALL SUMOV(K,PK)
J3=K+NV
CALL SUMOV(J3,PKV)
XSS1= (PK+PKV)* T2/ SK51/ SK52/ SK53/ SK54/ SK55/
$     SK56/ SK57/ SK58/ SK59/ SK510/ SK511/ SK512/
- $     SK513/ SK514/ SK515/ SK516/ SK517/ SK518/
$     SK519/ SK520
$     VKF8/ VKF9/ VKF11/ VKF12/ VKF13/ VKF14/
$     VKF15/ VKF16/ VKF17/ VKF18/ VKF19/ VKF10/
$     VKF20
XKST2=XSS1*XSS2
XKSUM=XKSUM+XKST2
IF(K.EQ.0) GO TO 2
DF=DABS(XKSUM-XKSUM1)/DABS(XKSUM)
IF(DF.LT.0.001) GO TO 3
2 CONTINUE
K=K+1
GO TO 130
3 CONTINUE
BB1=(-1)**(NV+1)*(DLOG(X/2.)+GAMA)*BESIV
BB2=XKST1/2.
BB3=(-1)**NV*(XKSUM/2.)
BESKV=BB1 + BB2 + BB3
SCL=1.
GO TO 330
210 MU=4*NV**2
SCL=1.
IF(X.GT.150.) SCL=1.D+50
PI=3.1415926
XLARG=0.
XKBES=0.
KJ=1
240 CALL SUBDEN (KJ, MU, DEN1, DEN2, DEN3, DEN4, DEN5, DEN6,
$     DEN7, DEN8, DEN9, DEN10, DEN11, DEN12, DEN13,
$     DEN14, DEN15)
CALL FACT (KJ, SF1, SF2, SF3, SF4, SF5, SF6, SF7, SF8,
$     SF9, SF10, SF11, SF12, SF13, SF14, SF15, SF16,
$     SF17, SF18, SF19, SF20)
XKBES1=XKBES
DS1=DEN1/SF1
DS2=DEN2/SF2
DS3=DEN3/SF3
DS4=DEN4/SF4
DS5=DEN5/SF5
DS6=DEN6/SF6
DS7=DEN7/SF7
DS8=DEN8/SF8
DS9=DEN9/SF9
DS10=DEN10/SF10
DS11=DEN11/SF11
DS12=DEN12/SF12
DS13=DEN13/SF13
DS14=DEN14/SF14
DS15=DEN15/SF15
XLARG=XLARG+DS1*DS2*DS3*DS4*DS5*DS6*DS7
$*DS8*DS9*DS10/(8.*X)**KJ*DS11*DS12*
$DS13*DS14*DS15/SF16/SF17/SF18/SF19/
$SF20
XKBES=(PI/X/2.)**.5*SCL*(1.+XLARG)
DF=DABS(XKBES-XKBES1)/DABS(XKBES)
IF(DF.LT..001) GO TO 325
KJ=KJ+1
GO TO 240
325 BESK V=XKBES
330 RETURN
END

C
C
SUBROUTINE SUBDEN (KJ, MU, DEN1, DEN2, DEN3, DEN4,
$DEN5, DEN6, DEN7, DEN8, DEN9, DEN10, DEN11, DEN12,
$DEN13, DEN14, DEN15)
IMPLICIT REAL*8 (A-H,O-Z)
DEN1=1.
DEN2=1.
DEN3=1.
DEN4=1.
DEN5=1.
DEN6=1.
DEN7=1.
DEN8=1.
DEN9=1.
DEN10=1.
DEN11=1.
DEN12=1.
DEN13=1.
DEN14=1.
DEN15=1.
LL=0
NLL=2*KJ-1
DO 1 L=1,NLL
  IF(LL.EQ.1) GO TO 560
  IF(DEN1.GT.1.D+20) GO TO 2
  DEN1=DEN1*(MU-L**2)
  GO TO 550
2 IF(DEN2.GT.1.D+20) GO TO 3
  DEN2=DEN2*(MU-L**2)
  GO TO 550
3 IF(DEN3.GT.1.D+20) GO TO 4
  DEN3=DEN3*(MU-L**2)
  GO TO 550
4 IF(DEN4.GT.1.D+20) GO TO 5
  DEN4=DEN4*(MU-L**2)
  GO TO 550
5 IF(DEN5.GT.1.D+20) GO TO 6
  DEN5=DEN5*(MU-L**2)
  GO TO 550
6 IF(DEN6.GT.1.D+20) GO TO 7
  DEN6=DEN6*(MU-L**2)
  GO TO 550
7 IF(DEN7.GT.1.D+20) GO TO 8
  DEN7=DEN7*(MU-L**2)
  GO TO 550
8 IF(DEN8.GT.1.D+20) GO TO 9
  DEN8=DEN8*(MU-L**2)
  GO TO 550
9 IF(DEN9.GT.1.D+20) GO TO 10
  DEN9=DEN9*(MU-L**2)
GO TO 550
10 IF(DEN10.GT.1.D+20) GO TO 11
    DEN10=DEN10*(MU-L**2)
    GO TO 550
11 IF(DEN11.GT.1.D+20) GO TO 12
    DEN11=DEN11*(MU-L**2)
    GO TO 550
12 IF(DEN12.GT.1.D+20) GO TO 13
    DEN12=DEN12*(MU-L**2)
    GO TO 550
13 IF(DEN13.GT.1.D+20) GO TO 14
    DEN13=DEN13*(MU-L**2)
    GO TO 550
14 IF(DEN14.GT.1.D+20) GO TO 15
    DEN14=DEN14*(MU-L**2)
    GO TO 550
15 DEN15=DEN15*(MU-L**2)
550 LL=1
    GO TO 570
560 LL=0
570 CONTINUE
1 CONTINUE
RETURN
END

SUBROUTINE FACT (N1, F1, F2, F3, F4, F5, F6, F7, F8, F9, F10, F11, F12, F13, F14, F15, F16, F17, F18, F19, F20)
IMPLICIT REAL*8 (A-H,O-Z)
F1=1.
F2=1.
F3=1.
F4=1.
F5=1.
F6=1.
F7=1.
F8=1.
F9=1.
F10=1.
F11=1.
F12=1.
F13=1.
F14=1.
F15=1.
F16=1.
F17=1.
F18=1.
F19=1.
F20=1.
IF(N1.EQ.0) RETURN
DO 1 II=1,N1
IF(F1.GT.1.D+15) GO TO 5
F1=F1*II
GO TO 1
5 IF(F2.GT.1.D+15) GO TO 6
F2=F2*II
GO TO 1
6 IF(F3.GT.1.D+15) GO TO 7
F3=F3*II
GO TO 1
7 IF(F4.GT.1.D+15) GO TO 8
F4=F4*II
GO TO 1
8 IF(F5.GT.1.D+15) GO TO 9
F5=F5*II
GO TO 1
9 IF(F6.GT.1.D+15) GO TO 10
F6=F6*II
GO TO 1
10 IF(F7.GT.1.D+15) GO TO 11
F7=F7*II
GO TO 1
11 IF(F8.GT.1.D+15) GO TO 12
   F8=F8*I1
   GO TO 1
12 IF(F9.GT.1.D+15) GO TO 13
   F9=F9*I1
   GO TO 1
13 IF(F10.GT.1.D+15) GO TO 14
   F10=F10*I1
   GO TO 1
14 IF(F11.GT.1.D+15) GO TO 15
   F11=F11*I1
   GO TO 1
15 IF(F12.GT.1.D+15) GO TO 16
   F12=F12*I1
   GO TO 1
16 IF(F13.GT.1.D+15) GO TO 17
   F13=F13*I1
   GO TO 1
17 IF(F14.GT.1.D+15) GO TO 18
   F14=F14*I1
   GO TO 1
18 IF(F15.GT.1.D+15) GO TO 19
   F15=F15*I1
   GO TO 1
19 IF(F16.GT.1.D+15) GO TO 20
   F16=F16*I1
   GO TO 1
20 IF(F17.GT.1.D+15) GO TO 21
   F17=F17*I1
   GO TO 1
21 IF(F18.GT.1.D+15) GO TO 22
   F18=F18*I1
   GO TO 1
22 IF(F19.GT.1.D+15) GO TO 23
F19 = F19 * I1
GO TO 1

23 F20 = F20 * I1
1 CONTINUE
RETURN
END

C
C
SUBROUTINE SUMOV(J3, PHI)
IMPLICIT REAL*8 (A-H,O-Z)
PHI = 0.
IF (J3.EQ.0) RETURN
DO 1 M = 1, J3
PS = 1. / M
PHI = PHI + PS
1 CONTINUE
RETURN
END
Vita

Turhan Yildiz, the son of Ayhan Yildiz and Naime Yildiz, was born in Arpacay, Turkey on November 10, 1962. He attended Istanbul Teknik Universitesi, receiving a B.S. degree in Petroleum Engineering in 1982. In 1984, he enrolled at Louisiana State University. He completed his M.S. degree in Fall 1985. Since Spring 1986, he has been in the Ph D program at the Louisiana State University.
DOCTORAL EXAMINATION AND DISSERTATION REPORT

Candidate: Turhan Yildiz
Major Field: Petroleum Engineering
Title of Dissertation: Analytical Studies of Wireline Formation Testing and Pressure Losses Across Gravel Packs

Approved:

[Signatures]

Major Professor and Chairman
Dean of the Graduate School

EXAMINING COMMITTEE:

[Signatures]

Date of Examination:

November 16, 1989