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BATSE GAMMA-RAY BURST LINE SEARCH:
III. LINE DETECTABILITY

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ABSTRACT

We evaluate the ability of the BATSE spectroscopy detectors to detect the absorption lines observed by *Ginga* in gamma ray burst spectra. We find that BATSE can detect the 20.6 keV line in the S1 segment of GB870303 with a detection probability of $\sim 1/4$ in nearly normally incident bursts, with the probability dropping off to nearly 0 at a burst angle of 50°; the lines at 19.4 and 38.8 keV lines in GB880205 have a high detection probability in BATSE up to burst angles of 75°. In addition, we calculate detection probabilities for these two line types as a function of signal-to-noise ratio and burst angle for use in detailed comparisons between BATSE and *Ginga*. Finally, we consider the probability averaged over the sky of detecting a line feature with the actual array of BATSE detectors on *CGRO*.

Subject headings: gamma rays: bursts

1. INTRODUCTION

What are the line detecting capabilities of the Burst and Transient Source Experiment (BATSE) on board the *Compton Gamma Ray Observatory (CGRO)*? As described by the first paper in this series (Palmer et al. 1994b), currently no absorption features have been

detected in the spectra observed by BATSE's spectroscopy detectors (SDs), while two sets of lines were discovered in the bursts observed by the *Ginga* gamma ray burst detector. In the second paper of this series (Band et al. 1994) we presented both Bayesian and "frequentist" methodologies for comparing the consistency of the *Ginga* and BATSE results. These comparisons require quantitative detection probabilities for lines in each spectrum accumulated by the two detectors; in the present paper we calculate these probabilities for the BATSE detectors. In addition, if lines are detected in the BATSE bursts, we will then estimate the rate at which lines occur, which also requires the line detectability probabilities.

The existence of absorption lines is one of the crucial remaining questions which BATSE can address in the study of GRBs. One of the key pieces of evidence supporting the Galactic neutron star origin of GRBs was the series of reported absorption features in the 15-75 keV band (Konus—Mazets et al. 1981; *HEAO 1*—Hueter 1987; *Ginga*—Murakami et al. 1988) interpreted as cyclotron absorption in a teragauss magnetic field (e.g., Wang et al. 1989); the only known astrophysical body with such strong fields is a neutron star. BATSE found that the burst distribution is isotropic yet radially non-uniform (Meegan et al. 1992), overturning the Galactic disk neutron star paradigm; whether BATSE also finds absorption lines is the subject of this series of papers. If these lines are discovered they will be an important probe of the burst environment; in particular, it will be difficult (but probably not impossible) for cosmological models to explain absorption features. However, in the absence of any conclusive BATSE detections (Teegarden et al. 1993; Band et al. 1993a; Palmer et al. 1993, 1994a,b) we have to consider the consistency between the BATSE and *Ginga* results (Band et al. 1994), the calculation of which requires the number of bursts observed by each instrument in which lines would be detectable.

To properly study the rate at which lines occur in the bursts observed by past and present missions, we need both a quantitative assessment of the reality of the claimed lines (as described below) and a list of the spectral parameters (e.g., the signal-to-noise ratio) which determine line detectability. Unfortunately, fit parameters were not provided for the line features reported in $\sim 20\%$ of the Konus bursts (Mazets et al. 1981); in addition, the searched spectra have not been described in quantitative detail. The two line detections reported among the 21 *HEAO 1* bursts (Hueter 1987) are only of moderate significance (by the methods described below) and few details have been provided concerning the spectra searched in this data set. However, the burst data necessary to evaluate both the line candidates and line detectability in the bursts observed by BATSE and *Ginga* are available. Consequently our quantitative comparison is between these detectors.

The absorption features are undoubtedly characterized by distributions of line

parameters. However, there have been too few definitive line detections to determine these distributions. Therefore we use the two *Ginga* detections as exemplars of the lines which might be present. While four sets of lines were reported, the sets in the S2 segment of GB870303 (Murakami et al. 1988) and in GB890929 (Yoshida et al. 1991) are not sufficiently significant (by the detection criteria discussed below) to be considered detections, while the lines in the S1 segment of GB870303 (Graziani et al. 1992) and in GB880205 (Murakami et al. 1988) are regarded as real. Therefore we study the detectability of the single line in the S1 segment of GB870303 (20.6 keV) and the harmonically spaced lines in GB880205 (19.4 and 38.8 keV) in the BATSE detectors as functions of the relevant parameters.

In this paper we explore the detectability of absorption lines as a function of various parameters by the BATSE SDs, both to demonstrate that BATSE can indeed detect lines similar to those observed by *Ginga*, and to derive detection probabilities which will be useful for future studies. After describing the detector (§2), we outline the methodology used for this study (§3). The presentation of our results follows (§4). Finally, we analyze the dependence of the detectability on various instrumental parameters, and consider the average detectability of *Ginga*-like lines in the BATSE detectors (§5). The implications of these detection probabilities for BATSE-*Ginga* consistency will be discussed later in this series of papers.

2. THE DETECTORS

Because more complete descriptions of the BATSE SDs (Fishman et al. 1989, 1995; Band et al. 1992) are available elsewhere, here we provide a brief overview of the detectors, emphasizing aspects which are important for understanding their line detection capabilities.

Each of BATSE's eight modules contains an SD, and therefore every burst is observed by a number of these detectors; however, not all the SDs from which burst data are available may have been configured to provide useful spectra (e.g., as a result of the discriminator and gain settings). The SDs are simple scintillation detectors built around a 5" diameter by 3" thick cylindrical NaI(Tl) crystal with no active shielding. The detectors are sensitive to gamma rays entering the side of the crystal above ~ 15 keV, and thus the maximum effective area, obtained for sources at an angle of $\sim 30°$ from the detector normal, is greater than the nominal 127 cm^2 area of the top of the crystal at energies where the aluminum case is transparent. To increase the response at low energy, the aluminum case has a 3"

diameter beryllium window in the top surface; thus the effective area at ~ 10 keV is greatest for sources normal to the detector.

The energy of a photon detected by the SDs is determined by a pulse height analyzer with 2752 linear channels which are then compressed to 256 quasi-logarithmic channels for transmission to the ground. The gain of the photomultiplier tube and the energy at which the lower level discriminator triggers the pulse height analyzer determine the energy range covered; in general a spectrum's 256 channels span two energy decades. Because of the priority accorded the line search, for most of the mission the gain of six of the eight SDs has been set to provide spectra as low as is technically feasible. As will be discussed below, because of detector-dependent effects, the low energy cutoff varies by detector. Unfortunately, after launch an electronic artifact was discovered which distorts the spectrum for a few channels just above the spectrum's low energy cutoff, effectively raising this cutoff. Although this artifact may be partially mitigated in the future by the calibration software, the channels of the artifact will probably never be trusted for more than determination of the continuum below a line candidate (Band et al. 1992). However, for the purpose of establishing consistency among all detectors we do search for features in the channels affected by this artifact when lines have been found at the same energy in other detectors; for example, in GB930506 the line candidate at 55 keV in one SD should have produced a feature just above the artifact in a second SD if it were a true absorption feature (Ford et al. 1994).

After a burst triggers BATSE, the experiment collects the data necessary to analyze the event. In particular, the SDs accumulate a series of 192 spectra from the four most brightly illuminated modules. The accumulation times are varied to provide comparable signal-to-noise ratios (SNRs) for each spectrum, and to cover a period of 4-10 minutes over which the burst may last (it is of course impossible *a priori* to determine when the burst will end); during analysis on the ground various averages of these spectra are considered. These spectra are the basic data which are searched; later papers in this series will consider how the search is conducted and how all possible averages should be treated in the statistical analysis.

The mapping of the input photon spectrum into the count spectrum is traditionally broken into the calibration, which assigns an energy to the detector channels, and the detector response, which models the detector efficiency, energy broadening, and processes which distribute counts over a range of channels. In the BATSE SD calibration the energy is a monotonic function of channel number (Band et al. 1992) while the detector response is written as a matrix equation relating photon and channel energies. Using Monte Carlo simulations, the response of a BATSE SD was calculated as a function of the burst angle,

the angle between the detector normal and the burst (Pendleton et al. 1989); the response was assumed to be the same for all detectors and also independent of azimuth (the angle around the detector normal). The response matrix equation is

$$C_i = D_{ij} F_j \tag{1}$$

where C_i is the count spectrum, F_j is the binned photon spectrum, and D_{ij} is the detector response matrix. The energy channels need not be the same for C_i and F_j, and indeed F_j covers an extended energy range (particularly on the high side) to model the scattering of photons into the detectors' nominal energy range. The BATSE response matrix has been separated into direct and Earth-scattered components. The direct component includes photons which scatter off the spacecraft, while the Earth-scattered components consist of photons which impinge on the spacecraft from different directions after scattering off the atmosphere. The calculations included here do not include the effects of Earth-scatter.

3. METHODOLOGY

We use computer simulations to calculate the detectability of various types of absorption lines. A model photon spectrum is convolved with the appropriate detector response to produce a count spectrum without noise. By adding Poisson noise to this simulated count spectrum we create a large number of realizations which we then fit with continuum and continuum+line models as though they were observed spectra. Figure 1 shows simulations of the two *Ginga* lines as they might have appeared in the BATSE SDs. Line significances are calculated from the fitted parameters. We are primarily interested in the fraction of the simulations for which the significance of the features exceeds various detection thresholds; this fraction is the probability that the feature would be considered a detection using a given threshold.

We map out the functional dependencies of the line detectability by varying the components of this procedure. First, we use response matrices corresponding to different burst angles. Second, we model the effect of varying the upper energy cutoff of the range over which the models were fit. We fix the low energy cutoff at 10 keV, a value which applies to some burst-detector data but is optimistic for most spectra. Third, we test the sensitivity to the complexity (i.e., the number of parameters) of the line model.

Fourth, we change the signal-to-noise ratio (SNR), one of the key parameters, by varying the live time. The signal is the source flux convolved with the detector response,

while the noise is assumed to result solely from the Poisson fluctuations of the signal and background counts (we use the background from an observed burst). The uncertainty in the background determination is not modeled and thus its contribution to the noise is not included in the SNR. Therefore the SNR is proportional to the square root of the live time. For spectra accumulated over the observed persistence times the SNR in the 25-35 keV band is ∼ 4.5 for GB870303 and ∼ 37 for GB880205. Note that varying the SNR by changing the live time is not exactly equivalent to using different signal-to-background ratios (SBRs) since the SNR does not vary uniformly across the spectrum as the SBR is changed: regions of the spectrum where the signal is much larger than the background will see a smaller change in the SNR than regions where the background dominates. The SBR for our GB870303 simulation varied between 0.04 in the middle of the absorption line to 0.5 at ∼ 150 keV, while for the GB880205 line the SBR ranged between ∼ 1 over $E = 10 - 40$ keV and 4.5 over $E = 100 - 200$ keV. However, simulations where we varied either the signal strength or the live time both show nearly the same dependence on SNR; therefore our method for calculating this dependence is robust.

We fit the simulated spectra with the standard Marquardt-Levenberg algorithm minimizing χ^2 (Bevington 1969, pp. 232-241; Press et al. 1992, pp. 678-683). In brief, the parameters of a model photon spectrum are varied to minimize the difference—quantified by χ^2—between the calculated count spectrum (the model photon spectrum folded through the instrument response) and the observed spectrum. We define χ^2 with model variances, requiring additional terms in the gradients used by the Marquardt-Levenberg algorithm (see the appendix of Ford et al. 1995).

We use the F-test to determine the line significance. Assume the continuum-only fit gives χ_1^2 with ν_1 degrees-of-freedom, while the continuum+line fit results in χ_2^2 with ν_2. We define the statistic F as

$$F = \left(\frac{\chi_1^2 - \chi_2^2}{\nu_1 - \nu_2} \right) \left(\frac{\nu_2}{\chi_2^2} \right) \quad . \tag{2}$$

We choose a maximum probability $P(> F)$ as the threshold for a candidate to be considered a line. This F-test probability is defined as the significance of the feature. Note that this is only the probability that the observed line is a fluctuation; it does not consider all the possible "trials", the range of parameters (e.g., line centroids, widths, etc.) in which line-like fluctuations could have occurred. The true probability that the line is a fluctuation is the product of the F-test probability and the number of trials; therefore we require a very small value of the F-test probability to conclude the line is real.

We parameterize burst detectability primarily by the SNR in a number of bands across the spectrum; the dependence on SNR will be discussed below. Since burst continua range in hardness (Band et al. 1993b), the SNR in a band near the line centroid should be used.

We generally characterize the BATSE spectra in the 25-35 keV band which is between the lines detected at 20 and 40 keV. We use only complete channels in these energy ranges, and therefore the actual energy width varies from burst to burst (the gain and consequently the energy-channel mapping changes with time). Since the SNR is proportional to the square root of the energy range ΔE over which it is calculated, we use $\mathrm{SNR}/\Delta E^{1/2}$ as the quantity characterizing the signal strength in each band.

As a consequence of statistical fluctuations, the line significance in the simulated spectra has a broad distribution, as shown by Figure 2. The line detection probability $p(\mathrm{SNR}, \theta)$ is the fraction of the simulations in which the lines were significant enough to be considered detections. Graphically, $p(\mathrm{SNR}, \theta)$ is the fraction of the area under the curve in Figure 2 to the left of the vertical lines indicating two different possible detection thresholds. Since this fraction is a number based on a finite number of simulations, there is an associated uncertainty which we approximate as $\sigma \sim \sqrt{p(1-p)/N_s}$, where p is the calculated probability and N_s is the number of simulations (usually $N_s = 200$). For $p = 0$ or 1 we calculate σ with $p = 1/N_s$ or $1 - 1/N_s$, respectively. To test this approximation we ran 12 sets of 200 simulations at two different burst angles and two different significance thresholds; we found that the actual dispersion ranged from 1 to 1.4 times σ.

The line models which are used to characterize the *Ginga* observations are either additive or multiplicative Gaussians (see eqns. [3] and [4] below). Consequently, lines are parameterized by three quantities: an intensity (e.g., an equivalent width), a line centroid and an intrinsic width. At times the line models can be simplified, and fewer than three parameters per line are required. Models with fewer parameters may result in more significant fits since both the F-test and Bayesian odds compare the improvement in the fit to the number of added parameters. Two lines were observed from GB880205 with harmonically spaced energy centroids at 19.4 and 38.8 keV. Thus the spectra from this burst can be fit by a line model with only five parameters since the energy of the second line can be fixed at twice the energy of the first. The intrinsic line width is often smaller than, or comparable to, the instrumental resolution and the fit may not be very sensitive to this width. For example, the GB880205 lines have FWHM of 3.0 and 13.4 keV where the resolution is 5.8 and 8.8 keV, and therefore the low energy line is unresolved while the high energy line will be somewhat resolved. Unresolved lines can also be modeled by a rectangular line profile where the flux is zero over an energy range (the equivalent width) centered on the line centroid; this model requires only two parameters. For the GB880205 lines we fit and compared the line significances in simulations with two independent three parameter lines (six parameters in total), two harmonically spaced three parameter lines (five parameters total), two independent lines with rectangular profiles (four parameters) and two harmonically spaced rectangular lines (three parameters). The issue is the tension

between the preference of statistical determinations of line significance (e.g., the F-test) for models with fewer parameters and the degradation in the fit when a spectrum is fit by an overly simplistic model.

4.   RESULTS

We use the spectra from the two *Ginga* detections to define the line types in our calculations. The first is based on the S1 segment of GB870303 (Graziani et al. 1992); we use a new fit provided by E. Fenimore (private communication, 1994),

$$N(E) = 5.76 \times 10^{-3} \left(\frac{E}{100} \right)^{-1.54} \exp \left[-42.5 \exp \left[-\left(\frac{E - 20.6}{2.5} \right)^2 \right] \right] \text{ ph-cm}^{-2}\text{-s}^{-1}\text{-keV}^{-1}, \quad (3)$$

where E is in keV. The line was observed to persist for 4s. The second line type models the lines in GB880205 (Murakami et al. 1988). Again, we use a new fit provided by E. Fenimore (private communication, 1994),

$$
\begin{aligned}
N(E) &= N_C(E) - \frac{0.7037}{\sqrt{2\pi}1.27} \exp \left[-\frac{1}{2} \left(\frac{E - 19.4}{1.27} \right)^2 \right] \\
&\quad - \frac{1.362}{\sqrt{2\pi}5.70} \exp \left[-\frac{1}{2} \left(\frac{E - 38.8}{5.70} \right)^2 \right] \text{ ph-cm}^{-2}\text{-s}^{-1}\text{-keV}^{-1} \quad (4) \\
\text{where} \quad N_C(E) &= 0.0819(E/100)^{-0.872} \exp \left[-\left(\frac{E}{250} \right) \right] \quad, \quad E \le 282 \text{ keV} \\
&= 0.0854(E/100)^{-2} \quad, \quad E > 282 \text{ keV} \quad ;
\end{aligned}
$$

note that we have expressed the Gaussians in terms of "standard deviations" σ (hence the numerical factors). This line persisted 5.5s. The continuum we use for GB880205 does not correspond to the best fit *Ginga* models above 100 keV, but is instead the four parameter functional form $N_C(E)$ (sometimes called the GRB model) which we find successfully describes burst spectra (Band et al. 1993b).

For each line type we ran simulations at burst angles of $\theta = 0°$, 15°, 30°, 46°, 60° and 75° for a variety of live times which were then translated into $\text{SNR}/\Delta E^{1/2}$; Figure 3 shows the results of these simulations. The spectra in GB870303 were fit over $E = 10 - 100$ keV (Fig. 3a) and $E = 10 - 1200$ keV (Fig. 3b), while the two lines in GB880205 were fit over $E = 10 - 1200$ keV with a variety of line models (Figs. 3c-f). In addition, we ran simulations every 5° for the time the features were observed to persist (4s for GB870303 and 5.5s for GB880205); the resulting dependencies are shown by Figure 4.

The results of the simulations at a given burst angle were modeled empirically by

$$p = \left[1 - \tanh \left(\frac{s - s_0}{\Delta s} \right) \right] / 2 \quad (5)$$

where p is the detection probability and s is the $\text{SNR}/\Delta E^{1/2}$ (as described in §3). At $s = s_0$ we have $p = 1/2$, while Δs characterizes the $\text{SNR}/\Delta E^{1/2}$ range over which p changes from

0 to 1. Here we use an F-test probability of less than 10^{-4} as the detection criterion, and the $\mathrm{SNR}/\Delta E^{1/2}$ in the 25-35 keV band. The resulting fits to the simulations are shown by Figure 3 and listed in Table 1. As can be seen, this model works remarkably well. It should be noted that these curves are fit to a small number of empirical data points, each of which is the average of a finite number of simulations. Consequently there is an uncertainty in the curves' true shape. We assume there is actually a monotonic increase in $\mathrm{SNR}/\Delta E^{1/2}$ with burst angle to achieve a given detection probability, yet the curves occasionally cross at small or large probabilities because of these uncertainties.

Figure 3 and Table 1 establish that there is a significant dependence of the detection probability not only on $\mathrm{SNR}/\Delta E^{1/2}$ but also on burst angle. Indeed the $\mathrm{SNR}/\Delta E^{1/2}$ necessary for a detection increases monotonically with burst angle; at small detection rates the empirical fits cross in a few cases, most likely because the $\mathrm{SNR}/\Delta E^{1/2}$ dependence is undersampled, and the points were calculated with a finite number of simulations. This dependence on burst angle can also be seen from Figure 4 where we show the detection probability for the two *Ginga* lines with the observed line persistence times. We use detection criteria of F-test probabilities less than 10^{-4} and 10^{-5}.

As Figures 3c-f and the listings for GB880205 in Table 1 also show, reducing the number of parameters by using a simpler line model does not change the calculated detectability appreciably. On the other hand, fitting the burst spectrum over a broader energy range makes lines more detectable; thus a line will be more significant when the spectrum is fit over a more extended energy range, even when the continuum model requires more parameters. To demonstrate this point, we fit the same 200 realizations of the model for GB880205 in eqn. (4) (except that in this case the break energy in the exponential of the GRB model N_C is 300 keV, not 250) from 10 to 100, 300, 900 and 1200 keV with different continuum models; the geometric mean of the F-test probabilities are presented in Table 2. As can be seen, the fits become more significant as the fitted spectrum is extended far above the high energy line at 38.8 keV. Note that the GRB continuum model N_C is "correct" since this is the model used to create the spectra. Yet the simpler, less "correct" continuum models give more significant line fits. Adding lines to incorrect, overly simple continua not only fits the lines but also compensates in part for the poor continuum model, and therefore the improvement in the fit when the lines are added to the continuum is greater, as is the apparent line significance. Thus this greater line significance is misleading. Of course, we do not know the correct continuum shape of real burst spectra; the GRB model is undoubtedly also too simple. Thus our fits may also overestimate the significance of line candidates.

5. DISCUSSION

Using a heuristic model of a spectrum with a line we can develop some approximate dependencies of $\langle \Delta \chi^2 \rangle$, the average improvement in χ^2 of the continuum+line model over the continuum model, on various line parameters; these dependencies explain our results, and provide guidance for future scalings (Ford et al. 1993). Let the observed count spectrum f_{obs} be divided into n channels of equal width ΔE, with the line falling in n_L channels. Assume the observed continuum is at a constant level f_c, and the line is at a level f_L (less than f_c): the line profile in this simplified example is rectangular. We now compare a continuum model to a continuum+line model, where once again the continuum is assumed to be at a constant flux, and the line has a rectangular profile. On average the best fit continuum+line model will be the same as the observed spectrum without noise, while the continuum-only fit will be $f_m = [(n - n_L)f_c + n_L f_L]/n$.

The noise has zero mean and non-zero variance σ^2. Using model variances and Poisson statistics we have for the ith channel $\sigma_i^2 = F_{M,i}/A\Delta t \Delta E$, where $F_{M,i}$ is the model flux averaged over the channel, A is the effective area of the detector, Δt is the live time, and ΔE is the channel width. The observed spectrum f_{obs} is assumed to be the model spectrum F_M with noise,

$$f_{obs,i} = F_{M,i} + \sigma_i \phi_i \tag{6}$$

where ϕ_i is the normalized noise distribution: $\langle \phi_i \rangle = 0$ and $\langle \phi_i^2 \rangle = 1$. The continuum+line model is "correct" and therefore

$$\langle \chi^2_{C+L} \rangle = \langle \sum_{i=1}^{n} \frac{(f_{obs,i} - F_{M,i})^2}{\sigma_i^2} \rangle = n \tag{7}$$

while for the continuum-only model

$$
\begin{aligned}
\langle \chi^2_C \rangle &= \langle \sum_{i=1}^{n} \frac{(f_{obs,i} - F_{M,i})^2}{\sigma_i^2} \rangle \\
&= n + \frac{A\Delta t \Delta E}{f_m} \left[\sum_{i=1}^{n-n_L} (f_c - f_m)^2 + \sum_{i=1}^{n_L} (f_L - f_m)^2 \right] \\
&= n + \frac{A\Delta t \Delta E}{(n - n_L)f_c + n_L f_L} n_L (n - n_L)(f_c - f_L)^2 \quad .
\end{aligned}
\tag{8}
$$

Identifying the observed line width as $\Delta E_{line} = n_L \Delta E$, the equivalent width of the line as $eW = \Delta E_{line}(f_c - f_L)/f_c$, and the SNR for a single continuum channel as SNR$= [f_c A \Delta t \Delta E]^{1/2}$, we find

$$\langle \Delta \chi^2 \rangle = \langle \chi^2_C - \chi^2_{C+L} \rangle = \frac{(n - n_L)f_c}{(n - n_L)f_c + n_L f_L} (eW)^2 \frac{(\text{SNR})^2}{\Delta E} \frac{1}{\Delta E_{line}} \quad . \tag{9}$$

Note that since SNR$\propto \Delta E^{1/2}$, (SNR)$^2/\Delta E$ is a function of the average continuum flux and not the width of the energy range; indeed we parameterize the line detectability by SNR/$\Delta E^{1/2}$. Since our heuristic calculation used the observed count spectrum and not the intrinsic photon spectrum, ΔE_{line} is the observed line width which results from the instrumental energy resolution and the intrinsic width.

As can be seen from eqn. (2), $F \propto \Delta\chi^2$, and we find that the negative of the logarithm of the probability $P(> F)$ is approximately proportional to $\Delta\chi^2$ for a large number of degrees-of-freedom. Thus the absolute value of the logarithm of the line significance is approximately proportional to $\Delta\chi^2$ (i.e., $-\log P(> F) \propto \Delta\chi^2$). In our case we are interested in the fraction of the distribution of $P(> F)$ which meets our detection criteria, and therefore the relationship between detectability and $\Delta\chi^2$ is more complicated, but monotonic. From our simulations we see that the line detectability is dependent on the SNR/$\Delta E^{1/2}$. In our simulations we fixed the line parameters, and therefore eW and ΔE_{line} were not varied. However, note that the line in GB870303 is detectable at lower SNR/$\Delta E^{1/2}$ than the two lines of GB880205 (compare Fig. 3b to Figs. 3c-f), which can be understood in terms of the equivalent widths and line widths. For the photon spectrum of the GB870303 line the eW and ΔE_{line} are both approximately 6 keV. The 19.4 keV line of GB880205 has $eW = 2.1$ keV and $\Delta E_{line} = 3$ keV while the 38.8 keV line has $eW = 7.3$ keV and $\Delta E_{line} = 13$ keV. However, the analysis above deals with the observed count spectrum, not the photon spectrum. Thus the GB880205 line at 19.4 keV would be unresolved, and the 20.4 keV line in GB870303 would be at best partially resolved, and thus both would be observed to have the width of the detectors' resolution of about 6 keV. Consequently the line in GB870303 would have a much greater equivalent width than, and comparable observed width to, the ~ 20 keV line in GB880205. Similarly, the line in GB870303 would have a comparable equivalent width to, but smaller observed width than, the ~ 40 keV line in GB880205. Thus the dependencies in eqn. (9) explain why the single line in GB870303 is detectable at a lower SNR/$\Delta E^{1/2}$ than the lines in GB880205.

The simulations show that extending the energy range over which the spectrum is fit increases the line significance, even if the continuum must be fit by a more complicated continuum model. This improvement in the significance results from two effects. First, increasing the number of degrees-of-freedom, ν_2, while holding F fixed (F is the statistic used for the F-test—eqn. 2) decreases $P(\geq F)$ (increases the line significance); for a large number of parameters (e.g., 6 for fitting GB880205) the factor of two increase in ν_2 resulting from raising the upper cutoff E_{max} from 100 to 1200 keV can decrease $P(\geq F)$ by more than an order of magnitude. Note that F is inversely proportional to the reduced $\chi^2_\nu = \chi^2_2/\nu_2$, which for a good continuum+line fit will be approximately 1 regardless of the value of ν_2. Thus if $\Delta\chi^2$ is fixed, F will most likely be fixed. Second, including more continuum in the

fit forces a larger difference between the continuum and continuum+line models, even when the added continuum is far from the energy of the candidate line feature. As eqn. (9) shows, $\Delta\chi^2$ increases with the ratio of the continuum counts, $(n - n_L)f_c$, to the counts in the line $n_L f_L$. The larger $\Delta\chi^2$ increases F and decreases $P(\geq F)$. The first effect is in some sense an artifact of the statistical method while the second effect results from more fully utilizing the available spectral information.

We can calculate the effect on the significance of merely changing ν_2. First we find for each continuum type the F_{1200} which gives the value of $P_{\nu_2 \sim 215}(\geq F_{1200})$ calculated from the simulations for $\nu_2 \sim 215$ when $E_{max} = 1200$ keV. Using this F_{1200} we then find $P_{\nu_2}(\geq F_{1200})$ for the values of ν_2 applicable to the different E_{max}. Table 2 shows ν_2 and the ratio $P(\geq F)/P_{\nu_2}(\geq F_{1200})$ for the different combinations of continuum type and E_{max}. As can be seen, $P_{\nu_2}(\geq F_{1200})$ decreases less rapidly than $P(\geq F)$ as ν_2 increases. Note that the COMP model is poorly constrained by a continuum up to only $E_{max} = 100$ keV, while the GRB model is poorly constrained for models to $E_{max} = 100$ and 300 keV.

Given the above dependencies on equivalent width and $\mathrm{SNR}/\Delta E^{1/2}$, the dependence on burst angle may seem surprising. However, the energy dependence of the detector efficiency changes with angle, particularly at low energy where the aluminum case and beryllium window have different absorption properties. Between ~ 10 and ~ 20 keV the effective area is a maximum along the detector normal, while at higher energies, where the aluminum is transparent, the effective area is a maximum at a burst angle of $\sim 30°$ (where the NaI crystal's projected area is greatest).

Our simulations reveal a large distribution of fitted line parameters and significances for the same input photon spectrum. Figure 2 shows that even a weak line, which would be considered a detection in only 15% of all realizations (with an F-test threshold of 10^{-4}), can occasionally appear to be extremely significant. This distribution of line significances is not related to fluctuations in the best fit line centroids, but clearly increases with the equivalent width of the fitted line, as demonstrated by Figure 5. The wide dispersion in the fitted line centroid and equivalent width are clear from Figures 5 and 6, respectively. Consequently, the fitted parameters to a line detection may not be an accurate description of the true line.

An interesting result of our simulations is that the line detectability is nearly the same whether the "correct" many-parameter line model or simpler models with fewer parameters are used to fit the GB880205 simulated spectra. As can be seen from the definition of the F statistic (eqn. [2]), the significance of a line increases with a reduction in the number of parameters (smaller $\Delta\nu$) or with an improvement of the fit (larger $\Delta\chi^2$). The width of the 19.4 keV line is $\sim 2/3$ of the energy resolution while the 38.8 keV line is $\sim 50\%$ larger than the resolution. Thus the fits are sensitive to the line profile, and eliminating the line width

degrades the fit sufficiently to balance the reduction in parameters. Surprisingly, fixing the ratio of the line centroids to be 2 does not increase the detectability appreciably. Note that reducing the number of line parameters from 6 to 5 increases F by only a factor of ~ 1.2 (since $\nu_2 \sim 200$, the difference in ν_2 for the two models is only $\sim 0.5\%$) which for line significances of order 10^{-4} increases the significance (i.e., decreases $P(> F)$) by a factor of only ~ 2.7. Because of fluctuations the best fit centroids are not at the input values, and therefore models with two independent line centroids result in better fits than models where the lines are constrained to be harmonic. The competition between simpler models and better fits is always present in modeling observations; with our sequence of models there is little preference for simplicity even when this simplicity appears to be justified on physical grounds. However, in other situations, such as fitting lines which are truly unresolved, simpler models may result in more significant fits.

While these detectability simulations were undertaken to support the comparison of the BATSE and *Ginga* observations, they also demonstrate BATSE's detection capabilities. As Figure 4 shows, the SDs could have detected the lines in GB880205 up to angles of $\sim 75°$ while the line in GB870303 would have been detected at small angles (less than $30°$) in about a quarter of the observations. BATSE consists of an array of detectors, and a burst at any point in the sky can be observed by a number of detectors at different burst angles. We therefore derived the average probability that the line would be detected in at least one detector.

Thus we calculated the detector array's set of burst angles for 100,000 "bursts" distributed randomly on the sky; these sets were binned by the cosine of the burst angles. The probability of a detection by at least one detector was computed for each set of burst angles using the detection probabilities as a function of burst angle (e.g., based on the data in Fig. 3). For most of the mission six SDs have been set at high gain (extending their sensitivity to low energies), while a pair of opposite detectors have been operated at low gain (effectively making them useless for line searches). However, the low energy cutoff of one high gain detector is only ~ 25 keV when operated at its highest gain; this detector is unable to detect lines at ~ 20 keV. Only one detector of any pair of opposite detectors will have a positive cosine which is necessary for the burst to fall in the detector's field-of-view (unless both detectors have a cosine of zero). We therefore model the average detection probability using five detectors, of which four are part of two detector pairs. For comparison we also calculated the probabilities for two, three and four detector pairs.

Of necessity we use a simplified model of the detector array in this calculation. The true energy cutoffs may differ from 10 and 1200 keV since without automatic gain control the gain of each detector (and therefore the energy covered) drifts with time; also, various

instrumental constraints cause the gain to vary. Next we assume that the data from all the SDs facing the burst are available. However, burst data are transmitted to Earth from only four detectors based on their LAD count rates, which may be affected by Earth scatter. In addition, the LAD and SD axes are offset by $\sim 18.5°$. Thus, data may be returned from one or two SDs with burst angles greater than $90°$ instead of from SDs with angles less than $90°$. Therefore, in reality data from less than four burst-facing SDs may be returned to Earth. Finally, Earth blockage was not included in the calculation since in *CGRO* coordinates the Earth's position should average out over many *CGRO* viewing periods.

Figure 7 shows the distribution of the cosine of the burst angle by detector rank assuming 4, 5, 6 and 8 detectors covered the energy range relevant for detecting lines. The detectors are ranked by the order of their burst angles; thus the first rank detector has the smallest angle, etc. As expected, operating more detectors at high gain results in a smaller average value of the smallest burst angle. However, the relevant quantity is the detection probability for the set of burst angles. For the detector array as a whole a given burst must be characterized by the persistence time and not by the $\mathrm{SNR}/\Delta E^{1/2}$ since a variety of $\mathrm{SNR}/\Delta E^{1/2}$ values will result for a given persistence time as a function of each detector's burst angle.

We calculated the average detection probability by averaging the net detection probability of the 100,000 bursts in each of our detector arrays, as shown by Fig. 8. This net probability is the probability that the line feature would be significant in at least one of the SDs; the probabilities for a single detector at a given angle were used to calculate the net probability for the entire detector array. Again as expected, the average detection probability increases when more detectors cover the energy range of the line features. The observed persistence times of the two *Ginga* bursts are indicated. Thus the line in GB870303 would be detected by BATSE with an average probability of $\sim 1/8$, while the lines in GB880205 would almost always be detected. Note that this calculation assumes the lines occur in spectra with the intensities reported by *Ginga*. The line in GB870303 is detectable at longer persistence times but at smaller values of $\mathrm{SNR}/\Delta E^{1/2}$ than the lines in GB880205 because the continuum in the GB870303 spectrum is much weaker than for GB880205, while the line equivalent width is greater. Thus a line with the equivalent width observed in GB870303 in a continuum as intense as GB880205 would be spectacular.

6. SUMMARY

By fitting simulated BATSE spectra of the two line sets observed by *Ginga* we find that BATSE should indeed be able to detect similar lines. These simulations provide us with the detectability as a function of a spectrum's signal-to-noise ratio, a relationship necessary for calculations of the number of lines which would have been detected if present in the BATSE bursts. We note that these are calculated instrumental capabilities.

These simulations have also provide guidance into the analysis of line candidates. We find that fitting spectra over a broad energy range increases the significance of line candidates. Decreasing the number of parameters by using physically motivated simplified models (e.g., assuming multiple lines are harmonically spaced or eliminating the intrinsic line width because it is smaller than, or comparable to, the instrumental resolution) may not result in the expected increase in significance, as we found in simulations with the pair of lines in GB880205. These lines have widths comparable to the instrument resolution, and therefore it is not surprising that simplifying the profile degrades the fit. Although the lines were harmonically spaced in the photon spectrum, forcing the fitted lines to be harmonic did not significantly improve the line detectability, presumably because fluctuations shift the best fit line centroids. We conclude that the degradation in the quality of the fit counterbalances the reduction in the number of line parameters.

Our simulations demonstrate that BATSE is able to detect the line in GB870303 in about an eighth of the possible realizations and positions on the sky relative to *CGRO*, while the lines in GB880205 are almost always detectable. For most of the mission five of the eight spectroscopy detectors have been operated at high gain to maximize the sensitivity to absorption lines; increasing the gain of two more detectors (one detector cannot be pushed to high enough gain) will increase the average detection probabilities but not by a large enough factor to qualitatively change BATSE's line detection abilities.

The implications of these line detectabilities for issues such as consistency between the BATSE and *Ginga* observations will be explored in future papers of this series.

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FIGURES

Fig. 1.—Simulated count spectra of (a) the S1 line in GB870303 and (b) the lines in GB880205 as they might have appeared in the BATSE SDs. The curve is the photon spectrum convolved with the detector response at a burst angle of 30°. The data points, rebinned into bins half a resolution element wide, are a possible realization of this count spectrum assuming Poisson statistics.

Fig. 2.—Distribution of F-test probabilities. Based on 2400 simulations of the S1 line of GB870303 which persisted 4s, observed at a burst angle of 15° and fit over $E = 10 - 100$ keV. The two vertical lines indicate the detection thresholds of F-test probabilities less than 10^{-4} (short dashes) and 10^{-5} (long dashes); the fraction of the distribution to the left of these lines is the detection probability.

Fig. 3.—Detection probability vs. $\mathrm{SNR}/\Delta E^{1/2}$ in the 25-35 keV band for a variety of burst angles. From left to right, the angles are 0°, 15°, 30°, 46°, 60° and 75°. An F-test probability less than 10^{-4} was used as the detection criterion. The data points are the results of the simulations and the solid curves are empirical fits to these points. Panels (a) and (b) are for the S1 segment of GB870303 fit over $E = 10 - 100$ keV and $E = 10 - 1200$ keV, respectively. Panels (c)–(f) show the detectability of the GB880205 line over $E = 10 - 1200$ keV where: (c) rectangular line profiles are used and the two line centroids are required to be harmonic, $E_2 = 2E_1$ (3 line parameters); (d) rectangular line profiles are used and the two line centroids are fit independently (4 parameters); (e) the three parameter line model is used with harmonic lines (5 parameters); and (f) the three parameter line model is used with the two centroids fit independently (6 parameters).

Fig. 4.—Detection probability vs. burst angle for the two *Ginga* lines at the observed persistence times. The upper data points are for an F-test probability less than 10^{-4} and the bottom set of points for 10^{-5}. The panels correspond to the same cases as in Fig. 3.

Fig. 5.—Fitted equivalent width vs. F-test probability for a set of 2400 simulations of the S1 line of GB870303 which persisted 4s at a burst angle of 15°. The fitted equivalent width has been normalized to the value for the input line.

Fig. 6.—Distribution of fitted line centroid energies for a set of 2400 simulations of the S1 line of GB870303 which persisted 4s at a burst angle of 15° fitted over $E = 10 - 100$ keV. The input line centroid is indicated by the vertical dashed line. The fitted energy was constrained to fall between 15 and 30 keV.

Fig. 7.—Distribution of the cosine of the burst angle by detector rank. The panels show the distribution for a) two detector pairs, b) 5 detectors including two pairs, c) three

pairs, and d) four pairs. The detectors are ranked by the size of their burst angles. For a detector to be included the burst must occur in its field-of-view (non-negative cosine), and the gain must be high enough to observe lines at ~ 20 keV.

Fig. 8—Average detection probability vs. the time a feature persisted for (a) the S1 line of GB870303 and (b) the lines in GB880205. From top to bottom, the curves are for four detector pairs (small dashes), three pairs (dash-dot-dot), 5 detectors including two pairs (solid), and two pairs (long dashes). The vertical lines indicate the observed persistence times. The burst intensities reported by *Ginga* are used.