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Mathematics Course for Elementary Teachers in an Alternate Certification Pathway

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A MATHEMATICS COURSE FOR ELEMENTARY TEACHERS IN AN ALTERNATE CERTIFICATION PATHWAY

A Thesis

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Abstract

In the current alternate certification program for elementary teachers at McNeese State University, students receive course work from only one of the elementary mathematics methods courses required in the traditional certification program. To assure that candidates in the alternate path learn all the math they need, I have created a course that combines the most important concepts from the two courses in one. In this thesis I will describe how the new course was designed, present an outline of the course, detail the content at the unit level and provide a template for the final exam.

Introduction

The purpose of this thesis is to describe a new mathematics course that will be required in an alternate certification program for elementary teachers offered at McNeese State University. McNeese State wishes to institute a new course specifically for the alternate certification students that combines the material covered in education courses currently used in the standard curriculum. This thesis includes a course description, outline, final exam, and a complete sample curriculum of ‘Unit 3’ for the new course. Additional materials and assignments will be created in the future, and it is my intention to make them publicly available.

Alternate certification candidates at McNeese are currently required to complete the course EDUC 334, Math Methods 1. The course, while it does act as an introduction to teaching elementary mathematics, does not include all topics essential for future elementary teachers. The course described in this thesis will provide candidates with a more adequate knowledge of mathematical concepts and algorithms, and their application to classroom instruction. Candidates will also receive guidance in areas such as lesson organization and effective teaching techniques.

This course, while designed for McNeese State, might also serve as an outline for a course in other certification programs. With alternative certification programs mushrooming across the country, a course such as this may be useful for the large group of candidates seeking certification. In order to improve the quality of alternate certification, the mathematics course work required of the candidate must also be improved.

The design of the new course was based on the following inputs: a review of textbooks currently being used at McNeese, an interview with a current, (but retiring), instructor for the methods course EDUC 335, a survey of textbooks used by other universities in the United States, and books written by Hung-Hsi Wu and Sybilla Beckman. These sources provided evidence concerning the contents and methods best incorporated in a math course for future elementary teachers.

The texts by Wu and Beckman were chosen to serve as the main readings for the course because they present the content in complementary ways. Beckmann's text is instruction-driven and concentrates on how to teach mathematics. Her writings cover various Common-Core- supported techniques and practices that teachers may use in classroom instruction. She includes a variety of approaches that allow teachers to address the diverse learning needs of their students. Her book is careful, detailed and explicit in describing instructional procedures.

Wu's book concentrates on explaining why techniques are taught and providing correct ways to explain mathematical concepts. Wu warns about deficient, misleading, or incorrect statements and explanations that are common in textbook school mathematics. He advocates definitions, precision, explanation, coherence, and purposefulness. He not only provides clear mathematical definitions, but concise explanations of content. He gives ample information as to why specific algorithms are performed, and not just how they are meant to be executed.

This course incorporates the material from Wu and Beckmann that is, according to my survey of courses and textbooks, essential for the preparation of alternate certification candidates. These topics include: Base 10 Number Systems and Place

Value, Addition and Subtraction Algorithms, Multiplication and Long Division Algorithms, Definitions of Fractions, Units, and Decimals, Fundamental Fraction Pairs: Equivalent Fractions, and the Addition and Subtractions of Fractions and Decimals. Material to support these topics was chosen from various chapters of Wu and Beckmann and reworked and combined. The course progresses from mathematics using whole numbers and integers to fractions and operations with them. It supplements the conceptual progression of Wu's book with the practical applications presented by Beckmann. Certification candidates will be given a wealth of classroom activities based on unit topics that they may apply to their own in-class instruction.

The course consist of seven units offered over 14 weeks, with each unit receiving two weeks of attention. Assignments will alternate between the texts of Wu and Beckmann throughout the 14 weeks. We make time for demonstrations, lesson plan critiques, and quizzes. This course is designed provide opportunities for student-led instruction and field experiences under the guidance of an instructor. In summary, the course includes a wide variety of learning opportunities: alternating textbook assignments, teaching demonstrations, lesson planning, lesson plan critiques, opportunities for candidate-led instruction, quizzes covering unit content, and field experiences.

Students will complete a final exam that has been carefully planned to align with the main content goals of the course. The exam measures the extent to which candidates have mastered the essential ideas. A bank of final-exam problems will be distributed at the beginning of the course in order to show clearly what the course expects. The sample consist of 38 questions, covering terminology, techniques, and

applications. In addition to these 38 questions, 25 discussion questions taken from Beckman's text are included. These questions are broad and general, covering more than one concept at a time, and not falling into a specific category.

Chapter Summaries

This thesis includes five chapters. Chapter 1 describes the of McNeese State University community, the existing programs for elementary teachers, the reasons why a new course is needed, and some background about my involvement in the project of creating a new course.

Chapter 2 describes the students who will be taking the new course. It also includes a summary of an interview with the current instructor of EDUC 335. The interview provided important insights into the audience and the conditions surrounding the course. An appendix supplementing this chapter contains the 2 year layout of the alt-cert curriculum. Chapter 3 introduces an overview of the course. I explain the design process as well as the new course plan. I also go into detail on the aspects of two textbooks that I have chosen as the basic resources for this course.

The brief Chapter 4 gives an overview the final exam and describes its grading rubric. Appendix 1 contains a bank of items for the final exam and Appendix 2 contains an answer key.

This thesis is founded on a belief in the necessity of proper mathematics education for those who aim to teach it. The new course provides prospective students with tasks and objectives that require serious use of rigor and understanding. In order to

improve the education of future students we must first better the education of those who will teach those future students.

Chapter 1. Setting and Motivation

McNeese State University is a public regional university based in Lake Charles, Louisiana. The campus is surrounded by a unique metropolitan area that is built on an industrial, cultural, and educational foundation, including the surrounding petrochemical refining plants and the university itself. While the university originally entered the Louisiana State University system in 1939 as a Junior college, today it offers four-year baccalaureate degrees and enrolls about 7500 students. Although the area is industrialized, it is home to many people coming from a rural background. Surrounding cities such as Sulphur, Bell City, Westlake, and Iowa act as gateways to areas of farmland and wildlife. In fact, many students find themselves drawn to McNeese due to its connections to the agricultural field. This “rural attitude” is embodied by the school’s mascot, Rowdy the Cowboy. “Cowboys” be they locals, students, or alumni yell “Geaux Pokes” at all athletic events, and the phrase acts as a unifier for all who look to the university as a beacon in the community.

The New Course Proposal: EDUC 327

McNeese State University offers a post-baccalaureate alternate certification program for elementary teachers. The alternate-certification students are required to take only the first of the two math methods courses that are required in the traditional undergraduate elementary education program. For the remainder of this section, I will refer to these courses as EDUC 334 and EDUC 335. In order to improve the preparation of the “alt-cert” candidates without increasing the time required, the

university has proposed to use a single math methods course that achieves the most important goals of EDUC 334 and EDUC 335.

My involvement with the alternate certification program at McNeese came about at the invitation of the department when they learned of my personal experience becoming a teacher through alternate certification and my teaching experience for over 20 years. I was referred to the Assistant Department Chair of the Burton College of Education at McNeese, who is primarily responsible for next semester's new curriculum and its guidelines. With the Chair's approval, the Assistant Director invited me to take on the responsibilities of creating a new course that would meet their new desired standards. She informed me of the changes they were hoping to make within the alternate certification program. The College provided me with information and material such as syllabi, lesson plan templates and rubrics, audience statistics, and other course data for EDUC 334 and EDUC 335.

The texts specified in the syllabi of EDUC 334 AND EDUC 335 were *Learning Math in Elementary and Middle School* by Prentice Hall and *Teaching Learners Who Struggle with Mathematics: Systematic Intervention and Remediation.* These texts were used in order to prepare students with current principles, practices, methods, and materials for designing and implementing math instruction in grades Pre-Kindergarten through Grade 5. Other widely used texts found for similar courses in various programs throughout the United States are *Young Mathematicians at Work* by Fosnot and Dolk, *Elementary and Middle School Mathematics: Teaching Developmentally* by Van de Walle, and *Helping Children Learn Mathematics* by Reys, Lindquist, Lambdin, and Smith, to name a few. I ultimately chose to use two books, one by Sybilla Beckmann

and the other by Hung-Hsi Wu. The qualities and selection of these books will be discussed in a later section.

The syllabi also listed and described the objective learning outcomes for each of the courses. EDUC 334 addresses six broad topics: 1) numbers and operations in base 10, 2) operations and algebraic thinking, 3) fractions, 4) measurement and data analysis, 5) algebra and geometry, and 6) math centers (an instructional method) and how to implement them. However, these topics were not always supported by readings from the text. Students and teachers reported to me that although reading assignments are on the syllabus, students typically participate in discussion regarding broad concepts covered in the chapter, rather than what the chapter describes explicitly. Video assignments are included in some the lessons. The videos varied and were from providers such as TED, Mathwire, Math Forum, as well as multiple media-streaming sources. Field experience for EDUC 334 consists of ten hours of tutoring, 4 hours of observations, and one hour of lesson practice with a cooperating teacher.

Alternate certification teachers are not currently required to take EDUC 335, and will not have the same opportunities as those enrolled in 335. According to the syllabus I received, 335 addresses the BESE Mathematics Teacher Competencies. The course has a stronger focus on lesson plan design when compared to EDUC 334. There are 15 hours of field experience required in the form of one-on-one tutoring with an individual child from a cooperating school of the instructors choosing, including 3 lessons taught in a cooperating classroom where lesson plans are created by the student.

In summary, the university felt that the alternate certification students needed a new means of acquiring math instruction knowledge. I was brought on to help with this endeavor and I was supported by the institution in a variety of ways.

Chapter 2. Students and Instructors

EDUC 327, the course created and described in this thesis, is designed specifically for students earning a Post Baccalaureate Alternate Certification in Elementary Education. It combines the requirements found in EDUC 334 and 335, with attention to the most important aspects of these courses, in order for alternate certification students to receive effective and beneficial instruction. The goal is to fill the gaps found in the understanding of elementary mathematics that exist among those who choose a path towards certification that does not require both EDUC 334 and 335. The present chapter describes the student audience, and the experiences of an instructor. Understanding these things provides important perspectives for course design.

The ages of the students in the alternate certification program range from recent college graduates to people desiring a career change after being in a different line of work. There is no limit to the time since their degree was earned. Many of the younger students planned to become elementary teachers but chose a different undergraduate degree for various reasons. Some of the older students retired from a previous career and decided to teach children, because they felt drawn to that work. Most of the students who will enroll in EDUC 327 will come from one of the five “feeder” districts of the university, those being Calcasieu, Jefferson Davis, Cameron, Allen, and Beauregard Parish. The expected number of students is based on typical numbers of students enrolling in similar classes each semester. This ranges from 7 to 12. Usually almost all are female.

To assure that the students possess basic educational knowledge, the College requires them to pass EDUC 499, which has the following course description:

EDUC 499 - Entrance into the Non-Master's Alternative Program

Students planning to be teachers through the non-master's alternative program must make application through the TeachSWLA office and the Graduate School. Upon admission, a grade of "S" (satisfactory) or "U" (unsatisfactory) will be assigned.

Prerequisite(s): Bachelor's degree with a minimum GPA of 2.20 and (passing scores on [Core Academic Skills for Educators Praxis Exam, ACT, or SAT] and Praxis content-specific exam).
Cr. 0

This description implies that admitted students will have passed certain exams that include some basic math skills. EDUC 327 must be designed so that people who have the minimum math knowledge required to pass these tests will have a chance of success. In summary, the audience will be small with some basic understanding of the necessary concepts.

Interview with Current Teacher: Student Character Traits

I interviewed one of the current teachers of the second math methods course, EDUC 335. He was very pleasant to meet with and his office showed his sense of humor. He is a collector of everything Peanuts and his office was packed full of paraphernalia. He immediately told me that he is not a fan of alternative certification programs, but understands that they are a necessity due to the lack of teachers in today's education system. He is a product of alternate certification, so his low opinion of it surprised me. He later mentioned that his undergraduate degree is in mathematics, and we recognized and appreciated our similar beginnings.

He spoke with me about his concerns regarding how most of the students entering the PBC (Post Baccalaureate Certification) program have only 6 to 9 credit hours of math. He feels that this puts students at a disadvantage, and leaves them under qualified as math instructors by the completion of their program. While the department receives the Praxis test results from the students' previous year in order to get an idea of the student's skill level, such a small amount of math during their time at the university does not supply them with the necessary skills required. He believes that the students should have in-class tutoring and teaching experiences prior to becoming certified. He finds that tutoring is invaluable to someone training to be a teacher.

Those who do participate in tutoring are able to research the testing results of an individual being tutored as opposed to the skill level in the classroom. Collaboration with the university is more between individual schools than with the local school board, as far as what to teach, how to teach, and when to teach it. They are also informed by the local school board regarding the Compass specific requirements for upcoming new teachers.

Nearby districts vary on the control of the schools' curriculum, and Calcasieu tends to encourage the use of Eureka. That is why the current alternate certification program requires 15 hours of mathematics. With many districts determining their own methods of instruction, it is important for instructors to be fluent in all aspects of required math. During one of our many conversations, the instructor spoke about how math is its own language, and that language is numerical. The program makes sure that even if the students entering this program don't like math, they will not leave gaps in math

understanding for the children they teach. He makes sure that new teachers can answer the “why” questions to build the foundation.

He has seen a change in who comes to the PBC program. They used to be older, but now they are generally younger. More are getting their degree in general studies (which only requires two math courses) or another less stringent degree, then they immediately join the alternate certification program. He finds the Praxis “too easy” and believes that it encourages students to take a simpler path to becoming an educator. Students are finding these loop-holes around an education degree because they find that a degree like General Studies can get them into education with arguably less stress than that of an education degree, and only a 2.2 GPA is required to enter the PBC program.

He is not a supporter of fast track programs. He believes that too many leaders in education are results of programs like Teach America, for example. He is definitely against online Math Methods courses and believes that face-to-face is a must. It doesn’t matter what their B.S. is in. Students just have to pass the Praxis and maintain a 3.0 to remain in the program.

He gave me the opportunity to look at his Methods II final exam so that I could have an idea of his testing style. It was very long and it required students to perform algorithms, provide explanations, and state rules. The level of rigor required to pass this exam was evident. It is the information found in examinations such as this that alternate certification students have been lacking.

After scheduling an interview with the Dean of Education I was surprised to find that he was actually the instructor for EDUC 335. From this interview, I walked away

feeling his passion about the creation of a new math methods course, but also his concerns about the students and how they enter the alternate certification program. He also shared concerns about instructor availability for the new EDUC 327 and the ability of the instructor to execute the desired plan of the course. I left me with concerns about the future of EDUC 327. I hoped that the new course would attract more individuals, and in particular, those that were more passionate about mathematics. I left the interview wanting to ensure the university that the quality of the material and instruction that I made available in this course would help to ensure its effective delivery.

Chapter 3. The New Course

This chapter first describes the course design and describes each unit. It provides an orientation of the weekly activities included in the classes, as well as the assignments for which the students will be responsible. The textbooks for the course are also described and the content from the textbooks that will be used. This chapter also references the appendix pages providing an outline of the course and the curriculum for the Alternate Certification program.

Course Structure

According the McNeese Calendar, the new course will meet once a week for 15 weeks. Each class meeting will last two and a half hours. The course will consist of seven two-week units and a comprehensive final exam in the fifteenth week.

The first week of each unit will utilize the text *Understanding Numbers in Elementary School Mathematics* (Wu 2011). Assignments for the first week of each unit will include readings, written reports on the readings (to be handed in), and end-of-chapter exercises from Wu’s book (also to be handed in). The assignments may cover more than one chapter. The second week will be based on Beckmann’s book (Beckmann 2014). Assignments will include readings, reports and exercises, as in the first week. Beckmann’s book also includes learning activities that the students will prepare and demonstrate in class. During the first half of the semester these will be selected by the instructor. In the second half of the semester, students will choose from a selection made by the instructor. The students will turn in lesson plans for their demonstrations.

Unit Descriptions

Unit 1: Base 10 Number System and Place Value. The course concentrates on whole numbers before more general parts of the number system. This allows students to get comfortable with using the properties and the algorithms and to develop an understanding of them before using them in more complicated number systems. The first unit of the course focuses on the following topics:

- number line,
- whole numbers
- the place-value system of number notation the meaning the digits according to their position in a number symbol, and
- the arithmetic operations in the case of whole numbers, using addition of lengths to model addition of numbers.

The following practices are emphasized:

- Instruction will progress from simple cases to complex cases to general ideas, for example, from examples with one-digit numbers to two-digit numbers, then to three-digit numbers and beyond.
- Students will work to state their reasoning clearly. The course is based on the idea that it is necessary to be able to clearly explain mathematics to another person who is at a much more rudimentary understanding in a way that makes sense to them as well.

Unit 2: Addition and Subtraction Algorithms. Unit 2 provides step-by-step instructions for the general multi-digit addition and subtraction algorithms and the ideas behind these algorithms. It concentrates on *why* these algorithms have the particular structure that they do, and how this structure works to produce correct results. The unit emphasizes the idea that the algorithmic procedures are not only to be mechanically followed, but need to be fully understood as well. With understanding, a student acquires the ability to deal confidently with a greater range of problems and a higher degree of complexity.

Terminology relating to subtraction is discussed in Unit 2. For example, words such as *take away*, *minus*, and *from*, are all words that are commonly used when discussing the process of subtraction. The unit also discusses student confusion about subtraction—in particular, its lack of the commutative property. The course will also examine interesting tricks and shortcuts for mental math and the reasons that they make sense and are effective.

Unit 3: Multiplication and Long Division Algorithms. Unit 3 first concerns the multiplication algorithm. Additional detail about this unit is in Appendix 3 Instruction on the multiplication algorithm in broken down into three parts: one-digit-by-one-digit, multi-digit-by-one-digit, and multi-digit-by-multi-digit. One-digit-by-one-digit examples need to be committed to memory. Once that's been done, the other cases can be handled by algorithms that utilize the memorized one-digit examples in a structure that makes use of place-value. If a student understands place value, or sees the number as the sum of multiples of powers of 10, the order of the steps and the "trading" (or "carrying") of an amount to the next digit to the left makes sense.

Addition and multiplication of whole numbers results in whole number answers. Subtraction of whole numbers introduces the possibility of negative results. Division of whole numbers can result in rational numbers that are not whole, leading to the concept of a remainder.

In the whole-number context, division needs to be understood as a means of expressing a number as a multiple of a divisor PLUS a whole-number remainder that is less than the divisor. This is expressed formally in the following:

Theorem: Given any two whole numbers *a* and *d*, with $d > 0$, there is one and only one way to express *a* as a multiple of *d* plus a whole number that strictly less than d. (Or, as mathematicians say, there are unique whole numbers q and r such that

$$a = q\,d + r \quad \text{and} \quad 0 <= r < d.$$

This unit also analyzes the long division algorithm, showing how it uses place value to break down division into problems that involve individual digits.

Unit 4: Definitions of Fraction, Unit, and Decimal. Fractions are sometimes said to have many definitions: as part of a whole, as a quotient, or as a ratio. But it is confusing to work with an idea that can change depending on context. Wu says: Definitions not only have to make sense, but they have to continue making sense when they are applied to more demanding situations.

The basic story we wish to tell (following Wu): Assuming the number one as the unit, one can easily understand dividing the single unit into a specified number (16, say) of equal parts. Once the standard unit on the number line has been divided into *k* equal parts, one of the parts can then be thought of as specifying a position between 0 and 1.

A single part can then be repeated---added to itself over and over again—to reach positions to the right. We approach fractions based on this mental picture.

This can make it understandable to actually see parts like $1\frac{1}{2}$ parts of a whole. Giving names to multiples of the parts naturally follows, and then to the concept of equivalent fractions as fractions that land on the same location on the number line. The unit also describes the use of a fraction “bar” as an instruction tool. When the fraction “bar” is seen as division and long division is carried out, a learner can see the fraction expressed in decimal notation as a string of digits created by the long-division process. Locating improper fractions on the number line can be made easier if you treat the fraction bars as division with remainder. The whole number can be found, and then the remainder will be a fraction, with the divisor specifying the unit. On the other hand, a decimal can be represented as a fraction if you consider the place values as fractions of powers of 10.

In introducing fractions, we draw a lot of attention to the difference between *fractional number* (aka rational number) and *fractional expression*, which is a way of specifying a position on the number line using a pair of whole numbers.

Unit 5: Fundamental Fraction Pairs: Equivalent Fractions. When considering fraction symbols involving small whole numbers as numerator and denominator, it is easy to understand that two different symbols may represent the same position on the number line. Two fraction symbols are said to be equivalent when they refer to the same point on the number line. Students are taught that multiplying or dividing the numerator AND denominator of a fraction symbol by the same number results in an “equivalent fraction.” But the fact that fractions (not fraction symbols) are points on the

number line is often not taught, and students never understand precisely why equivalence is an important (and basically simple) idea.

A fraction is considered to be reduced to its simplest form when the greatest common factor of the numerator and denominator is 1. Unit 5 explains how equivalent fractional expressions can be understood as either numbers that have the same location on the number line or as fractional symbols that can be reduced to the same number or amount.

Unit 5 focuses on the ways to determine whether two fractions are equivalent or represent the same amount. These are: (a) Find common denominators and compare the numerators, or (b) cross multiply and compare the products. If the cross products are equal, then the fractions are also equivalent. Though the cross product method is not uniformly supported by all text authors, it is very useful when dealing with large or unusual fractions.

<u>Unit 6: Addition & Subtraction of Fractions and Decimals.</u> The idea behind using the addition of lengths to model addition of numbers is not as easy to use with fractions as it is with whole numbers. The reason is that two different fraction symbols may use different units of measure (as expressed by their denominators). The unit of measure that is most suitable for expressing a sum might not be the unit used in the fraction symbols that express the summands. When the fraction symbols that are used to express the summands have the same denominator, of course, the task is very simple because one simply needs to add the numerators—we are adding two multiples of the same unit. In other cases, there are problems.

Most texts encourage teaching the procedure of finding a least common denominator. The cross multiplication strategy is a very beneficial one because it leaves out the decision making factor. Technically, stopping to think of the faster option just wastes more time.

The unit also addresses decimal fractions. The reason for lining up place value when adding decimals just makes sense. Lining up the decimal point is an organizational tool. Students look at the value of each digit instead of looking at the amount of digits in the number. Mixed number notation is often confused with multiplication. It actually represents the addition of an integer and a fraction. Mixed numbers should be introduced after fraction addition. It seems to take away the hesitancy of student use and encourages a more comfortable use with them.

Subtracting fractions has a very similar algorithm as addition does. If the fractions are in $\frac{a}{b}$ form, the same numerator and denominator rules apply, except that the numerators are subtracted. The real difference occurs with mixed numbers. Similar to the trading effect with whole numbers, a mixed number can be expressed as a whole number with a fraction that borrows a unit so the fraction part of the mixed number can be in descending order.

Unit 7: Multiplication and Division of Fractions**.** When it comes to whole numbers, multiplication may be understood as repeated addition. This is not easily transferred to fractions, what it might mean to add a quantity to itself a fractional number of times. The area model is a much better understandable way of seeing the true definition of fraction multiplication. The other model that can be used is the number line.

Using a fraction of a unit and then taking a fraction of that fraction of a unit leaves one with a solution of what fraction remains.

The area model again explains the idea of dividing fractions. By defining a fraction $\frac{A}{B}$ as the fraction C, a whole number, where A = CB, the laws of whole numbers can be applied to fractions. Division and multiplication are and stay directly connected to each other. The concept of using the inverse of a fraction to convert division to multiplication shouldn't feel foreign. The manipulation of complex fractions can be seen as a fraction built of fractions, and the same strategy can be used by turning it into a fraction division problem. There are several other strategies, like LCD multiplication, that are also useful.

Textbook Descriptions: *Mathematics for Elementary Teachers* by Sybilla Beckmann

The common attitude among most adults is that elementary teachers shouldn't have to take math classes in college because they are only teaching elementary math. They have obviously passed elementary math, right? What could they possibly learn? The major difference is the *why*. Teachers, especially elementary teachers, have to be ready to answer all the *why's*. Mathematics is not just a set of algorithms, though the algorithms are important. It is more like a process. It is often said that math is like a sport. It is not just something you know, it is something you do. Students are not going to like doing mathematics if they don't see the why. Beckmann's book helps give teachers explanations of how and why mathematical operations and concepts are what they are. It explains why mathematics works the way it does. Its progression is also one of its greater attributes. It is organized around the order of operations and not by types of numbers. The book treats all numbers as just that...numbers.

The specific book of Beckmann that is preferred is the “with Activities”. It not only has a MyMathLab Skill Review addition, but it has hundreds of class activities for new teachers to practice and perhaps even use in their future classrooms. Another notable is the illustrations. Many teachers fail to address the different learning styles of students. This is a reference to college professors, not to elementary teachers. Beckmann’s book is full of presented information in many forms, and thus can meet the needs of adults that learn in many different styles.

From a curriculum standpoint, it also meets several needs. It explains the how, the why, and the practice exercises that new teachers need. However, it doesn’t go quite in-depth into the *why* as perhaps Wu does.

Textbook Descriptions: *Understanding Numbers in Elementary School Mathematics* by Hung-Hsi Wu

Wu’s book is for K-6 mathematics curriculum. It focuses on numbers and operations. It is very concise, as it is for adult education, and does not shy away from higher level aspects of mathematical notation. If one does not understand mathematical notation, he cannot truly understand proofs and other mathematical concept definitions and explanations. These, however, appear slowly throughout the book and do not overwhelm the reader.

Another aspect of this book is that it addresses the need for coherency in the way we teach mathematics. For example, though most textbooks address fractions separately from other numbers and their operations, their operations are exactly the same. They are indeed numbers on the same number line, though that impression does not come across in most other textbooks.

Wu's book is divided into five parts, with each part having several chapters, and each chapter has several sections. This makes it a very technically sound book and great for teacher research. Another aspect that really separates this book from most others is the exercises at the end of each chapter. The questions in each exercise are not necessarily "practice" exercises that allow a teacher to practice their algorithm skills. They get the root of why we choose to do what we do to arrive at a solution. There are also many scenario questions that encourage deeper thinking skills than other texts.

Chapter 4. Final Exam

The objective of the Final Exam of EDUC 327 is to ensure the students are not only successful at computing mathematical operations correctly, but that they can explain the “why” behind their solutions. The test includes a brief statement of the notion tested in each problem. The answers are graded 60 percent on exposition and 40 percent on the solution. Upon successful completion the student will have provided evidence of their ability to execute and explain the most important math that appears in the elementary classroom. A complete problem bank is contained in Appendix 1, and solutions are provided in Appendix 2.

The final exam is divided into three parts. Part One focuses on the Arithmetic Skills and it addresses five specific mathematical operations. Part Two focuses on Application and Understanding and it is organized into seven sections aligning with the units of the course. Part Three is titled Discussion Questions and it contains a collection of questions from the Beckman’s chapter summaries. These questions test conceptual understanding of the operations on whole and rational numbers.

Part One, Arithmetic Skills, challenges the student on the elements of place value, operations with integers, rational number notation and arithmetic, rational number ordering and comparing and operation of decimal numbers. In part one, students are evaluated on their ability to perform basic math skills, but are also challenged within the questions for more than just an answer. The solution must include a clear execution of the process used to achieve that answer.

Part Two, Application and Understanding, aligns with the units within the course outline. In this section of the test questions include words such as: Explain, Identify, Modify, Write, to ensure that the student is being challenged on understanding and

applying operations, and not just on a skill set. Because this section covers all seven units of the course, it is the largest portion of the final exam. The questions within the section contain multiple parts, challenging the student with various scenarios in a given application.

Part three, Discussion Questions, is a selection from Beckman’s end-of-chapter “Key Skills and Understandings.” These questions use statements like describe, compare, draw, plot on a number line and state the property will not only be evaluating the students’ knowledge of the math skill, but their ability to teach the math skill. The questions on the final will be selected from a list that students will be able to review prior to the test. This will ensure that students are held accountable for the reading assignments in Beckman’s book. Thus, the questions will serve both as a study guide and to evaluate the students’ knowledge.

In many courses, development of the final exam tends to align with previous quizzes, chapter tests or computer-aided test generators. This final exam was created outside of any of these commonly used influences. The first two parts of the test contain unique questions to achieve authenticity of the examination. The structure of the exam specifically aligns the objectives of the course.

See the Appendices for the actual problems and solutions

Chapter 5. Conclusion

This thesis has presented materials to support an elementary math methods course for an alternate certification program. These materials includes a course description, textbook reviews, unit descriptions and a description of the final exam. Appendices to this thesis include a bank of final-exam problems with a grading rubric and an answer key, an outline to guide instructors through the 15-week timeline, an instructional presentation for one unit, and an example quiz.

The uniqueness of the course is the contribution that it makes, coming from the total package that it offers to both the student and the instructor. To the student it offers many different levels of learning math. To the teacher it offers a complete guideline for a rigorous course. In designing the course, I used evidence from existing courses, information supplied by the institution that will host the course, high-quality textbooks that focus on math and my own qualifications as a certified educator and mathematician.

I plan to maintain communication with the public, informing them of the progress on this course. Readers who have questions concerning anything pertaining to EDUC 327 should feel free to contact me through McNeese University.

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Appendix A. Final Exam

EDUC 327 ELEMENTARY MATH METHODS
FINAL EXAM (No calculators allowed)

Part I Arithmetic Skills

Place Value

1. Write the numbers as a sum of multiples of powers of 10.
   a. 540
   b. 12,017,289
   c. 721.259

2. Write "Twelve billion five hundred two thousand eight hundred four " as a numeral.

3. Write the following as a base-ten numeral.
   $6 \cdot 10^3 + 8 \cdot 10^2 + 4$

Operations with Integers

Answers will be graded 60% on exposition and 40% on the solution(s). Write out the solutions showing operations and steps in a way appropriate for classroom demonstration.

4. Understand the meaning of arithmetic expressions involving minus signs. Perform the indicated operations:
   a. 27 - 61
   b. 5 - (9 - 2)
   c. - 275 - 149
   d. 10 + (-3)(-10)
   e. -5(5 - 8)

5. Use the standard multiplication and long division algorithms to compute the following. Show clear execution.
   a. 837 x 782.
   b. $34961 \div 47$

Rational Number Notation and Arithmetic

6. Convert an improper fraction to a mixed number.
   Change the fraction $\frac{19}{3}$ to a mixed number and explain.

7. Change the following mixed numbers to a fraction in the form $\frac{a}{b}$, where a and b are integers and b ≠ 0 and explain..
   a. $4\frac{3}{5}$ b. $1\frac{5}{3}$ c. $7\frac{12}{4}$

8. Understand the meaning of arithmetic expressions involving fractions.
   Perform the indicated operations. Show steps and procedures and explain.
   a. $\frac{5}{12} - \frac{1}{7}$
   b. $4\frac{5}{8} \cdot 3\frac{9}{16}$
   c. $\frac{4}{5}(5 + \frac{2}{3}) + \frac{7}{8} - 1$

Rational Number Ordering and Comparing

9. Use an inequality symbol to compare the fractions:
   a. $\frac{5}{36}$ and $\frac{10}{12}$ b. $\frac{144}{89}$ and $\frac{89}{55}$

10. Place the fractions in order from smallest to largest: $\frac{6}{11}, \frac{4}{9}, \frac{1}{2}, \frac{2}{5}$

Operation of Decimal Numbers (Show clear execution.)

11. Understand how to add decimals by aligning them vertically according to place value.
    What is the sum of 200.45, 19.8, and 1.362?

12. Understand how to subtract decimals by aligning them vertically according to place value.
    Take away 0.37 from 1.065.

13. Multiplying decimals by considering the total number of decimal places.
    Multiply 0.03 and 0.0002.

14. Dividing decimals by multiplying each by 10 to the power of the larger number of decimal places.
    What is 1.53 divided by 0.425?

Part II. Application and Understanding

Unit 1 Base 10 Number System and Place Value

15. For a collection of 1238 sugar cubes, draw a simple picture and write a brief description for how to organize them in a way that corresponds to the way we use the base-ten system to write the number for that many objects.

16. Students are sometimes uncertain about which zeros in decimals can be dropped and which can't. Give examples of zeros in decimals that can be dropped and zeros that can't be dropped. Explain why they can or can't be dropped.

Unit 2 Addition and Subtraction Algorithms

(17 - 20) Modeling addition and subtraction of numbers connecting to various real-world situations.

17. Jack's checking account was overdrawn by $59. He deposited $77 dollars into his account. He later wrote a check for $150. How much money does he need to put in his account to not be overdrawn?

18. Write equations that correspond to the make-a-ten method for adding 6 + 8. Your equations should make careful and appropriate use of parentheses. Which property of arithmetic do your equations use?

19. Ashton is 4 feet 3 inches tall. Matthew is 3 feet 11 inches tall. How much taller is Ashton than Matthew? If someone comes up with a solution after the following method, are they right? If not, explain what is wrong with the method, and show how to modify the method of regrouping to make it correct..

$$\begin{array}{rr} \cancel{4\text{ ft}}^{3\text{ ft}} & \cancel{3\text{ in}}^{13\text{ in}} \\ \hline -\ 3\text{ ft} & 11\text{ in} \\ & 2\text{ in} \end{array}$$

20. Brian says that he can calculate 324 - 197 by adding 3 to both numbers and calculating 327 - 200 instead.
    a. Draw a number line to help you explain why Brian’s method is valid.
    b. Could you adapt Brian’s method to other subtraction problems? If so, give two more examples.
    c. Explain..

Unit 3 Multiplication and Long Division Algorithms

(21 - 24) Modeling with multiplication connecting multiplication of numbers to various real-world situations.

21. If a school’s classrooms can hold 28 children, what is the minimum number of classrooms needed for 210 children? After the classrooms are selected, how many students should be placed in each?

22. Rachael used 9 gallons of gas to travel 243 miles. She used 7 more gallons to travel 224 more miles. What is her average miles per gallon?

23. A group of 9 people wants to buy a boat. The boat costs $414. If they all pay the same amount, how much is each person's share?
    a. Solve and provide a teacher's solution.
    b. What are the important concepts in this problem?
    c. Make up and provide a teacher solution for a problem that tests the same concept, but at a higher level.

24. Tom earned $35 a week for washing cars. How much has he earned in the last 9 weeks?
    a. Solve and provide a teacher's solution.
    b. What are the important concepts in this problem?
    c. Make up and provide a teacher solution for a new problem that tests the same concept, but at a higher level.

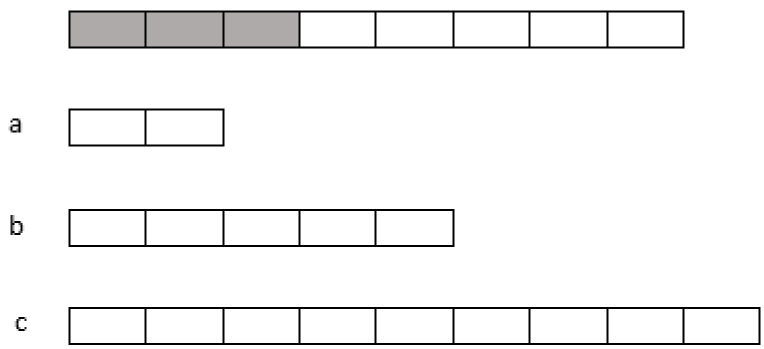
Understanding Divisibility Rules

25. For each of the listed numbers, determine whether it is prime. If it is not prime, factor the number into a product of prime numbers.
    a. 1081
    b. 1087
    c. 269,059
    d. 2081
    e. 1147

Unit 4 Definitions of Fractions, Unit, & Decimal

(25 - 26) Understanding the meaning of units

26. Name the number represented by the shaded area in the figure for each of the following widths, a, b, and c.



27. $\pi = 3.1415926\ldots.$

   a. Draw the part of the number line that goes from the largest multiple of $\frac{1}{10}$ smaller than π to the smallest multiple of $\frac{1}{10}$ larger than π. Mark the hundredths and show the location of π.

   b. Draw another number line showing the location of π marking the thousandths.

(27 - 28) Modeling with fractions

28. Kaitlyn gave half of her bag of candy to Arianna. Arianna gave one-third of the candy she got from Kaitlyn to Cameron. What fraction of the bag of candy did Arianna have? Explain your solution. Use our definition of fraction in your explanation and attend to the unit amount that each fraction is *of*.

29. Draw a number line and plot $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{5}{8}$ on your number line in such a way that each number falls on a tick mark between $\frac{1}{2}$ and $\frac{3}{4}$ (inclusive). Mark evenly-spaced tick marks so that $\frac{2}{3}$ and $\frac{5}{8}$ fall at tick marks. Label each tick mark in simplest form.

30. A student says that $\frac{1}{5}$ is halfway between $\frac{1}{4}$ and $\frac{1}{6}$. Use a carefully drawn number line to show that this is not correct. What fraction is halfway between them? Where exactly is $\frac{1}{5}$? Explain your reasoning.

Unit 5 Fundamental Fraction Pairs: Equivalent Fractions

31. For lunch, Jamie bought a 8-inch submarine sandwich.
    a. What fraction of a foot was Jamie's sandwich?
    b. What fraction of a yard?
    c. What fraction of a mile?
    d. What fraction of a mile is a yard?

    Note: 1 foot = 12 inches, 1 yard = 3 feet, 1 mile = 5280 feet

(31 - 33) Understanding and finding equivalent fractions

32. $\frac{9}{16} = \frac{?}{96}$

    What number belongs in the place of the question mark? Explain.

33. $\frac{1}{7}$ and $\frac{19}{133}$
    a. Are the two fractions equivalent?
    b. Why? What process do you use to determine your answer?

34. You may combine your answers to all three parts of this problem.

a. Is it valid to compare $\frac{30}{70}$ and $\frac{20}{50}$ by “cancelling” the 0’s and comparing $\frac{3}{7}$ and $\frac{2}{5}$?

b. Is it valid to compare $\frac{15}{25}$ and $\frac{105}{205}$ by “cancelling” the 5’s and comparing $\frac{1}{2}$ and $\frac{10}{20}$?

c.. Write a paragraph discussing the distinction between your answer in (a) and your answer in (b).

Unit 6 Addition and Subtraction of Fractions and Decimals

Recognizing and choosing appropriate mathematical operation.

35. The second tallest child in a class is $36\frac{2}{3}$ inches tall. The tallest child is $3\frac{11}{12}$ inches taller. How tall is the tallest child in the class?

Unit 7 Multiplication and Division of Fractions

36. A recipe for 8 servings calls for $1\frac{1}{3}$ cups of milk. What amount of milk would be needed in a recipe for

a. 12 servings?
b. 18 servings?
c. Convert $1\frac{1}{3}$cups to ounces.

Modeling with fractions

37. List the properties of arithmetic. For each property, provide an example of a PURPOSEFUL use of the property.

Understanding fraction operations appropriate for applications.

38. Compare the arithmetic needed to solve the following problems:
    a. What fraction of a ⅓ cup measure is filled when we pour in ¼ cup of water?
    b. What is ¼ of ⅓ cup?
    c. How much more is ⅓ cup than ¼ cup?
    d. If ¼ cup of water fills ⅓ of a plastic container, then how much water will the full container hold?

Part III. Discussion questions (from Beckmann)

Base-ten and Place Value

39. Describe base-ten units and describe how adjacent place values are related in the base-ten system.

40. Describe and draw rough pictures to represent a given positive decimal as a length in a way that fits with and shows the structure of the decimal system.

41. Explain the rationale for comparing positive numbers in base-ten by first comparing the place of greatest value (and then moving to places of lower value).

Rational Numbers

42. Given a fraction, use the definition of fractions, math drawings, and number lines to explain why multiplying the numerator and denominator by the same number produces an equivalent fraction.

43. Compare fractions that have the same numerator, and use math drawings and the definition of fraction to explain the rationale for this method of comparison.

44. Plot fractions, including improper ones, on number lines and explain why the location fits with the definition of fraction.

Addition and Subtraction

45. Explain how to use a number line to add and subtraction numbers (for non-negative numbers only).

46. Give examples to show how to use the associative property of addition to make problems easier to do mentally, including the make-a-ten strategy.

47. Describe how to view subtraction problems as unknown addend problems, explain how this can help young children with basic subtraction facts, and explain how this can be applied to other mental math problems.

48. Explain why we line up places and decimal points when we add and subtract decimals.

49. Explain how to add or subtract fractions, and explain why the process makes sense. In particular, why we give the fractions common denominators.

Multiplication

50. Describe multiplication by 10 as moving digits one place to the left.

51. Explain why we can multiply to find the area of a rectangle by describing rectangles as subdivided into groups of 1-unit by 1-unit squares.

52. State the associative and commutative properties of multiplication and explain why it makes sense by subdividing boxes or groups of objects two different ways.

53. State the distributive property, and explain why it makes sense (for counting numbers) by describing the total number of objects in a subdivided array in two different ways. Use simple situations to explain or illustrate the distributive property.

54. Describe how the multiplication algorithms can be explained in terms of the definitions of multiplication, place value, and properties of arithmetic.

Multiplication of Fractions, Decimals, and Negative Numbers

55. Write a word problem for a fraction multiplication problem, and solve the problems by using logic and a math drawing.

56. Explain why we put the decimal point where we do when we multiply decimals.

57. Describe decimal multiplication in terms of area.

58. Use properties of arithmetic to explain the rules for multiplying and negative numbers.

Division

59. Explain why we can't divide by 0, but why we *can* divide 0 by a non-zero number.

60. Describe how the whole-number-with-remainder answer to a division problem is related to the mixed number answer, and explain why this relationship holds.

61. Explain why we can divide fractions by dividing numerators and dividing denominators.

62. Explain in several different ways shy we move the decimals points the way we do when we divide decimals.

63. Explain why many division problems are equivalent, such as $6000 \div 2000$, $600 \div 200$, $60 \div 20$, $6 \div 2$, and $0.6 \div 0.2$.

Appendix B. Final Exam Key
EDUC 327 ELEMENTARY MATH METHODS
FINAL EXAM (No calculators allowed)

Grading Rubric (based on 6 points for each problem)

Scale I: Understanding the problem
2 Complete understanding of the problem
1 Part of the problem misunderstood or misinterpreted
0 Complete misunderstanding of the problem

Scale II: Planning a Solution
2 Plan could have led to a correct solution if implemented properly
1 Partially correct plan based on part of the problem being interpreted correctly
0 No attempt, or totally inappropriate plan

Scale III: Getting an Answer
2 Correct answer and correct label for the answer
1 Copying error; computational error; partial answer for a problem with multiple answers
0 No answer, or wrong answer based on an inappropriate plan

Part I Arithmetic Skills

Place Value

1. Write the numbers as a sum of multiples of powers of 10.
   a. 540 $5 \cdot 10^2 + 4 \cdot 10^1$
   b. 12,017,289 $1 \cdot 10^7 + 2 \cdot 10^6 + 0 \cdot 10^5 + 1 \cdot 10^4 + 7 \cdot 10^3 + 2 \cdot 10^2 + 8 \cdot 10^1 + 9 \cdot 10^0$
   c. 721.259 $7 \cdot 10^2 + 2 \cdot 10^1 + 1 \cdot 10^0 + 2 \cdot 10^{-1} + 5 \cdot 10^{-2} + 9 \cdot 10^{-3}$

2. Write "One thousand five hundredths seven thousandths and two millionths" as a numeral.

   1000.057002 (Commas are optional)

3. Write the following as a base-ten numeral.
   $6 \cdot 10^{10} + 5 \cdot 10^{6} + 2 \cdot 10^{4} + 0 \cdot 10^{3} + 1 \cdot 10 + 7$

   $6 \cdot 10000000000 + 5 \cdot 1000000 + 2 \cdot 10000 \ + \ 10 \ + 7$
   60,000,000,000+5,000,000+20,000+10+7
   60,005,020,017

Operations with Integers
Answers will be graded 60% on exposition and 40% on the solution(s). Write out the solutions showing operations and steps appropriate for classroom demonstration.

4. Perform the indicated operations:
   a. 27 - 61
   b. 5 - (9 - 2)
   c. - 275 - 149
   d. 10 + (-3)(-10)
   e. -5(5 - 8)

4a. 27 - 61

27 + (-61)	Definition of subtraction
61 - 27	Subtract the two as whole numbers, larger - smaller, and use
34	the sign of the larger number.
-34	

4b. 5 - (9 - 2)

5 - 7 Order of Operations
5 + (-7) ---> 7-5=2 (or number line movement can be used)
-2 Take the sign of the larger whole number

4c. - 275 -149

-275 + (-149)	Definition of Subtraction
- (275 +149)	distributive property, or combining integers with the same sign, the sign remains
- 424	

4d. 10 + (-3)(-10)	Order of operations
10 + 30	Multiplying integers with the same sign results in a poisitive integer

4e. -5(5-8) Order of Operations
-5(-3) Movement along number line
15 Multiplying integers with the same sign results in a positive integer

5. Use the standard multiplication and long division algorithms to compute the following.
   a. 837 x 782.
   b. $34961 \div 47$

a.
```
   837
 x 782
  1724
 66960
+585900
 654584
```

b. $47 \overline{)34961}$ with quotient $743\frac{40}{47}$
```
 -329
  206
 - 188
   181
 -  141
    40
```

Rational Number Notation and Arithmetic

6. Change the fraction $\frac{19}{3}$ to a mixed number and explain.

   Because the numerator is larger than the denominator, the fraction is considered complex, and should be considered as division with a remainder.

   $19 \div 3 = 6\ with\ a\ remainder\ of\ 1 = 6\frac{1}{3}$

7. Change the following mixed numbers to a fraction in the form $\frac{a}{b}$, where a and b are integers and b ≠ 0 and explain..
   a. $4\frac{3}{5}$ b. $1\frac{5}{3}$ c. $7\frac{12}{4}$

7a. $4\frac{3}{5} = 4 + \frac{3}{5} = \frac{20}{5} + \frac{3}{5} = \frac{23}{5}$
7b. $1\frac{5}{3} = 1 + \frac{5}{3} = \frac{3}{3} + \frac{5}{3} = \frac{8}{3}$
7c. $7\frac{12}{4} = 7 + \frac{12}{4} = 7 + 3 = 10$

Notice the common denominator on 7c. is unnecessary.

8. Perform the indicated operations and reduce to lowest terms. Show steps and procedures and explain.
   a. $\frac{5}{12} - \frac{1}{7}$
   b. $4\frac{5}{8} \cdot 3\frac{9}{16}$
   c. $\frac{4}{5}(5 + \frac{2}{3}) + \frac{7}{8} - 1$

8a. $\frac{5}{12} - \frac{1}{7} = \frac{35}{84} - \frac{12}{84} = \frac{23}{84}$ Find common denominators and subtract the numerators. Checked for simplest form.

8b. $4\frac{5}{8} \cdot 3\frac{9}{16} = \frac{45}{8} \cdot \frac{57}{16} = \frac{2109}{128}$

Mixed numbers are converted to improper fractions. Fraction multiplication multiplies the numerators and denominators and then put in simplest form.

8c. $\frac{4}{5}(5 + \frac{2}{3}) + \frac{7}{8} - 1$

$=\frac{4}{5}(5\frac{2}{3}) + \frac{7}{8} - 1$ Order of Operations

$=\frac{4}{5}(\frac{17}{3}) + \frac{7}{8} - 1$ mixed number changed to improper fraction

$=\frac{68}{15} + \frac{7}{8} - 1$ fraction multiplication

$=\frac{544}{120} + \frac{105}{120} - \frac{120}{120}$ Add and subtract numerators: 544+105-120=529

$=\frac{529}{120}$

Rational Number Ordering and Comparing

9. Use an inequality symbol to compare the fractions:
   a. $\frac{5}{36}$ and $\frac{10}{12}$
   $5 \cdot 12\ and\ 36 \cdot 10$
   $60\ and\ 360$
   b. $\frac{144}{89}$ and $\frac{89}{55}$
   $144 \cdot 55\ and\ 89 \cdot 89$
   7920 *and* 7921

Cross multiply to compare the fractions

60 < 360, so

7920 < 7921, so $\frac{144}{89} < \frac{89}{55}$

10. Place the fractions in order from smallest to largest: $\frac{6}{11}, \frac{4}{9}, \frac{1}{2}, \frac{2}{5}$

Find common denominators, or use the fact that a larger denominator with the same numerator is smaller. Example: $\frac{4}{9} < \frac{4}{8}$

Therefore, the correct order is $\frac{2}{5}, \frac{4}{9}, \frac{1}{2}, \frac{6}{11}$.

Operation of Decimal Numbers (Show computation.)

11. What is the sum of 200.45, 19.8, and 1.362?

$$\begin{array}{r} 200.450 \\ 19.800 \\ +\ \underline{1.362} \\ 221.612 \end{array}$$

The decimal points are lined up and 0's filled in blank spaces to add the appropriate columns beginning with the smallest place (right to left).

12. Take away 0.37 from 1.065.

Decimal points are lined up and trading is used when needed.

$$\begin{array}{r} {}^{9}\cancel{1.0}{}^{1}65 \\ -\ \underline{0.3\ 70} \\ 0.695 \end{array}$$

13. Multiply 0.03 and 0.0002.

$$\begin{array}{r} 0.0002 \\ \times\ \underline{0.03} \\ 0.00006 \end{array}$$

Count 0's to the right of the decimal place to determine how many decimal places belong in the solution.
2 x 3 =6 and there are 4 decimal places to the right of the decimal point.

14. What is 1.53 divided by 0.425?

$1.53 \div 0.425 = 1530 \div 425$

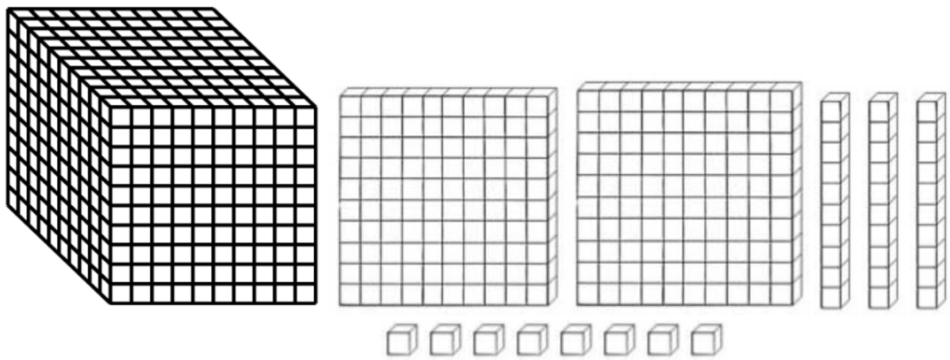
$$\begin{array}{r} 3.6 \\ 425\ \overline{)1530.0} \\ -\ \underline{1275} \\ 255.0 \\ -\ \underline{255.0} \\ 0 \end{array}$$

Part II. Application and Understanding

Unit 1 Base 10 Number System and Place Value

15. For a collection of 1238 sugar cubes, draw a picture and write a brief description for how to organize them in a way that corresponds to the way we use the base-ten system to write the number for that many objects.

$1238 = 1 \cdot 10^3 + 2 \cdot 10^2 + 3 \cdot 10^1 + 8$



What can also be used is one “bundle” of 1000 items and 2 “bundles” of 100 items and 3 “bundles” of 10 items, and then 8 single items. It doesn’t really matter what those items are, as long as they represent the right place value.

16. Students are sometimes uncertain about which zeros in decimals can be dropped and which can’t. Give examples of zeros in decimals that can be dropped and zeros that can’t be dropped. Explain why they can or can’t be dropped.

Only the zeros at the end of a decimal to the right of the decimal point can be dropped. Those zeros can be dropped because they are fractions of powers of 10 and can be reduced or dropped.

Examples:	200.0	Only the last zero can be dropped. The first 2 can’t.
	201.50	Only the last zero can be dropped.
	201.01	None of these zeros can be dropped.

17. Jack's checking account was overdrawn by $59. He deposited $77 dollars into his account. He later wrote a check for $150. How much money does he need to put in his account to not be overdrawn?

    Overdrawn implies a negative number and deposit implies addition. Writing a check also denotes subtraction or addition of a negative number.
    (-59) + 77 - 150 = (-59 - 150) +77 = -209 + 77 = -132   -(209 - 77)=132
    He needs to put in at least $132 into his account to not overdraw.

18. Write equations that correspond to the make-a-ten method for adding 6 + 8. Your equations should make careful and appropriate use of parentheses. Which property of arithmetic do your equations use?

6 + 8 = (4 + 2) + 8	re-expressing a number in terms of a sum
= 4 + (2 + 8)	associative property used to make-a-ten
= 4 + 10	addition property of equality
= 14	addition property of equality

19. Ashton is 4 feet 3 inches tall. Matthew is 3 feet 11 inches tall. How much taller is Ashton than Matthew? If someone comes up with a solution after the following method, are they right? If not, explain what is wrong with the method, and show how to modify the method of regrouping to make it correct..

~~4 ft~~ 3 ft ~~3 in~~ 13 in	No, they are not right. Inches should not borrow 10
- 3 ft 11 in	inches from a foot, but should instead borrow 12 because
2 in	there are 12 inches in one foot.

    How to modify:
    (3 + 1) ft   3 in = 3 ft   (12 + 3) in = 3 ft   15 in
    - 3 ft   11 in
    4 in

20. Brian says that he can calculate 324 - 197 by adding 3 to both numbers and calculating 327 - 200 instead.
    a. Draw a number line to help you explain why Brian's method is valid.
    b. Could you adapt Brian's method to other subtraction problems? If so, give two more examples.
    c. Explain.

    The distance on the number between 197 and 324 is 127, the solution to 324 - 197. If the focus moves 3 to the right on each number, the distance between 327 and 200 is still 127, which is the value of 327 - 200.

21. If a school's classrooms can hold 28 children, what is the minimum number of classrooms needed for 210 children? After the classrooms are selected, how many students should be placed in each?

    $210 \div 28 = 7\frac{1}{2}$, so the minimum number of classrooms is 8.

    $28 \cdot 8 = 224$, $27 \cdot 8 = 216$, $26 \cdot 8 = 208$, just two short of the necessary 210. Therefore 6 classrooms of 26 children and 2 classrooms of 27 handles the 210 and spreads the children out more evenly. $6 \cdot 26 + 2 \cdot 27 = 210$.

22. Rachael used 9 gallons of gas to travel 243 miles. She used 7 more gallons to travel 224 more miles on the highway. What is her average miles per gallon?

    Average mpg = $\frac{total\ miles}{total\ gallons} = \frac{243+224\ miles}{9+7\ gallons} = \frac{467}{16} = 29\frac{3}{16}$ miles per gallon

23. A group of 9 people wants to buy a boat. The boat costs $414. If they all pay the same amount, how much is each person's share?
    a. Solve and provide a teacher's solution.
    b. What are the important concepts in this problem?
    c. Make up and provide a teacher solution for a problem that tests the same concept, but at a higher level.

a. $\frac{\$414}{9\ people} = \$46\ per\ person$
b. Recognizing that "each" implies divided evenly among the 9 people.
c. An example problem that doesn't divide evenly, causing a round up to the next dollar amount would be at a higher level.

24. Tom earned $35 per week for washing cars. How much has he earned in the last 9 weeks?
    a. Solve and provide a teacher's solution.
    b. What are the important concepts in this problem?
    c. Make up and provide a teacher solution for a new problem that tests the same concept, but at a higher level.

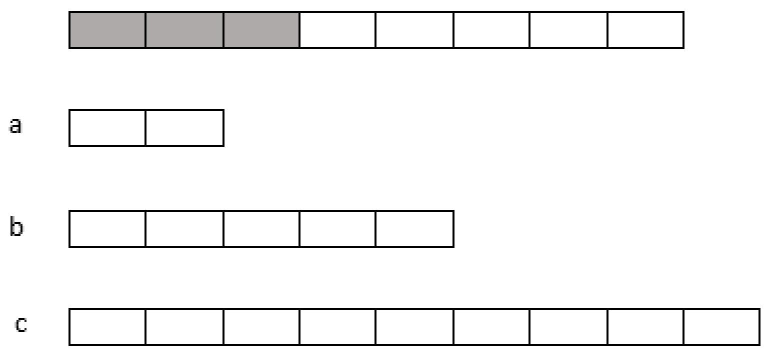
a. $35 for *each* of the 9 weeks implies multiply in this case. $35 times 9 weeks is $315.
b. Recognizing the "for each" implying multiplication is important.

25. For each of the listed numbers, determine whether it is prime. If it is not prime, factor the number into a product of prime numbers.

a.	1081	23 x 47
b.	1087	prime
c.	269,059	7 x 7 x 17 x 17 x 19
d.	2081	prime
e.	1147	31 x 37

Unit 4 Definitions of Fractions, Unit, & Decimal

26. Name the number represented by the shaded area when each of the quantities in a, b, and c is used as a unit of measure.

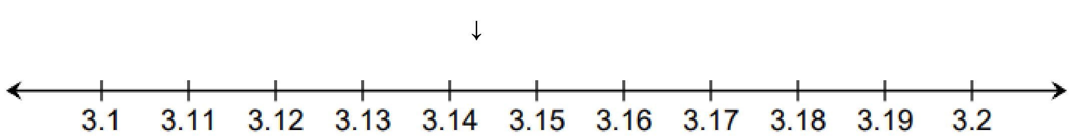


a. $1\frac{1}{2}$ b. $\frac{3}{5}$ c. $\frac{1}{3}$

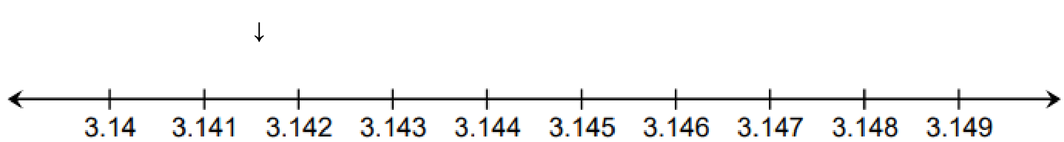
27. $\pi = 3.1415926\ldots.$

a. Draw the part of the number line that goes from the largest multiple of $\frac{1}{10}$ smaller than π to the smallest multiple of $\frac{1}{10}$ larger than π. Mark and label the hundredths and show the location of π.

b. Draw another number line showing the location of π marking the thousandths.



a.

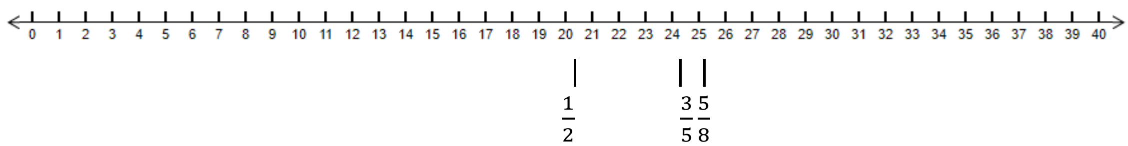


b.

28. Kaitlyn gave half of her bag of candy to Arianna. Arianna gave one-third of the candy she got from Kaitlyn to Cameron. What fraction of the bag of candy did Arianna keep? Explain your solution. Use our definition of fraction in your explanation and attend to the unit amount that each fraction is *of*.

Kaitlyn has one bag of candy. She gives half of it to Arianna, so Arianna has ½ of Kaitlyn's bag of candy. When Arianna gives ⅓ of her ½, she gives ⅓ times ½ of the original bag, which is ⅙. Arianna then has ½ - ⅙ of the original bag, which 3/6 - ⅙ which is ⅓ of the original bag.

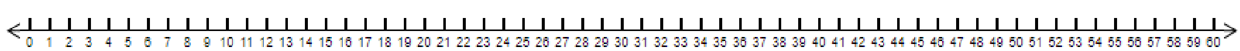
29. Draw a number line and plot ½, ⅗ and ⅝ on your number line in such a way that each number falls on a tick mark between 0 and 1 (inclusive). Mark evenly-spaced tick marks so that each fraction falls at a tick mark. Label each tick mark in simplest form. This line is labeled using $\frac{1}{20}$ as a unit.



The tick marks are fractions of 40 (the LCM of the denominators), as in the 7 tick mark represents $\frac{7}{40}$.

30. A student says that $\frac{1}{5}$ is halfway between $\frac{1}{4}$ and $\frac{1}{6}$. Use a carefully drawn number line to show that this is not correct. What fraction is halfway between them? Where exactly is ⅕? Explain your reasoning.

This line is labelled using $\frac{1}{60}$ as a unit.



The tick marks are fractions of 60 (the LCM of the denominators), as in the 10 tick mark represents $\frac{10}{40}$.

Halfway between $\frac{1}{4}$ and $\frac{1}{6}$ would be the average of the two fractions, which is $\frac{5}{24}$. This is close to ⅕, but not equivalent.

$$\frac{1}{2}(\frac{1}{4}+\frac{1}{6}) = \frac{1}{2}(\frac{6}{24}+\frac{4}{24}) = \frac{1}{2}(\frac{10}{24}) = \frac{5}{24}$$

31. For lunch, Jamie bought a 8-inch submarine sandwich.
    a. What fraction of a foot was Jamie's sandwich?
    b. What fraction of a yard?
    c. What fraction of a mile?
    d. What fraction of a mile is a yard?

   Note: 1 foot = 12 inches, 1 yard = 3 feet, 1 mile = 5280 feet

a. $8\,in \cdot \frac{1\,ft}{12\,in} = \frac{2}{3}ft$ b. $\frac{2}{3}\,ft \cdot \frac{1\,yd}{3\,ft} = \frac{2}{9}\,yd$ c. $\frac{2}{3}\,ft \cdot \frac{1\,mile}{5280\,ft} = \frac{2}{15840}mile = \frac{1}{7920}mile$

d. $1\,yd \cdot \frac{1\,mile}{5280\,ft} \cdot \frac{3\,ft}{1\,yd} = \frac{1}{1760}mile$

32. $\frac{9}{16} = \frac{?}{96}$

   What number belongs in the place of the question mark? Explain.

$\frac{9}{16} \cdot 1 = \frac{9}{16} \cdot \frac{6}{6} = \frac{54}{96}$ Multiplying a fraction by 1 does not change its value.

33. $\frac{1}{7}$ and $\frac{19}{133}$
    a. Are the two fractions equivalent?
    b. Why? What process do you use to determine your answer?

$\frac{1}{7}\ and\ \frac{19}{133}$

$1 \cdot 133\ and\ 7 \cdot 19$

$133\ and\ 133$

Yes, they are equivalent, because $\frac{1}{7} \cdot 1 = \frac{1}{7} \cdot \frac{19}{19} = \frac{19}{133}$.
I used the principle that the cross products of equivalent fractions are also equivalent.

34. You may combine your answers to all three parts of this problem.

a. Is it valid to compare $\frac{30}{70}$ and $\frac{20}{50}$ by "cancelling" the 0's and comparing $\frac{3}{7}$ and $\frac{2}{5}$?

Yes, it is. All 4 numbers can be reduced by, or divided by 10 and eliminate the zeros.

b. Is it valid to compare $\frac{15}{25}$ and $\frac{105}{205}$ by "cancelling" the 5's and comparing $\frac{1}{2}$ and $\frac{10}{20}$?

No, it is not. Dividing all four numbers by 5 does not eliminate them.

c.. Write a paragraph discussing the distinction between your answer in (a) and your answer in (b).

Cancelling the 0's was possible when dividing all of the numbers by 10. Dividing by 5, however, turns the last number into a 1, not an eliminated 0.

Unit 6 Addition and Subtraction of Fractions

35. The second tallest child in a class is $36\frac{2}{3}$ inches tall. The tallest child is $3\frac{11}{12}$ inches taller. How tall is the tallest child in the class?

$$36\frac{2}{3} + 3\frac{11}{12} = 36\frac{8}{12} + 3\frac{11}{12} = (36 + \frac{8}{12}) + (3 + \frac{11}{12}) = (36 + 3) + (\frac{8}{12} + \frac{11}{12}) = 39 + \frac{19}{12}$$
$$= 39 + \frac{12}{12} + \frac{7}{12} = 39 + 1 + \frac{7}{12} = 40 + \frac{7}{12} = 40\frac{7}{12}$$

Unit 7 Multiplication and Division of Fractions

36. A recipe for 8 servings calls for $1\frac{1}{3}$ cups of milk. What amount of milk would be needed in a recipe for

a. 12 servings?
b. 18 servings?
c. Convert $1\frac{1}{3}$cups to ounces.

a. $\frac{12}{8}(1\frac{1}{3}) = \frac{3}{2}(\frac{4}{3}) = \frac{12}{6} = 2\ cups$
b. $\frac{18}{8}(1\frac{1}{3}) = \frac{9}{4}(\frac{4}{3}) = \frac{36}{12} = 3\ cups$
c. $\frac{4}{3} cups(\frac{8oz}{1cup}) = \frac{32}{3}\ or\ 10\frac{2}{3} oz$

.

37. List the properties of arithmetic. For each property, provide an example of a PURPOSEFUL use of the property.

Understanding fraction operations appropriate for applications.

38. Compare the arithmetic needed to solve the following problems:
    a. What fraction of a ⅓ cup measure is filled when we pour in ¼ cup of water?
    b. What is ¼ of ⅓ cup?
    c. How much more is ⅓ cup than ¼ cup?
    d. If ¼ cup of water fills ⅓ of a plastic container, then how much water will the full container hold?

a. ¼ of ⅓ implies fraction multiplication
b. ¼ times ⅓ is equal to $\frac{1}{12}$.
c. Fraction subtraction: ⅓ - ¼
d. Fraction division: ¼ ÷⅓ = ¾

Part III. Discussion questions (from Beckmann)

Base-ten and Place Value

39. Describe base-ten units and describe how adjacent place values are related in the base-ten system.

40. Describe and draw rough pictures to represent a given positive decimal as a length in a way that fits with and shows the structure of the decimal system.

41. Explain the rationale for comparing positive numbers in base-ten by first comparing the place of greatest value (and then moving to places of lower value).

Rational Numbers

42. Given a fraction, use the definition of fractions, math drawings, and number lines to explain why multiplying the numerator and denominator by the same number produces an equivalent fraction.

43. Compare fractions that have the same numerator, and use math drawings and the definition of fraction to explain the rationale for this method of comparison.

44. Plot fractions, including improper ones, on number lines and explain why the location fits with the definition of fraction.

Addition and Subtraction

45. Explain how to use a number line to add and subtraction numbers (for non-negative numbers only).

46. Give examples to show how to use the associative property of addition to make problems easier to do mentally, including the make-a-ten strategy.

47. Describe how to view subtraction problems as unknown addend problems, explain how this can help young children with basic subtraction facts, and explain how this can be applied to other mental math problems.

48. Explain why we line up places and decimal points when we add and subtract decimals.

49. Explain how to add or subtract fractions, and explain why the process makes sense. In particular, why we give the fractions common denominators.

Multiplication

50. Describe multiplication by 10 as moving digits one place to the left.

51. Explain why we can multiply to find the area of a rectangle by describing rectangles as subdivided into groups of 1-unit by 1-unit squares.

52. State the associative and commutative properties of multiplication and explain why it makes sense by subdividing boxes or groups of objects two different ways.

53. State the distributive property, and explain why it makes sense (for counting numbers) by describing the total number of objects in a subdivided array in two different ways. Use simple situations to explain or illustrate the distributive property.

54. Describe how the multiplication algorithms can be explained in terms of the definitions of multiplication, place value, and properties of arithmetic.

Multiplication of Fractions, Decimals, and Negative Numbers

55. Write a word problem for a fraction multiplication problem, and solve the problems by using logic and a math drawing.

56. Explain why we put the decimal point where we do when we multiply decimals.

57. Describe decimal multiplication in terms of area.

58. Use properties of arithmetic to explain the rules for multiplying and negative numbers.

Division

59. Explain why we can't divide by 0, but why we *can* divide 0 by a non-zero number.

60. Describe how the whole-number-with-remainder answer to a division problem is related to the mixed number answer, and explain why this relationship holds.

61. Explain why we can divide fractions by dividing numerators and dividing denominators.

62. Explain in several different ways shy we move the decimals points the way we do when we divide decimals.

63. Explain why many division problems are equivalent, such as $6000 \div 2000$, $600 \div 200$, $60 \div 20$, $6 \div 2$, and $0.6 \div 0.2$.

Appendix C. Course Design Outline

COURSE DESIGN OUTLINE

Week 1	Unit 1 Chapter 1 & 2 - Base 10 Number Systems and Place Value **Unit 1 Key Points** Unit 1 will cover regarding place value of whole numbers, and the basic laws of operations. Key points of this unit include, but are not limited to: place value, digit, unit segment, powers, and equal sign. Concepts explored in this unit include, but are not limited to: how to count, the use of symbolic notation, numeral systems, the distributive laws, and the understanding and application of the associative and commutative laws of addition and multiplication. Key points and concepts will be explored through group discussion, chapter exercises, and in class exploration of skills.	● Syllabus (including Course Outline) ● Lesson Plan Template Instructions ● Lesson Plan Template Rubric ● Example Lessons Plans (Positive and Negative) ● Student assigned co-teaching partners and units ○ Students (individually or in groups) will choose a unit from unit 2-7 that they will demonstrate and execute a proper lesson in that field of mathematics to their peers. Each even beginning in Week 4 will have a demonstration per class period. ● Students will receive a copy of Chapters 1 & 2 of *Understanding Numbers in Elementary School Mathematics* ○ Students will be given until Week 3 meeting to have personal copy of text ● ***Student Assignment: Due on Moodle*** ○ ***Wu Chapter 1 Exercises 1-14*** ○ ***Wu Chapter 2 Exercises 1-16*** ○ ***Wu Read Chapters 1 & 2***
Week 2	Unit 1 Chapter 1 & 2 - Base 10 Number Systems and Place Value	● Discussion of Wu Chapter 1 & 2 Exercises and Readings ● Pre-Test Administered ● **DUE (Date)** ○ **Wu Chapter 1 & 2 Exercises** ● ***Student Assignment:*** ○ ***Beckman Exercises TBD*** ○ ***Lesson Plan 1 (Base 10 Number Systems and Place Value)***

Week 3	Unit 2
	Chapter 3, 4, & 5 - Addition and Subtraction Algorithms
	
	Unit 2 Key Points
	Unit 2 will cover concepts regarding the standard algorithms. Key points will include, but are not limited to: single and multi-digit computations, number line, properties, and algorithm. The concepts covered in this unit revolve around the definition and application of the addition and subtraction algorithms.
		● Unit 2 Instruction
		● Review Beckman Exercises from Week 2
		● **DUE (Date)**
		    ○ **Lesson Plan 1**
		    ○ **Beckman Exercises**
		● ***Student Assignments:***
		    ○ ***Wu Chapter 3 Exercises***
		    ○ ***Wu Chapter 4 Exercises***
		    ○ ***Wu Chapter 5 Exercises***

Week 4	Unit 2 Chapter 3, 4, & 5 - Addition and Subtraction Algorithms	● **Unit 2 Student Demo** ● Review and Discuss Wu Chapters 3, 4, & 5 Exercises ● ***Student Assignment:*** ○ ***Beckman Exercises TBD***

Week 5	Unit 3
	Chapter 6 & 7 - Multiplication and Long Division Algorithms
	
	Unit 3 Key Points
	Unit 3 will cover concepts regarding the multiplication and long division algorithms. Key points will include, but are not limited to: multiplication tables, arithmetic operations, multiple, dividend, divisor. Concepts for this unit include the parts of the multiplication and long division algorithms and how they affect numbers, division with remainders, and multiplication as division. The unit also emphasizes the importance of a fluent knowledge of the multiplication table in order to properly use algorithms for both multiplication and division.
		● **Quiz 1**
		● Unit 3 Instruction
		● Review Beckman Exercises from Week 4
		● **DUE (Date)**
		    ○ **Beckman Exercises**
		● ***Student Assignments:***
		    ○ ***Wu Chapter 6 Exercises***
		    ○ ***Wu Chapter 7 Exercises***

Week 6	Unit 3 Chapter 6 & 7 - Multiplication and Long Division Algorithms	● **Unit 3 Student Demo** ● Review and Discuss Wu Chapters 6 & 7 Exercises ● ***<u>Student Assignment:</u>*** ○ ***<u>Beckman Exercises TBD</u>***
Week 7	Unit 4 Chapter 12 - Definitions of Fractions, Unit, & Decimal **<u>Unit 4 Key Points</u>** Unit 4 will cover concepts such as decimals, the importance of the unit, the area model, and locating fractions on the number line. Key points of this unit include, but are not limited to: parts, whole, midpoint, standard representation of fractions, numerator, denominator, and congruence. This unit emphasizes the visual, numerical, and number line representation of fractions, units, and decimals.	● Unit 4 Instruction ● Review Beckman Exercises from Week 6 ● **DUE (Date)** ○ **Beckman Exercises** ● ***<u>Student Assignments:</u>*** ○ ***<u>Wu Chapter 12 Exercises</u>***
Week 8	Unit 4 Chapter 12 - Definitions of Fractions, Unit, & Decimal	● **Unit 4 Student Demo** ● Review and Discuss Wu Chapters 12 Exercises ● **<u>Quiz 2</u>** ● ***<u>Student Assignment:</u>*** ○ ***<u>Beckman Exercises TBD</u>***

Week 9	Unit 5
	Chapter 13 & 15 - Fundamental Fraction Pairs: Equivalent Fractions
	
	<u>Unit 5 Key Points</u>
	Unit 5 discusses equivalent fractions and their application. The unit covers topics such as the cancellation law, applications of said law to decimals, the cross-multiplication algorithm, whole number divisions, and comparing fractions. Key points touched in this unit include, but are not limited to: equivalent fractions, cancellation law, reducing fractions, Theorem 13.1, transitivity, and reciprocal.
		• Unit 5 Instruction
		• Review Beckman Exercises from Week 8
		• **DUE (Date)**
		    ○ **Beckman Exercises**
		• ***<u>Student Assignments:</u>***
		    ○ ***<u>Wu Chapter 13 Exercises</u>***
		    ○ ***<u>Wu Chapter 15 Exercises</u>***

Week 10	Unit 5 Chapter 13 & 15 - Fundamental Fraction Pairs: Equivalent Fractions	• **Unit 5 Student Demo** • Review and Discuss Wu Chapters 13 & 15 Exercises • ***<u>Student Assignment:</u>*** ○ ***<u>Beckman Exercises TBD</u>***

Week 11	Unit 6
	Chapter 14 & 16 - Addition & Subtraction of Fractions and Decimals
	<u>Unit 6 Key Points</u>
	Unit 6 will discuss concepts of the addition and subtractions of fractions as decimals. The unit covers topics such as the definition of addition, addition of decimals, mixed numbers. Unit 6 also touches on the use of calculators in a classroom setting. Key point touched on in this unit include, but are not limited to: mixed numbers, average, and proofs.
		• Unit 6 Instruction
		• Review Beckman Exercises from Week 10
		• **DUE (Date)**
		    ○ **Beckman Exercises**
		• ***<u>Student Assignments:</u>***
		    ○ ***<u>Wu Chapter 14 Exercises</u>***
		    ○ ***<u>Wu Chapter 16 Exercises</u>***

Week 12	Unit 6 Chapter 14 & 16 - Addition & Subtraction of Fractions and Decimals	• **Unit 6 Student Demo** • Review and Discuss Wu Chapters 14 & 16 Exercises • **Quiz 3** • ***<u>Student Assignment:</u>*** ○ ***<u>Beckman Exercises TBD</u>***

<table>
<tr><td>Week 13</td><td>Unit 7
Chapter 17, 18, & 19 - Multiplication and Division of Fractions

<u>Unit 7 Key Points</u>
Unit 7 covers the topics of multiplication of fractions and decimals, the use of rectangles as manipulatives, division of fractions, and complex fractions. Concepts covered in this unit include: ther definition and product of formulas, inequalities, and the importance of complex fractions. Key point for this unit include, but are not limited to: product formula, inequality, cancellation phenomena, invert and multiply, and inverse.</td><td>Unit 7 Instruction
Review Beckman Exercises from Week 12
DUE (Date)
Beckman Exercises
<u>Student Assignments:</u>
<u>Wu Chapter 17 Exercises</u>
<u>Wu Chapter 18 Exercises</u>
<u>Wu Chapter 19 Exercises</u></td></tr>
<tr><td>Week 14</td><td>Unit 7
Chapter 17, 18, & 19 - Multiplication and Division of Fractions</td><td>Unit 7 Student Demo
Review and Discuss Wu Chapters 17, 18, & 19 Exercises
Review for Final Exam</td></tr>
<tr><td>Week 15</td><td>Final Exam</td><td>Final Exam</td></tr>
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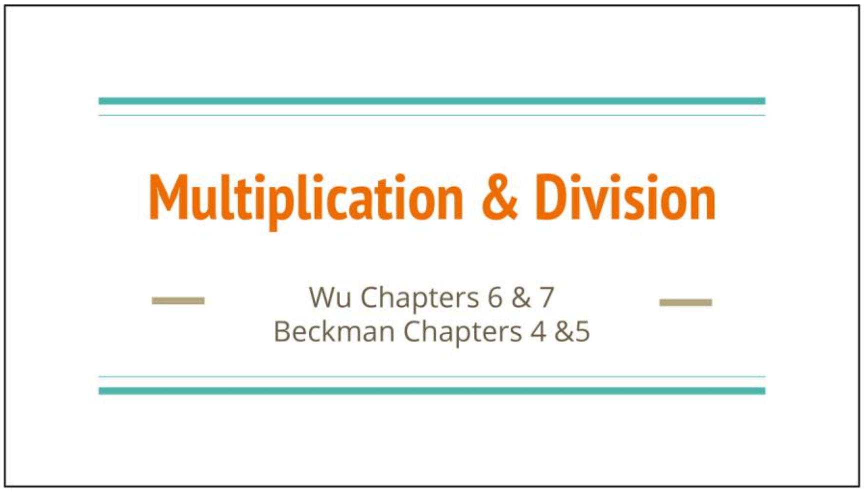
Appendix D. Sample Instructional Presentation

1. Sample presentation; modeled for Unit 3, Weeks 5 & 6, to use for in course instruction.



Multiplication

A x B

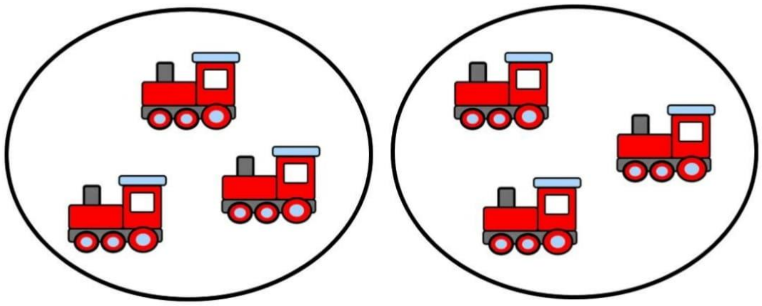
the total number objects in A groups if there are B objects in each group

The value of A x B is called a **product**, while the numbers A and B are called **factors**.

Multiplication

How is multiplication NOT addition, NOT subtraction, and NOT division?

In the beginning, we may explain multiplication by how it applies to **equal groups.**

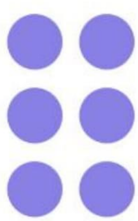


Here the figure shows 2 groups with 3 objects in each group. According the the **definition of multiplication**, there are **2x3** in all.

Array Problems

An **array** is a rectangular arrangement of horizontal and vertical columns.

How many soft drink case are in the case shown?



We can determine this by viewing each **row** (horizontal) in the array as a group.

3 groups

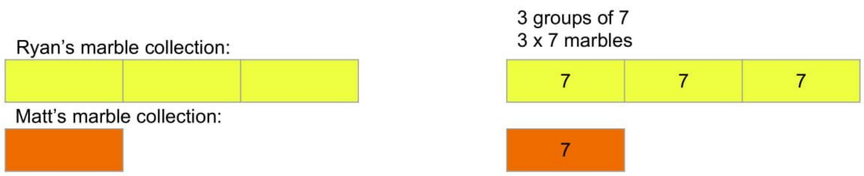
2 cans in each group

3 x 2

Comparison Problems

A **comparison problem** expresses a relationship between two quantities.

Ryan and Matt each have a marbles collection. Ryan has 3 times as many marbles as Matt. Matt has 7 marbles. How many marbles does Ryan have?



Ordered Pair Problems

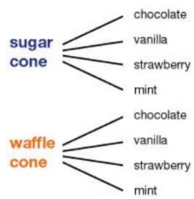
In some problems we want to determine how many pairs can be made.

A pair of things is an **ordered pair** if one object is designated as first and the other as second.

Ordered pair problems can be solved using an array or a **tree diagram**.

Tree Diagram

A **tree diagram** is a diagram consisting of line segments, or branches, that connect pieces of information where we start at the far left and work across to the right.



This diagram expresses two types of ice cream cones with 4 different flavors of ice cream as possible options.

2 types of cones
4 ice cream flavors
2 x 4 = 8 options

Properties of Multiplication

Commutative

AxB = BxA

Used to describe why expressions such a 3 groups of 5 marbles, is equal to expressions such as 5 groups of 3 marbles.

Associative

Ax(BxC) = (AxB)xC

Used to explain why equations are true.

Appendix E. Sample Quiz Week 6

Sample of content to be expected of quizzes; modeled for Unit 1 & 2 Quiz 1, Week 6

Name:______________________________ Date: ____________

Quiz 1 Units 1 and 2

1. Write the numbers as a sum of multiples of powers of 10.
   a. 730
   b. 21,015,249
   c. 425.289

2. Write “Fourteen billion three hundred five thousand seven hundred two” as a numeral.

3. Write the following as a base-ten numeral.
   $3 \cdot 10^3 + 2 \cdot 10^2 + 7$

4. Understand the meaning of arithmetic expressions involving minus signs. Perform the indicated operations:
   a. 27 - 81
   b. 4 - (9 - 2)
   c. - 274 - 148

5. For a collection of 1218 sugar cubes, draw a simple picture and write a brief description for how to organize them in a way that corresponds to the way we use the base-ten system to write the number for that many objects.

6. Students are sometimes uncertain about which zeros in decimals can be dropped and which can't. Give examples of zeros in decimals that can be dropped and zeros that can't be dropped. Explain why they can or can't be dropped.

7. Jack's checking account was overdrawn by $54. He deposited $127 dollars into his account. He later wrote a check for $50. How much money does he need to put in his account to not be overdrawn?

8. Write equations that correspond to the make-a-ten method for adding 5 + 8. Your equations should make careful and appropriate use of parentheses. Which property of arithmetic do your equations use?

9. Claire is 5 feet 2 inches tall. Ryan is 3 feet 10 inches tall. How much taller is Claire than Ryan? If someone comes up with a solution after the following method, are they right? If not, explain what is wrong with the method, and show how to modify the method of regrouping to make it correct.

$$\begin{array}{rr} \cancel{5\text{ ft}}^{4\text{ ft}} & \cancel{2\text{ in}}^{12\text{ in}} \\ \underline{-\ 3\text{ ft}} & \underline{10\text{ in}} \\ 1\text{ ft} & 2\text{ in} \end{array}$$

10. Stephen says that he can calculate 314 - 197 by adding 3 to both numbers and calculating 317 - 200 instead.
    a. Draw a number line to help you explain why Brian's method is valid.
    b. Could you adapt Brian's method to other subtraction problems? If so, give two more examples.
    c. Explain.

Appendix F. Alternate Certification Curriculum

The Curriculum of the Alternate Certification Program for Elementary Teachers at McNeese State University.

Year 1

<u>Summer Semester:</u>

EDUC 203 - Theories and Principles of Learning and Teaching (Lec. 3, Cr. 3)

Practical application of research based on learning theory. An overview of teaching in today's society and strategies of effective teaching.
Lec. 3 **Cr**. 3 **Field Experience** 10 hours

EDUC 216 - Introduction to Teaching Reading (Lec. 3, Cr. 3)

Introductory course on a balanced approach to reading instruction.
Prerequisite(s)/Corequisite(s): EDUC 217.
Lec. 3 **Cr.** 3 **Field Experience** 10 hours
Must turn in EDUC 499 to move into Fall Semester.

EDUC 499 - Entrance into the Non-Master's Alternative Program

Students planning to be teachers through the non-master's alternative program must make application through the TeachSWLA office and the Graduate School. Upon admission, a grade of "S" (satisfactory) or "U" (unsatisfactory) will be assigned.

Prerequisite(s): Bachelor's degree with a minimum GPA of 2.20 and (passing scores on [Core Academic Skills for Educators Praxis Exam, ACT, or SAT] and Praxis content-specific exam).
Cr. 0

<u>Fall Semester:</u>

EDUC 316 - Methods of Teaching Reading in the Elementary School

Principles, practices, methods, and materials for designing and implementing a balanced approach to teaching reading in elementary school.

Prerequisite(s): (EDUC 200 or EDUC 499), EDUC 203, and EDUC 216.
Lec. 3 **Cr.** 3 **Field Experience** 15 hours
WE

[3 cr.] Classroom Management

Spring Semester:

[2 cr.] EDUC 325 Science Methods

[3 cr.] EDUC 327 Math Methods

[2 cr.] EDUC 326 Social Studies Methods

Year 2

Summer Semester:

EDUC 416 - Diagnostic and Remedial Reading in the Elementary School Practicum (Lec. 2, Lab. 2, Cr. 3)

Practicum in a public K-5 setting, including characteristics of diagnosis and remedial teaching. Applications of diagnostic procedures and remedial techniques using special reading materials and evaluative devices.
Notes:
No duplicate credit for EDUC 416 and EDUC 503.
Prerequisite(s): (EDUC 200 or EDUC 499) and EDUC 316.
Lec. 2 **Lab**. 2 **Cr**. 3

Fall Semester:

[3 cr.] Residency 1 *Co-req FS & EDUC 351*

[2 cr.] Field Study *Co-req Res #1 and FS*

EDUC 351 - Educational Measurements (Lec. 3, Cr. 3)

Principles of tests and measurements in education.
Prerequisite(s): EDUC 200 or EDUC 499.
Lec. 3 **Cr.** 3

SPED Seminar

Spring Semester:

[3 cr.] Residency 2

Vita

Rachael Cramer Williams was born and raised in Branch, Louisiana, and attended high school at Notre Dame in Crowley. She is the fifth of eight children belonging to Lawrence and Mary Cramer. She is the proud wife of Brian Williams, also from Crowley, and the mother of three wonderful children, Ashton, Matthew, and Anna Claire. She received her undergraduate degree in Mathematics from McNeese State University in 1996. She received her teaching certification from Louisiana State University in …. She has taught mathematics, ranging from Algebra to Calculus, for over twenty years, and continues to enjoy it today.