Decentralized Optimal Control With Application In Power System

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DECENTRALIZED OPTIMAL CONTROL WITH APPLICATION IN POWER SYSTEM

A Dissertation

Submitted to the Graduate Faculty of the Louisiana State University and Agricultural and Mechanical College in partial fulfillment of the requirements for the degree of Doctor of Philosophy

in

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by

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ABSTRACT

An output-feedback decentralized optimal controller is proposed for power systems with renewable energy penetration. Renewable energy source is modeled similar to the classical generator model and is equipped with the unified power flow controller (UPFC). The transient performance of power system is considered and stability of the dynamical states are investigated. An offline decentralized optimal controller is designed that utilizes only the local states. The network comprises conventional synchronous generators as well as renewable sources with inverter equipped with UPFC. Subsequently, the optimal decentralized controller is compared to the initial stabilizing controller used to obtain the optimal controller. An online decentralized optimal controller is designed for discrete-time system. Two neuro networks are utilized to estimate value function and optimal control strategy.

Furthermore, a novel observer-based decentralized optimal controller is developed on small scale discrete-time power system. The system is trained followed by least square rules and successive approximation. Simulation results on IEEE 14-, 30-, and 118-bus power system benchmarks shows satisfactory performance of the online decentralized controller. And also, simulation results demonstrate great performance of the observer and the optimal controller compare to the centralized optimal controller.
CHAPTER 1. INTRODUCTION

The linear optimal control problem is well-known and studied in past several decades, which is formed as Riccati equation. In literatures, many method solving HJB has been developed. Neural Network (NN) is widely used because of its flexibility [1]-[7]. The cost function is estimated by NN and trained to be minimized offline and the convergence has been proved, either discrete-time or continuous time, finite-time or infinite-time full information or unknown dynamics. On the other hand, large-scale systems drawn more and more attention. A great portion of large system problem comes with interconnection terms. Different from centralized problem, decentralized system has to deal with effect between subsystems, which makes global optimal problem more complex. Jagannathan [8] proposes a decentralized controller which not only estimates unknown interconnection terms but the unknown dynamics, error and NN weight tracking guarantee the stability. In [9], it is shown that by choosing appropriate feedback gain for isolated systems, online NN will be trained to ensure the minimized cost function is uniformly ultimate bounded (UUB) with policy iteration. A smooth function is added to the system as an estimator in [10]. Other than ensuring the global stability a smaller bound is obtained with backstepping output tracking method.

Moreover, power system, which considered as a main application of decentralized control, has become a heated topic regarding to renewable energy integration, thermal storage, etc. Because of the uncertainty of renewable energy source, to maintain the stability of power grid, variety of devices and methods, such as excitation control, power system stabilizer, static VAR compensators.

During the last decade, microgrids (MGs) have emerged into the traditional AC grids due to their technical and economic advantages in presence of high penetration of renewable energy resource (RESs) for both grid-connected or islanded mode [11]. However, high penetration and increasing capacities of RESs can potentially lead to many challenges to the MG [12], [13], such
as power quality, frequency stability, and network reliability.

In different sets of literature, hierarchical control frameworks, that include primary, secondary, and tertiary control stages, have been widely adopted for MGs [14], [15]. The main focus of this paper is to develop a secondary control strategy for both grid-connected and islanded MG. The objective is to maintain the frequency to the appropriate value and also preserve exact active and reactive power sharing among all distributed renewable energy resources.

The secondary control strategies can be classified into three groups: centralized control [19], [20], distributed control [13], [18], [21], [22] and decentralized control [23], [24]. Centralized control of MGs are widely studied. In the centralized strategies, a central communication and computation unit is required to collect and process the information of all generators. In [19], a function is defined for each controllable generation unit of the microgrid and the dynamic set points are updated, using communication within the microgrid. The proposed controller is applied for the secondary voltage control of multi microgrids. In [20], a centralized control system is developed to coordinates parallel operations of several distributed generation (DG) inverters within a MG. Authors utilized the Model Predictive Control (MPC) algorithm to optimize the steady-state and the transient control problem, separately.

Centralized controllers are suffering from single point of failure problem (at the central node) and also may reduce the overall grid reliability. In order to overcome the centralized drawbacks, many research works on designing the distributed controllers for MGs. In [16], [17] the droop control strategies are proposed to dynamically preserve the supply-demand balance without any communication channel. However, these methods suffer from several drawbacks. For instance, voltage droop usually contains a poor reactive power sharing performance because of the dependence on output line impedance [18]. A multi-objective distributed control framework for
islanded AC MGs is proposed in [21]. The proposed controller includes two layers: The first layer regulates the voltage and frequency of voltage-controlled voltage source inverters (VCVSIs), while the second layer regulates the active and reactive powers of current-controlled voltage source inverters (CCVSIs). In [22], a new wireless-based robust communication algorithm is developed for the distributed secondary control of MGs. Although the reliability and security of the system is merit under the distributed controller, but failure of one controller effects other controllers and overall stability of the system [25]. Hence, it calls for very high Mean Time Between Failures (MTBF) and high degree of redundancy [25]. By applying the decentralized controllers, the grid security and reliability are maintaining and also the controllers are only depending on their local states and no need to have a knowledge of the neighbor’s states.

Unlike the pervious techniques, the secondary control of MGs using the decentralized techniques is not investigated properly yet. In [23], a two-layer agent-based decentralized control model is developed for islanded MGs, in which the upper level is the communication network collected of agent’s and the lower level is the electrical distribution MG. In [24] a decentralized controller is designed for the voltage regulation of islanded microgrid based on adaptive state estimator. To the best of authors’ knowledge, this paper is the first work that developing optimal offline decentralized controller for grid-connected and islanded microgrid operations based on the Hamiltonian-Jacobian-Bellman (HJB) equation with initial admissible controller. Furthermore, a novel online decentralized controller is developed for both grid-connected and islanded mode take advantages of two stage neural network (NN) technique to estimate the cost function and stabilizing the controller, respectively. In [21],[22], renewable energy sources are formed as synchronous generators with inertia controller, by using the inverter, terminal voltage of the source is well maintained, as well as the power output. [23], on the other hand, develop a novel unified
power quality conditioner (UPFC) device. By connecting the inverter described in [22], the stability of the power system and the quality of the power delivered to the system can be improved.

In addition to focus on the applications of optimal controller, lots of researchers turn their focus to online learning algorithm and output feedback control. Zhu et al., [32] developed an online learning algorithm to find the suboptimal output-feedback controller for linear systems using Integral Reinforcement Learning (IRF). The proposed method did not need the knowledge of the system drift dynamics while the adaptive observer was applied to collect the knowledge of states for IRL Bellman equation. The proposed method was validated on two real-world applications: The X-Y table and the F-16 flight control. Modares, Lewis and Jiang [33] developed a model-free $H_\infty$ tracker for nonlinear system. In order to get the solution of this problem, a generalized discounted $L_2$ gain condition was developed which did not require the complete knowledge of the system dynamics. The upper bound of the discount factor was found to ensure the stability of the solution. An online off-policy Reinforcement Learning (RL) method was developed to learn the solution. Simulation results proved the effectiveness of the proposed method. Hengster-Movric, Lewis, and Sebek [34] applied distributed static output-feedback (OPFB) control to study the synchronization of multi-agent systems. It is assumed that disturbances act on the agents. The study solve the cooperative bounded $L_2$ gain problem using OPFB. Specific conditions were found to ensure the global optimality that satisfied the quadratic performance criterion. Song et al. [35] developed an optimal control method for unknown systems with disturbance. The integral reinforcement learning (IRL) algorithm was applied to realize iterative control. In order to mitigate the influence of the unknown disturbance, a compensation controller was added to the system. The paper proved the uniformly ultimate bounded condition of weight errors and the convergence of Hamiltonian function. The proposed optimal control method for unknown systems have been
tested through simulation. Savkin and Petersen [36] developed an approach to solve the problem of linear quadratic optimal control and stabilization via decentralized control. There is no restriction of the finite dimension or time invariant of the controller. The paper present a framework on building a series of decentralized controllers to stabilized the systems and realizing the optimality with respect to the quadratic performance index. Xin et al. [37] developed a decentralized dynamic output feedback optimal guaranteed cost reliable controller for linear system. The method considered the special case of sensor failures. The optimal controller can ensure the asymptotic stability and minimize the upper bound of a given quadratic cost function. At the same time, the sufficient conditions of the existence of the solution are provided in the paper. Swigart and Lall [38] applied decentralize control on a system that consists of two subsystems and limited communication existed for only one direction. The assumption of the full state feedback was relaxed in this paper. In addition, the paper applied a partial output feedback architecture. One subsystem can measure the state directly. The other subsystem can only access the noisy measurement of his own states. They utilized spectral factorization approach to build the orders of the optimal controller and provided the corresponding state space equations. Lessard and Lall [39] present a two-player problem in the field of decentralized optimal control. The output feedback controllers were developed for each subsystem, which are interconnected. The solution to the general case was provided in the paper. Previous studies shown that associative gains were allowed to computed separately, which were not always the case in general. This paper accounts for the cases that some gains need to be solved simultaneously. Nayyar and Kalathil [40] developed an optimal decentralized controller for a plant with nested structure. The paper assumed that there exists a unit delay in communication in one direction and no transmission in the other direction. They combined linearity and summary statistics to formulate the problem as a deterministic convex
optimization problem where the optimal control law was tractable. Jalilvand and Safari [41] developed a unified power flow controller (UPFC) for output feedback control. They have transfer the problem of selection of the output feedback gains to an optimization problem. Swarm optimization algorithm was applied with a time domain-based objective function. They have considered various combinations of the operating conditions and system configurations. They have tested and validated the model with a nonlinear time simulation. Simulation results proved that the proposed model can effectively damp oscillations in power system.

Instead of using neural network, online learning algorithm, observer estimation is another way to approximate the unknown states for decentralized control. Ficklscherer and Muller [42] discussed three types of structures of decentralized systems. In their paper, they only focused on the structures which apply either data and signal of the subsystems themselves, global input/output, or the reduced order interaction models. The necessary and sufficient conditions were present for the existence of each type of decentralized observer. They also provided the instructions on how to design the corresponding systems. Abdollahi et al. [43] developed a stable nonlinear-in parameters neural network (NN)-based observer for nonlinear systems. The observer can be used to systems with high degree of nonlinearity under the condition that prior knowledge of system dynamics is unknown. They also developed a novel approach for learning parameters in NN based on the modified back propagation (BP) algorithm and e-modification. Lyapunov’s direct method was applied to evaluate the stability of the system. Simulation results confirmed the improved performance. Grip, Saberi, and Johansen [44] developed an observer design methodology to estimate the unknown output and states of the linear system. They also extended it to a more general method of feedback-interconnected systems. They restricted the observer to be a corresponding quadratic-type Lyapunov function. They further assumed that the output of the
nonlinear system is unknown while it is the input to the second linear system. Under these assumptions, they proved that the overall error dynamics is globally exponentially stable. They also illustrated the methodology by applying it on a navigation example which focused on the integration of inertial and satellite measurements. Zhao et al. [45] developed a decentralized tracking control (DTC) based on an observer-critic structure-based adaptive dynamic programming. The DTC can be applied on unknown large-scale nonlinear systems. The control system comprises of three parts: local desired control, local tracking error control, and compensator. The local desired control was developed based on a local Neural Network (NN) observer. The local error control was obtained by a critic NN to approximate the local value function. The compensator used an adaptive robustifying term to make up for the overall errors. Lyapunov’s direct method was applied to evaluate the stability of the closed-loop system. All the literatures and materials form this dissertation and make great contribution to the related area.

Briefly, the main contribution of this dissertation can be summarized as follows:

1. Developing a sophisticated mathematical model for the AC microgrid power flow along with the dynamics of the power sources.

2. Developing both online and offline decentralized controllers for grid-connected and islanded microgrid operation. The stability and optimality of the proposed method are proved and demonstrated.

3. Developing online decentralized optimal controller for discrete-time system with application on large-scale power system with renewable energy penetration.

4. Developing observer-based offline decentralized optimal controller for discrete-time system.

5. Microgrid in transient situation with multiple faults injection is tested to demonstrate the
effectiveness and merit of the proposed controller.

6. A realistic combination of different type of sources are proposed based on the optimal offline decentralized controller.

7. Multiple large-scale power system is investigated with proposed controllers to illustrate the adaptiveness of the controller.

The rest of the dissertation organized as follows: Chapter 2 introduces a power system modelling with renewable energy penetration. The system is linearized and remodeled with consideration of system dynamics. Chapter 3 expresses an offline continuous time decentralized controller and formulation. Chapter 4 shows an online non-optimal decentralized controller with neural network estimation. Chapter 5 illustrates an online optimal decentralized controller for discrete-time system using actor-critic neural network frame. Another offline decentralized observer-based optimal controller is then demonstrated in Chapter 6. The simulation results are shown in Chapter 7.
CHAPTER 2. RENEWABLE ENERGY SOURCE MODELLING WITH UPFC

This section presents the renewable energy source with UPFC model as well as power balance equations. The renewable energy source contains a photovoltaic (PV) source that is connected to a DC-AC grid-tied inverter (GTI) and be controlled by grid equipped UPFC, as shown in Figure 1 [16].

![Figure 1: Novel UPFC model with PV penetration](image)

2.1 GTI Model

A renewable energy source such as PV, wind power, biofuels, produce power as a DC source. Due to the nature of renewable source, a dc-link capacitor is required to prevent disturbances and maintain the stability of the income power source. Therefore, the capacitor at the dc-link stored energy and provide constant voltage as follows [17]:

\[ C_l V_{C,i} \dot{V}_{C,i} = P_{in,i} - P_{e,i} \]  

Therefore, the delivered power \( P_{e,i} \) to the grid through UPFC for the \( i \)th subsystem can be taken into account as follow:

\[ P_{e,i} = (1 + \epsilon_i) B_l V_{i} V_{s,i} \sin (\varphi_i - \theta_i) \]  

where \( \epsilon_i \leq 1 \), and \( V_i \) considered as a constant output (same as the terminal voltage of regular generator). Also, \( V_{i}^{'} \) and \( \theta_{i}^{'} \) represent voltage and angle as terminal voltage and angle of \( i \)th
inverter, which connects the UPFC through admittance \( B_i \), respectively. Therefore, \( P_{in,i} \) can be described as the output power from PV source, which is a function of PV output voltage as follows [18]:

\[
P_{in,i} = V_{in,i} \ast I_{in,i} = V_{in,i} \ast \kappa(V_{in,i}) = V_{in,i} \ast (n_{p,i}I_{ph,i} - n_{p,i}I_{rs,i}(e^{\mu_iV_{in,i}} - 1))
\]

where

\[
\mu_i = q/(n_{s,i}\varphi KT)
\]

Equation (1) can be defined by an auxiliary parameter \( \nu \) as follows:

\[
\frac{\dot{\nu}_i}{\tau} = V_{c,i} \ast \dot{V}_{c,i} = (1/C)(P_{in,i} - P_{e,i})
\]

Therefore, \( \dot{\varphi}_i = \nu \) and GTI’s output power is controlled by \( \nu \), that is derived from straightforward calculation as follows:

\[
\nu = \tau \ast (V_{c,i}^2 - V_{c0,i}^2)/2
\]

By defining these new sets of variables \( \varphi_i \) and \( \nu \), one can manipulate the GTI’s outputs by changing the value of capacitor voltage. Also, the PV source is formed as a synchronous generator-like source, where \( \varphi_i \) represents angle variation of a regular generator, \( \nu \) represents speed of a SG and \( C/\tau \) will be a projection of SG internal inertia.

A solar panel will be connected to a dc-dc buck converter, which controlled the input voltage of GTI. Therefore,

\[
V_{s,i} = Dt \ast Rt \ast V_{in,i}.
\]

### 2.2 UPFC Model

Instead of utilizing two transformers as conventional UPFC model shown in Figure 2[19], this paper used a novel UPFC model (see Figure 1). In this model the shunt transformer and an additional capacitor are ignored. Moreover, series voltage \( V_u \) which is generated by UPFC and also
can control the power flow, is decomposed as $V_{up}$ and $V_{uq}$ based on Figure 3. Therefore, $V_u$ will be mandatory set as $V_{up}$, for a single input subsystem. According:

$$\cos \theta_i' = \frac{(V_{ri} + V_{uq,i})}{V_i'}$$

(9)

$$\sin \theta_i' = \frac{V_{uq,i}}{V_i'}$$

(10)

Also, eq. (4) will be written as follows:

$$\begin{pmatrix} e_i \\ \tau_i \end{pmatrix} v_i = P_{in,i} - (1 + \epsilon_i) B_i V_{ri} V_{s,i} \sin (\phi_i + \varphi_{0,i}) + (1 + \epsilon_i) B_i V_{s,i} \cos (\phi_i + \varphi_{0,i}) u_1$$

(11)

Figure 2 Conventional UPFC scheme

According to eq. (2),(3), and (6) can be formed as follows:

$$\begin{pmatrix} \dot{\phi}_i \\ \dot{\psi}_i \end{pmatrix} = \begin{pmatrix} v_i \\ \frac{\tau_i}{\epsilon_i} (P_{in,i} - (1 + \epsilon_i) B_i V_{ri} V_{s,i} \sin (\phi_i + \varphi_{0,i})) \end{pmatrix} + \begin{pmatrix} \frac{\tau_i}{\epsilon_i} (1 + \epsilon_i) B_i V_{s,i} \cos (\phi_i + \varphi_{0,i}) \end{pmatrix} u_1$$

(12)

By linearizing eq. (12), we have:
\[
\left[ \frac{\Delta \phi_i}{\Delta \psi_i} \right] = 
\begin{bmatrix}
\frac{\tau_i}{C_i} \left( M - \frac{N * (\mu_i V_{z,i} e^{\Delta t} + RT - 1)}{e^{\Delta t} + RT} - B_i V_{r,i} V_{s,i} \cos(\phi_i + \varphi_{0,i}) \right) & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\Delta \phi_i \\
\Delta \psi_i
\end{bmatrix}
+F_y \begin{bmatrix}
V' \\
\theta'
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{\tau_i}{C_i} * B_i V_{s,i} \sin(\phi_i + \varphi_{0,i})
\end{bmatrix} \Delta u_1
\]

where \( M = \frac{n_{p,i} I_{ph,i}}{dt \times R_t} \) and \( N = n_{p,i} I_{rs,i} \mu_t \), and also \( F_y \) denotes the gradient matrix of swing-like equation takes partial derivative respect to each bus voltage and angle, which will be illustrated in next session. Finally, the system with output as voltage and angle from the power balance equations then can be simplified as follows:

\[
\Delta x = A \Delta x + B \Delta u + D y
\]

\[ (13) \]

where \( M = \frac{n_{p,i} I_{ph,i}}{dt \times R_t} \) and \( N = n_{p,i} I_{rs,i} \mu_t \), and also \( F_y \) denotes the gradient matrix of swing-like equation takes partial derivative respect to each bus voltage and angle, which will be illustrated in next session. Finally, the system with output as voltage and angle from the power balance equations then can be simplified as follows:

\[
\Delta x = A \Delta x + B \Delta u + D y
\]

\[ (14) \]

\[ \text{Figure 3 UPFC voltage vector diagram} \]

2.3 Power Balance Algebraic Equations

The power balance algebraic equations are modelled as follows:

\[
P_{li} + \sum_{j=1}^{ng} Y_{ij} V_i E_{mag} \sin(\theta_i - \phi_j) + \sum_{j=ng+1}^{N_b+ng} Y_{ij} V_i V_j \sin(\theta_i - \theta_j) = 0
\]

\[ (15) \]

\[
-Q_{li} + \sum_{j=1}^{ng} Y_{ij} V_i E_{mag} \cos(\theta_i - \phi_j) + \sum_{j=ng+1}^{N_b+ng} Y_{ij} V_i V_j \cos(\theta_i - \theta_j) = 0
\]

\[ (16) \]
Also, the non-generator bus power balance algebraic equations are defined as follows:

\[
P_{Li} + \sum_{j=n_g+1}^{N_b+n} Y_{ij} V_i V_j \sin(\theta_i - \theta_j) = 0
\]

\[
-Q_{Li} + \sum_{j=n_g+1}^{N_b+n} Y_{ij} V_i V_j \cos(\theta_i - \theta_j) = 0
\]

where \( N_b \) denotes the number of buses. The power balance equation can also be linearized as follows

\[
\frac{\partial \Delta P}{\partial x} = \frac{\partial \Delta P}{\partial V} \times \dot{V} + \frac{\partial \Delta P}{\partial \dot{\theta}} \times \dot{\theta} + \frac{\partial \Delta P}{\partial \dot{\phi}_i} \times \dot{\phi}_i = 0
\]

\[
\frac{\partial \Delta Q}{\partial x} = \frac{\partial \Delta Q}{\partial V} \times \dot{V} + \frac{\partial \Delta Q}{\partial \dot{\theta}} \times \dot{\theta} + \frac{\partial \Delta Q}{\partial \dot{\phi}_i} \times \dot{\phi}_i = 0
\]

The equations above can be convert to a set of dynamic variables as

\[
\begin{bmatrix}
\dot{V} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
A' & C' \\
B' & D'
\end{bmatrix}^{-1} \begin{bmatrix}
E & 0 \\
F & 0
\end{bmatrix} \times \begin{bmatrix}
\dot{\phi}_i \\
\dot{\psi}_i
\end{bmatrix}
\]

where \( A' : \frac{\partial \Delta P}{\partial V} \), \( C' : \frac{\partial \Delta P}{\partial \dot{\theta}} \), \( E : \frac{\partial \Delta P}{\partial \dot{\phi}_i} \), \( B' : \frac{\partial \Delta Q}{\partial V} \), \( D' : \frac{\partial \Delta Q}{\partial \dot{\theta}} \), \( F : \frac{\partial \Delta Q}{\partial \dot{\phi}_i} \) and it can be simplified as

\[
y = G^{-1} * H * x
\]

According to eq. (14), (19) and (20), the decentralized system state-space equation can be written as

\[
\Delta \dot{x} = A_c \Delta x + B \Delta u
\]

where \( A_c = A + D * G^{-1} * H \)
CHAPTER 3. OUTPUT-FEEDBACK DECENTRALIZED OPTIMAL CONTROLLER

3.1 System Formulation

Assume a conventional decentralized output-feedback linear system in canonical form as follows:

\[ \dot{x} = Ax + Bu \]  \hspace{1cm} (24)  
\[ u_i = C_i y_i, y_i = -K_i x_i \]  \hspace{1cm} (25)

where, overall system states and input can then be organized as \( x = [x_1, x_2, \ldots, x_N]^T \) and \( u = [u_1, u_2, \ldots, u_N]^T \) respectively, which \( 1 \leq i \leq N \) denotes the number of subsystem and \( m \) denotes the number of states for each subsystem.

Hence, state matrix can be converted into canonical form as follows:

\[
A_i = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 \\
a_{i,1} & a_{i,2} & a_{i,3} & \cdots & a_{i,mN-1} & a_{i,mN}
\end{bmatrix}_{m \times mN}  
\]  \hspace{1cm} (26)  

\[
B = [0 \ldots 1, 0 \ldots 1, \ldots 0 \ldots 1]^T  
\]  \hspace{1cm} (27)  

From the matrix \( A_i \), it can be deduced that for each subsystem, all states will show up in the last equation of \( i \) th subsystem’s state dynamics, while controlled by its own input controller \( u_i = -K_i C_i x_i \).
3.2 Offline Decentralized Optimal Controller Design

In order to reach the optimal solution of the system, the evaluation function of each subsystem should be minimized. A decentralized system cost function can be defined for each subsystem as follows:

\[
J_i = \int (x^T Q_i x + u_i^T R_{ii} u_i + u_j^T R_{ij} u_j) dt
\]  

(29)

where \( p = 1, 2, \ldots, N, q \neq p, Q_i \geq 0 \) is the state performance index, and \( R_{ii} > 0, R_{ij} > 0 \) are the input performance index.

For decentralized system, one can easily notice, \( J_i \) may be equal, if and only if proper indexes are selected as follows:

\[
Q = Q_1 = Q_2 = \cdots = Q_N
\]  

(30)

\[
R_{ii} = R_{ji}, R_{ij} = R_{jj}
\]  

(31)

Therefore, the performance criteria will be defined as follows:

\[
J_1 = J_2 = \cdots = J_N = \int (x^T Q_i x + C_i^T K_i^T R_{ii} K_i C_i + C_j^T K_j^T R_{ij} K_j C_j) dt = tr(M_l X)
\]  

(32)

where \( X \equiv x_0 x_0^T \), \( x_0 \) is the initial state, that is considered as a constant input. The evaluation function can be derived in Riccati equation form where \( M = M_l \) is the solution as follows:

\[
F = F_i \equiv A_c^T M_l + M_l A_c + Q_i + C_i^T K_i^T R_{ii} K_i C_i + C_j^T K_j^T R_{ij} K_j C_j
\]  

(33)

Therefore, the Hamilton function can be defined as follows:

\[
H = tr(M X) + tr(FS)
\]  

(34)

Consequently, a minimal solution can be determined by taking partial derivative of \( H \), respect to \( M, S, K_i, K_j \).
Table 1 Offline Controller Algorithm Procedure.

<table>
<thead>
<tr>
<th>Offline Controller Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Step 1:</strong></td>
</tr>
<tr>
<td><strong>2. Step 2:</strong></td>
</tr>
</tbody>
</table>

Solve for $M_k, S_k, K_1'$ based on (25)(26)(29)

Set $J_k = \text{tr}(M_kX) + \text{tr}(F_kS_k)$

Evaluate approaching direction with

$$\Delta K = K'_k - K_k$$

Update $K_k$ with proper approaching step $\alpha$

$$K_{k+1} = K_k + \alpha \times \Delta K$$

$\alpha$ is selected to guarantee:

- $A_{c,k+1} = A - B \times K_{k+1} \times C$ is asymptotically stable
- $J_{k+1} \leq J_k$

If error of $J_{k+1}$ and $J_k$ is small enough, go to 3,

else,

set $k = k + 1$, back to 2

| 3. | $J_{optimal} = J_{k+1}$, stop. |

Considering a 2 subsystem case as follows:

$$0 = F^T = A_c^T M + M A_c + Q + C_1^T K_1 R_{11} K_1 C_1 + C_2^T K_2 R_{22} K_2 C_2$$ (35)

$$0 = A_c^T S + S A_c + X$$ (36)

$$0 = -B_1^T M S C_1^T + R_{11} K_1 C_1 S C_1^T$$ (37)
\[ 0 = -B_2^T M S C_2^T + R_{22} K_2 C_2 S C_2^T \] (38)

Therefore, eq. (37) and eq. (38) can be simplified as a matrix form, by considering \( C_1 = [I \ 0], \ C_2 = [0 \ I], \ B_1 = [B_1^T \ 0]^T, \ B_2 = [0 \ B_2^T]^T \). Also, the feedback gain updated law can be taken into account as follows:

\[
\begin{bmatrix}
K_1 & 0 \\
0 & K_2
\end{bmatrix} = \begin{bmatrix}
R_{11} & 0 \\
0 & R_{22}
\end{bmatrix}^{-1} \begin{bmatrix}
B_1^T M & 0 \\
0 & B_2^T M
\end{bmatrix} * \begin{bmatrix}
S C_1^T & 0 \\
0 & S C_2^T
\end{bmatrix} \begin{bmatrix}
C_1 S C_1^T & 0 \\
0 & C_2 S C_2^T
\end{bmatrix}^{-1}
\] (39)

Finally, an output-feedback optimal decentralized control methodology has been developed with stabilizing controller as Table 1.

Remarks: The algorithm requires a stabilizing initial controller, which will later affect the performance of cost function. In this paper, the initial controller is derived and proved based on [30].
CHAPTER 4. ONLINE DECENTRALIZED CONTROLLER DESIGN

In this Chapter, an online decentralized controller is designed without knowing the dynamics of the system. A two layer NN is utilized to approximate the interconnected terms. Before the designed is introduced, define $X_{ij}$ as the all states of subsystem $i$ except the first state. 

$$e_{ij} = x_{ij} - x_{ijd} ,$$

where $x_{ijd}$ expresses the desired tracking target for the $j$th state of subsystem $i$, and so defined $e_{ij}$ as the tracking error. $y_{ij}$ denotes the intermediate error, which will be introduced later as an intermediate error that can be observed. These errors are defined and proven to approach to zero. In our case, all states target $x_{ijd}$ are sets to be zero, which means for each individual states of every subsystem, they will track to the origin. If the targets are nonzero, one may shift the targets and using this same approach.

Now, a two layer NN is defined as $\tilde{W}_{ij}^T \phi_{ij}(V_{ij}^T \Phi_{ij})$, the hidden layer weights are randomly chosen at the beginning of the algorithm and remain constant during the training process. The outer layer weights are updated as $\tilde{W}_{ij}$, and the input of the overall NN is $\Phi_{t1} = x_{t1}$, $\Phi_{ij} = [X_{ij}^T, e_{ij}, y_{ij}]^T$. Since the hidden layer weights are kept constant, the number of hidden layer neurons may be chosen by experience considering the tradeoff between computation complexity and estimation accuracy.

Thus, for all states in $i$th subsystem define

$$\tilde{x}_{i(j+1)} = u_i = e_{ij} \tilde{W}_{ij}^T \phi_{ij}(V_{ij}^T \Phi_{ij}) - K_{ij} e_{ij} + \dot{x}_{ijd}$$

(40)

where $\tilde{x}_{i(j+1)}$ is set as the desired input that stabilizes the $e_{ij}$ as $t \to \infty$. Furthermore, as mentioned above, the target tracking value $x_{t1d} = \dot{x}_{t1d} = 0$. The stabilizing input for subsequence states are defined as
\[ \sigma_{ij} \hat{x}_{ijd} = x_{ijd} - \bar{x}_{ij} \]  \hspace{1cm} (41)

where \( \sigma_{ij} \) is a designed stabilizing small constant.

Then, the observed output is defined

\[ y_{ij} = x_{ijd} - \bar{x}_{ij} \]  \hspace{1cm} (42)

Thus, the states are reformulated as

\[ x_{ij} = e_{ij} + y_{ij} + \bar{x}_{ij} \]  \hspace{1cm} (43)

The tracking error dynamics becomes

\[ \dot{e}_{ij} = A_{e} e_{i} + B_{i} y_{i(j+1)} - x_{ijd} + \Delta(x) \]  \hspace{1cm} (44)

Now, the stability proof is introduced as Theorem 1,

Theorem 1: The NN estimated weights will remain in the pre-defined bound \( W_{ij}^{Max} \) as long as the initial weights are chosen within \( W_{ij}^{Max} \), that is, \( \|\hat{W}_{ij}\| \leq W_{ij}^{Max} \) as \( t > 0 \). And also, \( \hat{W}_{ij} = W_{ij} - \bar{W}_{ij} \), which defined as the weights estimation error, will be bounded within the compact set.

In other words, \( \|W_{ij}\| \leq W_{ij}^{Max} \). If the NN weights is updated as

\[ \hat{W}_{ij} = -\gamma_{ij} e_{ij}^{2} \bar{W}_{ij}^{T} \Phi_{ij} (V_{ij}^{T} \Phi_{ij}) - \gamma_{ij} e_{ij}^{2} \bar{W}_{ij} + \gamma_{ij} e_{ij}^{2} \tau \frac{\|\hat{W}_{ij}\|}{\|\hat{W}_{ij}\|^{2}} \bar{W}_{ij}^{T} \Phi_{ij} (V_{ij}^{T} \Phi_{ij}) \]  \hspace{1cm} (45)

where

\[ \tau = \begin{cases} 0; & \text{if } \|\hat{W}_{ij}\| < W_{ij}^{Max} \text{ or } \\
 & \|\hat{W}_{ij}\| = W_{ij}^{Max}, \text{ and } \|\hat{W}_{ij}\|^{2} \bar{W}_{ij}^{T} \Phi_{ij} (V_{ij}^{T} \Phi_{ij}) \geq 0 \\
1; & \text{if } \|\hat{W}_{ij}\| = W_{ij}^{Max}, \text{ and } \|\hat{W}_{ij}\|^{2} \bar{W}_{ij}^{T} \Phi_{ij} (V_{ij}^{T} \Phi_{ij}) < 0 \\
1; & \text{if } \|\hat{W}_{ij}\| > W_{ij}^{Max} \end{cases} \]  \hspace{1cm} (46)
After re-formulate the system with error dynamics and tuning with estimation weights, the decentralized control system is asymptotically stable by Theorem 1, with inputs of local states and output tracking error. In addition, the activation function $\phi_{ij}$ can be selected by one’s experience. In our case, sigmoid function is chosen for all subsystem.

One should notice that the purposed online training method can guarantee the stability of the decentralized controller while estimating not only linear interconnection terms, but nonlinear terms. However, this error dynamics system may not lead to an optimal cost for overall system. Meanwhile, the bound $W_{ij}^{Max}$ plays a significant role during the estimation procedure. Narrow bounds selection can lead to no solution or large estimation errors.
CHAPTER 5. ONLINE DECENTRALIZED OPTIMAL DISCRETE-TIME CONTROLLER

In this chapter, an online decentralized optimal controller for discrete time system is proposed. The difficulties online learning algorithm face in contrary to the offline method are not only unknown system dynamics, but the unknown interconnection terms. Especially, it is a great challenge for the large-scale system to taken them into account, for example, the power grid. The chapter is organized as follows: interconnected system is stated first, secondly, the discrete time cost function is introduced, modified and approximated with a critic neural network. Finally, the action network is proposed to estimate the optimal controller.

5.1 Decentralized Linear Systems

Consider a decentralized linear discrete-time system composed by \( n \) subsystems:

\[
x_i(k + 1) = A_i x_i(k) + B_i u_i(x_i(k)) + A_{-i} x_{-i}(k)
\]

where \( A_{-i} \) denotes the interconnected term coefficient that \( 1 \leq i \leq n \) and \( x_{-i}(k) \) illustrates other subsystem states at step \( k \). In applications with canonical form dynamic system, \( A_{-i} x_{-i}(k) \) is the summation of linear combinations of all other subsystem states that adds up to the last state dynamics equation.

Assumption 1 [30]: The control input gain defined in eq. (47) is bounded by \( 0 < B_i < B_{l_{max}} \), where \( B_{l_{max}} \) are positive real constants.

Define the tracking error for \( i^{th} \) subsystem as \( d_i(k) = x_i(k) - x_{id}(k) \), where \( x_{id}(k) \) is the desired state trajectory for \( i^{th} \) subsystem. To stabilizing each subsystem, \( x_{id}(k) \) is set to be 0. The filtered error is then defined as \( e_i(k) = \lambda_{i,1} + \lambda_{i,2} d_i(k - n) + \cdots + \lambda_{i,n-1} d_i(k - 1) + d_i(k) \) where
the coefficients are selected to maintain the pole of polynomial $\lambda_i = [\lambda_{i,1}, \lambda_{i,2}, \ldots, \lambda_{i,n-1}]$ are within the unit circle in discrete time case.

Assumption 2 [31]: The interconnection terms $A_{-i}x_{-i}(k)$ are bounded by $A_{-i}x_{-i}(k) \leq \zeta_{lo} + \sum_{j=1}^{n} \zeta_{ij} \epsilon_i$, where $\zeta_{lo} \leq \zeta_{loMax}$, is a small positive number that $\zeta_{lo} = 0$ at origin. $\zeta_{ij}$ is a small positive constant that known and bounded by $\zeta_{ij} \leq \zeta_{ijMax}$.

Next, the quadratic cost function is introduced in discrete time form

$$J_i(x(k)) = \sum_{j=k}^{T} (x_i(j)^T Q_i x_i(j) + u_i^T (x_i(j)) R_i u_i (x_i(j)))$$

where $Q_i$ is a designed state gain semi-definite matrix and $R_i$ is a known control gain positive definite matrix, $1 \leq i \leq n$.

By stationary condition, one can get target optimal input as:

$$u_i^*(x(k)) = -\frac{1}{2} R_i^{-1} B_i^T (x_i(k)) \frac{\partial J_i^*(x(k+1))}{\partial x_i(k)} + \sum_{j=1; j \neq i}^{N} \frac{\partial J_j^*(x(k+1))}{\partial x_i(k+1)}$$

The difficulty of solving eq. (49) is, other than the complexity of solving partial derivatives, solving unknown individual cost of decentralized subsystem. Thus, a critic neuro network is created to estimate cost for each subsystem.

### 5.2 Actor-Critic Neuro Network Design

By utilizing the approximation property of NN, a subsystem cost function can be derived as:

$$J_i^*(x_i(k)) = W_{el}^T \sigma_{el} (x_i(k)) + \epsilon_{el}$$
where $W_{ci}$ denotes the number of weights for $i^{th}$ subsystem that $\|W_{ci}\| \leq W_{cimax}$, the $\sigma_{ci}(x_i(k))$ is the activation function that only composed by the local states $x_i(k)$. $\varepsilon_{ci}$ is the approximation error which is bounded by $\varepsilon_{cimax}$.

Next, the optimal controller updated law is defined as

$$\hat{u}_i(x_i(k)) = \hat{u}_{io}(x_i(k)) + u_{iF}(x_i(k))$$ (51)

where $\hat{u}_{io}(x_i(k))$ is estimated by an actor neural network which is formed by the action estimated weights $\hat{W}_{ai}$, actor estimation function $\sigma_{ai}(x_i(k))$ and the estimation error $\varepsilon_{ai}$ as:

$$\hat{u}_{io}(x_i(k)) = \hat{W}_{ai}^T \sigma_{ai}(x_i(k)) - \varepsilon_{ai}$$ (52)

$u_{iF}(x_i(k)) = G_i(x_i(k))$ indicates the feedback gain that corrects the estimation process by introducing the error as:

$$G_i(x_i(k)) = -B_i^{-1}(x_i(k))[0 \lambda_i]^T d_i(k)$$ (53)

Note that $\|W_{ai}\| \leq W_{aimax}$, and $\|\varepsilon_{ai}\| \leq \varepsilon_{aimax}$.

Augmenting all individual parameters in each subsystem, we have

$$\hat{W}_c = diag(W_{c1}, W_{c2} ..., W_{cn})$$ (54)

$$\vec{W}_c = diag(\hat{W}_{c1}, \hat{W}_{c2} ..., \hat{W}_{cn})$$ (55)

$$\kappa_c = diag(\kappa_{c1}, \kappa_{c2} ..., \kappa_{cn})$$ (56)

$$\vec{e}_c(k) = \begin{bmatrix} [e_{c1}(x_1(k)) \ e_{c1}(x_1(k-1)) \ e_{c1}(x_1(k-p))] \\
\vdots \\
[e_{cn}(x_n(k)) \ e_{cn}(x_n(k-1)) \ e_{cn}(x_n(k-p))] \end{bmatrix}$$ (57)
\[ \bar{\sigma}_{ci}(x_i(k)) = \begin{bmatrix} \sigma_{c1}(x_1(k)) & \sigma_{c1}(x_1(k-1)) & \sigma_{c1}(x_1(k-m+1)) \\ \vdots & \vdots & \vdots \\ \sigma_{cn}(x_n(k)) & \sigma_{cn}(x_n(k-1)) & \sigma_{cn}(x_n(k-m+1)) \end{bmatrix} \] (58)

\[ J_-(k) = [x_i(k)^T Q_i x_i(jk) + u_i^T(x_i(k)) R_i u_i(x_i(k)) ... x_i(k-m+1)^T Q_i x_i(k-m+1) + u_i^T(x_i(k-m+1)) R_i u_i(x_i(k-m+1))]^T \] (59)

\[ \bar{f}(k) = [J_{-1}(k), J_{-2}(k), ..., J_{-n}(k)]^T \] (60)

where \( m \) is the number of subsystem state. (assume all subsystems have the same number of states for simplicity)

Further, two neural network weights updated law is developed. First, define the critic neural network error as:

\[ \bar{e}_c(k) = \bar{f}(k-1) + \bar{W}_c^T \Delta \bar{\sigma}_c(x(k)) \] (61)

where \( \Delta \bar{\sigma}_{ci}(x_i(k)) = \bar{\sigma}_{ci}(x_i(k)) - \bar{\sigma}_{ci}(x_i(k-1)) \)

Thus,

\[ \bar{e}_c(k+1) = \bar{f}(k) + \bar{W}_c^T \Delta \bar{\sigma}_c(x(k+1)) \] (62)

Then one can easily derive the error updated law as:

\[ \bar{e}_c(k+1) = \kappa_c \bar{e}_c(k) \] (63)

where \( \kappa_c \) is a small positive design constant that guarantees the convergence of the critic error.

From eq. (63), the critic weights updated law can be expressed as:

\[ \bar{W}_c(k+1) = \Delta \bar{\sigma}_c(x(k)) \left( \Delta \bar{\sigma}_c^T(x(k)) \Delta \bar{\sigma}_c(x(k)) \right)^{-1} (\kappa_c \bar{e}_c(k) - \bar{f}(k))^T \] (64)

And also, the actor error is derived from above equations to evaluate the estimation of the optimal controller and the cost function that is approximate by the critic neural network.
\[
e_{al}(k) = \mathcal{W}_{ai}^T \sigma_{al}(x_i(k)) + G_i(x_i(k)) + \frac{1}{2} R_i^{-1} B_i^T (x_i(k)) \frac{\partial \sigma_{cl}(\hat{x}_i(k+1))}{\partial \sigma_{cl}(x_i(k))} \mathcal{W}_{cl}(k)
\] (65)

Furthermore, the actor weights update law is introduced as:

\[
\mathcal{W}_{ai}(k + 1) = \mathcal{W}_{ai}(k) - \kappa_{ai} \frac{\sigma_{al}(x_i(k)) e_{al}(k)}{\sigma_{al}(x_i(k)) \sigma_{al}(x_i(k)) + 1}
\] (66)

See [30] for more details of the convergence proof of the proposing decentralized online optimal controller.
CHAPTER 6. OFFLINE DECENTRALIZED OPTIMAL DISCRETE-TIME CONTROLLER

In this chapter, an observer-based offline decentralized optimal controller is proposed. Different from previous researchers’ work, after training the known linear system offline, an initial admissible control input is no longer needed. A decentralized observer is injected on each subsystem that utilizes the local states to estimate other subsystem states. The algorithm is introduced as follows: first of all, a discrete time cost function is formed and generalized, secondly, a centralized optimal controller is estimated and evaluated with a neural network, which approximates the cost function. After the training process, the controller is putting online with a decentralized observer.

6.1 Successive Approximation of Generalized HJB equation

Consider an affine discrete-time linear system

\[ x_{k+1} = Ax_k + Bu_k \]  \hspace{1cm} (67)

where \( x_k \) is a state column vector at step \( k \). Assume there exists an optimal controller \( u_k^* \) that asymptotically stabilized the system. Then in a finite horizon cost function can then be described as:

\[ J_k^* = \sum_{j=k}^{T} (x_k + u_k^{*T}Ru_k^*) \]  \hspace{1cm} (68)

Thus, define a Hamiltonian function in discrete-time version as:

\[ H_k(x_k, u_k) = J_{k+1} - J_k + Q(x_k) + u_k^T R u_k \]  \hspace{1cm} (69)

By using the stationary condition \( \frac{\partial J_k}{\partial u_k} = 0 \), the optimal controller is developed as:

\[ u_k^* = -\frac{1}{2} R^{-1} g_k + \frac{\partial J_{k+1}^*}{\partial x_{k+1}} \]  \hspace{1cm} (70)

Because of the complexity for solving partial derivative and the unknown cost function, a Taylor series expansion is deployed to the cost function in second order as
$\Delta j_k = J_{k+1} - J_k \approx \nabla J_k^T (x_{k+1} - x_k) + \frac{1}{2} * (x_{k+1} - x_k)^T \nabla^2 J_k (x_{k+1} - x_k)$ (71)

The generalized Hamiltonian equation can then be derived as:

$$H_k(x_k, J_k, u_k) = \nabla J_k^T (Ax_k + Bu_k - x_k) + \frac{1}{2} * (Ax_k + Bu_k - x_k)^T \nabla^2 J_k (Ax_k + Bu_k - x_k) + Q(x_k) + u_k^T R u_k$$ (72)

With the same stationary equality condition, a generalized optimal control input can be expressed as:

$$u_k^* = -[B^T \nabla^2 J_k B + 2 R]^{-1} B^T [\nabla J_k^T + \nabla^2 J_k (Ax_k - x_k)]$$ (73)

Let $u_k^{(0)}$ be an initial admissible control input in a compact set. The successive approximation starts with initial controller and update the cost function and evaluate the controller until the requirements of the cost is satisfied. The convergence proof is introduced in Theorem 2.

Theorem 2 [26]: Assume there is an initial admissible controller $u_k^{(i)}$, then for the system

$$x_{k+1} = Ax_k + Bu_k, J_k^i \geq J_k^{i+1} \geq J_k^* \text{ as } i \to \infty.$$ 

The statement of Theorem 2 indicates the convergence of the successive approximation and also relax the computation cost of solving the nonlinear partial derivative equations. And also, it shows the effects of higher order Taylor series can be ignored.

6.2 Cost Function Approximation Using Least Square Method

From the equations, above, we go through a process of transferring a non-solvable nonlinear partial derivative function to a linear solvable equation, and further cost function...
approximation is adopted. The overall cost is approximate by a neural network with augmented weight vector $W_L$ and activation vector $\overline{\sigma_l}(x)$.

$$J(x) = \sum_{i=1}^{L} w_i \sigma_i(x) = W_L^T \overline{\sigma_l}(x)$$  \hspace{1cm} (74)

Substitute the approximated cost to the generalized Hamiltonian equation, then comes with the Hamiltonian residual error, defined as:

$$e_L = \frac{\partial \overline{\sigma_l}^T(x_k)}{\partial x_k} W_L (Ax_k + Bu_k - x_k)$$

$$+ \frac{1}{2} * (Ax_k + Bu_k - x_k)^T W_L^T \frac{\partial \overline{\sigma_l}^T(x_k)}{\partial x_k} (Ax_k + Bu_k - x_k) + Q(x_k) + u_k^T R u_k$$  \hspace{1cm} (75)

To minimize the error, least square method is introduced as:

$$\langle \frac{\partial e_L(x)}{\partial W_L}, e_L(x) \rangle = 0$$  \hspace{1cm} (76)

The weights are evaluated by projecting the error to the partial derivative of error with respect to the weights itself. After reformat the equation, the weights are rewritten as:

$$W_L = -(X^T X)^{-1} XY$$  \hspace{1cm} (77)

$$X = [\nabla \overline{\sigma_l} \Delta x_1 + \frac{1}{2} \Delta x_1^T \nabla^2 \overline{\sigma_l} \Delta x_1, \nabla \overline{\sigma_l} \Delta x_p + \frac{1}{2} \Delta x_p^T \nabla^2 \overline{\sigma_l} \Delta x_p]^T$$  \hspace{1cm} (78)

$$Y = [x_1(k)^T Q x_1(k) + u^{(i)^T} R u^{(i)}, \ldots, x_p(k)^T Q x_p(k) + u^{(i)^T} R u^{(i)}]^T$$  \hspace{1cm} (79)

To calculate the inner product of this projection equation and reduce the computation burden of solving integration, we utilized an approximation of points summation inside the integration range. In order to guarantee a solution, $X^T X$ has to be full rank with rank size small or equal to the number of points in the approximation mesh.
6.3 Decentralized Observer Design

In this subsection, an adaptive observer for the linear CT system eq. (80) with known matrix $A$ is introduced to estimate the states, which are required during implementation of the decentralized optimal controller.

Considering the subsystem $i$

$$\dot{x}_i = A_i x_i + B_i u_i + \Delta(x_{-i})$$

$$y_i = C x_i$$

The observer for the system above is given by

$$\dot{\hat{x}} = A \hat{x}' + B \hat{u} + L_i (y_i - C \hat{x}_i)$$

where $\hat{x}$ contains all states estimation and $y_i - C \hat{x}_i$ denotes the output observation error. $\hat{x}'$ is composed by the local states $x_i$ and the estimated states of other subsystems.

On the other hand, $L_i$ is chosen so that $A - L_i C$ is Hurwitz. Note that, the observer error system will guarantee the convergence as time goes to infinity. However, different choice of $L_i$ may highly affect the performance of the observer.
CHAPTER 7. SIMULATION RESULTS

In order to demonstrate the performance of the proposed model, the IEEE 30-bus test system is selected and tested as shown in Figure 9. The network includes five distributed generators (DGs) and five photovoltaics (PVs) as it can see in Figure 1.

Figure 4 14-Bus system offline decentralized optimal controller training trajectory (SGs)

7.1 Offline Decentralized Optimal Controller Training Strategy

Figure 5 30-Bus system offline decentralized optimal controller training trajectory (SGs)
In this section, three cases are individually investigated to test the best performance of the proposed controller with different scales, including IEEE 14-bus power system, IEEE 30-bus system, IEEE 118-bus system. There are 5, 10 and 20 power sources in each case respectively. The training performance with our designed offline controller are shown in Figure 4-Figure 6. As the algorithm that is explained in Chapter 3, the training process is started with an initial stabilizing controller and stops when the difference of consecutive costs is reaching a small enough limit. And the training step, which indicates also in Chapter 3, is randomly generated in every iteration, thus causes the different number of iterations in three cases. The proposed algorithm trained the controller to minimize the iterative cost from 2.4047 to 2.3891, 6.9964 to 6.27, 17.6342 to 17.4760, in three cases respectively, which is 0.65%, 10.38%, 0.9% improvements. The performance largely depends on the initial control gain selection and the minimizing requirements. But also, one may notice that all generation sources are traditional synchronous generators and does not have much room to improve.

![Figure 6 118-Bus system offline decentralized optimal controller training trajectory (SGs)](image)

The Figure 7 and Figure 8 show the performance of the algorithm when dealing with
combination of power sources, which has 5 SGs and 5 PVs in 30-bus system and 10 SGs and 10 PVs in 118-bus system. The minimization process is better performed with renewable energy penetration clearly. In 30-bus system, the original cost is 3.7553, after 41.45% reduced, ends up with 2.1989. In 118-bus system, 6.2% improvements achieved from 62.1503 to 58.2999.

Moreover, it is worthy to mention that even the initial condition is generated randomly because of the different system formulation, the iterative cost is relatively smaller with only traditional generation sources, that is another reason the proposed algorithm can achieve better goal with renewable energy penetration.

![Image](image.jpg)

Figure 7 30-Bus offline decentralized optimal controller training trajectory (SGs and PVs)

7.2 Offline Decentralized Optimal Controller vs Online Optimal Controller

1. Scenario1: Grid-connected mode

In each case, different comparison between the initial controller, offline decentralized optimal controller and the online optimal controller is shown. The simulation description and details are presented in Table 2.
Figure 8 118-Bus offline decentralized optimal controller training trajectory (SGs and PVs)

Table 2 Simulation Description for Case 1.

SIMULATION DESCRIPTION

<table>
<thead>
<tr>
<th>GENERATION SOURCE TYPE</th>
<th>5 PVs and 5 DGs</th>
</tr>
</thead>
<tbody>
<tr>
<td>FAULTED BUS</td>
<td>Bus16 and Bus 6</td>
</tr>
<tr>
<td>FAULT VALUE</td>
<td>5j and 10j (reactance)</td>
</tr>
<tr>
<td>FAULT BEGINNING TIME</td>
<td>0.05s</td>
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<tr>
<td>FAULT DURATION</td>
<td>0.05s</td>
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<td>SIMULATION DURATION</td>
<td>3s</td>
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</table>

1) Case 1: Multi-fault comparison

In this scenario, the microgrid operates on the grid connected mode (and is connected from Bus 1), that means it can exchange power with the utility. Furthermore, two faults are injected on different locations (Buses 16 and 6), different magnitude (5j and 10j respectively), and fault duration is 0.05 second. Figure 10 represents the frequency errors within all PVs for both online and offline controllers.
Based on this figure, the error frequency in all PVs approach to zero after almost 1.5 sec. Also, in all PVs, the performance of the offline controller is faster than the online one and the offline controller can control the frequency of the PVs in less than 0.05 after fault appeared. It is worth to mention that the location of the PVs play a very significant role in the performance of the controllers. For instance, frequency control of PV5, which is located close to the decentralized controllers, is stabilized faster than PVs 1 and 2 that are far from distributed generators, more location simulations will be shown later.
Figure 11 Angle errors of PVs in grid connected mode

The PVs angle errors are demonstrated in Figure 11. As shown, all the angle errors converge to zero even after multiple large reactance faults are injected. Same as the frequency errors, the performance of the offline controller is more efficient than online one with less disturbances when faults occur. Furthermore, similar to frequency error, angle error of PV 4 and 5, which are close to the local controllers, approaches to the origin faster than others (see Figure 10 and Figure 11).

Figure 11 and Figure 12 represent the frequency and angle errors of DGs within the microgrid in the grid-connected mode respectively. Based on these figures, in both online and offline controllers, the frequency and angle control of DGs are faster than PVs. The traditional synchronous generator is more adaptive to the dynamic change of the system, as well as the proposed controller that only utilizes the local states.
Figure 12 Frequency errors of DGs in grid connected mode

2) Case 2: Source type portfolio analysis

Considering stability and possible isolated operation of MG, the volume of the penetration and the effect of different volume the renewable energy penetration cause comes to our notice. Within 10 generation source, different combination of traditional synchronous generators and renewable energy sources is investigated. From Figure 14, it is obviously seen that the cumulative cost of grid connected mode is the lowest when the number of PVs and the number of SGs is 9:1 with offline optimal decentralized controller. However, in reality, the renewable energy source is unpredicted and unreliable and may jeopardize the stability of the entire system. Thus, a large number of PVs in a microgrid is unacceptable and all the simulations are based on balanced power source with 5 PVs and 5 SGs.
Two variables are selected in this simulation analysis, the 6th state and the 19th state. One may notice that in the PV source type the states are considered as the inverter angle and the variable $\lambda$ that controls the angle, which states in Chapter 2. From Figure 15 and Figure 16, first of all, all the combinations of different numbers of source type are asymptotically stable, which proves the
effectiveness of the proposed offline decentralized optimal controller. Secondly, an optimized portfolio of different source type has been taken into account.

Figure 15 Angle error comparison on different source type analysis of 6th state

Figure 16 Frequency error comparison on different source type analysis of 19th state

It is clear that in both all traditional sources and all renewable energy sources situation, different variables performs relatively worse in oscillation perspective. And also, as we state above,
the ratio of renewable energy injection is critical in reality even we can minimize the iterative cost dramatically compare to all traditional sources situation in simulation.

3) Case 3: Fault analysis with different source combination

In this simulation, fault analysis is investigated under different source combination. Three types of combination are concerned, all traditional sources, all renewable energy sources, and half of both. It is well known that renewable energy can bring benefits to the system and our lives, energy wise and economic wise. However, it can bring instability too. Thus, it is worthwhile to analysis the performance under fault condition when multiple renewable energy injected. From Figure 17, all traditional sources and all renewable energy sources situations approach to the origin faster, because of the unity of the system formulation. However, from Figure 18, significant value changes when facing fault injection and clearance in generation sources is mixed. On the other hand, the system reaches stable point earlier than each other cases. That proves the adaptiveness of the proposing optimal controller, especially in contingency cases. Furthermore, all traditional case has almost no affects when faults happened and removed compared to all renewable energy case, which because of the nature of the unreliable renewable energy.

![Figure 17 Angle error comparison with different combination of sources](image-url)
Figure 18 Frequency error comparison with different combination of sources

4) Case 4: Fault duration analysis

In fault duration analysis, the same faults are injected but with a longer duration. This analysis is not only testing the designed optimal controller to validate, but to test the system durability to the faults. In most cases, contingencies are considered uncontrollable and unpredictable. Therefore, it is risky and dangerous even for 0.05 second. In a longer duration test, the faults last for 0.15s. From Figure 19 and Figure 20, even the faults do not affect the performance angle wise, the second state of each system increases greatly after 0.1s. This results show the system durability. If the maximum of the collapse point is reached during the fault injection, the entire microgrid breaks down.

And also, there is no control over the period of fault. That means the system is unpredicted and operates after the sudden change. In reality, a 0.5s ground fault has a significant impact on the overall system.
5) Case 5: Fault location analysis

It is critical to analysis fault location in decentralized power system. The system controller that is far away from the fault may delay the control processing because of the distance, the fault value and the fault duration. Especially in decentralized power system, the fault will be reflected in system as an interconnected term in most of the cases (if the fault is not happened on the generation bus). Thus, how the system behaves under different noise with respect to the system we train the controller is important. From Figure 21 and Figure 22, the system suffers with further
faults, which in our simulation are on bus 29 and bus 30 (compare to bus 16 and bus 6 in Figure 9), has a greater oscillation in a tolerable range. However, what needs to mention is the fault may have a greater impact on the system dynamics when it is further than the system we simulated and it should be kept in a practical range.

Figure 21 Angle error comparison in fault location analysis

Figure 22 Frequency error comparison in fault location analysis

6) Case 6: Islanded mode

In this case, the microgrid is disconnected from the main grid, that means it is unable to
exchange power with the utility. Indeed, the DGs within the microgrid are responsible for satisfying the microgrid total load demand. Hence, DGs are more prone to be unstable in comparison with the previous scenario. Moreover, two faults are injected simultaneously as same as the previous scenario to compare the effectiveness of the proposed controllers in islanded mode.

Figure 23 Frequency errors of PVs in islanded mode

Figure 23 presents the frequency errors of all PVs within the islanded microgrid. According to this figure, all errors are gone to zero point less than 0.05 second as designed after faults occur. Figure 24 demonstrates the angle error of PVs for the islanded mode.

Figure 25 and Figure 26 show the performance and effectiveness of the proposed model in controlling the frequency and angle of DGs in the islanded mode. According to these figures, both offline and online controllers are stabilized the network less than 0.01 second after faults are injected.
Figure 24 Angle errors of PVs in islanded mode

It surprises the author that the online decentralized controller can reach the target (origin in this case) faster than that the offline optimal controller. Since the online controller has no information of the system nor the interconnected term. On the other hand, the offline control feedback gain is obtained by training the system with full information of the system. Even during the fault, the online controller faces some challenges. It is reasonable it adapts itself to a new system model and bounce back to the origin. The overall performance of both controller indicates their ability in different fields.

Figure 25 Frequency errors of DGs in islanded mode
In addition, different combination of sources is considered as Figure 27, Figure 28 and Figure 29, unlike the grid connected mode, the system has a different portfolio because of different power flow throughout the system and the isolation to the main grid. The system with 6:4 PVs and SGs ratio is practically lowest cost and is stabilized faster with the proposed decentralized optimal controller.

Figure 26 Angle errors of DGs in islanded mode

Figure 27 Optimal cost for different combination of source in islanded mode.
In this section, the observer-based decentralized optimal controller for discrete-time system is simulated. IEEE 5-bus 2-generator system is tested considering each SG as a controller. Mesh is 4 dimension with range of 0.5. The inner product will be calculated as a summation of all the (10) points in a selected mesh. Convergence bound is set as 0.005. The NN is using 4th order polynomial, which is \((x_1 + x_2 + \cdots + x_n)^2 + (x_1 + x_2 + \cdots + x_n)^4\) as activation function. The initial admissible controller is needed to guarantee the convergence for the centralized controller.

However, in observer-based controller, it is not necessary to have the acknowledge of the initial stabilizing controller, which releases a hard constraint in majority of the research. In this simulation, the observer-based controller is initialized with the same controller as the centralized case for a better comparison, other states of the observer is initialized as zero. The number of points in the mesh has to be greater than the number of neurons to guarantee the full rank of X. The training process is shown in Figure 30.

Figure 28 Angle error comparison on different source type analysis of 4th state in island mode.

**7.3 Offline Observer-based Decentralized Optimal Controller**
Figure 29 Frequency error comparison on different source type analysis of 16th state.

Figure 30 Cost trajectory in Offline optimal controller training

As shown in the figure, the cost approaches to a relatively small range and remain unchanged at 27.1166, just after 9 iterations, considering 10000 mesh points approximation. The computation burden is huge with higher order polynomial. Thus, it is really important to select the mesh, choose the size, and evaluate the number of points.
Figure 31 Cumulative Cost of Observed-based Decentralized Optimal Controller.

The cumulative cost indicates the overall performance of the controller. In Figure 31, even the centralized controller is stabilizing faster than the observer-based decentralized one, the latter’s performance is better on the contrary. Again, even the improvement with respect to cumulative cost is not huge, one may notice this is the comparison between a decentralized controller and a centralized controller. The latter obtains the entire information of the system, while the other can only discover its own states.

Figure 32 Iterative Cost of Observed-based Decentralized Optimal Controller.
It is interesting to see from Figure 32 and Figure 33 that, with a lower oscillation of states, the iterative cost is much higher at the beginning of the injection. Considering the observer-based system has two observers in this case and are using the local states to observer other system states to formulate the optimal controller. It is clear that the centralized optimal controller will access more information as knowing the interconnection terms, thus has less vibration during the stabilizing process. However, the observer-based controller can achieve lower iterative cost and cumulative cost is impressing. From Figure 34, the answer reveals as the control input of the proposed controller takes lower effort to manipulate the states, so forth lower the cost, which is a quadratic summation of control input and the states.

The results show the contribution of our proposed controller, which is not only stabilizing the system with a lower cost, but control the system with less payoff.
7.4 Discrete-time Online Decentralized Optimal Controller

In this part of the simulation, two cases are considered: IEEE bus-30 system is considered as a small-scale system which has 10 generation sources. IEEE-118 bus system that represents a large-scale system that has 20 generation sources. Note that, each generation source acts as a decentralized controller and the size of the system is layered by the number of the individual controller, not the number of the buses, which in either case will be considered as a small system. The neural network training process is the same for both cases. For both NNs, activation is chosen as \( \rho(\cdot) = \frac{(1 + e^{-x})}{(1 - e^{-x})} \). 10 neurons are set for both outer layer and hidden layer network. The number of neurons and the accuracy of the approximation should be taken into account and selected by the experiences in order to balance the tradeoff between the computation cost and the time consuming. Hidden layer weights are selected randomly and kept constant throughout the simulation. States target at original. Sample time is chosen as 0.0001s.

1) Case 1: IEEE-bus 30
In this case, a system with 10 decentralized controllers is simulated. From Figure 35, the system is stabilized in just 0.0004 second as our sample time is 0.0001 second. For our simulation model, which is IEEE 30-Bus system, with 10 generators, the formulated dynamic system in classical model has 20 states. In other words, there are 18 interconnected terms within each subsystem that need the controller to estimate and cancel out, which is enormous when dealing with decentralized control system. The proposed optimal controller is proved to be very effective on discrete-time linear system considering fast response and less knowledge acquired.

![Iterative Cost for 30-Bus System of Online Decentralized Controller](image)

Figure 35 Iterative Cost for 30-Bus System of Online Decentralized Controller

From Figure 36, multiple states are selected to show the convergence of the proposing controller. With an initial admissible controller, the system is not only stabilized, but performs in a small range around the initial point, which largely because of the initial point is really close to the equilibrium in majority of the real-world application, especially for the vulnerable equipment, such as generators, flight navigation system, etc.

1) Case 2: IEEE-bus 118
In this scenario, we consider a relatively larger-scale system to show the adaptiveness of the proposed optimal controller. The difficulty of a large-scale system for decentralized optimal control has always been the interconnected terms. The unknown noise brings to the subsystem can usually cause the instability and collapse of the system. However, in our simulation, from Figure 37 to Figure 39, the system is well performed with 20 subsystem and decentralized controller.
For large-scale system as 118-bus power system, 20 subsystems are highly coupled. There are two states for each subsystem, thus, 38 interconnected terms affect each subsystem. The results shown in this subsection is really appealing because of the great performance when dealing with many interconnected terms and that gives the designer more flexibility in real application world.
CHAPTER 8. CONCLUSION

This dissertation proposes several decentralized optimal controllers. First, a new decentralized offline and online control framework for microgird operation in both grid-connected and islanded mode. The proposed networked is made up with distributed energy resources along with high penetration of PVs that are equipped with UPFC. The proposed controllers are tested under the fault condition in the network. Indeed, two simultaneously faults with different location and magnitude occur during the operation. According to the simulation results, the control action of the offline controller is quicker than the online one which is logical due to the dynamic of the system that is known. However, in the online method the dynamic of system is completely unknown. The performance of both online and offline controllers is compared with the initial controller to demonstrate the effectiveness and high performance of the proposed method. Second, an online optimal controller is developed and simulated on both small scale and large scale power systems. The adaptiveness is proven and the convergence is inspiring. The controller can respond to reach the equilibrium point in seconds with a minimized cumulative cost. Lastly, an observer-based offline decentralized optimal controller is innovated. After training the system offline by using successive approximation and least square method, the optimal controller is put online with isolated observer placed at each subsystem. The performance shows a lower cost compare to the centralized controller. The proposed controller can manipulate the system dynamics within a small range which is normally rare in real applications.
REFERENCES


VITA

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