The Interval Dissonance Rate in Chopin’s Études Op. 10, Nos. 1-4: Dissecting Arpeggiation, Chromaticism, and Linear Progressions

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THE INTERVAL DISSONANCE RATE IN CHOPIN’S ÉTUDES
OP. 10, NOS. 1-4: DISSECTING ARPEGGIATION,
CHROMATICISM, AND LINEAR PROGRESSIONS

A Dissertation
Submitted to the Graduate Faculty of the
Louisiana State University and
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in

School of Music

by
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PREFACE

This dissertation has been submitted to fulfill the graduation requirements for the degree of Doctor of Philosophy at the Louisiana State University. This study was conducted under the supervision of Professor Robert Peck in the Music Theory Area. I began my initial research for this project in the summer of 2016. Parts of my research have been presented in multiple conferences, which include 2016 International Chopinological Conference, 2016 and 2017 Bridges Conferences, 2017 Modus-Modi-Modality International Musicological Conference, and 2018 International Conference on New Music Concepts.
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ABSTRACT

Chopin’s twenty-seven piano études display the composer’s poetic musical language, uniting keyboard techniques, virtuosity, and artistic imagery, while preserving Romantic lyricism and songfulness. Each of these studies is unique in its set of pianistic challenges, compositional processes, and difficulty level. Schenkerian analysis provides an interpretation of relationships between the notes that constitute the harmony and the melody. This type of analysis allows one to understand the theoretical aspects that are necessary to play Chopin’s études. The Schenkerian theories can be used to amalgamate pianism and performance with harmony and analysis. Furthermore, the Schenkerian understanding of these études provides an analytical dissection of the compositions that can help explain certain pianistic techniques by the notions of musical elaborations, which include arpeggiation, as seen in Op. 10 No. 1, chromaticism, as seen in Op. 10 No. 2, and linear progressions, as seen in Op. 10 Nos. 3 and 4. Throughout these four compositions, Chopin employs different levels of harmonic dissonance to create tension and to move between any two harmonic structures.

This study traces the amount of dissonance in each of the études, focusing on the intervalllic makeup of Chopin’s harmonies. The notion of harmonic dissonance and consonance in music is established from two or more simultaneously played notes. There are multiple approaches into dissecting this concept, some of which are acoustical, mathematical, and psychological. This research uses the Interval Dissonance Rate (IDR) – a tool that integrates musical and mathematical analyses in non-monophonic Western music, using modified
interval-class vectors (modicv) and the frequency of recurrent pitches to determine the percentage of dissonant and consonant verticalities. The connection between pianism, Schenkerian analysis, and computation of dissonance is a vital aspect to consider when understanding these études on both artistic and analytical levels.
CHAPTER 1. INTRODUCTION

Frédéric Chopin’s works are among the most widely known compositions in the piano literature of the Romantic period. Chopin is responsible for the output of over two-hundred pieces of various forms and genres, including four ballades, two piano concerti, twenty-seven piano études, four scherzi, three sonatas, as well as multiple mazurkas, nocturnes, polonaises, and waltzes. While educated according to the music traditions of Mozart, Haydn, and Beethoven, Chopin falls into a unique category of composers who were equally virtuoso pianists, which is why his music is especially influential to other similar artists, such as Louis Gottschalk, Henri Herz, Franz Liszt, Anton Rubinstein, Alexander Scriabin, and Sergei Rachmaninoff. Chopin’s life bifurcates between Poland and France, with 1830 being the year of the composer’s immigration, due to the November Uprising. Despite the many changes occurring in his life, Chopin’s style is established early in his career and does not undergo significant stylistic evolution, even though some of his later works are more “ambitious” and “audacious.” After Chopin’s

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This is unlike Haydn, Beethoven, and Liszt, who went through major stylistic changes in their compositional careers.
immigration to France, the composer continued to produce music inclined by the
Polish character and Polish national traits.⁵ According to Poniatowska, Chopin is
a “musical poet,” who is “distinct amongst all the great masters of the keyboard
with his artistry being indeterminable in its beauty.”⁶ While this can be said
regarding multiple genres of works in Chopin’s compositional output, nowhere is
his poetics more evident than in his twenty-seven piano études.⁷

The purpose of this dissertation is to integrate the artistic and the technical
aspects of pianism and music analysis, based on Schenkerian theories and
theories of consonance and dissonance. For this project, I will be working with
Chopin’s Études Op. 10, Nos. 1-4, using the Interval Dissonance Rate (IDR), an
analytical system that allows one to measure the amount of harmonic dissonance
in a piece of music.⁸ I will situate my analytical tool in the context of what is already
known about the four études from analytical standpoints.⁹ Each étude carries its
own set of unique pianistic challenges that a performer must confront. Schenkerian
analysis is a theory that can define and represent these components with notated
elaborations. This dissertation will focus on the integration of Schenkerian studies,
its relation to the pianistic techniques that Chopin presents in each composition,
and how such unity of performance and analysis influences the harmonic
dissonance in these works. The chromatic harmony employed by Chopin in these

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⁶ Irena Poniatowska, *Fryderyk Chopin: A Poet of Sound*.
⁷ These include twelve études in Op. 10, twelve études in Op. 25, and *Trois Nouvelles Études*.
⁹ These standpoints include Schenkerian analysis, Chopin’s chromatic harmony, and pianism.
piano studies likewise impacts the level of harmonic dissonance. Furthermore, this research will look at methods, in which Chopin utilizes vertical tension, based on a Schenkerian reading of each work.

There are numerous composers in music history who showcased their compositional mastery in the creation of études – studies that are meant to perfect and exercise one or more sets of techniques or musical skills. Nevertheless, the elements of artistry and lyricism likewise play a crucial role. According to Rosen, Chopin is the first composer to give piano études a “complete artistic form,” in which the notions of “musical substance and technical difficulty coincide.”\(^\text{10}\) Chopin is one of the most significant composers of études in the Romantic era, providing innovative pianistic techniques and allowing the performer to enhance his or her musical abilities and present his or her own interpretation.

In his compositions, Chopin aligns the technical and the lyrical components of piano performance practice. This is especially evident in his piano études – exemplary works for analysis when it comes to combining the musical aspects of technique and artistry. Since the composer employs different levels of harmonic dissonance throughout his études to move between two or more harmonic structures, as evident from the analysis of Chopin’s music, his harmonic goals are not always certain.\(^\text{11}\) The Schenkerian approach is an example of theory that can be used to analyze such passages and allows one to trace harmony without

\(^{10}\) Charles Rosen, *The Romantic Generation* (Cambridge: Harvard University Press, 1998), 363. Chopin is one of the most significant composers of études in the Romantic era, providing innovative pianistic techniques and allowing the performer to enhance his or her musical abilities and present his or her own interpretation. Other notable composers of études are Charles-Valentin Alkan, Claude Debussy, Leopold Godowsky, Franz Liszt, Sergei Rachmaninoff, and Alexander Scriabin.

\(^{11}\) As per Schoenberg, a harmony that has a musical goal and either accepts or rejects the tonic is a *chord progression*, while a harmony that is aimless is a *chord succession*.
emphasis on Roman numerals, as seen in *Der freie Satz, Das Meisterwerk in der Musik*, and *Five Graphic Music Analyses*.\(^\text{12}\)

A Schenkerian reading can benefit from a quantifying system and the synthesis of both can be helpful in both performance and analysis.\(^\text{13}\) From the perspective of harmonic dissonance, an analyst or a performer can trace the level of vertical tension among multiple musical passages. When looking at the pianistic challenges presented by Chopin in his études, it is evident that the first four pieces of Op. 10 introduce the techniques of playing arpeggios, chromatic scales, and linear patterns in their purest form.\(^\text{14}\) Despite these techniques' being apparent from the pianist's point of view, the Schenkerian approach is the most effective theory to explain the technical aspects that Chopin presents from an analytical perspective.

The first étude, nicknamed *Waterfall*, is based on arpeggiation. This work is composed in the key of C major and contains a strong rhythmic balance with the bright sounds of the main arpeggiated theme that cover three or four octaves on every run. Each ascending arpeggio is followed by a descending arpeggio, where the left hand is generating the bass support. The fluency of the work is derived through maintaining a steady flow of harmonic tension at its dissonant passages. The difficulty of this étude lies in the execution of arpeggiated patterns with the

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\(^\text{12}\) *Analysis of Tonal Music: A Schenkerian Approach* by Cadwallader & Gagné.

*Introduction to Schenkerian Analysis: Instructor’s Manual* by Forte & Gilbert.

*Unfoldings: Essays in Schenkerian Theory and Analysis* by Schachter.

These are some of the notable sources on Schenkerian theory. For additional sources on Schenkerian approach to understanding Chopin, refer to the Bibliography section.

\(^\text{13}\) It is important to note that the theories of Schenker are purely analytical and not quantifiable. On the contrary, many theories of consonance and dissonance employ a computing metric.

right hand that cover intervals, as large as an eleventh, where an alteration of a single note can modify the harmony of the composition.¹⁵

Furthermore, this étude requires a particular fingering pattern, a strong wrist control in the right hand, and accurate extensions of the right elbow.¹⁶ The fourth finger is considered as the weakest finger on a pianist's hand according to Chopin's teaching philosophy, which likewise reflects his compositional approach to writing for the right hand in the opening étude and therefore, the emphasis of the study on scales and arpeggios should be directed to the fourth finger.¹⁷ Such an approach to playing arpeggios is the reason why Op. 10 No. 1 is considered as one of the most difficult pieces of the Op. 10 set.¹⁸ In addition, Rosen states that it is possible to alter Chopin’s fingering pattern to 1-2-4-5, which is particularly advantageous for the pianists with “moderate sized hands,” yet such a tactic generates issues in phrasing.¹⁹ An excerpt from the opening étude, found in mm. 59-64, can be seen in Figure 1.1.²⁰

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¹⁵ An example can be seen in Figure 1.1. Throughout m. 59, the combination of pitches, A, C, and F generate an F major harmony in first inversion. In the first half of m. 60, Chopin preserves notes A and C, yet introduces notes E and F in lieu of pitch F, creating an F♯ seventh chord in first inversion.


¹⁸ In Op. 10 No. 1, this can be seen in the predominance of 1-2-4-5 fingering pattern on most of the ascending and descending arpeggios.


The second étude, nicknamed *Chromatique*, is based on chromaticism. This étude is in the key of A minor, where Chopin utilizes ascending and descending chromatic scales in the right hand, played by the third, fourth, and fifth fingers, as the main voice varies among multiple triadic chordal structures. This approach allows Chopin to generate dissonance between the harmonies of the work, created by the left hand and by the first and the second fingers of the right hand. An example of this technique can be seen in the first phrase of the étude in Figure 1.2.²¹

Like Op. 10 No. 1, a certain fingering pattern must be used for proper execution throughout majority of the work. Typically, a chromatic scale is played with the use of the first, second, and third fingers in either right or left hands.²² In this étude, a pianist must employ the third, fourth, and fifth fingers for the chromatic

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²¹ Ibid., 7.
²² The chromatic scale between two Cs can be played using [1-3-1-3-1-2-3-1-3-1-3-1-2] in the right hand.
The chromatic scale between two Cs can be played using [1-3-1-3-2-1-3-1-3-1-3-2-1] in the left hand.
ascents and descents, as shown in Figure 1.2. Furthermore, the technical difficulty of this work lies in the generation of clear melody with very little use of sustaining pedal. Because of constant chromaticism in the right hand, each phrase generates a sense of a musical curvature, made of ascending and descending chromatic line, and Chopin makes sure that each arc interacts with the harmony provided by the left hand and right hand’s inner voices in a unique way.

Figure 1.2: Chopin’s Étude Op. 10 No. 2, mm. 1-4.

The third étude, nicknamed Tristesse, is one of the most well-known and most frequently analyzed studies by Chopin. It is based on linear patterns, which

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23 This pianistic approach creates difficulties and complexities for pianists in achieving the necessary speed and clarity.  
can be seen in each of the three sections and which occur in both left and right hands. While Étude No. 1 focuses on large arpeggiated stretches across multiple octaves and Étude No. 2 concentrates on chromaticism with multiple subordinary voices, the difficulty of Étude No. 3 lies in the musical contrast of hidden arpeggiation and chromaticism. This likewise defines the pianistic approach to performing this work.\textsuperscript{25} There are three unique characteristics that separate this étude from the opening two studies of Op. 10.

First, the slow and tranquil tempo of the A-section in the third étude creates disparity and contrast when compared to the musical characters of Op. 10 Nos. 1 and 2. Furthermore, Chopin generates a distinction between lyricism (as seen in the A-section of the third étude) and virtuosity (as seen in the étude’s B-section). Second, \textit{Tristesse} combines the pianistic concepts of polyphonic voice leading, \textit{cantabile} style, and constant legato, all of which integrate to create musical technical challenges.\textsuperscript{26} Third, there is an evident dissimilarity in the opening sections of the third étude, which is seen through tempo, musical character, harmony, and harmonic dissonance. A passage from Op. 10 No. 3 (mm. 36-44) can be seen in Figure 1.3.\textsuperscript{27}

\begin{footnotesize}
\begin{itemize}
\item[\textsuperscript{26}] Frederick Niecks, \textit{Frederick Chopin, as a Man and Musician} (London: Novello Inc., 1890), 253.
\item[\textsuperscript{27}] Frédéric Chopin, \textit{Douze Grandes Études} (New York: Schirmer Inc., 1934), 10.
\end{itemize}
\end{footnotesize}
The fourth étude, nicknamed *Torrent*, is based on linear patterns, in which a single melody must be executed at a quick tempo against an underlying harmony.\(^{28}\) This is also the first étude, in which Chopin deliberately alternates the main melody between the right and the left hands. Unlike *Tristesse*, this étude is played in *Presto* tempo and contains dueling linear patterns between the upper and the lower voices, which altogether generate perpetuum mobile motion. Like the third étude, but unlike the first two études, Chopin does not designate directions for these elaborations and therefore, no patterns to understanding the motion of the musical motives exist.\(^{29}\)

This aspect can be seen in mm. 13-21, shown in Figure 1.4.\(^{30}\) The main melody can be found in the right hand of mm. 13-16, as the left hand presents

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\(^{28}\) Throughout most of Chopin’s études, speed is one of the prominent components that pianists work to excel at.  
\(^{29}\) In Op. 10 Nos. 1 and 2, the respective ascending and descending arpeggios and chromatic lines are evident and part of compositional texture. This is not the case in Op. 10 No. 3.  
linear chromatic ascents of four-note segments, separated melodically by one, two, or three semitones. What serves as accompanimental support in mm. 13-16 becomes the main melodic material in mm. 17-18 of the right hand with the left hand’s utilizing a different accompanimental structure, employing eighth notes and descending stepwise from D♯ to A♯ before ascending to E. Both primary voices in right and left hands of this passage generate a contrasting linear motion. An unanticipated change occurs once more in mm. 19-20, where the right and the left hands switch roles. Finally, m. 21 is an example of an unforeseen key change, lasting only until m. 26 before Chopin returns to C♯ minor.

Figure 1.4: Chopin’s Étude Op. 10 No. 4, mm. 13-21.
Dissonance is a prominent component of both music performance and music theory.\textsuperscript{31} The study of harmonic dissonance and consonance is derived from combinations of two or more simultaneously played pitches. The notion of dissonance has always existed in Western classical music and throughout the history of music, different composers utilized dissonance in their own unique ways. Furthermore, composers have individual motivations for the application of dissonance and in many cases, the tactics for generating multiple levels of harmonic tension can reveal substantial information regarding the specifics of the musical style.\textsuperscript{32}

In my dissertation, I will trace Chopin’s use of harmonic dissonance in each of the opening four études, focusing on the intervallic makeup of their harmonies. The IDR technique that will be used throughout this dissertation combines the cognitivist and physicalist approaches to analyzing harmonic verticalities.\textsuperscript{33} The IDR will be used to dissect the level of harmonic dissonance from the perspective of modified interval-class vector, which will be based on eight interval-classes rather than the traditional six.\textsuperscript{34} Additionally, I will analyze the dissonance relations to approaching each work’s culmination points and how such instances of harmonic tension lead towards the apogees of each étude.

There are four parts in this dissertation that follow the Introduction. In the second chapter, I will discuss the Schenkerian readings of the four études.\textsuperscript{35} I will define how the Schenkerian approach combines the notions of pianism and analysis and how the performance grows out of the background structure.\textsuperscript{36} Furthermore, I will discuss the chromatic harmony that is found within these works, how such harmony relates to harmonic dissonance of each étude, and how Schenkerian analysis allows one to understand and interpret the dissonance from analytical and interpretational perspectives. I will consider the necessity to highlight and emphasize harmonic dissonance in Romantic music, as well as its importance as part of Chopin’s compositional style and its function in each of the four works. Finally, I will engage with the literature on empirical approaches to idiomatics and its relation to the introductory four compositions of Op. 10.

In the third chapter, I will cover the main methodologies for working with dissonance and will discuss the need for incorporating interdisciplinary studies for accurate analysis.\textsuperscript{37} I will also define the need for both subjective and objective dissonance analyses and the necessity for their integration when working with Western musical repertoire.\textsuperscript{38} Finally, I will conclude the chapter by discussing the significance behind quantification methods when analyzing tension in Chopin’s études.

\textsuperscript{35} The background and foreground graphs of Op. 10 Nos. 1-4 can be found in the Appendix section of this dissertation.
\textsuperscript{37} The interdisciplinary studies covered in this research are music and mathematics, as well as psychology and mathematics. In addition, the literature review can likewise be found in the third chapter.
The fourth chapter will be dedicated to the description of my analytical tool. I will explain the main components of the IDR and how such an approach to finding harmonic dissonance functions in a piece of music. This chapter will likewise present the mathematical calculations that are necessary to determine the harmonic dissonance rate and discuss the code that was used for generating the IDR calculations.\textsuperscript{39} In addition, the fourth chapter will focus on the perception of consonance and dissonance from the perspective of meter, define various rhythmic complexities that arise when using the IDR analysis, and discuss how these complexities can be resolved.

The fifth chapter will break down the mathematical results retrieved from the proposed analysis. I will emphasize the data related to harmonic dissonance, its influence from individual interval-classes, as well as other significant statistical patterns. Furthermore, I will explain the role that dissonance plays in Chopin’s tonal ambiguities and how the harmonic dissonance influences the overall structure of the four pieces examined in this dissertation. Each piece will likewise be dissected separately and compared with the numerical data retrieved from the Optimum Consonance Measure analysis – a dissonance metric developed by Huron.\textsuperscript{40}

In the sixth chapter, I will summarize my analytical method, my findings, and discuss how the knowledge of this information can be used for analysts and performers alike. I will likewise draw upon the musical connections that exist


\textsuperscript{40} David Huron, “Interval-Class Content in Equally Tempered Pitch-Class Sets: Common Scales Exhibit Optimum Tonal Consonance,” Music Perception 11, no. 3 (Spring 1994): 293.
among the data provided by the *IDR*, the Schenkerian interpretations of each work, and the performance practice. I will, in addition, propose future research that can be conducted with this study.
CHAPTER 2. PIANISM AND SCHENKERIAN APPROACH

2.1 Pianism and Influence

Music history plays a prominent role in the study of Chopin’s études. One of the key innovations in the Romantic era is the rise of compositions written particularly for the improvement of performer's technique.\(^4\)\(^1\) Chopin is one of the significant composers to focus on this genre and his artistic output is derived from compositional languages of his predecessors. Therefore, to fully understand Chopin’s compositional strategies, the works of other composers who influenced him need to be considered.

Chopin’s Op. 10, Op. 25, and Trois Nouvelles Études form the foundation and the basis for technique and artistry in the piano literature of the Romantic tradition. In Chopin’s earliest efforts at composing in this genre, the twelve pieces of Op. 10 encompass some of the most challenging and picturesque music, filled with poetic symbolism.\(^4\)\(^2\) These works are likewise considered as some of the most popular compositions in modern pianist’s concert repertoire and are frequently performed in recitals, festivals, and competitions. The musical characterizations that Op. 10 carries led to the drastic increase of appreciation and appearance of these works in the concert halls. In addition to the main goal of each composition to create a fixed musical training based on a specific technique, each of the pieces likewise provides a musical story, which is why Chopin’s études are considered as concert works rather than candid technical exercises. Guided by the compositional

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approaches of his predecessors, Chopin marks the new era in the history of études, offering innovation to performance practice.43

The works in Op. 10 are essential for professional concert artists from the perspective of pianism. These twelve short pieces allow one to tackle most of the major aspect of pianistic techniques. Throughout the history of piano performance practice, there have been multiple excellent interpretations of these works, each artistic in own way and each emphasizing on unique musical components. These include prewar recordings by Vladimir de Pachmann (1927) and Ignacy Jan Paderewski (1928), as well as postwar recordings by Sviatoslav Richter (1967, 1971, 1976), Maurizio Pollini (1985), Boris Berezovsky (1991), and Lang Lang (2012).44

Chopin’s compositional techniques in writing in this genre can be traced to composers of the previous generation, dating back to the early 19th century. Just like Chopin, early composers of études were interested in piano pedagogy, rather than refined expertise and virtuosity. Muzio Clementi (1752-1832) is one of the first such composers who took strides in developing works for enhancing keyboard techniques. This can be seen in his Gradus ad Parnassum – a set of one-hundred études, composed between 1817 and 1826.45 Although Clementi’s études are

45 Muzio Clementi, Gradus ad Parnassum (New York: Schirmer, 1908), 7.
highly repetitive in its motivic content, they are technical and challenging, when compared to compositions of his era.

In some of the études, Clementi provides more than one possible set of fingerings in which a technical challenge can be executed. Figure 2.1 shows mm. 1-4 of Étude No. 2, where Clementi presents the performer with three different fingering variations. As seen from the example, the main melody is presented in the right hand, while the technical aspect of this étude is presented in the left hand and is based on variations of a stepwise ascending and descending motive.

![Figure 2.1: Clementi's Étude No. 2, mm. 1-4.](image)

Chopin's rejection of the standard fingering technique in piano literature is widely known and in his études, Chopin develops own fingering patterns, based on the technical challenges imposed on the pianist. Similar to the Clementi's approach to determining the fingerings allow for more consistency, elegancy, and smoothness, when moving across the keyboard. The fingering provided in Chopin's études transformed the pianist's approach to studying Romantic repertoire.

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46 Ibid., 7.
example, Chopin’s Op. 10 No. 1 is a technical study, where the composer presents passages that can be played in more than one way. This can be seen in m. 28 and m. 30, shown in Figure 2.2, where the descending melody can be performed with two sets of fingerings. The fingering patterns that pianists apply are determined by multiple factors, such as speed, voicing, technique, and rhythm. The emphasis on fingering allows for various interpretations in performance practice, which include phrasing, dynamics, articulation, and memorization.

Figure 2.2: Chopin’s Étude Op. 10 No. 1, mm. 28-30.

English pianist and composer Johann Baptist Cramer (1771-1858) writes 

Studio per il Pianoforte – a set of 84 études between 1804 and 1808. It is

48 Frederick Chopin and Albert R. Parsons (ed.), Frederick Chopin’s Works: Instructive Edition with Explanatory Remarks and Fingerings by Dr. Theodore Kullak (New York: Schirmer, 1880), 3. In m. 28, a pianist may choose either a 5-3-2-1 pattern on all four descending arpeggios or 5-2-1-3 on the first three arpeggios with the return of 5-3-2-1 on the last set to prepare for m. 29. A similar concept can be traced in m. 30, only the primary fingering choice is 5-4-2-1 because of the three black keys (E♭, B♭, and E♭) in each arpeggio.


50 Ibid., 342. Parncutt et al. argue that one’s interpretation and the quality of one’s playing depends on the set of fingering that a pianist chooses. Furthermore, Parncutt et al. developed a model to predict the best recommended fingering pattern in performance, which depends on the degree of comfortability. A modified version of the system was presented by J. Pieter Jacobs.

51 Johann Baptist Cramer, Studio per il Pianoforte (Leipzig: C. F. Peters, 1890): 1-168. The first book of 42 études was completed in 1804 and the second book of 42 études was completed between 1807 and 1808.
unknown, yet possible that Chopin was aware of Cramer’s piano studies, since there are a few resemblances in the compositional style of both composers. Figure 2.3 shows the opening two bars of Cramer’s Étude No. 1. After the initial C major chord, Cramer continues with an ascending motivic structure moving in parallel motion in both hands. In Op. 10 No. 4, Chopin begins in similar fashion, only with a motivic ascent in the right hand in m. 1 and a motivic descent in the right hand in m. 2, shown in Figure 2.4. Figure 2.5 presents the same phrase in the left hand, repeated in mm. 5-6.

Figure 2.3: Cramer’s Étude No. 1, mm. 1-2.

Figure 2.4: Chopin’s Étude Op. 10 No. 4, mm. 1-2.

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Figure 2.5: Chopin’s Étude Op. 10 No. 4, mm. 5-6.

Carl Czerny (1791-1857), one of the most prolific composers and pedagogues in the history of Western music with over a thousand completed works, is one of the most significant early composer of piano études and has influenced Chopin in his works. Having taught some of the greatest piano technicians of the Romantic era, such as Franz Liszt, Theodor Leschetizky, and Theodor Kullak, Czerny completed multiple sets of technical studies, all of which vary in form, speed, technique, and difficulty level. An influence of Czerny’s Op. 740 No. 12 can be seen in Chopin’s Op. 10 No. 12. In Czerny’s work, mm. 7-8 serve as transition to return to the reinstatement of the main theme, where the left hand contains a stepwise descent, interrupted by a static repetition of pitch A, as seen in Figure 2.6. A similar pattern exists in mm. 25-27 of Chopin’s Op. 10 No. 12, only the left hand contains a stepwise ascent from pitch D to G♭ with a static repetition of B♭-C♭-B♭ in bar 25 and B♭-C♮-B♭ in bar 26, as seen in Figure 2.7.

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The closest precursor to Chopin’s études is the compositional output of Maria Szymanowska (1789-1831), known for her *Vingt Exercises et Préludes*, completed in 1820.\(^{54}\) Szymanowska is the first Polish-born composer to write a set of études.\(^{55}\) At the time when Chopin was attending the Warsaw Conservatory, Szymanowska already established her career as a composer and a pianist.\(^{56}\) Throughout the research of Polish music, there have been a lot of parallels drawn between both composers. For instance, it is a known fact that both artists knew

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each other and were aware of the similarities in their compositional careers.\(^{57}\) Furthermore, amalgamated by similar Polish traditions and driven by the notions of Polish nationalism, some of the genres in which Szymanowska composed align with Chopin’s compositional output and include études, mazurkas, nocturnes, polonaises, preludes, and waltzes.\(^{58}\) While Chopin never discussed or mentioned Szymanowska’s effect on his music, her influence on his works is non-debatable.\(^{59}\)

Szymanowska’s harmonic models and approaches to melodic developments have served as examples for Chopin’s études.\(^{60}\) In her Étude No. 1 in F major, Szymanowska begins with an anacrusis on note C that leads to a series of sixteenth notes in the right hand of the main melody, supported by the bass structure in the left hand, as seen in Figure 2.8.\(^{61}\) Chopin begins his Étude Op. 10 No. 8 in a similar manner with an anacrusis starting on pitch C, initializing the main melody with a technical passage in the right hand, accompanied by the steady supporting line in the left hand, as seen in Figure 2.9.\(^{62}\) Furthermore, the spacing on the downbeat between both hands is identical and the musical material, found in the right hand of both études, contains an evenly spaced melody of a broken chordal figuration of F major tonic.\(^{63}\)

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\(^{57}\) Ibid., 25.
\(^{61}\) Maria Szymanowska, *Vingt Exercices et Préludes pour le Pianoforte* (Warsaw: Première Livraison, 1819), 2.
Another of Szymanowska's influences can be seen in m. 32 of Chopin's Étude Op. 10 No. 8. In this instance, Chopin's rhythmic development of the melody in the left hand is comparable to the one of Szymanowska's Étude No. 8 in E♭ Major, where both composers use an identical dotted rhythm to generate contrast against the musical material in the right hand, as shown in Figures 2.10 and 2.11.64 In addition, similarly to Szymanowska, Chopin accents the fourth and the final beat of the bass support in the left hand.

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64 Maria Szymanowska, *Vingt Exercices et Préludes pour le Pianoforte* (Warsaw: Première Livraison, 1819), 16.
The composers, mentioned above, have added substantially to this genre, yet it was Chopin, who established the étude as a concert work, rather than a mere pedagogical exercise. While unique in ternary form, the twelve études of Op. 10 are all diverse in the level of difficulty. For instance, Études No. 3 in E Major and No. 6 in G♭ Minor are among the easier studies of the set. According to Robert Schumann, both études lack technical challenges related to speed, yet present musical artistry, where a pianist needs to have excellent control of melody and accompaniment with one hand at the same time. On the contrary, Études No. 1 in C Major, No. 2 in A Minor, and No. 4 in C♯ Minor, are among some of the most difficulty studies of the set and as the level of artistry and technique among

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professional pianists increased, these études were becoming essential for performers in order to improve their technical abilities. Throughout his compositional career, Chopin created his own unique musical style and while influenced by composers of previous generations, Chopin’s études present fundamental pianistic techniques in unique and innovative ways.

Finally, it is important to note that Chopin’s études have influenced composers of subsequent generations. For instance, Chopin’s Op. 10 and Op. 25 have carved the way for Alexander Scriabin’s set of twelve Op. 8 études, composed in 1894. Scriabin’s early études are based on the Chopinesque style, form, as well as melodic and accompanimental presentations. Figures 2.12 and 2.13 show the influence of Chopin’s Op. 25 No. 8 in D♭ Major on Scriabin’s Op. 8 No. 6 in A Major. Both études are based on the intervals of major and minor sixths, seen in right and left hands. In the right hand, these intervals function as a double primary melody, while in the left hand, these intervals are used for the purpose of accompaniment. Such texture in both works stays consistent until the very last measure. This example shows that Scriabin deliberately followed Chopin’s compositional approach.

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Another Polish composer and pianist who was influenced by Chopin’s études is Leopold Godowsky (1870-1938). He is most known for his Studies on Chopin’s Études – a set of fifty-three arrangements, based on Chopin’s Opp. 10, 25, and Trois Nouvelles Études, completed between 1894 and 1914. These compositions are considered as some of the most difficult and technical piano works in the literature, where Godowsky implements extra technical challenges on top of the already existing ones from Chopin’s études. Despite the difficulty level,

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Godowsky’s études clearly present his pedagogical philosophy at the keyboard, amalgamating mechanical, technical, and musical aspects of piano playing. Figure 2.14 shows mm. 25-26 of Chopin’s Op. 10 No. 1, where the phrase begins with a broken ascending and descending arpeggiated sequence in the right hand. Figure 2.15 shows mm. 25-26 of Godowsky’s transcription of the same étude, where the technique of playing broken arpeggios must be executed in both hands in contrary motion.\(^73\)

\[\text{Figure 2.14: Chopin’s Étude Op. 10 No 1, mm. 25-26.}\]

\[\text{Figure 2.15: Godowsky’s Étude No. 1, mm. 25-26.}\]

The various aspects of pianism are significant from the perspective of Schenkerian analysis. A composition’s pianism is seen in the musical path that a composer chooses for harmony and melody. Schenkerian analysis dissects such a path and presents analytical observations and hierarchical relationships between the prominent pitches located in the score. Furthermore, Schenkerian analysis allows one to view a composition by focusing on multiple musical layers, each consisting of its own analytical details. Schenkerian analysis additionally portrays the directions and the relations between the elements provided by pianism and reflects many musical nuances from the composer’s perspective. While the performance of a composition grows out of the background structure in Schenkerian analysis, it is nevertheless possible to apply such theory to graph the performer’s interpretations. According to Schenker, the performance of a musical work is based on one’s perception and one’s musical understanding and therefore, "musical punctuations" that a pianist provides in a work’s motives, themes, and phrases can be reflected via the means of a Schenkerian representation.74

2.2. The Schenkerian Approach, Op. 10 No. 1

The principles of Schenkerian analysis explain the structure of a musical work by employing the technique of prolongation. Schenkerian prolongations consist of various elaborations that allow one to connect the work through multiple linear units. Prolongation in Schenkerian analysis explains the development and the expansion of all melodies and harmonies throughout a composition. In addition,

understanding prolongation allows one to understand the basics of artistic consistency in performance.

Previous research history of Chopin’s music shows that there exist theoretical tools that help explain the composer’s analytical intentions and the analysis of Chopin’s music can be understood through the artistic meaning that the composer embodies in each of his works. For instance, Schenkerian theory is an appropriate approach that combines pianism and harmony, and explains the technical aspects that Chopin presents in his études. This occurs from the standpoint of three Schenkerian elaborations: arpeggiation, as seen in Étude No. 1, chromaticism, as seen in Étude No. 2, and linear patterns, as seen in Études Nos. 3 and 4. Schenkerian analysis allows one to dissect Chopin’s harmonic language and trace the pianistic proficiency that is expected of the performers.

Charles Rosen combines the notions of pianism with analysis in his book *The Romantic Generation* (1998). According to Rosen, Étude Op. 10 No. 1 is a prime example of a work, in which a change of harmony is evident and is based on phrasing of the broken ascending and descending arpeggios. The musical idea that Chopin presents is realized through the physical configuration of hand and arm. Therefore, the unification of the technique with musical thought generates a unique pianistic style of this étude. While the arpeggiated patterns continue

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throughout the whole work, it is the harmonic reduction that allows one to notice the shape, the structure, and the curvature of the composition.  

Figure 2.16 shows the harmonic reduction of mm. 1-10 of the étude, encompassing the first phrase and the opening two bars of the second phrase, as seen in Czerny’s *School of Practical Composition* (1840).  

![Figure 2.16: Harmonic reduction of Chopin’s Étude Op. 10 No. 1, mm. 1-10.](image)

Graph 2.1 illustrates the alterations of harmonies in the opening cycle. Chopin initializes and finalizes this cycle in an identical manner through the means of C major tonality in mm. 1-2 and mm. 9-10. The other harmonies presented in this cycle are F major in m. 3, F# diminished seventh in m. 4, G major in m. 5, D dominant seventh in m. 6, and G dominant seventh in mm. 7-8. The tonic C

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76 *Harmonic reduction* is the initial step taken in the generation of Schenkerian analyses. The harmonic reduction (chordal configuration) leads towards the creation of the *imaginary continuo*, which leads towards the generation of the Schenkerian graphs. Allen Cadwallader and David Gagné, *Analysis of Tonal Music: A Schenkerian Approach* (Oxford: Oxford University Press, 2011), 66.


78 The first phrase of Op. 10 No. 1 is found in bars 1-8; the second phrase of the A-section is initialized in mm. 9-10. Cohn’s term *closed harmonic cycle* can be used to define mm. 1-10. According to Cohn, the harmonies of a passage form a *cycle* if such a passage incorporates an ordered set of at least four elements, where the first and the last elements are identical, while the other elements are distinct. The first cycle of Op. 10 No. 1 contains six unique elements. Richard Cohn, “Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions,” *Music Analysis* 15, no. 1 (March 1996), 15.

79 Abbreviations used: M = major; m = minor; dim = diminished; 7 = seventh chord.
major, the subdominant F major, and the dominant G major are the three harmonies in the cycle shown in their pure triadic structure, where Chopin only incorporates the root, the third, and the fifth.

![Diagram of musical harmonies]

Graph 2.1: The opening harmonic cycle from Chopin’s Étude Op. 10 No. 1, mm. 1-10.

Schenkerian analysis shows that Op. 10 No. 1 is built entirely on chords that form combinations of rhythmically defined phrases in each of the three sections. According to Czerny, Chopin makes sure that each chord receives an appropriate musical answer, essential to sustain the melodic idea of the étude.\(^8^0\)

Even though Op. 10 No. 1 is based solely on arpeggiation, this is an example of Chopin’s use of “real melody” rather than “accidental melody.”\(^8^1\)

Rosen states that the theoretical notions of the work, seen in the phrasing through the arpeggios, can be amalgamated with Chopin’s intended pianistic technique. Musical interpretation can be reflected through Schenkerian analysis,

\(^8^0\) Carl Czerny, *School of Practical Composition, Volume I* (London: Robert Cocks & Co., 1848), 93.

\(^8^1\) Ibid., 94.

*Real melody* stays the same even without any accompaniment. The real melody is still “intelligible”, “full of meaning”, and is “capable of being sung”. *Accidental melody* is the type of melody that needs an accompaniment to be recognized.
which is the approach taken by Rosen to combine both theoretical and pianistic concepts of Op. 10 No. 1. The hand and the arm configurations about which Rosen writes are marked with arrows, representing the alternating ascending and descending arpeggio patterns. Each of the patterns is a Schenkerian arpeggiated elaboration, signified by a group of two slurred quarter notes, seen in the first phrase of Figure 2.17. This Schenkerian understanding of the passage likewise reveals the musical characterization that a pianist depicts in a performance.

Figure 2.17: The hand-arm motion effect from perspective of the Schenkerian analysis, seen in Chopin’s Étude Op. 10 No. 1, mm. 1-10.

Appendix 1 presents the background Schenkerian analysis of Op. 10 No. 1. The background graph of the work allows one to trace the form, the structure, and the central harmonic structure of the composition. The Ursatz of the étude consists of fundamental $^3-^2-/^-^3-^2^-1$ Urlinie with an interruption (labeled as //) occurring at the end of the B-section before the reinstatement of A’ at m. 49. The
interruption separates sections A and B from A’; while both A and B sections are twenty-four bars in length, section A’ consists of thirty-one bars. The symmetry in the measure length found in the opening forty-eight bars of the étude reveals Chopin’s intent to create a musical equilibrium between the opening two sections.

Appendix 2 presents the foreground Schenkerian graph of Op. 10 No. 1. Based on the foreground graph, it is evident that the identical pianistic technique is used throughout the piece and every two-measure unit incorporates the arpeggiation elaboration. The graph reveals that, while a variety of harmonies exist in the A-section, the highest pitch that Chopin includes in the right hand of the étude is \(^3\). In fact, the only three pitches used as the apogee of every arpeggiation pattern are D, D\(^\#\), and E (\(^2\), \(^\#2\), and \(^3\)). From the perspective of form, each of the composition’s three eight-measure phrases is generated by these two-measure units.\(^82\)

In the B-section, Chopin’s compositional approach to treating the apogee of each arpeggiation changes, as the composer utilizes a greater variety of pitches and harmonic regions. A unique choice of harmony to generate a cadential ending can be observed in mm. 23-24, where Chopin utilizes the E major mediant instead of the G major dominant, leading the B-section to begin with a resolution into A major harmony.\(^83\) A similar occurrence happens in mm. 47-48 at the end of the B-section. This time, however, the E major transforms into G\(^7\) before returning to C major tonic in section A’ at m. 49. Another significant aspect of the B-section is its

\(^{82}\) Such symmetry is interrupted in the final section of the étude.
\(^{83}\) Beethoven uses this technique in his Piano Sonata No. 21, Op. 53, where the opening theme in C major modulates to E major, the key of the secondary theme in the first movement. Ludvig Van Beethoven, Klaviersonaten (Munich: G. Henle Verlag, 1980), 88.
use of linear sequences in the bass of the left hand to lead a series of ascending and descending arpeggios in the right hand. This can be observed in three instances: two passages in mm. 25-33 and mm. 42-47, both containing the sequential descent of melodic perfect fourths, and one passage in mm. 35-41 that incorporates the sequential descent of melodic perfect fifths.84

2.3. The Schenkerian Approach, Op. 10 No. 2

Structurally, the second étude is similar to Op. 10 No. 1.85 While the technical challenges vary, the pianistic aspects of legato, accuracy, and voicing should still be implemented by the performer. Unlike the first étude, the main pianistic purpose of Op. 10 No. 2 is to achieve evenness, consistency, and strength in the right hand's three weakest fingers. Chopin uses very little innovation in this work, focusing on the chromaticism in the right hand and chordal harmony in the left hand, as well as in the secondary voices of the right hand.

Chopin’s ability to expand thematic material is evident in mm. 13-16, where an ascending chromatic run from A to F in mm. 13-14 develops and transforms into an ascending chromatic run from B♭ to A♭ in mm. 15-16, as shown in Figure 2.18. On the first chromatic rise, Chopin begins with a tonic harmony, moves towards the subdominant harmony, and returns to the tonic harmony, which generates an A minor → D minor → A minor progression. Chopin initializes the

84 Bars 25-33 (movement by fourths): (A-D) → (G-C) → (F-B♭);
Bars 42-47 (movement by fourths): (E-A) → (D-G) → (C-F) → (B-E);
Bars 35-41 (movement by fifths): (A-D) → (G-C) → (F-B♭).
85 Furthermore, the directions of Schenkerian elaborations are likewise evident throughout the second étude.
second chromatic ascent with the Neapolitan harmony before moving to the
dominant harmony. Instead of returning to the tonic, Chopin finalizes the musical
apex with the submediant harmony, which generates a B♭ major → E dominant
seventh → F major progression. The common characteristic of mm. 13-14 and
mm. 15-16 is the ascending and descending primary chromatic melody, yet Chopin
utilizes diverse harmonic paths and stresses different metric spots for the pitch
zeniths. The highest pitch in the initial ascent occurs on the first (strong) sixteenth
note of the second beat. On the contrary, the highest pitch in mm. 15-16 ensues
on the last (weak) sixteenth note of the fourth beat in m. 15.

Figure 2.18: Chopin’s Étude Op. 10 No. 2, mm. 13-16.

Appendix 3 presents the background Schenkerian analysis of Op. 10 No. 2.
The background graph reveals the untraditional path of the Urlinie throughout the
work. First, this is the first example of ^5^-^1 line in the Op. 10 set. Second, the
Urlinie occurs three times in the étude, each time amalgamating new melodic and
harmonic material. Third, the descent of the Urlinie in each of the final two
instances is based on a harmonic minor scale with a raised \(^7\), since A is followed by G\(^\#\), not a G\(^\natural\). Fourth, this is the only one of Chopin’s études to end on a Picardy third, where the final note of the melody is found in the right hand and ends on C\(^\#\) rather than C\(^\natural\). According to Schenker, the goal of the fundamental structure is to “represent tonality” and to “achieve musical coherence” through the means of stepwise descent from either \(^8\), \(^5\), or \(^3\).\(^66\) Even though \(^1\) is the final stated goal, it is achieved in m. 45 with mm. 45-49 serving as post-cadential closure, which includes an ascending chromatic rise and a descending chromatic return to the Picard third.\(^87\)

Appendix 4 presents the foreground Schenkerian graph of Op. 10 No. 2. Based on the foreground graph, it is evident that the main thematic passage in mm. 1-2 (mot. a\(^1\)) generates the melodic and harmonic expansion of each of the four phrases in Section A.\(^88\) The opening two measures are identical to mm. 5-6 (mot. a\(^2\)), mm. 9-10 (mot. a\(^3\)), and mm. 13-14 (mot. a\(^4\)). Each of these musical statements introduce a short phrase. A similar idea applies to section A’, where mm. 1-2 are identical to mm. 36-37 (mot. a\(^5\)), mm. 40-41 (mot. a\(^6\)), and mm. 45-46 (mot. a\(^7\)). The final three motives are responsible for all three phases in the final section of the étude. Based on the homogenous structure of the piece, the

\(^87\) It is important to mention that it is likewise possible for the Schenkerian Urlinie to fail reaching \(^1\), an example of which is presented Marston’s Schenkerian graph of Schumann’s Kinderszenen Op. 15 No. 1, where the Urlinie closes on \(^3\).
\(^88\) The key characteristic of Op. 10 No. 2 is that Chopin initializes multiple phrases in an identical manner. In this section, each such initialization will be labeled as mot. a\(^x\). Therefore, mm. 1-2 = mot. a\(^1\); mm. 5-6 = mot. a\(^2\); and so on. Since all mot. a\(^x\) share identical music material, it is to the benefit of a pianist to create artistic disparity in the way these motives are performed and the changes in performance practice can be traced through the use of Schenkerian analysis.
analytical representation of similarity at the beginning of each phrase is significant from the perspective of performer, since an identical playing of these motives is not encouraged. Therefore, a performer needs to decide on specific artistic characteristics that can enhance this work musically.

Figure 2.19: Schenkerian representation of Shishkin’s performance of Op. 10 No 2, mm. 1-16.
This can be seen and analyzed in Dmitry Shishkin’s performance of Étude Op. 10 No. 2 at the 17th International Fryderyk Chopin Piano Competition. Figure 2.19 shows the foreground graph of Shishkin’s interpretation of section A. In mot. a¹, there is a slight accent on note A in the left hand of the first beat on bar 2. As evident from the foreground graph, the pitch A is prolonged in mm. 1-2 and therefore, an identical note is not needed. Figure 2.19 likewise reveals that the primary prolonged pitch A is interconnected with a supplementary pitch A (as a quarter note), which Shishkin accents with the use of a dotted slur. Therefore, while the A minor harmony is prolonged, it is also emphasized.

In mot. a², no evident interpretational changes exist, although there is an emphasis on note E in the left hand of the second beat on m. 8. As in the previous example, no additional pitch E is necessary, since the E on the first beat of m. 8 represents the prolonged dominant harmony for the full bar. However, Shishkin’s interpretation requires an additional quarter note E on the second beat to emphasize the harmony.

The mot. a³ contains a similar execution as mot. a¹, except the accent on note A in the left hand of the first beat on m. 2 is stronger and more apparent. The Schenkerian analysis represents this by using a half note to define the additional note A, instead of a quarter note, as it was done in mot. a¹. Furthermore, Shishkin brings out the pitches D♯ and E of the secondary melody in the middle voices of the right hand, found on the second and third beat of m. 12. Since this does not happen in mot. a¹, the pitches D♯ and E (of mot. a³) are represented as individual

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quarter notes. Finally, mot. a⁴ has no evident interpretational changes, but the continuation of mot. a⁴ in mm. 15-17 incorporates a larger use of sustained pedal. Additionally, Shishkin creates a slight ritardando before the entrance into the B-section.

Each interpretation is unique and each pianist’s musical comprehension of a work can be represented by a distinctive Schenkerian depiction. The musical components that are idiosyncratic in each performance are based on accentuation and musical emphasis. Therefore, a series of dynamic accents can be highlighted by Schenkerian notation, such as filled in noteheads, quarter notes, and half notes. On the contrary, there are component of performance that Schenkerian analysis cannot graph. Examples of such components are purely interpretive, such as tempo alterations and pedaling.

2.4. The Schenkerian Approach, Op. 10 No. 3

Of the opening four works, the third étude has received the most research and thorough analysis with notable theoretical studies by Spicer and Parks.⁹⁰ Cadwallader and Gagné focus on the notion of melody and provide an analytical interpretation of the opening five measures in Op. 10 No. 3, as seen in Figure 2.20.⁹¹ The initial phrase of this composition allows one to outline the arch, where the tonic pitch E on the first beat of m. 1 ascends to the second scale degree (F♯)

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on the second beat of the same measure and to the third scale degree (G♯) on the second beat of m. 2.

![Figure 2.20: Chopin’s Étude Op. 10 No. 3, mm. 1-5.](image-url)

The apogee of the arch descends from G♯ back to tonic in mm. 4-5. The third measure contains a musical break between the two G♯ of the melodic arch, revealing the real apex of the phrase – the C♯ on the second beat of m. 3 preceded by the interval of a perfect fourth. The significance of m. 3 lies in its generation of dissonance on the dominant harmony that is resolved on the descent towards the E in m. 5. The tones E, F♯, and G♯ are heard as primary elements of the main melody of the étude.92

The harmonic reduction of this passage reveals the Chopinesque melodic arch in its highest voice, supported by the tonic on the first and the third scale degrees, and by the dominant on the second scale degree.93 Furthermore, the inclusion of pitch B in the pickup measure fulfills the outline the tonic triad through the means of “continuity” and “variety.”94 The consistence of lyrical and melodious

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92 Ibid., 17.
93 The complete background and foreground graphs of Étude Op. 10 No. 3 can be viewed in Appendices 5 and 6.
94 Ibid., 18.
qualities that Chopin presents in the third étude is the technical challenge in itself, seen in the analytical interpretation and the chordal configuration of the opening phrase. This instance of Chopin’s melodic development illustrates the meaning of the analysis in artistic pianism of the composition.

One of the distinct features of Chopin’s musical language is that his harmonic goals are not always certain and employ unexpected turns, as seen in the music analysis. This can be traced in mm. 13-17 of the third étude, shown in Figure 2.21, where Chopin incorporates chromatic intermediate chords to move from tonic to the cadential 6/4 that forms at the culmination point of the first section. In this passage, the three significant harmonies are found in mm. 15-16, where the goal is to generate an exaggerated motion towards the cadence at m. 17. The first intermediate harmony, C# minor, is seen in the second beat of m. 15. Relative to the tonic, it expands towards the second intermediate harmony – the second inversion of the dominant, seen on the first beat of m. 16.

This harmony is important from the perspective of both harmonic analysis and pianism. From the standpoint of analysis, the second intermediate harmony creates an unusual VI-V motion, while from the standpoint of pianism, there are two musical elaboration that need to be executed – con forza and ritenuto. The second intermediate harmony likewise resolves suddenly into the third intermediate harmony on the second beat of bar 16 – a French augmented chord, which cadences in m. 17.

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95 Ibid., 48.
Figure 2.21: Chopin’s Étude Op. 10 No. 3, mm. 13-17.

Another analytical phenomenon in the third étude occurs in the B-section, where a linear analysis can assist with revealing the tonal ambiguities of Chopin’s progressions.\(^{96}\) This happens in mm. 38-41 of Op. 10 No. 3, shown in Figure 2.22. This section of the work is highly dissonant, where chromatic harmony is embodied inside the linear patterns. In this passage, Chopin contradicts the traditional harmonic and melodic musical paths, generating a progression with high degree of tension that delays resolution until m. 42.\(^{97}\)

These several bars consist of chromatically descending fully-diminished seventh chords, contrasting drastically against the A-section of the étude, which is based on the prolongation of tonic and dominant harmonies.\(^{98}\) The passage in mm. 38-42 can be subdivided in four groups of eight descending chords.\(^{99}\) In each group, the left hand continues the chromatic descent of tritones, while the right

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\(^{97}\) Ibid., 141.


\(^{99}\) The third and the fourth groups can likewise be combined to represent one block of sixteen verticalities.
hand inverts the bottom note of the third, fifth, and the seventh elements (also represented by harmonic tritones). While such approach breaks off the descending motion of the thematic material, it preserves the parallel motion of harmonic tritones, generating a significant source of dissonance in the middle section of the étude.

An interesting harmonic phenomenon in the B-section incorporated by the composer can be traced with the return of the dominant B major harmony after a prolonged tonally adventurous excerpt. In Chopin's music, seventh chords are typically resolved to a consonant (major or minor) triadic harmony.\(^{100}\) In the instance of mm. 38-42 of Op. 10 No. 3, each seventh chord resolves into proceeding seventh chord and a musical progression, generated from a chain of dissonant verticalities, leads to the only consonance in the passage, found in m. 42.\(^{101}\)

![Figure 2.22: Chopin's Étude Op. 10 No. 3, mm. 38-41.](image)

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\(^{101}\) In the A-section, the dominant harmony serves the role of a dissonant harmonic region that seeks resolution to a consonant tonic harmony, generating a I-V-I progression. In the B-section, the dominant harmony is likewise prominent, yet this time as a consonant harmony, employed by Chopin as a harmonic region to accept the resolution.
Figure 2.23 presents a detailed Schenkerian representation of mm. 38-41 of the third étude. The Schenkerian analysis shows that the passage is prolonged on the G♯ fully-diminished seventh chord on the first beat of m. 38 leading towards the resolution of the B major dominant harmony in m. 42. The Schenkerian representation of mm. 38-41 enhances the performance in two unique ways.

First, the graph shows the primary harmonies of the passage, on which a pianist may focus. The primary emphasis is on the B major harmony (the dominant), as well as on the F♯ dominant seventh harmony (dominant of the B major dominant). Second, the Schenkerian graph illustrates the linear motivic descent, which serves as the basis of this work. Chopin generates four motives, where the first motive ends on the A♯ fully-diminished chord in m. 39, the second motive ends on the D♯ fully-diminished chord in second inversion in m. 40, the third motive ends on the G♯ fully-diminished chord in m. 41, and the fourth motive ends on the dominant harmony of B major. The four motives generate a strong drive towards the pure consonance, found in m. 42.

The interpretation of a work lies in the mixture of historical performance practices, based on the aspects of playing, such as tempo, articulation, dynamics, and pedaling. The third étude allows for a variety of interpretational performances. Similarly, the Schenkerian analysis builds an interpretation by utilizing pitches of different level of significance through multiple types of elaborations. The

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Schenkerian analysis allows one to see the voice leading and various fundamental designs in form and texture that make up the composition.

Figure 2.23: The Schenkerian analysis of Chopin’s Étude Op. 10 No. 3, mm. 38-42.

2.5. The Schenkerian Approach, Op. 10 No. 4

The background Schenkerian analysis of Op. 10 No. 4 can be viewed in Appendix 7. In this work, the background graph presents a set of interesting observations that are noteworthy in performances. The background graph reveals a ^5-^1 Urlinie descent with interruption occurring in m. 50 before a return to the ^5, the first such Urlinie in Op. 10. The A-section is in mm. 1-16 and consists of two phrases of equal length. Furthermore, the A-section ends on A major, generating a nontraditional transition to the upcoming section. The B-section is in mm. 17-50, spanning for a total of thirty-four measures. The B-section consists of two groups, found in mm. 17-33 and in mm. 33-45, as well as a transition, found in mm. 45-50. The first group of the B-section in mm. 17-33 transits through a key change in mm. 21-26. This passage ends with the E♯ fully-diminished seventh chord. The second group of the B-section primarily represents the ^2 of the Urlinie, cadencing on the first inversion
of the G♯. The B-section ends with a bridge that lead to the opening of the A'-section starting at m. 51. The A'-section reinstates the thematic material found in the A-section until m. 72, before presenting a coda that continues until the end of the étude.

The foreground graph, shown in Appendix 8, reveals multiple Schenkerian elaborations that constitute and disclose the compositional stylistic features of the fourth étude. First, Chopin encloses several ascending and descending ^1-^5 lines. This can be seen in mm. 1-3 in the right hand, where the ascent occurs between notes C# and G# before a descent to the original pitch. Another instance can be seen in mm. 17-19 in the right hand, where the ascent occurs between notes F♯ and D♯ before a descent to the original pitch. Second, the fourth étude contains multiple instances of scales that can be traced through the foreground graph, yet are not noticeable from the perspective of performance. One such example can be seen in mm. 10-12, where the descending line from G♯ to C♯ continues to F♯, ending on the subdominant of F♯ minor.

Third, this étude contains reversed motivic parallelism, which can be seen in mm. 25 and 26. The F-G-A♭ line in the right hand is reversed as an F-F♭-E♭ line in the left hand in m. 25, while the A-A♯-B line in the right hand is reversed as an A♭-G-F♯ line in the left hand in m. 26. Finally, while the background graph presents the strong cadential endings throughout the work, the foreground graph

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103 All of these are generated through the three primary Schenkerian techniques, described earlier: arpeggiation, chromaticism, and linear progressions.
104 These could be thought of as subordinary Urlinie structures.
presents their respective expansion, seen through motivic parallelism.\textsuperscript{106} This can be observed in m. 33, where Chopin uses the notes of the G$\#$ fully-diminished seventh chord to generate a melodic line in mm. 34-35. Similarly, Chopin uses the notes of the F$^X$ fully-diminished seventh chord, found in m. 37, to generate a melodic line in mm. 39-40.

\textsuperscript{106} Nicholas Cook, Beyond the Score: Music as Performance (Oxford: Oxford University Press, 2013), 36.
CHAPTER 3. DISSONANCE MEASUREMENT FOR ANALYSIS

3.1. Approaches to Dealing with Dissonance

The type of sound that is generated between two or more simultaneously played notes provides a piece of music with a sense of cohesiveness and adherence.\textsuperscript{107} This results in a generation of a series of harmonic intervals throughout a composition. The harmonic intervallic makeup of each musical work is unique and can define characteristics, such as style and genre. From the standpoint of performance practice, the notion of harmonic consonance and dissonance produces musical occurrences that evoke emotions in listeners and in performers themselves. Furthermore, the notions of consonance and dissonance allow one to generate own musical interpretation and areas of emphasis throughout the score.

The type of tension that exists between two notes depends on aspects such as pitch, register, intensity of the sound, and texture of the harmony.\textsuperscript{108} Likewise, the function for evaluating the relative dissonance of two tones is based on the roughness of its respective frequencies.\textsuperscript{109} There exist numerous sources of scholarship on consonance and dissonance, with the earliest treatises dating to the epoch of the Ancient Greeks, specifically focusing on harmonic intervals.\textsuperscript{110} Most scholarship on this topic can be divided based on particular methodology,

\textsuperscript{108} Ibid., 90.
with which one studies the notion of consonance and dissonance. Some of the interdisciplinary ways to subdivide this type of research are through the means of psychological, acoustical, and mathematical approaches.\footnote{The study of acoustics allows for the unique objective distinction between intervals. The context of acoustics, however, does not allow for such distinction and is bound for influence of stylistic perception. What is considered dissonant in one style can be considered consonant in another.}

Ian Cross considers three views in which music dissonance can be understood: \textit{cognitivist}, \textit{physicalist}, and \textit{immanentist}.\footnote{Ian Cross, "Music Science and Three Views," \textit{Revue Belge de Musicologie / Belgisch Tijdschrift voor Muziekwetenschap} 52 (1998): 207.} The \textit{cognitivist} approach to understanding music occurs through one’s psychological perception, where aspects, such as culture, musical training, and musical apprehension will decide how consonant or dissonant an interval sounds.\footnote{Paula Virtala and Mari Tervaniemi, "Neurocognition of Major-Minor and Consonance-Dissonance," \textit{Music Perception} 34, no. 4 (April 2017): 387.} This means that one’s awareness is an instance of “abstract representation” and “the structure and function of abstract representations, of their instantiations in cognition and in the material correlates of that instantiation in terms of musical behaviors, musical sound and neural activity.”\footnote{Ian Cross, "Music Science and Three Views," \textit{Revue Belge de Musicologie / Belgisch Tijdschrift voor Muziekwetenschap} 52 (1998): 208.} Furthermore, music is a "product of cultural convention" that relies on subjective reasoning rather than scientific (objective) explanation and therefore, the notion of \textit{cognitivist} point of view allows for multiple interpretations of consonance and dissonance.\footnote{Ibid., 207-208.} Likewise, the implementation of such view seeks to make an evident correlation between what one experience
when listening to dissonance and what (and how) one generalizes such dissonance to others.\textsuperscript{116}

A physicalist approach to understanding music is an example of objective methodology. Such philosophy of interpreting dissonance incorporates the comprehension that music "is out there" whereas the notions of music, sound, and one’s musical experiences are provided and determined by its mere existence.\textsuperscript{117} According to Cross, pure physicalist connection with music is problematic, since "if the physicalist view were to hold, then there would be no question of science’s efficacy for understanding music."\textsuperscript{118} The mathematical approach to working with dissonance derives from physicalist point of view and allows one to perform an objective data analysis, based on the nature of artistic subjectivity found in stylistic musical languages of various compositions. While statistical analysis does not eliminate the level of subjectivity, it diminishes it and allows for objective theoretical examination of musical tension in each work. On the contrary, the immanentist view of dissonance assumes that music has no physical reality. An example of such rejection of scientific premise as fundamental background for music can be seen in Scruton’s writings, who states that science deals with "material" which

\textsuperscript{116} From an acoustical perspective, multiple changing variables, such as tuning systems, can create diversity in the dissonance
Fauvel et. al. provide an example of two tuning systems. The Pythagorean tuning system is based on perfectly tuned fourths, fifths, and octaves, while equal temperament tuning system is based on equal distribution of pitches between every two neighboring semitones, hence the distance between any two pitches will vary (apart from the octaves), resulting in different acoustical calculations of dissonance.


\textsuperscript{118} Ibid., 208.
music is not. Nevertheless, the study of dissonance in music is a prominent component, which can be approached through multiple angles.

The proposed methodology in this dissertation assumes the cognitivist point of view. This occurs because the perception of vertical tension in this research is based on the unique grouping of harmonic intervals into consonances and dissonances. The IDR bifurcates all harmonic intervals into two distinct groups, according to the traditional principles of harmony in the Chopinesque Romantic era. Such understanding of vertical tension is suitable for the analysis of Chopin’s works, yet will require modification for analysis of other composers, such as Claude Debussy.

3.2. Evaluation of Existing Literature

There exists a vast amount of empirical literature on multiple approaches to understanding harmonic consonance and dissonance. This section of the chapter will discuss previous research that carved a path for the generation of the IDR tool. The sources mentioned in this section provided the foundations and the fundamentals for the development and the devising of a mathematical tool, which can be used to calculate the rate of harmonic dissonance in a piece of music.

The notion of consonance and dissonance depends on the musical fundamental building blocks of individual pitches. Composers organize pitches into

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121 Chapter 3.2 will discuss works by Huron, MacCallum & Einbond, Jensen, Kameoka & Kuriyagawa, Terhardt, and Isaacson.
collections, also known as pitch-class sets. Major and minor triads are two of many examples of pitch-class sets. Such sets considered with a combination of harmonic intervals and interval-classes. An interval is based on the notion of tonal context that is generated between two pitches and can be described with size and direction, while an interval-class defines and groups intervals; each interval-class represents several types of intervals. According to Huron, the frequency of occurrences of various interval-classes corresponds to the perceived consonance.

![Graph 3.1: Spectrum of piano, trumpet, and viola.](image)

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125 Ibid., 61.
According to Jensen, the sensory dissonance depends on the spectrum of the sounds, as shown in Graphs 3.1 and 3.2. Graph 3.1 displays the association between amplitude (dB) and frequency (Hz), seen in the sounds generated by piano, trumpet, and viola. On the contrary, Graph 3.2 presents the level of harmonic dissonance based on the frequency interval (represented in cents). Based on this data, the "more spectrally rich sounds" provide a higher level of sensory dissonance, as seen with the trumpet, while the "less spectrally rich sounds" present a lower level of sensory dissonance, as seen with piano and viola.

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126 Ibid., 61.
128 Ibid., 60.
Each phrase in music is generated from a unique series of harmonic intervals, meaning that each work incorporates distinct successions of musical verticalities and therefore, presents own individual level of harmonic dissonance. In addition, each musical composition can generate a variety of sensory dissonance levels, depending on performative interpretation. In the piano works of Chopin, the concepts of one’s interpretation that may alter the level of sensory dissonance may include tempo, articulation, voicing, timbre, and pedaling. From a performance perspective, different harmonic intervals evoke various types of emotions, which depend on elements, such as individual pitches, pitch registers, and dynamic levels. This means that one’s performance practice is guided by both sensory and perceptual dissonance. In Chopinesque Romantic music, while it is to the benefit of a pianist to “promote” a high degree of tonal consonance, most pianists prefer to “control” and “limit” the sensory dissonance of tonal tension. The perceptual dissonance, guided by one’s awareness, follows the sensory dissonance and allows one to know that something outside of the norm has just happened.

Kameoka and Kuriyagawa focused on analyzing harmonic dyad dissonance, based on the frequency ratios of two pitches. Such approach requires the use of intervallic ratios. When it comes to consonant intervals, the frequency

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129 Each interpretation is unique and performative components of one’s interpretation alter one’s perception of consonance and dissonance. Based on works of Richard Parncutt and Ernst Terhardt, John MacCallum and Aaron Einbond have designed an algorithm that calculates the sensory dissonance of a given performance. John MacCallum and Aaron Einbond, “Real-Time Analysis of Sensory Dissonance,” Lecture Notes in Computer Science 4969 (2007): 203-211.


ratios of 1:1, 2:1, 3:2, and 4:3 generate pure consonant intervals, while frequency ratios of 5:4, 5:3, 6:5, and 8:5 generate imperfect consonant intervals.\textsuperscript{132} All other ratios generate dissonant intervals.\textsuperscript{133} Furthermore, Kameoka’s and Kuriyagawa’s research is based on the notion that harmonic intervals cannot be measured by the degree of consonance or dissonance.\textsuperscript{134} This occurs because of deviations in tuning systems that generate different ratios and frequency rates for each interval.\textsuperscript{135}

Kameoka and Kuriyagawa define each interval as a distance (range) between two points of $f_x$, such as $f_1$ and $f_2$. The main variable in calculating for dissonance is the sound pressure level.\textsuperscript{136} If $f_1$ is fixed at 440 Hz and $f_2$ increases from 440 Hz to 880 Hz and if the sound pressure level is static, the consonance that begins at the maximum consonance point gradually decreases, generating a maximum dissonance point, before progressively increasing back to the maximum consonance point, as shown in Graph 3.3.\textsuperscript{137} Furthermore, Kameoka and Kuriyagawa introduce the concept of dissonance intensity, which can be applied to loudness of separate dyads.\textsuperscript{138}

\textsuperscript{132} By pure consonant intervals, the authors refer to perfect consonant intervals. Akio Kameoka and Mamoru Kuriyagawa, "Consonant Theory Part I: Consonance of Dyads," \textit{Journal of the Acoustical Society of America} 45, no. 6 (February 1969): 1451.

\textsuperscript{133} The author does not present possible groups of dissonant intervals. Ibid., 1452.

\textsuperscript{134} For more information on sound pressure level, refer to: Andrzej Miskiewicz and Andrzej Rakowski, "Loudness Level Versus Sound-Pressure Level: A Comparison of Musical Instruments," \textit{The Journal of the Acoustical Society of America} 96, no. 6 (1994): 203-216.

\textsuperscript{135} Ibid., 1452-1453.

Graph 3.3: The formed V-curve, where $f_1 = 440 \text{ Hz}$ and $440 \text{ Hz} \leq f_2 \leq 880 \text{ Hz}$.\(^{139}\)

Based on the intervallic analysis of tonal music, the pure consonant intervals with small frequency ratios are most common.\(^ {140}\) These include the perfect octave with a frequency ratio of 2:1, the perfect fifth with a frequency ratio of 3:2, and the perfect fourth with a frequency ratio of 4:3. In his research, Terhardt’s understanding of dissonance can be traced through the consolidation of smallest harmonic particles, such as intervals, to generate chords. According to Terhardt, chords are the “functional basis” of tonal music, since these chords are employed to produce melodies that one associates with respective harmonies.\(^ {141}\) Furthermore, Terhardt links the intervallic dissonance with existence of respective

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141 Ibid., 1067.
musical scales, as composers establish the keys and harmonic regions to realize such intervals. This leads to the inference that the consonance of chords is dependent on the harmonic structure and even completely consonant sonorities will generate different levels of consonance.

The research mentioned above provides scholarly insight on dissonance from perspective of performance practice. In Chopin’s études, interpretational deviations are frequent occurrences when looking at the history of performances and each epoch presents own set of artistic innovations. While different performances of the same work do not generate significant alterations to the level of dissonance, each pianist nevertheless chooses his or her own artistic approach and presents a reading of the work. Therefore, because of limitless possibilities to integrate tempo, dynamics, and pedaling, no two performances are ever identical.

As mentioned previously, many researchers (such as Terhardt) associate the derivation of dissonance with harmony, where chords are the fundamental building blocks of tension. Chordal progressions can be subdivided into smaller vertical elements – intervals, which can be stacked and combined to generate various types of harmonies. The fundamental way to dissect the intervallic makeup of a set of notes is to use the interval-class vector. The mathematical relationship between multiple interval-classes of such vectors can be measured and analyzed through Isaacson’s ICVSIM relations, a tool that measures the interval similarity of

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142 Ibid., 1068.
143 This is especially noticeable when comparing various prewar and postwar recordings of Chopin’s piano études.
two or more pitch-class sets based on their intervallic content. While the ICVSIM relations do not measure dissonance, there are three significant qualities to this interval relation that exist in the IDR calculations.

First, Isaacson bases the ICVSIM relations on values between 0.00 and 3.58, establishing a metric rate between two pitch-class sets. A rate of 0.00 specifies maximal similarity between two interval-class vectors and a rate of 3.58 generates a highest level of dissimilarity, which occurs between pitch-class sets in set classes 6-35 and 8-28. A metric rate is a special type of rate that contains a minimum and a maximum, allowing for mathematical comparison within specific numerical boundaries. Second, the ICVSIM and the IDR can be used equally with pitch collections of any cardinality. At least two simultaneously played notes must be present for a relation to exist. Third, similar to the IDR, the ICVSIM relations provide a distinct formula that is appropriate for any musical pitch collection.

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ICVSIM(x) = \sqrt{\frac{\sum (tdV_i - \overline{tdV})^2}{6}}
\]

In addition, the ICVSIM relations are relevant to IDR research for three reasons. First, both the ICVSIM and the IDR use a metric rate to generate a value for comparison. While the ICVSIM relations are based on the scale of 0.00 to 3.58,

145 Other interval-similarity measurements include Forte’s \( R_n \), Lord’s \( sf \), Morris’s \( SIM \), Rahn’s \( MEMB \), Lewin’s \( REL \), and Castren’s \( RECREL \) relations.
146 Ibid., 1.
The sets with identical interval-class content are maximally similar. The whole tone and octatonic collections are maximally diverse in their intervallic content.
147 The formula to calculate the Interval Dissonance Rate will be presented in Chapter 4 of the dissertation.
the IDR is based on the scale of 0 to 1. Both metrics contain the minima and the maxima that are responsible for providing the comparison parameters of all possible values. Second, both ICVSIM and IDR generate a value that represents rate by using an interval-class vector. While the ICVSIM relations are based on traditional interval-class vector, the IDR is based on the modified interval-class vector, which supplement the traditional vectors with the frequency of recurrent pitches and contains specific coordinate for tallies of unisons, octaves, and equivalent intervals.\textsuperscript{148} Third, both ICVSIM and IDR provide a rate to distinguish similarity in two or more musical structures.

There are two significant differences between the ICVSIM and IDR analyses. First, the ICVSIM can only be used to examine two separate interval-class vectors. This means that multiple harmonies will generate numerous values of comparison. On the contrary, the IDR does not carry any limitations regarding the number of analyzed harmonies and will provide one rate for musical passage of any length. Second, the primary purpose of the ICVSIM is to measure and compare interval-similarity in two harmonic contents. While the IDR can be used to examine various interval-classes and function of these interval-classes in music, its primary purpose is to analyze and compare harmonic dissonance.

Aside from intervallic similarity, the IDR system benefits from a segmentation methodology and the subdividing of each measure into multiple sectors, generating a series of verticalities. In his research, Quinn presented the

\textsuperscript{148} Chapter 4 goes in depth on modified interval-class vectors and its relation to the IDR. Furthermore, the modified interval-class vector is a derivation from Morris’s \textit{multiset}. Robert Morris, "Mathematics and the Twelve-Tone System: Past, Present, and Future," \textit{Perspectives of New Music} 45, no. 2 (Summer 2007): 98.
“salami slice” approach, allowing one to subdivide a musical passage based on its pitch simultaneities.\textsuperscript{149} While this type of research does not necessarily focus on interval-classes, the corpus allows for comparison of multiple variables of large data, such as normal form of verticalities, most frequent scale-degree sets, or ratios between dominant and dominant seventh chords as they would appear in each work through segmentation – a prominent feature of the IDR metric. Graph 3.4 presents the nineteen most frequent composers in the corpus study.

Graph 3.4: The nineteen most frequent composers of the corpus study.\textsuperscript{150}


Graph 3.5 presents the most frequent scale-degree sets among all the works in the corpus, as transposed to C major or C minor.\textsuperscript{151} This supports the notion that the tonic triad is the predominant phenomenon in tonal works, occurring 9\% of all times. Furthermore, this graph mathematically reveals that the works in the corpus studies are principally based on the sub-elements of either tonic or dominant. The nine most frequent sub-elements are the major tonic triad, $^5$, $^1$, dominant seventh chord, minor tonic triad, dominant triad, as well as harmonic intervals of C-E, C-G, and E-G, all of which constitute the tonic triad.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{graph3.5.png}
\caption{The most frequent scale-degree sets among all the works in the corpus, as transposed to C major or C minor.\textsuperscript{152}}
\end{figure}

\textsuperscript{151} Ibid., 53-55.
\textsuperscript{152} Ibid., 55.
3.3. Calculating Harmonic Dissonance

As stated previously, the accurate representation of harmonic dissonance can be achieved by combining the cognitivist and the physicalist methodologies. Numerical analysis that generates data is what constitutes the mathematical side of dissonance research. Dissonance that is measured can be ranked and compared based on statistical analysis. Huron derives the term *consonance index* to define a value that represents the perceived consonance between two concurrent tones. The consonance index likewise presents a particular weight to each interval-class. It is important to note that from a mathematical perspective, the accurate representation of consonance index would disregard aspects, such as sound pressure level and pitch register, even though these are essential components of acoustical and psychological dissonance.\(^{153}\)

Huron constructs an index of tonal consonance based on the experimental data from previous studies by Malmberg (1918), Kameoka and Kuriyagawa (1969), and Hutchinson and Knopoff (1978).\(^{154}\) All major, minor, and perfect intervals, except for unisons and octaves were analyzed. The results can be seen in Tables 3.1, 3.2, and 3.3.

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\(^{154}\) Ibid., 292.


Table 3.1: Index of tonal consonance, based on the experimental data from studies by Malmberg.\textsuperscript{155}

<table>
<thead>
<tr>
<th>Consonance</th>
<th>Interval</th>
<th>Level of Consonance</th>
</tr>
</thead>
<tbody>
<tr>
<td>minor second</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>major second</td>
<td>1.50</td>
<td></td>
</tr>
<tr>
<td>minor third</td>
<td>4.35</td>
<td></td>
</tr>
<tr>
<td>major third</td>
<td>6.85</td>
<td></td>
</tr>
<tr>
<td>perfect fourth</td>
<td>7.00</td>
<td></td>
</tr>
<tr>
<td>tritone</td>
<td>3.85</td>
<td></td>
</tr>
<tr>
<td>perfect fifth</td>
<td>9.50</td>
<td></td>
</tr>
<tr>
<td>minor sixth</td>
<td>6.15</td>
<td></td>
</tr>
<tr>
<td>major sixth</td>
<td>8.00</td>
<td></td>
</tr>
<tr>
<td>minor seventh</td>
<td>3.30</td>
<td></td>
</tr>
<tr>
<td>major seventh</td>
<td>1.50</td>
<td></td>
</tr>
</tbody>
</table>

Malmberg’s study is ranked on the increasing scale with the lowest value of 0.00 being highly dissonant. According to Malmberg’s work, the minor second is the most dissonant interval while the perfect fifth and the perfect fourth are the most consonant intervals. Hutchinson’s and Knopoff’s study is ranked on the decreasing scale with the lowest value of 0.0221 being highly consonant. As Malmberg, Hutchinson and Knopoff present the perfect fourth and the perfect fifth

\textsuperscript{155} David Huron, “Interval-Class Content in Equally Tempered Pitch-Class Sets: Common Scales Exhibit Optimum Tonal Consonance,” \textit{Music Perception} 11, no. 3 (Spring 1994): 293.
as a consonant interval. Unlike Malmberg, Hutchinson and Knopoff present the minor seventh as the most dissonant interval.

Table 3.2: Index of tonal dissonance, based on the experimental data from studies by Hutchinson and Knopoff.\textsuperscript{156}

<table>
<thead>
<tr>
<th>Interval</th>
<th>Level of Dissonance</th>
</tr>
</thead>
<tbody>
<tr>
<td>minor second</td>
<td>0.4886</td>
</tr>
<tr>
<td>major second</td>
<td>0.2690</td>
</tr>
<tr>
<td>minor third</td>
<td>0.1109</td>
</tr>
<tr>
<td>major third</td>
<td>0.0551</td>
</tr>
<tr>
<td>perfect fourth</td>
<td>0.0451</td>
</tr>
<tr>
<td>tritone</td>
<td>0.0930</td>
</tr>
<tr>
<td>perfect fifth</td>
<td>0.0221</td>
</tr>
<tr>
<td>minor sixth</td>
<td>0.0843</td>
</tr>
<tr>
<td>major sixth</td>
<td>0.0477</td>
</tr>
<tr>
<td>minor seventh</td>
<td>0.0998</td>
</tr>
<tr>
<td>major seventh</td>
<td>0.2312</td>
</tr>
</tbody>
</table>

Finally, Kameoka’s and Kuriyagawa’s study is based on the decreasing scale with the lowest value of 215 being highly consonant. According to Kameoka’s and Kuriyagawa’s research, the minor second is the most dissonant interval and the perfect fifth is the most consonant interval.

\textsuperscript{156} Ibid., 293.
Table 3.3: Index of tonal dissonance, based on the experimental data from studies by Kameoka and Kuriyagawa.\textsuperscript{157}

<table>
<thead>
<tr>
<th>Interval</th>
<th>Level of Dissonance</th>
</tr>
</thead>
<tbody>
<tr>
<td>minor second</td>
<td>285</td>
</tr>
<tr>
<td>major second</td>
<td>275</td>
</tr>
<tr>
<td>minor third</td>
<td>255</td>
</tr>
<tr>
<td>major third</td>
<td>250</td>
</tr>
<tr>
<td>perfect fourth</td>
<td>245</td>
</tr>
<tr>
<td>tritone</td>
<td>265</td>
</tr>
<tr>
<td>perfect fifth</td>
<td>215</td>
</tr>
<tr>
<td>minor sixth</td>
<td>260</td>
</tr>
<tr>
<td>major sixth</td>
<td>230</td>
</tr>
<tr>
<td>minor seventh</td>
<td>250</td>
</tr>
<tr>
<td>major seventh</td>
<td>255</td>
</tr>
</tbody>
</table>

Although all three studies show that the minor sixth is more consonant than the major third, such a difference is irrelevant, since both intervals are complementary and belong to the same interval-class.\textsuperscript{158} Huron compiles the three studies into one. In this instance, the complementary intervals are combined (such as minor second and major seventh, major second and minor seventh, minor third and major sixth, as well as perfect fourth and perfect fifth). Table 3.4 shows the

\textsuperscript{157} Ibid., 293.  
\textsuperscript{158} Ibid., 293.
combined intervals that generate interval-classes, as well as an appropriate consonance index for each interval-class.\textsuperscript{159}

Table 3.4: Normalized data set of combined interval-class index.

<table>
<thead>
<tr>
<th>Interval-Class</th>
<th>Consonance</th>
</tr>
</thead>
<tbody>
<tr>
<td>minor second / major seventh</td>
<td>-1.428</td>
</tr>
<tr>
<td>major second / minor seventh</td>
<td>-0.582</td>
</tr>
<tr>
<td>minor third / major sixth</td>
<td>+0.594</td>
</tr>
<tr>
<td>major third / minor sixth</td>
<td>+0.386</td>
</tr>
<tr>
<td>perfect fourth / perfect fifth</td>
<td>+1.240</td>
</tr>
<tr>
<td>augmented fourth / diminished fifth</td>
<td>-0.453</td>
</tr>
</tbody>
</table>

It is also important to mention LoPresto's work on measuring musical consonance because his research, similar to the IDR, bases the dissonance metric on the frequency of different intervals under equal temperament tuning system.\textsuperscript{160} Such a mathematical approach of calculating dissonance measures intervals according to the order of merit, ranking them from the most consonant to the most dissonant.\textsuperscript{161} This method predominantly focuses on harmonic dyads and does not combine them into larger harmonic structures, such as triads or seventh chords. The order of merit, as per LoPresto, can be seen in Table 3.5.

\textsuperscript{159} Ibid., 293.
\textsuperscript{161} Ibid., 226.
Table 3.5: LoPresto’s order of merit table.

<table>
<thead>
<tr>
<th>Interval</th>
<th>P8</th>
<th>P5</th>
<th>M6</th>
<th>M3</th>
<th>P4</th>
<th>m6</th>
<th>m3</th>
<th>trit.</th>
<th>m7</th>
<th>M2</th>
<th>M7</th>
<th>m2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order of merit</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

The scholarship mentioned above deals with various types of consonant and dissonant intervals. In addition, there has been research in the past that looks at dissonant intervals and the many ways of providing harmonic resolutions. In his works, Yavorksy deals with tritones – the most unstable harmonic interval in music.\(^\text{162}\) Yavorsky designs a music system that looks at all possible resolution of tritone towards consonance, or of the tritone towards dissonance, followed by consonance.\(^\text{163}\) Yavorsky focuses on the tritone as the nucleus of a harmonic progression.

Figure 3.1 provides two examples of “gravitational pulls,” which are possible for a tritone that requires resolution towards imperfect consonance.\(^\text{164}\) In the first example, the upper pitch of the tritone is resolved by a descending minor second, while the lower pitch of the tritone is resolved by an ascending minor second. This generates a resolution towards the interval of a major third. In the second example, the upper note of the tritone is resolved by an ascending minor second, while the lower pitch of the tritone is resolved by a descending minor second. This generates a resolution towards the interval of a minor sixth – the inverse of a major third. In

\(^\text{164}\) Ibid., 50.
both instances, the tritone only employs the resolution by a half step. Furthermore, both examples show the fundamental structure of either a major or a minor harmony.

![Figure 3.1: Two possible resolutions of a tritone towards imperfect consonance.](image)

This simple example of tritone resolution can be expanded. Figure 3.2 shows the instance of a “triple tritonal system.” In many cases, composers are either hesitant to resolve tritones immediately and many may want to create a longer passage of tension, or prefer to employ tritone as an element of contrast between two consonances. In the first example of Figure 3.2, a perfect fifth is resolved to a dissonant double-diminished fourth that is resolved to a minor third, where the upper voice descends by a half step and the lower voice ascends by a half step, generating a contracting wedge. In the second example, a perfect fourth is resolved to a dissonant double-augmented fourth that is resolved to a major sixth, where the upper voice ascends by a half step and the lower voice descends by a half step, generating an expanding wedge.

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165 In Yavorsky’s research, the notion of “тройная тритонная система” is translated as “triple tritonal system”.

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The theories of Yavorsky are based on the idea that harmonic intervals build harmonies, rather than chords. Furthermore, Yavorsky generates multiple common approaches how dissonance, such as tritone, can be used in the Western music. Chopin establishes the importance of tritone in many of his works.\(^{166}\) The tritone is an interval that exists in many chords used by Chopin to create dissonance, which include augmented sixth chords and seventh chords.

**3.4. Chopin’s Connection**

Chapter 2 covered the basics of Schenkerian methodology to understand the theory and the pianism of the four études. The various graphs of Schenkerian analysis can trace the vertical tension both on local and global levels, as seen in the foreground and the background readings of the four works. In music of Chopin, dissonance generates musical diversity and allows for necessity to gravitate towards consonance, since the dissonant inferences created by a composer in the melodic and harmonic processes must be resolved and finalized.\(^{167}\) One of the


ways to construe tension in music is through the means of implication and realization.

According to Meyer, the implicative relationships in music allow the listener to link multiple musical phenomena.\textsuperscript{168} Therefore, an implication, such as harmonic dissonance, is connected to a realization – its resolution. For composers such as Chopin, it is a matter of when to realize an implication and for how long such implicative relationship is to be held before proceeding onto the next one. Figure 3.3 presents example of \textit{implied dissonance}, taken from Meyer's \textit{Explaining Music: Essays and Explorations}. The example consists of a small passage of six measures and its two Schenkerian representations.

In this example, the middle graph is of most significance. In Graph B, the implied dissonance is indicated in the analysis by the tied note C, where “the falling seventh (C\# to D\#) creates an implied dissonance so that the following B acts as the resolution of an imagined suspension.”\textsuperscript{169} Another example of dissonance is \textit{morphological}, where a passage serves as a finale of one melody and the initialization of another.\textsuperscript{170} An instance of such phrasal intersection is where the “resolution enhances the sense of satisfaction and closure.”\textsuperscript{171}

\textsuperscript{169} Ibid., 168.
\textsuperscript{170} Meyer labels such occurrence as “elision.”
\textsuperscript{171} Ibid., 233.
The I-R model is likewise related to the notion of musical closure, since a strong implication weakens the closure and a strong closure weakens one’s perception of the implication.\textsuperscript{173} Dissonance, as a musical component, can determine and increase the strength of the implication, as well as have an influence on the type and quality of the resolution, which depends on the compositional strategy of a composer.\textsuperscript{174} Dissonance can likewise “displace the implied location of a melodic realization.”\textsuperscript{175} From perspective of music theory, dissonant elements can function in multiple ways, beneficial to reaching the resolution. An example can be seen in Chopin’s Mazurka Op. 17, no. 1, mm. 9-11 (Figure 3.4). This

\textsuperscript{172} Ibid., 168.
\textsuperscript{175} Ibid., 177.
passage begins with a large melodic leap from C to B♭. The next significant component of dissonance, the F♯, occurs on the first beat of m. 10; this pitch breaks off the descending pattern that was started in m. 9 (B♭-A-F♯) and initializes a melodic motion in an opposite direction towards (F♯-G-A), which ends on the second beat of m. 10 before reaching the closure with a stepwise descent to G on the third beat of m. 10.

![Figure 3.4: Chopin's Mazurka Op. 17 No. 1, mm. 9-10.](image)

From the perspective of harmony, the clearest and the simplest approach to realize a dissonant passage is to resolve such passage on a major or minor consonance. This can be seen in mm. 67-70 Op. 10 No. 3, shown in Figure 3.5. Chopin generates a string of dissonant implications that are realized through pure consonances. Three of the many instances, where a model of implication-realization can be applied, are in m. 67, m. 68, and mm. 69-70. In m. 67, the E7 harmony that spans through beats 1-4 is realized on A major harmony on beat 5, generating a V7/IV – IV progression. Similar concept exists in m. 68, where the G♯7 harmony on beats 1-4 is realized through the C♯ major harmony on beat 5, generating a V7/vi – vi progression. Finally, the dominant seventh and the French

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176 Ibid., 199.
augmented harmonies in m. 69 are realized through the E major harmony in m. 70.

![Image of Chopin's Étude Op. 10 No. 3, mm. 67-70]

Figure 3.5: Chopin's Étude Op. 10 No. 3, mm. 67-70.

These examples of implication and realization are likewise significant from a perspective of performance practice. One of the artistic elements that a pianist applies to signify the moments of dissonance and consonance is *rubato*. The methodology of rubato changed throughout the history of performance. In Chopin's works, the notion of rubato depends on one's "interpretation," "individuality," and the amount of musical meaning that a pianist wishes to apply to an element, such as consonance and dissonance.\(^77\) The flexibility of tempo that a pianist chooses for a passage can be understood through musical connotations and stylistic features of the work.\(^78\) The composer himself, for instance, preferred to play the melody with more rhythmic freedom while keeping the accompaniment close to instructed tempo.\(^79\) This is because Chopin was one of the first pianists to apply the word *rubato* as a "direction in a performance" and not simply as "tempo

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adjustment”.\textsuperscript{180} This occurred as a result of 19\textsuperscript{th}-century performance practice being more flexible and allowing for a more varied interpretational possibilities.\textsuperscript{181} Contrary to modern performance practice, Chopin’s innovative comprehension of tempo rubato represented a "slight delay of the melody with respect to the bass.\textsuperscript{182}

The preceding discussion summarizes the important elements on which the Interval Dissonance Rate will be based. The mathematical and acoustical research presented in previous scholarship allowing one to define and apply the notion of harmonic dissonance to the analysis of Chopin’s compositions. Furthermore, previous approaches to dissonance generate scholarly knowledge, on which the IDR tool is based on, and which leads to a better understanding of Chopin’s many contributions to harmony and vertical tension.

\textsuperscript{181} Nicholas Cook, Beyond the Score: Music as Performance (Oxford: Oxford University Press, 2013), 174.
CHAPTER 4. THE INTERVAL DISSONANCE RATE TECHNIQUE

4.1 About Interval Dissonance Rate

The computational approach to intervallic breakdown of a musical passage (or larger structures) provides beneficial results, as seen in works of Malmberg, Hutchinson & Knopoff, Kameoka & Kuriyagawa, LoPresto, and Huron. In their research, however, all intervals are ranked based on their respective level of consonance and dissonance. Nevertheless, the dissection of every possible harmonic interval presents an opportunity to focus on harmonies from perspective of any two concurrently played notes.\(^{183}\) Furthermore, the segmentation analysis allows for equal analytical comparison of any verticality that is found in the score. The IDR will employ segmentation to analyze for harmonic dissonance, dissecting all possible harmonic dyads in the opening four études of Op. 10. Such practice combines the musical analysis with a mathematical approach and the proposed methodology diminishes the level of subjectivity by emphasizing the objective theoretical examination of musical tension in Chopin’s compositions.\(^{184}\)

The IDR is an analytical system that measures the amount of harmonic dissonance and is based on a formula that can be applied to any piece of tonal music that contains vertical tension. The proposed system calculates the relative percentage of dissonant vertical intervals (that will be labeled as the DI group) and consonant vertical intervals (that will be labeled as the CI group). Furthermore, the IDR integrates analytical and scientific evaluations in non-monophonic Western music, using a modified interval-class vector (modicv) that incorporates the

\(^{183}\) The technique of IDR incorporates vertical dissection employing harmonic dyads.

\(^{184}\) It is important to note that musical tension in a composition is considered a subjective concept.
frequency of recurrent pitches ($p_n$), where the collection of $DI$ and $CI$ intervals contains two sets of four modified interval-classes ($ic$). The $DI$ and $CI$ groups combine to generate a set of all possible harmonic intervals ($TI$).

The traditional interval-class vector ($icv$) is a string of six digits, where each numeral represents one of the six interval-classes, based on the number of semitones ($s$).\footnote{Michiel Schuijer, \textit{Analyzing Atonal Music: Pitch-Class Set Theory and Its Contexts} (Rochester, NY: University of Rochester Press, 2008), 47.} The $icv$, with elements such as $<x_1, x_2, x_3, x_4, x_5, x_6>$, consists of the following:

- The element $<x_1>$ denotes the total amount of minor seconds (1$s$), major sevenths (11$s$), and equivalent intervals.
- The element $<x_2>$ denotes the total amount of major seconds (2$s$), minor sevenths (10$s$), and equivalent intervals.
- The element $<x_3>$ denotes the total amount of minor thirds (3$s$), major sixths (9$s$), and equivalent intervals.
- The element $<x_4>$ denotes the total amount of major thirds (4$s$), minor sixths (8$s$), and equivalent intervals.
- The element $<x_5>$ denotes the total amount of perfect fourth (5$s$), perfect fifths (7$s$), and equivalent intervals.
- The element $<x_6>$ denotes the total amount of tritones (6$s$) and equivalent intervals.
While the \textit{icv} accurately defines the intervallic content of pitch-classes in a set, employing such tool to summarize the level of harmonic dissonances in tonal music produces two main issues.\textsuperscript{186} First, the \textit{icv} functions under the assumption that all intervals are enharmonically equivalent.\textsuperscript{187} For instance, an augmented fifth (A5) is equivalent to a minor sixth (m6), where both intervals generate a distance of 5's, as shown in Figure 4.1. In another example, a major sixth (M6) is equivalent to a diminished seventh (d7), where both intervals generate a distance of 8's, as shown in Figure 4.2. While this is true from the perspective of acoustical perception, it is not true from perspective of notation.\textsuperscript{188} The distance in semitones between two notes does not reveal whether an interval is consonant or dissonant. Therefore, in a tonal context, an augmented fifth will be thought of as a dissonant interval, while a minor sixth will be thought of as a consonant interval. Furthermore, a major sixth is a consonant interval, while a diminished seventh is a dissonant interval.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{dissonant_consonant.png}
\caption{An example of dissonant augmented fifth and consonant minor sixth.}
\end{figure}

\textsuperscript{186} The interval-class vector is commonly used in atonal music. Such tool requires modification for employment in the tonal repertoire.


\textsuperscript{188} This is true from the perspective of perception, since one cannot tell whether an augmented third or a perfect fourth has been played merely from hearing two simultaneously played notes that are five semitones apart. This is, however, not true in the tonal context.
Figure 4.3 present a C augmented triad and an A♭ augmented triad in first inversion. Both triads employ the root, the third, and the fifth as chordal factors. Correspondingly, both triads contain identical pitch-classes and are represented by the same normal form (048). However, in a tonal context, these triads are notated in different ways and while a C augmented triad and an A♭ augmented triad in first inversion share an identical icv, both serve diverse functions in a tonal context. The initial triad in Figure 4.3 consists of two major thirds and an augmented fifth, while the latter triad consists of a major third, a diminished fourth, and a minor sixth. Therefore, enharmonic equivalence should not be utilized when working with dissonance in the Romantic period music of Chopin.

Figure 4.3: The C+ and A♭+6 triads with different intervallic makeups.\(^{190}\)

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\(^{189}\) The notion of prime and normal forms does not change when applying the modified interval-class vector.

\(^{190}\) The symbol “+” represents an augmented triad and the symbol “−” represents a diminished triad.
Second, the traditional icv does not incorporate recurrent pitches and therefore, does not present an interval-class that signifies unisons, octaves, and equivalent harmonic intervals. Since the icv is predominantly used for post-tonal music, the inclusion of such an interval-class is not necessary.\textsuperscript{191} Nevertheless, unisons and octaves are prominent intervals in tonal music, particularly in the works of Chopin.\textsuperscript{192} The significance of such interval-class in Chopin's Études Op. 10 Nos. 1-4 will be shown in Chapter 5. The unisons, octaves, and similar intervals generate major and minor triads, which are the fundamental musical elements for construction of tonal works. The number of times that a pitch repeats in a harmonic set is an important concept of calculating for dissonance, since the reinstated pitches create additional harmonic verticalities that must be included into the computational analysis.

Figure 4.4 shows two different sets of notes with identical icv. While the intervallic content of both sets is alike, the cardinality is not.\textsuperscript{193} As shown, the only source of dissonance in D-F-B is the single tritone between pitches F and B. On the contrary, the second set has a cardinality of five, with pitches D and F repeated. The second chord incorporates more tension with the supplementary notes, producing two tritones.

\textsuperscript{191} David Huron, "Interval-Class Content in Equally Tempered Pitch-Class sets: Common Scales Exhibit Optimum Tonal Consonance," \textit{Music Perception: An Interdisciplinary Journal} 11, no. 3 (Spring, 1994): 292.
\textsuperscript{192} Jim Samson, \textit{Chopin Studies, Volume 1} (Cambridge: Cambridge University Press, 1988), 145. An example can be seen in Chopin's Étude Op. 10 No. 1, where the composer utilizes only the unisons and octaves throughout the whole work in the left hand.
\textsuperscript{193} For more information on function of a set based on its cardinality, see Eric J Isaacson, "Similarity of Interval-Class Content Between Pitch-Class Sets: The ICVSIM Relation," \textit{Journal of Music Theory} 34, no. 1 (Spring 1990).
4.2. The modified icv

The *modicv* is derived from the Morris’s notion of a multiset. The *modicv* is a revised version of *icv*, modified to accurately represent the intervallic content of tonal music. The total number of intervals (*TI*) found in *modicv* consist of two intervallic groups – the consonant intervals groups (*CI*) and the dissonant intervals group (*DI*), where each group holds four modified interval-classes. Each interval must belong to a group and no interval may belong to more than one *icv*. The interval-classes of *modicv* are as follows:

- The element $<c_1>$ consists of consonant intervals, such as unisons ($0$s), octaves ($12$s), and equivalent intervals.
- The element $<c_2>$ consists of consonant intervals, such as minor third ($3$s), major sixth ($9$s), and equivalent intervals.
- The element $<c_3>$ consists of consonant intervals, such as major third ($4$s), minor sixth ($8$s), and equivalent intervals.

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194 Morris incorporates the use of multisets to include the multiple occurrences of pitch-classes in a set. Robert Morris, “Mathematics and the Twelve-Tone System: Past, Present, and Future,” *Perspectives of New Music* 45, no. 2 (Summer, 2007): 98.
• The element \(c_4\) consists of consonant intervals, such as perfect fourth (5s), perfect fifth (7s), and equivalent intervals.

• The element \(d_1\) consists of dissonant intervals, such as minor seconds (1s), major sevenths (11s), and equivalent intervals.

• The element \(d_2\) consists of dissonant intervals, such as major seconds (2s), minor sevenths (10s), and equivalent intervals.

• The element \(d_3\) consists of dissonant intervals, such as tritones (6s), and equivalent intervals.

• The element \(d_4\) consists of all dissonant intervals that are enharmonically equivalent to consonances.

It is important to mention that the ideas of consonance and dissonance are context-specific and many intervals can be thought of as both consonant and dissonant, depending on the music period and genre. The IDR focuses on defining consonant and dissonant intervals, based on the pre-20th century view of Western tonality. Therefore, while such system is appropriate for Chopin’s music and presents definite results, it would require modification and adjustment, when working with compositions of other composers, such as Debussy, or non-classical styles, such as jazz music.\(^{195}\)

\(^{195}\) For more information on dissonance in Debussy’s music, refer to: Robert Schmitz, The Piano Works of Claude Debussy (New York: Dover, 1966). For more information on dissonance in jazz music, refer to: Ajay Heble, Landing on the Wrong Note: Jazz, Dissonance, and Critical Practice (London: Routledge, 2000).
One of the notable examples is the understanding of musical tension in major and minor thirds, since such intervals were considered as dissonant in the pre-Odington era and as consonant in the post-Odington period. Another example is the perception of the harmonic ninths. According to the IDR, a minor and a major ninth is equivalent to a minor and a major second and therefore, both are to be considered as dissonant intervals. Such instance can be seen in Figure 4.5 in mm. 1-7 of Scriabin’s *Trois Études* Op. 65 No. 1, where a series of ninths generate a dissonant harmony in the need of resolution. Such resolution, however, never occurs.

![Figure 4.5: Scriabin's *Trois Études* Op. 65 No. 1, mm. 1-7.](image)

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197 This is an instance of Scriabin’s late period works, where the composer bases the whole composition on the interval of a ninth.
However, a ninth in the works of Debussy, whose compositional strategy includes consideration for and emphasis on sevenths, ninths, and eleventh chords, sounds consonant. This can be seen in Figure 4.6 in m 13-15 of *La Cathédrale Engloutie*. In this instance, the ninth is generated from the lowest and the highest pitches of the left hand in mm. 14-15. Such series of harmonies is tonally stable and is not in need of resolution.

![Figure 4.6: Debussy's *La Cathédrale Engloutie*, mm. 13-15.](image)

As mentioned in Chapter 2, one of the most common approaches to working with harmonic dissonance involves the ranking of harmony from least to most consonant. While the IDR does not rank the dissonances, it bifurcates them between two groups. The CI-group contains a total of eight intervals. These include four perfect intervals, two major intervals, and two minor intervals: perfect unison, minor third, major third, perfect fourth, perfect fifth, minor sixth, major sixth,

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198 Such a unique treatment of harmony is not a frequent occurrence in the common-practice period of the Western music, yet the use of sevenths, ninths, and elevenths is an important characteristic of Debussy’s compositional style.

199 The IDR can likewise calculate the frequency of a specific ic found in the score. For instance, Scriabin incorporates 16.5% of ic(d2) in Prelude Op. 11 No. 2 in A minor. This accounts for 56.16% of total dissonance found in the work.


200 There are four sub-groups of the \( c_x \), where each \( c_x \) contains two intervals.
and perfect octave. The $ DI $-group amalgamates minor, major, diminished, and augmented intervals: minor second, major second, tritone, minor seventh, major seventh, as well as all possible diminished and augmented intervals.\textsuperscript{201} Figure 4.7 shows the range of possible $ TI $ subdivisions.

![Diagram of Total Intervals (TI), Consonant Intervals (CI), and Dissonant Intervals (DI)]

Figure 4.7: The generation of the eight $ ic $ for construction of IDR.

There are two steps that need to be taken to calculate the IDR of a harmonic set. First, one needs to calculate the $ modicv $. Second, one needs to divide the $ DI $ group by the sum of all possible intervals ($ TI $). The IDR is calculated based on the following conditions:

\[
IDR \ (% \ ) = \frac{TI}{DI}, \text{ where } \{CI \ & DI\} = \modicv(x) + p_n \\
DI = \{d_1, d_2, d_3, d_4\}; \ CI = \{c_1, c_2, c_3, c_4\}
\]

4.3. The IDR of Trichords, Tetrachords, and Pentachords

Functional harmony in a tonal composition tends to heavily rely on the four main types of triads that appear in a diatonic scale, shown in Figure 4.8: major, minor, diminished, and augmented. The major and the minor triads are consonant, while

\textsuperscript{201} Every possible interval in both $ CI $ and $ DI $ groups can have a diminished and an augmented interval. Therefore, for each major, minor, or perfect interval, there exist two possible $ ic(d_i) $ intervals.
the diminished and the augmented triads are dissonant. Both major and minor triads have two similar features: an identical prime form (047) and an identical icv of <001110>. Furthermore, both major and minor triads in root position generate a total of three intervals: a minor third, a major third, and a perfect fifth. However, the order of the major and the minor thirds in each triad, which is influenced by the location of the third scale degree, makes the major and the minor triads different. The modicv of both triads is <01110000>. Since each triad consists of three pitches, one can conclude that $Cl = 3$ and $Di = 0$. Therefore, the IDR of C-E-G and C-E♭-G is 0.00%, (as $0/3 = 0$). It is also important to note that the number of $p_n$ will not affect either of these triads, since no dissonant intervals exist.

![Figure 4.8: The IDR of C major, C minor, C diminished, and C augmented triads.](image)

The diminished triad in root position that is made of three notes contains two stacked minor thirds and a diminished fifth. The prime form of a diminished triad is (036) and its icv is <002001>. The diminished fifth is the source of dissonance in this triad. The modicv of a diminished triad is <02000010>. Based on the modicv, it is evident that $Cl = 2$, $Di = 1$, and $IDR = 33.33\%$. A root-position augmented triad of identical cardinality contains two stacked major thirds and an augmented fifth. Its prime form is (048) and its icv is <000300>. The modicv of an augmented triad is <00200001>. Based on the modicv, it is evident that $Cl = 2$, $Di = 1$, and therefore, the $IDR = 33.33\%$. Since the IDR does not rank the intervals
on the level of dissonance, it can be concluded that the IDR of both diminished and augmented triads are identical.

An augmented triad is an example of a situation in which the $d_4$ interval-class is used. Enharmonically speaking, the augmented fifth is analogous to a minor sixth, and both are equal to eight semitones. This proves that the icv of a set cannot be used to accurately calculate the dissonance rate. Based on the icv of an augmented triad, its IDR would be identical to the IDR of purely consonant major and minor triads.

While tonal music is principally based on major, minor, diminished, and augmented triads, other three-note combinations likewise exist. Table 4.1 shows some of the trichords and their respective IDR. All trichords are ranked from most dissonant to most consonant. The following table also contains the notes of a triad, its icv, its modicv, as well as a possible musical location of such harmonic set. Based on the level of dissonance that they produce, all trichords can be perceived differently. However, the mathematical approach limits the IDR of all trichords with no recurrent pitches to four dissonance levels: 0.00%, 33.33%, 66.66%, and 100.00%. This occurs because any three notes can produce only

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202 There are multiple views on what constitutes a triad. Ronald Pen states that a triad is a set of three notes that must contain successive intervals of a third. Therefore, a triad requires a root, a third, and a fifth. On the contrary, Howard Hanson and Carlton Gamer state that a triad is a combination of any three unique pitches and there should not be any intervallic limitations in between such pitches. For clarity, any set of three consecutively played notes will be labeled as a trichord.
203 0.00% will be known as Pure Harmonic Consonance. This will be a significant term in the analysis of Chopin’s Études.
three intervals. The larger the cardinality of a harmonic set, the higher variety of dissonance there will be.

Table 4.1: Other common trichords with their respective IDR.

<table>
<thead>
<tr>
<th>Harmonic Trichord</th>
<th>&lt;icv&gt;</th>
<th>&lt;modicv&gt;</th>
<th>IDR%</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>[C-C♯-D]</td>
<td>&lt;210000&gt;</td>
<td>&lt;00001101&gt;</td>
<td>100.00%</td>
<td>Subset of a chromatic scale</td>
</tr>
<tr>
<td>[C-C♯-D♯]</td>
<td>&lt;111000&gt;</td>
<td>&lt;00000102&gt;</td>
<td>100.00%</td>
<td>Subset of an octatonic scale</td>
</tr>
<tr>
<td>[C-E-F♯]</td>
<td>&lt;100011&gt;</td>
<td>&lt;00100110&gt;</td>
<td>66.66%</td>
<td>Subset of a whole-tone scale</td>
</tr>
<tr>
<td>[C-D-E]</td>
<td>&lt;020100&gt;</td>
<td>&lt;00100200&gt;</td>
<td>66.66%</td>
<td>Subset of a whole-tone and diatonic scales</td>
</tr>
<tr>
<td>[C-D-F♯]</td>
<td>&lt;010101&gt;</td>
<td>&lt;00100110&gt;</td>
<td>66.66%</td>
<td>Subset of a dominant seventh chord</td>
</tr>
<tr>
<td>[C-D-F]</td>
<td>&lt;011010&gt;</td>
<td>&lt;01010100&gt;</td>
<td>33.33%</td>
<td>Subset of a diatonic scale</td>
</tr>
<tr>
<td>[C-C♯-E]</td>
<td>&lt;101100&gt;</td>
<td>&lt;01100001&gt;</td>
<td>33.33%</td>
<td>Subset of a hexatonic scale</td>
</tr>
<tr>
<td>[C-E-F]</td>
<td>&lt;100110&gt;</td>
<td>&lt;00111000&gt;</td>
<td>33.33%</td>
<td>Subset of a diatonic scale</td>
</tr>
<tr>
<td>[C-D-G]</td>
<td>&lt;010020&gt;</td>
<td>&lt;00020100&gt;</td>
<td>33.33%</td>
<td>Subset of a diatonic scale</td>
</tr>
<tr>
<td>[C-E♭-G♭]</td>
<td>&lt;002001&gt;</td>
<td>&lt;02000010&gt;</td>
<td>33.33%</td>
<td>A diminished triad</td>
</tr>
<tr>
<td>[C-E-G]</td>
<td>&lt;000300&gt;</td>
<td>&lt;00200001&gt;</td>
<td>33.33%</td>
<td>An augmented triad</td>
</tr>
<tr>
<td>[C-E-G]</td>
<td>&lt;001110&gt;</td>
<td>&lt;01110000&gt;</td>
<td>0.00%</td>
<td>Major triad</td>
</tr>
<tr>
<td>[C-E♭-G]</td>
<td>&lt;001110&gt;</td>
<td>&lt;01110000&gt;</td>
<td>0.00%</td>
<td>Minor triad</td>
</tr>
</tbody>
</table>

---

204 The amount of TI in a set can be calculated by combinatorial probability formula, such as

\[ C(n, r) = \frac{n!}{(n-r)!r!} \]

The variable \( n \) represents the number of notes in a harmonic set. Furthermore, \( r = 2 \), where \( r \) represents the possible combinations of an interval consisting of no more than two notes. Therefore, a harmonic set made of three notes produces three harmonic intervals; a harmonic set made of four notes produces six harmonic intervals; a harmonic set made of five notes produces ten harmonic intervals; and so on.

The trichords [C-C♯-D] and [C-C♯-D♯] produce an IDR of 100.00%. This means that such harmonic sets will incorporate zero consonance. Most of the trichords, however, will produce an IDR of either 33.33% or 66.66%, since there will be at least one or two dissonances in the set. There are only a handful of trichords that will produce a perfect harmonic consonance; an example of such trichords are all major and minor triads.205

In addition to trichords, there are multiple types of seventh chords that exist in Western music, all of which have distinct functions.206 The history of music theory reveals a rich tradition of various composers who utilize seventh chords in unique ways. Further, while the exact reasoning for the use of seventh chords lies in the compositional approach of each individual composer, it is a truism that one of the common purposes for the use of seventh chords is the generation of harmonic tension. Each of the seventh chords, made up of four notes with no recurrent pitches, will produce six harmonic intervals. Therefore, there are six possible IDR rates for pitch collections that consist of four unique notes: 0.00%, 16.66%, 33.33%, 50.00%, 66.66%, 83.33%.

It is important to note, however, that while the analysis of IDR presents a set of categorial variables when analyzing trichords and seventh chords, this is diversified with the inclusion of additional intervals. For instance, a musical passage with twenty harmonic intervals contains 190 dissonance possibilities. The

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205 The major and minor triads are not the only three-note harmonic sets that combine to form perfect harmonic consonance. However, a perfect harmonic consonance must be based on at least one of the scale degrees of the tonic triad.
first four Chopin’s études contain few thousand harmonic intervals. For comparison, the opening étude consists of 3196 harmonic intervals – the lowest of the four works. Based on the combinatorial probability formula, there are a total of 5,105,610 possible IDR values with such number of harmonic intervals.

Graph 4.1 presents the contrast between the harmonic dissonance according to IDR and Huron’s Optimum Consonance Measure. All values have been normalized on the scale of 0-1, where 0 is equivalent to maximum dissonance and 1 is equivalent to maximum consonance.

Graph 4.1: IDR and Optimum Consonance Measure comparison for trichords on a scale of 0 to 1, where 0 = highest level of dissonance and 1 = highest level of consonance.

The IDR of 0.00% is possible in a four-note collection. However, an IDR of 0.00% is impossible in a seventh chord, since at least one of the intervals of the seventh chord, (the seventh), will always be dissonant and be part of either interval-class $d_3$ or $d_4$. The most frequent types of seventh chords in tonal music are dominant seventh chord, minor seventh chord, leading tone seventh chord,
half-diminished seventh chord, and major seventh chord. Table 4.2 presents the most common seventh chords in tonal music, its interval-class vector, modified interval-class vector, and each chord’s respective IDR.

### Table 4.2: Most common seventh chords with their respective IDR.

<table>
<thead>
<tr>
<th>Harmonic Tetrachord</th>
<th>Example</th>
<th>(&lt;icv&gt;)</th>
<th>(&lt;modicv&gt;)</th>
<th>IDR%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major seventh chord</td>
<td>[C-E-G-B]</td>
<td>&lt;101220&gt;</td>
<td>&lt;0121000&gt;</td>
<td>16.66%</td>
</tr>
<tr>
<td>Minor seventh chord</td>
<td>[C-E♭-G-B♭]</td>
<td>&lt;012120&gt;</td>
<td>&lt;02120100&gt;</td>
<td>16.66%</td>
</tr>
<tr>
<td>Minor major seventh chord</td>
<td>[C-E♭-G-B]</td>
<td>&lt;101310&gt;</td>
<td>&lt;01211001&gt;</td>
<td>16.66%</td>
</tr>
<tr>
<td>Augmented major seventh chord</td>
<td>[C-E-G♯-B]</td>
<td>&lt;101310&gt;</td>
<td>&lt;01211001&gt;</td>
<td>16.66%</td>
</tr>
<tr>
<td>Dominant seventh chord</td>
<td>[C-E-G-B♭]</td>
<td>&lt;012111&gt;</td>
<td>&lt;02110110&gt;</td>
<td>33.33%</td>
</tr>
<tr>
<td>Half-diminished seventh chord</td>
<td>[C-E♭-G-B♭]</td>
<td>&lt;012111&gt;</td>
<td>&lt;02110110&gt;</td>
<td>33.33%</td>
</tr>
<tr>
<td>Fully-diminished seventh chord</td>
<td>[C-E♭-G-B♭♭]</td>
<td>&lt;004002&gt;</td>
<td>&lt;0200022&gt;</td>
<td>66.66%</td>
</tr>
<tr>
<td>Augmented minor seventh chord</td>
<td>[C-E-G♯-B♭]</td>
<td>&lt;020301&gt;</td>
<td>&lt;00200112&gt;</td>
<td>66.66%</td>
</tr>
<tr>
<td>Diminished major seventh chord</td>
<td>[C-E♭-G-B♭]</td>
<td>&lt;102111&gt;</td>
<td>&lt;02001021&gt;</td>
<td>66.66%</td>
</tr>
<tr>
<td>Dominant seventh flat five chord</td>
<td>[C-E-G-B♭]</td>
<td>&lt;020202&gt;</td>
<td>&lt;00200121&gt;</td>
<td>66.66%</td>
</tr>
</tbody>
</table>

Graph 4.2 shows the comparison between the harmonic dissonance of tetrachords according to IDR and Huron’s Optimum Consonance Measure. All values have been normalized on the scale of 0-1, where 0 is equivalent to maximum dissonance and 1 is equivalent to maximum consonance.

---

Graph 4.2: IDR and Optimum Consonance Measure comparison for tetrachords on a scale of 0 to 1, where 0 = highest level of dissonance and 1 = highest level of consonance.

As stated previously, the frequency of recurrent pitches likewise plays a role in the dissonance content of a harmony. This occurs because a supplemental note (or a set of notes) can create additional interval-class verticalities in the makeup of the TI. For instance, a dominant G\(^7\) chord, where each pitch is only used once, will generate an IDR of 33.33%. This happens because the harmonic relationship between pitches G, B, D, and F contains two dissonant and four consonant intervals. However, if a supplementary pitch G is implemented to the dominant G\(^7\) chord, the IDR decreases to 30.00%. This occurs because the harmonic relationship between pitches G, G, B, D, and F contains three dissonant and seven consonant intervals. As evident, a difference of 3.33% exists while no new pitch-classes are incorporated. This is shown in Figure 4.9.
Figure 4.9: The G\textsuperscript{7} chord with IDR of 33.33\% and 30.00\%.

The IDR of the two chords in Figure 4.10 has a difference of 3.33\% – a small change for a supplemental root note to generate. On the other hand, a similar differential rate between two full pieces of music is significant enough to reveal a stylistic change in the way both works generate harmonic dissonance. Typically, consonance and dissonance in a piece of music can be produced by pitch doubling and, therefore, the same notes can be employed to increase or decrease tension in music. As seen from previous example, the IDR of a dominant seventh chord that uses each of the pitches once is 33.33\%.

Figure 4.10: The C\textsuperscript{7} chord with IDR of 13.33\% and 40.00\%.

On the contrary, the first chord in Figure 4.10 is a dominant seventh chord that uses the fifth as a chordal factor three times, and the root, the third, and the seventh scale degrees once, generating the IDR of 13.33\%, creating a difference of 20.00\% when compared to a basic dominant seventh chord. The first chord in
Figure 4.10 contains thirteen dissonant and two consonant intervals. The second chord in Figure 4.10 is a dominant seventh chord that uses the seventh scale degree three times, and the root, the third, and the fifth scale degrees once, generating the IDR of 40.00%, creating a difference of 6.66% when compared to a basic dominant seventh chord and creating a difference of 26.66% when compared to the first chord. In the second scenario, there are nine consonant intervals and six dissonant intervals.

While not as frequent as the seventh chords, the ninth chords are likewise significant to the harmony of Chopin’s music. In tonal music, the common practice is to incorporate the 3rd, 7th, and the 9th scale degrees of the ninth chord, while omitting the 5th scale degree. In this instance, the ninth chord is to be analyzed as a tetrachord. However, there are occurrences, in which all five pitches of the ninth chord are employed in a harmonic verticality. Table 4.3 shows a series of ninth chords prominent to the analysis of Romantic music.

Table 4.3: The IDR analysis of ninth chords.

<table>
<thead>
<tr>
<th>Harmonic Pentachords</th>
<th>Example</th>
<th>&lt;icv&gt;</th>
<th>&lt;modicv&gt;</th>
<th>IDR%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minor ninth chord</td>
<td>[C-E♭-G B♭-D]</td>
<td>&lt;122230&gt;</td>
<td>&lt;02231200&gt;</td>
<td>30.00%</td>
</tr>
<tr>
<td>Major ninth chord</td>
<td>[C-E-G-B-D]</td>
<td>&lt;122230&gt;</td>
<td>&lt;02231200&gt;</td>
<td>30.00%</td>
</tr>
<tr>
<td>Dominant ninth chord</td>
<td>[C-E-G-B♭-D]</td>
<td>&lt;032221&gt;</td>
<td>&lt;02220310&gt;</td>
<td>40.00%</td>
</tr>
<tr>
<td>Dominant minor ninth chord</td>
<td>[C-E-G-B♭-D♭]</td>
<td>&lt;114112&gt;</td>
<td>&lt;03111121&gt;</td>
<td>50.00%</td>
</tr>
</tbody>
</table>

The dissonance range between the least dissonant chords (minor ninth and major ninth) and the most dissonant chord (dominant minor ninth) is 20%. However, two of the four chords share identical IDR. This occurs because the IDR does not rank the intervals based on their dissonance level and therefore, if a harmonic set has 10 interval-classes and three of them are dissonant, the IDR will always be 30%.

4.4. Meter and Rhythm

Meter plays a significant role in the perception of consonance and dissonance. The meter of a given work provides one with information regarding the strong and the weak beats in each measure, as well as presenting a suggested sequence of pulses. Furthermore, such emphasis on the strong and the weak beats changes depending on the nature of the rhythm and alters our awareness of music tension. Table 4.4 presents multiple examples of meter in music and provides an appropriate time signature for each.

<table>
<thead>
<tr>
<th>Simple Duple Meter</th>
<th>Simple Triple Meter</th>
<th>Simple Quadruple Meter</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/2; 2/4; 2/8</td>
<td>3/2; 3/4; 3/8</td>
<td>4/2; 4/4; 4/8</td>
</tr>
<tr>
<td>Compound Duple Meter</td>
<td>Triple Compound Meter</td>
<td>Quadruple Compound Meter</td>
</tr>
<tr>
<td>6/4; 6/8; 6/16</td>
<td>9/4; 9/8; 9/16</td>
<td>12/4; 12/8; 12/16</td>
</tr>
</tbody>
</table>

The main beats in each meter are emphasized by the metric accents. For instance, the first beat (the strong beat) of the 3/4 meter is accented, while the
second and the third beats (the weak beats) of 3/4 meter are unaccented. Other
types of accents, such as dynamic accent, tonic accent, and agogic accent are
likewise prominent. Whether part of melody or harmony falls on accentuated
division of the measure is an important conception both from theoretical and
performance perspectives, since accented beats will generate a higher level of
perceived dissonance. This occurs because of variance in the notions of internal
and external lengths of notes. The internal length of a note is how such note is
rhythmically represented in the score. From internal perspective, a metrically
accented quarter note is identical to metrically unaccented quarter note. However,
from external perspective, both notes would differ in duration, the concept that is
associated with performance practice. According to Caplin, this means that the
"position of the note within the measure is sufficient to impart accentuation in the
absence of any real durational or dynamic differentiation."

Another important component of consonance and dissonance can be traced
through non-chord tones, some of which are suspensions, passing tones, and
neighbor tones. The placement of these non-chord tones likewise affects one's
perception of consonance and dissonance. Figure 4.11 presents instances of two
harmonic successions. From the perspective of interval-class vector, both
examples can be represented identically: <001000>, <100000>, <001000>,
<010000>, and <000100>.

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210 Ibid., 662.
However, an evident difference exists in the way both passages sounds and therefore, both structures will produce different perceptions of dissonance. The second passage is generated by moving the top voice down the octave. Furthermore, the lower voice is responsible for generating dissonance in the first passage and the upper voice is responsible for generating dissonance in the second passage.

![Figure 4.11: Two passages with identical harmonic icv, but different perceptions of dissonance.](image)

Rhythm is one of the main characteristics of music that allows one to understand a compositional language. The notion of rhythm amalgamates the elements of pitch and duration, and therefore, influences the outcome of melodic and harmonic dissonance. One of the ways how a composer may vary and deviate the rhythm of a work is by using tuplets, allowing for division of a beat in different number of subdivisions. In Chopin’s works, tuplets are essential to create harmonic dissonance. The IDR treats tuplets the same way as other harmonic rhythms. Figure 4.12 shows how a triplet will be dissected in the IDR analysis. In this instance, the triplet is in the treble stave, while the regular rhythm is provided

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in the bass stave. Such a triplet generates six harmonic blocks, each with its own
ic. Therefore, there are seven possible IDR values in this measure.

Figure 4.12: The IDR analysis of a triplet (3 vs. 2).

Chopin’s Op. 10 No. 3 is the first étude of Op. 10 to utilize triplets. This happens in mm. 54-61, the transition before the entrance to A’ section. An important concept of this passage is the variety of rhythms that Chopin includes in the three voices. The primary melody in the right hand consist of eighth notes and sixteenth notes. The secondary melody in the left hand consists of quarter notes, eighth notes, and sets of sixteenth triplets. The accompanimental melody is in the middle voice and is played by the right hand, consisting of purely sixteenth notes.

In mm. 54-61, Chopin includes five sets of left hand triplets, creating music material depicting ornamental elaboration in m. 55, 57, 58, 59, and 60. This can be seen in Figure 4.13. The IDR of this passage is 36.72%.

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213 Other known works by Chopin that utilize tuplets are: 
*Fantaisie-Impromptu* in C♯ minor, Op. 66 (4 vs. 3), 
Nocturne in E minor, Op. 72 No. 1 (2 vs. 3).

Jim Samsons states that Chopin used triplets to create “formulas” for both the melody and the harmony.

While Op. 10 Nos. 1-4 only contain one instance of tuplets, similar concept can be applied to all tuplet subdivisions. Figure 4.14 presents how IDR analysis dissects an instance of quadruplet.

![Figure 4.14: The IDR analysis of a quadruplet (4 vs. 3).]


As mentioned earlier, the IDR can be used to test for dissonance of a specific passage in music. The texture of each of Chopin's études is dependent on the technical challenge that the composer chose for each work. The following
subchapter presents the IDR analysis of the four opening phrases of Études Nos. 1-4, each with its own unique harmonic dissonance rate.

The opening phrase of Op. 10 No. 1 occurs in mm. 1-8. The phrase ends with a half cadence that would go on to resolve to tonic in mm. 9-10. The IDR of this phrase is 15.96%. The passage begins with purely consonant harmonies in mm. 1, 2, and 3, using only the root, the third, and the fifth chordal factors of C major (tonic) and F major (subdominant) harmonies, before the harmonic dissonance begins to increase. It is also important to note that the octaves in the left hand generate the ic1 consonance at all times throughout the étude.

The opening phrase of Op. 10 No. 2 occurs in mm. 1-4. Much like the opening phrase of Op. 10 No. 1, this phrase ends on a half cadence (E7). The IDR of mm. 1-4 of the second étude is 16.51%. The harmonic consonance is created from the major and minor triadic structures, found in the left hand’s accompaniment and in the right hand’s inner voices. The harmonic dissonance, on the other hand, is produced from the chromatic ascent and descent of the main melody of the passage. For instance, the opening harmonic block in m. 1 contains the A minor triad with IDR of 0.00%.214 However, the following harmonic block of m. 1 contains the chromatic ascent of an augmented unison from A to A♯, generating dissonant interval-classes between the pitch A♯ and the eighth note A, found in the left hand. Another instance of an increase in dissonance occurs on the ninth harmonic block of m. 2, where the pitch C♯, being a part of the chromatic path in the main melody,

214 Each measure in Op. 10 No. 2 contains a total of sixteen harmonic blocks.
contradicts the A minor harmony in the measure, generated in the left hand, creating a harmonic tension between pitches C and C♯.

The opening phrase of Op. 10 No. 3 occurs in mm. 1-5. Similar to the opening phrase of Op. 10 No. 1, this phrase is made up of three essential musical layers: the main melody in the right hand, the secondary melody in the left hand, and the inner voices in both right and left hands. Unlike the opening phrases of Op. 10 Nos. 1 and 2, the opening phrase of the third étude consists of five measures and ends on tonic of E major (perfect authentic cadence). The unique characteristic of this phrase is the necessity of it to be performed unlike an étude, incorporating a *cantabile* style and creating polyphonic depiction among the three voices. The *IDR* of mm. 1-5 is 15.00%. This is a typical tonal Chopinesque melody that reveals small traces of harmonic dissonance on unstable harmonies that need resolution. This can be seen on the second beat of m. 1, where Chopin includes a V7 harmony; the pitch A (seventh scale degree) is dissonant against the pitch B (root) and the pitch D♯ (third). In this instance, the V7 harmony engulfs the full second beat of m. 1, consisting of harmonic blocks 5-8, and the first beat of m. 2, consisting of harmonic blocks 1-4. In both instances, the inner voice of the passage interchanges pitch A, which is part of a pure consonance, and pitch B that generates harmonic dissonance.

The opening phrase of Op. 10 No. 4 occurs in mm. 1-4. Like the opening phrase in Op. 10 No. 3, Chopin chooses to employ an anacrusis measure to create a stronger entrance into the tonic harmony. The harmonic dissonance is implemented in mm. 1-5 of Op. 10 No. 3 in the inner voice. On the contrary, the
harmonic dissonance is implemented in mm. 1-4 of Op. 10 No. 4 in the main melody in the right hand that consists of a series of sixteenth notes, which move against the subordinate musical material in the left hand. When looking at the IDR of the musical material found in the left hand, it is evident that the only harmonic dissonance exists on beat four of m. 2 and beat two of m. 3. Nevertheless, the inclusion of the right hand’s carrying the main melody of the étude generates chromatic harmony and increases the level of harmonic tension with the IDR of 17.93%.

4.6. Computing Components of IDR

There are four components of the proposed technique: (1) pitch representation; (2) interval-class representation; (3) range parameters; (4) the IDR calculations. The pitch representation component allows one to turn the pitch name from the music score into a number. The total of twelve recognized pitch-classes will generate twelve values from 0 to 12. However, since the concept of enharmonic equivalence does not exist in the IDR, this means that the same number cannot represent two enharmonically equivalent pitches. Therefore, each note (n) will contain five assigned values: double flat (♭♭), flat (♭), natural (♮), sharp (♯), and double sharp (xiv). Since there exist seven unique notes in a diatonic scale, this generates a total of thirty-five values for n. A variable (r) is used to represent a music rest and does not carry any mathematical weight towards the analysis. The script can be viewed in Appendix 9.

215 The script uses the “#” symbol to represent the “♯” and the “♭” symbol to represent the “♭.” The pitch is assumed as natural, if neither of the symbols are used.
Each harmonic interval is generated with the addition of two $n$ values (the combination of two pitch-classes). The sequence of all possible $n$ values, such as \{0,1,3,7…\} has an additive property, according to which the combination of any two $n$ values will never generate an identical answer, something that helps avoid the enharmonic equivalence.

\[ n_a + n_b = n_{a+b} \]
\[ n_c + n_d = n_{c+d} \]
\[ n_{a+b} - n_{c+d} \neq 0 \]

There are 630 unique values that exist when any two $n$ values are added. This means that there are that many different harmonic intervals that exist between any two notes of a diatonic scale, assuming octave equivalence. Each of the 630 values, which represent harmonic intervals, is assigned to one of the eight interval classes of the modicv. This is done using a $v$-lookup table that allows one to match $ic_x$ to one of the 630 values. Each value can only belong to one interval-class.

The range parameters in the IDR analysis exist in order for one to select the music that needs to be analyzed. The range parameter depends on the amount of voices in the piece (or the passage). For instance, a composition with a maximum of three musical layers, such as $a$, $b$, and $c$, will consist of three lines, each representing an interval class $ab$, $bc$, and $ac$. This is the case with Chopin’s Étude Op. 10 No. 1. On the contrary, Étude Op. 10 No. 2 contains a maximum of seven musical layers, such as $a$, $b$, $c$, $d$, $e$, $f$, and $g$, generating twenty-one lines. The parameters must state the beginning and the end of the chosen passage.
The IDR calculations are based on the parameters that are chosen for an analysis. The IDR calculations consist of interval-class calculations, interval-group calculations, and the final Interval Dissonance Rate calculations. The occurrence of interval-class such as $c_x$ or $d_x$ is computed with the use of countif and indirect functions. An indirect function is used because the range parameters are references rather than statements that are written in. The CI and DI calculations are computed by adding the $c_x$ and $d_x$. Finally, the IDR is computed with dividing the total number of dissonant intervals by the total number of consonant intervals.
CHAPTER 5. QUANTIFYING CHOPIN’S ÉTUDES NOS. 1-4

5.1 Overview

The IDR calculations provide one with a large amount of quantitative data, which allows for a broad assessable study of the four Chopin’s works. In addition, the retrieved data allows one to focus on specific interval-classes and their relations to Chopin’s music, his style of composition, and chromatic harmony. The IDR results consist of statistical information that clarifies multiple components of harmonic dissonance in the opening four études. These are:

1) Chopin’s most frequent dissonant interval-class is $\Delta_2$, which includes major seconds, minor sevenths, and equivalent intervals.

2) The IDR analysis supports the notion that the use of chromaticism does not result in a greater level of harmonic dissonance.

3) Each of the four B-sections are more dissonant than the average IDR of their respective études. Likewise, the middle section is the most dissonant part of each étude.\(^{216}\)

Table 5.1 presents the results retrieved from the IDR analysis of Chopin’s Op. 10 Nos. 1-4. For every étude, as well as each composition’s three sections of the A-B-A’ form, the table shows the percentile values of the total interval dissonance rate, as well as each distinct interval-class. The total percent of the IDR is equivalent to the sum of the all four dissonant interval-classes. Based on such an analysis, each interval-class in both consonant and dissonant groups is

\(^{216}\) The Average IDR of each étude consists of sections A, B, and A’.
ranked equally. The large amount of unisons, octaves, and similar intervals is likewise important to consider, specifically in the opening étude. Another IDR component to consider is the $d_4$ interval-class.

Table 5.1: The IDR analysis of Op. 10 Nos. 1-4.

<table>
<thead>
<tr>
<th>Op. 10 No. 1</th>
<th>IDR%</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
<th>$\sigma$ (TI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Composition</td>
<td>14.14</td>
<td>47.87</td>
<td>6.24</td>
<td>13.21</td>
<td>18.55</td>
<td>2.12</td>
<td>7.02</td>
<td>4.12</td>
<td>0.87</td>
<td>15.45</td>
</tr>
<tr>
<td>Section A (bars 1-24)</td>
<td>11.88</td>
<td>46.81</td>
<td>7.62</td>
<td>13.12</td>
<td>20.57</td>
<td>1.42</td>
<td>4.96</td>
<td>4.61</td>
<td>0.89</td>
<td>15.32</td>
</tr>
<tr>
<td>Section B (bars 25-48)</td>
<td>16.16</td>
<td>50.00</td>
<td>5.08</td>
<td>12.70</td>
<td>16.06</td>
<td>1.93</td>
<td>10.16</td>
<td>3.25</td>
<td>0.81</td>
<td>16.09</td>
</tr>
<tr>
<td>Section A' (bars 49-79)</td>
<td>14.55</td>
<td>47.19</td>
<td>5.93</td>
<td>13.65</td>
<td>18.68</td>
<td>2.85</td>
<td>6.45</td>
<td>4.35</td>
<td>0.90</td>
<td>15.20</td>
</tr>
<tr>
<td>$\sigma (A, B, A')$</td>
<td>2.16</td>
<td>1.74</td>
<td>1.30</td>
<td>0.48</td>
<td>2.27</td>
<td>0.73</td>
<td>2.68</td>
<td>0.72</td>
<td>0.05</td>
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<tr>
<th>Op. 10 No. 2</th>
<th>IDR%</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
<th>$\sigma$ (TI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section A (bars 1-17)</td>
<td>14.91</td>
<td>16.42</td>
<td>20.82</td>
<td>22.02</td>
<td>25.83</td>
<td>3.10</td>
<td>4.20</td>
<td>3.40</td>
<td>4.20</td>
<td>9.72</td>
</tr>
<tr>
<td>Section A' (bars 36-49)</td>
<td>14.53</td>
<td>22.91</td>
<td>17.23</td>
<td>21.66</td>
<td>23.68</td>
<td>3.46</td>
<td>3.85</td>
<td>2.31</td>
<td>4.91</td>
<td>9.69</td>
</tr>
<tr>
<td>$\sigma (A, B, A')$</td>
<td>8.70</td>
<td>4.42</td>
<td>3.46</td>
<td>4.24</td>
<td>4.64</td>
<td>2.15</td>
<td>3.11</td>
<td>2.21</td>
<td>1.35</td>
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</table>

<table>
<thead>
<tr>
<th>Op. 10 No. 3</th>
<th>IDR%</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
<th>$\sigma$ (TI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Composition</td>
<td>21.38</td>
<td>16.57</td>
<td>25.04</td>
<td>18.13</td>
<td>18.89</td>
<td>0.99</td>
<td>9.77</td>
<td>7.17</td>
<td>3.46</td>
<td>8.42</td>
</tr>
<tr>
<td>Section A (bars 1-24)</td>
<td>16.12</td>
<td>17.71</td>
<td>19.51</td>
<td>23.65</td>
<td>23.01</td>
<td>0.64</td>
<td>11.03</td>
<td>4.35</td>
<td>0.11</td>
<td>9.81</td>
</tr>
<tr>
<td>Section B (bars 25-48)</td>
<td>25.75</td>
<td>15.05</td>
<td>30.53</td>
<td>13.53</td>
<td>15.15</td>
<td>1.20</td>
<td>9.56</td>
<td>9.13</td>
<td>5.87</td>
<td>8.71</td>
</tr>
<tr>
<td>Section A' (bars 49-77)</td>
<td>17.27</td>
<td>18.88</td>
<td>18.47</td>
<td>22.49</td>
<td>22.89</td>
<td>0.94</td>
<td>8.70</td>
<td>5.89</td>
<td>1.74</td>
<td>9.19</td>
</tr>
<tr>
<td>$\sigma (A, B, A')$</td>
<td>5.26</td>
<td>1.96</td>
<td>6.68</td>
<td>5.54</td>
<td>4.50</td>
<td>0.28</td>
<td>1.18</td>
<td>2.44</td>
<td>2.97</td>
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</table>

<table>
<thead>
<tr>
<th>Op. 10 No. 4</th>
<th>IDR%</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
<th>$\sigma$ (TI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Composition</td>
<td>25.73</td>
<td>17.72</td>
<td>23.83</td>
<td>14.18</td>
<td>18.53</td>
<td>3.57</td>
<td>8.80</td>
<td>7.79</td>
<td>5.58</td>
<td>7.16</td>
</tr>
<tr>
<td>Section B (bars 25-50)</td>
<td>30.51</td>
<td>12.75</td>
<td>27.24</td>
<td>12.75</td>
<td>16.76</td>
<td>3.87</td>
<td>9.61</td>
<td>9.41</td>
<td>7.61</td>
<td>7.09</td>
</tr>
<tr>
<td>Section A' (bars 51-82)</td>
<td>22.92</td>
<td>21.43</td>
<td>21.63</td>
<td>15.07</td>
<td>18.95</td>
<td>3.43</td>
<td>8.40</td>
<td>6.51</td>
<td>4.57</td>
<td>7.64</td>
</tr>
<tr>
<td>$\sigma (A, B, A')$</td>
<td>4.04</td>
<td>4.36</td>
<td>2.89</td>
<td>1.21</td>
<td>1.77</td>
<td>0.27</td>
<td>0.70</td>
<td>1.45</td>
<td>1.74</td>
<td></td>
</tr>
</tbody>
</table>
Due to the predominance of major and minor triads in the first étude, the $d_4$ interval-class occurs less than 1% of the time. However, $d_4$ is a significant interval-class in the remaining three works and is especially emphasized in chromatic passages. In addition, the table provides the standard deviation of all interval-class rates and the standard deviation of each separate interval-class, based on its occurrence in A, B, and A’ sections. Throughout the opening four études, Chopin uses a varied set of harmonic interval-classes, as seen in diverse range of $\sigma(T_l)$. On the contrary, unique interval-class rates in each of the three sections of every étude tend to contain a lower range of $\sigma(T_l)$.

The average level of dissonance rises from the first to the fourth études. Such an increase in IDR can be explained by the larger variety of harmonies, something that can also be seen in the Schenkerian analysis. While there is no definite dissonance for each pianistic technique that Chopin presents in the études, it is evident that pure arpeggiation provides the least amount of harmonic dissonance. The arpeggios are based on the root, third, fifth, and octave chordal factors of each harmony. From a harmonic perspective, such a set will provide only consonant intervals. From a melodic perspective, the level of dissonance will depend on the supplementary musical material found in each harmonic slice. Figure 5.1 shows the generation of all possible interval-classes in a root-position major triad.
Figure 5.1: Modified interval-class vector generation of C major triad.

Figure 5.2 shows m. 51 of Op. 10 No. 1, as taken from the score – the example of pure harmonic consonance. Table 5.2 provides the intervallic breakdown of m. 51 that is generated form harmonic consolidation of notes F and F, F and A, as well as F and C. Based on all possible harmonies of this passage, only $c_1$, $c_3$, and $c_4$ interval-classes exist and no types of $ic(d_k)$. The pianist is expected to hold the octave in the left hand throughout the whole measure, which explains the 47.83% frequency of the $c_1$ interval-classes. Furthermore, m. 51 consists of sixteen beats, with the octaves and equivalent intervals generated on beats 5, 9, and 13. In this example, $ic(c_2)$ that generates the interval of a minor third or a major sixth does not exist.

![Figure 5.2: Chopin's Étude Op. 10 No. 1, m. 51.](image)

---

Table 5.2: The intervallic breakdown of Chopin’s Étude Op. 10 No. 1, m. 51.

<table>
<thead>
<tr>
<th>ic(c₁) of m. 51</th>
<th>22/46 (47.83%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ic(c₂) of m. 51</td>
<td>0/46 (0.00%)</td>
</tr>
<tr>
<td>ic(c₃) of m. 51</td>
<td>8/46 (17.39%)</td>
</tr>
<tr>
<td>ic(c₄) of m. 51</td>
<td>16/46 (34.78%)</td>
</tr>
<tr>
<td>ic(d₁) of m. 51</td>
<td>0/46 (0.00%)</td>
</tr>
<tr>
<td>ic(d₂) of m. 51</td>
<td>0/46 (0.00%)</td>
</tr>
<tr>
<td>ic(d₃) of m. 51</td>
<td>0/46 (0.00%)</td>
</tr>
<tr>
<td>ic(d₄) of m. 51</td>
<td>0/46 (0.00%)</td>
</tr>
<tr>
<td>CI to DI ratio</td>
<td>46:0</td>
</tr>
<tr>
<td>IDR%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

The level of dissonance will increase with the reduction of arpeggiation. This occurs because the arpeggios present outlines of major and minor triadic harmony, which lowers the average IDR rate. When the number of interval-classes that represent triadic harmony decreases, the ratio of dissonant to total intervals increases, resulting in a higher IDR. Figure 5.3 shows the excerpt of m. 25 from Op. 10 No. 4 and Table 5.3 presents the intervallic breakdown of the passage.

The IDR of m. 25 is 37.25%. The main reason for such a high level of harmonic dissonance is the inclusion of double neighbor tones. The IDR subdivides the measure into sixteen beats. Harmonic dissonance exists on beats 1-2, 5-6, 9-10, and 13-16. These beats contain multiple layers of music, as the main melody in the right hand is supported by the chordal structure in the left hand.
and in the inner voice of the right hand. Beats 3-4, 7-8, and 11-12 contain monophonic melody in the right hand, as the left hand and the inner voices do not carry any notes.

Figure 5.3: Chopin’s Étude Op. 10 No. 4, m. 25.218

Table 5.3: The intervallic breakdown of Chopin’s Étude Op. 10 No. 4, m. 25.

| ic(c1) of m. 25 | 12/51 (23.53%) |
| ic(c2) of m. 25 | 9/51 (17.65%)  |
| ic(c3) of m. 25 | 5/51 (9.80%)   |
| ic(c4) of m. 25 | 6/51 (11.76%)  |
| ic(d1) of m. 25 | 3/51 (5.88%)   |
| ic(d2) of m. 25 | 4/51 (7.84%)   |
| ic(d3) of m. 25 | 5/51 (9.80%)   |
| ic(d4) of m. 25 | 7/51 (13.72%)  |
| CI to DI ratio  | 32:19          |
| IDR%            | 37.25%         |

While the opening four études have an increasing rate of harmonic dissonance, as based on the calculations of IDR, this is not the case for the rest of

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218 Ibid., 12.
the opus, as the trend is broken at Op. 10 No. 5 – an étude in the key of G♭ Major, where Chopin employs the arpeggiation, albeit in a different way than in Op. 10 No. 1.\textsuperscript{219} Nevertheless, the increasing dissonance rate is an important component to consider from the perspective of performance practice, especially when performing all études in one concert.\textsuperscript{220} There are multiple pianistic approaches that a performer may choose to undertake, influenced by harmonic dissonance. This can be seen in musical deviations in dynamics, tempo, texture, or the use of pedals.

Graphs 5.1, 5.2, 5.3, and 5.4 present the frequency of all consonances and dissonances in the four études in order, as they appear according to the modified interval-class vector. The predominant interval-class in Op. 10 No. 1 is \( c_1 \), which occurs 47.87\% of the time. The second most frequent interval-class is \( c_4 \), which is seen only 18.55\% of the time. In the third and the fourth études, the interval-classes are more dispersed. The most common interval-class in Op. 10 Nos. 3 and 4 is \( c_2 \) and the least common interval-class is \( d_1 \). Likewise, the \( d_4 \) interval-class varies, based on the technical skill that Chopin presents in each work. More varied harmonies, as seen in Études Nos. 2 and 4 will generate a larger amount of \( d_4 \) interval-classes. On the contrary, triadic configuration, as seen in Op. 10 No. 1 present the \( d_4 \) interval-class less than 1\% of the time.

\textsuperscript{219} Also nicknamed as the \textit{Black Keys} étude, the arpeggiation model of Op. 10 No. 5 is based on the triplet configuration in the right hand, supported by the chordal accompaniment in the left hand. Another important characteristic of this work is that the melody in the right hand is played solely on black keys.

\textsuperscript{220} While an arduous task to accomplish, it is not an infrequent occurrence to find all works on the same program.
When it comes to harmonic dissonance in all four études, Chopin prefers to use the $d_2$ interval-class, which contains major seconds, minor sevenths, and equivalent intervals. This interval-class is used between 3.85% to 11.03% of the time throughout the analyzed works. The $d_2$ interval-class occurs 3.85% of the time in the A'-section of Op. 10 No. 1 and 11.03% of the time in the A-section of Op. 10 No. 3. This is a significant interval, since it can be used to generate multiple types of seventh chords, including dominant seventh chords, minor seventh chords, and half-diminished seventh chords.\footnote{Refer to Table 4.2 of Chapter 4 regarding the modicv of various seventh chords and the location of $ic(d2)$ in these chords.}

**Graph 5.1: All consonant and dissonant interval-classes in Op. 10 No. 1.**
Graph 5.2: All consonant and dissonant interval-classes in Op. 10 No. 2.

Graph 5.3: All consonant and dissonant interval-classes in Op. 10 No. 3.
In all of these seventh chords, at least one interval-class of $d_2$ exists when such chord is in root position or in inversion. An example of this phenomena can be seen in Figure 5.4 on the fourth beat of m. 54 of Étude Op. 10 No. 4. There are thirteen $d_2$ interval-classes out of possible sixty-six combinations. Table 5.4 reveals the intervallic breakdown of the fourth beat of m. 54 with the IDR of 21.21%. It is evident that on the final beat of the measure, $ic(d_2)$ is the most frequent dissonant interval. The only other source of harmonic dissonance is a sole $ic(d_1)$. Therefore, $ic(d_2)$ makes up a total of 92.86% of total dissonance in this excerpt. Moreover, $ic(d_2)$ is the third most frequent interval on the fourth beat of m. 54 after $ic(c_4)$ and $ic(c_2)$. 

Graph 5.4: All consonant and dissonant interval-classes in Op. 10 No. 4.
In Romantic music, composers use chromaticism to produce tension and generate musical variety, as well as to move between or replace two or more diatonic harmonies.\textsuperscript{222} However, a larger use of chromaticism does not result in a

\begin{table}[h]
\centering
\begin{tabular}{|l|c|}
\hline
\textit{ic}(c\textsubscript{1}) of m. 54, beat 4 & 11/66 (16.66\%) \\
\textit{ic}(c\textsubscript{2}) of m. 54, beat 4 & 16/66 (24.24\%) \\
\textit{ic}(c\textsubscript{3}) of m. 54, beat 4 & 7/66 (10.60\%) \\
\textit{ic}(c\textsubscript{4}) of m. 54, beat 4 & 18/66 (27.27\%) \\
\textit{ic}(d\textsubscript{1}) of m. 54, beat 4 & 1/66 (1.52\%) \\
\textit{ic}(d\textsubscript{2}) of m. 54, beat 4 & 13/66 (19.70\%) \\
\textit{ic}(d\textsubscript{3}) of m. 54, beat 4 & 0/66 (0.00\%) \\
\textit{ic}(d\textsubscript{4}) of m. 54, beat 4 & 0/66 (0.00\%) \\
\hline
CI to DI ratio & 52:14 \\
IDR\% & 21.21\% \\
\hline
\end{tabular}
\caption{The intervallic breakdown of Chopin’s Étude Op. 10 No. 4, m. 54, beat 4.}
\end{table}

greater level of dissonance.\textsuperscript{223} The comparison can be made between the second chromatic étude that is the second most harmonically consonant étude of the four and the fourth piano étude that is the most harmonically dissonant of the four. In this instance, the fourth étude is approximately on average 5.55\% more dissonant than the second étude. The most dissonant element of the second étude is the B-section, which contains 29.79\% of harmonic dissonance, while the most dissonant section (of the twelve) in the first four compositions of Op. 10 is the B-section of the fourth étude with the IDR of 30.51\%.

Based on the IDR results, the B-section of each étude is more dissonant than the average IDR of each respective étude. Likewise, the four middle sections are the most dissonant parts of the first four études. This occurs because the A-section seeks to present and establish the main thematic material of each composition. Given the expository functions of the A-sections, this is not surprising. The goal of middle sections is to develop the material presented in the opening sections, directly or non-directly.\textsuperscript{224} Therefore, Chopin incorporates a wider variety of harmonies and takes various motivic patterns through multiple tonal adventures. The B-sections need to have a greater diversity of dissonance, building towards a climax that can later be resolved in the closing A'-sections.

These results can be taken further and can be used to compare the contrast level between two or more sections, based on dissonance. An application of


\textsuperscript{224} Chopin develops the main musical material directly in the first, second, and the fourth études. This is not the case for the third étude.
standard deviation is appropriate in this instance. A low deviation value means that the sections have little or no contrast. For instance, 0.00 would mean that all three sections of an étude have a similar amount of dissonance.\footnote{A similar amount of dissonance means that such sections will have an identical IDR value. It is important to note, however, that the sections do not have to be identical. It is possible for two or more completely different pieces of music to achieve an identical IDR, due to similar ratio of dissonant and consonant harmonic intervals.} The higher the deviation value, the higher is the contrast between multiple sections of each étude. The second étude contains the most contrasting sections, based on the level of dissonance. A maximum difference of 15.26% exist between Section B and Section A’ with a total deviation of 8.70 among all three sections. On the other hand, the first étude provides the least contrast among dissonance level, seen in the deviation of 2.16. This occurs because Étude No. 1 is the work in which Chopin incorporates very little melodic development when comparing the opening and the middle sections.\footnote{Charles Rosen, \textit{The Romantic Generation} (Cambridge: Harvard University Press, 1998), 364.}

Another significant dissonance-related musical trait can be observed in the deviation of the interval-classes. In the interval-class analysis of each étude, a dissonant interval-class provides the lowest deviation: interval-classes $d_1$ and $d_3$ in the first étude; interval-class $d_1$ in the second and the third études; interval-class $d_2$ in the fourth étude. It is a truism that dissonance is one of the devices through which composers generate tension and move between altering harmonies in their works. The IDR confirms that Chopin relies on various sources for dissonance to create contrast and variety in his music.
5.2. Étude No. 1

As mentioned previously, the sections A and B of Op. 10 No. 1 consist of twenty-four measures each and section A’ consists of thirty-one measures. Each of the sections can be expanded into contour motives (CM) – the smallest building blocks of the work. Each contour motive is two measures in length (except for the final CM, which is three measures in length). In addition, each contour motive incorporates at least one ascending arpeggiation and a descending return. Furthermore, the musical content of every contour motive reflects the structure and the pianism of Chopin’s first étude. From an analytical perspective, contour motives add up to combine the three separate sections of the A-B-A’ form, while the merging of the three sections generates the entire composition. From the perspective of performance practice, each contour motive deserves separate work, due to the difference in distance between notes found inside of each CM.

The IDR analysis of this étude can be compared to the approach that is offered by Huron’s Optimum Consonance Measure, as seen in Graph 5.5. All values have been normalized between 0 and 1 with an increased level of harmonic dissonance. According to the IDR analysis, the lowest contour motive at 0.00 contains no harmonic tension. In these instances, all intervals are consonant and such contour motives represent consonant (major or minor) triads. This occurs multiple times throughout the work. On the contrary, the third contour motive contains the lowest dissonance, as per Optimum Consonance Measure. This is because by combining all notes in mm. 5-6, the interval vector of <254361> contains thirteen consonant and eight dissonant intervals. Out of the consonant
intervals, six of these are perfect fourths and perfect fifths and this particular interval-class contains the highest effect on the overall increase in consonance. According to the IDR, the highest dissonance in this étude is $CM_{32}$, where 34.04% of the interval-classes are dissonant. Based on Optimum Consonance Measure calculations $CM_{11}$ (mm. 21-22) and $CM_{17}$ (mm. 33-34) have the highest dissonance. This occurs because the pitches in mm. 21-22 and in mm. 33-34 generate interval-class vectors ($<322332>$ and $<122212>$) with predominance of dissonance intervals.

Graph 5.5: Comparison of IDR and Optimum Consonance Measure in Étude Op. 10 No. 1.

Some other interesting points of comparison between the IDR and Optimum Consonance Measure can be seen in $CM_6$, $CM_{22}$, and $CM_{30}$. $CM_6$ contains the juxtaposition of F major and F♯ diminished harmonies. While dissonance is evident and is reflected in the Optimum Consonance Measure analysis, the harmonic subdivision of each sixteenth beat nevertheless produces a series of consonant interval-classes, which is why the IDR of this motive is 0.00%. The generated
interval-classes are \( c_1 \) (A-A) \( c_2 \) (A-C), and \( c_3 \) (A-F) in m. 11 and \( c_1 \) (A-A), \( c_2 \) (A-F\#) and \( c_4 \) (A-E) in m. 12. \( CM_{21:22} \) contains a descending sequential pattern in the left hand, such as \([B\rightarrow E\rightarrow A\rightarrow D\rightarrow G\rightarrow C]\), accompanied by the broken arpeggios in the right hand. The higher IDR (in comparison to the Optimum Consonance Measure) is the result of ratio of dissonant to consonant intervals (15:48 in \( CM_{21} \) and 16:80 in \( CM_{22} \)). On the contrary, the Optimum Consonance Measure of these \( CMs \) has a highest consonant content that defines both interval-class vectors in \( CM_{21} \) and \( CM_{22} \) throughout the étude (4.755 for \( <254361> \)). Similarly, there are only 4 dissonant intervals in \( CM_{30} \), which includes two \( d_3 \) and two \( d_4 \) interval-classes, yet the interval-class vector of such \( CM \) (\( <435432> \)) generates an Optimum Consonance Measure of -0.130, one of the more dissonant hyper bars throughout Op. 10 No. 1.

When looking particularly at the IDR analysis, the graph notation of harmonic tension incorporates a total of thirty-nine \( CMs \) that can be found on the x-axis and the IDR percentage values that can be found on the y-axis. Graph 5.6 shows the spread of harmonic dissonance in the A-section (mm. 1-24). Graph 5.7 shows the spread of harmonic dissonance in the B-section (mm. 24-48). Finally, Graph 5.8 shows the spread of harmonic dissonance in the A’-section (mm. 49-79). The following graphs provides four significant observations, not evident to performers from the use of traditional notation.

First, this étude verifies Chopin’s intent to incorporate minimum harmonic dissonance and his commitment to resolving each harmonic tension as soon as possible. This is seen because of the following observations:
a) Twelve of thirty-nine CMs (≈31% of the entire work) incorporate perfect harmonic consonance.

b) Each increase in harmonic tension is typically followed by an immediate decrease in harmonic tension. This occurs fourteen times throughout the work, while an increase in harmonic tension is followed by another increase only three times (once in the A-section, once in the B-section, and once in the A’-section). While dissonance is essential in the arpeggiated sequences of Chopin’s compositional approach, the composer is not interested in prolonging it.

Graph 5.6: IDR of Étude Op. 10 No. 1, mm. 1-24.
Second, Chopin incorporates varied use of harmonic dissonance, ranging between 4.26% and 34.04%, which is found in the étude’s twenty-seven CMs, where the IDR is greater than 0.00%. Some IDR rates are identical. For instance,
$CM_1$ through $CM_5$ are identical to $CM_{25}$ through $CM_{29}$. This occurs because sections A and A’ share the same opening harmonic cycle (a general characterization of ternary form) with the turning point occurring at $CM_{26}$.

On the contrary, $CM_{14}$, $CM_{15}$, and $CM_{16}$ have no associations to each other from the perspective of musical form, and while their IDR is identical at 14.89%, their respective construction is not. The construction of the three $CM$s can be compared through $modicv$. The ratio of CI to DI is 48:14 in $CM_{14}$, $CM_{15}$, and $CM_{16}$. However, a different $modicv$ exists for seven (of the eight) different ic content, apart from the stationary ic($d_4$).

a) $modicv(CM_{14}) = [48,0,8,24,0,14,0,0]$;

b) $modicv(CM_{15}) = [42,16,14,8,0,6,8,0]$;

c) $modicv(CM_{16}) = [40,8,24,8,0,14,0,0]$.

Third, the three largest dissonances in this étude are the only three IDR values that are greater than 30%. All three dissonances resolve to perfect harmonic consonance. In this work, there is:

a) One occurrence in the A-section, where $CM_{11}$ with IDR of 31.91% resolves to $CM_{12}$, shown in Figure 5.5;

b) One occurrence in the B-section, where $CM_{17}$ with IDR of 31.75% resolves to $CM_{18}$, shown in Figure 5.6;

c) One occurrence in the A’-section, where $CM_{32}$ with IDR of 34.04% resolves to $CM_{33}$, shown in Figure 5.7.

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Figure 5.5: Chopin’s Étude Op. 10 No. 1, mm. 21-22.

Figure 5.6: Chopin’s Étude Op. 10 No. 1, mm. 33-34.

Figure 5.7: Chopin’s Étude Op. 10 No. 1, mm. 63-66.
Fourth, it is evident that each of the three sections contains a wide range of harmonic tension, yet each of the final cadences of sections A, B, and A′ ends on perfect authentic cadence. According to the analysis the IDR of the A-section fluctuates between 0.00% and 31.91% while cadencing on CM12, the IDR of the B-section fluctuates between 0.00% and 31.75% while cadencing on CM24, and the IDR of the A'-section fluctuates between 0.00% and 34.04% while cadencing on CM39. The IDR of CM12, CM24, and CM39 is 0.00%.

Chopin’s music provides theorists and musicologists with a variety of compositional innovations in the development of tonal harmony.228 Harmonic dissonance generated in Chopin’s compositions is a significant aspect of analysis and performance.229 Each point of tension in Op. 10 No. 1 signifies a musical climax that needs resolution and the graphical notation of harmonic dissonance allows one to trace such tension through the étude.

5.3. Étude No. 2

The use of chromaticism in Op. 10 No. 2 allows Chopin to generate both vertical and horizontal dissonance. The vertical dissonance is achieved through rhythm, since the performer is required to hold the left hand’s bass voice and right hand’s inner voices, both represented with eighth notes, at the time when the main melody in the right hand ascends and descends in chromatic motion, represented with

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sixteenth notes. The horizontal dissonance is achieved through constant half-step perpetuum mobile motion. While there are instances, in which Chopin incorporates alternate melodic intervals in the right hand, each note in the main melody is either preceded or proceeded by its half-step neighbor at all times. The employment of different levels of musical tension throughout the étude allows Chopin to move between two or more harmonic structures. The use of dissonance likewise shows Chopin's modified harmonic paths, which are generated through multiple turning points.

As seen previously, Huron’s Optimum Consonance Measure looks at the aggregate consonance values, combining pitches into a multiple interval-class vector. Due to the large amount of chromaticism in the second étude, such an approach to analyzing dissonance in each measure will result in interval-class vectors, generated from all possible pitch-classes. Such occurrence exists in the very first measure of the work, generating an interval-class vector of <12 12 12 12 12 6>. However, Huron’s interval-class ranking can be applied to the IDR of each measure, as seen in Graph 5.9. This creates a comparison of weighted interval-classes (as seen in Huron’s Optimum Consonance Measure) and non-weighted interval-classes (as seen in IDR). As seen from the analysis, the most consonant harmony is A major, located at the very last measure of the work.

The IDR analysis is helpful for the analysis of harmonic dissonance based on each measure, where the IDR is normalized between the values of 0 and 1 with 0 being equivalent to a perfect harmonic consonance. This type of analysis presents three unique observations. First, the maxima IDR value in the graph of
the A-section is located at m. 2, m. 6, m. 10, and m. 14, presenting the highest harmonic consonances. Each maxima is followed by two measures with a lesser IDR value with the exception for m. 14. Second, the variety of dissonance is evident in the B-section, particularly in mm. 20-26, which is the only consecutive passage in the work to drop below the value of 0.4. Furthermore, m. 20 contains the solitary harmonic dissonance with the negative value. Third, the final section of the étude presents an increase in harmonic tension at m. 45, followed by a steady resolution to the perfect harmonic consonance – the Picardy third, found in m. 49.

Graph 5.9: Combining the IDR and the Optimum Consonance Measure analyses in Étude Op. 10 No. 2

The main motivic segment of the work is located in mm. 1-2. Part of Chapter 1 of this dissertation focused on correlation between Chopin’s pianism and the Schenkerian approach. This work, however, provides an instance of disagreement between the Schenkerian understanding of the main motive and the pianistic understanding. According to Schenkerian theory, mm. 1-2 prolong the tonic harmony of A minor, even though the subdominant D minor harmony exists

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230 Refer to Chapter 2.3 – The Schenkerian Approach, Op. 10 No. 2.
in the second half of the first measure. On the other hand, based on the
performance practice of this étude, the D minor harmony cannot be ignored.\textsuperscript{231}
Since dissonance governs the main character of this work and since the way in
which Chopin utilizes chromaticism creates multiple technical complexities, a
pianist needs to apply extra importance on consonant harmonies, and the D minor
harmony on beat 9 of m. 1 is one of the examples of perfect harmonic consonance.
A unique characteristic of this étude is the emphasis that Chopin puts on mm. 1-2
in the A and A’ sections. Table 5.5 shows the IDR analysis of mm. 1-2. This is a
highly consonant motive, where $ic(d_4)$ is the prime source of dissonance.

\begin{table}[h]
\centering
\caption{The IDR analysis of Chopin’s Étude Op. 10 No. 2, mm. 1-2.}
\begin{tabular}{|l|l|}
\hline
$ic(c_1)$ of m. 1-2 & 17/114 (14.91\%) \\
\hline
$ic(c_2)$ of m. 1-2 & 27/114 (23.68\%) \\
\hline
$ic(c_3)$ of m. 1-2 & 29/114 (25.44\%) \\
\hline
$ic(c_4)$ of m. 1-2 & 32/114 (28.07\%) \\
\hline
$ic(d_1)$ of m. 1-2 & 1/114 (0.88\%) \\
\hline
$ic(d_2)$ of m. 1-2 & 2/114 (1.75\%) \\
\hline
$ic(d_3)$ of m. 1-2 & 0/114 (0.00\%) \\
\hline
$ic(d_4)$ of m. 1-2 & 6/114 (5.26\%) \\
\hline
CI to DI ratio & 105.9 \\
\hline
$IDR\%$ & 7.89\% \\
\hline
\end{tabular}
\end{table}

\footnotesize
\begin{itemize}
\item \textsuperscript{231} Kenneth Hamilton, \textit{After the Golden Age: Romantic Pianism and Modern Performance} (Oxford: Oxford University Press, 2008), 69.
\end{itemize}
All seven phrases of both sections in the second étude begin with an identical motive. Furthermore, the motive is responsible for generating multiple harmonic turning points. The A-section of Op 10 No. 2 (mm. 1-17) contains three small phrases of four measures (mm. 1-4), (mm. 5-8), and (mm. 9-12), as well as one phrase of five measures (mm. 13-17). The A’-section of Op. 10 No. 2 (mm. 36-49) contains one phrase of four measures (mm. 36-39) and two phrases of five measures (40-44) and (45-49). The seven phrases with identical motives generate six unique IDR rates, which are shown in Graph 5.10. Based on the IDR analysis, both the A and A’ sections begin with the most harmonically dissonant phrases and end with most harmonically consonant phrases. This is unlike Op. 10 No. 1, where the initial contour motives of the opening and the ending sections are perfect harmonic consonances. Finally, it is important to note that while the second étude resolves on tonic harmony of A major, the most consonant motive, nevertheless, is the penultimate.

Graph 5.10: The IDR analysis of all seven phrases in Étude Op. 10 No. 2 with identical opening motive at mm. 1-2.

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232 The music material found in mm. 1-4 and 9-12 is identical.
5.4. Étude No. 3

The third étude does not carry any contour motive or unique motivic thematic idea, on which the piece is based. The opening two measures of Op. 10 No. 3 initiate merely two other phrases in m. 9 and m. 62 (the start of A’-section). Instead, Chopin uses multiple levels of dissonance in several passages of the work. The A-section spans until the cadence on the start of m. 21 and can be subdivided into three phrases. These are mm. 1-5, mm. 6-13, and mm. 14-21. The B-section can be subdivided into four phrases. The first phrase spans until the subdominant harmony of A major at the start of m. 30. The second phrase continues until the E♯ fully-diminished seventh chord at the start of m. 38. The third phrase extends until the B major harmony at the beginning of m. 46. The fourth phrase spans until the half cadence at the start of m. 54. The passage in m. 54-61 serves as a transition into the A’-section. Comparable to Op. 10 No. 2, Chopin’s A’-section is significantly shorter than the opening section, based on identical thematic material, and consists of two phrases, found in mm. 62-66 and mm. 67-77.

The comparison between the Huron’s Optimum Consonance Measure and the IDR can be used to understand the harmonic makeup of the composition. Based on the harmonic and the Schenkerian analyses of this étude, the harmony of the opening phrase (mm. 1-5) consists of alternating tonic and dominant seventh chords. The Optimum Consonance Measure of m. 1 is 2.459, consisting of pitches D♯, E, F♯, G♯, A, and B with an interval-class vector of <233241>. The Optimum Consonance Measure of m. 2 is identical to m. 1 with identical pitches and interval-class vector. The Optimum Consonance Measure presents the same consonance
results in the opening of the first phrase. Such metric allows one to trace the harmonic plan of mm. 1-2 in terms of dissonance level, since m. 1 utilizes tonic moving towards the dominant and m. 2 returns the dominant back to the tonic. The third measure of the étude employs pitches C#, D#, E, G#, A, and B with an interval-class vector of <232341> and Optimum Consonance value of 2.251. In this instance, however, C# is used in lieu of F#. Measures 4 and 5 use pitches C#, D#, E, F#, G#, A, and B, generating an interval-class vector of <254361> and Optimum Consonance value of 4.755. The first five measures of the étude generate three Optimum Consonance values. On the contrary, the IDR generates four unique values because of consideration for repeated pitches as well as inclusion of unisons, octaves, and equivalent intervals. The average Optimum Consonance value of each eighth harmonic slices can also be compared to the IDR, as shown in Graph 5.11.

Graph 5.11: The IDR and the Optimum Consonance Measure analyses of Chopin’s Étude Op. 10 No. 3.

Similar to Op. 10 No. 2, certain passages in the third étude contain hidden chromaticism patterns in each measure and will therefore generate complex interval-class vectors. This can be seen in mm. 46-51. The Optimum Consonance
Measure of m. 46, m. 48, and m. 50 is 1.404 with the interval-class vector of <448444>, consolidating eight of the possible twelve pitches. The Optimum Consonance Measure of m. 47, m. 49, and m. 51 is -0.198 with the interval-class vector of <12 12 12 12 12 6>, consolidating all twelve pitches. This means that according to the Optimum Consonance Measure, m. 46, m. 48, and m. 50 are slightly consonant (as 1.404 > 0), while m. 47, m. 49, and m. 51 are slightly dissonant (as -0.198 < 0). This passage contains a descending series of harmonic alternating sixths (both major and minor) and diminished sevenths.

A disparity in analyses based on IDR and Optimum Consonance Measure can be seen in mm. 20-30 and mm. 70-77. This occurs because of certain Huron’s interval ranking indexes that generate a high level of harmonic dissonance. For instance, m. 22 has an IDR of 16.67% and only contains four dissonant intervals: A♯-E, F♯-E, E-A♯, and B-C♯. Two of these are ic6 on the standard interval-class vector with index value of -0.453 and two of these are ic2 on the standard interval-class vector with index value of -0.582. By applying Huron’s index values that generate ranking for interval-classes, the Optimum Consonance Measure of m. 22 is 1.4295, which is 0.7709 on a 0-1 scale and contains less consonance than similar passage, according to the IDR.

This étude is an example of Chopin’s ability to create a mix of unstable and unbalanced harmonies. The IDR analysis of Op. 10 No. 3 reveals that Chopin uses gradual harmonic dissonance to enhance musical tension towards the dissonant nucleus of the B-section, found at mm. 38-42. This passage, constructed from a

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series of fully-diminished chords, contains the musical climax of the entire work and requires resolution, which occurs on the B major harmony in the first beat of bar 42. Graph 5.12 shows the increase in harmonic dissonance, as one reaches mm. 38-42, as well as an increase in harmonic consonance, as the work comes to a close.

![Graph 5.12: The IDR analysis of all phrases in Étude Op. 10 No. 3.](image)

The three phrases of the A-section range between the IDR of 15.21% and 18.52%. Chopin makes sure to resolve any dissonant components of harmony. Each phrase of the A-section cadences on the tonic of E major. The B-section contains a larger variety of harmonic dissonance, ranging from 12.89% to 47.92%. In addition, the four cadential endings of each phrase never reach the harmony of E major. The third phrase of the B-section is of most interest. The phrase can be bifurcated into two subphrases. The opening of the third phrase is highly dissonant with the IDR of 47.92%. On the other hand, the closure of the phrase is highly consonant with the IDR of 12.89%, making it the most consonant passage in the étude.
The interpretation that one builds in the reading of a work is an essential component in studies of the performance practice. In the case of Op. 10 Nos. 1 and 2, the deviation in interpretation is not very large and the melodic motion is self-evident for majority of pianists. On the contrary, Op. 10 No. 3 differs in its departure from technical virtuosity and places emphasis on the poetic nature of its melody. The deviations in tempo, dynamics, and pedaling are some of the components on which pianists work to create an appropriate interpretation in this étude. Knowledge and familiarity with harmonic dissonance can allow pianists to incorporate such information into their own interpretations. This can be seen in performance analysis of mm. 38-54 of Maurizio Pollini, Lang Lang, and Valentina Lisitsa.  

The interpretational differences occur in three passages: mm. 38-42 and mm. 46-54, both of which contain a series of fully-diminished seventh chords, resolving on B major harmony, as well as mm. 42-46, which serves as the transition between the two dissonant passages.

Pollini establishes three groups in mm. 38-42, based on Chopin’s slurring recommendations, and stays very close to what the score suggests. Pollini likewise groups the passage in mm. 46-54, based on slight dynamic accents on the first beat of every measure. Lang Lang abruptly increases the tempo and the volume in mm. 38-42 and makes sure to emphasize the final B major harmony. Furthermore, Lang Lang continues to increase the tempo in mm. 42 and 46,

specifically on the consonant harmonic alterations of [D-B] and [A-F♯] in m. 42. Finally, Lang Lang gradually decreases the tempo in the last section of the passage. It is important to note that every chord is emphasized, making the harmonic dissonance very apparent. Similarly to Lang Lang, Lisitsa increases the tempo and crescendos towards the B major harmony in m. 42. Lisitsa employs aberrant pedaling in mm. 46-52 to emphasize dissonance that is created between alternating sevenths and sixths intervals before suddenly decreasing and returning to the original tempo in m. 53. In all performances, such dissonant passages are open to pianistic interpretation and can be understood musically in multiple ways.

5.5. Étude No. 4

The fourth étude is one of the most harmonically complex works in Op. 10. The high level of dissonance in this work depends on Chopin’s consolidation of arpeggiation, chromaticism, and linear patterns, which can be noticed in mm. 28-34. Beats 1, 5, 9, and 13 (of 16) in mm. 28-29 consist of harmonic triads or seventh chords, while the melody in the same passage consists of ascends and descends by a half (chromatic) or a whole step. On the contrary, the melodic motion in the inner voices of mm. 32-33 is consonant moving downward by harmonic major or minor thirds.

Another distinct characteristic of this study is Chopin’s subtle differences in alternating thematic material between section A and A’. This can be seen in m. 8 (Figure 5.8) and m. 58 (Figure 5.9). In the A-section, a series of half-step patterns on m. 8, represented by the eighth notes, is to be held in the inner voice of the right
hand while the main melody continues in perpetual motion, represented by the sixteenth notes. This creates an extra layer of harmonic dissonance, overlapping each of the sixteenth notes with the right hand’s main melody and the left hand’s bass support. On the contrary, Chopin avoids the inner voice in m. 58 of the A’-section and instead focuses on a two-voice contrary motion. The lack of secondary melody decreases the IDR from 18.33% (m. 8) to 15.91% (m. 58).

Figure 5.8: Chopin’s Étude Op. 10 No. 4, m. 8.

Figure 5.9: Chopin’s Étude Op. 10 No. 4, m. 58.

Graph 5.13 shows the comparison between the IDR and Huron’s Optimum Consonance Measure analyses. The highest dissonance in both analyses occurs in m. 39-40. In this passage, both right and left hands create a descending two-part harmony using broken seventh chords in the lower melody and linear pattern
with chromatic ascents in the main melody. A disparity in both analyses occurs at the end of A’-section, where the Optimum Consonance Measure is higher than the IDR. This occurs because of large amount of $d_1$ interval-classes on the modified interval-class vector, equivalent to $ic_1$ on the traditional interval-class vector in mm. 71-79. Out of 242 total intervals in this passage, 65 are dissonant and 76 belong to class $c_1$, which Huron’s Optimum Consonance Measure does not rank. Out of 65 harmonic dissonances, 19 of these (29.23% of total dissonance) belong to $d_1$ class and 27 of these (41.53%) belong to the $d_2$ class. Huron’s index values for $ic_1$ and $ic_2$ on traditional interval-class vector are -1.428 and -0.582. Such high dissonance occurrence is the reason why this passage is ranked highly dissonant according to the Optimum Consonance Value analysis.

Graph 5.13: The IDR and the Optimum Consonance Measure analyses of Chopin’s Étude Op. 10 No. 4.
CHAPTER 6. CONCLUSION

The notion of consonance and dissonance plays a prominent role in Western music. The generation of tension and its resolution is one of the largest fundamental principles that exists in tonal literature. Consonance and dissonance in each musical work provides a degree of contrast and ‘polarity’, which can be viewed from the stance of cognitivism, physicalism, and immanentism. In Chopin's music, harmonic dissonance can be interpreted as a musical function that requires detailed analysis and examination. It is up to each individual composer to enhance one’s composition by producing areas of tension and by deciding how such dissonant regions will be finalized from melodic and harmonic viewpoints.

Mathematically, the methods for computing for musical tension vary, depending on goals that one wishes to achieve by such calculations. The primary focus of IDR research is to compute the level of harmonic dissonance based on verticalities from the standpoint of interval-classes. In addition to the dissonance rates, the proposed methodology likewise generates data for the analytic investigation of intervallic structure. Each verticality can be expressed as a ratio of consonant and dissonant intervals and a series of such verticalities generate the harmonic and melodic formulae in a composition. The dissonance in a musical work that derives from such harmony and melody is an artistic component, which a pianist must execute in a performance. Therefore, the connection between pianism, Schenkerian analysis, and computation of dissonance is a vital aspect to consider when understanding a work on both artistic and analytical levels.

The intervallic makeup of a musical work relates to several features of its style and genre. In the case of Chopin’s music (and particularly in the case of Chopin’s études), such features are purely pianistic. Furthermore, the unique interval-classes generate harmonies, which in turn, produce chordal progressions and harmonic regions that Chopin arranges in a unique way. The details of such arrangement constitute a significant component of Chopin’s compositional language and lead to his idiomatic treatment of harmonic dissonance. Likewise, the aspects of the piano technique are significant to take into consideration in Études Op. 10 Nos. 1-4, since they also influence the harmonic path that the composer selects. A series of verticalities in the four études provides one with a data set for the evaluation of harmonic tension and intervallic distribution.

In Op. 10 No. 1, 47.87% of all interval-classes are perfect unisons and octaves, most of which are found in the left hand. The IDR analysis reveals that these interval-classes assist with producing a low dissonance ratio throughout the étude. Analytically, such intervals typically double the chordal factors of the major and minor triads, on which the work is based. Pianistically, the octaves in the left hand constitute a main melody that the right hand’s arpeggiations harmonize. In addition, supplementary octaves are produced when verticalities in the left hand’s melody correspond with the equivalent chordal factor in each right hand’s

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236 It is important to remember that Chopin himself was a pianist and pedagogue. Therefore, while many of his pieces are complex from the standpoints of artistry, musicianship, and technique, Chopin’s works are nevertheless pianistically comfortable. Irena Poniatowska, *Fryderyk Chopin: The Man and his Music* (Warsaw: Multico Oficyna Wydawnicza. 2010), 125.

237 The primary thematic material is in the right hand’s arpeggiation. However, the right hand does not generate a melody. On the contrary, the left hand does not carry technical difficulties, yet produces a melody that outlines the form of each of the three sections in the étude.
arpeggio. Since the octaves and equivalent intervals constitute almost half of the composition, it is important to highlight them pianistically. There are multiple methodologies to do so, one of which is employing accentuation, specifically on linear patterns and fully established phrases.

When it comes to comparing all possible contour motives in Op. 10 No. 1, only 14.14% of intervals are dissonant, with thirty-seven out of thirty-nine IDR values below 30%. This occurs because the arpeggiation patterns in this étude generate low amounts of dissonance, given the surplus of two-note dyads that arise between the first, the third, and the fifth chordal factors. Pianistically speaking, such phenomenon is only possible in predominantly triadic music, as shown in this étude, since the primary purpose of this work is to generate a series of practical components for enhancing arpeggio playing. Another intersection between the IDR and Chopin’s compositional approach can be seen in purely consonant contour motives, occurring in twelve separate passages, combining for a total of twenty-five measures of pure consonant verticalities.

From a pianistic standpoint, Op. 10 No. 1 is an appropriate work for the inclusion of these purely consonant passages, as Chopin is following a compositional plan related to a specific technique, alternating ascending and descending arpeggios. On the contrary, employing only unisons, thirds, fourths, fifths, sixths, octaves, and equivalent intervals may be considered too simplistic for Romantic era music in general, particularly for a work that is meant to provide

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238 In Chopin’s compositional output, there are several études that specifically focus on a particular interval.
challenges and complications for a pianist’s sense of artistry. In Chopin’s Op. 10, Op. 25, and *Trois Nouvelles Études*, Op. 10 No. 5 (*Black Key*) and Op. 25 No. 12 (*Ocean*) are the only other works, in which the composer incorporates pure arpeggiation for a substantial passage in a work. There are likewise three études that emphasize specific intervals and, therefore, are based on subsets of arpeggiation. This can be in Op. 25 No. 6, where a pianist must focus on thirds, Op. 25 No. 8, where a pianist must focus on sixths, and Op. 25 No. 10, where a pianist must focus on octaves. Such a compositional layout is not stylistic in more artistically-inclined Romantic compositions, or in works that are based on alternative pianistic techniques.

A musical composition can be dissected analytically, historically, or pianistically. In particular, music analysis allows one to separate and view the structural components of any work. From the perspective of theory, Schenkerian analysis presents a series of compositional rules that occur in the melody, harmony, and counterpoint. Furthermore, Schenkerian analysis defines and denotes a series of structural components with notated elaborations. Schenkerian prolongations reveal how various passages in a musical piece function and associate with one another. As pianistic techniques serve as the primary

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241 While the arpeggiation is used in both Op. 10 No. 5 and Op. 25 No. 12, the compositional strategy varies.
242 This is likewise known as holistic musical analysis. For more information on such musical evaluation, refer to: Daniil Zavlunov, “The ‘tselostnyi analiz’ (holistic analysis) of Zuckerman and Mazel,” *Music Theory Online* 20, no. 3 (2014).
foundations of Chopin’s études, Schenkerian analysis can represent and graph these components in analytical terms. In addition to pianism, the Schenkerian analysis can assist with identification of dissonance and the IDR application can be beneficial for mathematically calculating and classifying various dissonance levels.

During Chopin’s artistic period, the notion of tonality was the accustomed compositional tradition. Nonetheless, components such as chromaticism produce aspects of tonal ambiguities, as seen in Op. 10 No. 2. Chromaticism is present in the opening section of the work in the right hand’s primary voice in mm. 1-17. Throughout this passage, the level of vertical dissonance fluctuates between 12.5% and 31.25%. Whereas the inclusion of chromatic harmony produces higher levels of tonal tension, the IDR calculations allow one to see and compare various harmonically dissonant regions in the score regardless of their degree of chromaticism. For instance, the highest harmonic tension in the A-section occurs in m. 4, m. 8, and m. 12. These measures contain half cadences that generate tension and seek for respective resolutions into m. 5, m. 9, and m. 13. Pianistically, the opening three primary sub-phrases of mm. 1-17 are mm. 1-4, mm. 5-8, and mm. 9-12, each of which begins with identical thematic material.

In this instance through the use of technique of diminution, Schenkerian analysis represents such subphrases as a series of I-V progressions, each with identical half cadence that resolves to the tonic, generating three consecutive identical reoccurrences. By analyzing harmonic dissonance, it is evident that the most discordant regions in the A-section are the harmonies that seek to return to
tonic. As the analytical theory behind the construction of this section becomes relatively evident, the pianistic comprehension is not, since there are numerous interpretive and subjective aspects that an artist must decide for oneself. This is because the pianistic facets, such as rubato, the level of crescendo, accentuation, and the use of sustaining pedal must be carefully planned for a well-built artistic interpretation.

Another connection between pianism, Schenkerian analysis, and the dissonance range can be seen in the main melody of Étude Op. 10 No. 2. From the standpoint of pianism, this work has three layers: the main melody in the right hand, the bass accompaniment in the left hand, and the subordinate inner voices played by the left and the right hands. For an accurate performance, it is crucial that the main melody is emphasized in a smooth and graceful manner, while the bass and the inner voices are employed as harmonic support for the primary voice. Analytically, the chromatic passages range between pitch A (\(^1\)) and the apex – pitch F (\(^6\)). Each hyperbar that initializes every subphrase in the A-section of Op. 10 No. 2 contains an ascending and a descending chromatic passage. Schenkerian analysis emphasizes the lowest and the highest pitches of these melodies.

From the perspective of dissonance, the amount of harmonic tension decreases as the chromatic line ascends, while the amount of harmonic tension increases as the chromatic line descends. This generates an interesting

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244 In certain passages, some pianists may choose to emphasize parts of the bass accompaniment or subordinate voices for the purpose of musical contrast.
interpretive situation, since a crescendo occurs with the decrease in vertical dissonance and a diminuendo occurs with the increase of vertical dissonance. While employing chromaticism in Op. 10 No. 2, Chopin does not highlight any particular harmonic interval-classes. On the contrary, Op. 25 No. 5, nicknamed (Wrong Note) uses a large amount of melodic minor seconds, generating a high level of melodic dissonance.

Another interaction between the combined functionality of pianism, Schenkerian analysis, and IDR can be seen in the meaning of harmony in Étude Op. 10 No. 3. The most dissonant section of the work, found in mm. 38-41 is of most interested here. With an IDR of 47.92%, this passage is the only excerpt in the work to contain such a large amount of dissonance, due to a series of fully-diminished seventh chords. Pianistically and analytically speaking, Chopin deviates from his traditional use of harmony and chooses to integrate a prominent vertically dissonant section. The high IDR rate is significant both in terms of performance practice and music analysis. While no specific emphasis is needed to highlight a series of dissonant progressions, there are multiple interpretations how such passage should be played; in the history of performance practice, there have been many interpretations of this excerpt.

However, the amount of dissonance is not the only noteworthy musical component of this passage. Additionally, its resolution is likewise significant, as mm. 38-42 resolve to the dominant region of B Major. A typical phrase at its most simple level can be notated as I-V-I, generating a tonic region, a dominant region,

\[245\] This is shown in Figure 2.22.
and a return to tonic region. In the A-section, the dominant harmony serves the role of a dissonant harmonic region, as E major moves to its (dissonant) dominant B major, which seeks resolution by returning to the tonic E major, hence generating a I-V-I progression. On the contrary, the symbolism of the B major harmony in m. 42 relies on the fact that it serves as a stable region, employed as an area for resolution of mm. 38-41, rather than as a traditional dominant harmonic region that would seek such resolution. The IDR assists the Schenkerian understanding of such analytical component and it is up to the pianist to produce an interpretation that would fulfill Chopin’s approach to dealing with the B major harmonic region.

For music theorists, the IDR serves as an analytical technique that can examine and evaluate harmonic dissonance of a piece, based on the modified interval-vector, construed from eight interval-classes.\(^ {246}\) The IDR can be used to make comparisons between different stylistic periods of a composer and between different composers of the same stylistic era. For instance, the IDR can compare the amount of harmonic dissonance that Beethoven uses in his early period, middle period, and late period sonatas. Another example of hypothetical IDR application can be seen in assessment and evaluation of harmonic dissonance in romances of the same compositional period, such as of Cesar Cui and Nikolai Rimsky-Korsakov.\(^ {247}\) Finally, other works by Chopin can be tested for dissonance,

\(^{246}\) It is also important to note that the d4 interval-class can likewise be subdivided into multiple subgroups.

\(^{247}\) Stylistically, both composers were a part of the Mighty Five.
focusing on the relations and associations of interval-classes throughout the works, both small and large-scale compositions.\textsuperscript{248}

For pianists, the technique of IDR can be employed when shaping their own interpretations in performances.\textsuperscript{249} For instance, a unique dissonant section can be emphasized musically through dynamics and articulation. This leads to the amalgamation of performance and analysis, allowing musicians to generate an artistic reading of a work based on the extracted empirical data. Furthermore, the IDR can provide an insight into dissonance levels in works, which were influenced by historically earlier composers.

For instance, Scriabin, in his early compositional period, is inspired and influenced by Chopin and, therefore, many of Scriabin’s early works reveal traces of Chopin’s compositional approaches.\textsuperscript{250} The salient chromaticism in Chopin’s Op. 10 No. 2 was seen previously in Figure 1.2. In contrast, the chromaticism in Scriabin’s Op. 11 No. 2 is concealed between the primary melody and subordinary thematic material, as shown in in mm. 7-9 of the A-section in Figure 6.1.\textsuperscript{251}

\textsuperscript{248} For instance, the IDR can be used to calculate the harmonic dissonance of Chopin’s nocturnes or the four scherzi.
Scriabin’s work contains evident traits of Chopinesque lyricism, even though it incorporates different approaches to chromaticism and is tonally more complex. Considering harmonic dissonance, Chopin uses 4.19% of tritones and equivalent intervals, while Scriabin uses 10.58%. This is a significant percentile, as the combined IDR of Scriabin’s Op. 11 No. 2 is 29.37%. On the contrary, Chopin’s Op. 10 No. 2 utilizes 21.59% of unisons, octaves, and similar intervals – a significant amount, due to the predominance of chromatic scalar passages.

Unlike Chopin’s étude, Scriabin’s Op. 11 No. 2 employs merely 9.08% of octaves. It is evident that unlike Chopin, Scriabin makes sure to hide the chromatic passages, concealing them with alternative harmonic dissonances. From a pianistic standpoint, while both works carry many similarities, different performative approaches must be taken. In Chopin’s Op. 10 No. 2, the chromatic lines do not need to be emphasized and their clarity is evident nonetheless. On the contrary, a performer is required to uncover and emphasize the chromatic

elaborations in Scriabin’s Op. 11 No. 2. Such artistic elements are essential for pianists to notice and are prominent components of modern performance practice.

Western tonal music contains a repertoire of multiple styles and various theoretical analyses have been applied to music literature of different epochs. One of the ways to subdivide multiple centuries of compositions is by the concept of tonality. Roman numeral analysis and Schenkerian analysis, which are based on the principles of harmony and counterpoint, are two of the common methods to look at music that contains a tonal center. The IDR can be applied to any tonal piece that contains harmonic construction. By providing empirical data on how dissonant a piece of music is harmonically, the IDR can be used to identify and categorize patterns of dissonance and consonance in the music of various composers.
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APPENDIX 1. THE SCHENKERIAN BACKGROUND GRAPH:
OP. 10 NO. 1
APPENDIX 2. THE SCHENKERIAN FOREGROUND GRAPH:
OP. 10 NO. 1
APPENDIX 3. THE SCHENKERIAN BACKGROUND GRAPH:
OP. 10 NO. 2
APPENDIX 4. THE SCHENKERIAN FOREGROUND GRAPH:
OP. 10 NO. 2
APPENDIX 5. THE SCHENKERIAN BACKGROUND GRAPH:
OP. 10 NO. 3
APPENDIX 6. THE SCHENKERIAN FOREGROUND GRAPH:
OP. 10 NO. 3
APPENDIX 7. THE SCHENKERIAN BACKGROUND GRAPH:  
OP. 10 NO. 4
APPENDIX 8. THE SCHENKERIAN FOREGROUND GRAPH:
OP. 10 NO. 4
APPENDIX 9. THE SCRIPT TO COMPUTE IDR

=IF
(n = "Cbb",0, IF(n = "Cb",1, IF(n = "C",3, IF(n = "C#",7, IF(n = "C##",15, IF(n = "Db",31, IF(n = "D",63, IF(n = "D#",127, IF(n = "D##",255, IF(n = "Dbb",511, IF(n = "Db",1023, IF(n = "Eb",2047, IF(n = "E",4095, IF(n = "E#",8191, IF(n = "E##",16383, IF(n = "Fbb",32767, IF(n = "Fb",65535, IF(n = "F",131071, IF(n = "F#",262143, IF(n = "F##",524287, IF(n = "Gb",1048575, IF(n = "G",2097151, IF(n = "G#",4194303, IF(n = "G##",8388607, IF(n = "Ab",16777215, IF(n = "Abb",33554431, IF(n = "Ab",67108863, IF(n = "A",134217727, IF(n = "A#",268435455, IF(n = "A##",536870911, IF(n = "Bbb",1073741823, IF(n = "Bb",2147483647, IF(n = "B",4294967295, IF(n = "B#",8589934591, IF(n = "B##",17179869183, IF(n = "r","--"))))))))))))))))))))))))))))))))))))))))))

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VITA

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