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## Comment on "Photodetachment in combined static and dynamic electric fields"

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## Comment on “Photodetachment in combined static and dynamic electric fields”

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The theoretical analysis of C. Rangan and A. R. P. Rau [Phys. Rev. A **61**, 033405 (2000)] on the process of photodetachment of  $H^-$  in a strong static electric field, which calls into question the predictions of B. Gao and A. F. Starace [Phys. Rev. A **42**, 5580 (1990)] and also of M. Q. Bao *et al.* [Phys. Rev. A **58**, 411 (1998)], is shown in this Comment to be incorrect. First, we point out that a number of assumptions of Rangan and Rau’s analysis rest on tenuous theoretical grounds. Second, we adduce two completely independent and different analyses of the problem which precisely confirm the results of Gao and Starace. These independent analyses also provide the interpretation that Gao and Starace’s predicted strong-field effects are due to the exact account of the influence of the static electric field, not only on the final state of the detached electron, but also on the initial bound-electron state.

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A recent article by Rangan and Rau (RR) [1] presents a critique of the results of Gao and Starace (GS) [2] and of Bao, Fabrikant, and Starace (BFS) [3] on the photodetachment of  $H^-$  in a strong static field. GS and BFS predict that in a strong static field, the photodetachment cross section of  $H^-$  is significantly lower in the vicinity of the zero-field threshold energy than is predicted by calculations that ignore strong-field effects (e.g., see Refs. [4,5]). (In the limit of weak fields, of course, the results of GS and BFS reduce identically to those of Refs. [4,5].) In this Comment we point out that RR’s analysis of the problem of single-photon detachment of  $H^-$  in a strong static field rests on tenuous theoretical grounds. As one example, they insist that the phase of the laser field should be such that the laser field reduces to a static field in the zero-frequency limit, and that all terms in the wave function for the detached electron should be finite in the zero-frequency limit. By implication, they thus call into question the validity of the well-known Volkov solution for an electron in a laser field [6], including the existence of the well-known ponderomotive potential whose effects have been confirmed experimentally in above-threshold-ionization (ATI) experiments (see, e.g., Refs. [7,8]). As another example, they compute their final-state wave function using an evolution operator approach [9], employing an evolution operator  $U(t_1, t_0)$  that implies a sudden turn on of the laser and static fields at  $t=t_0$ , rather than the adiabatic turn on assumed by GS and BFS. Although we discuss in this Comment these and other problems we see with the theoretical treatment of RR, we also present a different, independent treatment of the problem according to the approach of Slonim and Dalidchik [10]. We show that this independent approach confirms precisely the results of GS and BFS. Finally, we discuss another completely independent approach to the problem that employs a quasistationary, quasienergy state approach [11], which has already confirmed independently the results of GS and BFS and that has also provided a physical interpretation of the terms in GS and BFS that RR find objectionable [12].

GS start with a well established approach [13] based on the quasienergy representation of the final-state wave func-

tion (i.e., a Volkov-type solution [6]). Integration in time of the  $S$ -matrix element with this function allows one to extract all  $\delta$  functions corresponding to one-photon, two-photon, etc., detachment amplitudes. In contrast, RR start with a function satisfying a certain initial condition at  $t=0$ . Their wave function is no longer a quasienergy eigenstate (as it includes terms having nonharmonic time dependence). Hence the standard  $S$ -matrix approach operating with in- and out-states having only harmonic time dependence defined at  $t \rightarrow \mp \infty$  becomes inapplicable. Nevertheless, they employ an  $S$ -matrix approach with their wave function. Consequently, it can be seen from Eq. (9) of their paper that the result of the time integration contains not only a “normal” term involving  $\delta(\epsilon_f - \epsilon_i - \omega)$ , but also several others, including  $\delta$  functions of other arguments and derivatives of  $\delta$  functions that have no physical meaning. In contrast to what RR claim, the second term in Eq. (9) gives a nonzero contribution, but it contains nonphysical terms. RR simply ignore these terms. Finally, in calculating the  $S$ -matrix amplitude, RR integrate over time from  $-T$  to  $+T$  and evaluate the limit  $T \rightarrow \infty$  numerically rather than analytically.

To justify their choice of the wave function, RR say that their solution remains well-behaved in the limit of zero frequency whereas that of GS “blows up.” In fact Volkov-type wave functions [6] simply exhibit rapid oscillations, which should be treated properly. However, it is unclear why the wave function, which is used in the calculation of a finite-frequency process ( $n$ -photon detachment), must have the correct limit at  $\omega=0$ . For instance, in the well-known Keldysh-Faisal-Reiss treatment of atomic ionization in a laser field (see, e.g., [14]), the final-state Volkov function also fails to have a static-field-limit as  $\omega \rightarrow 0$ . Moreover, the theoretical formulation in the limit of small frequencies must be changed, since in this limit the boundary conditions (in coordinate space) are different. Alternatively, using the  $S$ -matrix approach for this case, the sum over the whole range of  $n$  should be calculated. More generally, the physics of finite and zero-frequency fields are very different: one involves quantum objects, photons; the other involves a

static potential. The expectation that one can pass from one situation to the other simply by taking the zero limit of the parameter  $\omega$  is incorrect.

RR claim that their difference with GS might be due to a possible gauge dependence. There are several issues involved. First, GS have explicitly demonstrated that their results are gauge-consistent, i.e., their results are the same in length and velocity gauges. Moreover, they have derived their expression for the S matrix based on this equivalence. RR use exactly the same expression, Eq. (8) of the GS paper, but they never prove that it can be used with their wave function. Second, one can easily verify that if a phase,  $\varphi$ , is added to the vector potential factor,  $\sin(\omega t + \varphi)$ , then the GS cross section result is unaffected by  $\varphi$ . Specifically, the S-matrix amplitude has a change of the overall phase, which disappears in calculating the cross section. (Physically this means that the response of a system subjected to an infinitely long electromagnetic wave is independent of the phase of the wave at an arbitrarily chosen time,  $t=0$ .) Hence the  $t=0$  value of the GS vector potential is not important. Third, RR claim that the additional terms in the cross section obtained by GS may be obtained also by RR if they employ a different vector potential, i.e., the same vector potential as used by GS. This is very strange since it is well-known that a change of gauge of the vector potential also entails a change of phase of the wave function, but that physical observables are unaffected (see, e.g., [15]). A possible reason for their obtaining a physical change from making a change of gauge is that their addition of nonharmonic terms in their vector potential (as compared to that of GS) means that use of S-matrix formulas is not appropriate, as has already been noted.

Rather than continuing to discuss the many questionable points of the RR treatment of photodetachment in a strong static field, we switch our discussion now to two alternative, independent treatments of the problem that confirm the results of GS and BFS. First, we have calculated the one-photon detachment cross section with *exact* inclusion of static field effects in *both* the initial and the final state according to the method of Slonim and Dalidchik [10]. We used in this calculation the technique developed in Ref. [10] for the integration over the radial variables in the photodetachment amplitude in terms of the Airy function Ai. In this calculation, we neglect Ref. [10]'s treatment of the final-state interaction of the detached electron with the atomic residue (the so-called rescattering effect [16]) in order that our result here is comparable to those of GS and RR, as well as other previous studies [4,5] that neglect rescattering effects. We consider the initial state to have the binding energy  $\epsilon_i = -\kappa^2/2$  (in a.u.) in the presence of a static field  $\mathbf{F}$  and treat perturbatively the laser field with frequency  $\omega$  and a linear polarization parallel to the static field. We find that the photodetachment cross section can be written as [compare with Eqs. (13)–(15) of Ref. [10], which give the results including rescattering]:

$$\sigma = \sigma^{(0)} + \Delta\sigma, \quad (1)$$

where

$$\sigma^{(0)} = \frac{8\pi^2 F \kappa}{3c\omega^3} [\xi^2 \text{Ai}^2(\xi) - \xi \text{Ai}'^2(\xi) - 2 \text{Ai}(\xi) \text{Ai}'(\xi)], \quad (2)$$

$$\Delta\sigma = -\frac{2\pi^2 (2F)^{5/3} \kappa}{c\omega^4} \left[ \text{Ai}^2(\xi) - \frac{F^{2/3}}{2^{1/3}\omega} [\text{Ai}'^2(\xi) - \xi \text{Ai}^2(\xi)] \right], \quad (3)$$

where  $c$  is the speed of light,  $\xi = -k^2/(2F)^{2/3}$ , and  $k^2/2 = \epsilon_f = \epsilon_i + \omega$  is the detached electron energy.

The term  $\sigma^{(0)}$  can be rewritten as

$$\sigma^{(0)} = \frac{6\pi F}{k^3} \sigma_0 \int_{-\infty}^{\xi} dx \text{Ai}'^2(-x), \quad (4)$$

where  $\sigma_0$  is the photodetachment cross section for  $F=0$ . Calculation of the integral in Eq. (4) can be done using a known technique [10] and leads to Eq. (2). Equation (4) completely coincides with the photodetachment cross section obtained earlier by Rau and Wong [4] and Du and Delos [5]. The formulations leading to Eq. (4) in both works used the unperturbed initial state and also additional approximations: Rau and Wong calculated the photodetachment amplitude using the frame transformation theory, and Du and Delos used the stationary phase approximation. Therefore, the result (2) can be identified as the weak-static-field approximation, whereas the additional term (3) gives the strong-static-field effect. This effect is caused both by the static-field-induced distortion of the initial bound state and by the exact account of a static field in the final state (i.e., beyond the weak-static-field approximation). Note that Eq. (3),  $\Delta\sigma$ , completely coincides with the strong-field correction of GS [2] in their Eq. (64), after the integral in that equation is evaluated analytically.

A second, independent confirmation of the results of GS, as well as those of BFS, is presented in a recent review article [12], which develops a general approach to the description of negative ion decay in the presence of strong external fields, allowing for the calculation of higher-order processes. The approach is based on the quasistationary, quasienergy method for a short-range potential [17] for the case of both strong static and laser fields. When applied to one-photon detachment in strong static and weak laser fields, and neglecting the rescattering effect, it leads to exactly the same result as that presented by Eqs. (1)–(3) (cf. Sec. 9 and especially Sec. 9.2.3 of Ref. [12]). Note that Eqs. (1)–(3) do not give the exact result for the one-photon detachment cross section since the rescattering effect is not included. This effect was treated in Refs. [16,3]. Our recently developed approach confirms these results, too (cf. Sec. 9 and especially Sec. 9.2.2 of Ref. [12]).

We conclude that there exist two independent confirmations of the GS result for strong-field effects in photodetachment in the presence of a static field, presented in Refs. [2,3] and discussed here. Neither of these independent calculations

used either the GS or the RR final-state wave functions. A possible misunderstanding of the GS results could have occurred only because GS interpret their correction as a strong *laser field* effect. As the present discussion shows, it is more valid to interpret the GS correction term  $\Delta\sigma$  as a strong *static field* effect, namely as resulting from the exact account of a strong static field in both the initial bound state and the final state involving the detached electron. Finally, we have discussed in this Comment our numerous concerns regarding the theoretical approach of RR, who fail to obtain the strong static field term in the single photon detachment cross section that has now been predicted not only by GS, but also by two independent treatments that were given here using the method of Ref. [10] and that were presented in Ref. [12].

*Note added in proof.* In a recent paper [18], the approach of Ref. [12] is used for an exact analysis of two-photon detachment of a negative ion in the presence of a strong static electric field. The results of Ref. [18] are as follows: first, in the weak static-field limit the exact results reduce to what would be obtained following an approach similar to that in Refs. [4,5]; second, neglecting only rescattering effects, the results of Ref. [18] coincide precisely with what follows from the GS results for two-photon detachment.

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