What Motivates Children to Respond? Functional Analysis and Intervention of Math Computation Fluency

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WHAT MOTIVATES CHILDREN TO RESPOND?
FUNCTIONAL ANALYSIS AND INTERVENTION OF MATH COMPUTATION FLUENCY

A Dissertation

Submitted to the Graduate Faculty of
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Agricultural and Mechanical College
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by
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Abstract

An abundance of research exists supporting the use of functional analyses to assess and treat problem behavior. In contrast, little research has been conducted on the application of functional analysis (FA) procedures to academic behaviors. The current study attempted to fill this research gap by conducting an FA of academic responding for five elementary students with low math fluency. Sessions were conducted using both a traditional reinforcement schedule of a fixed ratio of one (FR-1) as well as a more practical reinforcement schedule of a fixed ratio of ten (FR-10) to examine the effects of the reinforcement schedule on the FA outcomes. In addition, the study assessed the applicability of the analysis results by designing an instructional intervention to teach the students novel computation facts. In Experiment 1, four of the five participants demonstrated differentiated responding in math fluency across reinforcement conditions, although differences were small resulting in partial crossovers. Responding more closely approximated differentiation under the FR-1 schedule compared to the FR-10 schedule for four of the five participants. In Experiment 2, three of the five participants demonstrated differentiated responding across intervention conditions, although results were contrary to expectation due to optimal responding occurring in the worst condition rather than the best. The implications of these results are discussed within the context of a need for further research on the application of FA procedures to academic interventions.
Introduction

Basic competency in mathematics is crucial for success in school and beyond. In 2007, Duncan and colleagues analyzed six large-scale longitudinal studies (two of which were nationally representative of U.S. children) to examine the predictive power of academic, attention, and socioemotional skills on later reading and math achievement. The researchers found that not only were early math skills a strong predictor for later math achievement, but they were also as strong of a predictor for later reading achievement as early reading skills. The surprising results from this study suggest that early math skills may be vital for later academic success in both math and reading. Students’ high school math competencies in turn predict both employment and wages (Bynner and Parsons, 1997; Rivera-Batiz, 1992). For instance, there is a strong correlation between being competent in math through Algebra II or higher and being admitted to college, graduating from college, and earning in the top quartile of income (National Mathematics Advisory Panel, 2008). Bynner and Parsons (1997) found that individuals with poor numeracy skills were more likely to be out of a job compared to those with reading problems. Furthermore, Rivera-Batiz (1992) demonstrated that math competence accounted for a person’s employment, income, and work productivity, even after controlling for intelligence level and reading achievement. Having strong math skills is especially important in today’s society with the increase in modern technological advancements. According to the National Science Board (2008), growth of mathematics-intensive science and engineering jobs has outpaced overall job growth by 3:1.

Given the importance of math skills, it is therefore concerning how widespread math difficulties are in America. According to the 2015 Nation’s Report Card, only 40% of 4th grade students and only 33% of 8th grade students performed at or above the proficient level in
mathematics (National Center for Education Statistics, 2015). Furthermore, these scores were 1 and 2 points, respectively, lower than those of 2013. Scores are more abysmal for minority groups, with only 19% and 13% of African American students and 26% and 19% Hispanic students scoring above proficient in the 4th and 8th grade, respectively. American students’ math performance has also lagged internationally. In the latest version of the cross-national test, the Program for International Student Assessment (PISA) in 2012, the United States was ranked 27th out of 34 countries in math performance (Organisation for Economic Co-operation and Development, 2014). Adults in America also show evidence of low math performance. According to the 2003 National Assessment of Adult Literacy (NAAL), 78% of adults cannot explain how to compute interest paid on a loan, 71% do not know how to calculate miles per gallon on a trip, and 58% cannot calculate a 10% tip on a bill (National Center for Education Statistics, 2003). American adults also lag in performance internationally, with the United States performing below the international average in both numeracy and problem solving on the 2012–2014 Program for the International Assessment of Adult Competencies (PIAAC; U.S. Department of Education, 2016).

Approximately 5–8% of students have severe enough math problems to warrant a diagnosis of a math learning disability (Clarke, Doabler, & Nelson, 2014). To understand the persistence of math disabilities, Morgan, Farkas, and Wu (2009) analyzed the data from a nationally representative sample of students from the Early Childhood Longitudinal Study—Kindergarten Cohort. The researchers found that students in the lowest 10% upon entrance and exit of kindergarten (which they used as the criteria for approximating learning disabilities) had a 70% chance of remaining in this bottom 10th quartile five years later in 5th grade. Math achievement for this group remained 2 standard deviations below that of students who did not
demonstrate a math disability in kindergarten. Nevertheless, for students who started in the 10th percentile upon entering kindergarten but no longer met criteria upon exiting kindergarten, only 30% demonstrated math difficulties in 5th grade. This study has two important implications for math disabilities. It first demonstrates that, in the absence of intervention, math disabilities are likely to persist across grade levels. However, it also demonstrates the promising impact of early intervention, since most students who met criteria for math disability upon entrance of kindergarten but not exit were able to maintain their achievement five years later.

**Educational Policies**

In response to the growing concern over America’s low math achievement, several educational policies have been enacted to increase student performance. One common theme highlighted throughout these policies is the notion of using evidence-based practice to improve math achievement and remediate educational problems. Both the No Child Left Behind Act of 2001 (NCLB) and the Individuals with Disabilities Education Improvement Act of 2004 (IDEIA) strongly emphasized the use of scientifically based research to guide educational practice. Furthermore, the response to intervention (RTI) model that was introduced in IDEIA 2004 provided a major framework of support for implementing evidence-based practice to all students based on their individual needs. Lastly, the 2008 Foundations for Success report released by the National Mathematics Advisory Panel continued to highlight the national importance of using research to guide educational reform. These policies and report were therefore important for educational reform because they argued for the increased importance of evidence-based practice to remediate the educational problems in America.

**No Child Left Behind Act of 2001.** In January 2002, the No Child Left Behind Act of 2001 was passed to close student-achievement gaps by creating a system of high educational
standards and accountability (No Child Left Behind, 2001). Under the law, states were required to set standards for math and reading achievement for students in grades 3 - 8 and then develop a system to measure students’ progress towards meeting those standards. To improve student achievement, NCLB required the use of evidence-based educational programs and practices. After there was debate about what constituted scientifically based research, the U.S. Department of Education prioritized research utilizing randomized control trials with random assignment to experimental and control groups (Dahlkemper, 2003). This emphasis on scientifically based research was significant because it was the first time that schools were specifically required to adopt programs backed by scientific evidence (Dahlkemper, 2003). Furthermore, the fact that the term “scientifically based research” was used over 100 times throughout the NCLB law highlights the critical importance placed on evidence-based educational practices (Zucker, 2004).

**IDEIA 2004.** In 2004, the Individuals with Disabilities Education Act (IDEA) of 1997 was reauthorized as the Individuals with Disabilities Education Improvement Act (IDEIA). The purpose of the original IDEA was to ensure that all children with disabilities received a free and appropriate public education with emphasis on receiving tools, modifications, and support to meet their educational needs (National Center on Secondary Education and Transition, 2004). Many of the original regulations of IDEA remained in IDEIA. Two of the biggest changes, however, were the modification in criteria concerning the diagnosis of a learning disability and the introduction of a response to intervention framework. Prior to 2004, schools were required to use a discrepancy model to identify learning disabilities. Children were considered to have a learning disability if there was a significant discrepancy between their intellectual ability and their achievement (U.S. Department of Education, 2007). Nevertheless, under the new IDEIA 2004 regulations, schools were not required to use the discrepancy model and instead had the
option to use other research-based methods to determine the presence of a learning disability. In addition, schools were encouraged to use a process to determine whether students responded to a scientifically based intervention, which is known as their response to intervention (RTI). The switch from the discrepancy model to RTI was based on the increasing recognition that the problems of many children labeled with a learning disability could be remediated with specific, scientifically-based general education interventions rather than requiring placement in a special education program.

One common method to implement RTI is to use a multi-tiered model of support, typically conceptualized with three tiers. Within the first tier, high-quality instruction is provided to all students using evidence-based practices to prevent academic problems from occurring (Stoiber, 2014). For students who do not respond to the general high-quality instruction, supplemental differential support is provided with increasing intensity based on the student’s needs. The second tier involves modification of the general curriculum and implementation of low intensity interventions to remediate academic problems. For the small percentage of students who continue to exhibit academic problems despite second tier interventions, tier three interventions (which are highly intensive) may be necessary. Two key components throughout the multi-tiered process are regular progress monitoring to quickly identify problems and the use of evidence-based practices to prevent and remediate any problems that arise. Therefore, like NCLB, RTI and IDEIA 2004 were important in continuing the national emphasis on using evidence-based practice to remediate and prevent academic problems.

their 2008 Foundations for Success report, the NMAP recommended the following six strategies:
1) streamlining the prekindergarten – 8th grade curriculum; 2) utilizing research about how children learn; 3) recognizing the critical role that mathematically knowledgeable teachers have in math education; 4) basing instructional practice on high quality research; 5) improving the National Assessment of Educational Progress and state assessments to emphasize the most critical skills and knowledge leading to Algebra; and 6) conducting more rigorous research in education. The report especially emphasized the last recommendation, citing that much more research is needed on multiple aspects of educational policy. For instance, the NMAP called for more on the following: effective instructional practices, materials, and principles, mechanisms of learning, how to enhance teacher effectiveness in a way that is linked to improved student achievement, and more effective ways to assess mathematical knowledge. Improved research would, in turn, help guide the modification of educational practice to attain higher student achievement. Thus, NMAP continued the close association between educational research and evidence-based practice in schools.

**Behavior Analysis**

Given the increasing attention to evidence-based practice and the inclusion of research to inform educational policy, it is important to consider the various areas of research that can be used to guide practice. One area of research that has affected evidence-based practice is applied behavior analysis. In their 1968 seminal article, Baer, Wolf, and Risley defined applied behavior analysis (ABA) as the application of behavioral principles to improve socially significant behavior by analyzing environmental variables controlling the behavior. One important part of this definition is the emphasis on analyzing controlling environmental variables. To analyze a behavior, a researcher must control the occurrence and nonoccurrence of the behavior. This is
achieved by manipulating environmental variables to demonstrate a functional relationship between the manipulated variables and a reliable change in behavior.

The notion that behavior is controlled by environmental variables stems from the early behavioral work of Watson and Skinner (Cooper, Heron, & Heward, 2007). In contrast to the focus on mental processes that dominated psychology in the early 1900s, John Watson heralded a new field of psychology (i.e., behaviorism) by proposing that psychologists should instead study observable behavior (Watson, 1913). Furthermore, Watson argued that this should be accomplished by analyzing the relationship between environmental stimuli and the evoked behavioral responses. In 1938, B. F. Skinner published his book *The Behavior of Organisms* introducing a new branch of science: experimental analysis of behavior. Skinner argued that behavior was less influenced by what preceded it (i.e., antecedents) and more influenced by what followed it (i.e., consequences). This theory became known as the three-term contingency: antecedent – behavior – consequence. This sequence was later updated to a four-term contingency to include a motivating operation (MO), a change in the environment that alters the effectiveness of a reinforcer, which in turn alters the frequency of the behavior that has been followed by that reinforcement (Laraway, Snycherski, Michael, & Poling, 2003; Michael, 1982).

Most of what ABA has discovered about predicting and controlling behavior involves this four-term contingency (motivating operation – antecedent – behavior – consequence), which is why it is considered the basic unit of analysis for ABA (Cooper et al., 2007). For instance, one way to change behavior is by manipulating MOs. As described above, MOs influence the effectiveness of the reinforcer. Two specific kinds of MOs are establishing operations (which increase the effectiveness of reinforcers) and abolishing operations (which decrease the effectiveness of reinforcers; Laraway et al., 2003). A classic example of the effect of MOs
involves food: food deprivation acts as an establishing operation, increasing the effectiveness of using food to reinforce a behavior, whereas food satiation acts as an abolishing operation, decreasing the effectiveness of food as a reinforcer.

Another way to change behavior is to manipulate the antecedents. When an antecedent stimulus is repeatedly paired with the availability of reinforcement (and its absence is associated with the absence of reinforcement), it increases the momentary frequency of a behavior and is referred to as discriminative stimulus. Practitioners can use discriminative stimuli to increase the frequency of appropriate behavior. For example, when a teacher tells students to raise their hands to answer questions, this serves as a discriminative stimulus because it signals the availability of reinforcement (e.g., being selected to answer the question and given praise) for engaging in a behavior that has been previously reinforced in the past (i.e., raising their hands).

The final variable that can be changed to influence behavior based on the four-term contingency is the consequence of the behavior. There are two main types of consequences that affect behavior: reinforcement and punishment. Reinforcement is a change in stimulus that follows a response and increases the likelihood of the response occurring again in the future, whereas punishment decreases the likelihood of the response occurring again in the future. Practitioners can therefore create an intervention that either increases positive behavior (through reinforcement) or decreases negative behavior (through punishment). Nevertheless, one issue that greatly complicates consequence-based interventions is that people have different histories of reinforcement and thus respond to the same consequences in different ways. One way that researchers have attempted to resolve this issue is to conduct a functional analysis to identify what environmental variables may be controlling a person’s behavior.
**Functional analyses.** Prior to 1977, behavioral interventions for severe behaviors such as self-injury had varied success. In his 1977 review of the self-injury literature, Carr postulated that perhaps treatment effectiveness was variable because people’s self-injurious behavior was under the control of different motivational variables, each of which required a different intervention to eliminate. He therefore emphasized the importance of identifying the motivational variables underlying a person’s behavior to develop appropriate and effective treatments. Specifically, he hypothesized that both extrinsic (e.g., positive reinforcement, such as attention, or negative reinforcement, such as escape) and intrinsic (e.g., self-stimulation) reinforcement factors can maintain self-injurious behavior. He went on to explain that the dichotomy of extrinsic versus intrinsic reinforcement has crucial implications for treatment selection because different treatment strategies need to be selected depending on the type of reinforcement. Carr concluded his review by suggesting a screening that clinicians could use to determine the motivation of a behavior.

In their seminal 1982 study, Iwata, Dorsey, Slifer, Bauman, and Richman introduced an assessment method to experimentally determine the environmental factor controlling a behavior, which became known as a functional analysis (FA). The researchers observed the self-injurious behavior of nine children and youth with developmental disabilities under repeated exposure to a series of analogue conditions: social disapproval, academic demand, unstructured play, and alone. During the social disapproval condition, the children had free access to toys, and attention was given in the form of disapproval or concern contingent upon self-injury. This condition was designed to assess whether caregivers unintentionally maintained self-injurious behavior through positive reinforcement of social attention. During the academic demand condition, the experimenter presented learning trials that were terminated for 30 seconds contingent upon self-
injury. This condition assessed whether self-injury was maintained through negative reinforcement by allowing the participant to escape from demands. During the unstructured play condition, the children had free access to toys and were not presented with any academic demands. In addition, they were given praise and physical contact contingent upon appropriate behavior every 30 seconds and their self-injury was ignored. This condition served as a control condition that functioned as an enriched environment, under which little self-injury was to be expected. The final condition was the alone condition, during which the child sat alone in a therapy room without access to any toys or materials. This condition was designed to approximate an impoverished environment, in which greater self-injury might be expected due to the low environmental stimulation.

For six of the nine children, higher self-injury was consistently associated with one of the stimulus conditions. Most importantly, the condition that produced higher behaviors varied between these six children: two had more self-injury during the academic sessions relative to the other sessions, one exhibited more self-injury during the social disapproval sessions, and four exhibited more self-injury during the alone sessions. These results supported Carr’s (1977) hypothesis that self-injurious behavior may be a function of different sources of reinforcement for different individuals. This, in turn, has important implications for interventions, although that was not tested within this study given the focus on developing the new assessment method. The results of the assessment provide key information about what motivates a particular child to engage in self-injurious behavior, which could then be used to develop an intervention to promote positive, rather than negative, behaviors.

Subsequent research was conducted to evaluate the ability of an FA to guide treatment selection. For instance, Iwata, Pace, Kalsher, Cowdery, and Cataldo (1990) conducted three
studies to evaluate the controlling variables of self-injurious behavior and used this information to develop function-based interventions. The first study replicated that of Iwata and colleagues (1982): the researchers observed the self-injurious behavior (SIB) of seven youth with developmental disabilities under four conditions: attention, alone, escape, and control. All seven participants engaged in more self-injurious behavior in the demand condition compared to other conditions, suggesting an escape function. In the second study, the researchers assessed the effect of an escape-extinction intervention on the behavior of six of the seven original subjects. In response to the intervention, all participants exhibited a significant reduction of SIB to zero or near zero levels and an increase in compliance. The third study evaluated an extinction plus reinforcement intervention for the seventh original participant. The treatment eliminated the SIB and results were generalized across multiple therapists and physicians. Taken together, these results demonstrated the utility of conducting an FA to inform treatment selection for problematic behavior. Since then, a plethora of research has been conducted supporting the use of an FA to inform treatment (Mace, 1994). Researchers have also adapted the FA methodology to assess and treat a wide variety of other behaviors beyond SIB such as aggression, destruction, disordered speech, stereotypy, pica, and tantrums (Dixon, Vogel, & Tarbox, 2012).

Although the original FA research focused on individuals with severe disabilities in inpatient hospitals, research gradually expanded to novel settings (e.g., outpatient clinics) and participants (e.g., those with average intelligence). In 1990, Cooper, Wacker, Sasso, Reimers, and Donn conducted a modified brief FA with typically developing children referred to an outpatient clinic for severe conduct problems. Parents were taught how to run the experimental analyses by manipulating task demand (easy versus difficult) and attention (attention versus no attention). Nevertheless, rather than analyzing contingencies that maintained problem behavior
and then applying the results to appropriate behavior, Cooper and colleagues directly analyzed the environmental variables controlling appropriate (on-task) behavior. With a one-day assessment, the researchers determined the contingencies that produced the highest level of appropriate behavior for all but one child, whose results were undifferentiated. The results of the assessment were then used to develop interventions for the children, which were rated as effective and acceptable both initially and at follow-up. Furthermore, problem behavior ratings at follow-up demonstrated overall improvement in the children’s behavior. This study was important in demonstrating that FAs could be effectively extended to the appropriate behavior of typically developing children.

In 1991, Northup and colleagues used the same brief FA procedures as Cooper and colleagues (1990) to assess the aggressive behavior of three individuals with severe disabilities. In addition, the researchers conducted a brief contingency reversal following the completion of the FA to evaluate whether the maintaining contingency for aggressive behavior could be used to maintain replacement (appropriate requesting) behavior. Within a single 90-minute outpatient evaluation, the researchers determined the function of each of the participants’ behaviors and successfully implemented a function-based intervention for alternative replacement behavior. In line with the results of Cooper and colleagues (1990), this study demonstrated the effectiveness of a more feasible, less time-intensive experimental analysis method. Additionally, the contingency reversal component of the study demonstrated the utility of FA results in designing effective interventions.

In 1992, Derby and colleagues evaluated the brief FA approach used by Cooper and colleagues (1990) and Northup and colleagues (1991) across 79 outpatient clients with varying levels of developmental disabilities. When clients engaged in problem behavior during the
assessment, the researchers could identify a maintaining contingency approximately 75% of the time. The application of the maintaining contingency to appropriate behavior resulted in decreased problem behavior during a little over half of the evaluations. However, only 63% of the clients engaged in the target problem behavior during the assessment. This reflects an important limitation of brief functional assessments that must be considered: they are most effective for high frequency behaviors. Nonetheless, the results demonstrate that the procedures are replicable and generalizable to a large proportion of individuals with developmental disabilities with high frequency problem behavior.

Cooper and colleagues (1992) extended the work of Cooper and colleagues (1990) by conducting two experiments that evaluated the effects of task preference, task demands, and adult attention on children’s appropriate (on-task) behavior. In the first experiment, the researchers conducted a brief FA for children seeking behavioral treatment at an outpatient clinic. The researchers identified a maintaining condition for the eight children and achieved replication by conducting a brief reversal. In the second experiment, the researchers conducted both extended classroom assessments and brief FAs for two children with borderline intelligence that displayed noncompliant behaviors in class. The researchers found that the results of the brief FA corresponded to those from the extended analysis, supporting their efficiency and effectiveness.

Harding, Wacker, Cooper, Millard, and Jensen-Kovalan (1994) extended the work of Cooper and colleagues (1992) by evaluating a designated hierarchy of antecedent- and consequence-treatment components to promote positive behavior (i.e. on-task behaviors) for children in an outpatient clinic. The researchers began by conducting a brief assessment of antecedent variables, given that they are typically easier for parents to implement. If the
antecedent variables were not successful in controlling behaviors, the researchers then assessed reinforcement procedures, followed by mild punishment procedures, in order to find the least intrusive treatment package. All seven children exhibited improved behavior with specific treatment components and experimental control was established via a brief reversal for six of the seven children (one child continued to behave appropriately during the contingency reversal). The results of this study extended the literature on brief FAs by demonstrating their effectiveness in selecting the least intrusive intervention package.

**Brief Experimental Analyses**

Recently, researchers have begun applying the basic principles of FAs to academic problems in a new assessment method known as brief experimental analysis (BEA). Similar to functional analyses, BEA also uses a single-case design with rapid alternation of experimental conditions to assess the environmental variables that control behavior (Daly & Martens, 1997). However, whereas FAs tend to focus on decreasing behavioral excesses, BEA instead focuses on increasing behavior that can be described as deficient (Daly et al., 2006). Additionally, interventions are assessed directly in a BEA instead of being inferred from maintaining variables, as they typically are in an FA. Furthermore, the conditions and data series are typically abridged to maximize time efficiency, similar to those used in the brief functional analyses literature (e.g. Northup et al., 1991).

BEA marks an important deviation from previous educational practices. Previously, under the “refer-test-place” model of school psychology, assessments of poor academic performance were typically limited to identifying the presence of a learning disability and placing the student in special education (Powers, Hagans, & Busse, 2008). Nevertheless, in line with the “research-based practice” focus of NCLB and IDEIA, BEA focuses on systematically
analyzing which evidence based intervention is the most effective in remediating a student’s academic problems. In line with the original three-term contingency of ABA, BEA examines academic performance in relation to various facets of classroom instruction that precede and follow student performance. The goal of using BEA is to test the impact of potential intervention strategies on academic behavior before they are recommended to a teacher, rather than wait for a selected intervention to fail (Daly, Hofstadter, Martinez, & Anderson, 2010).

**Intervention selection.** One useful method of intervention selection is based on Haring and Eaton’s (1978) Instructional Hierarchy (IH). The IH utilizes the basic principles of ABA to increase academic responding through the development of stimulus control and the use of differential reinforcement (Ardoin & Daly, 2007). According to the IH, skills progress through four phases: acquisition, fluency, generalization, and adaptation (Daly et al., 2010). Students first learn a new skill during the acquisition phase, so the goal is to promote accurate responding by developing stimulus control. Acquisition interventions typically require modeling, guided practice, and feedback. Once the student can respond accurately, the next goal is building fluency, or fast and accurate responding. Fluency interventions usually involve repeated practice with reinforcement provided for correct answers. Once the student can respond quickly and accurately, the focus then shifts to generalization, where the student learns to perform the skill under new conditions. Interventions for generalization include teaching multiple exemplars and programming common stimuli. The last stage of the hierarchy is adaptation, in which the student applies the new skill to new, increasingly complex conditions. Strategies for adaptation involve students applying the skills under new, higher order tasks. Four decades of research support using the IH to match interventions to students’ academic needs, making it an invaluable guide for treatment selection (Ardoin & Daly, 2007).
Another basis for guiding intervention selection is Daly and colleagues’ (1997) five hypotheses concerning low academic performance, which are presented in order of increasing intervention intensity. The first hypothesis is that a student’s performance may be low because (s)he does not want to complete the task. This hypothesis is in line with Lentz (1988)’s distinction between performance and skill deficits. According to Lentz, a performance deficit involves a lack of motivation, which can be remedied through stronger reinforcement contingencies, whereas a skill deficit involves lack of instructional control and cannot be remedied by providing reinforcement. To test for a performance deficit, researchers can provide incentives for increased academic responding and evaluate whether achievement improves. The second hypothesis concerning low performance is that the student may not have practiced the skill enough. To test this hypothesis, researchers can implement an intervention involving repeated practice, such as repeated readings (RR), to evaluate whether this improves the students’ performance. The third hypothesis for low performance is that the student has not had enough help to perform the skill. To test this hypothesis, researchers can use interventions such as modeling, instructional prompts, and explicit feedback. The fourth hypothesis for low achievement is that the student has not previously performed the skill in a certain way. This hypothesis considers the role of the instructional materials and their stimulus control over correct responding. To test this hypothesis, researchers can use specific instructional materials that produce student responses required for mastery of the skill. The final hypothesis concerning low performance is that the task might be too difficult. Research shows that students are more likely to generalize a skill to other instructional materials when they are instructed at their instructional level (Daly et al, 1996). Therefore, educators may need to evaluate whether the materials are at an appropriate instructional level. However, students’ varying skill levels within a classroom
makes changing instructional materials to match each student’s need difficult. Thus this hypothesis is presented last and is suggested only when the previous factors are ineffective at increasing student performance. Daly and colleagues’ (1997) article therefore provided an early conceptual model of testing empirically-based interventions to determine the most effective solution for a particular student’s needs.

**Reading BEA.** The early BEA literature began by assessing reading interventions. In the earliest BEA study, Daly and Martens (1994) compared the effects of three reading instruction interventions (subject passage preview, listening passage preview, and taped words) on the oral reading performance of four male students with learning disabilities. The researchers used a multi-element design to compare the effects of the three interventions to each other and to baseline over the course of several weeks. The results of the study demonstrated that all four students had the greatest increase in reading accuracy and fluency using the listening passage preview intervention. Since then, two decades of research has produced strong support for the utility of BEA to identify effective oral reading interventions. For instance, in a meta-analysis conducted in 2008, Burns and Wagner found that most of the analyzed BEA studies identified an intervention that was most effective for each participant. The average effect size for the most effective intervention, compared to other interventions, was 2.8, with 80% non-overlapping data. In addition, the most effective interventions resulted in an average fluency increase of approximately 30 words read correctly per minute.

**BEA methodology.** Three main methodologies have been used to conduct BEAs. Given that the majority of BEA research has been conducted with reading fluency interventions, the methodologies will be discussed within this topic. The first method, which was used in the initial BEA literature, is to evaluate individual treatment components to determine what produces the
strongest effect (Daly, Anderson, Gortmaker, & Turner, 2006). The study conducted by Daly and Martens (1994), for instance, utilized this methodology to compare the three reading interventions to each other and to baseline. Another study that utilized this method was Jones and Wickstrom (2002). The researchers analyzed the efficacy of four instructional strategies for five children with reading problems: incentives, repeated practice, increased learning trials, and easier materials. Each instructional strategy was tested once. The participants exhibited differentiated responding to treatments and an effective strategy was determined for each child. The most effective strategy was then alternated with the baseline condition in an extended analysis to assess the effects of the strategy across time. Four out of the five students had stable performance with the most effective strategy over time and demonstrated collateral effects on generalization passages.

The second BEA method that emerged combined intervention strategies sequentially to create more intricate interventions that could have stronger effects (Daly et al., 2006). Daly, Martens, Hamler, Dool, and Eckert (1999) evaluated the effects of combining instructional components to reading interventions for four students with reading problems. The instructional components were sequenced in order of increasing adult involvement. The purpose of this study was to determine the most effective treatment package that required the minimum amount of adult involvement. If student performance did not improve during an intervention, the treatment was enhanced by adding further components. The first instructional component used was a reward for rapid reading. If this intervention did not improve the student’s performance, then the student experienced repeated readings, followed by listening passage preview, sequential modification, and finally easier materials. All four students exhibited improved reading fluency in one of the treatment conditions, with some students responding better to simpler interventions.
and others requiring more complex interventions. Overall, the benefit of this method is that it allows for the identification of simple but effective interventions. This is important because practitioners are more likely to implement simple interventions compared to complex ones (Daly et al., 2006).

The third BEA method that can be used involves the reverse of the second method: components are removed from a strong treatment package until treatment effects disappear (Daly et al., 2006). One potential drawback to the second BEA method is the possibility that sessions may be terminated prematurely (Daly, Persampieri, McCurdy, & Gortmaker, 2005). In contrast, the third BEA method allows for the comparison between simple and comprehensive treatment packages to determine their relative benefits. The overall purpose of this method is therefore to identify the simplest treatment package that still enhances student performance. For instance, Daly and colleagues (2005) assessed the use of rewards, instruction, and a combined treatment package for two elementary students with reading problems in three phases. In the first phase, the students each experienced the combined treatment package, which consisted of a reward contingency and instruction (repeated reading, listening passage preview, phase drill, and syllable segmentation), as well as a control condition. In the second phase, a components analysis was conducted to separate and analyze the effects of the reward and instructional components. In the final condition, the best individual component (reward for one student and instruction for the other) was compared to the full treatment package and control condition. For one of the two students, the reward condition produced almost equivalent gains in reading fluency to the combined treatment package and was therefore selected as the preferred treatment given its simplicity. The other student, however, had considerably higher performance in the combined treatment package compared to the instructional condition (the next highest condition),
so the combined treatment package was selected despite its increased complexity. Based on the results of the experiment, a self-managed intervention was created for each student, which resulted in significant reading improvements for both students.

**Math BEA.** To date, comparatively less research has evaluated the application of BEA to math interventions. Hendrickson, Gable, Novak, and Peck conducted the first math BEA study in 1996. The researchers analyzed the effects of three skill-based interventions (time delay, number line, and decomposition) on improving a fourth-grade student’s math fact acquisition by testing one intervention at a time. The student had the greatest acquisition with the decomposition intervention. The researchers replicated this result by demonstrating that the decomposition intervention (the most effective intervention in the BEA) resulted in greater acquisition of new facts compared to the time delay intervention (the least effective intervention in the BEA). The results of this study showed that BEA could be used to determine an optimal math instructional intervention for a student struggling with math fact acquisition. This study was the first of several studies to examine the application of BEA to selecting effective math interventions.

Carson and Eckert (2003) conducted the second mathematics BEA study to assess the effects of student-selected versus empirically-selected interventions on the computational fluency of three elementary students. The researchers conducted a BEA in the first phase of the study to compare the effects of four interventions: contingent reinforcement, goal setting, feedback on digits correct, and timed-sprint. Prior to implementing the interventions, the students and experimenters discussed each intervention and each student ranked the interventions based on their perceived effectiveness. The researchers then compared the interventions in a multi-element design across several trials. In the second phase of the experiment, the most effective intervention from the BEA was compared to the student-selected intervention in an alternating
treatment design. The results of the study demonstrated that the BEA-selected intervention was more effective in increasing the students’ computational fluency compared to the student-selected intervention. Overall, the results of the study supported conducting a BEA to increase the probability of positive treatment outcomes.

In 2008, Gilbertson, Witt, Duhon, and Dufrene evaluated the efficacy of BEA to identify an effective intervention to improve both math fluency and on-task behavior. The experimenters conducted a BEA for four students referred for academic (math) and behavior problems. The BEA compared the effects of contingent reward versus contingent reward combined with instruction and was completed for each student within 15 minutes. The results demonstrated that the reward plus instruction intervention resulted in higher performance compared to the reward alone for all students. The researchers then developed and implemented an intervention that combined both instruction and rewards and analyzed the intervention’s effects on the students’ math fluency and on-task behavior using a multiple baseline across subjects. All four students demonstrated improved math fluency and on-task behavior with the implementation of the intervention. The results of the study support the use of BEA to identify effective interventions for children with behavior and academic problems.

Similar to Gilbertson and colleagues (2008), Codding and colleagues (2009) also conducted a BEA to compare skill versus performance-based interventions. In addition, they examined the effects of the selected intervention across time and generalization worksheets. The researchers conducted a BEA on four children grades 3 – 6 comparing four skill and performance interventions: incentive, performance feedback, goal setting, and cover-copy-compare. Each intervention was presented once and was analyzed based on the improvement of digits correct per minute as well as visual analysis. The BEA resulted in the selection of the most
effective strategy for each child, with the selected intervention varying between children. The researchers then employed an extended analysis to compare the selected intervention to baseline across both target and generalization worksheets. The generalization worksheets contained 50% of the problems used in the extended analysis, which were trained either with the most effective intervention or baseline conditions, and were not subjected to the selected treatment. Results of the study demonstrated that the effects of the selected intervention were stable across time compared to baseline. Performance for two of the students even reached mastery level upon implementation of the selected intervention. However, only one of the four students demonstrated generalized performance with the selected intervention condition compared to baseline on the generalization worksheets. For the other participants, performance was equivalent regardless of which problem type (baseline or intervention) was mixed with novel problems. This could be because problem overlap with targeted problems was only 50% whereas reading generalization research generally uses 80% overlap.

More recently, Mong and Mong (2012) evaluated the predictive ability of BEA to identify the most effective intervention for three elementary students struggling with math fluency. The researchers compared three math interventions (cover-copy-compare, taped problems, and math to mastery) by implementing them each once. Following this BEA, the most effective intervention for each child was selected and compared to the remaining interventions and baseline condition. The results of the study demonstrated that BEA correctly predicted the most effective intervention for increasing math fluency for each of the three students. Furthermore, generalization probes demonstrated that generalization was consistent with the BEA predicted intervention for two of the three students.
The most recent math BEA study was conducted by Reisener, Dufrene, Clark, Olmi, and Tingstrom (2016). The researchers conducted two studies, both of which involved an initial BEA comparing four conditions (reward, cover-copy-compare, constant time delay, and control), an extended analysis that compared the most and least effective interventions from the BEA, and an intervention phase, where the most effective intervention was implemented alone. Although only two of the eight participants demonstrated clearly differentiated responding between interventions, all participants responded favorably to at least one intervention. The authors posited that the lack of clear differentiation could be due to multiple treatment interference. In addition, the lack of differentiation could be due to limited exposure to the treatment conditions (each condition was run once in the BEA). For six of the eight students, the intervention that produced the greatest math fluency in the BEA also produced the greatest rate in the extended analyses. The remaining two students nonetheless both demonstrated a positive response in the experimental analyses to the predicted intervention. These results are promising because they indicate that students’ responses to a BEA intervention predict improved responses to that intervention in extended analyses. Finally, all students continued to make fluency gains when the predicted math intervention was implemented in isolation during the intervention phase.

**Functional Analysis of Academic Behavior**

Although considerable evidence exists to support BEA, only one study to date has used an experimental analysis to evaluate the naturalistic forms of reinforcement that were inherent in the traditional functional analyses. Hofstadter-Duke and Daly (2015) conducted a series of FAs to examine reinforcers that maintained academic responding (math computation) for three elementary school children. Their rationale for the experiment was that thus far functionally derived interventions for promoting academic behavior had typically generalized results from
functional analyses of problem behavior to identify consequence to maintain academic behaviors. Nevertheless, it is not always the case the variable maintaining problem behavior will generalize to positive behavior (Holden, 2002). Thus, the authors argued it was important to directly assess the environmental variables controlling academic responding.

The researchers began by conducting an FA of non-fluent math facts using adult attention, peer attention, escape, and control as the four reinforcement conditions. During the adult attention condition, the experimenter provided brief praise statements contingent upon academic completion. During the peer attention condition, a peer sat next to the student and provided praise statements similar to those in the adult attention condition. During the escape condition, the student received a 15-second break from work for each problem completed. For the control condition, the student completed problems alone while the experimenter monitored them nearby. The researchers found that when non-fluent math problems were used, results were undifferentiated between conditions across participants, which they attributed to their weak stimulus control. After they taught the children the math facts, they repeated the experimental analysis and found differentiated responding between conditions across all participants. This supported the researchers’ hypothesis that the academic material needed to be under stimulus control before differentiated responding could occur. Overall, this study demonstrated the possibility of directly analyzing functional reinforcers for academic behavior rather than indirectly determining reinforcers through analyzing problem behavior. Given that they were the only researchers thus far to assess the applicability of functional analyses to academic responding, more research is needed to replicate these results.
Purpose of the Current Study

Current practice in schools typically involves the utilization of praise to reinforce academic responding. But for some children, praise may not be sufficient to maintain high rates of accurate academic responding. At times, teachers may choose to utilize alternative sources of reinforcement such as tangibles (e.g., candy, toys) or escape (e.g., engagement in a fun activity contingent upon work completion), but their selection is often idiosyncratic rather than informed by assessment results. Teachers also start incorporating symbolic reinforcement in early grades that becomes more frequent as students progress throughout school (e.g., happy and sad faces, check marks, letter grades). Yet research has not evaluated the effectiveness of this reinforcement type compared to other forms of reinforcement (i.e., praise, escape, tangibles). This issue of reinforcement selection is particularly pertinent for children with low academic achievement.

Thus, the question at hand is how do teachers determine how to best motivate struggling learners? In the current age of evidence-based practice, it is vital that reinforcer analyses are conducted prior to intervention implementation to ensure optimal treatment outcomes. The purpose of the current study was to address this concern by replicating and extending the work of Hofstadter-Duke and Daly (2015) within the context of improving students’ math computational fluency. To accomplish this, two separate but related experiments were conducted.

The first experiment utilized an intervention based on the fluency stage of Haring and Eaton’s (1978) Instructional Hierarchy. Participants received repeated practice paired with different forms of reinforcement (i.e., the different FA conditions). The intervention was divided into two phases and was counterbalanced across participants. Phase A involved a direct replication of Hofstadter-Duke and Daly’s procedure to assess the most effective reinforcer for
fluent math problems. However, whereas Hofstadter-Duke and Daly reinforced problem completion, regardless of accuracy, only accurate responding was reinforced in the current study to promote computational fluency. In line with the results from Hofstadter-Duke and Daly (2015) and in parallel to the original functional analysis literature (e.g. Iwata et al. 1982/1994), it was hypothesized that there should be clear response differentiation between FA conditions for each participant.

During Phase B of Experiment 1, the FA procedure was modified to simulate a reinforcement schedule that is more representative of a classroom setting. Typical FA procedures provide reinforcement on a fixed ratio schedule of one (FR1), meaning that every targeted response is reinforced (e.g., Iwata, Dorsey, Slifer, Bauman, & Richman, 1982/1994). Nevertheless, it is not feasible for teachers to provide students with reinforcement this often, which can lead to poor treatment integrity and, in turn, decreased intervention effectiveness (Hanley, Iwata, & Thompson, 2001; Tiger, Hanley, & Bruzek, 2008). It is therefore important to examine whether a lean reinforcement schedule that more closely parallels actual classroom practice is still effective at producing differentiated results across reinforcers. Given that larger FR schedules generally produce higher rates of responding up until a certain point (Cooper, Heron, & Heward, 2007), it was hypothesized that academic responding in the optimal reinforcement condition in Phase B would be even higher than it was in the same condition for Phase A.

The purpose of the second experiment was to assess the applicability of the FA results. A constant time delay (CTD) intervention was selected based on the acquisition stage of the IH to teach students new math facts. Specifically, the CTD intervention utilized each student’s best and worst reinforcement conditions from Experiment 1 to reinforce correct responding. Although
there have not been any studies measuring the effectiveness of function-based interventions for academic behavior, there exists a multitude of research demonstrating the effectiveness of function-based interventions (Beavers, Iwata, & Lerman, 2013). In addition, there has recently been several studies conducted on the use of brief experimental analysis (BEA) to identify effective interventions for improving academic performance (e.g. Mong and Mong, 2012). Therefore, it was hypothesized that the CTD intervention based on the most effective reinforcer from Experiment 1 should be the most effective for teaching a new math skill in Experiment 2.
Method

Participants and Setting

Prior to participant recruitment, the study was approved by the LSU Institutional Review Board (see Appendix A). Participants were recruited from a local elementary school, where teachers were asked to nominate students with poor math fluency. A letter was sent home to the nominated students’ parents detailing the study and requesting permission to work with their children (see Appendix B). Once parental consent was obtained, students were asked to provide verbal assent before participating in the study (see Appendix C). Students were then screened individually to assess their eligibility to participate in the experiment. Five students were selected to participate in the study using the selection procedure described below. To maintain confidentiality, Participant 1 will be referred to Anna, Participant 2 as Beatrice, Participant 3 as Cameron, Participant 4 as Dominic, and Participant 5 as Eliza. Anna was a 9-year-old African-American female in the 4th grade. Beatrice was an 11-year-old White female in the 5th grade. Cameron was an 11-year-old African-American male in the 6th grade. Dominic was a 10-year-old African-American male in the 4th grade. Eliza was a 9-year-old African-American female in the 4th grade. All five children were enrolled in general-education math class, although Cameron did have a documented diagnosis of dyscalculia. Students individually completed a session three times a week in a quiet location at school with an experimenter.

Materials

A series of worksheets were used to present math stimuli to the students in both experiments. All worksheets were printed on 8.5 X 11 in. paper sheets that were pink, yellow, green, or blue to help students discriminate between the experimental conditions and contingencies. The worksheets for each student consisted of the same type of math problems
across Experiment 1, depending on their current skill level (addition, subtraction, multiplication, or division). The worksheets for Experiment 2 consisted of the next math problem type in the sequence (e.g., if multiplication was used in Experiment 1, division was used in Experiment 2). Based on the results of the screening (described below), all students worked on multiplication problems in Experiment 1 and division problems in Experiment 2. The worksheets for both experiments contained random problems using numbers 1 – 12 (or their multiples for division problems). To control difficulty across conditions, worksheets were created manually using a random number generator to select number pairings for each problem. Two separate sets of numbers were randomly assigned to the conditions in Experiment 2 to avoid carry over effects between conditions as the students learned the new problems. For instance, numbers 2, 4, 8, 1, 9, and 12 were randomly assigned to the best condition in Experiment 2 for each participant, so a set of 36 division problems were constructed from their multiples (e.g. $108 \div 9 = 8$ and $108 \div 8 = 9$). The numbers 3, 5, 6, 11, 10, and 7 were randomly assigned to the worst condition and a set of 36 division problems were constructed from their multiples (e.g. $15 \div 3 = 5$ and $15 \div 5 = 3$). A blank cover sheet was used in both experiments to allow the experimenter to control the rate of problem presentation. Multiplication flashcards were used with Beatrice for five minutes immediately prior to sessions 12 and 13 in Experiment 1 to enhance problem accuracy and improve response differentiation across conditions.

**Measurement of Dependent Variables**

**Math fluency.** The primary dependent variable for the both experiments was math fluency. Math fluency is defined as the rate of accurate skill production and was measured in digits correct per minute (DCPM), since this measure is more sensitive to detecting change than the number of correct answers (Hosp, Hosp, & Howell, 2007). Digits correct (DC) were
calculated by determining the total number of digits answered correctly for each problem. For instance, if a student answered the problem $12 + 6$ as 28, the “2” digit would be incorrect but the “8” digit would be correct, so the DC for this problem would be 1. During each experimental condition in Experiment 1, students wrote down the answer to math problems for 5 minutes. The experimenter followed along on a second copy of the worksheet that contained the correct answers, marking digits correct and incorrect as the student progressed. The experimenter kept track of time by using a stopwatch timer. For the attention, symbolic, and control conditions, the total number of digits correct was divided by the length of the session (in seconds) and then multiplied by 60 to calculate DCPM. For the escape condition, the experimenter first subtracted the total length of time that the student spent taking programmed breaks from the total session length, which yielded the time spent engaged in work. DCPM for this condition was calculated by dividing the DC by the time spent engaged in work (in seconds) and then multiplying this by 60. During Experiment 2, the participants worked on a probe worksheet at the beginning of each instructional session while the experimenter timed their progress. DCPM for each probe was calculated by dividing the digits correct by the time spent working on the probe (in seconds) and then multiplying this by 60.

**Problem accuracy.** Students’ problem accuracy was measured in Experiment 2 to assess their progress under each instructional intervention. Problem accuracy was assessed by calculating the percentage of problems answered correctly on the daily probe for each instructional condition.

**Visual and statistical analysis.** A combination of visual and statistical analysis was used to determine differentiation in responding for math fluency and problem accuracy across conditions. Visual inspection examined the following features in line with the What Works
Clearinghouse guidelines for single case design: consistency in level, trend, and variability per phase, immediacy of effect, overlap in data, and consistency of data patterns across phases (Kratochwill et al., 2010). Visual analysis was substantiated by summary statistics that presented the mean and standard deviation for each condition per phase.

**Procedural integrity.** To ensure procedural integrity, the main experimenter filled out a procedural checklist for each daily session in both experiments. The procedural checklist had 22 steps in Experiment 1 and 20 steps in Experiment 2 (see Appendices D and E). In addition, a second experimenter accompanied the main experimenter for approximately 33% of the sessions per experiment and completed a second checklist. Procedural integrity was calculated by dividing the number of steps implemented by the total number of steps and then multiplying by 100. Using this equation, procedural integrity was calculated for each participant, phase, and experiment (see Table 1). Procedural integrity was unable to be assessed for Phase A, Experiment 1 for Anna or Phase B, Experiment 1 for Eliza due to schedule conflicts.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Experiment 1 Phase A</th>
<th>Experiment 1 Phase B</th>
<th>Experiment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anna</td>
<td>---</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Beatrice</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Cameron</td>
<td>100%</td>
<td>82%</td>
<td>97.5%</td>
</tr>
<tr>
<td>Dominic</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Eliza</td>
<td>100%</td>
<td>---</td>
<td>100%</td>
</tr>
</tbody>
</table>

**Interscorer agreement.** To evaluate interscorer agreement, a second experimenter re-scored the worksheets from approximately 33% of the sessions in each experiment. Interscorer agreement was calculated by dividing the number of digits agreed upon by the total number of digits completed and multiplying by 100 (see Table 2).
Table 2

Interscorer Agreement

<table>
<thead>
<tr>
<th>Participant</th>
<th>Experiment 1</th>
<th>Experiment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anna</td>
<td>100%</td>
<td>99.7%</td>
</tr>
<tr>
<td>Beatrice</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Cameron</td>
<td>99.3%</td>
<td>100%</td>
</tr>
<tr>
<td>Dominic</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Eliza</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Experiment 1

Experimental Design

Experiment 1 consisted of an alternating treatment design with two phases. Each phase comprised a functional analysis (FA) with four alternating conditions: attention, escape, symbolic, and control. The FA conducted in Phase A mirrored the traditional FA reinforcement schedule of a fixed ratio of one (FR-1), meaning that every targeted response was reinforced (e.g., Iwata, Dorsey, Slifer, Bauman, & Richman, 1982/1994). The FA conducted in Phase B used a lean reinforcement schedule that more closely paralleled actual classroom practice. To this end, students received reinforcement after every 10 problems they answered correctly. In each phase, FA conditions were rapidly altered in a random order. Phases were counterbalanced across participants such that Anna and Cameron began with Phase A while Beatrice, Dominic, and Eliza began with Phase B. A phase changed occurred once stable responding across at least three data points was observed.

Phase A

Conditions. The following four experimental conditions were used: attention, escape, symbolic, and control. The attention, escape, and control procedures mirrored those used in the original functional analysis literature (e.g., Iwata et al., 1982/1984). The symbolic condition was added to parallel the frequent symbolic reinforcement students receive in school (e.g. grades,
checkmarks, smiley faces, etc.). A different colored worksheet was used in each of the four conditions to help facilitate discrimination across conditions. The four conditions are described below.

**Attention.** During the attention condition, the student worked on a pink math worksheet. The experimenter sat next to the student at the table so that she could see the student’s answers. The worksheet was covered so only one math problem could be seen at a time and the experimenter moved the coversheet as the student progressed through the worksheet. The student was told:

> When we use pink worksheets, I will give you praise for every problem you answer correctly. If you do not answer a problem correctly within 10 seconds or answer a question incorrectly, I will tell you the correct answer, which you will write down, and then we will move on to the next problem.

After explaining the instructions, the experimenter set the timer for five minutes and told the student to begin. If the student answered a problem correctly, (s)he was given a brief praise statement such as “good job” or “awesome”. If the participant answered the problem incorrectly or did not answer within 10 seconds, (s)he was told in a neutral tone “the answer is _______”, which (s)he wrote down. (S)he was not given any additional attention at this point. After the student wrote down the correct answer, either independently or with prompting, the next problem was uncovered. This procedure was repeated until the timer went off, signaling the end of the trial.

**Escape.** During the escape condition, the student worked on a yellow math worksheet using the same procedure as described above. The student was told:
When we use yellow worksheets, I will give you a 15 second break for every problem you answer correctly. If you do not answer a problem correctly within 10 seconds or answer a question incorrectly, I will tell you the correct answer, which you will write down, and then we will move on to the next problem.

After explaining the instructions, the experimenter set the timer for five minutes and told the student to begin. If the student answered a problem correctly, (s)he was told in a neutral tone that it was correct and received a 15 second break from work. During this break, the problem was covered and the experimenter turned away. After the break, the next question was presented. If the participant answered the problem incorrectly or did not answer within 10 seconds, (s)he was told in a neutral tone “the answer is ______”, which (s)he wrote down. After writing down the correct answer, the next problem was presented. This procedure was repeated until the timer went off, signaling the end of the trial.

**Symbolic.** During the symbolic condition, the student worked on a green math worksheet using the same procedure as described above. The student was told:

When we use green worksheets, I will give you a smiley face stamp next to every problem you answer correctly. If you do not answer a problem correctly within 10 seconds or answer a question incorrectly, I will tell you the correct answer, which you will write down, and then we will move on to the next problem.

After explaining the instructions, the experimenter set the timer for five minutes and told the student to begin. If the student answered a problem correctly, (s)he was told in a neutral tone that it was correct and then given a smiley face stamp next to the correct answer. If the participant answered the problem incorrectly or did not answer within 10 seconds, the experimenter told the student in a neutral tone “the answer is ______”, which (s)he wrote down.
After writing down the correct answer, the next problem was presented. This procedure was repeated until the timer went off, signaling the end of the trial.

Control. The control condition consisted of treatment as usual, where students worked on math problems without additional experimenter attention using the same procedure as described above. The student was told:

When we use blue worksheets, you will work on math problems. If you answer a problem correctly, I will uncover the next problem. If you do not answer a problem correctly within 10 seconds or answer a question incorrectly, I will tell you the correct answer, which you will write down, and then we will move on to the next problem.

After explaining the instructions, the experimenter set the timer for five minutes and told the student to begin. If the student answered a problem correctly, (s)he was told in a neutral tone that it was correct and then moved on to the next question. If the student answered a problem incorrectly, the experimenter told the student in a neutral tone “the answer is ______”, which (s)he wrote down. After the student wrote down the correct answer, either independently or with prompting, the next problem was uncovered. This procedure was repeated until the timer went off, signaling the end of the trial.

Phase B

Conditions. The conditions for Phase B were a replication of those from Phase A, apart from a different reinforcement schedule. Rather than reinforcing correct problem completion on an FR-1 schedule, a FR-10 schedule was used instead to mirror a leaner schedule of reinforcement that is used in a typical classroom environment. The students were told about the new reinforcement rate in the statement delivered in each condition.
Procedures

Screening. To screen for participation eligibility, each student was administered three math curriculum-based measures (CBMs) for each problem type that was appropriate for their grade level. On each CBM probe, students were told that they had two minutes to complete as many math problems as they could. Each math probe was scored using DCPM. Students’ math fluency score had to fall below the mastery range (49 DCPM for 4th and 5th graders or 40 DCPM for 6th graders; Wright, 2013) to participate within the study. Their average score on each problem type gave baseline estimates of their fact knowledge and math fluency.

Phase A. Phase A examined the impact of the FA conditions on students’ math fluency using an FR1 reinforcement schedule. Analog conditions were run for five minutes each at a table in an empty classroom. All four FA conditions (attention, escape, symbolic, control) were randomly presented to each student each day, with a 2-minute break in between each condition. Sessions were continued until stable responding in either level or trend was observed.

Phase B. Phase B examined whether differentiation of responding could still be produced when a lean schedule of reinforcement typical of actual classroom practice was used. All four FA conditions were presented in an identical manner to Phase A, with the exception that reinforcement was provided on an FR10 schedule instead of an FR1 schedule.

Experiment 2

Experimental Design

Experiment 2 consisted of an alternating treatment design using each student’s best and worst conditions from Experiment 1 to teach a new math skill using constant time delay. As mentioned above in the materials section, a separate set of numbers was randomly assigned to each condition to avoid carryover effects. Whichever reinforcement schedule produced more
promising results in Experiment 1 was used in Experiment 2. If both schedules from Experiment 1 produced equally clear results, then the FR-10 schedule was used since it better approximated natural contingencies in the students’ classroom. Both conditions were run each day in a random, counter balanced order with a two-minute break in between conditions.

**Constant Time Delay**

**Probe.** Prior to the start of instruction for each condition, a probe was conducted to assess the student’s progress on the assigned set of 36 math facts (see Appendices F and G). The students worked as quickly as they could on the problems while the experimenter timed their progress. No feedback was provided during this initial probe. After the probe, the experimenter set the timer for five minutes to begin the constant time delay instruction session.

**0-second delay.** Instruction began by the students completing each of the 36 assigned problem facts once using a 0-second prompt delay. As soon as each problem was uncovered, the experimenter immediately told the answer to the student, who wrote the answer down. The participant was reinforced for writing the correct answer based on the reinforcement condition.

**10-second delay.** After each assigned problem fact was completed once using the 0-second delay procedure, the experimenter used a 10-second delay for the reminder of the instruction time. Once each problem was uncovered, the student had ten seconds to write down the correct answer. If (s)he did not respond within ten seconds or wrote down the wrong answer, the experimenter prompted the student with the correct answer to write down. The student received the specific reinforcer (depending on the condition) for correct responding, regardless of whether it was prompted.
Results

Screening

Grade appropriate mathematics CBM probes were used to determine participation eligibility and baseline responding for each participant. Results are presented below in Table 2 for each participant’s average digits correct per minute (DCPM) for each problem type. Although Anna and Beatrice’s subtraction scores were lower than their multiplication scores, multiplication was chosen as the target problem type for all five participants due to teacher request.

Table 3

<table>
<thead>
<tr>
<th>Participant</th>
<th>Subtraction</th>
<th>Multiplication</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anna</td>
<td>12.3</td>
<td>15.5</td>
<td>4.17</td>
</tr>
<tr>
<td>Beatrice</td>
<td>13.5</td>
<td>14.8</td>
<td>2</td>
</tr>
<tr>
<td>Cameron</td>
<td>----</td>
<td>22.2</td>
<td>8.3</td>
</tr>
<tr>
<td>Dominic</td>
<td>20.5</td>
<td>15.7</td>
<td>4.75</td>
</tr>
<tr>
<td>Eliza</td>
<td>19.7</td>
<td>15.5</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Experiment 1

A separate functional analysis was run in Phases A and B using an FR-1 and FR-10 reinforcement schedule, respectively, to determine which reinforcement method would produce optimal academic responding. Participants’ results from both phases are presented below in Figures 1 through 5 and Tables 4 through 8.

Anna. Figure 1 displays the Anna’s FA results across Phases A and B, which are corroborated by summary statistics in Table 4. Visual inspection revealed largely undifferentiated responding in Phase A across conditions under the FR-1 reinforcement schedule due to variability within the conditions and a high percentage of overlap between conditions. Visual inspection revealed somewhat higher responding in the attention condition compared to
other conditions in Phase B under the FR-10 reinforcement schedule. While variability was still high within conditions, there was less overlap between the attention condition compared to the other conditions as well as the emergence of a steeper trend for the attention condition. Summary statistics confirmed that attention delivered on the FR-10 schedule produced the highest level of responding, although there was high variability within this condition. Overall, based on the results of visual analysis and summary statistics, attention delivered on an FR-10 reinforcement schedule was selected as the best reinforcer for Anna for Experiment 2. Responding was lowest when escape was delivered on an FR-10 schedule; therefore this was selected as Anna’s worst condition for Experiment 2.

![Figure 1. Anna’s math fluency performance across Experiment 1 FA phases and conditions](image)

**Figure 1.** Anna’s math fluency performance across Experiment 1 FA phases and conditions

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Mean and Standard Deviation for Anna’s Performance Across Experiment 1 FA Phases and Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FR-1 Schedule</td>
</tr>
<tr>
<td></td>
<td><strong>Att.</strong> <strong>Con.</strong> <strong>Esc.</strong> <strong>Sym.</strong> <strong>Att.</strong> <strong>Con.</strong> <strong>Esc.</strong> <strong>Sym.</strong></td>
</tr>
<tr>
<td>Mean DCPM</td>
<td>13.0 12.9 8.1 10.6 19.3 17.2 14.2 15.6</td>
</tr>
<tr>
<td>Mean DCPM</td>
<td>13.0 12.9 8.1 10.6 19.3 17.2 14.2 15.6</td>
</tr>
<tr>
<td>Mean DCPM</td>
<td>13.0 12.9 8.1 10.6 19.3 17.2 14.2 15.6</td>
</tr>
<tr>
<td>Mean DCPM</td>
<td>13.0 12.9 8.1 10.6 19.3 17.2 14.2 15.6</td>
</tr>
<tr>
<td>Mean DCPM</td>
<td>13.0 12.9 8.1 10.6 19.3 17.2 14.2 15.6</td>
</tr>
</tbody>
</table>
**Beatrice.** Figure 2 displays Beatrice’s FA results across Phases A and B, which are substantiated by summary statistics in Table 5. Visual inspection revealed largely undifferentiated responding across conditions in both phases due to high overlap between conditions with little difference in level or trend. Summary statistics demonstrated that math fluency was highest when attention was delivered on an FR-1 schedule, so this condition was selected as Beatrice’s best condition for Experiment 2. Although the lowest level of responding occurred when symbolic reinforcement was delivered on an FR-1 schedule, there was more overlap between conditions compared to when escape was delivered on an FR-1 schedule, so escape was selected as Beatrice’s worst condition for Experiment 2.

![Figure 2. Beatrice’s math fluency performance across Experiment 1 FA phases and conditions.](image)

**Table 5**

*Mean and Standard Deviation for Beatrice’s Performance Across Experiment 1 FA Phases and Conditions*

<table>
<thead>
<tr>
<th></th>
<th>FR-1 Schedule</th>
<th></th>
<th>FR-10 Schedule</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean DCPM</td>
<td>9.3</td>
<td>9.0</td>
<td>8.8</td>
<td>8.6</td>
</tr>
<tr>
<td>SD</td>
<td>2.1</td>
<td>1.8</td>
<td>2.1</td>
<td>1.5</td>
</tr>
</tbody>
</table>
**Cameron.** Figure 3 displays Cameron’s FA results across Phases A and B, which are corroborated by summary statistics in Table 6. Visual inspection revealed largely undifferentiated responding across conditions in Phase B under the FR-10 schedule due to high overlap between conditions and variability within conditions. Responding in Phase A under the FR-1 schedule similarly had some overlap between conditions and variability within conditions, however attention and symbolic appear to have the highest and lowest levels among conditions. These level differences were confirmed by summary statistics. Thus, FR-1 attention and symbolic were selected as Cameron’s best and worst conditions, respectively, for Experiment 2.

![Figure 3](image-url)  
**Figure 3.** Cameron’s math fluency performance across Experiment 1 FA phases and conditions.

<table>
<thead>
<tr>
<th></th>
<th><strong>FR-1 Schedule</strong></th>
<th></th>
<th><strong>FR-10 Schedule</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean DCPM</td>
<td>11.7</td>
<td>8.4</td>
<td>9.3</td>
</tr>
<tr>
<td>SD</td>
<td>2.9</td>
<td>1.9</td>
<td>3.6</td>
</tr>
</tbody>
</table>

*Table 6  
Mean and Standard Deviation for Cameron’s Performance Across Experiment 1 FA Phases and Conditions*
**Dominic.** Figure 4 displays Dominic’s FA results across Phases A and B, which are substantiated by summary statistics in Table 7. It should be noted that data are missing for the entire session 3 and the session 8 escape condition due to session interruptions. Visual inspection revealed differentiated responding in condition levels across both phases, with small overlap between conditions. Visual and summary statistics both demonstrated that the highest level of responding occurred when attention was delivered on an FR-1 schedule and lowest when escape was delivered on an FR-1 schedule, therefore these were selected as the best and worst conditions for Experiment 2.

![Figure 4](image-url)  
Figure 4. Dominic’s math fluency performance across Experiment 1 FA phases and conditions.

<table>
<thead>
<tr>
<th>Mean and Standard Deviation for Dominic’s Performance Across Experiment 1 FA Phases and Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Mean DCPM</td>
</tr>
<tr>
<td>SD</td>
</tr>
</tbody>
</table>
Eliza. Figure 5 displays Eliza’s FA results across Phases A and B, which are supported by summary statistics in Table 8. Visual inspection revealed undifferentiated responding across conditions in Phase B under the FR-10 schedule of reinforcement due to high overlap between conditions. Responding was more differentiated in Phase A, although there was still some overlap between conditions. Visual inspection and summary statistics both revealed that the highest and lowest levels of responding occurred when attention and escape, respectively, were delivered on an FR-1 schedule of reinforcement. Therefore, these conditions were selected as Eliza’s best and worst conditions for Experiment 2.

![Figure 5. Eliza’s math fluency performance across Experiment 1 FA phases and conditions.](image)

**Table 8**  
*Mean and Standard Deviation for Eliza’s Performance Across Experiment 1 FA Phases and Conditions*

<table>
<thead>
<tr>
<th></th>
<th>FR-1 Schedule</th>
<th></th>
<th>FR-10 Schedule</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean DCPM</td>
<td>24.5</td>
<td>21.8</td>
<td>18.2</td>
<td>22.4</td>
</tr>
<tr>
<td>SD</td>
<td>3.3</td>
<td>6.7</td>
<td>4.9</td>
<td>3.6</td>
</tr>
</tbody>
</table>
**Summary.** The most promising level of responding occurred across all five participants in the attention condition of the functional analysis, although differences were small due to largely undifferentiated analyses because of variability within conditions and a high percentage of overlap between conditions. Responding was lowest in the escape condition for four of the five participants. The remaining participant, Cameron, had lower performance during the symbolic condition compared to the other reinforcement conditions. Across four of the five participants, responding more closely approximated differentiation in Phase A, under an FR-1 reinforcement schedule, compared to Phase B, under an FR-10 schedule. In contrast, Anna’s responding was more differentiated under the FR-10 schedule rather than the FR-1.

**Experiment 2**

Instructional conditions based on the best and worst conditions from Experiment 1 were used to teach participants division facts in Experiment 2. Participants’ problem accuracy and fluency results on daily probes from each condition are presented below in Figures 6 through 15.

**Anna.** Figure 6 displays Anna’s daily probe results for math facts taught using interventions based on the best (FR-10 attention) and worst (FR-10 escape) conditions from Experiment 1. Visual inspection revealed differentiated responding in levels across conditions and small percentage overlap, with optimal responding occurring during the worst probe compared to the best. Figure 7 displays Anna’s problem accuracy on the daily best and worst condition probes. Visual inspection revealed that problem accuracy started off at or above 80% for both conditions in session 1 and rapidly reached 100% accuracy within several sessions. Visual inspection is corroborated by summary statistics in Table 8, which confirmed that responding was higher in the worst condition rather than the best.
Figure 6. Anna’s math fluency performance across Experiment 2 conditions

Figure 7. Anna’s problem accuracy across Experiment 2 conditions

Table 9

<table>
<thead>
<tr>
<th></th>
<th>Best Probe</th>
<th>Worst Probe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean DCPM</td>
<td>21.5</td>
<td>27.9</td>
</tr>
<tr>
<td>SD</td>
<td>7.5</td>
<td>8.9</td>
</tr>
<tr>
<td>Mean Accuracy</td>
<td>96.6</td>
<td>99.1</td>
</tr>
<tr>
<td>SD</td>
<td>6.7</td>
<td>1.8</td>
</tr>
</tbody>
</table>
**Beatrice.** Figure 8 displays Beatrice’s daily probe results for math facts taught using interventions based on the best (FR-1 attention) and worst (FR-1 escape) conditions from Experiment 1. Visual inspection revealed differentiated levels of responding across conditions with small percentage overlap, where optimal responding occurred during the worst probe rather than the best. Figure 9 displays Beatrice’s problem accuracy on the daily best and worst condition probes. Visual inspection revealed that problem accuracy started off near 80% for the worst condition and rapidly approached 100% accuracy within several sessions. Problem accuracy for the best condition, on the other hand, started off near 40% and consistently remained lower than that of the worst condition. Visual inspection is substantiated by summary statistics in Table 9, which confirmed that responding and accuracy were both higher in the worst condition rather than the best.

![Figure 8. Beatrice’s math fluency performance across Experiment 2 conditions](image-url)
Table 10
Means and Standard Deviations for Beatrice’s Performance Across Experiment 2 Conditions

<table>
<thead>
<tr>
<th></th>
<th>Best Probe</th>
<th>Worst Probe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean DCPM</td>
<td>7.6</td>
<td>11.4</td>
</tr>
<tr>
<td>SD</td>
<td>3.0</td>
<td>3.8</td>
</tr>
<tr>
<td>Mean Accuracy</td>
<td>68.4</td>
<td>90.6</td>
</tr>
<tr>
<td>SD</td>
<td>14.1</td>
<td>5.4</td>
</tr>
</tbody>
</table>

**Cameron.** Figure 10 displays Cameron’s daily probe results for math facts taught using interventions based on the best (FR-1 attention) and worst (FR-1 symbolic) conditions from Experiment 1. Data from the best probe for session 12 were invalidated due to behavioral problems during the session. Results from the best probe for session 13 are missing due to a misplaced probe worksheet. Visual inspection revealed differentiated responding in level and trend across conditions, with no overlap between conditions, where optimal responding occurred during the worst probe compared to the best. Figure 11 displays Cameron’s problem accuracy on the daily best and worst condition probes. Visual inspection revealed that problem accuracy started off near 85% for the worst condition and rapidly approached 100% accuracy within
several sessions. Problem accuracy for the best condition started off lower, near 70%, and consistently remained lower than that of the worst condition. Visual inspection is substantiated by summary statistics, which confirmed that responding and accuracy were both higher in the worst condition rather than the best.

Figure 10. Cameron’s math fluency performance across Experiment 2 conditions

Figure 11. Cameron’s problem accuracy across Experiment 2 conditions
Table 11
*Means and Standard Deviations for Cameron’s Performance Across Experiment 2 Conditions*

<table>
<thead>
<tr>
<th></th>
<th>Best Probe</th>
<th>Worst Probe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean DCPM</td>
<td>8.8</td>
<td>19.4</td>
</tr>
<tr>
<td>SD</td>
<td>3.7</td>
<td>10.1</td>
</tr>
<tr>
<td>Mean Accuracy</td>
<td>84.2</td>
<td>95.8</td>
</tr>
<tr>
<td>SD</td>
<td>13.2</td>
<td>6.3</td>
</tr>
</tbody>
</table>

**Dominic.** Figure 12 displays Dominic’s daily probe results for math facts taught using interventions based on the best (FR-1 attention) and worst (FR-1 escape) conditions from Experiment 1. Visual inspection revealed largely undifferentiated responding across conditions, with high overlap between conditions and little differentiation in level. Figure 13 displays Dominic’s problem accuracy results on the daily best and worst condition probes. Visual inspection revealed that problem accuracy started off near 100% for both conditions in session 1 and rapidly reached 100% accuracy within several sessions. Visual inspection is corroborated by summary statistics in Table 12, which shows little difference in response and accuracy levels between conditions.

![Figure 12. Dominic’s math fluency performance across Experiment 2 conditions](image-url)
Table 12
Means and Standard Deviations for Dominic’s Performance Across Experiment 2 Conditions

<table>
<thead>
<tr>
<th></th>
<th>Best Probe</th>
<th>Worst Probe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean DCPM</td>
<td>34.3</td>
<td>36.0</td>
</tr>
<tr>
<td>SD</td>
<td>10.1</td>
<td>12.5</td>
</tr>
<tr>
<td>Mean Accuracy</td>
<td>99.6</td>
<td>99.6</td>
</tr>
<tr>
<td>SD</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Eliza. Figure 14 displays Eliza’s daily probe results for math facts taught using interventions based on the best (FR-1 attention) and worst (FR-1 escape) conditions from Experiment 1. Visual inspection revealed partially differentiated responding, with higher responding occurring during the best condition across some sessions but with moderate percentage overlap and variability within conditions. Figure 15 displays Eliza’s problem accuracy results on the daily best and worst condition probes. Visual inspection revealed that problem accuracy started off at or above 70% for both conditions in session 1 and rapidly reached 100% accuracy within several sessions. Visual inspection is corroborated by summary statistics in Table 13, which confirmed that there were slight differences in responding between conditions but high variability within both conditions.
Figure 14. Eliza’s math fluency performance across Experiment 2 conditions

Figure 15. Eliza’s problem accuracy across Experiment 2 conditions

Table 13
Means and Standard Deviations for Eliza’s Performance Across Experiment 2 Conditions

<table>
<thead>
<tr>
<th></th>
<th>Best Probe</th>
<th>Worst Probe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean DCPM</td>
<td>22.5</td>
<td>20.9</td>
</tr>
<tr>
<td>SD</td>
<td>8.6</td>
<td>6.0</td>
</tr>
<tr>
<td>Mean Accuracy</td>
<td>94.4</td>
<td>92.5</td>
</tr>
<tr>
<td>SD</td>
<td>9.0</td>
<td>4.7</td>
</tr>
</tbody>
</table>
Summary. Clear differentiation in math fluency performance across intervention conditions occurred for three of the five students (i.e. Anna, Beatrice, and Cameron). These three students all demonstrated optimal responding in the worst condition compared to the best. Dominic’s math fluency results were largely undifferentiated, although responding was slightly better in the worst compared to the best condition across several sessions. Eliza’s math fluency was partially differentiated, with higher responding occurring in the best condition compared to the worst, with occasional path crossovers. Problem accuracy started out higher in the worst condition compared to the best for four of the five participants (i.e., Anna, Beatrice, Cameron, and Eliza), with initial accuracy in the worst condition falling at or above 80% for these four participants. Dominic was the only participant whose problem accuracy started near 100% for both conditions. Anna and Eliza’s problem accuracy rapidly reached 100% for both conditions within several sessions. Beatrice and Cameron’s problem accuracy during the best condition consistently remained lower than that of the worst.
Discussion

The purpose of the current study was to replicate and extend research on the functional analysis (FA) of academic responding (Hofstadter-Duke & Daly, 2015) to determine how to motivate struggling learners and optimize their academic responding. To this end, two related experiments were conducted. During the first experiment, an FA was conducted with five participants to evaluate which kind of reinforcement (attention, escape, or symbolic) resulted in the highest rate of correct responding for each participant. To determine whether reinforcement rate would impact the results of the functional analysis, sessions were conducted with both a traditional FR-1 reinforcement schedule and a lean FR-10 reinforcement schedule. The purpose of the second experiment was to assess the applicability of the FA results by designing an instructional intervention for each student based on their best and worst conditions from Experiment 1. The findings from the two experiments are discussed below.

Experiment 1

Four of the five participants (i.e., Anna, Cameron, Dominic, and Eliza) demonstrated differentiated responding to varying degrees in Experiment 1 across reinforcement conditions. While the differences between conditions were small as evidenced by partial crossovers, there were clear trends that emerged between conditions. In contrast, the fifth participant, Beatrice, displayed largely undifferentiated responding across conditions. Nevertheless, her responding in one condition (escape) was consistently lower than in the other conditions. It should be noted that Beatrice’s problem accuracy remained low throughout Experiment 1, whereas the other participants’ accuracy improved across sessions. Taken together, these outcomes are in line with the results from Hofstadter-Duke and Daly (2015), who found undifferentiated responding across conditions when unknown facts were used and differentiated responding when known facts were
used. Hofstadter-Duke and Daly attributed the difference in outcomes to stimulus control, postulating that items had to be under stimulus control to identify the function of the replacement behavior (i.e., academic responding). Thus, the results of the current study may not have been as differentiated as those of Hofstadter-Duke and Daly due to a lack of stimulus control. Math facts used in the “known facts” phase of the Hofstadter-Duke and Daly study were required to have 100% accuracy, whereas there was no similar accuracy requirement in the current study. Future research should be conducted to better understand the amount of stimulus control required to produce differentiated responding across FA conditions.

Another reason that the current results were less differentiated than those of the Hofstadter-Duke and Daly study are due to the different reinforcement contingencies between the studies. In the Hofstadter-Duke and Daly study, participants received reinforcement for responding, regardless of response accuracy. In contrast, responses in the current study had to be accurate to receive reinforcement. This reinforcement contingency was used to mirror typical classroom practice, where students only receive reinforcement for accurate responding. Nevertheless, perhaps the accuracy requirement, as opposed to response requirement alone, resulted in too high of a response effort, which in turn could mask the function of the behavior. Future research should be conducted on these two reinforcement contingencies to determine their effect on response differentiation and intervention selection.

Another notable difference between the current results and those documented by Hofstadter-Duke and Daly was that all current participants demonstrated optimal responding during the attention condition. In contrast, Hofstadter-Duke and Daly found that the optimal reinforcement condition varied between participants. Given that individuals are idiosyncratic in preferred reinforcers, the similarity of optimal reinforcers across the current set of participants is
likely due to coincidence. Alternatively, given the high frequency with which teachers use praise in the classroom, students may have an advanced learning history for this form of reinforcement. Previous meta-analytic research on FAs has found that problem-behaviors are most commonly maintained by attention (Beavers, Iwata, & Lerman, 2013; Hanley, Iwata, & McCord, 2003). Future researchers should therefore investigate whether attention is also the most common maintaining variable for academic responding as well.

A third outcome difference between the current study and the Hofstadter-Duke and Daly study was that the current students all had high levels of responding during the control condition. In contrast, participants in the Hofstadter-Duke and Daly study had significantly lower math fluency in the control condition compared to the other conditions. The difference in response outcomes between the studies is likely due to methodological variations. In Hofstadter-Duke and Daly’s study, participants completed the worksheet at their own pace and were told that they could complete as many or as few problems as they like. This methodology, however, adds a potential confounding variable, as this was the only FA condition in which participants could control problem pacing. In order to eliminate this potential confound, the methodology was changed for the current study, such that the experimenter controlled the rate of problem presentation in all four FA conditions. Given that the same problem pacing (10 seconds per question) was used in the control condition as the other conditions, it makes logical sense that responding would be high in this condition because no additional time was needed between problems to administer reinforcement (e.g., to deliver a praise statement or smiley face stamp). Another possible reason for the high rate of responding in the control condition could be a combination of rule-governed behavior and delayed reinforcement during the break in between sessions. Specifically, students’ responding was likely under control of the verbal contingency of
earning a two-minute break after the five-minute work period. Therefore, they may have persisted in answering math questions during the control condition, despite the lack of immediate reinforcement, to obtain the delayed reinforcement during the break, even though the break was not contingent on their performance. Future researchers should develop alternative control methodologies, particularly ones that reduce the likelihood of rule-governed behavior, to have a more accurate control condition in the analysis.

Beyond the direct replication of the Hofstadter-Duke and Daly (2015) study, Experiment 1 also aimed to evaluate the impact of manipulating the FA reinforcement schedule. Across four of the five participants, responding more closely approximated differentiation under the FR-1 schedule of Phase A compared to the FR-10 schedule of Phase B. This is counter to the original hypothesis that responding would be more differentiated under the FR-10 schedule because larger FR schedules produce higher rates of responding (Cooper, Heron, & Heward, 2007). To date, there has been minimal research on the use of alternative reinforcement schedules within functional analyses, thus comparison among studies is difficult. Currently, Rogers (2013) is the only study that has directly manipulated reinforcement schedules to analyze their impact on FA results. Rogers found that problem behavior occurrences were more differentiated under a continuous reinforcement schedule compared to an intermittent variable-ratio-of-2-schedule. Rogers posited that one reason for the lack of differentiation of responses under more lean schedules of reinforcement may be due to response extinction. The results of the current study parallel those obtained by Roger and lend additional support to her hypothesis regarding response extinction. Requiring ten correct answers to receive reinforcement in Phase B of the current study may have been too high of a response requirement. Perhaps requiring a smaller number, such as five correct answers, may have produced more optimal results by preventing
response extinction. Given the importance of determining practical reinforcement schedules, future research is needed to conclusively demonstrate the effect of reinforcement schedule manipulations on FA outcomes.

**Experiment 2**

Contradictorily, four of the five participants (i.e., Anna, Beatrice, Cameron, and Dominic) obtained higher fluency results during the intervention based on the worst condition from Experiment 1 rather than the best condition. Eliza was the only participant to obtain higher fluency results during the intervention based on the best condition from Experiment 1, although differences in response levels were minimal and there were several points of crossover across sessions. The results from Experiment 2 sharply contrast the plethora of research on the effectiveness of function-based interventions (Beavers et al., 2013). The reason for these contrary results is likely due to the participants’ differing levels of problem accuracy across the problem sets. Anna, Beatrice, and Cameron’s problem accuracy for the worst condition started off at least 20% higher than that of the best condition. While Anna’s problem accuracy on the best condition increased across sessions and eventually reached the same high level as that of the worst condition, Cameron and Beatrice’s problem accuracy remained consistently lower on the best condition compared to the worst. Although numbers were randomly assigned to the two conditions to equate for difficulty, it is possible that problems assigned to the worst condition may have been slightly easier compared to those of the best condition. Given the large quantity of research supporting the use of function-based interventions, future research on the applicability of academic FAs is warranted.

Another possible reason for these contradictory results could be due to the mismatch between the instructional intervention and the students’ skill level based on Haring and Eaton’s
(1978) Instructional Hierarchy (IH). A constant time delay intervention was selected to target students’ fact acquisition based on the premise that their division fact accuracy would be initially low. However, in contrast to participants’ low division performance during baseline, division problem accuracy began at 70% in session 1 for four of the five participants. Therefore an intervention that was specifically geared towards improving fluency, rather than fact acquisition, may have generated results that were more in line with those from Experiment 1. Future research is needed to clarify the applicability of FA results both within and across different IH skill levels.

**Limitations and Future Directions**

Future research should be conducted to address the limitations of the current study. The largest limitation from Experiment 1 was the lack of clear response differentiation across FA conditions. According to Tiger, Fisher, Toussaint, and Kodack (2009), FAs produce differentiated results in approximately 94% of cases. According to the authors, three potential reasons for undifferentiated results in the remaining 6% of cases are: 1) the behavior is maintained by idiosyncratic reinforcement not included in the original analysis; 2) a lack of establishing operation for the maintaining reinforcement variable; or 3) the behavior is under stimulus control of an event not included in the original analysis. Future studies should replicate and extend the findings of the current study by designing research methodology based on these considerations to produce more clearly differentiated results.

Idiosyncratic reinforcement, in particular, is an important consideration for motivating children academically given that individuals have varying histories of reinforcement. Future research should therefore include additional reinforcement conditions, such as contingent access to preferred books or electronic games, to assess their impact on academic responding. Researchers should also consider potential motivating operations when designing the FA
methodology. For instance, previous research has manipulated pre-session levels of attention or tangible items in order to increase response differentiation between FA conditions (McComas, Thompson, & Johnson, 2003; O’Reilly et al., 2009). Thus, future studies could investigate whether pre-session levels of reinforcement affect academic response differentiation across conditions. Finally, researchers should consider whether the desired behaviors are under stimulus control of an event included in the analysis. As discussed above, academic responding may not have been under full stimulus control of the selected problems. Future research should therefore examine varying levels of stimulus control to assess its impact on differentiated responding.

The largest limitation of Experiment 2 was the higher level of problem accuracy in the worst condition relative to the best condition. It is possible that this limitation could have been avoided by randomly assigning different number sets to the best and worst conditions for each student (i.e., the number 2 may be randomly assigned to the best condition for two of the students and to the worst condition for the other three students). This was not done in the current study because of the complexity of having to manually create each worksheet for each condition in every session. Nevertheless, given the large confound this methodology potentially produced, it is recommended that future studies attempt this methodological change.

Conclusion

Given recent educational reform and the growing emphasis on the utilization of evidence-based practice, it is crucial for researchers to determine how teachers can empirically optimize student academic behavior. In contrast to the large amount of research supporting the use of FAs to assess and treat problem behavior, only one study to date has been conducted on the direct application of FA procedures to academic responding (Hofstadter-Duke & Daly, 2015). The results of Experiment 1 thus extend the FA literature by demonstrating that FA procedures can
be used to identify the controlling variable that maintains high academic responding. However, much more research is needed to identify optimal methodology. For instance, researchers need to determine what schedules of reinforcement and what levels of problem accuracy are needed to produce response differentiation across conditions. Furthermore, as evidenced by the contradictory findings from Experiment 2, researchers need to further analyze the application of FA results to design function-based academic interventions. Once additional research is conducted and effective FA methods are developed for academic responding, teachers will be able to utilize this assessment method as part of typical classroom practice. Rather than randomly choosing reinforcement methods to optimize students’ academic responding, teachers will be able to empirically determine the most effective reinforcement method for each student.
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Appendix A
IRB Approval

ACTION ON EXEMPTION APPROVAL REQUEST

TO: Catherine Lark
Psychology

FROM: Dennis Landin
Chair, Institutional Review Board

DATE: June 15, 2016

RE: IRB# E9946


Review Date: 6/13/2016

Approved X Disapproved

Approval Date: 6/13/2016 Approval Expiration Date: 6/12/2019

Exemption Category/Paragraph: 1

Signed Consent Waived?: No

Re-review frequency: (three years unless otherwise stated)

LSU Proposal Number (if applicable):

Protocol Matches Scope of Work in Grant proposal: (if applicable)

By: Dennis Landin, Chair

PRINCIPAL INVESTIGATOR: PLEASE READ THE FOLLOWING – Continuing approval is CONDITIONAL on:

1. Adherence to the approved protocol, familiarity with, and adherence to the ethical standards of the Belmont Report, and LSU's Assurance of Compliance with DHHS regulations for the protection of human subjects*

2. Prior approval of a change in protocol, including revision of the consent documents or an increase in the number of subjects over that approved.

3. Obtaining renewed approval (or submittal of a termination report), prior to the approval expiration date, upon request by the IRB office (irrespective of when the project actually begins); notification of project termination.

4. Retention of documentation of informed consent and study records for at least 3 years after the study ends.

5. Continuing attention to the physical and psychological well-being and informed consent of the individual participants, including notification of new information that might affect consent.

6. A prompt report to the IRB of any adverse event affecting a participant potentially arising from the study.


8. SPECIAL NOTE: When emailing more than one recipient, make sure you use bcc. Approvals will automatically be closed by the IRB on the expiration date unless the PI requests a continuation.

*All investigators and support staff have access to copies of the Belmont Report, LSU's Assurance with DHHS, DHHS (45 CFR 46) and FDA regulations governing use of human subjects, and other relevant documents in print in this office or on our World Wide Web site at http://www.lsu.edu/irb
Appendix B
Parental Consent Form

**Project Title:** What Motivates Children to Respond? Functional Analysis and Intervention of Math Computation Fluency

**Performance Site:** Elementary Schools

**Investigator:** Catherine Lark, M.A., LSU School Psychology Doctoral Student
Catherine is available for questions M-F 8:00am – 4:30pm
By email: clark1@lsu.edu
By phone: (LSU psychology department #): 225-578-7792

**Supervisor:** Dr. George Noell, PhD, BCBA, Head of School Psychology, LSU
Dr. Noell is available for questions M-F 8:00am – 4:30pm
By email: gnoell@lsu.edu
By phone: 225-578-4119

**Purpose of the Study:** The purpose of this study is to examine the effects of different forms of reinforcement on improving students’ fluent completion of math problems.

**Inclusion Criteria:** Elementary students who would benefit from increased math fluency.

**Exclusion Criteria:** Elementary students who are fluent with math facts. Students who do not assent to participate.

**Description of the Study:** Students will work on mathematics assignments 1:1 with an experimenter for approximately 30 minutes outside of class several times a week, depending on student availability.

During the first part of the study, each student will work on a math assignment while receiving different forms of reinforcement that are typical of classroom practice (i.e., praise, breaks, and stamps). This will allow the experimenter to determine the optimal form of reinforcement for increasing the student’s math fact completion. During the second part of the study, the most effective reinforcer will be used to develop an intervention to teach a new math skill. This will examine the application of the assessment results.

**Benefits:** The study may identify a way to increase student performance on math problems as well as teach a new math skill.
Risks: There are no known risks for this study beyond those of daily living and education.

Right to Refuse: Student participation in the study is voluntary. A student will only be part of the study if both the student and parent agree to the student’s participation. A student may withdraw at any time, or a parent may withdraw the student at anytime without penalty.

Privacy: Results of the study may be published, but no identifying information or names will be included. Subject identity will remain confidential unless disclosure is required by law.

Financial Information: There is no cost for participation in the study, nor will students be compensated for participation.

Signatures:

I have read about the study and have no further unanswered questions at this time. I may direct additional questions regarding study specifics to the investigator. If I have questions about subjects’ rights or other concerns, I can contact Dennis Landin, Chairman, Institutional Review Board, (225) 578-8692, irb@lsu.edu, www.lsu.edu/irb.

I will allow my child ________________ to participate in the study described above and acknowledge the investigator’s obligation to provide me with a signed copy of this consent form.

Parent’s Signature: ___________________________ Date: ____________________

The parent/guardian has indicated to me that he/she is unable to read. I certify that I have read this consent form to the parent/guardian and explained that by completing the signature line above he/she has given permission for the child to participate in the study.

Signature of Reader: ___________________________ Date: ____________________
Appendix C
Student Assent Form

I, _____________________________, agree to be in a study that looks at assessing reinforcement for fluent math completion. I can decide to stop being in the study at any time without getting in trouble.

Child's Signature:_________________________________ Age:______ Date:__________________

Witness* ______________________________ Date:__________________

* (N.B. Witness must be present for the assent process, not just the signature by the minor.)
Appendix D
Experiment 1 Checklist

<table>
<thead>
<tr>
<th>Participant ID:</th>
<th>Condition Order:</th>
<th>Main Experimenter:</th>
</tr>
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<tbody>
<tr>
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</table>

<table>
<thead>
<tr>
<th>Date:</th>
<th>Reinforcement Rate:</th>
<th>IOA Experimenter:</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

**Prior to Session Start**

1. ________ Experimenter consulted the session order & reinforcement schedule and then arranged the worksheets in the correct order

**Condition 1**

2. ________ Experimenter explained the instructions

3. ________ Student worked on the worksheet for 5 minutes

4. ________ Specified reinforcer given for correct answers on specified schedule

5. ________ Corrective feedback given for incorrect / unanswered questions

6. ________ 2 minute break given after the student worked for 5 minutes

**Condition 2**

7. ________ Experimenter explained instructions

8. ________ Student worked on worksheet for 5 minutes

9. ________ Specified reinforcer given for correct answers on specified schedule

10. ________ Corrective feedback given for incorrect / unanswered questions

11. ________ 2 minute break given after the student worked for 5 minutes
Condition 3

12. __________ Experimenter explained instructions
13. __________ Student worked on worksheet for 5 minutes
14. __________ Specified reinforcer given for correct answers on specified schedule
15. __________ Corrective feedback given for incorrect / unanswered questions
16. __________ 2 minute break given after the student worked for 5 minutes

Condition 4

17. __________ Experimenter explained instructions
18. __________ Student worked on worksheet for 5 minutes
19. __________ Specified reinforcer given for correct answers on specified schedule
20. __________ Corrective feedback given for incorrect / unanswered questions
21. __________ 2 minute break given after the student worked for 5 minutes

After the Session

22. __________ Experimenter filled out session log
Appendix E
Experiment 2 Checklist

<table>
<thead>
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<th>Participant ID:</th>
<th>Condition Order:</th>
<th>Main Experimenter:</th>
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<table>
<thead>
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<th>Date:</th>
<th>Reinforcement Rate:</th>
<th>IOA Experimenter:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Prior to Session Start

1. ________ Experimenter consulted the session order & reinforcement schedule and then arranged the worksheets in the correct order

Probe 1

2. ________ Experimenter explained the instructions for the probe
3. ________ Student completed probe without feedback

Instruction Condition 1

4. ________ Experimenter explained the instructions for the condition
5. ________ Student worked on the worksheet for 5 minutes
6. ________ Experimenter taught the first presentation of each math fact using a 0-second time delay
7. ________ Additional fact presentations (after the bold line) were taught using a 10-second delay
8. ________ Corrective feedback given for incorrect / unanswered questions
9. _________ Specified reinforcer given on specified schedule after student wrote the correct answer, regardless of whether it was independent

10. _________ 2 minute break given after the student worked for 5 minutes

**Probe 2**

11. _________ Experimenter explained the instructions for the probe

12. _________ Student completed probe without feedback

**Instruction Condition 1**

13. _________ Experimenter explained the instructions for the condition

14. _________ Student worked on the worksheet for 5 minutes

15. _________ Experimenter taught the first presentation of each math fact using 0-second time delay

16. _________ Additional fact presentations (after the bold line) were taught using a 10-second delay

17. _________ Corrective feedback given for incorrect / unanswered questions

18. _________ Specified reinforcer given on specified schedule after student wrote the correct answer, regardless of whether it was independent

19. _________ 2 minute break given after the student worked for 5 minutes

**After the Session**

20. _________ Experimenter filled out session log
Appendix F
“Best” Probe Sample

<table>
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<th>Completion Time: _______</th>
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<th>Participant: _______</th>
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<td>64 ÷ 8</td>
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<tr>
<td>4 ÷ 2</td>
<td>16 ÷ 4</td>
<td>9 ÷ 1</td>
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<td>48 ÷ 12</td>
<td>72 ÷ 8</td>
<td>144 ÷ 12</td>
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<td>72 ÷ 9</td>
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<td>Probe B-1</td>
<td>Participant:</td>
</tr>
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<td>-----------------</td>
<td>-----------</td>
<td>--------------</td>
</tr>
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<td>12 ÷ 1</td>
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</tr>
<tr>
<td>9 ÷ 9</td>
<td>32 ÷ 4</td>
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<td>12 ÷ 12</td>
<td>8 ÷ 1</td>
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<td>16 ÷ 2</td>
<td>8 ÷ 4</td>
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</tr>
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<td>4 ÷ 4</td>
<td>81 ÷ 9</td>
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<td>1 ÷ 1</td>
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## Appendix G
### “Worst” Probe Sample

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<th>Participant:</th>
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<td>$66 \div 6$</td>
<td>$110 \div 11$</td>
</tr>
<tr>
<td>$36 \div 6$</td>
<td>$15 \div 3$</td>
<td>$21 \div 3$</td>
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<tr>
<td>$30 \div 5$</td>
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<td>$100 \div 10$</td>
</tr>
<tr>
<td>$77 \div 11$</td>
<td>$30 \div 3$</td>
<td>$21 \div 7$</td>
</tr>
<tr>
<td>33 ÷ 11</td>
<td>70 ÷ 10</td>
<td>49 ÷ 7</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>--------</td>
</tr>
<tr>
<td>55 ÷ 5</td>
<td>70 ÷ 7</td>
<td>77 ÷ 7</td>
</tr>
<tr>
<td>121 ÷ 11</td>
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<td>42 ÷ 7</td>
</tr>
<tr>
<td>18 ÷ 6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Vita

Catherine Rose Lark, a native of Dallas, Texas, received her bachelor’s degree in psychology with a minor in Spanish at Austin College in May 2013. During college, she worked as a behavioral therapist at two applied behavior analysis (ABA) clinics for children with Autism. She also conducted an honors thesis on the use of a functional analysis to reduce tantrum behavior for a child with Autism. These experiences contributed to her interest in applying ABA principles to treating children with disabilities. Based on this interest, she pursued graduate studies in the field of school psychology at Louisiana State University. She received her master’s degree in August 2015 and is now working towards her doctorate in School Psychology. She is working at the Marcus Autism Center in Atlanta, Georgia this year for her predoctoral internship before graduating with her PhD in School Psychology in August 2018.