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Evolution of Efimov states into the continuum in neutron rich (2n–core) nuclei — A general study

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ABSTRACT

The nuclear three-body system, with two halo neutrons very weakly coupled to a heavy core, is studied to investigate necessary conditions for the occurrence of Efimov states. Extending the analysis to the scattering sector, we find that these states evolve into Feshbach type resonances. This behaviour is very similar to the ^{20}C nucleus in which the occurrence of Efimov states evolving into resonances in the elastic scattering of n–^{19}C system has been investigated in recent publications. This work, thereby, extends the study of the Efimov effect beyond ^{20}C, showing that ^{32}Ne and ^{38}Mg exhibit a very similar dynamical structure. These nuclei are, therefore, also candidates for probing experimentally the Efimov effect.

Since the first experimental observation of Efimov states [1] in an ultracold gas of cesium [2], and the subsequent demonstration in other dilute atomic gases [3], there has been an upsurge of research in "Efimov physics" in general [4]. On the nuclear side, while there is no experimental observation yet of these states, attempts [5,6] have been made in recent years to identify 2n–rich halo nuclear systems that would be suitable candidates. The halo nucleus ^{20}C is an example. In recent publications [7–9], we have carried out a detailed study to provide evidence for the occurrence of Efimov states in this ^{20}C(n–n–^{18}C) system. It has been shown [7], through numerical analysis and also from analytical considerations that a non-Borromean halo nucleus like ^{20}C in which the halo neutron is supposed to be in the intruder low-lying bound state with the core, appear to be a promising candidate to have at least one Efimov state.

Recently, the possibility of excited Efimov states has also been studied using the traditional non-relativistic nuclear structure approaches [10] as well as the formalisms based on effective field theory [11], incorporating the effect of range corrections [12]. The criterion for the existence of Efimov states was initially analyzed in [6] and investigated in greater detail in [11] by depicting the boundary curves in the parametric region defined by the ratios of the two body (n–core and n–n) energies to the energies of the three-body excited states. It has generally been agreed that the only halo candidate for excited Efimov states is ^{20}C, consistent with the three-body ^{20}C energy and n–^{18}C bound state energy (having large uncertainty in the data[1]).

We have also extended the analysis to the scattering sector in order to address the problem of the evolution of the bound Efimov state (s) in ^{20}C with increasing n–core binding energy [8]. For weak pair binding, study of the elastic scattering of neutron–^{19}C shows the bound Efimov states evolving into Feshbach resonances with a characteristic asymmetric structure [8,9]. The evolution of such resonant structures, commonly seen in atomic and molecular systems as described by Fano [13], has also been the route to their recent observation in cesium and other trimers [2,3]. Our analysis of the Efimov state in neutron–rich nuclei moving above threshold into the continuum is thus in close analogy to the recent observations of a similar weakening of binding and consequent disappearance as reflected in the loss rate of cold atoms from an optical trap [2–4]. It should, however, be noted that there is an important difference between the nuclear and atomic systems. Unlike atomic systems, where the scattering length of the binary systems can be experimentally varied to approach the "Efimov region", the scattering length for nuclear systems cannot be tuned experimentally. The absence of any experimental handle to tune the two-body (n–core) nuclear interaction in atomic nuclei makes it imperative to search for the right candidates where the binary interactions (and scattering lengths) are appropriate to support Efimov states.

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[1] While the TUNL nuclear data evaluation quotes the energy $E_{nc} = 162 \pm 112$ keV, the most recent experimental work of T. Nakamura et al. finds $E_{nc} = 580 \pm 80$ keV which excludes the possibility of excited Efimov states in ^{20}C.

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Motivated by the above, we have undertaken a detailed study of the evolution of Efimov states into resonances for a three-body system consisting of one heavy core and two light particles of equal mass. To enlarge the scope of our ^{20}C study, scanning of nuclear data tables points to the nuclei ^{32}Ne and ^{38}Mg as exhibiting a structure dynamically similar to ^{20}C [14]. In both these nuclei, like ^{20}C, the n–core binary systems are marginally bound. Our numerical analysis presented in this work confirms this, opening up the possibility for experimentally studying Efimov states in these nuclei as well. In addition to these realistic cases that can possibly be studied at modern radioactive ion beam facilities, we have also carried out the calculations for a hypothetical case of two neutrons bound to a heavy (mass = 100) core to establish a generalised picture of the evolution of Efimov states in a system of one heavy and two light particles.

As mentioned before, several theoretical formalisms are in use to study 2-neutron halo nuclei and the possibility of Efimov states in such nuclei. These include both traditional non-relativistic nuclear structure approaches [10] and formalisms based upon effective field theory [11,12,15]. The following section presents a brief account of the salient features of our formalism.

The starting point of our model calculations is to consider the 2-neutron halo nucleus as a three-body system consisting of a compact and inert core and two loosely bound valence neutrons forming the halo. The calculations are carried out in momentum space with two-body separable interactions between the two valence neutrons (n–n) and the neutron–core (n–core) system. Labeling the two neutrons and the core as 1, 2, 3, with momenta $\vec{p}_1$, $\vec{p}_2$, and $\vec{p}_3$, respectively, the s-state interactions for the n–n and n–core are expressed as

$$V_{12} = -\frac{\lambda_n}{2\mu_{12}} g(p_{12}) g\big(p'_{12}\big),$$

$$V_{23} = -\frac{\lambda_c}{2\mu_{23}} f(p_{23}) f\big(p'_{23}\big),$$

$$V_{31} = -\frac{\lambda_c}{2\mu_{31}} f(p_{31}) f\big(p'_{31}\big),$$

where λ_n and λ_c are the interaction strength parameters for n–n and n–core potentials. The functions $g(p)$ and $f(p)$ have the Yamaguchi forms $g(p) = 1/(p^2+\beta^2)$, $f(p) = 1/(p^2+\beta_1^2)$, where β and β_1 are the range parameters for the n–n and n–core potentials. The values of λ_n and β can be estimated from low energy n–n scattering data reasonably well. In our analysis we fix $\lambda_n = 18.6\alpha^3$ and $\beta = 5.8\alpha$ where α is the deuteron binding energy parameter ($\alpha^2/m = 2.225$ MeV). These values of λ_n and β reproduce the spin singlet scattering length $a_{nn} = -23.69$ fm and the effective range $r_{nn} = 2.15$ fm [16].

The available two-body data for the n–core systems for 2-n halo nuclei are limited and at times ambiguous. In our analysis of different 2-n halo nuclei, we have used the experimentally determined n–core interaction strengths, wherever available. For cases where experimental data are unavailable, we have used 1 and 2n separation energies as derived from systematics by Audi and Wapstra [14]. In this connection, we also point to a more recent analysis and table of n–core interactions provided by Canham and Hammer based upon range corrections to the structural properties of halo nuclei within an effective quantum-mechanical framework [12].

Using the n–n and n–core potentials in the three-body Schrödinger equation,

$$(T - E)\psi = -(V_{12} + V_{23} + V_{31})\psi,$$

the solution (three-body wave function) is expressed as

$$\psi(\vec{p}_{12}, \vec{p}_{13}; E) = D^{-1}(\vec{p}_{12}, \vec{p}_3; E)\big[g(\vec{p}_{12})F(\vec{p}_3) + f(\vec{p}_{23})G(\vec{p}_1) + f(\vec{p}_{31})G(\vec{p}_2)\big], \quad (1)$$

where

$$D(\vec{p}_{12}, \vec{p}_3; E) \equiv \vec{p}_{12}^{\,2}/2\mu_{12} + \vec{p}_3^{\,2}/2\mu_{12,3} - E \quad (2)$$

represents the three-body energy term with total energy E in the three-body centre of mass system. Here $\vec{p}_{12}$ is the relative momentum of light particles labeled as 1 and 2, μ_{12} is their reduced mass, $\vec{p}_3$ is the relative momentum of the core (c, labeled as 3) with respect to the centre of mass of the other two, and μ_{12-3} is the corresponding reduced mass. The two-body functions $g(p_{ij})$ and $f(p_{ij})$ for the s-state separable potentials between the pair have already been defined. The spectator functions $F(\vec{p})$ and $G(\vec{p})$ describe, respectively, the dynamics of the core and of the light halo particles and satisfy the homogeneous coupled integral equations

$$\big[\Lambda_n^{-1} - h_n(p)\big]F(\vec{p}) = 2\int d\vec{q}\, K_1(\vec{p},\vec{q})G(\vec{q}), \quad (3)$$

$$\big[\Lambda_c^{-1} - h_c(p)\big]G(\vec{p}) = \int d\vec{q}\, K_2(\vec{p},\vec{q})F(\vec{q}) + \int d\vec{q}\, K_3(\vec{p},\vec{q})G(\vec{q}), \quad (4)$$

where $\Lambda_n \equiv \lambda_n/2\mu_{12}$, $\Lambda_c \equiv \lambda_c/2\mu_{13}$. The explicit expressions for the kernels K_1, K_2 and K_3 are given by,

$$K_1(\vec{p},\vec{q}; E) = \frac{mg(\vec{q}+\vec{p}/2) f(\vec{p}+a\vec{q})}{[q^2+\vec{q}\cdot\vec{p}+p^2/2a - mE]}, \quad (5)$$

$$K_2(\vec{p},\vec{q}; E) = \frac{mg(\vec{p}+\vec{q}/2) f(\vec{q}+a\vec{p})}{[p^2+\vec{q}\cdot\vec{p}+q^2/2a - mE]}, \quad (6)$$

$$K_3(\vec{p},\vec{q}; E) = \frac{mf(\vec{q}+b\vec{p}) f(\vec{p}+b\vec{q})}{[q^2/2a+\vec{q}\cdot\vec{p}+p^2/2a - mE]}, \quad (7)$$

with $a \equiv m_3/(m+m_3)$, $b \equiv m/(m+m_3)$ and

$$h_n(p) = m\int d\vec{q}\, g^2(\vec{q})/\big[q^2 + p^2/2a - mE\big], \quad (8)$$

$$h_c(p) = m\int d\vec{q}\, f^2(\vec{q})/\big[q^2 + p^2/2d - mE\big]. \quad (9)$$

with $d_3 \equiv (m+m_3)/(2m+m_3)$.

For our purpose of studying the sensitive computational details for the Efimov effect, these equations are suitably transformed in terms of $\phi(p)$ and $\chi(p)$ as defined below, involving only the dimensionless quantities.

$$\tau_n^{-1}(p)F(p) \equiv \phi(p) \quad \text{and} \quad \tau_c^{-1}(p)G(p) \equiv \chi(p) \quad (10)$$

where $\tau_n^{-1}(p) = \mu_n^{-1} - [\beta_r(\beta_r + \sqrt{\frac{p^2}{2a}+\epsilon_3})^2]^{-1}$, and $\tau_c^{-1}(p) = \mu_c^{-1} - 2a[1+\sqrt{2a(\frac{p^2}{4c}+\epsilon_3)}]^{-2}$, with $\beta_r = \frac{\beta}{\beta_1}$, and $\mu_n = \pi^2\lambda_n/\beta_1^3$ and $\mu_c = \pi^2\lambda_c/2a\beta_1^3$ are the dimensionless strength parameters. The two coupled equations are now reduced to one integral equation for $\chi(p)$ which is written as

$$\Lambda_i\chi(\vec{p}) = \int d\vec{q}\, K_3(\vec{p},\vec{q},\epsilon_3)\tau_c(\vec{q})\chi(\vec{q}) + 2\int d\vec{q}\, d\vec{q}'\, K_2(\vec{p},\vec{q},\epsilon_3)\tau_n(\vec{q}) \times K_1\big(\vec{q},\vec{q}',\epsilon_3\big)\tau_c\big(\vec{q}'\big)\chi\big(\vec{q}'\big). \quad (11)$$

Here the kernels K_1, K_2, K_3 are essentially the same as in Eqs. (3) and (4) except that the variables p, q etc. are now dimensionless quantities: $\frac{p}{\beta_1} \to p$, $\frac{q}{\beta_1} \to q$, and $\frac{-mE}{\beta_1^2} \equiv \epsilon_3$.

Table 1
Ground and excited states for a nucleus of mass 102 (columns 2, 3, 4) and for ^{38}Mg (columns 5, 6, 7) for different two-body input parameters.

n–core energy ϵ_2 keV	$\epsilon_3(0)$ keV	$\epsilon_3(1)$ keV	$\epsilon_3(2)$ keV	$\epsilon_3(0)$ keV	$\epsilon_3(1)$ keV	$\epsilon_3(2)$ keV
40	4020	53.6	44.4	3550	61.3	49.9
60	4080	70.4	61.7	3610	80.8	67.1
80	4130	86.9	(78.4)	3670	99.2	84.2
100	4170	103.1		3710	117	101.4
120	4220	(119.3)		3750	134.5	(118.9)
140	4260			3790	151.6	
180	4345			3860	185.6	
220	4410			3930	(219)	
250	4460			3980		
300	4530			4040		
350	4590			4120		

The integral equation (11), which is basically an eigenvalue equation in Λ_i is computed numerically to determine the three-body (core–n–n) ground-state energy as well as the Efimov states. For a given set of input parameters defining the n–n and n–core potentials appearing in the right-hand side of Eq. (11), we seek the solution of the equation for the three-body energy ϵ_3, when the eigenvalue $\Lambda_i \to 1$, accurate to at least four decimal places. Thus in this approach, we do not have the three-body energy parameter to be fixed from outside. Rather, it is being predicted by computation from the three-body equation once the parameters of the two-body potentials are fixed from the experimental data. It is to be noted that the factors τ_n and τ_c are quite sensitive particularly when the scattering lengths of the binary systems get large values.

We now discuss the results of the calculations for three different 2-n halo systems. Columns 2, 3 and 4 in Table 1 depict the three-body energy for the ground state and excited Efimov states for different two-body (n–core) binding energies for a system comprising a very heavy core (mass ~ 100) and two valence neutrons. It can be seen that when the n–core pair interaction just binds the two-body system with binding energy between 40 to 60 keV, the three-body system shows more than one Efimov state, but for binding energy at and beyond 80 keV there appears only one Efimov state, the second having moved into the unphysical region (the three-body energy becoming less than the two-body (n–core) energy). When the two-body binding increases further, say beyond 120 keV, even the first Efimov state disappears and moves over to the second unphysical sheet. Such solutions above the two-body breakup threshold while working within a bound-state formalism are spurious and are, therefore, not shown in the table beyond the very first value. The overall behaviour is very similar to the results obtained earlier for the ^{20}C nucleus [7,8] with minor differences in the binding energies of the ground state. In the present case, the ground state appears to be bound slightly more strongly in comparison to the 2-n separation energy in ^{20}C. This can most likely be attributed to the much heavier mass of the core. In addition, the first and second Efimov states move into the continuum at a lower two-body energy in comparison with ^{20}C.

Having studied the bound state problem, we next study the system in the scattering sector to probe the evolution of the Efimov states into the resonance states in the continuum. For the scattering of neutron (n) by a two-body bound (n–core) system, the spectator function $G(p)$ describing the behaviour of the light neutron in the presence of the other two is subjected to the boundary condition

$$G(\vec{p}) = (2\pi)^3\delta(\vec{p}-\vec{k}) + \frac{4\pi a_k(\vec{p})}{p^2-k^2-\iota\epsilon}, \tag{12}$$

where the first term represents the plane wave part and the second term is the outgoing spherical wave multiplied by an off-shell amplitude in momentum space for the scattering of neutron by the composite (n + core) system. The scattering amplitude is normalized such that for s-wave scattering,

$$a_k(\vec{p})_{|\vec{p}|=|\vec{k}|} \equiv f_k = \frac{e^{\iota\delta}\sin\delta}{k}. \tag{13}$$

The equation for the off-shell scattering amplitude is obtained as

$$\begin{aligned} 4\pi\left(\frac{a}{d}\right)h\left(p^2,k^2;\alpha_2^2\right)a_k(\vec{p}) &= (2\pi)^3K_3(\vec{p},\vec{k}) + 4\pi\int\frac{d\vec{q}\,K_3(\vec{p},\vec{q})a_k(\vec{q})}{q^2-k^2-\iota\epsilon} \\ &\quad + 2(2\pi)^3\int d\vec{q}\,K_3(\vec{p},\vec{q})K_1(\vec{q},\vec{k})\tau_n(q) \\ &\quad + 2(4\pi)\int d\vec{q}\,K_3(\vec{p},\vec{q})\tau_n(q)\int d\vec{q}'\,\frac{K_1(\vec{q},\vec{q}')a_k(\vec{q}')}{q'^2-k^2-\iota\epsilon}. \end{aligned} \tag{14}$$

This equation is solved numerically below the three-body breakup threshold and scattering cross sections are calculated for different values of the n–core two-body binding energies. For s-wave scattering and in the limit when $k \to 0$, the singularity in the two-body cut does not cause any problem; in fact the amplitude has only the real part. At incident energies different from zero, i.e. $k \neq 0$, the singularity in the two-body propagator is handled by complex scaling technique of continuing the kernel onto a second sheet originally proposed by Balslev and Combes [17]. According to this, the integral contour is deformed from its original position along the real positive axis to the position rotated by a fixed angle. In other words, we let the variables p, q etc., become complex by the transformation $p \to p_1e^{-i\phi}$ and $q \to q_1e^{-i\phi}$. The choice on the values of the parameter ϕ is restricted by the requirement that the imaginary part of the scattering amplitude calculated from the integral equation must be unitary, i.e., it must satisfy the condition $\mathrm{Im}(f_k^{-1}) = -k$. Our analysis has shown that for unequal mass particles, the Efimov states in the three-body system move over to produce a resonance near the scattering threshold for the scattering of particle by a bound pair.

The left panel of Fig. 1. depicts the behaviour of the elastic scattering cross section σ_{el} vs incident energy of the neutron for two different n–core binding energies of 250 and 150 keV. We find an asymmetric resonance structure with the centroid around 1.5 keV and width of approximately 0.5 keV. This behaviour is remarkably similar to that of n–(^{19}C) elastic scattering [8]. We have also checked the scattering length of the n–(core + n) system for incident energy near zero and found it to be positive and large, thereby, indicating a bound state. These results for a hypothetical nucleus with a very heavy core (mass = 100) and two valence halo neutrons show the same behaviour as that of a realistic 2-n halo nucleus, ^{20}C. This helps to reinforce the results obtained in [8] over a large mass range. It would be of interest to search for the same

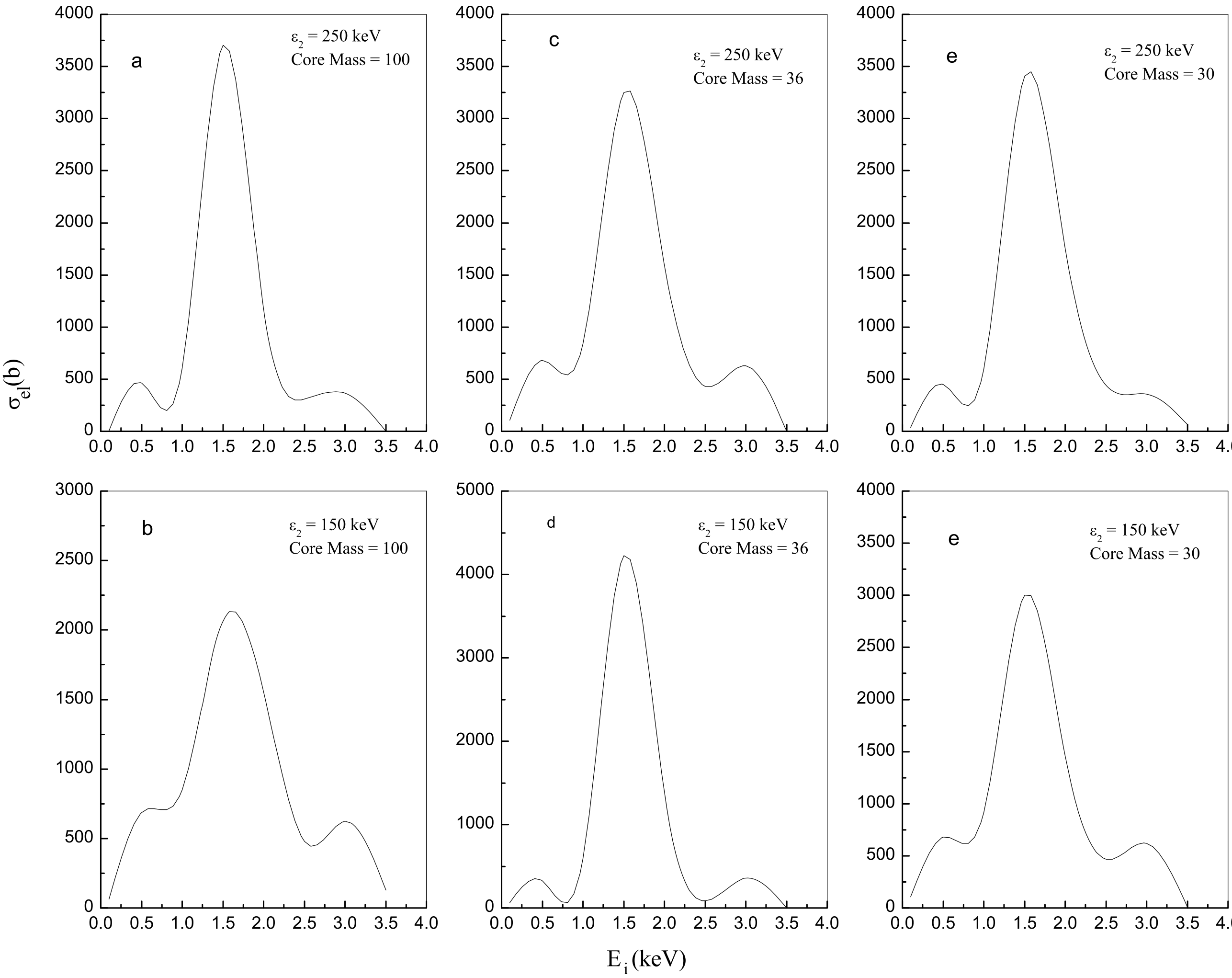

Fig. 1. Plots of elastic scattering cross sections for nuclei with a very heavy core of 100 (a and b), a ^{36}Mg core (c and d), and a ^{30}Ne core (e and f) for two different n–core interaction energies of 250 keV and 150 keV.

effect of movement of Efimov states to resonances in lighter 2-n halo nuclei.

While ^{20}C is by far the most promising case for experimental observation, we suggest a few others such as ^{38}Mg and ^{32}Ne. For both these nuclei the 2-n separation energies are comparable to that of ^{20}C (2570 and 1970 keV, respectively) as are the energies of the n–core systems of ^{37}Mg and ^{31}Ne which are, respectively, nominally bound by 250 and 330 keV [14]. We have carried out detailed calculations for both these nuclei assuming compact cores of ^{36}Mg and ^{30}Ne with two valence neutrons forming the halo in ^{38}Mg and ^{32}Ne, respectively. The results of the calculations are summarised in columns 5, 6, 7 for ^{38}Mg. For ^{38}Mg, the second excited state disappears in the continuum around 120 keV n–core energy and the first excited state disappears around 220 keV. In ^{32}Ne, a very similar trend continues with the first excited state vanishing around 220 keV and the second just beyond 120 keV. These bound state calculations are followed by probing the scattering sector as was done in the case of the heavy core of 100 and also in the case of ^{20}C [8]. The results for the elastic scattering cross sections for n and the bound n–core systems for ^{38}Mg and ^{32}Ne are shown in the middle and right panels of Fig. 1, respectively, for n–core interaction energies of 250 and 150 keV. These are the regions where the first and the second excited states disappear and show up as resonances in the two-body continuum. The figures show the resonance structures with centroids around 1.5–1.6 keV and widths of 0.5–0.6 keV. It is to be noted that 250 keV is also the n–core separation energy (ϵ_2, one neutron separation energy for ^{37}Mg) as given in the latest mass evaluation table of Audi and Wapstra [14]. Of course, the corresponding three body ground state energy that we get from the calculation is somewhat higher (by 50% or so) than the value given in the mass evaluation table. In this connection we would like two point out two issues. First the values for n–core separation energies given in the table are still very uncertain. Secondly, the discrepancy between the calculated and experimental values could possibly be due to our assumption of the point-like (inert) structure of the extended core. Presumably, this extended structure could play a sensitive role in extracting out the detailed information about the effective two-body (n–core) interaction. This would be a subject for future investigation.

In conclusion, we have generalized the study of the evolution of Efimov states into the scattering regime in nuclear systems where the two halo neutrons are weakly bound to the core. In particular, we have seen that in the field of 2n–rich halo nuclei, which are candidates for Efimov states, a resonant state can result from the coupling of an Efimov state in the closed channel to the open channel in scattering as the energy of the incident neutron approaches the excited level. The occurrence of such a resonance with an asymmetric profile should be a characteristic feature of 2n–rich halo nuclei. Thus, besides the ^{20}C nucleus, we have found that ^{32}Ne and ^{38}Mg are other possible candidates. These nuclei are non-Borromean with the valence neutron very weakly bound to the core. In a very recent publication [18], ^{22}C in Borromean configuration has been put forward as a suitable candidate to search for Efimov states. The rapid advancements in the production of secondary beams are making experimental investigations of highly neutron rich nuclei possible. It is worth noting that very recently ^{32}Ne has been produced and studied [19]. Production of ^{31}Ne and ^{37}Mg have also been reported in recent time [20,21]. In light of these experimental developments, it will be interesting as well as challenging to undertake experimental studies of Efimov states in nuclei like ^{32}Ne and ^{38}Mg.

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