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Bianchi Cosmologies: New Variables and a Hidden Supersymmetry

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Abstract

We find a supersymmetrization of the Bianchi IX cosmology in terms of Ashtekar's new variables. This provides a framework for connecting the recent results of Graham and those of Ryan and Moncrief for quantum states of this model. These states are also related with the states obtained particularizing supergravity for a minisuperspace. Implications for the general theory are also briefly discussed.

I. INTRODUCTION

Due to our inability to manage the canonical quantization of the full equations of General Relativity in the generic case, the minisuperspace approximation has been used several times to find results in the hope that they would illustrate behaviors of the general theory. The Bianchi cosmologies are the prime example. Even in this simplified case little progress has been achieved for the more generic model, the Bianchi IX cosmology. In fact, the classical model shows various signs of chaotic behavior [1] and this has usually been assumed to imply important complications for the quantization of the model. In particular, until just recently, not a single solution of the Wheeler-DeWitt (WDW) equation for the Bianchi IX cosmology was known, resembling the situation that one faces in the full theory.

In the years, the introduction of the Ashtekar variables [2] led to a change in this situation. Recently, Kodama [3] formulated Bianchi cosmologies in terms of these variables and found a solution to the WDW equation for the Bianchi IX case. The meaning of this possible quantum state remains unclear. Another interesting observation was made by Moncrief and Ryan [4]. They pointed out that in terms of the Ashtekar formulation (in a certain factor ordering), $\Psi = 1$ was a possible solution of the WDW equation. The point is that when translated into the traditional variables, this state had the form $\exp(-I)$, where I is a solution to the corresponding Euclidean Hamilton-Jacobi equation (with the sign of the potential changed). They explicitly checked that it solved the WDW equation, providing the first known example of a solution of the WDW equation in the traditional variables.

Almost simultaneously, Graham [5] found a closely related state using a very different technique. Exploiting what he calls a “hidden symmetry” (a point that will be discussed below) in the WDW equation he was able to supersymmetrize the model. The supersymmetry equations are complicated to solve for the full wavefunction. However, restricted to the bosonic sector, they yield a solution to the WDW equation.

On the other hand, in supergravity the Bianchi class A models seem to have the same kind of state as was pointed out by D’Eath, Hawking and Obregón [6] and for the Bianchi

IX case D'Eath [7] proved that only the term $\exp(-I)$ is permitted in the wave function. This behaviour is, as expected, present in the other Bianchi class A models, [8].

The three previous procedures virtually result in the same quantum state. This state appears as related to the wormhole ground state [6,7,9]. It is also present in the full theory of supergravity $N = 1$ [10]. The discussion of these points however, lies beyond the scope of this paper.

The main point of this paper is to point out a connection between the results of Moncrief and Ryan and those of Graham. We will show how one can supersymmetrize the Bianchi IX model in terms of Ashtekar's new variables and how the solution to the supersymmetry equations (again $\Psi = 1$) is the state that Moncrief and Ryan showed to be equivalent to that of Graham. This construction may allow to find other states for different factor orderings and it also works for other Bianchi models. We will not focus our attention on the supergravity methods. However, one expects that by solving the supergravity constraints in the Ashtekar's variables for the model in question, only the state $\Psi = 1$ may be permitted.

The plan of the paper is as follows, in section II we summarize the new variables for cosmology, the results of Ryan and Moncrief and of Graham are presented in sections III and IV respectively. In section V we draw the connection between them and we end with a discussion of the possible implications of these results.

II. NEW VARIABLES FOR BIANCHI MODELS: A SUMMARY

In this section we briefly summarize the results presented in [3,11]. The Ashtekar new variables are a canonically conjugate pair consisting of a triad \tilde{E}_i^a (we denote densities with a tilde) and a complex $SO(3)$ connection A_a^i . The constraint equations become,

$$\mathcal{G}^i = D_a \tilde{E}^{ai} \tag{1}$$

$$\mathcal{C}_b = \tilde{E}^{ai} F_{ab}^i \tag{2}$$

$$\mathcal{H} = \epsilon_{ijk} \tilde{E}^{ai} \tilde{E}^{bj} F_{ab}^k, \tag{3}$$

where F_{ab}^i is the curvature of A_a^i and D_a the covariant derivative formed with A_a^i . An important point in this formulation is that the variables are a priori complex. To retrieve real General Relativity, one has to impose “reality conditions”. One way of imposing them is to require that the metric and its Poisson bracket with the Hamiltonian be real. This ensures that the resulting formulation is equivalent to usual real General Relativity.

If one wants to restrict to Bianchi cosmologies one can separate the time dependence of the variables and the fixed spatial dependence. First introduce a fiducial basis of vectors X_i^a and one forms χ_a^i that implement the appropriate symmetry for the Bianchi model in question ($[X_i, X_j]^a = C^k{}_{ij} X_k^a$ and $2D_{[a}\chi_{b]}^i = -C^i{}_{jk}\chi_a^j\chi_b^k$). The indices a, b, \dots are spatial indices and the i, j, \dots label the vectors and forms in the basis and are raised and lowered with the Kronecker delta. $C^i{}_{jk}$ are the structure constants of Bianchi model in question, $C^i{}_{jk} = 0$ for Bianchi I, $C^i{}_{jk} = \epsilon^i{}_{jk}$ for Bianchi IX.

In terms of these fixed bases we can expand the new variables as,

$$\tilde{E}_j^a = E_j^i X_i^a \quad (4)$$

$$A_a^j = A_i^j \chi_a^i. \quad (5)$$

In doing this, we concentrate all the spatial dependence in the fiducial basis. The quantities E_j^i and A_i^j are constants in each three surface, that is, they only depend on “time”.

Inserting these substitutions into the constraint equations [12] one gets,

$$\mathcal{G}^i = C^k{}_{jk} E^{ij} + \epsilon^{ijk} A_{mj} E_k^m \quad (6)$$

$$C_k = -E_i^j A_m^i C^m{}_{jk} + \epsilon^{imn} E_j^i A_{jm} A_{kn} \quad (7)$$

$$\mathcal{H} = \epsilon^{ijk} C^p{}_{mn} E_i^m E_j^n A_{pk} + E_i^m E_j^n (A_m^i A_n^j - A_n^i A_m^j). \quad (8)$$

The reality conditions for Bianchi models read,

$$q^{ij} = (q^{ij})^* \quad (9)$$

$$\dot{q}^{pq} = -iC_{mn}^p E_i^m E_j^n E_k^q \epsilon^{ijk} + 2iE_j^i A_i^j q^{pq} + 2iE_i^p A_j^i q^{jq} + m \leftrightarrow n = (\dot{q}^{ij})^* \quad (10)$$

and can also be expressed in a nonpolynomial fashion by demanding that $-A_a^{*i} = A_a^i - 2\Gamma_a^i$, where Γ_a^i is the spin connection compatible with the triad.

If one further restricts to the diagonal models, the matrices can be assumed to be diagonal, and the only remaining constraint is the Hamiltonian [13]. Introducing the following notation for the variables,

$$E_j^i = \text{diag}(E^1, E^2, E^3) \quad (11)$$

$$A_j^i = \text{diag}(A_1, A_2, A_3), \quad (12)$$

the Hamiltonian constraint takes the form,

$$\text{Bianchi I : } \mathcal{H} = A_1 A_2 E^1 E^2 + A_1 A_3 E^1 E^3 + A_2 A_3 E^2 E^3 \quad (13)$$

$$\text{Bianchi II : } \mathcal{H} = A_1 A_2 E^1 E^2 + A_1 A_3 E^1 E^3 + (A_2 A_3 - A_1) E^2 E^3 \quad (14)$$

$$\text{Bianchi VIII : } \mathcal{H} = (A_1 A_2 - A_3) E^1 E^2 + (A_1 A_3 - A_2) E^1 E^3 + (A_2 A_3 + A_1) E^2 E^3 \quad (15)$$

$$\text{Bianchi IX : } \mathcal{H} = (A_1 A_2 - A_3) E^1 E^2 + (A_1 A_3 - A_2) E^1 E^3 + (A_2 A_3 - A_1) E^2 E^3. \quad (16)$$

At this point one could consider the quantization of these models. Start by choosing a realization, for instance $\Psi[A]$, with

$$\hat{E}_i \Psi[A] = \frac{\partial}{\partial A_i} \Psi[A] \quad (17)$$

$$\hat{A}_i \Psi[A] = A_i \Psi[A] \quad (18)$$

and a factor ordering for the Hamiltonian constraint. For example for the Bianchi IX model,

$$\hat{\mathcal{H}}\Psi[A] = (A_1 A_2 - A_3) \frac{\partial^2}{\partial A_1 \partial A_2} \Psi + (A_2 A_3 - A_1) \frac{\partial^2}{\partial A_2 \partial A_3} \Psi + (A_1 A_3 - A_2) \frac{\partial^2}{\partial A_1 \partial A_3} \Psi \quad (19)$$

In spite of the relatively simple appearance of this equation, there are few ideas about how to construct the physical space of states for the theory. Kodama explored some particular solutions of this equation in reference [3]. As can be seen, in this factor ordering $\Psi[A] = \text{constant}$ is a solution.

The interesting point for our purpose however, is that the constraint equations can be written in a unified fashion [11]. Introducing the variables,

$$Q^i_j = E^i_k A^k_j \quad (20)$$

the Hamiltonian constraint for all Bianchi class A models can be written as,

$$\mathcal{H} = Q^{*i}_k Q^k_i - Q^{*i}_i Q^j_j. \quad (21)$$

As can be readily seen, this version of the Hamiltonian constraint does not have any explicit reference to the Bianchi model in question! The dependence on the model appears in the diffeomorphism constraint and in the symplectic structure for the Q variables.

The diffeomorphism constraint for diagonal models is identically satisfied. This is important in connection with the unified rewriting of the Hamiltonian constraint for all Bianchi models, equation (21). Since the diffeomorphism constraint is not present for diagonal models, all the dynamics of all class A diagonal models is summarized in equation (21), which particularizes to,

$$\mathcal{H} = \overline{Q}_1 Q_2 + \overline{Q}_1 Q_3 + \overline{Q}_2 Q_1 + \overline{Q}_2 Q_3 + \overline{Q}_3 Q_1 + \overline{Q}_3 Q_2 = G_A^{ij} \overline{Q}_i Q_j, \quad (22)$$

where $i, j = 1 \dots 3$ and

$$G_A^{ij} = \frac{1}{2} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}. \quad (23)$$

The difference between different Bianchi models appears in the symplectic structure and the reality conditions of the theory.

III. THE RESULTS OF MONCRIEF AND RYAN

Moncrief and Ryan [4] recently explored amplitude-real-phase exact solutions to the quantum Hamiltonian constraint in terms of the more traditional Misner-type variables. We briefly summarize their results here.

The Misner type variables are obtained by parametrizing the three metric and for the diagonal cases taking the matrix β_{ij} as $\beta_{ij} = \text{diag}(\beta_+ + \sqrt{3}\beta_-, \beta_+ - \sqrt{3}\beta_-, -2\beta_+)$. In terms of the variables $(\alpha, \beta_+, \beta_-)$ and their conjugate momenta (p_α, p_+, p_-) the Hamiltonian constraint becomes,

$$\mathcal{H} = \exp(-3\alpha)(-p_\alpha^2 + p_+^2 + p_-^2 + \exp(4\alpha)V(\beta_\pm)), \quad (24)$$

where $V(\beta_\pm)$ is a function that depends on the particular type of Bianchi model considered.

For the Bianchi IX case it is given by,

$$V(\beta_\pm) = \frac{1}{3} \exp(-8\beta_+) - \frac{4}{3} \exp(-2\beta_+) \cosh(2\sqrt{3}\beta_-) + \frac{2}{3} \exp(4\beta_+) (\cosh(4\sqrt{2}\beta_-) - 1). \quad (25)$$

Equation (24) can be rewritten in an enlightening form,

$$\mathcal{H} = G^{ij} p_i p_j + U(q), \quad (26)$$

where the variables q^i , $i = 1, 2, 3$ refer to α, β_\pm and p_i their corresponding momenta. G^{ij} is just the flat Minkowski metric in 2+1 dimensions. One can therefore interpret, by considering $q^0 = \alpha$ to be a time coordinate, the dynamics of the Bianchi cosmologies as that of a massless particle moving in a (time dependent) potential in 2 + 1 dimensions. This permits a good qualitative understanding of the dynamics of the Bianchi cosmologies.

If one quantizes the model taking a representation $\Psi(\alpha, \beta_\pm)$, there is a factor ordering ambiguity in the first term of the Hamiltonian constraint. Hartle and Hawking [14] suggested the following ‘‘semigeneral’’ factor ordering, $-\exp(-3\alpha)\partial_\alpha^2 + B \exp(-3\alpha)\partial_\alpha$ with B an arbitrary constant. The resulting quantum constraint (WDW equation) is,

$$\left(-\frac{\partial^2}{\partial \alpha^2} + \frac{\partial^2}{\partial \beta_+^2} + \frac{\partial^2}{\partial \beta_-^2} + B \frac{\partial}{\partial \alpha} - \exp(4\alpha)V(\beta_\pm) \right) \Psi = 0 \quad (27).$$

Moncrief and Ryan set out to find solutions of the form

$$\Psi[\alpha, \beta_\pm] = W(\alpha, \beta_\pm) \exp(-\Phi(\alpha, \beta_\pm)). \quad (28)$$

If one inserts this ansatz into equation (27) one gets a complicated partial differential equation for Φ and W . Moncrief and Ryan make a further choice requiring that Φ satisfy,

$$\left(\frac{\partial \Phi}{\partial \alpha} \right)^2 - \left(\frac{\partial \Phi}{\partial \beta_+} \right)^2 - \left(\frac{\partial \Phi}{\partial \beta_-} \right)^2 + \exp(4\alpha)V(\beta_\pm) = 0. \quad (29)$$

Finding a solution to equation (29) one is left with solving a PDE for W . A solution to (29) can actually be found of the form,

$$\Phi = \frac{1}{6} \exp(2\alpha) (\exp(-4\beta_+) + 2 \exp(2\beta_+) \cosh(2\sqrt{3}\beta_-)) \quad (30)$$

and with this ansatz for Φ , choosing $B = -6, W = \text{const.}$ is a solution. What Moncrief and Ryan really demanded is Φ to be the solution of the ‘‘Euclidean’’ Hamilton-Jacobi equation (29) for this particular model.

What does this have to do with Ashtekar’s variables? Kodama proved that if one takes a wavefunction in terms of Ashtekar’s variables Ψ_A , one can reconstruct the wavefunction in terms of the traditional variables by choosing $\Psi_{\text{traditional}} = \exp(\pm i\Phi_A)\Psi_A$, with Φ_A given by $\Phi_A = 2i \int \tilde{E}_i^a \Gamma_a^i d^3x$. For the Bianchi IX case, Φ_A is equal to the $\mp i\Phi$ found by Moncrief and Ryan and therefore the solution $\Psi_A = \text{const}$ in terms of Ashtekar variables becomes, when transformed to the traditional variables, the wavefunction found by Moncrief and Ryan by direct substitution.

IV. THE WAVEFUNCTION OF GRAHAM

A totally (apparently, as we will see) independent result was found by Graham [5]. He considered a supersymmetrization of the Bianchi system using the fact that he was able to solve the ‘‘Euclidean’’ Hamilton-Jacobi equation (29) which he writes in the form,

$$U(q) = G^{ij} \frac{\partial \phi}{\partial q^i} \frac{\partial \phi}{\partial q^j}, \quad (31)$$

where ϕ is the Φ of equation (29). He finds the solution (30), the same as that given by Moncrief and Ryan. That this decomposition can be accomplished at all can be readily seen if one writes the potential in terms of the metric variables without using the Misner parametrization. In terms of those variables the potential term is just a second order polynomial.

With this observation, one can introduce a set of fermionic variables $\psi^i, \bar{\psi}^j$ satisfying the

spinor algebra,

$$\psi^i \psi^j + \psi^j \psi^i = 0 \quad (32)$$

$$\bar{\psi}^i \bar{\psi}^j + \bar{\psi}^j \bar{\psi}^i = 0 \quad (33)$$

$$\bar{\psi}^i \psi^j + \psi^i \bar{\psi}^j = G^{ij} \quad (34)$$

and define the following supercharges,

$$\mathcal{S} = \psi^i \left(p_i + i \frac{\partial \phi}{\partial q^i} \right) \quad (35)$$

$$\bar{\mathcal{S}} = \bar{\psi}^i \left(p_i - i \frac{\partial \phi}{\partial q_i} \right), \quad (36)$$

which satisfy $\mathcal{S}^2 = \bar{\mathcal{S}}^2 = 0$, such that the Hamiltonian can be written in the following form

$$\mathcal{H} = \frac{1}{2} (\mathcal{S} \bar{\mathcal{S}} + \bar{\mathcal{S}} \mathcal{S}). \quad (37)$$

So we see the \mathcal{S} 's work as ‘‘square roots’’ of the Hamiltonian. A quantum representation can also be introduced considering wavefunctions of the canonical coordinates and of three Grassmann variables η^i in terms of which one can represent the ψ algebra by, $\psi^i = \eta^i$, $\bar{\psi}^i = G^{ij} \frac{\delta}{\delta \eta^j}$. However if one writes the quantum Hamiltonian as $\hat{\mathcal{H}} = \frac{1}{2} (\hat{\mathcal{S}} \hat{\bar{\mathcal{S}}} + \hat{\bar{\mathcal{S}}} \hat{\mathcal{S}})$ there is a factor ordering discrepancy from simply quantizing the traditional Hamiltonian by a term $\hbar \frac{\partial^2 \phi}{\partial q^i \partial q^j} [\bar{\psi}^i, \psi^j]$.

Any solution of $\hat{\mathcal{H}}\Psi = 0$ can be written as

$$\Psi = A_+ + B_i \eta^i + C^k \epsilon_{kij} \eta^i \eta^j + A_- \epsilon_{ijk} \eta^i \eta^j \eta^k, \quad (38)$$

where the eight functions A_{\pm}, B_i, C^k depend on the canonical configuration variables $q^i = q^i(\alpha, \beta_{\pm})$. In the non-supersymmetric (bosonic) limit A_+ should be a solution of the traditional WDW equation. Therefore if one is able to find a Ψ solution to this model implicitly one finds a solution to the quantum constraints of the usual Bianchi IX cosmology.

Wavefunctions should be supersymmetric, that is they should satisfy $\hat{S}\Psi = 0, \hat{\bar{S}}\Psi = 0$. One can solve these equations in general for the bosonic part of the wavefunction. One gets,

$$A_+ = a_+ \exp(-\phi/\hbar) \quad (39)$$

where a_+ is a constant.

This in particular implies that this quantity should be a solution of the usual WDW equation for the Bianchi IX cosmology. This is readily seen. Recalling the form of ϕ , one sees that this wavefunction is modulo irrelevant constants the same as the one found by Ryan and Moncrief.

The reader may be surprised that in this formulation this appears as the general (pure bosonic) solution: of course, when one sees that equation (31), is the same as equation (29), different boundary conditions will give different solutions for ϕ (for the Taub model, that is, a diagonal Bianchi IX model with $\beta_- = 0$, there exist two distinct solutions for ϕ [15]). The procedure of Graham will then lead to a family of solutions, one for each ϕ . In fact they have been found, see [3]. What one encounters here is a common phenomenon in supersymmetry, that is, the kernel of an operator is usually smaller than that of its square.

In particular when, supergravity is used the constraint equations for the Bianchi models are so restrictive that only the functions A_+ and A_- remain as parts of the wave equation (38). They are the quantum states one would obtain by path integral methods [6,7].

Moreover, not all solutions to the Hamilton-Jacobi equations will be consistent with the desired boundary conditions and also the supergravity constraints [10]. However, we will not focus our attention on these issues here.

V. SUPERSYMMETRIZING THE ASHTEKAR FORMULATION

In spite of the fact that both the construction of Moncrief and Ryan and that of Graham make use of the same Hamilton-Jacobi function ϕ , it is somewhat remarkable that such radically different techniques yield the same wave function. Here we will seek to put them on a common ground. Let us start by recalling that in terms of the Ashtekar variables the Hamiltonian constraint can be written as,

$$\mathcal{H} = \bar{Q}_1 Q_2 + \bar{Q}_1 Q_3 + \bar{Q}_2 Q_1 + \bar{Q}_2 Q_3 + \bar{Q}_3 Q_1 + \bar{Q}_3 Q_2. \quad (40)$$

This is a “universal constraint” which is insensitive to the choice of a specific diagonal model. The dependence on the specific structure constants of the Bianchi model under consideration will appear by imposing the reality conditions.

This Hamiltonian immediately suggests to introduce the supercharges,

$$\mathcal{S} = \psi^i Q_i \tag{41}$$

$$\overline{\mathcal{S}} = \overline{\psi}^i Q_i^*. \tag{42}$$

The algebra of the ψ 's is the same as before except that now,

$$\overline{\psi}^i \psi^j + \psi^i \overline{\psi}^j = G_A^{ij} \tag{43}$$

with G_A given by equation (23).

In terms of these supercharges we can again write the Hamiltonian constraint as $\hat{\mathcal{H}} = \frac{1}{2}(\hat{\mathcal{S}}\hat{\overline{\mathcal{S}}} + \hat{\overline{\mathcal{S}}}\hat{\mathcal{S}})$. Also we can introduce a quantum representation. Since the Q variables are noncanonical, we will choose the more usual “connection” representation in terms of functions of A_i and η^i and we could seek for the bosonic part of the solution to the supersymmetry constraints. The equations become,

$$\hat{\mathcal{S}}\Psi[A, \eta] = \hat{Q}_i \eta^i \Psi[A, \eta] \tag{44}$$

$$\hat{\overline{\mathcal{S}}}\Psi[A, \eta] = \hat{Q}_i^* \frac{\partial}{\partial \eta^i} \Psi[A, \eta] \tag{45}$$

and using the reality conditions in terms of the Ashtekar connection, $-A^{*i}{}_a = A_a^i - 2\Gamma_a^i$ we get,

$$\eta^i A_i \frac{\partial}{\partial A_i} \Psi[A, \eta] = 0 \tag{46}$$

$$(A_k - 2\hat{\Gamma}_k) \frac{\partial}{\partial A_k} G_A^{ki} \frac{\partial}{\partial \eta^i} \Psi[A, \eta] = 0. \tag{47}$$

The only solution that this system of equations admits is $\Psi[A] = \text{constant}$. This may seem a trivial solution but as we saw in previous subsections, it is exactly the kind of solution that Ryan and Moncrief showed to have a very nontrivial form in terms of the traditional variables. It then turned that it was equivalent to the solution that Graham found for

the bosonic sector, supersymmetrizing the traditional formulation and the only component permitted by supergravity [10].

Notice that the supersymmetrization we have performed works for arbitrary Bianchi models. This seems quite natural, and in fact the Graham construction has already been generalized to the Bianchi II model [16], where the whole wavefunction was obtained.

VI. CONCLUSIONS

We have therefore shown that the Ashtekar variables provide a natural framework for seeking supersymmetric quantum states. We find that the only possible bosonic state is a constant, which translated in terms of the traditional variables corresponds to the result that Graham found, as Ryan and Moncrief proved and is also connected to the state obtained by means of supergravity [6,7]. This state could be understood as the wormhole quantum state [7,9,10].

What does this tell us about the general theory? One in general cannot formulate the Hamiltonian constraint in the Ashtekar formulation in terms of the Q variables. However taking a particular lapse gauge a similar formulation is possible and has found application in the asymptotically flat context [17]. There is however, the issue of the other constraints (diffeomorphism and gauss law) to take care of. It would be interesting to pursue this line of reasoning further to see if in some sense it simplifies finding solutions to the generic WDW equation. It is however, at first sight a bit disappointing to notice that even in the Bianchi context the only solution the technique yields is a constant. If one is to take seriously that somehow supersymmetry selects a preferred ground state [6] this could tell us something about the ground state of the general theory [10].

What does this formulation have to do with supergravity? In principle it is a different construction, i.e. supersymmetrizing a Bianchi model as a mechanical system is not the same as particularizing supergravity for a minisuperspace. It would however be interesting to compare both cases in terms of the new variables (supergravity has already been formulated

in terms of them by Jacobson [18]) and study similarities and differences. On the other hand, again one could study the implications for the general theory. For non-zero cosmological constant, a semi-classical WKB wave function [19] has been obtained for the full supergravity (N=1) in Ashtekar's formalism it has the form of the exponential of the Chern-Simons functional and has been particularized for the Robertson-Walker universe. Moreover the general, supergravity theory N=1 seems to select out the most symmetrical states [6], the whole the standard variables, the two expected bosonic states $\exp(\pm I)$ appear and all physical states are given by finite expressions. It is mostly interesting to look if these results can be obtained in terms of the Ashtekar variables and try to understand them in connection with the "ground state" of the theory.

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