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Three Essays on the Welfare Effects of Factor Immobility and Price Uncertainty for a Country Experiencing Growth, Entering a Customs Union and Giving or Receiving a Unilateral Transfer.

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Three Essays on the Welfare Effects of Factor Immobility and Price Uncertainty for a Country Experiencing Growth, Entering a Customs Union and Giving or Receiving a Unilateral Transfer

A Dissertation
submitted to the Graduate Faculty of the
Louisiana State University and
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in

The Department of Economics

by
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Abstract

This dissertation presents an intertemporal model with two degrees of endogenously determined labor mobility: complete immobility in the short run and partial mobility in the long run. It examines the impact on welfare when, in response to a change in an exogenous variable, the nation adjusts its output ratio. The movement of factors between sectors is necessary but in the case of labor, that does not take place at first because of labor's slowness in responding to a developing wage differential. The consequent contraction of the product transformation curve is welfare reducing. Uncertainty about the future terms of trade is introduced to see if that alters the results obtained in a certain environment.

Chapter III includes three types of growth: an increase in the labor supply, increase in the supply of capital and neutral technological innovation. Growth was found to be either welfare improving or immiserizing depending on the degree of labor mobility plus the sizes of the elasticities of factor substitution and the value and physical intensities of production. For a large country the terms of trade effect was incorporated in determining the total impact on welfare. Uncertainty about the future price ratio did not reverse the qualitative findings in a certain environment.

Chapter IV covers two customs union alternatives for the home country, one trade creating and the other trade diverting. It was found that labor immobility could cause a trade creating union to be welfare reducing while a welfare improvement might occur in an uncertain environment with a trade diverting union in spite of some labor immobility.

In Chapter V the impact of a unilateral transfer was analyzed from the perspective of the donor and recipient. With some degree of labor immobility there can be a normal result,

a weak or strong paradox. If the certainty of the transfer reduces uncertainty about the future terms of trade it may be welfare improving for both.

The objective of the intertemporal model is to add subtlety to trade theory in the treatment of immobility of labor and give it greater scope for utilization.

Chapter 1
Introduction

The focus of analysis in this paper is the Heckscher Ohlin model revised to accommodate an intertemporal difference in labor mobility. In the short run labor is immobile, so, if the economy attempts to adjust to a new output ratio, it develops an endogenous wage differential. In the long run once the critical differential is reached, labor becomes somewhat mobile. Then the development of the differential slows down and stabilizes when the equilibrium output mix is attained. At another stage in the analysis the assumption of uncertain terms of trade will be added to the model. The basic model has four parts:

1 Mobile Factors in a Certain Environment

2 Short Run Immobility of Labor, Long Run Partial Mobility in a Certain Environment

3 Mobile Factors in an Uncertain Environment

4 Short Run Immobility of Labor, Long Run Partial Mobility in an Uncertain Environment

Once developed, the basic model will be applied to three situations: the home country is growing; it enters a customs union; and it makes or receives a unilateral transfer.

While much work has been done on these subjects in a certain environment, there are still some areas which have not been fully explored, and this dissertation will attempt to do so. Less work has been done on an uncertain environment; so, there is more scope for investigation.

The criterion for evaluation is a change in welfare. Welfare will be defined as an increase in income, leaving distributional issues aside, since the potential exists for improvement of everyone's welfare.

The Classical model of international trade assumed most factors were tied to particular industries. The pattern of trade of a country depended upon the terms of trade. Each country concentrated on the production of those goods that it could produce relatively more cheaply than other countries and then exchanged the amount of that output above its own requirements.

The Modern theory assumes all factors are mobile among sectors and finds the basis for exchange to be in the difference between the factor endowments of the trading countries. According to this approach, delineated by Heckscher and Ohlin, a country has comparative advantage in the production of the commodity which uses more intensively that country's more abundant factor.

This paper will use the Modern theory but will incorporate a fixed factor in the short run (i.e. specific to a sector), a characteristic of the Classical model.

The first part of the basic model, mobile factors in a certain environment, is based on a presentation by Batra (1973). It shows the home country opening to trade and explains the mechanism as it adjusts the output mix to the international terms of trade. Here it will be assumed the terms of trade are an improvement over the autarkic price ratio. Even if the home country is large and, therefore, the previous terms of trade decline as it begins to trade, the home country will experience an increase in welfare over autarky.

The next part of the basic model involves incorporating labor immobility. It is important, however, first to examine why labor would not immediately respond to a change in wages. One reason may be that workers are unsure what the change means; mainly, they are uncertain as to whether the change is permanent or will be reversed in the short run. Before a worker relocates s/he wants to be sure that it will be worth the effort. In this

dissertation it will be assumed that when the wage differential goes beyond the critical value, some people are convinced and labor becomes mobile.

Labor economics has developed the concept of the reservation wage which can be used to amplify this explanation. The reservation wage is the lowest wage workers will accept rather than quit. Once the wage differential moves beyond the critical value, it will be assumed that the lower wage has fallen below the reservation wage for some workers; so, some labor begins to migrate. MacDonald (1988) pointed out recently that even in an uncertain environment, the primary determinant in intertemporal decision making affecting labor mobility is the difference in wages between sectors.

Grossman and Stiglitz (1980) provided an alternate explanation of this process which is particularly applicable to an uncertain environment. Prices convey information from the informed to the uninformed but do so imperfectly. That being so, whoever expends resources to obtain information receives compensation. How informative the price system is depends upon the number of individuals who are informed, but that number is itself an endogenous variable. As more participants become informed, the expected utility of being informed declines. An equilibrium is created when the expected utility of being informed is equal to the expected utility of being uninformed. Then those who are uninformed cannot improve their situation by taking on the cost of being informed. That equilibrium depends upon the cost of information, the amount of noise that interferes with the information conveyed by the price system and the quality of the information. These elements will affect the size of the variance of the price distribution when the future price is unknown. If the cost is high, as it may be in the short run, the number of informed is small and the informational quality of prices is low. The more noise in the system, the less precise is the price system. This may be true as the wage differential begins to develop. One can conclude, then, that when one factor is mobile, participants in that market face prices which fully reflect all available information. Participants in markets where factors are immobile face prices which do not, or they perceive do not, fully reflect all available information.

Since the return on capital is determined, by assumption, in the international capital markets, it will be clear and readily attainable. Information about wages, on the other hand, may not be so clear. The larger the variance of the expected price ratio distribution, the more tentative decision makers will be about reallocating factors. Uncertainty slows down and dampens the response to an expected change in the terms of trade. The issue to be addressed here is whether price uncertainty will reverse the findings in a certain environment.

Factor immobility has been recognized as an important element in international trade theory since Cairnes (1874) developed a theory of noncompeting groups of labor. Neo Classical writers such as Bastable (1903) and Haberler (1936) examined the implications for international trade among economies in which goods are produced using industry specific and non specific factors. Haberler showed that factor immobility within a county not associated with factor price rigidity, does not entail a distortion of the first order conditions of Pareto Optimality. He concluded that free trade is still beneficial and protection, by introducing a distortion, would reduce national welfare.

Jones's (1971) adaptation of the Heckscher Ohlin model allows interpretation as an economy with one mobile factor and one immobile factor. A return differential for the immobile factor was included. He showed that changes in the intensity with which specific factors are used depends upon the elasticities of substitution in each sector and changes in factor prices which, in turn, are linked to commodity prices and endowment changes. The role of elasticities of substitution in adjustment of the output mix is central to some of the analysis in this dissertation.

Casas (1984) carried forward the work. He assumed inertia in the labor market resulted in an intersectoral endogenous wage differential and brought forward the role of the elasticity of intersectoral labor mobility in determining the effects of exogenous disturbances. Throughout his analysis he assumes the elasticity of labor mobility is constant. The change in the wage ratio causes labor to move, if the differential has hit a

critical value. Importantly, Casas reinforced Haberler's conclusion when he found that with an endogenous wage differential, an improvement in the terms of trade always increases welfare. The value of the elasticity of labor mobility relative to the elasticities of factor substitution in the two sectors is critical in determining many of Casas's results. This approach is a valuable tool of analysis for this dissertation.

Reference was made to Yu's paper (1981) to solve some of the technical problems of incorporating labor immobility with a changing wage differential into this model. His work, however, allows for unemployment of labor which is not permitted here.

Batra and Pattanaik (1971) explored the implications of a change in the factor price differential for the terms of trade and welfare. They found that an increase in a wage differential does not necessarily reduce welfare if it is caused by an improvement in the terms of trade.

Beladi and Naqvi's paper (1987) constructed an intertemporal, two sector model of trade with a changing, exogenous wage differential in a certain environment. They assumed a small country; so, the terms of trade and return on capital were exogenously given. Then they proposed a change in the terms of trade and found a normal output response in both the short run and long run.

The model in this dissertation uses some aspects of the intertemporal model with an endogenous differential in both a certain and uncertain environment.

Hazari's recent book (1986) provided a general basis for the analysis of the wage differential; however, the assumption is made that an exogenous differential exists in spite of factors being perfectly mobile between sectors; so, it is a genuine domestic distortion.

The important point here is that this dissertation, unlike previous writings, uses the intertemporal approach to examine the implications of changing labor mobility. In the short run, with immobile labor, the elasticity of labor mobility is zero and that immobility causes the wage differential to develop. In the long run, labor becomes somewhat, not perfectly, mobile causing the differential eventually to stabilize.

As mentioned above, introducing uncertainty into the model is done by assuming future terms of trade are unknown. There is a distribution of possible prices with the mean being the expected price. The larger the variance of the distribution, the lower the confidence level in the mean. No financial markets or other risk sharing arrangements are allowed; so, firms are assumed to maximize the expected utility of future profits thereby also minimizing the cost of production. Sandmo (1971) and Ishii (1977) explored the optimal output under uncertainty when firms' decision makers are risk averse and demonstrated that with price uncertainty output is smaller than in a certain environment because risk premia are required.

Mayer (1974) developed a model with price uncertainty and firms maximizing the expected utility of profits. He assumed constant returns to scale with mobile factors and found that the Rybczynski Theorem held. When postulating a change in the expected price, he also found that the Stolper Samuelson and Factor Price Equalization Theorems held.

The approach used in this paper is to adjust the model to incorporate an uncertain terms of trade based on the Mayer paper and Batra's more complete model (1975), but replacing Batra's production uncertainty with price uncertainty.

Then uncertainty with immobility will be incorporated into the model. Reference was made to the article by Yu and Ingene (1985). They explored the effect of price uncertainty on sectoral capital labor ratios, factor employment, output levels, factor income and expected profits. They assumed both a wage and rental differential which was fixed while both factors are mobile. Their analysis included four types of changes in price uncertainty. The first type, a change in the mean has been adopted for this paper. Also, here in, full employment is assumed, and both sectors face an uncertain price.

Growth

Growth will take three forms: an exogenous increase in the labor supply, an exogenous increase in the supply of capital and disembodied, neutral technological innovation (Hicks neutral). Each will be analyzed separately with each of the four parts of

the basic model. In looking at the first two types of growth, it will be determined under what circumstances the Rybczynski Theorem will hold. Martin (1976) developed the concept of the Rybczynski loci which shows the change in the two outputs when a country grows and the terms of trade are fixed. That has proven to be a very useful tool in sorting out the impact of growth and the induced change in the terms of trade.

A paper by Choi and Yu (1987) developed a model with technical progress and variable returns. They found that Hicksian neutral technological improvement is ultra biased in production regardless of the direction and severity of returns to scale. Their model was adapted here to constant returns to determine the impact of immobility of labor in uncertainty.

Batra and Casas (1976) constructed a model which was a synthesis of the Heckscher Ohlin and Neoclassical models with one non-specific and two specific factors. They found that, if the growing factor is one of the specific factors, the output of that industry will increase and the output of the other industry will fall. If the increasing factor is mobile, output of both goods will increase, a conclusion at variance with the Rybczynski Theorem. Casas (1984) in a later paper with an endogenous wage differential also concluded that the Rybczynski Theorem may not always hold. More precisely with non reversal of factor intensities, an increase in the mobile factor will increase the output of the good intensive in that factor. The impact on the other good depends upon the relative sizes of the elasticity of labor mobility and the elasticity of factor substitution in that industry. This paper will expand his work by assuming an increase in the immobile factor in one or both sectors. The importance of the relative sizes of the elasticity of labor mobility and the elasticities of factor substitution in that each sector will be explored.

As mentioned above, growth in an uncertain environment was covered in Mayer's paper (1974) and he found that the Rybczynski Theorem held. This dissertation will evaluate the Rybczynski Theorem with price uncertainty and immobile labor when either the labor or capital supply increases. Then the analysis will continue on to the impact of

Hicksian neutral technological progress in the context of uncertainty about future price and immobile labor.

Customs Union

Viner (1950) developed the fundamental theoretical concepts of trade creation and trade diversion as the positive and negative effects of a customs union. Meade (1955), Gehrels (1956) and Lipsey (1957) have shown that even trade diversion may lead to a higher level of national utility. That is, if the customs union results in a net increase in volume of trade, welfare has increased. Johnson (1962) summed up the above findings and extended them. He found that after the home country removes its tariffs on imports from a union member but not the rest of the world, the price paid by the home country's consumers falls; so, unless their demand is perfectly inelastic, they will consume more imports, which expands trade. That is the consumption effect. Bhagwati (1971) concluded that trade diversion can be welfare improving for a nation where there is an increase in the output of exports and a decline in the output of importables.

There has been some controversy about the reasons for forming a customs union. A unilateral reduction in the home country's tariff has been shown to bring about a more efficient reallocation of factors than forming a customs union. Cooper and Massel (1965) pointed out that when there is a choice between lowering a unilateral tariff and entering a customs union, the customs union may be chosen if an important goal is access to another market. A unilateral reduction in its tariff will not guarantee that access. This argument was expanded by Wonnacott and Wonnacott (1960). They contended that a customs union can be used to insure the production of public goods. These arguments will be adapted to the context of this paper to explain the rankings of the alternatives available to the home country.

Traditionally customs union theory assumes flexible prices and mobile factors within each country. Some of Casas's work (1984) will be used to incorporate immobile labor

into the model. Yu and Parai (1987) set up a customs union model with a small home country having an endogenous factor price differential caused by imperfect labor mobility. They analyzed the interaction of a change in the terms of trade from forming the Customs Union and factor immobility. The elasticity of labor mobility was assumed constant. This paper will carry the discussion further with an intertemporal model having different elasticities of labor mobility.

Fries (1984) addressed the subject of the impact of uncertainty on customs union theory. He contended that the formation of a customs union may in itself be an attempt to reduce uncertainty and, therefore, increase welfare in an environment of uncertain terms of trade. Regardless of the change in the terms of trade, a member knows its exports to the other member(s) will have a price advantage over those goods coming from the rest of the world.

Grinols (1985) has developed a model incorporating uncertain terms of trade in the customs union context. It includes, however, securities traded on financial markets which are excluded from this model. This dissertation will develop the model without financial markets and incorporate immobile labor to determine the impact on welfare of joining the customs union.

Unilateral Transfer

A unilateral transfer occurs when commodities are shipped from the home country (A) to another (B) on concessionary terms rather than under normal market incentives. The impact of the transfer will be viewed from the standpoint of both the donor and the recipient.

This is, of course, a bilateral transfer in a multilateral context. If both participants are small, the rest of the world (C) provides perfectly elastic supplies of the two goods while having a perfectly elastic demand for both goods.

Only if one or both are large countries will the transfer induce a change in the terms of trade and, therefore, have an impact on the rest of the world.

Yano (1983) found that if the terms of trade do not change, then the real incomes of the giver and recipient change by the amount of the transfer.

His definitions of the three possible results of the transfer will be used here. A normal (orthodox) result occurs when the donor's welfare declines and the recipient's welfare improves. There is a weak paradox when the welfare of both move in the same direction: improve or decline. A strong paradox results when the donor's welfare improves and that of the recipient declines, an immiserizing transfer.

Leontief (1936) was the first to demonstrate the possibility of a strong paradox result. Samuelson (1952, 1954) showed that, in a two country world such a result was possible only if there were multiple and unstable equilibria.

McDougall (1965) observed that the terms of trade effect depended upon the substitution possibilities in both consumption and production. Jones (1970) expanded the argument by showing that since the trade pattern of the donor is independent of its output mix, then, with homothetic tastes, the terms of trade can improve for the donor.

Gale (1974), Chichilniskey (1980) and Brecher and Bhagwati (1982) showed under various circumstances that a strong paradox can occur even in a stable equilibrium when there is a three country world. Of particular interest here, Brecher and Bhagwati showed that a transfer could immiserize the recipient and enrich the donor when exogenous production distortions are created by tax cum subsidies in the home country.

This paper will address the impact of an endogenous wage differential which exhibits some of the characteristics of a distortion.

In a later paper Bhagwati, Brecher and Hatta (1983) found that a three agent model was critical to demonstrate the potential of an immiserizing transfer in the presence of market stability and a distortion in one economy. Herein, when both participants are small countries, then the two country model applies, since their actions have no effect on the rest

of the world. When one or both are large, then the three country model applies. The granting and receipt of the transfer will have an impact on the terms of trade thereby affecting the welfare of the rest of the world.

Choi and Yu (1987) showed how the paradox will occur under variable returns to scale. They concluded that sufficient condition for a normal outcome is that both have the same returns to scale. This model contains the assumption that both countries have constant returns to scale.

Finally, turning to uncertainty, Fries (1983) showed that in a two country world, the strong paradox can occur in the presence of uncertainty in production with a distortion in the form of incomplete asset markets. This dissertation, as explained above, will extend the discussion by allowing uncertainty about the future price ratio and an endogenously determined wage differential.

Having reviewed the relevant literature, let us continue by developing the basic model.

Chapter II
Basic Model

The Basic Model has the following characteristics.

There are two commodities: X_1, X_2; (outputs: $i = 1, 2$),
with two primary factors of production: capital (K), labor (L). Both outputs exhibit constant returns to scale.

Part 1 Mobile Factors in a Certain Environment

The production functions are:

$$X_1 = F_1\ (K_1, L_1) \tag{2.1}$$

$$X_2 = F_2\ (K_2, L_2) \tag{2.2}$$

Marginal products are positive but diminishing:

$$F_{Ki} > 0,\ F_{Li} > 0$$

$$F_{KKi} < 0,\ F_{LLi} < 0$$

$$F_{KLi} = F_{LKi} > 0$$

The functions are linearly homogeneous and concave:

$$F_{KKi}\, F_{LLi} - F^2_{KLi} > 0$$ [1]

X_1 is relatively capital intensive; X_2 is relatively labor intensive. The assumption is made throughout that there are no physical or value intensity reversals. (See Appendix 1).

---

[1] Along with two primary factors each sector has a minor input which is specific to that output allowing $F_{KKi}\, F_{LLi} - F^2_{KLi}$ to be positive. That follows Batra's approach, *Studies in the Pure Theory of International Trade* (New York: St. Martins Press, 1973), p.18.

To find the equilibrium conditions for factor markets, profits (Πi) are maximized. w is the return to labor; r is the return to capital.

$$\Pi_1 = X_1 - wL_1 - rK_1 \qquad \Pi_2 = PX_2 - wL_2 - rK_2$$

$$\frac{\partial \Pi_1}{\partial L_1} = FL_1 - w = 0 \qquad \frac{\partial \Pi_2}{\partial L_2} = PFL_2 - w = 0$$

$$\frac{\partial \Pi_2}{\partial K_1} = FK_1 - r = 0 \qquad \frac{\partial \Pi}{\partial K_2} = PFK_2 - r = 0$$

Note: $P = \frac{P_2}{P_1}$. Assume $P_1 = 1$

Therefore: $F_{L1} = w = PF_{L2}$ (2.3)

$F_{K1} = r = PF_{K2}$ (2.4)

These are the factor market equilibia conditions. Given perfect competition in the product markets, each factor is paid the value of its marginal product. Producers are price takers in output and input markets; so the unit cost of each good is equal to its price:

$$aK_1r + aL_1w = P_1 \tag{2.5}$$

$$aK_2r + aL_2w = P_2 \tag{2.6}$$

$aKi = \frac{Ki}{Xi}$: the amount of capital used per unit of output Xi, the input coefficient. Being variable, the input coefficients are functions of the factor price ratio (W = w/r) which in turn is affected by changes in the exogenous variable, the output price ratio.

$\frac{P_2}{P_1} = \frac{(w/r)\ aL_2 + aK_2}{(w/r)\ aL_1 + aK_1}$. w and r are flexible; so, full employment of K and L are assured. Supplies of inputs are fixed unless otherwise stated. The full employment conditions are:

$$L_1 + L_2 = L \tag{2.7}$$

$$K_1 + K_2 = K \text{ or } L_1(k_1) + L_2(k_2) = K \tag{2.8}$$

$k_i = K_i / L_i$: the capital labor ratio.

Demand

Consumers have a concave social utility function dependent upon consumption (Z_i) of the two commodities.

$$U = U(Z_1, Z_2) \quad U_i > 0; U_{ii} < 0 \tag{2.9}$$

Totally differentiating:

$$dU = \frac{\partial U}{\partial Z_1} dZ, + \frac{\partial U}{\partial Z_2} dZ_2 = 0$$

Note: $\frac{\partial U}{\partial Z_1} = U_1$; $\frac{\partial U}{\partial Z_2} = U_2$

$$dU = U_1 \left(dZ_1 + \frac{U_2}{U_1} dZ_2\right) = 0$$

$$\frac{dU}{U_1} = dZ_1 + \frac{U_2}{U_1} dZ_2 = 0 \tag{2.10}$$

When maximizing social utility for a given social budget: $\frac{U_2}{U_1} = \frac{P_2}{P_1} = P$

Therefore: $\frac{dU}{U_1} = dZ_1 + PdZ_2 = 0$ (2.11)

In a closed economy the budget constraint requires that the value of production be matched by the value of consumption:

$$X_1 + PX_2 = Z_1 + PZ_2 \tag{2.12}$$

Totally differentiating the production functions (2.1) and (2.2):

$$dX_1 = FK_1 dK_1 + FL_1 dL_1$$

$$dX_2 = FK_2 dK_2 + FL_2 dL_2$$

Using the factor market equilibia (2.3), (2.4) and the full employment conditions (2.7, (2.8), then:

$$dK_1 = -dK_2$$

$$dL_1 = -dL_2$$

Therefore: $$\frac{dX_1}{dX_2} = \frac{FK_1\ dK_1 + FL_1\ dL_1}{FK_2\ dK_2 + FL_2\ dL_2}$$ [2]

can be reinterpreted as: $$\frac{dX_1}{dX_2} = \frac{-PFK_2\ dK_2 - PFL_2\ dL_2}{FK_2\ dK_2 + FL_2\ dL_2}$$

so: $$\frac{dX_1}{dX_2} = -P$$

In equilibrium: $\frac{-dX_1}{dX_2} = P = \frac{U_2}{U_1}$, the marginal rate of transformation (i.e., the slope of the product transformation curve) equals the slope of the price line which in turn equals the marginal rate of substitution (i.e., the slope of the social indifference curve).

To demonstrate the mechanisms of the change in the output ratio, assume the economy opens to trade which results in incomplete specialization.

---

2 Since the minor inputs are specific to their sectors, they do not move between sectors.

The new price ratio facing the country represents an increase in the relative price of X_2 : $dP>0$.

To determine the impact on production, it is necessary to find $\frac{dX_1}{dP}$ and $\frac{dX_2}{dP}$ by differentiating the production functions (2.1), (2.2) with respect to the change in price:

$$\frac{dX_1}{dP} = FK_1 \frac{dK_1}{dP} + FL_1 \frac{dL_1}{dP} \tag{2.13}$$

$$\frac{dX_2}{dP} = FK_2 \frac{dK_1}{dP} + FL_2 \frac{dL_2}{dP} \tag{2.14}$$

As the price ratio changes, the producers will alter their input levels to satisfy the first order conditions of profit maximization. To show this, rewrite the production functions using the capital labor ratios.

$$X_1 = L_1 f_1(k_1) \tag{2.1'}$$

$$X_2 = PL_2 f_2(k_2) \tag{2.2'}$$

Using the profit equations and differentiating them with respect to the labor and capital inputs, yields the factor markets equilibria:

$$f_1 - k_1 f_1' = P\,(f_2 - k_2 f_2') = w \tag{2.3'}$$

$$f_1' = Pf_2' = r \tag{2.4'}$$

Recall the full employment conditions (2.7) and (2.8) to rewrite the capital equation:

$$K - k_1 L_1 - L_2(k_1 - k_2) = 0 \tag{2.8'}$$

as found in Yu and Ingene (1985).

Totally differentiate (2.3') and (2.4') and (2.8'):

$$-Pk_2 f_2''\, dk_2 + k_1 f_1'' dk_1 = -(f_2 - k_2 f_2')\, dP$$

$$Pf_2''\, dk_2 - f_1''\, dk_1 = -\, f_2'\, dP$$

$$(k_2 - k_1)\, dL_2 + L_1\, dk_2 + L_2\, dk_1 = dK - k_1 dL$$

Let $dK - k_1 dL = dR$ and find the impact on labor in Sector 2: L_2 and the capital labor ratios: k_1, k_2. [3]

As the relative price of X_2 increases, the output of that sector increases requiring more labor: $\frac{dL_2}{dP} > 0$. Both X_1 and X_2 become relatively more capital intensive: $\frac{dk_1}{dP} > 0$, $\frac{dk_2}{dP} > 0$.

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3 In matrix form: $$\begin{bmatrix} 0 & -Pk_2f_2'' & k_1f_1'' \\ 0 & Pf_2'' & -f_1'' \\ (k_2-k_1) & L_1 & L_2 \end{bmatrix} \begin{bmatrix} dL_2 \\ dk_2 \\ dk_1 \end{bmatrix} = \begin{bmatrix} -(f_2-k_2f_2')\,dP \\ -f_2'dP \\ dR \end{bmatrix}$$

$D = (k_2 - k_1)^2 \,[\, Pf_1''f_2''] > 0;$

$$\frac{dL_2}{dP} = \frac{\begin{vmatrix} -(f_2 - k_2f_2') & -Pk_2f_2'' & k_1f_1' \\ -f_2' & Pf_2' & f_1'' \\ 0 & L_1 & L_2 \end{vmatrix}}{D}$$

$$= \frac{-L_1 \,[\, f_2f_1'' - f_1''k_2f_2' + f_2'k_1f_1''] + L_2\,[-Pf_2f_2'']}{D}$$

Given $k_1 > k_2$ then both terms in numerator are positive. Since D is positive, then $\frac{dL_2}{dP} > 0$. When the relative price of X_2 increases, more labor is used in that sector.

$$\frac{dk_2}{dP} = \frac{\begin{vmatrix} 0 & -(f_2 - k_2f_2') & k_1f_1'' \\ 0 & -f_2' & -f_1'' \\ (k_2\text{-}k_1) & 0 & L_2 \end{vmatrix}}{D}$$

$$= (k_2\text{-}k_1)\,[f_1''f_2 + f_1''\,f_2'\,(k_1 - k_2)]/D > 0$$

$$\frac{dk_1}{dP} = \frac{\begin{vmatrix} 0 & -Pk_2f_2'' & -(f_2 - k_2f_2') \\ 0 & Pf_2'' & -f_2' \\ (k_2\text{-}k_1) & L_1 & 0 \end{vmatrix}}{D}$$

$$= (k_2\text{-}k_1)\,[Pf_2''f_2]/D > 0$$

This is because more capital is released from X_1 than labor whereas X_2 uses relatively more labor than capital. Since we know $\frac{dL_2}{dP} > 0$, then $\frac{dK_2}{dP}$ must also be positive bacause $\frac{dk_2}{dP} > 0$. Given full employment and fixed supplies of inputs, $\frac{dL_1}{dP} < 0$, $\frac{dK_1}{dP} < 0$.

Evaluating the production functions:

$$\frac{dX_1}{dP} = F_{K1} \frac{dK_1}{dP} + F_{L1} \frac{dL_1}{dP} < 0$$

$$\frac{dX_2}{dP} = F_{K2} \frac{dK_2}{dP} + F_{L2} \frac{dL_2}{dP} > 0$$

When the relative price of X_2 increases, more of X_2 is produced and less of X_1 is produced, so the output response is normal.

The Stolper Samuelson Theorem holds that an increase in the relative price of one good will cause an increase in the real return to the factor intensive in that good. Give $dP > 0$, the Theorem predicts $dw > 0$. Recall the factor price ratio:

$$W = \frac{w}{r} = \frac{fi - k_1 f_1'}{f_1'} \qquad (2.15)$$

It will be assumed that the return to capital (r) is constant.[4]

---

4 Once open to trade, r will be exogenously determined by the international capital market. While flexible, for purposes of sharpening the analysis, it will be assumed constant unless otherwise stated. Given a constant return to capital, the stock of capital is fixed, allowing the retention of the assumption of a fixed supply of capital in a static economy.

To determine the impact of a change in the price ratio on the factor return ratio (W) take the factor market equilibria (2.3'), (2.4') and the factor return ratio: (2.15) and totally differentiate:

$$-k_1f_1''\,dk_1 \quad + Pk_2f_2''\,dk_2 \quad -dw = -(f_2 - k_2f_2')dP$$

$$f_1''\,dk_1 \quad + Pf_2''\,dk_2 \quad -dr = -\,f_2'dP$$

$$\frac{f_1'(-k_1f_1'') - f_1''(f_1 - k_1f_1')}{(f_1')^2}\,dk_1 + \frac{f_2'(-k_2f_2'') - f_2''(f_2 - k_2f_2')}{(f_2')^2}\,dk_2 - dW = 0$$

The unknowns are dk_1, dk_2, and dW. [5]

---

5 $W = \frac{w}{r}$; $w = Wr$; $dw = Wdr + rdW$. $Wdr = 0$ by assumption as the economy begins to trade; therefore $dw = rdW$

Putting into matrix form:

$$\begin{bmatrix} -k_1f_1'' & PK_2f_2'' & -r \\ f_1'' & -Pf_2'' & 0 \\ \frac{f_1'(-k_1f_1'') - f_1''(f_1 - k_1f_1')}{(f_1')^2} & \frac{f_2'(-k_2f_2'') - f_2''(f_2 - k_2f_2')}{(f_2')^2} & -1 \end{bmatrix} \begin{bmatrix} dk_1 \\ dk_2 \\ dW \end{bmatrix} = \begin{bmatrix} -(f_2 - k_2f_2')dP \\ -f_2'dP \\ 0 \end{bmatrix}$$

$$D = (k_1 + k_2)(-Pf_1''\,f_2'') + \frac{rf_2f_1''f_2''}{(f_2')^2} + \frac{rPf_1f_1''f_2''}{(f_1')^2} > 0$$

$$\frac{dW}{dP} = \frac{\begin{vmatrix} -k_1f_1'' & -Pk_2f_2'' & -f_2+k_2f_2' \\ f_1'' & -Pf_2'' & -f'_2 \\ \frac{f_1'(-k_1f_1'') - f_1''(f_1-k_1f_1')}{(f_1')^2} & \frac{f_2'(-k_2f_2'')-f_2''(f_2-k_2f_2')}{(f_2')^2} & 0 \end{vmatrix}}{D}$$

$$= \frac{f_2^2f_1''f_2''}{(f_2)^2} - \frac{k_2f_2f_2'f_1''f_2''}{(f_2)^2} + \frac{Pf_2f_1f_1''f_2''}{(f_1)^2}$$

$$- \frac{Pk_2f_1f_2'f_1''f_2''}{(f_1)^2} + \frac{k_1f_2f_2'f_1''f_2''}{(f_2)^2} - \frac{Pk_2f_1f_2f_1''f_2''}{(f_1)^2} \,/\, D > 0$$

The impact on factor price ratio is as predicted by the Stopler Samuelson Theorem: $\frac{dW}{dP} > 0$.

The Impact on Welfare

As seen above, the opening to trade will cause an adjustment of the output mix, moving it along the product transformation curve to the new tangency. Recall $\frac{-dX_1}{dX_2} = P$

then $-dX_1 = PdX_2$; so, $dX_1 + PdX_2 = 0$

Turning to income (Y):

$$Y = Z_1 + PZ_2 = X_1 + PX_2 \tag{2.16}$$

and

$$\frac{dU}{U_1} = dY = dZ_1 + PdZ_2 + Z_2dP = dX_1 + PdX_2 + X_2dP \tag{2.17}$$

An increase in income will mean an increase in utility.

Since $dX_1 + PdX_2 = 0$, then $dY = X_2dP$.

Because the new terms of trade represent $dP > 0$, then $X_2dP > 0$.

The opening to trade causes an increase in income; therefore, welfare improves. More explicity: $\frac{dU}{U_1} = dX_1 + PdX_2 + (X_2 - Z_2)\, dP = dZ_1 + PdZ_2$.

It will be assumed that there is a balance of payments equilibrium:

$$IM_1 = PEX_2 \tag{2.18}$$

$$IM_1 = Z_1 - X_1 : \text{imports of } X_1 \tag{2.19}$$

$$EX_2 = X_2 - Z_2 : \text{exports of } X_2 \tag{2.20}$$

So $\frac{dU}{U_1} = dY = EX_2dP > 0$ because X_2 is exported and dP is positive

$$\frac{dY}{dP} = \frac{dX_1}{dP} + P\frac{dX_2}{dP} + EX_2 > 0 \tag{2.21}$$

To analyze the relationship between exports and imports in terms of welfare, the basic point to be remembered is that free trade income is a function of the terms of trade: $Y = Y(P)$. The demand for importables (Z_1) is determined by the terms of trade and real income; so, the balance of payments equilibrium can be rewritten: $IM_1(Y,P) = PEX_2$.

Therefore, rewriting (2.17), the change in income, becomes:

$$dY = dIM_1 + PdEX_2 + EX_2dP.$$

Differentiating with respect to the change in price:

$$\frac{dY}{dP} = \frac{\partial IM_1}{\partial Y}\frac{dY}{dP} + \frac{\partial IM_1}{\partial P}\frac{dP}{dP} + \frac{PdEX_2}{dP} + EX_2\frac{dP}{dP}$$

$1 > \frac{\partial IM_1}{\partial Y} > 0$. X_1 is a normal good but the elasticity of demand for X_1 is inelastic with respect to income: Let $IM_1y = \frac{\partial IM_1}{\partial Y}$

$\frac{\partial IM_1}{\partial P} > 0$. As the price ratio increases, imports will increase: Let $IM_1p = \frac{\partial IM_1}{\partial P}$

$\frac{dEX_2}{dP} > 0$ As the price ratio increases, exports increase.

$$\frac{dY}{dP} = IM_1y\frac{dY}{dP} + IM_1p + \frac{PdEX_2}{dP} + EX_2$$

$$\frac{dY}{dP} - IM_1y\frac{dY}{dP} = IM_1p + \frac{PdEX_2}{dP} + EX_2$$

$$\frac{dY}{dP}(1-IM_1y) = IM_1p + \frac{PdEX_2}{dP} + EX_2$$

$$\frac{dY}{dP} = \frac{1}{1-IM_1y}\left[IM_1p + \frac{PdEX_2}{dP} + EX_2\right] > 0$$

IM_1y is the marginal propensity to import; so, $\frac{1}{1-IM_1y}$ is a trade multiplier and greater than 1.

On the intuitive level, it is obvious that the country (assuming sovereignty) would not begin to trade unless welfare improved. By requiring it to adjust its output mix more in line with comparative advantage, the new price ratio will cause an increase in income.

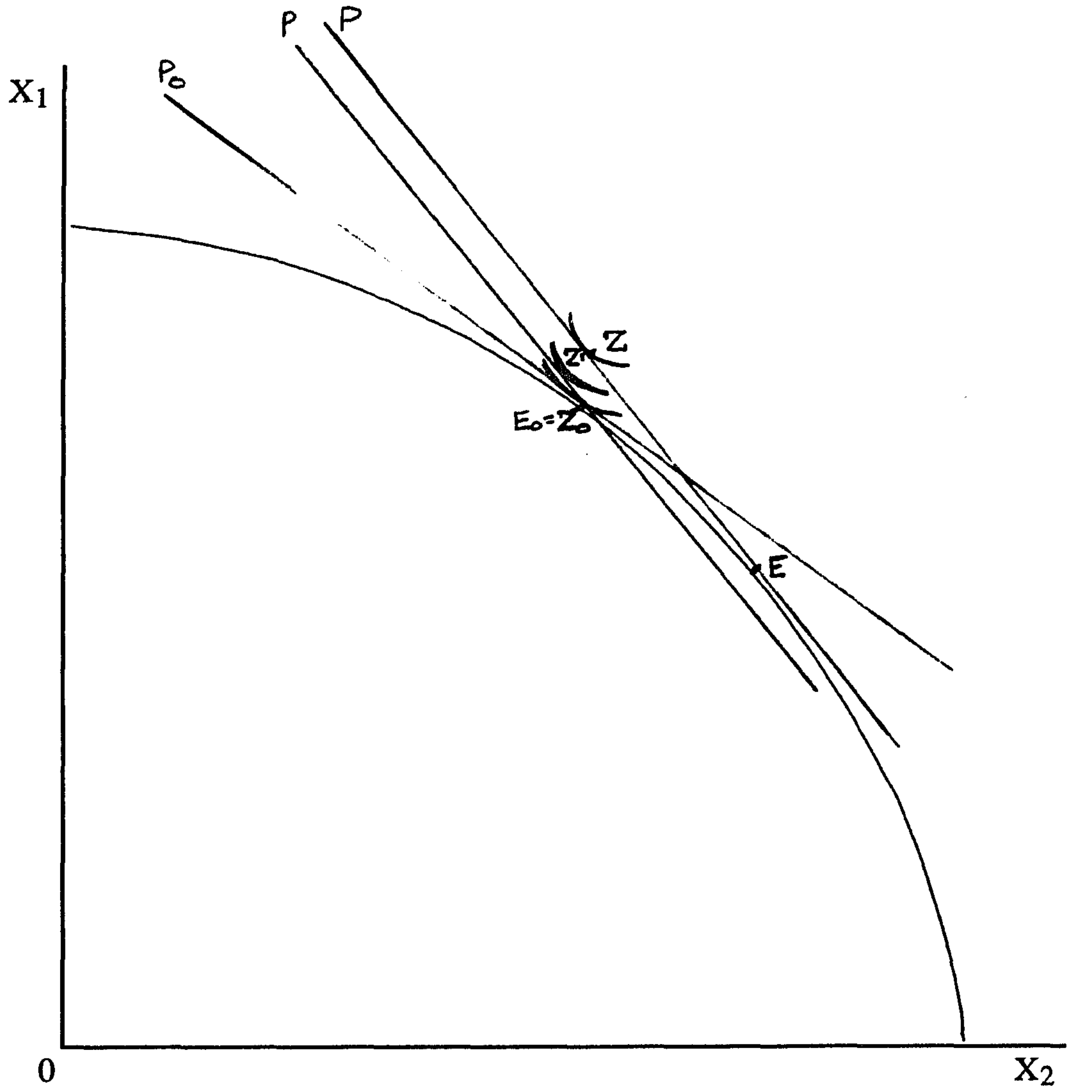

FIGURE 2.1: Opening to Trade with Mobile Factors

Before trade: Po, Zo = Eo

Opening to trade: Po to P, international terms of trade

Welfare gains from trade: Consumption gain: Z_0 to Z'

Production gain: Z' to Z

Figure 2.1 shows an increase in the consumption of both goods:

$dZ_1 > 0$, $dZ_2 > 0$; After trade opens more importables will be consumed but the consumption of the export may increase, decrease or not change. Even if it decreases, welfare will improve.

Next we will drop the assumption of perfectly mobile labor.

Part 2 Short Run Immobility of Labor; Long Run Partial Mobility in a Certain Environment

As explained in the introduction, labor may be slow to adjust to the allocation indicated by the new price ratio because workers are uncertain if it is a permanent change and because of the time and effort required to relocate and/or retrain for a new job. That makes labor immobile in the short run and causes the wage differential to develop: $\frac{d\mu}{dP} > 0$. Once the lower of the two wages drops below the reservation wage for some workers, some labor begins to move, which occurs in the long run. Then the development of the differential slows down and stabilizes when the equilibrium output mix is attained.

Using Casas's approach, the wage differential is defined as:

$$\mu = \frac{w_2}{w_1}$$

The labor ratio:

$$\frac{L_1}{L_2} = \Gamma\left(\frac{1}{b\mu}\right)^{\varepsilon}$$

Γ is positive and constant

b is positive and constant. It represents the critical wage differential. When μ moves beyond b, then labor will begin to improve.

ε is the elasticity of labor mobility. In the short run labor is immobile; so, $\varepsilon = 0$. Therefore in the short run, $\frac{L_1}{L_2} = \Gamma$. In the long run labor becomes somewhat mobile; the elasticity of labor mobility is positive: $0 < \varepsilon < \infty$, allowing further adjustment of the output mix.

Hazari [6] showed how the development of a differential will cause the contraction of the product transformation curve. As figures 2.2 and 2.3 show, it is no longer certain that the increase in the price ratio will improve welfare.

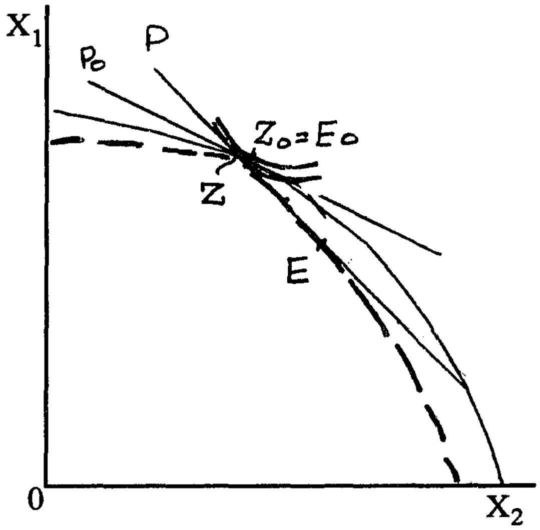

Figure 2.2: Decrease in Welfare

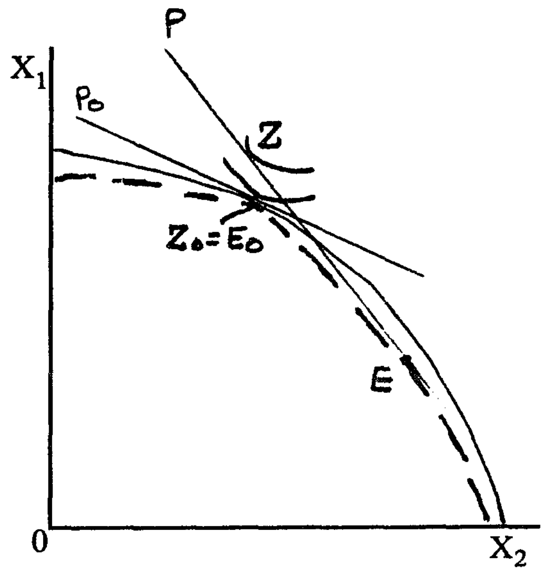

Figure 2.3: Increase in Welfare

In Figure 2.2 the opening to trade causes a decline in welfare: $Z < Z_0$. Figure 2.3 shows an improvement: $Z > Z_0$. Hazari points out that in the presence of the differential, the contracted product transformation curve may have any shape. Given a change in the price ratio, the output relationship can be normal even when the curve is convex to the origin. Throughout this dissertation we will assume the output response is normal.

The differential can be incorporated by defining:

$$\beta = \frac{FL_1 dL_1 + FK_1 dK_1}{-\mu FL_1 dL_1 - FK_1 dK_1} \tag{2.22}$$

---

[6] B. Hazari, *International Trade: Theoretical Issues* (New York: New York University, 1986) p. 116-117.

Then $\frac{dX_1}{dX_2} = P\beta$.

When labor is immobile, $dL_1 = 0$ so $\beta = 1$ and the price line is tangent to the new, contracted product transformation curve.[7]

To see the impact of the development of the wage differential in the short run, the factor markets equilibria (2.3), (2.4) will be adapted as suggested in the article by Yu (1981a).

$$w_2 = PFL_2\,(K_2,L_2)$$
$$w_1 = \mu FL_1(\bar{K}-K_2,L_1)$$
$$r = FK_1(K_1,L_1) = PFK_2(\bar{K}-K_1,L_2)$$

Totally differentiating

$$dw_2 = PFLK_2\, dK_2 + PFLL_2\, dL_2 = 0$$
$$dw_1 = \mu FLK_1\, dK_1 - \mu\, FLK_2\, dK_2 = -FL_1 d\mu$$
$$dr = FKK_1\, dK_1 - PFKL_2\, dK_2 - (PFKL_2 + FKL_1)dL_2 = 0$$

with respect to the differential to find $\frac{dK_1}{d\mu}, \frac{dK_2}{d\mu}, \frac{dL_2}{d\mu}$ [8]

---

[7] Yu and Parai also derived a formula for β in their equation (15) which is based on Casas's equations (20) and (21).

$$\beta = \left[\frac{\sigma_1\sigma_2 rK_1K_2L + \varepsilon\{\sigma_1L_1K_1P_2X_2 + \sigma_1L_1K_1L_2w_1[1-\mu] + \sigma_2K_2L_2P_1X_1\}}{\sigma_1\sigma_2 rK_1K_2L + \varepsilon\{\sigma_1L_1K_1P_2X_2 + \sigma_2L_2K_2P_1X_1 - \sigma_2L_2K_2L_1[1-\mu]w_1\}}\right] \quad (2.23)$$

When labor is immobile then $\varepsilon = 0$ so:

$$-\frac{dX_1}{dX_2} = P\left[\frac{\sigma_1\sigma_2 rK_1K_2L}{\sigma_1\sigma_2 rK_1K_2L}\right] = P.$$

This points up the crucial role of the elasticities of factor substitutions in each sector: σ_1,σ_2; in the adjustment of the output mix.

[8] In matrix form:

One finds that as the wage differential increases, the amount of capital used in X_1 declines: $\frac{dK_1}{d\mu} < 0$. This is because as the relative price of X_2 increases the economy attempts to adjust to a new output mix involving an increase in the output of X_2 and a decline in the output of X_1. The amount of capital used in X_2 increases: $\frac{dK_2}{d\mu} > 0$.

The differential develops because labor is immobile, therefore:

$$\frac{dL_2}{d\mu} = \frac{dL_1}{d\mu} = 0,$$

Plugging these findings into the production functions:

$$\frac{dX_1}{d\mu} = FL_1 \frac{dL_1}{d\mu} + FK_1 \frac{dK_1}{d\mu} < 0$$

$$\begin{bmatrix} 0 & PFLK_2 & PFLL_2 \\ \mu FLK_1 & -\mu FLK_2 & 0 \\ FKK_1 & -PFKL_2 & -(PFKL_2 + FKL_1) \end{bmatrix} \begin{bmatrix} dK_1 \\ dK_2 \\ dL_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -FL_1 d\mu \\ 0 \end{bmatrix}$$

$$D = -PFLK_2\,(-\mu PFLK_1 FKL_2 - \mu\, F^2 LK_1)$$
$$+ PFLL_2\,(-\mu PFLK_1 FKL_2) + PFLL_2\,(\mu FKK_1 FLK_1) > 0$$

$$\frac{dK_1}{d\mu} = \frac{\begin{vmatrix} 0 & PFLK_2 & PFLL_2 \\ -FL_1 & -\mu FLK_2 & 0 \\ 0 & -PFKL_2 & -(PFKL_2 + FKL_1) \end{vmatrix}}{D}$$
$$= FL_1 - (P^2 FKL_2^2 - PFKL_1 FKL_2) + FL_1\,(P^2 FLL_2 FKL_2) / D < 0$$

$$\frac{dK_2}{d\mu} = \frac{\begin{vmatrix} 0 & 0 & PFLL_2 \\ \mu FLK_1 & -FL_1 & 0 \\ FKK_1 & 0 & -(PFKL_2 + FKL_1) \end{vmatrix}}{D} = \frac{PFLL_2(FKK_1 FL_1)}{D} > 0$$

$$\frac{dX_2}{d\mu} = FL_2 \frac{dL_2}{d\mu} + FK_2 \frac{dK_2}{d\mu} > 0$$

The output mix adjusts to the increase in the price ratio in the same direction as it did when labor was mobile, but the adjustment is smaller.

To determine whether a given change in the price ratio will increase welfare in the short run, it is necessary to see how much the output mix will adjust without shifting labor. The greater the adjustment, the more likely there will be an increase in welfare. The elasticities of factor substitution (σ_1, σ_2) within each sector are crucial in determining this.

$$\sigma_2 = \frac{\frac{d(K_2/X_2)}{K_2/X_2} - \frac{d(L_2/X_2)}{L_2/X_2}}{dw_2/w - dr/r}$$

$dL_2 = 0$ by assumption

$dr = 0$ by assumption

Therefore

$$\sigma_2 = \frac{d(K_2/X_2) \bullet (X_2/K_2)}{dw_2/w} = \frac{dK_2/K_2 \bullet X_2/dX_2}{dw_2/w}$$

which becomes:

$$\sigma_2 = \left[\frac{dK_2}{K_2} \bullet \frac{X_2}{dX_2}\right] \frac{w}{dw_2}$$

$$\sigma_2 \left[\frac{dw_2}{w} \bullet \frac{dX_2}{X_2}\right] = \frac{dK_2}{K_2}$$

So $$\frac{dX_2}{X_2} = \frac{(dK_2/K_2 \bullet w/dw_2)}{\sigma_2}$$

and $$\frac{dX_1}{X_1} = \frac{(dK_1/K_1 \bullet w/dw_1)}{\sigma_1}$$

The adjustment of the outputs depend upon the elasticities of factor substitution (σ_1,σ_2) the developing wage differential ($d\mu = dw_2/dw_1$) and the reallocation of capital (dK_1/K_1, dK_2/K_2).

Recall: $\frac{dU}{U} = U_1 \left(\frac{dX_1}{X_1} + P\frac{dX_2}{X_2}\right)$

then $\frac{dU}{U} = U_1 \left[\frac{dK_1/K_1 \bullet w/dw_1}{\sigma_1} + P \left(\frac{dK_2/K_2 \bullet w/dw_2}{\sigma_2}\right)\right]$

Now let us determine under what circumstances welfare will improve: $\frac{dU}{U} > 0$.

Examining the terms:

$dw_2 > 0$. As sector 2 attempts to draw labor from sector 1, wages in sector 2 increase.

$K_2/dK_2 > 0$. Sector 2 draws capital from sector 1. Therefore the sector 2 term is positive.

$$\frac{dK_2/K_2 \bullet w/dw_2}{\sigma_2} > 0$$

With a fixed supply of capital the increase in capital allocated to sector 2 is matched by the decline of capital allocated to sector 1. Therefore: $dK_1/K_1 < 0$. The sign of dw_1 is not clear. It can be positive, negative or zero ($dw_1 \lesseqgtr 0$). If it is positive it will be less than the increase in wages in sector 2 since sector 1 is contracting and the differential is increasing:

$$dw_2 > dw_1 > 0$$

If wages in sector 1 do not change then $dw_1 = 0$. If wages in sector 1 decline then $dw_1 < 0$.

In Theorem 1B*iii* Casas[9] predicted that, when the elasticity of labor mobility is smaller than the elasticities of factor substitution, and the factor intensity rankings are not reversed then if the relative price of X_2 increases, the wage in sector 1 will decline.

The change in welfare will depend upon the relative sizes of the elasticities of factor substitution and the sign of dw_1/w. Let us examine this more closely.

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[9] "Imperfect Factor Mobility: A Generalization and Synthesis of Two Sector Models of International Trade" *Canadian Journal of Economics* (1984), p. 753.

If dw_1/w is positive then the sign of the welfare equation (2.24) is ambiguous. Since there is a decline in capital allocation, the sector 1 term is negative:

$$\frac{dK_1/K_1 \bullet w/dw_1}{\sigma_1} < 0$$

The sector 2 term is positive. Any difference in the sizes of the elasticities of substitution is important in this situation.

If dw_1/w is zero then the sector 1 term is also zero; so, welfare depends solely on the sector 2 term therefore there is an unambiguous increase in utility.

If dw_1/w is negative then the sector 1 term is positive and contributes to the welfare improvement from sector 2.

This allows the following conclusion:

Proposition 2.1: Welfare will improve if wages in the contracting sector decline or remain stable. Welfare will improve if the wages in the contracting sector increase and the adjustment in sector 1 $\left(\frac{dX_1}{X_1}\right)$ is smaller in absolute terms than the adjustment in sector 2 $\left(P\frac{dX_2}{X_2}\right)$.

Next let us turn to the long run because the final impact of the opening to trade can be seen then (* designates the long run). Once the wage differential has gone beyond the critical wage differential, then labor begins to move and in this model it will not occur until the second stage: the long run. The elasticity of labor mobility is now positive ($\varepsilon > 0$) and the development of the wage differential will slow down. The movement of labor ceases when equilibrium is established in terms of labor allocation and then the differential stabilizes. The size of the labor transfer will be determined by the adjustment in the wage differential ratio.

To determine the final impact on output totally differentiate the factor markets equilibria (2.3), (2.4) adapted to a wage differential which is now stable,

$$w_1{}^* = \mu^* FL_1(K_1L_1)$$

$$w_2^* = PFL_2(K-K_1,L_2)$$

$$r^* = FK_1(K_1,L_1) = PFK_2(K-K_1,L_2)$$

with respect to the change in price and find:

$$\frac{dK_1}{dP}, \frac{dL_1}{dP}, \frac{dL_2}{dP}$$ [10]

---

10 $$\begin{aligned} &\mu^*FLK_1\, dK_1 + \mu^*FLL_1\, dL_1 = 0 \\ &-P\, FLK_2\, dK_1 + PFLL_2\, dL_2 = -FL_2 dP \\ &(PFKK_2 + FKK_1)dK_1 + FKL_1\, dL_1 - PFKL_2\, dL_2 = FK_2\, dP \end{aligned}$$

In matrix form:

$$\begin{bmatrix} \mu^*FLK_1 & +\mu^*FLL_1 & 0 \\ -P\, FLK_1 & 0 & PFLL_2 \\ (PFKK_2 + FKK_1) & FKL_1 & -PFKL_2 \end{bmatrix} \begin{bmatrix} dK_1 \\ dL_1 \\ dL_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -FL_2 dP \\ FK_2\, dP \end{bmatrix}$$

$$D = \mu^*FLK_1\,(-P\, FKL_1\, FLL_2) - \mu^*FLL_1\,(P^2FK^2L_2 - P^2FLL_2FKK_2 - PFLL_2FKK_1) < 0$$

$$\frac{dK_1}{dP} = \frac{\begin{vmatrix} 0 & \mu^*FLL_1 & 0 \\ -FL_2 & 0 & PFLL_2 \\ FK_2 & FKL_1 & PFKL_2 \end{vmatrix}}{D} = \frac{\mu^*FLL_1(PFL_2FKL_2 - PFK_2FLL_2)}{D} < 0$$

$$\frac{dL_1}{dP} = \frac{\begin{vmatrix} \mu^*FLK_1 & 0 & 0 \\ -PFLK_2 & -FL_2 & PFLL_2 \\ (PFKK_2 + FKK_1) & FK_2 & -PFKL_2 \end{vmatrix}}{D}$$

$$= \frac{\mu^*FLK_1(PFL_2FKL_2 - PFK_2FLL_2)}{D} < 0$$

$\frac{dK_1}{dP} < 0$ This is the same result as in the short run. As the wage differential stabilizes the flow of capital from X_1 to X_2 does not stop or reverse. It still responds in the predicted way to the increase in the relative price of X_2.

$\frac{dL_1}{dP} < 0$ Labor is now moving in the expected way. Less labor is used in X_1.

Because of the fixed supply of labor, it is obvious that labor is moving into sector 2 in response to the higher wages in that sector.

$$-dL_1 = dL_2 \text{ so } \frac{dL_2}{dP} > 0.$$

The impact on output ratio in the long run, with a positive elasticity of labor mobility is:

$$\frac{dX_1}{dP} = FL_1 \frac{dL_1}{dP} + FK_1 \frac{dK_1}{dP} < 0$$

$$\frac{dX_2}{dP} = FL_2 \frac{dL_2}{dP} + FK_2 \frac{dK_2}{dP} > 0$$

Because of the fixed supply of capital: $-dK_1 = dK_2$.

The results are the same quality as in the short run, but the magnitude is greater since labor is also moving from sector 1 to sector 2.

$$\frac{dL_2}{dP} = \frac{\begin{vmatrix} \mu^*FLK_1 & \mu^*FLL_1 & 0 \\ -PFLK_2 & -0 & FL_2 \\ (PFKK_2 + FKK_1) & FKL_1 & FK_2 \end{vmatrix}}{D}$$

$$= \frac{\mu^*FLK_1(FL_2FKL_1) - \mu^*FLL_1(-PFK_2FLK_2 + PFL_2FKK_2 + FL_2FKK_1)}{D} > 0$$

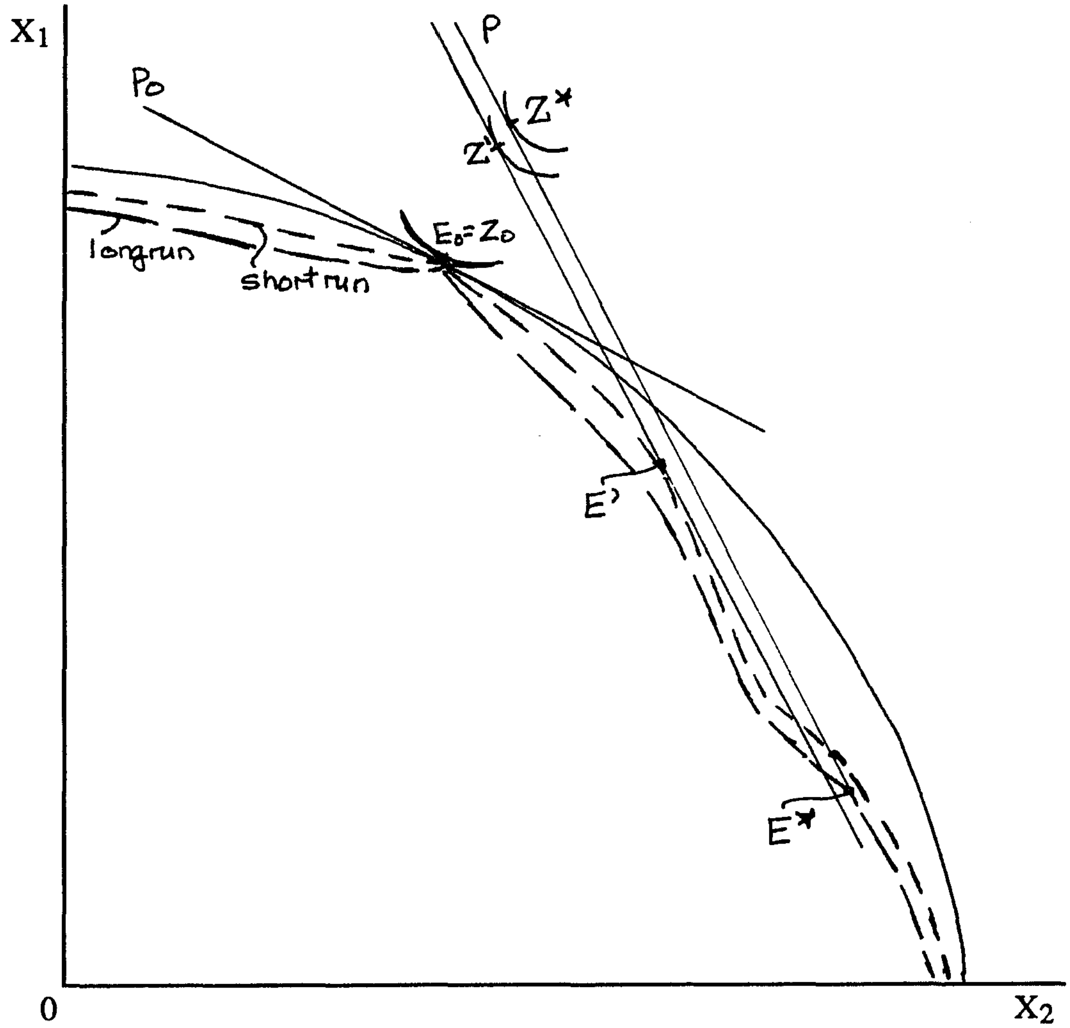

Figure 2.4

Short Run and Long Run Impacts of the Opening to Trade

Before trade: $E_0 = Z_0$; Open to trade: P_0 to P

In the short run most of the contraction of the product transformation curve takes place: E_0 to E' is the output adjustment; Z_0 to Z' shows the change in welfare. In the long run the curve stabilizes and output moves from E' to E* because labor is partially mobile. Welfare improves over the short run.

Since $\beta < 1$, at E* the marginal rate of transformation is less than the price ratio. This can happen if there is an inflection point on the curve. For that to be true the elasticity of factor substitution in sector 2 must be greater than in sector 1: $\sigma_2 > \sigma_1$. (See Appendix 2 for proof)

Figure 2.4 shows the short run and long run impacts of the opening to trade. In the long run, the price line is not tangent to the transformation curve. This is because labor is now somewhat mobile so the elasticity of labor mobility is positive. Then $dL_1 < 0$, so β no longer equals 1.

$$\beta = \frac{FL_1\, dL_1 + FK_1\, dK_1}{\mu FL_1 dL_1 + FK_1\, dK_1}$$

Therefore, $\frac{dX_1}{dX_2} = P\beta$. As long as the differential is greater than one ($\mu > 1$) then β is less than one ($\beta < 1$).

This is confirmed by referring back to Yu and Parai's formula for β (2.23)

$$\frac{-dX_1}{dX_2} = P\left[\frac{\sigma_1\sigma_2 rK_1K_2L+\varepsilon\{\sigma_1L_1K_1P_2X_2+\sigma_2K_2L_2P_1X_1+\sigma_1L_1K_1L_2w_1[1-\mu]\}}{\sigma_1\sigma_2 rK_1K_2L+\varepsilon\{\sigma_1L_1K_1P_2X_2+\sigma_2K_2L_2P_1X_1-\sigma_2L_1K_2L_2w_1[1-\mu]\}}\right]$$

$$=P\beta$$

As can be seen here, for a given value of μ, the greater the value of the elasticity of labor mobility, the more the marginal rate of transformation adjusts.

If welfare improves over the level in autarky (Z_0), the favorable impact of opening to trade is greater than the unfavorable impact of the developing and, in the long run, stable wage differential.

Next we will drop the assumption that the future terms of trade are known but reintroduce complete mobility of both factors..

Part 3 Mobile Factors in an Uncertain Environment

Uncertainty about the future international price ratio is shown by replacing P with $\hat{P}$, a probability distribution function of the future price ratio:

$$\hat{P} = \gamma P + \rho \tag{2.25}$$

γ and ρ are shift parameters. γ is multipicative.

Let us assume that when the economy opens to trade it goes from a certain environment to uncertainty. This will have an impact on national income. Producers are assumed to be risk averse with decreasing absolute risk aversion with respect to profits. Isoquants are still convex toward the origin implying that producers minimize the unit cost of the product:

$$\frac{FL_1}{FK_1} = \frac{-dK_1}{dL_1}, \; \frac{FL_2}{FK_2} = \frac{-dK_2}{dL_2}$$

but under uncertainty the value of the marginal product of each factor will be greater than its price. Output is at a level where the expected price: E(P), is greater than marginal cost but the combination of factors is optimum. The expected price ratio, E(P), is the mean of the price distribution. Given FLL_i, FKK_i, < 0, a risk averse producer employs smaller amounts of inputs than would a risk neutral producer: so, expected output is less than it would be in a certain environment because input output decisions are made prior to the resolution of uncertainty. Entering such an uncertain environment has a negative impact on income as seen by the contraction of the product transformation curve in Figure 2.5.

However, opening to trade also means by assumption that the country will probably face a price ratio different from that in autarky: $d\rho > 0$, an expected increase, will be assumed here. The expected relative price of X_2 will be greater than P_2 has been in autarky.

To demonstrate the process of adjusting the output mix, the procedure will be to maximize the expected utility of profits yielding factor markets equilibria which will be used to find the impact on output of the expected change in prices. See Appendix 3. The profit equations become:

$$\Pi_1 = \hat{P}_1 X_1 - wL_1 - rK_1$$

$$\Pi_2 = \hat{P}_2 X_2 - wL_2 - rK_2$$

Maximizing the expected utility of profits, yields the factor market equilibria:

$$w = \frac{E[U_1\ (\Pi_1)]\ \hat{P}_1\ FL_1}{E[U_1 \Pi_1]} \tag{2.26}$$

$$r = \frac{E[U_1\ (\Pi_1)]\ \hat{P}_2\ FL_1}{E[U_2 \Pi_2]} \tag{2.27}$$

Factor prices are certain.

Differentiating the production functions using Batra's approach (1975) yields the marginal rate of transformation:

$$\frac{dX_1}{dX_2} = \frac{FK_1 dL_1 + FK_1 dK_1}{FK_2 dL_2 + FK_2 dK_2}$$

becomes:

$$\frac{dX_1}{dX_2} = \frac{-E[U_2 P_2]\ E[U_1]}{E[U_1 P_1]\ E[U_2]}$$

$$= \frac{-E[U_1]\{E[U_2]\ E(P_2) + \xi_2\}}{E[U_2]\{E[U_1]\ E(P_1) + \xi_1\}}$$

ξ_2: covariance between U_2 and P_2

ξ_1: covariance between $U_1$and P_1

Therefore, the marginal rate of transformation does not equal the expected price ratio unless:

$$\frac{\xi_1}{E(P_1)\,E\,[U_1]} = \frac{\xi_2}{E(P_2)\,E\,[U_2]}$$

Then the marginal rate of transformation equals the expected price ratio: $\frac{-dX_1}{dX_2} = E(P)$,

when the marginal rate of risk premium is the same in both sectors. The risk premium is the difference between the expected return from the risky prospect and its certain equivalent. Risk premia are positive for risk averters. Given that producers have the same attitude toward risk in both sectors, then the risk premium of producers in each sector should be the same at the margin. Otherwise production would shift from the riskier to the less risky sector. Since producers are risk averse,

$$\frac{E[Ui\Pi i]}{E(Ui)} > 0; \quad Uii(\Pi) < 0$$

with decreasing absolute risk aversion.

Appendix 3 shows that uncertainty does not bring about counter intuitive output adjustments when there is a change in the expected price ratio. With an expected increase in the price ratio, labor and capital move from X_1 to X_2 causing the capital labor ratios for both goods to increase. This happens because X_1 is capital intensive and X_2 is labor intensive.

$$\frac{dX_1}{dE(P)} < 0; \frac{dX_2}{dE(P)} > 0\,.$$

The impact of uncertainty on production and consumption can be seen on Figure 2.5. Using a strong rational expectationist approach, consumers will, on average, predict the correct future terms of trade E (P) and thereby maximize their utility in an uncertain environment.

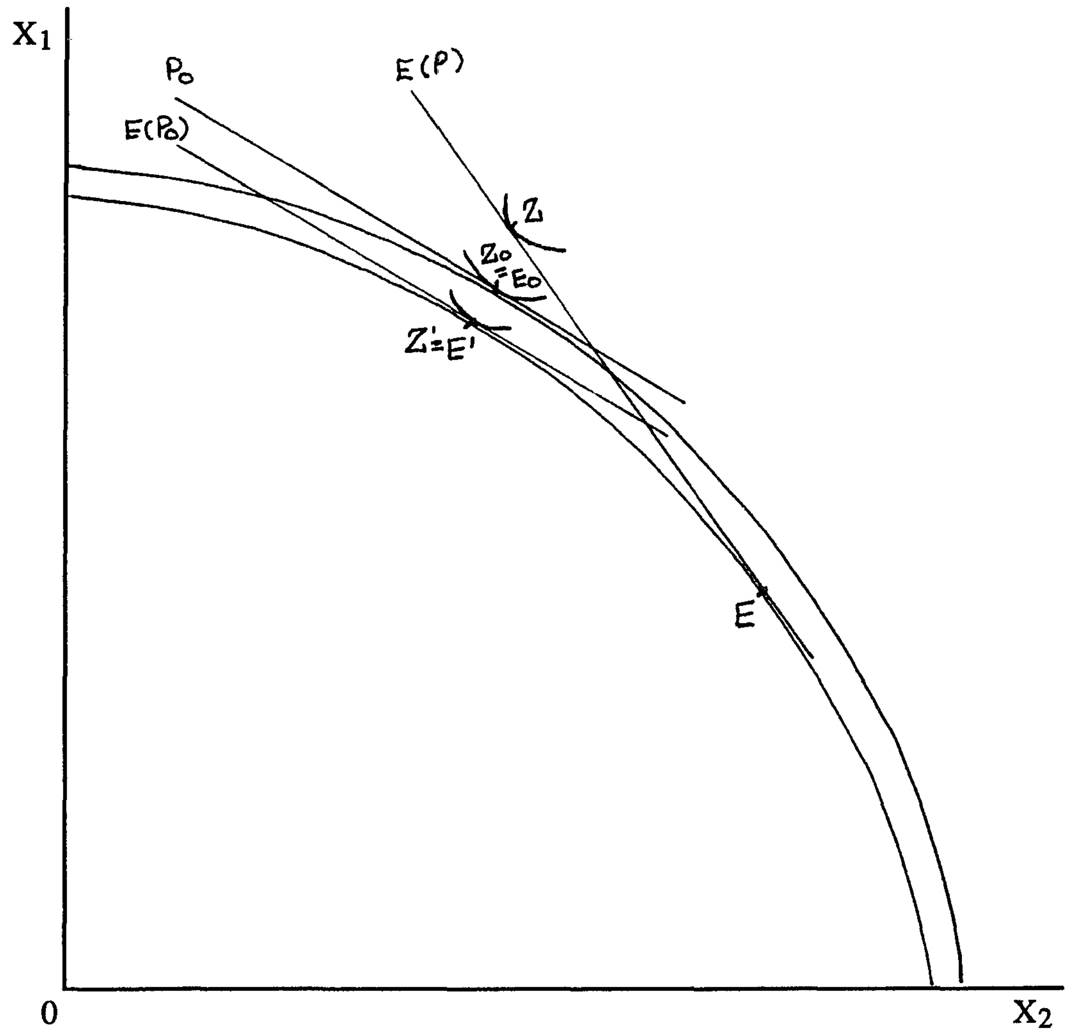

Figure 2.5: Entering an Uncertain Environment

Opening to trade, the nation faces an uncertain price ratio: P_0 to $\hat{P} = \gamma P_0$. There is a decline in output as producers require risk premia.

With the opening to trade, there is also an expected change in price over that in autarky: $dE(P) > 0$ because $d\rho > 0$. This causes the output mix to adjust along the shrunken product transformation curve: E' to E.

Figure 2.5 shows an increase in welfare in spite of operating in an uncertain environment. The benefits of an expected improvement in the price ratio outweigh the loss of production due to the introduction of uncertainty. The change in welfare would appear to

depend upon how large the expected improvement is in the price ratio (positive impact) and how risk averse producers are and, therefore, how large a risk premium will be demanded (negative).

To complete the basic model all that remains is to incorporate uncertainty into the wage differential calculations.

Part 4 Short Run Immobility of Labor; Long Run Partial Mobility in an Uncertain Environment

To determine the impact on output of the opening to trade under these circumstances, the expected utility of profits with a wage differential is maximized yielding the following factor markets equilibria:

$$w_2 = \frac{E\,[U_2(\Pi_2)]\hat{P}_2 FL_2}{E\,[U_2]}; \quad w_2 = w_2(K_2, L_2) \tag{2.28}$$

$$\mu w_1 = \frac{\mu E\,[U_1(\Pi_1)]\hat{P}_2 FL_1}{E\,[U_1]}; \quad \mu w_1 = \mu w_1\,(\bar{K} - K_2, L_1) \tag{2.29}$$

$$r = \frac{E\,[U_1(\Pi_1)]\hat{P}_1 FK_1}{E\,[U_1]} = \frac{E\,[U_2(\Pi_2)]\hat{P}_2 FK_2}{E\,[U_2]} \tag{2.30}$$

Differentiating with respect to the changing wage differential,[11] find $\frac{dK_1}{d\mu}, \frac{dK_2}{d\mu}, \frac{dL_2}{d\mu}$ for the short run.

11

$$dw_2 = E\left[\frac{U_2 \Pi_2}{U_2}\right] \hat{P}_2 FLK_2\, dK_2 + E\left[\frac{U_2 \Pi_2}{U_2}\right] \hat{P}_2 FLL_2\, dL_2 = 0$$

$$dw_1 = \mu E\left[\frac{U_1\Pi_1}{U_1}\right]\hat{P}_1FLK_1\ dK_1 - \mu E\left[\frac{U_2\Pi_2}{U_2}\right]\hat{P}_1FLK_2\ dK_2 = -E\left[\frac{U_1\Pi_1}{U_1}\right]\hat{P}_1FL_1\ d\mu$$

$$\begin{aligned} dr = {} & E\left[\frac{U_1\Pi_1}{U_1}\right]\hat{P}_1FKK_1\ dK_1 - E\left[\frac{U_2\Pi_2}{U_2}\right]\hat{P}_2FKL_2\ dK_2 \\ & - \left(E\left[\frac{U_2\Pi_2}{U_2}\right]\hat{P}_2FLK_2 + E\left[\frac{U_1\Pi_1}{U_1}\right]\hat{P}_1FKL_1\right)dL_2 = 0 \end{aligned}$$

$$\begin{bmatrix} 0 & E[\bullet]\hat{P}_2FLK_2 & E[\bullet]\hat{P}_2FLL_2 \\ \mu E[\bullet]\hat{P}_1FLK_1 & -\mu E[\bullet]\hat{P}_2FLK_2 & 0 \\ E[\bullet]\hat{P}_1FKK_1 & -E[\bullet]\hat{P}_2FKL_2 & [E[\bullet]\hat{P}_2FKL_2 + E[\bullet]\hat{P}_1FKL_1] \end{bmatrix} \begin{bmatrix} dK_1 \\ dK_2 \\ dL_2 \end{bmatrix} \begin{bmatrix} 0 \\ -E[\bullet]\hat{P}_1FL_1 d\mu \\ 0 \end{bmatrix}$$

$$\begin{aligned} D \quad & - E[\bullet]\hat{P}_2FLK_2\ (-\mu E[\bullet]\hat{P}_1FLK_1E[\bullet]\hat{P}_2FKL_2 - \mu\{E[\bullet]\hat{P}_1FLK_1\}^2) \\ & + E[\bullet]\hat{P}_2FLL_2\ (-\mu E[\bullet]\hat{P}_1FLK_1E[\bullet]\hat{P}_2FKL_2 + \mu E[\bullet]\hat{P}_2FLK_1E[\bullet]\hat{P}_1FKK_1) > 0 \end{aligned}$$

$$\begin{aligned} \frac{dK_1}{d\mu} & = \frac{\begin{vmatrix} 0 & E[\bullet]\hat{P}_2FLK_2 & E[\bullet]\hat{P}_2FLL_2 \\ -E[\bullet]\hat{P}_1FL_1 & -\mu E[\bullet]\hat{P}_2FLK_1 & 0 \\ 0 & -E[\bullet]\hat{P}_2FKL_2 & -(E[\bullet]\hat{P}_2FKL_2 + E[\bullet]\hat{P}_1FKL_1) \end{vmatrix}}{D} \\ & = E[\bullet]\hat{P}_1FL_1[-(E[\bullet]\hat{P}_2FLK_2)^2 - E[\bullet]\hat{P}_2FLK_2E[\bullet]\hat{P}_1FKL_1 \\ & \quad + E[\bullet]\hat{P}_2FLL_2E[\bullet]\hat{P}_2FKL_2]\ /D < 0 \end{aligned}$$

The development of the differential due to the expected increase in the terms of trade does not result in a counter intuitive flow of capital. The impact on the output mix:

$$\frac{dX_1}{d\mu} = FK_1\frac{dK_1}{d\mu} + FL_1\frac{dL_1}{d\mu} < 0$$

$$\frac{dX_2}{d\mu} = FK_2\frac{dK_2}{d\mu} + FL_2\frac{dL_2}{d\mu} > 0$$

In the short run there are two forces contracting the product transformation curve: the development of the wage differential and the risk premia. The issue is under what circumstances the benefits of opening to trade outweigh these two welfare reducing factors.

$$\frac{dK_2}{d\mu} = \frac{\begin{vmatrix} 0 & 0 & E[\bullet]\hat{P}_2FLL_2 \\ \mu E[\bullet]\hat{P}_1FLK_1 & -E[\bullet]\hat{P}_1FL_1 & 0 \\ -E[\bullet]\hat{P}_1FKK_1 & 0 & -(E[\bullet]\hat{P}_2FKL_2+E[\bullet]\hat{P}_1FKL_1) \end{vmatrix}}{D}$$

$$= E[\bullet]\hat{P}_2FLL_2\{E[\ ]\hat{P}_1(FL_1FKK_1)\} / D > 0$$

$$\frac{dL_2}{d\mu} = 0 \text{ by assumption.}$$

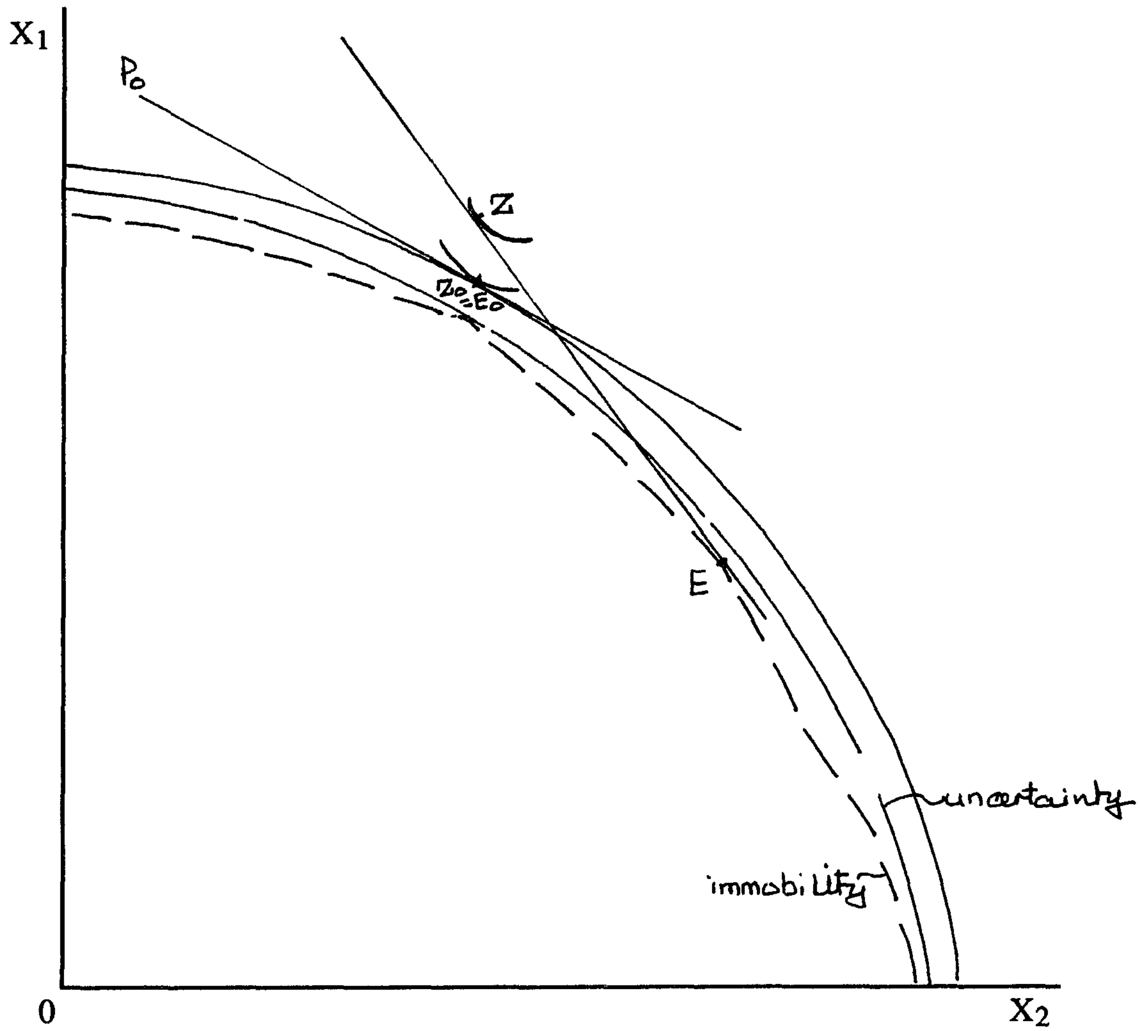

Figure 2.6: Uncertainty and Labor Immobility

At E, $\frac{dX_1}{dX_2} = E(P)$. $E(\beta) = 1$ because the elasticity of labor mobility is zero. Figure 2.6 shows that the impact of uncertainty and the developing wage differential overcomes the opening to trade which is welfare improving. It appears that the greater the adjustment of the output mix and the smaller the risk premia, the more likely there will be an increase in welfare.

As seen in Part 2, the adjustment of the output mix depends upon the elasticity of factor substitution in each sector and the size of the wage differential. The results in Part 2 apply here but are subject to modification by the size of the risk premia.

The marginal risk premia are the same in both sectors since this is the equilibrium situation:

$$\frac{E[U_1\Pi_1]}{E[U_1]} = \frac{E[U_2\Pi_2]}{E[U_2]} > 0$$

As seen in Part 3, then, the marginal rate of transformation equals the expected price ratio: $\frac{-dX_1}{dX_2} = E(P)$.

In the long run, once the wage differential hits the critical ratio and then w_1 falls below the reservation wage for some workers, some labor will move. Here, however, the variance of the price distribution comes into play. The greater the variance, the less certain decision makers are about their projection of the future price ratio and the more reluctant they are to commit today's resources based on their expectations. By implication, the larger the variance, the lower the reservation wage, the longer it takes before labor will move and the smaller the migration.

Once this does happen, then the elasticity of labor mobility is positive: $0 < \varepsilon < \infty$ and $E(\beta)$ is less than one. Adapting equation 2.23 to uncertainty:

$$E(\beta) = \frac{\sigma_1\sigma_2 rK_1K_2L+\varepsilon\{\sigma_1L_1K_1E(P_2)X_2+\sigma_2L_2K_2E(P_1)X_1+\sigma_1L_1K_1L_2w_1[1-\mu]\}}{\sigma_1\sigma_2 rK_1K_2L+\varepsilon\{\sigma_1L_1K_1E(P_2)X_2+\sigma_2L_2K_2E(P_1)X_1-\sigma_2L_1K_2L_2w_1[1-\mu]\}}$$

At the new production point, the price line is not tangent to the product transformation curve:

$$\frac{-dX_1}{dX_2} = E(P)E(\beta)$$

To determine the long run impact on the output mix, use the factor market equilibria (2.28), (2.29), (2.30) with a stabilizing wage differential and differentiate with respect to the expected change in the price ratio.[12]

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[12] Totally differentiating:

$$\mu^*E\left[\frac{U_1\Pi_1}{U_1}\right]FLK_1dK_1 + \mu^*E\left[\frac{U_1\Pi_1}{U_1}\right]FLL_1dL_1 = 0$$

$$-E\left[\frac{U_2\Pi_2}{U_2}\right]\hat{P}FLK_2dK_1 + E\left[\frac{U_2\Pi_2}{U_2}\right]\hat{P}FLL_2dL_2 = -E\left[\frac{U_2\Pi_2}{U_2}\right]FL_2dE(P)$$

$$\left(E\left[\frac{U_1\Pi_1}{U_1}\right]FKK_1 + E\left[\frac{U_2\Pi_2}{U_2}\right]\hat{P}FKK_2\right)dK_1 + E\left[\frac{U_1\Pi_1}{U_1}\right]FKL_1dL_1 - E\left[\frac{U_2\Pi_2}{U_2}\right]\hat{P}FKL_2dL_2 = E\left[\frac{U_2\Pi_2}{U_2}\right]FK_2dE(P)$$

In matrix form

$$\begin{bmatrix} \mu^*E[\bullet]FLK_1 & \mu^*E[\bullet]FLL_1 & 0 \\ -E[\bullet]\hat{P}FLK_2 & 0 & E[\bullet]\hat{P}FLL_2 \\ (E[\bullet]FKK_1+E[\bullet]\hat{P}FKK_2) & E[\bullet]FKL_1 & -E[\bullet]\hat{P}FKL_2 \end{bmatrix} \begin{bmatrix} dK_1 \\ dL_1 \\ dL_2 \end{bmatrix} = \begin{bmatrix} 0 \\ E[\bullet]FL_2dE(P) \\ E[\bullet]FK_2dE(P) \end{bmatrix}$$

$$D = \mu^*E[\bullet]FLK_1(-E[\bullet]\hat{P}FLL_2E[\bullet]FKL_1)$$
$$-\mu^*E[\bullet]FLL_1[(E[\bullet]\hat{P}FLK_2)^2 - E[\bullet]^2\hat{P}FLL_2FKK_1 - E[\bullet]^2\hat{P}^2FKK_2FLL_2] < 0$$

$$\frac{dK_1}{dE(P)} = \frac{\begin{vmatrix} 0 & \mu^*E[\bullet]FLL_1 & 0 \\ -E[\bullet]FL_2 & 0 & E[\bullet]\hat{P}FLL_2 \\ E[\bullet]FK_2 & E[\bullet]FKL_1 & -E[\bullet]\hat{P}FKL_2 \end{vmatrix}}{D}$$
$$= -\mu^*E[\bullet]FLL_1(E[\bullet]FL_2E[\bullet]\hat{P}FKL_2-E[\bullet]FK_2E[\bullet]\hat{P}FLL_2) / D < 0$$

The occurance of a wage differential in an uncertain environment does not bring about counter intuitive results:

$$\frac{dK_1}{dE(P)} < 0, \frac{dL_1}{dE(P)} < 0, \frac{dL_2}{dE(P)} > 0.$$

With a fixed supply of capital then $\frac{dK_2}{dE(P)} > 0$.

$$\frac{dL_1}{dE(P)} = \frac{\begin{vmatrix} \mu^*E[\bullet]FLK_1 & 0 & 0 \\ -E[\bullet]\hat{P}FLK_2 & -E[\bullet]FL_2 & E[\bullet]\hat{P}FLL_2 \\ (E[\bullet]FKK_1+E[\bullet]\hat{P}KK_2) & E[\bullet]FK_2 & -E[\bullet]\hat{P}FKL_2 \end{vmatrix}}{D}$$

$$= \mu^*E[\bullet]FLK_1(E[\bullet]FL_2E[\bullet]\hat{P}FKL_2-E[\bullet]FK_2E[\bullet]\hat{P}FLL_2)\ /\ D < 0$$

$$\frac{dL_2}{dE(P)} = \frac{\begin{vmatrix} \mu^*E[\bullet]FLK_1 & \mu^*E[\bullet]FLL_1 & 0 \\ -E[\bullet]\hat{P}FLK_2 & 0 & E[\bullet]FL_2 \\ (E[\bullet]FKK_1+E[\bullet]\hat{P}KK_2) & E[\bullet]FKL_1 & E[\bullet]FK_2 \end{vmatrix}}{D}$$

$$= \mu^*E[\bullet]FLK_1(E[\bullet]FL_2E[\bullet]FKL_1)-\mu^*E[\bullet]FLL_1(-E[\bullet]\hat{P}FLK_2E[\bullet]FK_2$$
$$-E[\bullet]FL_2E[\bullet]FKK_1-E[\bullet]FL_2E[\bullet]\hat{P}FKK_2)\ /\ D > 0$$

Plugging this into production functions:

$$\frac{dX_1}{dE(P)} = FK_1\frac{dK_1}{dE(P)} + FL_1\frac{dL_1}{dE(P)} < 0.$$

$$\frac{dX_2}{dE(P)} = FK_2\frac{dK_1}{dE(P)} + FL_2\frac{dL_2}{dE(P)} > 0.$$

Figure 2.7 shows what happens in the long run.

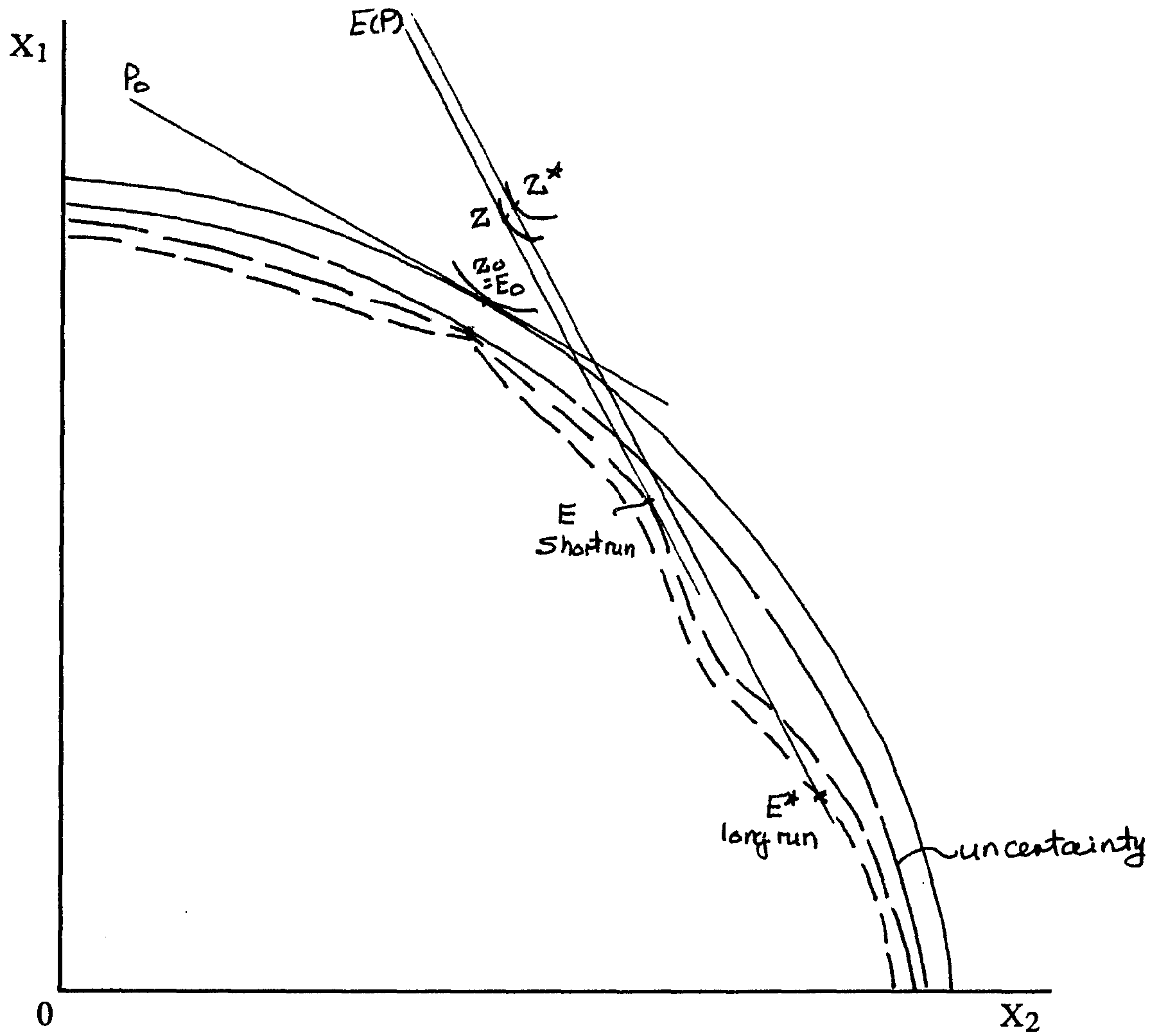

Figure 2.7: Uncertain Environment in the Long Run

Before trade: $E_0 = Z_0$

After trade and the long run adjustment has been made; so, the differential is stable, then output is at E^* and consumption at Z^*.

At E: $\frac{-dX_1}{dX_2} = E(P)E(\beta)$

The marginal rate of transformation does not equal the price ratio. Figure 2.7 shows a favorable impact on welfare. The actual outcome is not necessarily clear. Reworking the welfare equation (2.11) to incorporate uncertainty:

$$\frac{1}{U_1}\frac{dU}{dE(P)} = \frac{dZ_1}{dE(P)} + E(P)\frac{dZ_2}{dE(P)} \qquad (2.31)$$

$$= \frac{dX_1}{dE(P)} + E(P)\frac{dX_2}{dE(P)} + EX_2\frac{dE(P)}{dE(P)} \qquad (2.32)$$

While EX_2, the export term, is positive, the first two terms:

$$\frac{dX_1}{dE(P)} + E(P)\frac{dX_2}{dE(P)}$$

do not only represent a movement along a product transformation curve, but they also incorporate the impacts of uncertainty and the wage differential.

In the next chapters as growth, a customs union and a unilateral transfer are examined, these two output adjustment terms will be shown to sum to a negative number. The greater the risk premia and the larger the wage differential, the less likely that the export term can overcome those negative effects on welfare.

Chapter III
Growth

This chapter will discuss the welfare effects of three types of growth: an exogenous increase in the labor supply. $\left(\frac{dL}{L} > 0\right)$, an exogenous increase in the supply of capital $\left(\frac{dK}{K} > 0\right)$ and disembodied, neutral technological innovation $\left(\frac{d\delta}{\delta} > 0\right)$.

Reviewing the production functions (2.1), (2.2); they become:

$$X_1 = \delta_1 F_1(K_1, L_1) \tag{3.1}$$

$$X_2 = \delta_2 F_2(K_2, L_2) \tag{3.2}$$

Neutral disembodied technical progress ($\delta_i > 1$) occurs in an industry when increased output is obtained from given supplies of labor and capital. The isoquants of that industry shift inward and the marginal rate of substitution of labor for capital is the same before and after the technical change for all values of the capital labor ratio in the industry.

With constant returns, an increase in all inputs in the same proportion raises output in the same proportion; if inputs are doubled, outputs will double.

Recall the income equation (2.16). Totally differentiating it becomes:

$$dY \quad = \quad F_1(K_1,L_1)\, d\delta_1 + \frac{\partial X_1}{\partial K_1} dK_1 + \frac{\partial X_1}{\partial L_1} dL_1$$

$$+ P\left[F_2\,(K_2,L_2)\, d\delta_2 + \frac{\partial X_2}{\partial K_2} dK_2 + \frac{\partial X_2}{\partial L_2} dL_2\right].$$

Note that: $\frac{\partial X_i}{\partial K_i} = MP_{Ki} = FKi$

Dividing this equation throughout by Y yields:

$$\frac{dY}{Y} = \frac{d\delta_1}{\delta_1} + \frac{MP_{K1}}{Y} dK_1 + \frac{MP_{L1}}{Y} dL_1$$
$$+ P\left[\frac{d\delta_2}{\delta_2} + \frac{MP_{K2}}{Y} dK_2 + \frac{MP_{L2}}{Y} dL_2\right].$$

Which can be rewritten:

$$\frac{dY}{Y} = \frac{d\delta_1}{\delta_1} + \frac{K_1(MP_{K1})}{Y}\frac{dK_1}{K} + \frac{L_1(MP_{L1})}{Y}\frac{dL_1}{L}$$
$$+ P\left[\frac{d\delta_2}{\delta_2} + \frac{K_2(MP_{K2})}{Y}\frac{dK_2}{K} + \frac{L_2(MP_{L2})}{Y}\frac{dL_2}{L}\right].$$

Note that: $\frac{K_1(MP_{K1})}{Y}$ is the income share of capital from X_1: ΘK_1.

Therefore:

$$\frac{dY}{Y} = \frac{d\delta_1}{\delta_1} + \Theta K_1 \frac{dK_1}{K} + \Theta L_1 \frac{dL_1}{L} + P\left[\frac{d\delta_2}{\delta_2} + \Theta K_2 \frac{dK_2}{K} + \Theta L_2 \frac{dL_2}{L}\right] \quad (3.3)$$

The contribution of the growth of factor inputs is seen in:

$$\Theta L_1 . \frac{dL_1}{L} + \Theta K_1 \frac{dK_1}{K} + \Theta L_2 \frac{dL_2}{L} + \Theta K_2 \frac{dK_2}{K}.$$

Each factor contributes an amount equal to its individual growth rate multiplied by the share of that input in income.

$\frac{d\delta_1}{\delta_1} + \frac{d\delta_2}{\delta_2}$ is the rate of improvement in technology, that is, the growth of total factor productivity.

Let us focus now on the impact of an increase in just one input: labor.

Increase in the Labor Supply

Part 1 Mobile Factors in a Certain Environment

With the supply of capital constant: $\frac{dK_1}{K} = \frac{dK_2}{K} = 0$, and technology static: $\frac{d\delta_1}{\delta_1} = \frac{d\delta_2}{\delta_2} = 0$; $\delta_1 = \delta_2 = 1$, let us assume an exogenous increase in the supply of labor. The impact on income will be determined when there is no growth induced change in the terms of trade, as is true for a small country. Then the impact on income will be examined for a large country which does experience a growth induced change in the price ratio.

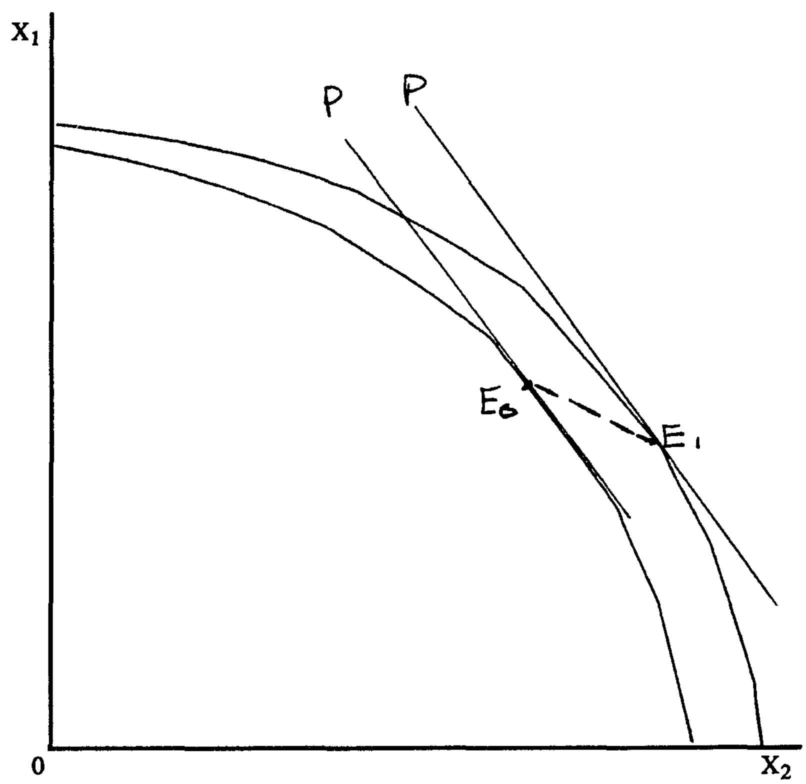

Figure 3.1 Increase in the Labor Supply

P: international terms of trade

E_0: initial output mix; E_1: output mix after an increase in the supply of labor.

The path traced out as the output mix moved from E_0 to E_1 is the Rybczynski loci as developed by Martin.

With a constant output price ratio, the Rybczynski Theorem[13] predicts that there will be an increase in the output of X_2, which is labor intensive, and a decline in output of X_1, which is capital intensive.

Figure 3.1 shows the impact of the increase in the labor supply as predicted by the Rybczynski Theorem. The slope of the product transformation curve at E_0: $\frac{-dX_1}{dX_2} = P$ is the

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13 The Rybczynski Theorem can be derived in the following manner:

Recall: $k = \frac{K}{L} = \frac{K_1}{L_1} \bullet \frac{L_1}{L} + \frac{K_2}{L_2} \bullet \frac{L_2}{L} = k_1 \bullet \frac{L_1}{L} + k_2 \bullet \frac{L_2}{L}$

When the labor supply increases, the capital labor ratio declines: $dk < 0$. However, k_1 and k_2 remain constant because, by assumption, the price ratio is constant; so, $\frac{L_1}{L}$ and $\frac{L_2}{L}$ must show the changes. Given that X_2 is labor intensive, then $\frac{L_2}{L}$ must rise. $\frac{L_1}{L} = 1 - \frac{L_2}{L}$ so $\frac{L_1}{L}$ must decline.

To determine the impact on outputs use the production functions:

$$X_1 = L_1 f_1(k_1) \tag{2.1'}$$

$$X_2 = L_2 f_2(k_2) \tag{2.2'}$$

Differentiating with respect to an increase in the labor supply:

$$\frac{dX_1}{dL} = L_1 f_1' \frac{dk_1}{dL} + f_1 \frac{dL_1}{dL}$$

Since the price ratio is constant, k_1 is constant and therefore $L_1 f_1' \frac{dk_1}{dL} = 0$.

As seen above $\frac{dL_1}{L}$ is negative; so, $\frac{dX_1}{dL}$ is negative. An increase in the labor supply causes a decline in the output of X_1.

$$\frac{dX_2}{dL} = L_2 f_2' \frac{dk_2}{dL} + f_2 \frac{dL_2}{dL}$$

The first term drops out as above, $\frac{dL_2}{dL}$ is positive; so, $\frac{dX_2}{dL}$ will be positive.

same as at E_1 because the price ratio has not changed. The marginal rates of transformation are the same.[14]

The increase in the labor supply will be assumed to be equal to the increase in population. Let us begin by examining its impact on income by differentiating the income equation (2.16) with respect to the change in the labor supply.

$$\frac{dY}{dL} = \frac{dZ_1}{dL} + P\frac{dZ_2}{dL} + Z_2\frac{dP}{dL} = \frac{dX_1}{dL} + P\frac{dX_2}{dL} + X_2\frac{dP}{dL} \tag{3.4}$$

Assume $\frac{dP}{dL} = 0$ for now.

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[14] Yu and Parai's (1987) development of β can be used to demonstrate what is occuring here. $\beta^0 = \beta^1 = 1$ because at E_0 $\frac{-dX_1}{dX_2} = P = \frac{-dX_1}{dX_2}$ at E_1.

$$\beta^0 = \left[\frac{\sigma_1\sigma_2 r^0 K_1^0 K_2^0 L^0 + \varepsilon\{\sigma_1 L_1^0 K_1^0 P_2 X_2^0 + \sigma_2 K_2^0 L_2^0 P_1 X_1^0 + \sigma_1 L_1^0 K_1^0 L_2^0 w^0\}}{\sigma_1\sigma_2 r^0 K_1^0 K_2^0 L^0 + \varepsilon\{\sigma_1 L_1^0 K_1^0 P_2 X_2^0 + \sigma_2 K_2^0 L_2^0 P_1 X_1^0 - \sigma_2 L_1^0 K_2^0 L_2^0 w^0\}}\right]$$

$$\beta^1 = \left[\frac{\sigma_1\sigma_2 r^0 K_1^1 K_2^1 L^1 + \varepsilon\{\sigma_1 L_1^1 K_1^1 P_2 X_2^1 + \sigma_2 K_2^1 L_2^1 P_1 X_1^1 + \sigma_1 L_1^1 K_1^1 L_2^1 w^1\}}{\sigma_1\sigma_2 r^0 K_1^1 K_2^1 L^1 + \varepsilon\{\sigma_1 L_1^1 K_1^1 P_2 X_2^1 + \sigma_2 K_2^1 L_2^1 P_1 X_1^1 - \sigma_2 L_1^1 K_2^1 L_2^1 w^1\}}\right]$$

The changes which take place can be seen by examining what happens to the elements that make up β. The allocation of the fixed supply of capital adjusts as capital moves from X_1 to X_2; so $K_1^0 > K_1^1$ and $K_2^0 < K_2^1$; however, since $dK_1 = -dK_2$, then $K_1^0 K_2^0 < K_1^1 K_2^1$. The total supply of labor increases: $L^0 < L^1$. The elasticities of factor substitution (σ_1,σ_2) and the return to capital (r) are assumed constant. The elasticity of labor mobility (ε) is infinite since labor is perfectly mobile; so, any changes within the brackets are inconsequential. The elements that have an impact, then, are the increase in the labor supply and movement of capital. That being so:

$$\sigma_1\sigma_2 r^0 K_1^0 K_2^0 L^0 < \sigma_1\sigma_2 r^0 K_1^1 K_2^1 L^1$$

To demonstrate the impact of the increased labor supply on the output mix, differentiate the production functions (2.1), (2.2) with respect to the increase in labor:

$$\frac{dX_1}{dL} = F_1\frac{d\delta_1}{dL} + \delta_1\left[F_{K1}\frac{dK_1}{dL} + F_{L1}\frac{dL_1}{dL}\right] \tag{3.5}$$

There is no impact on production technology by the increase in labor; so, the technical advance term: $F_1\frac{d\delta_1}{dL}$, drops out. The capital term: $\frac{dK_1}{dL}$ and labor term: $\frac{dL_1}{dL}$ are negative as explained by the Rybczynski Theorem. Output of X_1 declines as a result of the increase in the labor supply.

$$\frac{dX_2}{dL} = F_2\frac{d\delta_2}{dL} + \delta_2\left[F_{K2}\frac{dK_2}{dL} + F_{L2}\frac{dL_2}{dL}\right] \tag{3.6}$$

The technology term drops out as above. The capital and labor terms are positive as shown in the proof of the Rybczynski Theorem. As capital and labor move into sector 2, output of X_2 increases. The increase in labor in sector 2 represents not only the overall increase in the supply of labor (dL) but also labor transferred from sector 1 (dL_1).

$$L_2 = dL + dL_1$$

Chacholiades[15] points out that when the home country experiences an increase in the labor supply and exports the labor intensive good (X_2) welfare (and therefore income) may increase, decrease or be indeterminate.

Let us examine what the impact on income will be given the specifics of this model by using Casas's equations (27) and (28)[16] and adapting them to generate the following two equations:

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[15] M. Chacholiades, *International Trade Theory and Policy*, (New York: McGraw Hill, 1978) pp. 347-8.

[16] F. Casas, "Imperfect Factor Mobility: A Generalization and Synthesis of Two Sector Models of International Trade," *Canadian Journal of Economics* (1984) p. 758.

$$\frac{dX_1}{dL} = \frac{X_1}{L}\left[\frac{\Theta K_2 \Theta L_1 \sigma_1 - \lambda K_2 |\Theta| \varepsilon}{\lambda K_1 \Theta L_2 \sigma_1 + \lambda K_2 \Theta L_1 \sigma_2}\right] \tag{3.7}$$

All values in the denominator are positive. The first term in the numerator, the factor substitution elasticity term: $\Theta K_2 \Theta L_1 \sigma_1$, is positive whereas the second term, the elasticity of labor mobility term: $-\lambda K_2 |\Theta| \varepsilon$, is negative. The value intensity determinant: $|\Theta|$, is positive as shown in Appendix 1. Since $\varepsilon = \infty$ because labor is perfectly mobile by assumption, then the elasticity of labor mobility term is larger than the elasticity of factor substitution term. The value within the brackets is negative. When labor is perfectly mobile, the impact of an increase in the labor supply is to reduce the output of X_1.

Turning to sector 2:

$$\frac{dX_2}{dL} = \frac{X_2}{L}\left[\frac{\Theta K_1 \Theta L_2 \sigma_2 + \lambda K_1 |\Theta| \varepsilon}{\lambda K_1 \Theta L_2 \sigma_1 + \lambda K_2 \Theta L_1 \sigma_2}\right] \tag{3.8}$$

The denominator is the same as in $\frac{dX_2}{dL}$; so, it is positive. The elasticity of factor substitution term in the numerator is positive as is the elasticity of labor mobility term; therefore, the value within the brackets is positive. The Rybczynski Theorem is affirmed. The value inside the bracket is also greater than one, showing a magnification effect. The change in output of X_2 is greater than the change in the supply of labor.

These two equations can be used to show that the slope of the Rybczynski loci is negative as seen in Figure 3.1.

$$\frac{dX_1}{dX_2} = \frac{X_1}{X_2}\left[\frac{\Theta K_2 \Theta L_1 \sigma_1 - \lambda K_2 |\Theta| \varepsilon}{\Theta K_1 \Theta L_2 \sigma_2 + \lambda K_1 |\Theta| \varepsilon}\right] < 0 \tag{3.9}$$

This highlights the role of the elasticities of factor substitution (σ_1, σ_2) and the elasticity of labor mobility (ε) in the adjustment of the output mix.

Applying these findings to the income equation (3.4)

$$\frac{dY}{dL} = \frac{X_1}{L}\left[\frac{\Theta K_2 \Theta L_1 \sigma_1 - \lambda K_2 |\Theta| \varepsilon}{\lambda K_1 \Theta L_2 \sigma_1 + \lambda K_2 \Theta L_1 \sigma_2}\right] + P\frac{X_2}{L}\left[\frac{\Theta K_1 \Theta L_2 \sigma_2 + \lambda K_1 |\Theta| \varepsilon}{\lambda K_1 \Theta L_2 \sigma_1 + \lambda K_2 \Theta L_1 \sigma_2}\right].$$

Recalling these assumptions: $\sigma_2 > \sigma_1$

$$\Theta L_2 > \Theta L_1$$

$$\Theta K_1 > \Theta K_2$$

$\frac{dX_2}{dL}$ is greater than $\frac{dX_1}{dL}$ by:

$$\frac{X_2(\Theta K_1 \Theta L_2 \sigma_2) - X_1(\Theta K_2 \Theta L_1 \sigma_1)}{L(\lambda K_1 \Theta L_2 \sigma_1 + \lambda K_2 \Theta L_1 \sigma_2)}$$

This affirms that, given the assumptions of this model, income will increase when the labor supply increases. Aggregate welfare increases and, if the increase in income is greater than the increase in population, then per capita welfare improves.

Turning to a situation where the growing country is large, the increase in the labor supply will have a secondary impact on the terms of trade: the term $X_2\frac{dP}{dL}$ now has a value.

As has been seen above, an increase in the labor supply will cause an increase in the output of exportables: $\frac{dX_2}{dL} > 0$ and a decline in the output of importables: $\frac{dX_1}{dL} < 0$. This is an example of ultra pro trade biased growth as defined by Chacholiades (1979).

Since other countries' demand for imports and supply of exports remain the same at the initial terms of trade, there is a positive excess demand for X_1 and a positive excess supply of X_2 generated by the home country. That will cause the terms of trade to deteriorate for that country: $dP < 0$.

This can be further explored by showing the duality between the Rybczynski and Stolper Samuelson Theorems. Recalling from the proof of the Rybczynski Theorem:

$$\frac{dX_1}{dL} = f_1 \frac{dL_1}{dL}$$

$$\frac{dX_2}{dL} = f_2 \frac{dL_2}{dL}$$

and using the factor endowment equation (2.7) (2.8) Hazari found:[17]

$$\frac{dL_1}{dL} = \frac{k_2}{k_2 - k_1} \text{ so } \frac{dX_1}{dL} = f_1 \frac{k_2}{k_2 - k_1}$$

$$\frac{dL_2}{dL} = \frac{k_1}{k_2 - k_1} \text{ so } \frac{dX_2}{dL} = f_2 \frac{k_1}{k_2 - k_1}$$

Hazari also showed:[18]

$\frac{dw}{dP_1} = \frac{k_2 f_1}{k_2 - k_1} < 0$ Since X_1 is capital intensive, then $k_1 > k_2$. As predicted by the Stolper Samuelson Theorem, when the price of X_1 increases, wages will decline.

$\frac{dw}{dP_2} = \frac{-k_1 f_2}{k_2 - k_1} > 0$ X_2 is labor intensive. In conformity with the Stolper Samuelson Theorem, when the price of X_2 declines, so will wages.

By assumption, the return to capital (r) is constant. The wage rental ratio will increase when the price ratio increases, and decline when the price ratio declines. The marginal product of labor falls in terms of both commodities when the price ratio declines.

Establishing the duality:

$$\frac{dX_1}{dL} = \frac{k_2 f_1}{k_2 - k_1} = \frac{dw}{dP_1},$$

$$\frac{dX_2}{dL} = \frac{-k_1 f_2}{k_2 - k_1} = \frac{dw}{dP_2}$$

Moving on to the total impact on income from the increase in the labor supply, recall the income equation (3.4)

[17] B. Hazari, *International Trade* (New York: New York University Press, 1986) p. 47

[18] *Ibid,* p.53

$$\frac{dY}{dL} = \frac{dX_1}{dL} + P\frac{dX_2}{dL} + EX_2\frac{dP}{dL}$$

The terms of trade impact is found in the export term: $EX_2\frac{dP}{dL}$.

As shown above, the ouput adjustment terms sum to a positive: $\frac{dX_1}{dL} + P\frac{dX_2}{dL} > 0$

Given that the growth induced price effect is negative: $\frac{dP}{dL} < 0$, then the export term is negative. The outcome is now ambiguous. Income will increase if the impact on output of the larger labor supply outweighs the growth induced deterioration in the terms of trade. This is the standard result in the literature.

Now let us drop the assumption that labor is mobile and examine the consequences for output and income in both a small country and large country context.

Part 2 Short Run Immobility of Labor; Long Run Partial Mobility in a Certain Environment

In the short run labor is immobile; so, the elasticity of labor mobility is zero: $\varepsilon = 0$. Referring back to equation (3.7) the impact of the increase in the labor supply on X_1 becomes:

$$\frac{dX_1}{dL} = \frac{X_1}{L}\left[\frac{\Theta K_2 \Theta L_1 \sigma_1}{\lambda K_1 \Theta L_2 \sigma_1 + \lambda K_2 \Theta L_1 \sigma_2}\right] \tag{3.10}$$

This shows that the increase in labor will cause an increase in X_1, the capital intensive good. This is because the elasticity of factor mobility term has dropped out. The value inside the brackets is less than one. There is no magnification effect. Indeed, the change in output is smaller than the change in the labor supply. It should be noted here that the adjustment of output in this situation depends upon each sector's elasticity of factor substitution.

Equation (3.8) becomes:

$$\frac{dX_2}{dL} = \frac{X_2}{L}\left[\frac{\Theta K_1 \Theta L_2 \sigma_2}{\lambda K_1 \Theta L_2 \sigma_1 + \lambda K_2 \Theta L_1 \sigma_2}\right] \tag{3.11}$$

to show the impact on X_2 of the increase of the labor supply. It is positive as before but the magnification effect is lost here also.

Since both outputs increase, the Rybczynski Theorem does not hold when labor is immobile. The slope of the Rybczynski loci is positive since both outputs increase.

$$\frac{dX_1}{dX_2} = \frac{X_1}{X_2}\left[\frac{\Theta K_2 \Theta L_1 \sigma_1}{\Theta K_1 \Theta L_2 \sigma_2}\right] > 0$$

This expands Casas's results. He used a model where the supply of capital (a mobile input) increased with labor immobility and also found that both outputs increase. He explained it in terms of the sizes of the elasticities of factor substitution and the elasticity of labor mobility. When the elasticities of factor substitution in both sectors exceed the elasticity of factor mobility (as is true here) both outputs will increase. His finding is reaffirmed here when the elasticity of labor mobility is zero and the supply of labor increases. Both outputs increase because labor can be substituted somewhat for capital in both sectors and that is because the elasticites of factor substitution are both greater than zero. Implicit in this is the assumption that some of the new labor goes to each sector. In the following analysis we will explore the results when the labor is allocated to only one sector.

Examining the impact on income for a small country, recall (3.4)

$$\frac{dY}{dL} = \frac{dX_1}{dL} + P\frac{dX_2}{dL}$$

Outputs are now the function of labor, the developing wage differential, and capital.

Recall: $$\mu = \frac{w_2}{w_1}$$

$$X_1 = X_1\,[L_1(L), \mu(L), K_1\,(L)] \tag{3.12}$$

$$X_2 = X_2\,[L_2(L),\mu(L),\ K_2\,(L)] \tag{3.13}$$

Assume, to begin with that no new labor enters sector 1. Given $dL = dL_1 + dL_2$; if $dL_1 = 0$, then $dL = dL_2$. All of the new labor enters sector 2. The impact on the two outputs are shown by $\frac{dX_1}{dL}$ and $\frac{dX_2}{dL}$.

$$\frac{dX_1}{dL} = \frac{\partial X_1}{\partial L_1}\frac{dL_1}{dL} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial L} + \frac{\partial X_1}{\partial K_1}\frac{dK_1}{dL}$$

Given that $dL_1 = 0$, then $\frac{\partial X_1}{\partial L_1}\frac{dL_1}{dL} = 0$. This represents the primary impact of the increase in the labor supply and since all of the new labor is going to sector 2 this term is zero.

$\frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial L} = 0$ This represents the reallocation of the labor supply that occurs because the capital labor ratios differ. As the economy attempts to adjust the output mix as the Rybczynski Theorem predicts, labor does not move and a wage differential develops. As yet, the developing differential has no impact on the allocation of labor: $\frac{\partial L_1}{\partial \mu} = 0$, and the whole wage differential term is zero. With sector 2 output expanding, capital will shift to sector 2; so, this term is negative. As a result output in sector 1 declines. $\frac{\partial X_1}{\partial K_1}\frac{dK_1}{dL} < 0$

Turning to sector 2:

$$\frac{dX_2}{dL} = \frac{\partial X_2}{\partial L_2}\frac{dL_2}{dL} + \frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial L} + \frac{\partial X_2}{\partial K_2}\frac{dK_2}{dL}$$

$\frac{\partial X_2}{\partial L_2}\frac{dL_2}{dL} > 0$. This represents the impact of the increase in the supply of labor all of which goes to sector 2: $\frac{dL_2}{dL} = 1$.

$\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial L} = 0$ This represents the reallocation of the labor supply, and since labor is immobile it drops out.

$\frac{\partial X_2}{\partial K_2}\frac{dK_2}{dL} > 0$ Capital enters sector 2 from sector 1.

Plugging this into the income equation:

$$\frac{dY}{dL} = \frac{dX_1}{dL_1} + P\frac{dX_2}{dL} > 0$$

Income increases because the increase in output of X_2 due to the new labor and capital from X_1 is greater than the decline in X_1 due to the loss of some capital.

On the other hand, let us assume that sector 2 receives none of the new labor: $dL_2 = 0$; so, that, $dL = dL_1$. Then the results are reversed.

$$\frac{dX_1}{dL} = \frac{\partial X_1}{\partial L_1}\frac{dL_1}{dL} + \frac{\partial X_1}{\partial K_1}\frac{dK_1}{dL} > 0 \, ; \frac{dX_2}{dL} = \frac{\partial X_2}{\partial K_2}\frac{dK_2}{dL} < 0$$

The income will also increase in this situation.

$$\frac{dY}{dL} = \frac{dX_1}{dL} + P\frac{dX_2}{dL} > 0.$$

Finally, let us assume that some of the increase in supply of labor flows into each sector: $dL_1, dL_2 > 0$. $dL = dL_1 + dL_2$

Then $\frac{dX_1}{dL} = \frac{\partial X_1}{\partial L_1}\frac{dL_1}{dL}$ is positive, but $0 < \frac{dL_1}{dL} < 1$. The secondary impact of the developing wage differential is still zero since the elasticity of labor mobility is zero and, therefore, labor, does not move between sectors $\frac{\partial L_1}{\partial \mu} = 0$

The impact on sector 2 is also positive:

$\frac{dX_2}{dL} = \frac{\partial X_2}{\partial L_2}\frac{dL_2}{dL}; > 0$ The output of X_2 will increase, and $0 < \frac{dL_2}{dL} < 1$. The developing wage differential has no impact since labor is immobile. The impact on capital allocation is difficult to determine. Depending on the proportion of new labor each sector gets, capital could shift toward one or the other sectors or not move at all. Given, however, that the elasticities of factor substitution are both positive let us proceed on the basis that both outputs increase. Then it can be seen that the Casas based analysis using equations (3.10) and (3.11) can hold only if both sectors receive some of the new labor. Returning to income:

$$\frac{dY}{dL} = \frac{dX_1}{dL} + P\frac{dX_2}{dL} > 0.$$

Since labor cannot be reallocated, there is a loss in efficiency. The product transformation

curve contracts. The increase in income in these circumstances will be less than for a small country with perfectly mobile labor.

Moving on to the example of a large country, recall that increase in the supply of labor will cause a shift in the terms of trade. Since the economy is trying to adjust to a new price ratio, there is another force, beside the increase in the labor supply, causing a wage differential.

Following the pattern of analysis for a small country, let us assume all of the new labor goes to sector 2 which causes the price ratio to decline. Beginning with sector 1:

$$\frac{dX_1}{dL} = \frac{\partial X_1}{\partial L_1}\frac{dL_1}{dL} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial L} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{dL} + \frac{\partial X_1}{\partial K_1}\frac{dK_1}{dL} + \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial P}\frac{dP}{dL}$$

The primary impact of the labor supply: $\frac{\partial X_1}{\partial L_1}\frac{dL_1}{dL}$ and the reallocation of labor induced by the increase in the labor supply: $\frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial L}$, both drop out. Capital shifts to sector 2: $\frac{\partial X_1}{\partial K_1}\frac{dK_1}{dL} < 0$.

$\frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{dL} = 0$. This represents the induced, tertiary impact of the increase in the labor supply on the price ratio which drops out since labor is immobile.

$\frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial P}\frac{dP}{dL} > 0$. The increase in the relative price of X_1 would encourage expansion of that output, drawing capital from X_2. The final impact on X_1 in the short run will depend on which is greater; increase in X_2 output due to its increase in labor supply or the change in the price ratio. $\frac{dX_1}{dL} \lesseqgtr 0$.

Looking at sector 2 which receives all the new labor: $\frac{dL_2}{dL} > 0$.

$$\frac{dX_2}{dL}=\frac{\partial X_2}{\partial L_2}\frac{dL_2}{dL}+\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial L}+\frac{\partial X_2}{\partial L_2}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{dL}+\frac{\partial X_2}{\partial K_2}\frac{dK_2}{dL}+\frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial P}\frac{dP}{dL}$$

The primary impact of the increase in the labor supply is positive: $\frac{\partial X_2}{\partial L_2} > 0$. The secondary and tertiary impacts on labor reallocation drop out as explained above. As with X_1 capital reallocation is being pulled in both directions. Output of X_2 will increase because it receives the primary impact of the incease in labor and may receive some extra capital from X_1. Income will increase if the increase in X_2 outweights the reduction in exports due to the deterioration in the terms of trade plus the possible decline in X_1.

$$\frac{dY}{dL}=\frac{dX_1}{dL}+P\frac{dX_2}{dL}+EX_2\frac{dP}{dL}$$

In the opposing case, let all of the labor go into sector 1. As explained above, the immobility of labor causes the secondary and tertiary impacts on labor reallocation due to the increasing labor supply and improvement in the price ratio drop out in both sectors. The capital reallocation terms for each output move in opposite directions. Output of sector 1 increases and the change in output of sector 2 depends on which way capital moves.

Income will increase. Output of sector 1 increases, the export term is positive and output of sector 2 may increase.

In the case where some of the new labor is allocated to each sector (dL_1, $dL_2 > 0$), let both outputs increase. The secondary and tertiary effects will cause only capital to shift. Income is more likely to increase if the terms of trade improve because the export term is positive than if they deteriorate in which case the export term is negative.

In the long run labor becomes somewhat mobile because the wage differential has fallen beyond the critical ratio and labor is being paid a wage below the reservation wage for some workers in the lower wage sector. The change in the differential will slow down and stabilize when the equilibrium output ratio is attained.

Given that, it is possible to determine under what circumstances the Rybczynski Theorem will hold. Recall equation (3.10). Its sign is no longer clear.

$$\frac{dX_1}{dL} = \frac{X_1}{L}\left[\frac{\Theta K_2 \Theta L_1 \sigma_1 - \lambda K_2 |\Theta| \varepsilon}{\lambda K_1 \Theta L_2 \sigma_1 + \lambda K_2 \Theta L_1 \sigma_2}\right] \lesseqgtr 0 \qquad (3.14)$$

The impact of the increase in the labor supply is ambiguous. Having the elasticity of labor mobility greater than the elasticities of factor substitution is not sufficient to determine the sign, as Casas argued. Only if the elasticity of factor substitution term ($\Theta K_2 \Theta L_1 \sigma_1$) is greater than the elasticity of labor mobility term ($\lambda K_2 |\Theta| \varepsilon$) will the output of sector 1 increase. The following can now be stated.

Proposition 3.1 The relative sizes of value intensities ($\Theta K_2 \Theta L_1$), the size of the determinant of value intensities ($|\Theta|$) plus the physical intensity of capital in the export sector (λK_2) play a role along with the elasicity of labor mobility (ε) and the elasticities of factor substitution (σ_1,σ_2) in determining whether or not the Rybczynski Theorem holds.

However, the more mobile labor becomes, the larger is the elasticity of labor mobility; so, the more likely the output in sector 1 will decline and the Rybczynski Theorem will hold.

Turning to sector 2 and using equation (3.11)

$$\frac{dX_2}{dL} = \frac{X_2}{L}\left[\frac{\Theta K_1 \Theta L_2 \sigma_2 + \lambda K_1 |\Theta| \varepsilon}{\lambda K_1 \Theta L_2 \sigma_1 + \lambda K_2 \Theta L_1 \sigma_2}\right] > 0 \qquad (3.15)$$

Not surprisingly, since every term is positive inside the brackets, the output of X_2 increases when there is an increase in the supply of labor.

Comparing the short run equations (3.10) (3.11) with the long run equation (3.14) (3.15) it is clear that the role of the elasticity of labor mobility is pivotal in determining the final output ratio.

Approaching the matter from an alternative way, the impact on income can be determined. In the long run:

$$\frac{dX_1}{dL} = \frac{\partial X_1}{\partial L_1}\frac{dL_1}{dL} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{d\mu}{dL} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{dL} + \frac{\partial X_1}{\partial K_1}\frac{dK_1}{dL} + \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial P}\frac{dP}{dL}$$

The primary impact term will be zero and the secondary reallocation impact on labor and capital will be negative. For a small country that is the entire effect of the increase in the labor supply, an unambiguous decrease in the output of X_1. It should be kept in mind that the reallocation terms will be smaller than when labor is perfectly mobile. For a large country the tertiary terms of trade effect reallocating labor and capital are positive because the price ratio declines. Output of X_1 for a large country would increase only if the terms of trade reallocation effects overwhelm the reallocation effects of the increase in the labor supply.

Turning our attention to sector 2:

$$\frac{dX_2}{dL} = \frac{\partial X_2}{\partial L_2}\frac{dL_2}{dL} + \frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{d\mu}{dL} + \frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{dL} + \frac{\partial X_2}{\partial K_2}\frac{dK_2}{dL} + \frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial P}\frac{dP}{dL}$$

The primary impact term: $\frac{\partial X_2}{\partial L_2}\frac{dL_2}{dL}$ and the reallocation impacts on labor and capital of the increase in labor are positive. For a small country that yields an unambiguous increase in the output of X_2, but the reallocation is not as complete as it would be if labor were perfectly mobile. The economy is operating on a lower product transformation curve than one would with mobile labor due to the differential.

For a large country, the tertiary terms of trade effect on reallocation of labor and capital is negative. Output of X_2 will increase if the primary and reallocation terms outweigh the terms of trade effect.

Reviewing the income equation (3.4)

$$\frac{dY}{dL} = \frac{dX_1}{dL} + P\frac{dX_2}{dL} + EX_2\frac{dP}{dL}$$

In the long run income will increase for a small country if the primary and reallocation effects of the increase in the supply of labor are sufficient to overcome the impact of the wage differential.

For a large country, the change in the output mix: $\frac{dX_1}{dL} + P\frac{dX_2}{dL}$ is muted by the deterioration in the terms of trade. In addition, the export term is also negative: $EX_2\frac{dP}{dL} < 0$, due to the growth induced decline in the price ratio. It would appear, then, that in the long run a small country with partially mobile labor is more likely to experience an increase in income than a large country.

So far we have studied the impact of an increase in labor in a certain environment. In the next part that asssumption will be relaxed, while labor will again be assumed to be perfectly mobile.

Part 3 Mobile Factors in an Uncertain Environment

Recall from the basic model that the price ratio is replaced by a probability distribution: $\hat{P} = \gamma P + \rho$ and the mean of that distribution is the expected price: E(P). The economy is operating in an uncertain environment when the supply of labor increases.

Since labor is perfectly mobile, the elasicity of labor mobility is infinite ($\varepsilon = \infty$) and the Rybczynski Theorem holds. Rewriting the income equation to reflect the uncertainty:

$$\frac{dY}{dL} = \frac{dX_1}{dL} + E(P)\frac{dX_2}{dL} + EX_2\frac{dE(P)}{dL}.$$

For a small country the export term: $EX_2\frac{dE(P)}{dL}$, drops out, and income increases.

For a large country, the increase in the labor supply will cause the expected terms of trade to move against it. Since at the original expected price ratio, the increase in labor would generate an excess demand for imports (X_1) and an excess supply of exports (X_2) by the growing country, the price ratio would decline: $\frac{dE(P)}{dL} < 0$. Income for the large country will increase unless the impact of the growth induced change in the expected price ratio overwhelms the impact of the increase in labor on the output ratio.

The following result is established.

Proposition 3.2 Uncertainty about the future terms of trade has no qualitative impact on the results found when the labor input increases in a certain environment with mobile factors.

Next the model will be further amended to allow for the impact of immobility of labor in an environment where the future terms of trade are unknown.

Part 4 Short Run Immobility of Labor; Long Run Partial Mobility in an Uncertain Environment

Uncertainty about the future terms of trade may very well be a reason for labor immobility. Using the elasticity of labor mobility:

$$\varepsilon = \varepsilon\left[\mu(t), t, \sigma_p^2\right]$$

to show this, let us assume that the elasticity is a function of the wage differential (μ), time (t) and the variance of the price distribution $\left(\sigma_p^2\right)$. Totally differentiating the elasticity equation

$$d\varepsilon = \frac{\partial \varepsilon}{\partial \mu} \frac{\partial \mu}{\partial t} dt + \frac{\partial \varepsilon}{\partial t} dt + \frac{\partial \varepsilon}{\partial\left(\sigma_p^2\right)} d\left(\sigma_p^2\right)$$

The greater the wage differential, the larger the elasticity of labor mobility. As it initially develops in the short run, however, the elasticity of labor mobility remains zero because of

other factors: $\frac{d\varepsilon}{d\mu} = 0$. One of these is time. The differential continues to develop and in the long run it will go beyond the critical wage differential and trigger the movement of labor: $\frac{d\varepsilon}{d\mu} > 0$.

It takes time for people to sharpen their perceptions of what is going on, as Grossman and Stiglitz pointed out in their article. Uncertainty about the future terms of trade hampers this. The larger the variance of the price distribution, the greater the uncertainty and therefore the slower labor is to move: $\frac{d\varepsilon}{d\sigma_p^2} < 0$. Uncertainty may also work to reduce the reservation wage; so, it takes longer for the differential to reach and go beyond its critical value.

Finally, time in and of itself plays a part because people cannot instantaneously relocate between sectors: $\frac{d\varepsilon}{dt} > 0$.

In general, then, labor will be less mobile when there is uncertainty about the terms of trade than when the future price ratio is known.

Turning to the short run for a small economy with no growth induced price effect, the impact of the increase in the labor supply, regardless of the allocation of the new labor will be an increase in income if the contracting effect of the developing wage differential is overcome. The secondary impact of reallocation of labor in the two sectors is zero because labor is immobile: $\frac{\partial L_1}{\partial \mu} = \frac{\partial L_2}{\partial \mu} = 0$. Therefore, the Rybczynski Theorem will not hold.

For a large country the induced price effect on the output ratio will be due solely to the reallocation of capital between sectors. The impact of the increase of the labor supply on the output terms of the income equation is positive. The export term will be positive if the terms of trade improve:

$EX_2\frac{dE(P)}{dL} > 0$ and negative if they decline: $EX_2\frac{dE(P)}{dL} < 0$. Income will decline if and only if the expected terms of trade deteriorate and the export term overwhelms the output terms.

In the long run as labor becomes somewhat mobile there is greater adjustment possible in the output mix. As in a certain environment the sign of the slope of the Rybczynski loci depends upon the relationship of the elasticities of factor substitution and the elasticity of labor mobility terms in the numerator. Recall

$$\frac{dX_1}{dX_2} = \frac{X_1}{X_2}\left[\frac{\Theta K_2 \Theta L_1 \sigma_1 - \lambda K_2 |\Theta| \varepsilon}{\Theta K_1 \Theta L_2 \sigma_2 + \lambda K_1 |\Theta| \varepsilon}\right] \lesseqgtr 0$$

The denominator has a positive sign. This is the same result as found in the deterministic environment. Uncertainty here is reflected in the size of the elasticity of labor mobility. It will be smaller than in an environment where the future price ratio is known. Consequently, adjustment of the two outputs will be smaller, too. The reallocation of labor predicted by the Rybczynski Theorem will be less likely than in a certain environment.

For a small country, in the long run, income will increase. For a large country, income will also increase unless growth causes a deterioration in the terms of trade which overcomes the positive impact on output of the increased labor supply.

The following conclusion can be drawn:

Proposition 3.3: Uncertainty about the future terms of trade does not reverse the findings for large and small economies with immobile labor experiencing an increase in the labor supply in a certain environment.

Next we will turn to a situation where labor is held constant and the supply of capital increases and take it through the four part analysis.

Increase in the Supply of Capital

Part 1 Mobile Factors in a Certain Environment

Given constant output prices and a fixed supply of labor, the Rybczynski Theorem predicts that an increase in the supply of capital will cause an increase in the output of X_1 which is capital intensive, and a decline in the output of X_2 which is labor intensive. This can be confirmed using the output terms based on Casas analysis.[19]

$$\frac{dX_1}{dK} = \frac{X_1}{K}\left[\frac{\Theta L_2 \Theta K_1 \sigma_1 + \lambda L_2 |\Theta| \varepsilon}{\lambda L_1 \Theta K_2 \sigma_1 + \lambda L_2 \Theta K_1 \sigma_2}\right] > 0$$

All the values in the denominator are positive as are both the elasticity of factor substitution term ($\Theta L_2 \Theta K_1 \sigma_1$) and the elasticity of labor mobility term ($\lambda L_2 |\Theta| \varepsilon$) in the numerator. Also the value inside the brackets is greater than one because the elasticity of labor mobility is infinite. This is the magnification effect.

$$\frac{dX_2}{dK} = \frac{X_2}{K}\left[\frac{\Theta L_1 \Theta K_2 \sigma_2 - \lambda L_1 |\Theta| \varepsilon}{\lambda L_1 \Theta K_2 \sigma_1 + \lambda L_2 \Theta K_1 \sigma_2}\right] < 0$$

As above, the denominator is positive. The numerator is negative because the elasticity of labor mobility is infinite. The Rybczynski loci, then has a negative slope

$$\frac{dX_1}{dX_2} = \frac{X_1}{X_2}\left[\frac{\Theta L_2 \Theta K_1 \sigma_1 + \lambda L_2 |\Theta| \varepsilon}{\Theta L_1 \Theta K_2 \sigma_2 - \lambda L_1 |\Theta| \varepsilon}\right] < 0$$

Income increases when the supply of capital increases because the increase in sector 1 output is greater than the decline in the output in sector 2. X_1 receives all the new capital plus some labor and capital shift from sector 2. Recall the income equation for a small country (2.16) and differentiate with respect to an increase in the supply of capital

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[19] Casas, op.cit., p. 758-9.

$$\frac{dY}{dK} = \frac{dX_1}{dK} + P\frac{dX_2}{dK} > 0. \tag{3.16}$$

Since the population is assumed constant, when income increases it follows that welfare also increases.

This change in the output mix: increase in the output of importables (X_1) and decrease in the output of exports (X_2) is ultra antitrade biased growth as defined by Chacholiades. This will have an impact on the terms of trade if it takes place in a large country. It decreases the growing country's absolute demand for imports and reduces its supply of exports. Since the rest of the world's demand for imports and supply of exports are assumed constant at the initial terms of trade, there is an excess demand for the growing country's exports (X_2) and an excess supply of its importables (X_1) causing the relative price of its exports to rise: $\frac{dP}{dK} > 0$. For a large country, then, income increases because the output terms sum to a positive: $\frac{dX_1}{dK} + P\frac{dX_2}{dK} > 0$ and the export term is positive $EX_2\frac{dP}{dK} > 0$. There is an unambiguous increase in income.

$$\frac{dY}{dK} = \frac{dX_1}{dK} + P\frac{dX_2}{dK} + EX_2\frac{dP}{dK} > 0. \tag{3.17}$$

Also, the marginal improvement in income will be greater for a large country than for a small one because of the terms of trade effect.

Moving on let us drop the assumption of mobile labor and analyze the impact of an increase in the supply of capital in both the short run and long run.

Part 2 Short Run Immobility of Labor; Long Run Partial Mobility in a Certain Environment

In the short run labor is immobile by assumption; so, the elasticity of labor mobility is zero: $\varepsilon = 0$. Using Casa's equations (27) and (28) and adapting them to the assumptions of

this paper, it can be seen that both outputs will increase but the magnification effect is lost in sector 1.[20]

$$\frac{dX_1}{dK} = \frac{X_1}{K}\left[\frac{\Theta L_2 \Theta K_1 \sigma_1}{\lambda L_1 \Theta K_2 \sigma_1 + \lambda L_2 \Theta K_1 \sigma_2}\right] > 0$$

$$\frac{dX_2}{dK} = \frac{X_2}{K}\left[\frac{\Theta L_1 \Theta K_2 \sigma_2}{\lambda L_1 \Theta K_2 \sigma_1 + \lambda L_2 \Theta K_1 \sigma_2}\right] > 0$$

This replicates Casas's findings that the possibility that X_2 may increase is greater the smaller the elasticity of labor mobility (ε) is relative to the elasticity of factor substitution (σ_2) in sector 2. The slope of the Rybczynski loci is positive confirming the above finding that both outputs increase:

$$\frac{dX_1}{dX_2} = \frac{X_1}{X_2}\left[\frac{\Theta L_2 \Theta K_1 \sigma_1}{\Theta L_1 \Theta K_2 \sigma_2}\right] > 0$$

To examine this more closely, recall the production functions

$$X_1 = F_1\ (K_1,L_1) \tag{2.1}$$

$$X_2 = F_2\ (K_2,L_2) \tag{2.2}$$

Differentiating with respect to the increase in capital, we find that the prediction based on Casas's formulae does not hold for X_2.

$$\frac{dX_1}{dK} = \frac{\partial X_1}{\partial K} + \frac{\partial X_1}{\partial K_1}\frac{dK_1}{dK} > 0$$

$$\frac{dX_2}{dK} = \frac{\partial X_2}{\partial K_2}\frac{dK_2}{dK} < 0$$

Labor will not be reallocated: $\frac{dL_1}{dK} = \frac{dL_2}{dK} = 0$. Capital, however, is free to move.

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[20] *Ibid*, p 758.

For a small country, income increases

$$\frac{dY}{dK} = \frac{dX_1}{dK} + P\frac{dX_2}{dK} > 0$$

but, because labor cannot be reallocated, the increase is smaller than when labor is mobile.

The impact on a large country's income is less clear. The sign of the export term $\left(EX_2\frac{dP}{dL}\right)$ depends upon the direction of the change in the terms of trade. With both outputs increasing the price ratio could increase, decrease or not change depending upon if one output increases more than the other. If X_1 increases more than X_2 or X_2 declines, the terms of trade improve: $\frac{dP}{dK} > 0$. If X_2 increases more than X_1 the terms of trade deteriorate: $\frac{dP}{dK} < 0$. Because capital is mobile this is unlikely; however, output is also dependent upon the elasticities of factor substitution; so, if $\sigma_2 > \sigma_1$, that outcome is possible.

Recall income equation:

$$\frac{dY}{dK} = \frac{dX_1}{dK} + P\frac{dX_2}{dK} + EX_2\frac{dP}{dK}.$$

If the terms of trade deteriorate, income may still increase if the primary impact of the increase in capital on the two outputs is greater than the negative export term.

In the long run labor becomes somewhat mobile which is reflected in the Casas equations (27) and (28).

$$\frac{dX_1}{dK} = \frac{X_1}{K}\left[\frac{\Theta L_2 \Theta K_1 \sigma_1 + \lambda L_2 |\Theta| \varepsilon}{\lambda L_1 \Theta K_2 \sigma_1 + \lambda L_2 \Theta K_1 \sigma_2}\right] > 0$$

$$\frac{dX_2}{dK} = \frac{X_2}{K}\left[\frac{\Theta L_1 \Theta K_2 \sigma_2 - \lambda L_1 |\Theta| \varepsilon}{\lambda L_1 \Theta K_2 \sigma_1 + \lambda L_2 \Theta K_1 \sigma_2}\right] \lesseqgtr 0$$

The impact on X_1 when the supply of capital increases and labor has limited but positive mobility is unambiguously positive. The impact on X_2 is not as clear. It depends upon the relative sizes of the elasticity of factor substitution term ($\Theta L_1 \Theta K_2 \sigma_2$), which would be responsible for increasing the output of X_2, and the elasticity of labor mobility term ($\lambda L_1 |\Theta| \varepsilon$) which would cause the output of X_2 to decline because labor could migrate to

X_1. The larger the elasticity of labor mobility, the more likely that the output of X_2 will decline and the Rybczynski Theorem will hold.

The long run impact on income for a small country is positive; so, welfare improves.

$$\frac{dY}{dK} = \frac{dX_1}{dK} + P\frac{dX_2}{dK} > 0$$

This is true regardless of the sign of $\frac{dX_2}{dK}$ since even if it is negative, its absolute value is smaller than the increase in sector 1 output.

For a large country there is the additional export term in its income equation.

$$\frac{dY}{dK} = \frac{dX_1}{dK} + P\frac{dX_2}{dK} + EX_2\frac{dP}{dK}.$$

Examining the terms more closely:

$$\frac{dX_1}{dK} = \frac{\partial X_1}{\partial K_1}\frac{dK_1}{dK} + \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial K} + \frac{\partial X_1}{\partial P}\frac{dP}{dK} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{d\mu}{dK} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{dK} \qquad (3.18)$$

$\frac{\partial X_1}{\partial K_1}\frac{dK_1}{dK} > 0$. This represents the primary impact of the increase in the capital supply. Regardless of how immobile labor is, when the supply of capital increases it will flow into the capital intensive sector; so, $\frac{dK_1}{dK} = 1$, given no change in the output price.

$\frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial K} > 0$. This represents the reallocation of capital from X_2 to X_1 because X_1 is the expanding sector, given no change in the output price.

$\frac{\partial X_1}{\partial P}\frac{dP}{dK} \lesseqgtr 0$. This is the impact of the growth induced change in the terms of trade. If the price ratio increases which is the more likely case, the relative price of X_1 declines and the term is negative. If the price ratio decreases, the term is positive. The secondary and

tertiary reallocation effects on labor may also move in opposite directions if the terms of trade improve. To summarize, the impact of an increase in the supply of capital will cause the output of sector one to increase as long as the primary and reallocation effects are large enough to overcome an increase in the price ratio, should that occur.

Turning to sector 2:

$$\frac{dX_2}{dK} = \frac{\partial X_2}{\partial K_2}\frac{dK_2}{dK} + \frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial K} + \frac{\partial X_2}{\partial P}\frac{dP}{dK} + \frac{\partial X_2}{\partial K_2}\frac{\partial L_2}{\partial \mu}\frac{d\mu}{dK} + \frac{\partial K_1}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{dK} \quad (3.19)$$

The primary impact will be zero if all new capital goes to X_1 and the reallocation effects will be negative as capital and labor migrate to sector 1. This is what the Rybczynski Theorem would predict. However, prices are not constant. If the impact of the change in the terms of trade is positive, as would occur should the price ratio improve, the most likely case, then those terms will be positive.

Output in sector 2 will decline to the extent that the growth reallocation effects are larger, in absolute terms, than the price reallocation effects..

The sign of the export term: $EX_2\frac{dP}{dK}$ will depend upon which way the terms of trade move. If the price ratio increases, the export term is positive. If the price ratio declines, the export term is negative.

Returning to the evaluation of income, if the terms of trade improve, income will increase if the output adjustment in X_1 plus the export term are large enough to offset the negative impact of the price effect on X_1 and any decline in the output of X_2.

Proposition 3.4 For a small country, experiencing an increase in the capital supply,with less than perfect labor mobility, the greater the mobility of labor the larger the increase in income.

The analysis will continue by dropping the assumption that the future terms of trade are known.

Part 3 Mobile Factors in an Uncertain Environment

Given uncertainty about the future terms of trade, the question to be addressed is what impact that will have on the results in Part 1.

With labor being perfectly mobile the slope of the Rybczynski loci is negative:

$$\frac{dX_1}{dX_2} = \frac{X_1}{X_2}\left[\frac{\Theta L_2 \Theta K_1 \sigma_1 + \lambda L_2 |\Theta| \varepsilon}{\Theta L_1 \Theta K_2 \sigma_2 - \lambda L_1 |\Theta| \varepsilon}\right] < 0$$

The output of X_1 increases; the output of X_2 decreases; so, even with an uncertain environment, if factors are mobile, the Rybczynski Theorem holds.

Income of a small country increases because X_1 not only acquires all of the new capital but also some capital and labor from X_2. Therefore, the increase in X_1 is greater than the decline in X_2.

A large country will experience an improvement in the terms of trade so the export term: $EX_2\frac{dP}{dL}$, will be positive. However the tertiary impact of that on outputs [based on (3.18), (3.19)]

$$\frac{\partial X_1}{\partial E(P)} \frac{\partial E(P)}{\partial K} < 0$$

$$\frac{\partial X_2}{\partial E(P)} \frac{\partial E(P)}{\partial K} > 0$$

will move the output ratio in the opposite direction from that predicted by the Rybczynski Theorem. While it will dampen the adjustment it will not necessarily reverse it. With the addition of the positive export term, it is highly likely that income and, therefore, welfare will increase.

The following conclusion may be drawn:

Proposition 3.5 An increase in the supply of capital in an economy where both factors are mobile, taking place in a environment where there is uncertainty about the future terms of trade, will not change the quality of the findings in a deterministic environment.

As discussed above, however, uncertainty about future prices can be a contributor to labor immobility.

Part 4 Short Run Immobility of Labor; Long Run Partial Mobility in an Uncertain Environment

As in a deterministic setting, when labor is immobile an increase in capital may cause both outputs to increase to the extent that the elasticities of factor substitution allow. The slope of the Rybczynski loci is positive

$$\frac{dX_1}{dX_2} = \frac{X_1}{X_2}\left[\frac{\Theta L_2 \Theta K_1 \sigma_1}{\Theta L_1 \Theta K_2 \sigma_2}\right] > 0$$

If both outputs increase, the Rybczynski Theorem does not hold.

For a small country income increases. Modifying the income equation (3.17) for uncertainty:

$$\frac{dY}{dK} = \frac{dX_1}{dK} + E(P)\frac{dX_2}{dK} > 0.$$

For a large country all three terms may be positive. Since the terms of trade are expected to move in its favor, the export term is positive: $EX_2\frac{dE(P)}{dK}$. The tertiary impact on the outputs due to the price change [based on (3.18) (3.19)] will be more important than when both factors are mobile because the output adjustment with immobile labor is much smaller. $\frac{\partial X_1}{\partial E(P)}\frac{dE(P)}{dK}$ is negative and, if it outweighs the primary and reallocation impacts of the increase in capital: $\frac{\partial X_1}{\partial K} + \frac{\partial X_1}{\partial K_1}\frac{dK_1}{dK}$ then output of X_1 would decline.

The tertiary impact on sector 2: $\frac{\partial X_2}{\partial E(P)}\frac{dE(P)}{dK}$ would reinforce any short run increase in X_2 due to immobile labor.

Income for a large country will increase if the positive export and sector 2 output terms plus a possible positive impact of the increase in capital on the output of sector 1 outweigh the growth induced price change effect on the output of sector 1.

$$\frac{dY}{dK}=\frac{dX_1}{dK}+E(P)\frac{dX_2}{dK}+EX_2\frac{dE(P)}{dK}>0$$

In the long run labor becomes somewhat mobile, however given uncertainty about the future terms of trade, the elasticity of labor mobility will be smaller than in a certain environment.

There is greater adjustment of the output mix than in the short run. The sign of the slope of the Rybczynski loci depends upon the relationship of the elasticity of factor substitution and the elasticity of labor mobility terms in the denominator.

$$\frac{dX_1}{dX_2}=\frac{X_1}{X_2}\left[\frac{\Theta L_2\Theta K_1\sigma_1+\lambda L_2|\Theta|\varepsilon}{\Theta L_1\Theta K_2\sigma_2-\lambda L_1|\Theta|\varepsilon}\right]$$

The greater the elasticity of labor mobility, the more likely that output of sector 2 declines and the Rybczynski Theorem holds. This is the same result as in a deterministic setting. However, it can be said that over a given period of time (the long run) the Rybczynski Theorem is more likely to hold in a certain setting than in uncertainty because of the difference in the elasticity of labor mobility.

For a small country, income increases even if the output in sector 2 declines; so, welfare improves. For a large country there is the additional favorable impact of the improvement in the expected terms of trade; so, as explained above, income for it is also likely to improve. Recall from Part 2 that in a certain environment income could increase regardless of the size of the country and for the same reasons. Therefore the following conclusion holds:

Proposition 3.6: An increase in the supply of capital with some degree of labor immobility under uncertainty about the future terms of trade, will not reverse the findings in a deterministic setting.

Next the analysis focuses on the impact of neutral technological change.

Neutral Technological Progress

The third type of growth is based on neutral technological progress (δ) which may occur in one or both of the sectors. It is disembodied, a Hicksian neutral growth, which involves improving organization and skills and is independent of the age and type of capital the economy has.

Recall the production on functions:

$$X_1 = \delta_1 F_1(K_1,L_1) \tag{3.1}$$

$$X_2 = \delta_2 F_2(K_2,L_2) \tag{3.2}$$

which can also be written as:

$$X_1 = \delta_1 L_1 f_1(k_1) \tag{3.1'}$$

$$X_2 = \delta_2 L_2 f_2(k_2) \tag{3.2'}$$

The introduction of this technological progress affects marginal productivity; so, the new factor market equilibria are:

$$\delta_1 \, (f_1 - k_1 f'_1) = P\delta_2 \, (f_2 - k_2 \, f'_2) = w \tag{3.20}$$

$$\delta_1 f'_1 = P\delta_2 f'_2 = r$$

The factor price ratio is not affected.

First we will consider the case where technological progress affects both inputs equally; the impact on the two sectors is equivalent. The output ratio is constant. The product transformation curve shifts out in a parallel manner. The capital labor ratio remains constant. The marginal rate of technical substitution between labor and capital is the same before and after the technical progress takes place so factor intensities do not change. Thereafter we will go on to consider the cases where this technological progress occurs in each of the sectors while not in the other. Let us begin with perfect mobility of factors in a certain environment.

Part 1 Mobile Factors in a Certain Environment

When both sectors experience equivalent technical progress (i.e. $\delta_1 = \delta_2 > 1$), the impact on both outputs is positive. Differentiating the production functions (3.1') (3.2') with respect to technical progress will show what happens.

$$\frac{dX_1}{d\delta} = \delta_1 L_1 f'_1 \frac{dk_1}{d\delta_1} + \delta_1 f_1 \frac{dL_1}{d\delta_1} + L_1 f_1 \frac{d\delta_1}{d\delta_1}$$

Looking at each term to evaluate it:

$\delta_1 L_1 f'_1 \frac{dk_1}{d\delta_1} = 0$ Since the price ratio is constant and equivalent technical progress occurs in both sectors, factor intensities do not change because the output ratio remains constant without factor reallocation.

$\delta_1 f_1 \frac{dL_1}{d\delta_1} = 0$ The supply of labor is fixed by assumption, and labor does not shift between sectors because factor intensities do not change.

$L_1 f_1 \frac{d\delta_1}{d\delta_1} = L_1 f_1 > 0$

The change in output in sector 1, when it experiences neutral technical progress equals the production function relating the output to the inputs:

$$\frac{dX_1}{d\delta_1} = L_1 f_1 = F_1$$

A similar operation of sector 2 output with respect to technical progress yields:

$$\frac{dX_2}{d\delta_2} = L_2 f_2 = F_2$$

Since both outputs increase, income for a small country will increase. Totally differentiating the income equation (2.16) with respect to technical progress: $\delta_1 = \delta_2 = \delta$

$$\frac{dY}{d\delta} = \frac{dX_2}{d\delta_2} + P\frac{dX_2}{d\delta_2} > 0 \tag{3.22}$$

with a constant population, an increase in income improves welfare.

If the country is large, this type of technical progress occuring in both sectors may affect the country's demand for imports: $IM_1 = IM_1\ (P, Y, \delta_1)$

Recall: $IM_1 = Z_1\ (P, Y(\delta_1)) - X_1\ (P, \delta_1)$

Differentiating with respect to technical progress with the terms of trade held constant:

$$\frac{dIM_1}{d\delta_1} = \frac{\partial Z_1}{\partial Y}\frac{\partial Y}{\partial \delta_1} - \frac{\partial X_1}{\partial \delta_1}.$$

The income elasticity of demand for X_1 will be positive since it is a normal good. The output of sector 1 will increase due to technical progress in that sector. The impact on imports depends upon the relative strength of the increase in consumption $\frac{\partial Z_1}{\partial Y}\frac{\partial Y}{\partial \delta}$ and the increase in output of X_1, $\frac{\partial X_1}{\partial \delta_1}$.

If $\frac{\partial Z_1}{\partial Y}\frac{\partial Y}{\partial \delta_1} > \frac{\partial X_1}{\partial \delta_1}$, then imports will increase.

If $\frac{\partial Z_1}{\partial Y}\frac{\partial Y}{\partial \delta_1} = \frac{\partial X_1}{\partial \delta_1}$, then there is no change in imports.

If $\frac{\partial Z_1}{\partial Y}\frac{\partial Y}{\partial \delta_1} < \frac{\partial X_1}{\partial \delta_1}$, then imports will decline.

Turning to exports, technical progress may also have an impact: $EX_2 = EX_2\ (P, Y, \delta_2)$.

Recall $EX_2 = X_2\ (P, \delta_2) - Z_2\ (P, Y(\delta_2))$.

Differentiating with respect to technical progress with the terms of trade held constant:

$$\frac{dEX_2}{d\delta_2} = \frac{\partial X_2}{\partial \delta_2} - \frac{\partial Z_2}{\partial Y}\frac{\partial Y}{\partial \delta_2}.$$

Technical progress in sector 2 will cause output in that sector to increase. The income elasticity of demand for X_2 is positive because X_2 is a normal good. The impact on exports depends upon the relative strength of the increase in output of X_2 and the increase of consumption of X_2 .

If $\frac{\partial X_2}{\partial \delta_2} > \frac{\partial Z_2}{\partial Y}\frac{\partial Y}{\partial \delta_2}$, exports will increase.

If $\frac{\partial X_2}{\partial \delta_2} = \frac{\partial Z_2}{\partial Y}\frac{\partial Y}{\partial \delta_2}$, exports will not change.

If $\frac{\partial X_2}{\partial \delta_2} < \frac{\partial Z_2}{\partial Y}\frac{\partial Y}{\partial \delta_2}$, exports will decrease.

Recall that a balance of trade is assumed so:

$$\frac{dIM_1}{d\delta_1} = P\frac{dEX_2}{d\delta_2}$$

In both imports and exports increase $\left[\frac{dIM_1}{d\delta_1} = P\frac{dEX_2}{d\delta_2} > 0\right]$ that will result in a pro trade biased result as defined by Chacholiades. The excess supply of X_2 and excess demand for X_1 will cause the price ratio to decline. Now, incorporating the price effect:

$$\frac{dIM_1}{d\delta_1} = \frac{\partial Z_1}{\partial Y}\frac{\partial Y}{\partial \delta_1} + \frac{\partial Z_1}{\partial P}\frac{dP}{d\delta_1} - \left[\frac{\partial X_1}{\partial \delta_1} + \frac{\partial X_1}{\partial P}\frac{dP}{d\delta_1}\right]$$

Technical progress in X_1 causes a deterioration in the terms of trade: $\frac{dP}{d\delta_1} < 0$. An increase in the relative price of P_1 causes a decrease in consumption of X_1 : $\frac{\partial Z_1}{\partial P} < 0$ so $\frac{\partial Z_1}{\partial P}\frac{dP}{d\delta_1}$, the induced deterioration in the terms of trade has a negative impact on imports. It also causes an increase in output of X_1: $\frac{\partial X_1}{\partial P}\frac{dP}{d\delta_1} > 0$. Consumption of X_1 will increase only if

the income effect $\left(\frac{\partial Z_1}{\partial Y}\ \frac{\partial Y}{\partial \delta_1}\right)$ is greater than the induced price change effect: $\left(\frac{\partial Z_1}{\partial P}\ \frac{dP}{d\delta_1}\right)$. Output of X_1 will increase because the primary impact of technical progress $\left(\frac{\partial X_1}{\partial \delta_1}\right)$ is reinforced by the induced price change effect $\left(\frac{\partial X_1}{\partial P}\ \frac{dP}{d\delta_1}\right)$.

Accommodating exports to the induced change in the terms of trade

$$\frac{dEX_2}{d\delta_2} = \frac{\partial X_2}{\partial \delta_2} + \frac{\partial X_2}{\partial P}\frac{dP}{d\delta_2} - \left(\frac{\partial Z_2}{\partial Y}\frac{\partial Y}{\partial \delta_2} + \frac{\partial Z_2}{\partial P}\ \frac{dP}{d\delta_2}\right)$$

Retaining the assumption of a deterioration in the terms of trade: $\frac{\partial P}{\partial \delta_2} < 0$, that will cause a decline in output of sector 2, $\left(\frac{\partial X_2}{\partial P}\ \frac{dP}{d\delta_2} < 0\right)$ and an increase in consumption of X_2, $\left(\frac{\partial Z_2}{\partial P}\ \frac{dP}{d\delta_2} > 0\right)$. Exports will increase if the primary impact of technical progress $\left(\frac{\partial X_2}{\partial \delta_2}\right)$ outweighs the price effect on output and consumption and the income effect on consumption.

Reworking the income formula and remembering $PdEX_2 = dIM_1$, the trade adjustment term drops out so:

$$\frac{dY}{d\delta} = \frac{dX_1}{d\delta_1} + P\frac{dX_2}{d\delta_2} + EX_2\frac{dP}{d\delta}$$

The impact on the two outputs $\left(\frac{dX_1}{d\delta_1} + P\frac{dX_2}{d\delta_2}\right)$ is positive for the large country but the export term $\left(EX_2\ \frac{dP}{d\delta}\right)$ is negative because of the deterioration of the terms of trade. The impact of technical progress on income in this case will be positive as long as the increase in the two outputs outweighs the negative export term.

If there is no change in imports or exports because the increase in consumption in each sector caused by the rise in income from technical progress is exactly offset by the increase in output in each sector, no excess demand for one good and/or excess supply of the other will induce a movement in the terms of trade:

$$\frac{\partial Z_1}{\partial Y}\frac{\partial Y}{\partial \delta_1} = \frac{\partial X_1}{\partial \delta_1}$$

$$\frac{\partial X_2}{\partial \delta_2} = \frac{\partial Z_2}{\partial Y}\frac{\partial Y}{\partial \delta_2}$$

Therefore: $\frac{dIM_1}{d\delta_1} = \frac{dEX_2}{d\delta_2} = 0.$

Although this is theoretically possible for a large country, it is highly unlikely. The income formula, then, would be entirely dependent upon the primary impact of technical progress, as with a small country. There is an unambiguous improvement in income.

$$\frac{dY}{d\delta} = \frac{dX_1}{d\delta_1} + P\frac{dX_2}{d\delta_2} > 0$$

Moving on, let us assume the income effect of technical progress is less than the increase in output in sector 1 due to technical progress:

$$\frac{\partial Z_1}{\partial Y}\frac{\partial Y}{\partial \delta_1} < \frac{\partial X_1}{\partial \delta_1}$$

Imports will decline. In addition, if the increase in consumption in X_2, due to the income effect of technological progress is greater than the increase in output in sector 2 from technical progress, exports will decline:

$$\frac{\partial Z_2}{\partial Y}\frac{\partial Y}{\partial \delta_2} > \frac{\partial X_2}{\partial \delta_2}$$

Then with constant terms of trade there is an excess demand for X_2 and an excess supply of X_1, causing in turn an increase in the price ratio: $\frac{\partial P}{\partial \delta} > 0.$

Adding this induced price change to the import formula

$$\frac{dIM_1}{d\delta_1} = \frac{\partial Z_1}{\partial Y}\frac{\partial Y}{\partial \delta_1} + \frac{\partial Z_1}{\partial P}\frac{dP}{d\delta_1} - \left[\frac{\partial X_1}{\partial \delta_1} + \frac{\partial X_1}{\partial P}\frac{dP}{d\delta_1}\right]$$

The induced price change will cause an increase in consumption of X_1 $\left(\frac{\partial Z_1}{\partial P}\frac{\partial P}{\partial \delta_1} > 0\right)$. It will also cause a decline in output of sector 1 $\left(\frac{\partial X_1}{\partial P}\frac{\partial P}{\partial \delta_1} < 0\right)$. Imports will decrease only if the combined impact on consumption and the price effect on output are outweighed by the increase in output from technical progress.

Turning to exports:

$$\frac{dEX_2}{d\delta_2} = \frac{\partial X_2}{\partial \delta_2} + \frac{\partial X_2}{\partial P}\frac{\partial P}{\partial \delta_2} - \left[\frac{\partial Z_2}{\partial Y}\frac{\partial Y}{\partial \delta_2} + \frac{\partial Z_2}{\partial P}\frac{\partial P}{\partial \delta_2}\right]$$

The improvement in the terms of trade will cause an increase in output $\left(\frac{\partial X_2}{\partial P}\frac{\partial P}{\partial \delta_2} > 0\right)$ and consumption will decline $\left(\frac{\partial Z_2}{\partial P}\frac{\partial P}{\partial \delta_2} < 0\right)$. Exports will decline only if the income effect on consumption outweighs the price effect on consumption and the combined impact on output.

Since $dIM_1 = dEX_2$, these terms drop out of the income equation leaving it with only positive terms: the increase in both outputs $\left(\frac{dX_1}{d\delta_1} + P\frac{dX_2}{d\delta_2}\right)$ and the export term which is positive because the price ratio increased $\left(EX_2\frac{dP}{d\delta}\right)$.

$$\frac{dY}{d\delta} = \frac{dX_1}{d\delta_1} + P\frac{dX_2}{d\delta_2} + EX_2\frac{dP}{d\delta} > 0$$

Next we will turn to the situation where technical progress occurs in only one sector, X_1 in this example. ($\delta_1 > 1$; $\delta_2 = 1$). Since technical progress has an equivalent impact on both inputs, the isoquants are simply renumbered to reflect higher outputs and the input price ratio does not change:

$$\frac{\delta_1 (f_1 - k_1 f'_1)}{\delta_1 f'_1} = \frac{w}{r} = \frac{f_2 - k_2 f'_2}{f'_2}$$

Therefore the contract curve does not shift because every point along the curve implies the same marginal rate of substitution between factors in both industries before as well as after the change. Hazari showed that the capital labor ratios in both sectors would change when technical progress occurred in only one sector.[21]

If it took place in the capital intensive sector (X_1) with constant output prices, it would cause a substitution of capital for labor in both sectors. The process can be explained in this way. By lowering the cost of production for X_1, technical progress would increase the

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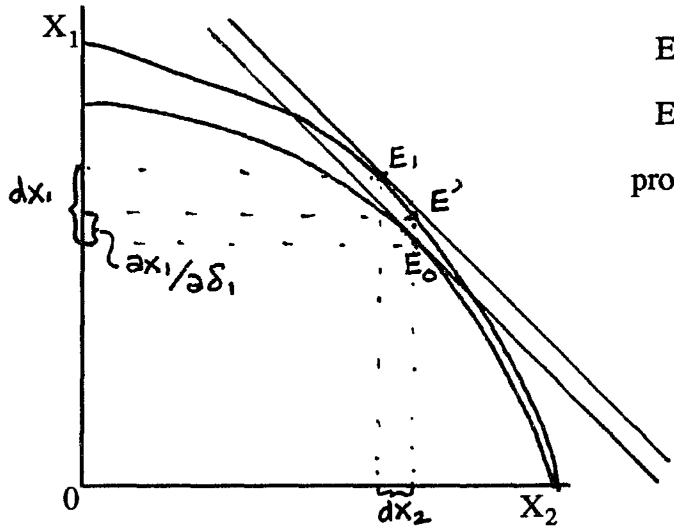
[21] Hazari, *op cit* p. 145

relative price of X_2. To maintain the constant output price ratio, there would have to be an adjustment in the factor price ratio so that the relative price of X_2 would remain at its original level. That would require the price of factors used in sector 1 to increase so that the price of X_1 rises to reestablish the original output price ratio. That would happen via an increase in the price of the factor intensive in sector 1: the return to capital would rise relative to the wage rate. Given the decline with the factor price ratio $\left(W = \frac{w}{r}\right)$, labor would be substitutied for capital causing the capital labor ratios in both sectors to decline. The product transformation curve shifts out as seen in Figure 3.2.



E_0 is the initial output mix

E_1 is the output mix after technical progress occurs in sector 1.

Figure 3.2 Technical Progress in X_1

For any point on the contract curve recall that the output of X_2 is the same before and after the change while the output of X_1 is greater. (On the graph that represents the shift from E_0 E'_1). Given the price ratio has not changed, output will move to E_1 not E'_1. This is true because at E'_1 the marginal rate of transformation is greater than the price ratio. Only if X_1 became cheaper relative to X_2 could the new output combination remain at E'_1. As Chacholiades showed,[22] when neutral technical progress occurs in one industry, the output

[22] M. Chacholiades, *op cit.* p. 355-356.

of the other industry declines absolutely. This can also be shown by using the income equation (3.22) and adjusting it for technical progress only in sector 1.

$$\frac{dY}{d\delta_1} = \frac{dX_1}{d\delta_1} + P\frac{dX_2}{d\delta_1}$$

Examining these terms more closely:

$$\frac{dX_1}{d\delta_1} = FK_1 \frac{dK_1}{d\delta_1} + FL_1 \frac{dL_1}{d\delta_1} + \frac{d\delta_1}{d\delta_1}$$

$FK_1 \frac{dK_1}{d\delta_1} + FL_1 \frac{dL_1}{d\delta_1}$ These input terms are both positive and represent the migration of factors from X_2 to X_1 as the economy adjusts to the new output mix where the marginal rate of transformation equals the price ratio [E'_1 to E_1 on Figure 3.2].

$\frac{d\delta_1}{d\delta_1}$. This is also positive and represents the pure expansion of output due to technical progress.

It can be seen, then, that the output of X_1 will increase by more than the technical progress.

Moving on to sector 2:

$\frac{dX_2}{d\delta_1} = FK_2 \frac{dK_2}{d\delta_1} + FL_2 \frac{dL_2}{d\delta_1}$. This is unambiguously negative as capital and labor migrate from X_2 to X_1 .

The impact on income will be unambigously positive.

If technical progress occurs in sector 2 but not sector 1($\delta_1 = 1$, $\delta_2 > 1$), then output of X_2 will increase because of the technical progress and the factors from X_1 . The output of X_1 will decline. In this case welfare will also increase because the impact on income is positive.

For a large country, if technological progress occurs in sector 1, increasing the supply of X_1, that would cause an excess world supply forcing down its price and resulting in the improvement in the terms of trade $\frac{dP}{d\delta_1} > 0$. Adding that to the income formula it becomes:

$$\frac{dY}{d\delta} = \frac{dX_1}{d\delta_1} + P\frac{dX_2}{d\delta_2} + EX_2\frac{dP}{d\delta_1}$$

This adds a tertiary impact on sector one: the growth induced change in the price ratio:

$\frac{dX_1}{d\delta_1} = 1 + FK_2\frac{dK_1}{d\delta_1} + FL_2\frac{dL_1}{d\delta_1} + \frac{\partial X_1}{\partial P}\frac{dP}{d\delta_1}$, as shown above the primary impact of technical progress and the reallocation effect are both positive, but the improvement in the terms of trade cause a negative impact on the output of X_1 because factors would move to higher priced output.

$\frac{dX_2}{d\delta_1} = FK_2\frac{dK_2}{d\delta_1} + FL_2\frac{dL_2}{d\delta_1} + \frac{\partial X_2}{\partial P}\frac{\partial P}{\partial \delta_1}$. The reallocation effect is negative as shown above; however, the improvement in the terms of trade has a positive impact on X_2. The induced change in the price ratio has blunted the impact of technical progress on the output combination.

$EX_2\frac{dP}{d\delta_1}$. The improvement in the terms of trade cause this export term to be positive.

If technical progress occurs in sector 2 there will be an excess supply of X_2 at the original terms of trade causing the price ratio to decline: $\frac{dP}{d\delta_2} < 0$.

Examining the terms of the income equation:

$$\frac{dY}{d\delta_2} = \frac{dX_1}{d\delta_2} + P\frac{dX_2}{d\delta_2} + EX_2\frac{dP}{d\delta_2}$$

$\frac{dX_1}{d\delta_2} = FK_1\frac{dK_1}{d\delta_2} + FL_2\frac{dL_1}{d\delta_2} + \frac{\partial X_1}{\partial P}\frac{dP}{d\delta_2}$,

The input terms are negative as seen above. The impact of the price change is positive on output of sector 1.

$\frac{dX_1}{d\delta_2} = 1 + FK_2\frac{dK_2}{d\delta_2} + FL_2\frac{dL_2}{d\delta_2} + \frac{\partial X_2}{\partial P}\frac{dP}{d\delta_2}$. The primary impact and the reallocation impact of technical progress is positive. The impact of the price change is negative. As above, the growth induced price change blunts the effect of technical progress in sector 2 on the output ratio.

$EX_2\frac{dP}{d\delta_2}$ The export term is negative because the price ratio declines.

In summary, given the induced change in the terms of trade, income is more likely to increase when there is technical progress in sector 1 than in sector 2 because the terms of trade improve when the progress occurs in the sector producing importables.

Next we will drop the assumption of mobile labor and determine the impact of technical progress in that context.

Part 2 Short Run Immobility of Labor; Long Run Partial Mobility in a Certain Environment

As explained earlier, in the short run labor is completely immobile because a developing wage differential will not yet have reached its critical value, beyond which labor begins to move.

Beginning with the case where equivalent technical progress occurs in both sectors ($\delta_1 = \delta_2 = \delta$), this type of growth leaves the output ratio unchanged, which implies that there is no reason for reallocation of inputs.

Since growth has no impact on the terms of trade for a small country, immobility will have no impact on welfare when this type of growth takes place, both in the short run and long run.

For a large country a growth induced improvement in the terms of trade will cause a change in the output mix which will require reallocation of inputs. To induce labor to move, a differential will develop: $\frac{d\mu}{dP} > 0$

Looking back to the production funtions (3.1) (3.2) and differentiating with respect to the differential:

$$\frac{\partial X_1}{\partial \mu} = FK1\frac{\partial K_1}{\partial \mu}$$

$$\frac{\partial X_2}{\partial \mu} = FK1\frac{\partial K_2}{\partial \mu}$$

The impact on X_1 is negative if the terms of trade improve and positive if the price ratio declines. The impact on X_2 is positive if the terms of trade improve and negative if the price ratio declines. Even though the wage differential is developing, labor is still immobile:

$$\frac{\partial L_1}{\partial \mu} = \frac{\partial L_2}{\partial \mu} = 0.$$

Since the supply of capital is fixed: $dK_1 = -dK_2$.

To integrate the developing wage differential into the analysis of technical progress let us rewrite output equations from the income formula (3.22)

$$\frac{dX_1}{d\delta_1} = \frac{\partial X_1}{\partial \delta_1} + \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial P}\frac{dP}{d\delta_1} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_1} \qquad (3.23)$$

$\frac{\partial X_1}{\partial \delta_1}$. represents the primary impact of the technical progress and, of course, is positive.

$\frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial P}\frac{dP}{d\delta_1}$, shows the movement of capital in response to a growth induced change in the terms of trade. It is negative if the terms of trade improve and positive if they deteriorate.

$\frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_1}$ shows the impact of the developing wage differential.

However, since labor is immobile, $\partial L_1 = 0$. Therefore the whole term drops out. Immobility of labor is demonstrated by the absence of a labor reallocation term.

Turning to sector 2:

$$\frac{dX_2}{d\delta_2} = \frac{\partial X_2}{\partial \delta_2} + \frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial P}\frac{dP}{d\delta_2} + \frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_2} \tag{3.24}$$

$\frac{\partial X_2}{\partial \delta_2}$ represents the primary impact of technical progress and is positive.

$\frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial P}\frac{dP}{d\delta_2}$ shows the capital reallocation term and it is positive if the terms of trade improve and negative if they decline.

$\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_2}$ shows labor reallocation term drops out because in the short run labor remains immobile in spite of the developing differential.

Outputs respond to technical progress and reallocation of capital in response to a change in the price ratio, but since labor is immobile the adjustment in the output ratio will not be as great as in Part 1 when labor was mobile.

The sign of the export term: $EX_2\frac{dP}{d\delta}$ will depend upon the direction of the change in the price ratio. If the terms of trade improve, the export term will be positive. If they deteriorate, the term is negative.

In the short run with immobile labor, the welfare of a large country experiencing equivalent technical progress in both sectors is more likely to improve if the terms of trade improve. Recall from Part 1, that an improvement in the terms of trade will occur when the income elasticity of demand for exportables (Z_2) is greater than the income elasticity of demand for importables (Z_1).

Let us move on to the situation where technical progress occurs only in sector 1 ($\delta_1 > 1$, $\delta_2 = 1$). As shown in Part 1, with mobile factors some of labor and capital would migrate

to that sector since output from X_2 would absolutely decline. With immobile labor only capital will adjust. Evaluating the income formula (3.22)

$$\frac{dX_1}{d\delta_1}=\frac{\partial X_1}{\partial \delta_1}+\frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial \delta_1}+\frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial \delta_1}+\frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial P}\frac{dP}{d\delta_1}+\frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_1} \quad (3.25)$$

$\frac{\partial X_1}{\partial \delta_1}$ This represents the increase in output from technical progress.

$\frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial \delta_1}$ This represents the reallocation of capital from sector 2

$\frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial \delta_1}$. Since labor is immobile this term drops out.

$\frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial P}\frac{dP}{d\delta_1}+\frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_1}$. These terms represent the reallocation of factors due to a growth induced change in the terms of trade. For a small country they do not appear. As shown in Part 1, technical progress in sector 1 will improve the large country's terms of trade. However only the capital reallocation term will appear. It will be negative. The labor reallocation term will drop out.

Turning to sector 2

$$\frac{dX_2}{d\delta_1}=\frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial \delta_1}+\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial \delta_1}+\frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial P}\frac{dP}{d\delta_1}+\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_1} \quad (3.26)$$

$\frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial \delta_1}+\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial \delta_1}$. They represent the reallocation of factors from X_2 to X_1. The capital term is negative. The labor term drops out because of the immobililty of labor.

$\frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial P}\frac{dP}{d\delta_1}+\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{\partial P}{\partial \delta_1}$ These represent the reallocation of factors due to the growth induced improvement in the terms of trade. The capital term is positive, but, here again, the labor term drops out. For a small country these terms do not appear.

For a large county the income formula also includes an export term which is positive when the terms of trade improve: $EX_2\frac{dP}{d\delta_2}$

Immobility of labor interferes with the adjustment of the output mix in response to technical progress in one sector and the induced change in the price ratio but does not reverse it.

In the case where technical progress occurs in the export sector (X_2) the same form of analysis applies. For output of sector 2 it will increase by more than the decline in sector 1 because of technical progress. The terms of trade, however, can be expected to move against a large country since there is an excess supply of its exports (X_2) and excess demand for its imports (X_1) at the original price ratio. As a result, the export term: $EX_2\frac{dP}{d\delta_2}$ is negative.

As seen in a situation where both factors are mobile; so, it is true here. The increase in income will be greater for a large country if technical progress occurs in its import competing industry (X_1) than in its export industry (X_2) since in the first instance the terms of trade will move in favor of the growing country. To a small country such a difference does not exist.

Let us turn now to the long run. The wage differential has reached the critical value and gone beyond it causing some labor to move from the lower wage sector to the higher wage sector because the wage they were receiving fell below their reservation wage. The development of the differential slows down and stabilizes when an equilibrium output mix is attained. As shown in the basic model this will be on a lower product transformation curve because of the wage differential.

Beginning with the case where equivalent technical progress occurs in both sectors ($\delta_1 = \delta_2 = \delta > 1$), as discussed earlier, immobility has no impact for a small country. For a large country, with a growth induced change in the price ratio, reallocation of factors will take place. Reviewing the output equations presented in the short run:

$$\frac{dX_1}{d\delta_1} = \frac{\partial X_1}{\partial \delta_1} + \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial P}\frac{dP}{d\delta_1} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_1} \qquad (3.23)$$

$\frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_1}$, In the long run the labor reallocation term does not drop out. The sign depends upon the direction of that price change. If the terms of trade improve, labor and

capital move out of the import substitution sector (X_1): If the price ratio declines, the reallocation terms are positive. Going on to sector 2:

$$\frac{dX_2}{d\delta_2} = \frac{\partial X_2}{\partial \delta_2} + \frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial P}\frac{dP}{d\delta_2} + \frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_2} \qquad (3.24)$$

Here again the labor reallocation term will have a value. The sign will depend upon the direction of the change in the price ratio. If the terms of trade improve, both factors will migrate into the export sector. If they deteriorate, the reallocation terms will be negative.

The sign of the export term will also depend upon the direction of the price change. An increase in the price ratio will cause the term to be positive; a decrease and the export term will be negtive.

This allows one to conclude the following:

Proposition 3.7 For a large country experiencing equivalent technical progress in both sectors, with some degree of labor immobility, the improvement in welfare will be greater if the terms of trade move in favor of the growing country than if they deteriorate.

It is the same result that was found when factors were assumed perfectly mobile. Immobility of labor dampens the impact of the technical progress induced price ratio change but does not reverse it.

Let us continue with the case where technical progress occurs in just one sector, in this example the import substitution sector (X_1), and see the long run impact of that technical progress on income. Referring back to the equations developed in the short run (3.25) (3.26):

$$\frac{dX_1}{d\delta_1} = \frac{\partial X_1}{\partial \delta_1} + \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial \delta_1} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial \delta_1} + \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial P}\frac{dP}{d\delta_1} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_1} \qquad (3.25)$$

Unlike the short run the labor reallocation terms have a value. The labor and capital terms showing the direct effect of technical progress in sector 1 are positive. The labor and capital terms showing the effect of the growth induced change in the price ratio are negative

because the terms of trade improve. Here can be seen two forces pulling the wage differential in opposite directions which will eventually contribute to its stability.

For a small country, however, there is no price effect so for the import competing sector, the primary impact and reallocation terms are positive resulting in an unambiguous increase in the output of sector 1. The equation (3.26) for output of sector 2 is

$$\frac{dX_2}{d\delta_1}=\frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial \delta_1}+\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial \delta_1}+\frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial P}\frac{dP}{d\delta_1}+\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_1} \qquad (3.26)$$

The price induced reallocation terms do not appear for a small country; so, there is an unambiguous decrease in output of the export sector. The adjustment of the output mix is greater with some labor mobility but in the same direction as in the short run. Income increases because the increase in output of sector 1 is greater than the decline in sector 2 since X_1 benefits from technical progress.

The situation in sector 2 for a large country is less clear. The terms representing reallocation of factors due to technical progress: $\frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial \delta_1}+\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{d\mu}{d\delta_1}$ are both negative as for the small country while the terms representing reallocation of factors due to the change in the price ratio are positive: $\frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial P}\frac{dP}{d\delta_1}+\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{d\delta_1}$. Output of X_2 may increase, decrease or not change depending upon the relative strengths of the two opposing forces.

Finally the large country has the export term $EX_2\frac{dP}{d\delta_1}$ in its income equation which is positive because the terms of trade improve. When the technical progress occurs in the export sector (X_2) the terms of trade deteriorate and the export term: $EX_2\frac{dP}{d\delta_2}$, will be negative.

This confirms the previous finding in the short run and allows the following conclusion:

Proposition 3.8 For a large country with limited mobility of labor, the improvement in welfare will be greater when technical progress occurs in the import competiting industry

than when it takes place in the export industry. This is due to the favorable turn in the price ratio when the output of import substitutes increases.

Now let us drop the assumption of certain terms of trade but reinstate mobility of both factors.

Part 3 Mobile Factors in an Uncertain Environment

Given uncertainty about the future terms of trade, the issue to be examined is whether this uncertainty reverses the findings in a certain environment.

To begin let us assume equivalent technical progress occurs in both sectors

$$\delta_1 = \delta_2 = \delta > 1$$

As in a certain environment both outputs would increase so that the output ratio does not change.

For a small country with no seconday impact on the expected terms of trade, the impact of technical progress on income is unambiguously positive.

For a large country there may be a change in the expected terms of trade. To analyse this, let us rewrite the income equation (3.22) to incorporate uncertainty:

$$\frac{dY}{d\delta} = \frac{dX_1}{d\delta_1} + E(P)\frac{dX_2}{d\delta_2} + EX_2\frac{dE(P)}{d\delta} \tag{3.27}$$

For sector 1: $\frac{dX_1}{d\delta_1} = \frac{\partial X_1}{\partial \delta_1} + \frac{\partial X_1}{\partial E(P)}\frac{dE(P)}{d\delta_1}$

The first term $\frac{\partial X_1}{\partial \delta_1}$, represents the primary impact of technical progress and is positive. The second term $\frac{\partial X_1}{\partial E(P)}\frac{dE(P)}{d\delta_1}$ shows the impact of the growth induced change in the expected price ratio. If the terms of trade are expected to deteriorate $\left(\frac{dE(P)}{d\delta_1} < 0\right)$ then the term is positive and there is an unambiguous increase in the output of X_1. If, however, the

future terms of trade are expected to improve then the price term is negative. While output of the import substitute might still increase, it will be less than in a situation where the terms of trade deteriorated.

For sector 2: $\frac{dX_2}{d\delta_2} = \frac{\partial X_2}{\partial \delta_2} + \frac{\partial X_2}{\partial E(P)} \frac{dE(P)}{d\delta_2}$

Again the impact of technical progress is positive $\left(\frac{\partial X_2}{\partial \delta_2} > 0\right)$. If, however, the terms of trade are expected to deteriorate, the price term is negative. If they improve, the term is positive. When the terms of trade are expected to improve that gives added impetus to the increase in output of the export sector.

The export term: $EX_2 \frac{dE(P)}{d\delta}$, will be positive if the terms of trade improve; negative if they deteriorate.

This allows the following conclusions:

Proposition 3.9 For a large country experiencing equivalent technical progress in both sectors facing an uncertain future terms of trade, there will be a greater increase in expected income if the expected terms of trade improve. Uncertainty about the future terms of trade has not affected the quality of the finding in a certain environment.

Likewise it can be readily ascertained that the results when technical progress occurs in one sector do not differ qualitatively from that in a certain environment.

Proposition 3.10 Regardless of which sector experiences technical progress, income may increase. However, when it occurs in the import competing industry, there is the added positive effect of the improvement in the expected terms of trade. Growth can be immiserizing only when it occurs in the export industry causing a deterioration in the expected terms of trade.

Let us now drop the assumption of mobile labor and reassess the impact on welfare of technical progress.

Part 4 Short Run Immobility of Labor; Long Run Partial Mobility in an Uncertain Environment.

As discussed above in some detail, uncertainty about the furture terms of trade can be a source of labor's reluctance to move from one sector to another. The larger the variance of the price ratio distribution the smaller the elasticity of labor mobility in the long run $\left(\frac{\partial \varepsilon}{\partial \sigma_p^2} < 0\right)$. In the short run, of course, labor is completely immobile by assumption: $\varepsilon = 0$.

To begin let us assume equivalent technical progress occurs in both sectors. For a small country with no seconday terms of trade effect, the output ratio will not adjust; so, immobility will have no impact in the short run or long run, just as in a certain environment.

For a large country, in the short run, there will probably be a secondary terms of trade effect. The economy, attempting to adjust to the new expected price ratio will have only one factor, capital which can be reallocated. Evaluating the terms of the expected income equation (3.27) in light of immobile labor:

$$\frac{dX_1}{d\delta_1} = \frac{\partial X_1}{\partial \delta_1} + \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial E(P)}\frac{dE(P)}{d\delta_1} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial E(P)}\frac{dE(P)}{d\delta_1}$$

The impact of technical progress is positive: $\frac{\partial X_1}{\partial \delta_1} > 0$. The capital reallocation term will be positive if the expected price ratio declines, and it will be negative if the expected price ratio increases. The labor reallocation term drops out because labor is immobile: $dL_1 = 0$.

$$\frac{dX_2}{d\delta_2} = \frac{\partial X_2}{\partial \delta_2} + \frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial E(P)}\frac{dE(P)}{d\delta_2} + \frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial E(P)}\frac{dE(P)}{d\delta_2}$$

Here also the impact of technical progress is positive: $\frac{\partial X_2}{\partial \delta_2} > 0$. The capital reallocation term will will be negative if the expected price ratio declines, and it will be positive if the expected price ratio increases. The labor reallocation term drops out.

For a large country, the third term in the income equation: $EX_2\frac{dP}{d\delta}$ will determine if this growth could be immiserizing. If the expected terms of trade move in its favor, growth cannot immiserize. If the expected price ratio declines, the export term will be negative and, if it outweighs the positive impact of technical progress in both sectors, then income would decline.

Moving on to the other case where technical progress occurs in only one sector, let us assume the import competing industry is growing: $\delta_1 > 1$; $\delta_2 = 1$.

A small country will experience an impact from labor immobility because, as shown in Part 1, some of both factors would be reallocated to sector 1 if both were mobile. For a large country there is the added reallocation of factors due to the expected growth induced change in the price ratio.

To examine this let us evaluate the terms of the income equation (3.27). For brevity the labor allocation terms will be excluded since they drop out in the short run.

$$\frac{dX_1}{d\delta_1} = \frac{\partial X_1}{\partial \delta_1} + \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial \delta_1} + \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial E(P)}\frac{dE(P).}{d\delta_1}$$

For a small country, both the technical progress term $\left(\frac{\partial X_1}{\partial \delta_1}\right)$ and the capital reallocation term $\left(\frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial \delta_1}\right)$ are positive. There is an unambiguous increase in the output of X_1. For a large country growth in the import competiting industry causes its expected terms of trade to improve which would cause capital to move out of X_1 and into X_2. Therefore the expected price term is negative. Turning to the export sector:

$$\frac{dX_2}{d\delta_1} = \frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial \delta_1} + \frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial E(P)}\frac{dE(P)}{d\delta_1}.$$

For a small country the capital reallocation term $\left(\frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial \delta_1}\right)$ is negative. Adjustment of the output ratio takes place in the small country but it is not as great as when both factors are mobile. For a large country, the term representing the growth induced change in the expected price ratio is positive because the expected terms of trade improve.

The export term in that case is positive: $EX_2\frac{dE(P)}{d\delta} > 0$. If technical progress occurs in the export sector (X_2), the expected terms of trade will deteriorate and the export term will be negative.

This allows one to conclude:

Proposition 3.11 When a large country faces an uncertain future terms of trade and immobile labor and is experiencing technical progress in only one sector, it is more likely to have an increase in expected income when the growth occurs in the import competing sector than when the technical progress happens in the export industry.

Let us now go on to the long run when labor becomes somewhat mobile. We begin with the first case where both sectors experience equivalent technical progress. While this will have no impact on a small country, for a large country with a growth induced change in the expected price ratio, reallocation of factors will be greater now that some labor is mobile; so, the adjustment of the output ratio is greater than in the short run. Using the terms of the income equation (3.27) this becomes clear.

$$\frac{dX_1}{d\delta_1} = \frac{\partial X_1}{\partial \delta_1} + \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial E(P)}\frac{dE(P)}{d\delta_1} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial E(P)}\frac{dE(P)}{d\delta_1}.$$

Given the expected price ratio increases, the capital and labor allocation terms are negative as those factors are drawn to the export sector. If the expected price ratio declines, capital and labor will flow into the import competing sector and there is an unambiguous increase in the output of X_1. The role of the wage differential is apparent. While both factors can move between sectors, labor's elasticity of mobility, being less than infinite, slows down and dampens the adjustment.

$$\frac{dX_2}{d\delta_2}=\frac{\partial X_2}{\partial \delta_2}+\frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial E(P)}\frac{dE(P)}{d\delta_2}+\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial E(P)}\frac{dE(P)}{d\delta_2}.$$

With an increase in the expected price ratio, the reallocation terms will be positive because capital and labor are moving into X_2. With a deterioration in the expected price ratio, the reallocation terms will be negative as capital and labor transfer to the import competing sector.

Finally, the sign of the export term: $EX_2\frac{dE(P)}{d\delta}$ will depend upon the movement in the expected terms of trade. If they improve, the term is positive. If they decline, exports will also decline.

Comparing the short run and long run for a large country with equivalent technical progress in both sectors, the following conclusion can be drawn:

Proposition 3.12 When a large country with less than perfect labor mobility, facing uncertain terms of trade, experiences equivalent technical progress in both sectors, the increase in expected income will be greater if the terms of trade turn in its favor.

Continuing on with an example of technical progress in just one sector let us assume it occurs in the import competing industry (X_1). As seen above the labor reallocation terms both in response to the technical progress and the induced price change will have a value but, because labor is not completely mobile and the wage differential continues to develop, the increase in expected income will not be as great as when labor is perfectly mobile as seen in Part 3.

Looking at the terms of the income equation (3.27), we can evaluate the response to this type of technical progress.

$$\frac{dX_1}{d\delta_1}=\frac{\partial X_1}{\partial \delta_1}+\frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial \delta_1}+\frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial \delta_1}+\frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial E(P)}\frac{dE(P)}{d\delta_1}+\frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial E(P)}\frac{dE(P)}{d\delta_1}$$

The labor reallocation term in response to technical progress is positive while the labor reallocation term in response to the improvement in the expected terms of trade is negative; so, while technical progress will cause the output of X_1 to increase, the reallocation terms move in opposite directions. Eventual reallocation will depend on which is stronger: the technical progress effect or the change in the expected price ratio effect.

In the export sector (X_2) there is no direct impact from technical progress since it takes place in X_1. The reallocation terms in response to technical progress are negative and those in response to the improvement in the expected terms of trade are positive. As above, the net effect will depend on the relative sizes of those two forces.

The export term will be positive because the expected terms of trade improve:

$$EX_2 \frac{dE(P)}{d\delta_1} > 0.$$

If technical progress occurs in sector 2, the expected terms of trade will deteriorate making the export term negative.

The following conclusion can be drawn in both the short run and long run:

Proposition 3.13 When a large country with less than perfect labor mobility and, facing an uncertain terms of trade, experiences technical progress in one sector, if it occurs in the import industry, it is more likely to have a rise in expected income than if the technical progress occurs in the export industry.

In general it may be concluded that, the presence of uncertainty about the future terms of trade do not affect the quality of the results found in a certain environment when the economy experiences technical progress.

Next we continue with an analysis of the impact on the economy of the home country when it joins a Customs Union.

Chapter IV
Customs Union

A customs union is formed when two or more countries abolish import duties on their mutual trade and adopt a common external tariff schedule on all imports from the rest of the world. This geographical discrimination can result in either trade creation or trade diversion. As defined by Viner, trade creation takes place when a lower cost source of supply replaces a higher cost source. This is welfare improving because it represents an increase in productive efficiency. Trade diversion occurs when the opposite happens: a higher cost source replaces the lower cost one. As discussed in the Introduction, Meade, Lipsey, Gehrels and Johnson have pointed out circumstances under which trade diversion can be welfare improving.

The model will have two commodities: X_1, X_2 and three countries: A, B, and C. Herein the home country (A) is small whereas a potential customs union partner (B) is large. The rest of the world (C) may also trade with A. The home country has comparative advantage in X_2 while B and C have comparative advantage in X_1. Since they have similar economies, they do not trade. Transportation costs will be ignored and incomplete specialization will be assumed. To avoid an impact on welfare from collecting tariff revenue, it will be assumed that the income distribution is maintained by the government making rebates to consumers in the form of lump sum transfers. Also, given that the utility function is homothetic over the consumption of X_1 and X_2, and assuming the income elasticity of substitution of demand between them is zero because the income elasticity of demand for X_1 equals that of X_2, then there will be no real income loss that exceeds the revenue returned as the rebate.

Initially A levies a prohibitive, uniform advalorem tariff on all imports. It's exports face prohibitive tariffs in both B and C. Recall $P = \frac{P_2}{P_1}$, the free trade terms of trade.

The domestic price ratio for A: $\frac{P}{1 + \tau_a} = P_a$

B's price ratio facing A: $P(1 + \tau_b) = P_b$, prior to a customs union

C's price ratio facing A: $P(1 + \tau_c) = P_c$, prior to a customs unon.

The home country wishes to get access for its exports; so, it offers to form a customs union with B and C. These two alternatives will be examined. A does not consider a reduction in its unilateral tariff as a way of increasing welfare. It's concern is with export penetration while sustaining a minimum impact on its domestic import substitution industry (X_1). This rationale for forming a customs union rather than a unilateral reduction in its tariff was first discussed by Cooper and Massell (1965) and expanded by Wonnecutt and Wonnecutt (1981).

Customs Union between the Home Country and the Rest of the World

Part 1 Mobile Factors in a Certain Environment

When A and C form a customs union, A will face C's domestic price ratio which will herein be assumed to be equivalent to the free trade price ratio (P). For A it represents an increase in the relative price of P_2: $dP > 0$. A will respond by increasing its output of X_2 because it now can sell its exports in C without a tariff. In turn its imports of X_1 will increase because it has abolished its tariff. The volume of trade will increase, pushing out the consumption possibility frontier for both countries. Therefore, this provides a source of improvement in welfare for both A and C. It conforms to Viner's definition of trade creation since imports of X_1 from C replace some of A's higher cost domestic output.

To examine this process more closely we will use the utility function (2.9) and the trade formulae (2.18) (2.19) (2.20). Totally differentiating these equations and using the consumer equilibrium condition: $\frac{U_2}{U_1} = P$, the utility formula can be derived.

$$\frac{dU}{U_1} = (dX_1 + P_a\, dX_2)\, dP_a + IM_1 - P_a\left[\left(dIM_1 - EX_2\, \frac{dP_a}{P_a}\right)\right] \tag{4.1}$$

Imports are a function of the tariff and the terms of trade

$$IM_1 = IM_1(\tau,P)$$

Totally differentiating

$$dIM_1 = \frac{\partial IM_1}{\partial \tau} d\tau + \frac{\partial IM_1}{\partial P} dP \tag{4.2}$$

The utility formula can then be rewritten:

$$\frac{dU}{U_1} = (dX_1 + PdX_2) + \frac{\tau}{1+\tau}\left(\frac{\partial IM_1}{\partial \tau}\right) d\tau + \left[\frac{\tau}{1+\tau}\left(\frac{\partial IM_1}{\partial P}\right) + \frac{IM_1}{P(1+\tau)}\right] dP \tag{4.3}$$

which is similar to the basic equation (4.24) developed by Batra.[23]

Since the marginal rate of transformation is equal to the domestic price ratio (now equal to the free trade terms of trade: $\frac{dX_1}{dX_2} = P$), the first term disappears: $dX_1 + PdX_2 = 0$.

$\frac{\tau}{1+\tau}\left(\frac{\partial IM_1}{\partial \tau}\right) d\tau$ represents the change in social welfare as a result of the change in tariff on imports. A trades only with C and tariffs are abolished so: $d\tau < 0$. Since X_1 is not inferior, when the tariff is reduced, imports increase: $\frac{\partial IM_1}{\partial \tau} < 0$. Therefore this whole tariff term is positive.

$\left[\frac{\tau}{1+\tau}\left(\frac{\partial IM_1}{\partial P}\right) + \frac{IM_1}{P(1+\tau)}\right] dP$ shows the change in welfare due to a change in the terms of trade facing A: P_a became P. The price ratio increased as the tariff dropped away: $dP > 0$. When the relative price of X_1 fell, more was purchased and some of that was imports. This price term is also positive.

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[23] R Batra, *Studies in the Pure Theory of International Trade*. (New York: St. Martins Press, 1973) p. 103

The increase in welfare in A from the reduction in the tariff and the change in the price ratio reflects the direction and size of the adjustment in both the output mix and consumption combination.

Looking at this more closely, the increase in imports can be divided into the price and income effects on demand and the substitution effect on production.

$$\frac{dIM_1}{IM_1} = \left(\frac{P_a}{IM_1}\frac{\partial Z_1}{\partial P_a}\right)\frac{dP_a}{P_a} + \frac{m_1}{P_a IM_1}dY - \left(\frac{P_a}{IM_1}\frac{\partial X_1}{\partial P_a}\right)P_a \qquad (4.4)$$

$\left(\frac{P_a}{IM_1}\frac{\partial Z_1}{\partial P_a}\right)\frac{dP_a}{P_a}$, the price effect. It shows the change in consumption of X_1 from the change in the price ratio facing A's consumers. It is positive

$\frac{m_1}{P_a IM_1}dY$, the income effect. It shows that as income increases after the formation of the customs union, more X_1 is consumed because the marginal propensity to consume (m_1) is positive.

$-\left(\frac{P_a}{IM_1}\frac{\partial X_1}{\partial P_a}\right)P_a$. the substitution effect on production. In response to a decrease in the relative price of X_1 facing A's producers, less X_1 is produced. With a minus preceding it, the substitution effect is also positive.

Next we will relax the assumption that labor is mobile and examine the impact of joining a customs union with C.

Part 2 Short Run Immobility of Labor; Long Run Partial Mobility in a Certain Environment

The increase in the price ratio facing producers in the home country caused by the formation of the customs union results in an adjustment in the output mix. In equation

(4.3), with mobile factors the adjustment balanced out: $dX_1 + PdX_2 = 0$. The movement was along the product transformation curve.

With immobile labor causing the development of a wage differential as the economy attempts to adjust, that adjustment term becomes negative. (As mentioned in the Basic Model, this was proven by Hazari). This reflects the fact that the product transformation curve contracts as the differential develops. Differentiating each output in the adjustment term with respect to the change in the price ratio will highlight the process in the short run.

$$\frac{dX_1}{dP} = \frac{\partial X_1}{\partial K_1}\frac{dK_1}{dP} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{d\mu}{dP}$$

$\frac{\partial X_1}{\partial K_1}\frac{dK_1}{dP}$ This shows the unimpeded response of capital to the change in the price ratio. With an increase in the price ratio; this will be negative. Capital leaves the sector where the relative price has fallen.

$\frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{d\mu}{dP}$. Since labor is immobile in the short run, $\frac{\partial L_1}{\partial \mu} = 0$.

Therefore the whole term drops out.

$$\frac{dX_2}{dP} = \frac{\partial X_2}{\partial K_2}\frac{dK_2}{dP} + \frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{d\mu}{dP}$$

$\frac{\partial X_2}{\partial K_2}\frac{dK_2}{dP}$ Capital will readily flow into this sector because of the relative increase in the price of X_2. This is positive.

$\frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{d\mu}{dP}$ This term drops out because labor is immobile: $\frac{\partial L_2}{\partial \mu} = 0$

The adjustment in the output mix depends upon the reallocation of capital and elasticities of factor substitution in each sector.

To determine the impact on welfare, reference is made to equation (4.3) but it is no longer unambiguously positive. The tariff and price terms remain positive, but as demonstrated, the adjustment term is negative. To the extent that the impact of the developing wage differential overwhelms the positive impacts of abolishing the tariff and the increase in the price ratio, welfare would decline in the short run.

This allows the following conclusion to be drawn:

Proposition 4.1 In the short run, welfare of the home country will decline as it enters the customs union with the rest of the world if the negative impact of the developing wage differential outweighs the positive impacts of abolishing the tariff and the increase in the price ratio.

In the long run greater adjustment of the output mix takes place as the critical wage differential is passed and labor begins to move. The rate of change in the wage differential slows down and, when the output equilibrium is attained, the differential stabilizes. As shown in the basic model the marginal rate of transformation no longer equals the price ratio:

$$\frac{-dX_1}{dX_2} = \beta P; \;\; \beta < 1 \text{ when } \mu > 1$$

The opportunity cost of increasing the output of X_2 in terms of foregone output of X_1 is less than the price ratio of the two outputs. Welfare is maximized, however, as long as the marginal rate of substitution in consumption is equal to that price ratio: $\frac{U_2}{U_1} = P$.

To examine the impact of β on welfare, Yu and Parai's approach will be adapted to the assumptions of this paper. That in turn was based on work done by Casas which has been previously discussed in this paper [equations 6-12, 20, 21, 27, 28].

Differentiating the utility function (2.9) and using the consumer equilibrium condition with the trade formulae (2.18) (2.19) (2.20) plus the new production equilibrium relation:

$$\frac{-dX_1}{dX_2} = \beta P$$

Then the utility equation (4.3) can be rewritten:

$$\frac{dU}{U_1} = (1-\beta)\, P_a\, dX_1 + P\tau\, dIM_1 - IM_1\, dP \tag{4.5}$$

which is essentially the same as equation (22) in Yu and Parai.[24]

Recall that $IM_1 = IM_1(\tau,P)$ and $X_1 = X_1(\tau,P)$. Substituting $dIM_1 = \frac{\partial IM_1}{\partial \tau} d\tau + \frac{\partial IM_1}{\partial P} dP$ and $dX_1 = \frac{\partial X_1}{\partial \tau} + \frac{\partial X_1}{\partial P} dP$ and using the partial differentiation:

$$P_a = P/1+\tau$$

$$\frac{\partial P_a}{\partial P} = \frac{1}{1+\tau}$$

Then:

$$\frac{dU}{U_1} = \left((1-\beta)\, PP_a \frac{\partial X_1}{\partial P_a} + P^2/\tau\ \frac{\partial IM_1}{\partial P_a}\right) d\tau$$

$$+ \frac{1}{1+\tau}\left((1-\beta)\, PP_a \frac{\partial X_1}{\partial P_a} + P^2/\tau\ \frac{\partial IM_1}{\partial P_a} + IM_1(1+\tau)\right) dP \tag{4.6}$$

which is similar to Yu and Parai's equation (23)[25] given the assumptions of this paper.

The tariff term represents the impact on welfare of the reduction in tariffs as A enters the customs union.

$(1-\beta)\, PP_a \frac{\partial X_1}{\partial P_a}$ shows the distortionary production effect of the reduction of the tariff with an endogenous wage differential. Assuming the system is stable, the output response is negative.

$P^2/\tau\ \frac{\partial IM_1}{\partial P_a}$ shows the effect of the reduction of the tariff rate on import demand. X_1 is normal so imports increase.

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[24] Eden S.H. Yu and Amark Parai, "Endogenous Wage Differentials, Imperfect Labor Mobility and Customs Union Theory" mimeo (1987), p. 5.

[25] *Ibid*

Since $d\tau$ is negative, the tariff reduction effect will be negative if the impact on imports is greater than the impact on output. The tariff reduction effect will be positive if the impact on imports is less than the impact on output.

The price term represents the welfare effect of the change in the price ratio facing A. $(1-\beta)\, PP_a \frac{\partial X_1}{\partial P_a}$ is again the distortionary effect on the production of X_1 and it is negative. $P^2/\tau \, \frac{\partial IM_1}{\partial Pd}$ shows the price effect on imports and is positive. $IM_1(1+\tau)$ shows the income effect of the change in the terms of trade and it is positive. Since the price ratio increases when A enters the customs union, $dP > 0$. The impact of the change in the price ratio will be positive if import and income effects outweigh the production effect.

The total impact on welfare depends upon the relative size of the distortionary impacts on the tariff and price terms.

If the distortionary impact of the tariff is greater than the impact on imports and the distortionary impact of the price is less than the price change's effect on imports and income, welfare will improve.

If the distortionary impact of the tariff is less than the impact on imports and the distortionary impact of the price is more than the price change's effect on imports and income, welfare will decline.

All other combinations are ambiguous. The first result is the more likely because wages are greater in the export sector than the import substitution sector. Yu and Parai come to the same conclusion in their model because economic inefficiency declines as the output structure moves closer to what it would have been in an undistorted economy. The reduction in inefficiency constitutes a production gain. Adding that to the favorable import demand effect of the tariff reduction contributes to the improvement in welfare in the long run over the short run.

Next we will drop the assumption of certain output prices but reinstitute complete mobility of factors.

Part 3 Mobile Factors in an Uncertain Environment

Since the future terms of trade are unknown the welfare equation (4.1) becomes

$$\frac{dU}{U_1} = (dX_1 + E(P)dX_2) + IM_1 - E(P_a)\left[dIM_1 - EX_2 \frac{dE(P)}{E(P)}\right] \tag{4.7}$$

At the point of entering the customs union with the rest of the world, which will actually bring about free trade, A does not know the future terms of world trade E (P); therefore, A does not know what its price ratio would be if it had kept its tariff. Rewriting 4.3 to accomodate uncertainty about the future terms of trade:

$$\frac{dU}{U_1} = (dX_1 + E(P)dX_2) + \frac{\tau}{1+\tau}\left(\frac{\partial IM_1}{\partial \tau}\right)d\tau + \left[\frac{\tau}{1+\tau}\left(\frac{\partial IM_1}{\partial E(P)}\right) + IM_1\,(1+\tau)\right]dE(P) \tag{4.8}$$

As in Part 1, the output adjustment term disappears in this situation too. The tariff term is positive and unaffected by price uncertainty. The price term does reflect uncertainty but is positive since the expected price ratio is greater than A's expected price ratio with the tariff.

Consequently in an environment of uncertainty about the future terms of trade, entering a customs union will improve welfare, which leads to this conclusion:

Proposition 4.2 The presence of uncertainty about the price ratio will not affect the quality of the results when a small country with mobile factors enters a customs union with the rest of the world.

Continuing the analysis, let us drop the assumption of mobile labor.

Part 4 Short Run Immobility of Labor; Long Run Partial Mobility in an Uncertain Environment

As the economy attempts to adjust its output ratio to the new expected terms of trade, the same process occurs as described in Part 2 in the short run.

Recalling equation 4.8, the output adjustment term is negative:

$$dX_1 + E(P)dX_2 < 0.$$

The tariff term is unaffected by uncertainty and remains positive. The price effect term does reflect the uncertainty about the expected price but it is also positive. As in Part 2, then, if the impact of the tariff reduction together with the impact of the expected price change outweigh the negative impact of the output adjustment with immobile labor, welfare will improve. This leads to the following conclusion:

Proposition 4.3 The presence of uncertainty about the price ratio will not affect qualitatively the results when a small country, with immobile labor in the short run, enters a customs union with the rest of the world.

In the long run labor becomes somewhat mobile and the wage differential stabilizes. To show this, when the future terms of trade are unknown, equation (4.6) should be revised accordingly

$$\frac{dU}{U_1} = \left[\{1-E(\beta)\}\ E(P)E(P_a)\frac{\partial X_1}{\partial E(P_a)} + E(P)^2/\tau\ \frac{\partial IM_1}{\partial E(P_a)} \right] d\tau$$

$$+ \frac{1}{1+\tau}\left[\{1-E(\beta)\}\ E(P)E(P_a)\frac{\partial X_1}{\partial E(P_a)} + E(P)^2/\tau\ \frac{\partial IM_1}{\partial E(P_a)} + IM_1(1+\tau) \right] dE(P) \quad (4.9)$$

Uncertainty about the future terms of trade is found in both the tariff reduction term and the expected price change term.

While uncertainty does not affect the qualitative results found in a certain environment with partially mobile labor in the long run, it does make an increase in welfare less likely. The greater the uncertainty, the less confidence decision makers have in their expectations; so, they will be more tentative in making decisions. The output adjustment will be smaller and more slowly done, which could result in the tariff term being negative instead of positive. That same adjustment effect, however, would make the price term more likely to be positive; so, without empirical data one cannot arrive at firm conclusions.

As we have seen so far, entering a customs union with C was, in effect, free trade. Going from a prohibitive tariff to free trade probably resulted in a sizable decline in the output of X_1. This, however, lead to the possibility of increased welfare in all four situations studied.

This decline in output of X_1 may have had too high a political price. For example, if X_1 represented the high tech sector and for national defense reasons it was unacceptable to be so reliant on the rest of the world for that output, then the home country might very well withdraw from the customs union with C and join in a union with B. A's goal was not necessarily a more efficient allocation of world resources but rather access to a foreign market while maintaining an acceptable level of X_1 output. In choosing to enter the customs union with B, a higher cost producer of X_1 and leaving the union with C, A's domestic industry will not face such stiff competition. Then A has made a trade diverting choice as defined by Viner.

In comparison with autarky, the alternative to a customs union with either B or C, there is economic gain from joining with B as factors are allocated away from the less efficient import competing sector (X_1) to the more efficient export sector. The Kruegar Sonneschein Theorem points out that a union with B over autarky which results in an improvement in its terms of trade implies such as gain.[26]

Johnson (1960) shows that A would be better off to choose free trade (the customs union with C) over the customs union with B and subsidize domestic production but for clarity and brevity, let us assume that was not a possible alternative. He goes on to contend that the public good, in this case national defense, should be included in A's social welfare function, to give an economic rationale for A's choice of a customs union with B rather than C. Then, if the output of X_1 equals or exceeds that required for national defense after

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[26] Anne O. Kruegar and H. Sonnerschein , "The Terms of Trade, the Gains from Trade and Price Divergence" *International Economic Review* 8,1; 1967. p. 121-127

joining with B, there will be a welfare gain. We will get around this by assuming such a calculation was made prior to forming the Customs Union and that is why B was chosen over C. It will be of interest to see if welfare, as measured here, can increase given the various assumptions made here in.

Having established the parameters and the motives of the choice of B let us analyze the impact on welfare as A withdraws from its union with C and forms a customs union with B.

Customs Union Between the Home Country and a Large Country

Part 1 Mobile Labor in a Certain Environment

Recalling (4.3) we will evaluate the change in utility when A and B form a customs union:

$$\frac{dU}{U_1} = (dX_1 + P'dX_2) + \frac{\tau}{1+\tau}\left(\frac{\partial IM_1}{\partial \tau}\right)d\tau + \left[\frac{\tau}{1+\tau}\left(\frac{\partial IM_1}{\partial P}\right) + \frac{IM_1}{P'(1+\tau)}\right] dP' \quad (4.10)$$

Let P' be the new price ratio facing A.

With mobile factors the adjustment term is zero. The tariff term is negative in order to sustain the union, the common tariff must keep out C's exports of X_1. The home country experiences an increase in tariffs ($d\tau > 0$). The price term is also negative . A faces a lower terms of trade than in the customs union with C. Welfare, as measured here, declines. For this to be a rational economic decision the utility from maintaining the X_1 industry must outweigh both the tariff effect and price effect.

Now let us drop the assumption of labor mobility.

Part 2 Short Run Immobility of Labor; Long Run Partial Mobility in a Certian Environment

The form of the analysis for A joining a customs union with B is the same as it is when A joined with C. However as shown above, the price ratio facing A declines.

To examine this more closely let us review equation (4.3) paying particular attention to the output adjustment term: $dX_1 + P'dX_2$. As seen above this term is negative as the wage differential is developing. However, in this case the adjustment is in the opposite direction. X_1 is the expanding sector; so, the goal of maintaining sector 1 at or above a politically acceptable level is more likely to be met than in the union with C. A has access to B's markets for X_2 but at a lower relative price so the output of sector 2 will decrease although constrained by immoble labor.

All three terms of the utility formula (4.10), adjustment $(dX_1 + P'dX_2)$, tariff $\left[\frac{\tau}{1+\tau}\left(\frac{\partial IM_1}{\partial \tau}d\tau\right)\right]$ and the price effect [•]dP' are negative. As stated above, for the union with B to be a rational economic decision, the utility of maintaining the defense industry must outweigh the impact of the wage differential, increase in tariffs and decline in the price ratio facing A for the union to be welfare improving over a customs union with A.

In the long run, as a critical wage differential is reached and labor becomes somewhat mobile, there is greater scope for adjustment of the output ratio. Reviewing equation (4.6) for a customs union with B:

$$\frac{dU}{U_1} = \left((1-\beta)\, P'P\,\frac{\partial X_1}{\partial P} + P'^2/\tau\ \frac{\partial IM_1}{\partial P}\right)d\tau$$

$$+\frac{1}{1+\tau}\left[(1-\beta)\, P'P\,\frac{\partial X_1}{\partial P} + P'^2/\tau\ \frac{\partial IM_1}{\partial P} + (1+\tau)IM_1\right] dP' \qquad (4.6)$$

Note: since X_1 is expanding $d\mu < 1$. μ becomes less that one so β becomes more than one.

With the tariff's increase ($d\tau > 0$) and the decline in the price ratio ($dP < 0$) the analysis follows the same form as it did in the Union with C.

Reviewing the tariff term: $\left[(1-\beta)\, P'P \frac{\partial X_1}{\partial P} + P'^2/\tau \; \frac{\partial IM_1}{\partial P}\right] d\tau$

This distortionary output effect is positive as is the effect on imports so the entire tariff term has a positive impact on welfare.

The price term: $\frac{1}{1+\tau}\left[(1-\beta)\, P'P \frac{\partial X_1}{\partial P} + P'^2/\tau \; \frac{\partial IM_1}{\partial P} + (1+\tau)IM_1\right] dP$. The distortionary output effect is positive as is the effect on imports. Since the price facing A declines, the price term is negative. In the long run the impact on welfare as measured here will increase only if the tariff effect outweighs the price effect.

This allows for the following conclusion:

Proposition 4.4: In the long run when labor is somewhat mobile, a trade diverting customs union can improve welfare over a trade creating customs union if the impact of the increase in tariffs outweighs the negative impact of the deterioration in the terms of trade.

Next, dropping the assumption that the future terms of trade are known but allowing labor to be mobile, we will evaluate the impact of the customs union with B.

Part 3 Mobility Factors in an Uncertain Environment

Again the analysis is the same as with the customs union with C except the expected price declines and the tariff increases. However, to the extent that the union with B rather than C is seen as preserving the sector 1 industry in an uncertain world and thereby reducing general uncertainty, the variance of the price distribution might decline since people have more confidence in their predictions. Fries (1984) makes this argument. If true then

choosing B over C as a customs union partner in an uncertain envrionment would be welfare increasing for A.

Finally we will drop the assumption of mobile labor to evaluate the customs union with B.

Part 4 Short Run Immobility of Labor; Long Run Partial Mobility in an Uncertain Environment

The procedure of analysis is the same as in Part 2 and the quality of the results are the same. However if the union with B does reduce uncertainty and that is one of the causes of labor immobility, then the union with B should result in a shorter period of complete immobility of labor and a greater adjustment in the long run because labor will become more mobile. In that case the union with B could result in a greater increase in welfare than a union with C.

In summary, the following conclusion can be drawn:

Proposition 4.5: A trade diverting customs union could increase welfare over that of a trade creating union if the trade diversion ensures the availability of a public good and, in the case where future terms of trade are unknown, that assurance reduces general uncertainty giving decision makers more confidence in their predictions thereby decreasing the variance of the price distribution.

We move now to an analysis of the impact on welfare of a unilateral transfer.

Chapter V
Unilateral Transfer

A unilateral transfer occurs when commodities are shipped from one country (A) to another (B) on concessionary terms rather than in resposne to normal market incentives. In this chapter it is assumed that A will ship some units of X_2 (UT_2) to B and receive nothing in return. The rest of the world (C) does not participate.

The orthodox presumption has been that the recipient will benefit and the donor will suffer a decline in income due to the transfer itself and, should there be a secondary impact on the terms of trade, it will be negative for the donor. Yano delineated two other possible welfare results: a weak paradox when the welfare of the donor and recipient both move in the same direction and the strong paradox when the welfare of the donor improves and that of the recipient declines.

While Lentief was the first to demonstrate a strong paradox result, Samuelson showed that it would occur only if there were multiple and unstable equilibria in a two agent world. Markets are stable when the law of demand holds: the quantity demanded varies inversely with the price. Income effects can be destabilizing if and only if the marginal propensity to consume the good, which experiences a price change, is greater for exporters than importers. As long as the substitution effect is greater than the income effect, markets are stable. Threfore, as he pointed out, the direction of change in the terms of trade do not depend upon the price elasticities of one or both offer curvers but rather the income elasticities of the donor and recipient countries.

Several writers cited in the introduction found that the strong paradox would occur in a stable equilibrium if a fixed coefficient framework were used thereby eliminating substitution effects. Others found that a three country model is necessary for the strong paradox to result. This dissertation will focus on two economies with variable coefficients

of production. They will first be examined in a two country model by assuming they are too small to affect the rest of the world. Then the assumption will be changed. At least one participant will be a large country to allow the transfer to affect the terms of trade thereby using a three country model. Also it should be kept in mind that for the donor's welfare to improve, not only must its terms of trade improve but also the effect must be strong enough to outweigh the loss in resources from the unilateral transfer.

Let us begin by reviewing the standard case of perfect factor mobility in a certain environment.

Part 1 Mobile Factors in a Certain Environment

If both the donor and recipient are small countries, the unilateral transfer will have no impact on the terms of trade. A's output mix, then, will not change. The impact on welfare is solely that of the transfer's effect on consumption. With no change in the price ratio, there is only the income effect on consumption; no substitution effect.

From the standpoint of the donor (A), net income (Y^a_n) equals the original income (Y^a) minus the transfer (UT_2). Using the income equation (2.16) from the basic model, in this context it becomes:

$$Y^a - UT_2 = Y^a_n = Z_1 + PZ_2 \tag{5.1}$$

Differentiating the net income equation:

$$dY^a - dUT_2 = dZ_1 + PdZ_2 \tag{5.2}$$

Clearly, the donor's income declines by the amount of the unilateral transfer. To examine the consumption terms, totally differentiate each.

$$dZ_1 = dX_1 + dIM_1 \tag{5.3}$$

Because the output ratio does not change, $dX_1 = 0$. Therefore, the impact of the unilateral transfer in sector 1 is seen solely in its effect on imports. Differentiating (5.3) with respect to the unilateral transfer: $\frac{dZ_1}{dUT_2} = \frac{dIM_1}{dUT_2}$

Recall that imports are a funtion of income and the price ratio. Since income has declined one would expect imports to decrease

$$\frac{\partial IM_1}{\partial Y} \frac{\partial Y}{\partial UT_2} < 0$$

Turning to exports, totally differentiate consumption in sector 2:

$$dZ_2 = dX_2 - dEX_2 - dUT_2 \tag{5.4}$$

Since the output ratio does not change, $dX_2 = 0$. Given that the marginal propensity to consume X_2 is less than one: $\frac{\partial Z_2}{\partial Y} < 1$, the decline in income triggered by the unilateral transfer:

$$\frac{\partial Z_2}{\partial Y} \frac{\partial Y}{dUT_2} < 0$$

will generate a smaller decline in the domestic consumption of X_2 than the size of the transfer. Rewriting (5.4)

$$dEX_2 = -dZ_2 - dUT_2$$

it is clear that exports decline.

The balance of payments equilibrium (2.18) is now modified by incorporating the unilateral transfer

$$IM_1 + UT_2 = PEX_2 \tag{5.5}$$

The consumer is in equilibrium when the marginal rate of substitution equals the price ratio: $\frac{U_2}{U_1} = P$ as can be seen in Figure 5.1 for the donor.

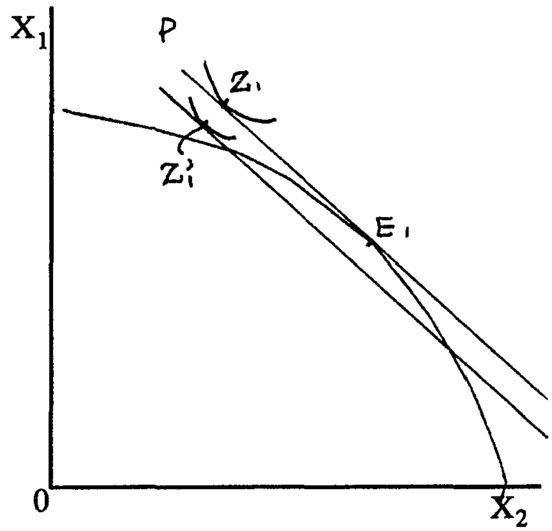

Figure 5.1 Donor (A)

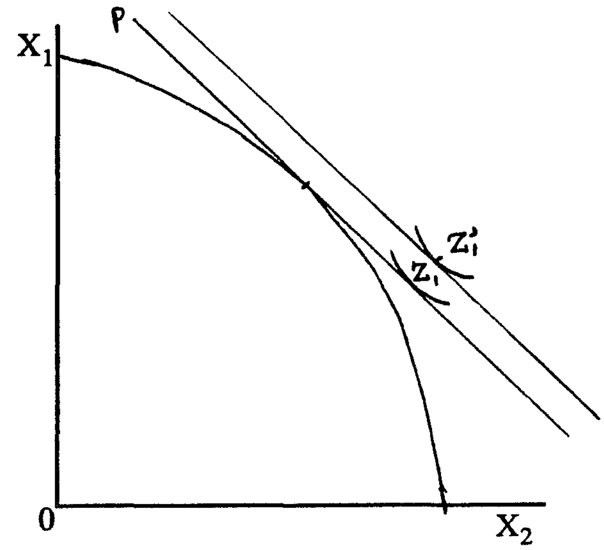

Figure 5.2 Recipient (B)

In both Figures E_1 is the output combination which does not change because the terms of trade do not change. Z_1 is the pre Transfer consumption ratio. Z_1' is the post Transfer level of utility.

To examine the effect of the unilateral transfer use the utility function:

$$\frac{dU^a}{U_1^a} = (dX_1 + PdX_2)\, dP + IM_1 dP - dUT_2 \tag{5.6}$$

Since there is no change in the output ratio, the adjustment term: $(dX_1 + PdX_2)$, drops out. With no change in the price ratio, the import term also drops out. Differentiating with respect to the unilateral transfer:

$$\frac{1}{U_1^a}\frac{dU^a}{dUT_2} = \frac{-dUT_2}{dUT_2}$$

which becomes: $\frac{dU^a}{dUT_2} = -U_1^a < 0$

Reworking (5.6) for the recipient

$$\frac{dU^b}{U_1^b} = (dX_1 + PdX_2)\, dP + IM_2 dP + dUT_2 \tag{5.7}$$

Differentiating with respect to the unilateral transfer and with manipulation:

$$\frac{dU^b}{dUT_2} = U_1^b > 0$$

Therefore it is clear that the loss in welfare for the donor is directly related to the improvement in welfare for the recipient.

Next, let us relax the assumption that the donor and recipient are both small countries. Now let one or both be sufficiently large; so, the transfer has an impact on the terms of trade.

Recalling the differentiation of the donor's utility function with respect to granting a transfer (5.6), the output adjustment term is still zero since at the new output ratio, the marginal rate of transformation equals the new price ratio:

$$dX_1 + PdX_2 = 0$$

because: $\frac{-dX_1}{dX_2} = P$.

With mobile factors output adjustment takes place along the product transformation curve.

Then $\frac{1}{U_1}\frac{dU^a}{dUT_2} = -1 + IM_1\frac{dP}{dUT_2}$ (5.8)

The first term: -1, represents the unilateral transfer.

The value of the import term depends upon the transfer induced change in the terms of trade: $IM_1\frac{dP}{dUT_2}$

Samuelson[27] discussed this and developed equations that showed the transfer induced impact using the stability criteria.

$$\frac{dP}{dUT_2} = \frac{1 - m_1^a - m_2^b}{IM_1(\alpha a + \alpha b - 1)}\ ^{28} \qquad (5.9)$$

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[27] Paul Samuelson, "The Transfer Problem and Transports Costs II: Analysis of the Effects of Trade Impediments," *Economic Journal,* June, 1954; p. 284

[28] αa: Price elasticity of demand for importables (X_1) in A.

For stability it is assumed $\alpha a + \alpha b - 1 > 0$; so, the denominator is positive. Samuelson proved that the terms of trade will improve for the donor if A's marginal propensity to consume importables (m_1^a) plus B's marginal propensity to consume importables (m_2^b) sum to less than one. Then the numerator of (5.9) is also positive; therefore: $\frac{dP}{dUT_2} > 0$. The unilateral transfer, by reducing A's exports, creates excess demand for X_2 in the rest of the world and that causes the price of X_2 to increase. Contrariwise, if $m_1^a + m_1^b > 1$, then the terms of trade will move against the donor.

Rewriting the welfare equation (5.8)

$$\frac{1}{U_1^a}\frac{dU^a}{dUT_2} = -\left[\frac{\left(d_1^a + s_1^a\right) + \left(d_2^b + s_2^b\right)}{\alpha a + \alpha b - 1}\right] < 0 \tag{5.10}$$

The welfare of the donor always declines regardless of the direction of the change in the terms of trade when there is no distortion. This reaffirms Samuelson's original finding: any improvement in the terms of trade will be insufficient to reverse the negative impact of the transfer, given stable markets.

αb: Price elasticity of demand for importables (X_2) in B

$\alpha a = d_1 + s_1 + m_1^a > 0$

$d_1 = \frac{P}{IM_1}\frac{\partial Z_1}{\partial P} > 0$, measure of substitution in consumption as the price ratio changes; as the relative price of X_2 increases ($dP > 0$) consumers buy more X_1.

$s_1 = -\left(\frac{P}{IM_1}\right)\left(\frac{\partial X_1}{\partial P}\right) > 0$, measure of substitution in production as the price ratio changes; as the relative price of X_2 increases less X_1.is produced

$0 < m_1^a < 1$, marginal propensity to consume importables (X_1) in A.

If the recipient country (B) is large, the change in the terms of trade will also depend upon the marginal propensities to consume importables:

$$\frac{dP}{dUT_2} = -\frac{\left(1 - m_2^b - m_1^a\right)}{IM_2(\alpha a + \alpha b - 1)} < 0 \tag{5.11}$$

As above, the denominator is positive. If $m_2^b + m_1^a > 1$, the terms of trade move in favor of the recipient: $\frac{dP}{dUT_2} < 0$. If $m_2^b + m_1^a < 1$, the terms of trade deteriorate for B. The symmetry with the donor is clear.

Welfare, however, will improve for the recipient regardless of the direction of the price ratio adjustment due to the transfer.

$$\frac{1}{U_1^b}\frac{dU^b}{dUT_2} = 1 + IM_2\frac{dP}{dUT_2} = \left[\frac{\left(d_2^b + s_2^b\right) + \left(d_1^a + s_1^a\right)}{\alpha b + \alpha a - 1}\right] > 0 \tag{5.12}$$

The result, then, is normal. Welfare of the donor (A) declines and, for the recipient, (B) welfare improves. This is true regardless of the direction of the transfer induced change in the price ratio.

Moving on, let us introduce an endogenous wage differential.

Part 2 Short Run Immobility of Labor; Long Run Partial Mobility in a Certain Environment

As shown above, for small countries, the terms of trade are not affected by the transfer; so, the output ratio does not change. That being so, immobility of a factor has no impact. The results in Part 1 hold here for a small country.

For a large donor country (A) the impact of immobility can be seen in the contraction of the product transformation curve as the economy attempts to adjust to the new transfer induced terms of trade and develops a wage differential: $\frac{d\mu}{dP} > 0$.

Differentiating (5.6) with respect to the unilateral transfer

$$\frac{1}{U_1^a}\frac{dU^a}{dUT_2} = (dX_1 + PdX_2)\frac{dP}{dUT_2} + IM_1\frac{dP}{dUT_2} - 1 \tag{5.13}$$

Evaluating each of the outputs will show us the impact of the wage differential.

$$\frac{dX_1}{dUT_2} = \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial P}\frac{dP}{dUT_2} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{dUT_2}$$

Capital will adjust, but labor is immobile, so the labor adjustment term drops out. If the price ratio increases, capital will leave sector 1. If the price ratio decreases, capital will enter sector 1. The size of adjustment then depends upon the elasticity of factor substitution in sector 1.

$$\frac{dX_2}{dUT_2} = \frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial P}\frac{dP}{dUT_2} + \frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{dUT_2}.$$

The capital term will have a value but, as above, the labor term will drop out because labor is immobile.

If the terms of trade turn in favor of the donor, then output of X_1 declines releasing capital to migrate to X_2 which increases. If the terms of trade deteriorate, then output of X_1 increases drawing capital from X_2 which is declining. The contraction of the product transformation curve, causes the output adjustment terms to sum a negative:

$$\frac{dX_1}{dUT_2} + P\frac{dX_2}{dUT_2} < 0.$$

Moving on the the imports $\left(IM_1 \frac{dP}{dUT_2}\right)$,if the terms of trade improve, then imports will increase. If the price ratio declines, imports decline.

This leads to the following conclusion:

Proposition 5.1 For a large donor country with immobile labor in the short run, welfare would improve only if the terms of trade move in its favor and the import term is

sufficiently large to outweigh the negative impact of the unilateral transfer and the developing wage differential.

For a large recipient country, with immobile labor in the short run recall the utility equation (5.7). The output adjustment term responding to the transfer induced change in the terms of trade is also negative:

$$\frac{dX_1}{dUT_2} + P\frac{dX_2}{dUT_2} < 0$$

The sign of the import term: $\left(IM_2 \frac{dP}{dUT_2}\right)$ will depend upon the direction of the change in the terms of trade. The sign of the unilateral transfer term is positive: $\frac{dUT_2}{dUT_2} = 1$. If the terms of trade move in favor of the recipient ($dP < 0$) then welfare will increase because of the positive unilateral transfer and import terms together with the output adjustment term. If the terms of trade move against the recipient then the import term becomes negative as does the output adjustment term. Welfare will decline if this negative impact outweighs the positive unilateral transfer.

This leads to the following conclusions:

Proposition 5.2 For a large recipient country, with immobile labor in the short run, welfare will decline if its terms of trade deteriorate making the import term negative and if the negative import term combined with the negative output adjustment term outweigh the positive unilateral transfer.

Now let us determine under what circumstances a normal result would occur. If the terms of trade deteriorate for the donor, or they improve and the positive import term is outweighted by the negative impacts of the unilateral transfer and output adjustment; then, welfare declines for the donor. For the recipient the terms of trade improve and the import term plus the unilateral transfer are sufficiently large to overcome the impact of the developing wage differential. If B's terms of trade deteriorate, then the unilateral transfer

must be sufficiently large to overcome the negative output adjustment and import terms. Either way, the welfare of the recipient improves.

A weak paradox will occur when the donor's welfare declines and also that of the recipient. Even when the terms of trade move in favor of the recipient, if the negative output adjustment outweighs the unilateral transfer and positive import term, its welfare will decline. A weak paradox can also occur if the donor's welfare improves and so does that of the recipient. In that case, the terms of trade improve for the donor ($dP > 0$) with the positive import term overcoming the negative unilateral transfer and output adjustment terms. For the recipient, even though the price ratio increases, the positive unilateral transfer term outweighs the negative impacts of the import and output adjustment terms.

This allows the following conclusion:

Proposition 5.3 In the short run, with both donor and recipient experiencing immobile labor, a strong paradox, weak paradox or normal result will occur depending upon the movement in the terms of trade, the size of the unilateral transfer and the size of negative impact of the wage differential on both economies.

In the long run, as labor becomes somewhat mobile, the wage differential eventually stabilizes. The endogenous wage differential takes on the familiar form of a domestic distortion. As shown in the basic model, the marginal rate of transformation does not equal the new transfer induced price ratio.

$$\frac{-dX_1}{dX_2} = P\beta\text{: } \textit{Note:} \text{ if } \mu > 1 \text{ then } \beta < 1\text{; if } \mu < 1 \text{ then } \beta > 1$$

Because of the limited mobility of labor greater adjustment takes place. Reevaluating equation (5.13), the output adjustment term shows the impact of the limited mobility of labor. The differential continues to develop but much more slowly than before.

$\frac{dX_1}{dUT_2} = \frac{\partial X_1}{\partial K_1}\frac{\partial K_1}{\partial P}\frac{dP}{dUT_2} + \frac{\partial X_1}{\partial L_1}\frac{\partial L_1}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{dUT_2}$. Now the labor term also has a value. It will be positive if the price ratio declines and negative if the price ratio increases.

$\frac{dX_2}{dUT_2} = \frac{\partial X_2}{\partial K_2}\frac{\partial K_2}{\partial P}\frac{dP}{dUT_2} + \frac{\partial X_2}{\partial L_2}\frac{\partial L_2}{\partial \mu}\frac{\partial \mu}{\partial P}\frac{dP}{dUT_2}$. Here, also the labor term has a value. It will be negative if the terms of trade deteriorate for the donor and factors move out of its export sector. If the terms of trade improve, factors will enter sector 2.

The entire adjustment of output, however, is still negative for both the donor and recipient, since the differential influenced product transformation curve is inside the original curve over the relevant range, as shown in the basic model. The situations under which a normal result, weak paradox or strong paradox may occur remain the same as in the short run.

The following general conclusion can now be drawn:

Proposition 5.4 In the long run with some degree of labor immobility creating a wage differential, a strong paradox, weak paradox or normal result may occur depending on the relative sizes of the negative impact of the output adjustment term, the size of the unilateral transfer and direction of the change of the terms of trade.

The assumption of a certain environment will now be dropped in favor of uncertinty in the future terms of trade. Factors will be assumed mobile.

Part 3 Mobile Factors in an Uncertain Environment

As discussed in the Introduction, Fries found that production uncertainty can cause a strong paradox result. The purpose of this and the following section is to explore the implications for welfare of both recipients when the future terms of trade are uncertain. Reworking the welfare equation (5.13) to reflect that uncertainty:

$$\frac{1}{U_1^a}\frac{dU^a}{dUT_2} = (dX_1 + E(P)\,dX_2)\frac{dE(P)}{dUT_2} + IM_1\frac{dE(P)}{dUT_2} - 1 \qquad (5.14)$$

Output decisions are made prior to the resolution of uncertainty. For a small country the output mix does not change because the transfer has no impact on the expected terms of trade; therefore, the output adjustment term drops out. The import term also drops out because there is no transfer induced change in the expected terms of trade. For a small country, then, as a donor:

$$\frac{dU^a}{dUT_2} = -U_1^a < 0$$

and as a recipient:

$$\frac{dU^b}{dUT_2} = U_1^b > 0$$

This is the same result as found in a certain environment; so, the following conclusion can be drawn:

Propositon 5.5 For a transfer between two small countries with mobile factors, uncertainty about the future terms of trade does not change the quality of the welfare results found in a certain environment.

For a large donor country the adjustment term in equation (5.14) drops out because with mobile factors, adjustment takes place along the product transformation curve:

$$dX_1 + E(P)dX_2 = 0$$

The sign of the import term: $IM_1\frac{dE(P)}{dUT_2}$, will depend upon which way the expected terms of trade are expected to move. If the negative impact of the unilateral transfer outweighs any impact on imports from an improvement in the expected terms of trade for A, the donor's welfare declines.

As shown in Part 1, a large recipient's welfare improves if the positive impact of the unilateral transfer, unaffected by uncertainty about the future terms of trade, outweighs any impact on imports of an expected deterioration in the terms of trade for B.

This fact that the unilateral transfer is unaffected by uncertainty may be a key to another source of welfare improvement for the recipient. If the transfer is a promise, regardless of what may happen to the terms of trade, then it becomes a certain thing in an uncertain world. That being so, if it reduces general uncertainty, it might work to reduce the variance of the price distribution and provide an additional souce of increasing welfare for the recipient when decision makers reduce their risk premia which, in turn, would cause the product transformation curve to shift out in a parallel manner.

Regardless, the following conclusion can be drawn:

Proposition 5.6 When the transfer causes a change in the expected terms of trade, and both the donor and recipient have mobile factors, this type of uncertainty does not affect the quality of the results found in a certain environment.

Continuing the analysis, allow labor to be immobile in the short run with a subsequent development of a wage differential and somewhat mobile in the long run causing the differential eventually to stabilize.

Part 4 Short Run Immobility of Labor; Long Run Partial Mobility in an Uncertain Environment

By referring to the analysis of the welfare equation (5.13) it is clear that the result for the donor and recipient in the short run and long run, when the future price ratio is known, are not reversed because of uncertainty about the future terms of trade. This includes the conditions under which a normal result, a weak paradox or strong paradox might occur.

One point should be stressed, however. The greater the variance of the expected price distribution, the more uncertainty of decision makers this reflects. That slows down the adjustment process. If the unilateral transfer reduces the variance for the recipient (or even the donor) as explained above, then it is a source of an improvement in welfare.

This final conclusion can be drawn:

Proposition 5.7 Existence of uncertainty about the future terms of trade does not affect the quality of the findings about the impact of a unilateral transfer on the welfare of the participants unless the existence of the unilateral transfer has the effect of reducing the variance of the price distribution, in which case it is welfare improving for both.

Chapter VI
Conclusions

The three essays that compromise this dissertation contain two central issues: the impact of a change in the elasticity of labor mobility and the impact of price uncertainty on welfare. The purpose of this chapter is to make clear the linkage between the essays.

Growth, whether due to an increase in one of the primary factors or neutral technical progress, causes the product transformation curve to shift out presenting the possibility of an improvement in aggregate welfare. If the country is large, growth will also result in a change in the terms of trade. The immobility of labor will retard any output adjustments due to the pattern of growth or a growth induced change in the price ratio. The development of the endogenous wage differential causes the product transformation curve to contract, a welfare reducing effect. For a small country in a certain environment, the net impact on welfare will depend upon the relative sizes of the positive impact of growth and the negative impact of the wage differential. For a large country there is the further complication of a growth induced change in the terms of trade. If the terms of trade improve, that is welfare improving. If they decline, that is welfare decreasing. The development of a wage differential adds an additional welfare decreasing element to the total effect of growth increasing the possibility that growth will be immiserizing. We found that when growth takes place in an environment of uncertainty about the future terms of trade, the qualitative results found in a certain environment are nor reversed.

Entering a customs union is welfare increasing if it is trade creating by Viner's definition. It may also be welfare increasing if it is trade diverting, by his definition, as long as the volume of trade increases. At first the home country joins the rest of the world in a customs union and enjoys free trade, a trade creating situation. Then only the wage differential has a negative impact on the home country's welfare. Next the home country

withdraws from that customs union and forms a customs union with a large country that is a higher cost producer than the rest of the world; so, this is a trade diverting customs union. That chapter explored the circumstances under which the trade diverting customs union could be welfare improving without changing the utility function and found that even with an endogenous wage differential there is a possibility for welfare improvement. Joining the customs union in an uncertain environment does not reverse the results found when the terms of trade are known; however, the trade diverting customs union may also have a source of welfare improvement beyond that of the trade creating union if, by assuring a minimum size of the import substitute industry, that reduces general uncertainty. That would cause the product transformation curve to shift out since smaller risk premia are required.

A unilateral transfer will cause the welfare of the donor to decrease and that of the recipient to improve unless the transfer causes a movement in the terms of trade. As the economies attempt to adjust to the new price, the developing wage differential will have a negative impact on welfare for both participants. Therefore, depending upon the impact of the unilateral transfer, output adjustment and the direction of the change in imports, there can be a strong paradox, weak paradox or normal result.

When a unilateral transfer takes place in an environment of uncertainty about the future terms of trade, the qualitative results of the certain environment still hold. The unilateral transfer can have an additional welfare increasing impact in an uncertain environment. If the certainty of the transfer reduces general uncertainty, that decreases the variance of the price ratio. The risk premia are then reduced causing the product transformation curve to shift out.

In conclusion, the endogenous wage differential has a welfare reducing impact in all three situations. The existence of uncertainty about the future terms of trade does not reverse the qualitative results found in a certain environment.

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Appendix 1

λ: portion of total supply of an input used to produce an output:

$$\lambda L_1 = \frac{aL_1X_1}{L}; \ \lambda L_1 + \lambda L_2 = 1 \ ; \quad \begin{vmatrix} \lambda K_1 & \lambda K_2 \\ \lambda L_1 & \lambda L_2 \end{vmatrix} > 0$$

$$\Theta L_1 = \frac{aL_1 w}{P_1}; \ \Theta L_1 + \Theta K_1 = 1 \ ; \quad \begin{vmatrix} \Theta K_1 & \Theta K_2 \\ \Theta L_1 & \Theta L_2 \end{vmatrix} > 0$$

therefore $|\lambda| \bullet |\Theta| > 0$, because physical and value intensities have the same sign.

Appendix 2

The slope of the product transformation curve : $\frac{-dX_1}{dX_2} = P\beta$

$$\beta = P\left[\frac{\sigma_1\sigma_2 rK_1K_2L+\varepsilon\{\sigma_1L_1K_1P_2X_2+\sigma_2K_2L_2P_1X_1+\sigma_1L_1K_1L_2w_1[1-\mu]\}}{\sigma_1\sigma_2 rK_1K_2L+\varepsilon\{\sigma_1L_1K_1P_2X_2+\sigma_2K_2L_2P_1X_1-\sigma_2L_1K_2L_2w_1[1-\mu]\}}\right]$$

$\mu > 1$ therefore $\beta < 1$. In the long run there is greater adjustment of the product mix than in the short run.

For this to be true, the product transformation must wiggle. That is, it must have a least one inflection point.

To find if it has an inflection, it is necessary to determine under what circumstances $\frac{d^2X_1}{dX_2^2} = 0$.

Let $\beta = \frac{\beta_1}{\beta_2}$; then $\frac{d^2X_1}{dX_2^2} = P\frac{d}{dX_2}\left(\frac{\beta_1}{\beta_2}\right) = -P\frac{\beta_2\left(\frac{d\beta_1}{dX_2}\right) - \beta_1\left(\frac{d\beta_2}{dX_2}\right)}{\beta_2^2}$

The constants are: r, L, ε, σ_1, σ_2,P_2, P_1, μ

$$\beta_2 \quad \left\{\sigma_1\sigma_2 rL\left(\frac{dK_1}{dX_2}K_2 + K_1\frac{dK_2}{dX_2}\right) + \varepsilon\sigma_1P_2\left(\frac{dL_1}{dX_2}K_1X_2 + L_1X_2\frac{dK_1}{dX_2} + L_1K_1\right)\right.$$

$$+ \varepsilon\sigma_2P_1\left(\frac{dL_2}{dX_2}K_2X_1 + L_2X_1\frac{dK_2}{dX_2} + L_2K_2\frac{dX_1}{dX_2}\right)$$

$$\left.+ \varepsilon\sigma_1(1-\mu)\left[\frac{dL_1}{dX_2}K_1L_2w_1 - \frac{dL_1}{dX_2}K_1L_1w_1 + L_1L_2w_1\frac{dK_1}{dX_2}L_1K_1L_2\frac{dw_1}{dX_2}\right]\right\}$$

$$\beta_1 \quad \left\{\sigma_1\sigma_2 rL\left(\frac{dK_1}{dX_2}K_2 + K_1\frac{dK_2}{dX_2}\right) + \varepsilon\sigma_1P_2\left(\frac{dL_1}{dX_2}K_1X_2 + L_1X_2\frac{dK_1}{dX_2} + L_1K_1\right)\right.$$

$$+ \varepsilon\sigma_2 P_1\left(\frac{dL_2}{dX_2} K_2X_1 + L_2X_1 \frac{dK_2}{dX_2} + L_2K_2 \frac{dX_1}{dX_2}\right)$$

$$- \varepsilon\sigma_2(1-\mu)\left[\frac{dL_1}{dX_2} K_2L_2w_1 - \frac{dL_1}{dX_2} K_2L_1w_1 - L_2L_1w_1 \frac{dK_1}{dX_2} L_1K_2L_2 \frac{dw_1}{dX_2}\right]\Big\}$$

The first three terms in $\beta_2\{\ \}$ and $\beta_1\{\ \}$ are the same; so attention is focused on the last terms:

$$\beta_2 \quad \Big\{\ldots + \varepsilon\sigma_1(1-\mu)\left[\frac{dL_1}{dX_2} K_1L_2w_1 - \frac{dL_1}{dX_2} K_1L_1w_1 + L_1L_2w_1 \frac{dK_1}{dX_2} L_1K_1L_2 \frac{dw_1}{dX_2}\right]\Big\}$$

$$\beta_1 \quad \Big\{\ldots - \varepsilon\sigma_2(1-\mu)\left[\frac{dL_1}{dX_2} K_2L_2w_1 - \frac{dL_1}{dX_2} K_2L_1w_1 - L_2L_1w_1 \frac{dK_1}{dX_2} L_1K_2L_2 \frac{dw_1}{dX_2}\right]\Big\}$$

We know $\beta_2 > \beta_1$and $K_1 > K_2$

For $\beta_2 \frac{d\beta_1}{dX_2} - \beta_1 \frac{d\beta_2}{dX_2} = 0; \frac{d\beta_1}{dX_2} < \frac{d\beta_2}{dX_2}$ sufficiently to offset $\beta_2 > \beta_1$

This can be accomplished by assuming $\sigma_2 > \sigma_1$, the elasticity of factor substitution in sector 2 is greater than the elasticity in Sector 1. Depending upon the relative sizes of K_1 and K_2, L_1 and L_2, $\beta_2 \frac{d\beta_1}{dX_2} \lesseqgtr \beta_1 \frac{d\beta_2}{dX_2}$.

Appendix 3

To determine the impact of the expected change in the price ratio on factor allocation and factor prices reformulate Batra's model (1975a) to reflect the assumptions in this paper.

Rewriting the factor markets equilibria:

$$w_1 = w_2; \quad \frac{E[U_1(\Pi_1)]\hat{P}_1 FL_1}{E[U'_1]} = \frac{E[U_2(\Pi_2)]\hat{P}_2 FL_2}{E[U'_2]}$$

$$r_1 = r_2: \frac{E[U_1(P_1)]\hat{P}_1 FK_1}{E[U'_1]} = \frac{E[U_2(\Pi_2)]\hat{P}_2 FK_2}{E[U'_2]}$$

The production functions: $X_1 = F_1(K_1 L_1) = L_1 f_1 (k_1)$

$$X_2 = F_2(K_2 L_2) = L_2 f_2 (k_2)$$

$FL_1 = f_1 - k_1 f'_1$; $FL_2 = f_2 - k_2 f'_2$; $FK_1 = f'_1$;$FK_2 = f'_2$

$K_1 = L_1 k_1$; $K_2 = L_2 k_2$

The profit equations: $\Pi_1 = \hat{P}_1 X_1 - w_1 L_1 - r_1 K_1$

$$\Pi_2 = \hat{P}_2 X_2 - w_2 L_2 - r_2 K_2$$

Rewriting the profit equation for X_1:

$$\Pi_1 = \hat{P}_1 \; [L_1 f_1(k_1)] - L_1 \left\{\frac{E[U_1(\Pi_1)]\hat{P}_1(f_1 - k_1 f'_1)}{E[U_1]}\right\} - L_1 \left\{k_1 f'_1 \frac{E[U_1 \Pi_1]}{E[U_1]}\right\}$$

$$\Pi_1 = L_1 \left\{\hat{P}_1 f_1(k_1) - \hat{P}_1(f_1 - k_1 f_1) \frac{E[U_1(\Pi_1)]}{E[U_1]} - k_1 \hat{P}_1 f'_1 \frac{E[U_1(\Pi_1)]}{E[U_1]}\right\}$$

$$= L_1 \left\{\hat{P}_1 f_1(k_1) - \hat{P}_1 f_1 \frac{E[U_1(\Pi_1)]}{E[U_1]} + k_1 \hat{P}_1 f'_1 \frac{E[U_1(\Pi_1)]}{E[U_1]} - k_1 \hat{P}_1 f'_1 \frac{E[U_1(\Pi_1)]}{E[U_1]}\right\}$$

$= L_1 \left\{\hat{P}_1 f_1(k_1) - \hat{P}_1 f_1 \frac{E[U_1(\Pi_1)]}{E[U_1]}\right\}$. Therefore Π_1 is a function of the labor input (L_1), the capital labor ratio (k_1) and expected price of X_1 ($\hat{P}_1$). Π_2 is a function of L_2, k_2, $\hat{P}_2$. k_1,k_2, L_1 and L_2 are dependent variables and can be solved by using the factor markets equilibria. Differentiating $w_1 = w_2$:

$$\hat{P}_1 (f_1 - k_1 f'_1) \left[\frac{E[U_1]E[U_{11}d\Pi_1] - E[U_1\Pi_1]E[U_{11}]}{(E[U_1])^2}\right] + \frac{\hat{P}_1 E[U_1\Pi_1]}{E[U_1]} (-k_1 f''_1 dk_1)$$

$$+ (f_1 - k_1 f'_1), \frac{E[U_1\Pi_1]}{E[U_1]} d\hat{P}_1 = \hat{P}_2 \; f_2 - k_2 f'_2, \left[\frac{E[U_2]E[U_{22}d\Pi_2] - E[U_2\Pi_2]E[U_{22}]}{(E[U_2])^2}\right]$$

$$+ \hat{P}_2 \frac{E[U_2\Pi_2]}{E[U_2]} (-k_2 f''_2 dk_2) + (f_2 - k_2 f'_2) \frac{E[U_2\Pi_2]}{(E[U_2])^2} d\hat{P}_2.$$

Simplifying

$$E\left[\frac{U_1U_{11}d\Pi_1 - U_1\Pi_1U_{11}}{(U_1)^2} (\hat{P}_1 FL_1) - \frac{U_2U_{22}d\Pi_2 - U_2\Pi_2U_{22}}{(U_2)^2} (\hat{P}FL_2)\right.$$

$$\left. -\hat{P}_1 \frac{U_1\Pi_1}{U_1} dFL_1 + \frac{U_1\Pi_1}{U_1} FL_1 d\hat{P}_1\right] = \frac{E[U_2\Pi_2]}{E[U_2]} \hat{P}_2 dFL_2 + \frac{E[U_2\Pi_2]}{E[U_2]} FL_2 d\hat{P}_2$$

becomes:

$$-\hat{P}_1 \frac{E[U_1\Pi_1]}{E[U_1]} dFL_1 + \frac{E[U_1\Pi_1]}{E[U_1]} FL_1 d\hat{P}_1 = \hat{P}_2 \frac{E[U_2\Pi_2]}{E[U_2]} dFL_2 + \frac{E[U_2\Pi_2]}{E[U_2]} FL_2 d\hat{P}_2$$

$dw_1 = dw_2$

Differentiating $r_1 = r_2$

$$\hat{P}FK_1\left[\frac{E[U_1]E[U_{11}d\Pi_1] - E[U_1\Pi_1]E(U_{11})}{(E[U_1])^2}\right] + \hat{P}_1\frac{E[U_1\Pi_1]}{E[U_1]}dFK_1 + FK_1\frac{E[U_1\Pi_1]}{E[U_1]}d\hat{P}_1$$

$$= \hat{P}_2 FK_2\left[\frac{E[U_2]E[U_{22}d\Pi_2] - E[U_2\Pi_2]E(U_{22})}{(E[U_2])^2}\right]$$

$$+ \hat{P}_2\frac{E[U_2\Pi_2]}{E[U_2]}dFK_2 + FK_2\frac{E[U_2\Pi_2]}{E[U_2]}d\hat{P}_2$$

Simplifying:

$$-\hat{P}_1\frac{E[U_1\Pi_1]}{E[U_1]}dFK_1 + FK_1\frac{E[U_1\Pi_1]}{E[U_1]}d\hat{P}_1 = \hat{P}_2\frac{E[U_2\Pi_2]}{E[U_2]}dFK_2 + FK_2\frac{E[U_2\Pi_2]}{E[U_2]}d\hat{P}_2$$

Therefore, $dr_1 = dr_2$

Differentiating the profits function, using $X_1 = L_1f_1(k_1)$

$\Pi_1 = L_1[\hat{P}_1f_1(k_1) - \hat{P}_2(f_2\text{-}k_2f'_2) - \hat{P}_2f'_2k_1]$ since $w_1 = w_2, r_1 = r_2$

$d\Pi_1 = L_1\{\hat{P}_1f_1(dk_1) + [f_1(k_1)]d\hat{P}_1 - (f_2\text{-}k_2f'_2)\, d\hat{P}_2 + \hat{P}_2k_2f''_2dk_2 - \hat{P}_2f'_2dk_1$

$- \hat{P}_2k_1f''_2dk_2 - f'_2k_1d\hat{P}_2\} + [\hat{P}_1(f'_1(k_1)\text{-}f_1)]\, dL_1$

$=L_1\{(\hat{P}_1f'_1 - \hat{P}_2f'_2)dk_1 + (\hat{P}_2k_2f''_2 - \hat{P}_2k_1f''_2)dk_2 + [f_1(k_1)]d\hat{P}_1$

$-(f_2\text{-}k_2f'_2 + f'_2k_1)\, d\hat{P}_2\} + [\hat{P}_1(f'_1(k_1)\text{-}f_1)]dL_1$

$=L_1(\hat{P}_1f'_1\text{-}\hat{P}_2f'_2)dk_1 + L_1(\hat{P}_2k_2f''_2 - \hat{P}_2k_1f''_2)\, dk_2 + [\hat{P}_1(f'_1(k_1)\text{-}f_1)]dL_1$

$+ L_1f_1(k_1)d\hat{P}_1 - (f_2 - k_2f'_2 + f_2k_1)d\hat{P}_2$

The dependent variables are: dL_1, dk_1, dk_2

The independent variables are: $d\hat{P}_1$, $d\hat{P}_2$, dK, dL; let $R = dK\text{-}k_2dL = 0$

let $\hat{P} = \hat{P}_2 / \hat{P}_1$; $\hat{P}_1 = 1$

Putting $d\Pi_1$, $dw_1 = dw_2$ and $dr_1 = dr_2$ into matrix form

$$\begin{array}{cccccc}
dL_1 & dk_1 & dk_2 & & d\hat{P}_1 & d\hat{P}_2 \\
f'_1(k_1)-f_1 & +L_1(f'_1-\hat{P}f'_2) & +L_1(\hat{P}k_2f''_2-\hat{P}k_1f''_2) & = & -L_1f_1(k_1) & +L_1(f_2-k_2f'_2+f'_2k_1) \\
A & B_1=0 & C_1>0 & & &
\end{array}$$

$$\begin{array}{cccccc}
\dfrac{-E[U_1\Pi_1]}{E[U_1]}k_1f''_1 & \hat{P}\dfrac{E[U_2\Pi_2]}{E[U_2]}k_2f''_2 & = & \dfrac{-E[U_1\Pi_1]}{E[U_1]}(f_1-k_1f'_1) & + \dfrac{E[U_2\Pi_2]}{E[U_2]}(f_2-k_2f'_2) \\
B_2>0 & C_2<0 & & &
\end{array}$$

$$\begin{array}{cccccc}
\dfrac{E[U_1\Pi_1]}{E[U_1]}f''_1 & \hat{P}\dfrac{E[U_2\Pi_2]}{E[U_2]}f''_2 & = & \dfrac{-E[U_1\Pi_1]}{E[U_1]}f'_1 & + \dfrac{E[U_2\Pi_2]}{E[U_2]}f'_2 \\
B_3<0 & C_3>0 & & &
\end{array}$$

Let $H_1 = -L_1f_1(k_1)\dfrac{E[U_1\Pi_1]}{E[U_1]}(f_1-k_1f'_1) - \dfrac{E[U_1\Pi_1]}{E[U_1]}f'_1 < 0$

$$H_2 = L_1(f_2-k_2f'_2+f'_2k_1) + \frac{E[U_2\Pi_2]}{E[U_2]}(f_2-k_2f'_2) + \frac{E[U_2\Pi_2]}{E[U_2]}f'_2 > 0$$

In matrix form: $\begin{bmatrix} A & B_1 & C_1 \\ 0 & B_2 & C_2 \\ 0 & B_3 & C_3 \end{bmatrix}\begin{bmatrix} dL_1 \\ dk_1 \\ dk_2 \end{bmatrix} = \begin{bmatrix} H_1d\hat{P} \\ H_2d\hat{P} \\ 0 \end{bmatrix}$

$$D = \begin{vmatrix} A & B_1 & C_1 \\ 0 & B_2 & C_2 \\ 0 & B_3 & C_3 \end{vmatrix} > 0$$

$$\frac{dL_1}{d\hat{P}} = \frac{\begin{vmatrix} H_1 & B_1 & C_1 \\ H_2 & B_2 & C_2 \\ 0 & B_3 & C_3 \end{vmatrix}}{D} = H_1\,(B_2C_3 - B_3C_2) - H_2(B_1C_3 - B_3C_1)\,/\,D < 0$$

As the expected relative price of X_2 increased, the amount of labor used in X_1 will decline. This is a logical result given that one would expect the output of X_2 to increase and, given full employment, the output of X_1 would decline.

$$\frac{dk_1}{d\hat{P}} = \frac{\begin{vmatrix} A & H_1 & C_1 \\ 0 & H_2 & C_2 \\ 0 & 0 & C_3 \end{vmatrix}}{D} = \frac{A(H_2C_3)}{D} > 0.$$ As the relative price of X_2 increases, the output of X_1 becomes relatively more capital intensive. This is not surprising since X_2 is relatively labor intensive and will draw from X_1 more labor than capital as output of X_2 expands.

$$\frac{dk_2}{d\hat{P}} = \begin{vmatrix} A & B_1 & H_1 \\ 0 & B_2 & H_2 \\ 0 & B_3 & 0 \end{vmatrix} = \frac{A(-B_3H_2)}{D} > 0.$$ As the relative price of X_2 increases, the output of X_2 uses relatively more capital, becoming less labor intensive because X_1 releases more capital than labor as its output declines.

The input price ratio: $W = \frac{w}{r} = \frac{EU_2(\Pi_2)]\hat{P}FL_2}{E[U_2]} \div \frac{EU_2(\Pi_2)]\hat{P}FK_2}{E[U_2]}$

$$dW = E\left[\frac{U_2U_{22}(d\Pi_2) - U_2(\Pi_2)U_{22}}{(U_2)^2}\right]\hat{P}FL_2 + \hat{P}E\left[\frac{U_2\Pi_2}{U_2}\right]dFL_2 + FL_2\,E\left[\frac{U_2\Pi_2}{U_2}\right]d\hat{P}$$

$$\div E\left[\frac{U_2U_{22}(\Pi_2) - U_2(\Pi_2)U_{22}}{(U_2)^2}\right]\hat{P}FK_2 + \hat{P}E\left[\frac{U_2\Pi_2}{U_2}\right]dFK_2 + FK_2\,E\left[\frac{U_2\Pi_2}{U'_2}\right]d\hat{P}$$

It can be see that the change in W is due to changes in factor intensities and profits besides the expected change in the output price ratio.

$dFL_2 = -k_2f''_2dk_2 > 0$

$dFK_2 = -f''_2dk_2 > 0$

Knowing $dP > 0$ and $dE\left[\frac{U_2\Pi_2}{U_2}\right] > 0$, then $dW > 0$; so, the Stolper Samuelson Theorem holds.

VITA

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