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## Large quantum gravity effects: backreaction on matter

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We reexamine the large quantum gravity effects discovered by Ashtekar in the context of 2 + 1 dimensional gravity coupled to matter. We study an alternative one-parameter family of coherent states of the theory in which the large quantum gravity effects on the metric can be diminished, at the expense of losing coherence in the matter sector. Which set of states is the one that occurs in nature will determine if the large quantum gravity effects are actually observable as wild fluctuations of the metric or rapid loss of coherence of matter fields.

One normally thinks that classical general relativity has a rather wide and well defined domain of applicability as a realistic theory of nature. This belief is based on the expectation that if one were to consider a full quantum treatment of the theory, one could find states that minimize uncertainties in all the basic fields and that obey macroscopically the classical Einstein equations of evolution. One expects that this picture would break down only when very high energies come into play.

Ashtekar [1] recently reexamined these beliefs by considering a model of quantum gravity coupled to matter in 2 + 1 dimensions. The specific kind of matter is not very relevant to the discussion, but for concreteness it is taken to be a scalar field. The theory is completely integrable classically. The metric is given by,

$$ds^2 = e^{G\Gamma(r,t)}(t,r)(-dt^2 + dr^2) + r^2 d\theta^2 \quad (1)$$

where  $\Gamma$  is completely determined by the scalar field,

$$\Gamma(r,t) = \frac{1}{2} \int_0^r dr' r' [\dot{\phi}^2 + \phi'^2]. \quad (2)$$

So one can picture the theory as that of a scalar field living in a fiducial flat spacetime that completely determines the metric through the above expressions. One can quantize the model by considering a Fock representation for the scalar field and thinking of the metric as a quantum operator. What Ashtekar observes is that if one considers a coherent state for the scalar field and computes the uncertainties in the metric one gets, in the high frequency regime (the frequency refers to the coherent state of the scalar field),

$$\left( \frac{\Delta g_{rr}}{\langle g_{rr} \rangle} \right)^2 \sim e^{\mathcal{N}(e^{2G\hbar\omega})} \quad (3)$$

where  $\mathcal{N}$  is the occupation number of the coherent state. The surprise here is that even if one considers a configuration with such a low classical field amplitude that it corresponds to a “one photon” ( $\mathcal{N} = 1$ ) state, one finds that the uncertainty in the metric is huge if the frequency of the photon is high. This is quite unexpected, ie, one expected that the classical theory would break down for large curvature effects, not that a field of low amplitude and high frequency could have such dramatic effects on the metric. The effect is largely due to the nonlinearity of gravity, which in turn implies the exponential dependence of metric and field. It is not present, for instance, for scalar electrodynamics [2]. The above effect is not necessarily an artifact of 2 + 1 dimensions, the above calculations also represent 3 + 1 gravity with one constant-norm Killing vector field. Similar effects could also be present in other, more realistic 3 + 1 contexts [2].

The issue we wish to raise in this note is up to what extent should one consider states for this model exactly as the ones considered above. Such states were precisely coherent states for the field, and therefore are clearly a good choice to represent a classical state of matter. In the quantization, since one thinks of the metric as a derived operator, one does not pay too much attention to its uncertainties while constructing the coherent states. But what if one did?

That is, what happens if we consider states that not only minimize the uncertainties in the scalar field variables but also simultaneously minimizes the uncertainties of the metric. Could one avoid the large quantum gravity effects? Or would the price of suppressing the large metric fluctuations translate in a rapid loss of coherence of the matter fields? A quantitative answer to these questions is the main purpose of this note.

For simplicity we will limit the discussion to one frequency mode, the extension to many modes is immediate. We seek for states  $|\psi\rangle$  that minimize<sup>1</sup>,

$$\mathcal{F} = \frac{(\Delta x)^2 + (\Delta p)^2}{\langle x \rangle^2 + \langle p \rangle^2} + \kappa \left( \frac{\Delta g}{\langle g \rangle} \right)^2 - \beta(\langle \psi | \psi \rangle - 1) + \lambda[\langle x \rangle^2 + \langle p \rangle^2 - 2|\alpha|^2]. \quad (4)$$

where  $\hat{x} = (a + a^\dagger)/\sqrt{2}$  and  $\hat{p} = i(a^\dagger - a)/\sqrt{2}$ . That is, we are seeking to minimize uncertainties in  $x$ ,  $p$  and  $g$  simultaneously. The parameter  $\kappa$  determines the relative weight of the uncertainties in the metric with respect to the field variables in the minimization process. The Lagrange multipliers  $\lambda$  and  $\beta$  are introduced to require that the states are normalized and that the expectation values of position and momenta have the correct scaling with the energy,  $|\alpha|$ . This latter condition is equivalent to imposing  $\alpha = (\langle x \rangle + i \langle p \rangle)/\sqrt{2}$ . If one takes  $\kappa = 0$  the states that minimize the above functional are the ordinary coherent states,

$$|\alpha\rangle = \sum_{n=0}^{\infty} e^{-\frac{|\alpha|^2}{2}} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \quad (5)$$

This suggests to consider as proposal for states that will minimize the above functional,

$$|\psi\rangle = \sum_{n=0}^{\infty} b_n |n\rangle \quad (6)$$

where  $|n\rangle$  are the  $n$ -photon states of frequency  $\omega$  for the scalar field in 2 + 1 dimensions, see [1].

In terms of the above states, we get,

$$\langle \psi | \psi \rangle = \sum_{n=0}^{\infty} b_n^* b_n = 1, \quad (7)$$

$$\frac{\sqrt{2}}{2} \langle x + ip \rangle = \sum_{m=0}^{\infty} \sqrt{m+1} b_m^* b_{m+1} = \alpha, \quad (8)$$

$$(\Delta x)^2 + (\Delta p)^2 = \langle x^2 \rangle + \langle p^2 \rangle - 2|\alpha|^2 = \sum_{m=0}^{\infty} (2m+1) b_m^* b_m - 2|\alpha|^2, \quad (9)$$

$$\langle g \rangle = \sum_{m=0}^{\infty} e^{m\Omega} b_m^* b_m \equiv g, \quad (10)$$

$$\langle g^2 \rangle = \sum_{m=0}^{\infty} e^{2m\Omega} b_m^* b_m \equiv C, \quad (11)$$

where the metric operator was defined as in [1]  $\hat{g} = \exp(\Omega a^\dagger a)$ ,  $\Omega$  being the ratio of the frequency  $\omega$  to the Planck frequency.

With the above relations we can proceed to minimize  $\mathcal{F}$ . We get as minimization condition

$$(2m+1)b_m^* + 2\kappa|\alpha|^2(g^{-2}e^{2\Omega m} - 2Cg^{-3}e^{\Omega m})b_m^* - \beta b_m^* - \lambda(2\sqrt{m}b_{m-1}^*\alpha^* + 2\sqrt{m+1}b_{m+1}^*\alpha) = 0 \quad (12)$$

where we have redefined  $\beta$  and  $\lambda$  to absorb a factor  $2|\alpha|^2$ . Given a value of  $\Omega$ , the solution depends on  $\kappa$  and  $\alpha$ . The joint solution of the above equation (12) with the constraint equations (7,8) imposed by the Lagrange multipliers completely determines the problem, giving values for  $b_m$ ,  $\beta$  and  $\lambda$ .

We have been unable to solve the above minimization condition in closed form. We will perform a perturbative analysis in terms of  $\Omega$ , which is a small parameter (it is the ratio of the frequency of the state to the Planck frequency) in

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<sup>1</sup>Our conventions are  $[a, a^\dagger] = 1$ ,  $[\hat{x}, \hat{p}] = i$ .

the classical, sub-Planckian domain. Before performing the perturbative analysis, we introduce new rescaled variables,  $\ell = \lambda^{-1}$  and

$$b_m = \frac{\alpha^m e^{-\frac{|\alpha|^2}{2}} \ell^m}{\sqrt{m!}} B_m. \quad (13)$$

In terms of these we can rewrite the minimization condition as

$$2m(B_m^* - \ell^{-2} B_{m-1}^*) + 2|\alpha|^2(B_m^* - B_{m+1}^*) - \beta B_m^* + 2\kappa|\alpha|^2(g^{-2} e^{2\Omega m} - 2Cg^{-3} e^{\Omega m}) B_m^* = 0. \quad (14)$$

Let us start by studying the case  $\Omega = 0$ , which we will consider the zeroth order in the perturbative analysis we wish to perform. The solution to the joint set of equations (7,8,14) is

$$B_m = 1, \quad \ell = 1 \quad \beta = -2\kappa|\alpha|^2, \quad (15)$$

and leads to the usual coherent states. This is equivalent to solving for  $\kappa = 0$  and  $\Omega$  arbitrary. For the usual coherent states  $(\Delta x)^2 + (\Delta p)^2 = 1$ .

We now study higher order perturbations in  $\Omega$ . It is straightforward to see that the term in  $\mathcal{F}$  dependent on the metric does not give contributions to first order in  $\Omega$ , so to see a nontrivial effect of the new proposal we need to go to second order. To compute to second order we write  $B_m = 1 + \Omega^2 B_m^{(2)}$ ,  $\ell = 1 + \Omega^2 \ell^{(2)}$  and  $\beta = -2\kappa|\alpha|^2 + \beta^{(2)}$ , and also,

$$\langle g \rangle = 1 + \Omega \sum_{m=0}^{\infty} m b_m^* b_m + \frac{\Omega^2}{2} \sum_{m=0}^{\infty} m^2 b_m^* b_m, \quad (16)$$

$$b_m = \frac{\alpha^m e^{-\frac{|\alpha|^2}{2}}}{\sqrt{m!}} + O(\Omega^2). \quad (17)$$

Now, noting that

$$\sum_{m=0}^{\infty} \frac{m|\alpha|^{2m}}{m!} e^{-|\alpha|^2} = e^{-|\alpha|^2} |\alpha|^2 \frac{\partial}{\partial |\alpha|^2} e^{|\alpha|^2} = |\alpha|^2, \quad (18)$$

$$\sum_{m=0}^{\infty} \frac{m^2 |\alpha|^{2m}}{m!} e^{-|\alpha|^2} = e^{-|\alpha|^2} |\alpha|^2 \frac{\partial}{\partial |\alpha|^2} |\alpha|^2 \frac{\partial}{\partial |\alpha|^2} e^{|\alpha|^2} = |\alpha|^4 + |\alpha|^2, \quad (19)$$

we get,

$$\langle g \rangle = 1 + |\alpha|^2 \Omega + \frac{|\alpha|^4 \Omega^2}{2} + \frac{|\alpha|^2 \Omega^2}{2} + O(\Omega^3), \quad (20)$$

$$\langle g^2 \rangle = 1 + 2|\alpha|^2 \Omega + 2|\alpha|^4 \Omega^2 + 2|\alpha|^2 \Omega^2 + O(\Omega^3), \quad (21)$$

and from here the recursion relation (14) reads,

$$2m(B_m^{(2)} - B_{m-1}^{(2)}) + 2|\alpha|^2(B_m^{(2)} - B_{m+1}^{(2)}) - \beta^{(2)} + 4m\ell^{(2)} + 2\kappa|\alpha|^2(m^2 - 2m|\alpha|^2 + |\alpha|^4) = 0. \quad (22)$$

One immediately sees that the  $B_m^{(2)}$ 's only depend on  $\alpha$  through  $|\alpha|^2$ , since  $g$  and  $C$  are real. This has important implications, it means that the states will not disperse in time, that is, under time evolution,

$$b_m(\alpha, \kappa) \longrightarrow b_m(\alpha e^{i\Omega t}, \kappa), \quad (23)$$

which in turn implies that  $\langle x \rangle$  and  $\langle p \rangle$  satisfy the classical equations of motion, ie, the states have all the required properties of coherent states [3].

The solution of the recursion relation is,

$$\ell = 1 - \Omega^2 \kappa |\alpha|^2, \quad (24)$$

$$\beta = -2\kappa|\alpha|^2 - 4\kappa|\alpha|^4 \Omega^2, \quad (25)$$

$$b_m = \frac{\alpha^m}{\sqrt{m!}} e^{-\frac{|\alpha|^2}{2}} \left( 1 - \frac{\kappa|\alpha|^6 \Omega^2}{2} + m\Omega^2 \left( \kappa|\alpha|^4 + \frac{\kappa|\alpha|^2}{2} \right) - \frac{\kappa|\alpha|^2}{2} m^2 \Omega^2 \right). \quad (26)$$

An analogous calculation yields the  $O(\Omega^3)$  corrections,

$$\ell^{(3)} = -\kappa|\alpha|^2, \quad (27)$$

$$b_m^{(3)} = \frac{\alpha^m}{\sqrt{m!}} e^{-\frac{|\alpha|^2}{2}} \left[ \frac{2}{3}\kappa|\alpha|^8 + \frac{5}{2}\kappa|\alpha|^6 + m(-\kappa|\alpha|^6 + 2\kappa|\alpha|^4 + \frac{5}{6}\kappa|\alpha|^2) + m^2(\kappa|\alpha|^4 - \frac{\kappa|\alpha|^2}{2}) - \frac{m^3\kappa|\alpha|^2}{3} \right] \quad (28)$$

From here we can compute the uncertainties in the quantities of physical interest,

$$(\Delta x)^2 + (\Delta p)^2 = 1 + \Omega^3 \left( \frac{2}{3}\kappa|\alpha|^{10} + 8\kappa|\alpha|^8 \right), \quad (29)$$

$$\left( \frac{\Delta g}{g} \right)^2 = \left( \frac{\Delta g}{g} \right)_{\kappa=0}^2 - 2\kappa|\alpha|^6\Omega^4 - \Omega^5\kappa \left( \frac{2}{3}|\alpha|^{12} + \frac{22}{3}|\alpha|^{10} - 8|\alpha|^8 + 6|\alpha|^6 \right). \quad (30)$$

So we see that as we increase the parameter  $\kappa$  the fluctuations in the matter fields increase and the fluctuations in the gravitational field decrease. This allows the large quantum gravity effects to manifest themselves in ways different from Ashtekar's original proposal, ie, they may arise as early losses of coherence of the matter fields, long before the gravitational perturbations are significant. Notice also the rather high powers of  $\alpha$  involved, which suggests that these kinds of effects might be more detectable than the ones due to ordinary coherent states.

The above results imply the existence of a one-parameter family of coherent states for the gravitational field coupled to matter. However, because we have pursued a perturbative power series approach, one may question if the results presented are rigorous, in the sense that the power series discussed may fail to converge. To show that one really has a family of states of physical interest, we need to discuss the convergence of the series discussed.

Therefore, we need to study the behavior of the coefficients for large values of  $m$ . We get,

$$b_m = \frac{e^{-\frac{|\alpha|^2}{2}}|\alpha|^m}{\sqrt{m!}} \left( 1 - \frac{\Omega^2\kappa|\alpha|^2m^2}{2} - \frac{\Omega^3\kappa|\alpha|^2m^3}{3} - \dots - \frac{\Omega^p(2^p-2)}{pp!}\kappa|\alpha|^2m^p - \dots \right), \quad (31)$$

so,

$$|b_m| < \frac{e^{-\frac{|\alpha|^2}{2}}|\alpha|^m}{\sqrt{m!}} \left( 1 + \sum_{p=1}^{\infty} \frac{\Omega^p 2^p m^p}{p!} \kappa |\alpha|^2 \right) = \frac{e^{-\frac{|\alpha|^2}{2}}|\alpha|^m}{\sqrt{m!}} [(e^{2m\Omega} - 1) \kappa |\alpha|^2 + 1] \quad (32)$$

and therefore the series  $\sum_m b_m^* b_m$  converges,

$$\sum b_m^* b_m < (1 - \kappa|\alpha|^2) + \kappa|\alpha|^2 e^{|\alpha|^2(e^{2\Omega} - 1)}. \quad (33)$$

Summarizing, we see that in the context of the 2 + 1 example analyzed by Ashtekar [1] the gravitational field coupled to matter admits a more general one-parameter family of coherent states. These are states that minimize the combined uncertainties of the matter fields and the gravitational field and are preserved under evolution. The ‘‘large quantum gravity effects’’ pointed out by Ashtekar are present for all these states. That is, if one has matter modes of high frequency, fluctuations in the physical quantities may become large, even if one has ‘‘one photon’’ of high frequency. However, depending on the value of the parameter  $\kappa$  that characterizes our family, the fluctuations can be concentrated on the metric (as in Ashtekar's original example) or in the matter fields. In this sense the ‘‘large quantum gravity effects’’ may not be necessarily manifested as in Ashtekar's original suggestion as large fluctuations in the metric, but as early losses of coherence of the matter fields. This also opens the possibility for the effects to become observable at lower values of the frequency.

A natural question is, which are the ‘‘correct’’ states to describe nature from within this one-parameter family. This will require a more detailed analysis of the situation in question, and how the material system under study interacts with its environment. That will ultimately determine which of these states are relevant and therefore in which range of physical parameters can one expect to see the large effects. One may think that these only manifest themselves for frequencies that are comparable with Planck's frequency, and therefore are mostly of academic interest. However, it is worthwhile pointing out that they not only depend on  $\Omega$  but also on the value of  $|\alpha|^2$ , which can be very large in realistic systems. Indeed, our perturbative calculations break down for  $\Omega \sim 10^{-25}$  if  $|\alpha|^2 \sim 10^{20}$ . In actual high power lasers, one can get  $\Omega \sim 10^{-28}$  with  $|\alpha|^2 \sim 10^{15}$  so the predicted decoherence is not necessarily that far from observability.

An intriguing possibility is if these effects might be present in the collapse of neutron stars. The outer core of neutron stars is in a superfluid state, which could be regarded as a coherent state of matter. The problem here is that the frequency in question is largely determined by the rotation rate of the star (the frequency is related with the kinetic energy of the matter). These rates are very low compared with the Planck frequency. However, when the star collapses and the outer regions become trapped within the horizon, local velocity gradients become very high. It is questionable however, if these local velocity gradients are the relevant ones for quantization. The detailed discussion of this situation clearly exceeds the scope and possibilities of the  $2 + 1$  example we have been discussing all along and application of the current results can only be considered at present as speculative. However, the possibility that large quantum gravity effects could imply a loss of coherence of matter or large fluctuations of the metric in neutron stars and therefore may affect the collapse scenario is attractive enough to merit further investigation of these kinds of effects.

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