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# An overview of canonical quantum gravity

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This is a summary of a talk delivered at the workshop “Quantum gravity in the Southern Cone II”. We present a very brief review of current results on canonical quantization of general relativity using Ashtekar’s variables and loop quantization.

## I. INTRODUCTION

Slightly over ten years ago, a new set of paths were opened in the quest to try to apply the rules of quantum mechanics to general relativity. Several researchers have contributed to this program, developing various points and lines of thought. The common threads of this work have to do with the use of a new set of variables to describe the gravitational field, introduced by Ashtekar [2], that make the canonical general relativity resemble a Yang–Mills theory.

At first some people might find the premise of the whole program questionable. After all, general relativity is known to be power-counting-non-renormalizable; string theory suggests extra structures are needed for a consistent theory; moreover, even quantum field theory in curved space-times encounters serious difficulties. Why waste effort attempting a program that appears surrounded by doom? It should be realized, however, that most of the above statements are made within the context of perturbation theory. Most of the detailed calculations also introduce artificial background structures (for instance, a Minkowski background in usual perturbative calculations) in the theory that conflict with the basic symmetry of general relativity: diffeomorphism invariance. In fact, if one takes seriously the idea that diffeomorphism invariance is a fundamental symmetry of nature, one might be led to believe that the proposed program is quite sensible. Diffeomorphism invariance is usually associated with non-perturbative, discrete objects and concepts. These are quite distinct from those normally used in quantum field theories. It might be the case that by attempting to impose diffeomorphism invariance, we will be forced to use techniques and ideas that are quite novel. In fact, the emergence of discrete structures and of fundamental dynamical length scales (like the Planck length), suggests that the ultraviolet problem of quantum field theories might be controlled. These ideas have been argued back and forth in generic terms for many years. However, the approach we will describe here actually puts them into practice and reaches conclusions about the points raised in a rather concrete fashion. In the end, the proof of the validity of what is being attempted will be given by the emergence of concrete physical predictions, hopefully testable, from the theory. We will show that although we still do not have complete control of the theory, some physical predictions are already emerging.

This paper will be a very quick and succinct review. It will not attempt to make a detailed case for the various aspects mentioned. It will be skimpy on explicit formulae. It will be incomplete in referring to previous work. It just attempts to be a quickly readable manuscript to introduce someone to some of the ideas in the subject. For a more comprehensive review see the article by Rovelli in Living Reviews [1].

## II. CANONICAL GRAVITY

Let us start with the basic setting. We will attempt a canonical quantization of gravity. This will require setting the theory in a Hamiltonian form. This has been studied by many authors (see [3] for references). The idea is that one foliates space-time into space and time and considers as fundamental canonical variables the three metric  $q^{ab}$  and as canonically conjugate momentum a quantity that is closely related to the extrinsic curvature  $K_{ab}$ . The time-time and the space-time portions of the space-time metric (known as the lapse and shift vector) appear as Lagrange multipliers in the action, which means that the theory has constraints. In total there are four constraints, that structure themselves into a vector and a scalar. These constraints are the imprint in the canonical theory of the diffeomorphism invariance of the four-dimensional theory. They also contain the dynamics of the theory, since the Hamiltonian identically vanishes. This is not surprising, it is the way in which the canonical formalism tells us that the split into space and time that we perform is a fiduciary one. If one attempts to quantize this theory one starts by choosing a polarization for the wavefunctions (usually functions of the three metric) and one has to implement the constraints as operator equations. These will assure that the wavefunctions embody the symmetries of the theory. The diffeomorphism constraint has a geometrical interpretation, demanding that the wavefunctions be functions of the “three-geometry” and not of the three-metric, that is, that they be invariant under diffeomorphisms

of the three manifold. The Hamiltonian constraint does not admit a simple geometric interpretation and should be implemented as an operatorial equation. Unfortunately, it is a complicated non-polynomial function of the basic variables and little progress had been made towards realizing it as a quantum operator ever since De Witt considered the problem in the 60's. Let us recall that in this context regularization is a highly non-trivial process, since most common regulators used in quantum field theory violate diffeomorphism invariance. Even if we ignore these technical details, the resulting theory appears as very difficult to interpret. The theory has no explicit dynamics, one is in the “frozen formalism”. Wavefunctions are annihilated by the constraints and observable quantities commute with the constraints. Observables are better described, as Kuchař emphasizes, as “perennials”. The expectation is that in physical situations some of the variables of the theory will play the role of “time” and in terms of them one would be able to define a “true” dynamics in a relational way, and a non-vanishing Hamiltonian. Unfortunately, this has never been fully implemented in practice and in fact several model examples show that the idea can quickly run into trouble [4]. This is the content of the “problem of time” in canonical quantization, which has been discussed extensively in the literature [5].

About ten years ago, Ashtekar [2] proposed the use of a new set of variables to describe canonical gravity. These new variables had the advantage that they made the theory resemble a Yang–Mills theory. This opened the possibility of importing techniques from the Yang–Mills context to the gravitational one. The easiest way to introduce the new variables (as pointed out by Barbero [6]) is to go through an intermediate step, reformulating ordinary canonical gravity in terms of triads instead of metrics. Canonical gravity has been described in terms of triads in the past (see for instance [7]). The canonical variables are three frame fields  $\tilde{E}_i^a$  and the conjugate variables  $K_a^i$  are again closely related to the extrinsic curvature. We use a tilde to denote density weights. The theory has the usual diffeomorphism and Hamiltonian constraints plus three additional constraints that state that the theory is invariant under triad rotations. The Hamiltonian constraint is, as in the usual metric variables, non-polynomial. If in this theory we now perform a canonical transformation defining a new variable  $A_a^i = \Gamma_a^i + \beta K_a^i$ , where  $\Gamma_a^i$  is the metric-compatible spin connection, and  $\beta$  is a parameter, and we re-scale the triads by  $1/\beta$ , the constraints of the theory read,

$$D_a \tilde{E}_i^a = 0 \tag{2.1}$$

$$\tilde{E}_i^a F_{ab}^i = 0 \tag{2.2}$$

$$\epsilon_{ijk} \tilde{E}_i^a \tilde{E}_j^b F_{ab}^k + 2 \frac{(1 + \beta^2)}{\beta} \tilde{E}_i^a \tilde{E}_j^b (A_a^i - \Gamma_a^i)(A_b^j - \Gamma_b^j) = 0. \tag{2.3}$$

As we see, the first constraint has exactly the form of a Yang–Mills type Gauss law ( $D_a$  is the derivative defined by the connection  $A_a^i$ ). The second constraint (the diffeomorphism or momentum constraint) states that the Poynting vector of the field vanishes. The last constraint (also called the Hamiltonian constraint, or, when promoted to a quantum operator, the Wheeler–DeWitt equation) is non-polynomial due to the presence of the  $\Gamma_a^i$  terms. If one were to choose the parameter  $\beta$  equal to the imaginary unit, the last term disappears and the constraints become polynomial. This was the original way in which the Ashtekar variables were introduced. The polynomiality of the constraint led quickly to a quantum representation and the discovery of solutions, for the first time ever, of the Wheeler–DeWitt equation [8]. Nowadays, the variables is slightly different. Thiemann [9] has shown that if one divides the Hamiltonian constraint by the square root of the determinant of the three metric, both pieces of the constraint can be made polynomial in terms of Poisson brackets of the connection and the volume of the space. This has led to a series of articles in which he has explored the quantum implementation of the constraints, that we will comment upon later. Therefore, there is no need, from the polynomiality standpoint, to set the value of  $\beta$  to the imaginary unit. This has additional advantages. If  $\beta$  is complex one is dealing with a real theory described in terms of complex variables. After quantization, reality has to be recovered. This led to lengthy discussions about how to implement the “reality conditions” that assured that the theory was a real one. If one takes real values for  $\beta$ , all variables in the theory are real. The resulting theory has a more complicated —yet polynomial— Hamiltonian constraint than the one with  $\beta = i$ , but it still retains the Gauss law and Poynting vector conditions (which are  $\beta$ -independent). We can still think of the phase space of the theory as a sub-manifold of the phase space of an  $SU(2)$  Yang–Mills theory, and apply several of the techniques we will discuss. We will therefore adopt the viewpoint from now on that the variables used are real.

### III. QUANTIZATION AND LOOPS

To proceed with the canonical quantization of the theory it is customary to pick a polarization where wavefunctions are functions of the connection  $A_a^i$ , this polarization would be the usual one used in a Yang–Mills context. The Gauss law requires that the wavefunction be  $SU(2)$  (gauge) invariant. The diffeomorphism constraint requires that the wavefunctions be diffeomorphism invariant. In this context it turns out to be useful to consider a particular

set of wavefunctions, parameterized by loops, called Wilson loops. These are constructed considering the trace of the holonomy of the connection  $A_a^i$  around a closed loop  $\gamma$ . These objects are gauge invariant and therefore solve automatically the Gauss law. In fact, there are a “basis” of gauge invariant functions in the sense that one can reconstruct all gauge invariant information in the connection from them [10], and therefore any function of the connection as well. These reasons favored their use for quite some time in the Yang–Mills context. Unfortunately, the Wilson loops  $W_\gamma(A)$  are not independent functions. Wilson loops based on different loops satisfy identities (called Mandelstam identities [11]) that constrain their values. Therefore they are not really a “basis” but an over-complete basis. We will address this point later on, at the moment we will loosely refer to Wilson loops as a basis.

If one expands a given wavefunction  $\psi(A)$  in terms of the Wilson loop basis,

$$\psi(\gamma) = \int DA \psi(A) W_\gamma(A) \tag{3.1}$$

the coefficients in the expansion will be functions of the loops  $\psi(\gamma)$  and will contain all the information  $\psi(A)$  contained. One can view these coefficients as wavefunctions in a new representation of the theory called the loop representation. As an analogy, one can consider the position and momentum representations in ordinary quantum mechanics and the Fourier transform between them. In the loop representation, wavefunctions are functions of loops, the Gauss law is already solved, and operators will have to be geometrical in nature, acting upon the loops. This representation had been studied by Gambini and Trias in the Yang–Mills context in the early eighties with some success, including lattice gauge theory calculations [11]. An added bonus in the gravitational context, first noted by Rovelli and Smolin [12], is that in this representation it is straightforward to satisfy the diffeomorphism constraint, simply by requiring that the wavefunctions be functions of loops invariant under smooth deformations of the loops. Such functions are called knot invariants in the mathematical literature. An unexpected connection between knot theory and quantum gravity had been uncovered.

#### IV. SPIN NETWORKS

Unfortunately, the fact that the Wilson loop basis is over-complete leaves an inconvenient imprint on the loop representation. For a function of a given loop to be admissible as a wavefunction in the loop representation, it has to satisfy the Mandelstam identities. Therefore not any knot invariant will be suitable for a wavefunction of the gravitational field. For years this problem somewhat blocked progress in the construction of states.

However, it was noted by Rovelli and Smolin [13] three years ago that there is a very natural way to label a basis of independent Wilson loops. The construction is based on the notion of spin network, introduced by Penrose [14] as a tool to quantize gravity in an unrelated context in the sixties. The spin networks of interest for quantum gravity,  $\Gamma$ , are graphs embedded in three dimensions with three or higher valence intersections. Each strand in the graph has associated with it an element of the gauge group in a given representation, labelled by a (half)integer  $j$ . The strands are “tied up together” at intersections using invariant tensors in the group. The resulting object, which we call “Wilson net”  $W_\Gamma(A)$  is a generalization to the spin network context of the trace of the holonomy of a single loop. It is a gauge invariant object, and it can be shown that they are free of Mandelstam identities. A way to believe this is to notice that Mandelstam identities are an imprint left on holonomies due to the use of a particular representation of the group in their definition [11]. Since Wilson nets contain all possible representations, there are no identities present.

Having labelled an independent set of Wilson loops allowed to make significant progress in other, apparently unrelated fronts. For many years the issue of what sort of measures would one use to perform integrals like (3.1) was an open question. These are measures on an infinite-dimensional space with nonlinear constraints (connections modulo gauge transformations). Having an independent basis allows to “do away with the nonlinearity” and construct examples of measures. This is a highly technical topic that was pioneered by Ashtekar and Isham [15] and further developed by Ashtekar, Baez, Lewandowski, Marolf, Mourao, Thiemann [16]. To help tame the infinite dimensionality of the space in questions, use is made of a special type of functions called “cylindrical functions” For this brief review, it will suffice to say that with the measures introduced, the cylindrical functions are orthogonal for different spin networks. That is, in the context of these functions, the basis of spin networks not only is independent, but in a sense it is an orthonormal basis. Moreover, these results can be straightforwardly translated to the diffeomorphism invariant context (where cylindrical functions are also defined). The statement there would be that spin networks in different diffeomorphism classes are orthogonal.

## V. PHYSICS AT THE KINEMATICAL LEVEL

Having a space of functions that solves both the Gauss law and the diffeomorphism constraint, endowed with an inner product preserved by both constraints, it is tempting to see if one can compute things that might have some physical interest. The quantities involved could be considered a “kinematical setting” in terms of which to discuss evolution. As we shall see, some interesting results arise that are independent of the evolution chosen.

Some attractive statements can be made about areas and volumes in terms of spin network states. If one is given a certain two dimensional surface  $S$  or a three dimensional one  $\Sigma$ , it is possible to compute the area or volume in terms of the canonical variables introduced. The expressions are,

$$A = \int_S d^2x \sqrt{\tilde{E}_i^a \tilde{E}_i^b n_a n_b}, \quad (5.1)$$

$$V = \int_\Sigma d^3x \sqrt{\epsilon^{ijk} \xi_{abc} \tilde{E}_i^a \tilde{E}_j^b \tilde{E}_k^c}, \quad (5.2)$$

where  $n_a$  is the normal of the surface. At first sight, the presence of the square roots forecasts a regularization nightmare if one wishes to promote these quantities to quantum operators. In fact, this is not so. Because the operators have well defined properties under diffeomorphisms and the correct density weights to be naturally integrated, it turns out that their expressions are particularly simple. For instance, the area of a surface evaluated on a spin network state is proportional to  $\sum_{j_i} \sqrt{j_i(j_i + 1)}$ , where  $j_i$  are the valences of the strands that pierce the surface. The proportionality factor involves Planck’s constant and the parameter  $\beta$ . So one sees that in the basis of states considered, and with the inner product we introduced in the last section, areas and volumes are actually quantized and have a discrete spectrum [13,17]. The elementary quantum of area involves the parameter  $\beta$  (also known as the Immirzi parameter). It appears therefore that different choices of  $\beta$  are associated with different quantum theories with distinct predictions [18].

The idea that areas are quantized and that the spin network strands arise as “elementary excitations” embodies the concept of “space-time foam” in a concrete setting, and has found an attractive use in attempts to explain black hole entropy. The idea is to view the horizon of a black hole as a boundary of spacetime. The “extra degrees of freedom” introduced by this boundary account for the entropy of the black hole. This idea had been pursued in detail in the 2+1 dimensional context by Carlip [19]. In the 3+1 context, using the formulation we are describing, the idea is that given a certain area  $A$  for the horizon, one could view the various possibilities to obtain the given value of  $A$  in terms of spin networks as the “degrees of freedom” of the given area. This requires a careful counting. Various countings have been proposed by Smolin [20], Krasnov [21], Rovelli [22], and more recently by Ashtekar, Baez, Corichi and Krasnov [23]. In the latter work, a certain geometric condition fulfilled by classical horizons (no outgoing radiation) is shown to imply the emergence of a Chern–Simons theory on the boundary, which in turn allows a quite precise counting. The end result is that the entropy, defined as proportional to the logarithm of the number of states ends up being proportional to the area of the surface, the constant of proportionality involving the parameter  $\beta$ . The usual Hawking–Bekenstein result of one-quarter the area can only be achieved for a particular value of  $\beta$ . At the moment this issue is still debated. If  $\beta$  could be determined by independent means, this would provide a check of the theory. It is encouraging that the same value of  $\beta$  is needed for different kinds of black holes. Another striking property of the result is that it is largely independent of the dynamics, which is only used at a classical level in the arguments presented. There is ongoing work by various people on all these issues.

## VI. DYNAMICS

As we mentioned before, one of the original motivations to use the Ashtekar variables was that with the preferred choice  $\beta = i$ , the Hamiltonian constraint became polynomial in the basic variables,

$$H = \epsilon^{ijk} \tilde{E}_i^a \tilde{E}_j^b F_{ab}^k. \quad (6.1)$$

However, we noted that the resulting operator is a density of weight +2, since it is quadratic in the momenta. This poses problems at the time of regularizing the operator. There are no naturally defined densities of weight two on a manifold. Therefore the end result of most regularization attempts ends up being dependent on fiducial backgrounds in terms of which one can construct the needed density weights. The explicit appearance in the regularized operator of artificial background structures implies very surely difficulties at the time of enforcing the constraint algebra. For instance, in the commutator of the Hamiltonian constraint and the diffeomorphism constraint, external structures are

not affected by the diffeomorphisms generated by the constraint and therefore are not covariant. This immediately leads to undesired anomalies.

Thiemann [9] realized recently that one can represent the apparently non-polynomial single-densitized Hamiltonian constraint,

$$H = \frac{\epsilon^{ijk} \tilde{E}_i^a \tilde{E}_j^b}{\sqrt{\det g}} F_{ab}^k. \quad (6.2)$$

through the following classical identity,

$$\frac{\epsilon^{ijk} \tilde{E}_i^a \tilde{E}_j^b}{\sqrt{\det g}} = \{A_c^i, V\} \tilde{\epsilon}^{abc} \quad (6.3)$$

where  $V$  is the volume we introduced in the previous section. The resulting Hamiltonian therefore has the form,

$$H = \tilde{\epsilon}^{abc} \text{Tr}(F_{ab} \{A_c, V\}), \quad (6.4)$$

which with some care can be promoted very cleanly to a quantum operator acting on diffeomorphism invariant cylindrical functions. Thiemann has pursued this goal in a series of papers [9], where he shows the operator is finite, well defined and commutes with itself, therefore satisfying the correct constraint algebra (if the space of functions were not diffeomorphism invariant, the commutator of two Hamiltonians should be proportional to a diffeomorphism, which automatically vanishes if the functions are diffeomorphism invariant). In fact, the same procedure can be applied to the other piece of the Hamiltonian constraint (if  $\beta$  is real) and to couplings to matter. Therefore Thiemann has constructed a finite, consistent regularization of real general relativity coupled to matter. This regularization implements the promise that diffeomorphism invariance cures the divergences of field theories: the resulting theory includes QCD and QED coupled to gravity with finite Hamiltonians acting on a well defined space of diffeomorphism invariant functions.

There is currently an active debate about the properties of Thiemann’s Hamiltonian. Most notably, Lewandowski and Marolf [24] have noted that one can define another “habitat” where Thiemann’s Hamiltonian is well defined. This is a space of non-diffeomorphism invariant functions that still share several properties of the usual diffeo invariant cylindrical functions. They show that Thiemann’s regularization can be implemented in this space, but unfortunately, the Hamiltonian that arises still commutes with itself, which is inappropriate. This casts doubts about the regularization Thiemann proposed, although no contradiction is proved about Thiemann’s original proposal. In collaboration with Gambini, Lewandowski and Marolf [25] we have also proved that several modifications one could propose to Thiemann’s Hamiltonian do not cure the problem.

A separate line of attack is being pursued in terms of a different space of functions. These functions are the knot invariants that come from Chern–Simons theories. Some important mathematical hurdles were cleared recently and progress is being made. But this topic will be covered in detail in Gambini’s lecture, so I will not describe it here.

To summarize the situation for the dynamics, let us say that there is a proposal (Thiemann’s) that is a concrete, consistent, well defined theory of canonical quantum gravity coupled to matter proposed by the first time ever. The proposal yields the correct dynamics in 2+1 gravity as well. It is yet to be seen if it encompasses the correct dynamics in 3+1 dimensions. Some people suspect that the results of Lewandowski and Marolf imply that it fails to do so, but there is no clear proof of this yet. In the meantime, a separate proposal is being worked out that might overcome these difficulties. Further studies will determine if this is so.

## VII. CONCLUSIONS

I have attempted to give a quick review of several issues associated with the current state of attempts to quantize gravity canonically. I have left out many things. In particular I have not even referred to many pieces of work that played crucial roles in the development of the subject but that have become obsolete by the understanding they themselves helped create. Among the current efforts I have failed to cover, a great deal of activity is taking place these days trying to find a “covariant” [26–28] formulation of the theory using path integrals. One of the rationales for this is that these kinds of formulations might offer a fresh look on what kind of modifications to make to Thiemann’s Hamiltonian to recover the correct dynamics. At the moment this work is largely exploratory.

The possibility to have for the first time ever a consistent, finite, well defined theory of quantum gravity coupled to matter should not be under-stressed. The payoff is big: not only is gravity quantized, but all divergences of quantum field theory disappear. It is quite natural and understandable that the first theories we might generate end up being

later considered with the hindsight of time “trivial” or “wrong”. However, it is obviously important then to pursue the current theories further to see if they corresponds to the quantum gravity we expect to see in nature or to inapplicable mathematical elaborations. This is what one expects from science.

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