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Canonical quantum gravity and consistent discretizations

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We review a recent proposal for the construction of a quantum theory of the gravitational field. The proposal is based on approximating the continuum theory by a discrete theory that has several attractive properties, among them, the fact that in its canonical formulation it is free of constraints. This allows to bypass many of the hard conceptual problems of traditional canonical quantum gravity. In particular the resulting theory implies a fundamental mechanism for decoherence and bypasses the black hole information paradox.

Keywords: canonical, quantum gravity, new variables, loop quantization

1. Introduction

Discretizations are very commonly used as a tool to treat field theories. Classically, when one wishes to solve the equations of a theory on a computer, one replaces the continuum equations by discrete approximations to be solved numerically. At the level of quantization, lattices have been used to regularize the infinities that plague field theories. This has been a very successful approach for treating Yang–Mills theories. The current approaches to non-perturbatively construct in detail a mathematically well defined theory of quantum gravity both at the canonical level¹ and at the path integral level² resort to discretizations to regularize the theory.

Discretizing general relativity is more subtle than what one initially thinks. Consider a $3 + 1$ decomposition of the Einstein equations. One has twelve variables to solve for (the six components of the spatial metric and the six components of the extrinsic curvature). Yet, there are *sixteen* equations to be solved, six evolution equations for the metric, six for the extrinsic curvature and four constraints. In the continuum, we know that these sixteen equations are *compatible*, i.e. one can find twelve functions that satisfy them. However, when one discretizes the equations, the resulting system of algebraic equations is in general incompatible. This is well known, for instance, in numerical relativity³. The usual attitude there is to ignore the constraints and solve the twelve evolution equations (this scheme is called “free evolution”). The expectation is that in the limit in which the lattice is infinitely

refined, the constraints will also be satisfied if one satisfied them initially. The situation is more involved if one is interested in discretizing the theory in order to quantize it. There, one needs to take into account all equations. In particular, in the continuum the constraints form an algebra. If one discretizes the theory the discrete version of the constraints will in many instances fail to close an algebra. Theories with constraints that do not form algebras imply the existence of more constraints which usually makes them inconsistent. For instance, it might be the case that there are no wavefunctions that can be annihilated simultaneously by all constraints. One can ask the question if this is not happening in the construction that Thiemann works out. To our knowledge, this issue has not been probed. What is clear, is that discretizing relativity in order to quantize it will require some further thinking.

The new proposal we have put forward ⁴, called *consistent discretization* is that, in order to make the discrete equations consistent, the lapse and the shift need to be considered as some of the variables to be solved for. Then one has 16 equations and 16 unknowns. This might appear surprising since our intuition from the continuum is that the lapse and the shift are freely specifiable. But we need to acknowledge that the discrete theory *is a different theory*, which may approximate the continuum theory in some circumstances, but nevertheless is different and may have important differences even at the conceptual level. This is true of any discretization proposal, not only ours.

We have constructed a canonical approach for theories discretized in the consistent scheme ⁵. The basic idea is that one does not construct a Legendre transform and a Hamiltonian starting from the discretized Lagrangian picture. The reason for this is that the Hamiltonian is a generator of infinitesimal time evolutions, and in a discrete theory, there is no concept of infinitesimal. What plays the role of a Hamiltonian is a canonical transformation that implements the finite time evolution from discrete instant n to $n + 1$. The canonical transformation is generated by the Lagrangian viewed as a type I canonical transformation generating functional. The theory is then quantized by implementing the canonical transformation as a unitary evolution operator. A discussion of an extension of the Dirac procedure to these kinds of systems can be seen in ⁶.

2. Examples

We have applied this discretization scheme to perform a discretization (at a classical level) of BF theory and Yang–Mills theories ⁵. In the case of BF theories this provides the first direct discretization scheme on a lattice that is known for such theories. In the case of Yang–Mills theories it reproduces known results. We have also studied the application of the discretization scheme in simple cosmological models. We find that the discretized models approximate general relativity well and avoid the singularity ⁷. More interestingly, they may provide a mechanism for explaining the value of fundamental constants ⁸. When the discrete models tunnel through the

singularity, the value of the lapse gets modified and therefore the “lattice spacing” before and after is different. Since in lattice gauge theories the spacing is related to the “dressed” values of the fundamental constants, this provides a mechanism for fundamental constants to change when tunneling through a singularity, as required in Smolin’s⁹ “life of the cosmos” scenario.

It is quite remarkable that the discrete models work at all. When one solves for the lapse and the shift one is solving non-linear coupled algebraic equations. It could have happened that the solutions were complex. It could have happened that there were many possible “branches” of solutions. It could have happened that the lapse turned negative. Although all these situations are possible given certain choices of initial data, it is remarkable that it appears that one can choose initial data and a convenient “branch” of solutions for which pathologies are avoided and the discrete theory approximates the continuum theory in a controlled fashion. For simple cosmological examples, the quantization implementing the evolution as a unitary operation has been worked out in detail⁷.

We are currently exploring the Gowdy models with this approach, initially at a classical level only. Here the problem is considerably more complex than in cosmological models. The equations to be solved for the lapse and the shift become a coupled system that couples all points in the spatial discretization of the lattice. The problem can only be treated numerically. Moreover, Gowdy models have a global constraint due to the topology that needs special treatment. We have written a fortran code to solve the system using iterative techniques (considerable care needs to be exercised since the system becomes almost singular at certain points in phase space) and results are encouraging. In the end the credibility of the whole approach will hinge upon us producing several examples of situations of interest where the discrete theories approximate continuum GR well.

3. Several conceptual advantages

The fact that in the consistent discrete theories one solves the constraints to determine the value of the Lagrange multipliers has rather remarkable implications. The presence of the constraints is one of the most significant sources of conceptual problems in canonical quantum gravity. The fact that we approximate the continuum theory (which has constraints) with a discrete theory that is constraint free allows us to bypass in the discrete theory many of the conceptual problems of canonical quantum gravity.

The reader may ask how is such an approximation possible. After all, if the continuum theory has constraints and the discrete version does not, the two theories do not even have the same number of degrees of freedom. This is true. What is happening is that in the discrete theory there will generically be several solutions that approximate a given solution of the continuum theory. As solutions of the discrete theory they are all different yet they represent the same solution in the continuum. Therefore it is not surprising that the discrete theory has more degrees

of freedom.

One of the main problems we can deal with due to the lack of constraints is the “problem of time”. This problem has generated a large amount of controversy and has several aspects to it. We cannot cover everything here, the definitive treatise on the subject is the paper by Kuchař¹⁰.

To simplify the discussion of the problem of time, let us consider an aspect of quantum mechanics that most people find unsatisfactory perhaps from the first time they encounter the theory as undergraduates. It is the fact that in the Schrödinger equation, the variables “ x ” and “ t ” play very different roles. The variable x is a quantum operator of which we can, for instance, compute its expectation value, or its uncertainty. In contrast t is assumed to be a continuous external parameter. One is expected to have a clock that behaves perfectly classically and is completely external to the system under study. Of course, such a construction can only be an approximation. There is no such thing as a perfect classical clock and in many circumstances (for instance quantum cosmology) there is no “external clock” to the system of interest. How is one to do quantum mechanics in such circumstances? The answer is: “relationally”. One could envision promoting *all* variables of a system to quantum operators, and choosing one of them to play the role of a “clock”. Say we call such variable t (it could be, for instance the angular position of the hands of a real clock, or it could be something else). One could then compute conditional probabilities for other variables to take certain values x_0 when the “clock” variable takes the value t_0 . If the variable we chose as our “clock” does correspond to a variable that is behaving classically as a clock, then the conditional probabilities will approximate well the probabilities computed in the ordinary Schrödinger theory. If one picked a “crazy time” then the conditional probabilities are still well defined, but they don’t approximate any Schrödinger theory well. If there is no variable that can be considered a good classical clock, Schrödinger’s quantum mechanics does not make sense and the relational quantum mechanics is therefore a generalization of Schrödinger’s quantum mechanics.

The introduction of a relational time in quantum mechanics therefore appears well suited as a technique to use in quantizing general relativity, particularly in cosmological situations where there is no externally defined “classical time”. Page and Wootters advocated this in the 1980’s¹¹. Unfortunately, there are technical problems when one attempts the construction in detail for general relativity. The problem arises when one wishes to promote the variables to quantum operators. Which variables to choose? In principle, the only variables that make sense physically are those that have vanishing Poisson brackets (or quantum mechanically vanishing commutators) with the constraints. But since the Hamiltonian is one of the constraints, then such variables are “perennials” i.e. constants of motion, and one cannot reasonably expect any of them to play the role of a “clock”. One could avoid this problem by considering variables that do not have vanishing Poisson brackets with the constraints. But this causes problems. Quantum mechanically one wishes to consider quantum states that are annihilated by the constraints. Variables that

do not commute with the constraints as quantum operators map out of the space of states that solve the constraints. The end result of this, as discussed in detail by Kuchař¹⁰ is that the propagators constructed with the relational approach do not propagate.

Notice that all the problems are due to the presence of the constraints. In our discrete theory, since there are no constraints, there is no obstruction to constructing the relational picture. We have discussed this in detail in¹².

Of great interest is the fact that the resulting relational theory will never entirely coincide with a Schrödinger picture. In particular, since no clock is perfectly classical, pure states do not remain pure forever in this quantization, but slowly decohere into mixed states. We have estimated the magnitude of this effect. In order to do this, we chose the “best possible classical clock” as constructed by Ng and Van Damme and Amelino-Camelia¹³ elaborating on the pioneering work of Salecker and Wigner¹⁴. The result is that the rate of decoherence is proportional to $\omega^2 T_{\text{Planck}}^{4/3} T^{2/3}$ where ω is the frequency associated with the spread in energy levels of the system under study, T_{Planck} is Planck’s time and T is the time that the system lives. The effect is very small. Only for systems that have rather large energy spreads (Bose–Einstein condensates are a possible example) the effect may be close to observability. With current technologies, the condensates do not have enough atoms to achieve the energy spreads of interest, but it might not be unfeasible as technology improves to observe the effect¹⁵.

The fact that a pure state evolves into a mixed state opens other interesting possibilities, connected with the black hole information puzzle. This puzzle is related to the fact that one could consider a pure quantum state that collapses into a black hole. The latter will start evaporating due to Hawking radiation until eventually it disappears. What one is left with at the end of the day appears to be the outgoing radiation, which is in a mixed state. Therefore a pure state appears to have evolved into a mixed state. There is a vast literature discussing this issue (see for instance¹⁶ for a short review). Possible solutions proposed include that the black hole may not disappear entirely or that some mechanism may allow pure states to evolve into a mixed state. But we have just discussed that the relational discrete quantum gravity predicts such decoherence! We have estimated that the decoherence is fast enough to turn the pure state into a mixed one before the black hole can evaporate completely,¹⁷ (an earlier manuscript we wrote did not use the optimal clocks and the rate of decoherence it predicted was not as decisive¹⁸). The result is quite remarkable, since the decoherence effect, as we pointed before, is quite small. It is large enough to avoid the information puzzle in black holes, even if one considers smaller and smaller black holes which evaporate faster since they also have larger energy spreads and therefore the decoherence effect operates faster.

4. Summary

Analyzing in detail how to discretize general relativity led us to develop a way to consistently discretize the theory, in the sense that all the resulting discrete equations can be solved simultaneously. Surprisingly, the consistent discretizations not only approximate general relativity well in several situations, but as theories are conceptually much simpler to analyze than the continuum theory, since they do not have constraints. This allows us to handle several of the hard conceptual problems of canonical quantum gravity. What is now needed is to demonstrate that the range of situations in which the discrete theory approximates general relativity well is convincingly large enough to consider its quantization as a route for the quantization of general relativity.

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