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Relational physics with real rods and clocks and the measurement problem of quantum mechanics

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Abstract

The use of real clocks and measuring rods in quantum mechanics implies a natural loss of unitarity in the description of the theory. We briefly review this point and then discuss the implications it has for the measurement problem in quantum mechanics. The intrinsic loss of coherence allows to circumvent some of the usual objections to the measurement process as due to environmental decoherence.

I. INTRODUCTION

The Copenhagen interpretation of quantum mechanics requires the existence of a classical macroscopic realm in order to explain the measurement process. In spite of this long held view, it is becoming increasingly clear that quantum mechanics should be understood entirely as a standalone quantum paradigm without having to refer to an external classical world. The issue is becoming more pressing as there exists a growing number of experiments showing the existence of superpositions of macroscopically distinct quantum states.

Even though we tend to consider that quantum mechanics is a universal scheme able to describe all the physical phenomena, it remains to be shown that quantum mechanics is sufficient to explain our observation of a classical world. There is an extended consensus among physicists that environment-induced decoherence may play an important role in the solution of the measurement problem of quantum mechanics. It allows us to understand how the interaction with the environment induces a local suppression of interference between a set of preferred states, associated with the “pointer basis”. More precisely, a) decoherence induces a fast suppression of the interference terms of the reduced matrix describing the system coupled to the measurement device, and b) it selects a preferred set of states that are robust in spite of their interaction with the environment. These facts are a direct consequence of the standard unitary time evolution of the total system-environment composition and therefore the global phase coherence is not destroyed but simply transferred from the system to the environment.

We have recently noticed that quantum mechanics also leads to other kinds of loss of coherence due to the quantum effects in real clocks [1, 2]. In fact, as ordinarily formulated, quantum mechanics involves an idealization. That is, the use of a perfect classical clock to measure times. Such a device clearly does not exist in nature, since all measuring devices are subject to some level of quantum fluctuations. The equations of quantum mechanics, when cast in terms of the variable that is really measured by a clock in the laboratory, differ from the traditional Schroedinger description. Although this is an idea that arises naturally in ordinary quantum mechanics, it is of paramount importance when one is discussing quantum gravity. This is due to the fact that general relativity is a generally covariant theory where one needs to describe the evolution in a relational way. This problem is most obvious in the context of quantum cosmology. Although space-time is usually described in terms

of fiduciary non-observable quantities like the value of the components of the metric in a coordinate system, the final physical questions end up being what are the values of certain physical quantities when others physical quantities taken as clocks and measuring rods take certain values at the same fiducial coordinate point. Contrary to what happens with the environment-induced decoherence this new type of decoherence is associated to a non-unitary evolution in the physical time. The origin of the lack of unitarity is the fact that in quantum mechanics the determination of the state of a system is only possible by repeating an experiment. If one uses a real clock, which has thermal and quantum fluctuations, each experimental run will correspond to a different value of the evolution parameter. The statistical prediction will therefore correspond to an average over several intervals, and therefore its evolution cannot be unitary. As has been observed by Salecker and Wigner [3] and more recently by Ng [4], quantum mechanics and general relativity impose fundamental limitations to how good a clock can be and therefore any physical system will suffer loss of quantum coherence. This is a fundamental inescapable limit. One may argue about the level at which the limitation arises, but the existence of a limitation at some level is non-controversial. A pure state inevitably will become a mixed state due to the impossibility of having a perfect classical clock in nature. In that sense it is important to stress that the strict application of quantum mechanics when the quantum nature of the clocks is taken into account leads to a modification of the Schroedinger evolution (a similar consideration of real “measuring rods” for space adds further corrections to quantum field theory as well).

Due to the extreme accuracy that real clocks can reach this effect is very small. A rough measure of the effect is given by the off diagonal terms in the evolution of the density matrix for a physical system in the energy eigenbasis. Taking into account the previously mentioned limits for time measurements, one gets that the off diagonal terms are depressed by the exponential of $\omega^2 t_{\text{Planck}}^{4/3} t^{2/3}$, where ω is the Bohr frequency associated to the levels, t_{Planck} is Planck’s time and t the elapsed time. So we conclude that any physical system that we study in the lab will suffer loss of quantum coherence at least at this rate. This is a fundamental inescapable limit. A pure state inevitably will become a mixed state due to the impossibility of having a perfect classical clock in nature. One can show that this effect, in absence of environment-decoherence, would not be relevant for most “Schroedinger cat” experiences involving less than 10^{10} atoms.

The aim of the present paper is to present a first discussion of the interplay between

these two types of decoherence and their consequences for the problem of measurements in quantum mechanics. In fact, standard decoherence by the environment leads to a reduced density matrix for the system coupled with the measurement apparatus that is approximately diagonal, this density matrix describes all the information that can be extracted by an observer when the correlations with the environment are ignored. That implies that a measurement of an observable that only pertains to the system plus the measurement device cannot discriminate between the total pure state and a mixed state. However, as it was extensively discussed by d’Espagnat [5] the formal identity between the reduced density matrix and a mixed-state density matrix is frequently misinterpreted as implying that the system is in a mixed state. As the system is entangled with the environment the total system is still described by a pure state and no individual definite state or set of possible states may be attributed to a portion of the total system. As it is well known, joint measurements of the environment and the system will always allow us, in principle, to distinguish between the reduced and mixed state density matrices.

The combined effect of these two forms of decoherence could allow to understand the physical transition from a reduced density matrix to a mixed state. In fact, the precise unitary evolution of the total system is broken by the clock-induced decoherence destroying the correlations. What remains to be studied is whether this effect is sufficiently fast to avoid any possibility of distinguishing, not only for all practical purposes but also on theoretical basis, between these two kinds of density matrices.

Understanding this transition may be crucial for determining which interpretations are compatible with quantum mechanics. In fact if there is not any deviation from the unitary dynamics, it would be compelling to consider the universal validity of the evolution of any system with the Schroedinger equation and therefore to single out a relative state-type Everett interpretation. In that case the resulting appearance of a classical regime should be considered as an apparent effect for observers in each branch.

Summarizing, when one takes into account that we are using real clocks, the standard postulates of quantum mechanics lead to the unexpected fact that the evolution in terms of a real time clock is not exactly unitary. This loss of exact unitarity may help to understand the transition of the reduced density matrix to a mixed state density matrix and consequently to put on equal footing the relative state interpretation with realistic interpretations of the measurement problem in which the observation of the system plus the apparatus in a definite

state is possible.

In the remaining sections of the paper we will give more details concerning these ideas. In the next section we outline how the loss of coherence arises in ordinary quantum mechanics when one considers the use of real clocks. In section 3 we discuss the application to the measurement problem. In section 4 we discuss the open problems that must be tackled before the proposed mechanism can be considered a completely satisfactory solution to the problem of measurement in quantum mechanics.

II. QUANTUM MECHANICS WITH REAL CLOCKS

We proceed to describe how is the appearance of ordinary quantum mechanics when cast in terms of real clocks. Given a physical situation of interest described by a (multi-dimensional) phase space q, p , we start by choosing a “clock”. By this we mean a physical quantity (more precisely a set of quantities, like when one chooses a clock and a calendar to monitor periods of more than a day) that we will use to keep track of the passage of *time*. An example of such a variable could be the angular position of the hand of an analog watch. Let us denote it by $T(q, p)$. We then identify some physical variables that we wish to study as a function of time. We shall call them generically $O(q, p)$ (“observables”). We then proceed to quantize the system by promoting all the observables and the clock variable to self-adjoint quantum operators acting on a Hilbert space. The latter is defined once a well defined inner product is chosen in the set of all physically allowed states. Usually it consists of squared integrable functions $\psi(q)$.

Notice that we are not in any way modifying quantum mechanics. We assume that the system has an evolution in terms of an external parameter t , which is a classical variable, given by a Hamiltonian and with operators evolving with Heisenberg’s equations (it is easier to present things in the Heisenberg picture, though it is not mandatory to use it for our construction). Then the standard rules of quantum mechanics and its probabilistic nature apply.

We will call the eigenvalues of the “clock” operator T and the eigenvalues of the “observables” O . We define the projector associated to the measurement of the time variable

within the interval $[T_0 - \Delta T, T_0 + \Delta T]$,

$$P_{T_0}(t) = \int_{T_0 - \Delta T}^{T_0 + \Delta T} dT \sum_k |T, k, t \rangle \langle T, k, t| \quad (1)$$

where k denotes the eigenvalues of the operators that form a complete set with \hat{T} (the eigenvalues can have continuous or discrete spectrum, in the former case the sum should be replaced by an integral). We have assumed a continuous spectrum for T therefore the need for the integral over an interval on the right hand side. The interval ΔT is assumed to be very small compared to any of the times intervals of interest in the problem, in particular the time separating two successive measurements. Similarly we introduce a projector associated with the measurement of the observable O ,

$$P_{O_0}(t) = \int_{O_0 - \Delta O}^{O_0 + \Delta O} dO \sum_j |O, j, t \rangle \langle O, j, t| \quad (2)$$

with j the eigenvalues of a set of operators that form a complete set with \hat{O} . These projectors have the usual properties, i.e., $P_a(t)^2 = P_a(t)$, $\sum_a P_a(t) = 1, \forall t$ and $P_a(t)P_{a'}(t) = 0$ if the intervals surrounding a and a' do not have overlap.

We would like now to ask the question “what is the probability that the observable O take a given value O_0 given that the clock indicates a certain time T_0 ”. One here is assuming therefore that one has access to a physical clock and that underlying behind the framework is a foliation of space-time constructed using an inaccessible idealized clock t so the clock reading and the apparatus measurement occur at the same idealized time t . Such question is embodied in the conditional probability,

$$\mathcal{P}(O \in [O_0 - \Delta O, O_0 + \Delta O] | T \in [T_0 - \Delta T, T_0 + \Delta T]) = \lim_{\tau \rightarrow \infty} \frac{\int_{-\tau}^{\tau} dt \text{Tr}(P_{O_0}(t)P_{T_0}(t)\rho P_{T_0}(t))}{\int_{-\tau}^{\tau} dt \text{Tr}(P_{T_0}(t)\rho)} \quad (3)$$

where we have used the properties of both projectors and the integrals over t in the right hand side are taken over all its possible values. The reason for the integrals is that we do not know for what value of the external ideal time t the clock will take the value T_0 . In this expression ρ is the density matrix of the system. One has to take some obvious cares, like for instance to choose a clock variable (or set of variables) that do not take twice the same value during the relevant lifetime of the experiment one is considering. It should be noted that in this context the role of the fiducial time t is similar to that of the use of coordinates to describe space-time in general relativity. At the end of the day physical questions are

embodied in relational measurements of quantities and no specific direct knowledge of the fiduciary parameters is needed. One may philosophically speculate as to why to assume that the evolution is unitary in the fiduciary parameter t . Here one could note that ordinary experience with quantum mechanics with real clocks suggests that evolution is at worst very approximately unitary. It is therefore natural to assume it is exactly unitary in the parameter t and approximately unitary in physical not absolutely accurate clocks T one may consider.

The above expression is general, it will apply to any choice of “clock” and “system” variables we make. The relational evolution of the conditional probabilities will be complicated and will bear little resemblance to the usual evolution of probabilities in ordinary quantum mechanics unless we make a “wise” selection of the clock and system variables. What we mean by this is that we would like to choose as clock variables a subsystem that interacts little with the system we want to study and that behaves semi-classically with small quantum fluctuations. Namely, the physical clock will be correlated with the ideal time in such a way to produce the usual notion of time. In such a regime one expects to recover ordinary Schroedinger evolution (plus small corrections) even if one is using a “real” clock. Let us consider such a limit in detail. We will assume that we divide the density matrix of the whole system into a product form between clock and system, $\rho = \rho_{\text{cl}} \otimes \rho_{\text{sys}}$ and the evolution will be given by a unitary operator also of product type $U = U_{\text{cl}} \otimes U_{\text{sys}}$.

Up to now we have considered the quantum states as described by a density matrix at a time t . Since the latter is unobservable, we would like to shift to a description where we have density matrices as functions of the observable time T . To do this, we recall the expression for the usual probability in the Schroedinger representation of measuring the value O at a time t ,

$$\mathcal{P}(O|t) \equiv \frac{\text{Tr}(P_O(0)\rho(t))}{\text{Tr}(\rho(t))} \quad (4)$$

where the projector is evaluated at $t = 0$ since in the Schroedinger picture operators do not evolve. We would like to get a similar expression in terms of the real clock. To do this we consider the conditional probability (3), and make explicit the separation between clock and system,

$$\mathcal{P}(O \in [O_0 \pm \Delta O] | T \in [T_0 \pm \Delta T]) = \lim_{\tau \rightarrow \infty} \frac{\int_{-\tau}^{\tau} dt \text{Tr}(U_{\text{sys}}(t)^\dagger P_O(0) U_{\text{sys}}(t) U_{\text{cl}}(t)^\dagger P_T(0) U_{\text{cl}}(t) \rho_{\text{sys}} \otimes \rho_{\text{cl}})}{\int_{-\tau}^{\tau} dt \text{Tr}(P_T(t) \rho_{\text{cl}}) \text{Tr}(\rho_{\text{sys}})} \quad (5)$$

$$= \lim_{\tau \rightarrow \infty} \frac{\int_{-\tau}^{\tau} dt \operatorname{Tr} \left(U_{\text{sys}}(t)^\dagger P_O(0) U_{\text{sys}}(t) \rho_{\text{sys}} \right) \operatorname{Tr} \left(U_{\text{cl}}(t)^\dagger P_T(0) U_{\text{cl}}(t) \rho_{\text{cl}} \right)}{\int_{-\tau}^{\tau} dt \operatorname{Tr} \left(P_T(t) \rho_{\text{cl}} \right) \operatorname{Tr} \left(\rho_{\text{sys}} \right)}. \quad (6)$$

We define the probability density that the resulting measurement of the clock variable takes the value T when the ideal time takes the value t ,

$$\mathcal{P}_t(T) \equiv \frac{\operatorname{Tr} \left(P_T(0) U_{\text{cl}}(t) \rho_{\text{cl}} U_{\text{cl}}(t)^\dagger \right)}{\int_{-\infty}^{\infty} dt \operatorname{Tr} \left(P_T(t) \rho_{\text{cl}} \right)}, \quad (7)$$

and notice that $\int_{-\infty}^{\infty} dt \mathcal{P}_t(T) = 1$. We now define the evolution of the density matrix,

$$\rho(T) \equiv \int_{-\infty}^{\infty} dt U_{\text{sys}}(t) \rho_{\text{sys}} U_{\text{sys}}(t)^\dagger \mathcal{P}_t(T) \quad (8)$$

where we dropped the “sys” subscript in the left hand side since it is obvious we are ultimately interested in the density matrix of the system under study, not that of the clock. Noting that

$$\operatorname{Tr}(\rho(T)) = \int_{-\infty}^{\infty} dt \mathcal{P}_t(T) \operatorname{Tr}(\rho_{\text{sys}}) = \operatorname{Tr}(\rho_{\text{sys}}), \quad (9)$$

one can equate the conditional probability (5) to the ordinary probability of quantum mechanics (4),

$$\mathcal{P}(O \in [O_0 \pm \Delta O] | T \in [T_0 \pm \Delta T]) = \frac{\operatorname{Tr} \left(P_{T_O}(0) \rho(T) \right)}{\operatorname{Tr}(\rho(T))}, \quad (10)$$

(and we omit the integrals needed to make the expression precise in the case of continuous spectra for brevity). We have therefore ended with the standard probability expression with an “effective” density matrix in the Schroedinger picture given by $\rho(T)$. By its very definition, it is immediate to see that in the resulting evolution unitarity is lost, since one ends up with a density matrix that is a superposition of density matrices associated with different t 's and that each evolve unitarily according to ordinary quantum mechanics.

Now that we have identified what will play the role of a density matrix in terms of a “real clock” evolution, we would like to see what happens if we assume the “real clock” is behaving semi-classically. To do this we assume that $\mathcal{P}_t(T) = f(T, T_{\text{max}}(t))$, where f is a function that decays very rapidly for values of T far from the maximum of the probability distribution T_{max} . To make the expressions as simple as possible, let us assume that $T_{\text{max}}(t) = t$, i.e. the peak of the probability distribution is simply at t . More general dependences can of course be considered, altering the formulas minimally (for a more complete treatment see [1]). We will also assume that we can approximate f reasonably well by a Dirac delta, namely,

$$f(T, t) = \delta(T - t) + a(T) \delta'(T - t) + b(T) \delta''(T - t) + \dots, \quad (11)$$

where the first term has a unit coefficient so the integral of the probability is unit and we assume $b(T) > 0$ so it represents extra width with respect to the Dirac delta.

We now consider the evolution of the density matrix,

$$\rho(T) = \int_{-\infty}^{\infty} dt \rho_{\text{sys}}(t) \mathcal{P}_t(T) = \int_{-\infty}^{\infty} dt \rho_{\text{sys}}(t) f(T, t) \quad (12)$$

and associating a Hamiltonian with the evolution operator $U(t) = \exp(iHt)$, we get,

$$\rho(T) = \rho_{\text{sys}}(T) + a(T)[H, \rho_{\text{sys}}(T)] - b(T)[H, [H, \rho_{\text{sys}}(t)]], \quad (13)$$

and we notice that there would be terms involving further commutators if we had kept further terms in the expansion of $f(T, t)$ in terms of the Dirac deltas.

We can now consider the time derivative of this expression, and get,

$$\frac{\partial \rho(T)}{\partial T} = i \left(-1 + \frac{\partial a(T)}{\partial T} \right) [H, \rho(T)] + \left(a(T) - \frac{\partial b(T)}{\partial T} \right) [H, [H, \rho(T)]]. \quad (14)$$

If we had considered a symmetric distribution (it is natural to consider such distributions since on average one does not expect an effect that would lead systematically to values greater or smaller than the mean value), we see that one would have obtained the traditional evolution to leading order plus a corrective term,

$$\frac{\partial \rho(T)}{\partial T} = i[\rho(T), H] + \sigma(T)[H, [H, \rho(T)]]. \quad (15)$$

and the extra term is dominated by the rate of change of the width of the distribution $\sigma(T) = \partial b(T)/\partial T$.

An equation of this form has been considered in the context of decoherence due to environmental effects, it is called the Lindblad equation [6],

$$\frac{d}{dt} \rho = -i[H, \rho] - \mathcal{D}(\rho), \quad (16)$$

with

$$\mathcal{D}(\rho) = \sum_n [D_n, [D_n, \rho]], \quad D_n = D_n^\dagger, \quad [D_n, H] = 0, \quad (17)$$

and in our case there is only one D_n that is non-vanishing and it coincides with H . This is a desirable thing, since it implies that conserved quantities are automatically preserved by the modified evolution. Other mechanisms of decoherence coming from a different set of effects of quantum gravity have been criticized in the past because they fail to conserve energy

[7]. It should be noted that Milburn arrived at a similar equation as ours from different assumptions [8]. Egusquiza, Garay and Raya derived a similar expression from considering imperfections in the clock due to thermal fluctuations [9]. It is to be noted that such effects will occur in addition to the ones we discuss here.

What is the effect of the extra term? To study this, let us pretend for a moment that $\sigma(T)$ is constant. That is, the distribution in the clock variable has a width that grows linearly with time. In that case, the evolution equation is exactly solvable. If we consider a system with energy levels, the elements of the density matrix in the energy eigenbasis is given by,

$$\rho(T)_{nm} = \rho_{nm}(0)e^{-i\omega_{nm}T}e^{-\sigma\omega_{nm}^2T} \quad (18)$$

where $\omega_{nm} = \omega_n - \omega_m$ is the Bohr frequency corresponding to the levels n, m . We therefore see that the off-diagonal elements of the density matrix go to zero exponentially at a rate governed by σ , i.e. by how badly the clock's wavefunction spreads. It is clear that a pure state is eventually transformed into a completely mixed state (“proper mixture” in d’Espagnat’s terminology) by this process.

The origin of the lack of unitarity is the fact that definite statistical predictions are only possible by repeating an experiment. If one uses a real clock, which has thermal and quantum fluctuations, each experimental run will correspond to a different value of the evolution parameter. The statistical prediction will therefore correspond to an average over several intervals, and therefore its evolution cannot be unitary.

In a real experiment, there will be decoherence in the system under study due to interactions with the environment, that will be superposed on the effect we discuss. Such interactions might be reduced by cleverly setting up the experiment. The decoherence we are discussing here however, is completely determined by the quality of the clock used. It is clear that if one does experiments in quantum mechanics with poor clocks, pure states will evolve into mixed states very rapidly. The effect we are discussing can therefore be magnified arbitrarily simply by choosing a lousy clock. This effect has actually been observed experimentally in the Rabi oscillations describing the exchange of excitations between atoms and field [10].

We have established that when we study quantum mechanics with a physical clock (a clock that includes quantum fluctuations), unitarity is lost, conserved quantities are still preserved, and pure states evolve into mixed states. The effects are more pronounced the

worse the clock is. Which raises the question: is there a fundamental limitation to how good a clock can be? This question was first addressed by Salecker and Wigner [3]. Their reasoning went as follows: suppose we want to build the best clock we can. We start by insulating it from any interactions with the environment. An elementary clock can be built by considering a photon bouncing between two mirrors. The clock “ticks” every time the photon strikes one of the mirrors. Such a clock, even completely isolated from any environmental effects, develops errors. The reason for them is that by the time the photon travels between the mirrors, the wavefunctions of the mirrors spread. Therefore the time of arrival of the photon develops an uncertainty. Salecker and Wigner calculated the uncertainty to be $\delta t \sim \sqrt{t/M}$ where M is the mass of the mirrors and t is the time to be measured (we are using units where $\hbar = c = 1$ and therefore mass is measured in 1/second). The longer the time measured the larger the error. The larger the mass of the clock, the smaller the error.

So this tells us that one can build an arbitrarily accurate clock just by increasing its mass. However, Ng and Van Damme [4] pointed out that there is a limit to this. Basically, if one piles up enough mass in a concentrated region of space one ends up with a black hole. Some readers may ponder why do we need to consider a concentrated region of space. The reason is that if we allow the clock to be more massive by making it bigger, it also deteriorates its performance. For instance, in the case of two mirrors and a photon, if one makes the mirror big, there will be uncertainty in its position due to elastic effects like sound waves traveling across it, which will negate the effect of the additional mass (see the discussion in [11] in response to [12]).

A black hole can be thought of as a clock (as we will see it turns out to be the most accurate clock one can have). It has normal modes of vibration that have frequencies that are of the order of the light travel time across the Schwarzschild radius of the black hole. (It is amusing to note that for a solar sized black hole the frequency is in the kilohertz range, roughly similar to that of an ordinary bell). The more mass in the black hole, the lower the frequency, and therefore the worse its performance as a clock. This therefore creates a tension with the argument of Salecker and Wigner, which required more mass to increase the accuracy. This indicates that there actually is a “sweet spot” in terms of the mass that minimizes the error. Given a time to be measured, light traveling at that speed determines a distance, and therefore a maximum mass one could fit into a volume determined by that distance before one forms a black hole. That is the optimal mass. Taking this into account

one finds that the best accuracy one can get in a clock is given by $\delta T \sim T_{\text{Planck}}^{2/3} T^{1/3}$ where $T_{\text{Planck}} = 10^{-44} \text{s}$ is Planck's time and T is the time interval to be measured. This is an interesting result. On the one hand it is small enough for ordinary times that it will not interfere with most known physics. On the other hand is barely big enough that one might contemplate experimentally testing it, perhaps in future years.

With this absolute limit on the accuracy of a clock we can quickly work out an expression for the $\sigma(T)$ that we discussed in the previous section [2, 13]. It turns out to be $\sigma(T) = \left(\frac{T_{\text{Planck}}}{T_{\text{max}} - T}\right)^{1/3} T_{\text{Planck}}$. With this estimate of the absolute best accuracy of a clock, we can work out again the evolution of the density matrix for a physical system in the energy eigenbasis. One gets

$$\rho(T)_{nm} = \rho_{nm}(0) e^{-i\omega_{nm}T} e^{-\omega_{nm}^2 T_{\text{Planck}}^{4/3} T^{2/3}}. \quad (19)$$

So we conclude that *any* physical system that we study in the lab will suffer loss of quantum coherence at least at the rate given by the formula above. This is a fundamental inescapable limit. A pure state inevitably will become a mixed state due to the impossibility of having a perfect classical clock in nature.

Given these conclusions, one can ask what are the prospects for detecting the fundamental decoherence we propose. If one would like to observe the effect in the lab one would require that the decoherence manifest itself in times of the order of magnitude of hours, perhaps days at best. That requires energy differences of the order of 10^{10}eV in the Bohr frequencies of the system. Such energy differences can only be achieved in ‘‘Schroedinger cat’’ type experiments. Among the best candidates today are Bose–Einstein condensates, which can have 10^6 atoms in coherent states. These states can have energy differences for which the fundamental decoherence exponents become of order unity only in times larger than the age of the universe. These effects could be observed sooner if one could build larger coherent states, or if loss of coherence could be monitored with high levels of precision. But the challenge of eliminating faster acting environmental decoherence effects is huge and at present it remains unclear if any experiment could be proposed in the near future that could detect the fundamental decoherence.

A point that could be raised is that atomic clocks currently have an accuracy that is less than a decade of orders of magnitude worse than the absolute limit we derived in the previous section. Couldn't improvements in atomic clock technology actually get better

than our supposed absolute limit? This seems unlikely. When one studies in detail the most recent proposals to improve atomic clocks, they require the use of entangled states [14] that have to remain coherent. Our effect would actually prevent the improvement of atomic clocks beyond the absolute limit!

Another point to be emphasized is that our approach has been quite naive in the sense that we have kept the discussion entirely in terms of non-relativistic quantum mechanics with a unique time across space. It is clear that in addition to the decoherence effect we discuss here, there will also be decoherence spatially due to the fact that one cannot have clocks perfectly synchronized across space and also that there will be fundamental uncertainties in the determination of spatial positions (“use of real measuring rods”). We have not studied this in great detail yet, but it appears that this type of decoherence could be even more promising from the point of view of experimental detection (see [15]). For a brief discussion of the possible effects see [16].

It is interesting to notice that the presence of fundamental loss of coherence has implications for the black hole information puzzle. We will not expand on this issue here, but we refer the reader to our treatment of this issue in references [2, 13]. Implications for quantum computing were also discussed in [17].

III. IMPLICATIONS FOR THE MEASUREMENT PROBLEM OF QUANTUM MECHANICS

A potential conceptual application of the fundamental decoherence that we discussed that has not been exploited up to now is in connection with the measurement problem in quantum mechanics. The latter is related to the fact that in ordinary quantum mechanics the measurement apparatus is assumed to be always in an eigenstate after a measurement has been performed. The usual explanation [18] for this is that there exists interaction with the environment. This selects a preferred basis, i.e., a particular set of quasi-classical states often referred to as “pointer states” that are robust, in the sense of retaining correlations over time inspite of their immersion in the environment. These states are determined by the form of the interaction between the system and its environment and correspond to the classical states of our everyday experience. Decoherence then quickly damps superpositions between the localized preferred states when only the system is considered. This is taken as

an explanation of the appearance to a local observer of a “classical” world of determinate, “objective” (robust) properties.

The main problem with such a point of view is how is one to interpret the local suppression of interference in spite of the fact that the total state describing the system-environment combination retains full coherence. One may raise the question whether retention of the full coherence could ever lead to empirical conflicts with the ascription of definite values to macroscopic systems. The usual point of view is that it would be very difficult to reconstruct the off diagonal elements of the density matrix in practical circumstances. However, at least as a matter of principle, one could indeed reconstruct such terms (the evolution of the whole system remains unitary [19]).

Our mechanism of fundamental decoherence could contribute to the understanding of this issue. In the usual system-environment interaction the off-diagonal terms of the density matrix oscillate as a function of time. Since the environment is usually considered to contain a very large number of degrees of freedom, the common period of oscillation for the off-diagonal terms to recover non-vanishing values is very large, in many cases larger than the life of the universe. This allows to consider the problem solved in practical terms. When one adds in the effect we discussed, since it suppresses exponentially the off-diagonal terms, one never has the possibility that the off-diagonal terms will see their initial values restored, no matter how long one waits.

To analyze the implications of the use of real clocks in the measurement problem, we will analyze an example. In spite of the universality of the loss of coherence we introduced, it must be studied in specific examples of increasing level of realism. The simplest example we can think of is due to Zurek [20]. This simplified model does not have all the effects of a realistic one, yet it exhibits how the information is transferred from the measuring apparatus to the environment. The model consists of taking a spin one-half system that encodes the information about the microscopic system plus the measuring device. A basis in its two dimensional Hilbert space will be denoted by $\{|+ \rangle, |- \rangle\}$. The environment is modeled by a bath of many similar two-state systems called atoms. There are N of them, each denoted by an index k and with associated two dimensional Hilbert space $\{|+ \rangle_k, |- \rangle_k\}$. The dynamics is very simple, when there is no coupling with the environment the two spin states have the same energy, which is taken to be 0. All the atoms have zero energy as well in the absence of coupling. The whole dynamics is contained in the coupling, given by the

following interaction Hamiltonian

$$H_{\text{int}} = \hbar \sum_k \left(g_k \sigma_z \otimes \sigma_z^k \otimes \prod_{j \neq k} I_j \right). \quad (20)$$

In this notation σ_z is analogous to a Pauli spin matrix. It has eigenvalues $+1$ for the spin eigenvector $|+\rangle$ and -1 for $|-\rangle$; it acts as the identity operator on all the atoms of the environment. The operators σ_z^k are similar, each acts like a Pauli matrix on the states of the specific atom k and as the identity upon all the other atoms and the spin. I_j denotes the identity matrix acting on atom j and g_k is the coupling constant that has dimensions of energy and characterizes the coupling energy of one of the spins k with the system. In spite of the abstract character of the model, it can be thought of as providing a sketchy model of a photon propagating in a polarization analyzer.

Starting from a normalized initial state

$$|\Psi(0)\rangle = (a|+\rangle + b|-\rangle) \prod_{k=1}^N \otimes [\alpha_k |+\rangle_k + \beta_k |-\rangle_k], \quad (21)$$

it is easy to solve the Schroedinger equation and one gets for the state at the time t ,

$$\begin{aligned} |\Psi(t)\rangle = & a|+\rangle \prod_{k=1}^N \otimes [\alpha_k \exp(ig_k t) |+\rangle_k + \beta_k \exp(-ig_k t) |-\rangle_k] \\ & + b|-\rangle \prod_{k=1}^N \otimes [\alpha_k \exp(-ig_k t) |+\rangle_k + \beta_k \exp(ig_k t) |-\rangle_k]. \end{aligned} \quad (22)$$

Writing the complete density operator $\rho(t) = |\Psi(t)\rangle \langle \Psi(t)|$, one can take its trace over the environment degrees of freedom to get the reduced density operator,

$$\rho_c(t) = |a|^2 |+\rangle \langle +| + |b|^2 |-\rangle \langle -| + z(t) a b^* |+\rangle \langle -| + z^*(t) a^* b |-\rangle \langle +|, \quad (23)$$

where

$$z(t) = \prod_{k=1}^N \left[\cos(2g_k t) + i \left(|\alpha_k|^2 - |\beta_k|^2 \right) \sin(2g_k t) \right]. \quad (24)$$

The complex number $z(t)$ controls the value of the non-diagonal elements. If this quantity vanishes the reduced density matrix ρ_c would correspond to a totally mixed state (“proper mixture”). That would be the desired result, one would have several classical outcomes with their assigned probabilities. However, although the expression we obtained vanishes quickly assuming the α ’s and β ’s take random values, it behaves like a multiperiodic function, i.e. it is a superposition of a large number of periodic functions with different frequencies.

Therefore this function will retake values arbitrarily close to the initial value for sufficiently large times. This implies that the apparent loss of information about the non-diagonal terms reappears if one waits a long enough time. This problem is usually called “recurrence of coherence”. The characteristic time for these phenomena is proportional to the factorial of the number of involved frequencies. Although this time is usually large, perhaps exceeding the age of the universe, at least in principle it implies that the measurement process does not correspond to a change from a pure to a mixed state in a fundamental way.

The above derivation was done using ordinary quantum mechanics in which one assumes an ideal clock is used to measure time. If one redoes the derivation using the effective equation we derived for quantum mechanics with real clocks one gets the same expression for $z(t)$ except that it is multiplied by $\prod_k \exp\left(- (2g_k)^2 T_{\text{Planck}}^{4/3} t^{2/3}\right)$. That means that asymptotically the off diagonal terms indeed vanish, the function $z(t)$ is not periodic anymore. Although the exponential term decreases slowly with time, the fact that there is a product of them makes the effect quite relevant. Therefore we see that including real clocks in quantum mechanics offers a mechanism to turn pure states into mixed states in a way that is desirable to explain the problem of measurement in quantum mechanics.

IV. DIRECTIONS FOR FUTURE WORK

As we showed in the previous section, at least for simple models, the loss of coherence that arises in quantum mechanics when one considers real clocks may open a possibility to endow quantum mechanics with a fundamental process of measurement without the usual conceptual problems. It is clear, however, that more work is needed before one considers this a definitive answer to the problem.

The first aspect of this approach that should be further studied is to consider more realistic models. Examples of such models can be found, for instance, in the book by Omnés [19].

It is clear that to consider only the use of real clocks without introducing real measuring rods is insufficient to completely address the issues of the measurement process. d’Espagnat [5] has proposed certain observables that are preserved by the unitary evolution that involve the system and the environment. Such observables would change drastically in value if the reduced density matrix were to turn into an proper mixture. Therefore a measurement on

the system plus environment of the observable would allow to test if the reduced density matrix was in a pure or mixed state and would be incompatible with any process of reduction of the wavefunction. Such observables are very difficult to measure since they imply the simultaneous measurement of a very large number of microsystems, so for practical purposes again this is not a problem. It is however, a conceptual objection. In quantum mechanics with real clocks, the d’Espagnat observables are preserved upon evolution, since all quantities that commute with the Hamiltonian are automatically preserved in the evolution with respect to the real clock. It is to be noted that the measurement of the observable requires to measure many systems that are in different positions in space. A proper treatment therefore requires to consider not only the use of real clocks in quantum mechanics but also the use of “real measuring rods”. This is more involved, since it implies dealing with quantum field theory, and to do it in a way that is not conventional (in quantum field theory one computes correlation functions and not the evolution of density matrices). We have carried out a sketch of the problem in reference [16]. We intend to continue this analysis in more depth and to relate it to the problem of measurement and in particular to an analysis of the observables of d’Espagnat.

We also plan to study the impact of the fundamental loss of coherence in the various interpretations of quantum mechanics. For instance, in Everitt’s relative state (“many worlds”) interpretation, a key element is that the many worlds all evolve unitarily. If the evolution is fundamentally non-unitary this interpretation loses its compelling nature.

It should be noted that even after the inclusion of both temporal and spatial loss of coherence as argued above, problems still persist with the interpretation of quantum mechanics. As is well known in traditional decoherence solutions to the problem, the fact that the reduced density matrix at the end is in a diagonal form is not necessarily a completely satisfactory solution to the measurement problem. This is known as the “and-or” problem. As Bell [21] put it “if one were not actually on the lookout for probabilities, ... the obvious interpretation of even ρ' [the reduced density matrix] would be that the system is in a state in which various $|\Psi_m\rangle$ ’s coexist:

$$|\Psi_1\rangle\langle\Psi_1| \quad \text{and} \quad |\Psi_2\rangle\langle\Psi_2| \quad \text{and} \quad \dots \quad (25)$$

This is not at all a *probability* interpretation, in which the different terms are seen not as *coexisting* but as *alternatives*.” It is not obvious how our contribution to the problem changes

anything in the discussion of this point.

Even though the proposed research listed above is self-contained and within the context of ordinary quantum mechanics and quantum field theory, it is worthwhile mentioning its connection with a broader line of work taking place in the context of quantum gravity. The authors have been involved in the last four years in constructing an approach to quantum gravity called “consistent discretizations” (see [22] for a recent review). The philosophy of this approach is similar to that of lattice researchers who deal with QCD. It consists in discretizing general relativity and using the resulting discrete theory to probe issues of interest to quantum gravity. The discrete theories are constructed in such a way that is very advantageous, in particular the theories are constraint free. This allows to work out many proposals that have run into technical problems in quantum gravity. One such proposal is the use of real clocks and measuring rods to analyze the theory and therefore “solve the problem of time”. The use of relational ideas has long been sought as a means of dealing properties with the physics of diffeomorphism invariant theories like general relativity. Up to now it has always been hampered by technical problems associated with the presence of the constraints in the canonical theory. The discrete approach proposed, being constraint free allows to do away with these technical problems and to formulate the relational description. The fact that this relational description is central to dealing with quantum gravity and inspires a point of view that can have implications for the conceptual problems of ordinary quantum mechanics brings together two of the most fascinating areas modern theoretical physics in a remarkably unexpected way.

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