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Rodolfo Gambini
Facultad de Ciencias

Jorge Pullin
Louisiana State University

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Holography from loop quantum gravity

Rodolfo Gambini[1], Jorge Pullin[2]

1. Instituto de Física, Facultad de Ciencias, Iguá 4225,

esq. Mataojo, Montevideo, Uruguay rgambini@fisica.edu.uy

2. Department of Physics and Astronomy, Louisiana State University,

Baton Rouge, LA 70803-4001 pullin@lsu.edu

Abstract

We show that holography follows directly from the basic structure of spherically symmetric loop quantum gravity. The result is not dependent on detailed assumptions about the dynamics of the theory being considered. It ties strongly the amount of information contained in a region of space to the tight mathematical underpinnings of loop quantum geometry.

Physical principles usually represent facts partially collected by observation that end up guiding the development of physical theories. When the underlying theories are completely understood in detail, the principles can be explained as consequences of the theory they guided to create. The holographic principle has guided the construction of some of the leading physical theories of space time in the last few years. In a nutshell, holography establishes a limit to the amount of information contained in a space-time region [1]. In its simplest form, for spherical symmetry and weak gravity the principle establishes that the entropy of a region of space is limited by the area surrounding it and was first formulated by t'Hooft and Susskind [2]. Any successful theory of quantum gravity that incorporates holography should be able to derive it as a consequence of its framework. We would like to argue that holography does indeed follow from the framework of loop quantum gravity in spherical symmetry and that the result is robust: it does not depend on the details of the dynamics of the theory nor the type of matter included but rather on its kinematical structure and elementary dynamical considerations independent of the details of the Hamiltonian. That holography in its simple and straightforward *spatial* form is materialized in the spherical case is appropriate, since it is known that in non-spherical cases more care is needed (in particular involving spatiotemporal regions) in its definition in order not to run into counterexamples (see [1] for a discussion of this point).

The argument we will present can be summarized as follows: holography follows from the dependence of the volume operator in spherical loop quantum gravity on the radial distance, yielding an uncertainty in the determination of volumes that grows radially. Such a dependence for the uncertainty of spatial measurements had already been postulated in heuristic treatments relating limitations of space-time measurements to holography by Ng [3] and with alternative reasonings by Ng and Lloyd [4]). In this article we show that such a dependence can be derived from the kinematical structure of spherical loop quantum gravity.

We consider spherically symmetric loop quantum gravity. Its kinematic structure is well established and was discussed in detail by Bojowald and Swiderski [5]. There is only one non-trivial spatial direction (the radial) which we call x since it is not necessarily parameterized by the usual radial coordinate. The only non-trivial components of the metric are given in terms of the Ashtekar triads by $g_{xx} = (E^\varphi)^2/|E^x|$ and $g_{\theta\theta} = |E^x|$. It is convenient to gauge fix the radial direction (for further details see [6]) to the usual Schwarzschild coordinate $E^x - (x + 2M)^2 = 0$ with M the mass of the space-time and $x \in \mathbb{R}^+$ and the horizon at

$x = 0$. In terms of these variables the volume of shell of radial interval I is given by,

$$V(I) = 4\pi \int dx |E^{\varphi}(x)|(x + 2M). \tag{1}$$

One can introduce a loop representation. The "loops" consist of intervals in the radial direction and the "vertices" are the ends of the intervals. Since one has gauge fixed the radial direction, variables behave as scalars and the loop representation resembles the one introduced for loop quantum cosmology, except that there is one such variable per vertex. Essentially the variables conjugate to the triads are represented through their exponentiated form and depend on a parameter that appears in the exponent per vertex, μ_v. States in the loop representation are labeled by collections of real valued parameters $|\vec{\mu}\rangle = |\mu_1, \ldots, \mu_n\rangle$. The volume operator can be readily quantized to give,

$$\hat{V}(I)|\vec{\mu}\rangle = \sum_{v \in I} 4\pi |\mu_v|(x_v + 2M)\gamma \ell^2_{\rm Planck}|\vec{\mu}\rangle \tag{2}$$

where γ is the Immirzi parameter. On this kinematical arena one will have to build the dynamics of the theory of interest, be it general relativity or some other theory, including possible matter couplings. In order to build the dynamics we need the action of the exponentiated variable that plays the role in this reduced context of the holonomies of the loop representation. As is customary, the action of these elementary operators in the loop representation is particularly simple,

$$\hat{h}_{\varphi}(v_i, \rho)|g, \vec{\mu}>=|g, \mu_{v_1}, \ldots, \mu_{v_i} + \rho, \ldots > . \tag{3}$$

Any candidate for a Hamiltonian of the theory one may wish to build will involve the action of the elementary operator $\hat{h}$. Usually the Hamiltonian one starts with will be a function of the connection, which can be expressed as a limit of the elementary operator when $\rho \to 0$. In the loop representation the unique measure that arises [7] prevents one from taking the limit and one has to take a minimum value for ρ.

The crucial observation is to note that the action of the elementary operator, on which all possible Hamiltonians will be based, takes an eigenstate of the volume operator and produces a new eigenstate where the volume has increased by

$$\Delta V = 4\pi 2\gamma\rho\ell^2_{\rm Planck}(x + 2M) \tag{4}$$

where $(x + 2M)$ is the usual Schwarzschild coordinate. If we take for ρ the minimum value possible, this quantity plays a role of minimum volume for the models we are considering.

To get a handle on the minimum possible value of ρ, we use a reasoning similar to the one used in loop quantum cosmology. This estimate comes from the fact that in the full theory areas have a minimum quantum and this leaves an imprint on the variables of the symmetry reduced models. The estimate [8] from loop quantum cosmology [9] is $\rho = \sqrt{3}/4$. We can then evaluate the number ΔN of such elementary volumes in a shell of width Δx (in the asymptotic region where we assume the metric is flat, otherwise one would have to add a finite correction and substitute x by $x + 2M$),

$$\Delta N = \frac{4\pi x^2 \Delta x}{4\pi 2\gamma\rho x \ell_{\text{Planck}}^2} = \frac{x\Delta x}{2\gamma\rho\ell_{\text{Planck}}^2}, \tag{5}$$

We can now compute the entropy in a shell as the one discussed,

$$\Delta S = \nu_v \frac{x\Delta x}{2\gamma\rho\ell_{\text{Planck}}^2}, \tag{6}$$

where ν_v is the mean entropy per unit volume.

The Immirzi parameter $\gamma = c_A/(\pi\sqrt{3})$ with c_A is a quantity that is to be determined by comparing physical predictions of the theory with reality. For instance, calculations of the entropy of black holes suggests it is of order unity. We can therefore write for the entropy of an infinitesimal shell,

$$\Delta S = \frac{\nu_v}{c_A} 4\pi \frac{x\Delta x}{2\ell_{\text{Planck}}^2}, \tag{7}$$

so for a finite shell of inner radius r_a and outer radius r_b one would have,

$$S = \frac{\nu_v}{c_A} 4\pi \frac{r_b^2 - r_a^2}{4\ell_{\text{Planck}}^2} = \frac{1}{4} \frac{\nu_v}{c_A} \frac{\Sigma_b - \Sigma_a}{\ell_{\text{Planck}}^2}, \tag{8}$$

where Σ is the area of the shell of the given radius which implies that the entropy is proportional to the area. This quantity is a upper bound for the entropy, since we have used the minimum value of ρ, obviously choosing larger ρ's one would obtain lower values for S.

We have therefore established that the kinematical structure of loop quantum gravity in spherical symmetry implies the holographic principle irrespective of the dynamics of the theory being studied. It is therefore a very general result. It stems from the fact that the elementary volume that any dynamical operator may involve goes as $x\ell_{\text{Planck}}^2$ (as suggested by previous heuristic estimates [3]). We have assumed a finite amount of information per elementary volume, as is usually argued in this context [1]. This implies that the information in a spatial region is bounded by the area, contrary to what happens if one assumes the elementary volume goes as ℓ_{Planck}^3. This is usually justified by thinking that the fields are collections of harmonic oscillators and the energy in each oscillator is bounded by the Planck energy and therefore has a finite number of states. Although a complete quantum gravity analysis has not been done, studies of the harmonic oscillator [10] and of linearized gravity [11] suggest that this bound is even tighter in loop quantum gravity.

Holography is therefore naturally built into the elementary framework of loop quantum gravity with spherical symmetry. The calculation we showed also implies for the first time a derivation from first principles of equation (4) which had been heuristically proposed [3] as a fundamental limit on the measurement of space and time and the ultimate limits of computability in nature and which may even be tested observationally in the near future in astronomical settings [12]. We can therefore consider that we have taken the first steps to unravel the mystery of holography using some of the most well established elements of traditional canonical quantization. It is yet to be seen if these results are only a coincidence in the spherical case or if they can be found as well in more general settings.

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[1] R. Bousso, Rev. Mod. Phys. **74**, 825 (2002) [arXiv:hep-th/0203101].

[2] G. 't Hooft, "Dimensional reduction in quantum gravity," arXiv:gr-qc/9310026; L. Susskind, J. Math. Phys. **36**, 6377 (1995) [arXiv:hep-th/9409089].

[3] Y. J. Ng, Phys. Rev. Lett. **86**, 2946 (2001) [Erratum-ibid. **88**, 139902 (2002)] [arXiv:gr-qc/0006105].

[4] S. Lloyd, Y. J. Ng, Scientific American, **281**, 52 (2004).

[5] M. Bojowald and R. Swiderski, Class. Quant. Grav. **23**, 2129 (2006) [arXiv:gr-qc/0511108].

[6] M. Campiglia, R. Gambini, J. Pullin Class. Quant. Grav. **24**, 3649 (2007) [arXiv:gr-qc/0703135].

[7] J. Lewandowski, A. Okolow, H. Sahlmann and T. Thiemann Comm. Math. Phys. **267**, 3

(2006) [gr-qc/0504147] and references therein.

[8] The most recent work on loop quantum cosmology (A. Ashtekar, T. Pawlowski, P. Singh and K. Vandersloot, Phys. Rev. D **75**, 024035 (2007) [arXiv:gr-qc/0612104]) uses an estimate for ρ that is dependent on the metric . If one adopts that point of view, it does not change our results far away from the horizon. When one gets to the region where the gravitational fields are intense, our predictions would change. This is in line with the fact that the spatial Bekenstein bound is known to have problems with strong gravitational fields.

[9] A. Ashtekar, M. Bojowald and J. Lewandowski, Adv. Theor. Math. Phys. **7**, 233 (2003) [arXiv:gr-qc/0304074].

[10] A. Corichi, T. Vukasinac and J. A. Zapata, Class. Quant. Grav. **24**, 1495 (2007) [arXiv:gr-qc/0610072].

[11] M. Varadarajan, Phys. Rev. D **66**, 024017 (2002) [arXiv:gr-qc/0204067].

[12] W. A. Christiansen, Y. J. Ng and H. van Dam, Phys. Rev. Lett. **96**, 051301 (2006) [arXiv:gr-qc/0508121].